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EDEXCEL INTERNATIONAL GCSE FURTHER PURE MATHS: QUESTIONS BY TOPIC

Series (5)

International GCSE Further Pure Mathematics Topic 5

62 questions · 561 marks · Mark schemes included

Pearson Edexcel International GCSE (9–1) Further Pure Mathematics 4PM1, Papers 1, 1R, 2 & 2R, June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions run from the lowest mark tariff to the highest, so similar questions sit together.

Click any heading or question row below to jump straight to it.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

62 questions · 561 marks · lowest tariff first



4-MARK QUESTIONS

2 questions in this booklet



- 1 The n th term of an arithmetic series is a_n where

$$a_{10} + a_{11} + a_{12} = 129 \quad \text{and} \quad a_{19} + a_{20} + a_{21} = 237$$

Find a_1

(4)



- 3 An arithmetic series has first term 16 and common difference -5

The sum to n terms of this series is S_n

Given that $S_n < -450$

find the least value of n

(4)



5-MARK QUESTIONS

5 questions in this booklet



3 (a) Expand

$$(1 + px)^{-5} \quad p \neq 0$$

in ascending powers of x , up to and including the term in x^4
Give each term in its simplest form.

(3)

The coefficient of x^r in the expansion is c_r

Given that $c_4 = 2c_3$

(b) find the value of p .

(2)

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- 4 The n th term of an arithmetic series is u_n where

$$u_n = (n + 1)\ln 4$$

Given that the sum of the first n terms of the series is S_n

show that $S_n = \ln 2^{(n^2 + an)}$ where a is an integer whose value is to be found.

(5)



- 1 An arithmetic series has 5th term 16 and 100th term 301

Find the sum of the first 50 terms of the series.

(5)



1 (a) Show that $\sum_{r=1}^n (3r + 2) = \frac{n}{2}(3n + 7)$

(3)

(b) Hence, or otherwise, evaluate $\sum_{r=10}^{40} (3r + 2)$

(2)



- 2 (a) Expand $\frac{2}{\sqrt{1+3x}}$ in ascending powers of x up to and including the term in x^3
Express each coefficient as a fraction in its simplest terms where appropriate.

(4)

- (b) State the range of values of x for which the expansion is valid.

(1)

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6-MARK QUESTIONS

5 questions in this booklet



1 The n th term of an arithmetic series A is a_n

The n th term of a geometric series G is t_n

For these two series

$$a_1 = t_1 \quad a_{10} = t_3 = 48 \quad a_{10} = 4t_2$$

Find

(i) the common ratio of G ,

(ii) the common difference of A .

(6)

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- 3 A geometric series has first term a and common ratio r , where $r > 0$

Given that the 3rd term of the series is 5 and that the 5th term of the series is $\frac{5}{2}$

(a) find

(i) the exact value of r

(ii) the value of a

(4)

(b) Find the sum to infinity of this series.

Give your answer in the form $p + q\sqrt{2}$ where p and q are integers.

(2)

- 1 (a) Expand $\left(1 + \frac{x}{4}\right)^8$ in ascending powers of x up to and including the term in x^3

Give each coefficient in its simplest terms.

(3)

- (b) Use your expansion with a suitable value of x to obtain an approximation, to 4 decimal places, of $(1.035)^8$

(3)

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- 2 The n th term of a convergent geometric series is $8^{(1-2n)}$

Find the sum to infinity of the series.

Give your answer in the form $\frac{p}{q}$ where p and q are integers to be found.

(6)



- 4 A geometric series G has first term a and common ratio r

The third term of G is 10 and the seventh term of G is 40

Given that the second term of G is negative,

(a) find

(i) the exact value of r

(ii) the value of a

(5)

(b) Explain why G does not have a sum to infinity.

(1)

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7-MARK QUESTIONS

7 questions in this booklet



- 3 The n th term of an arithmetic series is u_n such that

$$u_n = \ln a + (n - 1) \ln b$$

where a and b are positive integers.

Given that $u_2 = \ln 12$ and that $u_5 = \ln 768$

find the value of a and the value of b .

(7)

5 The n th term of a geometric series with common ratio r is u_n

Given that $u_2 + u_4 = 212.5$ and that $u_3 + u_4 = 62.5$

(a) find the two possible values of r .

(5)

Given that the series is convergent with sum to infinity S ,

(b) find the exact value of S .

(2)

- 3 A geometric series G has first term a and common ratio r

The 2nd term of G is $\frac{5}{16}$ and the 5th term of G is $\frac{135}{1024}$

(a) Find

- (i) the value of r
- (ii) the value of a

(5)

Given that G is convergent with sum to infinity S

(b) find the exact value of S

(2)

- 4 The common ratio of a geometric series G is positive.

The sum of the first 4 terms of G is 80

The sum to infinity of G is 81

Show that the sum of the first 7 terms of G differs from the sum to infinity of G by $\frac{1}{27}$

(7)

- 6 Given that $(8 + 3x)^{\frac{1}{3}}$ can be expressed in the form $p(1 + qx)^{\frac{1}{3}}$ where p and q are constants,

(a) find the value of p and the value of q

(2)

(b) Hence, expand $(8 + 3x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2 , expressing each coefficient as an exact fraction in its lowest terms.

(3)

Using the expansion found in part (b) with a suitable value of x

(c) show that $\sqrt[3]{9} \approx \frac{599}{288}$

(2)

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- 7 (a) Expand $(1 + 2x^2)^{-\frac{3}{4}}$ in ascending powers of x up to and including the term in x^6

Express each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{(2 + kx)^{\frac{3}{4}}}{(1 + 2x^2)^{\frac{3}{4}}} \quad \text{where } k \neq 0$$

- (b) Obtain a series expansion for $f(x)$ in ascending powers of x up to and including the term in x^5

Give each coefficient in terms of k where appropriate.

(2)

The coefficient of the term in x^5 is fourteen times the coefficient of the term in x^2

- (c) Find the value of k

(2)

- 2 An arithmetic series F has third term 8 and fifth term 20

Given that the sum to n terms of F is greater than 220

find the least value of n

(7)

8-MARK QUESTIONS

10 questions in this booklet



6 Given that $\sqrt{9-x}$ can be expressed in the form $p(1+qx)^{\frac{1}{2}}$ where p and q are constants

(a) find the value of p and the value of q .

(2)

(b) Hence expand $\sqrt{9-x}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms.

(3)

Using the expansion you found in part (b) with a suitable value of x ,

(c) find an estimate to 5 decimal places for the value of $\sqrt{\frac{31}{4}}$

(3)

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- 9 (a) Expand $\sqrt{1-4x}$ in ascending powers of x up to and including the term in x^3 , giving each coefficient as an integer.

(3)

- (b) Use your expansion with a suitable value for x to obtain an estimate of $\sqrt{0.76}$. Give your answer correct to 4 decimal places.

(3)

- (c) Hence find, to 3 decimal places, an estimate of $\sqrt{19}$

(2)

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- 5 (a) Expand $\sqrt{1-x}$ in ascending powers of x up to and including the term in x^3
Give each coefficient as an exact fraction in its lowest terms.

(3)

- (b) Using your expansion with a suitable value of x , obtain an approximation,
to 6 decimal places, of $\sqrt{0.92}$

(3)

- (c) Hence find an approximation, to 5 decimal places, of $\sqrt{23}$

(2)

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5 (a) Show that $\sum_{r=1}^n (3r + 5) = \frac{1}{2}n(3n + 13)$

(3)

(b) Hence evaluate $\sum_{r=35}^{50} (3r + 5)$

(2)

Given that $\sum_{r=1}^n (3r + 5) = 385$

(c) find the value of n .

(3)

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2 The n th term of an arithmetic series is u_n

Given that $u_5 = 46$ and that $u_{20} = 181$

(a) find

(i) the common difference of this series,

(ii) the first term of this series.

(4)

(b) Evaluate $\sum_{n=21}^{50} u_n$

(4)

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$$f(x) = (1 + ax)^n \text{ where } n > 2$$

- (a) Write down the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3
Give each coefficient in terms of n and a

(2)

In the expansion of $f(x)$, the coefficient of x is 8 and the coefficient of x^2 is 30

- (b) Find the value of n and the value of a

(4)

- (c) Calculate the coefficient of x^3 in $f(x)$

(2)



- 6 The n th term of a geometric series G is U_n

The first three terms of G are given by

$$U_1 = q(4p + 1) \quad U_2 = q(2p + 3) \quad U_3 = q(2p - 3)$$

- (a) Find the possible values of p

(5)

Given that G is convergent with sum to infinity 250

- (b) find the value of q

(3)

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3 (a) Show that $\sum_{r=1}^n (5r - 3) = \frac{n}{2}(5n - 1)$ (3)

(b) Hence, or otherwise, evaluate $\sum_{r=31}^{60} (5r - 3)$ (2)

Given that $\sum_{r=1}^n (5r - 3) = 3783$

(c) find the value of n (3)

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2 Given that

$$1 - \frac{1}{3}x + \frac{5}{36}x^2 + \dots$$

is the binomial expansion, in ascending powers of x , of $(1 + Ax)^n$

where A and n are rational numbers,

(a) find the value of A and the value of n

(6)

(b) Hence find the value of the coefficient of x^3

Give your answer in the form $-\frac{p}{q}$ where p is a prime number and q is an integer.

(2)

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- 6 (a) Show that $\sqrt{4-x} = A\left(1 - \frac{x}{B}\right)^{\frac{1}{2}}$ where A and B are integers to be found. (2)
- (b) Hence expand $\sqrt{4-x}$ in ascending powers of x up to and including the term in x^3 .
Give each coefficient as an exact fraction in its lowest terms. (3)
- (c) Use your expansion with a suitable value of x to obtain an estimate of $\frac{\sqrt{305}}{10}$.
Give your answer correct to 5 decimal places. (3)

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9-MARK QUESTIONS

4 questions in this booklet



7 The sum of the first n terms of an arithmetic series is A_n where

$$A_n = \sum_{r=1}^n (4r + 5)$$

(a) For this arithmetic series, find

- (i) the first term,
- (ii) the common difference.

(2)

The sum of the first n terms of a geometric series is G_n where

$$G_n = \sum_{r=1}^n 4(3)^{r-1}$$

(b) For this geometric series, find

- (i) the first term,
- (ii) the common ratio.

(2)

(c) Find the value of n for which $A_{14} - 6 = G_n$

(5)

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- 1 The n th term of an arithmetic series is t_n and the common difference of the series is d .

Given that $t_2 + t_9 = 0$ and that $t_4 + t_6 + t_{10} = 14$

(a) (i) show that $d = 4$

(ii) find the first term of this series.

(4)

A different arithmetic series A has first term 24 and common difference 6

For series A , the sum of the first $2n$ terms is 3 times the sum of the first n terms.

(b) Find the value of n .

(5)

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- 9 (a) Write $\frac{3}{(3-x)^3}$ in the form $a(1-bx)^{-3}$
where a and b are fractions in their lowest terms. (2)
- (b) Expand $\frac{3}{(3-x)^3}$ in ascending powers of x up to and including the term in x^3
Express each coefficient as a fraction in its lowest terms. (3)
- (c) (i) Use a suitable value of x with your expansion in part (b), to obtain an
approximation for $\frac{24}{125}$ to 5 decimal places.
- (ii) Find the percentage error, to 2 decimal places, of your approximation from the
actual value. (4)

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7 A geometric series G with common ratio r , has first term 16 and third term $\frac{2704}{625}$

(a) Find the two possible values of r

(2)

Given that $r > 0$

(b) find the sum to infinity of G

(2)

The sum to n terms of G is greater than 33

(c) Find, using logarithms, the least possible value of n
Show your working clearly.

(5)

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10-MARK QUESTIONS

8 questions in this booklet



6 (a) Show that $\sum_{r=1}^n (4r - 3) = n(2n - 1)$

(3)

(b) Hence, or otherwise, find the least value of n such that $\sum_{r=1}^n (4r - 3) > 1000$

(3)

Given that $S_n = n(2n - 1)$, $t_n = (4n - 3)$ and that $18 + 3t_{n+7} = S_{n+4}$

(c) find the value of n .

(4)

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9 Given that $\frac{1}{(2-x)^3}$ can be written as $p(1-qx)^{-3}$

(a) find the value of p and the value of q .

(2)

(b) Expand $\frac{1}{(2-x)^3}$ in ascending powers of x up to and including the term in x^3 and express each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{a+bx}{(2-x)^3} \text{ where } a \text{ and } b \text{ are integers}$$

The first three terms of the expansion of $f(x)$ are $\frac{3}{8} - \frac{43}{16}x + cx^2$

(c) Find the value of a and the value of b .

(3)

(d) Find the exact value of c .

(2)

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- 6 An arithmetic series A has first term a and common difference d .

The sum S_n of the first n terms of A is given by $S_n = n(15 + 2n)$

- (a) Find the value of a and the value of d .

(4)

- (b) Find the 20th term of A .

(2)

Given that $S_{2p} - 2S_p = 1 + S_{(p-1)}$

- (c) find the value of p .

(4)

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- 5 (a) Expand $(1 + ax)^n$ in ascending powers of x up to and including the term in x^3

Express each coefficient of x in terms of a and n where a and n are constants and $n > 2$

(2)

The coefficient of x is 15 and the coefficient of x^2 is equal to the coefficient of x^3

- (b) Find the value of a and the value of n .

(6)

- (c) Find the coefficient of x^3

(2)

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7 A geometric series G has first term a and common ratio r

The sum of the first three terms of G is $\frac{61}{6}$

The sum to infinity of G is $\frac{125}{6}$

(a) (i) Show that $r = \frac{4}{5}$

(ii) Find the value of a

(6)

The sum of the first n terms of G is S_n

Given that $S_n > 19.8$

(b) show that $n \lg\left(\frac{4}{5}\right) < \lg\left(\frac{31}{625}\right)$

(2)

(c) Hence find the least value of n

(2)

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5 A geometric series G has first term 12 and common ratio $\frac{3}{8}$

(a) Find the sum to infinity of G

(2)

(b) Show that the 6th term of G can be written as $\frac{3^6}{2^{13}}$

(3)

The n th term of G is u_n

(c) By finding an expression for u_n in terms of n , show that

$$\log_2 u_n = n \log_2 3 - 3n + 5$$

(5)

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- 8 The n th term of a geometric series G is U_n and the sum of the first n terms of G is S_n

Given that $U_n = \frac{25}{4} \left(\frac{3}{5} \right)^n$

- (a) find the exact value of U_5

(1)

- (b) Show that $S_n = \sum_{r=1}^n \frac{A}{B} \left(\frac{3}{5} \right)^{r-1}$ where A and B are integers to be found.

(3)

The sum to infinity of G is S

- (c) Find the least value of n such that $S - S_n < 0.045$

(6)



8 The sum to n terms of an arithmetic series A is S_n

The sum of the first four terms of A is 42 and the fifth term of A is 23

(a) Show that $S_n = \sum_{r=1}^n (Pr - Q)$ where P and Q are prime numbers.

(6)

$S_{2n} - 3U_n = 1062$ where U_n is the n th term of A

(b) Find the value of n

(4)



11-MARK QUESTIONS

9 questions in this booklet



7 The n th term of a geometric series G is u_n

The first term of G is a and the common ratio of G is r , where $r > 0$

Given that $u_3 = 4$ and that $u_7 = 16$

(a) (i) show that $r = \sqrt{2}$

(ii) find the value of a .

(3)

(b) Find the least value of n for which $u_n > 500$

(4)

The sum of the first n terms of G is S_n

(c) Find S_{20}

Give your answer in the form $p(1 + \sqrt{2})$ where p is an integer.

(4)

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7 An arithmetic series P has first term a , common difference d and n th term u_n

Given that $u_5 = 4x + 6$ and that $u_8 = 7x + 3$

(a) (i) show that $d = x - 1$

(ii) find the value of a

(4)

Given further that $u_9 = 42$

(b) find the value of x

(2)

The sum of the first n terms of P is S_n

(c) Find the value of n for which $S_{(n+1)} = 12u_n + 18$

(5)

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9 (a) Show that $\sum_{r=1}^n (5r - 1) = \frac{n}{2} (3 + 5n)$ (3)

(b) Hence, or otherwise, evaluate $\sum_{r=10}^{20} (5r - 1)$ (3)

The sum of the first n terms of an arithmetic series is S_n where $S_n = \sum_{r=1}^n (5r - 1)$

The r th term of this series is u_r

Given that $S_n = 12u_{n+1} + 52$

(c) find the value of n (5)

- 9 (a) Expand $\frac{1}{\sqrt{1-2x}}$ in ascending powers of x up to and including the term in x^3

Simplify each term as far as possible.

(3)

- (b) Show that $\frac{1}{\sqrt{0.96}} = \frac{5\sqrt{6}}{12}$

Show all your working clearly.

(2)

- (c) Express $\frac{1}{5\sqrt{6}-12}$ in the form $\frac{a\sqrt{6}}{c} + b$ where a , b and c are integers.

Show all your working clearly.

(2)

Using the expansion you found in part (a) with a suitable value of x

- (d) find an estimate, to 5 decimal places, of $\frac{9}{5\sqrt{6}-12}$

(4)

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- 5 Given that $(2 + 3x)^{-1}$ can be expressed in the form $p(1 + qx)^{-1}$ where p and q are constants,

(a) find the value of p and the value of q

(2)

(b) Hence expand $(2 + 3x)^{-1}$ in ascending powers of x up to and including the term in x^3 , expressing each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{1 + x}{2 + 3x}$$

(c) Obtain a series expansion for $f(x)$, in ascending powers of x up to and including the term in x^3 , expressing each coefficient as an exact fraction in its lowest terms.

(2)

(d) Hence use algebraic integration to obtain an estimate, to 4 decimal places, of

$$\int_0^{0.5} f(x) \, dx$$

(4)

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- 7 (a) Expand $\left(1 + \frac{x}{3}\right)^{-3}$ in ascending powers of x up to and including the term in x^3

Where appropriate express each coefficient as an exact fraction in its lowest terms.

(3)

- (b) Write down the range of values of x for which your expression is valid.

(1)

- (c) Express $(3 + x)^{-3}$ in the form $P(1 + Qx)^{-3}$ where P and Q are rational numbers whose values should be stated.

(2)

$$f(x) = \frac{(1 + 4x)}{(3 + x)^3}$$

- (d) Obtain a series expansion for $f(x)$ in ascending powers of x up to and including the term in x^2

(2)

- (e) Hence, using algebraic integration, obtain an estimate of $\int_0^{0.2} f(x) \, dx$

Give your answer to 5 significant figures.

(3)

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- 9 (a) Expand $(1-8x^2)^{-\frac{1}{2}}$ in ascending powers of x , up to and including the term in x^6 giving each coefficient as an integer.

(3)

$$g(x) = \frac{a+bx}{\sqrt{1-8x^2}} \quad \text{where } a \text{ and } b \text{ are prime numbers}$$

Given that the fourth and fifth terms, in ascending powers of x , in the series expansion of $g(x)$ are $20x^3$ and $48x^4$ respectively,

- (b) find the value of a and the value of b

(4)

Using the first five terms, in ascending powers of x , in the series expansion of $g(x)$

- (c) obtain an estimate, to 4 significant figures, of $\int_0^{0.2} g(x) \, dx$

(4)

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5 The sum of the first 10 terms of an arithmetic series A is $36k + 1$ where k is a constant.

The 6th term of A is $4k + 1$

(a) (i) Find an expression in terms of k for the common difference of A

(ii) Show that the first term of A is -8

(5)

Given that the 4th term of A is 7

(b) show that $k = 4$

(2)

The sum of the first n terms of A is S_n and the n th term of A is U_n

(c) Find the value of n such that $S_n = 5U_{n+10} + 105$

(4)

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4 (i) Find the value of $\sum_{k=6}^{17} 3(2)^{k-1}$

(4)

(ii) The common ratio of a geometric series G is positive.

The third term of G is $\frac{7}{13}$ and the ninth term of G is $\frac{448}{9477}$

G is convergent with sum to infinity S

Find the exact value of S

(7)

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12-MARK QUESTIONS

5 questions in this booklet



7 The 7th term of a geometric series is 192 and the 8th term of this geometric series is 1152

(a) Find, as a fraction in its simplest form, the 4th term of this geometric series.

(3)

A different geometric series G has a common ratio r and n th term t_n

Given that $t_3 = 24$ and $t_2 + t_3 + t_4 = -36$

(b) show that r satisfies the equation

$$2r^2 + 5r + 2 = 0$$

(5)

Given further that G is convergent with sum to infinity S ,

(c) find the value of S .

(4)

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7 A geometric series has first term $(x - 3)$, second term $(x + 1)$ and third term $(4x - 2)$.

(a) Find the two possible values of x .

(5)

Given that $x < 1$,

(b) show that the series is convergent.

(2)

The sum to infinity of the series is S .

(c) Find the value of S .

(2)

The sum of the first n terms of the series is S_n

Given that $\frac{S}{S_n} = \frac{256}{255}$

(d) find the value of n .

(3)

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- 9 A geometric series G has common ratio r where $r > 0$

The third term of G is $\frac{27}{2}$ and the sum of the first three terms of G is $\frac{57}{2}$

Given that the sum to n terms of G is S_n

(a) show that $S_n = \sum_{j=1}^n 4\left(\frac{3}{2}\right)^j$

(8)

Given that $S_k > 50\,000$

- (b) show that the least value of k is given by

$$k > \frac{\lg\left(\frac{12503}{3}\right)}{\lg\left(\frac{3}{2}\right)}$$

(3)

- (c) Hence find the least value of k

(1)

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8 The sum of the first and second terms of a geometric series G is 400

The sum of the second and third terms of G is 100

(a) Show that the common ratio of G is $\frac{1}{4}$ (4)

(b) Show that the first term of G is 320 (2)

(c) Find the sum to infinity of G (2)

The sum to n terms of G is S_n

(d) Find, using logarithms, the least value of n such that

$$S_n > 426.6$$
 (4)

- 6 (a) Show that

$$\frac{a}{\sqrt{4+bx}} = \frac{a}{2} \left(1 + \frac{bx}{4} \right)^{\frac{1}{2}} \text{ where } a \text{ and } b \text{ are positive integers.} \quad (2)$$

The expansion of $\frac{a}{\sqrt{4+bx}}$ in ascending powers of x can be written as

$$P + Qx + Rx^2 + Sx^3$$

where P , Q , R and S are rational numbers.

- (b) Show that $Q = -\frac{ab}{16}$ and $S = -\frac{5ab^3}{2048}$
and find P and R in terms of a and b , as fractions in their lowest terms. (4)

Given that $Q = \frac{128}{5}S$ and $R = \frac{9}{256}$

- (c) show that $a = 3$ and $b = 1$ (3)

- (d) Hence, using an appropriate value of x , find, to 3 decimal places, an approximate value for $\frac{\sqrt{6}}{2}$ (3)

13-MARK QUESTIONS

4 questions in this booklet



7 The geometric series G has first term a , common ratio r and n th term u_n

Given that $u_4 = e^{x+2}$ and that $u_7 = e^{\frac{2x+1}{2}}$

(a) show that $r = e^{-\frac{1}{2}}$

(3)

(b) Hence find a in terms of e and x .

(3)

Given that the sum to infinity of G can be written as $\frac{e^p}{e^{\frac{1}{2}} - 1}$

(c) find an expression for p in terms of x .

(3)

Given that $u_{18} > 1.6$ and that x is an integer,

(d) find the least value of x .

(4)

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10 (a) Show that $\frac{3}{\sqrt{9-3x}} = \left(1 - \frac{x}{3}\right)^{-\frac{1}{2}}$ (2)

(b) Hence expand $\frac{3}{\sqrt{9-3x}}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms. (3)

$$f(x) = \frac{1+2x}{\sqrt{9-3x}}$$

(c) Find the expansion of $3f(x)$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms. (4)

(d) Hence, using algebraic integration, obtain an approximation to 6 significant figures for

$$\int_{0.1}^{0.2} \frac{1+2x}{\sqrt{9-3x}} dx$$
 (4)

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6 The sum of the first n terms of an arithmetic series is S_n where $S_n = n(3 + 2n)$

(a) Find the value of S_{20}

(2)

Given that $S_n = \sum_{r=1}^n (Ar + B)$

(b) find the value of A and the value of B

(6)

A different arithmetic series has first term 7 and common difference 4

The sum of the first n terms of this series is T_n

(c) Use algebra to find the value of n for which $T_n = S_n + 252$

(5)

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- 2 The sum of the fifth, sixth and seventh terms of an arithmetic series A is nine times the sum of the first and second terms.
The third term of A is 12

(a) Find the first term and common difference of A

(5)

The n th term of A is u_n

(b) Find the value of $\sum_{r=15}^{60} u_r$

(4)

The sum to n terms of A is S_n

Given that $2S_n - 5u_n = 10$

(c) find the value of n

(4)

14-MARK QUESTIONS

3 questions in this booklet



10 (a) Expand $(1 + 2x^2)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^6 , expressing each coefficient as an exact fraction in its lowest terms. (3)

(b) State the range of values of x for which your expansion is valid. (1)

$$f(x) = \frac{2 + kx^2}{(1 + 2x^2)^{\frac{1}{3}}} \quad \text{where } k \neq 0$$

(c) Obtain a series expansion for $f(x)$ in ascending powers of x up to and including the term in x^6 . Give each coefficient in terms of k where appropriate. (3)

Given that the coefficient of x^4 in the series expansion of $f(x)$ is zero

(d) find the value of k . (2)

(e) Hence use algebraic integration to obtain an estimate, to 4 decimal places, of

$$\int_0^{0.5} f(x) dx \quad (5)$$

- 9 (a) Expand $(1 + 2x)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as a fraction in its lowest terms.

(3)

- (b) Find the range of values of x for which your expansion is valid.

(1)

$$f(x) = \frac{2 + kx^2}{(1 + 2x)^{\frac{1}{3}}}$$

- (c) Obtain a series expansion of $f(x)$ in ascending powers of x up to and including the term in x^3

Give your coefficients in terms of k where appropriate.

