



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL INTERNATIONAL GCSE FURTHER PURE MATHS: QUESTIONS BY TOPIC

Graphs (4)

International GCSE Further Pure Mathematics Topic 4

36 questions · 408 marks · Mark schemes included

Pearson Edexcel International GCSE (9–1) Further Pure Mathematics 4PM1, Papers 1, 1R, 2 & 2R, June 2019 - June 2025

 [guidemygrades.com](https://www.guidemygrades.com)

 [@Guidemygrades](https://www.instagram.com/Guidemygrades)

 [@Guidemygrades](https://twitter.com/Guidemygrades)

 [@Guidemygrades](https://www.youtube.com/Guidemygrades)

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions run from the lowest mark tariff to the highest, so similar questions sit together.

Click any heading or question row below to jump straight to it.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
4-MARK QUESTIONS		2 questions · 8 marks	4
1. Q2	Paper 2R June 2019	4	5
2. Q4	Paper 2 June 2024	4	6
5-MARK QUESTIONS		1 questions · 5 marks	7
3. Q3	Paper 1 June 2023	5	8
6-MARK QUESTIONS		1 questions · 6 marks	9
4. Q4	Paper 2R January 2023	6	10
7-MARK QUESTIONS		4 questions · 28 marks	11
5. Q4	Paper 1 November 2020	7	12
6. Q3	Paper 2 November 2021	7	13
7. Q5	Paper 2R June 2022	7	14
8. Q6 <i>also Topic 1</i>	Paper 2R June 2024	7	17
8-MARK QUESTIONS		1 questions · 8 marks	18
9. Q4	Paper 2 November 2024	8	19
9-MARK QUESTIONS		6 questions · 54 marks	22
10. Q6	Paper 1R January 2020	9	23
11. Q7 <i>also Topic 1</i>	Paper 2 June 2021	9	26
12. Q8	Paper 2 November 2021	9	29
13. Q9	Paper 1R January 2022	9	32
14. Q3	Paper 1 January 2023	9	35
15. Q4	Paper 1R June 2023	9	36
10-MARK QUESTIONS		4 questions · 40 marks	39
16. Q3 <i>also Topic 8</i>	Paper 1 June 2021	10	40
17. Q7 <i>also Topic 1</i>	Paper 1 January 2023	10	41
18. Q7 <i>also Topic 1</i>	Paper 1R June 2025	10	44
19. Q7 <i>also Topic 1</i>	Paper 2 June 2025	10	47
11-MARK QUESTIONS		4 questions · 44 marks	50
20. Q8 <i>also Topic 1</i>	Paper 1R June 2019	11	51
21. Q4	Paper 2R November 2020	11	54
22. Q8 <i>also Topic 9</i>	Paper 2 January 2022	11	57
23. Q6	Paper 1 June 2022	11	60



14-MARK QUESTIONS			2 questions · 28 marks	63
24. Q11		Paper 2R January 2020	14	64
25. Q7	<i>also Topic 8</i>	Paper 2R June 2024	14	65
15-MARK QUESTIONS			1 questions · 15 marks	66
26. Q9	<i>also Topic 8</i>	Paper 2R June 2023	15	67
16-MARK QUESTIONS			3 questions · 48 marks	69
27. Q9	<i>also Topic 8</i>	Paper 2 January 2020	16	70
28. Q10	<i>also Topic 8</i>	Paper 2 June 2023	16	71
29. Q10		Paper 1 June 2024	16	72
17-MARK QUESTIONS			2 questions · 34 marks	73
30. Q11	<i>also Topic 8</i>	Paper 2 June 2022	17	74
31. Q10	<i>also Topic 8</i>	Paper 2 June 2025	17	76
18-MARK QUESTIONS			5 questions · 90 marks	78
32. Q9	<i>also Topic 2</i>	Paper 2 November 2020	18	79
33. Q10		Paper 2 January 2022	18	80
34. Q10	<i>also Topic 8</i>	Paper 2R June 2022	18	83
35. Q10	<i>also Topic 8</i>	Paper 2 November 2024	18	85
36. Q10	<i>also Topic 8</i>	Paper 2R June 2025	18	87



QUESTIONS

36 questions · 408 marks · lowest tariff first



4-MARK QUESTIONS

2 questions in this booklet



2

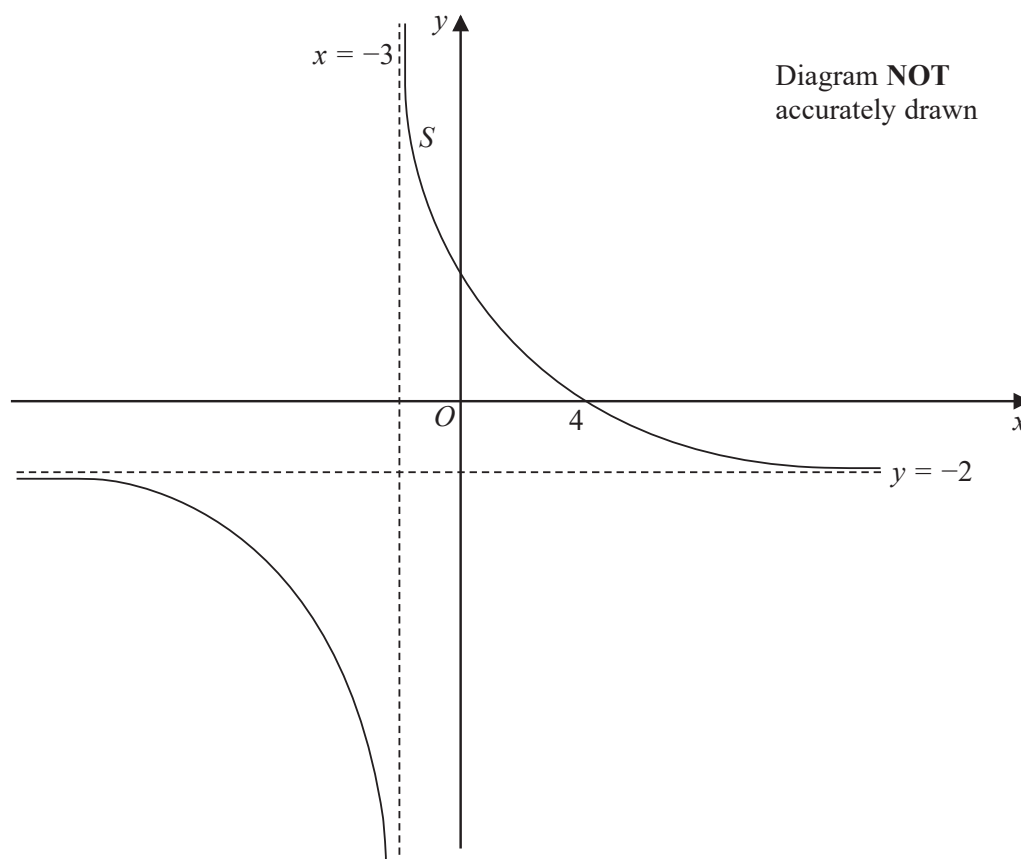


Figure 1

Figure 1 shows part of the curve S with equation $y = \frac{ax + b}{x + c}$ where a , b and c are integers.

The asymptote to S that is parallel to the x -axis has equation $y = -2$

The asymptote to S that is parallel to the y -axis has equation $x = -3$

The curve crosses the x -axis at the point with coordinates $(4, 0)$

The curve crosses the y -axis at the point with coordinates $(0, p)$ where p is a rational number.

Find

- (i) the value of a ,
- (ii) the value of b ,
- (iii) the value of c ,
- (iv) the value of p .

(4)



4

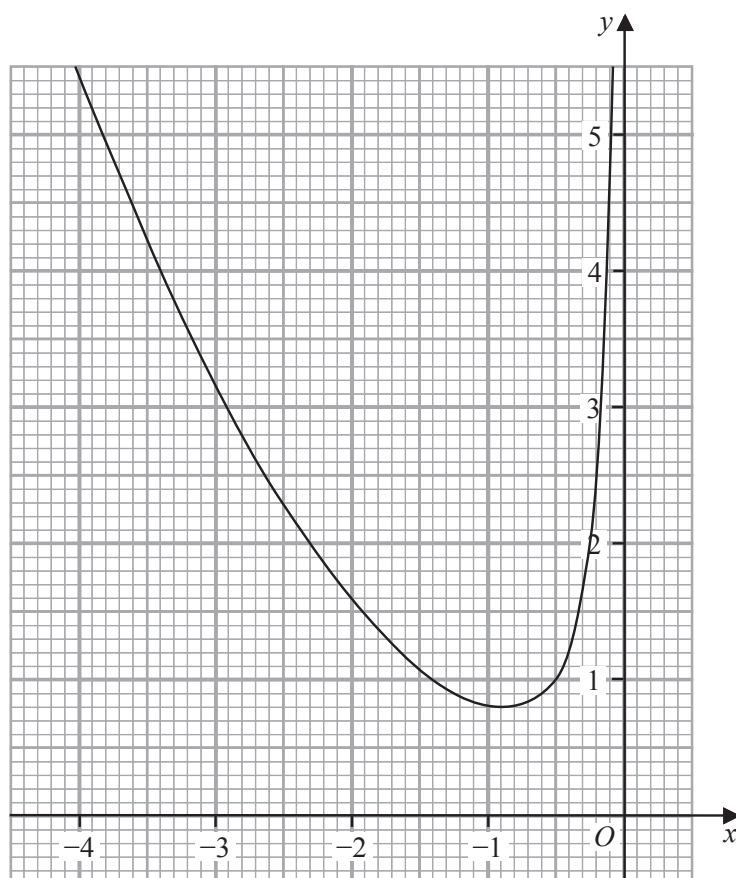
**Figure 2**

Figure 2 shows part of the curve with equation $y = \frac{x^2}{3} - \frac{1}{2x}$ for $-4 < x < 0$

By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $4x^3 + 3x^2 - 36x - 6 = 0$ in the interval $-4 < x < 0$

(4)

.....

.....

.....

.....

.....

5-MARK QUESTIONS

1 question in this booklet



3

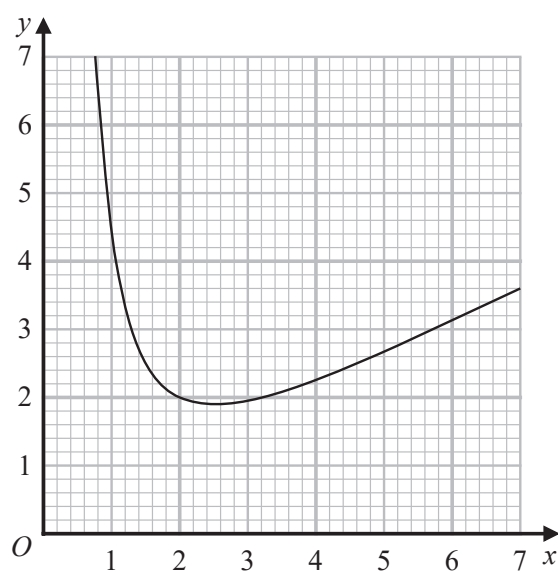


Figure 1

Figure 1 shows part of the curve with equation $y = \frac{x}{2} + \frac{4}{x^2}$ in the interval $0.8 < x < 7$

By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the roots of the equation $3x^3 - 12x^2 + 8 = 0$ in the interval $0.8 < x < 7$

(5)



6-MARK QUESTIONS

1 question in this booklet



4

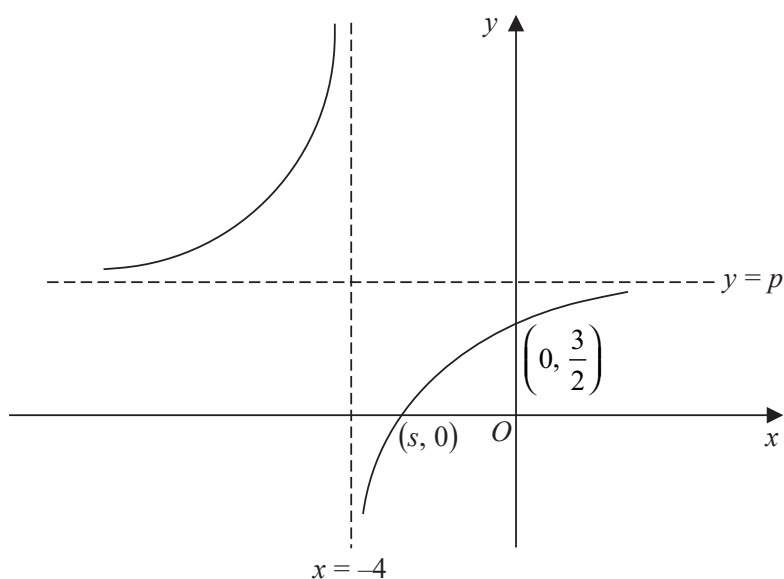


Figure 1

Figure 1 shows part of the curve C with equation

$$y = \frac{2x + q}{x + r} \quad x \neq -r$$

where q and r are integers.

The asymptote to C that is parallel to the y -axis has equation $x = -4$

The asymptote to C that is parallel to the x -axis has equation $y = p$

(a) Write down

(i) the value of p

(ii) the value of r

(2)

Given that C crosses the y -axis at the point with coordinates $\left(0, \frac{3}{2}\right)$

(b) find the value of q

(2)

Given that C crosses the x -axis at the point with coordinates $(s, 0)$

(c) find the value of s

(2)

.....

.....

.....

.....



7-MARK QUESTIONS

4 questions in this booklet



4



Figure 2

Figure 2 shows the graph of $y = x + \frac{5}{x^2}$ for $1 \leq x \leq 4$ drawn on a grid.

- (a) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, for the roots of the equation

$$x^3 - 4x^2 + 5 = 0$$

in the interval $1 \leq x \leq 4$

(3)

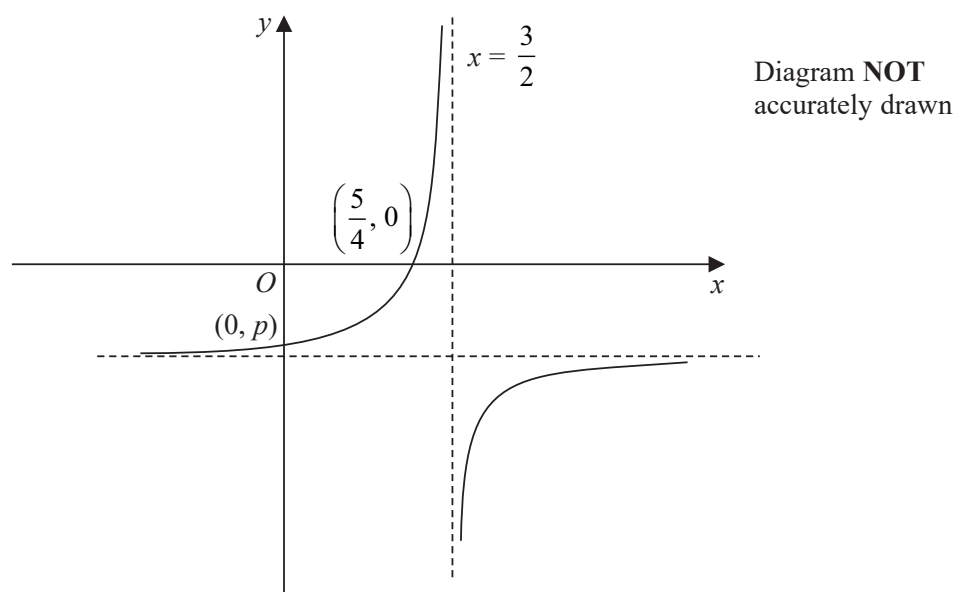
- (b) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, for the root of the equation

$$x^3 - x^2 - 5 = 0$$

in the interval $1 \leq x \leq 4$

(4)

3



$$y = \frac{a - bx}{cx - d} \quad x \neq \frac{d}{c}$$

where a , c and d are prime numbers and b is an integer.

The asymptote to the curve that is parallel to the y -axis has equation $x = \frac{3}{2}$

- (a) Write down the value of c and the value of d (2)

The curve crosses the x -axis at the point $\left(\frac{5}{4}, 0\right)$

- (b) Find the value of a and the value of b (2)

The curve crosses the y -axis at the point $(0, p)$ where p is a rational number.

- (c) Find the value of p (2)

- (d) Find an equation of the asymptote to the curve that is parallel to the x -axis. (1)

.....

.....

.....

- 5 (a) Complete the table of values for $y = e^{3x-2}$ giving your answers to 2 decimal places.

x	0	0.25	0.5	0.75	1
y	0.14				2.72

(2)

- (b) On the grid opposite, draw the graph of $y = e^{3x-2}$ for $0 \leq x \leq 1$

(2)

- (c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $3x = 2 + \ln(3 - x)$

(3)

.....

.....

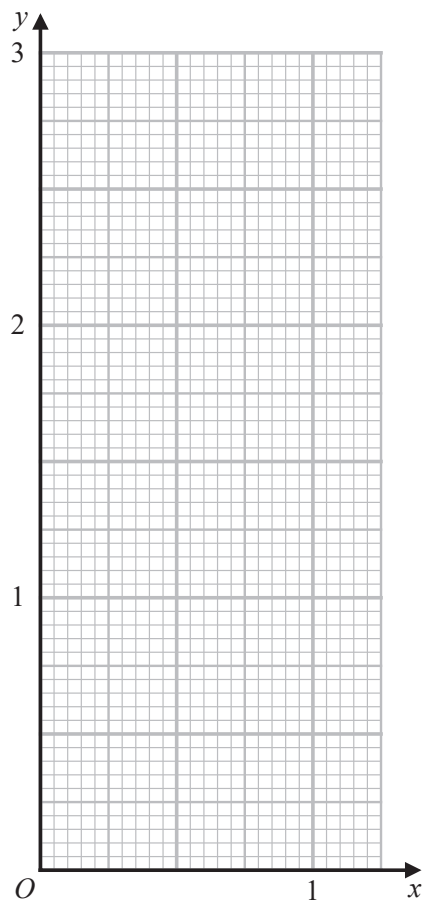
.....

.....

.....



Question 5 continued



.....

.....

.....

.....

.....

.....

.....

.....

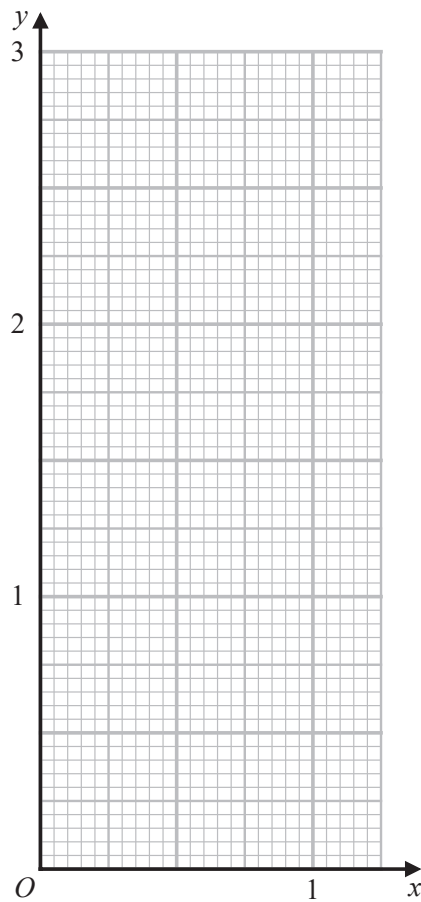
.....

.....

Turn over for a spare grid if you need to redraw your graph.

Question 5 continued

Only use this grid if you need to redraw your graph.



.....

.....

.....

.....

.....

6

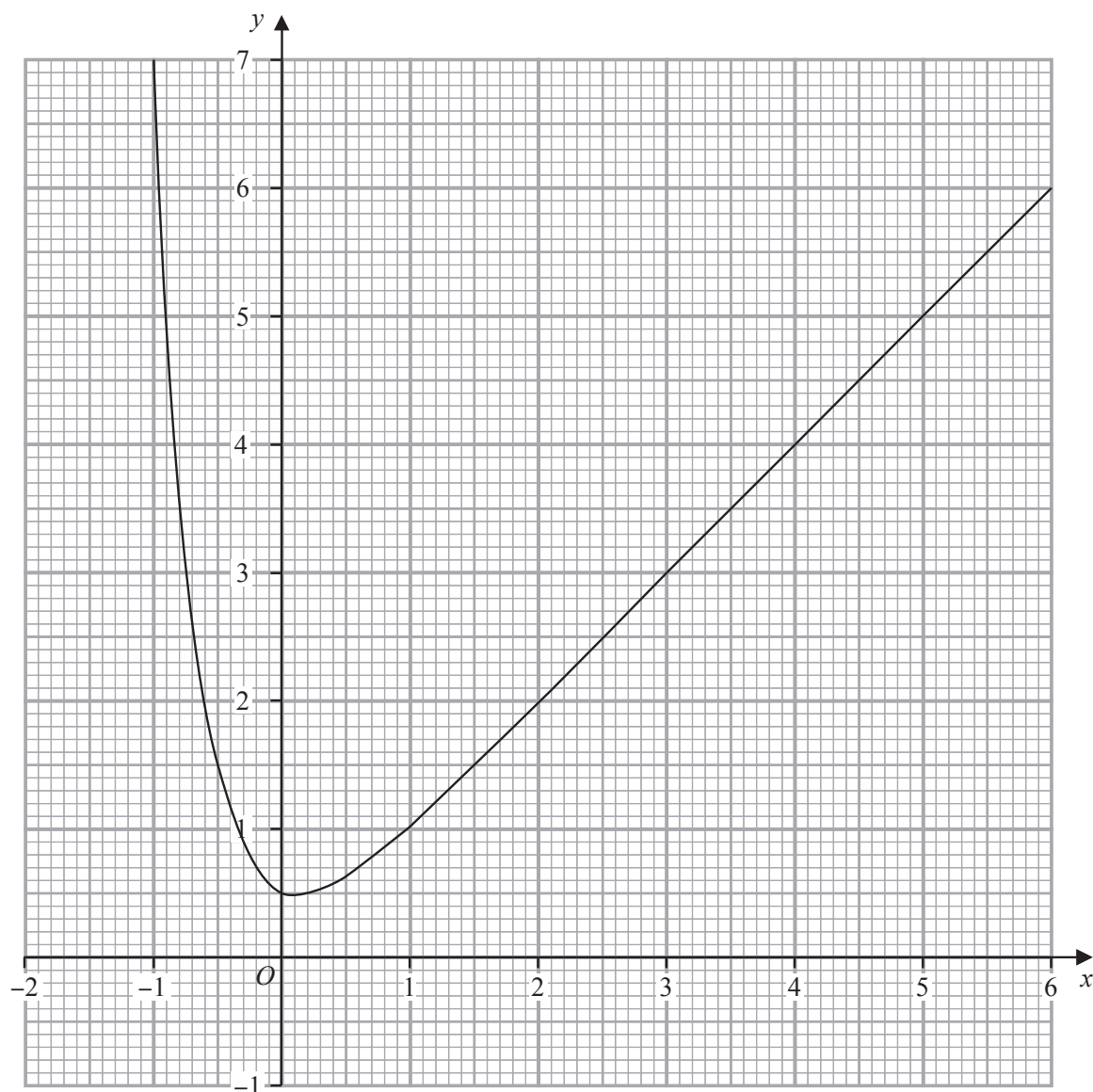
**Figure 1**

Figure 1 shows part of the graph of the curve with equation $y = x + 2^{-(4x+1)}$

By drawing a suitable straight line on the graph, obtain an estimate, to one decimal place, of the roots of the equation $\log_2(8 - 3x) + 4x = 0$ in the interval $-2 \leq x \leq 6$

(7)

.....

.....

.....

.....

8-MARK QUESTIONS

1 question in this booklet



- 4 (a) Complete the table of values for $y = \log_{10}(6x - 1) - x$ giving your answers to 2 decimal places.

x	0.25	0.5	1	1.5	2	2.5	3
y		-0.20	-0.30	-0.60			

(2)

- (b) On the grid opposite, draw the graph of $y = \log_{10}(6x - 1) - x$ for $0.25 \leq x \leq 3$

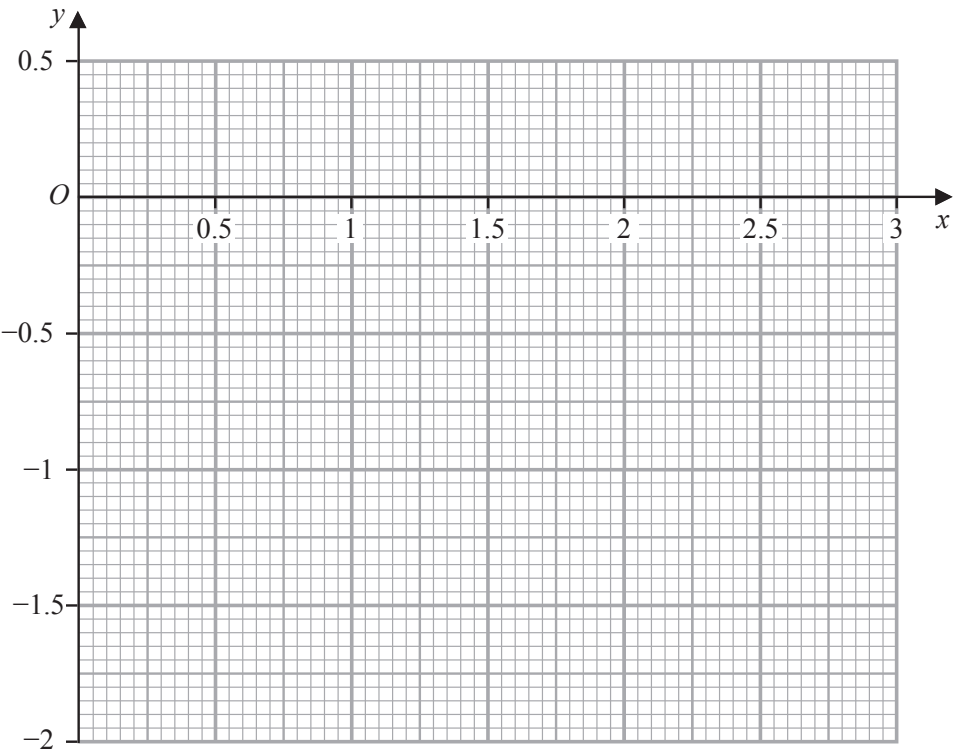
(2)

- (c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$10^{\frac{3x-4}{2}} = 6x - 1$ in the interval $0.25 \leq x \leq 3$

(4)

Question 4 continued



.....

.....

.....

.....

.....

.....

.....

.....

.....

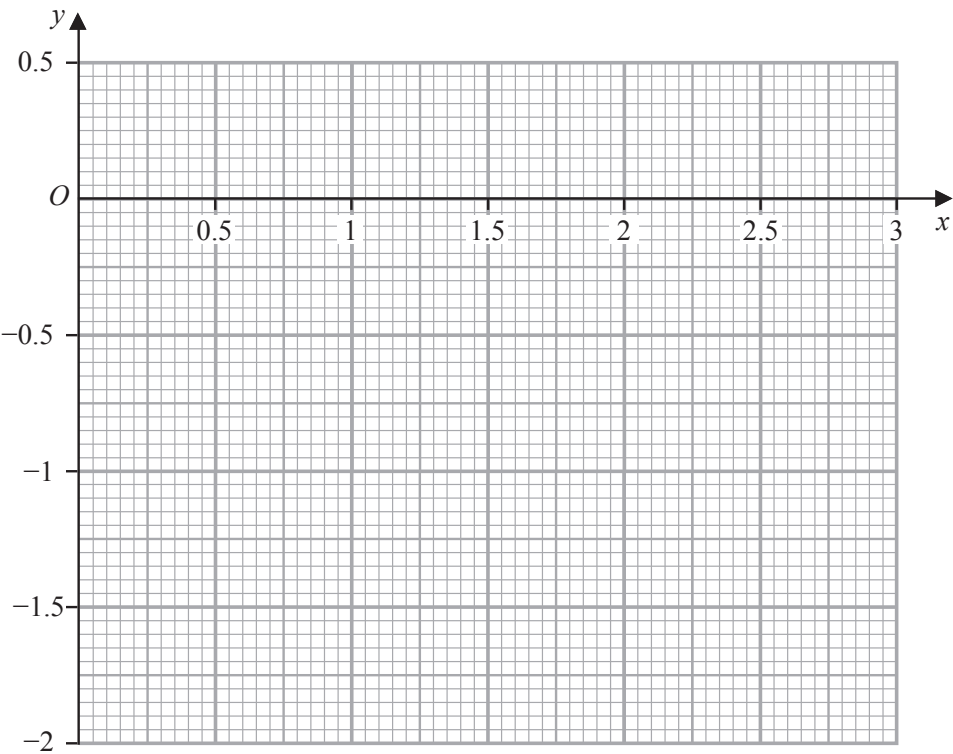
.....

.....

Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

Only use this grid if you need to redraw your graph.



9-MARK QUESTIONS

6 questions in this booklet



- 6 (a) Complete the table of values for

$$y = x - \frac{3}{x^2}$$

giving your answers to one decimal place where appropriate.

x	0.5	1	1.5	2	3	4	5	6
y	-11.5			1.3	2.7			5.9

(2)

- (b) On the grid opposite, draw the graph of $y = x - \frac{3}{x^2}$ for $0.5 \leq x \leq 6$

(2)

- (c) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of each of the two roots of the equation

$$2x^3 - 6x^2 + 3 = 0$$

in the interval $0.5 \leq x \leq 6$

(5)

.....

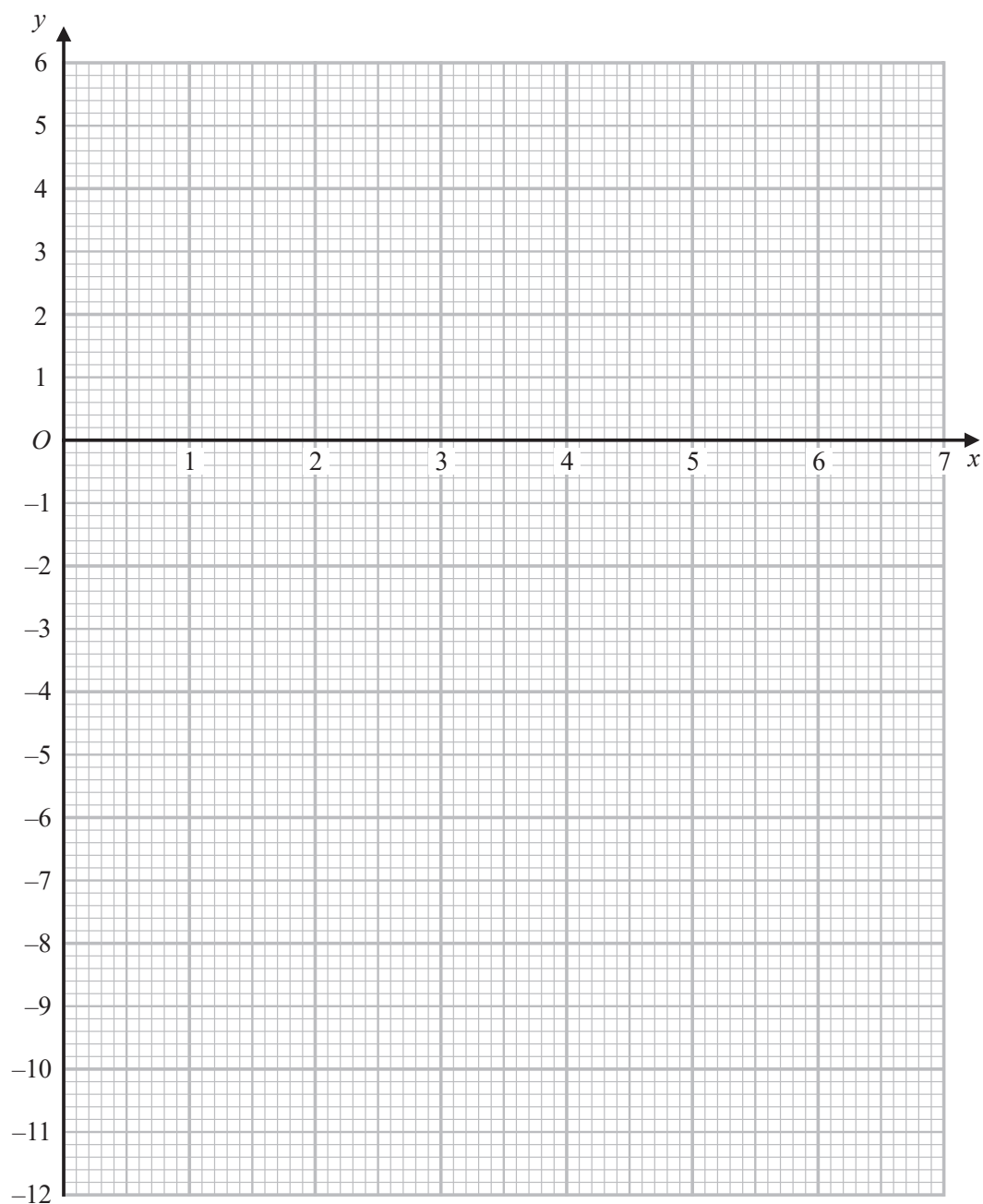
.....

.....

.....

.....

Question 6 continued



.....

.....

.....

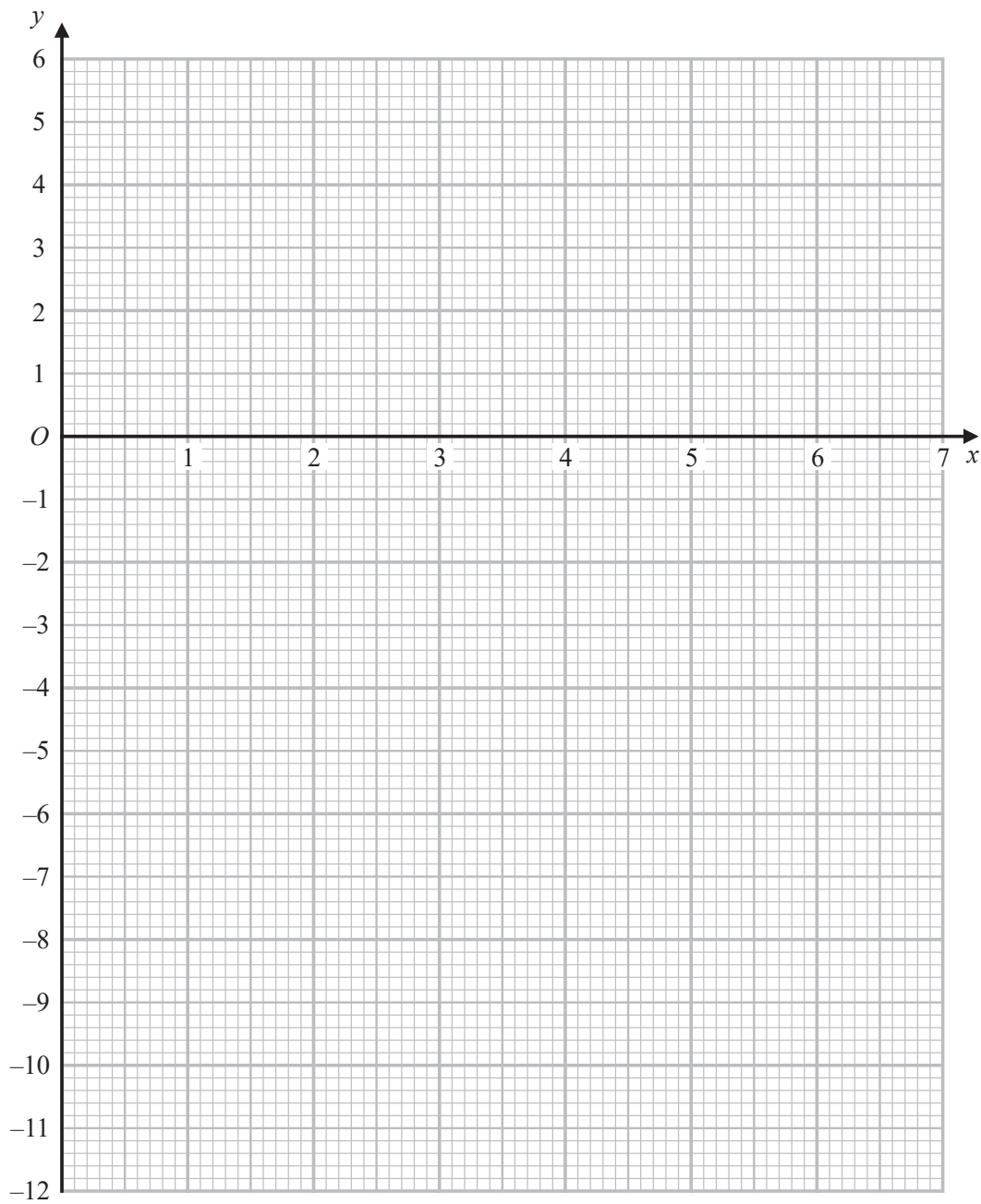
.....

Turn over for a spare grid if you need to redraw your graph.



Question 6 continued

Only use this grid if you need to redraw your graph.



.....

.....

.....

(Total for Question 6 is 9 marks)

- 7 (a) Complete the table of values for $y = 3^{\frac{x}{4}} + 2$

Give your answers to 2 decimal places where appropriate.

(2)

x	0	1	2	3	4	5
y	3	3.32				5.95

- (b) On the grid opposite, draw the graph of

$$y = 3^{\frac{x}{4}} + 2 \quad \text{for } 0 \leq x \leq 5$$

(2)

- (c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$\log_3(6 - 2x)^4 - x = 0$$

in the interval $0 \leq x \leq 5$

(5)

.....

