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EDEXCEL INTERNATIONAL GCSE FURTHER PURE MATHS: QUESTIONS BY TOPIC

Calculus (8)

International GCSE Further Pure Mathematics Topic 8

126 questions · 1123 marks · Mark schemes included

Pearson Edexcel International GCSE (9–1) Further Pure Mathematics 4PM1, Papers 1, 1R, 2 & 2R, June 2019 - June 2025

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HOW TO USE THIS BOOKLET

Questions run from the lowest mark tariff to the highest, so similar questions sit together.
Click any heading or question row below to jump straight to it.
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Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

126 questions · 1123 marks · lowest tariff first

3-MARK QUESTIONS

1 question in this booklet

1 Differentiate with respect to x

$$6e^{3x^2}\cos 2x$$

(3)

4-MARK QUESTIONS

4 questions in this booklet

1 A particle P is moving along a straight line that passes through the fixed point O.
At time t seconds, $t \geqslant 0$, the displacement, s metres, of P from O is given by

$$s = t^3 + 4t^2 - 27t + 4$$

Find the value of t at the instant when the velocity of P is 8 m/s.

(4)

1 A particle P is moving in a straight line. At time t seconds, $t \geqslant 0$, the displacement, s metres, of P from a fixed point O of the line is given by

$$s = 3 + 8t + t^2 - \frac{1}{3}t^3$$

Find the distance of P from O when P is instantaneously at rest.

(4)

2 The region enclosed by the curve with equation $y = e^{3x}$, the x-axis, the y-axis and the line with equation $x = 3$ is rotated through 360° about the x-axis.

Use algebraic integration to find, in terms of π and e, the volume of the solid generated.

(4)

1 Using calculus find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 4\theta \, \mathrm{d}\theta$

Give your answer in the form $-\frac{\sqrt{a}}{b}$ where a is a prime number and b is an integer.

(4)

5-MARK QUESTIONS

8 questions in this booklet

5 A circle has radius $3r$ cm and area A cm^2

Given that the value of r increases by 0.05%

use calculus to find an estimate for the percentage increase in the value of A.

(5)

9 Find an equation of the normal to the curve with equation

$$y = (x^3 - 2x)e^{(1-x)}$$

at the point on the curve with coordinates $(1, -1)$

(5)

4 $y = \dfrac{\sin 2x}{\sqrt{x^2 - 9}} \qquad |x| > 3$

Show that $\dfrac{dy}{dx} = \dfrac{2(x^2 - 9)\cos 2x - x\sin 2x}{\sqrt{(x^2 - 9)^3}}$

(5)

3 The curve C has equation $y = 9 - x^2$

Use algebraic integration to find the area of the finite region bounded by C and the x-axis.

(5)

2 $y = (\sin 2x)\sqrt{3 + 2x}$

Show that $\frac{dy}{dx} = \frac{\sin 2x + (A + Bx)\cos 2x}{\sqrt{3 + 2x}}$ where A and B are integers to be found. **(5)**

3 The curve C has equation $y = e^{3x}(2x-1)^4$

Using calculus, find the exact value of the gradient of the tangent to C when $x = 1$

(5)

5 The height of liquid in a vessel P is h
The volume, V, of the liquid in P is given by $V = 6h^3$
Liquid is leaking from P at a constant rate of $36\,\text{cm}^3/\text{s}$

Find the exact rate of change, in cm/s, of h when $V = 384\ \text{cm}^3$

(5)

8 Use calculus to find the gradient of the curve with equation

$$y = e^{3x}(9x+2)^{\frac{1}{3}}$$

at the point on the curve where $x = 1$
Give your answer to 3 significant figures.

(5)

6-MARK QUESTIONS

17 questions in this booklet

2 Oil is leaking from a pipe and forms a circular pool on a horizontal surface. The area of the surface of the pool is increasing at a constant rate of $8\,cm^2/s$. Find, in cm/s to 3 significant figures, the rate at which the radius of the pool is increasing when the area of the pool is $50\,cm^2$

(6)

3 A particle P moves in a straight line. At time t seconds, the velocity, v m/s, of P is given by

$$v = t^2 - 4t + 7$$

(a) Find the acceleration of P, in m/s^2, when $t = 3$

(2)

(b) Find the distance, in m, that P travels in the interval $0 \leqslant t \leqslant 6$

(4)

3

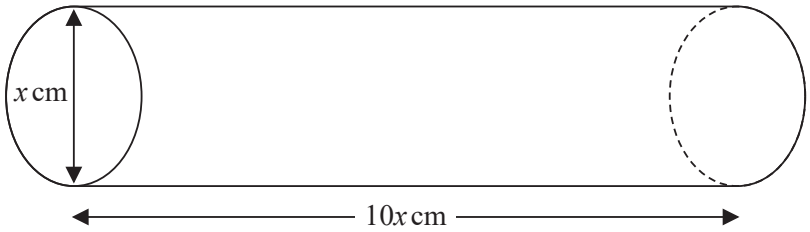


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a solid right circular cylindrical metal rod.

The diameter of the rod is x cm and the length of the rod is $10x$ cm.

The rod is being heated so that the length of the rod is increasing at a rate of 0.005 cm/s.

Find the rate of increase, in cm^3/s to 2 significant figures, of the volume of the rod when $x = 3$

(6)

3 Given that $y = e^{2x}(x^2 + 1)$

(a) find $\frac{dy}{dx}$

(3)

The straight line l is the tangent to the curve with equation $y = e^{2x}(x^2 + 1)$ at the point on the curve where $x = 0$

(b) Find an equation for l in the form $y = mx + c$

(3)

6

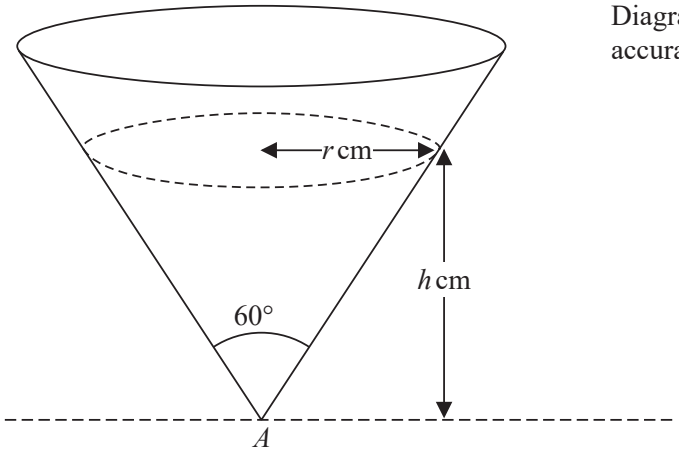


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows a hollow right circular cone fixed with its axis of symmetry vertical.

The cone is inverted and contains liquid, which is dripping out of a small hole at the vertex A of the cone at a constant rate of $0.9\,\text{cm}^3/\text{s}$.

At time t seconds after the liquid starts to drip from the cone, the height of the liquid is h cm above A. The volume of liquid in the cone at time t seconds is $V\,\text{cm}^3$

The vertical angle of the cone is 60°

(a) Show that $V = \frac{1}{9}\pi h^3$

(2)

(b) Find, in cm/s to 3 significant figures, the rate at which the height of the liquid is decreasing when the height of the liquid in the cone above the vertex is 1.2 cm.

(4)

4 The curve C has equation

$$y = x^3 - 3x^2 - 24x + 6$$

(a) Use calculus to find the coordinates of each of the stationary points on C.

(4)

(b) Determine the nature of each of these stationary points.
Justify your answers.

(2)

6 The volume of a sphere with radius r cm is increasing at a constant rate of $3\,\text{cm}^3/\text{s}$.

Find the rate, in cm^2/s, at which the surface area of the sphere is increasing when $r = 10$

(6)

2 The finite region enclosed by the curve with equation $y = 4 - x^2$ and the line with equation $y = x + 2$ is rotated through 360° about the x-axis.

Use algebraic integration to find the exact volume of the solid formed.

(6)

5 y and x vary so that $y = xe^{-2x}$

Given that the value of x increases by 3%, use calculus to find, in terms of x, an estimate for the percentage change in y
Give your answer in the form $a(b - cx)$ where a, b and c are integers.

(6)

3 Differentiate with respect to x

(a) $e^{2x}\sqrt{5x-3}$

(3)

(b) $\dfrac{x^3}{\cos 3x}$

(3)

2 A particle P moves along the x-axis. At time t seconds, the displacement, x metres, of P from the origin O is given by

$$x = t^4 - 13.5t + 12$$

(a) Find the velocity, in m/s, of P when $t = 3$

(2)

(b) Find the value of t for which P is instantaneously at rest.

(2)

(c) Find the acceleration, in m/s^2, of P when $t = 2$

(2)

2 When poured from a pipe, concrete is formed into the shape of a cuboid with a square base of side x and with a height of $3x$

The volume of the cuboid increases at a constant rate of $8\,m^3/s$

Find the rate of increase, in m/s, of x when $x = 2$ metres.

(6)

6

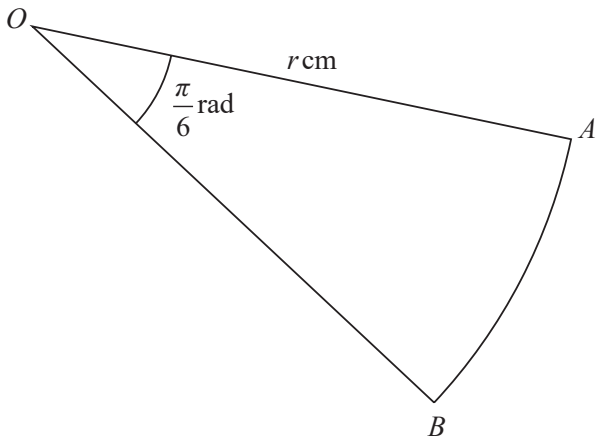


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows the sector OAB of a circle with centre O and radius r cm.

$$\angle AOB = \frac{\pi}{6} \text{ radians} \qquad OA = OB = r \text{ cm}$$

The area of the sector is increasing in such a way that the size of $\angle AOB$ remains constant, and the lengths OA and OB are both increasing at a constant rate of $0.2\,\text{cm/s}$

Find the exact rate of change, in cm^2/s, of the area of the sector when the length of arc AB is $\frac{5\pi}{2}$

(6)

6

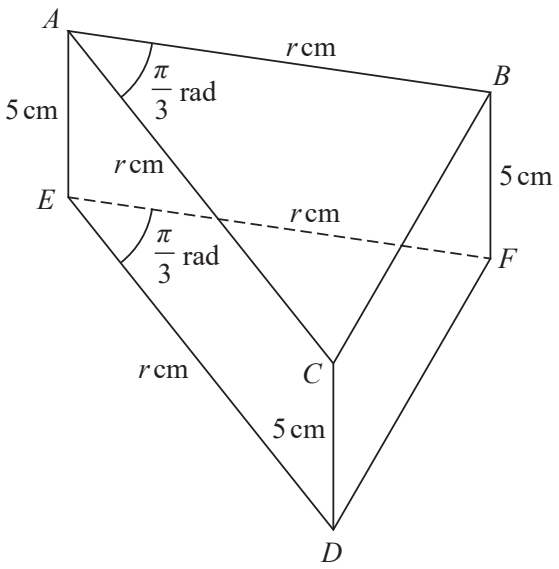


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a right triangular prism $ABCDEF$. A cross section ABC of the prism is a triangle in which $AB = AC = r\,\text{cm}$ and $\angle CAB = \frac{\pi}{3}$ radians.

In the prism

$$AE = BF = CD = 5\,\text{cm} \qquad ED = EF = r\,\text{cm} \text{ and } \angle DEF = \frac{\pi}{3} \text{ radians}$$

(a) Show that the volume of the prism is $\frac{5\sqrt{3}}{4} r^2\ \text{cm}^3$

(1)

The volume of the prism is increasing in such a way that the size of $\angle CAB$ and the size of $\angle DEF$ remain constant and the length of AE, the length of BF and the length of CD remain constant.
The lengths of AB, AC, ED and EF are each increasing at a constant rate of $0.2\,\text{cm/s}$

(b) Find the exact rate of increase, in cm^3/s, of the volume of the prism when the area of the rectangular face $BCDF$ is $60\,\text{cm}^2$

(5)

6 A particle P is moving in a straight line. The displacement s of P, in metres, at time t seconds, $t \geqslant 0$, is given by

$$s = e^{2t} \sin 3t + 2$$

At time $t = 0$, P is at the point A and at time $t = \frac{\pi}{6}$, P is at the point B

(a) Find the exact distance AB

(2)

(b) Find the exact velocity of P when $t = \frac{\pi}{3}$

(4)

5 The force F newtons between two magnetic poles is given by the formula

$$F = \frac{3}{20r^2}$$

where r is the distance, in centimetres, between the poles.

The distance between the two poles is increasing at a constant rate of 0.7 cm/s

Find the rate of change of F, in newtons/s to 3 significant figures, when the distance between the poles is 2.8 cm

(6)

7

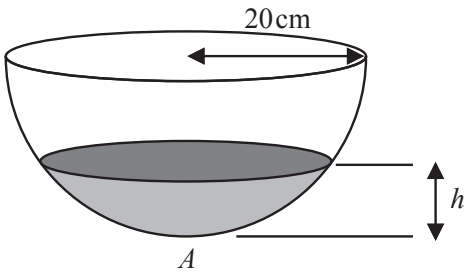


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a hollow hemisphere with radius 20 cm

The hemisphere contains liquid, which is dripping out of a small hole at the lowest point A at a constant rate of k cm^3/s

At time t seconds after the liquid starts to drip from the hemisphere, the height of the liquid is h cm above A

The volume V cm^3 of liquid in the hemisphere is given by

$$V = \frac{\pi}{3}h^2(60 - h)$$

When $h = 12$, the height of the liquid is decreasing at a rate of $\frac{1}{60}$ cm/s

Find the value of k
Give your answer in terms of π

(6)

7-MARK QUESTIONS

13 questions in this booklet

12

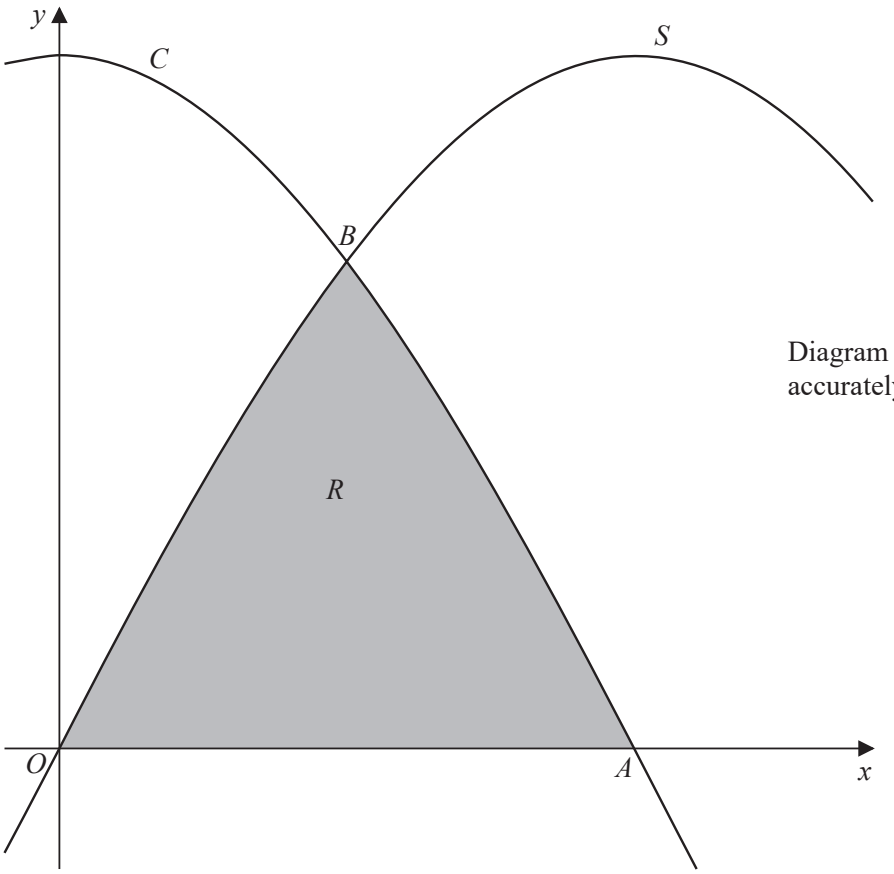


Diagram **NOT** accurately drawn

Figure 2

The region R, shown shaded in Figure 2, is bounded by the x-axis, the curve S with equation $y = 2\sin x$ and the curve C with equation $y = 2\cos x$.
As shown in Figure 2, C crosses the x-axis at the point A.

(a) Write down the x coordinate of A.

(1)

As shown in Figure 2, C and S intersect at the point B.

(b) Find the x coordinate of B.

(2)

(c) Using calculus, find the area of the shaded region R.
Give your answer in the form $a - \sqrt{b}$ where a and b are integers.

(4)

8 Liquid drips onto a large horizontal flat cloth, forming a circular stain.

The liquid starts to drip onto the cloth at time $t = 0$

The area of the stain increases at a constant rate of $1.5\,\text{cm}^2/\text{s}$

(a) Find, in terms of π, the radius of the stain at time $t = 4$ seconds.

(3)

(b) Find, in cm/s to 3 significant figures, the rate at which the radius of the stain is increasing at time $t = 4$ seconds.

(4)

8 The volume of a sphere is $500\,cm^3$

(a) Calculate the radius, in cm to 3 significant figures, of the sphere.

(2)

The surface area of the sphere is increased by $20\,cm^2$

(b) Using calculus, find an estimate for the increase in the radius, in cm to 2 significant figures, of the sphere.

(5)

2 A particle P is moving along a straight line, which passes through the fixed point O.

At time t seconds ($t \geqslant 0$), the velocity, v m/s, of P is given by

$$v = t^2 - 3t + 4$$

At time t seconds the acceleration of P is a m/s^2

(a) Find an expression for a in terms of t

(2)

The displacement of P from O is 7 m when $t = 2$

(b) Find the exact displacement of P from O when $t = 4$

(5)

4 The equation of a curve is $y = x^3 \sin x$

Find an equation of the tangent to the curve at the point on the curve where $x = \frac{1}{2}\pi$

Give your answer in the form $y = mx + c$

(7)

9 A cube has edges of length x cm.

The total surface area, A cm^2, of the cube is increasing at a constant rate of 0.45 cm^2/s

Find the rate of increase, in cm^3/s, of the volume of the cube at the instant when the total surface area of the cube is 384 cm^2

(7)

5 A particle P is moving along the x-axis.

At time t seconds, $t \geqslant 0$, the velocity, v m/s, of P is given by

$$v = 2t^2 - 19t + 35$$

(a) Find the acceleration of P when $t = 5$

(2)

The particle comes to instantaneous rest at the points A and B at times t_1 seconds and t_2 seconds respectively, where $t_1 < t_2$

(b) Find the value of t_1 and the value of t_2

(2)

(c) Use calculus to find the distance AB

(3)

7

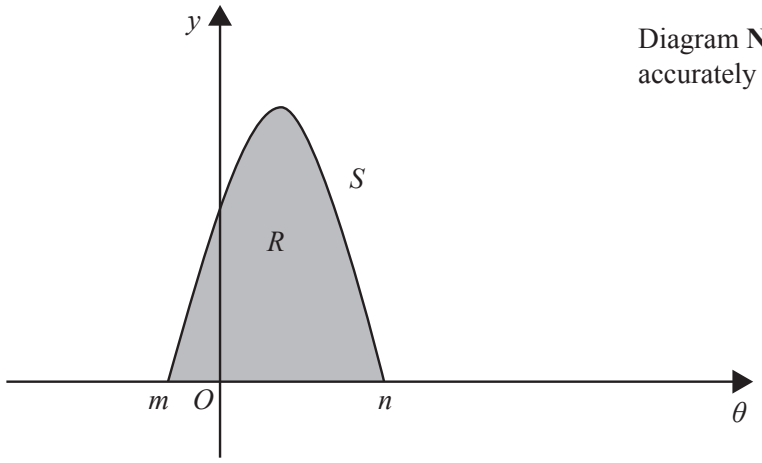


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows part of the curve S with equation $y = \left(\cos 3\theta + \sqrt{3}\sin 3\theta\right)^{\frac{1}{2}}$

where $m \leqslant \theta \leqslant n$

The curve S meets the x-axis at the point with coordinates $(m, 0)$ and at the point with coordinates $(n, 0)$

(a) Find the exact value of m and the exact value of n

(3)

The finite region R, shown shaded in Figure 2, is bounded by the curve S, and the x-axis in the region $m \leqslant \theta \leqslant n$

The region R is rotated through 2π radians about the theta-axis.

(b) Use calculus to find the exact volume of the solid generated.

(4)

3 A particle P moves along the x-axis.

At time t seconds $(t \geqslant 0)$ the acceleration, a m/s^2, of P is given by $a = 6t - 16$

When $t = 0$, P is at the origin and is moving with velocity 12 m/s.

(a) Find an expression in terms of t for

(i) the velocity of P at time t seconds

(ii) the displacement of P at time t seconds.

(4)

(b) Hence find the time at which P first returns to the origin.

(3)

6 The volume of oil in a container is V cm^3 when the height of the oil is h cm.
Oil is pouring into the container at a constant rate of 12 cm^3/s.
Given that $V = 3h^3$

find the exact rate, in cm/s, at which the height of the oil is increasing when $V = 1536$ cm^3

(7)

5

$$y = e^{2x}\left(x^2 - 5x\right)$$

Show that $2e^{2x} = \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$

(7)

1 A particle P moves in a straight line. At time t seconds, the velocity, v m/s, of P is given by

$$v = 3t^2 - 9t + 7$$

(a) Show that P never comes to rest.