(3)

The coefficient of x^3 in the series expansion of $f(x)$ is $-\frac{8}{3}$

- (d) Find the exact value of k

(2)

- (e) Hence, using algebraic integration, estimate the value of

$$\int_{0.1}^{0.2} f(x) \, dx$$

Give your answer to 4 decimal places.

(5)

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- 8 The sum of the first 2 terms of a geometric series G is 360
The sum of the 2nd and 3rd terms of G is 288

The n th term of G is U_n

(a) Show that $U_n = A\left(\frac{4}{5}\right)^{n-1}$ where A is an integer to be found. (7)

(b) Explain why G is convergent. (1)

(c) Hence find the sum to infinity of G (2)

(d) Find the least number of terms for which the sum is greater than 978 (4)

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MARK SCHEMES

official Pearson Edexcel mark schemes



Question number	Scheme	Marks
1	$a + 9d + a + 10d + a + 11d$ or $3a + 30d = 129$ oe $a + 18d + a + 19d + a + 20d = 237$ or $3a + 57d = 237$ oe $27d = 108$ oe ($d = 4$) $(a_1 =)3$	M1 M1 M1 A1 (4)
ALT	$a_{10} + a_{10} + d + a_{10} + 2d$ or $3a_{10} + 3d = 129$ oe $a_{10} + 9d + a_{10} + 10d + a_{10} + 11d = 237$ or $3a_{10} + 30d = 237$ oe $27d = 108$ oe ($d = 4$) $(a_1 =)3$	M1 M1 M1 A1
Total 4 marks		

Mark	Additional Guidance
M1	For fully correct use of $a + (n-1)d$ in an equation for the 10 th , 11 th and 12 th terms OR the 19 th , 20 th and 21 st terms simplified or unsimplified oe
M1	For fully correct use of $a + (n-1)d$ in an equation for the 10 th , 11 th and 12 th terms AND the 19 th , 20 th and 21 st terms simplified or unsimplified oe
M1	For an attempt to solve their equations simultaneously, allow one processing error. Must eliminate one variable and arrive at a value for a or d .
A1	For $(a / a_1 =)3$
	Allow mixed use of a or a_1 throughout

Question	Scheme	Marks
3	$S_n < -450 \Rightarrow \frac{n}{2}(2 \times 16 + [n-1](-5)) < -450$ $\Rightarrow 37n - 5n^2 < -900 \Rightarrow 5n^2 - 37n - 900 > 0$ $n = \frac{-(-37) \pm \sqrt{(-37)^2 - 4 \times 5 \times (-900)}}{2 \times 5} \Rightarrow n = 17.617.. \text{ so } n = 18$	M1A1 M1A1 [4]
Total 4 marks		

Question	Notes	Marks
3	Uses the correct summation formula and sets <, > or = to -450 $S_n < -450 \Rightarrow \frac{n}{2}(2 \times 16 + [n-1](-5)) < -450$	M1
	Forms a correct 3TQ with their expression $37n - 5n^2 < -900 \Rightarrow 5n^2 - 37n - 900 > 0$ Accept <, > or = 0 and accept terms in any order.	A1
	Attempts to solve their 3TQ using a valid method. [See General Guidance] $n = \frac{-(-37) \pm \sqrt{(-37)^2 - 4 \times 5 \times (-900)}}{2 \times 5} \Rightarrow n = ...$	M1
	$n = 17.617...$ so $n = 18$ [Other root is -10.217....]	A1
Total 4 marks		

Question Number	Scheme	Marks
3(a)	$(1+px)^{-5} = 1 + (-5)(px) + \frac{(-5)(-6)(px)^2}{2!} + \frac{(-5)(-6)(-7)(px)^3}{3!}$ $+ \frac{(-5)(-6)(-7)(-8)(px)^4}{4!} + \dots$ $= 1 - 5px + 15p^2x^2 - 35p^3x^3 + 70p^4x^4 + \dots$	M1 A1A1 (3)
(b)	$70p^4 + 2 \times 35p^3 = 0$ $p = -1$	M1 A1 (2) [5]
(a) M1	Attempt the binomial expansion up to and including the term in x^4 . Must start with 1 and (px) must appear in at least one term. Ignore terms beyond x^4 . 2! or 2, 3! or 6, 4! or 24 accepted.	
A1	Any 2 correct algebraic terms, simplified (1 is not algebraic) Numbers must be simplified but $(px)^n$, $n = 2, 3, 4$ allowed	
A1	Fully correct simplified expansion as shown but allow terms such as $+(-5px)$ etc	
(b) M1	Use their coefficients and the given equation to form an equation in p (If powers of x included give M0)	
A1	Correct value of p $p = -1$ only Must have come from correct working	

Question number	Scheme	Marks
4	$u_1 = (1+1)\ln 4 = 2\ln 4$ and $d = \ln 4$	M1
	$S_n = \frac{n}{2}(2 \times '2\ln 4' + (n-1)' \ln 4')$ or $S_n = \frac{n}{2}('2\ln 4' + (n+1)' \ln 4')$	M1
	$\ln 4$ to either $2\ln 2$ or $\ln 2^2$ at any stage.	M1
	$S_n = \frac{n}{2}(2n+6)\ln 2$	
	or	
	$S_n = \frac{n}{2}(n+3)\ln 4$	
	or	M1
	$S_n = \frac{n}{2}(\ln 2^6 + \ln 2^{2n})$	
	or	
	$S_n = \frac{n}{2}(\ln 4^3 + \ln 4^n)$	
	$S_n = \ln 2^{n^2+3n}$	A1 cso
Total 5 marks		

Note: You may see the use of $\ln 4 \sum_1^n (r+1)$

Solution

$$\begin{aligned}
 S_n &= \ln 4 \sum_1^n (r+1) \\
 &= \ln 4 \left(\sum_1^n r + \sum_1^n 1 \right) \\
 &= \ln 4 \left(\frac{n}{2}(n+1) + n \right) \\
 &= 2\ln 2 \left(\frac{n^2}{2} + \frac{3n}{2} \right) \\
 &= (n^2 + 3n)\ln 2 \\
 S_n &= \ln 2^{n^2+3n}
 \end{aligned}$$

If this is a full and correct solution (no errors) as shown – award full marks – otherwise, please send to Review.



Mark	Notes
M1	Finds the first term and the common difference. $u_1 = (1+1)\ln 4 = 2\ln 4 \text{ and } d = \ln 4$ Both must be correct for this mark.
The general principle of marking this question is as follows: Note: Their a and d must be in terms of $\ln 4$ or $2\ln 2$ - Second M1 for a correct substitution of their a and their d into either $\frac{n}{2}(2a + (n-1)d)$ or $\frac{n}{2}(a + L)$ - Third M1 is for dealing correctly with all terms in $\ln 4$ at any stage ($\ln 4 = 2\ln 2$ or $\ln 2^2$) seen anywhere in the solution. - Fourth M1 for attempting to simplify the sum to the required form using their a and d - Final mark is for obtaining the given answer with no errors seen.	
M1	Uses either form of the summation formula for an arithmetic series with their a and d provided both are in terms of $\ln 4$ or $2\ln 2$ There must be no errors in the use of and substitution of their values into the formula for this question – it is given on page 2. $S_n = \frac{n}{2}(2 \times '2\ln 4' + (n-1)' \ln 4')$ or $S_n = \frac{n}{2}('2\ln 4' + (n+1)' \ln 4')$
M1	For correctly changing all terms in $\ln 4$ to either $2\ln 2$ or $\ln 2^2$ at any stage. You may see this step at the end of the solution.
M1	Simplifies their expression in either $\ln 2$ or $\ln 4$ to obtain one of the following. $S_n = \frac{n}{2}(2n+6)\ln 2$ or $S_n = \frac{n}{2}(n+3)\ln 4$ or $S_n = \frac{n}{2}(\ln 2^6 + \ln 2^{2n})$ or $S_n = \frac{n}{2}(\ln 4^3 + \ln 4^n)$
A1cso*	For obtaining the given answer in full with no errors. $S_n = \ln 2^{n^2+3n}$

Question number	Scheme	Marks
1	Method 1 $16 = a + (5 - 1)d \quad (= a + 4d) \quad \text{or}$ $301 = a + (100 - 1)d \quad (= a + 99d)$ $95d = 285$ $d = 3 \quad a = 4$ Method 2 $301 - 16 = 295 \Rightarrow 295 = (100 - 5)d$ $\Rightarrow d = \frac{285}{95} = (3)$ $d = 3 \quad a = 4$ Sum to 50 terms <u>Uses</u> $S_n = \frac{n}{2}(2a + (n - 1)d)$ $S_{50} = \frac{50}{2}[2 \times "4" + "3"(50 - 1)] = 3875$ ALT <u>Uses</u> $S_n = \frac{n}{2}(a + l)$ $50\text{th term} = "4" + 49 \times "3" (=151) \quad \text{and} \quad S_{50} = \frac{50}{2}("4" + "151") = 3875$	M1 M1 A1 [M1 M1 A1] M1A1 [5] [M1A1]
Total 5 marks		

Mark	Additional Guidance
Method 1	
M1	At least, one fully correct equation.
M1	Solves their linear equations simultaneously, allow one arithmetical or processing slip to eliminate a (or possibly d).
A1	Both a and d correct.
Method 2	
M1	Uses the difference of the two terms $(301 - 16)$ together with the difference in position $(100 - 5)$ and equates to form the equation $285 = 95d$ This may be implied by their working.
M1	Solves the equation, allow one arithmetical or processing slip to find a value for d
A1	Both a and d correct.
NOTE: If you see the correct values of a and d that come from a method without errors, award M1M1A1	
Summation	
M1	Substitution of their values for a and d into the correct summation formula. OR They must use the formula for the 50th term correctly using their values and use the correct formula for first plus last with their values.
A1	For 3875

Question	Scheme	Marks
1(a)	$d = 3$ and $a = 5$ $\sum_{r=1}^n (3r + 2) = \frac{n}{2} (2 \times 5 + (n-1)3) = \frac{n}{2} (3n + 7) *$ ALT $\sum_{r=1}^n (3r + 2) = 3 \sum_{r=1}^n r + 2 \sum_{r=1}^n 1$ $\sum_{r=1}^n (3r + 2) = \frac{3}{2} n(n+1) + 2n = \frac{3n^2 + 3n}{2} + \frac{4n}{2} = \frac{3n^2 + 7n}{2} = \frac{n}{2} (3n + 7)$	B1 M1A1 CSO [3] [B1 M1A1]
(b)	$\sum_{r=10}^{40} (3r + 2) = \frac{40}{2} (3 \times 40 + 7) - \frac{9}{2} (3 \times 9 + 7) = [2540 - 153] = 2387$	M1A1 [2]
Total 5 marks		

Note: Part (b) can be answered simply by entering the expression in the question into a permissible calculator. A solution seen with no working is M0.



Part	Mark	Notes
(a)	B1	For the correct value of a and d
	M1	For using the correct summation formula with their values of a and d
	A1	For achieving $\sum_{r=1}^n (3r+2) = \frac{n}{2}(3n+7)$ with no errors
	ALT 1 – Uses first plus last formula	
	B1	For the correct value of $a = 5$ and the final value $l = 3n + 2$
	M1	For the correct use of the first plus last summation formula $\sum_{r=1}^n (3r+2) = \frac{n}{2}(5 + [3n+2]) = \frac{n}{2}(3n+7)$
	A1	For achieving $\sum_{r=1}^n (3r+2) = \frac{n}{2}(3n+7)$ with no errors
	ALT 2 – Uses standard results	
	B1	For stating $\sum_{r=1}^n (3r+2) = 3\sum_{r=1}^n r + 2\sum_{r=1}^n 1$
	M1	For using the standard expressions $\sum_{r=1}^n (3r+2) = \frac{3}{2}n(n+1) + 2n$
	A1	For simplifying to the required form. $\sum_{r=1}^n (3r+2) = \frac{3n^2+3n}{2} + \frac{4n}{2} = \frac{3n^2+7n}{2} = \frac{n}{2}(3n+7) \quad *$
(b)	M1	Uses the given formula with $n = 40$ and with $n = 9$ (allow $n = 10$ for this mark) [Note: use of $n = 10$ will give a value of 2355] Allow alternative correct methods
	A1	For the correct sum of 2387
	ALT 1 – Uses first + last summation formula	
	M1	$U_{10} = 32, \quad U_{40} = 122, \quad n = 31$ (allow 30 for this mark) $\sum_{r=10}^{40} (3r+2) = \frac{31}{2}(32+122) = \dots$ [Note: Use of $n = 30$ will give a value of 2310]
	A1	For the correct sum of 2387
	ALT 2 – Finds new first term and uses the summation formula	
	M1	$U_{10} = 32 \quad n = 31$ (allow 30 for this mark) $\sum_{r=10}^{40} (3r+2) = \frac{31}{2}(2 \times 32 + (31-1)3) = \dots$ [Note: Use of $n = 30$ will give a value of 2265]
	A1	For the correct sum of 2387



e.g. $kx^2 - (2k + 6)x + 16 \rightarrow 2kx - 2k - 6 = 0 \rightarrow x = 1 + \frac{3}{k}$



Question number	Scheme	Marks
1 (i)/(ii)	$ar^2 = a + 9d = 48$ or $a + 9d = 4ar$ $\frac{4ar}{ar^2} = 1$ $\frac{4}{r} = 1$ $r = 4$ $a = 3$ $d = 5$ Alternative method $a_1 + 9d = t_1 r^2$ or $a_1 + 9d = 4t_1 r$ $r^2 = 4r$ $r = 4$ $t_1 r = 12$ So $t_1 = a_1 = 3$ $a_1 r + 9dr = 48r \Leftrightarrow 12 + 36d = 192 \Leftrightarrow 36d = 180$ $d = 5$	B1 M1 M1 A1 A1 [6] B1 M1 A1 A1 M1 A1 [6]
Notes		
B1	For either $ar^2 = a + 9d = 48$ or $a + 9d = 4ar$ oe	
M1	For solving simultaneously	
M1	For simplifying to $\frac{4}{r} = 1$ oe	
A1	$r = 4$	
A1	$a = 3$	
A1	$d = 5$	
Alternative		
B1	For either $a_1 + 9d = t_1 r^2$ or $a_1 + 9d = 4t_1 r$ oe	
M1	For solving simultaneously	
A1	$r = 4$	
A1	$a = 3$	
M1	For $a_1 r + 9dr = 48r \Leftrightarrow 12 + 36d = 192 \Leftrightarrow 36d = 180$	
A1	$d = 5$	

Question number	Scheme	Marks
3 (a) (i)	$ar^2 = 5$ or $ar^4 = \frac{5}{2}$ or $5r^2 = \frac{5}{2}$ $\left(\frac{ar^4}{ar^2}\right) = \frac{\frac{5}{2}}{5}$ or $\frac{5}{r^2} = \frac{\frac{5}{2}}{r^4}$ oe $\rightarrow r$ or $r = \sqrt{\frac{\frac{5}{2}}{5}}$ $r = \frac{\sqrt{2}}{2}$ oe	B1 M1 A1
(ii)	$a = 10$	A1 [4]
(b)	$S_{\infty} = \frac{"10"}{1 - \frac{\sqrt{2}}{2}}$ $20 + 10\sqrt{2}$	M1 A1 [2]
Total 6 marks		

Part	Mark	Additional Guidance
(a)		Ignore labelling and mark parts (i) and (ii) together.
(i)	B1	One correct equation as shown. This is an M mark in epen.
	M1	Attempts to solve simultaneously. Must be working with correct equations or with $ar^3 = 5$ and $ar^5 = \frac{5}{2}$ Minimum attempt to correctly divide their equations or rearrange for a and equate as shown or to correctly rearrange and eliminate r , must achieve a value for r or for a . OR attempts to solve $5r^2 = \frac{5}{2}$ to obtain r Allow errors in arithmetic but not mathematically incorrect process.
	A1	Value as shown. Allow this mark for correct answer from working with $ar^3 = 5$ and $ar^5 = \frac{5}{2}$ Must reject negative if seen. isw attempt to convert to decimal.
	A1	Value as shown.
	A1	Value as shown.
(b)	M1	Correctly substitutes their values for a and r into the formula provided $ r < 1$
	A1	Correct value.

Question	Scheme	Marks
1(a)	$\left(1 + \frac{x}{4}\right)^8 = 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$ oe	B1M1A1 [3]
(b)	$1 + \frac{x}{4} = 1.035 \Rightarrow x = 0.14$ oe $(1.035)^8 = 1 + 2(0.14) + 1.75(0.14)^2 + 0.875(0.14)^3 = 1.3167$ [NB: the calculator value is 1.316809037]	B1 M1A1 [3]
Total 6 marks		

Question	Notes	Marks
1	$\left(1 + \frac{x}{4}\right)^8$	
(a)	For an attempt at the binomial expansion of $\left(1 + \frac{x}{4}\right)^8$ $\left(1 + \frac{x}{4}\right)^8 = 1 + 2x + \dots$	B1
	For either correct third or fourth term. Need not be simplified. $\dots + C_2^8 \times \left(\frac{x}{4}\right)^2$ or $C_3^8 \times \left(\frac{x}{4}\right)^3$	M1
	For the correct expansion fully simplified. $\left(1 + \frac{x}{4}\right)^8 = 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$ OR $\left(1 + \frac{x}{4}\right)^8 = 1 + 2x + 1.75x^2 + 0.875x^3$ Do not ignore subsequent incorrect simplification.	A1 [3]
(b)	Finds the value of x to be substituted. $1 + \frac{x}{4} = 1.035 \Rightarrow x = 0.14$ oe	B1
	Substitutes their value of x into their expansion provided it has at least 2 terms in x . <i>Their</i> $x \neq 1.035$ $(1.035)^8 = 1 + 2(0.14) + 1.75(0.14)^2 + 0.875(0.14)^3$	M1
	For the correct value of $(1.035)^8 = 1.3167$ rounded correctly. [NB: the calculator value is 1.316809037]	A1 [3]
Total 6 marks		

Question	Scheme	Marks
2	$n = 1 \Rightarrow a = 8^{(1-2)} = 8^{-1}$ oe eg $\frac{1}{8}$	B1
	$n = 2 \Rightarrow ar = 8^{(1-4)} = 8^{-3} \Rightarrow r = \frac{8^{-3}}{8^{-1}} = 8^{-2}$ oe eg $\frac{1}{64}$	M1A1
	$(S_{\infty}) = \frac{8^{-1}}{1-8^{-2}} = \frac{2^{-3}}{\frac{63}{2^6}} = \frac{8}{63}$ oe eg $\frac{\frac{1}{8}}{1-\frac{1}{64}} = \frac{\frac{1}{8}}{\frac{63}{64}} = \frac{1}{8} \times \frac{64}{63}$	M1dM1
	$\frac{8}{63}$ oe or $p = 8, q = 63$ oe	A1
Total 6 marks		

Mark	Notes
B1	For $a = 8^{-1}$ oe
M1	For substituting $n = 2$ into the expression for n th term to find a value for ar and dividing by a to find r . This mark can be implied by a correct value for r .
A1	For $r = 8^{-2}$ oe
M1	For applying the correct formula for the sum to infinity of a convergent geometric series for their values of a and r , providing $ r < 1$ They must be using values they've attained or stated for a and r .
dM1	For a correct attempt to use an index law with their expression to obtain the required form or a correct attempt to divide their fractions. Dependent on previous method mark.
A1	For the correct answer in the required form any equivalent with p and q integers is acceptable.
In this question, the final dM1 may implied from a correct substitution of their values of r and a, evaluated correctly, if working isn't shown. You may have to check their final answer. Eg $r = \frac{1}{4}, a = \frac{1}{8}$ $(S_{\infty}) = \frac{\frac{1}{8}}{1-\frac{1}{4}} = \frac{1}{6}$ is M1 dM1 A0 because the $\frac{1}{6}$ is correct for their a and their r	

Question number	Scheme	Marks
4 (a) (i)	$\frac{ar^6}{ar^2} = \frac{40}{10}$ $r = \sqrt[4]{\frac{40}{10}} = (\pm\sqrt{2})$ $r = -\sqrt{2}$	M1 M1 A1
(ii)	$ar^2 = 10 \Rightarrow a = \frac{10}{r^2}$ $a = 5$	M1 A1 (5)
(b)	$ r > 1$ so the series is not convergent.	B1FT (1)
		Total 6 marks

Part	Mark	Additional Guidance
(a)(i)	M1	For $\frac{ar^6}{ar^2} = \frac{40}{10}$ or $\frac{ar^2}{ar^6} = \frac{40}{10}$
	M1	For rearranging to find a value for r $r = \sqrt[4]{\frac{40}{10}} = (\pm\sqrt{2})$ or $(\pm\sqrt[4]{4})$
	A1	For the correct value of $r = -\sqrt{2}$ Accept $-\sqrt[4]{4}$
(a)(ii)	M1	For attempting to find a value for a using their r $ar^2 = 10 \Rightarrow a = \frac{10}{r^2}$
	A1	For the correct value of $a = 5$ Allow this mark from the use of $r = \sqrt{2}$
(b)	B1FT	$ r > 1$ so the series is not convergent. Follow through their value of r provided it is greater than 1.



Question number	Scheme	Marks
3	$\ln 12 = \ln a + (2-1) \ln b$ oe $ab = 12$ oe $\ln 768 = \ln a + (5-1) \ln b$ oe $ab^4 = 768$ oe $\frac{768}{12} = \frac{ab^4}{ab} \quad (b^3 = 64)$ $b = 4 \quad a = 3$	M1 A1 M1 A1 ddM1 A1 A1
	ALT 1 $\ln a + (2-1) \ln b = \ln 12$ oe $\ln a + (5-1) \ln b = \ln 768$ oe $3 \ln b = \ln b^3 \quad \ln 768 - \ln 12 = \ln 64$ $b^3 = 64$ $b = 4 \quad a = 3$	M1 A1 M1 A1 ddM1 A1 A1
	ALT 2 $d = \ln 12 - \ln a$ $(d = \ln b = \ln 12 - \ln a \Rightarrow \ln b = \ln \left(\frac{12}{a} \right) \Rightarrow b = \frac{12}{a}$ $\ln 768 = \ln a + \ln \left(\frac{12}{a} \right)^4$ $\ln 768 = \ln \left(\frac{12^4}{a^3} \right)$ $a^3 = \frac{20736}{768}$ $b = 4 \quad a = 3$	M1 A1 M1 A1 ddM1 A1 A1
	ALT 3 $(u_2 \Rightarrow) u_1 + d = \ln 12$ $(u_5 \Rightarrow) u_1 + 4d = \ln 768$ $3d = \ln 768 - \ln 12$ $d = \ln 4$ $u_1 = \ln 12 - \ln 4 = \ln 3 (= \ln a)$ $b = 4 \quad a = 3$	M1 A1 M1 A1 ddM1 A1 A1 [7]

Part	Mark	Additional Guidance
	M1	Correct equation as shown oe
	A1	Correct equation as shown oe
	M1	Correct equation as shown oe
	A1	Correct equation as shown oe
	ddM1	Dependent on both previous method marks , uses any clear, valid method to reduce to an equation in a (or less likely, b)
	A1	For correct b
	A1	For correct a
ALT 1	M1	One correct equation as shown oe
	A1	Both correct equations as shown oe
	M1	Clear valid attempt to subtract one equation from the other
	A1	Achieves the two terms shown
	ddM1	Dependent on both previous method marks , uses a valid method to eliminate the logs and achieves an equation in b only
	A1	For correct b
	A1	For correct a
ALT 2	M1	Finds a correct equation as shown for the common difference, d
	A1	Correct equation as shown oe
	M1	Correct equation as shown oe (subs to get u_5)
	A1	Correct equation as shown oe
	ddM1	Dependent on both previous method marks , eliminates the logs and achieves an equation in a only
	A1	For correct b
	A1	For correct a
ALT 3	M1	One correct equation as shown oe
	A1	Both correct equations as shown oe
	M1	Clear attempt to subtract one equation from the other
	A1	Achieves the correct value for d in any single ln form
	ddM1	Dependent on both previous method marks , arrives at a single term log for u_1
	A1	For correct b
	A1	For correct a
		Allow full marks in general for just $b = 4$ and $a = 3$

Question number	Scheme	Marks
5 (a)	$u_2 + u_4 = ar + ar^3 = 212.5$ $u_3 + u_4 = ar^2 + ar^3 = 62.5$ $\frac{(1+r^2)}{(r+r^2)} = \frac{17}{5}$ $12r^2 + 17r - 5 = 0$ $(4r-1)(3r+5) = 0$ $r = \frac{1}{4} \quad r = -\frac{5}{3}$	M1 M1 M1 dM1 A1 (5)
(b)	$r = \frac{1}{4} \Rightarrow a = 800$ So $\frac{a}{1-r} = \frac{800}{\frac{3}{4}} = \frac{3200}{3}$	M1 A1 (2)
Total 7 marks		

Part	Mark	Guidance
(a)	M1	For either $ar + ar^3 = 212.5$ or $ar^2 + ar^3 = 62.5$ correct
	M1	For attempting to eliminate a or ar either by division or substitution: e.g. $\frac{ar(1+r^2)}{ar(r+r^2)} = \frac{212.5}{62.5} \Rightarrow \frac{(1+r^2)}{(r+r^2)} = \frac{212.5}{62.5} = \left(\frac{17}{5}\right)$ An attempt involves some factorisation to eliminate a or ar
	Method 1 – finds a 3TQ	
	M1	For forming a 3TQ in r only using their expressions. $(12r^2 + 17r - 5 = 0 \text{ oe})$ Accept for example $150r^2 + 212.5r - 62.5 = 0$
	dM1	For an attempt to solve their 3TQ to give two values of r See General Guidance for the definition of an attempt. For example: $(4r-1)(3r+5) = 0 \Rightarrow r = \dots, \dots$ This mark is dependent on the FIRST M mark being awarded
	Method 2 – finds a cubic equation	
	M1	For forming a cubic with a common factor of r in each term. e.g. $12r^3 + 17r^2 - 5r = 0$
	dM1	For factorising their cubic equation to achieve $r(12r^2 + 17r - 5) = 0$ and for an attempt to solve their 3TQ to give two values of r Ignore $r = 0$ if also given. See General Guidance for the definition of an attempt. For example: $(4r-1)(3r+5) = 0 \Rightarrow r = \dots, \dots$ This mark is dependent on the FIRST M mark being awarded
(b)	A1	For the correct values; $r = \frac{1}{4}$ and $r = -\frac{5}{3}$ (reject $r = 0$ if seen earlier)
	M1	Uses their $r = \frac{1}{4}$ [where $ r < 1$] to find the value of a (800) with the correct formula for the sum of a geometric series to infinity. Condone an incorrect value of a even if they have used $r = \frac{1}{4}$ [The formula is given on page 2 of this booklet]. $S_{\infty} = \frac{a}{1-r} = \frac{'800'}{1 - \frac{1}{4}} = \dots$
	A1	For the correct value, $S_{\infty} = \frac{3200}{3}$ or $1066\frac{2}{3}$ Do not accept for example 1066.67 unless the stated value is 1066. $\dot{6}$

Question number	Scheme	Marks
3(a)(i)	$\frac{ar^4}{ar} = \frac{\frac{135}{5}}{\frac{16}{5}}$ $r = \sqrt[3]{\frac{\frac{135}{5}}{\frac{16}{5}}} = \left(\sqrt[3]{\frac{135 \times 16}{5 \times 1024}} \right) = \dots$ $r = \frac{3}{4} \text{ oe}$	M1 M1 A1
(a)(ii)	$ar = \frac{5}{16} \Rightarrow a = \frac{\frac{5}{16}}{\frac{3}{4}} = \left(\frac{5}{12} \right)$ $a = \frac{5}{12}$	M1 A1 [5]
(b)	$S = \frac{\frac{5}{12}}{1 - \frac{3}{4}} = \dots$ $S = \frac{5}{3}$	M1 A1 [2]
Total 7 marks		

Part	Mark	Notes
(a)(i)	M1	For $\frac{ar^4}{ar} = \frac{\frac{135}{5}}{\frac{16}{5}}$ or $\frac{ar}{ar^4} = \frac{\frac{5}{16}}{\frac{135}{1024}}$
	M1	For rearranging to find a value for r $r = \sqrt[3]{\frac{\frac{135}{5}}{\frac{16}{5}}} = \left(\sqrt[3]{\frac{135 \times 16}{5 \times 1024}} \right) = \dots$
	A1	For the correct value of $r = \frac{3}{4}$ oe
(a)(ii)	M1	For attempting to find a value for a using their r $ar = \frac{5}{16} \Rightarrow a = \frac{\frac{5}{16}}{\frac{3}{4}} = \left(\frac{5}{12} \right)$
	A1	For the correct value of $a = \frac{5}{12}$

(b)	M1	For using the correct formula for the sum to infinity using their a and r provided $ r < 1$ $S = \frac{\frac{5}{12}}{1 - \frac{3}{4}} = \dots$ $\frac{5'}{12} \neq \frac{5}{16}, \frac{5'}{12} \neq \frac{135}{1024}$
	A1	For the correct value of $S = \frac{5}{3}$ Note: Must be the exact value.