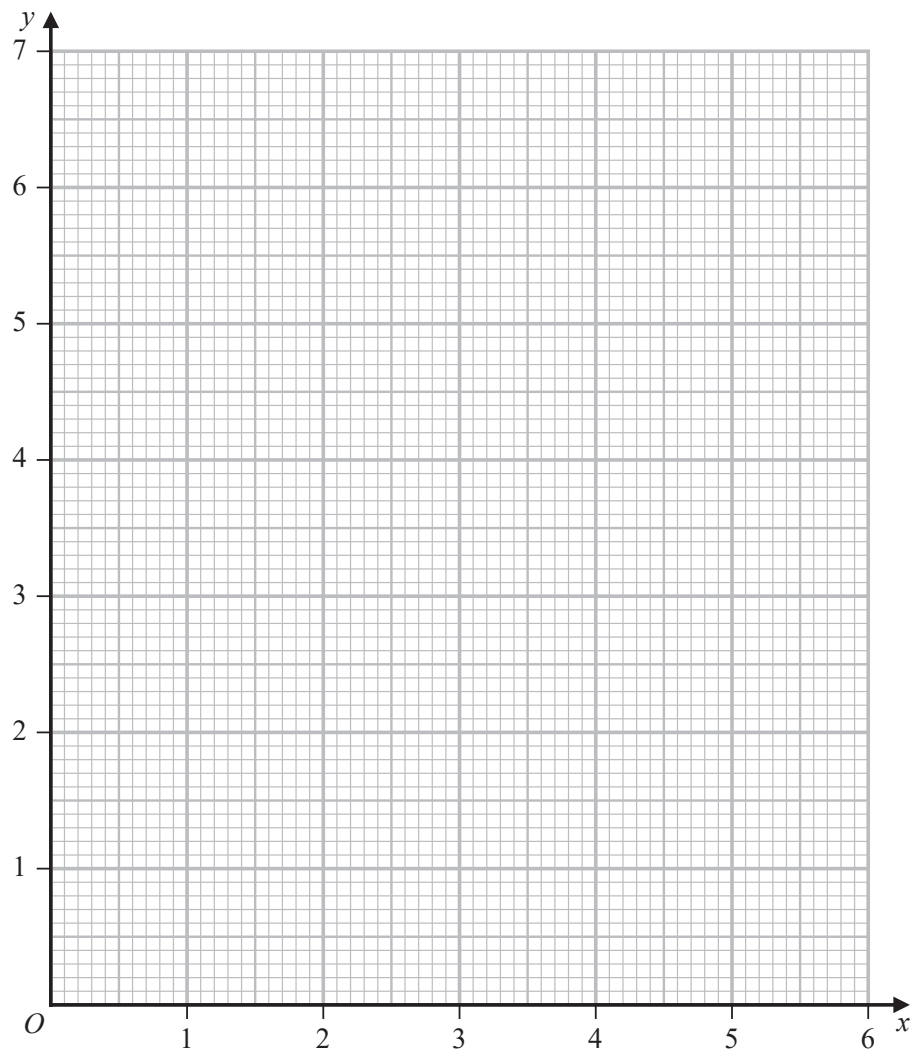
.....

.....

.....

.....

Question 7 continued



.....

.....

.....

.....

.....

.....

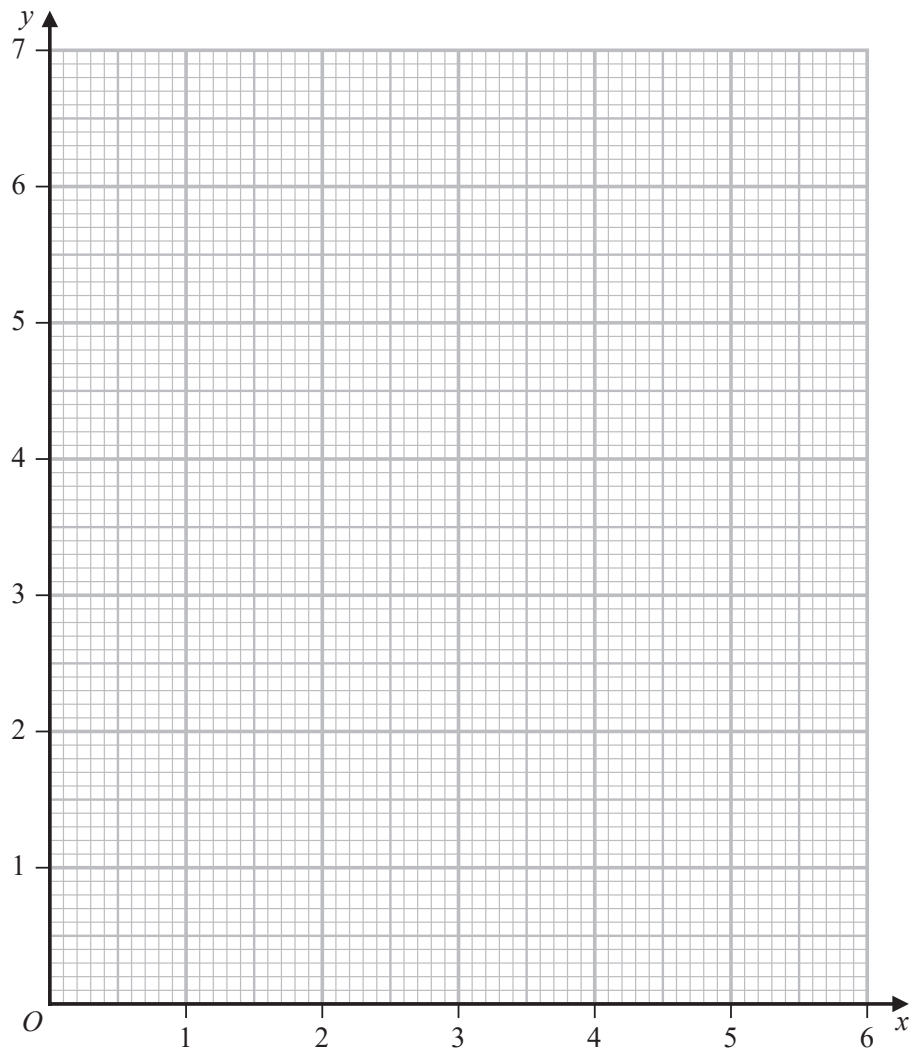
.....

.....

Turn over for a spare grid if you need to redraw your graph.

Question 7 continued

Only use this grid if you need to redraw your graph.



.....

.....

.....

.....

.....

- 8 (a) Complete the table of values for $y = 2x + \frac{3}{x^2} - 3$ giving your answers to 2 decimal places where appropriate.

x	0.5	0.75	1	1.5	2	3	4	5
y	10		2				5.19	7.12

(2)

- (b) On the grid opposite, draw the graph of $y = 2x + \frac{3}{x^2} - 3$ for $0.5 \leq x \leq 5$

(2)

- (c) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $4x^3 - 10x^2 + 3 = 0$ for $0.5 \leq x \leq 5$

(5)

.....

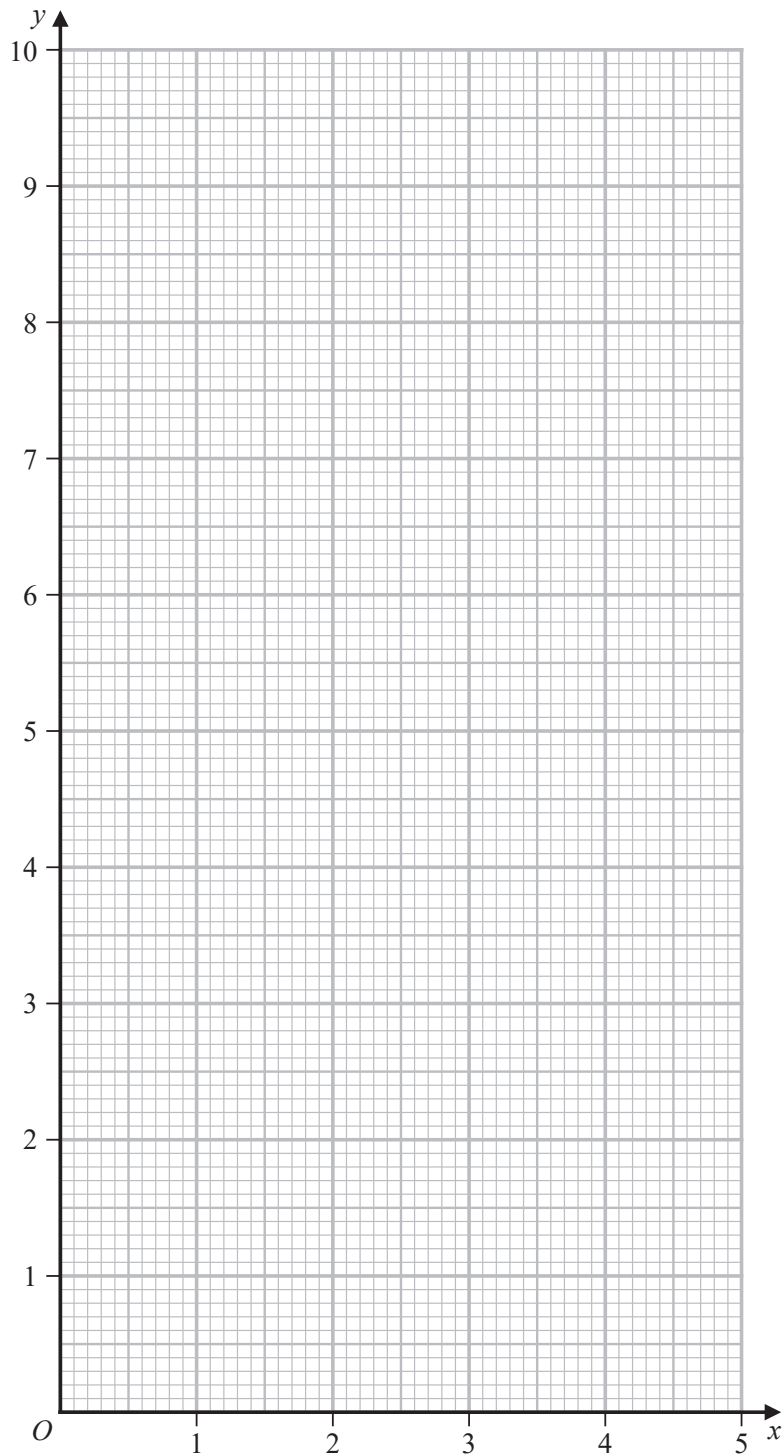
.....

.....

.....

.....



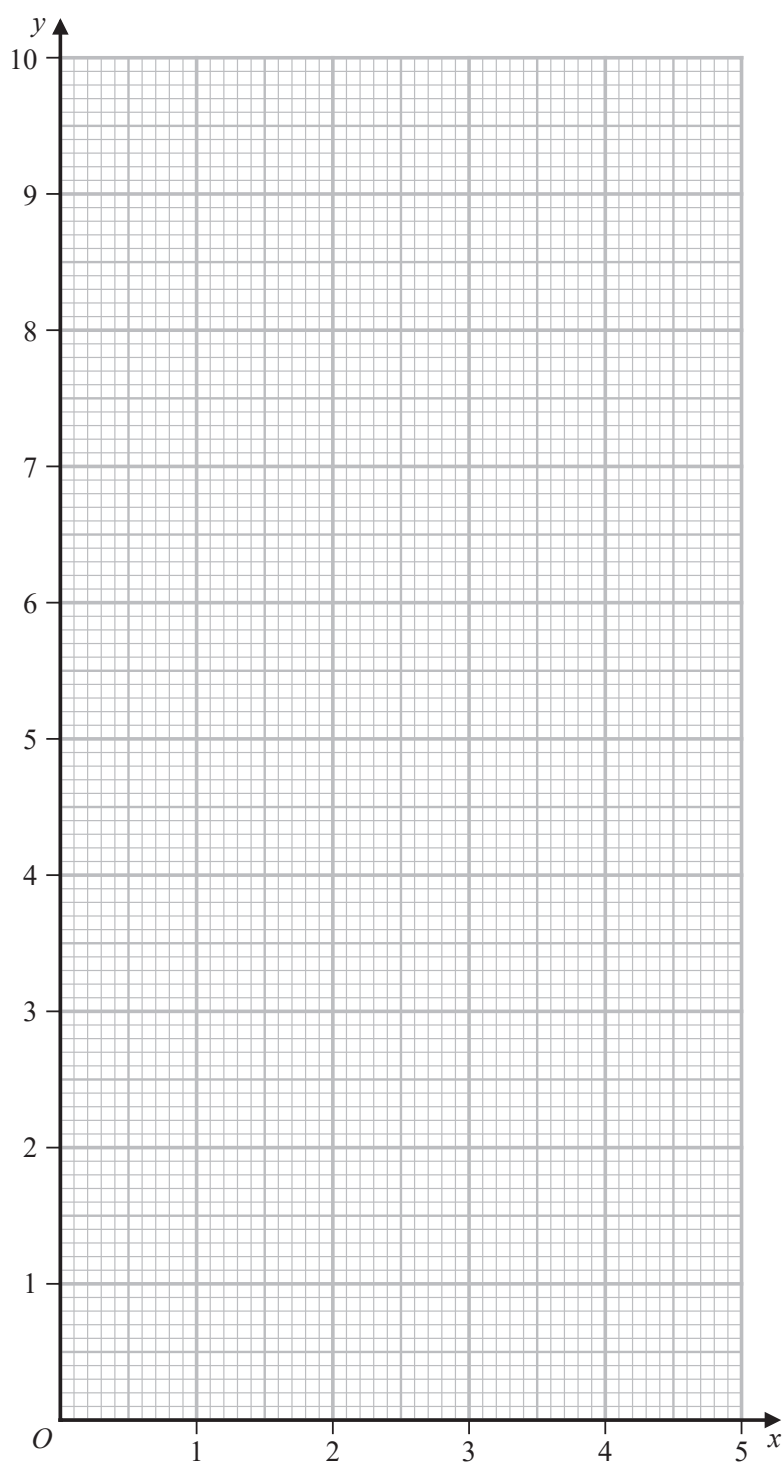
Question 8 continued

Turn over for a spare grid if you need to redraw your graph.



Question 8 continued

Only use this grid if you need to redraw your graph.



(Total for Question 8 is 9 marks)



- 9 (a) Complete the table of values for

$$y = \frac{x^3 + 4}{5 - x}$$

giving your answers to 2 decimal places.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	-0.57	0.10	0.5		0.8		1.25		4

(2)

- (b) On the grid opposite, draw the graph of

$$y = \frac{x^3 + 4}{5 - x} \quad \text{for } -2 \leq x \leq 2$$

(2)

- (c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $x^3 - x^2 + 8x - 11 = 0$ in the interval $-2 \leq x \leq 2$

(5)

.....

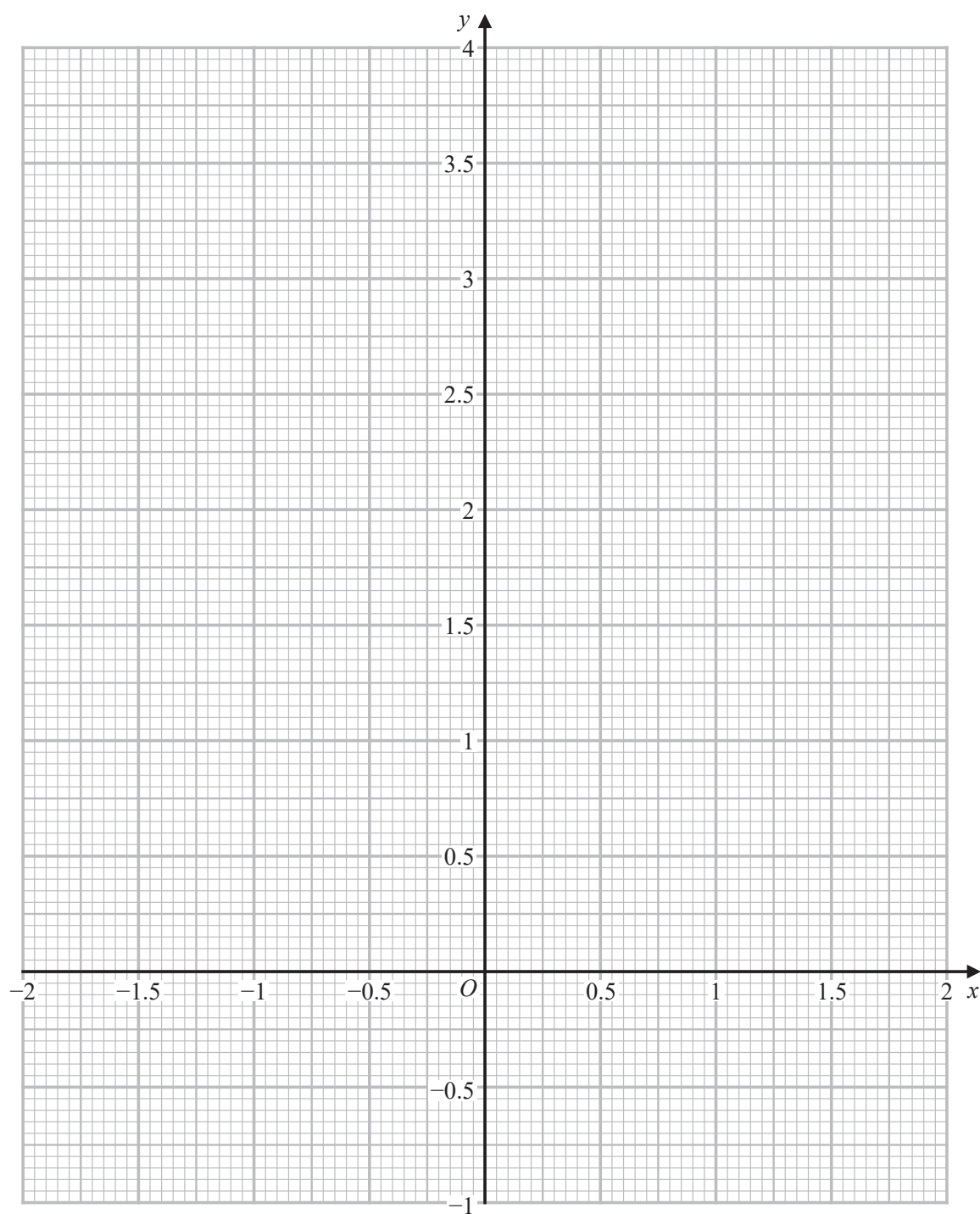
.....

.....

.....

.....

Question 9 continued



.....

.....

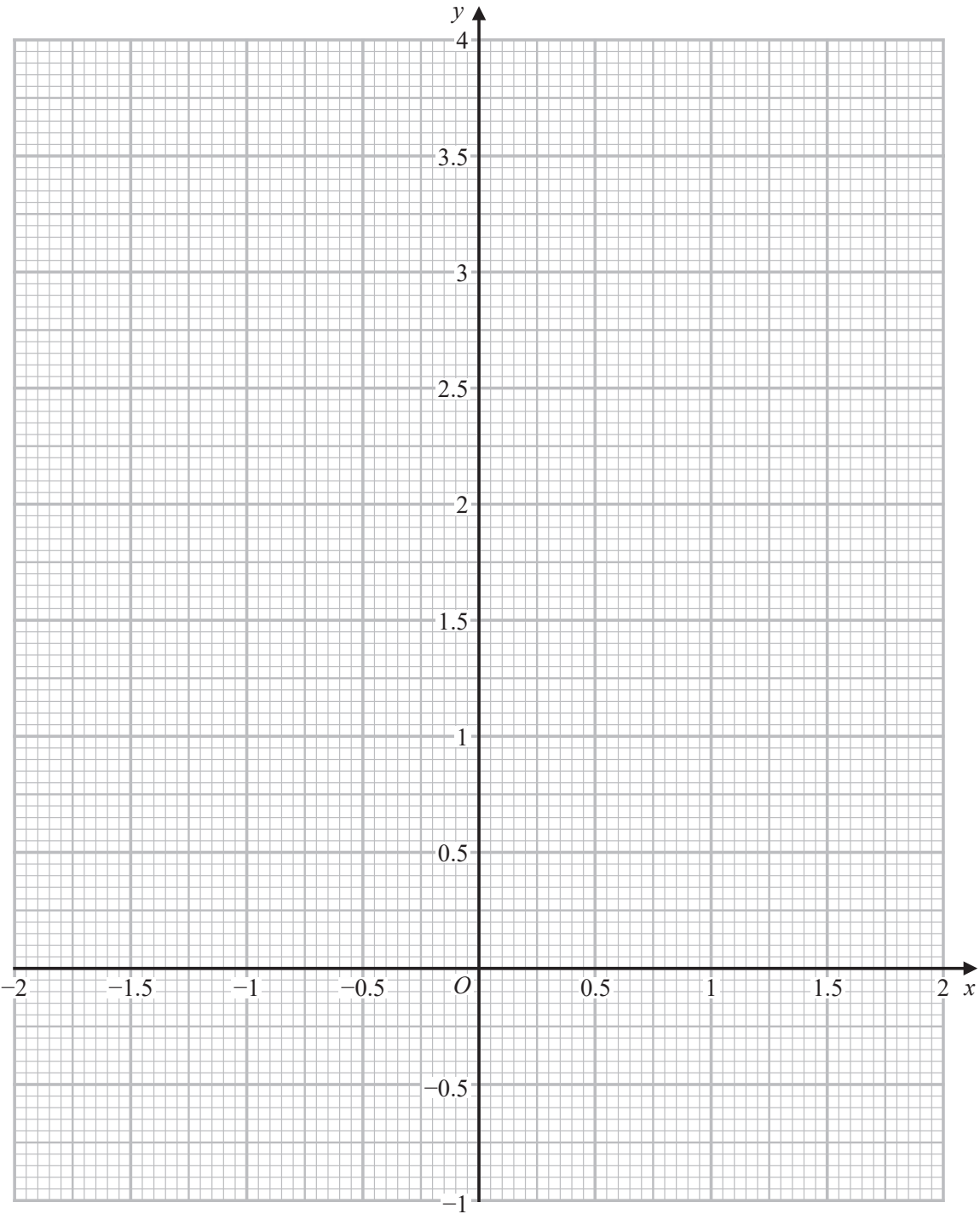
.....

Turn over for a spare grid if you need to redraw your graph.



Question 9 continued

Only use this grid if you need to redraw your graph.



.....

.....

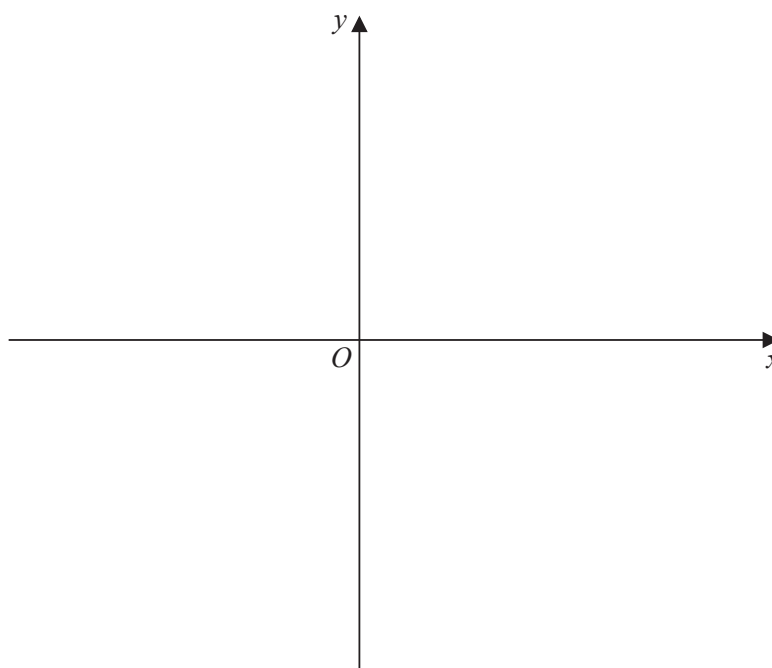
(Total for Question 9 is 9 marks)



- 3 Curve C has equation $y = \frac{ax + 3}{1 - 2x}$ where $x \neq \frac{1}{2}$ and a is a constant.

The asymptote to C that is parallel to the x -axis has equation $y = 4$

- (a) Find the value of a (2)
- (b) Write down the equation of the asymptote to C that is parallel to the y -axis. (1)
- (c) Find the coordinates of the point where C crosses
- (i) the x -axis, (ii) the y -axis. (2)
- (d) Using the axes below, sketch C , showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes. (4)



- 4 (a) Complete the table of values for $y = \frac{x}{2} + 6e^{-2x} + 1$ giving your answers to one decimal place.

x	0	1	1.5	2	3	4	5	6
y	7		2.0			3.0		4.0

(2)

- (b) On the grid opposite, draw the graph of $y = \frac{x}{2} + 6e^{-2x} + 1$ for $0 \leq x \leq 6$

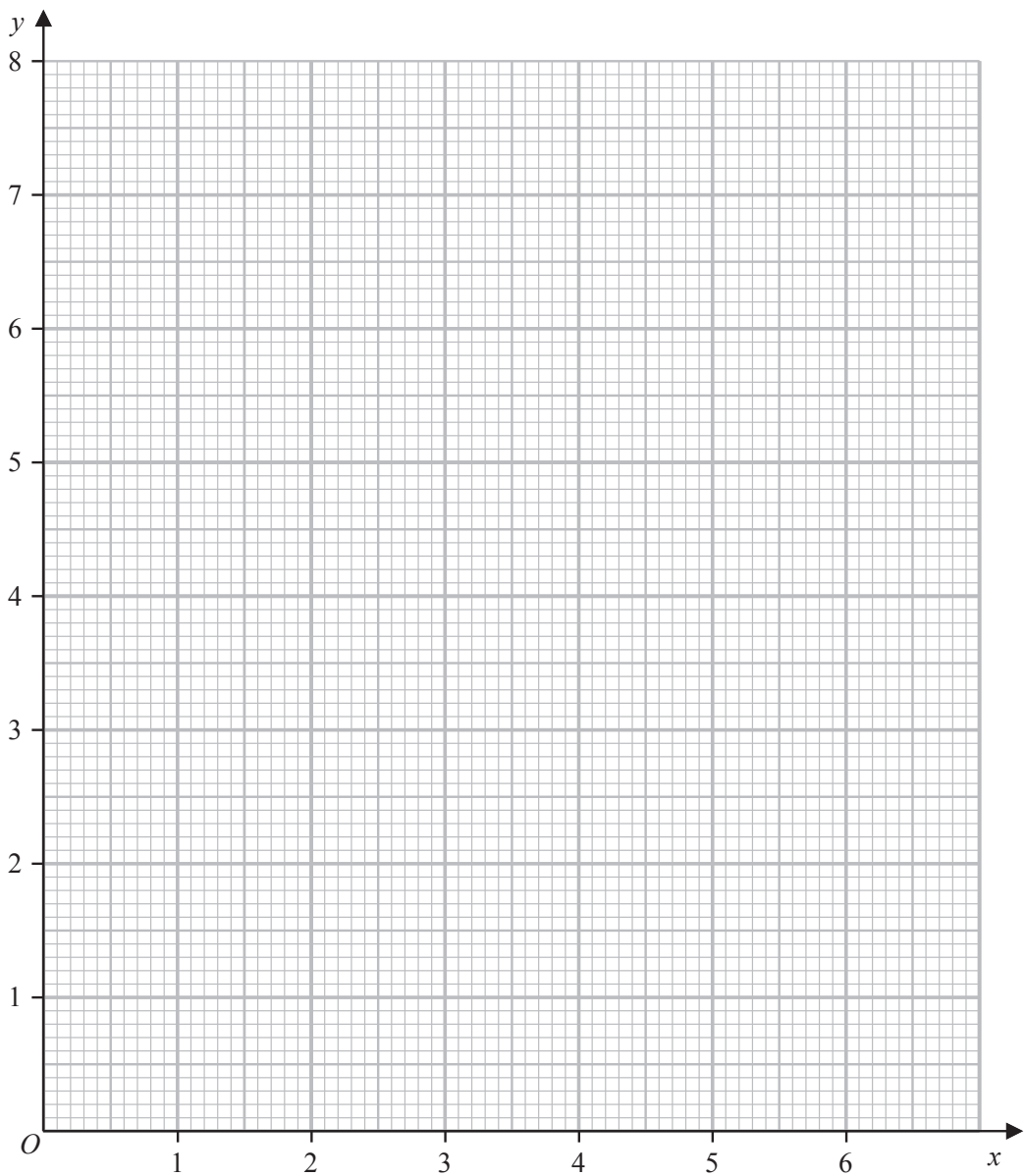
(2)

- (c) By drawing a suitable straight line on your graph, obtain estimates, to one decimal place, of the roots of the equation

$$2x + \ln(24 - 5x) = \ln 36$$

(5)

Question 4 continued



.....

.....

.....

.....

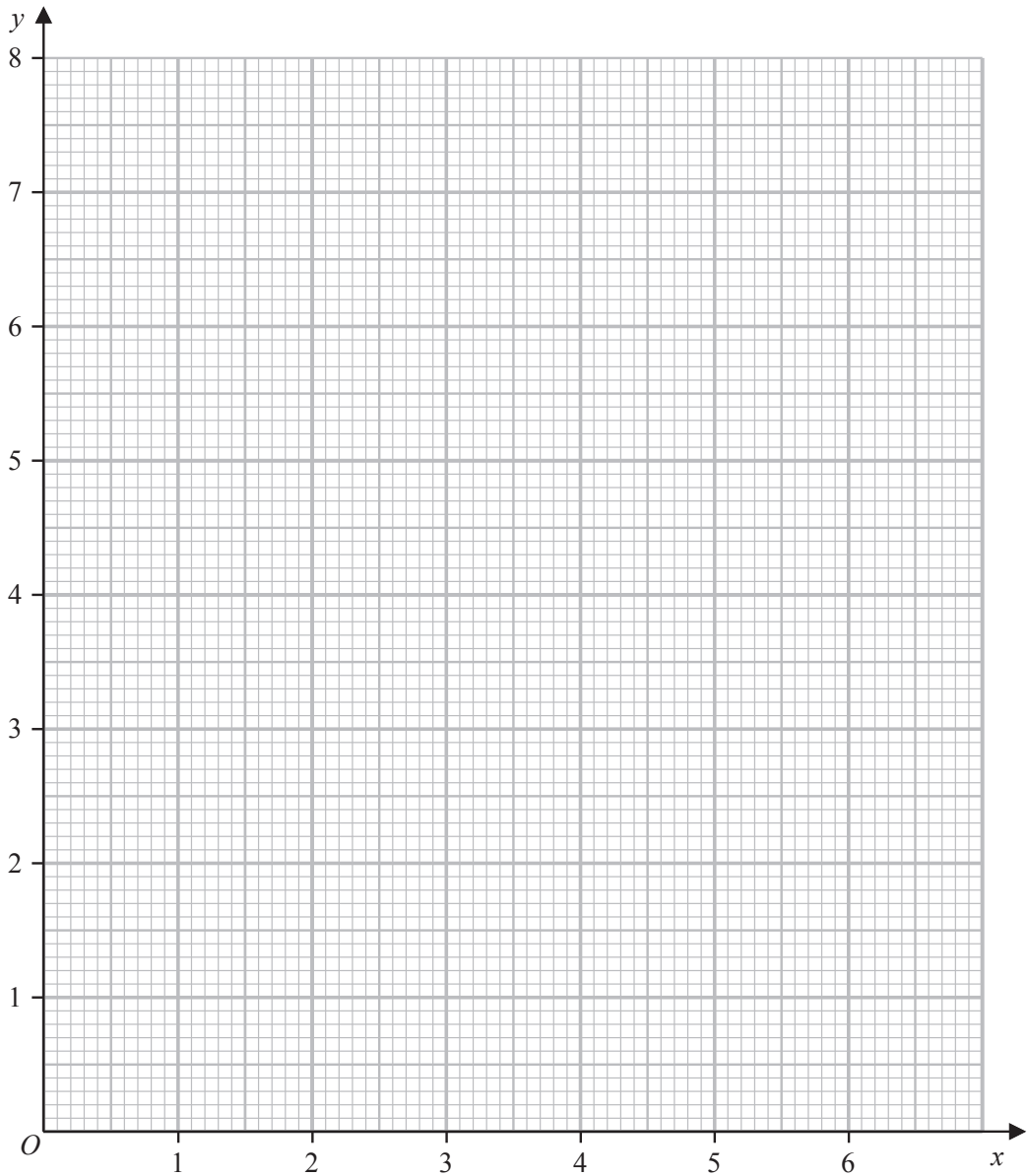
.....

.....

Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

Only use this grid if you need to redraw your graph.



.....

.....

.....

.....

.....



10-MARK QUESTIONS

4 questions in this booklet



- 3 A curve C has equation $y = \frac{ax - 3}{x + 5}$ where a is a constant and $x \neq -5$

The gradient of C at the point on the curve where $x = 2$ is $\frac{18}{49}$

- (a) Show that $a = 3$

(3)

Hence

- (b) write down an equation of the asymptote to C that is

- (i) parallel to the x -axis,
(ii) parallel to the y -axis,

(2)

- (c) find the coordinates of the point where C crosses

- (i) the x -axis,
(ii) the y -axis.

(2)

- (d) Sketch the curve C , showing clearly its asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

.....

.....

.....

.....

.....



- 7 (a) Complete the table of values for

$$y = 0.5 \left(\frac{x}{3} + 1 \right) + 2$$

giving each value to 2 decimal places where appropriate.

x	-6	-5	-4	-3	-2	-1	0
y	4	3.59	3.26				2.5

(2)

- (b) On the grid opposite, draw the graph of $y = 0.5 \left(\frac{x}{3} + 1 \right) + 2$ for $-6 \leq x \leq 0$

(2)

- (c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$\log_2(2x + 2)^3 + x + 3 = 0 \text{ in the interval } -6 \leq x \leq 0$$

(6)

.....

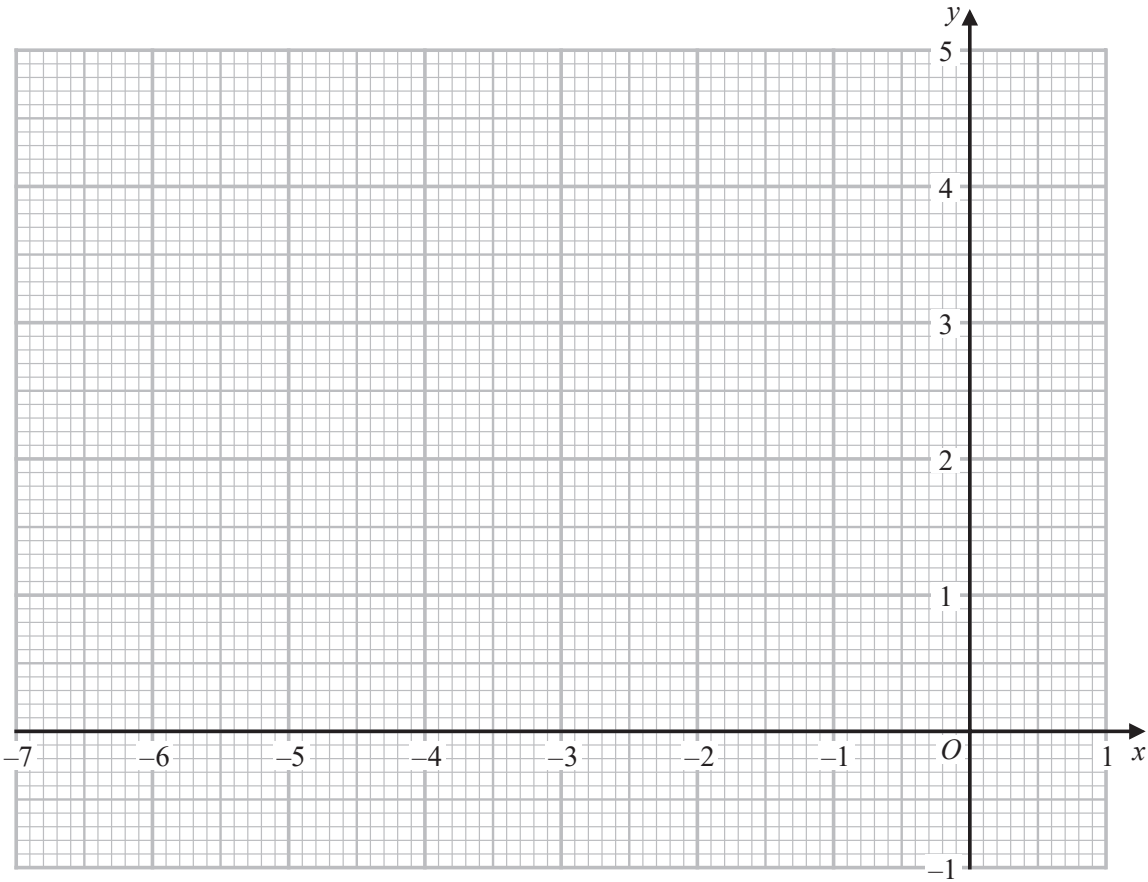
.....

.....

.....

.....

Question 7 continued



.....

.....

.....

.....

.....

.....

.....

.....

.....

