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EDEXCEL INTERNATIONAL GCSE FURTHER PURE MATHS: QUESTIONS BY TOPIC

Vectors (6)

International GCSE Further Pure Mathematics Topic 6

29 questions · 293 marks · Mark schemes included

Pearson Edexcel International GCSE (9–1) Further Pure Mathematics 4PM1, Papers 1, 1R, 2 & 2R, June 2019 - June 2025

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 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions run from the lowest mark tariff to the highest, so similar questions sit together.

Click any heading or question row below to jump straight to it.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

29 questions · 293 marks · lowest tariff first



4-MARK QUESTIONS

2 questions in this booklet



- 1 Referred to a fixed origin O , the point A has position vector $(4\mathbf{i} + 3\mathbf{j})$ and the point B has position vector $(\mathbf{i} + 7\mathbf{j})$

(a) Find $\overrightarrow{AB}$ as a simplified expression in terms of $\mathbf{i}$ and $\mathbf{j}$ (2)

(b) Find a unit vector that is parallel to $\overrightarrow{AB}$ (2)

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- 1 The points A , B , C and D are the vertices of a quadrilateral $ABCD$ such that

$$\overrightarrow{AB} = 3\mathbf{a} + 2\mathbf{b} \quad \overrightarrow{AD} = 2\mathbf{a} - 5\mathbf{b} \quad \overrightarrow{AC} = 5\mathbf{a} - 3\mathbf{b}$$

Show that $ABCD$ is a parallelogram.

(4)



5-MARK QUESTIONS

1 question in this booklet



- 1 The position vector of the point A is $(3\mathbf{i} - 2\mathbf{j})$, referred to a fixed origin O .

The point B is such that $\overrightarrow{AB} = (6\mathbf{i} + 8\mathbf{j})$

- (a) Find the position vector of B as a simplified expression in terms of $\mathbf{i}$ and $\mathbf{j}$

(2)

- (b) Find the magnitude of vector $\overrightarrow{AB}$

(1)

- (c) Find a unit vector, in terms of $\mathbf{i}$ and $\mathbf{j}$, that is parallel to $\overrightarrow{AB}$

(2)

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6-MARK QUESTIONS

2 questions in this booklet



- 4 O, A and B are fixed points such that

$$\vec{OA} = 5\mathbf{i} + 7\mathbf{j} \quad \vec{AB} = a\mathbf{i} + 16\mathbf{j} \quad \text{and} \quad |\vec{OB}| = 5\sqrt{29}$$

- (a) Find the possible values of a

(4)

Given that $a > 0$

- (b) find a unit vector that is parallel to $\vec{AB}$

(2)

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3 O, A and B are fixed points such that

$$|\vec{OA}| = 3\sqrt{5} \quad \vec{AB} = \mathbf{i} + 3a\mathbf{j} \quad \vec{OB} = 7\mathbf{i} + 2a\mathbf{j}$$

Given that $a > 0$

(a) find the value of a

(4)

(b) Hence find a unit vector parallel to $\vec{OA}$

(2)

7-MARK QUESTIONS

1 question in this booklet



- 4 The points A , B , C and D are the vertices of a quadrilateral $ABCD$ such that

$$\overrightarrow{AB} = 7\mathbf{i} + p\mathbf{j} \quad \overrightarrow{AC} = 11\mathbf{i} - p\mathbf{j} \quad \overrightarrow{AD} = 4\mathbf{i} - 2p\mathbf{j}$$

- (a) Show that, for all values of p , $ABCD$ is a parallelogram.

(3)

Given that $|\overrightarrow{BD}| = 3\sqrt{10}$

- (b) find the possible values of p .

(3)

Given that $p > 0$

- (c) find a unit vector which is parallel to $\overrightarrow{BD}$.

(1)

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8-MARK QUESTIONS

1 question in this booklet



7 O, A, B and C are fixed points such that

$$\overrightarrow{OA} = 8\mathbf{i} - 6\mathbf{j} \quad \overrightarrow{OB} = 15\mathbf{i} - 6\mathbf{j} \quad \overrightarrow{OC} = 8\mathbf{i} + \mathbf{j}$$

(a) Find $\overrightarrow{BC}$ as a simplified expression in terms of $\mathbf{i}$ and $\mathbf{j}$

(2)

(b) Find a unit vector parallel to $\overrightarrow{BC}$

(2)

The point M is the midpoint of OA and the point N lies on OB such that $ON:NB = 1:2$

(c) Show that the points M, N and C are collinear.

(4)

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9-MARK QUESTIONS

3 questions in this booklet



- 3 O, A and B are fixed points such that

$$\vec{OA} = (p\mathbf{i} - 4\mathbf{j}) \qquad \vec{OB} = \mathbf{i} + (2p + 1)\mathbf{j}$$

Given that $\sqrt{2} |\vec{OA}| = |\vec{OB}|$ and $p > 0$

- (a) find the value of p

(4)

Using this value of p

- (b) find a unit vector that is parallel to $\vec{AB}$

(5)

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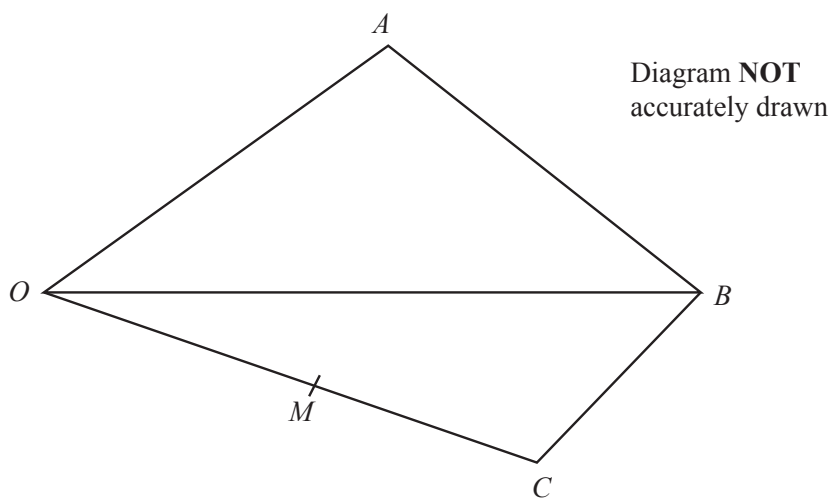


Figure 3

Figure 3 shows quadrilateral $OABC$ where

$$\vec{OA} = 4\mathbf{p} + 5\mathbf{q} \quad \vec{OB} = 3\mathbf{p} + \mathbf{q} \quad \vec{OC} = 2\mathbf{p} - 4\mathbf{q}$$

The point M is the midpoint of OC

(a) Find $\vec{MA}$ as a simplified expression in terms of $\mathbf{p}$ and $\mathbf{q}$

(3)

The point N lies on OB such that M , N and A are collinear.

(b) Find the ratio $MN : NA$

(6)

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6 Referred to a fixed origin O , the position vector of the point A is $(9\mathbf{a} + 12\mathbf{b})$

and the position vector of the point B is $(2\mathbf{a} + p\mathbf{b})$ where $|\mathbf{a}| = |\mathbf{b}| = 1$

Given that $|AB| = 25$

(a) show that the two possible values of p are -12 and 36

(4)

For $p = -12$

(b) find a unit vector parallel to $\overrightarrow{OB}$

Give your answer in terms of $\mathbf{a}$ and $\mathbf{b}$ in its simplest form.

(2)

Point C lies on OB such that $OC : CB = 2 : 3$

For $p = 36$

(c) find $\overrightarrow{CA}$

Give your answer in terms of $\mathbf{a}$ and $\mathbf{b}$ in its simplest form.

(3)

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10-MARK QUESTIONS

5 questions in this booklet



- 6 The points P , Q , R and S are the vertices of a quadrilateral $PQRS$ such that

$$\vec{PQ} = 2\mathbf{i} + 3\mathbf{j} \quad \vec{PR} = -\mathbf{i} + 18\mathbf{j} \quad \vec{PS} = -3\mathbf{i} + 15\mathbf{j}$$

- (a) Show that $PQRS$ is a parallelogram.

(4)

- (b) Find a unit vector parallel to $\vec{QS}$ as a simplified expression in terms of $\mathbf{i}$ and $\mathbf{j}$

(4)

The point T lies on QS such that $QT:TS = 5:8$

- (c) Find $\vec{PT}$ as a simplified expression in terms of $\mathbf{i}$ and $\mathbf{j}$

(2)

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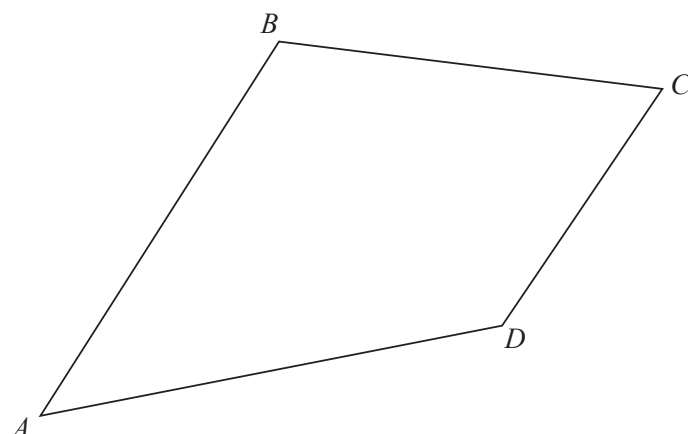
Diagram **NOT**
accurately drawn**Figure 3**

Figure 3 shows quadrilateral $ABCD$ such that

$$\overrightarrow{AD} = 2\mathbf{a} + \mathbf{b} \qquad \overrightarrow{BC} = \frac{1}{3}\mathbf{b} \qquad \overrightarrow{BD} = -4\mathbf{a} - \mathbf{b}$$

- (a) Prove that $\overrightarrow{AB}$ is parallel to $\overrightarrow{DC}$ (4)

The diagonals, AC and BD , of the quadrilateral intersect at the point Y .

- (b) Using a vector method, find $\overrightarrow{AY}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$ (6)

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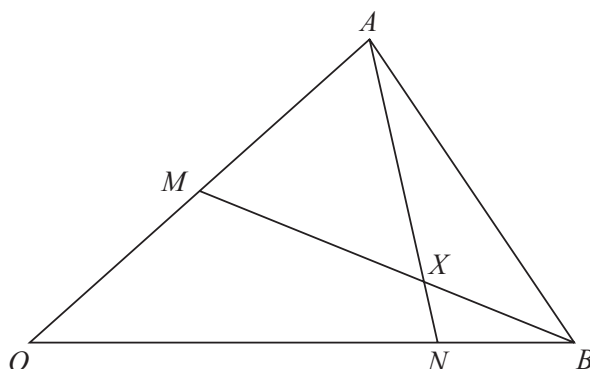
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Figure 3

Figure 3 shows triangle OAB with $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$

M is the midpoint of OA .

N is the point on OB such that $ON:NB = 3:1$

The lines AN and BM intersect at the point X .

(a) Find expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for

(i) $\vec{AN}$

(ii) $\vec{BM}$

(3)

(b) Using a vector method, find $AX:XB$

(7)

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10 O, A and B are fixed points such that

$$\vec{OA} = (b + 1)\mathbf{i} + b\mathbf{j}$$

$$\vec{AB} = 3\mathbf{i}$$

The unit vector parallel to $\vec{OB}$ is $\frac{\sqrt{17}}{34} [(3a + 2)\mathbf{i} + b\mathbf{j}]$

Given that a and b are constants where $a > 0$ and $b > 0$

find the exact value of

(i) a

(ii) b

(10)

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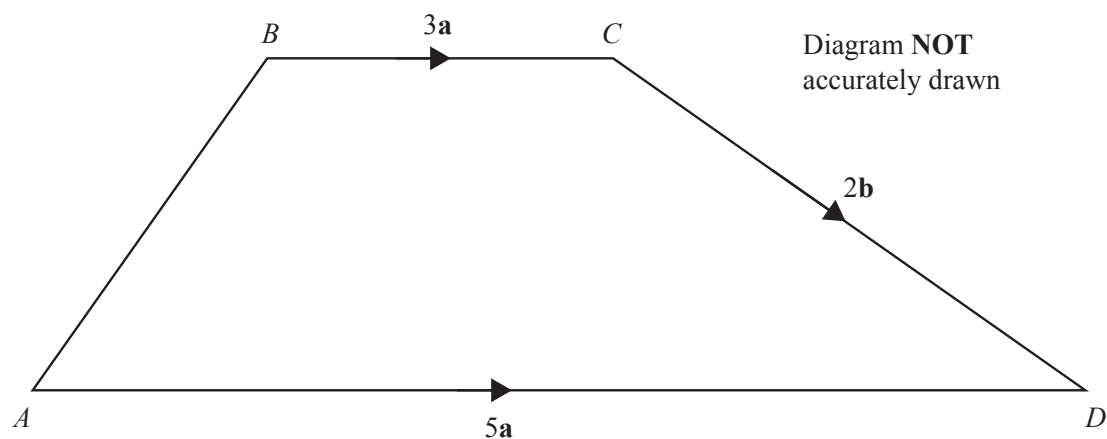


Figure 3

Figure 3 shows a trapezium $ABCD$

$$\vec{BC} = 3\mathbf{a} \quad \vec{AD} = 5\mathbf{a} \quad \vec{CD} = 2\mathbf{b}$$

- (a) Find $\vec{AB}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(1)

The diagonals BD and AC intersect at point X where $\vec{BX} = k \vec{BD}$

- (b) Using a vector method, find the value of k

(5)

- (c) Find the ratio of the area of triangle CXD : area of the trapezium $ABCD$

(4)

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11-MARK QUESTIONS

4 questions in this booklet



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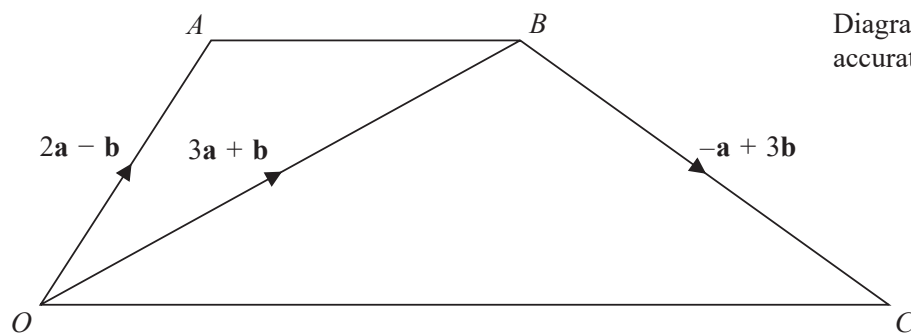


Figure 1

Figure 1 shows quadrilateral $OABC$ with

$$\overrightarrow{OA} = 2\mathbf{a} - \mathbf{b} \quad \overrightarrow{OB} = 3\mathbf{a} + \mathbf{b} \quad \overrightarrow{BC} = -\mathbf{a} + 3\mathbf{b}$$

(a) Find $\overrightarrow{AB}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$.

(2)

(b) Prove that $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$

(2)

The diagonals, OB and AC , intersect at the point X .

(c) Using a vector method find the ratio $AX:XC$

(7)

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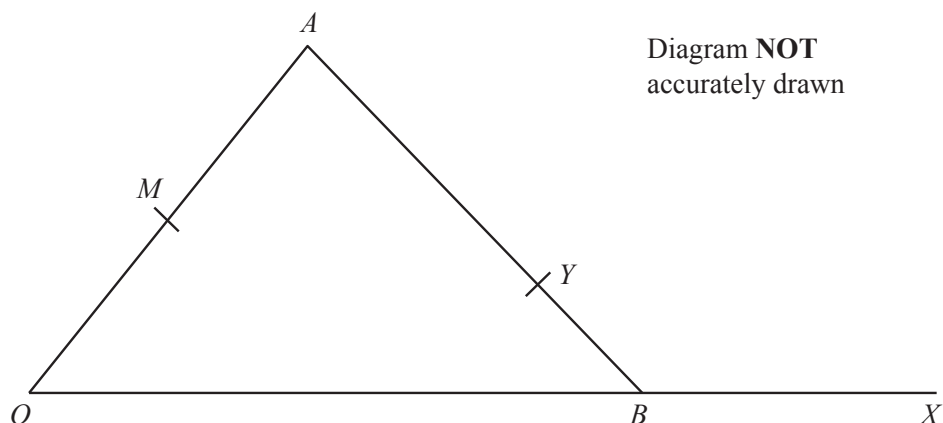


Figure 5

In Figure 5, $\vec{OA} = 2\mathbf{a}$, $\vec{OB} = 4\mathbf{b}$ and M is the midpoint of OA .

The point Y lies on AB such that $AY : YB = 3 : 1$

The point X lies on OB produced.

(a) Find as simplified expressions in terms of $\mathbf{a}$ and $\mathbf{b}$

- (i) $\vec{AB}$ (ii) $\vec{MY}$

(3)

The points M , Y and X are collinear.

(b) Find the ratio $OB : OX$

(5)

(c) Find the ratio of (Area $\triangle YBX$) : (Area $\triangle OAX$)

(3)

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10 The points A , B , C and D are the vertices of a quadrilateral such that

$$\vec{AB} = 3\mathbf{a} + 4\mathbf{b} \quad \vec{AC} = 7\mathbf{a} + 9\mathbf{b} \quad \vec{AD} = 4\mathbf{a} + 5\mathbf{b}$$

(a) Show that $ABCD$ is a parallelogram.

(3)

BC is extended to the point E such that BCE is a straight line.

Point F lies on CD such that $CF : FD = 1 : 2$

Given that A , F and E are collinear,

(b) find the vector $\vec{AE}$ in the form $X\mathbf{a} + Y\mathbf{b}$ where X and Y are rational numbers to be found.

(8)

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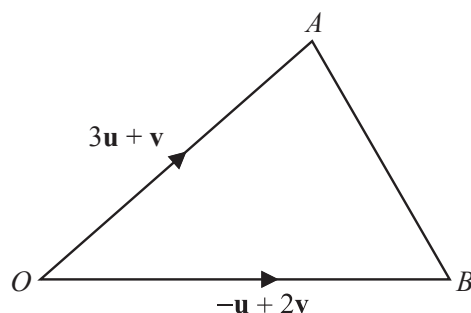
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accurately drawn

Figure 2

Figure 2 shows triangle OAB with

$$\vec{OA} = 3\mathbf{u} + \mathbf{v} \quad \vec{OB} = -\mathbf{u} + 2\mathbf{v}$$

- (a) Find $\vec{AB}$ as a simplified expression in terms of $\mathbf{u}$ and $\mathbf{v}$

(2)

Point C is such that AC is parallel to OB Given that $\vec{OC} = \mu\mathbf{v}$

- (b) find the value of μ

(4)

The lines OC and AB intersect at point X Point D lies on $\vec{OC}$ such that

$$\text{area of triangle } BOX : \text{area of triangle } BXD = 2 : 3$$

Using a vector method,

- (c) find $\vec{BD}$ as a simplified expression in terms of $\mathbf{u}$ and $\mathbf{v}$

(5)

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12-MARK QUESTIONS

2 questions in this booklet



- 4 O, A and B are fixed points such that

$$\overrightarrow{OA} = p\mathbf{i} + 2p\mathbf{j} \quad \overrightarrow{OB} = 5\mathbf{i} + 9p\mathbf{j}$$

Given that $\overrightarrow{AB}$ is parallel to $(\mathbf{i} - 2\mathbf{j})$

- (a) find the value of p

(6)

- (b) Hence find $\overrightarrow{AB}$ as a simplified expression in terms of $\mathbf{i}$ and $\mathbf{j}$

(2)

- (c) Find a unit vector parallel to $\overrightarrow{OA}$

Give your answer in the form $\frac{\sqrt{a}}{5}(b\mathbf{i} + c\mathbf{j})$ where a, b and c are integers to be found.

(4)

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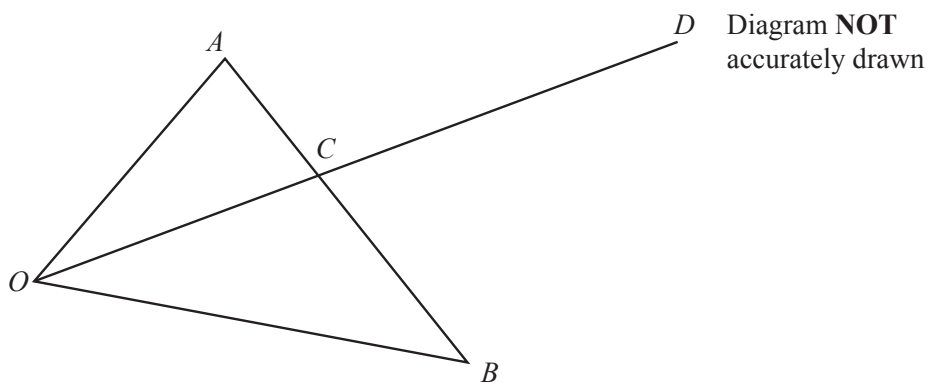


Figure 2

Figure 2 shows triangle AOB

$$\vec{OA} = 4\mathbf{a} + 5\mathbf{b} \quad \vec{OB} = 8\mathbf{a} - \mathbf{b} \quad \vec{OD} = 15\mathbf{a} + 10\mathbf{b} \quad \text{where } |\mathbf{a}| = |\mathbf{b}| = 1$$

(a) (i) Find $\vec{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(ii) Find, in its simplest form, the exact value of $|\vec{AB}|$

(3)

(b) Find the area of triangle AOB

(4)

The point C lies on AB and OD such that O , C and D are collinear.

(c) Use a vector method to find vector $\vec{OC}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(5)

13-MARK QUESTIONS

5 questions in this booklet



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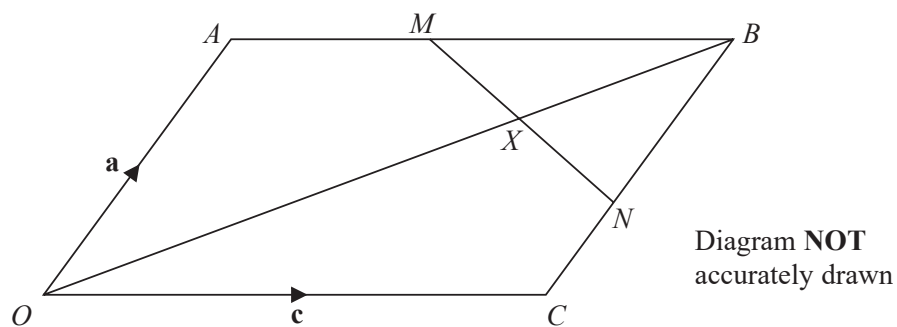


Figure 3

Figure 3 shows the parallelogram $OABC$

$$\overrightarrow{OA} = \mathbf{a} \quad \overrightarrow{OC} = \mathbf{c}$$

The midpoint of AB is M and the midpoint of BC is N .

The line OB intersects MN at the point X .

(a) Find in terms of $\mathbf{a}$ and $\mathbf{c}$,

(i) $\overrightarrow{OB}$

(ii) $\overrightarrow{MN}$

(2)

Given $\overrightarrow{MX} = \lambda \overrightarrow{MN}$ and that $\overrightarrow{OX} = \mu \overrightarrow{OB}$,

(b) use a vector method to find the value of λ and the value of μ .