Question number	Scheme	Marks
4	$(S_4 =) \frac{a(1-r^4)}{1-r} = 80 \quad \text{or} \quad (S_\infty =) \frac{a}{1-r} = 81 \quad \text{or} \quad a + ar + ar^2 + ar^3 = 80$ $\frac{81(1-r)(1-r^4)}{1-r} = 80 \Rightarrow (81(1-r^4) = 80)$ or $81(1-r) = \frac{80(1-r)}{1-r^4}$ $(\Rightarrow (81-81r)(1-r^4) = 80(1-r) \Rightarrow 81r^5 - 81r^4 - r + 1 = 0 \Rightarrow (81r^4 - 1)(r-1) = 0)$ or $81(1-r) + 81(1-r)r + 81(1-r)r^2 + 81(1-r)r^3 = 80$ $r^4 = \frac{1}{81}$ $r = \pm \frac{1}{3}$ [a = 54] $S_7 = \frac{54 \left(1 - \left(\frac{1}{3} \right)^7 \right)}{1 - \frac{1}{3}} \text{ or } \frac{2186}{27} \text{ or } 81 \left(1 - \left(\frac{1}{3} \right)^7 \right)$ $81 - \frac{2186}{27} = \frac{1}{27} *$	B1 M1 M1 A1 M1 dM1 A1* cso (7)
ALT	FINAL THREE MARKS $S_7 = \frac{54 \left(1 - \left(\frac{1}{3} \right)^7 \right)}{1 - \frac{1}{3}} \text{ or } \frac{2186}{27}$ $81 - \frac{1}{27} = \frac{2186}{27}$	M1 ddM1 A1
Total 7 marks		

Marks	Notes
B1	For $\frac{a(1-r^4)}{1-r} = 80$ or $\frac{a}{1-r} = 81$ or $a + ar + ar^2 + ar^3 = 80$
M1	For substituting S_{∞} into S_4 and eliminating a Students may also rearrange to make a the subject of both and then substitute to eliminate a . Allow one error in manipulation. Must use a valid method to eliminate. Using their S_{∞} and S_4
M1	For reaching $r^4 = \dots$ or for reaching $81r^5 - 81r^4 - r + 1 = 0$
A1	For $r = \frac{1}{3}$ (accept $\pm$)
M1	Correct substitution into $\frac{a(1-r^7)}{1-r}$ or $a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6$ or $81(1-r^7)$ - using their a and their r . r must be positive. Or $\frac{2186}{27}$
dM1	For $81 - \frac{2186}{27}$ Dependant on previous method mark. Their r must be positive. If a and r are correct, the evaluation of each part of these calculations need not be shown (can be done on a calculator). If either a or r are incorrect, their $\frac{2186}{27}$ must have been evaluated and 81 used.
A1 cso*	Obtains the given answer. No incorrect work.
ALT	Final 3 marks
M1	Correct substitution into $\frac{a(1-r^7)}{1-r}$ or $a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6$ - using their a and their r . r must be positive.
dM1 A1	Must state $81 - \frac{1}{27} = \frac{2186}{27}$ (for this ALT, candidates must evaluate and get $\frac{2186}{27}$) It isn't possible to get dM1 A0 on the ALT Dependent on previous method mark.

Question number	Scheme	Marks
6 a	$p = 2 \quad q = \frac{3}{8}$	B1 B1 (2)
b	$(2) \left[1 + \frac{1}{3} \left(\frac{3}{8}x \right) + \frac{1}{2!} \left(\frac{3}{8}x \right)^2 \right] = 2 + \frac{1}{4}x - \frac{1}{32}x^2$	M1 A1ft A1 (3)
c	$\left(\sqrt[3]{9} = \sqrt[3]{8+1} \Rightarrow x = \frac{1}{3} \right)$ So $2 + \frac{1}{4} \times \frac{1}{3} - \frac{1}{32} \times \left(\frac{1}{3} \right)^2 = \frac{576 + 24 - 1}{288} = \frac{599}{288} *$	M1 A1 cso (2)
Total 7 marks		

Part	Mark	Additional Guidance
(a)	B1	For $p = 2$ or $q = \frac{3}{8}$ condone $8^{\frac{1}{3}}$
	B1	For $p = 2$ and $q = \frac{3}{8}$ condone $8^{\frac{1}{3}}$
(b)	M1	For an attempt to expand $(1 + qx)^{\frac{1}{3}}$ with their value of q up to the term in x^2 It is not necessary to see p at this stage. The definition of an attempt is as follows: • The first term must be 1 • The next term must be correct for their value of q • The powers of qx must be correct eg $(qx)^2$ • The denominators must be correct Simplification not required. Do not allow missing brackets unless recovered later – this is a general point of marking. Ignore any terms with powers higher than 2.
	A1ft	For both algebraic terms fully correct and unsimplified in the expansion of $(1 + qx)^{\frac{1}{3}}$ for their value of q . It is not necessary to see p at this point. Ignore any terms with powers higher than 2.
	A1	For all 3 terms correct, all simplified. Ignore any terms with powers higher than 2
If there are any other methods used – send this to review please.		
(c)	M1	For correct substitution of $x = \frac{1}{3}$ into their expansion, which must have at least 2 terms, including a term in x^2
	A1cso*	Obtains the given result

Question	Scheme	Marks
7(a)	$(1+2x^2)^{-\frac{3}{4}} = 1 + \left(-\frac{3}{4}\right)(2x^2) + \frac{\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)(2x^2)^2}{2!} + \frac{\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)\left(-\frac{11}{4}\right)(2x^2)^3}{3!}$ $= 1 - \frac{3}{2}x^2 + \frac{21}{8}x^4 - \frac{77}{16}x^6$	M1 A1A1 [3]
(b)	$f(x) = (2+kx)\left(1 - \frac{3}{2}x^2 + \frac{21}{8}x^4 - \frac{77}{16}x^6\right)$ $= 2 + kx - 3x^2 - \frac{3k}{2}x^3 + \frac{21}{4}x^4 + \frac{21k}{8}x^5$	M1A1 [2]
(c)	$14 \times (-3) = \frac{21k}{8} \Rightarrow k = \frac{14 \times (-3) \times 8}{21} = -16$	M1A1 [2]
Total 7 marks		

Part	Mark	Notes
(a)	M1	For an attempt at the binomial expansion. - The expansion must begin with 1 and there must be an attempt at terms up to $(2x^2)^3$ - The denominators must be correct. - The powers of $(2x^2)$ must be correct.
	A1	For at least one term in x correct and simplified, subject to the conditions for the M mark!
	A1	For a fully correct and simplified expansion in the correct order of ascending powers.
(b)	M1	For multiplying their expansion by $(2+kx)$ provided their expansion has at least 2 terms in x with differing powers.
	A1	For a fully correct expansion.
(c)	M1	For setting $14 \times (\text{their coeff of } x^2) = (\text{their coeff of } x^5)$ and attempting to solve to find a value for k
	A1	For the correct value of k

Question	Scheme	Marks
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Question number	Scheme	Marks
2	$a + 2d = 8 \quad a + 4d = 20$ $2d = 12 \Rightarrow d =$ $(a =) - 4 \quad (d =) 6$ $S_n = \frac{n}{2}(2 \times -4 + (n-1)6) (> 220)$ $\left(\frac{n}{2}(-8 + 6n - 6) > 220 \right) \text{ or } 3n^2 - 7n > 220 \text{ oe}$ Solution using 3TQ: $6n^2 - 14n - 440 > 0 \text{ or } 3n^2 - 7n - 220 > 0 \text{ oe}$ $n = \frac{7 \pm \sqrt{(-7)^2 - 4(3)(-220)}}{2 \times 3} \Rightarrow n = (9.80925, -7.47592)$ $n = 10$ Solution using trial and improvement: $n = 9: 3 \times (9)^2 - 7 \times (9) = 180 < 220$ $n = 10: 3 \times (10)^2 - 7 \times (10) = 230 > 220$ Hence minimum value is $n = 10$	B1 M1 A1 M1 M1 M1 A1 [7] [M1] [A1]
Total 7 marks		

Part	Mark	Additional Guidance
2	B1	For both correct equations
	M1	For a complete attempt to solve their 2 linear simultaneous equations, allow one processing error.
	A1	For both correct: $(a =) -4$ and $(d =) 6$. Values may be implied by subsequent working.
	ALT for first 3 marks: Uses the third term	
	B1	Let the 3 rd term be T $20 - T = d$ $T - 8 = d$
	M1	Solves the above equations simultaneously by any method to find d AND finds a value for a
	A1	For both correct $a = -4$ and $d = 6$. Values may be implied by subsequent working
	M1	Correct substitution of their values for a and d into the correct formula for S_n (doesn't need to be placed > 220 at this point)
	M1	For the correct inequality shown with their values oe
	M1	For a full and correct method to solve their 3TQ leading to at least one value for n . Award this mark for the positive root [9.809...] without explicit working seen [use of calculator] following a correct 3TQ seen. If the 3TQ is incorrect and there is no evidence of explicit working to solve their 3TQ do not award this mark.
	A1	For choosing only positive root, and rounding to $n = 10$ If a candidate offers a positive and negative value without rejecting the negative [-7.47...] do not award this mark.
	ALT: uses T & I Final two marks	
	M1	Attempts $n = 10$ AND $n = 9$ When $n = 10$ $S = 230$ and when $n = 9$ $S = 180$ These values must be seen
	A1	For choosing $n = 10$

Question number	Scheme	Marks
6 (a)	$\sqrt{9-x} = 3\left(1-\frac{x}{9}\right)^{\frac{1}{2}} \Rightarrow p=3, q=-\frac{1}{9}$	B1B1 [2]
(b)	$3\left(1-\frac{x}{9}\right)^{\frac{1}{2}} =$ $3\left\{1 + \left(\frac{1}{2}\right)\left(-\frac{x}{9}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{x}{9}\right)^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{9}\right)^3}{3!} + \dots\right\}$ $= 3\left(1 - \frac{x}{18} - \frac{x^2}{648} - \frac{x^3}{11664}\right) = 3 - \frac{x}{6} - \frac{x^2}{216} - \frac{x^3}{3888}$	M1 A1A1 [3]
(c)	$\frac{31}{4} = 9 - x \Rightarrow x = \frac{5}{4}$ $\sqrt{\frac{31}{4}} \approx 3 - \frac{1.25}{6} - \frac{1.25^2}{216} - \frac{1.25^3}{3888} = 2.783930523... \approx 2.78393$	B1 M1A1 [3]
Total 8 marks		

(a)	B1	$p = 3$ or $\sqrt{9}$ or $9^{\frac{1}{2}}$	May be shown in their $3\left(1-\frac{x}{9}\right)^{\frac{1}{2}}$ (allow isw)
	B1	$q = -\frac{1}{9}$	
(b)	M1	Attempt to use the binomial expansion for their $(1+qx)^{\frac{1}{2}}$. Must have first term 1, three more terms with ascending powers of x , 2 or 2! and 6 or 3! Seen, and their $\left(-\frac{x}{9}\right)$ used at least once. No simplification needed. Ignore terms beyond x^3	
	A1	Two algebraic terms correct in the expansion of $3\left(1-\frac{x}{9}\right)^{\frac{1}{2}}$. Must be single fractions, not necessarily in lowest terms. Ignore terms beyond x^3	
	A1	All four terms correct and in lowest terms. Ignore terms beyond x^3	
(c)	B1	Identify $x = \frac{5}{4}$ as the value needed to make $\sqrt{9-x} = \sqrt{\frac{31}{4}}$	
	M1	Substitute their x into each of the algebraic terms of their expansion of $\sqrt{9-x}$	
	A1	2.78393 given to 5DP.	

Question number	Scheme	Marks
9 (a)	$1 + \frac{1}{2}(-4x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)(-4x)^2}{2!} + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-4x)^3}{3!}$ $1 - 2x - 2x^2 - 4x^3$	M1 A1 A1 (3)
(b)	$x = 0.06$ $1 - 0.12 - 0.0072 - 0.000864$ 0.8719	B1 M1 A1 (3)
(c)	$\sqrt{\frac{76}{100}} = \frac{1}{5}\sqrt{19}$ $\sqrt{19} = 0.8719 \times 5$ 4.360	M1 A1 (2)
		[8]

Part	Mark	Additional Guidance
(a)	M1	For an attempt at a Binomial expansion. A attempt is defined as the following - The expansion must start with 1 - The powers of x must be correct $-4x$ must be used at least once - The denominators (2! And 3!) must be seen. Accept 2 and 6 $(1-4x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(-4x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(-4x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-4x)^3$
	A1	For at least one term in x correct and fully simplified. $1 - 2x - 2x^2 - 4x^3$
	A1	For the expansion fully correct and simplified
(b)	B1	For finding the value of $x = 0.06$
	M1	For substituting their value of x into the expansion provided $ x \leq 0.25$ Use of their expansion or the correct expansion must be seen explicitly here
	A1	0.8719
(c)	M1	For using their value from (b) in $\sqrt{0.76} = \frac{\sqrt{19}}{5} \Rightarrow \sqrt{19} = 5\sqrt{0.76} = 5 \times '0.8719'$
	A1	For 4.360 rounded correctly
Penalise rounding once only in this question. Answers must round to the given answers.		

Question number	Scheme	Marks
5 (a)	$1 + \frac{1}{2}(-x) + \frac{\frac{1}{2}(-\frac{1}{2})}{2!}(-x)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!}(-x)^3$ $1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$	M1 A1 A1 (3)
(b)	$x = 0.08$ $1 - \frac{1}{2}(0.08) - \frac{1}{8}(0.08)^2 - \frac{1}{16}(0.08)^3$ 0.959168 cao	B1 M1 A1 cao (3)
(c)	$(\sqrt{0.92} = \frac{\sqrt{23}}{5} \Rightarrow \sqrt{23} =) "0.959168" \times 5$ 4.79584	M1 A1 (2) [8]

Part	Mark	Additional Guidance
(a)	M1	For an attempt at a Binomial expansion. An attempt is defined as the following - The expansion must start with 1 - The powers of their $-x$ must be correct $-x$ must be used at least once - The denominators 2! And 3! must be seen. Accept 2 and 6 Can be implied by at least 2 correct terms in an expansion
	A1	For at least one term in x correct and fully simplified.
	A1	For the expansion fully correct and simplified. Ignore terms in higher powers of x .
(b)	B1	For finding the value of $x = 0.08$
	M1	For correctly substituting their value of x into the expansion provided $ x < 1$ Use of their expansion or the correct expansion must be seen explicitly here
	A1	cao
(c)	M1	For use of their value from (b) in $\sqrt{0.92} = \frac{\sqrt{23}}{5} \Rightarrow \sqrt{23} = "0.959168" \times 5$
	A1	Cao
		Allow the final M1 A1 if $\sqrt{0.92} \times 5$ is clearly written and 4.79584 is clearly indicated as the answer.

Question Number	Scheme	Marks
5(a)	$\sum_{r=1}^n (3r+5) = 8+11+14+\dots+(3n+5) = \frac{1}{2}n(8+3n+5) = \frac{1}{2}n(3n+13) *$	M1M1A1cso (3)
ALT	$\sum_{r=1}^n (3r+5) = \sum_1^n 3r+5n = \frac{n}{2}(3+3n)+5n = \frac{n}{2}(13+3n) *$	M1M1A1cso
(b)	$\sum_{r=35}^{50} (3r+5) = \frac{50}{2}(13+150) - \frac{34}{2}(13+102)$ $= 2120$	M1 A1 (2)
(c)	$\frac{n}{2}(13+3n) = 385$ $3n^2 + 13n - 770 = 0$ $(3n+55)(n-14) = 0 \quad n = 14$	M1 M1A1 (3)
[8]		
(a) M1 M1 A1cso ALT M1 M1 A1 (b) M1	Evaluate either first and last terms or first and common difference Use either sum formula. Can be shown explicitly or implied by a correct, full substitution of n and their a and their d or l Reach the given result with no errors in the working. Must be the complete result, not just the RHS or there must be a conclusion eg “shown” Split the $(3r+5)$ into 2 parts and deal with the 5 correctly Sum $(3r)$ either by using a summation formula or by using the standard result Reach the given result with no errors in the working	
(b) M1	Express the required sum as the difference of 2 sums. Second sum must have 34 terms. Use the result given in (a). Using a standard formula with first term and either last term or common difference scores 0/2 as question states “hence”. Calculator solutions (without showing the difference of the 2 sums first) score M0	
A1	Correct answer.	
(c) M1	Use the result in (a) or some other valid method to form a 3 term quadratic in n	
M1	Solve their 3TQ by any valid means. Must reach $n = \dots$ Negative value need not be seen	
A1	Correct answer. $n = 14$ and no other Correct quadratic followed by correct answer scores 3/3 Incorrect quadratic solved by calculator M0A0	

Question number	Scheme	Marks
2 (a)(i)	$a + 4d = 46$ $a + 19d = 181$ oe	B1
	" $15d = 135$ " $\Rightarrow d = 9$	M1 A1 [3]
(a)(ii)	$a = 10$	B1 [1]
(b)	$\sum_{n=1}^{50} u_n = 25(2 \times "10" + (50 - 1) \times "9") = 11525$	M1
	$\sum_{n=1}^{20} u_n = 10(2 \times "10" + (20 - 1) \times "9") = 1910$	M1
	" 11525 " – " 1910 " = 9615	ddM1 A1 [4]
ALT	(First term =) " 10 " + " 9 " $\times (21 - 1) = 190$	{M1}
	(Last term =) " 10 " + " 9 " $\times (50 - 1) = 451$	{M1}
	$\frac{30}{2} ("190" + "451") = 9615$	{ddM1} {A1} [4]
Total 8 marks		

Part	Marks	Notes
(a) (i)	B1	For $a + 4d = 46$ and $a + 19d = 181$, both correct oe.
	M1	For solving their equations simultaneously, valid correct method with one slip only (algebraic or sign error), leading to $d =$
	A1	For $d = 9$
(a) (ii)	B1	For $a = 10$
(b)	M1	For use of their a and d in the correct formula for the sum to n terms $\frac{n}{2}(2"a" + (n - 1)"d")$ with $n = 50$
	M1	For use of their a and d in the correct formula for the sum to n terms $\frac{n}{2}(2"a" + (n - 1)"d")$ with $n = 20$
	ddM1	For their " $\sum_{n=1}^{50} u_n$ " – " $\sum_{n=1}^{20} u_n$ " Dependent on both previous method marks
	A1	For 9615
	ALT	
	M1	For correct use of " a " + " $(n - 1)"d$ " with $n = 21$ with their a and their d
	M1	For correct use of " a " + " $(n - 1)"d$ " with $n = 50$ with their a and their d
	ddM1	For correct use of the formula $\frac{n}{2}(a_{21} + a_{50})$ Dependent on both previous method marks
	A1	For 9615

Question number	Scheme	Marks
5 (a)	$1 + anx + \frac{n(n-1)}{2}a^2x^2 + \frac{n(n-1)(n-2)}{3!}a^3x^3$	M1 A1 [2]
(b)	$an = 8 \quad \frac{n(n-1)}{2}a^2 = 30$	
	$\frac{n(n-1)}{2}\left(\frac{8}{n}\right)^2 = 30 \quad \text{or} \quad \frac{\left(\frac{8}{a}\right)\left(\frac{8}{a}-1\right)}{2}a^2 = 30$	M1
	$32n - 32 = 30n \rightarrow n = \quad \text{or} \quad 64 - 8a = 60 \rightarrow a =$	dM1
	$n = 16 \quad a = \frac{1}{2}$	A1 A1 [4]
(c)	$\frac{16 \times (16-1)(16-2)}{3!} \times \left(\frac{1}{2}\right)^3 = 70$	M1 A1 [2]
Total 8 marks		

Part	Mark	Notes
(a)	M1	For an attempt to find the binomial expansion - The expansion must begin with 1 or 1^n - The denominators must be correct (ie. 2! And 3!) on the third and fourth terms. The power of x must be correct (ie. Must see x , x^2 and x^3 , with the correct corresponding denominators). Simplification not necessary – may see $(ax)^2$ and $(ax)^3$
	A1	For a fully correct expression (Allow 1^n for 1) - must see a^2x^2 and a^3x^3 , but 2! And 3! is acceptable
(b)	M1	For correct substitution of either $\frac{8}{n}$ or $\frac{8}{a}$ into their coefficient of x^2
	dM1	For rearranging and forming a linear equation, leading to $a =$ or $n =$
	A1	For $n = 16$
	A1	For $a = \frac{1}{2}$
(c)	M1	For correct substitution seen of their n and a into their coefficient of x^3
	A1	For 70

Question number	Scheme	Marks
6 a	$r = \frac{q(2p-3)}{q(2p+3)} = \frac{q(2p+3)}{q(4p+1)}$ $(4p+1)(2p-3) = (2p+3)^2$ $2p^2 - 11p - 6 = 0$ $(2p+1)(p-6) = 0$ $p = -\frac{1}{2} \quad \text{or} \quad p = 6$	M1 dM1 A1 M1 A1 (5)
b	When $p = 6$ $r = \frac{3}{5}$ and $U_1 = q(4 \times '6' + 1) = (25q)$ $S_\infty = \frac{25q}{\frac{2}{5}} = 250 \Rightarrow q = 4$	M1 dM1 A1 (3)
Total 8 marks		

Part	Mark	Notes
(a)	M1	For $r = \frac{\text{3rd term}}{\text{2nd term}} = \frac{\text{2nd term}}{\text{1st term}}$
	dM1	For attempting to remove the denominators/simplifying the expression to attempt to obtain a 3TQ. Allow one processing error. Allow a minimally acceptable $4p^2 \pm Xp \pm Y = (0)$ or $2p^2 \pm Pp \pm Q = (0)$ $X, Y, P, Q \neq 0$ Note: This mark is dependent on the previous M mark.
	A1	For obtaining $2p^2 - 11p - 6 = (0)$ or equivalent. Their working will give them $4p^2 - 22p - 12 = (0)$ before further simplification
	M1	For a valid attempt to solve the 3TQ. Note this is an independent mark for their 3TQ. See General Guidance. The 3TQ must have come from some attempted manipulation involving $q(2p-3)$, $q(2p+3)$ and $q(4p+1)$
	A1	For $p = -\frac{1}{2}$ or $p = 6$
(b)	M1	For finding: - A value of r for their p. Allow any value of r $r \neq 0$ E.g. $r = \frac{(2 \times '6' - 3)}{(2 \times '6' + 3)} = \dots$ (Note: for $p = -\frac{1}{2}$ $r = -2$) - A value for U_1 $U_1 = q(4 \times '6' + 1) = \dots$ (Note: for $p = -\frac{1}{2}$, $U_1 = -q$) You may see a = 100 after later working to find the sum to infinity. If their values of p are incorrect, we must see working here. NOTE: This is a B mark in Epen.
	dM1	For use of $S_\infty = \frac{a}{1-r} = 250$ with their r and their U_1 provided $ r < 1$ If they use the formula for the sum to infinity on an $ r > 1$ withhold this mark even if they use it on one valid and one invalid attempt. Note: This mark is dependent on the previous M mark.
	A1	For $q = 4$

Question	Scheme	Marks
3(a)	$\left(\sum_{r=1}^n (5r-3)\right) \Rightarrow a=2, \quad d=5$ $\left(\sum_{r=1}^n (5r-3)\right) = \frac{n}{2}(2 \times 2 + (n-1)5) \text{ or } \frac{n}{2}(4+5n-5)$ $\text{or } \frac{n}{2}(2+2+(n-1)5) \text{ or } \frac{n}{2}(2+5n-3)$ $= \frac{n}{2}(5n-1)^*$ ALT (Using standard results) $\left(\sum_{r=1}^n (5r-3)\right) = 5\sum_1^n r - 3\sum_1^n 1$ $\left(\sum_{r=1}^n (5r-3)\right) = 5\left[\frac{n}{2}(n+1)\right] - 3n = \frac{5n^2+5n-6n}{2} = \frac{n}{2}(5n-1)^*$	B1 M1 A1 cso [3] B1 M1A1 cso [3]
(b)	$\left(\sum_{31}^{60} (5r-3)\right) = \frac{60}{2}(5 \times 60 - 1) - \frac{30}{2}(5 \times 30 - 1) = 6735$ ALT $\left(\sum_{31}^{60} (5r-3)\right) = \frac{30}{2}(152+297) = 6735$ $\text{or } \left(\sum_1^{60} (5r-3) - \sum_1^{30} (5r-3)\right) = \frac{60}{2}(2 \times 2 + (60-1) \times 5) - \frac{30}{2}(2 \times 2 + (30-1) \times 5) = 6735$	M1A1 [2] M1A1 [2]
(c)	$\frac{n}{2}(5n-1) = 3783 \Rightarrow 5n^2 - n - 7566 = 0$ $\Rightarrow (5n+194)(n-39) = 0$ $\Rightarrow n = 39 \quad \left[-\frac{194}{5}\right]$	M1 M1 A1 [3]
Total 8 marks		

Part	Mark	Notes
(a)	B1	For the values of a and d , these may be explicitly stated or implicitly used in a formula.
	M1	Correctly substitutes their values of a and d into the correct summation formula
	A1 cso	For the correct expression, minimum steps as shown, no errors or omissions.
	ALT	
	B1	For writing the given expression as $5\sum_{1}^n r - 3\sum_{1}^n 1$
	M1	For the sum shown, using standard results for the series.
	A1 cso	For the correct expression, minimum steps as shown, no errors or omissions.
(b)	M1	Uses the given expression with both $n = 60$ and $n = 30$ and subtracts. Indicated by $8970 - 2235$. As a concession, allow substitution of $n = 31$ and 60 for the mark. ALT Correct substitution of the correct ‘first’ and ‘last’ value into the correct summation formula ie $\frac{30}{2}(152 + 297)$. Allow as a concession $\frac{29}{2}(152 + 297)$ or correct substitution of both $n = 30$ and $n = 60$ into the correct sum to n terms formula and a subtraction. Indicated by $8970 - 2235$. As a concession, allow substitution of $n = 31$ and 60 for the mark.
	A1	For 6735
(c)	M1	For correctly placing the given expression = 3783 and rearranging (allow one error) to get a $3TQ = 0$.
	M1	For solving their $3TQ$ and achieving at least one value. Minimum attempt to solve required (see general guidance). A correct value of 39 can imply this mark
	A1	For 39 only. If $-\frac{194}{5}$ is also given as a solution, withhold this mark.