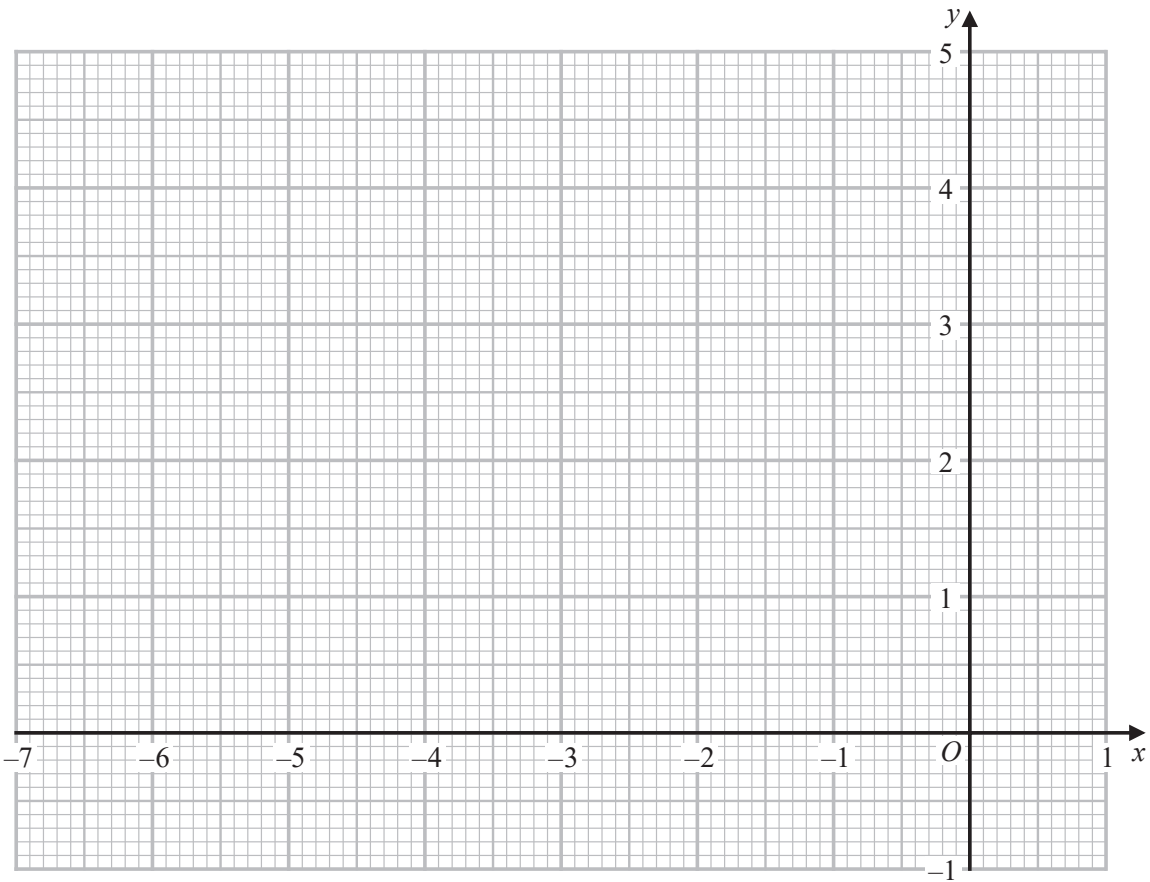
.....

.....

Turn over for a spare grid if you need to redraw your graph.

Question 7 continued

Only use this grid if you need to redraw your graph.



.....

.....

.....

.....

.....

- 7 (a) Complete the table of values for $y = 3\log_3(x^2 - x) - 2x$ giving your answers to 2 decimal places.

x	1.3	1.5	2	2.5	3	3.5
y		-3.79	-2.11			

(2)

- (b) On the grid opposite, draw the graph of $y = 3\log_3(x^2 - x) - 2x$ for $1.3 \leq x \leq 3.5$

(2)

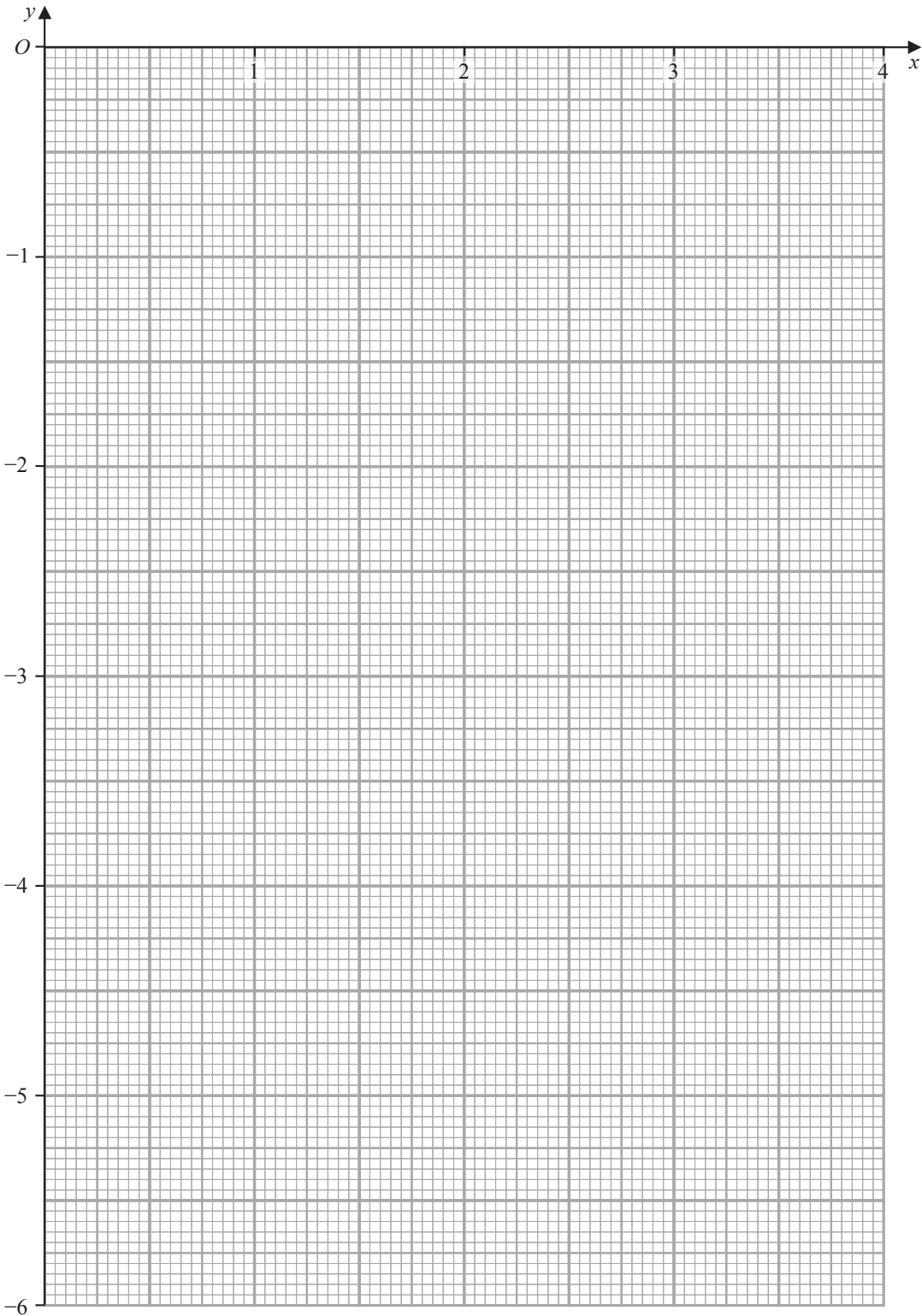
- (c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$3^{\frac{4}{3}x} = 9(x^2 - x)^3 \quad \text{in the interval } 1.3 \leq x \leq 3.5$$

(6)



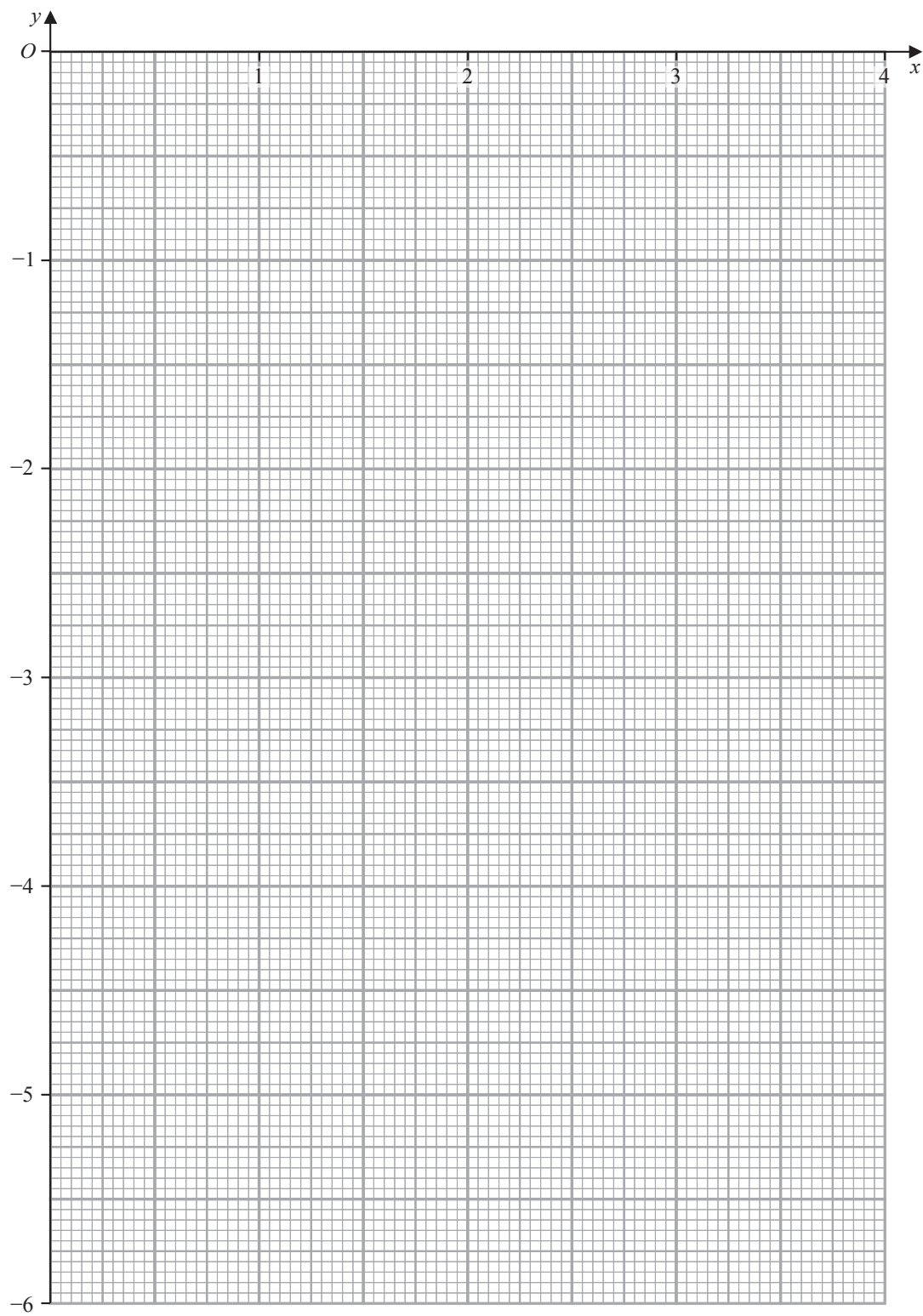
Question 7 continued



Turn over for a spare grid if you need to redraw your graph.

Question 7 continued

Only use this grid if you need to redraw your graph.



(Total for Question 7 is 10 marks)

7 (a) Complete the table of values for $y = \log_3(4 - x) + 3$ giving your answers to 2 decimal places.

x	0	0.5	1	1.5	2	2.5	3	3.5
y	4.26		4		3.63		3	2.37

(2)

(b) On the grid opposite, draw the graph of $y = \log_3(4 - x) + 3$ in the interval $0 \leq x \leq 3.5$

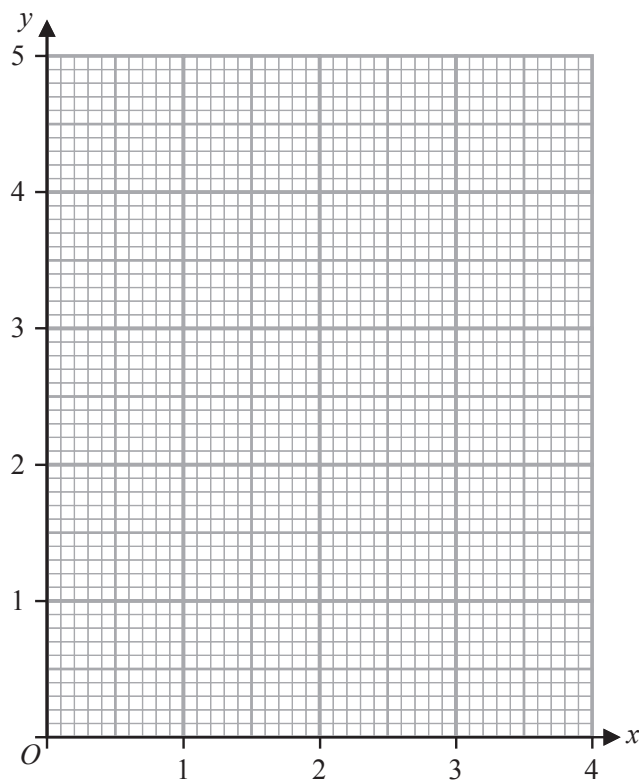
(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $3^{2x-5} - (4 - x)^3 = 0$ in the interval $0 \leq x \leq 3.5$

(6)



Question 7 continued



.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

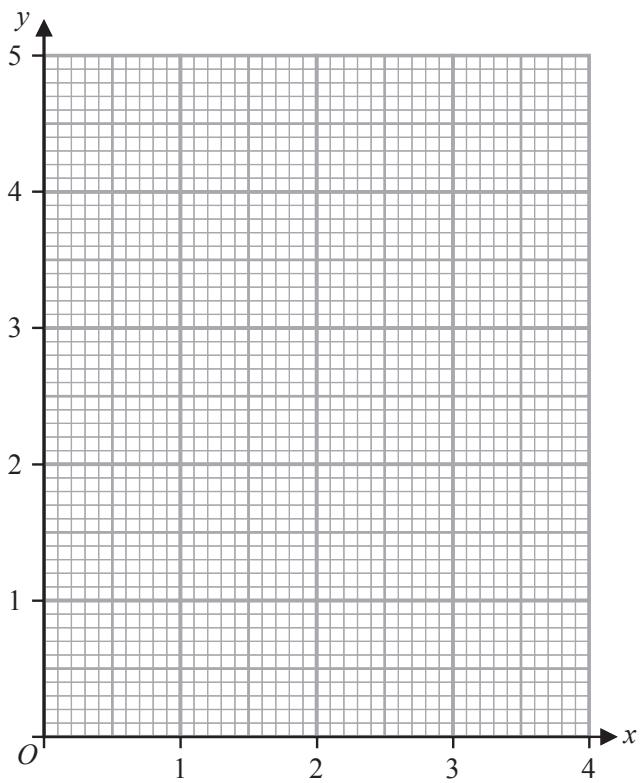
.....

.....

Turn over for a spare grid if you need to redraw your graph.

Question 7 continued

Only use this grid if you need to redraw your graph.



.....

.....

.....

.....

.....

11-MARK QUESTIONS

4 questions in this booklet



- 8 (a) Complete the table of values for $y = 2 + \ln(2x + 1)$ giving your answers to 2 decimal places.

(2)

x	0	0.25	0.5	1	1.5	2	3
y	2			3.10	3.39	3.61	

- (b) On the grid opposite, draw the graph of $y = 2 + \ln(2x + 1)$ for $0 \leq x \leq 3$

(2)

- (c) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $\ln(2x + 1) = 3x - 4$ in the interval $0 \leq x \leq 3$

(3)

- (d) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $e^{(6-x)} - (2x + 1)^2 = 0$ in the interval $0 \leq x \leq 3$

(4)

.....

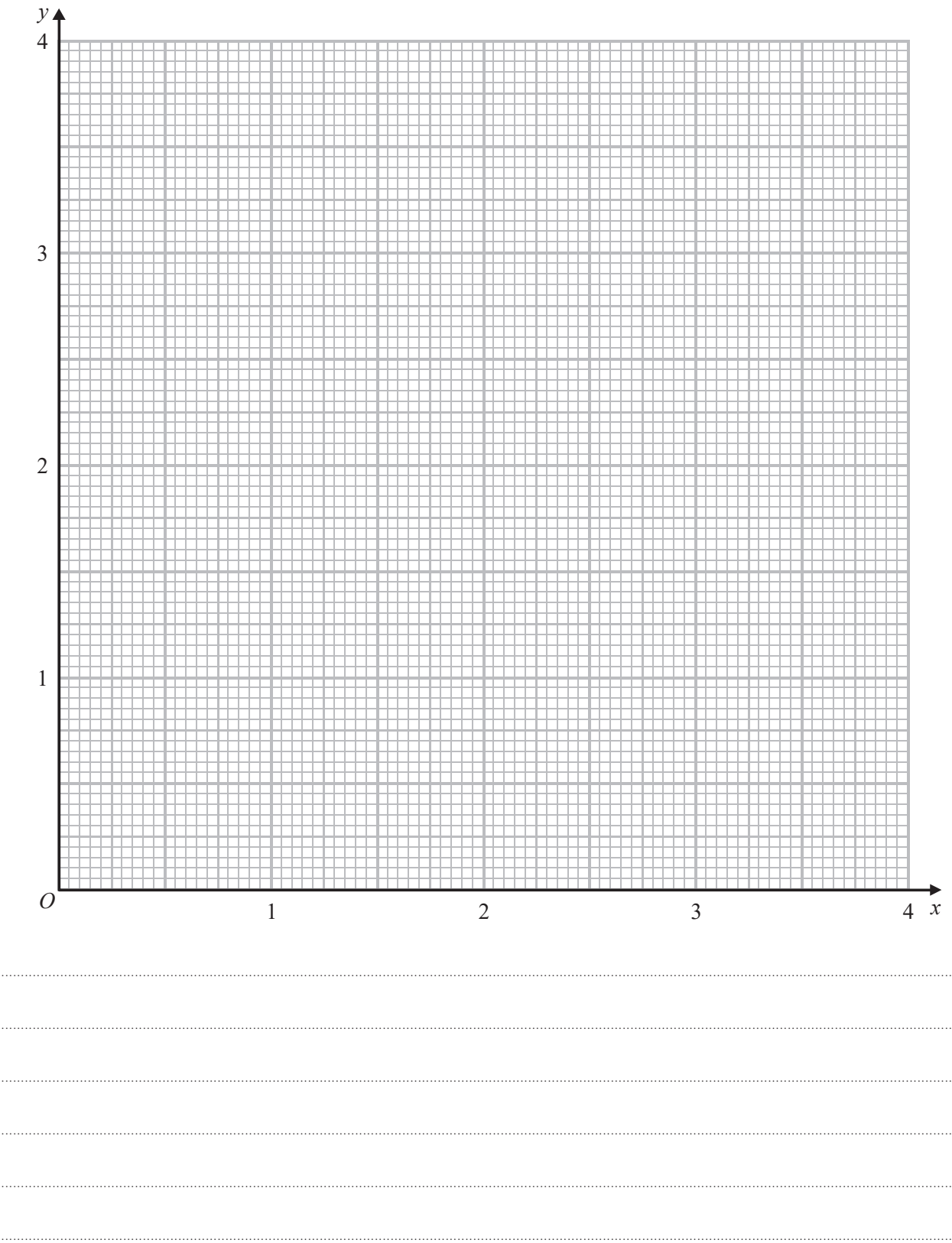
.....

.....

.....

.....

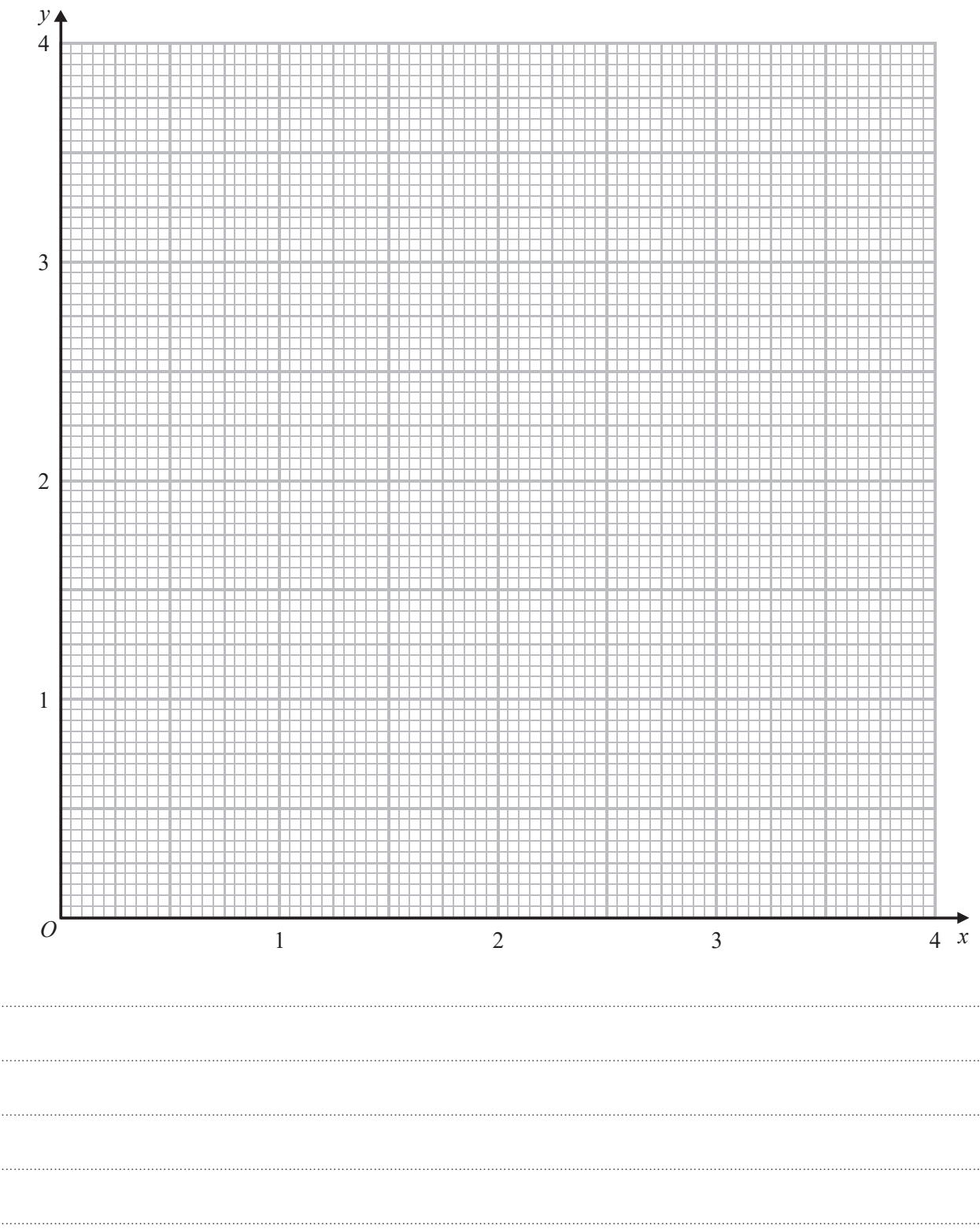
Question 8 continued



Turn over for a spare grid if you need to redraw your graph.

Question 8 continued

Only use this grid if you need to redraw your graph.



- 4 (a) Complete the table of values for $y = 2x + 1 + \frac{2}{x^2}$

Give your answers to 2 decimal places where appropriate.

x	0.5	1	1.5	2	2.5	3	3.5
y		5			6.32		8.16

(2)

- (b) On the grid opposite, draw the graph of $y = 2x + 1 + \frac{2}{x^2}$ for $0.5 \leq x \leq 3.5$

(2)

- (c) Use your graph to obtain estimates, to 1 decimal place, of the roots of the equation

$$2x + \frac{2}{x^2} = 7 \quad \text{in the interval } 0.5 \leq x \leq 3.5$$

(2)

- (d) By drawing a suitable straight line on the grid, obtain estimates, to 1 decimal place, of the roots of the equation

$$\frac{3x}{2} + \frac{2}{x^2} = 5 \quad \text{in the interval } 0.5 \leq x \leq 3.5$$

(5)

.....

.....

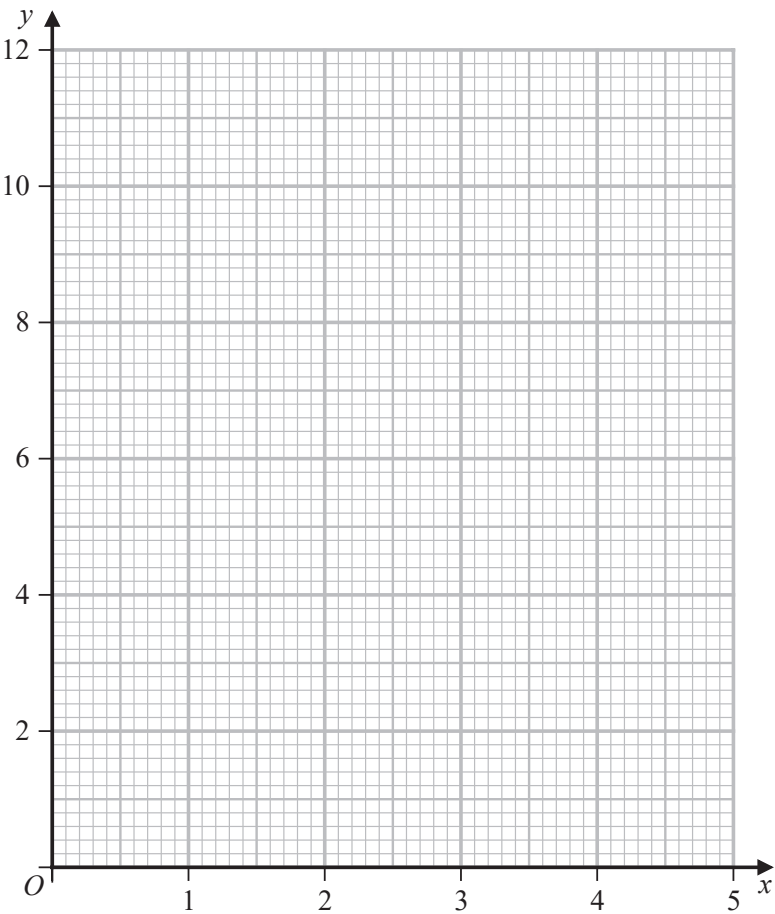
.....

.....

.....



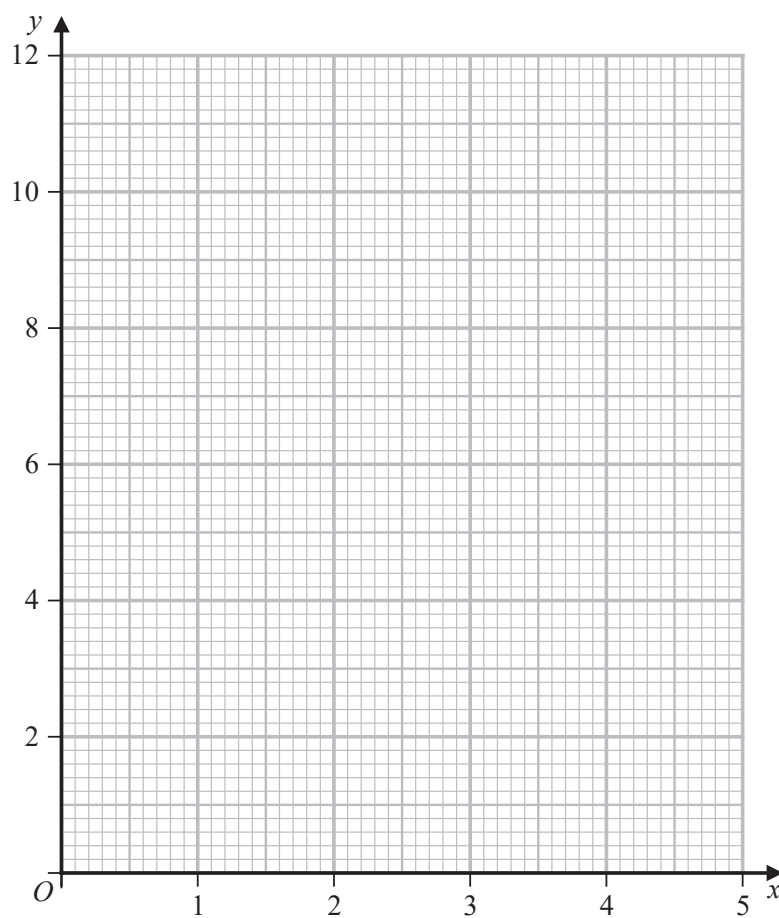
Question 4 continued



Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

Only use this grid if you need to redraw your graph.



.....

.....

.....

.....

- 8 (a) Given that x is measured in radians, complete the table of values for

$$y = \sin x - 3 \cos 2x - 0.5$$

Give your answers to one decimal place.

x	0	0.5	0.8	1	1.6	2	2.5	3
y			0.3	1.6			-0.8	-3.2

(2)

- (b) On the grid opposite, draw the graph of $y = \sin x - 3 \cos 2x - 0.5$ for $0 \leq x \leq 3$

(2)

- (c) Use a formula from page 2 to show that $\cos 2A = 1 - 2 \sin^2 A$

(2)

$$f(x) = 2 \sin x + 12 \sin^2 x - x - 5$$

- (d) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $f(x) = 0$ in the interval $0 \leq x \leq 3$

(5)

.....

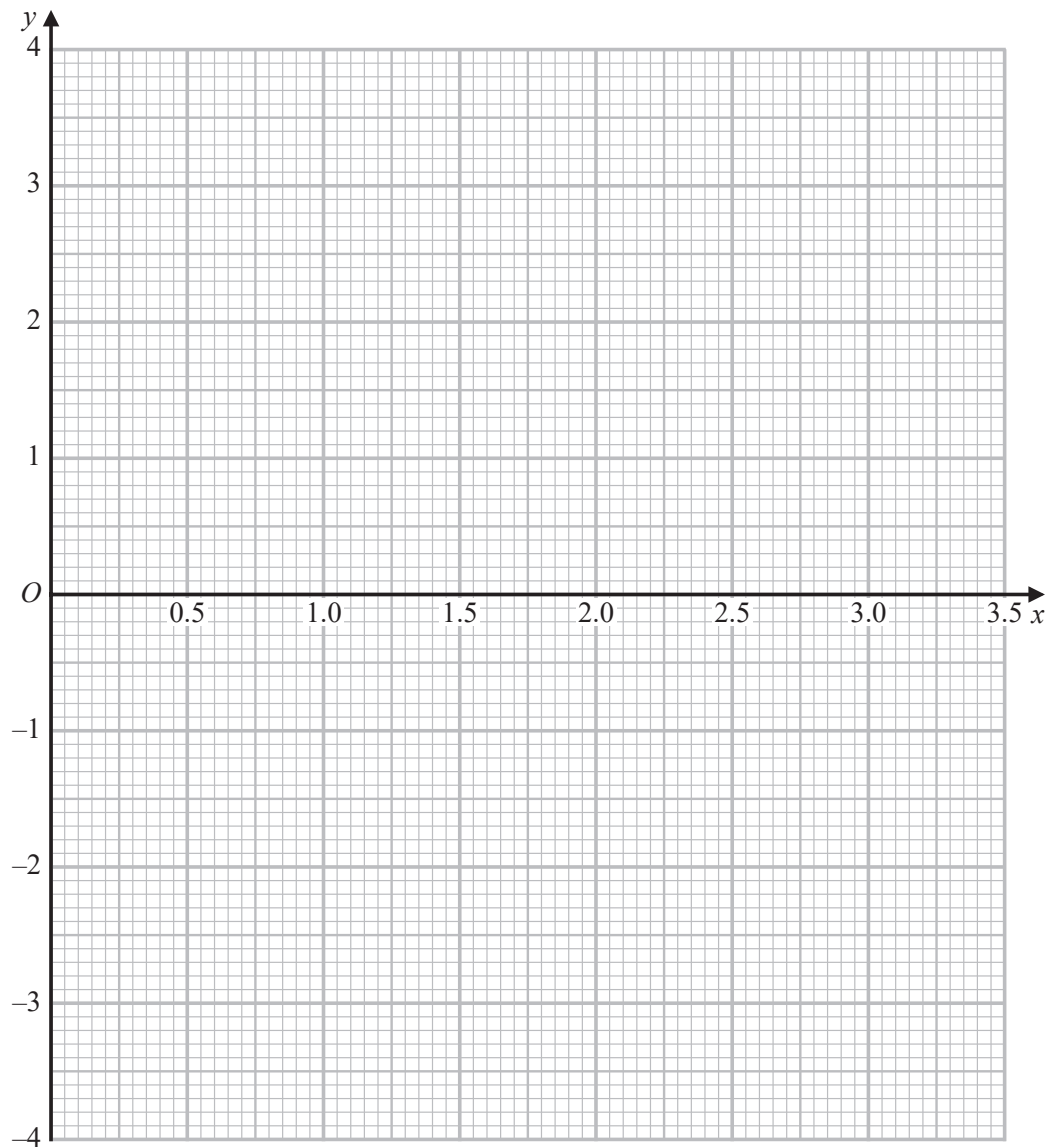
.....

.....

.....

.....

Question 8 continued



.....

.....

.....

.....

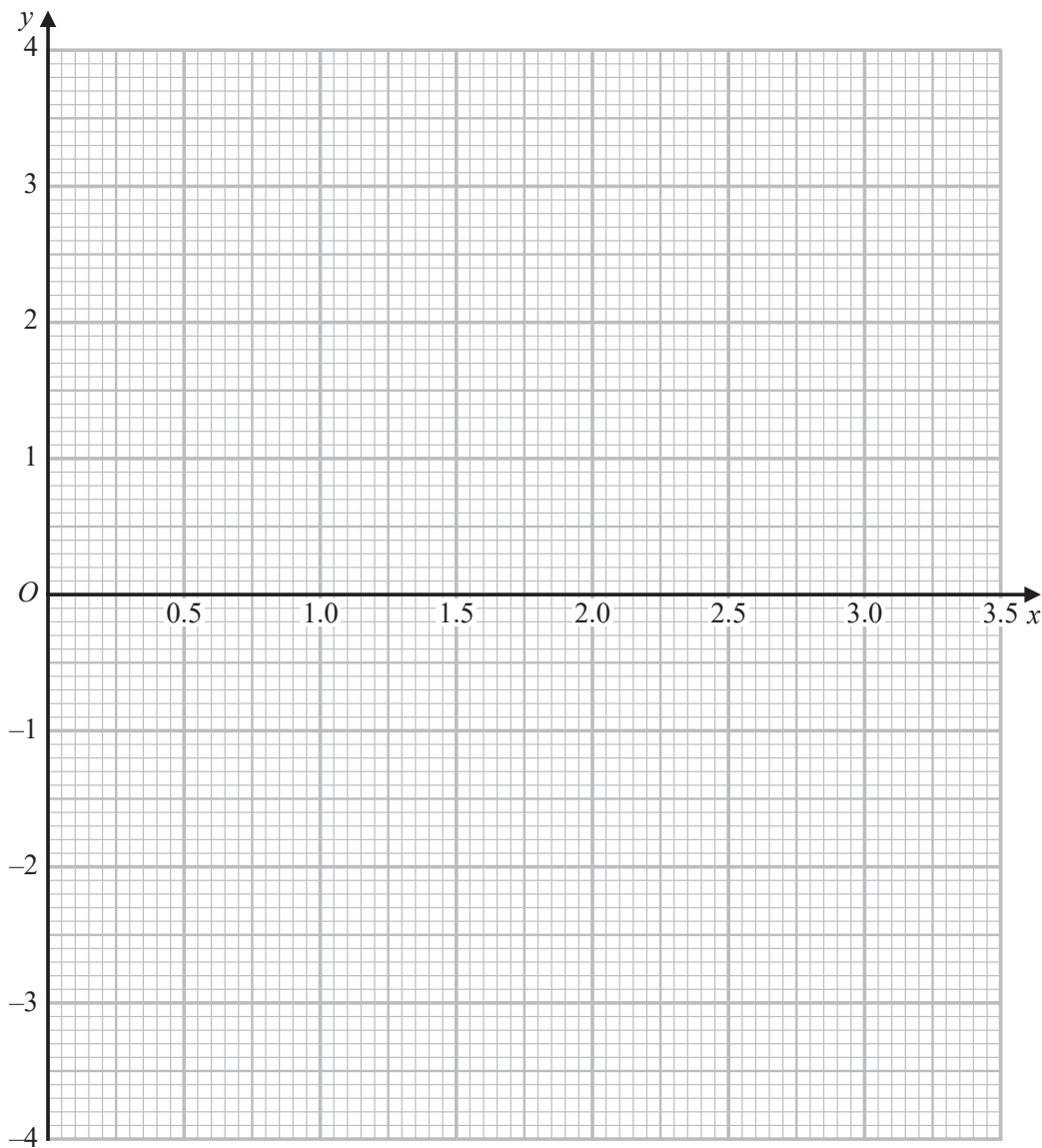
.....

.....

Turn over for a spare grid if you need to redraw your graph.

Question 8 continued

Only use this grid if you need to redraw your graph.



- 6 (a) Complete the table of values for $y = 1 + 3e^{-x}$ giving your answers to 2 decimal places where appropriate.

(2)

x	0	0.25	0.5	1	1.5	2	3
y		3.34	2.82		1.67		1.15

- (b) On the grid opposite, draw the graph of $y = 1 + 3e^{-x}$ for $0 \leq x \leq 3$

(2)

- (c) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$x = e^{-x} \quad \text{in the interval } 0 \leq x \leq 3$$

(3)

- (d) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$\ln(x-1)^3 = -3x \quad \text{in the interval } 0 \leq x \leq 3$$

(4)

.....