(8)

(c) Hence find, in its simplest form, the ratio

Area of quadrilateral $OXNC$: Area of parallelogram $OABC$.

(3)

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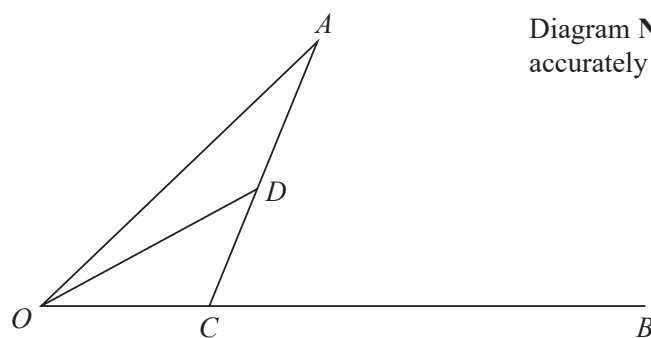
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Figure 5

In Figure 5, $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$

The point C divides OB in the ratio 1 : 3

The point D is the midpoint of AC

(a) Find, as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(i) $\vec{AC}$

(ii) $\vec{OD}$

(iii) $\vec{BD}$

(5)

The point E is such that $\vec{OE} = \lambda \vec{OA}$

Given that E , D and B are collinear

(b) find the value of λ

(4)

Given that $\frac{\text{area } \triangle OAC}{\text{area } \triangle OEB} = \mu$

(c) find the value of μ

(4)

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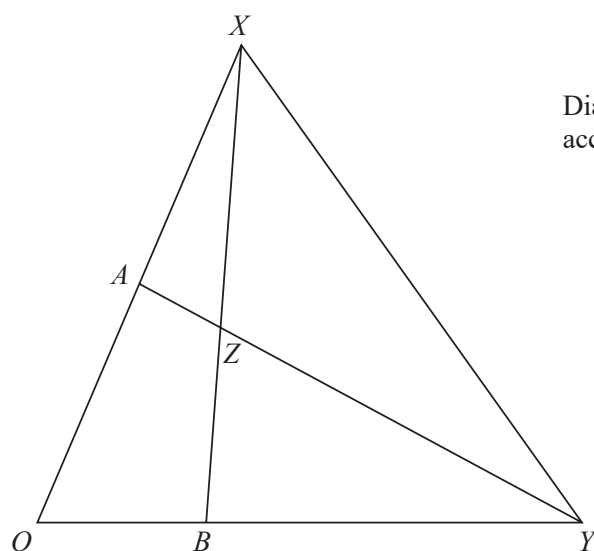
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Figure 1

Figure 1 shows a triangle OXY

$$\overrightarrow{OX} = 2\mathbf{a} \text{ and } \overrightarrow{OY} = 3\mathbf{b}$$

A is the midpoint of OX and B is the point on OY such that $OB : BY = 1 : 2$
The lines XB and AY intersect at Z .

(a) Find $\overrightarrow{AB}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(1)

(b) Using a vector method, find $\overrightarrow{OZ}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(9)

The point M on XY is such that O , Z and M are collinear.

(c) Find $\overrightarrow{OM}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(3)

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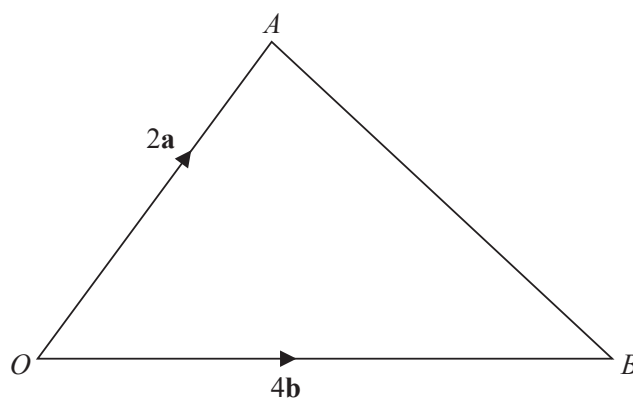


Figure 3

Figure 3 shows the triangle OAB with

$$\vec{OA} = 2\mathbf{a} \text{ and } \vec{OB} = 4\mathbf{b}$$

- (a) Find $\vec{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(2)

The point P is the midpoint of AB

- (b) Find $\vec{OP}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(2)

The point Q lies on OP such that $OQ : QP = 3 : 1$

- (c) Find $\vec{AQ}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(3)

The point R lies on OB such that AQR is a straight line.

- (d) Find in its simplest form the ratio $OR : RB$

(6)

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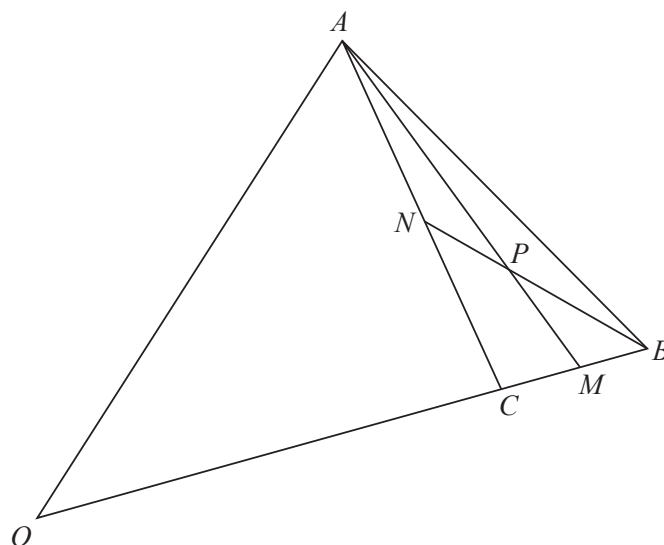
Diagram **NOT**
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Figure 3 shows triangle OAB with $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$

The point C lies on OB such that $OC:CB = 2:1$

The point M is the midpoint of CB and the point N is the midpoint of AC

The lines AM and NB intersect at the point P

(a) Using a vector method, find $\vec{OP}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(9)

The point Q is the midpoint of AB

(b) Using a vector method, show that C , P and Q are collinear.

(4)

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14-MARK QUESTIONS

2 questions in this booklet



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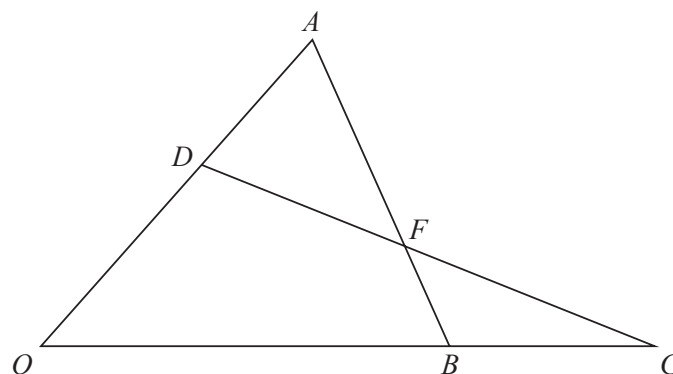
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Figure 1

Figure 1 shows triangle OAB and triangle OCD .

$$\vec{OA} = 5\mathbf{p} \quad \vec{AB} = 3\mathbf{q} \quad \vec{OC} = \frac{3}{2}\vec{OB} \quad \vec{OD} = \frac{3}{5}\vec{OA}$$

- (a) Find $\vec{DC}$ as a simplified expression in terms of $\mathbf{p}$ and $\mathbf{q}$.

(3)

The line DC meets the line AB at F .

- (b) Using a vector method, find $\vec{OF}$ as a simplified expression in terms of $\mathbf{p}$ and $\mathbf{q}$.

(7)

The point G lies on OB such that FG is parallel to AO .

- (c) Using a vector method, find $\vec{OG}$ as a simplified expression in terms of $\mathbf{p}$ and $\mathbf{q}$.

(4)

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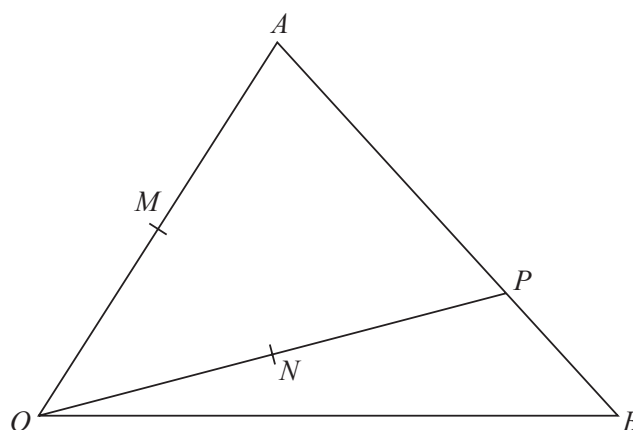
Diagram **NOT**
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Figure 4

Figure 4 shows triangle OAB in which

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}$$

The point P lies on AB such that $AP:PB = 3:1$

The point M is the midpoint of OA and the point N is the midpoint of OP .

(a) Find, as simplified expressions in terms of $\mathbf{a}$ and $\mathbf{b}$, the vector

(i) $\overrightarrow{OP}$ (ii) $\overrightarrow{MN}$

(4)

The point C lies on OB such that ANC is a straight line.

(b) Using a vector method, find the vector $\overrightarrow{OC}$ as a simplified expression in terms of $\mathbf{b}$

(6)

Given that $\frac{\text{area of quadrilateral } AMNP}{\text{area of triangle } OAB} = K$

(c) find the exact value of K

(4)

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15-MARK QUESTIONS

1 question in this booklet



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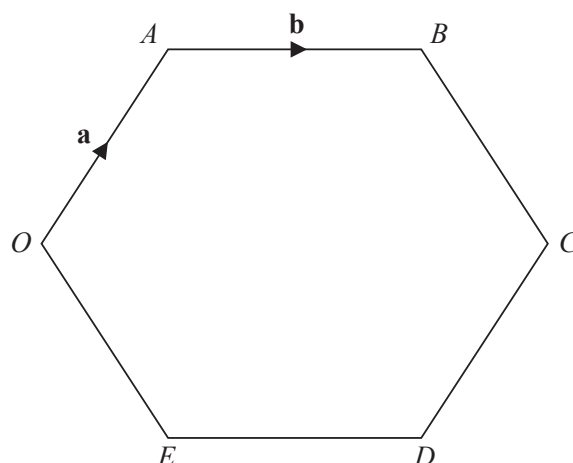
Diagram NOT
accurately drawn

Figure 3

Figure 3 shows the regular hexagon $OABCDE$ with $\vec{OA} = \mathbf{a}$ and $\vec{AB} = \mathbf{b}$

(a) Find $\vec{OB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(1)

(b) Find $\vec{BC}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(3)

The point M divides BC in the ratio $2:1$

(c) Find $\vec{OM}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

(2)

The point Y is such that OMY and ABY are straight lines.

(d) Use a vector method to find $AB:BY$

(5)

The area of hexagon $OABCDE$ is 60 cm^2

(e) Find the area of triangle OAY

(4)

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MARK SCHEMES

official Pearson Edexcel mark schemes



Question Number	Scheme	Marks
1(a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}, = -3\mathbf{i} + 4\mathbf{j}$	M1,A1 (2)
(b)	$ \overrightarrow{AB} = 5$ Unit vector $= \frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$ or $-\frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$ oe Accept column vectors	M1A1 (2)
		[4]
(a)	For $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ seen, or $\overrightarrow{AB} = (\mathbf{i} + 7\mathbf{j}) - (4\mathbf{i} + 3\mathbf{j})$ or equivalent in column form Correct simplified answer as shown or equivalent but NOT a column vector Correct modulus of their $\overrightarrow{AB}$ and divide $\pm$ their $\overrightarrow{AB}$ by it Correct unit vector, as shown or equivalent inc column vector $\pm \frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$ scores M1A0 $\frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$ scores M1A0 If $\overrightarrow{BA}$ is found in (a) both (b) marks are still available	
M1		
A1		
(b)		
M1		
A1		
NB:		

Question number	Scheme	Marks
1	$\vec{BC} = \vec{BA} + \vec{AC} = -3\mathbf{a} - 2\mathbf{b} + 5\mathbf{a} - 3\mathbf{b}$ oe $\vec{DC} = \vec{DA} + \vec{AC} = -2\mathbf{a} + 5\mathbf{b} + 5\mathbf{a} - 3\mathbf{b}$ oe $\vec{BC} = 2\mathbf{a} - 5\mathbf{b}$ $\vec{DC} = 3\mathbf{a} + 2\mathbf{b}$ $\vec{BC} = \vec{AD}$ therefore parallelogram * or $\vec{DC} = \vec{AB}$ therefore parallelogram *	M1 M1 A1 A1 cso* [4]
Total 4 marks		

Part	Mark	Additional Guidance
	M1	For a correct vector path for at least one of $\vec{BC}$ or $\vec{DC}$ or $\vec{CB}$ or $\vec{CD}$ Eg $\vec{BC} = \vec{BA} + \vec{AC}$ or $\vec{DC} = \vec{DA} + \vec{AC}$ This mark may be implied by the next method mark – ie by correctly adding vectors in terms of a and b. Accept poor notation Eg for $\vec{BC}$ accept BC or $\vec{BC}$ or BC
	M1	For a correct unsimplified vector in terms of a and b for $\vec{BC}$ or $\vec{CB}$ or $\vec{DC}$ or $\vec{CD}$ Don't accept $\vec{BC} = +3\mathbf{a} + 2\mathbf{b} - 5\mathbf{a} + 3\mathbf{b}$ (as this is $\vec{CB}$) Don't accept unlabelled vectors Accept poor notation Eg for $\vec{BC}$ accept BC or $\vec{BC}$ or BC
	A1	For a correct simplified vector in terms of a and b for $\vec{BC}$ or $\vec{CB}$ or $\vec{DC}$ or $\vec{CD}$ In order to gain this accuracy mark we must see the first or second M mark (ie the adding together of the paths in vector notation or in terms of the vectors in a and b) Do not allow poor notation. A correct simplified vector alone does not imply previous marks.
	A1 cso*	For a correct simplified vector in terms of a and b for $\vec{BC}$ or $\vec{CB}$ or $\vec{DC}$ or $\vec{CD}$ and a clear (brief) conclusion. Eg $\vec{BC} = -\vec{DA}$ $\vec{BC} = \vec{AD}$ $\vec{DC} = -\vec{BA}$ $\vec{DC} = \vec{AB}$ shown or # * or Candidates may also work with 2 correctly paired vectors and state $\vec{DC} = \vec{AB}$ and $\vec{BC} = \vec{AD}$ or $\vec{DC}$ is parallel to $\vec{AB}$ and $\vec{BC}$ is parallel to $\vec{AD}$ to gain this mark, with also a brief conclusion, if all their work is correct for the vectors involved. Minimum steps shown, ignore any incorrect vectors that they are not using to draw their conclusion.

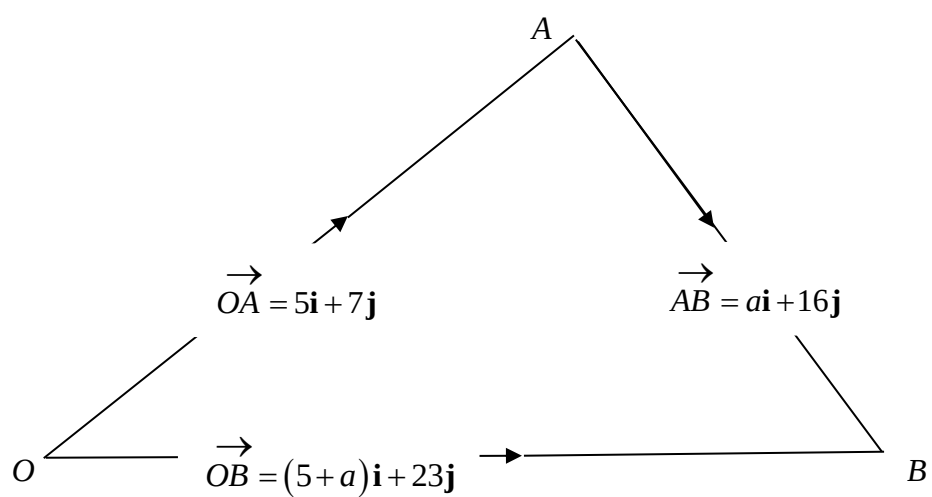


Question number	Scheme	Marks
1(a)	$\vec{OB} = \vec{OA} + \vec{AB}$ or $6\mathbf{i} + 8\mathbf{j} = \vec{OB} - (3\mathbf{i} - 2\mathbf{j})$ oe $9\mathbf{i} + 6\mathbf{j}$	M1 A1 [2]
(b)	$\sqrt{6^2 + 8^2}$ or 10 (from Pythagorean triple)	B1 [1]
(c)	$(\pm) \frac{1}{10} (6\mathbf{i} + 8\mathbf{j})$	M1 A1 [2]
Total 5 marks		

Part	Mark	Additional Guidance
(a)	M1	Correct vector path written, can be implied by correct addition of vectors OR correct vector statement together with correct substitution of the given vectors (where $\vec{OB}$ is not the subject)
	A1	$9\mathbf{i} + 6\mathbf{j}$
(b)	B1	Need not be simplified
(c)	M1	Correctly uses their magnitude from part (b)
	A1	Correct vector Penalise column vector notation for answer first time only

Question	Scheme	Marks
4(a)	$\vec{OB} = \vec{OA} + \vec{AB} = 5\mathbf{i} + 7\mathbf{j} + a\mathbf{i} + 16\mathbf{j} = (5+a)\mathbf{i} + 23\mathbf{j}$	M1A1
	$(5\sqrt{29})^2 = (5+a)^2 + 23^2 \Rightarrow 5+a = \pm\sqrt{196} \Rightarrow a = 9, -19$	M1A1
		[4]
(b)	$\vec{AB} = 9\mathbf{i} + 16\mathbf{j} \Rightarrow \vec{AB} = \sqrt{9^2 + 16^2} = \sqrt{337}$	M1
	Unit vector: $\frac{1}{\sqrt{337}}(9\mathbf{i} + 16\mathbf{j})$ oe.	A1
		[2]
Total 6 marks		

Part	Mark	Notes
(a)	M1	For the correct vector statement for $\vec{OB}$ For example, accept $\vec{AB} = \vec{AO} + \vec{OB}$ This mark can be implied by a correct vector for $\vec{OB}$
	A1	For the correct vector in terms of a [simplified or unsimplified].
	M1	For using Pythagoras theorem with $(5\sqrt{29})$ and their vector for $\vec{OB}$ and solving the equation to find two values of a
	A1	For the two correct values. $a = 9, -19$ seen in their working
(b)	M1	For finding $ \vec{AB} $ by using a correct Pythagoras and writing down the unit vector where $\vec{AB} = k\mathbf{i} + 16\mathbf{j}$ where k is a positive value. Award for $\frac{9\mathbf{i} + 16\mathbf{j}}{\sqrt{9^2 + 16^2}}$ $\sqrt{9^2 + 16^2}$ can be implied by sight of $\sqrt{337}$ If they have an incorrect value for a , full substitution using Pythagoras theorem must be seen for the award of this mark.
	A1	For the correct unit vector. Accept any correct equivalent unit vectors. For example; $\frac{\sqrt{337}}{337}(9\mathbf{i} + 16\mathbf{j})$ or $\frac{9\mathbf{i}}{\sqrt{337}} + \frac{16\mathbf{j}}{\sqrt{337}}$ Accept decimal answers. Eg., $0.054(9\mathbf{i} + 16\mathbf{j})$ for awrt 0.054 Or $0.49\mathbf{i} + 0.87\mathbf{j}$ or better. Allow $\pm \frac{1}{\sqrt{337}}(9\mathbf{i} + 16\mathbf{j})$

Useful Sketch

$$\rightarrow k\left(1 + \frac{3}{k}\right)^2 - (2k+6)\left(1 + \frac{3}{k}\right) + 16 = 0$$

$$\rightarrow k = \dots$$

Send to review if not sure

Question	Scheme	Marks
2(a)	$\frac{2}{\sqrt{1+3x}} = 2(1+3x)^{\left(-\frac{1}{2}\right)}$ $\{2\} \left\{ 1 + \left(-\frac{1}{2}\right)(3x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(3x)^2}{2!} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(3x)^3}{3!} + \dots \right\}$ $2 - 3x + \frac{27x^2}{4} - \frac{135x^3}{8} + \dots$	B1 M1A1 A1 [4]
(b)	$-\frac{1}{3} < x < \frac{1}{3} \quad \left(\text{Accept } x < \frac{1}{3}\right)$	B1 [1]
Total 5 marks		

Part	Mark	Notes
(a)	B1	Correct simplification of the given expression. This mark may be implied by a correct expansion.
	M1	Attempts at the binomial expansion $(1+3x)^{\pm\frac{1}{2}}$ or $2(1+3x)^{\pm\frac{1}{2}}$ For an attempt at the binomial expansion. - The first term is 1 or 2 - The powers of $3x$ are correct in all terms, e.g. $(3x)^2$ - The correct denominators are used, 2! and 3! oe
	A1	Allow this mark for at least 1 correct algebraic term, correctly simplified, from $-3x + \frac{27x^2}{4} - \frac{135x^3}{8}$ or $-\frac{3}{2}x + \frac{27x^2}{8} - \frac{135x^3}{16}$
	A1	Correct simplified expansion in ascending order, coefficients must be in simplest fractions . (Ignore extra terms with powers > 3) Do not isw
(b)	B1	For the correct range of values of x . $-\frac{1}{3} < x < \frac{1}{3}$ or $ x < \frac{1}{3}$, do not accept $ 3x < 1$

ALT for (a) Uses **Maclaurin's expansion** (If seen send to review)

$$f(x) = f(0) + f'(0)x + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + \dots$$

B1 Correct simplification of the given expression.