Question	Scheme	Marks
2(a)	$(1+Ax)^n = 1+n(Ax) + \frac{n(n-1)(Ax)^2}{2} + \dots$ $\Rightarrow nA = -\frac{1}{3} \quad \frac{n(n-1)A^2}{2} = \frac{5}{36}$ $\Rightarrow A = -\frac{1}{3n}$ $\Rightarrow \frac{n(n-1)\left(-\frac{1}{3n}\right)^2}{2} = \frac{5}{36} \Rightarrow n = -\frac{2}{3} \Rightarrow A = \frac{1}{2}$ ALT 1 $\Rightarrow nA = -\frac{1}{3} \quad \frac{n(n-1)A^2}{2} = \frac{5}{36}$ $\Rightarrow n = -\frac{1}{3A}$ $\Rightarrow \frac{-\frac{1}{3A}\left(-\frac{1}{3A}-1\right)(A)^2}{2} = \frac{5}{36} \Rightarrow A = \frac{1}{2} \Rightarrow n = -\frac{2}{3}$ ALT 2 $\Rightarrow nA = -\frac{1}{3} \Rightarrow \left[(nA)^2 = \frac{1}{9}\right]$ $\frac{n(n-1)A^2}{2!} = \frac{5}{36} \Rightarrow \left[\frac{(nA)^2 - A(An)}{2!} = \frac{5}{36}\right]$ $\Rightarrow \frac{\frac{1}{9} - A\left(-\frac{1}{3}\right)}{2} = \frac{5}{36} \Rightarrow A = \frac{1}{2} \Rightarrow n = -\frac{2}{3}$	B1 M1 M1dM1A1A1 [6] [M1] M1dM1A1A1] [M1] M1dM1A1A1]
(b)	$\text{Coefficient of } x^3 \Rightarrow \frac{\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)\left(\frac{1}{2}\right)^3}{3!} = -\frac{5}{81}$	M1A1 [2]
Total 8 marks		

Part	Mark	Notes
(a)	B1	For the correct expansion of $(1 + Ax)^n$ This can be implied by correct work in the term in x and x^2
	M1	Equates their second term to $-\frac{1}{3}$ and their third term to $\frac{5}{36}$ Allow presence of x and x^2 , provided it is on BOTH sides. eg Allow $nAx = -\frac{1}{3}x$ and $\left[\frac{n(n-1)A^2}{2}\right]x^2 = \frac{5}{36}x^2$
	M1	Substitutes A into n to form an equation in n
	dM1	For solving their equation in n If they make errors in simplification to this step e.g., $\left(-\frac{1}{3n}\right)^2 \Rightarrow \left(\frac{1}{9n}\right)$ or $\left(-\frac{1}{3n}\right)^2 \Rightarrow -\left(\frac{1}{9n^2}\right)$ then this is M0 Their equation must be either a quadratic or linear if they cancel an n This is dependent on the previous M mark only.
	A1	For the correct value of n or A $n = -\frac{2}{3}, A = \frac{1}{2}$ isw $n = 0$ seen. This is equivalent to cancelling n 's
	A1	For the correct value of n and A
	ALT 1 First two marks same as main method	
	M1	Substitutes n in terms of A into an equation in A $\frac{\left(-\frac{1}{3A}\right)^2 - \left(-\frac{1}{3A} - 1\right)(A)^2}{2} = \frac{5}{36}$ Do not allow a substitution of $n = -\frac{1}{3}$ for this mark
	dM1	For solving their equation in A to find a value for A Their equation must be either a quadratic or linear if they cancel an A This is dependent on the previous M mark only.
	A1	For the correct value of n or A $n = -\frac{2}{3}, A = \frac{1}{2}$
	A1	For the correct value of n and A
	ALT 2 First two marks same as main method	
	M1	Substitutes nA into $\frac{(nA)^2 - A(An)}{2!} = \frac{5}{36}$
	dM1	Solves their equation to find a value for A provided no errors introduced. This is dependent on the previous M mark only.
	A1	For the correct value of n or A $n = -\frac{2}{3}, A = \frac{1}{2}$
	A1	For the correct value of n and A

(b)	M1	Uses the correct form for the fourth term of a binomial expansion with their A and their n . You may see this in terms of A and n in part (a) Accept as a minimum, the correct power of $\left(\frac{1}{2}\right)^3$ with the correct denominator i.e. $3!$ Allow the presence of x^3 for this mark.
	A1	For the correct value of $-\frac{5}{81}$ If you see the correct value following the correct values of A and n with no working, award M1A1

Question number	Scheme	Marks
6 (a)	$\sqrt{4-x} = 2\left(1 - \frac{x}{4}\right)^{\frac{1}{2}}$ or $A = 2, B = 4$	B1B1 [2]
(b)	$2 \left\{ 1 + \left(\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{4}\right)^3}{3!} + \dots \right\}$ $= 2 - \frac{x}{4} - \frac{x^2}{64} - \frac{x^3}{512} + \dots$	M1A1ft A1 [3]
(c)	$\frac{\sqrt{305}}{10} = \sqrt{4-x} \Rightarrow x = 0.95$ $2 - \frac{0.95}{4} - \frac{0.95^2}{64} - \frac{0.95^3}{512} = 1.7467238 \dots \approx 1.74672$	B1 M1A1 [3]
Total 8 marks		

Part	Mark	Additional Guidance
(a)	B1B1	B1B0 for one correct value, B1B1 For the correct values of both A and B A and B must be integers from correct working Do not accept $A = 4^{\frac{1}{2}}$ or $\sqrt{4}$
(b)	M1	Attempt to expand their $2\left(1 - \frac{x}{4}\right)^{\frac{1}{2}}$ up to x^3 term (with or without '2') Must have: - First term is 1 for their $\left(1 - \frac{x}{4}\right)^{\frac{1}{2}}$ expansion - Correct powers of $\pm \frac{x}{4}$ for each term No simplification needed. Ignore terms beyond x^3
	A1ft	Two correct algebraic terms in their expansion, with or without '2'
	A1	All terms correct and in lowest terms. Ignore terms beyond x^3 .
(c)	B1	Identifies $x = 0.95$ oe. e.g. $\frac{19}{20}$
	M1	Substitutes their x which must come from some working (provided $ x < 4$) into each of the algebraic terms of their expansion.
	A1	1.74672 (must be 5dp) Note: Calculator value for $\frac{\sqrt{305}}{10}$ is 1.74642492

Question Number	Scheme	Marks
7(a)	(i) $a = 9$	B1
	(ii) $d = 4$	B1 (2)
(b)	(i) $a = 4$	B1
	(ii) $r = 3$	B1 (2)
(c)	$A_{14} = \frac{14}{2}(2 \times 9 + 13 \times 4)$ or $\frac{14}{2}(9 + 61), = 490$	M1, A1
	$"490" - 6 = \frac{4(3^n - 1)}{3 - 1}$	M1
	$3^n = 243 \quad n = 5$	ddM1A1 (5) [9]
(a) B1 B1 (b) B1 B1 (c) M1 A1 M1 ddM1 A1 ALT M1 ddM1 A1	Correct value, no working or explanation needed Correct value, no working or explanation needed Correct value, no working or explanation needed Correct value, no working or explanation needed Use either formula for the sum of an arithmetic series with their a and d (if needed) and obtain a value for the sum of the first 14 terms Correct value for the sum Subtract 6 from their sum (explicitly or implicitly) and equate to the sum of the first n terms of the geometric series obtained using their a and r Solve their equation by a correct method. No method need be shown but must reach $n = \dots$ Depends on both M marks above Correct value for n obtained For the last 3 marks: Subtract 6 from their sum and generate at least the first 5 terms of the geometric series. Sum their terms until at least "484" is reached Correct answer (5) obtained from correct work.	

Question number	Scheme	Marks
1 (a) (i)	$a + d + a + 8d = 0$ $a + 3d + a + 5d + a + 9d = 14$ Solve simultaneously $d = 4$	M1 M1 A1
(ii)	$a = -18$	A1 (4)
(b)	$\frac{3n}{2}[48 + 6(n - 1)] = \frac{2n}{2}[48 + 6(2n - 1)]$ $3n(42 + 6n) = 2n(42 + 12n) \Rightarrow 6n^2 - 42n = 0$ $6n(n - 7) = 0 \Rightarrow n = [0, 7]$ $n = 7$	M1 A1 A1 M1 A1 (5) [9]

Part	Mark	Additional Guidance
(a)	M1	For writing down both correct expressions in terms of a and d $a + d + a + 8d = 0$ $a + 3d + a + 5d + a + 9d = 14$
	M1	For attempting to solve their simultaneous equations for a and d $2a + 9d = 0$ $3a + 17d = 14$
	A1 (i)	For $d = 4$ * This is a show question – there must be no errors for the award of this mark
	B1 (ii)	For $a = -18$ This is an A mark in Epen
(b)	M1	For the correct use of the correct summation formula on one of the LHS or the RHS of the following equation. $\frac{3n}{2}(2 \times 24 + 6[n - 1]) = \frac{2n}{2}(2 \times 24 + 6[2n - 1])$ No simplification is required for this mark.
	A1	For a fully correct equation as shown above – simplified or unsimplified
	A1	For reaching a correct 2TQ equation in n $126n + 18n^2 = 84n + 24n^2 \Rightarrow 6n^2 - 42n = 0$
	M1	For attempting to solve their quadratic (See General Guidance for the definition of an attempt) $6n^2 - 42n = 0 \Rightarrow 6n(n - 7) = 0 \Rightarrow n = [0, 7]$
	A1	$n = 7$ Condone the value of 0 for this mark
	ALT	
	M1	For the correct use of the correct summation formula on one of the LHS or the RHS of the following equation. $\frac{3n}{2}(2 \times 24 + 6[n - 1]) = \frac{2n}{2}(2 \times 24 + 6[2n - 1])$ No simplification is required for this mark.
	A1	For a fully correct equation as shown above – simplified or unsimplified
	A1	Divides through n to reach a linear equation to give $6n = 42$ oe
	M1	Solves their linear equation in n
	A1	$n = 7$

Question number	Scheme	Marks
9(a)	$(3(3-x))^{-3} = \frac{1}{9}\left(1-\frac{x}{3}\right)^{-3} \quad a = \frac{1}{9} \quad b = \frac{1}{3}$	B1 B1 [2]
(b)	$\left(1-\frac{x}{3}\right)^{-3} =$ $\left[1 + (-3)\left(-\frac{x}{3}\right) + \frac{(-3)(-4)\left(-\frac{x}{3}\right)^2}{2!} + \frac{(-3)(-4)(-5)\left(-\frac{x}{3}\right)^3}{3!}\right]$ $\frac{1}{9} + \frac{1}{9}x + \frac{2}{27}x^2 + \frac{10}{243}x^3$	M1 A1ft A1 [3]
(c) (i)	$\frac{24}{125} = \frac{3}{(3-x)^3} \quad \text{or} \quad \frac{125}{8} = (3-x)^3 \Rightarrow \frac{5}{2} = 3-x$ $x = 0.5$ oe $\frac{1}{9} + \frac{1}{9}("0.5") + \frac{2}{27}("0.5")^2 + \frac{10}{243}("0.5")^3 \quad (=0.19033)$	B1 B1ft
(ii)	$\pm \left(\frac{\frac{24}{125} - "0.19033"}{\frac{24}{125}} [\times 100] \right)$ oe $0.87\% \text{ or } -0.87\%$	M1 A1 [4]
Total 9 marks		

Part	Mark	Additional Guidance
(a)	B1	Correct a , can be left embedded
	B1	Correct b , can be left embedded
(b)	M1	An attempt to use the binomial expansion for their $(1-bx)^{-3}$ The minimally acceptable attempt is as follows: - The power of x must be correct in each term. - The first term is 1 (or "$\frac{1}{9}$" $\times$ 1 ...) - The 2!, 3! are correct (may be unevaluated) - Their "$-\frac{x}{3}$" must appear in at least one term of the expansion. a does not need to be present to attain this mark.
	A1ft	Any two (unsimplified) algebraic terms fully correct in their expansion. Follow through their value for b . a does not need to be present to attain this mark.
	A1	Fully simplified correct expression.
Mark parts (i) and (ii) together. If you see $\frac{24}{125} = 8 \times \frac{3}{125}$ leading to $x = -2$ for part (c) then send to review.		
(c) (i)	B1	Correct identification of $x = 0.5$.
	B1ft	Correct use of their value of x in their expansion. If their x and / or their expansion is incorrect then must show the substitution.
	M1	Uses the correct formula, with their value from part (i) to calculate a percentage error.
	A1	0.87% or -0.87% Awrt 0.87% or awrt -0.87%

Question	Scheme	Marks
7(a)	$\left(\frac{ar^2}{a} = r^2 = \frac{2704}{16}\right) \text{ oe}$ $(r =) \pm \frac{13}{25} \text{ oe}$	M1 A1 [2]
(b)	$(S_{\infty} =) \frac{16}{1 - \frac{13}{25}} = \frac{100}{3} \text{ oe}$	M1A1 [2]
(c)	$\frac{16\left(1 - \left(\frac{13}{25}\right)^n\right)}{1 - \frac{13}{25}} > 33 \left(\Rightarrow 1 - \left(\frac{13}{25}\right)^n > 0.99\right) \text{ oe}$ $\Rightarrow \left(\frac{13}{25}\right)^n < 0.01 \text{ oe}$ $n \lg\left(\frac{13}{25}\right) < \lg(0.01) \text{ oe}$ $\Rightarrow n > \frac{\lg(0.01)}{\lg\left(\frac{13}{25}\right)} \Rightarrow n > 7.04... \text{ oe} \Rightarrow (n =) 8$	M1 dM1 M1 dddM1A1 [5]
Total 9 marks		

Part	Mark	Notes
(a)	M1	For any correct unsimplified expression for r or r^2 .
	A1	For $r = \pm \frac{13}{25}$ oe
(b)	M1	Uses the correct formula for the sum to infinity of a geometric series using their positive $r < 1$ and the given first term.
	A1	For $\frac{100}{3}$ oe. Allow $33.33\dots$, 33.3^r , $33.\dot{3}$ etc. The question demand is for an exact answer so there must be some indication of recurrence. This could be min 1 dp with dots after ie $33.3\dots$, but cannot be a terminating decimal.
(c)	M1	Uses the correct formula for the sum of a geometric series to set up an inequality or equation in terms of n , ft their positive $r < 1$, must be using $a = 16$ Allow $<$ or $>$ or $=$
	dM1	For simplifying (allow errors in simplification) their inequality or equation in n to the form $\left(\frac{13}{25}\right)^n < d$ $d \neq 0$. Allow $\frac{13}{25}$ to be their positive $r < 1$. Allow $<$ or $>$ or $=$. Dependent on the 1 st method mark.
	M1	Takes logarithms (any base) of their exponential equation or inequality correctly on both sides and correctly uses the power law to reach $n \lg(a) < \lg(b)$. or ‘de-logs’ their exponential equation or inequality correctly to get $\log_e d < e$ This isn’t a dependent mark, but the candidate must be able to correctly take logs of both sides and use the power law or correctly de-log their equation/inequality. r can be any value carried through from part (a) for this mark. Allow $<$ or $>$ or $=$, the inequality doesn’t need to have been reversed at this point.
	dddM1	For finding a value for n setting up the correct inequality (with their positive $r < 1$) from the beginning and where the inequality sign has been reversed at the appropriate point during their work. Dependent on all previous method marks. If candidates give a final answer of $(n =) 8$ – this mark can be implied even if the inequality sign is not correctly reversed.
	A1	For $(n =) 8$

Question number	Scheme	Marks
6 (a)	$a = 4 \times 1 - 3 = 1, \quad (d = 4)$ $\sum_{r=1}^n 4r - 3 = \frac{n}{2}(2 \times 1 + (n-1)4) = n(2n-1)^*$	B1 M1A1 [3]
(b)	$n(2n-1) > 1000 \Rightarrow 2n^2 - n - 1000 > 0$ $\frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 2 \times (-1000)}}{2 \times 2} \Rightarrow n > 22.612... \Rightarrow n = 23$	M1 M1A1 [3]
(c)	$3t_{(n+7)} + 18 = S_{(n+4)}$ $\Rightarrow 3[4(n+7) - 3] + 18 = (n+4)[2(n+4) - 1]$ $\Rightarrow 2n^2 + 3n - 65 = 0$ $2n^2 + 3n - 65 = (2n+13)(n-5) = 0 \Rightarrow n = 5$	M1 A1 depM1A1 [4]
Total 10 marks		
(a) B1 M1 A1 (b) M1 M1 A1 (c) M1 A1 depM1 A1	$a = 1$ Use of $S = \frac{n}{2}(2a + (n-1)d)$ or $S = \frac{n}{2}(a + L)$ Obtains the given expression Sets up a 3 term quadratic from the given information (Condone = rather than >) Solve their 3 term quadratic (May be implied by 22.6 ...) $n = 23$ Substitution of $n + 7$ and $n + 4$ A correct 3 term quadratic Solve their 3 term quadratic (Dependent on previous M mark) $n = 5$ (must reject other answer if offered)	

Question number	Scheme	Marks
9 (a)	$\frac{1}{(2-x)^3} = (2-x)^{-3} = \frac{1}{8} \left(1 - \frac{x}{2}\right)^{-3} \Rightarrow p = \frac{1}{8}, q = \frac{1}{2}$	B1B1 [2]
(b)	$\frac{1}{8} \left(1 - \frac{x}{2}\right)^{-3} = \frac{1}{8} \left[1 + (-3) \left(-\frac{x}{2}\right) + \frac{(-3)(-4) \left(-\frac{x}{2}\right)^2}{2!} + \frac{(-3)(-4)(-5) \left(-\frac{x}{2}\right)^3}{3!} \right]$ $= \frac{1}{8} + \frac{3}{16}x + \frac{3}{16}x^2 + \frac{5}{32}x^3 + \dots$	M1 A1A1 [3]
(c)	$(a+bx) \left(\frac{1}{8} + \frac{3}{16}x + \frac{3}{16}x^2 \right) = \frac{a}{8} + x \left(\frac{2b}{16} + \frac{3a}{16} \right) + \left\{ x^2 \left(\frac{3a}{16} + \frac{3b}{16} \right) \right\}$ $\Rightarrow \frac{3}{8} = \frac{a}{8} \Rightarrow a = 3$ $\Rightarrow -\frac{43}{16} = \frac{2b+3a}{16} \Rightarrow 2b = -43 - 9 = -52 \Rightarrow b = -26$	M1 A1 A1 [3]
(d)	$\frac{3a+3b}{16} = \frac{9-78}{16} = -\frac{69}{16} \text{ oe}$	M1A1 [2]
Total 10 marks		
(a)	NB If p and q are stated then they must be correct but if p and q are not stated then $\frac{1}{8} \left(1 - \frac{x}{2}\right)^{-3}$ scores B1B1	
B1	$p = \frac{1}{8}$ (Allow $p = 2^{-3}$)	
B1	$q = \frac{1}{2}$	

(b) M1 A1 A1	Attempts to use the binomial expansion for their $(1 - qx)^{-3}$. Must have first term 1, three more terms with ascending powers of x, 2 or $2!$ and 6 or $3!$ seen, and their $\left(-\frac{x}{2}\right)$ used at least once. No simplification needed. Ignore terms beyond x^3 Two algebraic terms correct in the expansion for their $(1 - qx)^{-3}$.. Must be single fractions, not necessarily in lowest terms. Ignore terms beyond x^3 All four terms correct and in lowest terms. Ignore terms beyond x^3
(c) M1 A1 A1	For either their $\frac{1}{8}a = \frac{3}{8}$ or their $\frac{3}{16}a$ + their $\frac{1}{8}b = -\frac{43}{16}$ May be implied by a correct value of a or b. $a = 3$ $b = -26$ NB answers $a = 3$ and $b = -26$ scores 3/3
(d) M1 A1	Substituting their a and their b into their $\frac{3a + 3b}{16}$ $-\frac{69}{16}$ oe NB If $p = 2$, $q = \frac{1}{2}$ is used then $a = \frac{3}{16}$ and $b = -\frac{13}{8}$ which substituted into $3a + 3b$ gives an answer of $-\frac{69}{16}$ but scores A0

Question Number	Scheme	Marks
6(a)	$a = S_1 = 1(15 + 2 \times 1) = 17$	B1
	$S_2 = 2(15 + 2 \times 2)(= 38) = 2a + d$	M1A1
	$2 \times 17 + d = 38 \Rightarrow d = 4$	A1 (4)
(b)	20th term $= a + 19d = 17 + 19 \times 4 = 93$	M1A1 (2)
(c)	$S_{2p} - 2S_p = 1 + S_{p-1}$	
	$2p(15 + 4p) - 2p(15 + 2p) = 1 + (p-1)(13 + 2p)$	M1
	$2p^2 - 11p + 12 = 0$	A1
	$(2p-3)(p-4) = 0 \Rightarrow p = 4 \left(p \neq \frac{3}{2}; \text{ may not be seen } \right)$	M1A1 (4)
		[10]

Part	Mark	Notes
(a)	B1	For the first term $a = 17$ $[a = S_1 = 1(15 + 2 \times 1) = 17]$
	M1	For the second term. Uses the given summation formula to form a linear equation in a and d for a minimally acceptable response of $k = 2a + d$ where k is a positive integer.
	A1	For the correct linear equation $38 = 2a + d$
	A1	For the correct value of $d = 4$
	ALT 1	
	B1	For the first term $a = 17$
	M1	For using a correct summation formula $n(15 + 2n) = \frac{n}{2}(2a + [n-1]d) \Rightarrow 30 + 2n = 2a - d + nd$ and equates coefficients
	A1	For equating coefficients of n $4n = dn \Rightarrow d = \dots$ and $30 = 2a - 4 \Rightarrow a = \dots$
		For the correct value of $d = 4$

	ALT 2	
	B1	For the first term $a = 17$
	M1	Uses two values of n to set up a pair of simultaneous equations.e.g. $S_4 = 4(15 + 2 \times 4) = 92 \text{ and } 92 = \frac{4}{2}(2a + 3d) \Rightarrow 46 = 2a + 3d$ $S_5 = 5(15 + 2 \times 5) = 125 \text{ and } 125 = \frac{5}{2}(2a + 4d) \Rightarrow 50 = 2a + 4d$
	A1	Attempts to solve the pair of equations
	A1	$d = 4$
(b)	M1	For using the correct n th term formula with their a and their d $U_{20} = '17' + 19 \times '4' = \dots$
	A1	For the correct 20 th term = 93
(c)	M1	Uses the given summation formula with the correct substitution $2p(15 + 4p) - 2p(15 + 2p) = 1 + (p - 1)(13 + 2p)$
	A1	For achieving the correct 3TQ $2p^2 - 11p + 12 = 0$
	ALT	
	M1	Uses the summation formula: Follow through their a and d $S_{2p} = \frac{2p}{2}(2 \times 17 + (2p - 1)4) = p(30 + 8p)$ $2S_p = 2 \times \frac{p}{2}(2 \times 17 + (p - 1)4) = p(30 + 4p)$ $S_{p-1} = \frac{p-1}{2}(2 \times 17 + (2[p-1] - 1)4) = (p-1)(13 + 2p)$ For a correct substitution into the given expression $p(30 + 8p) - p(30 + 4p) = 1 + (p - 1)(13 + 2p)$ oe
	A1	For achieving the correct 3TQ $2p^2 - 11p + 12 = 0$
	M1	For attempting to solve their 3TQ (provided it is a 3TQ) by any valid method. $2p^2 - 11p + 12 = (2p - 3)(p - 4) = 0 \Rightarrow p = \dots, \dots$
	A1	For $p = 4$ If they give both roots of their 3TQ as an answer without rejecting $p = 1.5$ A0