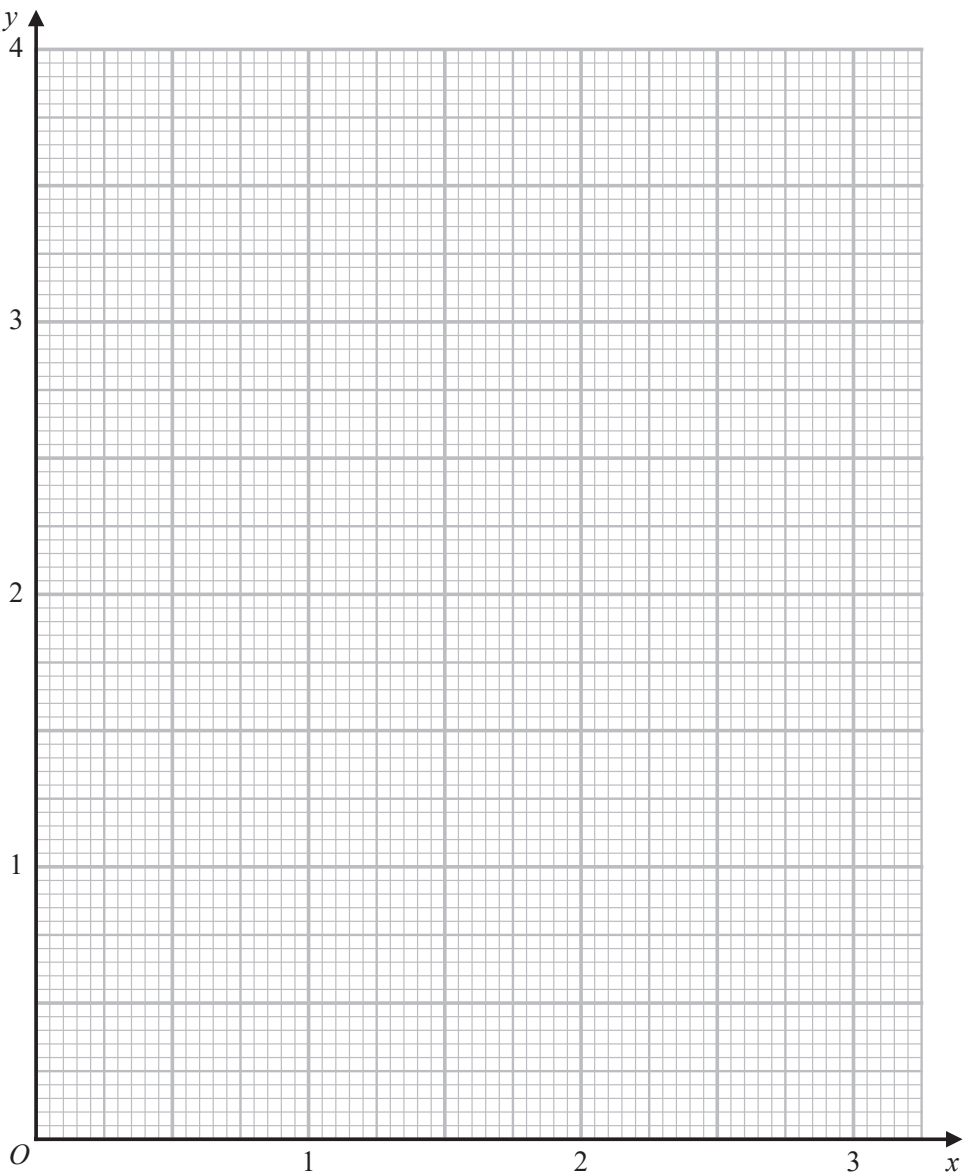
.....

.....

.....

.....

Question 6 continued



.....

.....

.....

.....

.....

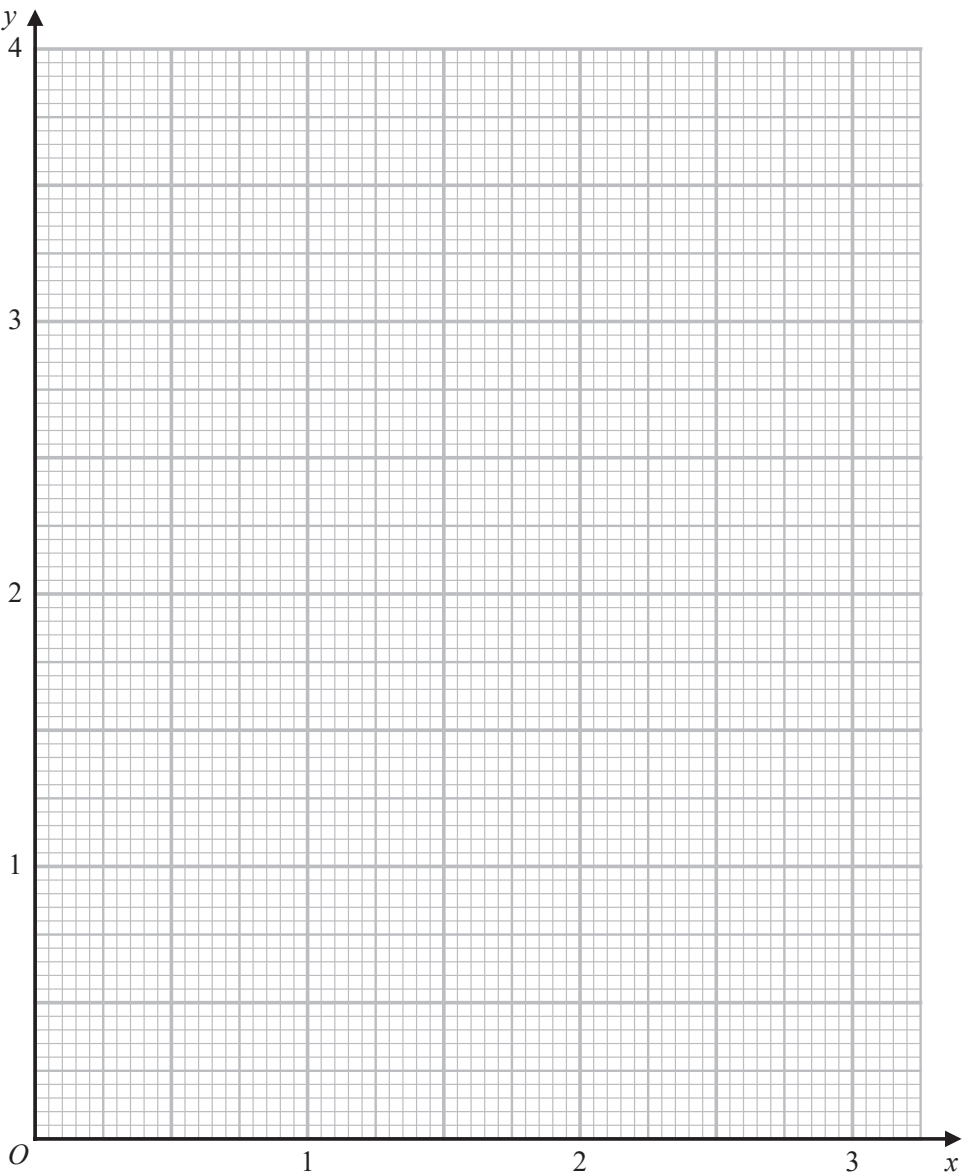
.....

Turn over for a spare grid if you need to redraw your graph.



Question 6 continued

Only use this grid if you need to redraw your graph.



.....

.....

.....

.....

.....



14-MARK QUESTIONS

2 questions in this booklet



11 The curve C has equation $y = \frac{3x - 2}{x + 1}$

(a) Write down an equation of the asymptote to C which is parallel to the

- (i) x -axis (ii) y -axis

(2)

(b) Find the coordinates of the point where C crosses the

- (i) x -axis (ii) y -axis

(2)

(c) Sketch C , showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

The straight line l has equation $y = mx + 4$

Given that there are **no** points of intersection between l and C ,

(d) show algebraically that the range of possible values of m can be written as

$$a - 2\sqrt{b} < m < a + 2\sqrt{b}$$

where a and b are integers whose values need to be found.

(7)

.....

.....

.....

.....

.....

7

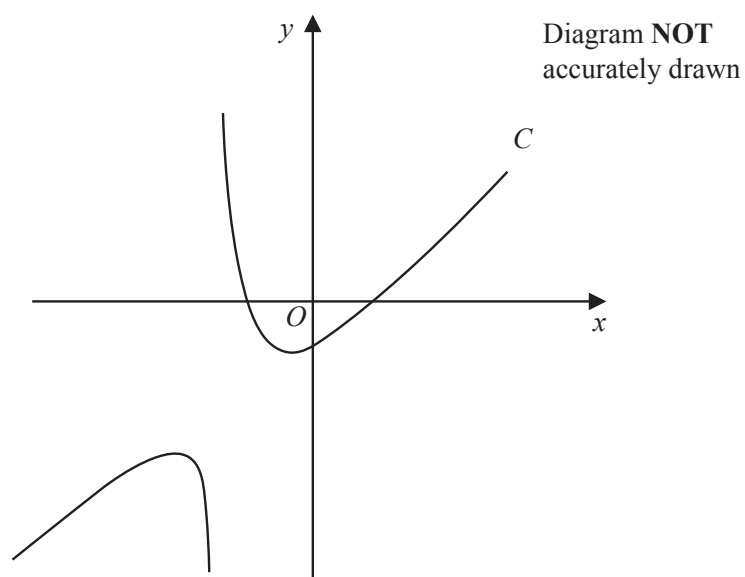
**Figure 2**

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 - 1}{4x + 5} \quad \text{where } x \neq -\frac{5}{4}$$

- (a) Write down the equation of the asymptote to C that is parallel to the y -axis.

(1)

The line l is the normal to C at the point where $x = -1$

- (b) Find an equation of l

(7)

The line l meets C again at the point D

- (c) Find the coordinates of D

(6)

.....

.....

.....

.....

.....

15-MARK QUESTIONS

1 question in this booklet



9 A curve C has equation $y = \frac{3-2x}{x+6}$ where $x \neq -6$

(a) Write down an equation of the asymptote to C that is parallel to the

- (i) x -axis (ii) y -axis

(2)

(b) Find the coordinates of the point where C crosses the

- (i) x -axis (ii) y -axis

(2)

(c) Using the axes opposite, sketch the graph of C , showing clearly its asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

(d) Show that the gradient of the tangent to C is always negative.

(3)

A tangent to C has equation $y = -\frac{3}{5}x + k$ where $k > 0$

(e) Find the value of k

(5)

.....

.....

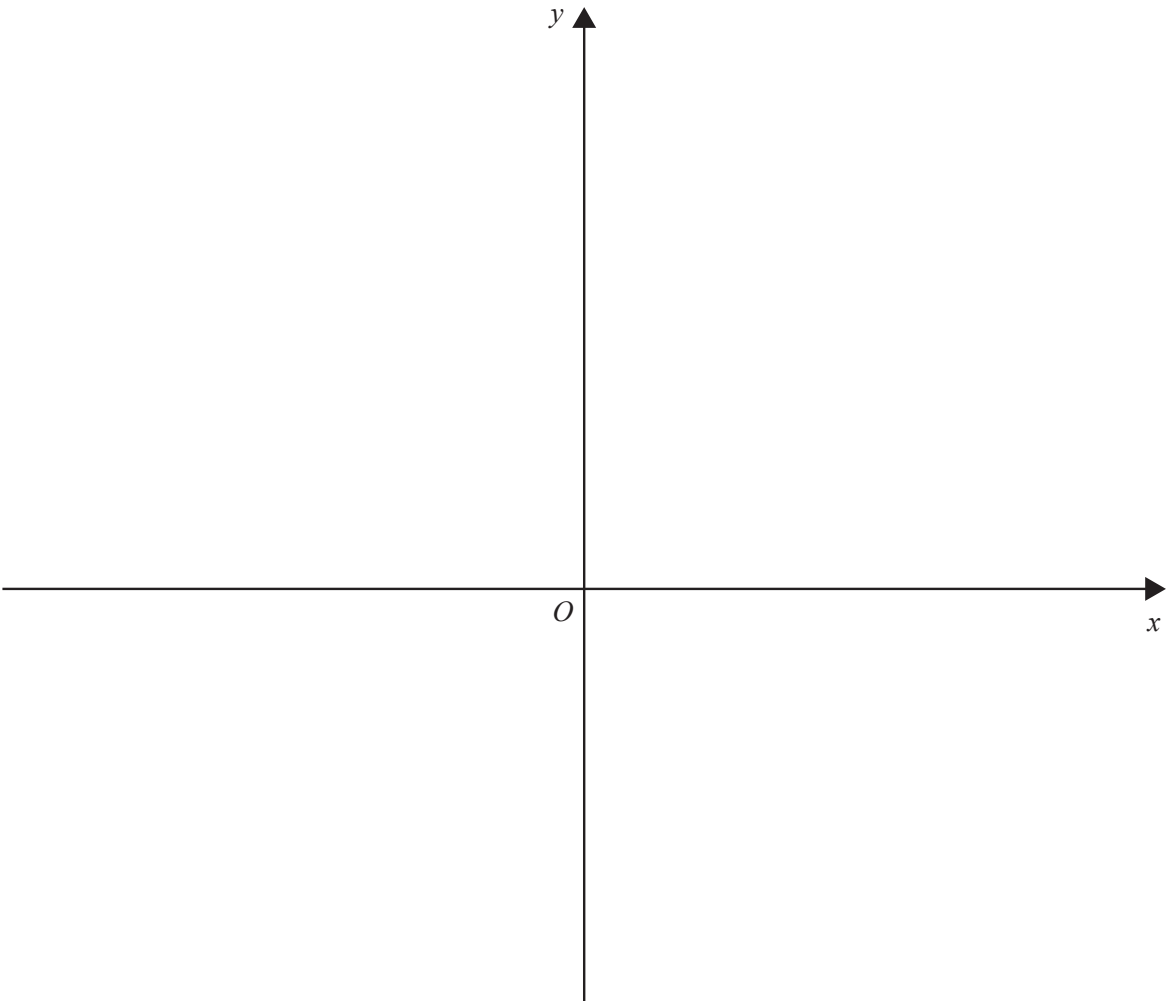
.....

.....

.....



Question 9 continued



16-MARK QUESTIONS

3 questions in this booklet



9 A curve C has equation

$$y = \frac{qx - 2}{x - p} \quad x \neq p$$

The curve crosses the y -axis at the point A .

The line l with equation $y = x + 2$ is the normal to C at A .

(a) (i) Show that $p = 1$

(ii) Find the value of q .

(7)

(b) Using the axes on the opposite page, sketch C , showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

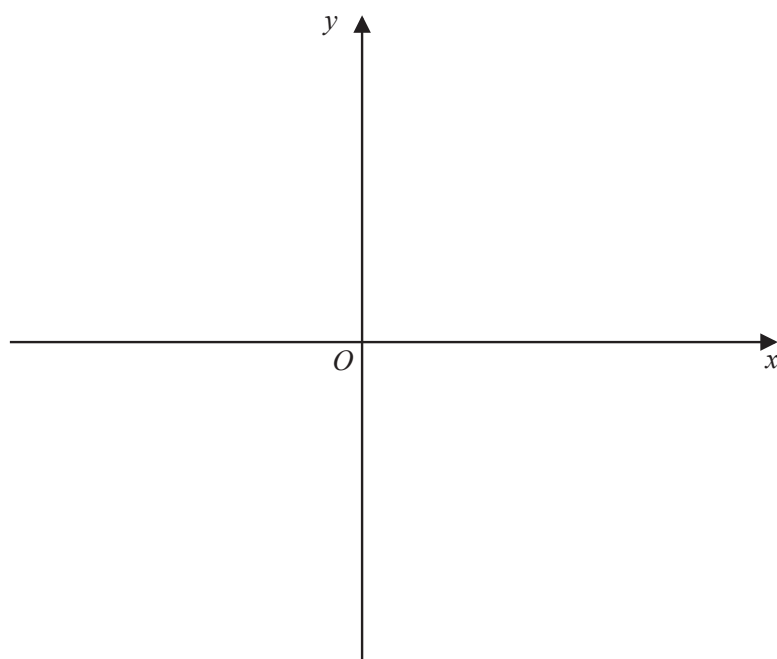
(5)

The line l meets C again at the point D .

(c) Find the x coordinate of D .

(4)

Question 9 continued



10 The curve C with equation $y = \frac{6-3x}{x-4}$ where $x \neq 4$, crosses the x -axis at the point P and the y -axis at the point Q

(a) Find the coordinates of

(i) P (ii) Q

(2)

(b) Write down an equation of the asymptote to C which is

(i) parallel to the y -axis (ii) parallel to the x -axis

(2)

(c) Sketch C showing clearly the asymptotes and the coordinates of the points P and Q

(3)

The line L is the normal to C at the point on C where $x = 2$

(d) Find an equation of L

(6)

The line L intersects C again at the point R

(e) Find the x coordinate of R

(3)

.....

.....

.....

.....

.....

- 10** The curve C has equation $y = \frac{ax - 5}{b - x}$ where a and b are integers and $x \neq b$
 One intersection of C with the coordinate axes is at the point with coordinates $\left(\frac{5}{4}, 0\right)$

The asymptote parallel to the y -axis has equation $x = 3$

- (a) Find the value of a and the value of b (2)
- (b) Sketch C , showing clearly the asymptotes with their equations and the coordinates of the points of intersection with the coordinate axes. (5)

The straight line l with equation $4y - 7x = k$ has no points of intersection with C

- (c) Show, using algebra, that the range of possible values of k can be written as

$$m < k < n$$

where m and n are integers to be found.

(9)

17-MARK QUESTIONS

2 questions in this booklet



11 A curve C has equation

$$y = \frac{(2a-1)x+1}{ax-6} \quad \text{where } a \text{ is a constant and } x \neq \frac{6}{a}$$

(a) Find $\frac{dy}{dx}$

(3)

The curve crosses the y -axis at the point A .

The normal to C at the point A is the line l with equation $66y - 72x + 11 = 0$

Show that

(b) (i) $a = 3$

(4)

(ii) the equation of C is $y = \frac{5x+1}{3x-6}$ where $x \neq 2$

(1)

(c) Using the axes on the opposite page, sketch C , showing clearly the asymptotes with their equations and the coordinates of the points where C crosses the coordinate axes.

(5)

The line l meets C again at the point D .

(d) Find the x coordinate of D .

Give your answer as an improper fraction.

(4)

.....

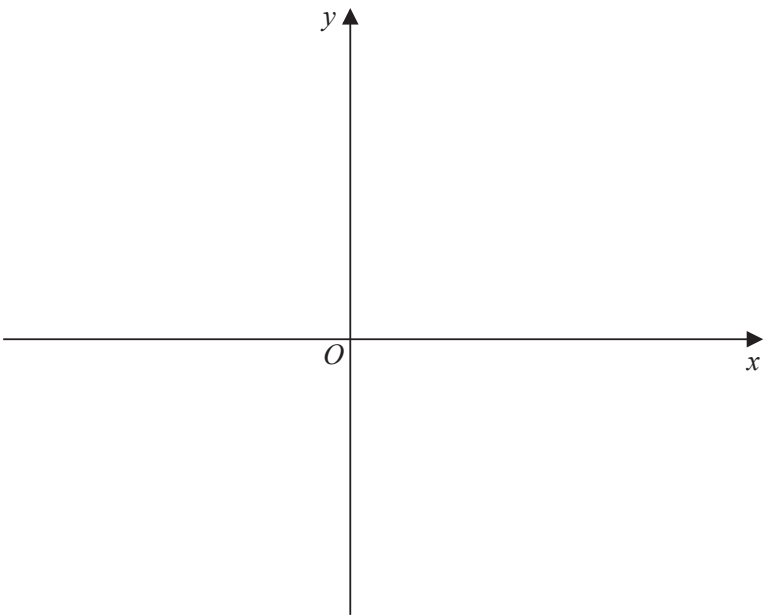
.....

.....

.....

.....

Question 11 continued



.....



10 A curve C has equation

$$y = \frac{7-x}{2x+3} \quad x \neq -\frac{3}{2}$$

(a) Write down an equation of the asymptote to C that is

(i) parallel to the y -axis

(ii) parallel to the x -axis

(2)

(b) Find the coordinates of the points of intersection of C with the coordinate axes.

(2)

(c) Using the axes on the opposite page, sketch C , showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

C passes through the point A with coordinates $\left(\frac{1}{2}, \frac{13}{8}\right)$

(d) Show that the gradient of C at A is $-\frac{17}{16}$

(3)

C also passes through the point P

Given that the tangent to C at P is parallel to the tangent to C at A

(e) find an equation of the tangent to C at P

Give your answer in the form $ax + by + c = 0$ where a , b and c are integers.

(7)

.....

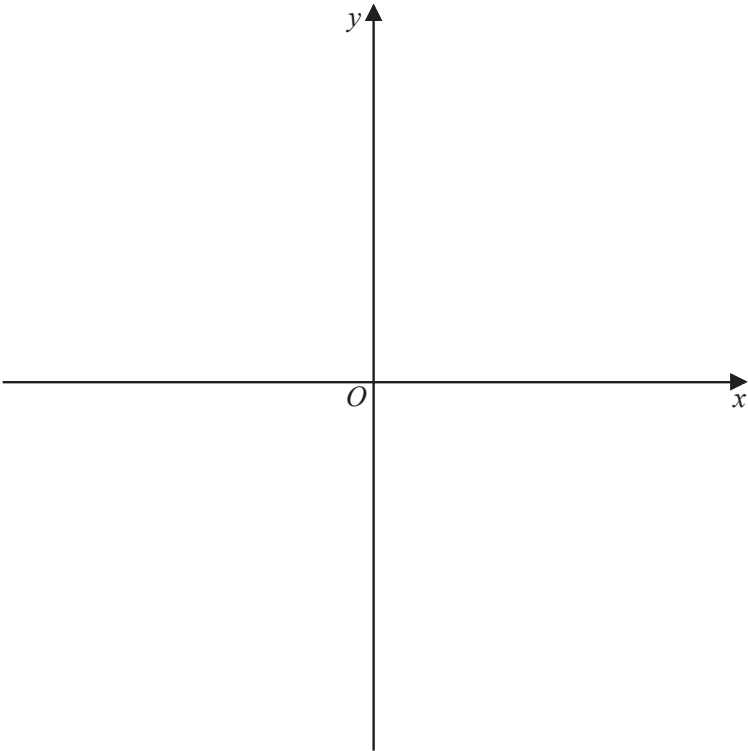
.....

.....

.....

.....

Question 10 continued



18-MARK QUESTIONS

5 questions in this booklet



9 A curve C has equation

$$y = \frac{2 + 4x - x^2}{2x + 1} \quad x \neq -\frac{1}{2}$$

- (a) Write the equation of C in the form $ax^2 + (by - 4)x + (y - c) = 0$, where a , b and c are integers whose values are to be found. (3)
- (b) Hence show that x is real when $y \leq 2$ and when $y \geq 3$ (4)
- (c) Find the coordinates of the stationary points on C . (6)
- (d) Sketch C showing clearly
- (i) the exact coordinates of the points where C crosses the x -axis,
 - (ii) the asymptote to C that is parallel to the y -axis,
 - (iii) the coordinates of the stationary points. (5)

.....

.....

.....

.....

.....

10 The curve C has equation $y = \frac{2x-1}{x+4} \quad x \neq -4$

(a) Write down an equation of the asymptote to C that is

- (i) parallel to the x -axis,
- (ii) parallel to the y -axis.

(2)

(b) Find the coordinates of the points of intersection of C with the coordinate axes.

(2)

(c) Using the axes on the opposite page, sketch C , showing clearly the asymptotes and the coordinates of the points of intersection of C with the coordinate axes.

(3)

The line with equation $y = x + k_1$ is the tangent to C at the point P

The line with equation $y = x + k_2$ is the tangent to C at the point Q

Given that the x coordinate of P is greater than the x coordinate of Q

(d) using calculus, find the coordinates of

- (i) P
- (ii) Q

(8)

(e) Hence find the value of

- (i) k_1
- (ii) k_2

(3)

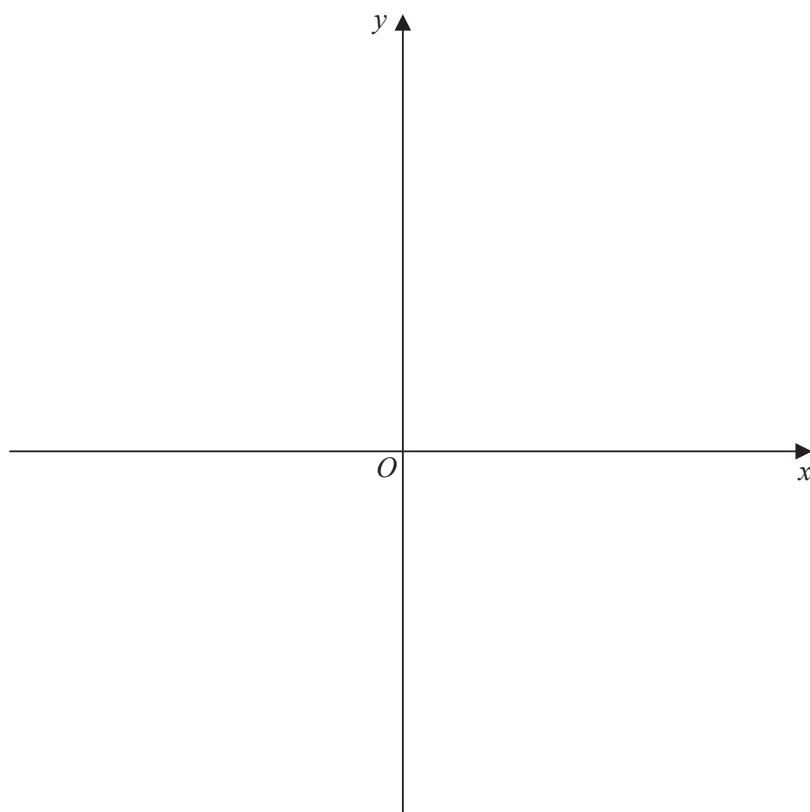
.....

.....

.....

.....

.....

Question 10 continued

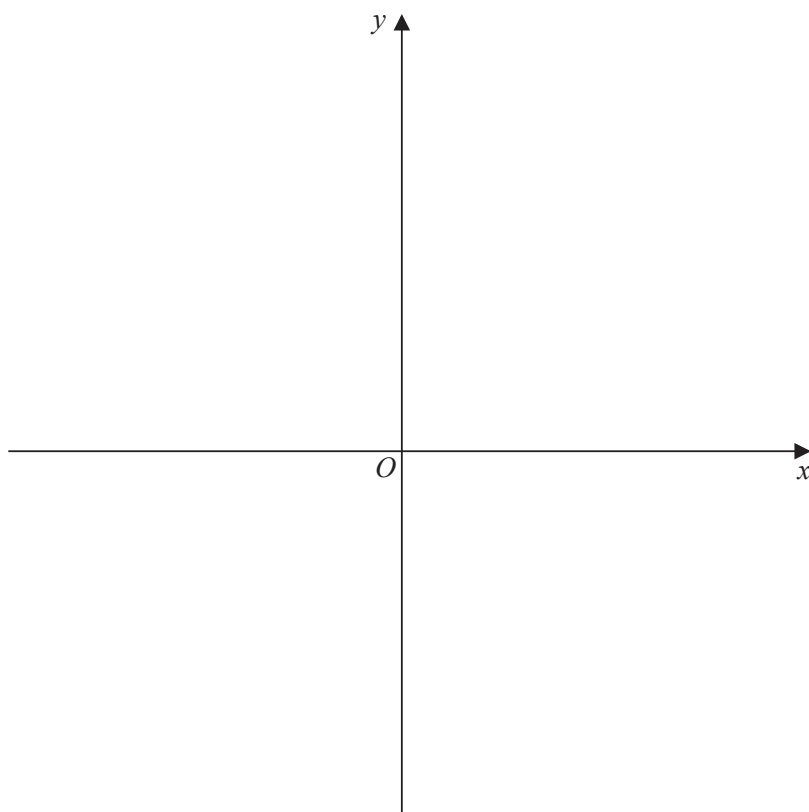
Area for drawing the graph, consisting of 12 horizontal dotted lines.

Turn over for spare axes if you need to redraw your sketch.



Question 10 continued

Only use these axes if you need to redraw your sketch.



10 A curve C has equation

$$y = \frac{7x - 2}{2x - 3} \quad x \neq \frac{3}{2}$$

(a) Write down an equation of the asymptote to C that is

- (i) parallel to the y -axis,
- (ii) parallel to the x -axis.

(2)

(b) Find the coordinates of the points of intersection of C with the coordinate axes.

(2)

(c) Using calculus, show that at every point on the curve, the gradient of C is negative.

(4)

(d) Using the axes on the opposite page, sketch C .

Show clearly and label with their equation any asymptotes and the coordinates of the points of intersection of C with the coordinate axes.

(3)

The straight line l is the normal to C at the point A .

The x coordinate of A is positive and the gradient of l is 17

The line l also intersects C at the point B .

(e) Find the exact coordinates of B .

(7)

.....

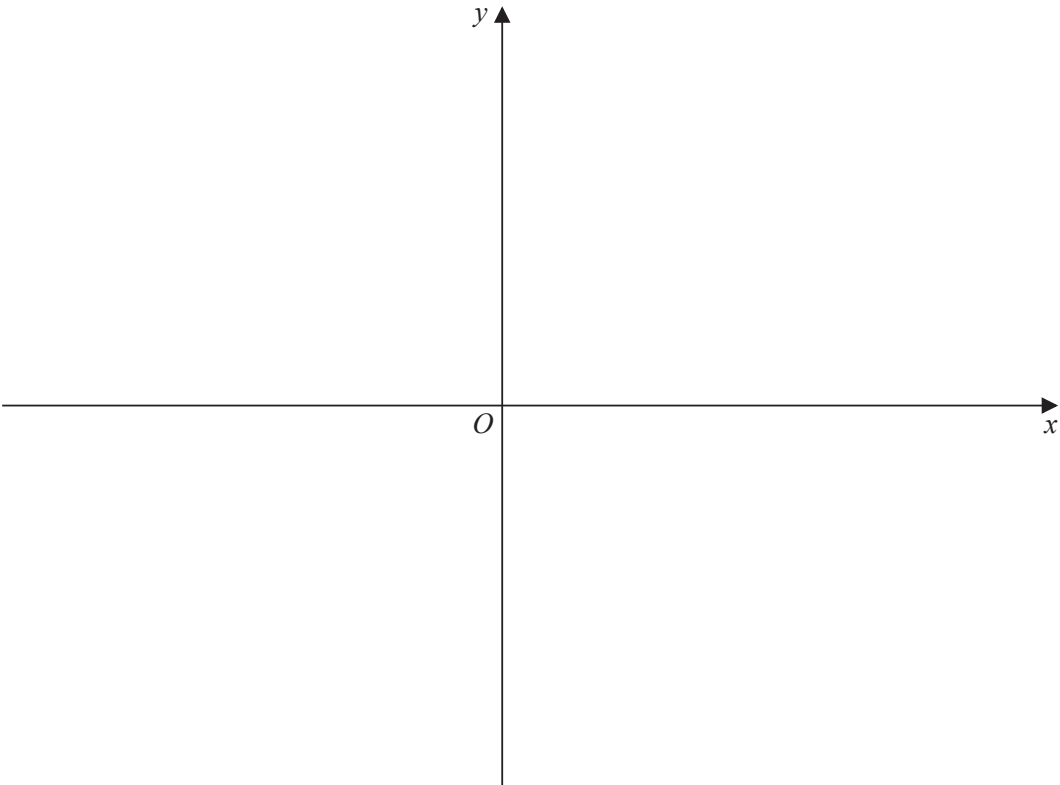
.....

.....

.....

.....

Question 10 continued



10 A curve C has equation

$$y = \frac{5x - 2}{3x + 2} \qquad x \neq -\frac{2}{3}$$

- (a) Find the coordinates of the point where C intersects the
- (i) x -axis

(ii) y -axis
- (2)
- (b) Write down an equation of the asymptote to C that is
- (i) parallel to the x -axis

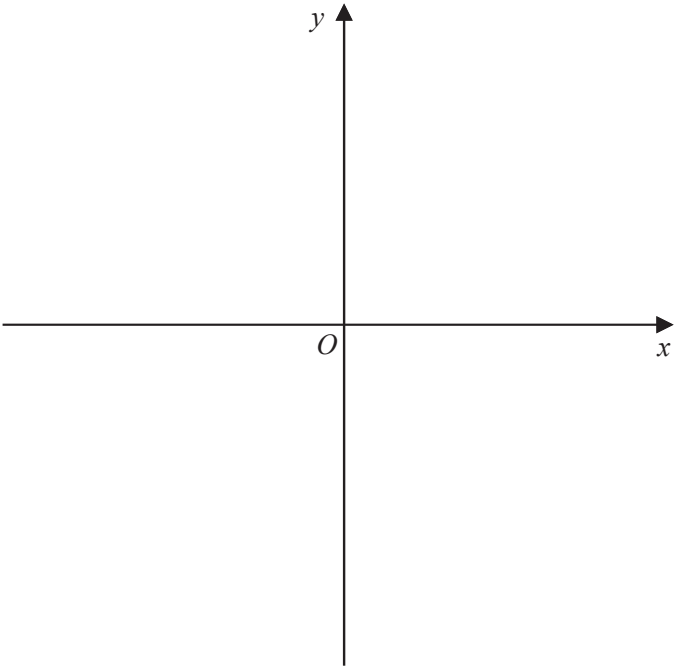
(ii) parallel to the y -axis
- (2)
- (c) Sketch C on the opposite page.
- Show and label the asymptotes and the coordinates of the points where C crosses the coordinate axes.
- (3)

Point A lies on C such that the gradient of C at A is parallel to the line with equation $4y - x = 7$

The normal to C at A intersects the x -axis at point D and the y -axis at point E
Given that the x coordinate of A is positive,

- (d) find, in its simplified form, the exact length of line DE
- (11)

Question 10 continued



10 The curve C has equation

$$y = \frac{5x-2}{ax+b} \quad x \neq -\frac{b}{a}$$

The asymptote to C that is parallel to the y -axis has equation $x = -\frac{4}{3}$

C crosses the y -axis at the point with coordinates $\left(0, -\frac{1}{4}\right)$

(a) (i) Show that $b = 8$

(ii) Find the value of a

(3)

(b) Write down the equation of the asymptote to C that is parallel to the x -axis.

(1)

(c) Find the coordinates of the point where C crosses the x -axis.

(1)

(d) Using calculus, show that at every point on C , the gradient is positive.

(4)

(e) Using the axes on the next page, sketch C

Clearly label the asymptotes and the coordinates where C crosses the coordinate axes.

(3)

The gradient of C is $\frac{13}{100}$ at the point E and at the point F

(f) Find the length of the line EF , giving your answer to 2 decimal places.

(6)

.....

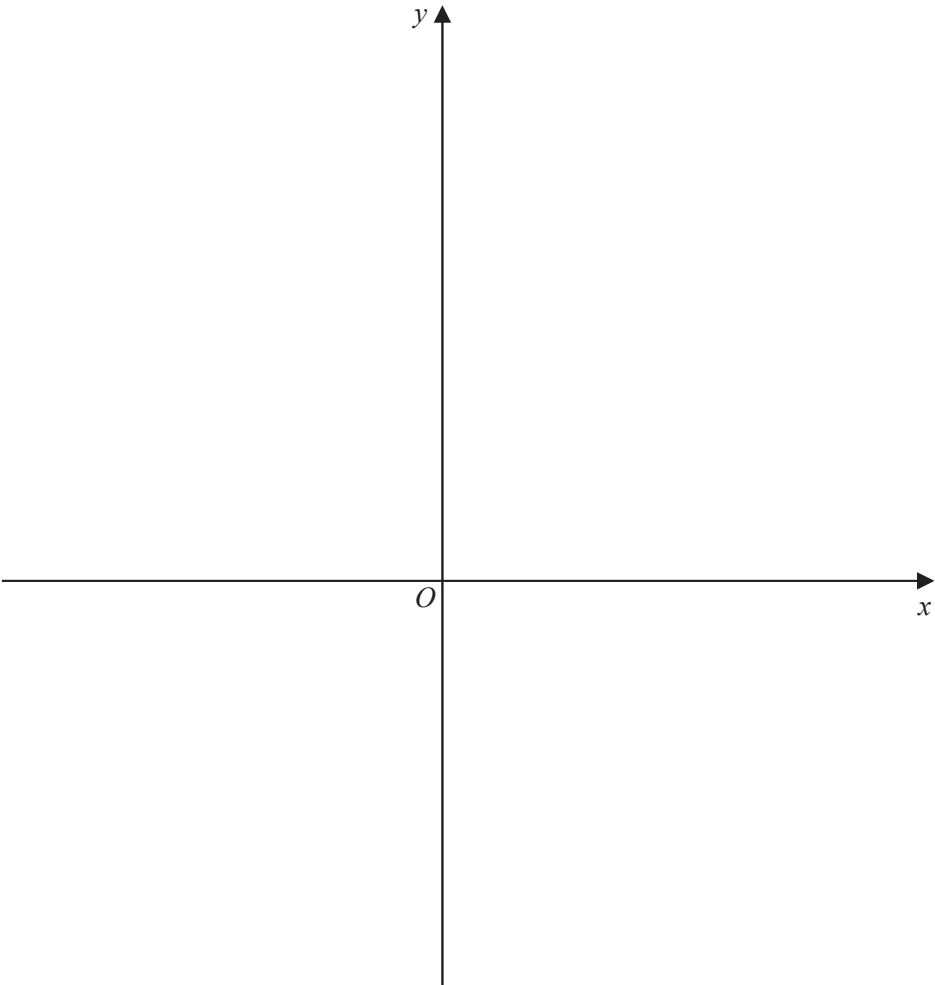
.....