(2)

(b) Find the acceleration of P, in m/s^2, when $t = 4$

(2)

Using algebra

(c) find the distance, in m, that P travels in the interval $0 \leqslant t \leqslant 5$

(3)

11

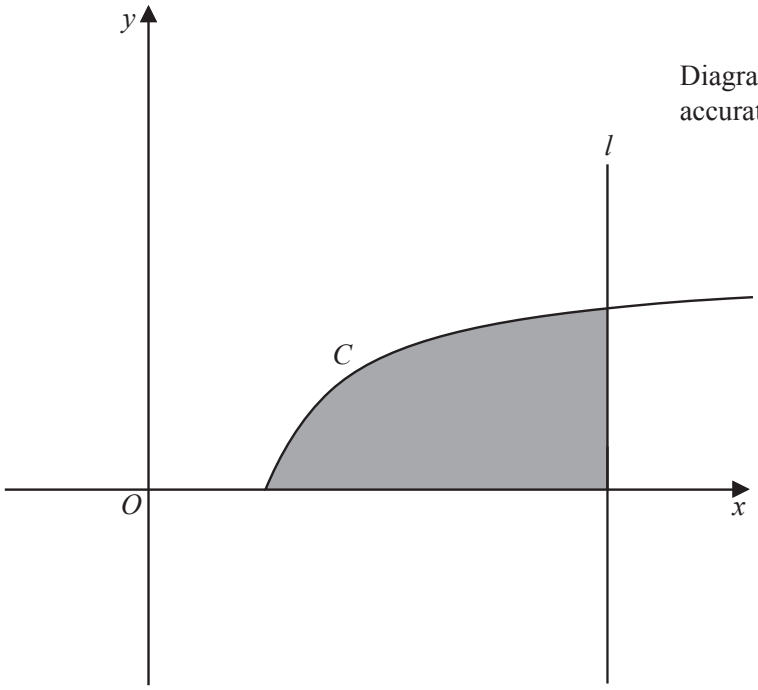


Diagram **NOT** accurately drawn

Figure 5

Figure 5 shows part of the curve C with equation $y = \sqrt{4x - 8}$ and the line l with equation $x = b$ where $b > 0$

The finite region bounded by the curve C, the x-axis and the line l, shown shaded in Figure 5, is rotated through $360°$ about the x-axis.

Given that the volume of the solid formed is 50π units^3

find the value of b

(7)

8-MARK QUESTIONS

22 questions in this booklet

6 (a) Given that $y = (4x - 3)e^{2x}$

(i) find $\frac{dy}{dx}$

(3)

(ii) show that $(4x - 3)\frac{dy}{dx} = (8x - 2)y$

(2)

(b) Differentiate $\frac{\sin 5x}{(x-3)^2}$ with respect to x

(3)

4 A particle P moves along the x-axis. At time t seconds ($t \geqslant 0$) the acceleration, a m/s^2, of P is given by $a = 6t - 12$

When $t = 0$, P is at rest at the origin.

(a) Find the velocity of P when $t = 2$

(3)

At time T seconds, $T > 0$, P is instantaneously at rest.

(b) Find the value of T.

(2)

(c) Find the distance travelled by P in the first 8 seconds of its motion.

(3)

8 A particle P moves along the positive x-axis. At time t seconds ($t \geqslant 0$) the velocity, v m/s, of P is given by $v = 3 + 5t - 2t^2$

At time t seconds, P is at the point with coordinates $(x, 0)$.

Given that at time $t = 0$, P is at the point with coordinates $(5, 0)$, find the maximum value of x, justifying that this is a maximum value.

(8)

6

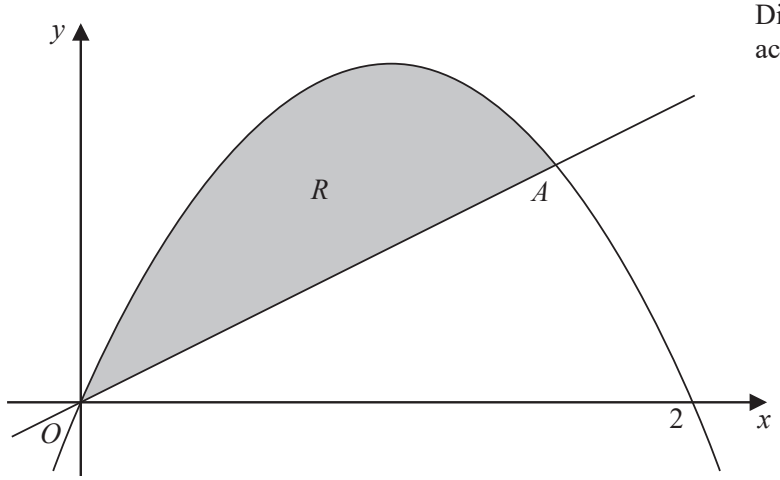


Diagram **NOT** accurately drawn

Figure 2

The region R, shown shaded in Figure 2, is bounded by the curve with equation $y = 2x - x^2$ and the line with equation $2y - x = 0$

The curve and the line intersect at the origin O and the point A.

(a) Show that the point A has coordinates $\left(\frac{3}{2}, \frac{3}{4}\right)$.

(2)

The region R is rotated through 360° about the x-axis.

(b) Use algebraic integration to find, in terms of π, the volume of the solid formed.

(6)

8 Given that $y = e^{3x} \sin 2x$

show that $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$

(8)

9 Given that

$$x = e^{-t} \sin 2t$$

show that

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0$$

(8)

9

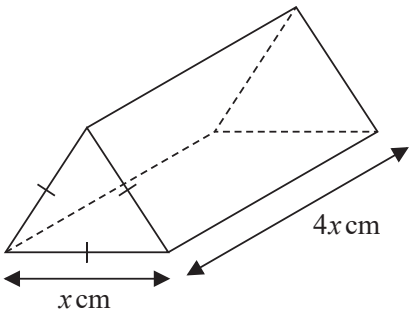


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a metal solid S.

The solid is a right triangular prism.

The cross section of S is an equilateral triangle with sides of length x cm.
The length of S is $4x$ cm.

The prism is being heated so that the cross sectional area is increasing at a constant rate of $0.03\,\text{cm}^2/\text{s}$.

(a) Find, giving your answer to 3 significant figures, $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $x = 2$

(5)

(b) Find the rate of increase, in cm^3/s, of the volume of S when $x = 2$

(3)

11

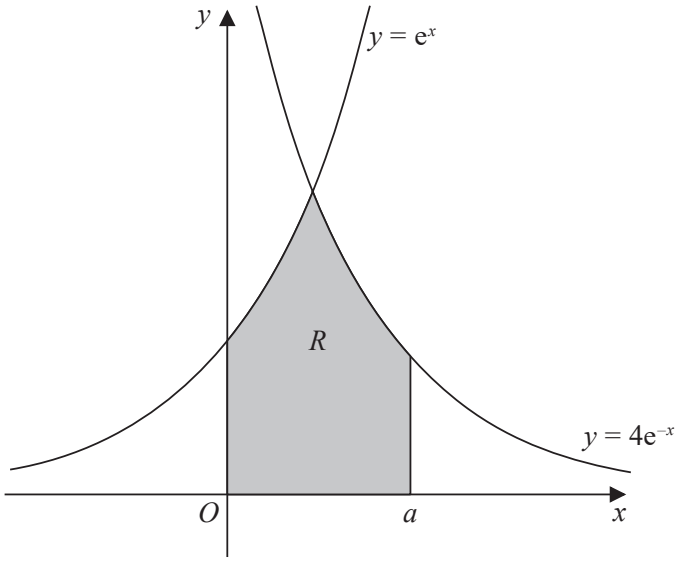


Diagram **NOT** accurately drawn

Figure 4

The region R, shown shaded in Figure 4, is bounded by the curve with equation $y = e^x$, the curve with equation $y = 4e^{-x}$, the straight line with equation $x = a$, the x-axis and the y-axis.

When the region R is rotated through 360° about the x-axis, the volume of the solid generated is

$$k - 8\pi e^{-4}$$

where k is a constant.

Using algebraic integration, find a possible value of a and the exact corresponding value of k.

(8)

6 A particle P is moving along the x-axis. At time t seconds ($t \geqslant 0$) the displacement, s metres, of P from the origin O, is given by $s = t^3 - 4t^2 - 16t - 8$

(a) Find the distance of P from O when $t = 0$

(1)

(b) Find the value of t for which P is instantaneously at rest.

(4)

(c) Find the value of t for which P is accelerating at $10\,\text{m/s}^2$ in the positive x direction.

(3)

8 The curve C has equation $y = 2x^2 - \sin x$

The point A on C has x coordinate π

Show that an equation of the normal to C at the point A is

$$x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0$$

(8)

5 Differentiate with respect to x

(a) $e^{4x}(6x+2)^{\frac{3}{2}}$

Give your answer in the form $e^{4x}\left(\sqrt{6x+2}\right)(Ax+B)$ where A and B are integers.

(5)

(b) $\dfrac{\sin 3x}{(2x-4)^3}$

(3)

6 (a) Using a formula from page 2, show that $\cos 2A = 1 - 2\sin^2 A$

(2)

The finite region R is bounded by the curve with equation $y = 3 + 2\sin x$, the x-axis, the y-axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 360° about the x-axis.

(b) Use calculus to find the volume of the solid generated.
Give your answer to the nearest integer.

(6)

3 Given that $y = e^{3x} \sin 2x$

show that $13y + \frac{d^2y}{dx^2} = 6\frac{dy}{dx}$

(8)

7

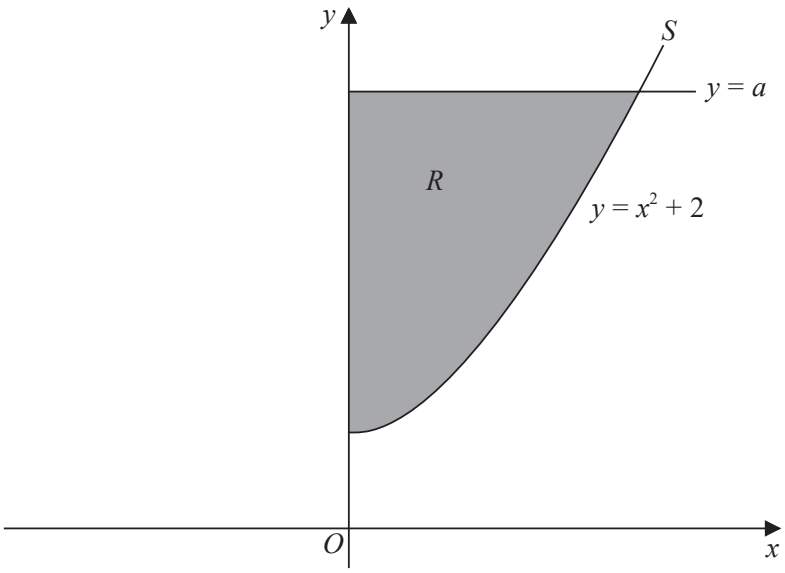


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows part of the curve S with equation $y = x^2 + 2$

The finite region R, shown shaded in Figure 3, is bounded by S, the y-axis and the line with equation $y = a$ where $a > 2$

The region R is rotated through 360° about the y-axis to generate a solid with volume 18π

Use algebraic integration to find the value of a

(8)

7 The curve with equation $y = mx^2 + 64\sqrt{x} + 39$ has a stationary point with coordinates $(4, n)$ where m and n are integers.

Using calculus

(a) find the value of m and the value of n

(6)

(b) determine the nature of the stationary point.

(2)

4

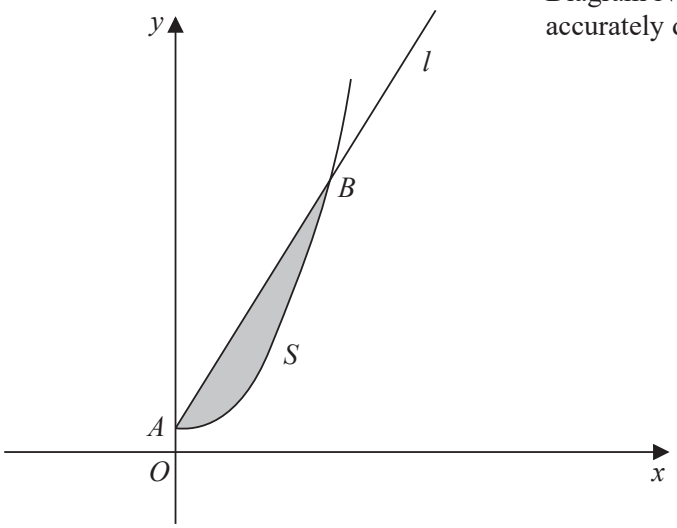


Diagram **NOT** accurately drawn

Figure 2

The curve S with equation $y = \frac{x^2}{4} + 2$ where $x \geqslant 0$ and the line l with equation $2y - x - 4 = 0$ where $x \geqslant 0$ intersect at the points A and B, as shown in Figure 2.

(a) (i) Show that the coordinates of point A are $(0, 2)$

(ii) Find the coordinates of the point B

(4)

The finite region bounded by S and l, shown shaded in Figure 2, is rotated through 2π radians about the y-axis.

(b) Use algebraic integration to find the volume of the solid generated. Give your answer in terms of π

(4)

5

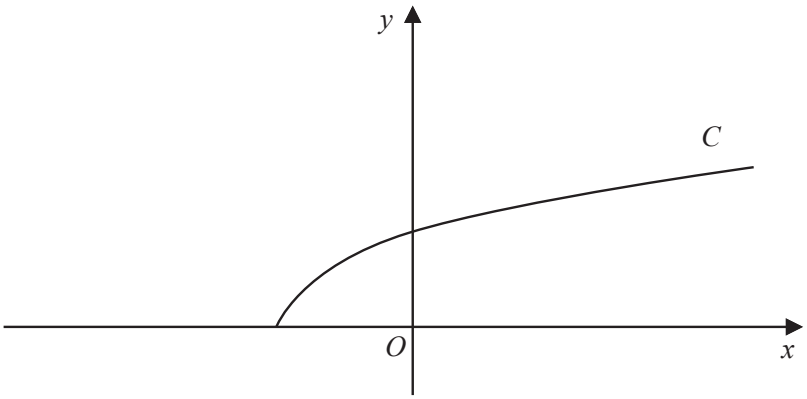


Figure 2

Figure 2 shows the graph of part of the curve C with equation $y = \sqrt{2x+6}$
The finite region enclosed by the curve C and the straight line with equation $3y - x = 3$ is rotated through 360° about the x-axis.

Use algebraic integration to find the exact volume of the solid generated.
Give your answer in terms of π

(8)

8

$$y = \frac{2e^{3x+1}}{5x^2}$$

(a) Find $\frac{dy}{dx}$

Give your answer in the form $\frac{Ae^{3x+1}(Bx - A)}{Cx^3}$ where A, B and C are prime numbers to be found.

(5)

The value of x increases by 2%

(b) Use your answer to part (a) to find an estimate, in terms of x, for the percentage change in y
Give your answer in the form $(Px - Q)$ where P and Q are integers.

(3)

4 The surface area of a sphere with radius r cm is increasing at a constant rate of 50π cm^2/s

Find, in cm^3, the exact volume of the sphere at the instant when the rate of increase of r is $\frac{5}{12}$ cm/s

(8)

9

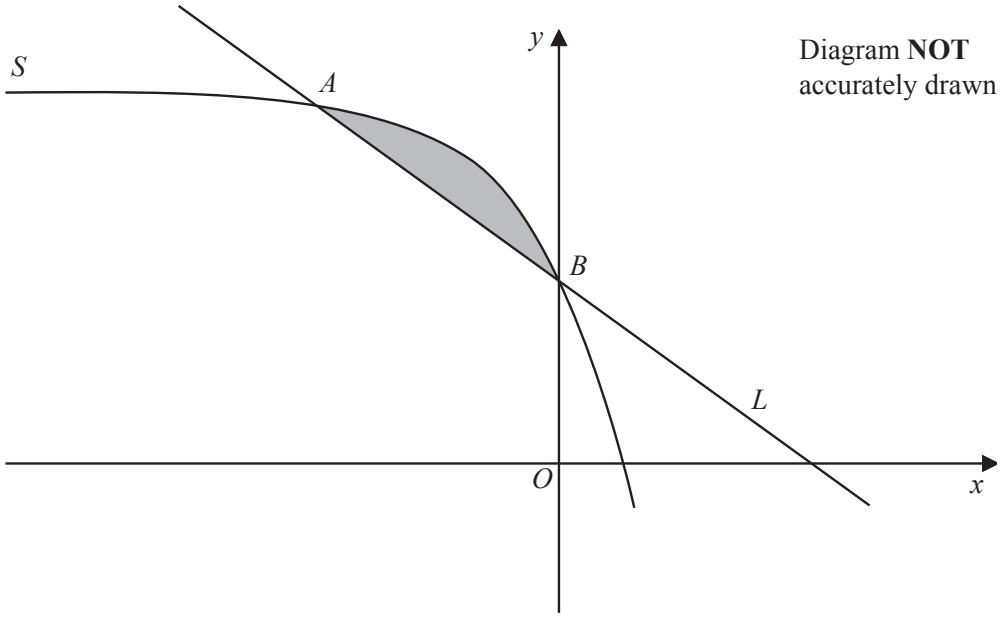


Figure 3

Figure 3 shows a sketch of part of the curve S with equation $y = -2e^{3x} + 4$ and the line L

The curve S has intersections with the line L at the points A and B with x coordinates $x = -1$ and $x = 0$ respectively.

The finite region bounded by S and L is shown shaded in Figure 3

Use calculus to find the exact area of this region.

Give your answer in the form $\frac{a + be^{-c}}{c}$ where a, b and c are integers to be found.

(8)

11

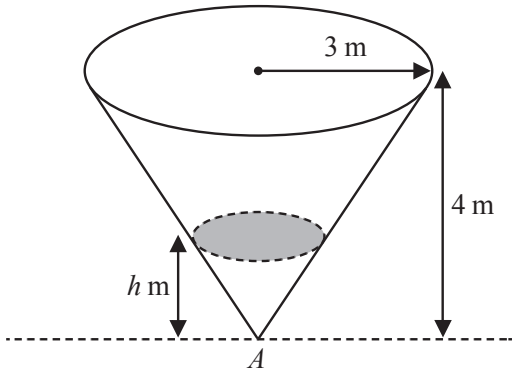


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a hollow right circular cone with radius 3 metres and height 4 metres above the vertex A

The cone is fixed with its axis of symmetry vertical.
The cone is initially empty.

Water pours into the cone at a constant rate of 9π m^3/s
At time t seconds after the water starts to pour into the cone, the height of the water is h metres above A

Find, in m/s, the rate at which the height of the water is increasing at the instant when $h = 2$

(8)

9

$$y = e^{-4t}\cos 2t$$

(a) Show that $2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4y$

(3)

Given that $\frac{d^2y}{dt^2} + M\frac{dy}{dt} + Ny = 0$ where M and N are integers

(b) find the value of M and the value of N

(5)

9-MARK QUESTIONS

11 questions in this booklet

9 The finite region R enclosed by the y-axis, the straight line with equation $y + 2x = 13$ and the curve with equation $y = x^2 - 2$, is defined for points with coordinates (x, y) with $x \geqslant 0$

The region R is rotated through 360° about the y-axis.

Use algebraic integration to find the volume of the solid generated.
Give your answer in terms of π.

(9)

10 The volume of a sphere is increasing at a constant rate of $40\,cm^3/s$.

Find the rate of increase, in cm^2/s, of the surface area of the sphere at the instant when the radius is 4 cm.

(9)

7 The length of each side of a cube S_1 is increasing at a constant rate of 0.1 m/s.

(a) Find, in m^3/s, the rate of increase of the volume of the cube S_1 when the length of each side of the cube is 2 m.

(4)

The total surface area of a different cube S_2 is increasing at a constant rate of $0.05\,m^2/s$.

(b) Find in m^3/s, the rate of increase of the volume of the cube S_2 when the length of each side of the cube is 6 m.

(5)

11 Given that

$$y = \frac{e^{4x}}{32}(8x^2 - 4x + 1)$$

(a) show that $\frac{dy}{dx} = x^2e^{4x}$

(5)

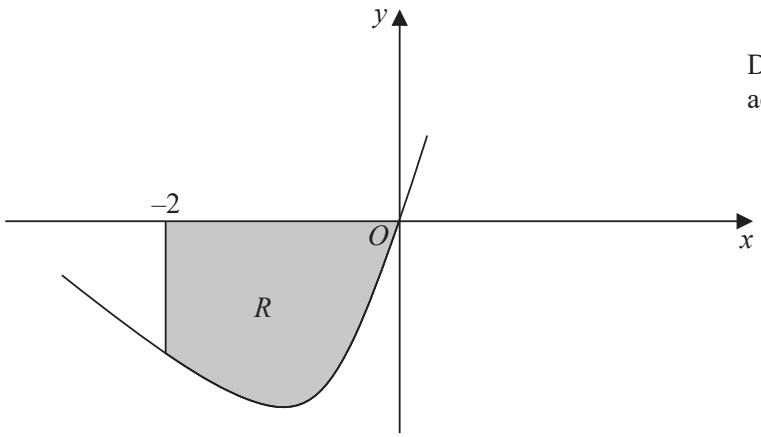


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows part of the curve C with equation $y = 3xe^{2x}$

The finite region R bounded by C, the straight line with equation $x = -2$ and the x-axis, shown shaded in Figure 2, is rotated though 360° about the x-axis.

(b) Using part (a), find the volume, to 2 significant figures, of the solid formed.

(4)

11

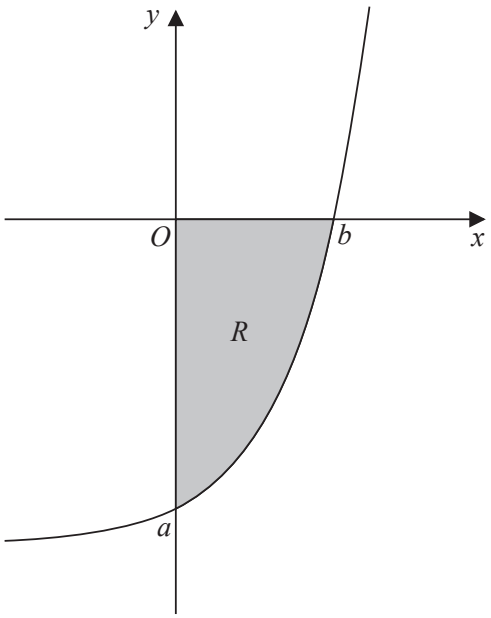


Diagram **NOT** accurately drawn

Figure 3

The finite region R, shown shaded in Figure 3, is bounded by the curve with equation $y = e^{2x} - 9$ and the coordinate axes.

The curve crosses the coordinate axes at the points with coordinates $(0, a)$ and $(b, 0)$

(a) (i) Find the value of a

(ii) Show that $b = \ln 3$

(3)

The region R is rotated through 360° about the x-axis.

(b) Use calculus to find the volume of the solid generated.

Give your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers.

(6)

5 A particle P is moving along the x-axis. At time t seconds ($t \geqslant 0$) the velocity, v m/s, of P is given by $v = 3t^2 - 23t + 30$

(a) Find the values of t when P is instantaneously at rest.