M1 Achieves $f'(x) = P(1+3x)^{-\frac{3}{2}}$, $f''(x) = Q(1+3x)^{-\frac{5}{2}}$, $f'''(x) = R(1+3x)^{-\frac{7}{2}}$, $P, Q, R \neq 0$ and attempts to find the values of $f'(0)$, $f''(0)$ and $f'''(0)$

A1 Correct unsimplified expansion OR at least 2 correct simplified terms

A1 Fully correct simplified expansion. (Ignore extra terms with powers higher than 3)

For reference, the correct derivatives are:

$$f'(x) = -3(1+3x)^{-\frac{3}{2}}, f''(x) = \frac{27}{2}(1+3x)^{-\frac{5}{2}}, f'''(x) = \frac{-405}{4}(1+3x)^{-\frac{7}{2}}$$



Question	Scheme	Marks
3(a)	$\vec{OA} = \vec{OB} - \vec{AB} = 7i + 2aj - (i + 3aj) = 6i - aj$ $ \vec{OA} = \sqrt{6^2 + a^2} = 3\sqrt{5} \Rightarrow 6^2 + a^2 = 45 \Rightarrow a^2 = 9 \Rightarrow a = 3$	M1A1 M1A1 [4]
(b)	$\vec{OA} = 6i - '3'j$ {Unit vector parallel to $\vec{OA}$ is} $\{\pm\} \frac{1}{3\sqrt{5}}(6i - 3j)$ oe ,isw	B1ft B1 [2]
Total 6 marks		

Part	Mark	Notes
(a)	M1	For the correct vector statement with vector $\vec{OA}$ or $\vec{AO}$ e.g. $\vec{OA} = \vec{OB} - \vec{AB}$ or $\{\vec{OA} = \} 7i + 2aj - (i + 3aj)$ oe
	A1	For the correct simplified or unsimplified vector $\vec{OA}$ or $\vec{AO}$
	M1	For using the correct Pythagoras theorem on their $\vec{OA}$ or $\vec{AO}$ with $3\sqrt{5}$, leading to a value for a^2 or a
	A1	For the correct value of $a=3$ only (must reject $a=-3$ if present) NB:A value of $a=3$ must come from a correct vector of OA with no incorrect work seen.
(b)	B1ft	For the correct vector for $\vec{OA}$ or $\vec{AO}$ using their a (which $a>0$), (Substitutes their a value to $\vec{OA} = 6i - aj$ or $\vec{AO} = -6i + aj$)
	B1	Correct unit vector $\frac{1}{3\sqrt{5}}(6i - 3j)$ oe, can be in either direction, so accept + or - isw

Extra notes:

- Allow working in column vectors throughout.
- Condone missing arrow on their vectors.
- Special Case for part a:** Correct answer with minimum work, no incorrect working shown
e.g. $\sqrt{6^2 + a^2} = 3\sqrt{5}$ M1A1M1 $\rightarrow 6^2 + a^2 = 45 \rightarrow a^2 = 9 \rightarrow a = 3$ A1



Question Number	Scheme	Marks
4(a)	$\overrightarrow{DC} = (11\mathbf{i} - p\mathbf{j}) - (4\mathbf{i} - 2p\mathbf{j}) = 7\mathbf{i} + p\mathbf{j} = \overrightarrow{AB}$ OR: $\overrightarrow{BC} = (11\mathbf{i} - p\mathbf{j}) - (7\mathbf{i} + p\mathbf{j}) = 4\mathbf{i} - 2p\mathbf{j} = \overrightarrow{AD}$ Parallel and equal in length $\therefore$ Parallelogram	M1A1 A1cso (3)
(b)	$\overrightarrow{BD} = (4\mathbf{i} - 2p\mathbf{j}) - (7\mathbf{i} + p\mathbf{j}) = -3\mathbf{i} - 3p\mathbf{j}$ (or $3(-\mathbf{i} - p\mathbf{j})$) oe $\sqrt{9 + (3p)^2} = 3\sqrt{10}$ ($\Rightarrow 9 + 9p^2 = 90$) $p = \pm 3$	B1 M1 A1 (3)
(c)	$(\pm) \frac{1}{3\sqrt{10}}(-3\mathbf{i} - 9\mathbf{j})$ oe	B1ft (1)
		[7]
(a)	Accept column vectors throughout.	
M1	Attempt $\pm \overrightarrow{DC}$ or $\pm \overrightarrow{BC}$ using the difference of 2 appropriate vectors in component form.	
A1	Show that $\pm \overrightarrow{DC}$ or $\pm \overrightarrow{BC} = \pm \overrightarrow{AB}$ or $\pm \overrightarrow{AD}$	
A1cso	Suitable conclusion with reason from correct working. One pair of vectors only needed if reason is “parallel and equal”. Both pairs needed if reason is “2 pairs of sides parallel/equal”.	
(b)		
B1	For a correct $\overrightarrow{BD}$ or $\overrightarrow{DB}$. No simplification needed.	
M1	Use the given length of $\overrightarrow{BD}$ with the length of their $\overrightarrow{BD}$ to form an equation	
A1	Obtain correct values for p . Both needed.	
(c)		
B1ft	Use their positive value for p to obtain a unit vector (no simplification needed)	

Question number	Scheme	Marks
7 (a)	$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$ $\overrightarrow{BC} = -(15\mathbf{i} - 6\mathbf{j}) + 8\mathbf{i} + \mathbf{j} = -7\mathbf{i} + 7\mathbf{j}$	M1 A1 [2]
(b)	$ \overrightarrow{BC} = \sqrt{98} = (7\sqrt{2})$ Unit vector is $\frac{1}{\sqrt{98}}(-7\mathbf{i} + 7\mathbf{j})$ oe	B1 B1 [2]
(c)	$(\overrightarrow{OM} = 4\mathbf{i} - 3\mathbf{j}) \quad \overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j}$ $\Rightarrow \overrightarrow{MN} = -(4\mathbf{i} - 3\mathbf{j}) + 5\mathbf{i} - 2\mathbf{j} (= \mathbf{i} + \mathbf{j})$ $\Rightarrow \overrightarrow{MC} = -(4\mathbf{i} - 3\mathbf{j}) + 8\mathbf{i} + \mathbf{j} (= 4\mathbf{i} + 4\mathbf{j})$ Conclusion: $\overrightarrow{MN}$ and $\overrightarrow{MC}$ are parallel oe (and have same point of origin (M)) hence they are collinear. ALT 1 $(\overrightarrow{OM} = 4\mathbf{i} - 3\mathbf{j}) \quad \overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j} \quad \text{or} \quad \overrightarrow{NB} = 10\mathbf{i} - 4\mathbf{j}$ $\Rightarrow \overrightarrow{MN} = -(4\mathbf{i} - 3\mathbf{j}) + 5\mathbf{i} - 2\mathbf{j} (= \mathbf{i} + \mathbf{j})$ $\Rightarrow \overrightarrow{NC} = 10\mathbf{i} - 4\mathbf{j} - 7\mathbf{i} + 7\mathbf{j} (= 3\mathbf{i} + 3\mathbf{j})$ or $\Rightarrow \overrightarrow{NC} = -(5\mathbf{i} - 2\mathbf{j}) + 8\mathbf{i} + \mathbf{j} (= 3\mathbf{i} + 3\mathbf{j})$ Conclusion: $\overrightarrow{MN}$ and $\overrightarrow{NC}$ are parallel oe (and share the same point (N)) hence they are collinear. ALT 2 $(\overrightarrow{OM} = 4\mathbf{i} - 3\mathbf{j}) \quad \overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j} \quad \text{or} \quad \overrightarrow{NB} = 10\mathbf{i} - 4\mathbf{j}$ $\Rightarrow \overrightarrow{MC} = -(4\mathbf{i} - 3\mathbf{j}) + 8\mathbf{i} + \mathbf{j} (= 4\mathbf{i} + 4\mathbf{j})$ $\Rightarrow \overrightarrow{NC} = 10\mathbf{i} - 4\mathbf{j} - 7\mathbf{i} + 7\mathbf{j} (= 3\mathbf{i} + 3\mathbf{j})$ Conclusion: $\overrightarrow{MC}$ and $\overrightarrow{NC}$ are parallel oe (and share the same point (C)) hence they are collinear.	B1 M1 M1 A1 [4] {B1} {M1} {M1} {A1} [4] {B1} {M1} {M1} {A1} [4]
Total 8 marks		

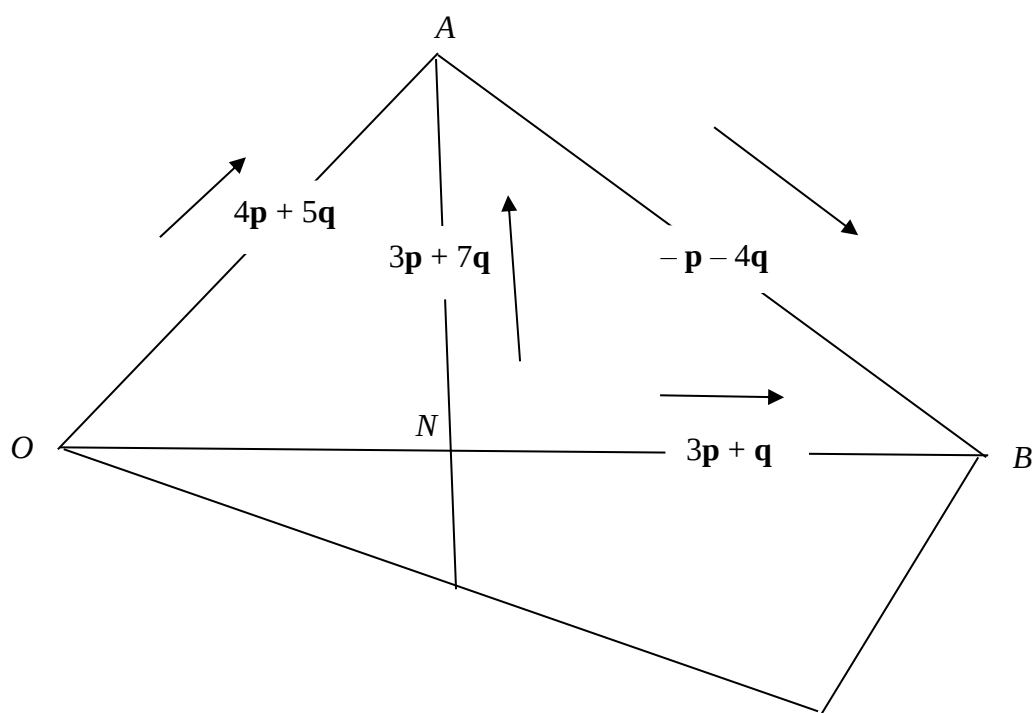
(a)	
M1	$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$
A1	$\overrightarrow{BC} = -7\mathbf{i} + 7\mathbf{j}$
(b)	
B1	$\sqrt{98}$ oe
B1	$\frac{1}{\sqrt{98}}(-7\mathbf{i} + 7\mathbf{j})$ oe
(c)	
B1	$\overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j}$ (may be implied by $\overrightarrow{MN}$)
M1	$\overrightarrow{MN} = -(4\mathbf{i} - 3\mathbf{j}) + 5\mathbf{i} - 2\mathbf{j} (= \mathbf{i} + \mathbf{j})$
M1	$\overrightarrow{MC} = -(4\mathbf{i} - 3\mathbf{j}) + 8\mathbf{i} + \mathbf{j} (= 4\mathbf{i} + 4\mathbf{j})$
A1	Correct conclusion from correct working e.g. $\overrightarrow{MC} = 4\overrightarrow{MN}$
ALT 1	
B1	$\overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j}$ or $\overrightarrow{NB} = 10\mathbf{i} - 4\mathbf{j}$ (may be implied by $\overrightarrow{MN}$ or $\overrightarrow{NC}$)
M1	$\overrightarrow{MN} = -(4\mathbf{i} - 3\mathbf{j}) + 5\mathbf{i} - 2\mathbf{j} (= \mathbf{i} + \mathbf{j})$
M1	$\overrightarrow{NC} = 10\mathbf{i} - 4\mathbf{j} - 7\mathbf{i} + 7\mathbf{j} (= 3\mathbf{i} + 3\mathbf{j})$ or $-(5\mathbf{i} - 2\mathbf{j}) + 8\mathbf{i} + \mathbf{j} (= 3\mathbf{i} + 3\mathbf{j})$
A1	Correct conclusion from correct working e.g. $\overrightarrow{NC} = 3\overrightarrow{MN}$
ALT 2	
B1	$\overrightarrow{NB} = 10\mathbf{i} - 4\mathbf{j}$ (may be implied by $\overrightarrow{NC}$)
M1	$\overrightarrow{MC} = -(4\mathbf{i} - 3\mathbf{j}) + 8\mathbf{i} + \mathbf{j} (= 4\mathbf{i} + 4\mathbf{j})$
A1	$\overrightarrow{NC} = 10\mathbf{i} - 4\mathbf{j} - 7\mathbf{i} + 7\mathbf{j} (= 3\mathbf{i} + 3\mathbf{j})$
	Correct conclusion from correct working e.g. $\overrightarrow{NC} = \frac{3}{4}\overrightarrow{MC}$
For part c: Send any geometrical solutions to review	

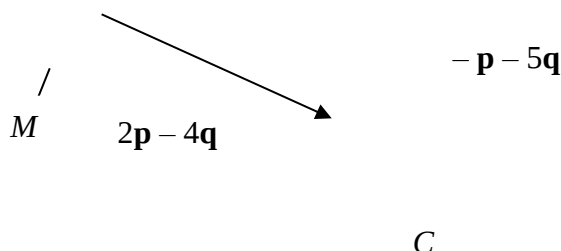
Question number	Scheme	Marks
3 a	$\left \vec{OA} \right = \sqrt{p^2 + 16} \quad \text{and} \quad \left \vec{OB} \right = \sqrt{4p^2 + 4p + 2}$ $\sqrt{2} \left \vec{OA} \right = \left \vec{OB} \right \Rightarrow 2p^2 + 32 = 4p^2 + 4p + 2$ $2p^2 + 4p - 30 = 0 \Rightarrow p^2 + 2p - 15 = 0$ $(p+5)(p-3) = 0$ $p = 3$	M1 M1 M1 A1 (4)
b	$\vec{AB} = -'3'\mathbf{i} + 4\mathbf{j} + \mathbf{i} + (2 \times '3' + 1)\mathbf{j} = -2\mathbf{i} + 11\mathbf{j}$ $\left \vec{AB} \right = \sqrt{'4' + '121'} \quad [= 5\sqrt{5}]$ $\left[\frac{1}{5\sqrt{5}} \right] (-2\mathbf{i} + 11\mathbf{j})$ $(\pm) \frac{1}{5\sqrt{5}} (-2\mathbf{i} + 11\mathbf{j})$	M1 A1 M1 dM1 A1 (5)
Total 9 marks		

Part	Mark	Notes
(a)		For use of $\sqrt{2} \left \vec{OA} \right = \left \vec{OB} \right $
	M1	i.e., $\sqrt{2} \times \sqrt{p^2 + (-4)^2} = \sqrt{1 + (2p+1)^2} \Rightarrow (\sqrt{2} \times \sqrt{p^2 + 16} = \sqrt{4p^2 + 4p + 2})$ They may find $\left \vec{OA} \right $ and $\left \vec{OB} \right $ separately. Award when combined with $\sqrt{2}$ and condone arithmetical slips.
	M1	For forming a 3TQ in any order. [The correct 3TQ is $2p^2 + 4p - 30 = 0$ or $p^2 + 2p - 15 = 0$]
	M1	For a correct attempt to solve their 3TQ by any valid method. They must reach a value of p for this mark.
	A1	For $p = 3$ If they also give $p = -5$ without evidence of rejecting this solution, withhold the A mark
(b)	M1	For the vector statement $\vec{AB} = \vec{AO} + \vec{OB}$ o.e. This can be implied by sight of $\vec{AB} = '-2'\mathbf{i} + '11'\mathbf{j}$ or $\vec{AB} = \begin{pmatrix} '-2' \\ '11' \end{pmatrix}$ If there is no vector statement you must check their vector for substitution of their p .
	A1	For the correct $\vec{AB}$ (allow unsimplified) $\left(\vec{AB} = -2\mathbf{i} + 11\mathbf{j} \right)$ and also allow $\vec{AB} = \begin{pmatrix} -2 \\ 11 \end{pmatrix}$ Award for sight of $-2\mathbf{i} + 11\mathbf{j}$ only.
	M1	For using Pythagoras theorem on their $\vec{AB}$ i.e., $\sqrt{(-2)^2 + 11^2}$
	dM1	For correct method to find a unit vector using their values and their $\vec{AB}$ NB: this mark is dependent on the previous M mark
	A1	For $\frac{1}{5\sqrt{5}}(-2\mathbf{i} + 11\mathbf{j})$ oe $\left[\text{Allow } -\frac{1}{5\sqrt{5}}(-2\mathbf{i} + 11\mathbf{j}) \right]$ OR $\vec{AB} = \pm \frac{1}{5\sqrt{5}} \begin{pmatrix} -2 \\ 11 \end{pmatrix}$

Question	Scheme	Marks
11(a)	$\vec{OM} = \frac{1}{2}\vec{OC} = \mathbf{p} - 2\mathbf{q}$ $\vec{MA} = -\vec{OM} + \vec{OA}$ $\vec{MA} = -(\mathbf{p} - 2\mathbf{q}) + (4\mathbf{p} + 5\mathbf{q}) = 3\mathbf{p} + 7\mathbf{q}$	B1 M1 A1 [3]
(b)	$\vec{MN} = \lambda(3\mathbf{p} + 7\mathbf{q})$ $\vec{MN} = -(\mathbf{p} - 2\mathbf{q}) + \mu(3\mathbf{p} + \mathbf{q})$ $\lambda(3\mathbf{p} + 7\mathbf{q}) = -(\mathbf{p} - 2\mathbf{q}) + \mu(3\mathbf{p} + \mathbf{q})$ $\Rightarrow 3\lambda\mathbf{p} + 7\lambda\mathbf{q} = (-1 + 3\mu)\mathbf{p} + (2 + \mu)\mathbf{q}$ $3\lambda = -1 + 3\mu$ $7\lambda = 2 + \mu$ $\Rightarrow 18\lambda = 7 \Rightarrow \lambda = \frac{7}{18}$ $MN : NA = 7 : 11$	M1 M1 M1 dM1 A1 A1 [6]
Total 9 marks		

Useful sketch



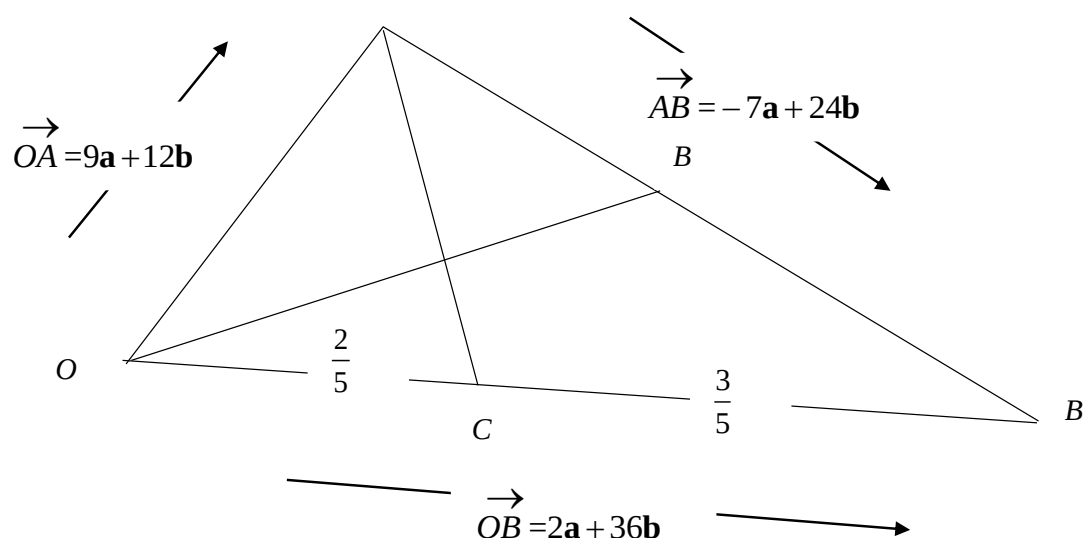


Part	Mark	Notes
(a)	B1	For finding either vector $\vec{OM} = \mathbf{p} - 2\mathbf{q}$ or $\vec{MO} = -\mathbf{p} + 2\mathbf{q}$ This may be embedded in their working for $\vec{MA}$
	M1	For the correct vector statement $\vec{MA} = -\vec{OM} + \vec{OA}$ or $\vec{MA} = -\frac{1}{2}\vec{OC} + \vec{OA}$
	A1	For the correct simplified vector
(b)	General principle of marking part (b)	
		- First two M marks are for two vector statements for $\vec{MN}$ or $\vec{AN}$ that will allow them to be equated. Note that they can use any path that involves either of these two vectors. - The third M mark is for equating coefficients and forming a pair of simultaneous equations. - The final M mark is for solving their simultaneous equations.
	M1	For one vector for $\vec{MN}$ or $\vec{AN}$ involving a constant One example is: $\vec{AN} = K \vec{AM} = K(-3\mathbf{p} - 7\mathbf{q})$
	M1	For a second vector for $\vec{MN}$ or $\vec{AN}$ following a different path involving a different constant. One example is: $\vec{AN} = \vec{AO} + L \vec{OB} = -4\mathbf{p} - 5\mathbf{q} + L(3\mathbf{p} + \mathbf{q})$
	M1	For equating the two vectors and forming a pair of simultaneous linear equations, both of which must be in terms of λ and μ
	dM1	For an attempt to solve their linear equations. Allow up to one processing error. This mark is dependent on the previous M mark.
	A1	For the value of $\lambda = \frac{7}{18}$ μ is not required but the value is $\frac{13}{18}$
	A1	For the correct ratio $MN : NA = 7 : 11$

Question number	Scheme	Marks
6 (a)	$\begin{pmatrix} \vec{AB} \end{pmatrix} = \vec{AO} + \vec{OB} = -9\mathbf{a} - 12\mathbf{b} + 2\mathbf{a} + p\mathbf{b}$ $(-7)^2 + (p-12)^2 = 25^2$ $(p-12)^2 = 576$ $p = 12 \pm \sqrt{576}$ $p = -12, 36^*$	M1 M1 M1 A1* [4]
(b)	$\sqrt{12^2 + 2^2} \Rightarrow (\sqrt{148} = 2\sqrt{37})$ $\pm \frac{1}{\sqrt{37}} (\mathbf{a} - 6\mathbf{b}) \text{ or } \pm \frac{\sqrt{37}}{37} (\mathbf{a} - 6\mathbf{b})$	M1 A1 [2]
(c)	$(\vec{OC}) = \frac{2}{5} (2\mathbf{a} + 36\mathbf{b})$ $(\vec{CA}) = -\frac{2}{5} (2\mathbf{a} + 36\mathbf{b}) + 9\mathbf{a} + 12\mathbf{b}$ $= \frac{41}{5} \mathbf{a} - \frac{12}{5} \mathbf{b}$	M1 M1 A1 [3]
Total 9 marks		

Part	Mark	Additional Guidance
6 (a)	M1	For a correct vector path to find $\pm \vec{AB}$ or $-9\mathbf{a} - 12\mathbf{b} + 2\mathbf{a} + p\mathbf{b}$ or $9\mathbf{a} + 12\mathbf{b} - 2\mathbf{a} - p\mathbf{b}$
	M1	For using Pythagoras' Theorem with the $\vec{AB}$
	M1	For any complete and minimally acceptable attempt to solve for p See General Guidance.
	A1*	For both correct values of p – no errors seen. Note: This is a show question and the answers are given
(b)	M1	For a correct vector path to find the magnitude.
O	A1	For a correct vector as shown.
(c)	M1	For a correct vector for $\vec{CA}$
	M1	For the correct vector for $\vec{CA}$ using their $\vec{CB} + \vec{BA}$
	A1	For $\frac{41}{5}\mathbf{a} - \frac{12}{5}\mathbf{b}$
	ALT: Uses the vector path $\vec{CA} = \vec{CB} + \vec{BA}$	
	M1	For a correct vector for $\vec{CB} = \frac{3}{5}(2\mathbf{a} + 36\mathbf{b})$ or $\vec{BA} = 7\mathbf{a} - 24\mathbf{b}$
	M1	For the correct vector for $\vec{CA}$ using their $\vec{CB}$ and $\vec{BA}$ $\vec{CA} = \frac{3}{5}(2\mathbf{a} + 36\mathbf{b}) + (7\mathbf{a} - 24\mathbf{b})$
	A1	$\vec{CA} = \frac{41}{5}\mathbf{a} - \frac{12}{5}\mathbf{b}$