.....

.....

.....

Question 10 continued



MARK SCHEMES

official Pearson Edexcel mark schemes

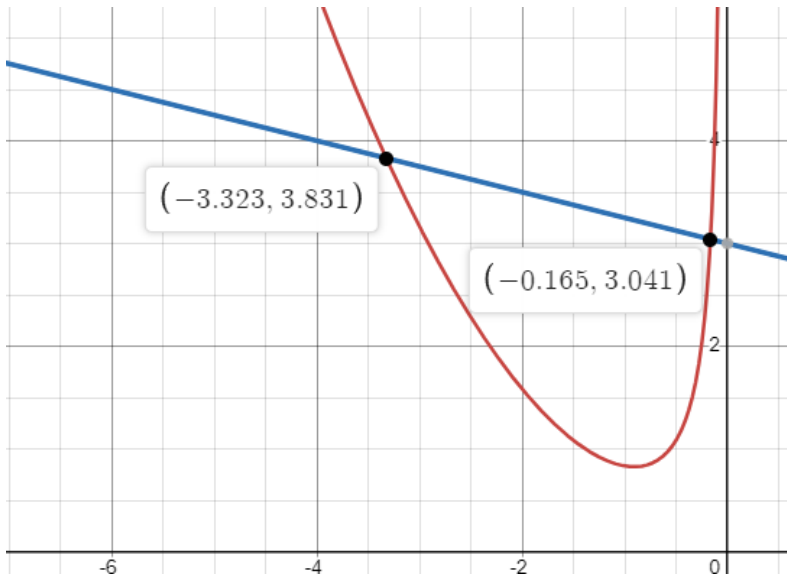


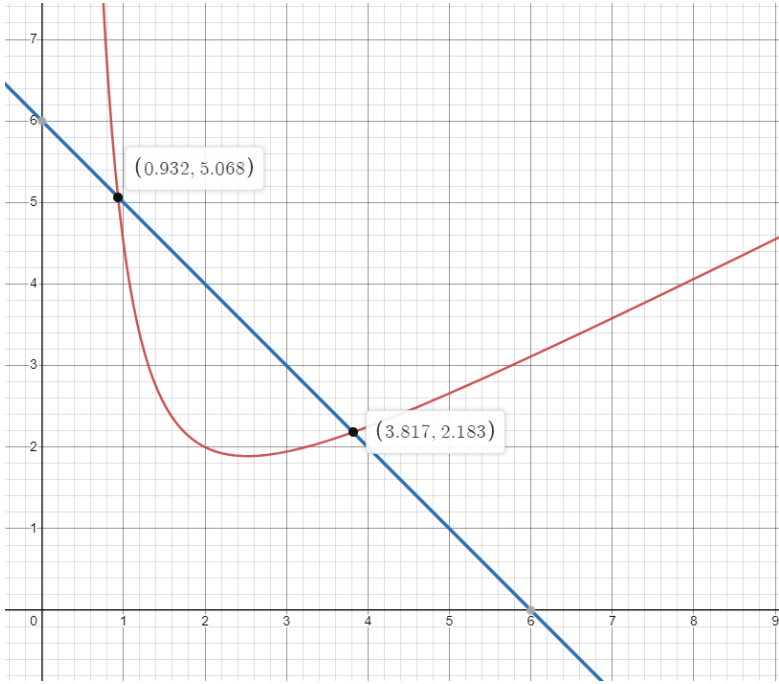
Question number	Scheme	Marks
2 (i)	$\frac{a}{1} = -2$	B1
(ii)	$ax + b = 0 \Rightarrow -2 \times 4 + b = 0$	B1ft
(iii)	$-3 + c = 0$	B1
(iv)	So the equation of S is $y = \frac{8-2x}{x+3}$	
	When $x = 0$	
	$y = \frac{8-2 \times 0}{0+3} = \frac{8}{3}$	B1ft
	$p = \frac{8}{3}$	[4]
Total 4 marks		

		In parts (i), (ii) and (iii), values must be clearly stated or shown correctly substituted in the equation of the curve.
(ii)	B1ft	ft (i) $b = -4 \times$ their a
(iv)	B1ft	ft (ii) and (iii) $p = \frac{\text{their } b}{\text{their } c}$ Accept 2.67 or better, $\left(0, \frac{8}{3}\right)$ or $\frac{8}{3}$ marked on the diagram at the point where the curve crosses the y axis.



Question	Scheme	Marks
4	$4x^3 + 3x^2 - 36x - 6 = 0 \rightarrow \div 12x$ $\Rightarrow \frac{x^2}{3} + \frac{x}{4} - 3 - \frac{1}{2x} = 0 \Rightarrow \frac{x^2}{3} - \frac{1}{2x} = 3 - \frac{x}{4}$ ALT $\frac{x^2}{3} - \frac{1}{2x} = ax + b \Rightarrow 2x^3 - 6ax - 6bx - 3 = 0$ $\Rightarrow 4x^3 - 12ax - 12bx - 6 = 4x^3 + 3x - 36x - 6$ $\Rightarrow a = -\frac{1}{4} \quad b = 3$ $\Rightarrow$ line required is $y = 3 - \frac{x}{4}$ Draws the line with equation $y = 3 - \frac{x}{4}$ $x = -0.2 \quad x = -3.3$ or -3.4	M1A1 [M1 A1] M1 A1 [4]
Total 4 marks		

Mark	Notes
	Note - Correct values without any evidence of valid working or a line is M0A0M0A0 - Correct values with a correct line drawn without valid working is M0A0M0A0 - Correct values with a line drawn and the correct equation of a line without evidence of any valid working is SC M0A0M1A0
M1	For dividing through by $12x$ [you may well see this in stages – e.g., first by 4, then by 3, then by x etc] and attempting to rearrange the equation to give as a minimum $y = k \pm \frac{x}{4} \quad \text{or} \quad y = 3 \pm \frac{x}{m} \quad 2 \leq m \leq 6$ OR Sets the given equation of the curve $= ax + b$ and solves for a and b to find as a minimum to give as a minimum $y = k \pm \frac{x}{4} \quad \text{or} \quad y = 3 \pm \frac{x}{m} \quad 2 \leq m \leq 6$
A1	For the correct straight line.
M1	Draws their line correctly on the grid. Look for $(0, 3)$ and $(-4, 4)$ provided it is of the form $y = k \pm \frac{x}{4}$ You MUST see the equation of the line WITH the drawn line. A correct line without an equation is M0 
A1	For both $x = -0.2$ and $x = -3.3$ Answers must be given to 1 dp only [2 or more dp is A0] NB: Calculator values are -0.165, -3.323

Question	Scheme	Marks
3	$\frac{x}{2} + \frac{4}{x^2} = Ax + B \Rightarrow x^3(1-2A) - 2Bx^2 + 8 = 0$ $x^3(1-2A) - 2Bx^2 + 8 = 3x^3 - 12x^2 + 8 \Rightarrow A = -1, B = 6 \Rightarrow y = 6 - x$ Draws line $y = 6 - x$ Roots are $x = 0.9, 3.8$ 	M1 M1A1 M1 A1 [5]
Total 5 marks		

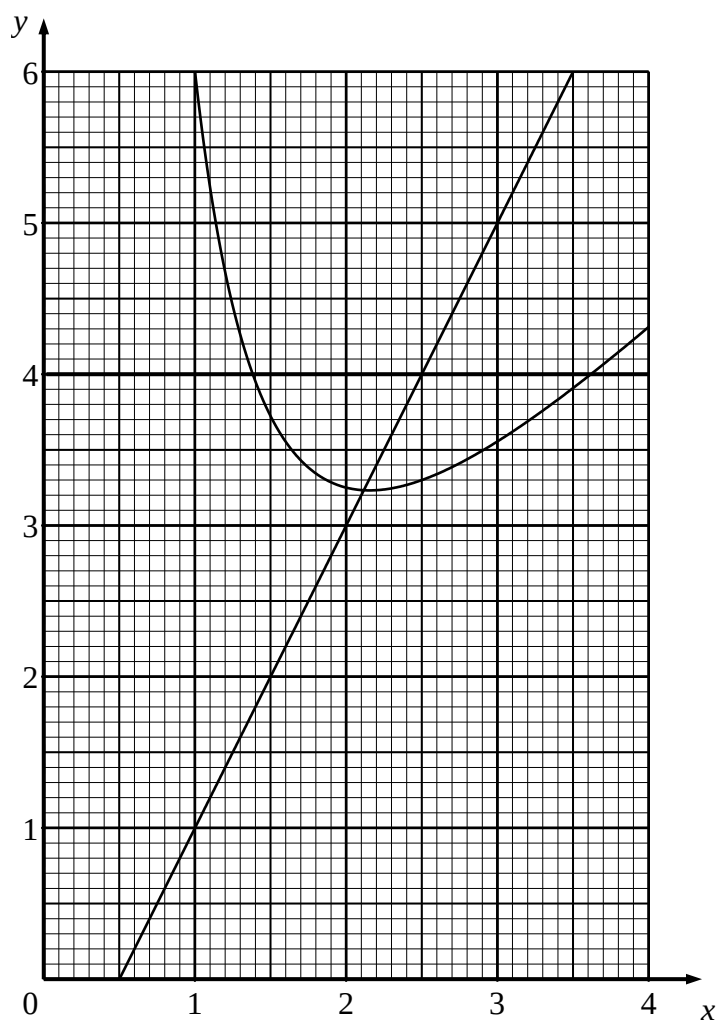
Mark	Notes
M1	Removes the denominators by multiplying through by $2x^2$ This can be carried out separately by multiplying by 2 and then x^2 or vice versa. Allow one slip in their working. $\frac{x}{2} + \frac{4}{x^2} = Ax + B \Rightarrow x^3 + 8 = 2Ax^3 + 2Bx$
M1	Collects like terms and equates components. $x^3(1-2A) - 2Bx^2 + 8 = 3x^3 - 12x^2 + 8$ and finds the equation of the straight line.
A1	The straight line is $y = 6 - x$
ALT 1	
M1	Divides through by $2x^2$ This can be carried out separately by dividing by 2 and then x^2 or vice versa.

	Allow one slip in their working. $3x^3 - 12x^2 + 8 = 0 \quad (\div 2x^2) \Rightarrow \frac{3x}{2} - 6 + \frac{4}{x^2} = 0$
M1	Adds $(6 - x)$ to both sides. $\Rightarrow \frac{x}{2} + \frac{4}{x^2} = 6 - x$
A1	The straight line is $y = 6 - x$
ALT 2	
M1	Multiplies through $\frac{x}{2} + \frac{4}{x^2} = y \Rightarrow x^3 + 8 = 2x^2y$ and subtracts this from $3x^3 - 12x^2 + 8 = 0$ as follows to obtain $2x^3 - 12x^2 = -2x^2y$ $3x^3 - 12x^2 + 8 = 0$ $\begin{array}{r} x^3 \qquad \qquad + 8 = 2x^2y \\ \hline 2x^3 - 12x^2 \qquad = -2x^2y \end{array}$
M1	Divides $2x^3 - 12x^2 = -2x^2y$ by $2x^2$ to obtain a linear equation $x - 6 = -y$
A1	The straight line is $y = 6 - x$
SC: No or incorrect working to find the equation of the line. - If fully correct answers are seen without correct or any working at all and without the correct equation of the line seen [even if a line is drawn], award M0M0A0M0A0 - If the rounded answers to 1 dp are seen with the correct equation $y = 6 - x$, and the line drawn, award M0M0A0M1A1 - If rounded answers are seen to a greater degree of accuracy with the correct equation $y = 6 - x$ seen, and the line drawn, award M0M0A0M1A0	
M1	For drawing their straight line correctly onto the grid, provided it is of the form $y = k - x$
A1	For a value of 0.9 [allow 1.0] and 3.8 (do not award without the evidence of the line) Allow seen as coordinates (0.9, [5.1]) (3.8, [2.2]) ignore y values. Do not accept awrt in this question. Rounding must be to one decimal place.

Question	Scheme	Marks
4(a)(i)	$p = 2$	B1
(ii)	$r = 4$	B1 [2]
(b)	$\frac{3}{2} = \frac{2 \times 0 + q}{0 + '4'} \Rightarrow q = 6$	M1A1 [2]
(c)	$0 = \frac{2s + '6'}{s + '4'} \Rightarrow s = -3$	M1A1 [2]
Total 6 marks		

Question	Notes	Marks
4(a)(i)	$p = 2$	B1 [1]
(ii)	$r = 4$	B1 [1]
(b)	For using the equation for C, substituting $x = 0$ and $y = \frac{3}{2}$ and attempt to rearrange to find the value of q $\frac{3}{2} = \frac{2 \times 0 + q}{0 + 4} \Rightarrow q = \dots$	M1
	$q = 6$	A1 [2]
(c)	For using the equation of C to find s $0 = \frac{2s + 6}{s + 4} \Rightarrow s = \dots$	M1
	$s = -3$	A1 [2]
Total 6 marks		

4



(a)

$$x^3 - 4x^2 + 5 = 0 \Rightarrow x - 4 + \frac{5}{x^2} = 0$$

$$x + \frac{5}{x^2} = 4$$

Draw " $y = 4$ ", $x = 1.4, 3.6$

M1

M1, A1 (3)

(b)

$$x^3 - x^2 - 5 = 0 \Rightarrow x - 1 - \frac{5}{x^2} = 0$$

$$x - 1 - \frac{5}{x^2} = 0 \Rightarrow 2x - 1 = x + \frac{5}{x^2}$$

Draw " $y = 2x - 1$ ", $x = 2.1$

M1A1

dM1A1
(4)

[7]

Part	Mark	Notes
(a)	M1	Divides through $x^3 - 4x^2 + 5 = 0$ by x^2 and rearranges to achieve as a minimum $x + \frac{5}{x^2} = k$ where k is a constant $\left[x - 4 + \frac{5}{x^2} = 0 \Rightarrow x + \frac{5}{x^2} = 4 \right]$
	M1	Draw the line $y = k$ following through their value for k No line is M0
	A1	For the two values of $x = 1.4$ and $x = 3.6$ Condone answers given as coordinates provided they are completely correct. (1.4, 4) and (3.6, 4) Require both M marks for this mark.
(b)	M1	For setting $x + \frac{5}{x^2} = Ax + B \Rightarrow x^3 + 5 = Ax^3 + Bx^2 \Rightarrow Ax^3 - x^3 + Bx^2 - 5 = 0$, and equating coefficients with $x^3 - x^2 - 5$ $x^3(A-1) + Bx^2 - 5 \equiv x^3 - x^2 - 5$ to achieve as a minimum $A = (\pm 2), B = (\pm 1)$
	A1	For the correct straight line $y = 2x - 1$
	ALT – to find the line $y = 2x - 1$	
	M1	Divides through $x^3 - x^2 - 5 = 0$ by x^2 and rearranges the equation to achieve as a minimum $\Rightarrow \pm 2x \pm 1 = x + \frac{5}{x^2}$
	A1	For the correct straight line $y = 2x - 1$
	dM1	Draws their $y = 2x - 1$ on the graph and locates the point of intersection. Please check that they draw their line correctly. Coordinates for you to check are (0.5, 0) and (2.5, 4) No line is M0 This mark is dependent on the first M mark in (b)
	A1	For the correct value of $x = 2.1$ [allow $x = 2.2$] Can only score this mark from M1A1M1 Do not accept the answer given as coordinates.

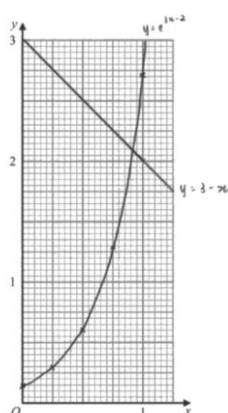


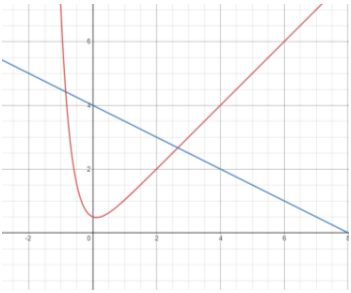
Question number	Scheme	Marks
3 (a)	$c = 2, d = 3$	B1B1 [2]
(b)	$a - bx = 0 \Rightarrow x = \frac{a}{b} = \frac{5}{4} \Rightarrow a = 5, b = 4$	M1A1 [2]
(c)	When the curve crosses the y -axis $x = 0$ $y = \frac{5 - 4 \times 0}{2 \times 0 - 3} = -\frac{5}{3} \Rightarrow p = -\frac{5}{3}$	M1A1 [2]
(d)	$y = -2$	B1ft [1]
Total 7 marks		

Part	Mark	Notes
(a)	B1	For either $c = 2$ OR $d = 3$
	B1	For both $c = 2$ AND $d = 3$
(b)	M1	For setting the numerator = 0, making x the subject and equating the result to $\frac{5}{4}$
	A1	For both $a = 5$ and $b = 4$
	ALT	
	M1	Substitutes $x = \frac{5}{4}$ into the equation, sets the numerator = 0 and finds the value of $\frac{a}{b}$
	A1	For both $a = 5$ and $b = 4$
(c)	M1	For setting the value of $x = 0$ and finding a value for y ft their values of a and d
	A1	For the value of $p = -\frac{5}{3}$
(d)	B1ft	For $y = -2$ Ft their values of b and c such that $y = \frac{-b}{c}$

Question number	Scheme						Marks												
5 (a)	<table border="1"><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>y</td><td>0.14</td><td>0.29</td><td>0.61</td><td>1.28</td><td>2.72</td></tr></table>						x	0	0.25	0.5	0.75	1	y	0.14	0.29	0.61	1.28	2.72	first B1 2 values correct
	x	0	0.25	0.5	0.75	1													
	y	0.14	0.29	0.61	1.28	2.72													
						second B1 all 3 values correct [2]													
(b)	Each point plotted correctly within the correct small square Smooth curve through the points						B1ft B1ft [2]												
(c)	$e^{3x-2} = 3 - x$ $y = 3 - x$ 0.9						M1 M1 A1ft [3]												
Total 7 marks																			

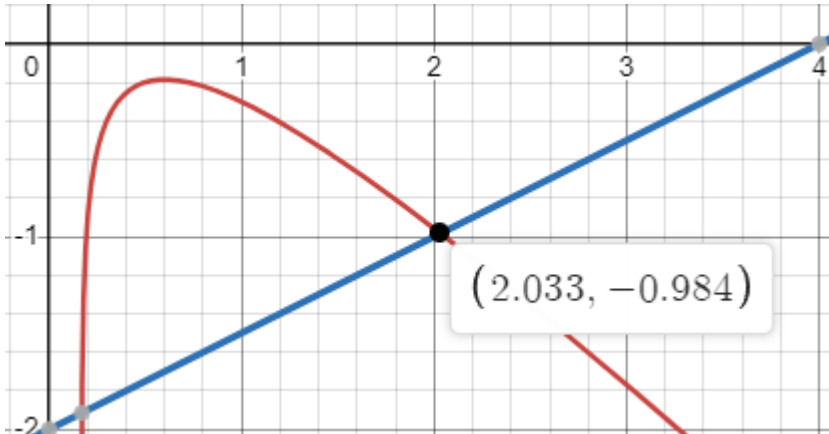
Part	Mark	Additional Guidance
(a)	B1	SC1 – allow 0.29 and/or 0.61 to be truncated to 0.28 and/or 0.60 with 1.28 correct to gain this mark OR for all three values correct but given to greater than 2 decimal places
	B1	For all 3 values rounded correctly as shown.
(b)	B1ft	ft the correct plotting of their points.
	B1ft	ft a curve “sensibly” plotted through their points, need not have the correct shape. Must pass through all of the points they have plotted. Minimum 4 points.
(c)	M1	Rearranges the equation must be of form $e^{3x \pm 2} = 3 - x$
	M1	$y = 3 - x$ drawn correctly on the graph paper. Passing through the points (0,3) and (1,2) as a minimum.
	A1ft	“0.9” must be given to 1 decimal place. Not as part of a coordinate. Follow through an appropriately shaped curve (or line segments) and correct $y = 3 - x$. With answer given to 1 decimal place.



Question	Scheme	Marks
6	$\log_2(8 - 3x) - 1 = -4x - 1 \Rightarrow \log_2(8 - 3x) - \log_2 2 = -4x - 1$ $\Rightarrow \log_2\left(\frac{8 - 3x}{2}\right) = -4x - 1$ $\Rightarrow \frac{8 - 3x}{2} = 2^{-4x-1}$ $\Rightarrow 4 - \frac{3}{2}x + x = x + 2^{-(4x+1)}$ $\Rightarrow 4 - \frac{1}{2}x = x + 2^{-(4x+1)}$ Draws the line $y = 4 - \frac{x}{2}$ For the values $x = -0.8/0.9$ and $x = 2.6/2.7$ 	M1 M1 M1 M1A1 M1 A1 [7]
Total 7 marks		

Mark	Notes
	General principles of marking this question for the first 5 marks Award the first two marks when you see them.
M1	Award the first M mark for dealing with the log wherever you see it For example: $\log_2(8 - 3x) = -4x \Rightarrow 8 - 3x = 2^{-4x}$
M1	Award the second M mark for either adding -1 to both sides and converting $-1 = \log_2 2$, or by dividing throughout by 2 and dealing with the power of -1 . For example: $\frac{8-3x}{2} = \frac{2^{-4x}}{2} \Rightarrow 4 - \frac{3x}{2} = 2^{-4x-1}$
M1	Award the third M mark for adding x to both sides
M1	For simplifying and obtaining $(y) = x + 2^{-4x-1}$ or $(y) = x + 2^{-(4x+1)}$ on one side and a linear equation of the form $y = K \pm \frac{1}{2}x$ (may be unsimplified) where K is a constant, on the other side.
A1	For obtaining the correct equation $y = 4 - \frac{1}{2}x$ (may be unsimplified)
M1	For drawing their line correctly • Their line is in the form $y = k - \frac{x}{2}$, k nonzero constant (may be unsimplified) • There must be at least one intersection point with the given curve.
A1	Correct x values (both rounded to 1dp), ignore if $y=...$ is present, accept also given as coordinates, even if their y coordinates are incorrect.

ALT I for first 5 marks**M1:** Correctly removes log, achieves $2^{-4x} = 8 - 3x$ **M1:** For dividing by 2 throughout $\frac{2^{-4x}}{2} = \frac{8-3x}{2} \Rightarrow 2^{-(4x+1)} = 4 - \frac{3}{2}x$ **M1:** For adding x to both sides**M1:** Rearranges leading to $x + 2^{-4x-1} = 4 - \frac{x}{2}$ **A1:** For obtaining the correct equation $y = 4 - \frac{1}{2}x$ (may be unsimplified)**ALT II** for first 5 marks**M1:** Correctly removes log, achieves $2^{-4x} = 8 - 3x$ **M1:** Rearrange the given curve to $y - x = 2^{-4x} \times 2^{-1}$ **M1:** Attempts to make 2^{-4x} the subject from $y - x = 2^{-4x} \times 2^{-1} \Rightarrow 2^{-4x} = \dots$ **M1:** Equates the two resulting equations, then attempts to make $y = \dots$ **A1:** For obtaining the correct equation $y = 4 - \frac{1}{2}x$ (may be unsimplified)For reference the two correct equations are: $\begin{cases} 2^{-4x} = 8 - 3x \\ 2^{-4x} = 2y - 2x \end{cases} \Rightarrow 8 - 3x = 2y - 2x \Rightarrow y = 4 - \frac{x}{2}$ **ALT III** for first 5 marks**M1:** Correctly removes log, achieves $2^{-4x} = 8 - 3x$ **M1:** For $2^{-(4x+1)} = 2^{-4x} \times 2^{-1}$ (may be embedded)**M1:** Replaces 2^{-4x} with their $8 - 3x$ **M1:** Rearranges leading to $y = \dots$ **A1:** For obtaining the correct equation $y = 4 - \frac{1}{2}x$ (may be unsimplified)

Question number	Scheme								Marks
4 (a)	x	0.25	0.5	1	1.5	2	2.5	3	B1
	y	-0.55	-0.20	-0.30	-0.60	-0.96	-1.35	-1.77	B1 [2]
(b)	Points plotted.								B1ft
	Smooth curve.								B1ft [2]
(c)	$\frac{3x-4}{2} = \log_{10}(6x-1)$								M1
	$\frac{x}{2} - 2 = \log_{10}(6x-1) - x$								M1
	Draws the line $y = \frac{x}{2} - 2$								M1
	2.0 / 2.1								A1 [4]
									
Total 8 marks									

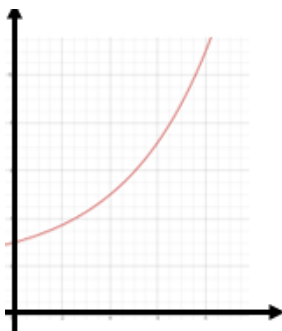
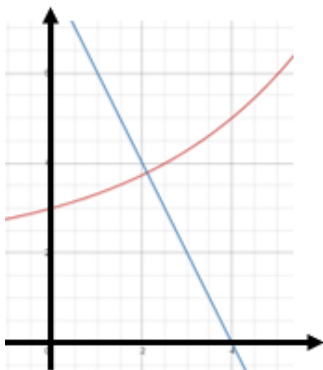
Part	Mark	Additional Guidance
(a)	B1	For 2 values correct. 2dp only.
	B1	For all 4 values correct. 2dp only.
(b)	B1ft	For all 7 points plotted within half a square ft their values from the table.
	B1ft	For points joined with a smooth curve ft all their 7 points plotted, their points do not have be plotted correctly.
(c)	M1	Correctly takes logs to give the expression shown. Accept $\frac{3x-4}{2} = \log(6x-1)$
	M1	Rearranges to give an equation of the form $\pm kx - 2 = \log_{10}(6x-1) - x$ (their left-hand side can be unsimplified equivalent)
	M1	Draws a line of the form $y = \pm kx - 2$ oe, correctly.
	A1	Accept answer given as 2.0 or 2.1
ALT (c)	M1	Correctly rearranges $y = \log_{10}(6x-1) - x$ to $10^{y+x} = 6x-1$
	M1	Equates $y + x$ to $\frac{3x-4}{2}$, and obtains $y = \pm kx - 2$ (can be unsimplified)
	M1	Draws a line of the form $y = \pm kx - 2$ oe, correctly. There must be at least one intersection point with the curve.
	A1	Accept answer given only as either 2.0 or 2.1, Do not allow 2



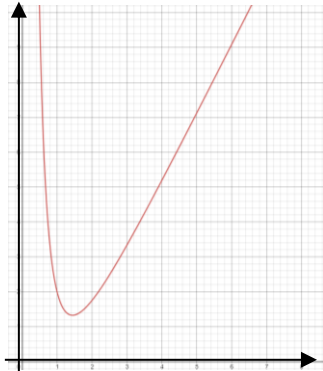
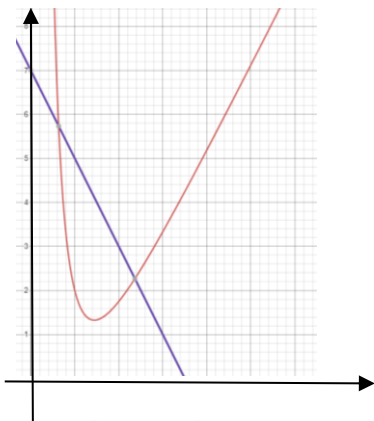
Question number	Scheme	Marks																		
6 (a)	<table><tr><td>x</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>-11.5</td><td>-2</td><td>0.2</td><td>1.3</td><td>2.7</td><td>3.8</td><td>4.9</td><td>5.9</td></tr></table>	x	0.5	1	1.5	2	3	4	5	6	y	-11.5	-2	0.2	1.3	2.7	3.8	4.9	5.9	B2 [2]
x	0.5	1	1.5	2	3	4	5	6												
y	-11.5	-2	0.2	1.3	2.7	3.8	4.9	5.9												
(b)	Points plotted within half of a square Points joined together in a smooth curve	B1ft B1ft [2]																		
(c)	$\frac{x^3-3}{x^2}=ax+b \Rightarrow x^3-3=ax^3+bx^2 \Rightarrow 0=x^3(a-1)+bx^2+3$ Comparing coefficients $x^3(a-1)-bx^2+3=2x^3+6x^2+3$ $\Rightarrow a=3, b=-6$ so line required is $y=3x-6$ Draws the line $y=3x-6$ and identifies two intersections with the curve when $x=0.8/0.9$ and $x=2.8/2.9$	M1 M1A1 M1 A1 (both) [5]																		
	ALT																			
	$2x^3-6x^2+3=0 \Rightarrow 2x-6=-\frac{3}{x^2}$	{M1}																		
	$3x-6=x-\frac{3}{x^2}$ so line required is $y=3x-6$	{M1}{A1}																		
	Draws the line $y=3x-6$ and identifies two intersections with the curve when $x=0.8/0.9$ and $x=2.8/2.9$	{M1} {A1} both [5]																		
Total 9 marks																				

(a)	
B2	All 4 points correct (B1 for 3 points correct)
(b)	
B1ft	Points plotted fit their table allow half a square tolerance
B1ft	Points joined together with a smooth curve fit their table
(c)	
M1	Setting $x - \frac{3}{x^2} = ax + b$ and simplifying to $x^3(a-1) + bx^2 + 3$
M1	Comparing coefficients
A1	Identifying that the line required is $y = 3x - 6$
M1	$y = 3x - 6$ drawn intersecting the curve in two places
A1	$x = 0.8/0.9$ and $x = 2.8/2.9$
ALT	
M1	Subtracting 3 from both sides and dividing by x^2
M1	Adding x to both sides
A1	Identifying that the line required is $y = 3x - 6$
M1	$y = 3x - 6$ drawn intersecting the curve in two places
A1	$x = 0.8/0.9$ and $x = 2.8/2.9$

Question number	Scheme			Marks
7 (a)	2	3	4	B1 B1 (2)
	3.73	4.28	5	
(b)	Points plotted Joined up with a smooth curve			B1ft B1ft (2)
(c)	$\log_3(6-2x) = \frac{x}{4}$			M1
	$6-2x = 3^{\frac{x}{4}}$			M1
	$8-2x = 3^{\frac{x}{4}} + 2$			A1
	$y = 8-2x$ drawn			M1
	$x = 2.1$			A1 (5)
Total 9 marks				

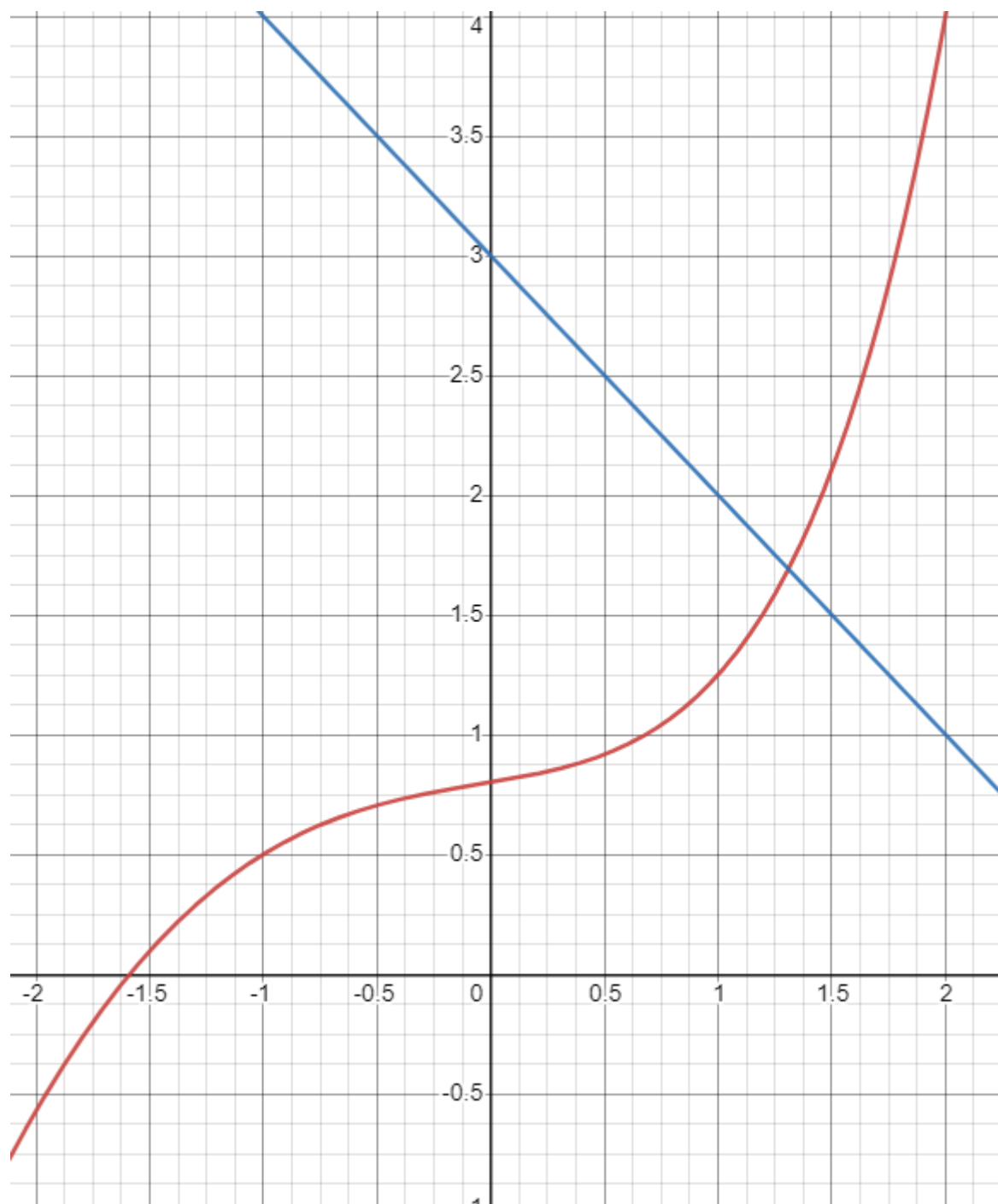
Part	Mark	Guidance												
(a)	B1	For two points (rounded correctly) correct from; <table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>3</td><td>3.32</td><td>3.73</td><td>4.28</td><td>5</td><td>5.95</td></tr></table>	0	1	2	3	4	5	3	3.32	3.73	4.28	5	5.95
	0	1	2	3	4	5								
3	3.32	3.73	4.28	5	5.95									
	B1	All three points correct and correctly rounded. Penalise rounding only once here. Condone 5.00												
(b)	B1ft	All points plotted within half of one square. Ft their values of y for x = 2,3,4 respectively												
	B1ft	All drawn points joined up in a smooth curve 												
(c)	M1	For use of power law to obtain $\log_3(6 - 2x) = \frac{\pm x}{4}$												
	M1	For removing the $\log_3$ to obtain: $6 - 2x = 3^{\frac{\pm x}{4}}$ Allow $(6 - 2x)^4 = 3^{\pm x}$ for this mark.												
	A1	For obtaining the equation $8 - 2x = 3^{\frac{x}{4}} + 2$ oe (eg., $-2x + 8 = 2 + 3^{\frac{\pi}{4}}$)												
	M1	For drawing their straight line, provided it is of the form $y = k - 2x$ where k is a constant and $k \neq 6$  [Check coordinates (1, 6) (2, 4) (3, 2) (4, 0)]												
	A1	For the intersection point (x =) 2.1												

Question number	Scheme									Marks
8 (a)	x	0.5	0.75	1	1.5	2	3	4	5	B2 [2]
	y	10	3.83	2	1.33	1.75	3.33	5.19	7.12	
(b)	Graph drawn: All points to within half of a square All points joined together in a smooth curve									B1ft B1ft [2]
(c)	$2x + \frac{3}{x^2} - 3 = ax + b \Rightarrow 2x^3 + 3 - 3x^2 = ax^3 + bx^2$ $\Rightarrow x^3(2 - a) - x^2(3 + b) + 3 \equiv 4x^3 - 10x^2 + 3$ $2 - a = 4 \Rightarrow a = -2$ $3 + b = 10 \Rightarrow b = 7$ So line is $y = 7 - 2x$ $x = 0.6, 2.4$ [Accept $x = 0.7$ and $x = 2.3$] ALT $\frac{4x^3 - 10x^2 + 3}{x^2} = 0 \Rightarrow 4x - 10 + \frac{3}{x^2} = 0$$2x + \frac{3}{x^2} = 10 - 2x \Rightarrow 2x + \frac{3}{x^2} - 3 = 7 - 2x$ so line is $y = 7 - 2x$ $x = 0.6, 2.4$ [Accept $x = 0.7$ and $x = 2.3$]									M1 M1A1 M1A1 [5] {M1 M1A1 M1A1 [5]}
Total 9 marks										