(3)

At time t seconds the acceleration of P is a m/s^2

(b) Find the range of values of t for which $a > 0$

(2)

When $t = 0$, P is at the point with coordinates $(d, 0)$

Given that, when $t = 8$, P is at the point with coordinates $(26, 0)$

(c) find the value of d

(4)

10

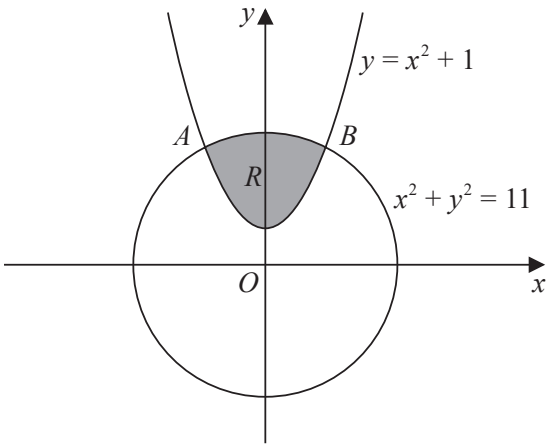


Diagram **NOT** accurately drawn

Figure 2

The region R, shown shaded in Figure 2, is bounded by the curve with equation $y = x^2 + 1$ and the curve with equation $x^2 + y^2 = 11$

The two curves intersect at the point A and at the point B.

(a) Find the x coordinate of the point A and the x coordinate of the point B.

(4)

The region R is rotated through 360° about the x-axis.

(b) Use algebraic integration to find the volume, to 2 decimal places, of the solid generated.

(5)

7 $y = e^{2x} \cos 2x$

(a) Show that

$$\frac{dy}{dx} = 2y - 2e^{2x} \sin 2x$$

(4)

(b) Hence show that

$$\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y$$

(5)

7 Two numbers x and y are such that $3x - y = 4$

$S = 5x^3 + y^2$

(a) Show that $S = 5x^3 + 9x^2 - 24x + 16$

(2)

Given that x can vary,

(b) use calculus to find the value of x for which S is a minimum, justifying that this value of x gives a minimum value of S

(5)

(c) Find the minimum value of S

(2)

5 A particle P is moving along the x-axis.
At time t seconds $(t \geqslant 0)$ the acceleration, a m/s^2, of P is given by $a = 3t - 4$

When $t = 0$, P is at rest.

(a) Find the velocity of P when $t = 4$

(3)

At time T seconds, $T > 0$, P is instantaneously at rest.

(b) Find the value of T

(2)

When $t = 0$, P is at the point with coordinates $(-10, 0)$

(c) Find the displacement of P from the origin when $t = 3$

(4)

9 Given that $f(x) = (1 + x^2)^4$

(a) show that $f'(x) = 8x(1 + x^2)^3$

(2)

The curve C has equation $y = \dfrac{\sin 3x}{(1 + x^2)^4}$

The point A on C has x coordinate $\dfrac{2\pi}{3}$

(b) Show that the gradient of the normal to C at A is

$$-\frac{1}{3}\left(1 + \left(\frac{2\pi}{3}\right)^2\right)^4$$

(7)

10-MARK QUESTIONS

15 questions in this booklet

4 $f(x) = e^{3x}\sqrt{1+2x}$

(a) Show that $f'(x) = \dfrac{2e^{3x}(2+3x)}{\sqrt{1+2x}}$

(4)

(b) Find an equation of the normal to the curve with equation $y = f(x)$ at the point on the curve where $x = 0$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

9

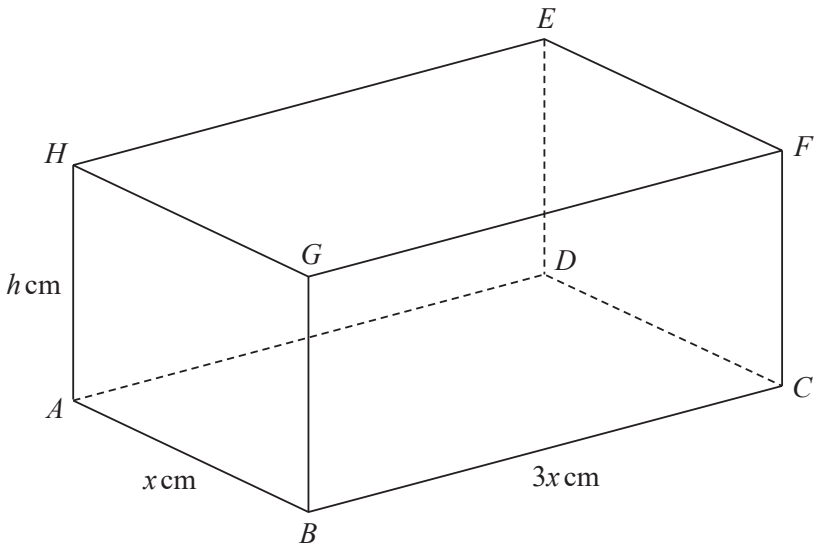


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a solid cuboid $ABCDEFGH$

$$AB = x\,\text{cm} \qquad BC = 3x\,\text{cm} \qquad AH = h\,\text{cm}$$

The volume of the cuboid is $540\,\text{cm}^3$

The total surface area of the cuboid is $S\,\text{cm}^2$

(a) Show that $S = 6x^2 + \dfrac{1440}{x}$

(4)

Given that x can vary,

(b) use calculus to find, to 3 significant figures, the value of x for which S is a minimum. Justify that this value of x gives a minimum value of S.

(5)

(c) Find, to 3 significant figures, the minimum value of S.

(1)

8 Given that $2xy + 5y = e^x$

(a) show that $\frac{dy}{dx} = \frac{y(2x+3)}{2x+5}$

(5)

(b) find the value of $\frac{dy}{dx}$ when $x = 0$

(2)

(c) find an equation of the normal to the curve with equation $2xy + 5y = e^x$ at the point where $x = 0$
Give your answer in the form $px + qy + r = 0$ where p, q and r are integers.

(3)

8

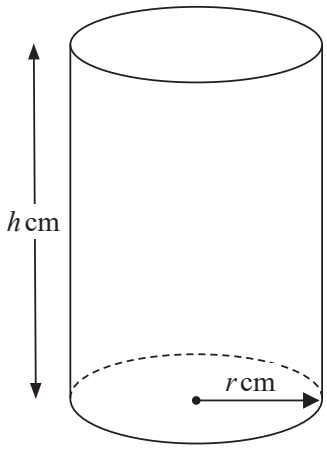


Diagram **NOT** accurately drawn

Figure 1

A solid right circular cylinder has base radius r cm and height h cm as shown in Figure 1.

The cylinder has a volume of $90\pi\,\text{cm}^3$ and a total surface area of $S\,\text{cm}^2$

(a) Show that $S = 2\pi r^2 + \dfrac{180\pi}{r}$

(3)

Given that r can vary,

(b) use calculus to find, to 3 significant figures, the value of r for which S is a minimum, justifying that this value of r gives a minimum value of S

(5)

(c) Find, to 3 significant figures, the minimum value of S

(2)

6 The curve C has equation $y = \dfrac{e^{(x^2+1)}}{x^2+1}$

(a) Show that $\dfrac{dy}{dx} = \dfrac{Kx^3e^{(x^2+1)}}{(x^2+1)^2}$ where K is a constant whose value is to be found.

(5)

(b) Find an equation of the tangent to C at the point on C where $x = -1$
Simplify your answer.

(5)

11

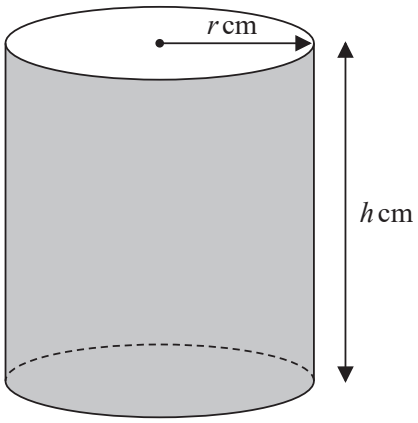


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows an open container in the shape of a cylinder with radius r cm and height h cm.

Given that the total surface area of the container is $625\pi\,\text{cm}^2$

(a) show that

$$h = \frac{625 - r^2}{2r}$$

(3)

The volume of the container is $V\,\text{cm}^3$

Given that r can vary,

(b) use calculus to find the value, to 3 significant figures, of r for which V is a maximum.
Justify that this value of r gives a maximum value of V

(6)

(c) For the value of r found in part (b), find the corresponding value, to 3 significant figures, of h

(1)

4 A particle P is moving along the x-axis. At time t seconds, $t \geqslant 0$, the velocity, v m/s, of P is given by $v = 2t^2 - 16t + 30$

(a) Find the acceleration, in m/s^2, of P when $t = 5$

(2)

P comes to instantaneous rest at the points M and N at times t_1 seconds and t_2 seconds where $t_2 > t_1$

(b) Find the exact distance MN

(8)

7 The equation of a curve is $y = \sqrt{\dfrac{e^{4x}}{2x-3}}$

When x is increased to $(x + \delta x)$, y increases to $(y + \delta y)$ where δx and δy are small.

(a) Show that $\delta y \approx \dfrac{e^{2x}(4x-7)}{(2x-3)^{\frac{3}{2}}}\,\delta x$

(7)

Given that $x = 2.5$

(b) find an estimate, to 2 significant figures, of the value of δy when the value of x increases by 0.2%

(3)

3

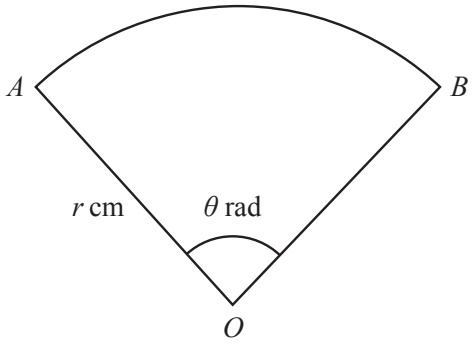


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the sector OAB of a circle with centre O.

The radius of the circle is r cm and the angle AOB is θ radians.

The area of the sector is 675 cm^2

(a) Show that the perimeter of the sector, P cm, is given by

$$P = 2r + \frac{1350}{r}$$

(3)

Given that r can vary,

(b) find, using calculus, the minimum value of P
Give your answer in the form $a\sqrt{b}$ where a is an integer and b is a prime number.

(5)

(c) Justify that the value of P you found in (b) is a minimum.

(2)

6

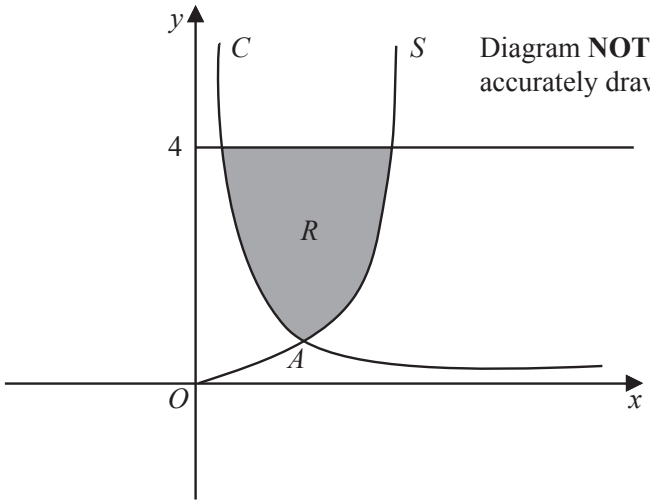


Figure 3

Figure 3 shows part of the curve C with equation $y = \frac{1}{4x}$, $x > 0$ and part of the curve S with equation $y = 2x^2, x \geqslant 0$

The curve C and the curve S intersect at the point A

(a) Find the coordinates of point A

(3)

The finite region R, shown shaded in Figure 3, bounded by the curve C, the curve S and the straight line $y = 4$ is rotated through 360° about the y-axis.

(b) Find, using algebraic integration, the exact volume of the solid formed.

(7)

8

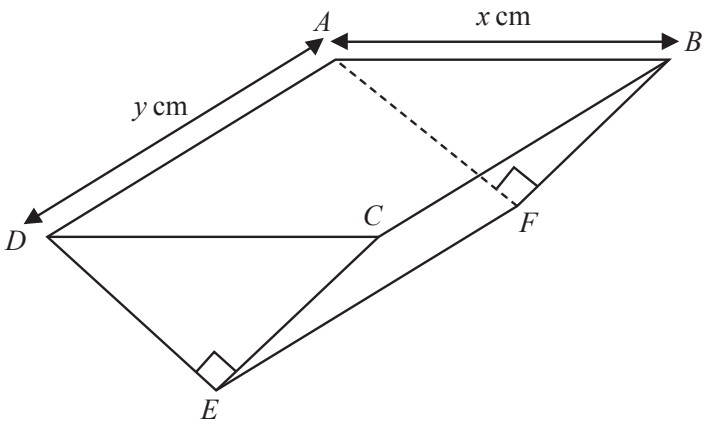


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows a solid right triangular prism $ABCDEF$

The cross section of the prism is an isosceles triangle.

- $\angle DEC = \angle AFB = 90^\circ$
- $AB = DC = x$ cm
- $AD = BC = FE = y$ cm
- $AF = BF = DE = CE$

The triangular faces of the prism are vertical and the edges AD, BC and FE are horizontal.
The volume of the prism is 3.6 cm^3
The total external surface area of the prism is S cm^2

(a) Show that S satisfies the equation

$$S = \frac{x^2}{2} + \frac{72\left(\sqrt{2}+1\right)}{5x}$$

(4)

Given that x can vary,

(b) use calculus, to find to 3 significant figures, the value of x for which S is a minimum.
Justify that this value of x gives a minimum value of S

(4)

(c) Hence find, to 2 significant figures, the minimum value of S

(2)

5 A particle P is moving along a straight line.

At time t seconds $(t \geqslant 0)$, the velocity, v m/s, of P is given by

$$v = 3t^2 - 16t + 5$$

(a) Find the values of t when P is instantaneously at rest.

(3)

At time t seconds the acceleration of P is a m/s^2

(b) Find the range of values of t for which $a > 0$

(2)

(c) Find the distance, in m, that P travels in the interval $1 \leqslant t \leqslant 4$

(5)

9

Diagram **NOT** accurately drawn

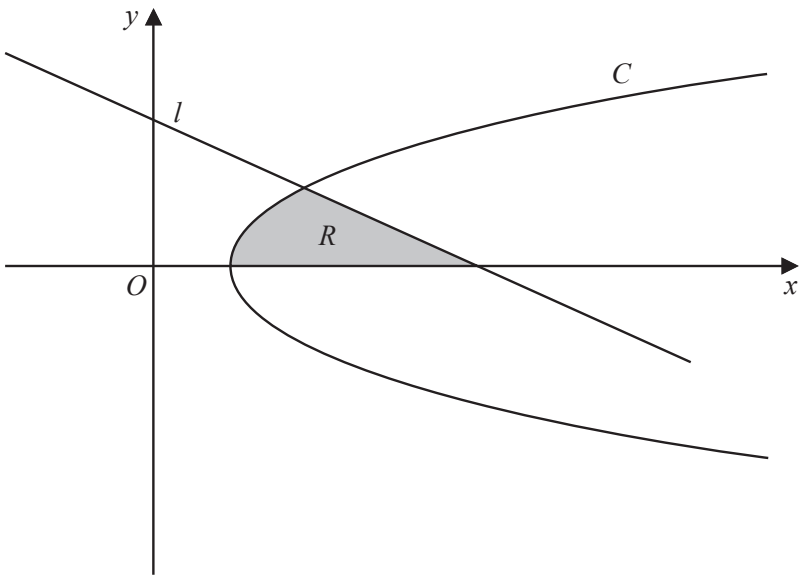


Figure 3

Figure 3 shows part of the curve C with equation $y^2 = x - 1$ and part of the line l with equation $2y + x - 4 = 0$

The region R, bounded by the x-axis, the curve C and the line l, is rotated through 360° about the x-axis.

Using algebraic integration, find the exact value of the volume of the solid generated.

(10)

3 A particle P is moving along a straight line, from a fixed origin O

At time t seconds $(t \geqslant 0)$, the velocity, v m/s, of P is given by

$$v = 2t^2 - 19t + 35$$

At time t seconds the acceleration of P is a m/s^2

(a) Find the value of t for which $a = 0$

(3)

P is instantaneously at rest at time T_1 seconds and at time T_2 seconds where $T_2 > T_1$

(b) Find the value of T_1 and the value of T_2

(3)

(c) Find the exact distance, in metres, that P travels between the times T_1 and T_2
Show your working clearly.

(4)

6

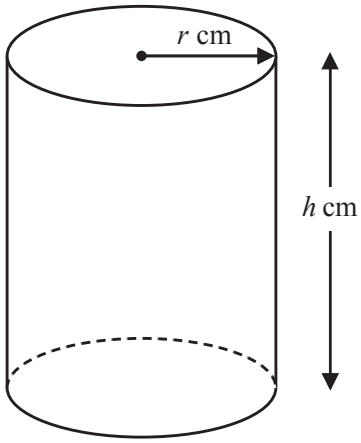


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a solid right circular cylinder with radius r cm and height h cm

The total surface area of the cylinder is 700π cm^2

The volume of the cylinder is V cm^3

(a) Show that $V = \pi r\left(350 - r^2\right)$

(4)

Given that r can vary and using calculus,

(b) find, in cm to 3 significant figures, the value of r for which V is a maximum. Justify that this value of r gives a maximum value of V

(5)

(c) Find, to 3 significant figures, the height h cm for which V is a maximum.

(1)

11-MARK QUESTIONS

14 questions in this booklet

6

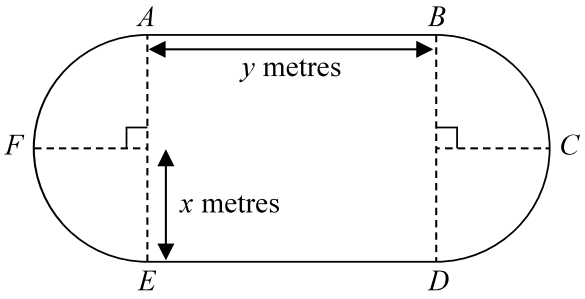


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a lawn $ABCDEF$, where $ABDE$ is a rectangle of length y metres and width $2x$ metres. Each end of the lawn is a semicircle of radius x metres. The lawn has perimeter 90 m and area S m^2

(a) Show that S can be written in the form

$$S = kx - \pi x^2$$

where k is a constant.

State the value of k.

(4)

(b) Use calculus to find, to 4 significant figures, the value of x for which S is a maximum, justifying that this value of x gives a maximum value of S.

(5)

(c) Find, to the nearest whole number, the maximum value of S.

(2)

5

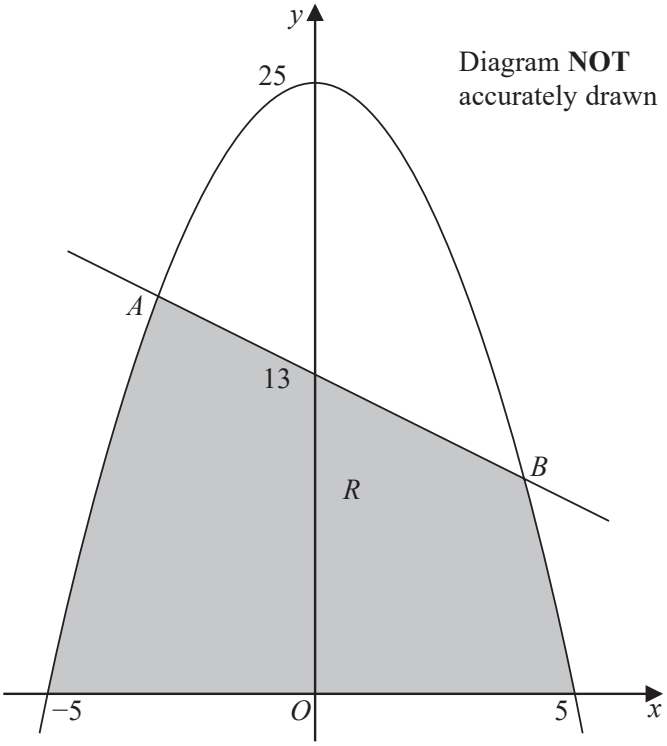


Figure 2

Figure 2 shows part of the curve with equation $y = 25 - x^2$ and part of the line with equation $y + x = 13$

The curve and the line intersect at the points A and B.

(a) Use algebra to find the coordinates of A and the coordinates of B.

(4)

The region R, shown shaded in Figure 2, is bounded by the curve, the straight line and the x-axis.

(b) Use algebraic integration to find the area of R.

(7)

11

Diagram **NOT** accurately drawn

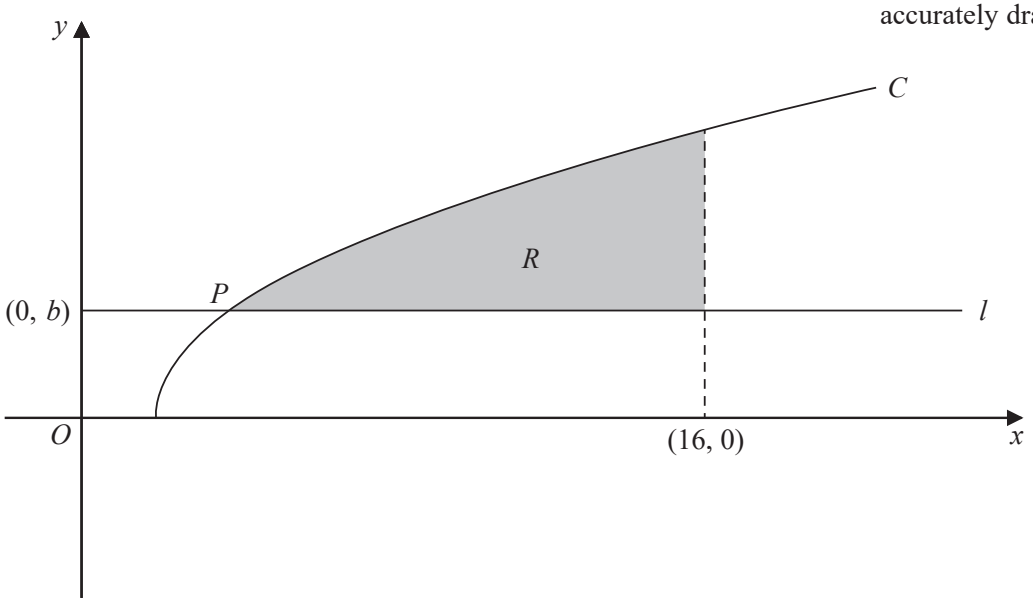


Figure 2

Figure 2 shows part of the curve C with equation $y = \sqrt{x - 2}$
Figure 2 also shows the straight line l with equation $y = b$ for $x > 0$ where $b > 0$

Given that C and l intersect at the point P with coordinates (a, b), where $2 < a < 16$

(a) show that $b^2 = a - 2$

(2)

The finite region R bounded by C, the straight line with equation $x = 16$ and l, shown shaded in Figure 2, is rotated through 360° about the x-axis to form a solid S.