+

Question number	Scheme	Marks
5 (a)	$1 + anx + \frac{n(n-1)}{2}a^2x^2 + \frac{n(n-1)(n-2)}{3!}a^3x^3$	M1 A1 (2)
(b)	$an = 15$ $\frac{n(n-1)}{2}a^2 = \frac{n(n-1)(n-2)}{6}a^3 \Rightarrow (3 = (n-2)a)$ Solving simultaneously leading to $a = \dots$ or $n = \dots$ $a = 6$ and $n = 2.5$	B1 M1A1 M1 A1 A1 (6)
(c)	$\frac{'2.5' \times 1.5 \times 0.5 \times '6'^3}{6} = 67.5$	M1 A1 (2)
Total 10 marks		

Part	Mark	Notes
(a)	M1	For an attempt to expand the given expression up to the term in x^3 The definition of an attempt is as follows: - The first two terms must be correct $[1 + anx]$ - The powers of x must be correct - The denominators must be correct - Simplification not required e.g accept $(ax)^2$ or $(ax)^3$ $(1 + ax)^n = 1 + anx + \frac{n(n-1)}{2}a^2x^2 + \frac{n(n-1)(n-2)}{3!}a^3x^3$
	A1	For a fully correct expansion. Simplification not required.
(b)	B1	For setting $an = 15$
	M1	For setting their coefficient of x^2 equal to their coefficient of x^3 $\frac{n(n-1)}{2}a^2 = \frac{n(n-1)(n-2)}{6}a^3$ Do not condone the presence of either x^2 or x^3 for this mark unless there is later recovery.
	A1	For the fully correct equation simplified or unsimplified. $\frac{n(n-1)}{2}a^2 = \frac{n(n-1)(n-2)}{6}a^3 \Rightarrow (3 = (n-2)a)$
	M1	For simplifying the correct equation above to an equation where the powers of a and n are 1 and attempting to solve the two equations simultaneously leading to $a = \dots$ or $n = \dots$ Condone one arithmetical slip at any point.
	A1	For either $a = 6$ or $n = 2.5$
	A1	For both $a = 6$ and $n = 2.5$
(c)	M1	For substituting their values of a and n into their coefficient of x^3 $\frac{'2.5' \times 1.5 \times 0.5 \times '6'^3}{6} = 67.5$ where $a, n \neq 1$ or 0
	A1	For 67.5

Question number	Scheme	Marks
7 (a)	$\frac{61}{6} = \frac{a(1-r^3)}{1-r} \rightarrow 1$ $\frac{125}{6} = \frac{a}{1-r} \rightarrow 2$	M1
(i)	$1 \div 2 \Rightarrow \frac{6}{125} = \frac{\frac{a(1-r^3)}{6}}{\frac{a}{6}} \Rightarrow \frac{61}{125} = 1-r^3 \Rightarrow r = \frac{4}{5}^*$	dM1A1 cso
(ii)	$\frac{125}{6} = \frac{a}{1-\frac{4}{5}} \Rightarrow a = \frac{25}{6}$	M1A1
	ALT	[6]
	$a + ar + ar^2 = a(1+r+r^2) = \frac{61}{6} \text{ and } \frac{a}{1-r} = \frac{125}{6}$ $\Rightarrow \frac{125}{6} (1-r) (1+r+r^2) = \frac{61}{6} \Rightarrow 125(1-r^3) = 61$	[M1M1 dM1
(i)	$\Rightarrow 1-r^3 = \frac{61}{125} \Rightarrow r^3 = \frac{64}{125} \Rightarrow r = \frac{4}{5}^*$	A1 cso
(ii)	$\frac{125}{6} = \frac{a}{1-\frac{4}{5}} \Rightarrow a = \frac{25}{6}$	M1A1]
(b)	$19.8 < \frac{\frac{25}{6}(1-0.8^n)}{1-0.8} \Rightarrow \frac{19.8 \times 6 \times 0.2}{25} < 1-0.8^n$ $\frac{594}{625} < 1-0.8^n \Rightarrow 0.8^n < \frac{31}{625}$ $n \lg(0.8) < \lg\left(\frac{31}{625}\right)^*$	M1
(c)	$n > \frac{\lg\left(\frac{31}{625}\right)}{\lg(0.8)} \Rightarrow n > 13.461... \Rightarrow n = 14$	A1 cso [2] M1A1 [2]
Total 10 marks		

Part	Mark	Notes
(a)(i)	M1	Forms a correct equation for the sum of the first 3 terms or the sum to infinity. Either $\frac{61}{6} = \frac{a(1-r^3)}{1-r}$ or $\frac{125}{6} = \frac{a}{1-r}$ These formulae must be correct as they are given in the Formulae sheet

(i)	M1	For both $\frac{61}{6} = \frac{a(1-r^3)}{1-r}$ and $\frac{125}{6} = \frac{a}{1-r}$ correct
	dM1	For a valid method to eliminate a from both equations by division or substitution and attempting to re-arrange to find a value for r^3 . E.g., $\frac{125}{6} = \frac{a}{1-r} \Rightarrow a = \frac{125(1-r)}{6} \Rightarrow \frac{61}{6} = \frac{\left[\frac{125(1-r)}{6}\right](1-r^3)}{1-r} \Rightarrow r^3 = 1 - \frac{61}{125}$ This mark is dependent on both previous M marks.
	A1*	For $r = \frac{4}{5}$ This value is given, there must be no errors for the award of this mark
	ALT	
	M1	For either $a + ar + ar^2 = \left[a \frac{1+r+r^2}{1-r}\right] = \frac{61}{6}$ or $\frac{125}{6} = \frac{a}{1-r}$ correct
	M1	For both $a + ar + ar^2 = \left[a \frac{1+r+r^2}{1-r}\right] = \frac{61}{6}$ and $\frac{125}{6} = \frac{a}{1-r}$ correct
	dM1	For a valid method to eliminate a from both equations by division or substitution and attempting to re-arrange to find a value for r^3 . E.g. $\Rightarrow \frac{125}{6} \frac{1-r}{1+r+r^2} = \frac{61}{6} \Rightarrow 125 \frac{1-r}{1+r+r^2} = 61 \Rightarrow r^3 = 1 - \frac{61}{125}$ This mark is dependent on both previous M marks.
	A1	For $r = \frac{4}{5}$ This value is given, there must be no errors for the award of this mark
	M1	For substituting $\frac{4}{5}$ into either their $\frac{61}{6} = \frac{a(1-r^3)}{1-r}$ or their $\frac{125}{6} = \frac{a}{1-r}$ or their $a + ar + ar^2 = \frac{61}{6}$ (where their expression for a sum has been seen earlier) to find a value of a
	A1	For $a = \frac{25}{6}$
(b)	M1	For using the correct formula for the sum of a geometric series with $r = 0.8$ [oe] and their a , setting up an inequality (allow $<$ or $>$ for this mark) using the value of 19.8 and attempting to achieve the given result.
	A1*	For the correct inequality as shown with no errors as this is a given result. $n \lg(0.8) < \lg\left(\frac{31}{625}\right)$
(c)	M1	For solving the given inequality in n using logarithms. They must achieve a value for n for this mark. Allow use of $<$, $>$, or $=$ for this mark.
	A1	For $n = 14$

Question number	Scheme	Marks
5 (a)	$[S_{\infty}] = \frac{12}{1 - \frac{3}{8}} = \frac{96}{5}$	M1 A1 (2)
(b)	$ar^5 = 12\left(\frac{3}{8}\right)^5$	M1
	$= \frac{2^2 \times 3 \times 3^5}{2^{15}} = \frac{3^6}{2^{13}} \quad *$	M1 A1cso (3)
(c)	e.g. $u_n = 12\left(\frac{3}{8}\right)^{n-1}$	M1
	$\log_2 u_n = \log_2 12 + \log_2 \left(\frac{3}{8}\right)^{n-1}$	M1
	$\log_2 u_n = \log_2 12 + (n-1)[\log_2 3 - \log_2 8]$	M1
	$\log_2 u_n = \log_2 3 + 2\log_2 2 + (n-1)[\log_2 3 - 3\log_2 2]$	M1
	$\log_2 u_n = n\log_2 3 - 3n + 5 \quad *$	A1 cso (5)
Total 10 marks		

Part	Mark	Notes
(a)	M1	For the correct use of $\frac{a}{1-r}$
	A1	For the correct value $\frac{96}{5}$
(b)	M1	For the correct use of ar^{n-1}
	M1	For expressing 12 ($2^2 \times 3$) and 8 (2^3) as powers of 2
	A1cso	For obtaining the given expression with no errors seen.
(c)	M1	For the correct use of ar^{n-1} and the given values of a and r to give $u_n = 12\left(\frac{3}{8}\right)^{n-1}$
	M1	For taking logs [base 2] of both sides and applying the addition law.
	M1	For applying the power law and subtraction laws to $\log_2\left(\frac{3}{8}\right)^{n-1}$ $\log_2\left(\frac{3}{8}\right)^{n-1} = (n-1)[\log_2 3 - \log_2 8]$
	M1	For obtaining $\log_2 12 = \log_2 3 + 2\log_2 2$
	A1cso	For obtaining the given equation with no errors seen.
	ALT	
	M1	For the correct use of ar^{n-1} and the given values of a and r to give $u_n = 12\left(\frac{3}{8}\right)^{n-1}$
	M1	For rearranging the equation to obtain: $U_n = 12\left(\frac{3}{8}\right)^{n-1} = 2^2 \times 3 \times \frac{3^{n-1}}{2^{3(n-1)}} = \frac{3^n}{2^{3n-5}}$
	M1	For taking logs of both sides and applying the addition (subtraction) law. $\log_2 U_n = \log_2 3^n - \log_2 (3n-5)$
	M1	For applying the power law to obtain: $\log_2 U_n = n \log_2 3 - \log_2 (3n-5)$
	A1cso	For obtaining the given equation with no errors seen.

Question	Scheme	Marks
8(a)	$U_5 = \left(\frac{25}{4}\right)\left(\frac{3}{5}\right)^5 = \frac{243}{500}$ oe	B1 [1]
(b)	$U_1 = \left(\frac{25}{4}\right)\left(\frac{3}{5}\right)^1 = \frac{15}{4}$ oe $S_n = \left(\frac{15}{4}\right)\left(\frac{3}{5}\right)^0 + \left(\frac{15}{4}\right)\left(\frac{3}{5}\right)^1 + \dots + \left(\frac{15}{4}\right)\left(\frac{3}{5}\right)^{n-1}$ $\Rightarrow S_n = \sum_{r=1}^n \left(\frac{15}{4}\right)\left(\frac{3}{5}\right)^{r-1}$ oe	B1 M1 A1 [3]
	ALT 1	
	$U_1 = \left(\frac{25}{4}\right)\left(\frac{3}{5}\right)^1 = \frac{15}{4}$ $r = \frac{3}{5} \quad U_n = \frac{15}{4} \times \left(\frac{3}{5}\right)^{n-1}$ $S_n = \sum_{r=1}^n \frac{15}{4} \times \left(\frac{3}{5}\right)^{r-1}$	[B1 M1 A1]
	ALT 2	
	$S_n = \sum_{r=1}^n \frac{25}{4} \left(\frac{3}{5}\right)^r$ $= \sum_{r=1}^n \frac{25}{4} \times \frac{3}{5} \times \left(\frac{3}{5}\right)^{r-1}$ $= \sum_{r=1}^n \frac{15}{4} \times \left(\frac{3}{5}\right)^{r-1}$	[B1 M1 A1]
(c)	$S_\infty = \frac{\frac{15}{4}}{1 - \frac{3}{5}} = \frac{75}{8}$ or 9.375 $S_n = \frac{\frac{15}{4} \left(1 - \left[\frac{3}{5}\right]^n\right)}{1 - \frac{3}{5}}$ $9.375 - \frac{\frac{15}{4} \left(1 - \left[\frac{3}{5}\right]^n\right)}{1 - \frac{3}{5}} < 0.045 \Rightarrow \frac{\frac{15}{4} \left(1 - \left[\frac{3}{5}\right]^n\right)}{1 - \frac{3}{5}} > 9.33$ $\frac{\frac{15}{4} \left(1 - \left[\frac{3}{5}\right]^n\right)}{1 - \frac{3}{5}} > 9.33 \Rightarrow 1 - \left[\frac{3}{5}\right]^n > 0.9952 \Rightarrow \left[\frac{3}{5}\right]^n < \frac{3}{625}$ $n \lg \left[\frac{3}{5}\right] < \lg \frac{3}{625} \Rightarrow n > 10.45$ $n = 11$	M1M1 dM1 ddM1A1 A1 [6]
Total 10 marks		



Part	Mark	Notes
Mark (a) and (b) together		
(a)	B1	For the correct value only. Decimal value is 0.486.
(b)	B1	Finds the value of the first term. Must be identified as U_1 / first term.
	M1	Forms the sum of the series with at least the first term and the n th term.
	A1	For the correct expression.
	ALT 1	
	B1	Finds the value of the first term. Must be identified as U_1 / first term.
	M1	For finding r and writing $U_n = \frac{15}{4} \times \left(\frac{3}{5}\right)^{n-1}$
	A1	For the correct expression.
	ALT 2	
	B1	For correct summation statement using U_n from the question
	M1	For showing process to change index from r to $r - 1$ and find the value of $\frac{A}{B}$ Minimum working is $\frac{25}{4} \times \left(\frac{3}{5}\right)^r = \frac{25}{4} \times \frac{3}{5} \times \left(\frac{3}{5}\right)^{r-1}$. May be with n as index.
	A1	For the correct expression.
(c)	Trial and improvement approaches for (c) should be sent to review.	
	M1	Correct method to find the sum to infinity using the correct formula with their $a = \frac{15}{4}$ and $r = \frac{3}{5}$. Their a should be $\frac{A}{B}$ if found in (b). Condone if working with $a = \frac{25}{4}$, r must be correct.
	M1	Using the correct formula, forms the sum to n terms using their $a = \frac{15}{4}$ and $r = \frac{3}{5}$. Their a should be $\frac{A}{B}$ if found in (b). Condone if working with $a = \frac{25}{4}$, r must be correct.
	dM1	Forms the inequality in terms of n with their S_∞ and S_n and simplifies to $\left[\frac{3}{5}\right]^n < \frac{3}{625}$, Must have dealt with negative. Allow for working with equation if inequality sign reinstated later. Reinstatement can be implied by rounding up to the next integer value once n found. Dep M1M1
	ddM1	Uses logarithms correctly to attempt to find the value of n They must reverse the inequality for this mark. Allow for working with equation if inequality sign reinstated later. Reinstatement can be implied by rounding up to the next integer value once n found. Dep previous M mark. Allow use of log base $\frac{3}{5}$.
	A1	For correct n awrt 10.5 Can be implied by $n = 11$ if decimal not seen.
	A1	For $n = 11$

Question	Scheme	Marks
8(a)	$42 = \frac{4}{2}(2a + (4-1)d) \Rightarrow 42 = 4a + 6d$ oe $23 = a + 4d$ $\Rightarrow a = 3, \quad d = 5$ $n\text{th term} = 3 + (n-1)5 = 5n - 2 \Rightarrow S_n = \sum_{r=1}^n (5r - 2)$ $[P = 5, Q = 2]$	B1 B1 M1A1 M1A1 [6]
(b)	$S_{2n} - 3U_n = 1062$ $\Rightarrow \frac{2n}{2}(2 \times '3' + (2n-1)'5') - 3['3' + (n-1)'5'] = 1062$ $\Rightarrow 10n^2 + n - 15n + 6 = 1062 \Rightarrow 10n^2 - 14n - 1056 = 0$ $10n^2 - 14n - 1056 = (5n + 48)(2n - 22) = 0$ $\Rightarrow n = 11$	M1 M1 M1 A1 [4]
Total 10 marks		

Part	Mark	Notes
(a)	B1	Forms a correct equation in a and d for either the sum of the first 4 terms or for the 5 th term This must be correct.
	B1	Forms correct equations in a and d for both the sum of the first 4 terms and for the 5 th term Both must be correct.
	M1	Solves their two equations simultaneously by any method. Accept a pair of equations as follows: $pa + qd = 21$ or 42 $ra + sd = 23$ where $p \neq r$ and $q \neq s$ Allow a maximum of one arithmetical error in the solution of their SE.
	A1	For both $a = 3$ and $d = 5$
	M1	Attempts to form the required expression using their values of a and d They must use the n th term for this.
	A1	For the correct expression as written. This must be exactly as written in the question with the inclusion of the Σ . However, allow the omission of $S_n = \dots$.
	ALT for last 2 marks	
	M1	$S_n = \sum_{r=1}^n (Pr - Q) \Rightarrow 3 = P - Q \quad 5 = (2P - Q) - (P - Q) = P$ $\Rightarrow 3 = 5 - Q \Rightarrow Q = 2$
	A1	For the correct expression as written. This must be exactly as written in the question with the inclusion of the Σ . However, allow the omission of $S_n = \dots$ Allow also $S_n = \sum_{r=1}^n (Pr - Q) \quad (\Rightarrow) \quad P = 5, Q = 2$ for this mark.
(b)	M1	Applies the summation formula and the n th term formula correctly to form an equation in n using their values for a and d For this mark you must see an equation, e.g., $\frac{2n}{2}(2 \times 3 + (2n - 1)5) - 3[3 + (n - 1)5] = 1062$ However, allow recovery in the next step.
	M1	Forms a 3TQ $10n^2 - 14n - 1056 = 0$ o.e. Condone missing $= 0$ if it is clear they are solving this equation.
	M1	Attempts to solve their 3TQ Accept any method. If their 3TQ is incorrect, only award this mark when you can see their method. If a calculator is used with an incorrect 3TQ and no method is seen, this mark is not available.

		Note: We do not need to see $-\frac{48}{5}$. Some candidates will automatically reject this solution.
	A1	For the correct value of n Do not award if $-\frac{48}{5}$ is included as a value.

Question number	Scheme	Marks
7 (a)	(i) $\frac{U_7}{U_3} = \frac{ar^6}{ar^2} = r^4 \Rightarrow \frac{16}{4} = r^4 \Rightarrow 4 = r^4 \Rightarrow 2 = r^2 \Rightarrow r = \sqrt{2} *$ (ii) $ar^2 = 4 \Rightarrow a = \frac{4}{(\sqrt{2})^2} = 2$	M1 A1cso A1 [3]
Alternative		
(a)	(i) $ar^2 = 4, ar^6 = 16 \Rightarrow a\left(\frac{4}{a}\right)^3 = 16 \Rightarrow a^2 = 4 \Rightarrow a = 2$ $r^2 = \frac{4}{2} \Rightarrow r = \sqrt{2} *$ (ii) $a = 2$	M1 A1cso A1
(b)	$500 < 2 \times \sqrt{2}^{(n-1)} \Rightarrow 250 < \sqrt{2}^{(n-1)}$ or $500 < 2^{\frac{1}{2}(n+1)}$ $\lg 250 < (n-1) \lg \sqrt{2}$ or $\lg 500 < \frac{1}{2}(n+1) \lg 2$ $n-1 > \frac{\lg 250}{\lg \sqrt{2}} \Rightarrow n-1 > 15.93...$ or $\frac{1}{2}(n+1) > \frac{\lg 500}{\lg 2} \Rightarrow \frac{1}{2}(n+1) > 8.96...$ $n > 16.93... \Rightarrow n = 17$	M1 M1 M1A1 [4]
(c)	$S_{20} = \frac{2\left((\sqrt{2})^{20} - 1\right)}{\sqrt{2} - 1} = \frac{2046}{\sqrt{2} - 1}$ $\frac{2046}{(\sqrt{2} - 1)} \times \frac{(\sqrt{2} + 1)}{(\sqrt{2} + 1)} = 2046\sqrt{2} + 2046 = 2046(1 + \sqrt{2})$	M1A1 M1A1 [4]
Total 11 marks		

(a)(i)	M1	$ar^2 = 4$ and $ar^6 = 16$
	A1 cso	Correct working to eliminate a and show that $r = \sqrt{2}$ (Answer given) Must see $r^4 = 4$.
(ii)	A1	$a = 2$ (NB: This is not a B mark; must have scored the first M mark.)
Alternative		
(a)(i)	M1	$ar^2 = 4$ and $ar^6 = 16$
	A1 cso	Correct working to find a and use it to show that $r = \sqrt{2}$ (Answer given) Must see a correct substitution to eliminate r which is processed correctly to give $a = 2$, and $r^2 = \frac{4}{2}$ or $r^6 = \frac{16}{2}$.
(ii)	A1	$a = 2$ (Must have scored the first M mark.)

(b)	M1	$500 < 2 \times (\sqrt{2})^{(n-1)}$ Accept an equation or $>$. Accept $500 < 2 \times \sqrt{2}^{n-1}$ ft their value for a
	M1	$n-1 > \frac{\lg 250}{\lg \sqrt{2}}$ or $\frac{1}{2}(n-1) > \frac{\lg 250}{\lg 2}$ or $\frac{1}{2}(n+1) > \frac{\lg 500}{\lg 2}$ or $\lg_{\sqrt{2}} 250 < n-1$ Accept equation or $>$, and logs with any base. ft $500 < 2 \times (\sqrt{2})^n$
	M1	Evaluate and divide logs and attempt to find a value (at least 1DP) or numerical expression for n . Ignore incorrect inequality sign. eg $n > 16.9$ or $n = 16.9$ or $n > 1 + 15.93$ or $n = 1 + 2 \times 7.96$ ft $500 < 2 \times (\sqrt{2})^n$
	A1	$n = 17$ Do not award after incorrect inequalities e.g. $n < 16.9$
Alternative		
(b)	M1	$500 < 2 \times (\sqrt{2})^{(n-1)}$ Accept an equation or $>$. Accept $500 < 2 \times \sqrt{2}^{n-1}$ ft their value for a
	M1	Attempt to use trial and improvement, showing two trials that can be used to confirm the required value of n , eg $(\sqrt{2})^{15} = 181.0...$ and $(\sqrt{2})^{16} = 256$ or $2(\sqrt{2})^{15} = 362.0...$ and $2(\sqrt{2})^{16} = 512$ or $2^8 = 256$ and $2^9 = 512$ ft $500 < 2 \times (\sqrt{2})^n$ if clearly stated
	M1	Use the higher power from these trials in an appropriate equation eg $n-1 = 16$ or $\frac{1}{2}(n+1) = 9$ and attempt to find a value for n . ft $500 < 2 \times (\sqrt{2})^n$ if clearly stated
	A1	$n = 17$
(c)	M1	Attempt to use $S_n = \frac{a(1-r^n)}{(1-r)}$ with their $a, r = \sqrt{2}$ and $n = 20$ Allow notation or sign errors if formula is stated, otherwise the substitution must be correct.
	A1	$S_{20} = \frac{2046}{\sqrt{2}-1}$ or $S_{20} = -\frac{2046}{1-\sqrt{2}}$ Correct expression with numerator evaluated.
	M1	Attempt to rationalise the denominator. Must use $\sqrt{2}+1$ and attempt to simplify the denominator.
	A1	$2046(1+\sqrt{2})$ or $2046(\sqrt{2}+1)$ Accept $p = 2046$. Award full marks for a correct answer if first M1 has been awarded.