Part	Mark	Notes
(a)	B1	For two out of 4 values rounded correctly. Penalise rounding only once for awrt the required values.
	B1	For all four values correctly rounded
(b)	B1ft	For their points plotted correctly to within half of a square.
	B1ft	For their points joined up in a smooth curve. 
(c)	M1	For setting $2x + \frac{3}{x^2} - 3 = ax + b$ and attempting to simplify to the form $x^3(2-a) - x^2(3+b) + 3$ where the coefficient of each of x^3 and x^2 is in terms of a and/or b
	M1	For equating coefficients and attempting to find the value of a and b
	ALT	
	M1	For dividing through by x^2 to give $4x - 10 + \frac{3}{x^2} = 0$
	M1	For adding 7 and subtracting $2x$ from both sides to give on the LHS $2x + \frac{3}{x^2} - 3$
	A1	For the correct straight line $y = 7 - 2x$
	M1	For their line drawn provided it is of the form $y = A - 2x$ where $A \neq 0$ 
	A1	The line intersects the coordinates axes at (0, 7) and (3.5, 0) For both values of 0.6 and 2.4 Accept 0.7 and 2.3 - without working these roots score no marks. NB: Calculator roots are 2.366 and 0.634

Question number	Scheme	Marks						
9 (a)	<table border="1"> <tr> <td>-0.5</td><td>0.5</td><td>1.5</td></tr> <tr> <td>0.70</td><td>0.92</td><td>2.11</td></tr> </table>	-0.5	0.5	1.5	0.70	0.92	2.11	B1 B1 [2]
-0.5	0.5	1.5						
0.70	0.92	2.11						
(b)	Points plotted	B1ft						
	Joined with a smooth curve	B1ft [2]						
(c)	$x^3 + 4 = x^2 - 8x + 15$	M1						
	$x^3 + 4 = (x - 5)(x - 3)$ oe	M1						
	$\frac{x^3 + 4}{5 - x} = 3 - x$	A1						
	$y = 3 - x$ drawn	M1						
	$x = 1.3/1.4$	A1 [5]						
ALT	$\frac{x^3 + 4}{5 - x} = ax + b \Leftrightarrow x^3 + 4 = 5ax - ax^2 + 5b - bx$	{M1}						
	$\Leftrightarrow x^3 + ax^2 + b - 5a x + 4 - 5b \equiv x^3 - x^2 + 8x - 11$	{M1}						
	$\Leftrightarrow a = -1 \quad b = 3$	{A1}						
	$y = 3 - x$ drawn	{M1}						
	$x = 1.3/1.4$	{A1} [5]						
Total 9 marks								

Part	Marks	Notes
(a)	First B1	1 point correct
	Second B1	For all 3 points correct
(b)	B1ft	For points plotted within half a square ft their points
	B1ft	For points joined with a smooth curve ft their table
(c)	M1	For correctly rearranging the given equation to give $x^3 + 4 = x^2 - 8x + 15$
	M1	For factorising their quadratic, minimum attempt, see general guidance.
	A1	For $\frac{x^3 + 4}{5 - x} = 3 - x$
	M1	For $y = 3 - x$ drawn
	A1	$x = 1.3/1.4$
	ALT	
ALT	M1	For $\frac{x^3 + 4}{5 - x} = ax + b$ and an attempt to multiply both sides by $(5 - x)$, allow two errors only (sign or algebraic).
	M1	For correctly stating $x^3 + ax^2 + b - 5a x + 4 - 5b \equiv x^3 - x^2 + 8x - 11$
	A1	For $a = -1 \quad b = 3$
	M1	For $y = 3 - x$ drawn
	A1	$x = 1.3/1.4$

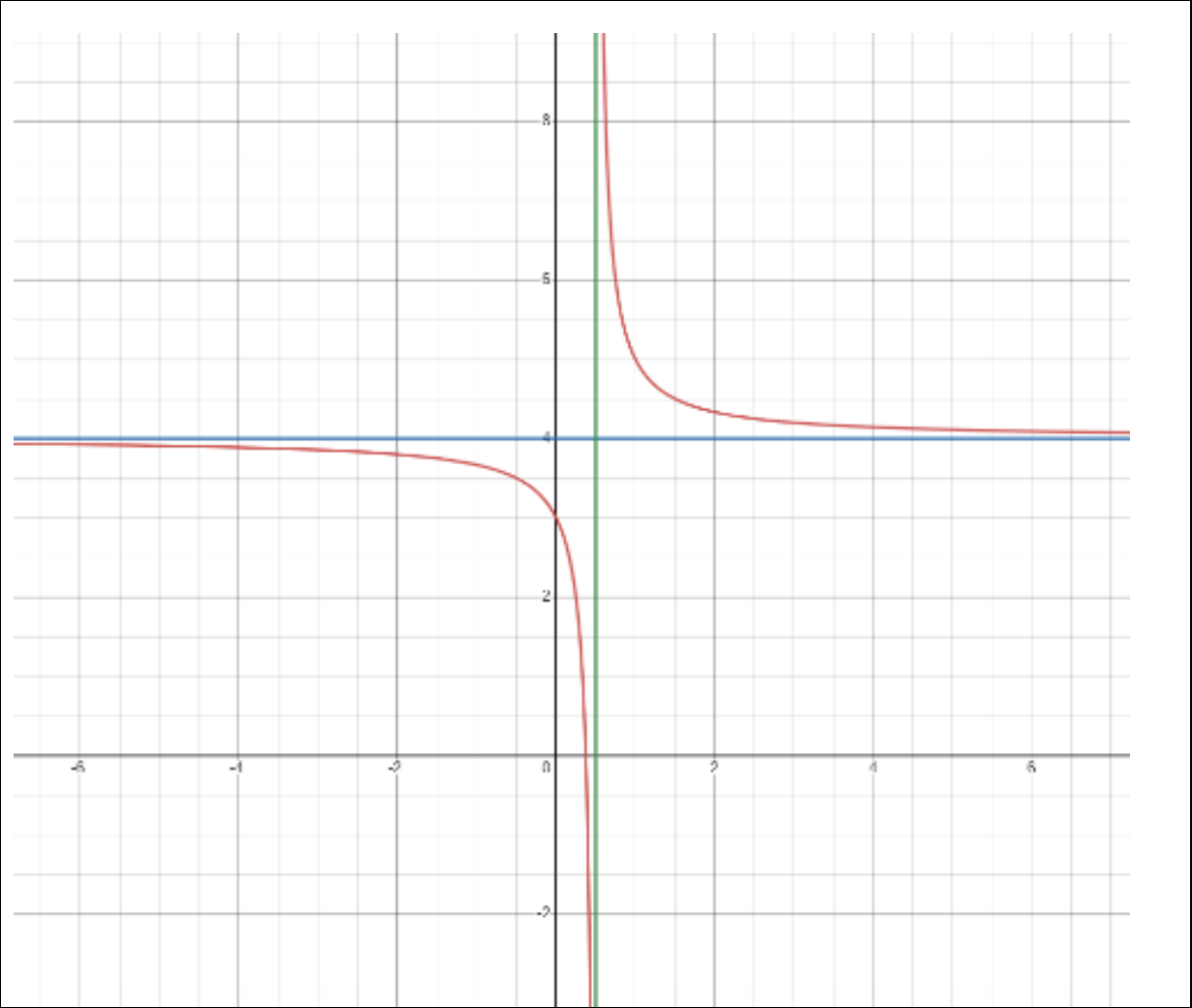


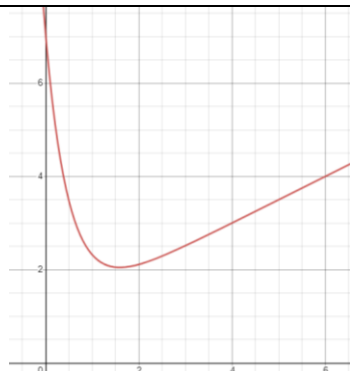
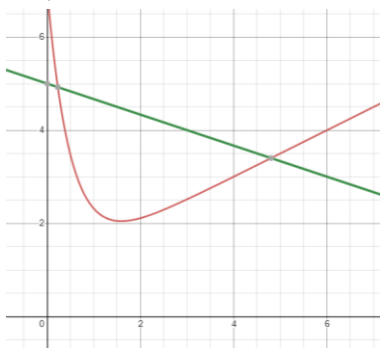
Question number	Scheme	Marks
3 a	$\frac{a}{-2} = 4 \Rightarrow a = -8$	M1 A1 (2)
b	$x = \frac{1}{2}$	B1 (1)
c (i)	$\left(\frac{3}{-8}, 0\right) \left(= \left(\frac{3}{8}, 0\right)\right)$	B1 ft
c (ii)	(0,3)	B1 (2)
d	Asymptote drawn at $y = 4$. The asymptote must either be clearly labelled with its equation or pass through the y axis with 4 clearly labelled. Asymptote with equation drawn at $x = \frac{1}{2}$ The asymptote must either be clearly labelled with its equation or pass through the x axis with $\frac{1}{2}$ labelled. Correct curve drawn with two branches “ $\frac{3}{8}$ ” and “3” labelled on x and y axes respectively	B1 B1 ft B1 ft (4)
Total 9 marks		

Part	Mark	Additional Guidance
(a)	M1	For $\frac{a}{-2} = 4$
	A1	For $a = -8$
		Note: it is acceptable if a candidate uses a suitably large value for x and deduces the correct value for a. M1 A1
(b)	B1	For $x = \frac{1}{2}$
(c)(i)	B1 ft	For $\left(\frac{3}{-8}, 0\right) \left(= \left(\frac{3}{8}, 0\right)\right)$ ft their value from (a). Allow omission of brackets. This mark is awarded for the point at which you see $\left(\frac{3}{-their\ a}, 0\right)$ May be written $x =$
(ii)	B1	For (0,3). Allow omission of brackets. May be written $y =$
If there is no labelling of (i) and (ii), it must be unambiguously indicated which coordinate crosses the y axis and which crosses the x axis. If any doubt, do not award the marks.		
(d)	B1	For asymptote with equation drawn at $y = 4$. The line must be labelled with the equation or must clearly pass through the y-axis labelled as 4. There must be at least one branch of the curve present, which must not cross or bend back from the asymptote.
	B1 ft	For asymptote with equation drawn at $x = \frac{1}{2}$, ft their answer from part b The line must be labelled with the equation or must clearly pass through the x-axis labelled as 0.5. There must be at least one branch of the curve drawn, which must not cross or bend back from the asymptote.

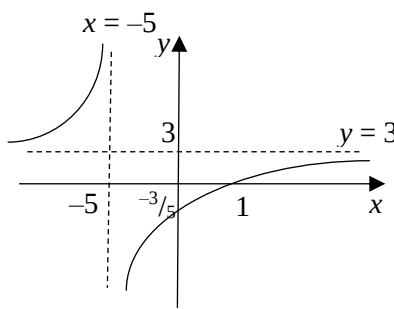
	B1 ft	For correct curve drawn with two branches, in the correct quadrants, ft their answers from part a and b. The curve must be drawn in quadrants 1 and 3.
	B1 ft	For a curve passing through “ $\frac{3}{8}$ ” and “3” labelled on x and y axes respectively, ft their answers from part a, b and c.

Note: answers for parts a, b and c cannot be awarded marks for correct values labelled on the sketch for part d. For all parts where coordinates are requested, allow $x =$ and $y =$

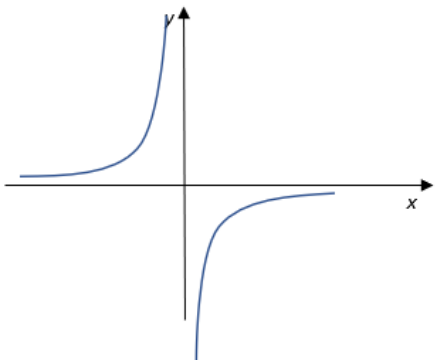
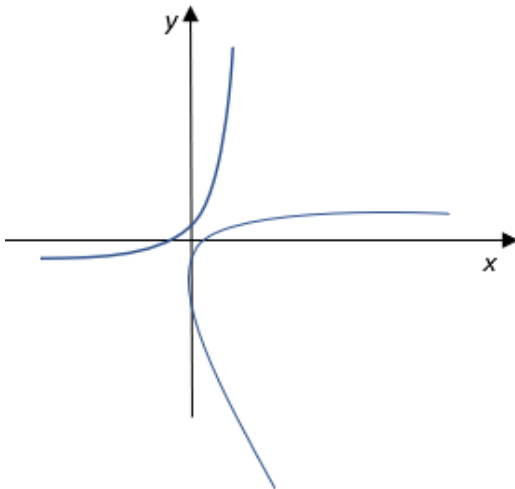


Question	Scheme	Marks																		
4(a)	<table><tr><td>x</td><td>0</td><td>1</td><td>1.5</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>7</td><td>2.3</td><td>2.0</td><td>2.1</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr></table>	x	0	1	1.5	2	3	4	5	6	y	7	2.3	2.0	2.1	2.5	3	3.5	4	B2 [2]
x	0	1	1.5	2	3	4	5	6												
y	7	2.3	2.0	2.1	2.5	3	3.5	4												
(b)		B2ft [2]																		
(c)	$2x + \ln(24 - 5x) - \ln 36 = 0 \Rightarrow \ln\left(\frac{24 - 5x}{36}\right) = -2x$ $\Rightarrow \frac{24 - 5x}{36} = e^{-2x} \Rightarrow \frac{2}{3} - \frac{5x}{36} = e^{-2x}$ $\Rightarrow 4 - \frac{5x}{6} = 6e^{-2x}$ $\Rightarrow 5 - \frac{x}{3} = \frac{x}{2} + 6e^{-2x} + 1 \Rightarrow y = 5 - \frac{x}{3}$ $\Rightarrow x = 0.2, \quad 4.8$ 	M1 M1A1 M1A1FT [5]																		
Total 9 marks																				

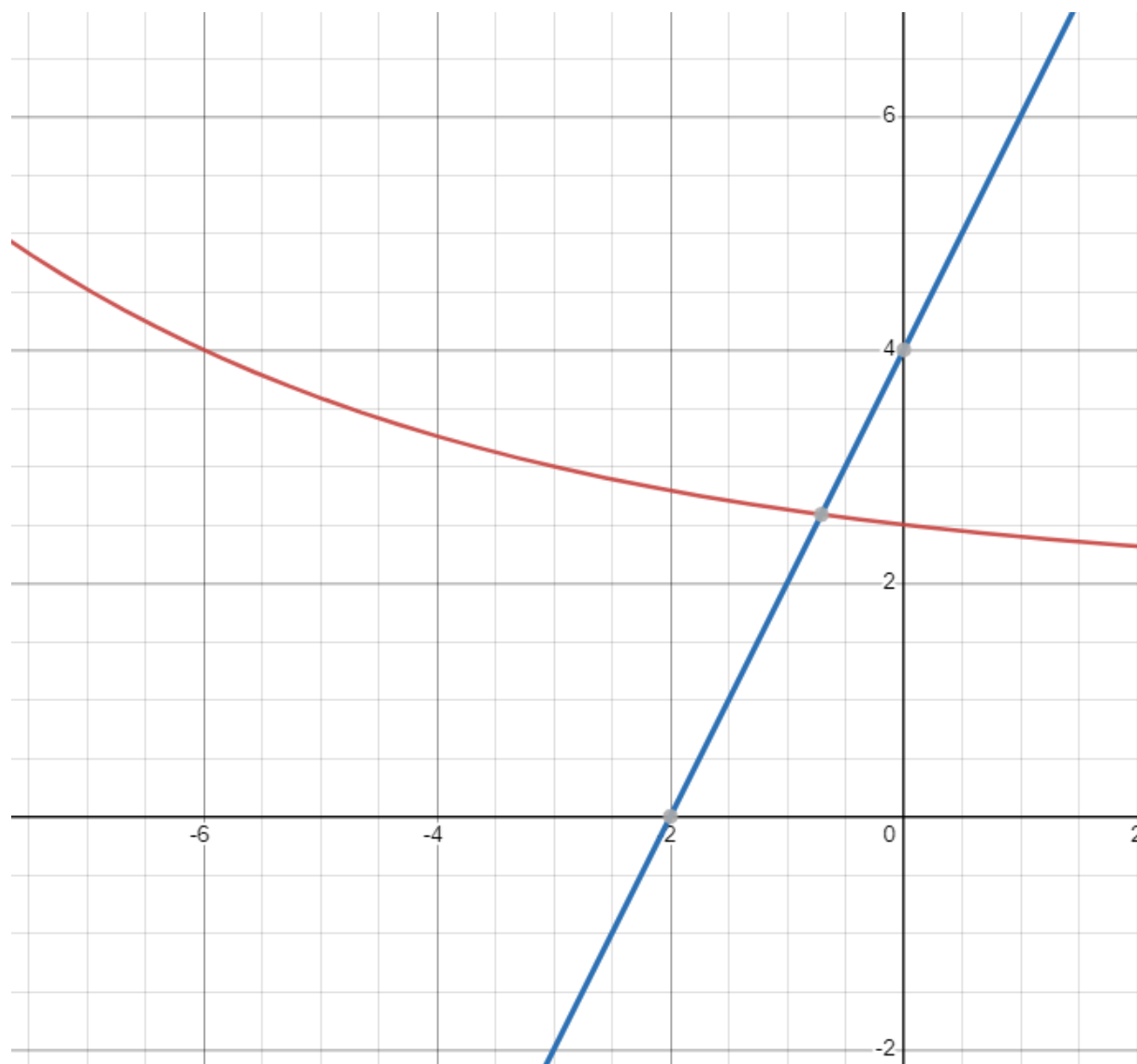
Part	Mark	Notes
(a)	B1	At least two correct values rounded correctly.
	B1	All values correct, rounded correctly.
(b)	B1ft	Their points plotted within half of one square.
	B1ft	Their points joined up in a smooth curve. Note: do not allow ruled sections.
(c)	M1	Combines the logs correctly and raises both sides to the power of base e
	M1	Rearranges the equation to obtain the equation of the curve on one side and the equation of the straight line on the other.
	A1	Achieves a correct equation of the straight line. Note: equivalent forms are acceptable.
	M1	Draws their line provided it is of the form $y = k \pm \frac{x}{3}$ when simplified.
	A1FT	Provided M1M1A1M1 awarded. For both correct values of x to 1 decimal place, their values must follow through from their graph. 0.2, 0.3, 0.4 and 4.8 Condone coordinates. Note: correct values seen with no working scores M0M0A0M0A0

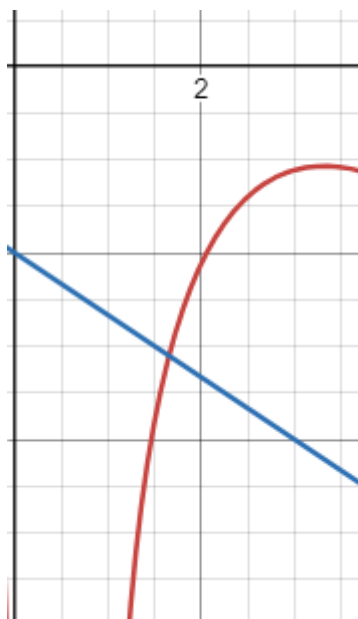
Question number	Scheme	Marks
3 (a)	$\frac{dy}{dx} = \frac{a(x+5) - (ax-3)}{(x+5)^2}$ When $x = 2$ $\frac{dy}{dx} = \frac{7a-2a+3}{49} = \frac{18}{49} \Rightarrow a = \dots$ $a = 3^*$	M1 M1 A1cso (3)
3 (b)(i)	$y = 3$	B1
(b)(ii)	$x = -5$	B1
(c)(i)	$(1, 0)$	(2) B1
(c)(ii)	$\left(0, \frac{-3}{5}\right)$	B1 (2)
(d)	 Curve drawn Asymptotes drawn and labelled $-3/5, 1$ labelled on axes	B1 B1ft B1ft (3) [10]
Total 10 marks		

Part	Mark	Notes
(a)	M1	For an attempt at Quotient rule. The definition of an attempt is that there must be a correct attempt to differentiate both terms and the denominator must be squared. Allow the terms in the numerator to be the wrong way around, but the terms must be subtracted. $ax - 3 \Rightarrow a$ and $x + 5 \Rightarrow 1$ must be correct. $\frac{dy}{dx} = \frac{a(x+5) - (ax-3)}{(x+5)^2}$ [allow $\frac{dy}{dx} = \frac{(ax-3) - a(x+5)}{(x+5)^2}$]
	M1	For substituting $x = 2$ into their differentiated expression, setting it equal to $\frac{18}{49}$ and attempting to solve the linear equation leading to a value for a $\frac{dy}{dx} = \frac{7a-2a+3}{49} = \frac{18}{49} \Rightarrow 5a+3=18 \Rightarrow a = \dots$ Allow one slip in their method.
	A1 cso	For $a = 3^*$ No errors in working.

(b)(i)	B1	For $y = 3$ This must be an equation of a line. Do not award for just 3.
(ii)	B1	For $x = -5$ This must be the equation of a line. Do not award for just -5
(c)(i)	B1	For (1, 0) or clearly listing $x = 1, y = 0$ as a pair.
(ii)	B1	For $\left(0, -\frac{3}{5}\right)$ oe. or clearly listing $x = 0, y = -\frac{3}{5}$ as a pair.
(d)	B1	For the curve drawn with two branches anywhere on the grid provided it is a negative reciprocal curve. The ends of the curves must be asymptotic and must not turn back on themselves. Do not allow any obvious overlap across the ends of the curve with evidence of the presence of asymptotes.
		For example, accept: Negative reciprocal curve in incorrect position  Do not accept: • Overlap of asymptotes • Ends turning back on themselves 
	B1ft	For their asymptotes correctly drawn and clearly labelled with their equation. At least one branch of the curve is required. It must be a negative reciprocal and it must be in the correct position for their asymptotes. The follow through is available for their answers in part b. If correct asymptotes appear on the sketch, do not award marks retrospective marks in part b.
	B1ft	The curve must be drawn going through their two points of intersection. It must be a negative reciprocal, in the correct position for their intersections and clearly marked on the axes. The follow through is available for their answers in part c. Allow the other branch to even be missing. If correct coordinates appear on the sketch, do not award marks retrospective marks in part c.

Part (a)	Mark	Additional Guidance
	B2	For all 3 values in the table correct to 2 d.p. Allow 3.0 or 3.00 for “3” (B1 for 2 values)
(b)	B1 ft	For all of the points plotted within half a square, allow use of their values. Points must be checked carefully, including using the zoom tool on ePen if necessary.
	B1 ft	For all of their points joined with a smooth curve. The curve must pass through each of their points to within half a square. Be cautious to not award this mark if straight lines are drawn between the points plotted.
(c)	M1	For use of $\log_a x^k = k \log_a x$ and an attempt to rearrange. Accept an equation of the form $\pm 3 \log_2 (2x + 2) = \pm x \pm 3$ or $\pm \log_2 (2x + 2) = \frac{\pm x}{3} \pm 1$
	M1	For correctly dividing by 3 (if not already done) and correct use of $\log_b a = \frac{\log_{0.5}(a)}{\log_{0.5}(b)}$ with their equation to get an expression of the form $\frac{\log_{0.5}(2x + 2)}{\log_{0.5} 2} = \pm \frac{x}{3} \pm 1$ or For correctly dividing by 3 (if not done) and correctly converting their equation to exponential form get an equation of the form $2x + 2 = 2^{\pm \frac{x}{3} \pm 1}$ or $\frac{1}{2x + 2} = 2^{\pm \frac{x}{3} \pm 1}$
	M1	For reaching an equation of the form $2x + 2 = 0.5^{\pm \frac{x}{3} \pm 1}$
	A1	For $2x + 4 = 0.5^{\left(\frac{x}{3} + 1\right)} + 2$
	M1	For $y = 2x + 4$ drawn. Correct line drawn can imply any of the 4 previous marks. Allow any line of $y = 2x + c$ to gain this mark. If the candidate draws any line where c is other than 4, they must have gained the previous 3 method marks.
	A1	For $x = -0.8 / -0.7$
ALT	M1	For use of $\log_a x^k = k \log_a x$ and an attempt to rearrange. Accept an equation of the form $\pm 3 \log_2 (2x + 2) = \pm x \pm 3$ or $\pm \log_2 (2x + 2) = \frac{\pm x}{3} \pm 1$
	M1	For correctly dividing by 3 (if not already done) and correct use of $\log_b a = \frac{\log_{0.5}(a)}{\log_{0.5}(b)}$ with their equation to get an expression of the form $\frac{\log_{0.5}(2x + 2)}{\log_{0.5} 2} = \pm \frac{x}{3} \pm 1$
	M1	For an attempt to rearrange and converting to log form to reach $\log_{0.5}(y \pm 2) = \pm \frac{x}{3} \pm 1 \Rightarrow \log_{0.5}(y \pm 2) = \pm \log_{0.5}(2x + 2)$
	A1	For the correct equation show.
	M1 A1	As main scheme.



Question number	Scheme	Marks														
7 (a)	<table><tr><td>x</td><td>1.3</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td><td>3.5</td></tr><tr><td>y</td><td>-5.17</td><td>-3.79</td><td>-2.11</td><td>-1.39</td><td>-1.11</td><td>-1.08</td></tr></table>	x	1.3	1.5	2	2.5	3	3.5	y	-5.17	-3.79	-2.11	-1.39	-1.11	-1.08	B1 B1 [2]
x	1.3	1.5	2	2.5	3	3.5										
y	-5.17	-3.79	-2.11	-1.39	-1.11	-1.08										
(b)	Points plotted. Smooth curve. 	B1ft B1ft [2]														
(c)	General principle for marking M1 – Deal with $9 = 3^2$ or apply addition law and change $\log_3 9 = 2$ M1 – Deal with logs M1 – Apply power law A1 – Correct line $\frac{3^{\frac{4}{3}x}}{3^2} = (x^2 - x)^3$ $3^{\frac{4}{3}x-2} = (x^2 - x)^3$ $\frac{4}{3}x - 2 = \log_3 (x^2 - x)^3$ $\frac{4}{3}x - 2 = 3\log_3 (x^2 - x)$ $-\frac{2}{3}x - 2 = 3\log_3 (x^2 - x) - 2x$	M1 M1 M1 A1														

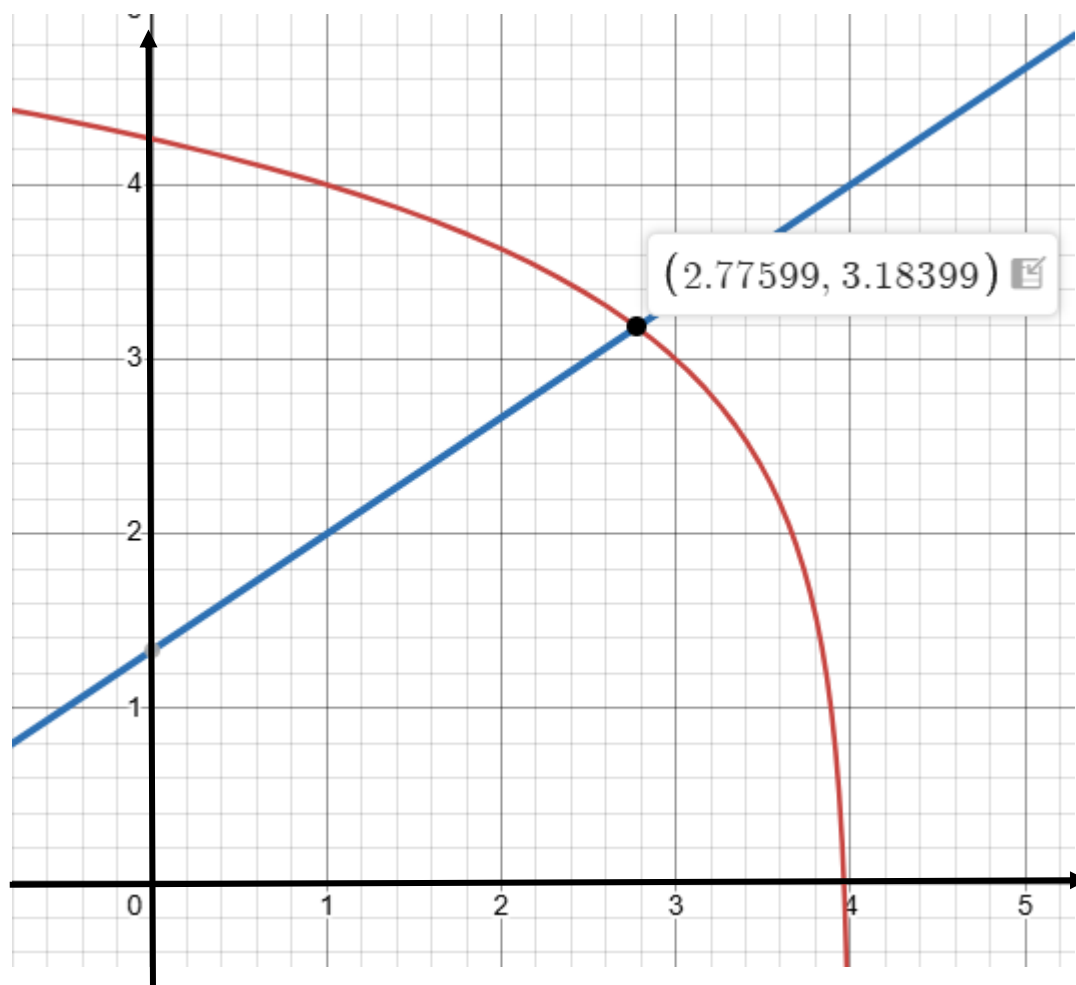
	OR $3^{\frac{4}{3}x} = 9(x^2 - x)^3$ $\log_3 3^{\frac{4}{3}x} = \log_3 9(x^2 - x)^3 \Rightarrow \frac{4}{3}x = \log_3 9(x^2 - x)^3$ $\frac{4}{3}x = \log_3 9 + \log_3 (x^2 - x)^3 \Rightarrow \frac{4}{3}x = 2 + \log_3 (x^2 - x)^3$ $\frac{4}{3}x - 2 = \log_3 (x^2 - x)^3 \Rightarrow \frac{4}{3}x - 2 = 3\log_3 (x^2 - x)$ $-\frac{2}{3}x - 2 = 3\log_3 (x^2 - x) - 2x$ Draws the line $y = -\frac{2}{3}x - 2$ (x =) 1.6 or 1.7	2nd M1 1st M1 3rd M1 A1 M1 A1 [6]
ALT (c)	Set $3\log_3 (x^2 - x) - 2x = Ax + B$ $\log_3 (x^2 - x)^3 = Ax + 2x + B$ $\Rightarrow (x^2 - x)^3 = 3^{(A+2)x+B}$ Drawing on given equation $9(x^2 - x)^3 = 9 \times 3^{(A+2)x+B} = 3^{(A+2)x+B+2}$ Equating exponents $\frac{4}{3}x = (A+2)x \quad 0 = B+2$ $A = \frac{4}{3} - 2 = -\frac{2}{3} \quad B = -2$ Hence $y = -\frac{2}{3}x - 2$ Draws the line $y = -\frac{2}{3}x - 2$ (x =) 1.6 or 1.7	M1 M1 M1 A1 M1 A1*cso [6]
Total 10 marks		

Part	Mark	Additional Guidance
7 (a)	B1	For 1 value correct to 2dp
	B1	For all 4 values correct.
(b)	B1ft	For points plotted within half a square ft their values from the table.
	B1ft	For points joined with a smooth curve ft their points plotted.
(c)	General principles using this method	
	M1 – Deal with 9 or apply addition law and change $\log_3 9 = 2$	
	M1 – Deal with logs	
	M1 – Apply power law	
	A1 – Correct line	
	M1	Correct rearrangement and use of index rule to give the equation shown. [Deals correctly with $9 = 3^2$]
	M1	Correctly takes logs with their expression to eliminate the exponential.
	M1	Applies the power law for logs correctly with their expression
ALT (c)	A1	For the equation seen oe
	M1	Draws a line of the form $y = \frac{a}{b}x - 2$ Check the intercept on the y-axis or otherwise (0, -2) (1.5, -3) (3, -4)
	A1*cs0	1.6 or 1.7, 1 decimal place only.
	M1	For equating the given formula for y with a generic linear equation and setting out to find the coefficients and uses power law correctly.
	M1	Correctly takes exponential from their expression to eliminate the log
	M1	Substitutes from their expression into the given equality.
	A1	For finding the coefficients of the linear equation oe
	M1	Draws a line of the form $y = \frac{a}{b}x - 2$ Check the intercept on the y-axis or otherwise (0, -2) (1.5, -3) (3, -4)
	----- A1*cs0	1.6 or 1.7, 1 decimal place only.