Useful Sketch



Question number	Scheme	Marks
6 (a)	$\vec{SR} = \vec{SP} + \vec{PR} = (3\mathbf{i} - 15\mathbf{j} - \mathbf{i} + 18\mathbf{j})$ $= 2\mathbf{i} + 3\mathbf{j}$ or $\vec{QR} = \vec{QP} + \vec{QR} = (-2\mathbf{i} - 3\mathbf{j} - \mathbf{i} + 18\mathbf{j})$ $= -3\mathbf{i} + 15\mathbf{j}$	M1 A1
	$\vec{PQ} = \vec{SR}$ or $\vec{PS} = \vec{QR}$ oe	M1
	(As opposite sides are equal in length and parallel) PQRS is a parallelogram *	A1cso [4]
(b)	$\vec{QS} = \vec{QP} + \vec{PS} = (-2\mathbf{i} - 3\mathbf{j} - 3\mathbf{i} + 15\mathbf{j})$ $= -5\mathbf{i} + 12\mathbf{j}$	M1 A1
	Unit vector = $(\pm) \frac{1}{13}(-5\mathbf{i} + 12\mathbf{j})$	M1 A1 [4]
(c)	$\vec{PT} = 2\mathbf{i} + 3\mathbf{j} + \frac{5}{13}(-5\mathbf{i} + 12\mathbf{j}) = \frac{1}{13}\mathbf{i} + \frac{99}{13}\mathbf{j}$	M1 A1 [2]
Total 10 marks		

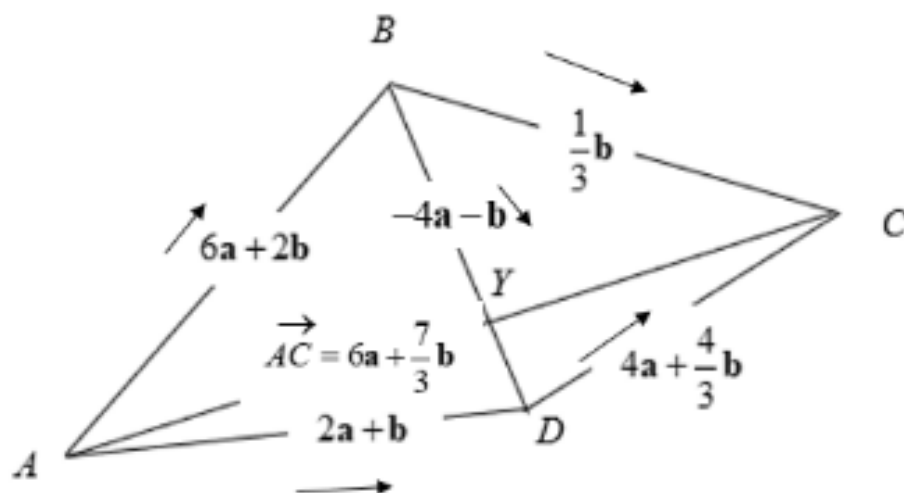
Part	Mark	Notes
(a)	M1	For stating a correct valid vector path, e.g. $\vec{SR} = \vec{SP} + \vec{PR}$
	A1	For $\pm(2\mathbf{i} + 3\mathbf{j})$ or $\pm(-3\mathbf{i} + 15\mathbf{j})$
	M1	For a correct vector statement leading to a conclusion eg $\vec{PQ} = \vec{SR}$
	A1*cso	For some form of conclusion.
(b)	M1	For using e.g. $\vec{QS} = \vec{QP} + \vec{PS}$ or $\vec{SQ}$
	A1	For $-5\mathbf{i} + 12\mathbf{j}$
	M1	For an attempt to use Pythagoras with a plus sign, allow use of their vector $\pm \vec{QS}$
	A1	For $(\pm) \frac{1}{13}(-5\mathbf{i} + 12\mathbf{j})$ oe Must be one vector only ie can't list as $\pm$
(c)	M1	For using e.g. $\vec{PT} = \vec{PQ} + \frac{5}{13}(\text{their } \vec{QS})$
	A1	For $\frac{1}{13}\mathbf{i} + \frac{99}{13}\mathbf{j}$ oe

Question number	Scheme	Marks
9 (a)	$\vec{AB} = \vec{AD} + \vec{DB} = 2\mathbf{a} + \mathbf{b} - (-4\mathbf{a} - \mathbf{b}) = 6\mathbf{a} + 2\mathbf{b} \text{ or } 2(3\mathbf{a} + \mathbf{b}) \text{ oe}$ $\vec{DC} = \vec{DB} + \vec{BC} = 4\mathbf{a} + \mathbf{b} + \frac{1}{3}\mathbf{b} = \left(4\mathbf{a} + \frac{4}{3}\mathbf{b}\right) \text{ or } \frac{4}{3}(3\mathbf{a} + \mathbf{b})$ $\vec{AB} = \frac{3}{2}\vec{DC} \text{ so } \vec{AB} \text{ is a multiple of } \vec{DC}$ Conclusion: Therefore, DC parallel to AB)	B1 B1 M1 A1 [4]
(b)	$\vec{AC} = 2\mathbf{a} + \mathbf{b} + \left(4\mathbf{a} + \frac{4}{3}\mathbf{b}\right) = \left[6\mathbf{a} + \frac{7}{3}\mathbf{b}\right]$ $\vec{AY} = \lambda \vec{AC} \quad (= \lambda(6\mathbf{a} + \frac{7}{3}\mathbf{b}))$ $\vec{AY} = \vec{AD} + \mu \vec{DB} \quad (= 2\mathbf{a} + \mathbf{b} + \mu(4\mathbf{a} + \mathbf{b}))$ or $\vec{AY} = \alpha \vec{AB} + \beta \vec{BD} \quad (= \alpha(6\mathbf{a} + 2\mathbf{b}) + \beta(-4\mathbf{a} - \mathbf{b}))$ $2 + 4\mu = 6\lambda \quad \text{or} \quad 6 - 4\alpha = 6\lambda$ $1 + \mu = \frac{7}{3}\lambda \quad \text{or} \quad 1 + \mu = \frac{7}{3}\lambda$ $\lambda = \frac{3}{5} \quad \text{or} \quad \mu = \frac{2}{5} \quad \text{or} \quad \alpha = \frac{3}{5}$ $\vec{AY} = \frac{18}{5}\mathbf{a} + \frac{7}{5}\mathbf{b} \text{ or } \vec{AY} = \frac{1}{5}(8\mathbf{a} + 7\mathbf{b})$	M1 M1 M1 M1 A1 A1cao [6]
Total 10 marks		

Part	Mark	Additional Guidance
(a)	B1	For stating a valid vector for $\vec{AB} = 6\mathbf{a} + 2\mathbf{b}$ [accept simplified or unsimplified] NB: This is an M mark in Epen
	B1	For stating a valid vector for $\vec{DC} = \frac{4}{3}(3\mathbf{a} + \mathbf{b})$ [accept simplified or unsimplified] NB: This is an A mark in Epen
	M1	For comparing their two vectors and establishing that $\vec{DC}$ and $\vec{AB}$ are multiples of each other .i.e. $\vec{AB} = \frac{3}{2}\vec{DC}$ oe NB This statement must be correct! <u>Incorrect directions.</u> Award this mark if they are comparing for example; $\vec{CD}$ and $\vec{AB}$ or $\vec{CD}$ and $\vec{BA}$ and ft their incorrect direction for the M mark.

	A1ft	For a conclusion that the vectors are therefore parallel. Allow for example $\vec{AB} = -6\mathbf{a} - 2\mathbf{b}$ and $\vec{DC} = \frac{4}{3}(3\mathbf{a} + \mathbf{b})$ are therefore parallel Accept a very brief conclusion even if it is just the word 'shown' or 'QED' or even '#' or a tick.
(b)		Part (b) can be done in a number of different ways – use the following as general principles to mark, referring also to the scheme and additional guidance for the example given, to determine when each mark should be awarded
	M1	For any simplified or un-simplified vector along AC $\vec{AC} = 2\mathbf{a} + \mathbf{b} + \left(4\mathbf{a} + \frac{4}{3}\mathbf{b}\right) = \left[6\mathbf{a} + \frac{7}{3}\mathbf{b}\right]$ For this mark allow $\vec{CA} = -6\mathbf{a} - \frac{7}{3}\mathbf{b}$ as it is possible to use this to find $\vec{AY}$ This mark can be implied by a correct vector for $\vec{AY}$ e.g. $\vec{AY} = \lambda \left[6\mathbf{a} + \frac{7}{3}\mathbf{b}\right]$
	M1	Uses their vector for $\vec{AC}$ to write a vector for $\vec{AY}$ by introducing a parameter e.g. $\vec{AY} = \lambda \left[6\mathbf{a} + \frac{7}{3}\mathbf{b}\right]$
	M1	States a second valid vector path for $\vec{AY}$ introducing a second parameter that can be used with the first vector path [to solve an equation to find their parameters]. Note: this mark is awarded only for stating the vector path, but it must be valid – i.e. able to be used to find the parameters. This second parameter cannot be 1 – their first parameter.
	dddM1	Equating components and reaching a value for any of their parameters. Allow arithmetical slips in processing their simultaneous equations. This is dependent on all previous method marks.
	A1	Any one correct value for one of their two parameters.
	A1	For $\vec{AY} = \frac{18}{5}\mathbf{a} + \frac{7}{5}\mathbf{b}$

USEFUL SKETCH



Question number	Scheme	Marks
11 a i	$\left(\vec{AN} = \vec{AO} + \vec{ON} \right) \text{ or } \left(\vec{AN} = \vec{AO} + \frac{3}{4} \vec{OB} = -\mathbf{a} + \frac{3}{4} \mathbf{b} \right)$	M1 A1
ii	$\left(\vec{BM} = \vec{BO} + \vec{OM} = \right) -\mathbf{b} + \frac{1}{2} \mathbf{a}$	B1 (A1 on ePen) (3)
b	$\left(\vec{AX} = \lambda \left(\pm \left(\text{their } \vec{AN} \right) \right) \right) = \lambda \left(\pm -\mathbf{a} + \frac{3}{4} \mathbf{b} \right)$	B1 ft
	$\left(\vec{AX} = \vec{AM} + \mu \left(\pm \left(\text{their } \vec{MB} \right) \right) \right) = -\frac{1}{2} \mathbf{a} + \mu \left(\pm -\frac{1}{2} \mathbf{a} + \mathbf{b} \right)$	M1
	Equating components gives $-\lambda = -\frac{1}{2} - \frac{1}{2} \mu$	M1
	and $\frac{3}{4} \lambda = \mu$	M1
	$-\lambda = -\frac{1}{2} - \frac{1}{2} \left(\frac{3}{4} \lambda \right) \Rightarrow \lambda = \frac{4}{5} \text{ or } \mu = \frac{3}{5}$	ddM1 A1
	So $AX : XN = 4 : 1$	A1 (7)
ALT	$\left(\vec{AX} = \vec{AB} + \vec{BN} + \lambda \left(\pm \left(\text{their } \vec{AN} \right) \right) \right) = \mathbf{b} - \mathbf{a} - \frac{1}{4} \mathbf{b} + \lambda \left(\pm -\mathbf{a} + \frac{3}{4} \mathbf{b} \right)$	B1ft
	$\left(\vec{AX} = \vec{AM} + \mu \left(\pm \left(\text{their } \vec{MB} \right) \right) \right) = -\frac{1}{2} \mathbf{a} + \mu \left(\pm -\frac{1}{2} \mathbf{a} + \mathbf{b} \right)$	M1
	Equating components gives $-\frac{1}{2} - \frac{1}{2} \mu = -1 + \lambda$	M1
	and $\mu = \frac{3}{4} - \frac{3}{4} \lambda$	M1
	$\mu = \frac{3}{4} - \frac{3}{4} \left(\frac{1}{2} - \frac{1}{2} \mu \right) \Rightarrow \lambda = \frac{1}{5} \text{ or } \mu = \frac{3}{5}$	ddM1 A1
	So $AX : XN = 4 : 1$	A1 (7)

General principles for marking part b

B1 ft states a valid vector, with a parameter, that could be used in a solution to find the ratio required.

M1 states a second valid vector path, with a second distinct parameter that could be used to find the ratio required. **This must be a distinct route, different to that used already, not travelling along the same line.**

M1 This mark is obtained by correctly equating their components for **a**. **They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ**

M1 This mark is obtained by correctly equating their components for **b**. **They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ**

ddM1 For an attempt to solve their simultaneous equations. There can be errors, but it must be clear they are solving simultaneous equations to arrive at a value for λ or μ or their parameters.

Dependent on both previous method marks.

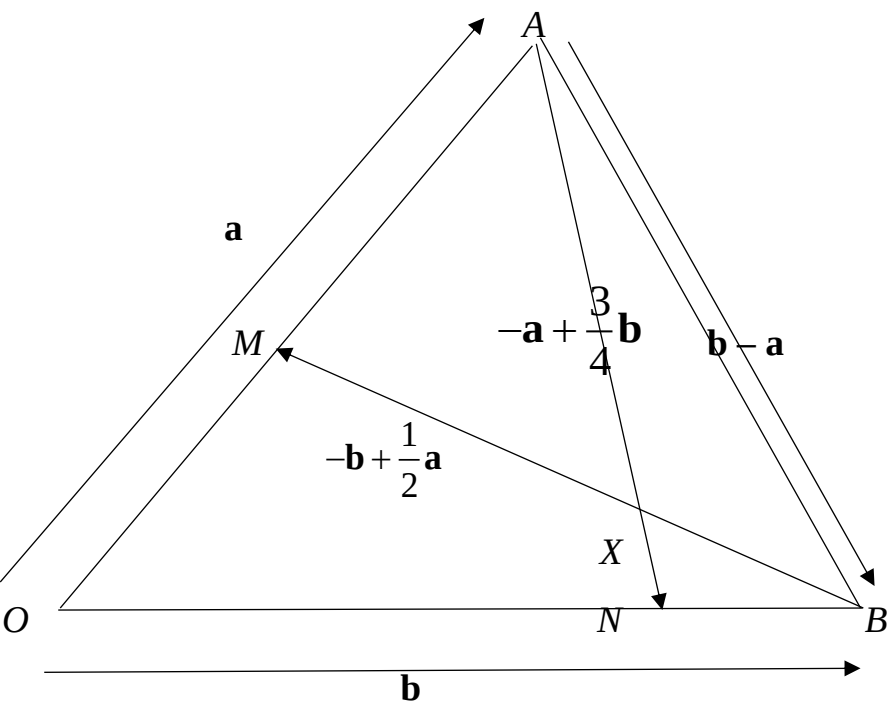
A1 correct value for one of their parameters

A1 correct ratio



Part	Mark	Additional Guidance
(a)	M1	For a correct vector path stated, a fully correct answer implies this mark.
(i)	A1	For $-\mathbf{a} + \frac{3}{4}\mathbf{b}$
(ii)	B1 (A1 ePen)	For $-\mathbf{b} + \frac{1}{2}\mathbf{a}$
(b)	B1 ft	For $\vec{AX} = \lambda \pm$ their $\vec{AN}$ written in terms of vectors $\mathbf{a}$ and $\mathbf{b}$ $\left(\vec{AX} = \right) \lambda \left(\pm \left(-\mathbf{a} + \frac{3}{4}\mathbf{b} \right) \right)$ This may be seen embedded in working.
	M1	For e.g. $\vec{AX} = \vec{AM} + \mu \pm$ their $\vec{MB}$ written in terms of vectors $\mathbf{a}$ and $\mathbf{b}$ $\left(\vec{AX} = \right) -\frac{1}{2}\mathbf{a} + \mu \left(\pm \left(-\frac{1}{2}\mathbf{a} + \mathbf{b} \right) \right)$ This may be seen embedded in working. This must be a distinct route, different to that used already, not travelling along the same line.
	M1	For $-\lambda = -\frac{1}{2} - \frac{1}{2}\mu$ This mark is obtained by correctly equating their components for $\mathbf{a}$, the example given is when their equations are set up using $\vec{AX} = \lambda \vec{AN}$ and $\vec{AX} = \vec{AM} + \mu \vec{MB}$ They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ.
	M1	For $\frac{3}{4}\lambda = \mu$ This mark is obtained by correctly equating their components for $\mathbf{b}$, the example given is when their equations are set up using $\vec{AX} = \lambda \vec{AN}$ and $\vec{AX} = \vec{AM} + \mu \vec{MB}$ They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ.
	ddM1	For an attempt to solve their simultaneous equations. There can be errors, but it must be clear they are solving simultaneous equations to arrive at a value for λ or μ or their parameters. Dependent on both previous method marks.
	A1	There are a number of different correct answers here, depending on how they've set up their equations: For $\lambda = \frac{4}{5}$ or $\mu = \frac{3}{5}$ using $\vec{AX} = \lambda \vec{AN}$ and $\vec{AX} = \vec{AM} + \mu \vec{MB}$ For $\lambda = -\frac{4}{5}$ or $\mu = \frac{3}{5}$ using $\vec{AX} = \lambda \left(-\vec{AN} \right)$ and $\vec{AX} = \vec{AM} + \mu \vec{MB}$ For $\lambda = \frac{4}{5}$ or $\mu = -\frac{3}{5}$ using $\vec{AX} = \lambda \vec{AN}$ and $\vec{AX} = \vec{AM} + \mu \left(-\vec{MB} \right)$ For $\lambda = -\frac{4}{5}$ or $\mu = -\frac{3}{5}$ using $\vec{AX} = \lambda \left(-\vec{AN} \right)$ and $\vec{AX} = \vec{AM} + \mu \left(-\vec{MB} \right)$
	A1	For $AX : XN = 4 : 1$

(b) ALT	B1ft	For $\vec{AX} = \vec{AB} + \vec{BN} + \lambda \left(\pm \text{their } \vec{AN} \right)$ written in terms of a and b $\left(\vec{AX} \right) = \mathbf{b} - \mathbf{a} - \frac{1}{4}\mathbf{b} + \lambda \left(\pm "-\mathbf{a} + \frac{3}{4}\mathbf{b}" \right)$ This may be seen embedded in working.
	M1	For e.g. $\vec{AX} = \vec{AM} + \mu \pm \text{their } \vec{MB}$ written in terms of vectors a and b $\left(\vec{AX} \right) = -\frac{1}{2}\mathbf{a} + \mu \left(\pm "-\frac{1}{2}\mathbf{a} + \mathbf{b}" \right)$ This may be seen embedded in working. This must be a distinct route, different to that used already, not travelling along the same line.
	M1	For $-\frac{1}{2} - \frac{1}{2}\mu = -1 + \lambda$ This mark is obtained by correctly equating their components for a, the example given is when their equations are set up using $\vec{AX} = \vec{AB} + \vec{BN} + \lambda \vec{AN}$ and $\vec{AX} = \vec{AM} + \mu \vec{MB}$ They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ.
	M1	For $\mu = \frac{3}{4} - \frac{3}{4}\lambda$ This mark is obtained by correctly equating their components for b, the example given is when their equations are set up using $\vec{AX} = \vec{AB} + \vec{BN} + \lambda \vec{AN}$ and $\vec{AX} = \vec{AM} + \mu \vec{MB}$ They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ.
	ddM1	For an attempt to solve their simultaneous equations. There can be errors, but it must be clear they are solving simultaneous equations to arrive at a value for λ or μ Dependent on both previous method marks.
	A1	There are a number of different correct answers here, depending on how they've set up their equations: For $\lambda = \frac{1}{5}$ or $\mu = \frac{3}{5}$ using $\vec{AX} = \vec{AB} + \vec{BN} + \lambda \vec{AN}$ and $\vec{AX} = \vec{AM} + \mu \vec{MB}$ For $\lambda = -\frac{1}{5}$ or $\mu = \frac{3}{5}$ using $\vec{AX} = \vec{AB} + \vec{BN} + \lambda \left(-\vec{AN} \right)$ and $\vec{AX} = \vec{AM} + \mu \vec{MB}$ For $\lambda = \frac{1}{5}$ or $\mu = -\frac{3}{5}$ using $\vec{AX} = \vec{AB} + \vec{BN} + \lambda \vec{AN}$ and $\vec{AX} = \vec{AM} + \mu \left(-\vec{MB} \right)$ For $\lambda = -\frac{1}{5}$ or $\mu = -\frac{3}{5}$ using $\vec{AX} = \vec{AB} + \vec{BN} + \lambda \left(-\vec{AN} \right)$ and $\vec{AX} = \vec{AM} + \mu \left(-\vec{MB} \right)$
	A1	For $AX : XN = 4 : 1$



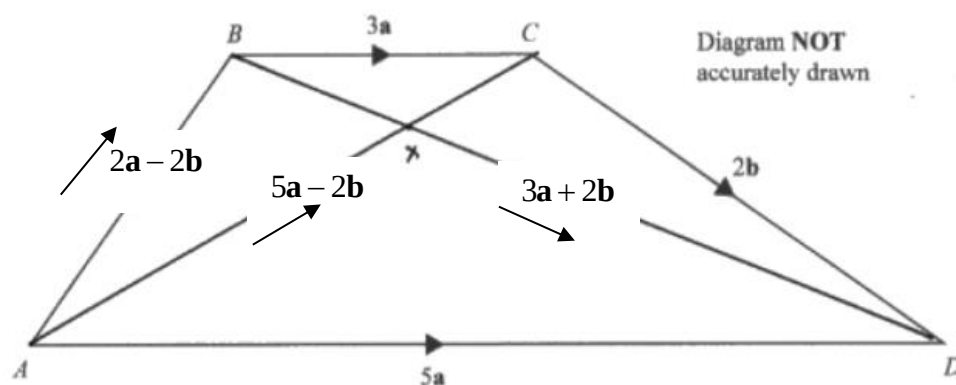
Question	Scheme	Marks
10(i)	$\vec{AB} = \vec{OB} - \vec{OA} = [(3a+2)\mathbf{i} + b\mathbf{j}] - [(b+1)\mathbf{i} + b\mathbf{j}]$ $\vec{AB} = [(3a+2) - (b+1)]\mathbf{i} = 3\mathbf{i}$ $(3a+2) - (b+1) = 3 \Rightarrow 3a - b = 2 \text{ or } b = 3a - 2$ $\frac{\sqrt{17}}{34} = \frac{1}{\sqrt{68}}$ $68 = (3a+2)^2 + b^2$ $\Rightarrow 68 = (3a+2)^2 + (3a-2)^2 \Rightarrow 68 = 18a^2 + 8$ $\Rightarrow 60 = 18a^2 \Rightarrow a = \frac{\sqrt{30}}{3} \text{ oe}$	M1 A1 M1A1 B1 M1 M1 A1
(ii)	$b = \frac{3\sqrt{30}}{3} - 2 = \sqrt{30} - 2$	M1A1 [10]
Total 10 marks		

Part	Mark	Notes
	M1	For the vector equation $\vec{AB} = \vec{OB} - \vec{OA}$ This can be implied by the next correct step.
	A1	For the correct unsimplified vector $\vec{AB} = [(3a+2)\mathbf{i} + b\mathbf{j}] - [(b+1)\mathbf{i} + b\mathbf{j}]$
	M1	For setting their vector $\vec{AB} = 3\mathbf{i}$ and attempting to find an expression in terms of a and b only
	A1	For a correct expression in any form. e.g. $3a - b = 2$ or $b = 3a - 2$ or equivalent
	ALT	
	M1	For the vector equation $\vec{OB} = \vec{AB} + \vec{OA}$
	A1	For the correct unsimplified vector $\vec{OB} = [(b+1)\mathbf{i} + b\mathbf{j}] + 3\mathbf{i} = \{(b+4)\mathbf{i} + b\mathbf{j}\}$
	M1	For setting their $\vec{OB}$ equal to $(3a+2)\mathbf{i} + b\mathbf{j}$ and equating coefficients of $\mathbf{i}$ and $\mathbf{j}$ $(b+4)\mathbf{i} + b\mathbf{j} = (3a+2)\mathbf{i} + b\mathbf{j} \Rightarrow b+4 = 3a+2$
	A1	For a correct expression in any form.