Given that the volume of the solid formed is 50π

(b) use algebraic integration to find the value of a and the value of b.

(9)

7 The curve C has equation $y = \dfrac{x}{x^2 + 4}$

(a) Using calculus, find the coordinates of the stationary points on C.

(5)

(b) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{2x(x^2 - 12)}{(x^2 + 4)^3}$

(4)

(c) Hence, or otherwise, determine the nature of each of these stationary points.

(2)

4 The curve C has equation $y = 2x^2 + px + q$ where p and q are integers.

The curve C has a stationary point at $(3, -5)$

(a) Show that $p = -12$ and find the value of q

(4)

(b) State, giving a reason, the nature of the stationary point.

(1)

(c) Find an equation of the normal to C at the point on C where $x = 1$

Give your answer in the form $ax + by + c = 0$

(6)

7

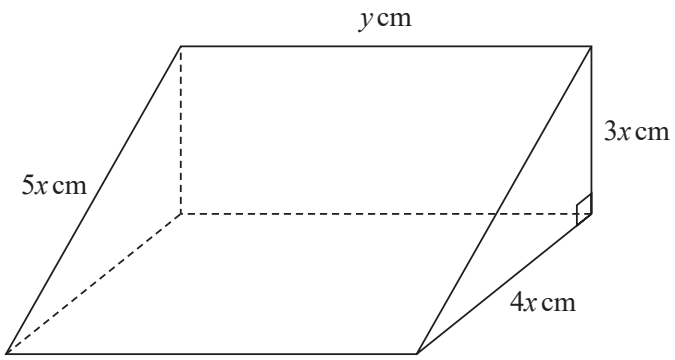


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a block of wood in the shape of a right triangular prism.

The cross section of the prism is a right-angled triangle with sides of length $3x$ cm, $4x$ cm and $5x$ cm.

The length of the prism is y cm.

The total surface area of the five faces of the prism is $144\,\text{cm}^2$

The volume of the prism is $V\,\text{cm}^3$

(a) Show that

$$V = 72x - 6x^3$$

(5)

Given that x can vary,

(b) use calculus to find the value of x for which V is a maximum, justifying that this value gives a maximum value of V

(4)

(c) Find the maximum value of V

(2)

6 The curve C has equation

$$y = 4\sqrt{x}$$

The point A on C has coordinates (9, 12)

The tangent to C at the point A meets the x-axis at the point T

(a) Find the coordinates of T

(5)

The normal to C at the point A meets the x-axis at the point N

(b) Find the coordinates of N

(4)

(c) Calculate the area of triangle ATN

(2)

10 Using suitable results for $\sin(A + B)$ and $\sin(A - B)$ from the formulae page,

(a) show that $2\sin 4x \cos x = \sin 5x + \sin 3x$

(3)

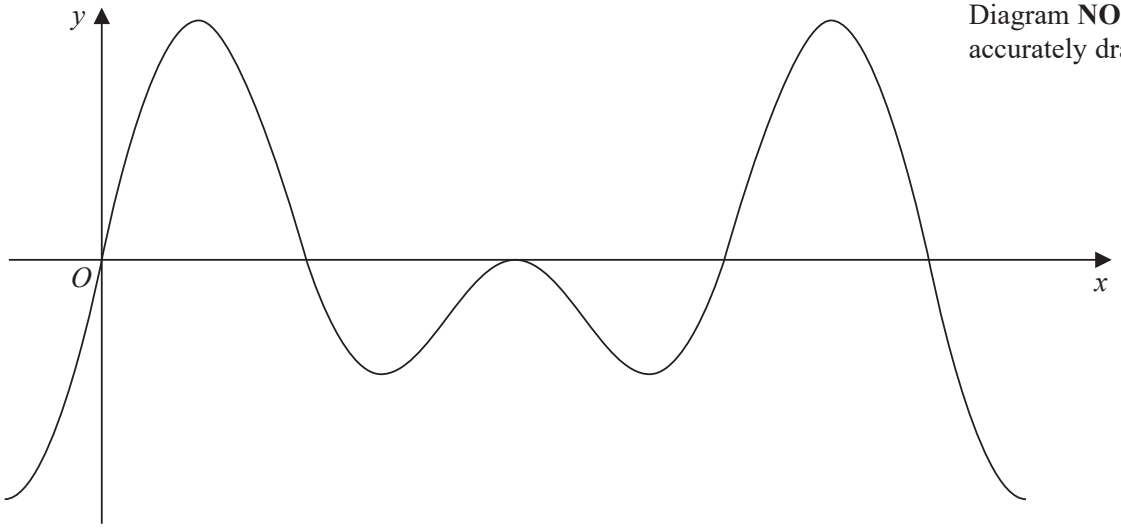


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows part of a sketch of the curve $y = 6\sin 4x \cos x$

(b) Using calculus, find the total area bounded by the curve and the x-axis between $x = 0$ and $x = \frac{\pi}{2}$

Give your answer to 3 significant figures.

(8)

8

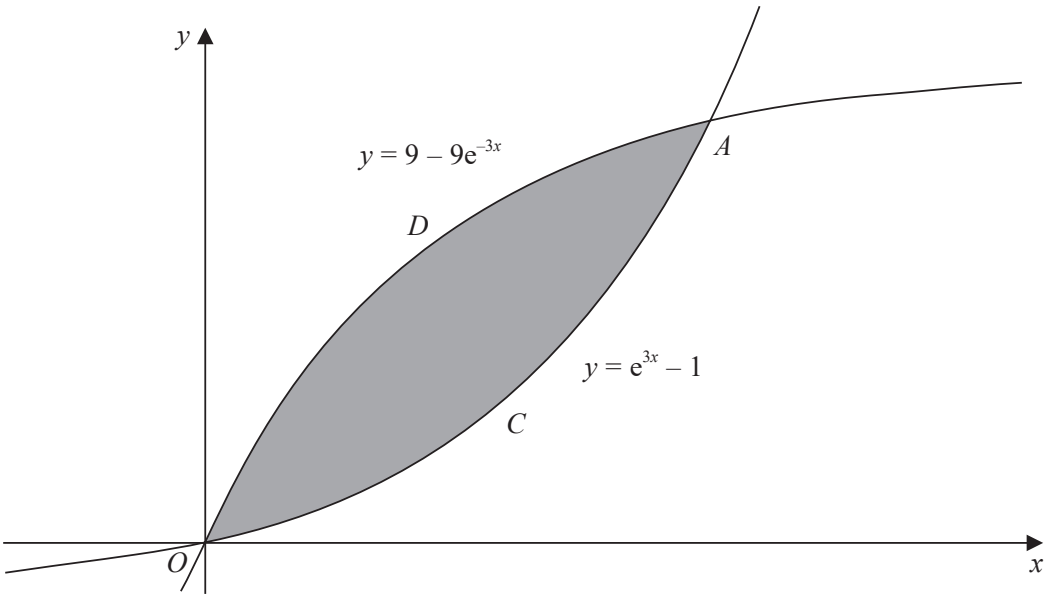


Figure 2

Figure 2 shows part of the curve C with equation $y = e^{3x} - 1$ and part of the curve D with equation $y = 9 - 9e^{-3x}$

The curves intersect at the origin O and the point A.

(a) (i) Show that the x coordinate of the point A satisfies the equation

$$(e^{3x})^2 - 10e^{3x} + 9 = 0$$

(ii) Hence, show that the x coordinate of the point A is $\frac{1}{3}\ln 9$

(5)

The finite region bounded by C and by D is shown shaded in Figure 2.

(b) Use calculus to find the exact area of this region.

(6)

7 A particle P is moving along the x-axis. At time t seconds $(t \geqslant 0)$ the velocity of P is v m/s where

$$v = t^2 - 10t + 28$$

(a) Find the velocity of P when $t = 1$

(1)

Given that the distance of P from the origin is 24 m when $t = 3$

(b) find the distance of P from the origin when $t = 5$

(5)

(c) Find the acceleration of P when $t = 9$

(2)

(d) (i) Show that there are no values of t for which P is instantaneously at rest.

(ii) Find the least magnitude of the velocity of P

(3)

5 A solid cuboid has width x cm, length $4x$ cm and height h cm.
The volume of the cuboid is 75 cm^3 and the surface area of the cuboid is S cm^2

(a) Show that $S = 8x^2 + \frac{375}{2x}$

(4)

Given that x can vary, using calculus,

(b) (i) find to 3 significant figures, the value of x for which S is a minimum,

(ii) justify that this value of x gives a minimum value of S

(5)

(c) Find, to 3 significant figures, the minimum value of S

(2)

6

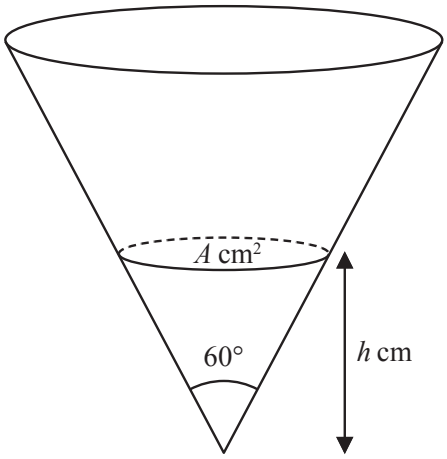


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows a container in the shape of a right circular cone.
The container is fixed with its axis of symmetry vertical.
The vertical angle of the container is 60° as shown in the diagram.

At time t seconds, $t > 0$, the height of oil in the container is h cm and the volume of oil in the container is V cm^3

(a) Show that $V = \frac{1}{9}\pi h^3$

(3)

At time t seconds the surface area of oil in the container is A cm^2, as shown in Figure 4

Oil is dripping out of the bottom of the container at a constant rate of 4 cm^3/s.

(b) Find the exact rate of change, in cm^2/s, of the surface area of oil in the container when $h = 24$

(8)

7

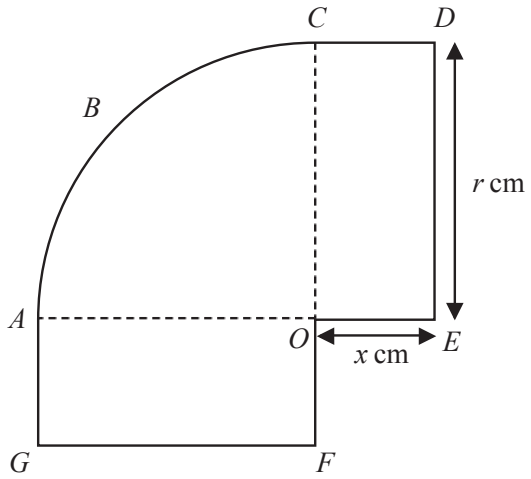


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a shape *ABCDEOFG*

ABCO is a quarter circle with radius r cm
CDEO and *AOFG* are congruent rectangles of length r cm and width x cm

The total area of the shape is $100\,\text{cm}^2$
The perimeter of the shape is P cm

(a) Show that $P = \dfrac{200}{r} + 2r$

(4)

(b) Use calculus to find the value of r for which P is a minimum, justifying that this value of r gives a minimum value of P

(5)

(c) Find the minimum value of P

(2)

7

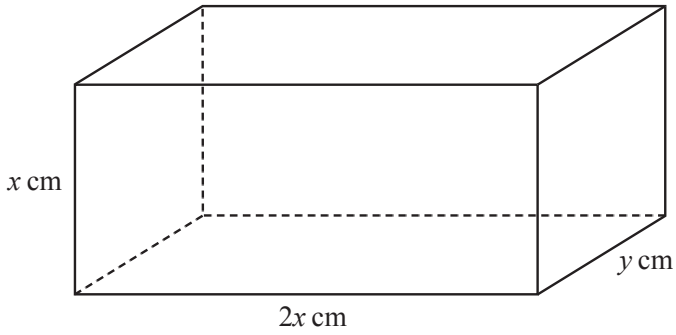


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a solid cuboid with sides of length x cm, $2x$ cm and y cm

The volume of the cuboid is 72 cm^3

The total surface area of the cuboid is A cm^2

(a) Show that $A = \frac{216}{x} + 4x^2$

(4)

(b) Use calculus to find the minimum value of A
Justify that your value is a minimum value of A

(7)

12-MARK QUESTIONS

5 questions in this booklet

10 The equation of a curve C is $y = f(x)$ where $f'(x) = 3x^2 - 4x - p$ and $p \neq 0$

The points with coordinates $(2, 0)$ and $(-1, 9)$ lie on C.

(a) Show that C has equation $y = x^3 - 2x^2 - 4x + 8$

(6)

The straight line l has equation $y = 8 - 4x$

(b) Use algebraic integration to find the exact area of the finite region bounded by C and l.

(6)

10

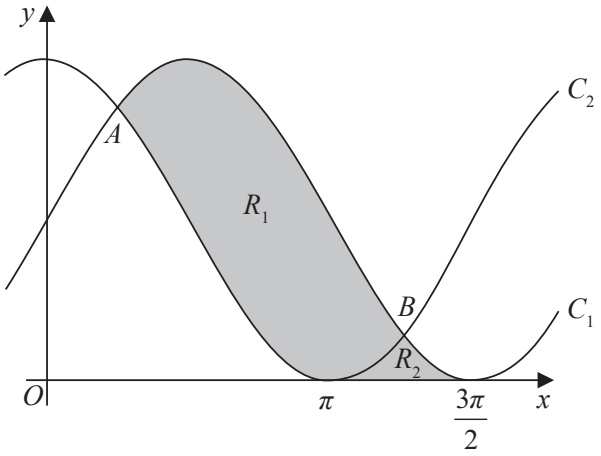


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows part of the curve C_1 with equation $y = \sin x + 1$ and part of the curve C_2 with equation $y = \cos x + 1$

As shown in Figure 2, C_1 and C_2 intersect at the point A and at the point B

(a) Find the exact value of the x coordinate of A and the exact value of the x coordinate of B

(3)

The shaded finite region R_1 shown in Figure 2 is bounded by C_1 and C_2

The shaded finite region R_2 shown in Figure 2 is bounded by the x-axis, C_1 and C_2

(b) Use calculus to find the ratio

$$\text{area of } R_1 : \text{area of } R_2$$

Give your answer in the form $a : \left(\frac{\pi\sqrt{2}}{b} - c\right)$ where a, b and c are integers.

(9)

8

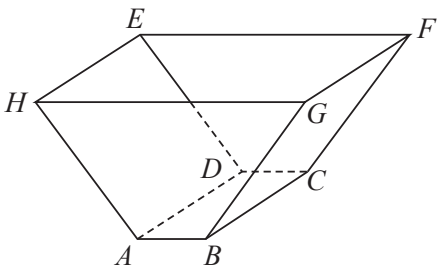


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a waste paper basket in the shape of a right prism with 5 faces and a cross section that is a trapezium. The top, *EFGH*, of the waste paper basket is open.

The base of the prism *ABCD* is a rectangle with

$$AB = DC = 2x\,\text{cm} \text{ and } AD = BC = h\,\text{cm}$$

The cross sections *HGBA* and *EFCD* are such that

$$EF = HG = 8x\,\text{cm} \text{ and } AH = BG = CF = DE = 5x\,\text{cm}$$

The top, *EFGH*, of the waste paper basket is such that

$$EH = FG = h\,\text{cm}$$

The volume of the waste paper basket is $2250\,\text{cm}^3$

The total surface area of the 5 faces of the waste paper basket is $S\,\text{cm}^2$

(a) Show that $S = 40x^2 + \dfrac{1350}{x}$

(5)

Given that x can vary,

(b) use calculus, to find, to 3 significant figures, the value of x for which S is a minimum.
Justify that this value of x gives a minimum value of S

(5)

(c) Find, to 3 significant figures, the minimum value of S

(2)

11

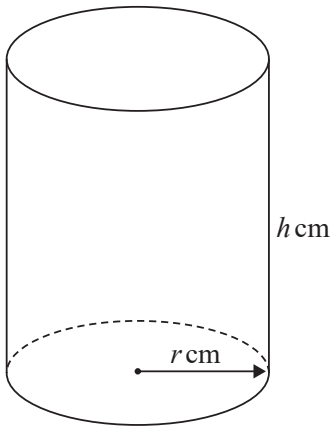


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a solid metal right circular cylinder of radius r cm and height h cm.

The total surface area of the cylinder is $600\,\text{cm}^2$

The volume of the cylinder is $V\,\text{cm}^3$

(a) Show that $V = 300r - \pi r^3$

(4)

Given that r can vary,

(b) (i) use calculus to show that the exact value of r for which V is a maximum is

$$r = \sqrt{\frac{100}{\pi}}$$

(ii) justify that this value of r gives a maximum value of V

(5)

The cylinder is melted down and reformed into a sphere of radius p cm.

(c) Find, to one decimal place, the greatest possible value of p

(3)

11

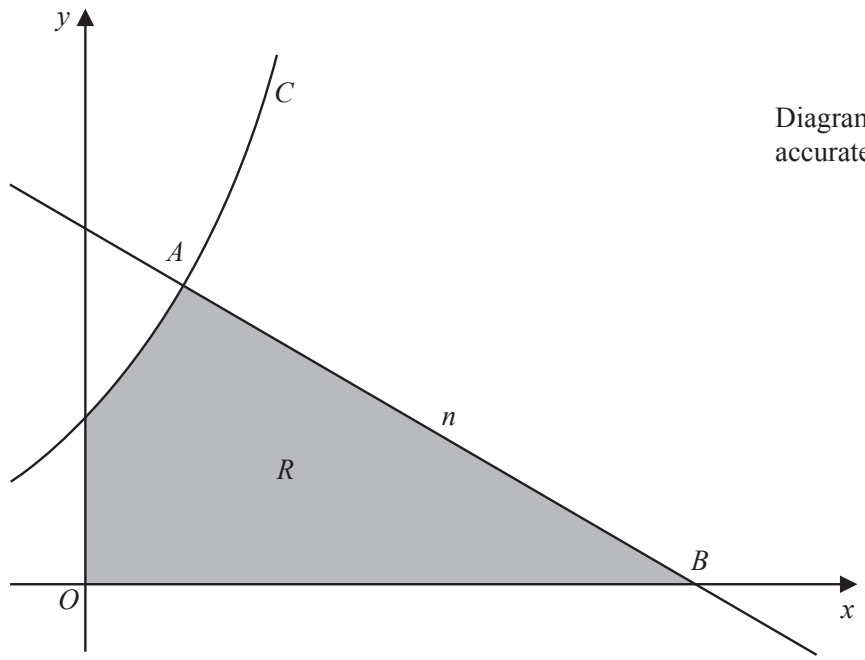


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows part of a curve C with equation $y = 3e^{\frac{x}{3}}$
The point A with coordinates $(2, p)$ lies on C

(a) Write down the exact value of p

(1)

The straight line n, shown on Figure 4, is the normal to C at A and crosses the x-axis at point B

The region R, shown shaded in Figure 4, is bounded by C, n, the x-axis and the y-axis.

(b) Use algebraic integration to find the area of R

Give your answer in the form $\frac{W}{V}e^2 + We^{\frac{2}{3}} - W$

where W and V are integers to be found.

(11)

13-MARK QUESTIONS

4 questions in this booklet

11

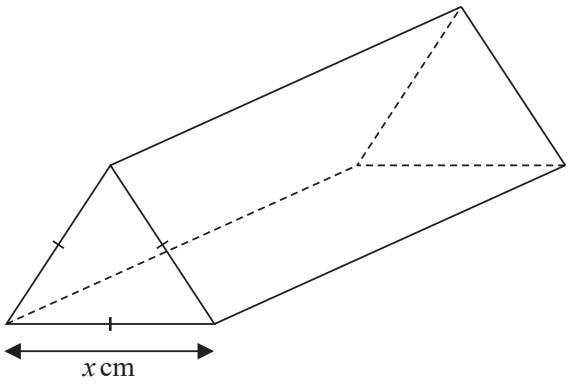


Diagram **NOT** accurately drawn

Figure 4

A company manufactures chocolate bars that are inside packaging that is in the shape of a right triangular prism.

The cross section of the prism is an equilateral triangle with sides of length x cm, as shown in Figure 4.

The volume of the prism is $72\,\text{cm}^3$

The total surface area of the prism is $S\,\text{cm}^2$

(a) Show that

$$S = \frac{\sqrt{3}x^2}{2} + \frac{288\sqrt{3}}{x}$$

(6)

Given that x can vary,

(b) use calculus to find, to 4 significant figures, the value of x for which S is a minimum, justifying that this value gives a minimum value of S.

(5)

(c) Find, to 3 significant figures, the minimum value of S.

(2)

8

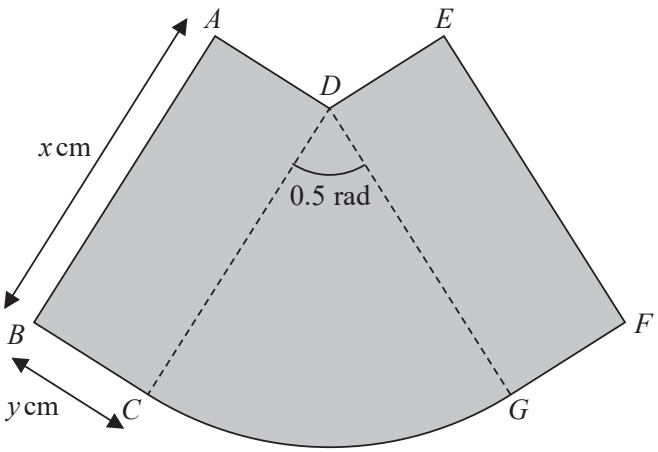


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a badge, shown shaded, made from two identical rectangles, *ABCD* and *DEFG*, and a sector *DCG* of a circle with centre *D*.

Each rectangle measures x cm by y cm.
The radius of the sector is x cm and the angle *CDG* is 0.5 radians.

The area of the badge is 50 cm^2
The perimeter of the badge is P cm.

(a) Show that

$$P = 2x + \frac{100}{x}$$

(5)

Given that x can vary,

(b) use calculus, to find the exact value of x for which P is a minimum.
Justify that this value of x gives a minimum value for P

(6)

(c) Find the minimum value of P
Give your answer in the form $k\sqrt{2}$, where k is an integer to be found.