Question number	Scheme	Marks
7 (a)(i)	$a + 4d = 4x + 6$ $a + 7d = 7x + 3$ $\Rightarrow 3d = 3x - 3 \Rightarrow d = x - 1^*$	M1A1cso
(ii)	$a + 7(x - 1) = 7x + 3 \Rightarrow a = 10$ or $a + 4(x - 1) = 4x + 6 \Rightarrow a = 10$	M1A1 [4]
(b)	$42 = 10 + 8(x - 1) \Rightarrow x = 5$	M1A1 [2]
(c)	$d = x - 1 \Rightarrow d = 5 - 1 = 4$ $S_{n+1} = 12U_n + 18 \Rightarrow \frac{n+1}{2}(2 \times 10 + [(n+1) - 1]4) = 12[10 + (n-1)4] + 18$ $\Rightarrow n^2 - 18n - 40 = 0$ $\Rightarrow (n - 20)(n + 2) = 0 \Rightarrow n = 20$	B1 M1 M1A1 [5]
Total 11 marks		
(a) (i) M1 A1 cso (a) (ii) M1 A1 (b) M1 A1 (c) B1 M1 M1 M1 A1	$a + 4d = 4x + 6$ and $a + 7d = 7x + 3$ Obtains the given answer with no errors in the working Substitution of $d = x - 1$ $a = 10$ Use of $a + 8d = 42$ $x = 5$ $d = 4$ Use of $\frac{n}{2}(2a + (n - 1)d)$ Simplifying to $n^2 - 18n - 40 = 0$ Solving the 3TQ $n = 20$ if shown must reject $n = -2$	

Question number	Scheme	Marks
9 (a)	$a = 5 \times 1 - 1 = 4$ $d = 5$ $S_n = \frac{n}{2}(2 \times 4 + [n-1]5) = \frac{n}{2}(3 + 5n) * \text{cso}$ ALT $a = 4$ $l = 5n - 1$ $S_n = \frac{n}{2}(a + l) = \frac{n}{2}(4 + 5n - 1) = \frac{n}{2}(3 + 5n) * \text{cso}$	B1 M1A1 [3] [B1 M1A1]
(b)	$\sum_{r=10}^{20} (5r-1) = \sum_{r=1}^{20} (5r-1) - \sum_{r=1}^9 (5r-1)$ $\sum_{r=10}^{20} (5r-1) = \frac{20}{2}(3 + 5 \times 20) - \frac{9}{2}(3 + 5 \times 9)$ $= 1030 - 216 = 814$ ALT $a = 4 + 9 \times 5 = 49, l = 4 + 20 \times 5 = 99, n = 20 - 10 + 1 = 11$ $\sum_{r=10}^{20} (5r-1) = \frac{11}{2}(49 + 99) = 814$ ALT $a = 5 \times 10 - 1 = 49$ $d = 5$ $n = 11$ $S_n = \frac{11}{2}(2 \times 49 + (11 - 1) \times 5) = 814$	B1 M1 A1 [3] [B1 M1A1] [B1 M1A1]
(c)	$\frac{n}{2}(3 + 5n) = 12(4 + 5n) + 52 \Rightarrow 5n^2 - 117n - 200 = 0$ $\Rightarrow (n - 25)(5n + 8) = 0 \Rightarrow n = 25, \left(n \neq -\frac{8}{5}\right)$	M1M1A1 M1A1 [5]
Total 11 marks		

Part	Mark	Notes
(a)	B1	For finding the first term and common difference. $a = 5 \times 1 - 1 = 4$ $d = 5$ May be implied by correct values seen in summation formula.
	M1	Uses a correct form of the summation formula for an arithmetic series with their a and their d provided their a and their d are stated. $S_n = \frac{n}{2}(2 \times' 4' + [n - 1] \times' 5')$
	A1 cso	For obtaining the given answer in full with no errors. $\sum_{r=1}^n (5r - 1) = \frac{n}{2}(3 + 5n)$
Alternative method		
	B1	For finding the first term and an expression for the last term. $a = 4$ $l = 5n - 1$

		May be implied by correct values seen in summation formula.
	M1	Uses a correct form of the summation formula for an arithmetic series with their a and their l provided their a and their l are stated. $S_n = \frac{n}{2}(a + l) = \frac{n}{2}('4' + '5n - 1')$
	A1 cs0	For obtaining the given answer in full with no errors. $\sum_{r=1}^n (5r - 1) = \frac{n}{2}(3 + 5n)$
Note: If standard summation results are correctly used award B1M1A1, if not fully correct send to review.		
(b)	B1	For correctly giving the required summation as the difference between two summations starting at $r = 1$. $\sum_{r=10}^{20} (5r - 1) = \sum_{r=1}^{20} (5r - 1) - \sum_{r=1}^9 (5r - 1)$
	M1	For substitution of $n = 20$ and $n = '9'$ into the result from part (a) and subtracting. $\frac{20}{2}(3 + 5 \times 20) - \frac{'9'}{2}(3 + 5 \times '9')$ Allow for use of 9 or 10.
	A1	For the correct summation 814
Alternative method		
	B1	For finding the first term, last term and number of terms for the arithmetic sequence. $a = 49, l = 99, n = 11$
	M1	Uses a correct summation formula for an arithmetic series with their a , their l and their n provided these are stated. $\frac{'11'}{2}('49' + '99')$
	A1	For the correct summation 814
Alternative method – considering this as a series starting at the 10th term of the original series		
	B1	For finding the first term, common difference and number of terms. $a = 5 \times 10 - 1 = 49$ $d = 5$ $n = 11$
	M1	Uses a correct form of the summation formula for an arithmetic series with their a , their d , and their n provided their a and their d and their n are stated and their $n \neq 20$ $S_n = \frac{'11'}{2}(2 \times '49' + ('11' - 1) \times '5')$
	A1	For the correct summation 814
(c)	M1	Uses $5r - 1$ with $n + 1$ to find an expression for u_{n+1} in terms of n . $5r - 1 = 5(n + 1) - 1 = 5n + 4$
	M1	Forms a correct equation for n using the result given in part (a) and their expression for $5r - 1$ in terms of n . $\frac{n}{2}(3 + 5n) = 12('4 + 5n') + 52$
	A1	Obtains a correct 3TQ $5n^2 - 117n - 200 = 0$ oe
	M1	For an attempt to solve their 3TQ. See General Guidance for the definition of an attempt. $(n - 25)(5n + 8) = 0 \Rightarrow n = 25, (n = -\frac{8}{5})$
	A1	For correct value: $n = 25$ If $n = -\frac{8}{5}$ is seen it must be rejected.

Question	Scheme	Marks
9 (a)	$(1-2x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2} \times -2x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2}{2!} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-2x)^3}{3!} + \dots$ $(1-2x)^{-\frac{1}{2}} = 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3 + \dots$	M1 A1A1 [3]
(b)	$\frac{1}{\sqrt{0.96}} = \frac{1}{\sqrt{\frac{96}{100}}} = \frac{10}{4\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \dots$ $\frac{10\sqrt{6}}{4 \times 6} = \frac{5\sqrt{6}}{12} *$	M1 A1 cso [2]
ALT – confirming given result		
	$\frac{1}{\sqrt{0.96}} = \frac{5\sqrt{6}}{12} \Rightarrow 12 = \sqrt{0.96} \times 5\sqrt{6}$ $12^2 = 0.96 (5\sqrt{6})^2 = 0.96 \times 5^2 \times 6 *$	[M1 A1cso]
(c)	$\frac{1}{(5\sqrt{6}-12)} \times \frac{(5\sqrt{6}+12)}{(5\sqrt{6}+12)}$ $= \frac{5\sqrt{6}+12}{150-12^2} = \frac{5\sqrt{6}+12}{6} = \frac{5\sqrt{6}}{6} + 2$	M1 A1 [2]
(d)	$1-2x=0.96 \Rightarrow 2x=0.04 \Rightarrow x=0.02$ $\frac{9}{5\sqrt{6}-12} = 9 \left(2 \times \left[\frac{5\sqrt{6}}{12} \right] + 2 \right) = 9 \times \left[2 \left(1 + 0.02 + \frac{3}{2} \times 0.02^2 + \frac{5}{2} \times 0.02^3 \right) + 2 \right] = \dots$ 36.37116	B1 M1:M1 A1 [4]
Total 11 marks		

Part	Mark	Notes
(a)	M1	For an attempt to use the Binomial Expansion The minimally acceptable attempt is as follows; - The power of x must be correct in each term. $[x, x^2 \text{ and } x^3]$ - The first term is 1 - The denominators are correct $-2x$ correct in each term $(1-2x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2} \times -2x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2}{2!} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-2x)^3}{3!} + \dots$
	A1	The first term and one algebraic term correct and simplified

		$(1-2x)^{-\frac{1}{2}} = 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3 + \dots$
	A1	Fully correct simplified expansion as shown above.
(b)	M1	For changing 0.96 to $\frac{96}{100}$ or equivalent fraction and attempting to multiply numerator and denominator by either $\sqrt{6}$ (or $\sqrt{96}$) $\frac{1}{\sqrt{0.96}} = \frac{1}{\sqrt{\frac{96}{100}}} = \frac{10}{4\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \dots$
	A1 cso	For the correct answer as shown with no errors $\frac{10\sqrt{6}}{4 \times 6} = \frac{5\sqrt{6}}{12} *$
ALT – confirming given result		
	M1	For rearranging and squaring OR for squaring on both sides
	A1 cso	For showing that the two sides of the result are equal $12^2 = 0.96 (5\sqrt{6})^2 = 0.96 \times 5^2 \times 6 = 144$
(c)	M1	For multiplying numerator and denominator by $5\sqrt{6} + 12$ $\frac{1}{(5\sqrt{6} - 12)} \times \frac{(5\sqrt{6} + 12)}{(5\sqrt{6} + 12)}$ [Can be implied by $\frac{(5\sqrt{6} + 12)}{(150 - 12^2)}$ seen]
	A1	For a correct expansion of brackets throughout. $\frac{1}{(5\sqrt{6} - 12)} \times \frac{(5\sqrt{6} + 12)}{(5\sqrt{6} + 12)} \left[= \frac{5\sqrt{6} + 12}{150 - 12^2} \right] = \frac{5\sqrt{6} + 12}{6} = \frac{5\sqrt{6}}{6} + 2$
(d)	B1	For finding the required value of x $1 - 2x = 0.96 \Rightarrow 2x = 0.04 \Rightarrow x = 0.02$
	M1	For substituting their value of x provided it is $-\frac{1}{2} < x < \frac{1}{2}$ into the expansion as follows: $\frac{9}{5\sqrt{6} - 12} = 9 \left(2 \times \left[\frac{5\sqrt{6}}{12} \right] + 2 \right) =$ Note: Must show substitution if x is incorrect.
	M1	For substituting their expansion for $\frac{5\sqrt{6}}{12}$ $9 \times \left[2 \left(1 + 0.02 + \frac{3}{2} \times 0.02^2 + \frac{5}{2} \times 0.02^3 \right) + 2 \right] = \dots$
	A1	For the value of 36.37116 [The calculator value is 36.37117]

Question number	Scheme	Marks
5 a	$(2+3x)^{-1} = \frac{1}{2} \left(1 + \frac{3}{2}x \right)^{-1}$ so $p = \frac{1}{2}$ and $q = \frac{3}{2}$ oe	B1 B1 (2)
b	$\left(\frac{1}{2} \right) \left[1 + (-1) \left(\frac{3}{2}x \right) + \frac{(-1)(-2)}{2!} \left(\frac{3}{2}x \right)^2 + \frac{(-1)(-2)(-3)}{3!} \left(\frac{3}{2}x \right)^3 + \dots \right]$ $\frac{1}{2} - \frac{3}{4}x + \frac{9}{8}x^2 - \frac{27}{16}x^3 + \dots$	M1 A1ft A1 (3)
c	$\frac{1}{2} - \frac{3}{4}x + \frac{9}{8}x^2 - \frac{27}{16}x^3 + \frac{1}{2}x - \frac{3}{4}x^2 + \frac{9}{8}x^3 + \dots$ $\frac{1}{2} - \frac{1}{4}x + \frac{3}{8}x^2 - \frac{9}{16}x^3$	M1 A1 (2)
d	$\int_0^{0.5} \left(\frac{1}{2} - \frac{1}{4}x + \frac{3}{8}x^2 - \frac{9}{16}x^3 \right) dx = \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{8}x^3 - \frac{9}{64}x^4$ $\frac{1}{2} \left(\frac{1}{2} \right) - \frac{1}{8} \left(\frac{1}{2} \right)^2 + \frac{1}{8} \left(\frac{1}{2} \right)^3 - \frac{9}{64} \left(\frac{1}{2} \right)^4 = 0.2256 \text{ for awrt } 0.2256$	M1 A1 M1 A1 (4)
Total 11 marks		

Part	Marks	Notes
(a)	B1	For $p = \frac{1}{2}$ or 2^{-1} or $q = \frac{3}{2}$
	B1	For $p = \frac{1}{2}$ or 2^{-1} and $q = \frac{3}{2}$
		NB $\frac{1}{2}\left(1 + \frac{3}{2}x\right)^{-1}$ scores B1 B1
(b)	M1	For an attempt to expand $(1 + qx)^{-1}$ with their value of q up to the term in x^3 It is not necessary to see p at this stage. The definition of an attempt is as follows: • The first term must be 1 • The next term must be correct for their value of q • The powers of qx must be correct eg $(qx)^2$ • The denominators must be correct Simplification not required. Do not allow missing brackets unless recovered later – this is a general point of marking.
	A1ft	For at least 3 terms fully correct and unsimplified for their value of q . It is not necessary to see p at this point.
	A1	For all 4 terms correct, all simplified.
		If there are any other methods used – send this to review please.
(c)	M1	For multiplying their expansion by $(1 + x)$ There must be a clear attempt to multiply to get 7 terms, allow up to 2 errors. Ignore terms with powers higher than 3.
	A1	For all 4 terms correct, ignore terms with powers higher than 3.
(d)	M1	For a minimally acceptable attempt to integrate an expression with at least 3 terms, which must include a term in x^3 See general guidance for definition of minimally acceptable attempt.
	A1ft	For a fully correct integration of their expression (defined as above for the M mark).
	M1	For substitution of limits into a changed expression, the correct way round – this can be implied if the final answer is correct. We don't need to see 0 substituted in.
	A1	For awrt 0.2256 (Note the calculator value is 0.2288461.....)

7(a)	$\left(1 + \frac{x}{3}\right)^{-3} = \left[1 + (-3)\left(\frac{x}{3}\right) + \frac{(-3)(-3-1)}{2!}\left(\frac{x}{3}\right)^2 + \frac{(-3)(-3-1)(-3-2)}{3!}\left(\frac{x}{3}\right)^3 \dots\right]$ $= 1 - x + \frac{2}{3}x^2 - \frac{10}{27}x^3$	M1A1 A1 [3]
(b)	$-3 < x < 3$ or $ x < 3$	B1 [1]
(c)	$\left((3+x)^{-3} = 3^{-3} \times \left(1 + \frac{x}{3}\right)^{-3} = \frac{1}{27} \times \left(1 + \frac{x}{3}\right)^{-3}\right)$ $P = \frac{1}{27}, Q = \frac{1}{3}$	B1, B1 [2]
(d)	$\frac{(1+4x)}{(3+x)^3} = (1+4x) \times \frac{1}{27} \times \left(1 - x + \frac{2}{3}x^2\right)$ $= \frac{1}{27} \times \left(1 - x + \frac{2}{3}x^2 + 4x - 4x^2 + \dots\right)$ $= \frac{1}{27} \left(1 + 3x - \frac{10x^2}{3}\right) \quad \text{or} \quad \frac{1}{27} + \frac{x}{9} - \frac{10x^2}{81}$	M1 A1 [2]
(e)	$\int_0^{0.2} \left(\frac{1}{27} + \frac{x}{9} - \frac{10x^2}{81}\right) dx = \left[\frac{x}{27} + \frac{x^2}{18} - \frac{10x^3}{243}\right]_0^{0.2}$ $= \left(\frac{0.2}{27} + \frac{0.2^2}{18} - \frac{10 \times 0.2^3}{243}\right) - [0]$ $= 0.0093004$	M1 M1 A1 [3]
Total 11 marks		

Part	Mark	Notes
(a)	M1	For applying a correct binomial expansion in unsimplified form. Minimum required: - The expansion begins with 1 - The next term is correct - The powers of $\frac{x}{3}$ must be correct eg $\left(\frac{x}{3}\right)^2$ - The denominators are correct. Do not allow missing brackets unless recovered later – this is a general point of marking. Ignore any terms with powers higher than 3.
	A1	Following M1 (this is a general point of marking, A marks can only follow M marks), all conditions above met, must see $1 - x$ and at least the term in x^2 or x^3 correct and simplified. Ignore any terms with powers higher than 3.
	A1	A fully correct and simplified expansion. Ignore any terms with powers higher than 3..
(b)	B1	For the correct validity.
(c)	B1	For the correct value of P or Q explicitly written or embedded in $\frac{1}{27} \times \left(1 + \frac{x}{3}\right)^{-3}$
	B1	For the correct values of P and Q explicitly written or embedded in $\frac{1}{27} \times \left(1 + \frac{x}{3}\right)^{-3}$
(d)	M1	For attempting to multiply their expansion, which must be of the form " P " $\times$ (their expansion from part (a), with a minimum of 3 terms, by $(1 + 4x)$. An attempt must include 3 correctly multiplied out terms of their expansion before simplification. The " P " may remain factorised. Ignore any terms which to powers of x higher than 2.
	A1	For the correct expansion in either form shown. Allow equivalent coefficients. Ignore powers of x higher than 2.
(e)	M1	For an attempt to integrate their expression from part (d), provided it has at least one constant term and at least one algebraic term. See general guidance, no power of x must decrease.
	M1	For substituting in the value of 0.2 into a changed expression and subtracting the correct way. Must see the explicit substitution of 0.2 at least once, if the final answer is not correct. Can be implied if final answer correct. Substitution of 0 does not need to be seen.
	A1	For the value of 0.0093004 [The calculator value is 0.0093316] Accept the value of $\frac{113}{12150}$ if seen.

Question	Scheme	Marks
9(a)	$(1-8x^2)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-8x^2) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)(-8x^2)^2}{2!}$ $+ \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)(-8x^2)^3}{3!}$ $(1-8x^2)^{-\frac{1}{2}} = 1 + 4x^2 + 24x^4 + 160x^6 + \dots$	M1 A1A1 [3]
(b)	$\left((a+bx)(1+4x^2+24x^4+160x^6)\right) = (a+bx+4ax^2) + 4bx^3 + 24ax^4$ $4b = 20 \Rightarrow b = 5$ $24a = 48 \Rightarrow a = 2$	M1A1ft A1 A1 [4]
(c)	$\left(\int_0^{0.2} g(x) \, dx = \int_0^{0.2} \left(\frac{2+5x}{\sqrt{1-8x^2}}\right) dx = \right)$ $\int_0^{0.2} ("2" + "5"x + "8"x^2 + "20"x^3 + "48"x^4) \, dx$ $\left(\int_0^{0.2} g(x) \, dx = \right) \left["2x + \frac{5x^2}{2} + \frac{8x^3}{3} + \frac{20x^4}{4} + \frac{48x^5}{5} " \right]_0^{0.2}$ $\left(\int_0^{0.2} g(x) \, dx = \right) "2 \times 0.2 + \frac{5 \times 0.2^2}{2} + \frac{8 \times 0.2^3}{3} + \frac{20 \times 0.2^4}{4} + \frac{48 \times 0.2^5}{5} "$ $= 0.532405333 \approx 0.5324$ Note: The calculator value is 0.5347698	M1 M1 M1 A1 [4]
Total 11 marks		

Part	Mark	Notes
(a)	M1	For an attempt at the binomial expansion. The expansion must: • Begin with 1 • The denominators must be correct • The powers of $-8x^2$ must be correct eg $(-8x^2)^2$ Do not allow missing brackets unless recovered later – this is a general point of marking. Ignore any terms with powers higher than 3. Allow a misread of $\frac{1}{2}$ for $-\frac{1}{2}$ but not $8x^2$ for $-8x^2$ - see gen guidance for misreads.
	A1	Following M1 (this is a general point of marking, A marks can only follow M marks). All conditions above met and at least one algebraic term correct and simplified.
	A1	Fully correct and simplified
(b)	M1	For attempting to expand $(a + bx)(1 + cx^2 + dx^4 + \dots)$ to reach at least the correct terms in x^3 and x^4 for their expansion from part (a) ie $4bx^3$ and $24ax^4$ or cbx^3 and dax^4 Ignore terms with powers higher than 4. It is only necessary to see the correct terms (for their expansion) in x^3 and x^4 . They must be using an expansion from (a) of the form $1 + cx^2 + dx^4 + \dots$ This mark may be implied from any following working.
	A1ft	For a fully correct expansion in terms of x , a and b , up to the term in x^4 or for $4b = 20$ AND $24a = 48$. or if using an incorrect expansion from part (a) of the form $1 + cx^2 + dx^4 + \dots$ for a fully correct expansion or $cb = 20$ AND $da = 48$. Ignore terms with powers higher than 4.
	A1ft	Either a or b correct ft their expansion from (a) of the form $1 + cx^2 + dx^4 + \dots$ and $cb = 20$ AND $da = 48$.
	A1ft	Both a and b correct. ft their expansion from (a) of the form $1 + cx^2 + dx^4 + \dots$ and $cb = 20$ AND $da = 48$.
(c)	M1	For using their expansion from (b) with the correct limits. Although their expression does not need to be fully correct, it must be clear it has arisen from attempting to find a and b in part (b) and using these values. The expression must include the first 5 terms in their expansion of the form $p + qx + rx^2 + sx^3 + tx^4$ from part (b) using their a and b and may include more for this mark. It is not necessary to see the integral sign, if the candidate clearly integrates later. Similarly, if the limits are not present until substitution, this mark can be awarded.
	M1	For an attempt to integrate an expression (see general guidance) – no power of x to decrease. This is not a dependent method mark and the mark is for an attempt to integrate, but they must integrate all their terms and there must be a minimum of 2 algebraic terms.
	M1	For substitution of the given limits the correct way around. There must be at least one correct substitution of limits into their changed expression with a minimum of 2 algebraic terms (ie this again is not a dependent method mark). 0 need not be substituted if substitution into the changed expression gives an evaluation of 0. This may be implied by a correct answer, but must be shown if the final answer is not correct.
	A1	For awrt 0.5324 ie condone more decimal places.
The question does not demand that students use algebraic integration or show their working clearly and therefore a correct answer of 0.5324.... may imply the method marks and full marks should be awarded. A value of 0.5347.....will score 0 marks.		

Question	Scheme	Marks
5(a)	Mark both parts together $4k + 1 = a + (6 - 1)d \Rightarrow [4k + 1 = a + (6 - 1)d]$ $36k + 1 = \frac{10}{2}(2a + 9d) \Rightarrow [36k + 1 = 10a + 45d]$ Solves the simultaneous equations by any method:	M1 M1 M1
(i)	$d = \frac{4k + 9}{5}$	A1
(ii)	$a = -8^*$	A1 CSO [5]
(b)	$7 = -8 + 3d \Rightarrow 7 = -8 + 3\left(\frac{4k + 9}{5}\right) \Rightarrow 25 = 4k + 9 \Rightarrow k = 4^*$	M1A1 CSO [2]
(c)	$\left[d = \frac{4 \times 4 + 9}{5} = 5 \text{ you may see this in part (c)} \right]$ $\frac{n}{2}(2 \times -8 + (n - 1) \times 5) = 5[-8 + (n + 10 - 1)5] + 105$ $\Rightarrow -21n + 5n^2 = 370 + 50n + 210 \Rightarrow 5n^2 - 71n - 580 = 0$ $\Rightarrow (5n + 29)(n - 20) = 0 \Rightarrow n = 20$	M1 dM1 M1A1 [4]
Total 11 marks		

Part	Mark	Notes
(a)	Mark parts (i) and (ii) together. If they find a first score M1M1M1-	
	M1	For using the correct formula to write down: $4k + 1 = a + (6 - 1)d$
	M1	For using the correct formula to write down: $36k + 1 = \frac{10}{2}(2a + 9d)$
	M1	For an explicit, complete method to solve their two simultaneous equations to find a value for d in terms of k , and a value for a Allow one processing error. Note: This is a show question
	A1	For the correct expression for d
	A1 cso	For finding $a = -8$ With no errors, this is a given value. Award this if they find only a but not d
(b)	M1	Uses a correct n th term with their expression for d to form a linear equation and attempts to solve it.
	A1 cso	For $k = 4$ With no errors, this is a given value.
(c)	M1	For setting up the equation as required using the given a and their d No simplification is required for this mark.
	dM1	For forming a 3TQ using the given a and their d Allow one error: $5n^2 - 71n - K = 0$ Or $5n^2 - Ln - 580 = 0$ Or $Mn^2 - 71n - 580 = 0$ Where K , L and M are constants. Note: this mark is dependent on the previous mark being scored.
	M1	For solving their 3TQ using any valid, correct method If they use a calculator and the 3TQ is correct together with the correct final value, award this mark. If the 3TQ is NOT correct, they must show us a correct valid method for solving the 3TQ for the award of this mark.
	A1	For $n = 20$ with no other value.

Question number	Scheme	Marks
4 (i)	$a = 3$ or $r = 2$ $\left(\sum_{k=1}^{17} 3(2)^{k-1}\right) = \frac{3(1-(2)^{17})}{1-2} = (393213) \quad \left(\sum_{k=1}^5 3(2)^{k-1}\right) = \frac{3(1-(2)^5)}{1-2} = (93)$ "393213" – "93" 393120	B1 M1 dM1 A1 [4]
ALT	$a = 3 \times 2^5 (= 96), r = 2$ $\left(\sum_{k=6}^{17} 3(2)^{k-1}\right) = \frac{(3 \times 2^5)(1-(2)^{11 \text{ or } 12})}{1-2}$ $\frac{(3 \times 2^5)(1-(2)^{12})}{1-2}$ 393120	B1 M1 M1 A1 [4]
(ii)	$ar^2 = \frac{7}{13}$ or $ar^8 = \frac{448}{9477}$ $\frac{ar^8}{ar^2} = \frac{\frac{448}{9477}}{\frac{7}{13}} \left(= \frac{64}{729} \right) \Rightarrow r^6 =$ $r = \sqrt[6]{\left(\frac{\frac{448}{9477}}{\frac{7}{13}}\right)}$ $r = \frac{2}{3} \quad a = \frac{63}{52}$ $S_{\infty} = \frac{\frac{63}{52}}{1 - \frac{2}{3}}$ $= \frac{189}{52}$ oe	M1 dM1 M1 A1 A1 M1 A1 [7]
Total 11 marks		



Part	Mark	Additional Guidance
(i)	B1	For either value of a or r correct, stated or implicit in working.
	M1	For correctly substituting into the formula for either sum. Allow use of their a and their r with any numbers it is clear they intend to be a and r
	dM1	For their 2 sums subtracted, the correct way round. Dependent on the previous method mark. They must be using $a = 3$ for this mark.
	A1	For 393120
ALT	B1	For either value of a or r correct, stated or implicit in working.
	M2	For correctly substituting into the formula for the sum to n terms. Must be using $n = 12$ and $a = 96$, may use their r. For M1 students may use $n = 11$ or 12 and allow use of their a and their r with any numbers it is clear they intend to be a and r
	A1	For 393120
		Students who complete this by adding every term B1 for the first term correct M2 for all terms written out and added (we are not expected to evaluate them), there is no M1 mark A1 correct answer and as usual, can imply the M and B marks. A fully correct answer with little or no working will score 4 marks
(ii)	M1	For either correct equation.
	dM1	For correctly using their equations to eliminate a (or r) We will condone $\frac{ar^9}{ar^3} \left(\frac{\frac{448}{9477}}{\frac{7}{13}} = \frac{64}{729} \right) \Rightarrow r^6 =$ For this mark if the candidate works correctly to find r
	M1	Rearranges their equation, allowing one processing error, to find a value for r (or a). Do not award the mark if a student moves from r^6 to r without showing what they are doing if r is incorrect. We will condone correct r from $\frac{ar^9}{ar^3} \left(\frac{\frac{448}{9477}}{\frac{7}{13}} = \frac{64}{729} \right) \Rightarrow r =$ if the candidate works correctly to find r
	A1	Either r or a correct.
	A1	Both r and a correct.
	M1	Correct substitution of their values for a and r into a correct formula for S_∞
	A1	$\frac{189}{52}$ oe

Question Number	Scheme	Marks
7(a)	$\frac{ar^7}{ar^6} = \frac{1152}{192} \quad (= 6) = r$ $4\text{th term} = \frac{192}{6^3} \quad \text{or} \quad \frac{1152}{6^4} = \frac{8}{9}$	B1 M1A1 (3)
(b)	$\frac{t_3}{r} + t_3 + rt_3 \Rightarrow \frac{24}{r} + 24 + 24r = -36$ $24 + 24r + 24r^2 = -36r$ $24r^2 + 60r + 24 = 2r^2 + 5r + 2 = 0 \quad *$	M1A1 NB B1B1 on e-PEN dM1 ddM1A1cso (5)
(c)	$2r^2 + 5r + 2 = 0 \Rightarrow (2r + 1)(r + 2) = 0 \Rightarrow r = -\frac{1}{2}$ $S = \frac{a}{1-r} = \frac{24 \div \left(-\frac{1}{2}\right)^2}{1 - \left(-\frac{1}{2}\right)}, = 64$	M1A1 M1,A1 (4)
[12]		
(a) B1 M1 A1 ALT	Obtain a correct value for r . Fraction need not be simplified Use their r and either the 7th or 8th term divided by the appropriate power of r to obtain the 4th term as a fraction – no need to simplify $\frac{8}{9}$ M1 Find a ($=1/243$) and use ar^3 A1 Correct answer	
(b) M1 A1 dM1 ddM1 A1cso	Use the given information to obtain an equation in r Correct equation Eliminate the fraction Obtain a 3TQ, terms in any order Reach the given result with no errors in the working	
(c) M1 A1 M1 A1	Solve the given quadratic by any valid method. Must reach a value of r Correct value of r (Ignore second answer if given) Use the formula for the sum to infinity with their r provided $ r < 1$. a must be 24 divided by (their r) ² Correct answer.	