Question number	Scheme										Marks
7 (a)	x	0	0.5	1	1.5	2	2.5	3	3.5		B2
	y	4.26	4.14	4	3.83	3.63	3.37	3	2.37		(2)
(b)	Points plotted within half a square. Points joined with a smooth curve.										B1ft B1ft (2)
(c)	$3^{2x-5} = (4-x)^3 \Rightarrow 2x-5 = 3\log_3(4-x)$ $\frac{2x-5}{3} + 3 = \log_3(4-x) + 3 \Rightarrow \frac{2x+4}{3} = \log_3(4-x) + 3$ ALT $\log_3(4-x) + 3 = mx + c$ $\Rightarrow \log_3(4-x) = mx + (c-3)$ $\Rightarrow (4-x) = 3^{mx+(c-3)}$ $\Rightarrow (4-x)^3 = 3^{(mx+(c-3))3} = 3^{3mx+3c-9}$ $\Rightarrow 2x-5 = 3mx+3c-9 \Rightarrow 3m = 2 \text{ and } -5 = 3c-9$ $\Rightarrow m = \frac{2}{3} \quad c = \frac{4}{3}$ Graph of $y = \frac{2x+4}{3}$ drawn. Intersection point is $x = 2.8$										M1M1 M1A1 <

Part	Mark	Additional Guidance
(a)	B2	All three points correct (B1 for 2 points correct)
(b)	B1ft	Points plotted ft their table. Allow half a square tolerance.
	B1ft	Points joined together with a smooth curve. Allow half a square tolerance on joining up their points. FT their table
(c)	M1	For taking logs base 3 of both sides $2x - 5 = \log_3(4 - x)^3$
	M1	For applying the power law $2x - 5 = 3\log_3(4 - x)$
	ddM1	For attempting to rearrange to the required form An attempt is defined as dividing through by 3 and adding 3 to both sides.
	A1	For the correct line, in any form $\frac{2x-5}{3} + 3 = \log_3(4-x) + 3 \Rightarrow \left[(y =) \frac{2}{3}x + \frac{4}{3} \right]$
	ALT for first 4 marks	
	M1	Sets up the equation to obtain $\log_3(4-x) + 3 = mx + c$ And removes the log to obtain $(4-x) = 3^{mx+(c-3)}$
	M1	Raises both sides to a power of 3. Allow one error in this on the LHS
	M1	Compares coefficients to obtain an equation in m and an equation in c
	A1	Obtains the values $m = \frac{2}{3}$ $c = \frac{4}{3}$
	Draws the line	
	M1	For their line drawn provided it is of the form $y = kx \pm \frac{4}{3}$ for $k \neq 0$, 1
	A1	$x = 2.8$ or 2.7 Calculator value is 2.776

USEFUL GRAPH



Question number	Scheme	Marks																
8 (a)	<table><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td><td>3</td></tr><tr><td>y</td><td>2</td><td>2.41</td><td>2.69</td><td>3.10</td><td>3.39</td><td>3.61</td><td>3.95</td></tr></table>	x	0	0.25	0.5	1	1.5	2	3	y	2	2.41	2.69	3.10	3.39	3.61	3.95	B2 [2]
x	0	0.25	0.5	1	1.5	2	3											
y	2	2.41	2.69	3.10	3.39	3.61	3.95											
(b)	Points plotted within half of a square Points joined together in a smooth curve	B1ft B1ft [2]																
(c)	$\ln(2x+1) = 3x-4 \Rightarrow \ln(2x+1)+2 = 3x-2$ Graph of $y = 3x-2$ drawn. Intersection point is at $x = 1.8$ or 1.9 (Accept either)	M1 M1A1 [3]																
(d)	$e^{(6-x)} = (2x+1)^2 \Rightarrow 6-x = \ln(2x+1)^2 \Rightarrow 6-x = 2\ln(2x+1)$ $\Rightarrow \ln(2x+1)+2 = 5-\frac{x}{2}$ Graph of $y = 5-\frac{x}{2}$ drawn. Intersection point is at $x = 2.4$ or 2.5 (Accept either)	M1 M1 M1A1 [4]																
Total 11 marks																		
(a) B2	All 3 points correct (B1 for 2 points correct)																	
(b) B1ft B1ft	Points plotted fit their table allow half a square tolerance Points joined together with a smooth curve fit their table																	
(c) M1	$\ln(2x+1)+2 = 3x-2$																	
M1	$y = 3x-2$ drawn.																	
A1	Intersection point is at $(x =) 1.8$ or 1.9 Accept either																	
(d) M1	$6-x = 2\ln(2x+1)$																	
M1	$\ln(2x+1)+2 = 5-\frac{x}{2}$																	
M1	Graph of $y = 5-\frac{x}{2}$ drawn																	
A1	Intersection point is at $(x =) 2.4$ or 2.5 Accept either																	

Question number	Scheme								Marks
4 (a)	x	0.5	1	1.5	2	2.5	3	3.5	B2
	y	10	5	4.89	5.5	6.32	7.22	8.16	(2)
(b)	Drawn								B1ft B1ft (2)
(c)	$2x+1+\frac{2}{x^2}=8$ $x=0.6 \quad x=3.4$								M1 A1 (2)
(d)	$2x+1+\frac{2}{x^2}=\frac{1}{2}x+6$ $y=\frac{1}{2}x+6$ drawn $x=0.7 \quad x=3.2$								M1 A1 M1 A1 A1 (5)
[11]									
Notes									
(a) B2	For all 4 values correct (B1 for at least 2 values correct)								
(b) B1ft B1ft	For points plotted ft their table (allow half square tolerance) For points joined together with a smooth curve ft their table								
(c) M1	For $2x+1+\frac{2}{x^2}=8$ may be implied by [y =] 8 identified on the graph								
A1 (d)	For 0.6 and 3.4 (Allow 0.7 for 0.6)								
M1	For adding $\frac{1}{2}x$ to both sides of the equation or adding 1 to both sides of the equation								
A1	For $2x+1+\frac{2}{x^2}=\frac{1}{2}x+6$								
M1	For $y=\frac{1}{2}x+6$ drawn								
A1	For 0.7 (allow 0.8)								
A1	For 3.2 (allow 3.3)								

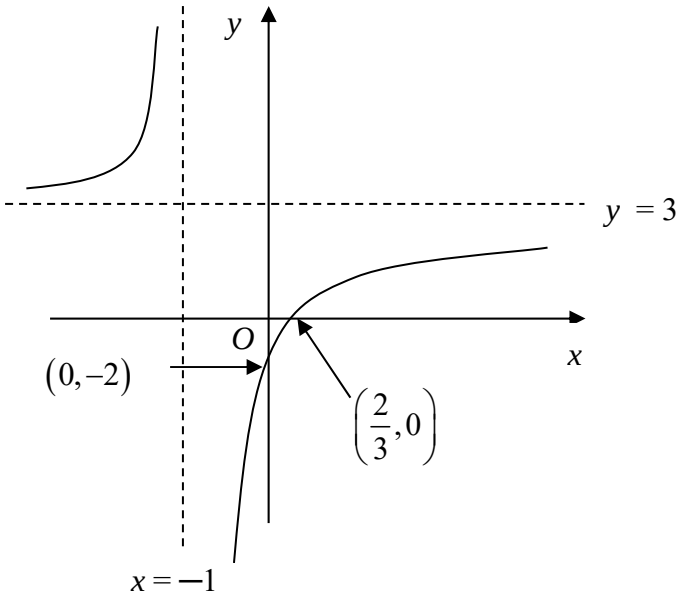
Question	Scheme	Mark																		
8(a)	<table><tr><td>x</td><td>0</td><td>0.5</td><td>0.8</td><td>1</td><td>1.6</td><td>2</td><td>2.5</td><td>3</td></tr><tr><td>y</td><td>−3.5</td><td>−1.6</td><td>0.3</td><td>1.6</td><td>3.5</td><td>2.4</td><td>−0.8</td><td>−3.2</td></tr></table>	x	0	0.5	0.8	1	1.6	2	2.5	3	y	−3.5	−1.6	0.3	1.6	3.5	2.4	−0.8	−3.2	B2
x	0	0.5	0.8	1	1.6	2	2.5	3												
y	−3.5	−1.6	0.3	1.6	3.5	2.4	−0.8	−3.2												
(b)	Their points plotted correctly on the graph Points are joined to form a smooth curve	B1ft B1ft [2]																		
(c)	$\cos(A+B)=\cos A\cos B-\sin A\sin B\Rightarrow\cos 2A=\cos^2 A-\sin^2 A=1-\sin^2 A-\sin^2 A$ $\cos(2A)=1-2\sin^2 A$	M1 A1 cso [2]																		
(d)	$2\sin x+6(1-\cos 2x)-x-5=0$ $\sin x-3\cos 2x-\frac{1}{2}=\frac{x}{2}\pm k$ where k is a constant $\Rightarrow y=\frac{x}{2}-1$ Draws line $y=\frac{x}{2}-1$ $x=0.6$ or $x=0.7$ AND $x=2.3$ or $x=2.4$	M1 dM1A1 M1 A1 [5]																		
Total		11 marks																		

Part	Mark	Scheme																		
(a)	B2	<table><tr><td>x</td><td>0</td><td>0.5</td><td>0.8</td><td>1</td><td>1.6</td><td>2</td><td>2.5</td><td>3</td></tr><tr><td>y</td><td>-3.5</td><td>-1.6</td><td>0.3</td><td>1.6</td><td>3.5</td><td>2.4</td><td>-0.8</td><td>-3.2</td></tr></table> For all 4 values correct OR If B2 not scored then award B1 for any 2 values correct Note: These values must be rounded to one decimal place only. Accept only the above values.	x	0	0.5	0.8	1	1.6	2	2.5	3	y	-3.5	-1.6	0.3	1.6	3.5	2.4	-0.8	-3.2
x	0	0.5	0.8	1	1.6	2	2.5	3												
y	-3.5	-1.6	0.3	1.6	3.5	2.4	-0.8	-3.2												
(b)	B1ft B1ft	 All points plotted correctly (ft their values) on the graph within half of one square. All of their points are joined to form a smooth curve within half of a square of a point. There must be at least six points plotted and joined.																		
(c)	M1	For using the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B = \cos^2 A - \sin^2 A$ correctly and the identity $\sin^2 A + \cos^2 A = 1$ also correctly.																		
	A1 cso	For simplifying to the required identity $\cos 2A = 1 - 2 \sin^2 A$ This is a given result so there must be no errors in their solution.																		
(d)	M1	$2 \sin^2 x = 1 - \cos 2x$ For substituting the above into the given $f(x)$ with no errors. e.g., $2 \sin x + 6(1 - \cos 2x) - x - 5 = 0$																		
	dM1	For reaching the equation of the curve on one side and the equation of a straight line on the other. $\sin x - 3 \cos 2x - \frac{1}{2} = \frac{x}{2} \pm k$ where k is a constant o.e Note: This mark is dependent on the previous M mark.																		
	A1	For the correct equation of the equation of the straight line required $y = \frac{x}{2} - 1$																		
	M1	For drawing their line which must be in the form $y = \frac{x}{2} \pm k$ Correct coordinates for $y = \frac{x}{2} - 1$ are; $(0, -1), \left(1, -\frac{1}{2}\right), (2, 0), \left(3, \frac{1}{2}\right)$																		
	A1	For both correct roots of $x = 0.6$ or $x = 0.7$ AND $x = 2.3$ or $x = 2.4$																		

Question number	Scheme								Marks
6 a	0	0.25	0.5	1	1.5	2	3		B2
	4(.00)	3.34	2.82	2.1(0)	1.67	1.41	1.15		(2)
b	Points plotted within half a square Points joined with a smooth curve								B1 ft B1 ft (2)
c	$x = e^{-x} \Rightarrow 3x + 1 = 1 + 3e^{-x}$ or sight of $y = 3x + 1$ and $y = 1 + 3e^{-x}$ $y = 3x + 1$ drawn. Intersection is at $x = 0.5$ or 0.6								M1 M1 A1 (3)
d	$\ln(x - 1)^3 = -3x \Rightarrow \ln(x - 1) = -x$ $\Rightarrow 3x - 3 = 3e^{-x} \Rightarrow 3x - 2 = 1 + 3e^{-x}$ $y = 3x - 2$ drawn. Intersection is at $x = 1.2$ or 1.3								M1 M1 M1 A1 (4)
ALT – first 2 marks	$(x - 1)^3 = e^{-3x} \Rightarrow \left(\sqrt[3]{(x - 1)^3} = \sqrt[3]{e^{-3x}}\right) \Rightarrow x - 1 = e^{-x}$ $3x - 3 = 3e^{-x} \Rightarrow 3x - 2 = 1 + 3e^{-x}$								M1 M1
Total 11 marks									

Part	Marks	Notes
(a)	B2	B2 for all 3 values correct (condone 2.1 for 2.10) (B1 for 2 values correct)
(b)	B1ft	For all of the points plotted within half a square, allow use of their values. Points must be checked carefully, including using the zoom tool on ePen if necessary.
	B1ft	For all of their points joined with a smooth curve. Be cautious to not award this mark if straight lines are drawn between the points plotted.
(c)	M1	For multiplying both sides by 3 and adding 1 to both sides or sight of $y = 3x + 1$
	M1	For $y = 3x + 1$ drawn – the line must intersect the curve and pass through a minimum of 2 correct points – eg (0, 1) and (1, 4) M1 M1 if the correct straight line is drawn, without working.
	A1	$x = 0.5$ or 0.6
(d)	M1	For use of $\log_a x^k = k \log_a x$ and simplifying to give the expression shown in the scheme.
	M1	For removing logs, multiplying both sides by 3 and subtracting 2 to give the expression shown. Award M1 M1 if $y = 3x - 2$ or any equivalent form is seen eg $y = 1 + 3(x - 1)$
	M1	$y = 3x - 2$ drawn – the line must intersect the curve and pass through a minimum of 2 correct points M1 M1 M1 if the correct straight line is drawn, without working.
	A1	$x = 1.2$ or 1.3
	M1	For removing logs and cube rooting each side to arrive at the expression shown in the scheme.
	M1	For multiplying by 3 and subtracting 2 to give the expression shown Award M1 M1 if $y = 3x - 2$ or any equivalent form is seen eg $y = 1 + 3(x - 1)$
ALT first 2 marks		



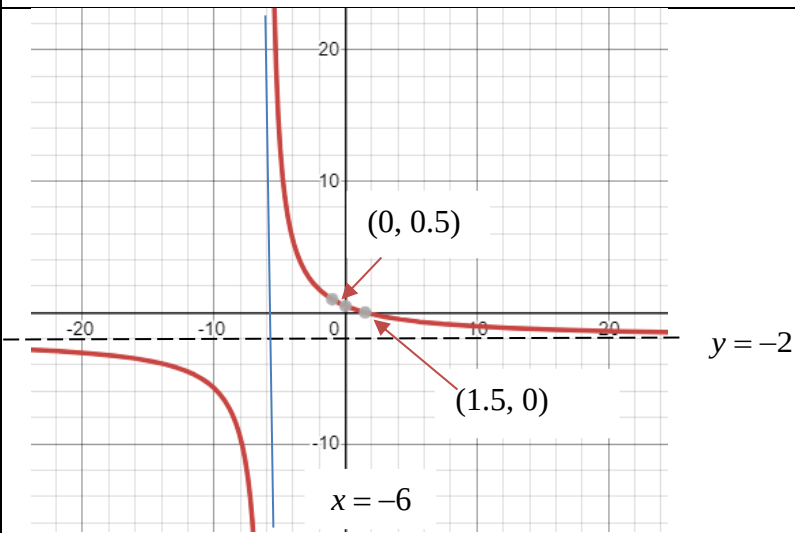
Question number	Scheme	Marks
11 (a)	(i) $y = 3$ (ii) $x = -1$	B1B1 [2]
(b)		B1
(i)	$0 = \frac{3x-2}{x+1} \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$	B1
(ii)	$y = \frac{3 \times 0 - 2}{0+1} = -2$	[2]
(c)		B1 (shape and position) B1 (asymptotes) B1 (intersections) [3]
(d)	$mx + 4 = \frac{3x-2}{x+1} \Rightarrow mx^2 + x(m+1) + 6 = 0$ $b^2 - 4ac < 0 \Rightarrow (m+1)^2 - 4 \times m \times 6 < 0 \Rightarrow m^2 - 22m + 1 < 0$ $(m-11)^2 - 120 = 0 \Rightarrow m = 11 \pm \sqrt{120}$ Defines region $11 - 2\sqrt{30} < m < 11 + 2\sqrt{30}$	M1 M1 M1A1 M1 depM1A1 [7]
Total 14 marks		

(a) (i) B1	$y = 3$
(a) (ii) B1	$x = -1$
(b) (i) B1	$x = \frac{2}{3}$
(b) (ii) B1	$y = -2$
(c) B1	Two curves in the correct quadrant (Condone poor shape as long as intention is clear)
B1	Two asymptotes drawn and correctly labelled (Allow -1 and 3 correctly labelled on axes to count as labelled). There must be at least one part of the curve drawn
B1	Intersections labelled
(d) M1	Equates C and l (Condone use of $\neq$ and $<$)
M1	Forms a 3TQ (Condone the use of $\neq$ and $<$) (allow 1 error in the expansion of correct brackets or 1 error in simplifying)
M1	Use of $b^2 - 4ac < 0$ to form a 3TQ
A1	Simplify to $m^2 - 22m + 1 < 0$
M1	Solving the 3TQ If the quadratic is incorrect then the method for solving must be shown
depM1	Simplifying to the form $a \pm b\sqrt{2}$ Dependant on previous M1
A1	$11 - 2\sqrt{30} < m < 11 + 2\sqrt{30}$ (Allow $a = 11, b = 30$)

Question	Scheme	Marks
7(a)	$x = -\frac{5}{4}$	B1 [1]
(b)	$\left\{\frac{dy}{dx}\right\} = \frac{2x(4x+5) - 4(x^2-1)}{(4x+5)^2}$ $y = 0$ $\left\{\frac{dy}{dx}\right\} = \frac{2(-1)(4(-1)+5) - 4((-1)^2-1)}{(4(-1)+5)^2} = -2$ $\rightarrow \{gradient\ of\ normal\} = \frac{1}{2}$ $y - 0 = \frac{1}{2}(x - (-1))\text{oe}$	M1A1A1 B1 M1 M1A1cso [7]
(c)	$\frac{x+1}{2} = \frac{x^2-1}{4x+5}$ $2x^2 + 9x + 7 = 0 \text{ OR } (x+1)(4x+5) = 2(x+1)(x-1)$ $(2x+7)(x+1) = 0 \Rightarrow x = \dots \text{ OR } (4x+5) = 2(x+1) \Rightarrow x = \dots$ $x = -\frac{7}{2}, \{x = -1\}$ $y = \frac{-\frac{7}{2} + 1}{2} = \dots$ $\left(-\frac{7}{2}, -\frac{5}{4}\right)$	M1 dM1 ddM1 A1 M1 A1 [6]
Total 14 marks		

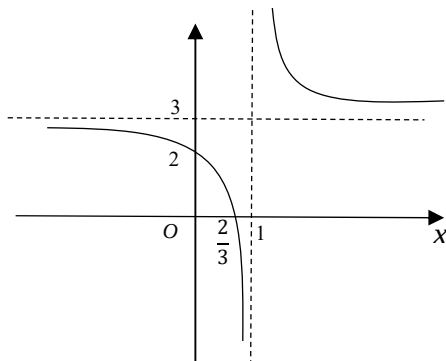


Part	Mark	Notes
(a)	B1	$x = -\frac{5}{4}$ or $x = -1.25$, do not accept $4x+5=0$
(b)	M1	Attempts to differentiate y. Quotient Rule: Look for $\frac{Px(4x+5) - Q(x^2-1)}{(4x+5)^2}, \text{ where } P > 0, Q > 0$ If the quotient rule is quoted, it must be correct. Condone invisible bracket for this mark. Product Rule: Look for $Px(4x+5)^{-1} \pm Q(x^2-1)(4x+5)^{-2}$, where $P > 0, Q > 0$
	A1	Quotient rule: Correct denominator $(4x+5)^2$ and one correct term in the numerator, $2x(4x+5)$ or $-4(x^2-1)$. Product rule: One term correct
	A1	Fully correct. $\left\{\frac{dy}{dx} = \right\} \frac{2x(4x+5) - 4(x^2-1)}{(4x+5)^2}$ or $\left\{\frac{dy}{dx} = \right\} 2x(4x+5)^{-1} - 4(x^2-1)(4x+5)^{-2}$ oe
	B1	$y = 0$ seen or implied
	M1	Correct method finding gradient of normal to the curve where $x = -1$
	M1	Correct method of forming the equation of a straight line using the point (-1, their y value) with a “changed” gradient. (“changed” means a different gradient from their tangent gradient, their normal could be found by an incorrect method) If using $y = mx + c$, a value for c must be found and the equation formed.
	A1cso	For the correct line in any form, simplified or unsimplified. (isw) Once correct equation is seen, award the mark, ignore later incorrect simplification. e.g. $2y = x + 1$ or $2y - x - 1 = 0$ or $y = 0.5x + 0.5$ oe or even $y - 0 = \frac{1}{2}(x - (-1))$
(c)	M1	For equating the equation of their line with the equation of the curve.
	dM1	For forming an equation with all like terms collected. OR cross multiplies then factorises e.g. $2(x^2 - 1) = 2(x + 1)(x - 1)$
	ddM1	Solves their equation with a correct method and finds at least one real value of x which is not -1 , method of solving could be implied by correct answers if calculator used. OR cancels $(x+1)$ and solves their linear equation.
	A1	$x = -\frac{7}{2}, \quad \{x = -1\}$
	M1	For substituting their x value (not -1) to either the equation of their normal or the equation of the given curve to find a value for y
	A1	For the correct exact coordinates of $D\left(-\frac{7}{2}, -\frac{5}{4}\right)$ oe Allow $x = -3.5, y = -1.25$ instead of given as coordinates.

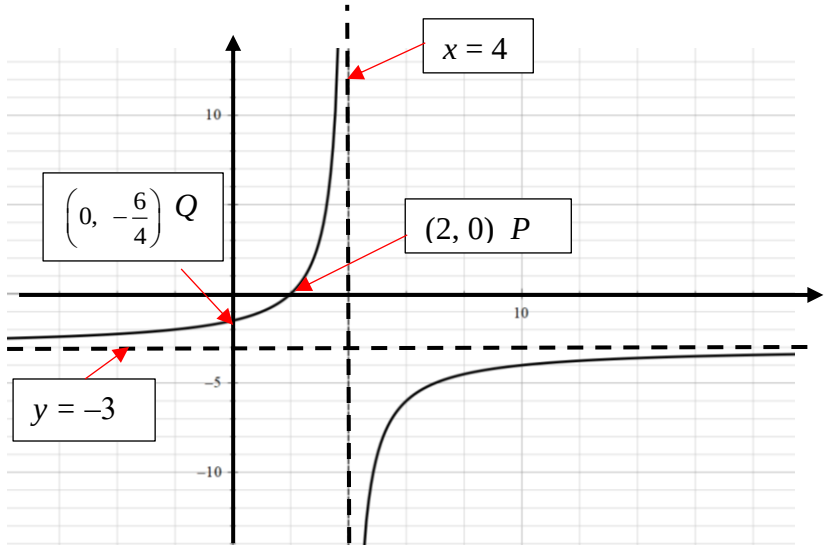
Question	Scheme	Marks
9(a)	(i) $y = -2$ (ii) $x = -6$	B1 B1 [2]
(b)	(i) $\left(\frac{3}{2}, 0\right)$ (ii) $\left(0, \frac{1}{2}\right)$	B1 B1 [2]
(c)		B1 – shape B1ft – Asymptotes B1ft – Intersections [3]
(d)	$\frac{dy}{dx} = \frac{(x+6)(-2) - (3-2x)(1)}{(x+6)^2}$ $\frac{dy}{dx} = \frac{-15}{(x+6)^2} \text{ with conclusion; numerator negative, denominator always positive, } \frac{-}{+} \Rightarrow \text{negative}$	M1A1 A1 [3]
(e)	$\frac{-15}{(x+6)^2} = -\frac{3}{5} \Rightarrow 25 = (x+6)^2 \Rightarrow x = -6 \pm 5 = -11, -1$ $y = \frac{3-2(-1)}{-1+6} = 1$ $y-1 = -\frac{3}{5}(x-[-1]) \Rightarrow y = -\frac{3}{5}x + \frac{2}{5} \Rightarrow k = \frac{2}{5}$	M1A1 B1 M1A1 [5]
Total 15 marks		

Part	Mark	Notes
(a)	B1	For the correct equation.
(i)		If their equations are not labelled (i) and (ii) accept them in the order given only. For example, the following presentation is B0B0 $x = -6$ $y = -2$
(ii)	B1	For the correct equation.
(b)		If their coordinates are not labelled (i) and (ii) or are given in the incorrect place [for example, (b)(i) $y = \frac{1}{2}$ is B0] accept them in the order given only. For example, the following presentation is B0B0 $\left(0, \frac{1}{2}\right)$ $\left(\frac{3}{2}, 0\right)$
(i)	B1	For the correct coordinates. Accept $x = \frac{3}{2}$
(ii)	B1	For the correct coordinates. Accept $y = \frac{1}{2}$
(c)	B1	For the correct shape with two branches the correct way around anywhere in the grid. It must be asymptotic in nature. The ends of the curve must not come back on themselves. Whilst you need to be fairly generous, any obvious turning back is B0
	B1ft	For the correct asymptotes drawn and labelled with at least one branch of the curve [which must be asymptotic in nature in at least one branch] in the correct place seen. Ft their asymptotes Accept the vertical line drawn shown passing through their -6 and the horizontal line drawn shown passing through their -2
	B1ft	For the correct coordinates of intersections with the relevant branch of the curve drawn correctly seen. The curve must go through the points. Do not accept touching the axis. Allow $\frac{1}{2}$ marked on y-axis and $\frac{3}{2}$ marked on the x-axis. Ft their coordinates of intersections.
(d)	M1	For an attempt at quotient rule. - The denominator must be squared, or accept $(x+6)(x+6)$ or $x^2 + 12x + 36$ - Both $(x+6)$ and $(3-2x)$ differentiated correctly. - The two terms in the numerator subtracted either way around.
	A1	Fully correct derivative (simplification not required for this mark).
	A1	For a correct simplified derivative with a correct conclusion. For example: $(x+6)^2 \geq 0$, -15 is negative, $\frac{\text{negative}}{\text{positive}}$ is always negative. [Accept also $(x+6)^2 > 0$]

(e)	M1	For setting the value of $-\frac{3}{5}$ = their $\frac{dy}{dx}$ with an attempt to find at least one value of x . Allow this even if their derivative results in a linear equation.
	A1	For both correct values of x
	B1	For $y=1$ using $x=-1$ OR For $y=-5$ using $x=-11$
	M1	For forming an equation of the line with either $x=-1$, $y=1$ or $x=-11$, $y=-5$ or their x and their y
	A1	For the correct value of k (accept an embedded value). $y = -\frac{3}{5}x + \frac{2}{5} \Rightarrow k = \frac{2}{5}$ You can award this mark even if the previous A mark has not been scored. So, for a correct solution without showing that $x = -11$ score M1A0B1M1A1
	ALT for last 3 marks	
	B1	Sets $-\frac{3}{5}x + k = \frac{3-2x}{x+6}$
	M1	Substitutes $x = -1$ or -11 into the above equation
	A1	For $k = \frac{2}{5}$ You can award this mark even if the previous A mark has not been scored. So, for a correct solution without showing that $x = -11$ score M1A0B1M1A1

Question Number	Scheme	Marks
9(a)(i)	$\frac{2}{p} = 2$ so $p = 1$ *	M1A1cso
(ii)	$\frac{dy}{dx} = \frac{q(x-1) - (qx-2)}{(x-1)^2}$	M1A1
(b)	When $x = 0$ $\frac{dy}{dx} = \frac{-q+2}{1} = -1, \Rightarrow q = 3$	M1A1,A1 (7)
		B1ft B1ft B1 B1ft B1ft (5)
(c)	$x + 2 = \frac{3x - 2}{x - 1}$	M1
	$x^2 - 2x = 0$	M1
	$x(x - 2) = 0$ $x = 2$	dM1A1cao(4)
[16]		
(a)(i)M1 A1cso (ii)M1 A1 ALT M1 A1 A1	Set $x = 0$ in the curve equation and equate result to 2. Obtain a value for p . Correct value of p obtained from a correct equation. Attempt the quotient rule. (formula is given on formula page). Denominator must be $(x-1)^2$. Numerator to be $q(x-1) - (qx-2)$ or $(qx-2) - q(x-1)$ Must use $p = 1$ now or later. Fully correct derivative Use product rule: $\frac{dy}{dx} = q(x-1)^{-1} - (qx-2)(x-1)^{-2}$ M1 for attempt with 2 terms similar to above, either term to be correct A1 Both terms correct Set $x = 0$ in their derivative and equate to -1 Correct equation $q = 3$	

Question Number	Scheme	Marks
(b)	No value for q: B0B1B0B1B1 available. Incorrect q: B1B1B0B1B1 available.	
B1ft	Equations of asymptotes seen or lines parallel to axes passing through $x = 1$, $y = 3$ drawn. $y = 3$ or their q . Must have a value for q.	
B1ft	Coordinates of crossing points seen explicitly or marked on the sketch. Must have $y = 2$; may have $x = 2/q$ (value for q not needed)	
B1	Two branches in the correct “quadrants” Must have $q = 3$ for this mark.	
B1ft	Asymptotes drawn.	
B1ft	There must be at least one branch of the curve drawn and 2 asymptotes drawn and labelled on the diagram by showing the coords of the points where they cross the axes or with their equations.	
B1ft	The curve must not touch (or cross) either asymptote. ft their asymptotes, inc $y = q$	
(c)	Both crossing points clearly marked on their diagram. ft their crossing points .	
M1	Eliminate y between the line and the curve equation. May use q or their value for q	
M1	Obtain a 2 or 3 term quadratic. May use q or their value for q .	
dM1	Solve their equation to obtain 1 or 2 values of x Depends on both M marks above.	
A1cao	$x = 2$ from a correct equation. If $x = 0$ is seen it must be clear that $x = 2$ is the only answer	
	If x is eliminated: M1 elimination M1 obtain quadratic in y M1 solve for y	
	A1 complete to a single value of x	

Question	Scheme	Marks
10(a) (i) (ii)	When $y = 0$, $x = 2$ or $(2, 0)$ When $x = 0$, $y = -\frac{6}{4}$ oe or $(0, -\frac{6}{4})$ oe If not labelled part (i) and (ii), do not award marks unless the candidate has presented in the correct order or has made it clear which coordinate or pair of values is P and which is Q	B1 B1 [2]
(b)	(i) $x = 4$ (ii) $y = -3$ If not labelled part (i) and (ii), do not award marks unless the candidate has presented in the correct order or has identification of horizontal (or parallel to x-axis) and vertical (or parallel to y-axis) is clear.	B1 B1 [2]
(c)		B1 B1 ft B1 ft [3]
(d)	$\left(\frac{dy}{dx}\right) \frac{-3(x-4) - (6-3x) \times 1}{(x-4)^2} = \left[\frac{6}{(x-4)^2}\right] \text{ oe}$ where $x = 2$ $\frac{dy}{dx} = \frac{6}{(2-4)^2} = \frac{6}{4} \Rightarrow \text{Gradient of normal} = -\frac{4}{6} \text{ oe}$ Equation of normal: $y - 0 = -\frac{4}{6}(x - 2) \Rightarrow [3y = -2x + 4] \text{ oe}$	M1A1A1 M1 dM1A1 [6]
(e)	$\frac{6-3x}{x-4} = \frac{-2x+4}{3} \Rightarrow 2x^2 - 21x + 34 = 0$ $\Rightarrow (2x-17)(x-2) = 0$ At R $\Rightarrow x = \frac{17}{2}, (2)$	M1 M1 A1 [3]
Total 16 marks		

Part	Mark	Notes
(a) (i)	B1	Must state $y = 0$, $x = 2$ or $(2, 0)$ or clearly stating $x = 2$
(ii)	B1	Must state $x = 0$, $y = -\frac{6}{4}$ oe or $\left(0, -\frac{6}{4}\right)$ oe or clearly stating $y = -\frac{6}{4}$
(b)(i)	B1	For $x = 4$
(ii)	B1	For $y = -3$
(c)	B1	For a negative reciprocal curve drawn anywhere in the grid – there must be two branches present, they must not cross any asymptotes drawn and must not obviously ‘bend back’ on themselves. Mark intention.
	B1ft	For the asymptotes drawn, follow through their (b)(i) and (ii). There must be at least one branch of a negative reciprocal curve present in the correct place for their work in (a) and (b), which must not cross or obviously bend back from the asymptotes. The asymptotes must be labelled with their equation or shown as passing through 4 on the x-axis and -3 on the y-axis, both clearly labelled.
	B1ft	For at least one branch of a negative reciprocal curve in the correct place for their work in (a) and (b), passing through their intercepts on the axes. The intercepts should be correctly labelled with the coordinates or the axes labelled correctly with $-\frac{6}{4}$ (oe) and 2 (or their ft values) but condone labelling to be P and Q .
(d)	M1	For an expression of the form. $\frac{-a(x-4)-(6-3x) \times b}{(x-4)^2}$ oe
	A1	For an expression of the form. $\frac{-3(x-4)-(6-3x) \times b}{(x-4)^2}$ or $\frac{-a(x-4)-(6-3x) \times 1}{(x-4)^2}$.oe
	A1	Fully correct – need not be simplified.
	M1	For substituting $x = 2$ into their $\frac{dy}{dx}$ and finding the gradient of the normal. This is not a dependent method mark, but the substitution must be into a changed function. If their expression does not allow substitution of $x = 2$, this mark cannot be awarded.
	dM1	For a complete and correct method to find the equation of the normal using their (changed gradient), $y = 0$ and $x = 2$. Dependent on the previous mark. If $y = mx + c$ is used they must find a value for c .
	A1	For any correct equation. This can be in any form and may be left unsimplified.
(e)	M1	For equating their equation of the normal to C and attempting to form a 3TQ. The attempt must involve correctly removing both denominators of the equation as a minimum and any attempt to collect terms.
	M1	For a minimally acceptable (see general guidance) and complete attempt to solve their quadratic equation, leading to a value of x .
	A1	For the x coordinate of point R $x = \frac{17}{2}$

Question	Scheme	Marks
10(a)	$\left(ax - 5 = 0, x = \frac{5}{4} \Rightarrow\right) a = 4$ $(\text{Line parallel to the } y\text{-axis is } x = 3 \Rightarrow) b = 3$	B1 B1 {2]
(b)		B1 B1ftB1 B1ftB1ft [5]
(c)	$\left(\frac{dy}{dx} = \frac{(3-x)(4) - (4x-5)(-1)}{(3-x)^2} = \left[\frac{7}{(3-x)^2}\right]\right.$ $(\text{Gradient of } l =) \frac{7}{4}$ $\frac{7}{4} = \frac{7}{(3-x)^2} \Rightarrow (3-x)^2 = 4 \Rightarrow 3-x = \pm 2 \Rightarrow x = 1, 5$ When $x = 1$, $y = -\frac{1}{2}$ and when $x = 5$, $y = -\frac{15}{2}$ Equation of line when $x = 1$: $y - -\frac{1}{2} = \frac{7}{4}(x-1) \Rightarrow 4y - 7x = -9$ Equation of line when $x = 5$: $y - -\frac{15}{2} = \frac{7}{4}(x-5) \Rightarrow 4y - 7x = -65$ $\Rightarrow -65 < k < -9$	M1 B1 M1A1 M1 A1 (B1B1 on ePen) ddM1 ddM1 A1 [9]

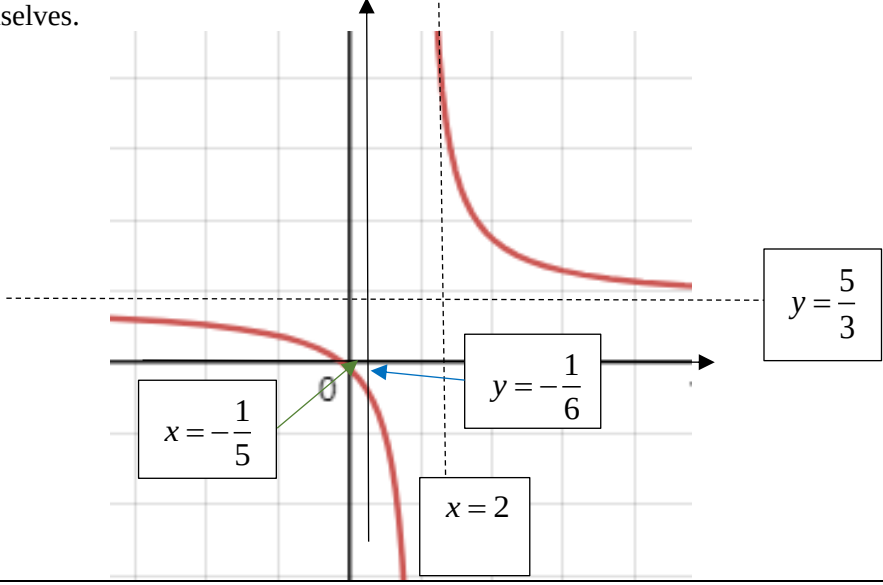
ALT	$(y =) \frac{7x+k}{4} = \frac{"4"x-5}{"3"-x} \quad \text{oe} \quad 4\left(\frac{"4"x-5}{"3"-x}\right) - 7x = k \quad \text{oe}$ $16x - 20 = (7x+k)("3"-x) \quad \text{oe}$ $16x - 20 = 21x - 7x^2 + 3k - kx \Rightarrow 7x^2 + kx - 5x - 20 - 3k (=0) \quad \text{oe}$ $(\Rightarrow 7x^2 + (k-5)x - 20 - 3k (=0)) \quad \text{oe}$ $(k-5)^2 - 4(7)(-20-3k) (<0)$ $k^2 + 74k + 585 (<0)$ $(k =) \frac{-74 \pm \sqrt{(74)^2 - 4 \times (1) \times 585}}{2} \quad \text{or} \quad (k-9)(k-65) (=0)$ $-65 < k < -9$	M1 A1(B1 on ePen) M1 A1 M1A1 (B1 B1 on ePen) ddM1 ddM1 A1 [9]
-----	--	---

Part	Mark	Notes
(a)	B1	For the value of $a = 4$ or $b = 3$
	B1	For the value of $a = 4$ and $b = 3$
Note: These are independent of method marks, not given answers. Unless the answers come from very obviously incorrect working, these marks should be awarded for sight of the correct values.		
(b)	B1	For a negative reciprocal curve drawn anywhere in the grid – there must be two branches present, they must not cross any asymptotes drawn and must not obviously ‘bend back’ on themselves. Mark intention.
For the following marks, where candidates have used incorrect values of a and b , examiners will find using desmos.com or similar packages useful. Type into desmos $y = (ax - b)/(b - x)$ and add ‘sliders’ for a and b		
	B1ft	For the horizontal asymptote of $y = -4$ drawn in the correct place and labelled with its equation or where the line passes through the axis. There must be at least one branch of a reciprocal curve present which must not cross or obviously bend back from the asymptote(s) ignore any other curves present. Allow follow through only of $y = -$ (their a) and only if other conditions met.
	B1	For the vertical asymptote $x = 3$ drawn in the correct place. There must be at least one branch of a reciprocal curve present which must not cross or obviously bend back from the asymptote(s), ignore any other curves present.
	B1ft	A single curve (or even line) passing through $\left(0, -\frac{5}{3}\right)$ marked clearly on the graph as a coordinate or as a crossing point on the axis. Ignore any other branches present. Allow follow through only of $\left(0, -\frac{5}{\text{their } b}\right)$
	B1ft	A single curve (or even line) passing through both $\left(\frac{5}{4}, 0\right)$ and $\left(0, -\frac{5}{3}\right)$ both marked clearly on the graph as a coordinate or as crossing points on the axes. Ignore any other branches present. Allow follow through only of $\left(0, -\frac{5}{\text{their } b}\right)$

(c)	M1	For applying the quotient (or product rule). - With two terms in the numerator subtracted either way around. - The differentiation on both terms must be correct. - The denominator must be squared. $\frac{(3-x)(4)-(4x-5)(-1)}{(3-x)^2} \text{ or } \frac{(4x-5)(-1)-(3-x)(4)}{(3-x)^2} \text{ (if using quotient rule)}$ Or $4(3-x)^{-1} \pm (4x-5)(3-x)^{-2}$ (if using product rule)
	B1	For (Gradient of $l =$) $\frac{7}{4}$
	M1	For correctly setting their gradient of $l =$ to their derivative and an attempt to solve to find 2 values of x . Allow one error in rearrangement. Their derivative must be of the form $\frac{rx+s}{(3-x)^2}$ s or r could be 0, but not both
	A1	For $x = 1$ and 5
	M1 (B1 on ePen)	For substitution of either of their x values into the equation of their curve.
	A1 (B1 on ePen)	For $y = -\frac{1}{2}$ and $y = -\frac{15}{2}$
	ddM1	For correctly forming an equation of the line for either pair of their values and their gradient and rearranging (allow errors in rearrangement) to the form $4y - 7x = k$. Dependent on 1 st 2 method marks.
	ddM1	For correctly forming an equation of the line for both pairs of their values and their gradient and rearranging (allow rearrangement errors) to the form $4y - 7x = k$ Dependent on 1 st 2 method marks.
ALT	A1	For the correct inequality as shown.
	M1	For rearrangement of l to the form $y = \frac{7}{4}x + c$ $c \neq 0$ and placing equal to their equation of C or substitution of the equation for curve C in the equation for l . Allow $4\left(\frac{"4"x-5}{"3"-x}\right) = k \pm 7x$ Condone using an inequality sign, follow through their a and b
	A1 (B1 on ePen)	For a correct unsimplified equation. DO NOT condone using an inequality sign
	M1	For an attempt to rearrange their equation of the form $mx + c = \frac{4x-5}{3-x}$ $m, c \neq 0$, to allow them to reach a "3TQ", allow one error in rearrangement. Condone using an inequality sign By "3TQ", we means terms in x , x^2 and constant, all on 'one side' even if the equals sign is missing or an incorrect inequality sign is shown, terms simplified where possible.
	A1	For the correct "3TQ" shown (oe), the term in x does not have to be factorised and $= 0$ may be omitted. "3TQ" defined as above
	M1 (B1 on ePen)	For the correct formulation of the discriminant from their "3TQ". Inequality sign doesn't need to be shown or can be incorrect.
	A1 (B1 on ePen)	For the correct unsimplified discriminant. Inequality sign doesn't need to be shown or can be correct.
	ddM1	For a correct method to solve their quadratic equation in k , leading to a value of k . Dependent on first 2 method marks. Mark awarded for the correct method .
	ddM1	For a correct method to solve their quadratic equation in k , leading to two values of k . Dependent on 1 st 2 method marks. Mark awarded for the correct method .
	A1	For the inequality shown.