(2)

7

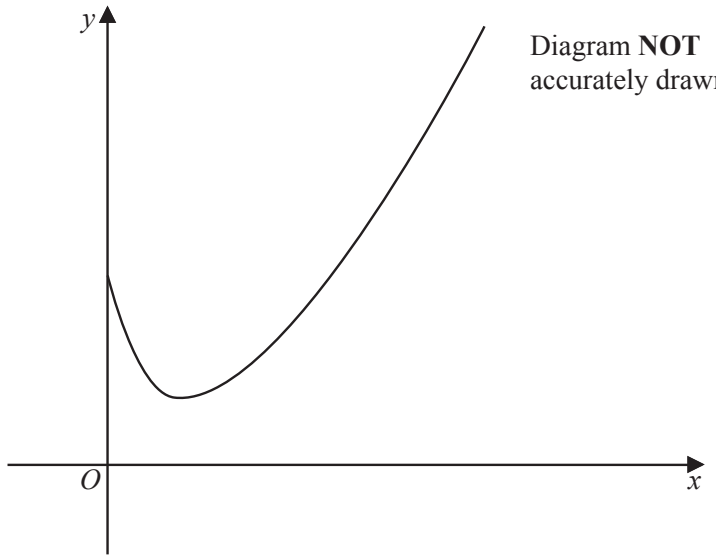


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{x^2}{4} - 3\sqrt{x} + 8$$

The point P lies on C and has coordinates $(4, a)$

(a) Show that $a = 6$

(1)

The line L is the normal to C at the point P

(b) Show that an equation of L is $5y + 4x - 46 = 0$

(6)

The finite region R is bounded by the curve C, the line L, the x-axis and the line with equation $x = 1$

(c) Use calculus to find the exact area of R

(6)

9

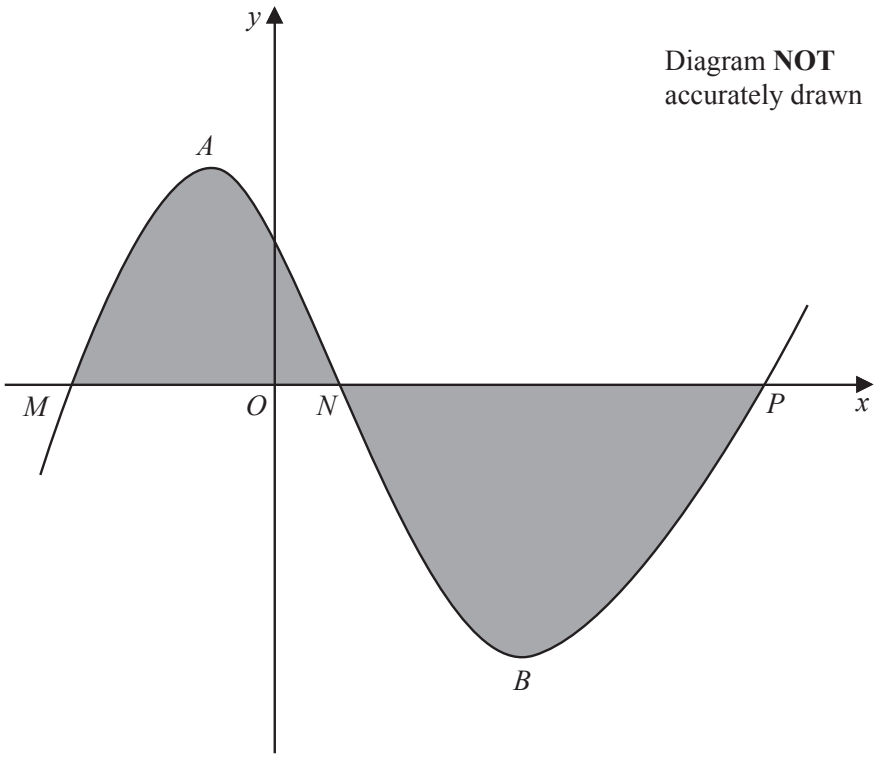


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^3 + ax^2 + bx + c$$

The curve C has a maximum at the point A with coordinates $\left(-\frac{1}{3}, \frac{100}{27}\right)$ and a minimum at the point B with coordinates $(2, -9)$

Given that a, b and c are integers

(a) show that $a = -5, b = -4$ and $c = 3$

(5)

(b) (i) Show that $(x+1)$ is a factor of $\mathrm{f}(x)$

(1)

(ii) Hence, or otherwise, use algebra to factorise $\mathrm{f}(x)$ completely.

(3)

The curve C crosses the x-axis at the points M, N and P
The finite regions shown shaded in Figure 4 are bounded by the curve C and parts of the x-axis from M to N and from N to P

(c) Use algebraic integration to determine the total area of the shaded regions. Give your answer as an exact fraction.

(4)

14-MARK QUESTIONS

3 questions in this booklet

9 The curve C, with equation $y = f(x)$, passes through the point with coordinates $\left(-2, -\frac{28}{3}\right)$

Given that $f'(x) = x^3 - x^2 - 4x + 4$

(a) show that C passes through the origin.

(4)

(b) (i) Show that C has a minimum point at $x = 2$ and a maximum point at $x = 1$

(ii) Find the exact value of the y coordinate at each of these points.

(7)

The curve has another turning point at A.

(c) (i) Find the coordinates of A.

(ii) Determine the nature of this turning point.

(3)

10 $f(x) = 32x^3 - 33x + 1$

(a) Show that $f(1) = 0$

(1)

(b) Hence using an algebraic method solve $f(x) = 0$

(4)

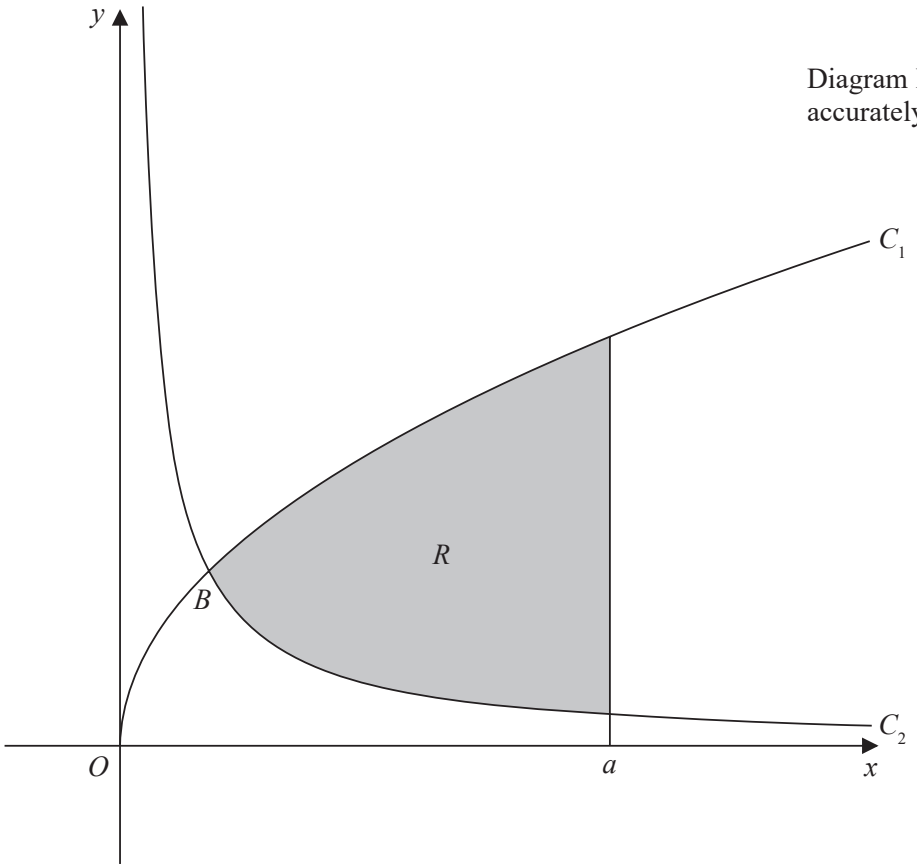


Diagram **NOT** accurately drawn

Figure 4

The region R, shown shaded in Figure 4, is bounded by the curve C_1 with equation $y = \sqrt{x}$, by the curve C_2 with equation $y = \frac{1}{8x}$ and by the line with equation $x = a$

The curves C_1 and C_2 intersect at the point B, with x coordinate p, where $p < a$

(c) Find the value of p.

(2)

The region R is rotated through 360° about the x-axis to generate a solid with volume $\frac{27\pi}{64}$

(d) Use algebraic integration to find the value of a.

(7)

11

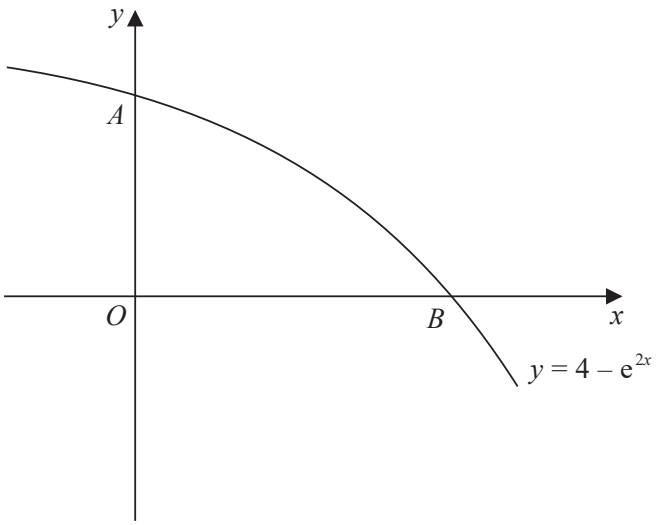


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows part of the curve C with equation $y = 4 - e^{2x}$
The curve C crosses the y-axis at the point A and the x-axis at the point B.

(a) (i) Write down the y coordinate of point A.

(ii) Show that the x coordinate of B is $x = \ln 2$

(3)

The line l is the normal to C at the point B.

(b) Find an equation for l, giving your answer in the form $y = mx + c$

(4)

The finite region R is bounded by C, l and the y-axis.

(c) Using calculus, find the area of R.
Give your answer to one decimal place.

(7)

15-MARK QUESTIONS

7 questions in this booklet

8 The curve C_1 has equation $y = 5e^{-2x} + 4$

The curve C_2 has equation $y = e^{2x}$

The curves C_1 and C_2 intersect at the point A.

(a) Find the exact coordinates of A.

(4)

The tangent at A to C_1 intersects the x-axis at the point B.

(b) Show that the x coordinate of B is $\frac{1}{2}(5 + \ln 5)$

(5)

The tangent at A to C_2 intersects the x-axis at the point D.

(c) Find the area of ΔABD.

(6)

6

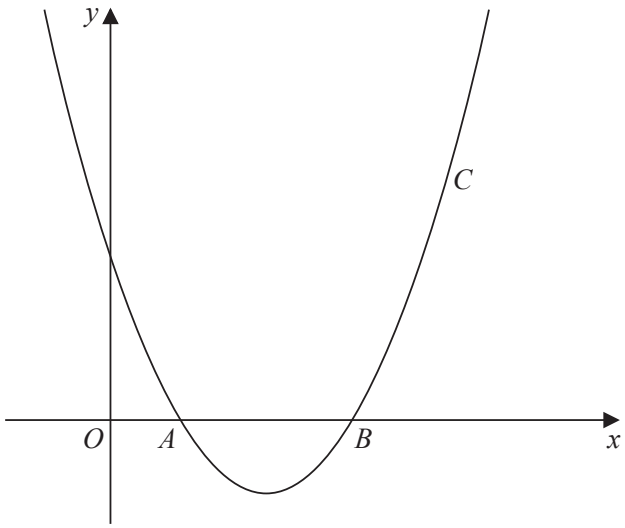


Diagram **NOT** accurately drawn

Figure 3

The curve C with equation $y = x^2 - 5x + 4$ crosses the x-axis at the points A and B, as shown in Figure 3

(a) Find the coordinates of A and the coordinates of B.

(3)

The tangent to C at A meets the tangent to C at B at the point T.

(b) Find the coordinates of T.

(6)

The normal to C at A meets the normal to C at B at the point N.

(c) Find the coordinates of N.

(3)

(d) Find the area of the quadrilateral $ATBN$.

(3)

10

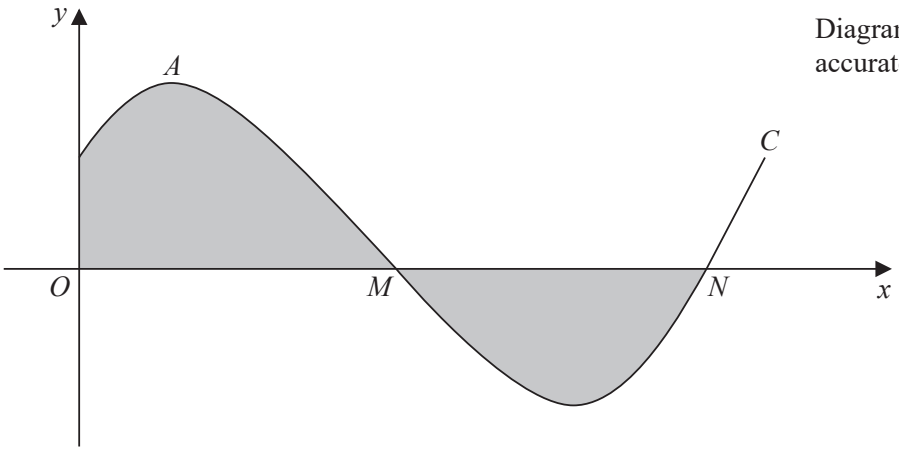


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows the curve C with equation $y = \frac{1}{2} + \sin 3x$ where $0 \leqslant x < \frac{2\pi}{3}$

The curve C crosses the x-axis at the points M and N

(a) Show that the coordinates of M are $\left(\frac{7\pi}{18}, 0\right)$ and find the coordinates of N

(3)

The curve C has a maximum at the point A

(b) Find the coordinates of A

(4)

(c) Find an equation of the tangent to C at M

Give your answer in the form $ay + b\sqrt{3}x - c\sqrt{3}\pi = 0$ where a, b and c are integers to be found.

(4)

The finite region, shown shaded in Figure 4, is bounded by the curve C, the y-axis and the part of the x-axis from O to N

(d) Use algebraic integration to find, to 3 significant figures, the total area of the shaded region.

(4)

7 The point with coordinates (4, –104) lies on the curve C with equation $y = f(x)$

Given that $f'(x) = 4x^3 - 12x^2 - 19x + 12$

(a) (i) show that C passes through the origin,

(4)

(ii) show that C has a maximum at the point on the curve where $x = 0.5$

(3)

The curve C has another turning point at A and another turning point at B.

Given that the x coordinate of A is negative,

(b) (i) find the coordinates of A and the coordinates of B,

(5)

(ii) determine the nature of these turning points.

(3)

8

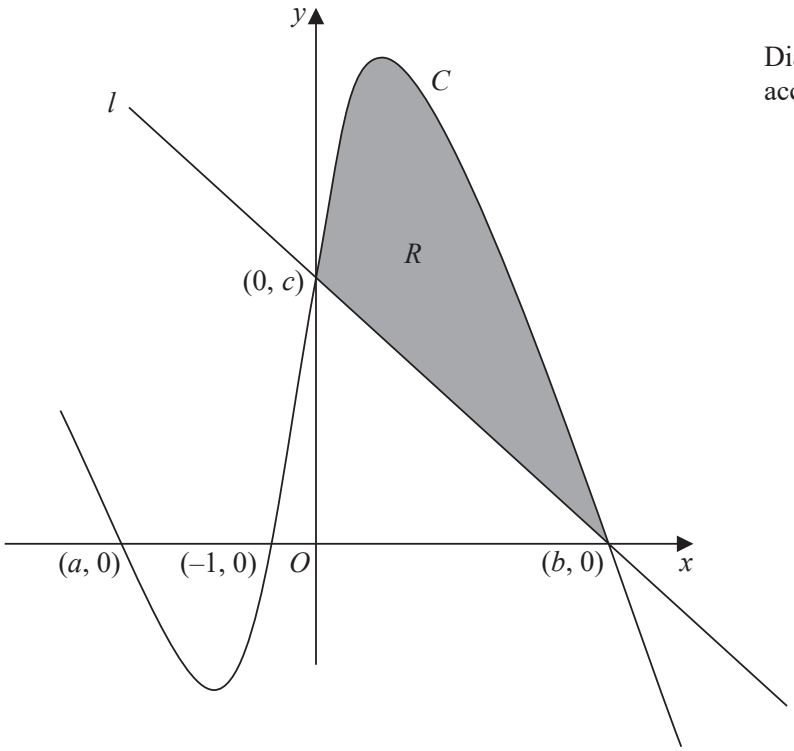


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows part of the curve C with equation $y = f(x)$

The curve C passes through the points with coordinates

$$(a, 0), (-1, 0), (b, 0) \text{ and } (0, c)$$

Given that $f'(x) = 17 + 2x - 3x^2$

(a) show that the equation of C is $y = 15 + 17x + x^2 - x^3$

(4)

(b) Find the value of a, the value of b and the value of c

(6)

The straight line l intersects C at the points with coordinates $(b, 0)$ and $(0, c)$

The region R, shown shaded in Figure 1, is bounded by l and C

(c) Use algebraic integration to find the exact area of region R

(5)

5

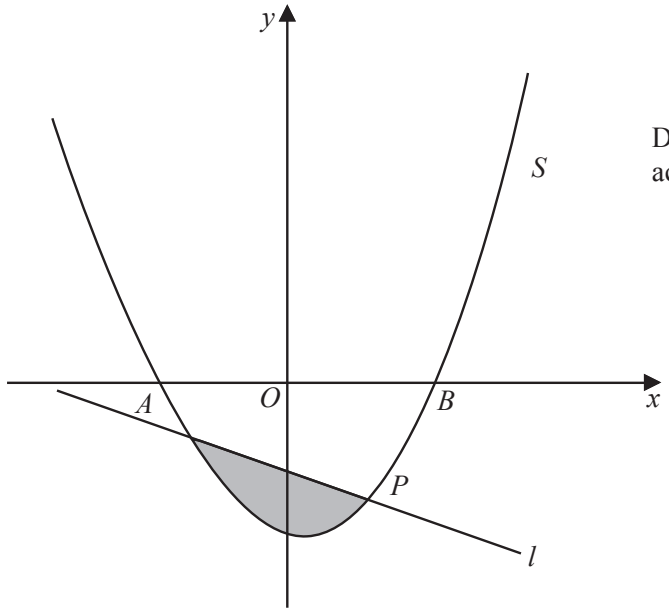


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows part of the curve S with equation $y = px^2 + qx + r$ where p, q and r are constants.

The points A, B and P with coordinates $(-2, 0)$, $(6, 0)$ and $(4, -6)$ respectively lie on S

(a) Show that an equation of S is $y = \frac{x^2}{2} - 2x - 6$

(3)

The line l is the normal to S at the point P

(b) Show that an equation of l is $2y + x + 8 = 0$

(5)

The finite region shown shaded in Figure 1 is bounded by S and l

(c) Use algebraic integration to find the exact area of the shaded region.

(7)

8 $\mathrm{f}(x) = 2x^2 - 4x + 9$

The curve C has equation $y = \mathrm{f}(x)$
The line l with equation $y - 8x + 9 = 0$ is a tangent to C
The line k is perpendicular to l and is also the tangent to C at the point where $x = p$

(a) Find the value of p

(5)

(b) Show that

(i) $(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$

(ii) $\alpha^4 + \beta^4 = \left((\alpha + \beta)^2 - 2\alpha\beta\right)^2 - 2(\alpha\beta)^2$

(2)

The quadratic equation $\mathrm{f}(x) = 0$ has roots α and β
Without solving the equation and using your results from part (b)

(c) form a quadratic equation with integer coefficients, that has roots

$\alpha^3 - \beta$ and $\beta^3 - \alpha$

(8)

16-MARK QUESTIONS

1 question in this booklet

9 $f(x) = 3x^4 + 4x^3 - 36x^2 + 64$

Given that $f(x)$ can be written in the form $(x-2)^2(ax^2 + bx + c)$

(a) find the value of a, the value of b and the value of c

(4)

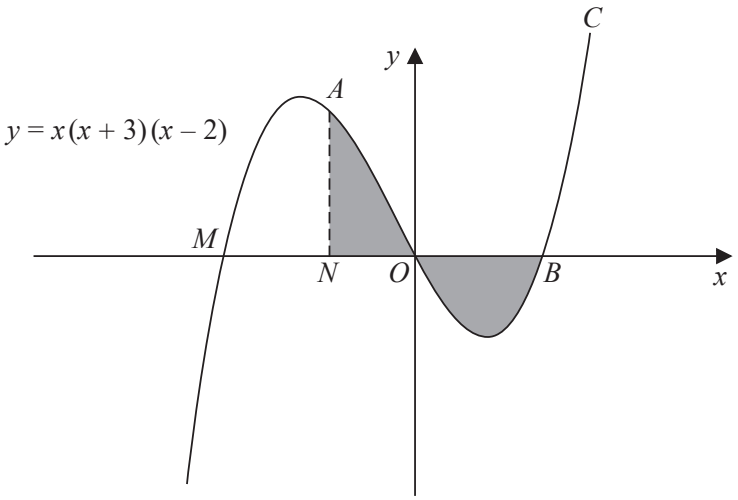


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a sketch of part of the curve C with equation $y = x(x+3)(x-2)$

The curve C crosses the x-axis at the point M, the origin and the point B.
The point N lies on the x-axis between M and O.
The point A lies on C such that AN is parallel to the y-axis.

The area of the shaded region bounded by the curve and OB is numerically equal to the area of the shaded region bounded by the curve, ON and NA.

Given that the coordinates of N are $(n, 0)$,

(b) use algebraic integration to show that n satisfies the equation

$$(x-2)^2(3x^2 + 16x + 16) = 0$$

(7)

(c) Hence find the exact coordinates of A.

(5)

17-MARK QUESTIONS

1 question in this booklet

11 The curve C has equation $3y = x^2 + 2$

The point P lies on C and has x coordinate 4

The line k is the tangent to C at P.

(a) Find an equation for k, giving your answer in the form $ay = bx + c$ where a, b and c are integers.

(6)

The line l is the normal to C at P.

(b) Find an equation for l, giving your answer in the form $dy = ex + f$ where d, e and f are integers.

(2)

(c) Find the area of the triangle bounded by the line k, the line l and the x-axis.

(3)

The finite region bounded by C, the line l, the x-axis and the y-axis is rotated through 360° about the x-axis.

(d) Use algebraic integration to find, to the nearest whole number, the volume of the solid generated.