(i)		e.g. $3a - b = 2$ or $b = 3a - 2$ or equivalent
	B1	For simplifying $\frac{\sqrt{17}}{34}$ into $\frac{1}{\sqrt{68}}$ or any equivalent. For example, $\left(\frac{\sqrt{17}}{34}\right)^2 = \frac{1}{68}$ Accept seen anywhere in their working. If this step is not explicitly, correct a or b implies correct work.
	Way 1	
	M1	For using Pythagoras to form an equation with their $\sqrt{68}$ or 68 and $(3a + 2)\mathbf{i} + b\mathbf{j}$ and forming an expression in terms of a and b [These values must be applied correctly. Do not accept $\frac{1}{68} = (3a + 2)^2 + b^2$ for this mark]
	M1	For eliminating b from their equation and solving the resulting equation to find a value for a or a^2 .
	A1	For $a = \frac{\sqrt{30}}{3}$ oe This does not have to be simplified.
	(ii)	M1 For finding the value of b using their expression in terms of a and b and their a provided their $a > 0$
		A1 For $b = \sqrt{30} - 2$ o.e. This does not have to be simplified.
(ii)	Way 2	
	M1	For using Pythagoras to form an equation with their $\vec{OB} = (b + 4)\mathbf{i} + b\mathbf{j}$ $\sqrt{68}$ or 68 and forming an expression in terms of b [This must be applied correctly. Do not accept $\frac{1}{68} = (b + 4)^2 + b^2$ for this mark]
	M1	For finding a value for b or b^2 by solving their 3TQ by any valid method. E.g., by completing the square. $68 = (b + 4)^2 + b^2$ $\Rightarrow 68 = 2b^2 + 8b + 16 \Rightarrow b^2 + 4b - 26 = 0$ $\Rightarrow b^2 + 4b - 26 = (b + 2)^2 - 30 = 0$ $\Rightarrow b = \sqrt{30} - 2$
	A1	For $b = \sqrt{30} - 2$ o.e. This does not have to be simplified.
	(i)	M1 For finding the value of a using their expression in terms of a and b and their b provided their $b > 0$.
		A1 For $a = \frac{\sqrt{30}}{3}$ oe This does not have to be simplified.

Question	Scheme	Marks
8(a)	$\vec{AB} = \vec{AD} + \vec{DC} + \vec{CB} = 5\mathbf{a} - 2\mathbf{b} - 3\mathbf{a} = 2\mathbf{a} - 2\mathbf{b}$	B1 [1]
(b)	$\vec{BX} = k \vec{BD} = k(3\mathbf{a} + 2\mathbf{b})$ $\vec{BX} = \vec{BA} + \lambda \vec{AC} = -2\mathbf{a} + 2\mathbf{b} + \lambda(5\mathbf{a} - 2\mathbf{b})$ $\Rightarrow k(3\mathbf{a} + 2\mathbf{b}) = -2\mathbf{a} + 2\mathbf{b} + \lambda(5\mathbf{a} - 2\mathbf{b})$ $\Rightarrow 3k\mathbf{a} + 2k\mathbf{b} = (5\lambda - 2)\mathbf{a} + (2 - 2\lambda)\mathbf{b}$ $\Rightarrow 3k = 5\lambda - 2 \quad 2k = 2 - 2\lambda$ $\Rightarrow k = \frac{3}{8} \quad \left[\lambda = \frac{5}{8} \right]$	M1 M1 M1 M1A1 [5]
(c)	$\Delta CXD = \frac{5}{8} \Delta BCD$ $\Delta BCD = \frac{3}{8} \Delta ABCD$ $\frac{\Delta CXD}{\Delta ABCD} = \frac{\frac{5}{8}}{\frac{3}{8}} = \frac{5}{3}$ Ratio of area of triangle CXD : Trapezium ABCD = 15 : 64	M1 M1 M1 A1 [4]
Total 10 marks		

Useful sketch



Part	Mark	Notes
(a)	B1	For the correct vector for $\overrightarrow{AB}$
(b)	Note carefully. For the first two marks in part (b) you must follow through their working using their $\overrightarrow{AB}$ or $\overrightarrow{AC}$ or $\overrightarrow{BD}$	
	M1	For one correct vector statement for $\overrightarrow{BX}$
	M1	For a second correct and different vector statement for $\overrightarrow{BX}$ For example: $\overrightarrow{BX} = \overrightarrow{BA} + \lambda \overrightarrow{AC} = -2\mathbf{a} + 2\mathbf{b} + \lambda(5\mathbf{a} - 2\mathbf{b})$ OR $\overrightarrow{BX} = \overrightarrow{BC} + \lambda \overrightarrow{AC} = 3\mathbf{a} - \lambda(5\mathbf{a} - 2\mathbf{b})$
	ddM1	For equating coefficients of a and b and forming two equations in <i>k</i> and another parameter. This must be correct Dependent on first two M marks
	dddM1	For solving their simultaneous equations to find <i>k</i> Allow one arithmetical error in processing. Dependent on all three previous M marks
	A1	For $k = \frac{3}{8}$
(c)	M1	For using their <i>k</i> to find the ratio of the areas of for example, $CXD : CAD \Rightarrow \frac{CXD}{CAD} = \frac{3}{8} \text{ or } \frac{CXD}{CAD} = \frac{\frac{1}{2} \times \frac{3}{8} \times 2b \times \sin ACD}{\frac{1}{2} \times \frac{8}{8} \times 2b \times \sin ACD} = \frac{3}{8}$ OR $CXD : BCD \Rightarrow \frac{CXD}{BCD} = \frac{5}{8} \text{ or } \frac{CXD}{CAD} = \frac{\frac{1}{2} \times \frac{5}{8} \times 2b \times \sin BDC}{\frac{1}{2} \times \frac{8}{8} \times 2b \times \sin BDC} = \frac{5}{8}$
	M1	For using the given lengths of <i>BC</i> and <i>AD</i> [3 and 5], to find the ratio of the area of triangle <i>ACD</i> : trapezium <i>ABCD</i> $\frac{CAD}{ABCD} = \frac{\frac{1}{2} \times h \times 5}{\frac{1}{2} \times h \times (5+3)} = \frac{5}{8} \quad \text{OR} \quad \frac{BCD}{ABCD} = \frac{\frac{1}{2} \times h \times 3}{\frac{1}{2} \times h \times (5+3)} = \frac{3}{8}$
	Note carefully. You must fit their ratios above and check to see if they are being combined correctly for the final M mark here.	
	M1	Combines the areas above to obtain: $\frac{CXD}{CAD} \times \frac{CAD}{ABCD} = \frac{3}{8} \times \frac{5}{8} = \left(\frac{15}{64}\right) \quad \text{OR} \quad \frac{CXD}{BCD} \times \frac{BCD}{ABCD} = \frac{5}{8} \times \frac{3}{8} = \left(\frac{15}{64}\right)$
	A1	For the correct ratio 15 : 64

Question number	Scheme	Marks
10 (a)	$\vec{AB} = \vec{AO} + \vec{OB} = -(2\mathbf{a} - \mathbf{b}) + 3\mathbf{a} + \mathbf{b} = \mathbf{a} + 2\mathbf{b}$	M1A1 [2]
(b)	$\vec{OC} = \vec{OB} + \vec{BC} = 3\mathbf{a} + \mathbf{b} - \mathbf{a} + 3\mathbf{b} = 2\mathbf{a} + 4\mathbf{b} = 2(\mathbf{a} + 2\mathbf{b})$ Conclusion required; same direction and $\vec{OC}$ is a multiple of $\vec{AB}$ therefore $\vec{OC}$ is parallel to $\vec{AB}$.	M1 A1 [2]
(c)	$\vec{AC} = \vec{AB} + \vec{BC} = \mathbf{a} + 2\mathbf{b} + (-\mathbf{a} + 3\mathbf{b}) = 5\mathbf{b}$ $\vec{AX} = \mu \vec{AC} = \mu 5\mathbf{b}$ $\vec{AX} = \vec{AO} + \vec{OX} = -(2\mathbf{a} - \mathbf{b}) + \lambda(3\mathbf{a} + \mathbf{b})$ $\Rightarrow \mu 5\mathbf{b} = -(2\mathbf{a} - \mathbf{b}) + \lambda(3\mathbf{a} + \mathbf{b})$ $\Rightarrow -2 + 3\lambda = 0 \Rightarrow \lambda = \frac{2}{3}$ $\Rightarrow 5\mu = 1 + \lambda \Rightarrow \mu = \frac{1}{3}$ $\Rightarrow AX : XC = 1 : 2$	B1 B1 M1 M1 M1 A1 A1 [7]
Total 11 marks		
(a) M1 A1	Use of $\vec{AB} = \vec{AO} + \vec{OB}$ $\mathbf{a} + 2\mathbf{b}$	
(b) M1 A1	Use of $\vec{OC} = \vec{OB} + \vec{BC}$ Correct conclusion i.e. $\vec{OC} = 2\vec{AB} \therefore \vec{OC}$ is parallel to $\vec{AB}$	
(c) B1 B1	$\vec{AC} = 5\mathbf{b}$ may be implied by 2 nd B1 $\vec{AX} = \mu 5\mathbf{b}$ or $\vec{XC} = \lambda 5\mathbf{b}$	
M1	A correct vector for $\vec{AX}$ or $\vec{OX}$ or $\vec{BX}$ or $\vec{CX}$ or $\vec{BC}$ including an unknown multiple of a vector e.g. $\vec{AX} = -(2\mathbf{a} - \mathbf{b}) + \lambda(3\mathbf{a} + \mathbf{b})$	
M1	Equate 2 forms of the same vector e.g. $\mu 5\mathbf{b} = -(2\mathbf{a} - \mathbf{b}) + \lambda(3\mathbf{a} + \mathbf{b})$	
M1	Comparing coefficients	
A1	$\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$	
A1	$AX : XC = 1 : 2$	

Question	Scheme	Marks
10(a)	(i) $\vec{AB} = -\vec{OA} + \vec{OB} \Rightarrow \vec{AB} = -2\mathbf{a} + 4\mathbf{b}$	B1
	(ii) $\vec{MY} = \vec{MA} + \frac{3}{4}(\vec{AB}) = \mathbf{a} + \frac{3}{4}(-2\mathbf{a} + 4\mathbf{b}) = -\frac{\mathbf{a}}{2} + 3\mathbf{b}$	M1A1 [4]
(b)	$\vec{OX} = \mu \vec{OB} = \mu 4\mathbf{b}$	M1
	$\vec{OX} = \vec{OM} + \vec{MX} = \vec{OM} + \lambda \vec{MY} = \mathbf{a} + \lambda \left(-\frac{\mathbf{a}}{2} + 3\mathbf{b}\right) = \mathbf{a} \left(1 - \frac{\lambda}{2}\right) + 3\lambda\mathbf{b}$	M1
	$\Rightarrow \mu 4\mathbf{b} = \mathbf{a} \left(1 - \frac{\lambda}{2}\right) + 3\lambda\mathbf{b}$	dM1
	$\Rightarrow 1 - \frac{\lambda}{2} = 0 \Rightarrow \lambda = 2$	ddM1
	$\Rightarrow 4\mu = 3\lambda \Rightarrow \mu = \frac{6}{4} = \frac{3}{2}$	
	$OB:OX = 2:3$ oe	A1 [5]
	ALT – working with alternative vector within triangle OMX	
	$\vec{MX} = \vec{MO} + \vec{OB} + \mu \vec{OB} = -\mathbf{a} + 4\mathbf{b} + \mu 4\mathbf{b}$	[M1 M1]
	$\vec{MX} = \lambda \left(-\frac{\mathbf{a}}{2} + 3\mathbf{b}\right)$	
	$\Rightarrow -\mathbf{a} = -\frac{\lambda\mathbf{a}}{2} \Rightarrow \lambda = 2$	dM1
	$\Rightarrow 4\mathbf{b} + \mu 4\mathbf{b} = 3\lambda\mathbf{b} \Rightarrow \mu = \frac{1}{2}$	ddM1
	$OB:OX = 2:3$ oe	A1]
(c)	$\frac{\Delta YBX}{\Delta ABX} = \frac{1}{4}$	M1
	$\frac{\Delta ABX}{\Delta OAX} = \frac{1}{3}$	
	$\Rightarrow \frac{\Delta YBX}{\Delta OAX} = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12} \Rightarrow \Delta YBX : \Delta OAX = 1:12$	M1A1 [3]
	ALT – working with relative areas of triangles	
	Area $\Delta YBX = a$	[M1]
	Area $\Delta ABX = 4a$	
	Area $\Delta AYX = 3a$	
	Area $\Delta OYB = 2a$	
	Area $\Delta OAX = 12a$	M1
	$\Delta YBX : \Delta OAX = 1:12$	A1]
Total 11 marks		

Part	Mark	Notes
(a)	B1	For the correct simplified vector $\vec{AB}$
	M1	For the correct vector statement for $\vec{MY}$
	A1	For the correct simplified vector for $\vec{MY}$
(b)	M1	For the statement $\vec{OX} = \mu 4\mathbf{b}$ Note: this is a B mark on open.
	M1	For the correct vector for $\vec{OX}$ (ft their $\vec{MY}$)
	dM1	For equating both vectors for $\vec{OX}$ and for comparing coefficients of $\mathbf{a}$ and $\mathbf{b}$ Dep on M1M1
	ddM1	For finding a value for their parameter for μ Note: there is no mark for only finding λ , they must find μ Dep on M1M1M1
	A1	For the correct ratio $OB:OX = 2:3$ Allow equivalent ratios e.g. 4 : 6, 1 : 1.5
	ALT – working with alternative vector within triangle OMX	
	M1	For a vector statement which includes $\mu 4\mathbf{b}$ (for OX or BX) (ft their $\vec{MY}$) Note: this is a B mark on open.
	M1	For a correct second vector equation for the same vector (ft their $\vec{MY}$)
	dM1	For equating both vectors and comparing coefficients of $\mathbf{a}$ and $\mathbf{b}$ Dep on M1M1
	ddM1	For finding a value for their parameter for μ Note: there is no mark for only finding λ , they must find μ Dep on M1M1M1
	A1	For the correct ratio $OB:OX = 2:3$ Allow equivalent ratios e.g. 4 : 6, 1 : 1.5 Condone $OX:OB = 3:2$ if clearly stated, but not just 3:2
	ALT – working with relative areas	
(c)	M1	For either the relationship between the of areas of triangles BYX and ABX or the relationship between the areas of triangles ABX and OAX
	M1	For finding the relationship between the of areas of triangles BYX and OAX
	A1	For the correct ratio [1:12] Allow equivalent ratios. Note: do not penalise answer given as a fraction i.e. $\frac{1}{12}$ if already penalised in (b).
	ALT – working with relative areas	
	M1	For assigning a value to one triangle area and writing a second area in terms of this. Note: This could also follow from working with area of a triangle $= \frac{1}{2}ab \sin C$

		e.g. $\Delta YBX = \frac{1}{2} yz \sin B$ and $\Delta ABX = \frac{1}{2} y(4z) \sin B$
	M1	For finding the relationship between the of areas of triangles BYX and OAX Note: This could also follow from working with area of a triangle $= \frac{1}{2} ab \sin C$ e.g. $\Delta YBX = \frac{1}{2} yz \sin B$ and $\Delta ABX = \frac{1}{2} y(4z) \sin B$
	A1	For the correct ratio [1:12] Allow equivalent ratios. Note: do not penalise answer given as a fraction i.e. $\frac{1}{12}$ if already penalised in (b).

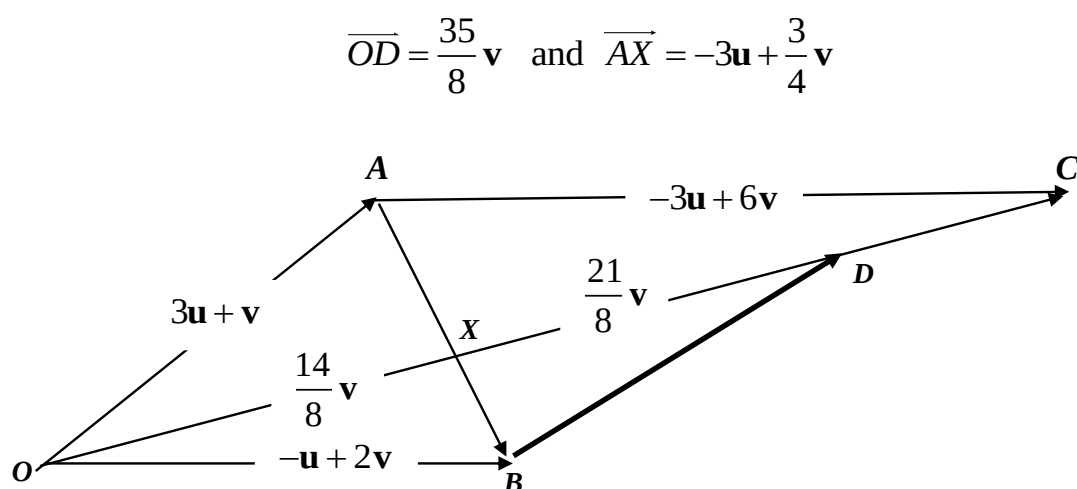
Question	Scheme	Marks
10(a)	$\vec{BC} = -\vec{AB} + \vec{AC} = -(3\mathbf{a} + 4\mathbf{b}) + 7\mathbf{a} + 9\mathbf{b} = 4\mathbf{a} + 5\mathbf{b}$ $\vec{DC} = -\vec{DA} + \vec{AC} = -(4\mathbf{a} + 5\mathbf{b}) + 7\mathbf{a} + 9\mathbf{b} = 3\mathbf{a} + 4\mathbf{b}$ $AB \parallel DC \text{ and } BC \parallel AD \text{ hence the quadrilateral is a parallelogram}$	B1 B1 B1ft [3]
(b)	$\vec{CF} = -\frac{1}{3}(3\mathbf{a} + 4\mathbf{b})$ $\vec{AF} = \vec{AC} + \vec{CF} = 7\mathbf{a} + 9\mathbf{b} - \frac{1}{3}(3\mathbf{a} + 4\mathbf{b}) = \left[6\mathbf{a} + \frac{23}{3}\mathbf{b}\right]$ $\vec{AE} = \mu \vec{AF} = \mu \left(6\mathbf{a} + \frac{23}{3}\mathbf{b}\right)$ $\vec{AE} = \vec{AB} + \vec{BE} = 3\mathbf{a} + 4\mathbf{b} + \lambda(4\mathbf{a} + 5\mathbf{b}) = [(3 + 4\lambda)\mathbf{a} + (4 + 5\lambda)\mathbf{b}]$ $\mu \left(6\mathbf{a} + \frac{23}{3}\mathbf{b}\right) = (3 + 4\lambda)\mathbf{a} + (4 + 5\lambda)\mathbf{b}$ $\Rightarrow 6\mu = 3 + 4\lambda \quad \frac{23}{3}\mu = 4 + 5\lambda$ $\Rightarrow \lambda = \frac{3}{2} \quad \left(\mu = \frac{3}{2}\right)$ $\vec{AE} = \frac{3}{2} \left(6\mathbf{a} + \frac{23}{3}\mathbf{b}\right) = 9\mathbf{a} + \frac{23}{2}\mathbf{b}$	B1 B1 M1 M1 ddM1 M1A1 B1ft [8]
Total 11 marks		

Part	Mark	Notes
(a)	B1	For the correct vector for $\overrightarrow{BC}$ or $\overrightarrow{CB}$
	B1	For the correct vector for $\overrightarrow{DC}$ or $\overrightarrow{CD}$
	B1ft	For the correct complete conclusion including stating that the quadrilateral is a parallelogram. Ft their vectors for reverse directions.
	ALT	
	B1	For the correct lengths of $\overrightarrow{AD}$ AND $\overrightarrow{BC}$ $ AD = BC = \sqrt{41}$
	B1	For the correct lengths of $\overrightarrow{AB}$ AND $\overrightarrow{DC}$ $ AB = DC = 5$
	B1ft	Opposite sides are of equal length so the quadrilateral is a parallelogram.
(b)	B1	For the correct vector for $\overrightarrow{CF}$ or $\overrightarrow{DF}$ or correct reverse directions $\left[\overrightarrow{CF} = -\frac{1}{3}(3\mathbf{a} + 4\mathbf{b}) \right] \left[\overrightarrow{DF} = \frac{2}{3}(3\mathbf{a} + 4\mathbf{b}) \right]$ this may be embedded in $\overrightarrow{AF}$
	B1	For the correct vector for $\overrightarrow{AF}$ using their $\overrightarrow{CF}$ or $\overrightarrow{DF}$ e.g., $\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF} = 7\mathbf{a} + 9\mathbf{b} - \frac{1}{3}(3\mathbf{a} + 4\mathbf{b}) = \left[6\mathbf{a} + \frac{23}{3}\mathbf{b} \right]$
	M1	For one correct vector for $\overrightarrow{AE}$ using their $\overrightarrow{AF}$ and a parameter For example: $\overrightarrow{AE} = \mu \overrightarrow{AF} = \mu \left(6\mathbf{a} + \frac{23}{3}\mathbf{b} \right)$
	M1	For a second vector for $\overrightarrow{AE}$ using their vectors and a second unique parameter For example; $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = 3\mathbf{a} + 4\mathbf{b} + \lambda(4\mathbf{a} + 5\mathbf{b}) = [(3 + 4\lambda)\mathbf{a} + (4 + 5\lambda)\mathbf{b}]$
	ddM1	For setting their two vectors for $\overrightarrow{BF}$ equal and setting up two equations involving two different parameters. $\mu \left(6\mathbf{a} + \frac{23}{3}\mathbf{b} \right) = (3 + 4\lambda)\mathbf{a} + (4 + 5\lambda)\mathbf{b}$ $\Rightarrow 6\mu = 3 + 4\lambda \quad \frac{23}{3}\mu = 4 + 5\lambda$ This mark is dependent on the previous two M marks
	dddM1	For solving their two simultaneous equations by any method to find a value for λ This mark is dependent on the previous three M marks
	A1	For $\mu = \frac{3}{2}$ or $\lambda = \frac{3}{2}$
	B1ft	For writing down a vector for $\overrightarrow{AE}$ using their $\overrightarrow{AF}$ and their λ in the required form, provided $1 < \lambda < 2$