Question number	Scheme	Marks
11 a	$\frac{dy}{dx} = \frac{(2a-1)(ax-6) - a((2a-1)x+1)}{(ax-6)^2}$	M1A1A1 (3)
b (i)	At $x = 0$ $\frac{dy}{dx} = -\frac{11}{12}$ So $\frac{dy}{dx} = \frac{-6(2a-1) - a}{36} = -\frac{11}{12}$ $-13a + 6 = -33 \Rightarrow a = 3^*$	B1 M1 dM1 A1 cso (4)
b (ii)	$y = \frac{5x+1}{3x-6}^*$	B1 cso (1)
c	Asymptote with equation drawn at $y = 2$ Asymptote with equation drawn at $x = \frac{5}{3}$ Correct curve drawn with two branches $-\frac{1}{5}$ and $-\frac{1}{6}$ labelled on x and y axes	B1 B1 B1 B1 B1 (5)
d	$\frac{12}{11}x - \frac{1}{6} = \frac{5x+1}{3x-6}$ $(3x-6)(72x-11) = 66(5x+1)$ $x(72x-265) = 0$ oe $x = \frac{265}{72}$ oe	M1 M1 M1 A1 (4)
Total 17 marks		

Part	Mark	Notes
(a)	M1	For attempting to use a correct quotient rule: The definition of an attempt is as follows: - The denominator must be correct - The terms in the numerator must be subtracted either way around - The attempt to differentiate both terms must be as follows: $(2a-1)x+1 \rightarrow \pm(2a-1)$ and $ax-6 \rightarrow \pm a$
	A1	For one term in the numerator correct
	A1	For a fully correct differentiated expression. $\frac{dy}{dx} = \frac{(2a-1)(ax-6) - a((2a-1)x+1)}{(ax-6)^2}$ No simplification is required.

Uses Product Rule		
	M1	For attempting to use a correct product rule: The definition of an attempt is as follows: - The terms in the numerator must be added. - The attempt to differentiate both terms must be as follows: $(2a-1)x+1 \rightarrow \pm(2a-1)$ and $(ax-6)^{-1} \rightarrow \pm a(ax-6)^{-2}$
	A1	For one term correct
	A1	For a fully correct differentiation $\frac{dy}{dx} = [(2a-1)x+1](a)(ax-6)^{-2} + (ax-6)^{-1}(2a-1)$
(b)(i)	B1	For finding the gradient of the tangent $= -\frac{11}{12}$
	M1	For using that at the point A $x=0$ and setting their $\frac{dy}{dx} = \left(-\frac{11}{12}\right)$ which must be the gradient of their tangent to obtain $\frac{-6(2a-1)-a}{36} = -\frac{11}{12} \Rightarrow \left(\frac{-13a+6}{36} = -\frac{11}{12}\right)$
	dM1	For simplifying to a linear equation and attempting to solve.
	A1	For $a=3$ with no errors seen
(b)(ii)	B1	For $y = \frac{5x+1}{3x-6}$ exactly in that form. You must see evidence of substitution of $a=3$ for this mark. There must be $y = \dots$ [Condone missing $x \neq 2$]
(c)		This is a sketch of the curve. The ends must be asymptotic in nature and not turn back on themselves. 
	B1	For asymptote with equation drawn at $y = \frac{5}{3}$ OR $x=2$
	B1	For both asymptotes with equations drawn at $y = \frac{5}{3}$ AND $x=2$
	B1	Correct curve drawn with two branches with one branch in 1 st Quadrant and another branch in Quadrants 2, 3 and 4 with the graph drawn the correct way around. The graph must be asymptotic in nature.

	B1	$-\frac{1}{5}$ labelled on x -axis or $-\frac{1}{6}$ labelled on y -axis
	B1	$-\frac{1}{5}$ labelled on x axis and $-\frac{1}{6}$ labelled on y -axis
(d)	M1	For setting $\frac{12}{11}x - \frac{1}{6} = \frac{5x+1}{3x-6}$ [eliminating y from both equations]
	M1	For forming a quadratic equation, 2TQ or 3TQ in any order. $(3x-6)(72x-11) = 66(5x+1) \Rightarrow 72x^2 - 265x = 0$
	M1	For solving their QE provided it is either a 2TQ or a 3TQ
	A1	For $x = \frac{265}{72}$ oe

Question number	Scheme	Marks
10 (a)	(i) $x = -\frac{3}{2}$ (ii) $y = -\frac{1}{2}$	B1 B1 [2]
(b)	$(7, 0)$ $\left(0, \frac{7}{3}\right)$	B1 [2]
(c)		B1ft curve B1ft asymptotes B1ft (intersections with x- and y-axes) [3]
(d)	$\frac{-1(2x+3) - 2(7-x)}{(2x+3)^2} = \left[\frac{-17}{(2x+3)^2} \right]$	M1
	$\frac{-1\left(2 \times \frac{1}{2} + 3\right) - 2\left(7 - \times \frac{1}{2}\right)}{\left(2 \times \frac{1}{2} + 3\right)^2} = -\frac{17}{16}$	M1A1* cso [3]
	ALT – product rule	
	$-1(2x+3)^{-1} + (7-x)(-1)(2)(2x+3)^{-2} = \left[\frac{-17}{(2x+3)^2} \right]$	M1
	$-1\left(2 \times \frac{1}{2} + 3\right)^{-1} + \left(7 - \frac{1}{2}\right)(-1)(2)\left(2 \times \frac{1}{2} + 3\right)^{-2} = -\frac{17}{16}$	M1A1* cso

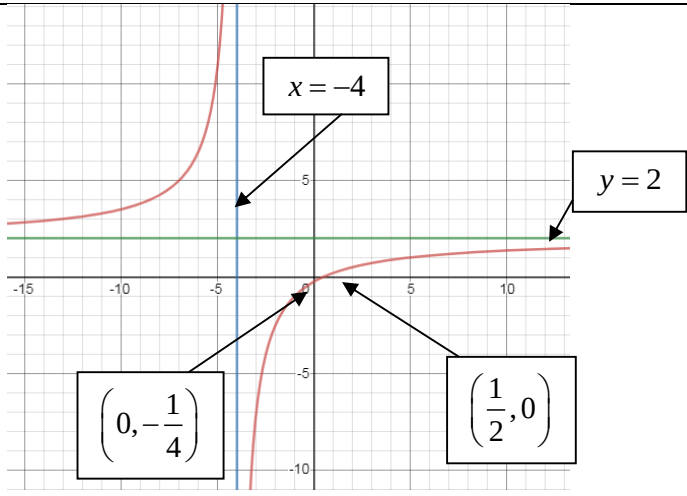
(e)	$-\frac{17}{(2x+3)^2} = -\frac{17}{16}$ $(2x+3)^2 = 16 \Rightarrow 2x+3 = \pm\sqrt{16} \Rightarrow x = 0.5 \text{ or } -3.5$ $y = \frac{7 - -3.5}{2 \times -3.5 + 3} \left(= -\frac{21}{8} \right)$ $\left(-\frac{7}{2}, -\frac{21}{8} \right)$ $y - -\frac{21}{8} = -\frac{17}{16} \left(x - -\frac{7}{2} \right)$ $34x + 32y + 203 = 0$	M1 dM1 ddM1 A1 M1A1 A1 [7]
	Total 17 marks	

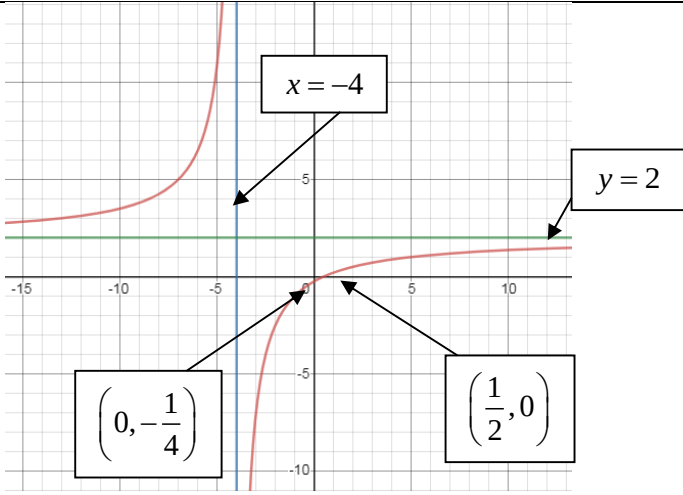
Part	Mark	Additional Guidance
For parts (a) and (b) look for their responses in the body of the question. In part (a), if the responses are not labelled (i) and (ii) accept them in order.		
(a)	(i) B1	For $x = -\frac{3}{2}$ oe
	(ii) B1	For $y = -\frac{1}{2}$ oe
(b)	B1 B1	First B1 for either correct, second B1 for both correct Condone if not given as coordinates e.g. $x = 7$ and/or $y = \frac{7}{3}$ given
(c)	B1ft	Two branches drawn in the correct two “quadrants” created by the two asymptotes. Mark intention, allow poor curves, but do not allow the curve to bend back on itself or touch any asymptotes.
	B1ft	Two clearly marked asymptotes, ft their (a), labelled as described, there must be one section of the curve present, tending towards these asymptotes.
	B1ft	Two clearly labelled intersections with the axes, ft their (b), at least one section of their curve must pass through one of these intersections. Intersections must be labelled correct way around. If additional intersections seen then B0
(d)	M1	Attempt the quotient rule. Numerator must be of the form $A(2x+3) - B(7-x)$, $B > 1$. Denominator must be of the form $(2x+3)^2$
	M1	Substitutes $x = \frac{1}{2}$ into their derivative NB: This is an A mark in Epen
	A1* cso	Obtains the required value of gradient $\left(-\frac{17}{16}\right)$ with no errors NB: This is a B mark in Epen
	ALT – product rule	
	M1	For an attempt at Product Rule. Must be a sum of two products. Must have the form $c(2x+3)^{-1} + d(7-x)(2x+3)^{-2}$ for constants c, d .
	M1	Substitutes $x = \frac{1}{2}$ into their derivative NB: This is an A mark in Epen
(e)	A1* cso	Obtains the required value of gradient $\left(-\frac{17}{16}\right)$ with no errors NB: This is a B mark in Epen
	M1	For equating the gradient of the tangent to their $\frac{dy}{dx}$ from (d)
	dM1	For rearranging to obtain $(2x+3)^2 = 16$ and attempt to solve or for rearrangement to a 3TQ and method to solve the 3TQ. See general guidance for acceptable methods to solve.

	ddM1	For substitution of their x into the original equation to obtain y . This must not be $x = \frac{1}{2}$ This M mark is dependent on the two previous M marks
	A1	For correct coordinate of $P \left(-\frac{7}{2}, -\frac{21}{8} \right)$
	M1	For substitution of their coordinates of P and the gradient $-\frac{17}{16}$ to obtain an equation of a straight line using any valid method. If $y = mx + c$ is used a value of c must be found and an equation needs to be formed. For example; $-\frac{21}{8} = -\frac{17}{16} \times -\frac{7}{2} + c \Rightarrow c = -\frac{203}{32} \Rightarrow y = -\frac{17}{16}x - \frac{203}{32}$ [
	A1	For any correct equation of the tangent with the correct values of x and y in any form.
	A1	For the correct equation of tangent given in the required form. $34x + 32y + 203 = 0$

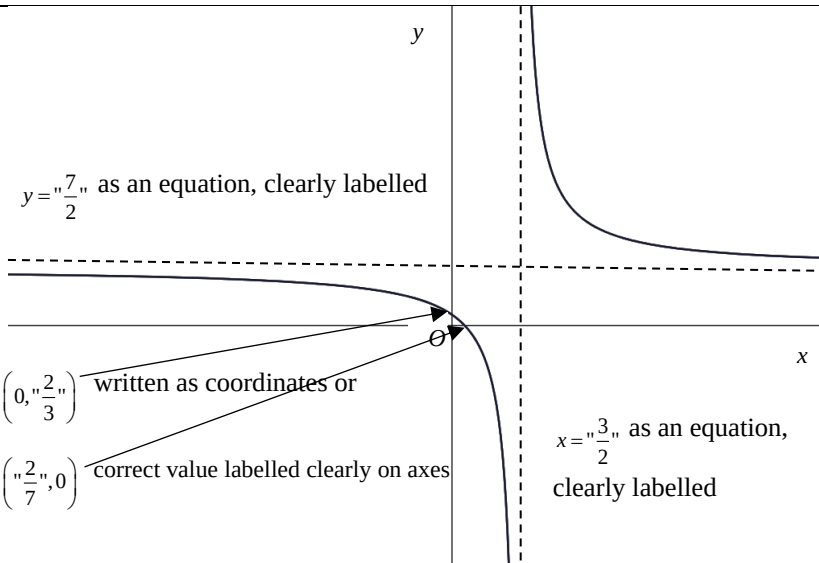
Question Number	Scheme	Marks
9(a)	$y = \frac{2+4x-x^2}{2x+1} \Rightarrow x^2 - 4x - 2 + 2yx + y (=0)$ $x^2 + (2y-4)x + (y-2) = 0$	M1A1A1 (3)
(b)	$(2y-4)^2 \geq 4(y-2)$ $4y^2 - 16y + 16 = 4y - 8 \Rightarrow 4y^2 - 20y + 24 = 0$ $y \leq 2 \text{ or } y \geq 3 \quad *$	M1 M1A1 A1cso (4)
(c)	$y = \frac{2+4x-x^2}{2x+1}$ $\frac{dy}{dx} = \frac{(4-2x)(2x+1) - 2(2+4x-x^2)}{(2x+1)^2} \quad \text{See notes for product rule method}$ $\frac{dy}{dx} = 0 \Rightarrow (4-2x)(2x+1) - 2(2+4x-x^2) = 0$ $2x(x+1) = 0 \Rightarrow x = 0, -1$ stationary points are (0,2) (-1,3)	M1A1A1 M1 A1 A1 (6)
ALT	$x^2 + (2y-4)x + (y-2) = 0 \Rightarrow 2x + (2y-4) + 2\frac{dy}{dx}x + \frac{dy}{dx} = 0$ $\frac{dy}{dx} = 0 \Rightarrow x + y = 2$ (using (b)) stationary points are (0,2) (-1,3)	M1A1A1 M1 A1A1
(d)	The graph shows a curve with a vertical asymptote at $x = -\frac{1}{2}$. The curve passes through the points $(-1, 3)$ and $(0, 2)$. The x-axis is labeled with $2 - \sqrt{6}$ or -0.45 and $2 + \sqrt{6}$ or 4.45. The y-axis is labeled with 1. The origin is marked O.	B1 curve (i) M1A1 (M1 finding coords, A1 correct (oe or min 2dp) and on diagram (ii) B1 (iii) B1ft (5) [18]

(a)M1	Re-write the equation of C without fractions and rearrange to the required form.
A1	Correct value for a and either b or c . These values need not be stated explicitly.
A1	Fully correct equation. Values of a , b and c need not be stated explicitly. Condone missing brackets round " $y - 2$ ". Award this mark when the equation is reached – isw any listing of values with incorrect signs.
(b)	
M1	Use " $b^2 \geq 4ac$ " for their equation
M1	Re-arrange their inequality to a 3TQ in y . Allow an equation here.
A1	Correct 3TQ, as shown or any equivalent
A1cso	Deduce the CVs (need not be shown explicitly) and state the (given) inequalities. There must be no errors in the working but the equation, if correct, can be solved easily so no working need be shown. Condone use of "and" instead of "or".
(c)	
M1	Differentiate the equation of C using the quotient rule. The denominator must be correct and the numerator must consist of the difference of 2 terms of the type shown. The product rule can be used.
A1	Either numerator term correct
A1	Fully correct numerator
ALT:	Product Rule $y = (2 + 4x - x^2)(2x + 1)^{-1}$
M1A1	$\frac{dy}{dx} = (4 - 2x)(2x + 1)^{-1} - 2(2 + 4x - x^2)(2x + 1)^{-2}$
A1	M1: rewrite without denominator and attempt product rule. Difference of 2 terms of the form shown needed (Difference because of the negative power)
	A1 Either term correct A1 Second term correct
M1	Equate the numerator of their derivative to 0 and solve to $x = \dots$ (any valid method of solving a quadratic with 2 or 3 terms) If product rule used must multiply through by $(2x + 1)^2$
A1	Both x values correct
A1	Both stationary points correct
ALT	
M1	Use implicit differentiation on the re-arranged equation
A1	Correct derivative of $(2y - 4)x$ (inc use of product rule)
A1	Fully correct derivative
M1	Set $\frac{dy}{dx} = 0$ and use the result from (b) to obtain solutions
A1	One correct stationary point
A1	Both correct stationary points
(d)	
B1	Shape: Two parts, one above $y = 3$ and the other below $y = 2$
(i)M1	Attempt to find the x coordinates of the crossing points
A1	Correct coordinates shown on their sketch, 2 crossing points only. $y = 0$ need not be seen. There must be a curve through these points.
(ii)B1	The asymptote must be drawn and labelled (by its equation or by showing the x coordinate of the point where it crosses the x -axis). There must be at least one part of the curve which is asymptotic to the line $x = -\frac{1}{2}$. No part of the curve should touch/cross the asymptote or curve dramatically away from the line.
(iii)B1ft	Label the stationary points with their coords. Follow through provided the result is sensible.

Question	Scheme	Mark
10(a)	(i) $y = 2$ (ii) $x = -4$	B1B1 [2]
(b)	$\left(\frac{1}{2}, 0\right), \left(0, -\frac{1}{4}\right)$	B1B1 [2]
(c)		B3 [3]
(d)	$\frac{dy}{dx} = \frac{2(x+4) - (2x-1)}{(x+4)^2}$ $m = 1$ $\frac{dy}{dx} = 1 \Rightarrow \frac{9}{(x+4)^2} = 1 \Rightarrow (x+4)^2 = 9 \Rightarrow x = -7, -1$ $P(-1, -1), Q(-7, 5)$	M1A1 B1 dM1ddM1A1 A1A1 [8]
(e)	$y = x + k$ $'-1' = '-1' + k_1 \Rightarrow k_1 = 0 \quad '5' = '-7' + k_2 \Rightarrow k_2 = 12$	M1A1A1 [3]
Total		18 marks

Part	Mark	Notes
(a)	B1 B1	For correct equations only (i) $y = 2$ (ii) $x = -4$ These must be clearly labelled.
(b)	B1	For either $\left(\frac{1}{2}, 0\right)$ OR $\left(0, -\frac{1}{4}\right)$ accept $y = -\frac{1}{4}$ OR $x = \frac{1}{2}$
	B1	For both $\left(\frac{1}{2}, 0\right)$ AND $\left(0, -\frac{1}{4}\right)$ accept $y = -\frac{1}{4}$ AND $x = \frac{1}{2}$
(c)	B1 B1ft B1ft	 For the correct shape in the correct ‘quadrants’. Do not allow the curves to turn back on themselves, but be reasonable in your judgement. If they all turn back on themselves clearly – withhold the mark. If there is doubt over just one end, allow the mark. If you are really not sure, then please send to Review. Their asymptotes drawn and labelled and there must be at least one part of the curve present that is asymptotic in nature. Any curve that crosses the asymptotes does not score this mark. Accept -4 written on the x-axis and 2 written on the y-axis. Their intersections labelled. Accept $y = -\frac{1}{4}$ AND $x = \frac{1}{2}$ labelled correctly and their curve must pass through these points.
(d)	M1	For attempting to use Quotient rule: An attempt is defined as both $(2x-1)$ and $(x+4)$ differentiated CORRECTLY and the correct formula used (subtracted either way around in the numerator) with the denominator squared. $\frac{dy}{dx} = \frac{2(x+4) - (2x-1)}{(x+4)^2}$ [Correct] or $\frac{dy}{dx} = \frac{(2x-1) - 2(x+4)}{(x+4)^2}$ [Incorrect] Or for an attempt to use product rule: Both terms differentiated correctly with the correct formula used. Allow a maximum of one sign error.

		$\frac{dy}{dx} = (2x-1)(1)(-1)(x+4)^{-2} + (2)(x+4)^{-1} \Rightarrow \left[\frac{dy}{dx} = \frac{-(2x-1)+2(x+4)}{(x+4)^2} \right]$
	A1	$\frac{dy}{dx} = \frac{2(x+4)-(2x-1)}{(x+4)^2}$ Fully correct
	B1	For $m = 1$
		ALT 1 for next two marks only
	dM1	For setting their $\frac{dy}{dx} = 1$ and rearranging to reach a 3TQ. If there is no squared term in their $\frac{dy}{dx}$ then this is M0. $x^2 + 8x + 7 = 0$
	ddM1	For attempting to solve their 3TQ $x^2 + 8x + 7 = 0 \Rightarrow (x+1)(x+7) = 0 \Rightarrow x = \dots, \dots$
		ALT 2 for next two marks only
	dM1	For setting their $\frac{dy}{dx} = 1 \Rightarrow \frac{9}{(x+4)^2} = 1 \Rightarrow (x+4)^2 = 9$
	ddM1	$(x+4)^2 = 9 \Rightarrow x+4 = \pm 3 \Rightarrow x = \dots, \dots$
	A1	For BOTH correct values of x , -7 and -1
	A1	For using their values of x to find either the coordinates of P or Q , they must be given as coordinates but allow missing or incorrect labels i.e, if they label P as Q or vice versa, allow the marks. At P $y = \frac{2 \times (-1) - 1}{-1 + 4} = -1 \Rightarrow$ Coordinates are $(-1, -1)$ OR At Q $y = \frac{2 \times (-7) - 1}{-7 + 4} = 5 \Rightarrow$ Coordinates are $(-7, 5)$
	A1	At P $y = \frac{2 \times (-1) - 1}{-1 + 4} = -1 \Rightarrow$ Coordinates are $(-1, -1)$ BOTH At Q $y = \frac{2 \times (-7) - 1}{-7 + 4} = 5 \Rightarrow$ Coordinates are $(-7, 5)$
(e)	M1	For substituting either of their coordinates into $y = x + k$ $'-1' = '-1' + k_1 \Rightarrow k_1 = \dots$ $'5' = '-7' + k_2 \Rightarrow k_2 = \dots$
	A1	$k_1 = 0$
	A1	$k_2 = 12$

Question number	Scheme	Marks
10 (a)		
(i)	$x = \frac{3}{2}$	B1
(ii)	$y = \frac{7}{2}$	B1 [2]
(b)	$\left(\frac{2}{7}, 0\right) \quad \left(0, \frac{2}{3}\right)$	B1 B1 [2]
(c)	$\frac{7(2x-3)-2(7x-2)}{(2x-3)^2}$ $\frac{-17}{(2x-3)^2} \text{ or } \frac{-17}{4x^2-12x+9}$ Correct conclusion ALT – product rule $7(2x-3)^{-1} + (7x-2)(-1)(2)(2x-3)^{-2}$ $\frac{-17}{(2x-3)^2} \text{ or } \frac{-17}{4x^2-12x+9}$ Correct conclusion	M1 A1 A1 B1 [4] M1 A1 A1 B1
(d)	 $y = -\frac{17}{(2x-3)^2}$ as an equation, clearly labelled $\left(0, \frac{2}{3}\right)$ written as coordinates or $\left(\frac{2}{7}, 0\right)$ correct value labelled clearly on axes $x = \frac{3}{2}$ as an equation, clearly labelled	B1 (curve) B1ft (asymptotes) B1ft (intersection s with x- and y-axes) [3]
(e)	$-\frac{1}{17} = \frac{-17}{(2x-3)^2}$ $(2x-3)^2 = 17^2 \quad \text{or} \quad 4x^2 - 12x - 280 = 0 \quad \text{oe}$ $x = 10 \quad y = 4 \quad \text{or} \quad (10, 4)$ $y - 4 = 17(x - 10) \quad \text{or} \quad 4 = -17 \times 10 + c$ leading to $c =$ $y = 17x - 166 \quad \text{oe}$	M1 dM1 A1 M1 A1

	$\text{"}17x - 166\text{"} = \frac{7x - 2}{2x - 3} \quad \rightarrow \quad 17x^2 - 195x + 250$ $x = \frac{25}{17} \quad y = -141 \quad \text{or} \quad \left(\frac{25}{17}, -141\right)$	M1 A1 [7]
Total 18 marks		



Part	Mark	Additional Guidance
		If a candidate gives no response to (a) and/or (b) but shows the correct answers on the graph we will award the marks. Where answers are given in (a) and/or (b) these should be marked as they stand with no reference to the graph. Ignore labelling of (i) and (ii) and mark (a) together.
(a)(i)	B1	For $x = \frac{3}{2}$ oe
(a)(ii)	B1	For $y = \frac{7}{2}$ oe
(b)	B1 B1	First B1 for either correct, second B1 for both correct Condone if not given as coordinates e.g. $x = \frac{2}{7}$ and/or $y = \frac{2}{3}$ given
(c)	M1	Attempt the quotient rule. Numerator must be the difference of two terms (either way round) of the form $A(2x-3) - B(7x-2)$, A and $B > 1$. Denominator must be of the form $(2x-3)^2$
	A1	Either term on the numerator correct (either way round), dependent on previous method mark.
	A1	Obtains $\frac{-17}{(2x-3)^2}$ or $\frac{-17}{4x^2-12x+9}$
	B1	Correct conclusion based on correct working only, for example, (the numerator is a negative number and) the denominator is always positive and therefore the fraction/gradient is always negative.
	ALT – product rule	
	M1	For an attempt at Product Rule. Must be a sum of two products. Must have the form $c(2x-3)^{-1} + d(7x-2)(2x-3)^{-2}$ for constants c, d .
	A1	Either term correct, dependent on previous method mark.
	A1	Obtains $\frac{-17}{(2x-3)^2}$ or $\frac{-17}{4x^2-12x+9}$
	B1	Correct conclusion based on correct working only, for example, (the numerator is a negative number and) the denominator is always positive and therefore the fraction/gradient is always negative.
(d)	B1	Two branches drawn in the correct two “quadrants” created by the two asymptotes. Mark intention, allow poor curves, but do not allow the curve to bend back on itself or touch any asymptotes. Allow BOD if intention is for curve to run alongside asymptote but there is a slight deviation back on itself.
	B1ft	Two clearly marked asymptotes, ft their (a), labelled as described, there must be one section of the curve present, tending towards these asymptotes.
	B1ft	Two clearly labelled intersections with the axes, ft their (b), at least one section of their curve must pass through one of these intersections. Intersections must be labelled correct way around. If additional intersections seen then B0
(e)	M1	Sets their differentiated function from part (c) = $-\frac{1}{17}$
	dM1	Rearranges to get to an equation of the form shown with no denominators or a 3TQ and solves using an acceptable method to obtain $x = \dots$ Dependent on previous method mark
	A1	Correct values for point A (10, 4)
	M1	Uses their values for x and y (from an attempt at working with gradient of the curve) with gradient 17 to find an equation for l (if using $y = mx + c$, must be a complete method arriving at $c =$) If correct $c = -166$.
	A1	Correct equation, any form

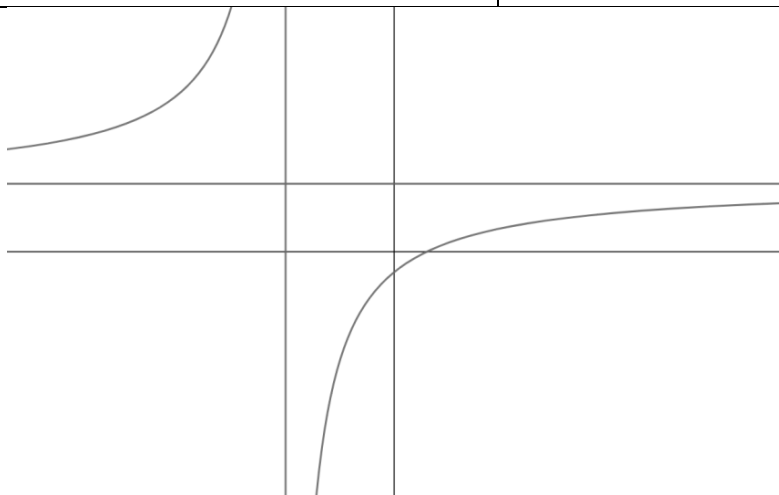


	M1	Sets their equation for the normal equal to the curve, makes a correct rearrangement to remove any denominator and forms a 3TQ Note this method mark is not dependant.
	A1	Correct exact values for x and y



Question number	Scheme	Marks
10 (a)	(i) $\left(\frac{2}{5}, 0\right)$ (ii) $(0, -1)$	B1 B1 [2]
(b) (i) (ii)	$y = \frac{5}{3}$ $x = -\frac{2}{3}$	B1 B1 [2]
(c)	$x = -\frac{2}{3}$ as an equation or clearly labelled $y = \frac{5}{3}$ as an equation or clearly labelled $\left(\frac{2}{5}, 0\right)$ written as coordinates $(0, -1)$ or clearly labelled	B1 B1ft B1ft [3]
(d)	$\left\{ y = \frac{1}{4}x + \frac{7}{4} \right\} \Rightarrow \frac{1}{4}$ $\left\{ \frac{dy}{dx} = \frac{5(3x+2) - 3(5x-2)}{(3x+2)^2} \right\} = \frac{16}{(3x+2)^2}$ $\frac{1}{4} = \frac{16}{(3x+2)^2} \Rightarrow x = \dots$ $x = 2$ and $y = \frac{5 \times 2 - 2}{3 \times 2 + 2} = 1$ $y - 1 = -4(x - 2)$ $(0, 9)$ and $\left(\frac{9}{4}, 0\right)$ $\{DE\} = \sqrt{9^2 + \left(\frac{9}{4}\right)^2} = \frac{9\sqrt{17}}{4}$	B1 M1 dM1A1 M1 A1 M1 dM1 A1ft M1A1 [11]
Total 18 marks		

Part	Mark	Additional Guidance
(a)(i)	B1	$\left(\frac{2}{5}, 0\right)$ Accept $x = \frac{2}{5}, \{y = 0\}$
(a)(ii)	B1	$(0, -1)$ Accept $\{x = 0\}, y = -1$, if not labelled, we score by their order
(b)(i)	B1	Correct equation $y = \frac{5}{3}$
(b)(ii)	B1	Correct equation $x = -\frac{2}{3}$
(c)	B1	Two branches drawn in the correct two “quadrants” created by the two asymptotes. Mark intention, allow poor curves, but do not allow the curve to bend back on itself or touch any asymptotes.
	B1ft	Two clearly labelled intersections with the axes, ft their (a), at least one section of their curve must pass through one of these intersections.
	B1ft	Two clearly marked asymptotes, ft their (b), labelled as described, there must be at least one section of the curve present, not crossing the curve.
(d)	B1	$\frac{1}{4}$ seen any where in part (d)
	M1	Attempt to differentiate y Quotient rule: Numerator must be the difference of two terms (either way round) of the form $P(3x+2) - Q(5x-2)$, $P, Q > 0$. Denominator must be correct. Product rule: obtains form $P(3x+2)^{-1} - Q(5x-2)(3x+2)^{-2}$, $P, Q > 0$
	dM1	Quotient rule: Either term in the numerator correct Product rule: Either term correct Dependent on previous method mark.
	A1	Correct $\frac{dy}{dx}$. May be unsimplified.
	M1	Sets their $\frac{1}{4}$ equal to their $\frac{dy}{dx}$ and attempts to solve for x
	A1	$x = 2$ and $y = 1$
	M1	Finds the gradient of the normal using $-\frac{1}{\text{their } \frac{1}{4}}$
	dM1	A complete and correct method to find the equation of the normal. If using $y = mx + c$, c must be found. Dependent on previous method mark.
	A1ft	$(0, 9)$ and $\left(\frac{9}{4}, 0\right)$, accept $y = 9, x = \frac{9}{4}$ Ft their normal equation.
	M1	Correct use of the formula for the length of a line segment, using their values from their points of D and E , may not come from the normal.
	A1	Correct exact length in simplest form

Question number	Scheme		Marks
10(a)	$-\frac{1}{4} = -\frac{2}{b} \rightarrow b = 8$ *		B1*cs0
Mark (i) and (ii) together	$-\frac{4}{3}a + 8 = 0$ oe or $-\frac{b}{a} = -\frac{4}{3}$ $a = 6$		M1 A1 [3]
(b)	$y = \frac{5}{\text{"6"}}$		B1ft [1]
(c)	$\left(\frac{2}{5}, 0\right)$		B1 [1]
(d)	$\frac{5(6x+8) - 6(5x-2)}{(6x+8)^2}$ $\frac{52}{(6x+8)^2}$ oe eg $\frac{52}{36x^2 + 96x + 64}$ $\frac{52}{(6x+8)^2}$ and correct conclusion	ALT using product rule $(5x-2)(6x+8)^{-1}$ $5(6x+8)^{-1} + (-1)(6)(5x-2)(6x+8)^{-2}$ $\frac{52}{(6x+8)^2}$ oe eg $\frac{52}{36x^2 + 96x + 64}$ $\frac{52}{(6x+8)^2}$ and correct conclusion	M1 dM1 A1 A1 [4]
(e)			B1 (curve) B1ft (asymptotes) B1ft (intersections with axes) [3]
(f)	$\frac{13}{100} = \frac{52}{(6x+8)^2}$ oe $(6x+8)^2 = 400$ $3x^2 + 8x - 28 = 0$ oe $x = 2, y = \frac{2}{5}$ $x = -\frac{14}{3}, y = \frac{19}{15}$ oe $\sqrt{\left(\left(\text{"2"}\right) - \left(\text{"-14/3"}\right)\right)^2 + \left(\left(\frac{2}{5}\right) - \left(\frac{19}{15}\right)\right)^2}$ 6.72		M1 dM1 A1 A1 dM1 A1 [6]
Total 18 marks			

Part	Mark	Additional Guidance
(a)	B1*cs0	For a minimum of the steps shown in the MS to find b , no errors.
(i)	M1	For correct substitution of $x = -\frac{4}{3}$ and $b = 8$ into $ax + b = 0$
(ii)	A1	Correct a .
		Students can complete this in a different order $-\frac{4}{3}a + 8 = 0 \Rightarrow a = 6$ $-\frac{b}{a} = -\frac{4}{3} \left(\text{or } -\frac{b}{6} = -\frac{4}{3} \right) \Rightarrow b = 8$ B1 M1 A1
(b)	B1ft	Follow through their value of a . $y = \frac{5}{\text{"their 6"}}$
(c)	B1	$\left(\frac{2}{5}, 0\right)$ allow $x = \frac{2}{5}$ $y = 0$ $x = \frac{2}{5}$ alone is insufficient
(d)	M1	Attempt the quotient rule. Numerator must be the difference of two terms of the form $A \times (ax + 8) - B \times (5x - 2)$ or $B \times (5x - 2) - A \times (ax + 8)$, A and $B > 1$. Denominator must be of the form $(ax + 8)^2$ $a \neq 0$ Allow follow through of their value of a (general principle of method mark).
	dM1	Either term on the numerator fully correct (either way round), dependent on previous method mark, doesn't need to be simplified. Denominator must be of the form $(ax + 8)^2$ Allow follow through of their value of a (general principle of method mark).
	A1	$\frac{52}{(6x+8)^2}$ oe eg $\frac{52}{36x^2 + 96x + 64}$ eg $\frac{13}{(3x+4)^2}$ A simplified expression (denominator expanded acceptable) Dependent on both previous method marks – question states must use calculus, so some method must be shown.
	A1	$\frac{52}{(6x+8)^2}$ and correct conclusion, for example, (the numerator is a positive number) eg the denominator is always positive eg the $(ax + 8)^2 > 0$ and therefore (the fraction) is always positive. No incorrect work for this mark eg don't accept incorrectly simplified gradient.
ALT	M1	Attempt the product rule. $C \times (ax + 8)^{-1} + (5x - 2)(-1)(D) (ax + 8)^{-2}$, C and $D > 1$. Allow follow through of their value of a (general principle of method mark).
	M1	Either term correct (either way round), dependent on previous method mark, doesn't need to be simplified
	A1 A1	As main scheme

(e)	B1	For a negative reciprocal curve drawn anywhere in the grid – there must be two branches present, they must not cross any asymptotes drawn or implied or cross each other and must not obviously ‘bend back’ on themselves. Mark intention.
	B1ft	Two clearly marked asymptotes $x = -\frac{8}{\text{their } a}$, $y = \frac{5}{\text{their } a}$, ft their value for a , labelled as equations or clearly passing through the correctly labelled numbers on the axis. There must be one section of a negative reciprocal curve present, tending towards these asymptotes. This branch must not obviously cross or bend back from any asymptotes drawn or implied Ignore any other branch/curve/line drawn if there is one branch fulfilling these conditions.
	B1ft	Single curve (or even line) passing through $\left(0, -\frac{1}{4}\right)$ and $\left(\frac{2}{5}, 0\right)$ marked clearly on the graph as coordinates or crossing points on the axis. Ignore any other branch/curve/line present. Allow follow through only of their $(c, 0)$ $c \neq 0$ from part (c)
(f)	M1	Sets their differentiated function from part (d) = $\frac{13}{100}$ Allow any function = $\frac{13}{100}$ coming from working in part (d) or re-started work to find the derivative. The function may not be the original function.
	dM1	For correctly rearranging an equation of the form $\frac{13}{100} = \frac{Q}{(cx+d)^2}$ to $(cx+d)^2 = R$ using their a , or to the form $px^2 + qx + r = 0$ $p, q, r \neq 0$ with 2 out of 3 of p, q, r correct. leading to $x =$
	A1	For $x = 2, y = \frac{2}{5}$ or $x = -\frac{14}{3}, y = \frac{19}{15}$ The pairing must be clear (may be implied in further working)
	A1	For $x = 2, y = \frac{2}{5}$ and $x = -\frac{14}{3}, y = \frac{19}{15}$ The pairing must be clear (may be implied in further working)
	dM1	For correct use of the formula to find the length of a line, using their values for x and y . Dependent on first method mark only.
	A1	For awrt 6.72