(6)

MARK SCHEMES

official Pearson Edexcel mark schemes

Question Number	Scheme	Marks
1	$36xe^{3x^2}\cos 2x - 12e^{3x^2}\sin 2x$	M1A1A1 (3) [3]

Mark	Notes
	$6e^{3x^2}\cos 2x$
M1	For applying the Product rule • There must be an attempt to differentiate both terms. Accept as a minimum **either** $e^{3x^2} \Rightarrow \pm axe^{3x^2}$ **or** $\cos 2x \Rightarrow -b\sin 2x$ • A correct application of product rule – accept e.g $36xe^{3x^2}\cos 2x \pm 12e^{3x^2}\sin 2x$ $\left[36xe^{3x^2}\cos 2x - 12e^{3x^2}\sin 2x\right]$
A1	For either $36xe^{3x^2}\cos 2x$ or $-12e^{3x^2}\sin 2x$ Need not be simplified
A1	For the fully correct expression $36xe^{3x^2}\cos 2x - 12e^{3x^2}\sin 2x$ Need not be simplified. Accept for example: $6\times 6xe^{3x^2}\cos 2x - 6\times 2\times e^{3x^2}\sin 2x$

Question Number	Scheme	Marks
1	$\frac{ds}{dt} = 3t^2 + 8t - 27 = 8$ $3t^2 + 8t - 35\ (= 0)$ $(3t - 7)(t + 5) = 0$ $t = \frac{7}{3}$	M1 A1 M1A1cao **[4]**
M1 A1 M1 A1cao	Attempt the differentiation and equate their result to 8. Power of at least one term to decrease and none to increase' Obtain the correct 3TQ. Terms can be in any order and = 0 may be omitted. Attempt to solve their 3TQ by any valid method. Must reach t = ... For $t = \frac{7}{3}$ (negative answer must be omitted or eliminated) or t = 2.33 or better	

Question Number	Scheme	Marks
1	$(v=)8+2t-t^2$ $8+2t-t^2=(2+t)(4-t)=0\Rightarrow t=4$ Distance $=3+8\times4+4^2-\frac{1}{3}4^3=29\frac{2}{3}$ m (accept 29.7 or better or a recurring decimal)	B1 M1A1 A1 (4) [4]

B1	Correct differentiation
M1	Equate their differentiated expression (min 2 correct terms) to 0 (= 0 may be implied by their solution) and attempt to solve the 3 TQ by any valid method. Must reach t = ... Calculator solution: Allow M1A1 if their equation and its roots are correct , otherwise M0A0
A1	Correct value of t (Ignore t = -2 if shown)
A1	Correct distance, exact or min 3 s f Award A0 if value when t = -2 is also offered (and not excluded) If there is an error in the solution of their equation but t = 4 is used to obtain the correct answer this mark cannot be awarded.

Question Number	Scheme	Marks
2	$\text{Vol} = \pi\int_0^3 \left(e^{3x}\right)^2 dx \left(= \pi\int_0^3 e^{6x} dx\right)$ $\pi\left[\frac{1}{6}e^{6x}\right]_0^3, = \left(\frac{1}{6}e^{18} - \frac{1}{6}\right)\pi$ oe	M1 dM1A1,A1 (4) [4]

M1 Use $\text{Vol} = \pi\int y^2 dx$

Award if pi missing here but reappears later. Limits not needed, ignore any shown.
dx may be missing.

dM1 Square correctly and attempt the integration. $e^{6x} \to ke^{6x}$ where $k = \pm\frac{1}{6}$ or ± 1 Limits and dx may be missing. Award if pi missing here but reappears later.

A1 Correct integration including correct limits

A1 Substitute the limits and obtain the correct answer

Question	Scheme	Marks
1	$\int \cos 4\theta \ \mathrm{d}\theta = \left[\frac{\sin 4\theta}{4}\right]$ For an attempt to evaluate **their** integral using the given values and reach a value $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 4\theta \ \mathrm{d}\theta = \left[\frac{\sin 4\theta}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\sin\left(4\times\frac{\pi}{3}\right)}{4} - \frac{\sin\left(4\times\frac{\pi}{4}\right)}{4} = \ldots$ $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 4\theta \ \mathrm{d}\theta = -\frac{\sqrt{3}}{8}$	M1A1 M1 A1 [4]
		Total 4 marks

Mark	Notes
M1	For an attempt to integrate $\cos 4\theta$ obtaining: $\pm\frac{\sin 4\theta}{4}$ For this mark ignore incorrect / absent limits
A1	For the correct integrated expression: $\frac{\sin 4\theta}{4}$
M1	For an attempt to evaluate **their** integral using the given values and reach a value. Must be substituting into $k \sin 4\theta$ Condone candidates who convert to working in degrees to evaluate.
A1	For the correct value $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 4\theta \ \mathrm{d}\theta = -\frac{\sqrt{3}}{8}$ Note: question requires answer to be given in the form $-\frac{\sqrt{a}}{b}$ and therefore equivalent answers are not acceptable.

Question number	Scheme	Marks
5	$A=\pi(3r)^2=9\pi r^2 \Rightarrow \frac{dA}{dr}=18\pi r$	M1
	$\delta A \approx \frac{dA}{dr}\times\delta r = 18\pi r(\delta r)$	B1
	$\frac{\delta A}{A}\approx\frac{18\pi r}{A}\delta r=\frac{18\pi r}{9\pi r^2}\delta r=2\frac{\delta r}{r}$	M1
	So when $\frac{\delta r}{r}=0.05\% \Rightarrow \frac{\delta A}{A}\approx 0.1\%$ so the area increases by about 0.1%	M1A1
	ALT	
	Radius (after increase) = $3r\times\left(1+\frac{0.05}{100}\right)$	{M1}
	$=3.0015r$	{B1}
	Area before increase = $\pi(3r)^2=9\pi r^2$ Area after increase =	
	$A=\pi(3.0015r)^2=9.00900225\pi r^2$	{M1}
	Percentage increase = $\frac{9.00900225\pi r^2-9\pi r^2}{9\pi r^2}\times 100=0.100025\approx 0.1\%$ so the area increases by about 0.1%	{M1} {A1}
		Total 5 marks

M1	Differentiate A wrt r
B1	Use of $\delta A \approx \frac{dA}{dr}\times\delta r$
M1	Use of $\frac{\delta A}{A}$
M1	Use of $\frac{\delta r}{r}=0.05\%$
A1	Area increases by about 0.1%
ALT:	
M1	Finding the radius after the increase (may be implied by $3.0015r$)
B1	$3.0015r$ (may be implied by a correct area after the increase)
M1	Finding the area after the increase
M1	Use of $\frac{\text{Area (new)-Area (original)}}{\text{Area (original)}}\times 100$
A1	Area increases by about 0.1%

Question number	Scheme	Marks
9	$\frac{dy}{dx} = -(x^3 - 2x)e^{1-x} + (3x^2 - 2)e^{1-x}$	M1 A1
	When $x = 1$ $\frac{dy}{dx} = 2$ $\Rightarrow$ Gradient of **normal** = $-\frac{1}{"2"}$	M1
	$(y+1) = -\frac{1}{2}(x-1)$ oe and isw once seen	M1 A1 (5)
		Total 5 marks

Mark	Notes
M1	For the use of product rule. This is not given on page 2 so please mark as follows: • There must be an acceptable attempt to differentiate both terms. For this question $x^3 - 2x \to ax^2 + b \quad a, b \neq 0$ $e^{1-x} \to \pm e^{1-x}$ • Allow their $u\frac{dv}{dx} \pm v\frac{du}{dx}$ (as long as it fulfils these minimum conditions) $\frac{dy}{dx} = -(x^3 - 2x)e^{1-x} + (3x^2 - 2)e^{1-x}$
A1	For the correct simplified or unsimplified $\frac{dy}{dx}$ as shown above.
M1	For substituting $x = 1$ correctly into their $\frac{dy}{dx}$ to obtain a value for the gradient of the **normal**. When $x = 1$ $\frac{dy}{dx} = "2" \Rightarrow m_n = -\frac{1}{"2"}$ (must come from their $\frac{dy}{dx}$)
M1	For correctly forming an equation using the given coordinates with their gradient of the normal which is the negative reciprocal of their value of $\frac{dy}{dx}$ $(y+1) = -\frac{1}{2}(x-1)$ If $y = mx + c$ is used, then they must find a value for c and find an equation. $c = -\frac{1}{2}$ so $y = -\frac{x}{2} - \frac{1}{2}$ oe
A1	For the correct equation as shown above in any form.

Question number	Scheme	Marks
4	$\dfrac{dy}{dx}=\dfrac{2\cos 2x\left(x^2-9\right)^{\frac{1}{2}}-\dfrac{1}{2}\times 2x\sin 2x\left(x^2-9\right)^{-\frac{1}{2}}}{\left(x^2-9\right)}$	M1A1A1
	$\left\{\dfrac{dy}{dx}=\dfrac{\dfrac{2\cos 2x\left(x^2-9\right)^{\frac{1}{2}}\left(x^2-9\right)^{\frac{1}{2}}-\dfrac{1}{2}\times 2x\times\sin 2x}{\left(x^2-9\right)^{\frac{1}{2}}}}{\left(x^2-9\right)}\right\}$	
	$\dfrac{dy}{dx}=\dfrac{2\left(x^2-9\right)\cos 2x-x\sin 2x}{\sqrt{\left(x^2-9\right)^3}}$ *	dM1A1 cso [5]
	ALT	
	$y=\sin(2x)\,(x^2-9)^{-\frac{1}{2}}$ $\dfrac{dy}{dx}=2\cos(2x)(x^2-9)^{-\frac{1}{2}}+\sin(2x)\left(-\frac{1}{2}\right)(2x)(x^2-9)^{-\frac{3}{2}}$	[M1A1A1
	$\left\{\dfrac{dy}{dx}=\dfrac{2\cos(2x)(x^2-9)-x\sin(2x)}{(x^2-9)^{\frac{3}{2}}}\right\}$	
	$\dfrac{dy}{dx}=\dfrac{2(x^2-9)\cos(2x)-x\sin(2x)}{\sqrt{(x^2-9)^3}}$ *	dM1A1 cso]
		Total 5 marks

Mark	Notes
M1	For an attempt at Quotient rule. The definition of an attempt is that there must be a correct attempt to differentiate at least one term and the denominator must be $\left(\sqrt{x^2-9}\right)^2$. Allow the terms in the numerator to be the wrong way around, but the terms must be subtracted. Attempt at differentiation of the terms: $\sin(2x)\rightarrow k\cos(2x)$ where k is an integer $(x^2-9)^{\frac{1}{2}}\rightarrow lx(x^2-9)^{-\frac{1}{2}}$
A1	For correct differentiation of at least one term. $2\cos(2x)\,(x^2-9)^{\frac{1}{2}}$ or $-\frac{1}{2}\times 2x\sin 2x\,(x^2-9)^{-\frac{1}{2}}$
A1	For a fully correct Quotient rule $\dfrac{dy}{dx}=\dfrac{2\cos 2x(x^2-9)^{\frac{1}{2}}-\frac{1}{2}\times 2x\sin 2x(x^2-9)^{-\frac{1}{2}}}{(x^2-9)}$

dM1	For an attempt to rearrange to the given form. Dependent on M1 being scored. An attempt requires obtaining a single fraction and multiplying numerator and denominator by $(x^2-9)^{\frac{1}{2}}$ (see {} in mark scheme).
A1 **cso**	Fully correct method to show $\frac{dy}{dx} = \frac{2(x^2-9)\cos 2x - x\sin 2x}{\sqrt{(x^2-9)^3}}$ Allow with $\sqrt{(x^2-9)^3}$ given as $(x^2-9)^{\frac{3}{2}}$ or $\sqrt{(x^2-9)}^3$
ALT – product rule	
M1	For an attempt at Product rule. The definition of an attempt is that there must be a correct attempt to differentiate at least one term. Attempt at differentiation of the terms: $\sin(2x)\left(x^2-9\right)^{-\frac{1}{2}} \rightarrow k\cos(2x)\left(x^2-9\right)^{-\frac{1}{2}}$ where k is an integer $\sin(2x)(x^2-9)^{-\frac{1}{2}} \rightarrow lx\sin(2x)(x^2-9)^{-\frac{3}{2}}$
A1	For correct differentiation of at least one term. Either $2\cos(2x)(x^2-9)^{-\frac{1}{2}}$ or $\sin(2x)\left(-\frac{1}{2}\right)(2x)(x^2-9)^{-\frac{3}{2}}$
A1	For a fully correct Product rule $\frac{dy}{dx} = 2\cos(2x)(x^2-9)^{-\frac{1}{2}} + \sin(2x)\left(-\frac{1}{2}\right)(2x)(x^2-9)^{-\frac{3}{2}}$
dM1	For an attempt to rearrange to the given form. Dependent on M1 being scored. An attempt requires obtaining a single fraction and multiplying numerator and denominator by $(x^2-9)^{\frac{3}{2}}$ (see {} in mark scheme).
A1 **cso**	Fully correct method to show $\frac{dy}{dx} = \frac{2(x^2-9)\cos 2x - x\sin 2x}{\sqrt{(x^2-9)^3}}$ Allow with $\sqrt{(x^2-9)^3}$ given as $(x^2-9)^{\frac{3}{2}}$ or $\sqrt{(x^2-9)}^3$

Question	Scheme	Mark
3	$9-x^2=0\Rightarrow x=\pm 3$	B1
	$A=\int_{'-3'}^{'3'} 9-x^2\ \mathrm{d}x=\left[9x-\frac{x^3}{3}\right]_{'-3'}^{'3'}$	M1A1
	$A=\left(9\times 3-\frac{3^3}{3}\right)-\left(9\times -3-\frac{[-3]^3}{3}\right)=36$	M1A1 [5]
	Total	**5 marks**

Mark	Notes
You may see the working without A = …. . Please accept for full marks.	
B1	Find the intersections of C with the x-axis. $9-x^2=0\Rightarrow x=\pm 3$
M1	Attempt to integrate the given expression only i.e., $9-x^2$ A squared expression integrated is M0. See General Guidance for the definition of an attempt. $(A)=\int_{'-3'}^{'3'} 9-x^2\ \mathrm{d}x=\left[9x-\frac{x^3}{3}\right]_{'-3'}^{'3'}$ Ignore limits for this mark, even if they are completely missing.
A1	For the correct integral $(A)=\left[9x-\frac{x^3}{3}\right]$ [ignore limits for this mark].
M1	For substituting **their** limits into **their** integrated expression provided it is changed from $9-x^2$ and is not a differentiated expression. $(A)=\left(9\times '3'-\frac{'3'^3}{3}\right)-\left(9\times '[-3]'-\frac{['-3']^3}{3}\right)=...$ This mark can be implied by the correct final answer following correct integration. Allow also double 18 for this mark. If the final answer is incorrect and their integrated expression is incorrect, do not award this mark unless substitution is explicitly seen.
A1 [5]	For the correct answer only $(A)=36\ \left(\mathrm{cm}^2\right)$

Question	Scheme	Marks
2	$\frac{dy}{dx} = \sin 2x \times \frac{1}{2} \times 2 \times (3+2x)^{-\frac{1}{2}} + (3+2x)^{\frac{1}{2}} \times 2 \times \cos 2x$	M1A1A1
	$\frac{dy}{dx} = \frac{\sin 2x}{\sqrt{3+2x}} + 2 \times \sqrt{(3+2x)} \cos 2x = \frac{\sin 2x + 2(3+2x)\cos 2x}{\sqrt{3+2x}} = ...$	dM1
	$\frac{dy}{dx} = \frac{\sin 2x + (6+4x)\cos 2x}{\sqrt{3+2x}}$ *	A1 cso
	Where $A = 6$ and $B = 4$	[5]
		Total 5 marks

Mark	Notes
M1	For an attempt to apply product rule: An attempt is as follows. • The formula used must be correct. $\frac{dy}{dx} = uv' + vu'$ • Minimally acceptable differentiation used within a correct formula. $\sin 2x \to k\cos 2x$ and $\sqrt{3+2x} \to l(3+2x)^{-\frac{1}{2}}$ where k and l are a positive integers and $k, l \neq 0$ (You will see $l = 1$ without other working which is correct)
A1	One term correct – (simplification not required) and the other minimally acceptable. Either $\sin 2x \times \frac{1}{2} \times 2 \times (3+2x)^{-\frac{1}{2}}$ Or $(3+2x)^{\frac{1}{2}} \times 2 \times \cos 2x$
A1	Fully correct – simplification not required $\frac{dy}{dx} = \sin 2x \times \frac{1}{2} \times 2 \times (3+2x)^{-\frac{1}{2}} + (3+2x)^{\frac{1}{2}} \times 2 \times \cos 2x$
dM1	For **correct** use of $\sqrt{3+2x}$ or $(3+2x)^{\frac{1}{2}}$ as a common denominator. **NB:** This mark is dependent on the first M mark scored.
A1 cso	For the correct $\frac{dy}{dx}$ as shown with no erroneous or missing working. Allow embedded values. E.g., $\frac{dy}{dx} = \frac{\sin 2x + (6+4x)\cos 2x}{\sqrt{3+2x}}$ is dM1A1

Question number	Scheme	Marks
3	$4\times2(2x-1)^3e^{3x}+3e^{3x}(2x-1)^4$ *oe* $4\times2(2(1)-1)^3e^3+3e^3(2(1)-1)^4$ $11e^3$ cao	M1A1A1 dM1 A1
		Total 5 marks

Part	Mark	Additional Guidance
	M1	Use of product rule to give an expression of the form $pe^{3x}(2x-1)^3+qe^{3x}(2x-1)^4$. There must be a + sign between terms. $p > 0$, $q > 0$ If they have expanded $(2x-1)^4$ first, look for the form $e^{3x}(ax^3+bx^2+cx+d)+qe^{3x}(2x-1)^4$, where $q > 0$, and a, b, c, d are nonzero constants FYI, the correct expanded form of $(2x-1)^4$ is $16x^4-32x^3+24x^2-8x+1$, and its correct derivative is $64x^3-96x^2+48x-8$, so you might see the correct $\frac{dy}{dx}$ as $\frac{dy}{dx}=e^{3x}(64x^3-96x^2+48x-8)+3e^{3x}(2x-1)^4$
	A1	Either term correct.
	A1	Both terms correct.
	dM1	Substitution of $x = 1$ into their derivative. If no explicit substitution seen, provided their derivative is correct, we award this mark if awrt 221 seen. Dependent on previous method mark
	A1	Correct exact value, cao

Question number	Scheme	Marks
5	$\left\{\frac{dV}{dh}\right\}18h^2$	M1
	$\frac{dV}{dt}=(\pm)36$	B1
	$384=6h^3 \Rightarrow h^3=64$ or $h=4$	B1
	$\left\{\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}\right\}$	
	$\left\{\frac{dh}{dt}\right\}=\frac{1}{18\times4^2}\times(\pm)36$ oe	M1
	$=-\frac{1}{8}$ oe	A1
		Total 5 marks

Part	Mark	Additional Guidance
	M1	Correct differentiation, $18h^2$ seen award the mark
	B1	For $\frac{dV}{dt}=-36$ or 36 correctly stated explicitly or used implicitly in a chain rule.
	B1	For $h^3=64$ or $h=4$ or $h=\sqrt[3]{\frac{384}{6}}$
	M1	For correctly substituting their derivative and their numbers into a correct chain rule, implied or stated.
	A1	For $-\frac{1}{8}$ oe e.g. stating the height is decreasing at the rate of $\frac{1}{8}$ is also correct

Question number	Scheme	Marks
8	$3e^{3x}(9x+2)^{\frac{1}{3}}+\frac{1}{3}.9.e^{3x}(9x+2)^{-\frac{2}{3}}$	M1 A1 A1
	$3e^{3}(9+2)^{\frac{1}{3}}+\frac{1}{3}.9.e^{3}(9+2)^{-\frac{2}{3}}$	M1
	$=146$	A1 [5]
		Total 5 marks

Part	Mark	Additional Guidance
	M1	Use of product rule to give an expression of the form $ke^{3x}(9x+2)^{\frac{1}{3}}+le^{3x}(9x+2)^{-\frac{2}{3}}$ where k = 1 or 3, $l \neq 1$. There must be a + sign between terms.
	A1	For $+\frac{1}{3}.9.e^{3x}(9x+2)^{-\frac{2}{3}}$ or $+3e^{3x}(9x+2)^{\frac{1}{3}}$ simplification not necessary.
	A1	Both terms correct – simplification not necessary.
	M1	Substitution of x = 1 into any changed expression Allow substitution into an expression which has been simplified incorrectly after an expression of the required form. Can be implied by a correct answer or by 12.18…… + 134.0…….
	A1	Awrt 146

Question Number	Scheme	Marks
2	$\frac{dA}{dt}=8$	B1
	$\frac{dA}{dr}=2\pi r$	M1
	$A=50 \quad r=\sqrt{\frac{50}{\pi}}\ (3.989...)$	M1
	$\frac{dr}{dt}=\frac{dr}{dA}\times\frac{dA}{dt},=\frac{1}{2\pi\sqrt{\frac{50}{\pi}}}\times 8,=0.319\ (\text{cm/s})$	M1,A1ft,A1 [6]
NB	For either method, accept A or S for area, r for radius. Any other letters used for area and/or radius must be defined.	
B1	$\frac{dA}{dt}=8$ seen explicitly or used	
M1	Attempt to differentiate πr^2 to obtain $\frac{dA}{dr}$ Power of r must decrease	
M1	Attempt to obtain r when $A = 50$ cm^2 (ie solve $50=\pi r^2$)	
M1	For a correct, useful, chain rule. Derivatives can appear in any order	
A1ft	Substitute their known quantities and rearrange to $\frac{dr}{dt}=...$ if not in this form already.	
	All 3 M marks needed	
A1	Correct answer, **must be 3 sf**	
ALT	$\frac{dA}{dt}=8$	B1
	$r=\sqrt{\frac{a}{\pi}}$ oe	M1
	$\frac{dr}{dA}=\frac{1}{2\sqrt{\pi}}A^{-\frac{1}{2}}$	M1
	$\frac{dA}{dt}\times\frac{dr}{dA}=\frac{dr}{dt}$	M1
	$=8\times\frac{1}{2\sqrt{\pi}}A^{-\frac{1}{2}}=8\times\frac{1}{2\sqrt{\pi}}\times\frac{1}{\sqrt{50}}$	A1ft
	$= 0.3191\ldots = 0.319$ (cm/s)	A1
B1	$\frac{dA}{dt}=8$ seen explicitly or used	
M1	Attempt to find r in terms of A	
M1	Attempt to differentiate their expression for r to obtain $\frac{dr}{dA}$ power of A must decrease	
M1	For a correct, useful, chain rule. Derivatives can appear in any order	
A1ft	Substitute their known quantities and rearrange to $\frac{dr}{dt}=...$ if not in this form already. All 3 M marks needed	
A1	Correct answer, **must be 3 sf**	

Question Number	Scheme	Marks
3(a)	$\frac{dv}{dt} = 2t - 4$	M1
	Accel $= 2\ \left(\text{m/s}^2\right)$	A1 (2)
(b)	$s = \int_0^6 \left(t^2 - 4t + 7\right) dt = \left[\frac{t^3}{3} - 2t^2 + 7t\right]_0^6$	M1A1
	$= \frac{6^3}{3} - 2\times 6^2 + 7\times 6 = 42\ \left(\text{m}\right)$	dM1 A1cao (4)
		[6]

(a)

M1 Attempt to differentiate the expression for v. Power of t to decrease in at least one term and increase in none.