ALT	
B1	For the correct vector for $\vec{CF}$ or $\vec{DF}$ $\left[\vec{CF} = -\frac{1}{3}(3\mathbf{a} + 4\mathbf{b}) \right] \quad \left[\vec{DF} = \frac{2}{3}(3\mathbf{a} + 4\mathbf{b}) \right]$
B1	For the correct vector for $\vec{AF}$ using their $\vec{CF}$ or $\vec{DF}$ e.g., $\vec{AF} = \vec{AC} + \vec{CF} = 7\mathbf{a} + 9\mathbf{b} - \frac{1}{3}(3\mathbf{a} + 4\mathbf{b}) = \left[6\mathbf{a} + \frac{23}{3}\mathbf{b} \right]$
M1	For one correct vector for $\vec{AE}$ using their $\vec{AF}$ and a parameter For example: $\vec{AE} = \mu \vec{AF} = \mu \left(6\mathbf{a} + \frac{23}{3}\mathbf{b} \right)$
M1	For a correct vector for $\vec{BE}$ using their $\vec{AE}$ $\vec{BE} = -3\mathbf{a} - 4\mathbf{b} + \mu \left(6\mathbf{a} + \frac{23}{3}\mathbf{b} \right) = \left[(6\mu - 3)\mathbf{a} + \left(\frac{23}{3}\mu - 4 \right)\mathbf{b} \right]$
ddM1	For forming the correct ratio using the vector $\vec{BC}$ $\frac{(6\mu - 3)}{\left(\frac{23}{3}\mu - 4 \right)} = \frac{4}{5}$
dddM1	For solving the equation to find a value for μ
A1	For $\mu = \frac{3}{2}$
B1ft	For writing down a vector for $\vec{AE}$ using their $\vec{AF}$ and their λ in the required form For writing down a vector for $\vec{AE}$ using their $\vec{AF}$ and their λ in the required form, provided $1 < \lambda < 2$

Question number	Scheme	Marks
8 (a)	$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}, = -(3\mathbf{u} + \mathbf{v}) + (2\mathbf{v} - \mathbf{u}) = -4\mathbf{u} + \mathbf{v}$	M1,A1
(b)	$\overrightarrow{AC} = \lambda(2\mathbf{v} - \mathbf{u})$	(2) M1
	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = (3\mathbf{u} + \mathbf{v}) + \lambda(2\mathbf{v} - \mathbf{u}) = \mu\mathbf{v}$	M1
	$\mathbf{u}: 0 = 3 - \lambda \quad \mathbf{v}: \mu = 1 + 2\lambda$	M1
	$\mu = 7 \quad [\lambda = 3]$	A1 (4)
(c)	$\overrightarrow{OX} = \overrightarrow{OA} + \alpha \overrightarrow{AB} = 3\mathbf{u} + \mathbf{v} + \alpha(-4\mathbf{u} + \mathbf{v})$	M1
	or $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AC} + \beta \overrightarrow{CO} = 3\mathbf{u} + \mathbf{v} + 3(2\mathbf{v} - \mathbf{u}) + \beta(-7\mathbf{v})$	
	$\overrightarrow{OX} = \frac{7}{4}\mathbf{v}$	A1
	$\overrightarrow{OD} = \frac{5}{2}\overrightarrow{OX}$ or $\overrightarrow{XD} = \frac{3}{2}\overrightarrow{OX}$	M1
	$\overrightarrow{OD} = \frac{35}{8}\mathbf{v}$ or $\overrightarrow{XD} = \frac{21}{8}\mathbf{v}$	M1
	$\overrightarrow{BD} = \mathbf{u} - 2\mathbf{v} + \frac{35}{8}\mathbf{v}$ or $\overrightarrow{BD} = \mathbf{u} - 2\mathbf{v} + \frac{7}{4}\mathbf{v} + \frac{21}{8}\mathbf{v}$	A1
	$\overrightarrow{BD} = \mathbf{u} + \frac{19}{8}\mathbf{v}$	(5)
Total 11 marks		

USEFUL SKETCH



Part	Mark	Additional Guidance
(a)	M1	Use of $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
	A1	$-4\mathbf{u} + \mathbf{v}$
(b)	M1	For use of $\overrightarrow{AC}$ is parallel to $\overrightarrow{OB}$ to form an expression for $\overrightarrow{AC}$
	M1	For use of their expression for $\overrightarrow{AC}$ to form an expression for $\overrightarrow{OC}$
	M1	For equating for $\mathbf{u}$ and for $\mathbf{v}$
	A1	For $\mu = 7$
(c)	General principles for marking part (c) For example: You may see the vector path for $\overrightarrow{BD}$ first - score the 4 th M mark - Candidates will need to find vector $\overrightarrow{OX}$ [M1A1] - Candidates will need to use the ratio of bases to find $\overrightarrow{OD} = \frac{5}{2}\overrightarrow{OX}$ or $\overrightarrow{XD} = \frac{3}{2}\overrightarrow{OX}$ leading to establish $\overrightarrow{OD} = \frac{35}{8}\mathbf{v}$ or $\overrightarrow{XD} = \frac{21}{8}\mathbf{v}$ [M1] - Check any path they choose on the diagram below. For example, $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OX} + \overrightarrow{XD}$ or [M1] $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OA} + \overrightarrow{AX} + \overrightarrow{XD}$ If a candidate chooses to take a longer path and need to find for example, $\overrightarrow{AX}$ they will need to find a vector. There is no separate mark for this. - For the correct vector	
	M1	For an expression for $\overrightarrow{OX}$ including an unknown constant
	A1	$\overrightarrow{OX} = \frac{7}{4}\mathbf{v}$ For
	M1	For recognising that triangle OBX and triangle OBD share a base OB and using this to obtain a ratio for $\overrightarrow{OX}$ to $\overrightarrow{OD}$ together with an attempt to find $\overrightarrow{OD}$ OR For recognising that triangle OBX and triangle BXD share a base BX and using this to obtain a ratio for $\overrightarrow{OX}$ to $\overrightarrow{XD}$ together with an attempt to find $\overrightarrow{XD}$
	M1	For method to find $\overrightarrow{BD}$ Award this for any valid path given
	A1	$\overrightarrow{BD} = \mathbf{u} + \frac{19}{8}\mathbf{v}$

Question	Scheme	Marks
4(a)	$\vec{AB} = \vec{OB} - \vec{OA} \Rightarrow \vec{AB} = (5\mathbf{i} + 9p\mathbf{j}) - (p\mathbf{i} + 2p\mathbf{j})$ $\mathbf{i}(5 - p) + \mathbf{j}(7p) = Q(\mathbf{i} - 2\mathbf{j}) \Rightarrow 5 - p = Q \text{ and } 7p = -2Q$ $7p = -2(5 - p) \Rightarrow p = -2$	M1A1 M1M1 M1A1 [6]
(b)	$7(-2) = -2Q \Rightarrow Q = 7, \vec{AB} = 7(\mathbf{i} - 2\mathbf{j}) = 7\mathbf{i} - 14\mathbf{j}$ OR $\vec{AB} = (5\mathbf{i} + 9(-2)\mathbf{j}) - ((-2)\mathbf{i} + 2(-2)\mathbf{j}) = 7\mathbf{i} - 14\mathbf{j}$	M1A1ft [2] [M1A1ft]
(c)	$\vec{OA} = -2\mathbf{i} - 4\mathbf{j} \Rightarrow \left \vec{OA} \right = \sqrt{(-2)^2 + (-4)^2} = \sqrt{20}$ Unit vector is $\frac{1}{\sqrt{20}}(-2\mathbf{i} - 4\mathbf{j}) = \frac{\sqrt{5}}{5}(-\mathbf{i} - 2\mathbf{j})$	M1A1ft M1A1 [4]
Total 12 marks		

Question	Notes	Marks
4(a)	For the basic vector statement $\vec{AB} = \vec{OB} - \vec{OA}$	M1
	For the correct vector (simplified or unsimplified) $\vec{AB} = (5\mathbf{i} + 9p\mathbf{j}) - (p\mathbf{i} + 2p\mathbf{j}) = [\mathbf{i}(5 - p) + \mathbf{j}(7p)]$	A1
	For setting their $\vec{AB} = Q(\mathbf{i} - 2\mathbf{j})$ where $Q \neq 1, Q \neq 0$ $\mathbf{i}(5 - p) + \mathbf{j}(7p) = Q(\mathbf{i} - 2\mathbf{j})$	M1
	For equating components of $\mathbf{i}$ and $\mathbf{j}$ $\mathbf{i} \quad 5 - p = Q$ $\mathbf{j} \quad 7p = -2Q$	M1
	Solving the simultaneous equations by any method to find the value of p $7p = -2(5 - p) \Rightarrow p = \dots$	M1
	For the value of $p = -2$	A1 [6]
(b)	For finding the value of k and using it to find the vector $\vec{AB}$ $7(-2) = -2Q \Rightarrow Q = 7$	M1
	For the correct vector $\vec{AB} = 7(\mathbf{i} - 2\mathbf{j}) = 7\mathbf{i} - 14\mathbf{j}$	A1ft [2]
	ALT	
	For substituting their value of p to find the vector $\vec{AB}$ $\vec{AB} = \mathbf{i}(5 - [-2]) + \mathbf{j}(7[-2]) = \dots$	M1
	$\vec{AB} = \mathbf{i}(5 - p) + \mathbf{j}(7p) \Rightarrow \vec{AB} = 7\mathbf{i} - 14\mathbf{j}$	A1ft [2]
(c)	$\vec{OA} = -2\mathbf{i} - 4\mathbf{j} \Rightarrow \vec{OA} = \sqrt{(-2)^2 + (-4)^2} = \dots$	M1
	$ \vec{OA} = \sqrt{20}$	A1ft
	Unit vector in the direction of $\vec{OA}$ is $\frac{1}{\sqrt{20}}(-2\mathbf{i} - 4\mathbf{j})$	M1
	Unit vector in the required form $\frac{\sqrt{5}}{5}(-\mathbf{i} - 2\mathbf{j})$ Allow $\frac{\sqrt{5}}{5}(\mathbf{i} + 2\mathbf{j})$ provided no processing errors seen.	A1 [4]
Total 12 marks		

Question number	Scheme	Marks
8 (a) (i)	$\vec{AB} = \vec{AO} + \vec{OB} = -4\mathbf{a} - 5\mathbf{b} + 8\mathbf{a} - \mathbf{b} = 4\mathbf{a} - 6\mathbf{b}$	M1A1 [2]
(ii)	$\left\{ \vec{AB} = \sqrt{52} \right\} = 2\sqrt{13}$	B1 [1]
(b)	$ \vec{OA} = \sqrt{4^2 + 5^2} (= \sqrt{41}) \quad \text{or} \quad \vec{OB} = \sqrt{8^2 + 1^2} (= \sqrt{65})$ $(\sqrt{52})^2 = (\sqrt{41})^2 + (\sqrt{65})^2 - 2 \times \sqrt{41} \times \sqrt{65} \cos(AOB)$ $\left\{ \text{angle } AOB = \cos^{-1} \left(\frac{41 + 65 - 52}{2 \times \sqrt{41} \times \sqrt{65}} \right) = 58.465... \right\}$ $\text{Area triangle } AOB = \frac{1}{2} \times \sqrt{41} \times \sqrt{65} \times \sin 58.465...$ $= 22 \text{ awrt}$	M1 M1 M1 A1 [4]
ALT 1	$\text{Area} = \frac{1}{2} \begin{vmatrix} 0 & 4 & 8 & 0 \\ 0 & 5 & -1 & 0 \end{vmatrix} \text{ oe}$ $= \frac{1}{2} (0 \times 5 + 4 \times (-1) + 8 \times 0) - (0 \times 4 + 5 \times 8 + (-1) \times 0) \text{ oe}$ $\left\{ = \frac{1}{2} -44 \right\} = 22$	M1 M1 M1A1 [4]
ALT 2	Use Heron's formula $ \vec{OA} = \sqrt{4^2 + 5^2} (= \sqrt{41}) \quad \text{or} \quad \vec{OB} = \sqrt{8^2 + 1^2} (= \sqrt{65})$ $p = \frac{\sqrt{41} + \sqrt{65} + \sqrt{52}}{2}$ $\text{Area} = \sqrt{p(p - \sqrt{41})(p - \sqrt{65})(p - \sqrt{52})}$ $= 22 \text{ exact}$	M1 M1 M1 A1 [4]
(c)	$\vec{OC} = \lambda \vec{OD} \{ = \lambda(15\mathbf{a} + 10\mathbf{b}) \}$ $\vec{OC} = \vec{OA} + \mu \vec{AB} \{ = 4\mathbf{a} + 5\mathbf{b} + \mu(4\mathbf{a} - 6\mathbf{b}) = (4 + 4\mu)\mathbf{a} + (5 - 6\mu)\mathbf{b} \}$ $4 + 4\mu = 15\lambda \quad \text{and} \quad 5 - 6\mu = 10\lambda \Rightarrow \mu = ... \text{ or } \lambda = ...$ $\mu = \frac{7}{26} \quad \text{or} \quad \lambda = \frac{22}{65}$ $\vec{OC} = \frac{66}{13}\mathbf{a} + \frac{44}{13}\mathbf{b} \quad \text{or} \quad 5\frac{1}{13}\mathbf{a} + 3\frac{5}{13}\mathbf{b} \text{ cao}$	M1 M1 ddM1 A1 A1 [5]
Total 12 marks		



Part	Mark	Additional Guidance
(a)(i)	M1	For a correct vector path to find $\vec{AB}$ or $\vec{BA}$ or for $\pm(-4\mathbf{a} - 5\mathbf{b} + 8\mathbf{a} - \mathbf{b})$
	A1	For the correct vector $4\mathbf{a} - 6\mathbf{b}$
(ii)	B1	For the correct exact simplest magnitude $2\sqrt{13}$
(b)	M1	For the correct simplified or unsimplified $ \vec{OA} $ or $ \vec{OB} $
	M1	For correct substitution into the cosine rule. e.g. $(\sqrt{65})^2 = (\sqrt{41})^2 + (\sqrt{52})^2 - 2 \times \sqrt{41} \times \sqrt{52} \cos(OAB)$ or $(\sqrt{41})^2 = (\sqrt{52})^2 + (\sqrt{65})^2 - 2 \times \sqrt{52} \times \sqrt{65} \cos(ABO)$
	M1	For correct substitution into the area of a triangle formula, use their values. e.g. $\frac{1}{2} \times \sqrt{41} \times \sqrt{52} \times \sin 72.349\dots$ or $\frac{1}{2} \times \sqrt{65} \times \sqrt{52} \times \sin 49.184\dots$
	A1	awrt 22
ALT1	M1	For a correct statement for the area, must have $\frac{1}{2}$ (Start with any point, go clockwise or anticlockwise, but must repeat the first point at the end.)
	M1	For a correct evaluation of their determinant, ignore $\frac{1}{2}$
	M1	$\frac{1}{2}$ multiplies their determinant value, correct answer implies this mark.
	A1	Correct exact value 22, not awrt
ALT2	M1	For the correct simplified or unsimplified $ \vec{OA} $ or $ \vec{OB} $
	M1	For finding half of the perimeter
	M1	Substitute the perimeter and 3 lengths to a correct Heron's formula
	A1	Correct exact value 22, not awrt
(c)	M1	For $\vec{OC} = \lambda \vec{OD}$ (can be implied in their later work)
	M1	For correct vector statement $\vec{OC} = \vec{OA} + \mu \vec{AB}$ or $\vec{OC} = \vec{OB} + \mu \vec{BA}$ or Can be implied for later substitution of their vectors Please note: $\vec{OC} = \vec{OA} + \mu \vec{BA}$ or $\vec{OC} = \vec{OB} + \mu \vec{AB}$ is also correct, provided they have found a negative value of μ
	ddM1	Equating components and reaching a value for either of their parameters. Dependent on both previous method marks
	A1	Either correct value of their parameters. They do not need to work out both values. Note: $\vec{OC} = \vec{OA} + \mu \vec{AB} \Rightarrow \mu = \frac{7}{26}$, $\vec{OC} = \vec{OB} + \mu \vec{BA} \Rightarrow \mu = \frac{19}{26}$
	A1	Correct vector in simplified expression

Question number	Scheme	Marks
10 (a) (i)	$\mathbf{a} + \mathbf{c}$	B1
(ii)	$\frac{1}{2}(\mathbf{c} - \mathbf{a})$	B1 (2)
(b)	$\overrightarrow{OX} = OA + AM + \lambda MN$ $\mathbf{a} + \frac{1}{2}\mathbf{c} + \lambda(\frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{a})$ $\mu(\mathbf{a} + \mathbf{c})$ $\mathbf{a} + \frac{1}{2}\mathbf{c} + \lambda(\frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{a}) = \mu(\mathbf{a} + \mathbf{c})$ $1 - \frac{1}{2}\lambda = \mu \quad \frac{1}{2} + \frac{1}{2}\lambda = \mu$ $1 - \frac{1}{2}\lambda = \frac{1}{2} + \frac{1}{2}\lambda$ $\lambda = \frac{1}{2} \quad \mu = \frac{3}{4}$	M1 A1 B1 M1 M1 M1
(c)	Triangle $XBN = \frac{1}{8}$ of $\frac{1}{2}$ the parallelogram Quadrilateral $OXNC = \frac{7}{8}$ of $\frac{1}{2}$ the parallelogram So Quadrilateral $OXNC = \frac{7}{16}$ of the parallelogram $\therefore 7 : 16$	A1 A1 (8) M1 M1 A1 (3) [13]

Part	Mark	Additional Guidance	
(a)(i)	B1	For the correct vector $\mathbf{a} + \mathbf{c}$	
(ii)	B1	For the correct vector $\frac{1}{2}(\mathbf{c} - \mathbf{a})$	
(b)	M1	For the correct vector statement $\overrightarrow{OX} = OA + AM + \lambda MN$	
	A1	For the correct vector (need not be simplified) $\overrightarrow{OX} = \mathbf{a} + \frac{1}{2}\mathbf{c} + \frac{\lambda}{2}(\mathbf{c} - \mathbf{a})$ or $\overrightarrow{OX} = \mathbf{a} + \frac{1}{2}\mathbf{c} + \lambda\left(\frac{\mathbf{c}}{2} - \frac{\mathbf{a}}{2}\right)$	
	B1ft	For $\overrightarrow{OX} = \mu(\mathbf{a} + \mathbf{c})$ ft their $OB = 'a + c'$	
	M1	For equating their two vector statements for $\overrightarrow{OX}$ $\mathbf{a} + \frac{1}{2}\mathbf{c} + \frac{\lambda}{2}(\mathbf{c} - \mathbf{a}) = \mu(\mathbf{a} + \mathbf{c})$	
	M1	For equating coefficients of $\mathbf{a}$ and $\mathbf{c}$ $\mathbf{a} + \frac{1}{2}\mathbf{c} + \frac{\lambda}{2}(\mathbf{c} - \mathbf{a}) = \mu(\mathbf{a} + \mathbf{c}) \Rightarrow \mathbf{a}\left(1 - \frac{\lambda}{2}\right) + \mathbf{c}\left(\frac{1}{2} + \frac{\lambda}{2}\right) = \mu\mathbf{a} + \mu\mathbf{c}$ $\Rightarrow \mu = 1 - \frac{\lambda}{2}, \mu = \frac{1}{2} + \frac{\lambda}{2}$	
	M1	For attempting to solve their two simultaneous equations in terms of λ and μ .	
		$1 - \frac{\lambda}{2} = \frac{1}{2} + \frac{\lambda}{2} \Rightarrow \lambda = \dots \Rightarrow \mu = \dots$	$1 - \mu = \mu - \frac{1}{2} \Rightarrow \mu = \dots \Rightarrow \lambda = \dots$

	A1	For either $\lambda = \frac{1}{2}$ or $\mu = \frac{3}{4}$
	A1	For both $\lambda = \frac{1}{2}$ and $\mu = \frac{3}{4}$
	ALT	
	M1	For the correct vector statement $\overrightarrow{MX} = \overrightarrow{MO} + \overrightarrow{OX}$
	A1	For the correct vector (need not be simplified) $\overrightarrow{MX} = -\frac{\mathbf{c}}{2} - \mathbf{a} + \mu(\mathbf{a} + \mathbf{c})$
	B1ft	$\overrightarrow{MX} = \frac{\lambda}{2}(\mathbf{c} - \mathbf{a})$ ft their $\overrightarrow{MN} = \frac{1}{2}(\mathbf{c} - \mathbf{a})$
	M1	For equating the two vector statements for $\overrightarrow{MX}$ $-\frac{\mathbf{c}}{2} - \mathbf{a} + \mu(\mathbf{a} + \mathbf{c}) = \frac{\lambda}{2}(\mathbf{c} - \mathbf{a})$
	M1	For equating coefficients of $\mathbf{a}$ and $\mathbf{c}$ $-\frac{\mathbf{c}}{2} - \mathbf{a} + \mu(\mathbf{a} + \mathbf{c}) = \frac{\lambda}{2}(\mathbf{c} - \mathbf{a}) \Rightarrow \mathbf{c}\left(-\frac{1}{2} + \mu\right) + \mathbf{a}(\mu - 1) = \mathbf{c}\frac{\lambda}{2} - \mathbf{a}\frac{\lambda}{2}$ $\Rightarrow \frac{\lambda}{2} = \mu - \frac{1}{2} \quad \text{and} \quad -\frac{\lambda}{2} = \mu - 1$
	M1	For attempting to solve their two simultaneous equations in terms of λ and μ . $\mu - \frac{1}{2} = -(\mu - 1) \Rightarrow \mu = \left(\frac{3}{4}\right) \quad \frac{\lambda}{2} = 1 - \frac{3}{4} \Rightarrow \lambda = \left(\frac{1}{2}\right)$
	A1	For either $\lambda = \frac{1}{2}$ or $\mu = \frac{3}{4}$
	A1	For both $\lambda = \frac{1}{2}$ and $\mu = \frac{3}{4}$
(c)	M1	For area of $\triangle XBN = \frac{1}{8}\triangle OBC$ so $\frac{1}{8}$ of $\frac{1}{2}$ of the area of parallelogram $OABC$ $\left(\begin{array}{l} \triangle OBC = \frac{1}{2} \times OB \times BC \times \sin \angle XBN \\ \triangle XBN = \frac{1}{2} \times \frac{1}{4} OB \times \frac{1}{2} BC \times \sin \angle XBN = \frac{1}{8} \triangle OBC \end{array} \right)$
	M1	Therefore Quadrilateral $OXNC = \frac{7}{8}$ of $\frac{1}{2}$ of the area of parallelogram $OABC$ ft their fraction from the first M mark provided it is $< \frac{1}{2}$
	A1	Quadrilateral $OXNC = \frac{7}{16}$ of the area of parallelogram $OABC$ so ratio is 7:16