Question Number	Scheme	Marks
7(a)	$\frac{(x+1)}{(x-3)} = \frac{(4x-2)}{(x+1)} \text{ or } (x+1)^2 = (x-3)(4x-2)$ $(x+1)^2 = (4x-2)(x-3) \Rightarrow 3x^2 - 16x + 5 (=0)$ $(x-5)(3x-1) = 0 \Rightarrow x = 5, \frac{1}{3}$	M1 A1 M1A1A1 (5)
(b)	$x = \frac{1}{3} \Rightarrow r = \frac{x+1}{x-3} = \frac{\frac{4}{3}}{-\frac{8}{3}}$ $r = -\frac{1}{2} \therefore \text{convergent as } -1 < r < 1 \quad *$	M1 A1cso (2)
(c)	$S = \frac{a}{1-r} = \frac{-\frac{8}{3}}{1+\frac{1}{2}} = -\frac{16}{9} \text{ oe}$	M1A1 (2)
(d)	$\frac{S}{S_n} = \frac{a}{1-r} \times \frac{1-r}{a(1-r^n)} = \frac{1}{(1-r^n)} = \frac{256}{255}$ $255 = 256(1-r^n)$ $256r^n = 1 \quad \left(-\frac{1}{2}\right)^n = \frac{1}{256} \text{ oe } n = 8$	M1 dM1A1 (3) [12]
(a)M1 A1 M1 A1 A1 (b) M1 A1cso (c) M1 A1 (d) M1 dM1 A1	Form an equation using the given information about the terms Simplify their equation to a correct 3TQ, terms in any order Condone missing = 0 Attempt to solve their 3TQ by any valid method. Must reach $x = \dots$ (at least one root) Calculator solutions: <i>Both</i> roots correct from a correct equation scores M1A1A1 Incorrect equation or incorrect roots scores M0A0A0 One correct value of x Both correct values of x Use either of their values of x , provided it is < 1 , to find the corresponding value of r . No need to simplify Correct value of r and the conclusion including the reason Use their value of r (not x) provided $-1 < r < 1$ (as found in (b) or here) and the formula for the sum to infinity to obtain a value for S Correct value Obtain an equation in r and n . May use the formulae to cancel a or may sub values of a and r in the formulae for the LHS Must equate to $256/255$ Value of r not needed for this mark so allow any value used. Solve their equation of the form $r^n = \dots$ where $-1 < r < 1$ (r not x). May use trial and improvement or logs. This mark can be given if the equation and value of r are incorrect. Evidence of method needed if final answer is incorrect. If logs used condone $\log(-1/2)^n$ Correct value from correct working	
Question Number	Scheme	Marks

Question	Scheme	Marks
9(a)	$ar^2 = \frac{27}{2}$ $a + ar + \frac{27}{2} = \frac{57}{2} \left(\Rightarrow a + ar = 15 \Rightarrow a(1+r) = 15 \Rightarrow a = \frac{15}{1+r} \right)$ $\left(\frac{15}{1+r} \right) r^2 = \frac{27}{2} \Rightarrow 30r^2 - 27r - 27 = 0 \Rightarrow (10r^2 - 9r - 9 = 0)$	B1 M1 M1
	ALT $ar^2 = \frac{27}{2}$ $\frac{57}{2} = \frac{a(r^3-1)}{r-1}$ $\frac{57}{2} = \frac{27}{2r^2} \times \frac{(r^3-1)}{r-1} \Rightarrow 30r^3 - 57r^2 + 27 = 0$ $(\Rightarrow 10r^3 - 19r^2 + 9 = 0)$	B1 M1 M1
	$r: \quad 10r^2 - 9r - 9 = 0 \Rightarrow (5r+3)(2r-3) = 0 \Rightarrow r = \frac{3}{2}$ $a: \quad a = \frac{15}{1+\frac{3}{2}} = 6$ $S_n = \sum_{r=1}^n \left(6 \cdot \frac{3}{2} \right) \left(\frac{3}{2} \right)^r \Rightarrow S_n = \sum_{r=1}^n 4 \left(\frac{3}{2} \right)^r *$	M1A1 A1 M1A1 cso [8]
(b)	$\frac{6(1.5^k - 1)}{1.5 - 1} > 50\,000$ $1.5^k > \frac{12\,503}{3} \Rightarrow \lg 1.5^k > \lg \frac{12\,503}{3}$ $k > \frac{\lg \frac{12\,503}{3}}{\lg \frac{3}{2}} \text{ or } k > \frac{\lg \frac{12\,503}{3}}{\lg 1.5} *$	M1 M1 A1 [cso] [3]

(c)	For $k > 20.556...$ so $k = 21$	B1 [1]
Total 12 marks		

Question	Scheme	Marks
9(a)	For stating, $ar^2 = \frac{27}{2}$	B1
	For summing the first three terms; $a + ar + \frac{27}{2} = \frac{57}{2} \left(\Rightarrow a + ar = 15 \Rightarrow a(1+r) = 15 \Rightarrow a = \frac{15}{1+r} \right)$ OR $\frac{57}{2} = \frac{a(r^3-1)}{r-1}$	M1
	For attempting to form a 3TQ using their two expressions for U_3 and S_3 $\left(\frac{15}{1+r} \right) r^2 = \frac{27}{2} \Rightarrow 30r^2 - 27r - 27 = 0 \Rightarrow (10r^2 - 9r - 9 = 0)$ OR For attempting to form a cubic using their two expressions for U_3 and S_3 $\frac{57}{2} = \frac{27}{2r^2} \times \frac{(r^3-1)}{r-1} \Rightarrow 30r^3 - 57r^2 + 27 = 0$	M1
	For attempting to solve their 3TQ (see general guidance). $10r^2 - 9r - 9 = 0 \Rightarrow (5r+3)(2r-3) = 0 \Rightarrow r = ...$ OR If following the alt method then this mark is awarded for factorising the cubic into a linear expression and a 3TQ and attempting to solve their 3TQ. $(r-1)(10r^2 - 9r - 9) = 0 \Rightarrow (r-1)(5r+3)(2r-3) = 0$ $\Rightarrow r = ...$	M1
	For the correct value of $r = \frac{3}{2}$	A1
	For the correct value of $a = \frac{15}{1+\frac{3}{2}} = 6$	A1
	For attempting to use their a and r to find the correct expression for S_n	

	$S_n = \sum_{r=1}^n \left('6' \div \frac{3}{2} \right) \left(\frac{3}{2} \right)^r \Rightarrow S_n = \dots$ OR For demonstrating that a particular value of r gives the correct term in the sequence.	M1
	For the correct expression $S_n = \sum_{r=1}^n 4 \left(\frac{3}{2} \right)^r$ * OR For demonstrating that a particular value of r gives the correct term in the sequence and commenting on a correct common ratio.	A1 cso [8]
(b)	For using the summation formula $> 50\,000$ $\frac{6(1.5^k - 1)}{1.5 - 1} > 50\,000$	M1
	For rearranging the inequality to achieve, $1.5^k > \frac{12\,503}{3}$ and takes logs base 10 of both sides $\lg 1.5^k > \lg \frac{12\,503}{3}$	M1
	For using the laws of logs to make k the subject. $k > \frac{\lg \frac{12\,503}{3}}{\lg \frac{3}{2}} \text{ or } k > \frac{\lg \frac{12\,503}{3}}{\lg 1.5} *$	A1 cso [3]
(c)	For $k > 20.556... \Rightarrow k = 21$	B1 [1]
Total 12 marks		

Question	Scheme	Marks
8 (a)	$a + ar = 400$ or $\frac{a(1-r^2)}{1-r} = 400$, $ar + ar^2 = 100$ oe Common methods 1) $r(a + ar) = 100 \Rightarrow r(400) = 100$ $r = \frac{1}{4}$ * 2) $a = \frac{400}{1+r} \Rightarrow \left(\frac{400}{1+r}\right)r + \left(\frac{400}{1+r}\right)r^2 = 100$ $\Rightarrow 400r + 400r^2 = 100 + 100r \Rightarrow 4r^2 + 3r - 1 = 0 \Rightarrow (4r-1)(r+1) (=0)$ $r = \frac{1}{4}$ * 3) $r = \frac{400-a}{a} \Rightarrow a\left(\frac{400-a}{a}\right) + a\left(\frac{400-a}{a}\right)^2 = 100 \Rightarrow$ $\Rightarrow 400 - a + \frac{(400-a)^2}{a} = 100 \Rightarrow$ $400 - a^2 + 160000 - 800a + a^2 = 100a \Rightarrow a = 320 \Rightarrow 320 + 320r = 400$ $r = \frac{1}{4}$ *	B1B1 M1 A1cso M1 A1cso M1 A1cso [4]
ALT	(Let G_1, G_2, G_3 be the first 3 terms) $G_2 + G_3 = 100$ $(G_1 + G_2 = 400) \Rightarrow rG_1 + rG_2 = 100$ $rG_1 + rG_2 = 100 \Rightarrow (r(G_1 + G_2) = 100) \Rightarrow r(400) = 100$ $r = \frac{1}{4}$ *	B1 B1 M1 A1*cso [4]
(b)	$(a =) \frac{300}{1 - \left(\frac{1}{4}\right)^2}$ or $\frac{400}{1 + \frac{1}{4}}$ or $\frac{100}{\frac{1}{4} + \left(\frac{1}{4}\right)^2} = 320$ *	M1A1 cso [2]
(c)	$S_\infty = \frac{320}{1 - \frac{1}{4}} = \frac{1280}{3}$	M1A1 [2]
Total 12 marks		

Part	Mark	Notes
(a)	B1	For either equation shown correct a and r can be any letters throughout.
	B1	For both equations shown correct a and r can be any letters throughout.
	M1	For forming an equation eliminating a or r Allow one error in processing such as a sign or arithmetical error, but not a 'cancellation'/simplification error . Must be working with 2 correct equations. This mark can be awarded as soon as a or r are eliminated. Doesn't need simplification at this stage.
	A1 CSO	For correctly solving and attaining $r = \frac{1}{4}$ minimum steps shown, no errors/omissions, ignore $r = -1$
ALT	B1	For either equation shown correct
	B1	For both equations shown correct
	M1	For multiplication of the first equation by r and formation of an equation in r Allow one error in processing. Must be working with 2 correct equations.
	A1	For $r = \frac{1}{4}$ minimum steps shown, no errors or omissions. Ignore work on any negative values
There are a number of different methods to do this, the four most commonly anticipated are shown. Mark to the following principles to gain the method mark: - One processing error only in any method (M mark only, not A mark). - Rearrange for r or a and correctly substitute into the other equation or such as method 1 to reach an equation in one variable only. - Rearrange the resulting equation so that an equation of the form $br = c$ is reached. Note for the quadratic option, a factorisation will suffice. Note, if eliminating r, a must be found and the value of a then substituted into an appropriate equation. Methods where these principles can't be applied and thought worthy of credit – send to review please.		
(b)	M1	For using their expression for a with the correct r , to find a value for a Note, for this question only , work in (a) may be credited for this mark – only if they eliminated r in their solution for part (a) and this is then used .
	A1CSO	For 320, no errors.
(c)	M1	For using the correct formula for the sum to infinity of a convergent series with the given values of a and r to find a value.
	A1	For the exact value of $\frac{1280}{3}$ or 426.67 or better (ie correctly rounded to more decimal places) or 426.6... (minimum 3 dots) or 426.66 ^r or 426.6 [.]
(d)	M1	Uses the correct formula for the sum of a geometric series, to set up an inequality or equation , allow $<$ or $>$ or $=$ using the given values of r and a . Condone $\frac{1^n}{4}$
	dM1	For simplifying (allow errors in simplification) their inequality or equation in n to the form $\left(\frac{1}{4}\right)^n < d$ $d \neq 0$ or $4^n < d$ Allow $<$ or $>$ or $=$. Dependent on the 1 st method mark. Condone poor bracketing with powers again.
	ddM1	For the correct use of logs and correct use of an inequality sign throughout, including the reversal at the appropriate point. Dependent on both previous method marks. This mark may not be awarded if 'd' is negative. If candidates give a final answer of $(n =) 7$ – this mark can be implied even if the inequality sign is not correctly reversed.
	A1	For $(n =) 7$ Note although $n = 7$ can imply ddM1 as described, it is unlikely to imply the first 2 marks as there must be some logs work (directed by the question).



(d)	$426.6 < \frac{320 \left(1 - \left(\frac{1}{4} \right)^n \right)}{1 - \frac{1}{4}} \Rightarrow \left(\frac{1}{4} \right)^n < \frac{1}{6400} \text{ or } 4^n < 6400$ $\Rightarrow n > \frac{\log \left(\frac{1}{6400} \right)}{\log \left(\frac{1}{4} \right)} \text{ or } n > \log_{\frac{1}{4}} \left(\frac{1}{6400} \right) \text{ oe}$ $\Rightarrow n > 6.32... \Rightarrow n = 7$	M1dM1 ddM1 A1 [4]
Total 12 marks		

Question number	Scheme	Marks
6(a)	eg $\frac{a}{\sqrt{4\left(1+\frac{bx}{4}\right)}}$ or $\frac{a}{\sqrt{4}\sqrt{\left(1+\frac{bx}{4}\right)}}$ or $\frac{a}{4^{\frac{1}{2}}\left(1+\frac{bx}{4}\right)^{\frac{1}{2}}}$ or $a\left(4^{-\frac{1}{2}}\right)\left(1+\frac{bx}{4}\right)^{-\frac{1}{2}}$ or $a\left(4\left(1+\frac{bx}{4}\right)\right)^{-\frac{1}{2}}$ $=\frac{a}{2}\left(1+\frac{bx}{4}\right)^{-\frac{1}{2}}$	M1 A1*cso [2]
(b)	$\left(\frac{a}{2}\right)\left[1+\left(-\frac{1}{2}\right)\left(\frac{bx}{4}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{bx}{4}\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(\frac{bx}{4}\right)^3\right]$ oe $\left(\left(\frac{a}{2}\right)\left[1+\left(-\frac{bx}{8}\right)+\frac{3}{128}b^2x^2+\left(-\frac{5}{1024}b^3x^3\right)\right]\right)$ Not necessary - but must be checked if seen All intermediate steps must be checked for errors $\frac{a}{2}-\frac{ab}{16}x+\frac{3ab^2}{256}x^2-\frac{5ab^3}{2048}x^3$ $P=\frac{a}{2}, Q=-\frac{ab}{16}^*, R=\frac{3ab^2}{256}, S=-\frac{5ab^3}{2048}^*$	M1 M1 A1 A1*cso [4]
(c)	$-\frac{ab}{16}=\frac{128}{5}\cdot\left(-\frac{5ab^3}{2048}\right)$ or $\frac{5}{128}\cdot\left(-\frac{ab}{16}\right)=-\frac{5ab^3}{2048}$ or $-\frac{5ab}{2048}=-\frac{5ab^3}{2048}$ or $-\frac{ab}{16}=-\frac{ab^3}{16}$ $(b^2=1)b=1^*$ $\frac{3a\times 1^2}{256}=\frac{9}{256}\Rightarrow a=3^*$	M1 A1* B1*cso [3]
(d)	$\frac{\sqrt{6}}{2}=\frac{3}{2}\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$ or $\frac{\sqrt{6}}{2}=\frac{3}{\sqrt{4+x}}$ or $\frac{3}{\sqrt{6}}=\frac{3}{\sqrt{4+x}}$ or $\sqrt{6}=\sqrt{4+x}$ oe $\Rightarrow x=(2)$ $\frac{3}{2}-\frac{3\times 1}{16}\times 2+\frac{3\times 3\times 1^{(2)}}{256}\times 2^2-\frac{5\times 3\times 1^{(3)}}{2048}\times 2^3$ 1.207	M1 dM1 A1 [3]

Total 12 marks



Part	Mark	Additional Guidance
(a)	M1	For correctly taking out 4 as a factor (must be explicitly seen as in the mark scheme).
	A1*cs0	For the correct answer, minimum steps as shown, no errors seen or omissions .
(b)	M1	For an attempt to find the binomial expansion for the given expression. - The expansion must begin with 1 - The denominators must be correct (ie. 2! And 3! oe) on the third and fourth terms. - The power of x must be correct (ie. Must see $\frac{bx}{4}$, $\left(\frac{bx}{4}\right)^2$ and $\left(\frac{bx}{4}\right)^3$ with the correct corresponding denominators). Simplification not necessary.
	M1	For any two terms correct and unsimplified in the binomial expansion. It is not necessary to see the term in a .
	A1	For a fully correct unsimplified binomial expansion. It is not necessary to see the term in a .
	A1*cs0	For the correct P , Q , R and S , stated explicitly or embedded, with no omissions or errors seen.
It is not necessary to see this step to gain the mark, but if seen, must be checked to ensure no errors: $\left(\frac{a}{2}\right)\left[1 + \left(-\frac{bx}{8}\right) + \frac{3}{128}b^2x^2 + \left(-\frac{5}{1024}b^3x^3\right)\right] \quad \text{oe}$ As must any intermediate steps, as this is a show question.		
(c)	M1	For $(Q) = \frac{128}{5}(S)$ or $\frac{5}{128}(Q) = (S)$ in terms of a and b .
	A1*	For b correct, minimum steps as shown, no errors or omissions.
	B1*cs0	For a correct equation as shown leading to the correct value of a , no errors or omissions. Although B marks are independent of method, this is a show question and this mark won't be awarded if there are errors in the work.
(d)	M1	For equating either form to $\frac{\sqrt{6}}{2}$ or $\frac{3}{\sqrt{6}}$, leading to a value for x (ie an incorrect rearrangement may be seen, but candidate must reach a value of x)
	dM1	For correctly substituting their value of x into their expansion with the given values of a and b , the given expressions for Q and S and their expressions for P and R Dependent on previous method mark. 1.207 will imply this mark.
	A1	For awrt 1.207 (note calculator value is 1.2247...)

Question number	Scheme	Marks
7 (a)	$\frac{ar^6}{ar^3} = r^3 = \frac{e^{\frac{2x+1}{2}}}{e^{x+2}} = \frac{e^x \times e^{\frac{1}{2}}}{e^x \times e^2} = e^{-\frac{3}{2}} \Rightarrow r = e^{-\frac{1}{2}} *$ ALT $\frac{e^{x+2}}{r^3} r^6 = e^{\frac{2x+1}{2}} \Rightarrow r^3 = e^{\frac{x+1}{2}-x+2} \Rightarrow r^3 = e^{-\frac{3}{2}} \Rightarrow r = e^{-\frac{1}{2}} *$	M1M1A1 cso [3]
(b)	$ar^3 = ae^{-\frac{3}{2}} = e^{x+2} \Rightarrow a = \frac{e^{x+2}}{e^{-\frac{3}{2}}} = \frac{e^x \times e^2}{e^{-\frac{3}{2}}} = e^{x+\frac{7}{2}} \text{ oe}$ ALT $a = \frac{e^{x+2}}{e^{-\frac{3}{2}}} = e^{x+2+\frac{3}{2}} = e^{x+\frac{7}{2}} \text{ oe}$	M1M1A1 [3]
(a) M1 M1 A1 cso ALT M1 M1 A1 cso (b) M1 M1 A1 ALT M1 M1 A1	Use of $\frac{ar^6}{ar^3} = r^3$ Using $e^{a+b} = e^a \times e^b$ to simplify leading to $r^3 = e^c$ where c is a number Obtains the given answer with no errors in the working For rearranging to make a subject and substituting into the other equation Using $e^{a-b} = e^a \div e^b$ to simplify leading to $r^3 = e^c$ where c is a number Obtains the given answer with no errors in the working For $a = \frac{e^{x+2}}{e^{-\frac{3}{2}}}$ Using $e^{a+b} = e^a \times e^b$ to simplify to $a = \frac{e^x \times e^2}{e^{-\frac{3}{2}}}$ $e^{x+\frac{7}{2}} \text{ oe}$ For $a = \frac{e^{x+2}}{e^{-\frac{3}{2}}}$ Using $e^{a+b} = e^a \times e^b$ to simplify to $e^{x+2+\frac{3}{2}}$ $e^{x+\frac{7}{2}} \text{ oe}$	

(c)	$S_{\infty} = \frac{e^{x+\frac{7}{2}}}{1-e^{-\frac{1}{2}}} = \frac{e^{x+\frac{7}{2}}}{\frac{1}{e^2-1}} = \frac{e^{x+4}}{e^2-1} \Rightarrow p = x+4$ ALT 1 $S_{\infty} = \frac{e^{x+\frac{7}{2}}}{1-e^{-\frac{1}{2}}} = \frac{e^{x+\frac{7}{2}}}{1-e^{-\frac{1}{2}}} \times \frac{e^{\frac{1}{2}}}{e^{\frac{1}{2}}} = \frac{e^{x+4}}{e^2-1} \Rightarrow p = x+4$ ALT 2 $S_{\infty} = \frac{e^{x+\frac{7}{2}}}{1-e^{-\frac{1}{2}}} = \frac{e^p}{e^{\frac{1}{2}}-1} \Rightarrow e^{x+\frac{7}{2}} \left(e^{\frac{1}{2}}-1 \right) = e^p \left(1-e^{-\frac{1}{2}} \right)$ $\Rightarrow e^{x+4} - e^{x+\frac{7}{2}} = e^p - e^{p-\frac{1}{2}} \Rightarrow p = x+4$	M1M1A1 [3] {M1} {M1} {A1} [3] {M1} {M1} {A1} [3]
(c) M1 M1 A1 ALT 1 M1 M1 A1 ALT 2 M1 M1 A1	Use of $S_{\infty} = \frac{a}{1-r}$ $1-e^{-\frac{1}{2}} = \frac{e^{\frac{1}{2}}-1}{e^{\frac{1}{2}}}$ $p = x+4$ Use of $S_{\infty} = \frac{a}{1-r}$ Multiplying S_{∞} by $\frac{e^{\frac{1}{2}}}{e^{\frac{1}{2}}}$ $p = x+4$ Use of $S_{\infty} = \frac{a}{1-r}$ For simplifying to $e^{x+4} - e^{x+\frac{7}{2}} = e^p - e^{p-\frac{1}{2}}$ oe $p = x+4$	

(d)	$e^{x+\frac{7}{2}} \times \left(e^{-\frac{1}{2}}\right)^{17} > 1.6 \Rightarrow e^x \times e^{\left(\frac{7}{2}-\frac{17}{2}\right)} > 1.6 \Rightarrow e^x > \frac{1.6}{e^{-5}}$ $\Rightarrow e^x > 237.46105....$ $\Rightarrow x > \ln(237.46105) \Rightarrow x > 5.4700...$ $\Rightarrow x = 6$ ALT $e^{x+\frac{7}{2}} \times \left(e^{-\frac{1}{2}}\right)^{17} > 1.6 \Rightarrow e^{x-5} > 1.6$ $\Rightarrow x-5 > \ln(1.6)$ $\Rightarrow x > \ln(1.6) + 5 \Rightarrow x > 5.4700...$ $\Rightarrow x = 6$	M1 M1 M1 A1 [4] {M1} {M1} {M1} {A1} [4]
Total 13 marks		
(d)	NB Allow use of = for M marks M1 Use of $ar^{17} > 1.6$ M1 Simplifying to $e^x > ...$ M1 Using logs to reach $x > \ln ...$ A1 $x = 6$ ALT M1 Use of $ar^{17} > 1.6$ M1 Simplifying to $e^{x-5} > ...$ ft their a M1 Using logs to reach $x > \ln ... + 5$ ft their a A1 $x = 6$	