A1 Substitute $t = 3$ and obtain correct acceleration – units may be missing

(b)

M1 Attempt to integrate the expression for v. Power of t to increase in at least one term and decrease in none. Ignore limits if shown. Constant not needed for indefinite integration.

A1 Correct integration. Limits/constant not needed.

dM1 **Either** substitute the limits 0 and 6 **or** use $s = 0$, $t = 0$ to obtain a value for the constant and substitute $t = 6$ in the complete expression. (Substitution of 0 can be implied if the result would have been 0) Depends on the previous M mark
If more values of t are substituted **and results used** award M0

A1cao $S = 42$ (m)

NB Ans 42 w/o working scores 4/4 (Done on a calculator)

Question number	Scheme	Marks
3	Let l be the length. $\frac{dl}{dt}=0.005$	
	$l=10x \Rightarrow \frac{dl}{dx}=10$ or $\frac{dl}{dt}=10\frac{dx}{dt}=0.005 \Rightarrow \frac{dx}{dt}=0.0005$	B1
	$r=\frac{x}{2}, \quad V=\pi\left(\frac{x}{2}\right)^2\times 10x \Rightarrow V=\frac{5\pi x^3}{2}$	B1
	$\frac{dV}{dx}=\frac{15\pi x^2}{2}$	M1
	$\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dl}\times\frac{dl}{dt}$ or $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$	M1
	$\frac{dV}{dt}=\frac{15\pi\times 3^2}{2}\times\frac{1}{10}\times 0.005$ or $\frac{dV}{dt}=\frac{15\pi\times 3^2}{2}\times 0.0005$ $=0.106028...$	dM1
	$\frac{dV}{dt}\approx 0.11\ (\text{cm}^3/\text{s})$	A1 [6]
Alternative		
	$x=\frac{l}{10}$ or $r=\frac{l}{20}$ stated or used	B1
	$V=\pi\left(\frac{l}{20}\right)^2\times l=\frac{\pi l^3}{400}$	B1
	$\frac{dV}{dl}=\frac{3\pi l^2}{400}$	M1
	$\frac{dV}{dt}=\frac{dV}{dl}\times\frac{dl}{dt}$	M1
	$\frac{dV}{dt}=\frac{3\pi\times 30^2}{400}\times 0.005=0.106028...$	dM1
	$\frac{dV}{dt}\approx 0.11\ \ (\text{cm}^3/\text{s})$	A1 [6]
		Total 6 marks

Apart from x, which is defined in the question, different variables may be used providing that they are consistent throughout the working. Score the best mark using either alternative but not a mixture of both.		
	B1	$\frac{dl}{dx}=10$ or $\frac{dx}{dt}=0.0005$
	B1	Correct expression for the volume of the cylinder. Simplification not needed. Brackets for $\left(\frac{x}{2}\right)^2$ are required unless implied by subsequent working.
	M1	Attempt to differentiate to find $\frac{dV}{dx}$. You need to see a multiple of x^2 from differentiating x^3. ft their V if it is a multiple of x^3. Mark the differentiation and ignore any error in the label, $\frac{dV}{dx}$, for this mark only.
	M1	State, or use correctly, the chain rule $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dl}\times\frac{dl}{dt}$ or $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$
	dM1	Substitute to find a numerical expression for $\frac{dV}{dt}$ dep on both M marks. ft their $\frac{dV}{dx},\frac{dl}{dx},\frac{dx}{dt}$ if clearly stated.
	A1	$\frac{dV}{dt}\approx 0.11$ **given to 2SF**

Alternative		
	B1	$x=\frac{l}{10}$ or $r=\frac{l}{20}$ stated or used
	B1	Correct expression for the volume of the cylinder. Simplification not needed. Brackets for $\left(\frac{l}{20}\right)^2$ are required unless implied by subsequent working.
	M1	Attempt to differentiate to find $\frac{dV}{dl}$. You need to see a multiple of l^2 from differentiating l^3. ft their V if it is a multiple of l^3. Mark the differentiation and ignore any error in the label, $\frac{dV}{dl}$, for this mark only.
	M1	State, or use correctly, the chain rule $\frac{dV}{dt}=\frac{dV}{dl}\times\frac{dl}{dt}$
	dM1	Substitute to find a numerical expression for $\frac{dV}{dt}$ dep on both M marks. ft their $\frac{dV}{dl}$ if clearly stated.
	A1	$\frac{dV}{dt}\approx 0.11$ **given to 2SF**

Question number	Scheme	Marks
3(a)	$\frac{dy}{dx} = 2e^{2x}\left(x^2+1\right)+e^{2x}\left(2x\right)$	M1A1A1 [3]
(b)	When $x=0$ $\frac{dy}{dx} = 2\times1\times1+1\times0=2$ $\quad y=e^{2\times0}\left(0+1\right)=1$ $y-1=2\left(x-0\right)\Rightarrow y=2x+1$	B1B1 B1 [3]
		Total 6 marks

(a)	
M1	Attempted use of the product rule. Sum of two terms (either way round) with $x^n \rightarrow x^{n-1}$ (Condone e^{2x} instead of $2e^{2x}$) Once the correct answer is seen ISW. This mark may be implied by the sum of two terms with one of the two terms correct.
A1	Either term correct
A1	Both terms correct
(b)	
B1	When $x=0$ $\frac{dy}{dx}=2$
B1	When $x=0$ $y=1$
B1	$y=2x+1$

Question number	Scheme	Marks
6 (a)	$\left[V=\frac{1}{3}\pi r^2 h\right]$ $\tan 30^\circ=\frac{r}{h}\Rightarrow r=\frac{h}{\sqrt{3}}$ $V=\frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h\Rightarrow V=\frac{1}{9}\pi h^3$ *	M1 A1 cso [2]
(b)	$\frac{dV}{dh}=\frac{\pi h^2}{3}$ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}\Rightarrow\frac{dh}{dt}=\frac{3}{\pi h^2}\times -0.9=\frac{3}{\pi\times 1.2^2}\times -0.9=-0.59683...$ ≈ -0.597 cm/s [Accept an answer of ± 0.597 cm/s]	M1 M1M1 A1 [4]
		Total 6 marks

(a)	
M1	Finding $r=\frac{h}{\sqrt{3}}$ and substituting into $V=\frac{1}{3}\pi r^2 h$ (Allow $r=h\tan 30^\circ$)
A1 cso	Obtains the **given** answer with no errors in the working
(b)	
M1	$\frac{dV}{dh}=\frac{\pi h^2}{3}$
M1	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$
M1	$\frac{dh}{dt}=\frac{3}{\pi h^2}\times\pm 0.9$
A1	± 0.597 (cm/s)

Question number	Scheme	Marks
4 (a)	$\frac{dy}{dx} = 3x^2 - 6x - 24$	M1
	$"(x+2)(x-4)" = 0$	M1
	$x = -2$ or $x = 4$	A1
	(–2, 34) and (4, –74)	A1 (4)
(b)	$\frac{d^2y}{dx^2} = "6x - 6"$ and substitution of their –2 or 4 or consider values of $\frac{dy}{dx}$ either side of their –2 or 4 or use properties of a cubic*	M1
	(–2, 34) maximum and (4, –74) minimum	A1cso (2) **[6]**

Part	Mark	Additional Guidance
(a)	M1	General guidance – an attempt to differentiate, power of at least one term must decrease by one. Also, no power must increase.
	M1	Equates their derivative = 0 and attempts to solve by any method **– see general guidance.**
	A1	Both correct values for x
	A1	All values correct, listed as coordinates or correctly paired $x = \ldots\ldots$ $y = \ldots\ldots$
(b)	M1	**Correctly differentiates their derivative** from part a and substitutes one of their x values. Allow sight of 18 or -18 following a correct differentiation to imply this mark.
	A1 cso	Allow this mark if there is an incorrect y value from part a Must correctly make the argument they've chosen and identify the points as shown. If substituting x = -2 and x = 4 or values either side of the these, their evaluations of the substitutions must be correct. * A convincing argument about the shape of a positive cubic curve and position of the minimum and maximum point must be made. Send to review, if in doubt. In the conclusion, it must be clearly stated which coordinate or value of x is a maximum and which is a minimum. Just a substitution of a value and stated max or min is insufficient.

Question number	Scheme	Marks
6	$\left[\frac{dV}{dt} = 3\ (\text{cm}^3/\text{s})\right]$ $V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dr} = 4\pi r^2, \quad A = 4\pi r^2 \quad \frac{dA}{dr} = 8\pi r$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$ oe $\frac{dA}{dt} = [8\pi \times 10] \times \left[\frac{1}{4\pi \times 10^2}\right] \times 3 = 0.6\ (\text{cm}^2/s)$	M1A1,A1 (M1 for any one) M1 dM1A1 [6]
		Total 6 marks

Mark	Notes
M1	For using the **correct** formula for volume of a sphere or for surface area of a sphere and attempt to differentiate their expression. [See General Guidance for definition of attempt to differentiate]
A1	For one correct $\frac{dV}{dr} = 4\pi r^2$ or $\frac{dA}{dr} = 8\pi r$
A1	For both correct $\frac{dV}{dr} = 4\pi r^2$ and $\frac{dA}{dr} = 8\pi r$
M1	For applying a **correct** Chain rule using their $\frac{dV}{dr}$, their $\frac{dA}{dr}$ and $\frac{dV}{dt} = 3$ to obtain $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt} = '\ 8\pi r' \times '\frac{1}{4\pi r^2}' \times 3$ May be seen in two stages.
dM1	For substitution of $r = 10$ into their expression for $\frac{dA}{dt}$ to obtain $\frac{dA}{dt} = '\ 8\pi \times 10' \times '\frac{1}{4\pi \times 10^2}' \times 3$
A1	$\frac{dA}{dt} = 0.6\ (\text{cm}^2\ /\ \text{s})$

Question number	Scheme	Marks
2	$4-x^2=x+2\Rightarrow x^2+x-2=0\Rightarrow (x-1)(x+2)=0\Rightarrow x=1,\ -2$	M1A1
	$V=\pi\int_{-2}^{1}\left(4-x^2\right)^2\ \mathrm{d}x-\pi\int_{-2}^{1}(x+2)^2\ \mathrm{d}x$	M1
	$V=\pi\int_{-2}^{1}x^4-9x^2-4x+12\ \mathrm{d}x=\pi\left[\frac{x^5}{5}-\frac{9x^3}{3}-2x^2+12x\right]_{-2}^{1}$	M1
	$V=\pi\left\{\left(\frac{1^5}{5}-3\times1^3-2\times1^2+12\times1\right)-\left(\frac{[-2]^5}{5}-3\times[-2]^3-2\times[-2]^2+12\times[-2]\right)\right\}$	M1
	$V=\frac{153\pi}{5}-9\pi$	
	$V=\frac{108\pi}{5}$	A1
	ALT	[M1
	$V=\pi\int_{-2}^{1}\left(4-x^2\right)^2\ \mathrm{d}x-\frac{\pi}{3}\times3^2\times3$	
	$V=\pi\int_{-2}^{1}\left(16-8x^2+x^4\right)\mathrm{d}x-[9\pi]=\pi\left[16x-\frac{8x^3}{3}+\frac{x^5}{5}\right]_{-2}^{1}-[9\pi]$	M1
	$V=\pi\left\{\left(16\times1-\frac{8(1)^3}{3}+\frac{(1)^5}{5}\right)-\left(16\times(-2)-\frac{8(-2)^3}{3}+\frac{(-2)^5}{5}\right)\right\}-[9\pi]$	M1
	$V=\frac{153\pi}{5}-9\pi$	
	$V=\frac{108\pi}{5}$	A1]
		Total 6 marks

Mark	Notes
M1	For equating the equation of the curve and the straight line, form a 3TQ with an acceptable attempt to solve the equation to find the x coordinates of the **two** points of intersection. [See General Guidance for the definition of an attempt] This mark is awarded for a complete method.
A1	For $x = 1$ and -2
Method 1	
M1	Uses the **correct** form for the volume of rotation. Ft their x coordinates used correctly. Everything must be correct for this mark including π
M1	For an attempt to integrate their expression for the volume of rotation for either the curve or the line (or even a combined expression). Do not accept a mixture of integration and differentiation. [See general guidance for minimum requirements for integration]. Award even if their expression is not squared. Ignore the absence of π or limits for this mark.
M1	For attempting to evaluate their integrated expression (which must be a changed expression) with their limits the correct way round. Substitution must be seen. Ignore the absence of π and ft their values of x for this mark
A1	For the correct exact value of $\frac{108\pi}{5}$ or exact equivalent.
Method 2	
M1	Uses the **correct** form for the volume of rotation of the curve minus the volume of the cone. The volume of the cone is: $\frac{1}{3}\times\pi\times('1'+2)^2\times('1'-'-2')$ Ft their x coordinates for the limits and for the dimensions of the cone. Everything must be correct for this mark including π
M1	For an attempt to integrate their expression for the curve. Do not accept a mixture of integration and differentiation. [See general guidance for minimum requirements for integration]. Award even if their expression is not squared, Ignore the absence of π or limits for this mark.
M1	For attempting to evaluate their integrated expression (which must be a changed expression) with their limits the correct way round. Substitution must be seen. Ignore the absence of π and ft their values of x for this mark
A1	For the correct exact value of $\frac{108\pi}{5}$ or exact equivalent.

Question number	Scheme	Marks
5	$\frac{dy}{dx} = e^{-2x} - 2xe^{-2x} = e^{-2x}(1-2x) \Rightarrow \delta y = e^{-2x}(1-2x)\delta x$	M1A1A1
	$\frac{\delta y}{y} \times 100 = \frac{e^{-2x}(1-2x)\delta x}{xe^{-2x}} \times 100 = \frac{e^{-2x}(1-2x)\times 0.03x}{xe^{-2x}} \times 100$	dM1ddM1
	% change in $y = 3(1-2x)[\%]$	A1
		Total 6 marks

Mark	Notes
M1	For an attempt at Product Rule. The definition of an attempt is as follows. • Both terms must be differentiated as follows: $\frac{d(x)}{dx} = 1 \qquad \frac{d(e^{-2x})}{dx} = \pm Be^{-2x}$ • The correct form of product rule must be used. $u\frac{dv}{dx} + v\frac{du}{dx}$ or the other way around.
A1	For a fully correct product rule $\frac{dy}{dx} = e^{-2x} - 2xe^{-2x}$
A1	For $\delta y = e^{-2x}(1-2x)\delta x$ or $\delta y = (e^{-2x} - 2xe^{-2x})\delta x$ or $\delta y = (e^{-2x} - 2xe^{-2x}) \times \frac{3x}{100}$ or $\delta y = (e^{-2x} - 2xe^{-2x}) \times 0.03x$ oe
dM1	For writing $\frac{\delta y}{y} = \frac{e^{-2x}(1-2x)\delta x}{xe^{-2x}}$ or $\frac{\delta y}{y} = \frac{(e^{-2x} - 2xe^{-2x})\delta x}{xe^{-2x}}$ Ft their differentiated expression provided it is of the form: $\delta y = (Ae^{-2x} - Bxe^{-2x})\delta x$ This mark is dependent on the first M mark.
ddM1	For substituting in the value of $\delta x = 0.03x$ and attempting to simplify the **percentage change** to the required form. i.e. $\frac{\delta y}{y} \times 100 = \frac{e^{-2x}(1-2x)\times 0.03x}{xe^{-2x}} \times 100$ or $\frac{\delta y}{y} \times 100 = \frac{(e^{-2x} - 2xe^{-2x})\times 0.03x}{xe^{-2x}} \times 100$ This mark is dependent on the previous two M marks The minimum required for an attempt is a correct cancellation of xe^{-2x} from the numerator and denominator.
A1	For the correct answer in the required form $y = 3(1-2x)[\%]$

Question number	Scheme	Marks
3 (a)	$\frac{dy}{dx} = 2e^{2x}\sqrt{5x-3} + \frac{5e^{2x}}{2\sqrt{5x-3}}$	M1 A1 A1 (3)
(b)	$\frac{dy}{dx} = \frac{3x^2\cos 3x - x^3(-3\sin 3x)}{(\cos 3x)^2}$	M1 A1 A1 (3)
		Total 6 marks

Part	Mark	Notes
(a)	**M1**	For use of the product rule. Sum of two terms (either way round) There must be an attempt to differentiate both terms. See below. $(5x-3)^{\frac{1}{2}} \Rightarrow \frac{1}{2}\times k\times(5x-3)^{-\frac{1}{2}} \quad k\neq 0$ $e^{2x} \Rightarrow le^{2x} \quad l\neq 0$
	A1	For either term correct $2e^{2x}\sqrt{5x-3}$ or $\frac{5e^{2x}}{2\sqrt{5x-3}}$
	A1	For the correct derivative. $\frac{dy}{dx} = 2e^{2x}\sqrt{5x-3} + \frac{5e^{2x}}{2\sqrt{5x-3}}$ Or $\frac{dy}{dx} = 2e^{2x}(5x-3)^{\frac{1}{2}} + \frac{5}{2}e^{2x}(5x-3)^{-\frac{1}{2}}$ oe
(b)	**M1**	For an attempt at the use of the quotient rule. - There must be an acceptable attempt to differentiate both terms $x^3 \Rightarrow 3x^2$ $\cos(3x) \Rightarrow -m\sin 3x \quad m\neq 0$ - The denominator must be squared. - The terms in the numerator must be subtracted in either order.
	A1	For one term correct $3x^2\cos 3x$ or $x^3(-3\sin 3x)$
	A1	For the fully correct derivative.

Question number	Scheme	Marks
2 a	$v = \frac{dx}{dt} = 4t^3 - 13.5$	M1
	When $t = 3$ $v = 4(3)^3 - 13.5 = 94.5$ ms^{-1}	A1 (2)
b	$4t^3 - 13.5 = 0 \Rightarrow t^3 = \frac{27}{8} \Rightarrow t = 1.5$	M1 A1ft (2)
c	$a = \frac{dv}{dt} = 12t^2$	M1
	When $t = 2$ $a = 12 \times 2^2 = 48$ ms^{-2}	A1 (2)
		Total 6 marks

Part	Mark	Notes
Ignore incorrect/spurious notation through this question. e.g. ignore $\frac{dy}{dx} = ...$ or the LHS		
(a)	**M1**	For an attempt to differentiate the given expression [with no terms integrated] See General Guidance for the definition of an attempt to differentiate.
	A1	For substituting the value of $t = 3$ into their differentiated expression and obtains 94.5 (units not required)
(b)	**M1**	For setting their $\frac{dx}{dt} = 0$ which must be at a minimum of the form $\pm kt^3 \pm l$ and attempting to find a value for t
	A1ft	For $t = 1.5$ Ft their expression for v which must have come from an acceptable attempt to differentiate the given x
(c)	**M1**	For differentiating their $\frac{dx}{dt}$ which must be of the form $\frac{dx}{dt}$ (or v) $= \pm kt^3 \pm l$ where k and l are constants, [with no term integrated]
	A1	For substituting $t = 2$ into their differentiated expression and obtains 48 (units not required)

Question number	Scheme	Marks
2	$(V=)3x^3$	B1
	$\frac{dV}{dx}=9x^2$	M1
	$\left(\frac{dx}{dt}=\right)\frac{dV}{dt}\times\frac{dx}{dV}$ or $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ oe	M1
	$\left(\frac{dx}{dt}=\right)\frac{8}{9x^2}$ oe	A1
	$\left(\frac{dx}{dt}=\right)\frac{8}{9\times2^2}$ oe	dM1
	$\frac{2}{9}$ oe	A1 [6]
		Total 6 marks

Part	Mark	Additional Guidance
	B1	Correct simplified expression for Volume
	M1	Minimally acceptable attempt at differentiation– see general guidance (kx^2 where $k \neq 0$ if working from correct V)
	M1	A correct chain rule that could be used to find $\frac{dx}{dt}$ Condone absence of $\frac{dx}{dt}$ unless $\frac{dx}{dt}$ is not the subject.
	A1	As shown oe
	dM1	Substitution of $x = 2$ into their $\frac{dx}{dt}$, dependent on second method mark.
	A1	Correct answer (exact or correct to 2dp or better)
	For all marks condone poor notation e.g. use of dy/dx as long as not ambiguous.	