Question Number	Scheme	Marks
11(a)	(i) $\overrightarrow{AC} = -\mathbf{a} + \frac{1}{4}\mathbf{b}$	B1
	(ii) $\overrightarrow{OD} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC} = \mathbf{a} + \frac{1}{2}\left(-\mathbf{a} + \frac{1}{4}\mathbf{b}\right) = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b}$	M1A1
	(iii) $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD} = -\mathbf{b} + \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} = \frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}$	M1A1 (5)
	(b) $\overrightarrow{BE} = k\overrightarrow{BD} = k\left(\frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}\right)$	M1
	$\overrightarrow{BE} = -\mathbf{b} + \overrightarrow{OE} = -\mathbf{b} + \lambda\mathbf{a}$	M1
	$\frac{7}{8}k = 1 \Rightarrow k = \frac{8}{7}$	M1
	$\frac{k}{2} = \lambda \Rightarrow \lambda = \frac{4}{7}$	A1 (4)
	(c) $\Delta OAC = \frac{1}{4}\Delta OAB$	M1
	$\Delta OEB = \frac{4}{7}\Delta OAB$	A1 ft
	$\frac{\Delta OAC}{\Delta OEB} = \frac{1}{4} \times \frac{7}{4} = \frac{7}{16}$	M1
	$\mu = \frac{7}{16}$	A1 (4)
[13]		

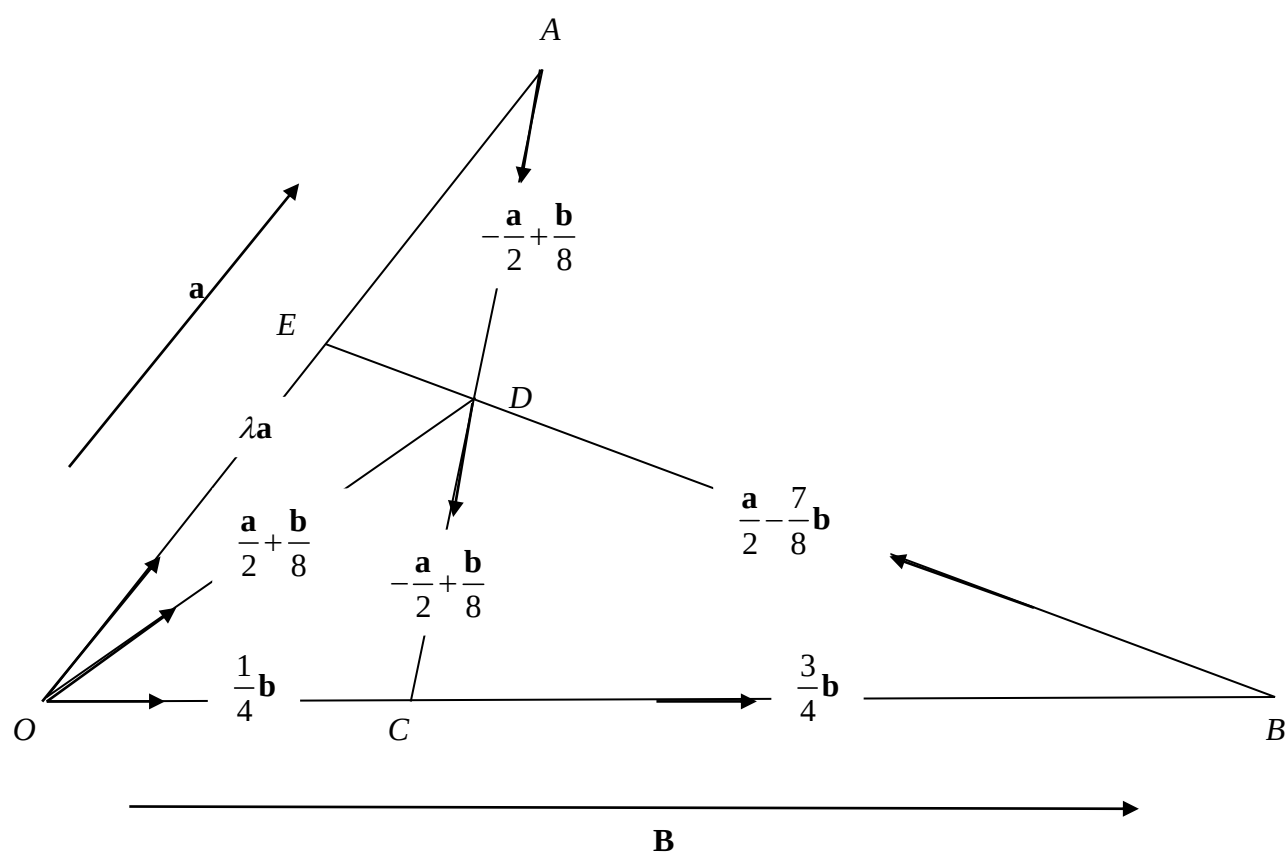
Part	Mark	Notes
(a)(i)	B1	For the correct vector $\overrightarrow{AC} = -\mathbf{a} + \frac{1}{4}\mathbf{b}$
(ii)	M1	For the correct vector statement $\overrightarrow{OD} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$ or $\overrightarrow{OD} = \overrightarrow{OC} + \frac{1}{2}\overrightarrow{CA}$
	A1	For the correct simplified vector $\overrightarrow{OD} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b}$
(iii)	M1	For the correct vector statement $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$ or $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$
	A1	For the correct simplified vector $\overrightarrow{BD} = \frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}$



In parts (b) and (c) you must follow through the vectors they have found in part (a)		
(b)	M1	States or implies two paths that will lead to a solution. For example: using triangle OEB $\vec{BE} = k\vec{BD}$ and $\vec{BE} = \vec{BO} + \vec{OE}$ or using triangle OED $\vec{OE} = \lambda\mathbf{a}$ and $\vec{OE} = \vec{OD} + \vec{DE}$ This is a B mark in Epen
	M1	For writing their paths as vectors in terms of $\mathbf{a}$, $\mathbf{b}$ λ and another constant (e.g. k or μ) For example: using triangle OEB $\vec{BE} = k\vec{BD} = k\left(\frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}\right)$ and $\vec{BE} = -\mathbf{b} + \lambda\mathbf{a}$ or using triangle OED $\vec{OE} = \lambda\mathbf{a}$ and $\vec{OE} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} + k(-\mathbf{b} + \lambda\mathbf{a})$ OR $\vec{OE} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} + k\left(\frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}\right)$
	dM1	Equates coefficients of their two expressions and attempts to find the value of λ : In triangle OEB $k\frac{1}{2}\mathbf{a} - k\frac{7}{8}\mathbf{b} = \lambda\mathbf{a} - \mathbf{b} \Rightarrow k\frac{7}{8} = 1 \Rightarrow k = \frac{8}{7}$ and $\frac{k}{2} = \lambda \Rightarrow \lambda = \dots$ or in triangle OED , there are two possible expressions for $\vec{DE}$ $\lambda\mathbf{a} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} + k(-\mathbf{b} + \lambda\mathbf{a}) \Rightarrow k = \frac{1}{8}$ and $\lambda = \frac{1}{2} + \frac{1}{8}\lambda \Rightarrow \lambda = \dots$ Or $\lambda\mathbf{a} = \frac{1}{2}\mathbf{a} + \frac{1}{8}\mathbf{b} + k\left(\frac{1}{2}\mathbf{a} - \frac{7}{8}\mathbf{b}\right) \Rightarrow \frac{7}{8}k = \frac{1}{8} \Rightarrow k = \frac{1}{7}$ and $\lambda = \frac{1}{2} + \frac{1}{7}k \Rightarrow \lambda = \dots$
	A1	For the correct value of λ $\lambda = \frac{4}{7}$
(c)	M1	For stating that $\Delta OAC = \frac{1}{4}\Delta OAB$ or $4\Delta OAC = \Delta OAB$ 1
	M1	For stating $\Delta OEB = \lambda\Delta OAB \Rightarrow \Delta OEB = \frac{4}{7}\Delta OAB$ 2 This is an A mark in Epen
	M1	For dividing 1 by 2 $= \frac{\Delta OAC}{\Delta OEB} = \frac{1}{4} \times \frac{7}{4} = \dots$
	A1	For $\mu = \frac{7}{16}$



USEFUL SKETCH



Question number	Scheme	Marks
11 (a)	b – a	B1 (1)
(b)	$\vec{OZ} = \vec{OB} + \lambda \vec{BX} (= \mathbf{b} + \lambda(-\mathbf{b} + 2\mathbf{a}))$ $= (1 - \lambda)\mathbf{b} + 2\lambda\mathbf{a}$ $\vec{OZ} = \vec{OA} + \mu \vec{AY} (= \mathbf{a} + \mu(-\mathbf{a} + 3\mathbf{b}))$ $= (1 - \mu)\mathbf{a} + 3\mu\mathbf{b}$ $(1 - \lambda)\mathbf{b} + 2\lambda\mathbf{a} = (1 - \mu)\mathbf{a} + 3\mu\mathbf{b}$ $2\lambda = 1 - \mu$ $3\mu = 1 - \lambda$ $3(1 - 2\lambda) = 1 - \lambda \quad \text{or} \quad 2(1 - 3\mu) = 1 - \mu$ $\lambda = \frac{2}{5} \quad \text{or} \quad \mu = \frac{1}{5}$ $\vec{OZ} = \frac{1}{5}(4\mathbf{a} + 3\mathbf{b}) \quad \text{See notes regarding alternatives}$	M1 A1 M1 A1 ddM1 A1 M1 A1 A1 (9)
(c)	$\textcircled{\text{R}} \quad \vec{OM} = p \cdot \frac{1}{5}(4\mathbf{a} + 3\mathbf{b}) \quad \text{and} \quad \textcircled{\text{R}} \quad \vec{OM} = 2\mathbf{a} + q(-2\mathbf{a} + 3\mathbf{b})$ $\frac{4p}{5} = 2 - 2q \quad \text{and} \quad \frac{3p}{5} = 3q$ $(\text{Solving these equations leads to } p = \frac{5}{3})$ $\vec{OM} = \frac{1}{3}(4\mathbf{a} + 3\mathbf{b})$	M1 M1 A1 (3) [13]

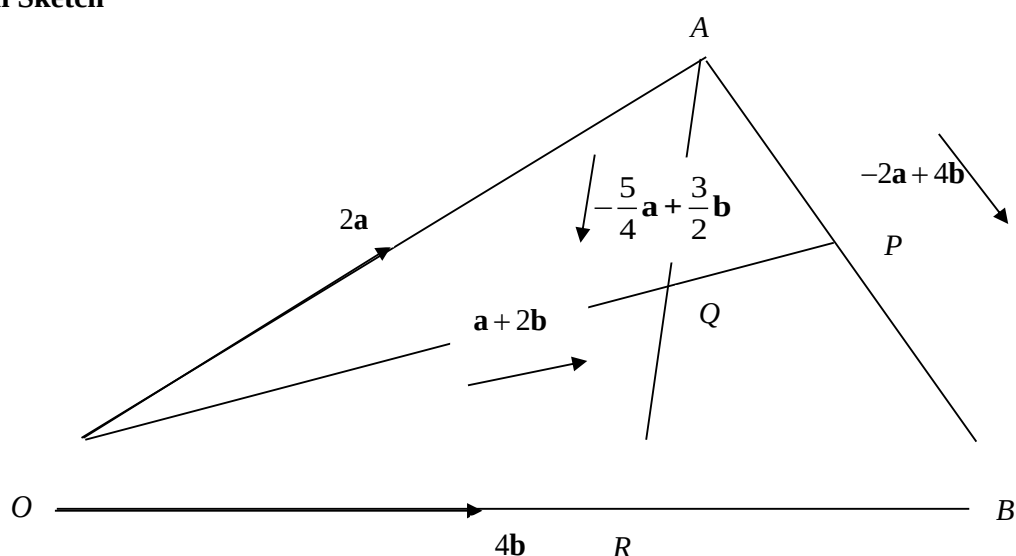
Part	Mark	Additional Guidance
(a)	B1	For the indicated vector
(b)	M1	For any correctly written vector path, must include a parameter
	A1	For the vector shown
	M1	For any correctly written vector path, must include a parameter
	A1	For the vector shown
	ddM1	Equates their 2 vectors – this mark may be implicit in the candidate equating the two components of their 2 vectors, dependent on the first two method marks.
	A1	Correct equations as shown
	ddM1	Full and correct method to solve their two simultaneous equations, either by substitution as shown or by elimination. There must be no errors in the method to eliminate λ or μ , dependent on the first two method marks.
	A1	Correct value for λ or μ
	A1	Correct vector.
	There are a number of alternatives for part b, all marked in the same way. Examples: ALT1 $\mu \vec{XB} \text{ and } \lambda \vec{AY} \text{ (must use to get } \vec{AB} \text{ for 2nd A1)}$ equate 2 vectors for $\vec{AB}$ ALT2 $\vec{AZ} = \vec{AX} + \mu \vec{XB}$ $\vec{AZ} = \lambda \vec{AY}$ equate 2 vectors for $\vec{AZ}$ M1A1 M1A1 ddM1 The A1 ddM1 A1 A1 following these marks should all be marked in the same way as the main mark scheme.	
(c)	M1	For the two correct vectors shown, allow use of their $\vec{OZ}$
	dM1	Correctly equating the components of their vectors for $\vec{OZ}$ and arriving at a value for p or q
	A1	For the correct vector, as shown.
	Can also be done using other vectors eg finding two alternatives for $\vec{OM}$ Mark in the same way as main scheme.	

Question	Scheme	Mark
9(a)	$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB} \Rightarrow \overrightarrow{AB} = -2\mathbf{a} + 4\mathbf{b}$	M1A1 [2]
(b)	$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} \Rightarrow \overrightarrow{OP} = 2\mathbf{a} + \frac{1}{2}(4\mathbf{b} - 2\mathbf{a}) = \mathbf{a} + 2\mathbf{b}$	M1A1 [2]
(c)	$\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ}$ $\overrightarrow{OQ} = \frac{3}{4}(\overrightarrow{OP})$ $\overrightarrow{AQ} = -2\mathbf{a} + \frac{3}{4}(\mathbf{a} + 2\mathbf{b}) = -\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b}$	M1 B1 A1 [3]
(d)	$\overrightarrow{OR} = \lambda 4\mathbf{b}$ or $\overrightarrow{OR} = \phi \mathbf{b}$ $\overrightarrow{OR} = \overrightarrow{OA} + \mu \overrightarrow{AQ} = 2\mathbf{a} + \mu \left(-\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b} \right)$ $2\mathbf{a} + \mu \left(-\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b} \right) = \lambda 4\mathbf{b}$ $\mathbf{a} \left(2 - \frac{5\mu}{4} \right) + \frac{3\mu}{2}\mathbf{b} = \lambda 4\mathbf{b} \Rightarrow \frac{3\mu}{2} = 4\lambda$ and $2 - \frac{5\mu}{4} = 0$ $2 - \frac{5\mu}{4} = 0 \Rightarrow \mu = \left(\frac{8}{5} \right)$ $\frac{3}{2} \times \frac{8}{5} = 4\lambda \Rightarrow \lambda = \frac{12}{20}, \left(\frac{3}{5} \right)$ $OR : RB = 3 : 2$	M1 M1 M1 M1A1 A1 [6]
Total		13 marks

Part	Mark	Scheme
(a)	M1	For the correct vector statement $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$
	A1	$\overrightarrow{AB} = -2\mathbf{a} + 4\mathbf{b}$
(b)	M1	For a correct vector statement e.g. $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$
	A1	For the correct simplified vector $\overrightarrow{OP} = 2\mathbf{a} + (2\mathbf{b} - \mathbf{a}) = \mathbf{a} + 2\mathbf{b}$
(c)	M1	For a correct vector statement e.g., $\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ}$ or $\overrightarrow{AQ} = \overrightarrow{AP} + \overrightarrow{PQ}$
	B1	For the correct vector statement for $\overrightarrow{OQ}$ $\overrightarrow{OQ} = \frac{3}{4}(\overrightarrow{OP})$ or $\overrightarrow{PQ} = -\frac{1}{4}(\overrightarrow{OP})$
	A1	For the correct simplified vector

		$\overrightarrow{AQ} = -2\mathbf{a} + \frac{3}{4}(\mathbf{a} + 2\mathbf{b}) = -\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b}$ or $\overrightarrow{AQ} = \frac{1}{2}(-2\mathbf{a} + 4\mathbf{b}) - \frac{1}{4}(\mathbf{a} + 2\mathbf{b}) = -\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b}$
(d)	In this part there is more than one route to finding the required ratio, but any route must involve $\overrightarrow{OR}$ or $\overrightarrow{RB}$ As the general principle: The first M mark is for one vector using a parameter. The second M mark is for a second vector using a different parameter. The third M mark is for equating coefficients and forming two equations in both parameters. The fourth M mark is for solving their equations. Check their working and do not allow an erroneous method. The following is using one path. Please trace their path on the sketch to check that it is valid. The path they use must involve the vector $\overrightarrow{OR}$ or $\overrightarrow{RB}$ eg., $\overrightarrow{AR} = \overrightarrow{AO} + \overrightarrow{OR}$ or $\overrightarrow{AR} = \overrightarrow{AB} + \overrightarrow{BR}$	
M1	For the correct statement $\overrightarrow{OR} = \lambda 4\mathbf{b}$ or $\overrightarrow{OR} = \phi \mathbf{b}$	
M1	For the correct statement for $\overrightarrow{OR}$ $\overrightarrow{OR} = \overrightarrow{OA} + \mu \overrightarrow{AQ} = 2\mathbf{a} + \mu \left(-\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b} \right)$	
M1	For equating the two statements for $\overrightarrow{OR}$ and equating coefficients $2\mathbf{a} + \mu \left(-\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b} \right) = \lambda 4\mathbf{b}$ $\mathbf{a} \left(2 - \frac{5\mu}{4} \right) + \frac{3\mu}{2}\mathbf{b} = \lambda 4\mathbf{b} \Rightarrow \frac{3\mu}{2} = 4\lambda$ and $2 - \frac{5\mu}{4} = 0$	
M1	For solving their two simultaneous equations correctly to find the value of λ (the parameter for OR) $2 - \frac{5\mu}{4} = 0 \Rightarrow \mu = \frac{8}{5}$ $\frac{3}{2} \times \frac{8}{5} = 4\lambda \Rightarrow \lambda = \dots$	
A1	For the correct value of λ $\lambda = \frac{12}{20} = \left(\frac{3}{5} \right)$	
A1	For the correct ratio $OR : RB = 3 : 2$	

Useful Sketch



Question	Scheme	Marks
11(a)	$\vec{AB} = -\mathbf{a} + \mathbf{b} \quad \vec{AC} = -\mathbf{a} + \frac{2}{3}\mathbf{b} \quad \vec{AM} = -\mathbf{a} + \frac{5}{6}\mathbf{b}$ $\vec{NB} = -\frac{1}{2}\vec{AC} + \vec{AB} = -\frac{1}{2}\left(-\mathbf{a} + \frac{2}{3}\mathbf{b}\right) + (-\mathbf{a} + \mathbf{b}) = \left[-\frac{\mathbf{a}}{2} + \frac{2\mathbf{b}}{3}\right]$ OR $\vec{NB} = -\frac{1}{2}\vec{AC} + \vec{AB} = \frac{1}{2}\left(-\mathbf{a} + \frac{2}{3}\mathbf{b}\right) + \frac{1}{3}\mathbf{b} = \left[-\frac{\mathbf{a}}{2} + \frac{2\mathbf{b}}{3}\right]$ Two statements for $\vec{OP}$ using two parameters $\vec{OP} = \vec{OA} + \lambda \vec{AM} = \mathbf{a} + \lambda\left(-\mathbf{a} + \frac{5}{6}\mathbf{b}\right) = \left[\mathbf{a}(1-\lambda) + \frac{5}{6}\lambda\mathbf{b}\right]$ $\vec{OP} = \vec{OB} - \mu \vec{NB} = \mathbf{b} - \mu\left(-\frac{\mathbf{a}}{2} + \frac{2\mathbf{b}}{3}\right) = \left[\mathbf{a}\left(\frac{\mu}{2}\right) + \mathbf{b}\left(1 - \frac{2\mu}{3}\right)\right]$ Equates coefficients $\Rightarrow 1 - \lambda = \frac{\mu}{2} \quad \frac{5}{6}\lambda = 1 - \frac{2\mu}{3}$ Solves S. E. $\Rightarrow \lambda = \frac{2}{3} \quad \mu = \frac{2}{3}$ $\Rightarrow \vec{OP} = \mathbf{a} + \frac{2}{3}\left(-\mathbf{a} + \frac{5}{6}\mathbf{b}\right) = \frac{1}{3}\mathbf{a} + \frac{5}{9}\mathbf{b}$	B2 M1 M1 M1 M1 M1A1 [9]
(b)	Any two from $\vec{CP} = \frac{1}{3}\mathbf{a} - \frac{1}{9}\mathbf{b} \quad \vec{PQ} = \frac{1}{6}\mathbf{a} - \frac{1}{18}\mathbf{b} \quad \vec{CQ} = \frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}$ $\vec{CQ} = \frac{3}{2}\vec{CP} \quad \text{OR} \quad \vec{PQ} = 2\vec{CP} \quad \text{OR} \quad \vec{CQ} = 3\vec{PQ}$ with a conclusion	B2 M1 A1 [4]
Total 13 marks		

Part	Mark	Notes
(a)	B1	For at least one from $\vec{AB} = -\mathbf{a} + \mathbf{b}$ $\vec{AC} = -\mathbf{a} + \frac{2}{3}\mathbf{b}$ $\vec{AM} = -\mathbf{a} + \frac{5}{6}\mathbf{b}$
	B1	All three correct.
	M1	For a correct vector statement for $\vec{NB}$
		There are several paths to $\vec{OP}$. Those below are examples. For the award of each of the two M marks, there must be two different paths using two distinct parameters.
	M1	For either vector statement for $\vec{OP}$ using a parameter e.g. $\vec{OP} = \vec{OA} + \lambda \vec{AM} = \mathbf{a} + \lambda \left(-\mathbf{a} + \frac{5}{6}\mathbf{b} \right) = \left[\mathbf{a}(1-\lambda) + \frac{5}{6}\lambda\mathbf{b} \right]$
	M1	For a second vector statements for $\vec{OP}$ using a different parameter e.g. $\vec{OP} = \vec{OB} - \mu \vec{NB} = \mathbf{b} - \mu \left(-\frac{\mathbf{a}}{2} + \frac{2\mathbf{b}}{3} \right) = \left[\mathbf{a} \left(\frac{\mu}{2} \right) + \mathbf{b} \left(1 - \frac{2\mu}{3} \right) \right]$
	M1	For equating coefficients of $\mathbf{a}$ and $\mathbf{b}$
	dM1	For solving the simultaneous equations This mark is dependent on the previous 3 M marks $\left[\lambda = \frac{2}{3} \quad \text{OR} \quad \mu = \frac{2}{3} \right]$
	M1	For substituting their value of λ or μ into their $\vec{OP} = \vec{OB} - \mu \vec{NB}$ or $\vec{OP} = \vec{OA} + \lambda \vec{AM}$
	A1	For the correct vector $\vec{OP} = \frac{1}{3}\mathbf{a} + \frac{5}{9}\mathbf{b}$
		SC – If $\vec{CQ}$ is assumed to be a straight line and is used to find $\vec{OP}$, award all the marks as above but withhold the final A mark.
(b)	B2	For any two vectors from $\vec{CP} = \frac{1}{3}\mathbf{a} - \frac{1}{9}\mathbf{b}$ or $\vec{PQ} = \frac{1}{6}\mathbf{a} - \frac{1}{18}\mathbf{b}$ or $\vec{CQ} = \frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}$ (B1 for one of the above)
	M1	For $\vec{CQ} = \frac{3}{2}\vec{CP}$ oe or $\vec{PQ} = 2\vec{CP}$ oe or $\vec{CQ} = 3\vec{PQ}$
	A1cso	For a correct conclusion with supporting working