Question number	Scheme	Marks
10 (a)	$\frac{3}{\sqrt{9-3x}} = \frac{1}{\sqrt{9}} \times \frac{3}{\sqrt{1-\frac{3x}{9}}} = \frac{1}{\sqrt{1-\frac{x}{3}}} = \left(1-\frac{x}{3}\right)^{-\frac{1}{2}} *$	B1B1* CSO [2]
(b)	$\left(1-\frac{x}{3}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{3}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{3}\right)^2}{2!} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{x}{3}\right)^3}{3!}$ $\left(1-\frac{x}{3}\right)^{-\frac{1}{2}} = 1 + \frac{x}{6} + \frac{x^2}{24} + \frac{5x^3}{432}$	M1 A1A1 [3]
(c)	$3f(x) = \frac{3(1+2x)}{\sqrt{9-3x}} = \frac{3(1+2x)}{3\sqrt{1-\frac{x}{3}}} = (1+2x)\left(1-\frac{x}{3}\right)^{-\frac{1}{2}}$ $3f(x) = (1+2x)\left(1 + \frac{x}{6} + \frac{x^2}{24} + \frac{5x^3}{432}\right)$ $3f(x) = (1+2x)\left(1 + \frac{x}{6} + \frac{x^2}{24} + \frac{5x^3}{432}\right) = 1 + \frac{13}{6}x + \frac{3}{8}x^2 + \frac{41}{432}x^3$	B1 M1 M1A1 [4]
(d)	$\int_{0.1}^{0.2} \frac{3+6x}{\sqrt{9-3x}} dx = \int_{0.1}^{0.2} \left(1 + \frac{13}{6}x + \frac{3}{8}x^2 + \frac{41}{432}x^3\right) dx$ $= \left[x + \frac{13}{12}x^2 + \frac{3}{24}x^3 + \frac{41}{1728}x^4\right]_{0.1}^{0.2}$ $\int_{0.1}^{0.2} \frac{1+2x}{\sqrt{9-3x}} dx = \frac{1}{3} \left[\left(0.2 + \frac{13}{12} \times 0.2^2 + \frac{3}{24} \times 0.2^3 + \frac{41}{1728} \times 0.2^4\right) - \left(0.1 + \frac{13}{12} \times 0.1^2 + \frac{3}{24} \times 0.1^3 + \frac{41}{1728} \times 0.1^4\right) \right] = 0.044\ 470\ 2$	M1A1ft dM1A1 [4]
Total 13 marks		

Part	Mark	Notes
(a)	B1	For taking out the common factor of 9 from the denominator to achieve $\frac{3}{\sqrt{9-3x}} = \frac{1}{\sqrt{9}} \times \frac{3}{\sqrt{1-\frac{3x}{9}}} \quad \text{NB The common factor of 9 must be seen for this mark.}$

	B1*	For simplifying to the required form $\frac{3}{\sqrt{9-3x}} = \left(1 - \frac{x}{3}\right)^{-\frac{1}{2}}$
(b)	M1	For an attempt to find the binomial expansion for the given expression. - The expansion must begin with 1 - The denominators must be correct (ie. 2! And 3!) on the third and fourth terms. - The power of x must be correct (ie. $\left(-\frac{x}{3}\right)$ $\left(-\frac{x}{3}\right)^2$ and $\left(-\frac{x}{3}\right)^3$ with the correct corresponding denominators.
	A1	For one algebraic term correct and simplified
	A1	For the whole expansion correct up to x^3 Ignore extra terms beyond these. $\left(1 - \frac{x}{3}\right)^{-\frac{1}{2}} = 1 + \frac{x}{6} + \frac{x^2}{24} + \frac{5x^3}{432}$
(c)	B1	For writing down $3f(x) = (1+2x)\left(1 - \frac{x}{3}\right)^{-\frac{1}{2}}$
	M1	For replacing $\left(1 - \frac{x}{3}\right)^{-\frac{1}{2}}$ with their $\left(1 + \frac{x}{6} + \frac{x^2}{24} + \frac{5x^3}{432}\right)$ provided their expansion has at least two terms one of which is an algebraic term.
	M1	For an attempt to expand $(1+2x)\left(1 + \frac{x}{6} + \frac{x^2}{24} + \frac{5x^3}{432}\right)$ up to and including the term in x^3 . Accept the expansion of $(3+6x)\left(1 + \frac{x}{6} + \frac{x^2}{24} + \frac{5x^3}{432}\right)$
	A1	For the correct expansion $3f(x) = 1 + \frac{13}{6}x + \frac{3}{8}x^2 + \frac{41}{432}x^3$ ignore extra terms
(d)	M1	For an attempt to integrate their expansion of $3f(x)$ or $f(x)$ which must have as a minimum two terms, one of which is a term in x , x^2 or x^3 Ignore limits for this mark. They may multiply by $\frac{1}{3}$ at this stage so accept e.g. $\left[\frac{x}{3} + \frac{13}{36}x^2 + \frac{3}{72}x^3 + \frac{41}{5184}x^4\right]$
	A1ft	For a correct integrated expression ft their expression.
	dM1	For substituting both values of 0.2 and 0.1 the correct way around seen. This mark can be implied by the value of 0.044 470 2 seen. If their expansion from (c) has not been divided by 3 for the integration, it must be divided by 3 here for the award of this mark.
	A1	For the correct value of 0.044 470 2 NB: The calculator value is 0.044 470 77

Question	Scheme	Mark
6(a)	$S_{20} = 20(3 + 2 \times 20) = 860$	M1A1 [2]
(b)	$S_1 = 3 \times 1 + 2 \times 1^2 = 5$ $S_2 = 3 \times 2 + 2 \times 2^2 = 14$ $14 = 5 + U_2 \Rightarrow U_2 = 9$ $d = 9 - 5 = 4$ $U_n = '5' + (n-1)'4'$ $S_n = \sum_{r=1}^n (4r + 1) \Rightarrow A = 4, \quad B = 1$	B1 M1 M1 A1 M1 A1 [6]
(c)	$T_n = \frac{n}{2}(2 \times 7 + (n-1)4) = \left[\frac{n}{2}(10 + 4n) \right] \Rightarrow T_n = \frac{n}{2}(10 + 4n) \text{ or } T_n = n(5 + 2n)$ $n(5 + 2n) = 3n + 2n^2 + 252, \quad 5n = 2n + 252$ $5n = 3n + 252 \Rightarrow n = 126$	M1 M1,dM1 ddM1A1 [5]
Total		13 marks

Part	Mark	Notes
6(a)	M1	For substituting 20 into the given $S_{20} = 20(3 + 2 \times 20) = 860$
	A1	For $S_{20} = 860$ Sight of 860 with no working scores M1A1
(b)	B1	For finding the first term $S_1 = 3 \times 1 + 2 \times 1^2 = 5$ or $a = 5$ Award this mark even if it is not clear that they understand that S_1 is the first term.
	M1	For a complete method to find the second term $S_2 = 3 \times 2 + 2 \times 2^2 = 14$ $14 = 5 + U_2 \Rightarrow U_2 = 9$
	M1	For finding the common difference, they must reach a value for this mark. $d = '9' - '5' = (4)$
	A1	For $d = 4$
	M1	For either $A = 4$ or $B = 1$ Accept embedded values.
	A1	For both $A = 4$ and $B = 1$ Accept embedded values.
	For the final correct answer seen without any or minimal working, award full marks in part (b)	
(c)	M1	For an expression for T_n using the given values $T_n = \frac{n}{2}(2 \times 7 + (n-1)4) = \left[\frac{n}{2}(10 + 4n) \right]$
	M1	For equating their expression for T_n in $T_n = n(3 + 2n) + 252$ The correct expression for S_n must be used here. $\Rightarrow \frac{n}{2}(10 + 4n) = n(3 + 2n) + 252 \Rightarrow (5n = 3n + 252)$ This is an A mark in Epen
	dM1	For forming a linear equation. e.g. $5n = 3n + 252$ o.e. This mark is dependent on the previous M mark.
	ddM1	For solving their linear equation. $5n = 3n + 252 \Rightarrow n = (126)$ This mark is dependent on both previous M marks.
	A1	For $n = 126$

Question number	Scheme	Marks
2 (a)	$9(a + a + d) = a + 4d + a + 5d + a + 6d$ oe $(5a = 2d)$ oe $a + 2d = 12$ oe $5(12 - 2d) = 2d$ oe $d = 5 \quad a = 2$	M1 B1 M1 A1 A1 [5]
For the main scheme, ALT1 and ALT2, give the first 1 or 2 M marks, regardless of if students carry on to use the method for the 3 rd M mark. Favour the ALT which gives the most marks. ALT3 may not be used except in the circumstances described below – clarify with your team leader if this is not clear.		
(b)	$\sum_{n=1}^{60} u_n = \frac{60}{2}(2 \times "2" + (60 - 1) \times "5") (= 8970)$ $\sum_{n=1}^{14} u_n = \frac{14}{2}(2 \times "2" + (14 - 1) \times "5") (= 483)$ or $(u_{14}) = "2" + (14 - 1) \times "5" (= 67)$ $"8970" - "483" = 8487$	M1 M1 ddM1 A1 [4]
ALT1	(First term = $u_{15} =$) $"2" + "5" \times (15 - 1) (= 72)$ (Last term = $u_{60} =$) $"2" + "5" \times (60 - 1) (= 297)$ or $"72" + "5" \times (46 - 1) (= 297)$ $\frac{46}{2}("72" + "297") = 8487$	M1 M1 ddM1 A1 [4]
ALT2	$(u_{14} =) "2" + "5" \times (14 - 1) (= 67)$ $(u_{60} =) "2" + (60 - 1) \times "5" (= 297)$ $\frac{60}{2}("2" + "297") - \frac{14}{2}("2" + "67") = 8487$	M1 M1 ddM1 A1 [4]
This following ALT, may <u>only</u> be used if candidates are clearly and obviously going on to use $\left(\sum_{r=15}^{60} u_r = \right) \frac{46}{2}(2 \times "72" + (46 - 1) \times "5")$ Allow $n = 45$ for first 3 method marks.		
ALT3	(First term =) $"2" + "5" \times (15 - 1) (= 72)$ $\left(\sum_{r=15}^{60} u_r = \right) \frac{46}{2}(2 \times "72" + (46 - 1) \times "5")$	M2 ddM1 A1 [4]
(c)	$2 \times \frac{n}{2}(2 \times 2 + (n - 1) \times 5) - 5(5n - 3) = 10$ oe $(4n + 5n^2 - 5n - 25n + 15 = 10)$ $5n^2 - 26n + 5 = 0$ $(5n - 1)(n - 5) = 0 \rightarrow n = \frac{1}{5}, 5$ $n = 5$	M1 A1 M1 A1 [4]
Total 13 marks		



Part	Mark	Additional Guidance
(a)	M1	Correctly forms the equation shown. Any equivalent unsimplified equation. Note: an alternative seen is (the mark is awarded for the second line below or any unsimplified equivalent) $(S_7 - S_4 = 9(a + a + d))$ $\frac{7}{2}(2a + (7 - 1)) - \frac{4}{2}(2a + (4 - 1)) = 9(a + a + d)$ $(\Rightarrow 15a = 6d)$
	B1	For the correct equation.
	M1	For a complete and valid method to solve the equations simultaneously, leading to either $a =$ or $d =$. Allow one processing error only . Can be awarded for a complete and valid method to solve any two linear equations, both in a and d , one processing error only. Must lead to either $a =$ or $d =$
	A1	For either the correct value of a or the correct value of d .
	A1	For the correct values of both a and d .
If candidates use a different method, follow the same principles to award marks or send to Review if this is not clear. Candidates may use any letters or symbols to denote a and d		
(b)	M1	Use of their a and d in the correct formula $\frac{n}{2}(2"a" + (n - 1)"d")$ with $n = 60$
	M1	Use of their a and d in the correct formula $\frac{n}{2}(2"a" + (n - 1)"d")$ with $n = 14$ Allow $n = 15$ as a concession.
	ddM1	For their " $\sum_{n=1}^{60} u_n$ " - " $\sum_{n=1}^{14/15} u_n$ ". Dependent on both previous method marks. Follow through their value from use of $n = 15$ if used. The presence of $8970 - 483$ will imply M3 here as will 8487.
	A1	For 8487
ALT1	M1	For correct use of " a " + $(n - 1)"d"$ with $n = 15$ with their a and their d
	M1	For correct use of " a " + $(n - 1)"d"$ with $n = 60$ with their a and their d or correct use of " a_{15} " + $(n - 1)"d"$ with $n = 46$ with their d and correctly using their term for a_{15} from the first M mark. Allow $n = 45$ as a concession.
	ddM1	For correct use of the formula $\frac{n}{2}(u_{15} + u_{60})$. Dependent on both previous method marks. Allow $n = 45$ as a concession. $\frac{46}{2}("72" + "297")$ or 8487 implies M3
	A1	For 8487
ALT2	M1	For correct use of " a " + $(n - 1)"d"$ with $n = 14$ with their a and their d Allow $n = 15$
	M1	For correct use of " a " + $(n - 1)"d"$ with $n = 60$ with their a and their d
	ddM1	For correct use of the formula $\frac{n}{2}(u_1 + u_{60}) - \frac{n}{2}(u_{14} + u_{60})$. Dependent on both previous method marks. Allow $n = 15$ The presence of $8970 - 483$ will imply M3 here as will 8487
	A1	For 8487

ALT3	M2	For correct use of " a " + $(n-1)d$ " with $n = 15$ with their a and their d
	ddM1	For correct use of the formula $\frac{n}{2}(2 \times "u_{15}" + (n-1)d)$. Dependent on both previous method marks. Allow $n = 45$ as a concession.
	A1	8487
(c)	M1	For correct use of $2S_n - 5(an + b) = 10$ Substitution into S_n must be correct for their values of a and d . and candidates must correctly use their a in an expression of the form $an + b$ b can be any integer, $b \neq 0$ $2S_n - 5(an + b) = 10$ does not have to be simplified to be awarded this mark.
	A1	For the correct 3 term quadratic equation = 0. Equivalent coefficients are possible.
	M1	For any valid, complete method to solve their quadratic (see general guidance), which must be of the form $pn^2 + qn + r$ $p, q, r \neq 0$ leading to $n =$ $n = 5$ will imply this mark.
	A1	$n = 5$ $n = 0.2$ must be dismissed.

Question	Scheme	Marks
10 (a)	$(1+2x^2)^{-\frac{1}{3}} = 1 - \frac{1}{3}(2x^2) + \frac{-\frac{1}{3} \times \left(-\frac{4}{3}\right)}{2!}(2x^2)^2 + \frac{-\frac{1}{3} \times \left(-\frac{4}{3}\right) \left(-\frac{7}{3}\right)}{3!}(2x^2)^3$ $= 1 - \frac{2x^2}{3} + \frac{8x^4}{9} - \frac{112x^6}{81}$	M1 A1A1 [3]
(b)	$ x < \frac{1}{\sqrt{2}} \quad \text{or} \quad -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}} \quad (\text{But accept } \leq \text{ for either or both})$	B1 [1]
(c)	$f(x) = \left(2 + kx^2\right) \left(1 - \frac{2x^2}{3} + \frac{8x^4}{9} - \frac{112x^6}{81}\right)$ $= 2 - \frac{4x^2}{3} + \frac{16x^4}{9} - \frac{224x^6}{81} + kx^2 - \frac{2kx^4}{3} + \frac{8kx^6}{9} + \dots$ $= 2 + \left(k - \frac{4}{3}\right)x^2 + \left(\frac{16}{9} - \frac{2k}{3}\right)x^4 + \left(\frac{8k}{9} - \frac{224}{81}\right)x^6$	M1 dM1 A1 [3]
(d)	$\frac{16}{9} = \frac{2k}{3}, \quad k = \frac{8}{3}$	M1,A1 [2]
(e)	Coeffs of $x^2 = \frac{4}{3}, \quad x^6 = -\frac{32}{81}$ $\int_0^{0.5} f(x) dx \approx \int_0^{0.5} \left(2 + \frac{4}{3}x^2 - \frac{32}{81}x^6\right) dx$ $= \left[2x + \frac{4x^3}{9} - \frac{32x^7}{567}\right]_0^{0.5}$ $= 2 \times 0.5 + \frac{4}{9}(0.5)^3 - \frac{32}{567}(0.5)^7, \approx 1.0551$	B1 (for both) M1A1ft dM1,A1 [5]
Total 14 marks		

Additional Guidance		
(a)	M1	For an attempt to use the Binomial Expansion The minimally acceptable attempt is as follows; - The power of x must be correct in each term. $\left[(2x^2), (2x^2)^2 \text{ and } (2x^2)^3\right]$ - The first term is 1 - The denominators in terms 2, 3 and 4 must be correct.
	A1	For the first term and at least one term in x correct and simplified.
	A1	All terms correct and simplified $1 - \frac{2x^2}{3} + \frac{8x^4}{9} - \frac{112x^6}{81}$
(b)	B1	For the correct inequality. $ x < \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ Accept $ x $ „ $\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ „ x „ $\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} < x$ „ $\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ „ $x < \frac{1}{\sqrt{2}}$
(c)	M1	Shows that they intend to multiply their expansion from part(a) by $(2 + kx^2)$
	dM1	Multiplies out the two brackets to at least 4 terms up to and including the term in x^6 with a constant, term in x^2 , term in x^4 and a term in x^6
	A1	For the expansion as shown, simplified and the coefficient of each term in x in terms k $2 + \left(k - \frac{4}{3}\right)x^2 + \left(\frac{16}{9} - \frac{2k}{3}\right)x^4 + \left(\frac{8k}{9} - \frac{224}{81}\right)x^6$
(d)	M1	For $\frac{16}{9} = \frac{2k}{3}$ Allow the presence of x^4 provided it is on both sides of the equality.
	A1	For $k = \frac{8}{3}$
(e)	B1	Finds the coefficients of x^2 and x^6 $x^2 = \frac{4}{3}, \quad x^6 = -\frac{32}{81}$
In this part the question clearly states using algebraic integration No evidence of algebraic integration – no marks		
	M1	For an attempt to integrate their expansion in (c) with their value for k provided it has as a minimum; a constant term and at least two algebraic terms. Ignore the limits for this mark
	A1	For a fully correct integrated expression as shown (ignore limits)
	M1	Attempts to evaluate their integrated expression using the correct limits Substitution of 0 need not be seen
	A1	For a value of 1.0551 rounded correctly.

Question	Scheme	Marks
9(a)	$(1+2x)^{-\frac{1}{3}} = 1 + \left(-\frac{1}{3}\right)(2x) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)(2x)^2}{2!} + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)(2x)^3}{3!} + \dots$ $= 1 - \frac{2x}{3} + \frac{8x^2}{9} - \frac{112x^3}{81}$	M1 A1A1 [3]
(b)	$-\frac{1}{2} < x \leq \frac{1}{2}$ accept $-\frac{1}{2} < x < \frac{1}{2}$	B1 [1]
(c)	$f(x) = (2 + kx^2) \left(1 - \frac{2x}{3} + \frac{8x^2}{9} - \frac{112x^3}{81}\right)$ $= 2 - \frac{4x}{3} + \frac{16}{9}x^2 + kx^2 - \frac{224}{81}x^3 - \frac{2k}{3}x^3$	M1 M1A1 [3]
(d)	$-\frac{224}{81} - \frac{2k}{3} = -\frac{8}{3} \Rightarrow k = -\frac{4}{27}$	M1A1 [2]
(e)	$\int_{0.1}^{0.2} \left(2 - \frac{4x}{3} + \frac{44x^2}{27} - \frac{8}{3}x^3\right) dx = \left[2x - \frac{4x^2}{2 \times 3} + \frac{44x^3}{3 \times 27} - \frac{8x^4}{4 \times 3}\right]_{0.1}^{0.2}$ $= \left(2 \times 0.2 - \frac{2 \times 0.2^2}{3} + \frac{44 \times 0.2^3}{81} - \frac{2}{3} \times 0.2^4\right) - \left(2 \times 0.1 - \frac{2 \times 0.1^2}{3} + \frac{44 \times 0.1^3}{81} - \frac{2}{3} \times 0.1^4\right)$ $= 0.1828$ NB: Calculator value is 0.18301744	B1FT M1A1ft M1 A1 [5]
Total 14 marks		

Part	Mark	Notes
(a)	M1	For an attempt at binomial expansion. - The first term is 1 - The denominators are correct. - The powers of x are correct.
	A1	For at least one algebraic term correct and simplified.
	A1	For a fully correct and simplified expansion.
(b)	B1	For the correct range. Accept $ x < \frac{1}{2}$ Do not accept $-\frac{1}{2} \leq x \leq \frac{1}{2}$
(c)	M1	For showing the intent to multiply their expansion by $(2 + kx^2)$
	M1	For expanding the two brackets up to terms in x^3 . Ignore terms in higher powers.
	A1	For the correct expansion. This does not have to be simplified. Must be in ascending powers, but accept $\frac{16}{9}x^2$ and kx^2 in either order and accept $-\frac{224}{81}x^3$ and $-\frac{2k}{3}x^3$ in either order. $f(x) = 2 - \frac{4x}{3} + x^2 \left(\frac{16}{9} + k \right) + x^3 \left(-\frac{224}{81} - \frac{2k}{3} \right)$ ISW errors in collection of coefficients once correct expansion in ascending powers seen.
(d)	M1	For setting their coefficient of x^3 equal to $-\frac{8}{3}$
	A1	For the correct value of k
(e)	B1FT	For substituting in their k into coefficient for x^2 in their expansion from (c) and having the coefficient of x^3 as $-\frac{8}{3}$. If the coefficient of x^2 is not correct then the substitution must be seen to award this mark. Note: this is an M mark in open.
	M1	For an attempt to integrate their expansion which must have 4 terms. At least two powers of x to increase by 1 (including $c \rightarrow cx$). No power of x to reduce.
	A1ft	For a correctly integrated expression follow through their 4 term polynomial. Simplified integrated expression is $2x - \frac{2x^2}{3} + \frac{44x^3}{81} - \frac{2}{3}x^4$
	M1	For substituting in 0.2 and 0.1 the correct way around in their integrated expression. Expression should not have been differentiated and needs to have changed. Not dependent on first M. Must show the substitution if their integral or their limits are incorrect. Correct awrt 0.1828 following correct integral with correct limits implies this mark.
	A1	For the correct value of awrt 0.1828 NB: The calculator value is 0.18301744

Question	Scheme	Marks
8(a)	$a + ar = 360 \quad \text{oe} \quad \text{e.g.} \quad \frac{a(1-r^2)}{1-r} = 360$ $ar + ar^2 = 288 \quad \text{oe}$ $r = \left\{ \frac{a(1+r)}{ar(1+r)} \right\} \frac{288}{360} = \frac{4}{5}$ $a = \frac{360}{1 + \frac{4}{5}} = 200$ $U_n = 200 \left(\frac{4}{5} \right)^{n-1} *$	M1M1 (B1B1 on ePen) dM1A1 M1A1 A1cso(B1 on ePen) [7]
8(a) ALT	$a + ar = 360 \quad \text{oe} \quad \text{e.g.} \quad \frac{a(1-r^2)}{1-r} = 360$ $ar + ar^2 = 288 \quad \text{oe}$ $a \left(\frac{360}{a} - 1 \right) + a \left(\frac{360}{a} - 1 \right)^2 = 288$ $\Rightarrow a = 200$ $r = \frac{360}{200} - 1 = \frac{4}{5}$ $U_n = 200 \left(\frac{4}{5} \right)^{n-1} *$	M1M1 (B1B1 on ePen) dM1A1 M1A1 A1cso(B1 on ePen) [7]
(b)	{S is convergent because} $\left \frac{4}{5} \right < 1$	B1 [1]
(c)	$S_\infty = \frac{200}{1 - \frac{4}{5}} = 1000$	M1A1cao [2]
(d)	$\frac{200 \left(1 - \left(\frac{4}{5} \right)^n \right)}{1 - \left(\frac{4}{5} \right)} > 978$ $\left(\frac{4}{5} \right)^n < \frac{11}{500}$ $n \log \left(\frac{4}{5} \right) < \log \left(\frac{11}{500} \right)$ $\Rightarrow n > 17.104 \dots$ $n = 18$	M1 dM1 ddM1 A1 [4]
Total 14 marks		

Part	Mark	Notes
(a)	M1	For either correct equation. Allow use of U_1 as the first term instead of a
	M1	For both correct equations. Allow use of U_1 as the first term instead of a
	dM1	For eliminating a from their equations, reaches $r = \dots$ (Depends on previous mark)
	A1	For the correct value of r
	M1	Uses their value of r to find a value for a
	A1	For $a = 200$
	A1 cso	For the correct required expression of $U_n = 200 \left(\frac{4}{5}\right)^{n-1}$ (must have U_n)
(a)ALT	M1	For either correct equation.
	M1	For both correct equations
	dM1	For eliminating r from their equations, reaches $a = \dots$ (Depends on previous mark)
	A1	For the correct value of a
	M1	Uses their value of a to find a value for r
	A1	For the correct value of r
	A1 cso	For the correct required expression of $U_n = 200 \left(\frac{4}{5}\right)^{n-1}$ (must have U_n)
(b)	B1	For stating the correct reason, $ r < 1$ or $-1 < r < 1$ or " 0.8 " < 1 or $\left \frac{4}{5}\right < 1$ Do not accept just $r < 1$ without a correct reason
(c)	M1	For using their value of A/a /first term of their geometric sequence and their r (provided their $ r < 1$) in a correct formula for the sum to infinity. $S_{\infty} = \frac{\text{their } A}{1 - (\text{their } r)}$
	A1cao	For the correct value of 1000
(d)	M1	Uses their a and r (their r can be $ r < 1$ or $ r > 1$) to set the correct formula for the sum of a geometric series > 978 or $=978$ or ≥ 978 (>978 or $=978$ or ≥ 978 may be implied by later work.)
	dM1	Attempts to rearrange and achieves ($\text{their } r$) $^n < k$ (k is non zero positive constant) Allow strict or non-strict inequalities or equation: $<, >, \geq, \leq$ or $=$
	ddM1	For correctly takes logs (any base)/ln, and achieves a positive value for n , it maybe implied by 17.1...but not 18 ALT: trial and error to find a value of n (at least tries $n=17$ and $n=18$)
	A1	For $n = 18$ Must come from correct working, achieves $n=18$ with no incorrect inequalities in their working.

If a candidate does not score the dM1 mark but arrives at an answer of 17.1... this scores a maximum of M1 as clear use of calculator is not condoned.