Question number	Scheme	Marks
10 (a)	$\left(\vec{OB} = \vec{OA} + \vec{AB} \right) = 5\mathbf{p} + 3\mathbf{q}$	M1
	$\vec{DC} = \left(\vec{DO} + \vec{OC} \right) = \vec{DO} + \frac{3}{2} \vec{OB}$	M1
	$\vec{DC} = \frac{9}{2}(\mathbf{p} + \mathbf{q}) = \frac{9\mathbf{p}}{2} + \frac{9\mathbf{q}}{2}$	A1 (3)
(b)	$\vec{OF} = \vec{OD} + \lambda \vec{DC}$	M1
	$= 3\mathbf{p} + \frac{9}{2} \lambda (\mathbf{p} + \mathbf{q})$	A1
	$\vec{OF} = \vec{OA} + \mu \vec{AB}$	M1
	$= 5\mathbf{p} + 3\mu\mathbf{q}$	A1
	$3 + \frac{9}{2} \lambda = 5$	M1
	$\lambda = \frac{4}{9} \quad \left(\frac{9}{2} \lambda = 3\mu \right) \quad \text{or} \quad \mu = \frac{2}{3}$	A1
	$\vec{OF} = 5\mathbf{p} + 2\mathbf{q}$	A1 (7)
(c)	$\vec{OG} = \alpha(5\mathbf{p} + 3\mathbf{q})$	M1
	$\vec{OG} = 3\mathbf{p} + 2(\mathbf{p} + \mathbf{q}) + 5\beta\mathbf{p} = 5\mathbf{p} + 5\beta\mathbf{p} + 2\mathbf{q}$	M1
	ft their $\vec{OF}$	
	$3\alpha = 2 \Rightarrow \alpha = \frac{2}{3} \quad \left(\beta = -\frac{1}{3} \right)$	M1
	$\vec{OG} = \frac{10}{3}\mathbf{p} + 2\mathbf{q}$	A1 (4)
	ALT (c)	
	$\vec{OG} = \alpha(5\mathbf{p} + 3\mathbf{q})$	{M1}
	$\vec{FG} = -5\mathbf{p} - 2\mathbf{q} + \alpha(5\mathbf{p} + 3\mathbf{q}) \quad \text{ft their } -\left(\vec{OF} \right)$	{M1}
	As FG is parallel to AO there can be no $\mathbf{q}$ component in $\vec{FG}$	
	$3\alpha = 2 \Rightarrow \alpha = \frac{2}{3}$	{M1}
	$\vec{OG} = \frac{10}{3}\mathbf{p} + 2\mathbf{q}$	{A1}
		(4)
Total 14 marks		

Part	Mark	Notes
(a)	M1	For finding the vector $\vec{OB}$: $\left(\vec{OB} = \vec{OA} + \vec{AB}\right) = 5\mathbf{p} + 3\mathbf{q}$ This must be correct.
	M1	For the correct vector statement for $\vec{DC}$: $\vec{DC} = \left(\vec{DO} + \vec{OC}\right) = \vec{DO} + \frac{3}{2}\vec{OB}$ This mark can be implied by a correct (unsimplified) vector.
	A1	For the correct simplified $\vec{DC}$ in terms of a single $\mathbf{p}$ and a single $\mathbf{q}$ only $\vec{DC} = -3\mathbf{p} + \frac{3}{2}(5\mathbf{p} + 3\mathbf{q}) = \frac{9}{2}(\mathbf{p} + \mathbf{q})$ accept $\vec{DC} = \frac{9\mathbf{p}}{2} + \frac{9\mathbf{q}}{2}$ oe
(b)	M1	For stating $\vec{OF} = \vec{OD} + \lambda \vec{DC}$ (or any other variable in place of λ)
	A1	For the vector $\vec{OF} = 3\mathbf{p} + \frac{9}{2}\lambda(\mathbf{p} + \mathbf{q})$
	M1	For stating $\vec{OF} = \vec{OA} + \mu \vec{AB}$ (or any other variable in place of μ)
	A1	For the vector $\vec{OF} = 5\mathbf{p} + 3\mu\mathbf{q}$
	M1	Collects like terms correctly, equates coefficients correctly and attempts to solve their two equations in λ and μ They must achieve a value for μ or λ for this mark $3\mathbf{p} + \frac{9}{2}\lambda(\mathbf{p} + \mathbf{q}) = 5\mathbf{p} + 3\mu\mathbf{q} \Rightarrow \mathbf{p}\left(3 + \frac{9}{2}\lambda\right) + \frac{9}{2}\lambda\mathbf{q} = 5\mathbf{p} + 3\mu\mathbf{q}$ $3 + \frac{9}{2}\lambda = 5$ and $\frac{9}{2}\lambda = 3\mu \Rightarrow \mu = \frac{2}{3}$ or $\lambda = \frac{4}{9}$
	A1	For the correct $\mu = \frac{2}{3}$ or $\lambda = \frac{4}{9}$
	A1	For the correct vector $\vec{OF} = 5\mathbf{p} + 2\mathbf{q}$
		Part (b) can be solved in different ways. These are the general principles of marking this part question. M1 – for any correct vector statement leading to $\vec{OF}$ which introduces a parameter. A1 – for one correct vector $\vec{OF}$ M1 – For any correct second vector statement for $\vec{OF}$ which can be used with the first vector and introduces a second parameter. A1 – For a second correct vector $\vec{OF}$ M1A1A1 – as main scheme above
(c)	M1	For one vector for $\vec{OG} = \alpha(5\mathbf{p} + 3\mathbf{q})$

	M1	For a second vector for $\vec{OG} = \left(\vec{OF} + \vec{FG} \right) = 3\mathbf{p} + 2(\mathbf{p} + \mathbf{q}) + 5\beta\mathbf{p} = (5\mathbf{p} + 5\beta\mathbf{p} + 2\mathbf{q})$ ft their $\vec{OF}$ Accept simplified or unsimplified.
	M1	Collects like terms correctly, equates coefficients correctly to find the value of α $\alpha(5\mathbf{p} + 3\mathbf{q}) = 5\mathbf{p} + 5\beta\mathbf{p} + 2\mathbf{q} \Rightarrow 5\mathbf{p}\alpha + 3\mathbf{q}\alpha = (5 + 5\beta)\mathbf{p} + 2\mathbf{q}$ $\Rightarrow 3\alpha = 2 \Rightarrow \alpha = \frac{2}{3}$
	A1	For the correct vector $\vec{OG} = \frac{10}{3}\mathbf{p} + 2\mathbf{q}$
	ALT	
	M1	For the vector for $\vec{OG} = \alpha(5\mathbf{p} + 3\mathbf{q})$
	M1	For the vector $\vec{FG} = \left(\vec{FO} + \vec{OG} \right) = -5\mathbf{p} - 2\mathbf{q} + \alpha(5\mathbf{p} + 3\mathbf{q})$ Note: ft their $-\left(\vec{OF} \right)$ from part (b)
	M1	As FG is parallel to OA there is no $\mathbf{q}$ component in $\vec{FG} \Rightarrow 3\alpha = 2 \Rightarrow \alpha = \frac{2}{3}$
	A1	For the correct vector $\vec{OG} = \frac{10}{3}\mathbf{p} + 2\mathbf{q}$

Question number	Scheme	Marks
10 a (i)	$\vec{OP} = \vec{OA} + \frac{3}{4} \vec{AB} = \mathbf{a} + \frac{3}{4}(-\mathbf{a} + \mathbf{b}) \text{ or } \vec{OB} + \frac{1}{4} \vec{BA} = \mathbf{b} + \frac{1}{4}(\mathbf{a} - \mathbf{b})$ $\vec{OP} = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$	M1 A1
(ii)	$\vec{MN} = \vec{MO} + \frac{1}{2} \vec{OP} = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right) \text{ or } \frac{1}{2} \vec{AO} + \frac{1}{2} \vec{OP}$ $\vec{MN} = -\frac{3}{8}\mathbf{a} + \frac{3}{8}\mathbf{b}$	M1 A1 (4)
b	$\vec{OC} = \lambda \mathbf{b}$ $\vec{AN} = \left(\vec{AO} + \vec{ON} \right) = -\mathbf{a} + \frac{1}{2}\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right) = -\frac{7}{8}\mathbf{a} + \frac{3}{8}\mathbf{b} \text{ or } \vec{AN} = \vec{AP} + \vec{PN} =$ $\vec{OC} = \vec{OA} + \mu \vec{AN} = \mathbf{a} + \mu\left(-\mathbf{a} + \frac{1}{2}\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right)\right)$ $= \mathbf{a} - \frac{7}{8}\mu\mathbf{a} + \frac{3}{8}\mu\mathbf{b} \text{ or } \left(1 - \frac{7}{8}\mu\right)\mathbf{a} + \frac{3}{8}\mu\mathbf{b}$ $\therefore \lambda = \frac{3}{8}\mu \quad \text{and} \quad 0 = 1 - \frac{7}{8}\mu \Rightarrow \mu = \frac{8}{7} \quad \therefore \lambda = \frac{3}{7}$ $\vec{OC} = \frac{3}{7}\mathbf{b}$	B1 M1 M1 (A1 on ePen) A1 (M1 on ePen) M1 (A1 on ePen) A1 (6)
ALT	$\vec{AC} = -\mathbf{a} + \lambda \mathbf{b}$ $\vec{AN} = \left(\vec{AO} + \vec{ON} \right) = -\mathbf{a} + \frac{1}{2}\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right) = -\frac{7}{8}\mathbf{a} + \frac{3}{8}\mathbf{b} \text{ or } \vec{AN} = \vec{AP} + \vec{PN} =$ $\vec{AC} = \mu \vec{AN} = \mu\left(-\mathbf{a} + \frac{1}{2}\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right)\right)$ $= -\frac{7}{8}\mu\mathbf{a} + \frac{3}{8}\mu\mathbf{b}$ $\therefore \lambda = \frac{3}{8}\mu \quad \text{and} \quad 0 = 1 - \frac{7}{8}\mu \Rightarrow \mu = \frac{8}{7} \quad \therefore \lambda = \frac{3}{7}$ $\vec{OC} = \frac{3}{7}\mathbf{b}$	B1 M1 M1 (A1 on ePen) A1 (M1 on ePen) M1 (A1 on ePen) A1 (6)
General principles for marking part (b) – if in any doubt about allocating marks – send to review B1 Writes a valid vector with a parameter in terms of $\mathbf{a}$ or $\mathbf{b}$ which leads to finding $\vec{OC}$ M1 M1 A1 Writes a second valid vector with a different parameter, in terms of $\mathbf{a}$ and $\mathbf{b}$, following a distinct different route, which leads to finding $\vec{OC}$ M1 Compares components with two different parameters and arrives at a value for μ or λ A1 correct vector		

c	$(\text{Area triangle}) OAP = \frac{3}{4} (\text{Area triangle}) OAB$	B1
	$(\text{Area triangle}) OMN = \frac{1}{4} (\text{Area triangle}) OAP = \frac{3}{16} (\text{Area triangle}) OAB$	B1
	$(\text{Area quadrilateral}) AMNP =$ $\frac{3}{4} (\text{Area triangle}) OAB - \frac{3}{16} (\text{Area triangle}) OAB$	M1
	$= \frac{9}{16} (\text{Area triangle}) OAB \quad k = \frac{9}{16}$	A1 (4)
	ALT $\frac{(\text{Area triangle}) OAP}{(\text{Area triangle}) OAB} = \frac{3}{4}$	B1
	$\left(\frac{\text{Area triangle } OMN}{\text{Area triangle } OAP} = \frac{1}{4} \right) \Rightarrow \frac{(\text{Area quadrilateral}) MNAP}{(\text{Area triangle}) OAP} = \frac{3}{4}$	B1
	$\frac{(\text{Area quadrilateral}) MNAP}{(\text{Area triangle}) OAP} \times \frac{(\text{Area triangle}) OAP}{(\text{Area triangle}) OAB} = \frac{3}{4} \times \frac{3}{4}$	M1
	$k = \frac{9}{16}$	A1 (4)
Total 14 marks		

Part	Marks	Note
(a) (i)	M1	For stating or using $\vec{OP} = \vec{OA} + \frac{3}{4} \vec{AB}$ or for $\mathbf{a} + \frac{3}{4}(-\mathbf{a} + \mathbf{b})$ or $\vec{OB} + \frac{1}{4} \vec{BA}$ or $\mathbf{b} + \frac{1}{4}(\mathbf{a} - \mathbf{b})$ (can be implied by correct vector)
	A1	For $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$ or valid alternative form such as $\frac{\mathbf{a} + 3\mathbf{b}}{4}$ or $\frac{1}{4}(\mathbf{a} + 3\mathbf{b})$
(ii)	M1	For stating or using $\vec{MN} = \vec{MO} + \frac{1}{2} \vec{OP}$ or $\frac{1}{2} \vec{AO} + \frac{1}{2} \vec{OP}$ or $-\frac{1}{2}\mathbf{a} + \frac{1}{2}(\text{their } \vec{OP})$
	A1	$-\frac{3}{8}\mathbf{a} + \frac{3}{8}\mathbf{b}$ or valid alternative form such as $\frac{-3\mathbf{a} + 3\mathbf{b}}{8}$ or $\frac{1}{8}(-3\mathbf{a} + 3\mathbf{b})$ (can be implied by correct vector)
(b)	B1	$\vec{OC} = \lambda \mathbf{b}$ or any equivalent statement involving a parameter, in terms of $\mathbf{b}$.
	M1	A fully correct method to find $\vec{AN}$ using their $\vec{OP}$ $\vec{AN} = -\mathbf{a} + \frac{1}{2}\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right)$
	M1 (A1 on ePen)	Using $\vec{OC} = \vec{OA} + \mu(\text{their } \vec{AN})$ in terms $\mathbf{a}$ and $\mathbf{b}$ $\vec{OC} = \vec{OA} + \mu \vec{AN} = \mathbf{a} + \mu\left(-\mathbf{a} + \frac{1}{2}\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right)\right)$ Simplification not required
	A1 (M1 on ePen)	$= \mathbf{a} - \frac{7}{8}\mu\mathbf{a} + \frac{3}{8}\mu\mathbf{b}$ or $\left(1 - \frac{7}{8}\mu\right)\mathbf{a} + \frac{3}{8}\mu\mathbf{b}$ either of these forms, ready for comparing coefficients
	M1 (A1 on ePen)	For correctly equating their components with two different parameters and attempting to solve, reaching values for μ or λ We need to see two equations here, leading to a value for one of the parameters.
	A1	$\vec{OC} = \frac{3}{7}\mathbf{b}$
ALT	B1	$\vec{AC} = -\mathbf{a} + \lambda \mathbf{b}$ or any equivalent statement involving a parameter, in terms of $\mathbf{a}$ and $\mathbf{b}$.
	M1	A fully correct method to find $\vec{AN}$ using their $\vec{OP}$ $\vec{AN} = -\mathbf{a} + \frac{1}{2}\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right)$
	M1 (A1 on ePen)	$\vec{AC} = \mu\left(-\mathbf{a} + \frac{1}{2}\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right)\right)$ using their $\vec{AN}$
	A1 (M1 on ePen)	Correct vector for $\vec{AC}$
	M1 (A1 on ePen) A1	Marks allocated as main scheme
(c) BOTH SCHEMES	B1	Correct statement
	B1	Correct statement
	M1	Uses their statements to carry out a relevant calculation
	A1	Correct value for k

Question number	Scheme	Marks
8 (a)	$\vec{OB} = \vec{OA} + \vec{AB} = \mathbf{a} + \mathbf{b}$	B1 [1]
(b)	$\vec{OC} = 2\mathbf{b}$ $\vec{BC} = \vec{BO} + \vec{OC} = -(\mathbf{a} + \mathbf{b}) + 2\mathbf{b} = \mathbf{b} - \mathbf{a}$	B1 M1A1 [3]
(c)	$\vec{OM} = \vec{OB} + \vec{BM} = \mathbf{a} + \mathbf{b} + \frac{2}{3}(\mathbf{b} - \mathbf{a}) = \frac{\mathbf{a}}{3} + \frac{5\mathbf{b}}{3}$ or $\frac{1}{3}(\mathbf{a} + 5\mathbf{b})$	M1A1ft [2]
(d)	$\vec{OY} = \mu \left(\frac{\mathbf{a}}{3} + \frac{5\mathbf{b}}{3} \right) = \frac{\mu\mathbf{a}}{3} + \frac{5\mu\mathbf{b}}{3}$ $\vec{OY} = \vec{OA} + \vec{AY} = \mathbf{a} + \lambda\mathbf{b}$ $\Rightarrow \frac{\mu}{3} = 1$ and $\frac{5\mu}{3} = \lambda$ Solves simultaneous equations by any method $\mu = 3, \lambda = 5$ $AB : BY = 1 : 4$ ALT $\vec{BY} = \lambda \vec{AB} = \lambda\mathbf{b}$ $\vec{BY} = \vec{BO} + \vec{OY} = \vec{BO} + \mu\vec{OM} = -(\mathbf{a} + \mathbf{b}) + \mu \left(\frac{1}{3}\mathbf{a} + \frac{5}{3}\mathbf{b} \right)$ $= \left(-1 + \frac{1}{3}\mu \right)\mathbf{a} + \left(-1 + \frac{5}{3}\mu \right)\mathbf{b}$ $\Rightarrow -1 + \frac{1}{3}\mu = 0$ and $\lambda = -1 + \frac{5}{3}\mu$ $\mu = 3, \lambda = 4$ $AB : BY = 1 : 4$	M1 M1 M1 M1 A1 [5] [M1 M1 M1 M1 A1]
(e)	$10 = \frac{1}{2}ab \sin 60^\circ \Rightarrow ab = \frac{40}{\sqrt{3}} \Rightarrow a = \frac{40}{b\sqrt{3}}$ $\text{Area} = \frac{1}{2}a \times 5b \sin 120^\circ = \frac{1}{2} \times \frac{40}{b\sqrt{3}} \times 5b \sin 120^\circ$ $\text{Area} = 50$ ALT $\frac{\text{Area } OAY}{\text{Area } OAB} = \frac{\frac{1}{2} \times h \times 5}{\frac{1}{2} \times h \times 1} = \frac{5}{1}$ $\text{Area } OAY = 5 \times \text{Area } OAB$ $\text{Area } OAB = 10$ $\text{Area} = 50$	M1A1 dM1 A1 [4] [M1 A1 dM1 A1]
Total 15 marks		

Part	Mark	Notes
(a)	B1	For a correct expression for $\vec{OB}$ in terms of a and b
(b)	B1	For a correct expression for $\vec{OC}$
	M1	For a correct vector statement for $\vec{BC}$: $\vec{BC} = \vec{BO} + \vec{OC}$ This mark can be implied by a correct (unsimplified) vector using their $\vec{OB}$. Vector statement must be suitable for substitution to find $\vec{BC}$
	A1	For the correct simplified $\vec{BC}$ in terms of a single a and b only. $\vec{BC} = \mathbf{b} - \mathbf{a}$ If answer $\vec{BC} = \mathbf{b} - \mathbf{a}$ seen without wrong working then award B1M1A1.
(c)	M1	For a correct vector statement for $\vec{OM}$: $\vec{OM} = \vec{OB} + \frac{2}{3}\vec{BC}$ This mark can be implied by a correct (unsimplified) vector using their $\vec{OB}$ and their $\vec{BC}$.
	A1ft	For the correct simplified using their $\vec{OM}$ in terms of a single a and b only. $\vec{OM} = \frac{\mathbf{a}}{3} + \frac{5\mathbf{b}}{3}$ or $\frac{1}{3}(\mathbf{a} + 5\mathbf{b})$
(d)	M1 M1	M1 for one correct statement of route for $\vec{OY}$ M1 for second correct statement of route for $\vec{OY}$ $\vec{OY} = \mu \vec{OM}$ (or any other variable in place of μ) $\vec{OY} = \left(\vec{OA} + \vec{AY}\right) = \mathbf{a} + \lambda \mathbf{b}$ (or any other variable in place of λ , provided this is different to their μ). Allow use of their vectors from earlier parts of the question.
	M1	For equating their coefficients of a and b to obtain two equations. Mark intent – one must be correct, condone slips in second.
	M1	Solving their simultaneous equations by any method. Only the value for their λ is required for this mark.
	A1	For $AB:BY = 1:4$
	ALT – use of $\vec{BY}$ route, use also for $\vec{AY}$ route	
	M1 M1	M1 for one correct statement of route for $\vec{BY}$ M1 for second correct statement of route for $\vec{BY}$ $\vec{BY} = \lambda \vec{AB} = \lambda \mathbf{b}$ (or any other variable in place of λ) $\vec{BY} = \vec{BO} + \vec{OY} = \vec{BO} + \mu \vec{OM} = -(\mathbf{a} + \mathbf{b}) + \mu \left(\frac{1}{3}\mathbf{a} + \frac{5}{3}\mathbf{b}\right)$ $= \left(-1 + \frac{1}{3}\mu\right)\mathbf{a} + \left(-1 + \frac{5}{3}\mu\right)\mathbf{b}$ (or any other variable in place of μ provided this is different to their λ). Allow use of their vectors from earlier parts of the question.
	M1	For equating their coefficients of a and b to obtain two equations. Mark intent – one must be correct, condone slips in second.
	M1	Solving their simultaneous equations by any method. Only the value for their λ is required for this mark.
	A1	For $AB:BY = 1:4$

(e)	M1	For use of the correct formula for area of a triangle with 60° and correct value of $\sin 60^\circ$ $10 = \frac{1}{2}ab \sin 60^\circ = ab \times \frac{\sqrt{3}}{4}$
	A1	For correct expression for ab or a $ab = \frac{40}{\sqrt{3}}$ or $a = \frac{40}{b\sqrt{3}}$
	dM1	For use of the correct formula for area of a triangle with 120° and attempt to substitute for ab or a . $\text{Area} = \frac{1}{2}a \times '5'b \sin 120^\circ = \frac{1}{2} \times ' \frac{40}{b\sqrt{3}} ' \times '5'b \sin 120^\circ$ or $\text{Area} = \frac{1}{2}a \times '5'b \sin 120^\circ = \frac{1}{2} \times ab \times '5' \sin 120^\circ = \frac{1}{2} \times ' \frac{40}{\sqrt{3}} ' \times '5' \sin 120^\circ$ Dependent on the first M awarded. Allow use of their λ from part (d).
	A1	For the correct area Area = 50
ALT – use of ratios of areas		
	M1	For use of their ratio $AB:BY$ to write an equation linking area OAY and area OAB $\frac{\text{Area } OAY}{\text{Area } OAB} = \frac{\frac{1}{2} \times h \times (1 + '4')}{\frac{1}{2} \times h \times 1} = \frac{'5'}{1}$
	A1	For correct relationship between area OAY and area OAB
	dM1	For a correct method to find the area of OAB Dependent on first M mark being awarded.
	A1	For the correct area Area = 50