Question	Scheme	Marks
6	$A=\frac{r^2}{2}\times\frac{\pi}{6}=\left(\frac{\pi r^2}{12}\right)$	B1
	$\frac{\mathrm{d}A}{\mathrm{d}r}=\frac{2\times\pi r}{12}=\left(\frac{\pi r}{6}\right)$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$	M1
	Length of arc $AB=\frac{5\pi}{2}=r\frac{\pi}{6}\Rightarrow r=15$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\pi\times15}{6}\times0.2=\frac{\pi}{2}\ \left(\mathrm{cm}^2/\mathrm{s}\right)$	M1A1 [6]
		Total 6 marks

Question	Notes	Marks
6	For using the correct formula for the area of the sector $A=\frac{r^2}{2}\times\frac{\pi}{6}=\left(\frac{\pi r^2}{12}\right)$	B1
	For an attempt to differentiate their expression for the area provided it is in the form $A=kr^2$ where $k\neq1$ (see general guidance) $\frac{\mathrm{d}A}{\mathrm{d}r}=\frac{2\times\pi r}{12}=\left(\frac{\pi r}{6}\right)$	M1
	For a correct statement of chain rule to achieve $\frac{\mathrm{d}A}{\mathrm{d}t}$ $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ may be seen stated or implied by substitution.	M1
	For using the length of arc when the length of arc is $\frac{5\pi}{2}$ $\frac{5\pi}{2}=r\frac{\pi}{6}\Rightarrow\left(r=15\right)$	M1
	For attempting to find the rate of change of area using a correct chain rule, their $\frac{\mathrm{d}A}{\mathrm{d}r}$, the given $\frac{\mathrm{d}r}{\mathrm{d}t}$ and their value for r $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{'\pi'}{6}\times'15'\times0.2=\ldots$	M1
	For the correct value of $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\pi}{2}\ \left(\mathrm{cm}^2/\mathrm{s}\right)$	A1 [6]
		Total 6 marks

Question	Scheme	Marks
6 (a)	$(V=)5\times\frac{1}{2}\times r^2\times\sin\left(\frac{\pi}{3}\right)=\frac{5\sqrt{3}}{4}r^2$ *	B1*cso [1]
(b)	$\left(\frac{dV}{dr}=\right)\frac{5\sqrt{3}}{2}r$ oe	M1
	(When the area of $BCDF$ = 60 cm^2, $BC = DF = r$ =)12 cm	B1
	$\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ oe	M1
	$\left(\frac{dV}{dt}=\right)\frac{5\sqrt{3}}{2}\times12\times0.2=6\sqrt{3}$ oe $\left(\text{cm}^3/\text{s}\right)$	dM1A1 [5]
		Total 6 marks

Part	Mark	Notes
(a)	B1* cso	For a correct expression for the volume of a prism $V=5\times\frac{1}{2}\times r^2\times\sin\left(\frac{\pi}{3}\right)$ followed by the given answer stated. No errors. Use of $60°$ for $\frac{\pi}{3}$ is fine.
(b)	M1	For differentiating the expression for V to given an expression of the form pr where p is a positive constant. We don't need to see $\left(\frac{dV}{dr}=\right)$ if it's clear the candidate has attempted to differentiate the given volume.
	B1	For finding the length BC or DF when the area of the face $BCDF$ = 60 and deducing that r = 12 cm.
	M1	For a correct chain rule involving $\frac{dV}{dt}$, $\frac{dV}{dr}$ and $\frac{dr}{dt}$ [Can be implied by later correct substitution.]
	dM1	For substituting their $\frac{dV}{dr}$, their r and the given value of $\frac{dr}{dt}=0.2$ Dependent on the first method mark.
	A1	For $\left(\frac{dV}{dt}=\right)6\sqrt{3}$ $\left(\text{cm}^3/\text{s}\right)$

Question	Scheme	Marks

Question	Scheme	Marks
6(a)	$t=0 \Rightarrow s=\mathrm{e}^{2\times 0}\sin\left(3\times 0\right)+2=2$ $t=\dfrac{\pi}{6} \Rightarrow s=\mathrm{e}^{2\times\frac{\pi}{6}}\sin\left(3\times\dfrac{\pi}{6}\right)+2=\mathrm{e}^{\frac{\pi}{3}}+2$ $AB=\mathrm{e}^{\frac{\pi}{3}}+2-2=\mathrm{e}^{\frac{\pi}{3}}$ (m)	M1 A1 [2]
(b)	$v=2\mathrm{e}^{2t}\sin 3t+3\mathrm{e}^{2t}\cos 3t$ When $t=\dfrac{\pi}{3}$ $v=2\mathrm{e}^{\frac{2\pi}{3}}\sin\left(\dfrac{\pi}{3}\times 3\right)+3\mathrm{e}^{\frac{2\pi}{3}}\cos\left(\dfrac{\pi}{3}\times 3\right)=-3\mathrm{e}^{\frac{2\pi}{3}}$ (m/s)	M1A1A1 B1ft [4]
		Total 6 marks

Part	Mark	Notes
(a)	M1	For an attempt to find the displacements of A **and** B as simplified values. At least one must be correct and simplified for this mark and there must be an attempt at the other.
	A1	For finding the distance AB
(b)	M1	For an attempt at product rule. • The formula must be correct. i.e., $\left[v\right]=k\mathrm{e}^{2t}\times\sin 3t+\mathrm{e}^{2t}\times l\cos 3t$ • There must be an acceptable attempt to differentiate both e^{2t} and $\sin 3t$ $\mathrm{e}^{2t}\rightarrow k\mathrm{e}^{2t}$ and $\sin 3t\rightarrow l\cos 3t$ $k,l\neq 0$ and integers • The terms must be added.
	A1	One term fully correct.
	A1	Both terms fully correct.
	B1ft	For using $t=\dfrac{\pi}{3}$ in their v to find a value for v The minimally acceptable expression for v must be a **changed** expression from the given s in terms of e^{2x}, $\sin 3x$ and $\cos 3x$ Note: This is a ft mark. Please check their substitution and the final result. Please isw a value of $-24.36....$ if seen **together** with the exact value.

Question	Scheme	Marks
5	$\frac{dr}{dt} = 0.7$	B1
	$\{\frac{dF}{dr}\} = -2 \times \frac{3}{20} r^{-3}$ oe	M1A1
	$\{\frac{dF}{dt} = \frac{dF}{dr} \times \frac{dr}{dt} =\} -\frac{0.3}{2.8^3} \times 0.7 = -0.0095663 \ldots$ (newtons/s)	M1dM1A1
	Accept **awrt** -0.00957 or -9.57×10^{-3}	[6]
		Total 6 marks

Mark	Notes
B1	For writing $\frac{dr}{dt} = 0.7$ can be implied or embedded in later work. Condone using letter ***d*** or ***D*** or ***R*** only instead of ***r.***
M1	For an attempt to differentiate the formula for *F*, for $\ldots r^{-2} \rightarrow \ldots r^{-3}$ oe Attempts quotient rule, look for $\frac{-Pr}{Qr^4}$ ($P > 0$, $Q > 0$) oe, if the quotient rule is quoted, it must be correct, if clearly they use incorrect quotient rule M0
A1	For a correct differentiated expression simplified or unsimplified
M1	For a **correct** chain rule, accept in any order. (correct chain rule stated or implied by their working) Condone using letter d instead of r. For example, accept $\frac{dF}{dr} = \frac{dF}{dt} \times \frac{dt}{dr}$ *or* $\frac{dF}{dr} = \frac{dF}{dt} \div \frac{dr}{dt}$ $\left(\frac{dF}{dt}\right.$ does not need to be the subject$\left.\right)$
dM1	For substituting the value of $r=2.8$ into their dF/dr in a **correct** chain rule, leading to $\frac{dF}{dt} = \ldots$ (a value, not a numerical expression) , the value can be a fraction or decimal. If their final answer is not correct, explicit substitution must be seen. **Depends on the previous method mark only.**
A1	For the correct value of $\frac{dF}{dt}$ (Condone lack of units or incorrect units) isw

Question number	Scheme	Marks
7	$\frac{dV}{dt} = -k$	B1
	$\left\{\frac{dV}{dh}\right\} = 40\pi h - \pi h^2$	M1
	$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = -\frac{k}{40\pi h - \pi h^2}$	M1A1
	$-\frac{1}{60} = -\frac{k}{40\times\pi\times12 - \pi\times12^2}\left\{= -\frac{k}{336\pi}\right\} \Rightarrow k = \frac{28}{5}\pi\left(\text{cm}^3/\text{s}\right)$	dM1A1 [6]
		Total 6 marks

Part	Mark	Additional Guidance
	B1	For identifying that $\frac{dV}{dt} = -k$ seen/embedded in a correct chain rule, accept $\pm k$
	M1	Attempt to find $\frac{dV}{dh}$, condone if $\frac{dV}{dh}$ is not present or incorrect If attempted by expanding the bracket first, they need to achieve $Ph \pm Qh^2$ If attempted by product rule, score for $Ah(60-h) \pm Bh^2$
	M1	A correct chain rule formed by using $\frac{dV}{dh}, \frac{dh}{dt}$ and $\frac{dV}{dt}$ (in any order) stated or implied
	A1	Obtains the correct chain rule having at least two terms with values/expressions substituted in, no errors seen. e.g. $-k = \left(40\pi h - \pi h^2\right) \times \frac{dh}{dt}$ or $\frac{dV}{dt} = \left(40\pi h - \pi h^2\right) \times -\frac{1}{60}$
	dM1	Substitutes $h = 12$, $\frac{dV}{dt} = \pm k$, $\frac{dh}{dt} = \pm\frac{1}{60}$ and their $\frac{dV}{dh} = '40\pi h - \pi h^2'$ to a correct chain rule to find $k = ...$ Dependent on the 2nd M mark only
	A1	Correct answer only for using correct signs throughout $\frac{28}{5}\pi$ or 5.6π, ignore unit

Question number	Scheme	Marks
12 (a)	$(2\cos x = 0)$ $(x =)\frac{\pi}{2}$ or 90 °	B1 (1)
(b)	$(2\cos x = 2\sin x)$ $\tan x = 1$	M1
	$x = \frac{\pi}{4}$ or 45°	A1 (2)
(c)	$\int_{(0)}^{("\frac{\pi}{4}")} (2\sin x)dx + \int_{("\frac{\pi}{4}")}^{("\frac{\pi}{2}")} (2\cos x)dx$	M1
	$\left[-2\cos x\right]_0^{\frac{\pi}{4}} + \left[2\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	A1
	$\left(-\sqrt{2}+2\right)+\left(2-\sqrt{2}\right)=4-2\sqrt{2}$	dM1
	$= 4-\sqrt{8}$ cao	A1cao cso (4) **[7]**

Part	Mark	Additional Guidance
(a)	B1	For $\frac{\pi}{2}$ or 90 degrees. Can also be shown as a coordinate – ignore any incorrect y coordinate.
(b)	M1	For $\tan x = 1$
	A1	For $\frac{\pi}{4}$ or 45 degrees. Can also be shown as a coordinate – ignore any incorrect y coordinate.
(c)	M1	For both integrals correctly shown, with an addition sign between them. Limits need not be shown. Must be 2 integrals shown. Can't be shown as one integral with incorrect limits.
	A1	For both functions correctly integrated. Limits need not be shown.
	dM1	For their limits clearly and correctly substituted in or for the numerical expression(s) shown in the MS. If mark awarded for substitution, both integrated expressions must have both limits correctly substituted. 0 must be the lower limit on the first integral. Allow ft of their $\frac{\pi}{4}$ (must be the upper limit on the first integral and the lower limit on the second and can be in degrees) and their $\frac{\pi}{2}$ (must be the upper limit on the second integral and can be in degrees).
	A1	cao cso A0 if degrees used in part c
		Note, can also be completed as either integral doubled – symmetry. M1 – correct integral stated with multiply by 2 evident or implicit later. A1 correctly integrated dM1 – as for main scheme, the multiply by 2 must be clearly shown. A1 as main scheme

Question number	Scheme	Marks
8 (a)	$4\times1.5=6$	B1
	$\pi r^2=6$	M1
	$r=\sqrt{\frac{6}{\pi}}$	A1
		(3)
(b)	$\frac{dA}{dr}=2\pi r$	M1
	$\frac{dr}{dt}=\frac{dr}{dA}\times\frac{dA}{dt}\left[=\frac{1}{2\pi r}\times1.5\right]$	M1
	$=\frac{1}{2\pi\sqrt{\frac{6}{\pi}}}\times1.5$	M1
	$=0.173$	A1
		(4)
		Total 7 marks

Part	Mark	Notes
(a)	**B1**	For $(4\times1.5=)6$
	M1	For $\pi r^2=6$ an attempt to rearrange to make r the subject
	A1	For $r=\sqrt{\frac{6}{\pi}}$
(b)	**M1**	For $\frac{dA}{dr}=2\pi r$ This must be correct.
	M1	For application of $\frac{dr}{dt}=\frac{dr}{dA}\times\frac{dA}{dt}$
	M1	For substituting their r into $\frac{dr}{dt}$
	A1	For awrt 0.173

Question number	Scheme	Marks
8 a	$\frac{4}{3}\pi r^3 = 500$ so $r^3 = \frac{3\times 500}{4\pi}$ therefore $r = 4.92$ accept awrt 4.92	M1 A1 (2)
b	$\delta A = 20$	B1
	$(A = 4\pi r^2) \rightarrow \left(\frac{\mathrm{d}A}{\mathrm{d}r}\right) = 8\pi r$	M1
	$\delta r \approx \frac{\mathrm{d}r}{\mathrm{d}A}\delta A = 20 \times \frac{1}{8\pi r}$	M1
	When r = 4.92 $\delta r \approx \frac{20}{8\pi(4.92)} = 0.16(16204507)$	dM1
	or 0.1617428283 if 4.92 used 0.16 cm accept awrt to 0.16	A1 (5)
		Total 7 marks

Part	Marks	Notes
(a)	**M1**	For correct substitution into the formula for volume of a sphere and correct rearrangement to give r or r^3.
	A1	For $r = 4.92$
(b)	**B1**	For $\delta A = 20$ - may be stated explicitly or implicit in working. Accept $\frac{\mathrm{d}A}{\mathrm{d}t} = 20$
	M1	For $8\pi r$ A may be replaced with another variable, eg S
	M1	For $\delta r \approx \frac{\mathrm{d}r}{\mathrm{d}A}\delta A$ and substitution of 20 and their expression for $\frac{\mathrm{d}A}{\mathrm{d}r}$ Condone poor notation if substitution and their expression for $\frac{\mathrm{d}A}{\mathrm{d}r}$ is correct. Eg accept $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}A} \times \frac{\mathrm{d}A}{\mathrm{d}t}$ if using their expression for $\frac{\mathrm{d}A}{\mathrm{d}r}$
	dM1	For substitution of their value for r into their expression (as given above, poor notation condoned) for δr Dependent on previous method mark.
	A1	For 0.16 cm (units not required)
	Without appropriate calculus – no marks may be awarded for this question.	

Question number	Scheme	Marks
2(a)	$(a=)2t-3$	M1 A1 [2]
(b)	$(s=\int(t^2-3t+4)\mathrm{d}t)$	
	$(s=)\frac{t^3}{3}-\frac{3}{2}t^2+4t(+C)$ oe	M1 A1
	$(t=2, s=7)$	
	$\frac{2^3}{3}-\frac{3}{2}\times2^2+4\times2+C=7\Rightarrow C=\frac{7}{3}$	dM1
	$\frac{4^3}{3}-\frac{3}{2}\times4^2+4\times4+'\frac{7}{3}'=\frac{47}{3}$ oe	M1A1
		[5]
	ALT	
	Displacement $-7=\int_2^4\left(x^2-3x+4\right)\mathrm{d}x$	
	$=\left[\frac{x^3}{3}-\frac{3x^2}{2}+4t\right]_2^4$	[M1A1
	$=\left[\frac{4^3}{3}-\frac{3\times4^2}{2}+4\times4\right]-\left[\frac{2^3}{3}-\frac{3\times2^2}{2}+4\times2\right]$	dM1M1
	$=\frac{47}{3}$	A1]
		Total 7 marks

Part	Mark	Additional Guidance
2(a)	M1	For a minimally acceptable attempt at differentiation (see general guidance), no power of t must increase.
	A1	Correct expression with or without $a = \ldots$
(b)	M1	For a minimally acceptable attempt at integration (see general guidance), no power of t must decrease. + C is not necessary for this mark.
	A1	Fully correct integration. C is not necessary for this mark.
	dM1	Correct substitution of $t = 2$ into **their** integrated expression, correctly equated to 7 and an attempt to rearrange to find C. **Note: This mark is dependent on the first M mark in (b)**
	M1	Substitution of $t = 4$ correctly into their expression for s, provided it is a changed expression from v
	A1	Correct value oe must be an exact value.
	ALT	
	M1	For a minimally acceptable attempt at integration (see general guidance), no power of t must decrease.
	A1	Fully correct integration
	dM1	For the correct limits between $t = 4$ and $t = 2$ **and** equates to $d - 7$ or equivalent
	M1	Substitutes the limits to evaluate the integral
	A1	Correct value oe must be an exact value.

Question number	Scheme	Marks
4	When $x=\frac{\pi}{2}$ $y=\frac{\pi^3}{8}$ So $\left(\frac{\pi}{2},\frac{\pi^3}{8}\right)$	B1
	$\frac{dy}{dx}=3x^2\sin x+x^3\cos x$	M1 A1 A1
	When $x=\frac{\pi}{2}$ $\frac{dy}{dx}=3\left(\frac{\pi}{2}\right)^2\sin\left(\frac{\pi}{2}\right)+\left(\frac{\pi}{2}\right)^3\cos\left(\frac{\pi}{2}\right)=\left['\frac{3\pi^2}{4}'\right]$	M1
	$y-\frac{\pi^3}{8}=\frac{3\pi^2}{4}\left(x-\frac{\pi}{2}\right)$	M1
	$y=\frac{3}{4}\pi^2x-\frac{1}{4}\pi^3$	A1
		Total 7 marks

Mark	Notes
Note: In this question, all substitution of angle values must be in Radians only.	
B1	For obtaining $y=\frac{\pi^3}{8}$ [allow $\left(\frac{\pi}{2}\right)^3$ and also awrt y = 3.88]
M1	For an attempt to use the product rule. The definition of an attempt is as follows: • There must be a correct attempt to differentiate both terms. $\sin x\Rightarrow\cos x$ $x^3\Rightarrow ax^2$ where $x\neq 0$ • The correct formula must be used. i.e., it must be a sum of their two terms.
A1	At least one term must be correct. Either $3x^2\sin x$ or $x^3\cos x$
A1	For $3x^2\sin x+x^3\cos x$ oe Ignore any subsequent simplification once you have seen the correct answer – even if the simplification is incorrect.
M1	For substitution of $\frac{\pi}{2}$ into their $\frac{dy}{dx}=\left['\frac{3}{4}\pi^2'\right]$ provided it is a changed expression. Allow a value of awrt 7.4(0) **NOTE:** You must see a full substitution of $\frac{\pi}{2}$ into their $\frac{dy}{dx}$ if their expression for $\frac{dy}{dx}$ is incorrect.
M1	For a correct method for finding the equation of a line using their value of y, dy/dx and the given x [allow x = 1.57…]. This must be applied correctly. Either uses the formula with their values, or if uses $y=mx+c$ they must reach a value for c before this mark can be awarded. Do **not** allow processing errors to find the value of c
A1	For $y=\frac{3}{4}\pi^2x-\frac{1}{4}\pi^3$ Allow $y=7.4(0)x-7.75$ or better values. Do not allow a mixture of decimals and exact values.

Question number	Scheme	Marks
9	$\frac{dA}{dt} = 0.45$	B1
	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	B1
	$A = 6x^2 \Rightarrow \frac{dA}{dx} = 12x$	B1
	$384 = 6x^2 \Rightarrow x = 8$	M1
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dA} \times \frac{dA}{dt}$	M1
	$\frac{dV}{dt} = 3x^2 \times \frac{1}{12x} \times 0.45 \left[= \frac{9x}{80} \right]$ oe	dM1
	When $x = 8$ $\frac{dV}{dt} = 0.9\,\text{cm}^3/\text{s}$	A1
		Total 7 marks

Mark	Notes
B1	For $\frac{dA}{dt} = 0.45$ seen anywhere in their working Accept other letters, for example S for the area $\frac{dS}{dt} = 0.45$
B1	For $\frac{dV}{dx} = 3x^2$ Accept also other letters in place of x such as r for example.
B1	For $\frac{dA}{dx} = 12x$ Accept also other letters in place of x such as r for example.
M1	For setting $384 = 6x^2$ and proceeding to a correct method leading to a value of x Award this mark when they obtain $x^2 = 64 \Rightarrow x = ...$
M1	For a correct expression of the chain rule seen or implied. i.e., $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dA} \times \frac{dA}{dt}$ They may complete this in two stages. So you may see for example: $\frac{dx}{dt} = \frac{1}{\frac{dA}{dx}} \times \frac{dA}{dt}$ **AND** $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$
dM1	For substituting their values into a **correct** chain rule. $\frac{dV}{dt} = 3(8)^2 \times \frac{1}{12(8)} \times 0.45$ This mark is dependent on the previous M mark scored.
A1	For 0.9 (cm^3/s)

Question	Scheme	Marks
5(a)	$\frac{dv}{dt} = 4t - 19$ $\frac{dv}{dt} = 4\times 5 - 19 = 1\left(\text{m/s}^2\right)$	M1 A1 [2]
(b)	$2t^2 - 19t + 35 = 0 \Rightarrow (2t-5)(t-7) = 0 \Rightarrow t = \frac{5}{2}, 7$ $t_1 = \frac{5}{2} \quad t_2 = 7$	M1 A1 [2]
(c)	$D = \int_{\frac{5}{2}}^{7} \left(2t^2 - 19t + 35\right) dt = \left[\frac{2t^3}{3} - \frac{19t^2}{2} + 35t\right]_{\frac{5}{2}}^{7}$ $\left(\frac{2\times 7^3}{3} - \frac{19\times 7^2}{2} + 35\times 7\right) - \left(\frac{2\times 2.5^3}{3} - \frac{19\times 2.5^2}{2} + 35\times 2.5\right)$ $= \left[\frac{49}{6} - \frac{925}{24}\right] = [8.167 - 38.542] = \left[-\frac{243}{8} \text{ or } -30.375\right]$ $\Rightarrow D = \frac{243}{8}$ (m) oe e.g. 30.375	M1 M1 A1 [3]
		Total 7 marks

Part	Mark	Notes
(a)	M1	For differentiating the given v and substituting in $t = 5$ into the derivative. Both differentiation and substitution must be correct for this mark.
	A1	For the correct acceleration of 1 m/s^2. Units are not required.
(b)	M1	For attempting to solve the given 3TQ for the velocity. A correct method must be used, and they must reach **two** values of t for the award of this mark. See General Guidance for the definition of an attempt to solve a 3TQ. If there is no visible method seen, both values of t must be seen for the award of this mark.
	A1	For the two correct values of t They do not need to ne identified as t_1 or t_2 for this mark. Accept $\frac{5}{2}$ and 7 seen.
(c)	M1	For an attempt to integrate the given expression for v. At least one term must be correct and no terms are to be differentiated. If the value of 35 'disappears' it is M0. The question states 'use calculus' so integration must be seen.
	M1	For substituting the **two** values of t into their integrated expression. and subtracting the result (either way) of both substitutions. Allow for any changed expression from v A correct answer of $\pm\frac{243}{8}$ or ± 30.375 implies correct substitution into a correct integral. If the integration is incorrect, or the values are incorrect, full correct substitution must be seen for the award of this mark.
	A1	For the correct distance (which must be a positive value).

Question	Scheme	Marks
7(a)	$\left(\cos 3\theta + \sqrt{3}\sin 3\theta\right)^{\frac{1}{2}} = 0 \Rightarrow \cos 3\theta = -\sqrt{3}\sin 3\theta \Rightarrow \tan 3\theta = -\dfrac{1}{\sqrt{3}}$	M1
	$\Rightarrow 3\theta = -\dfrac{\pi}{6}$ **or** $\dfrac{5\pi}{6}$	A1
	$\Rightarrow m = -\dfrac{\pi}{18} \quad n = \dfrac{5\pi}{18}$	A1 [3]
(b)	$V = \pi\int_{-\frac{\pi}{18}}^{\frac{5\pi}{18}} \left(\cos 3\theta + \sqrt{3}\sin 3\theta\right) \mathrm{d}\theta$	M1
	$V = \pi\left[\dfrac{\sin 3\theta}{3} - \dfrac{\sqrt{3}\cos 3\theta}{3}\right]_{-\frac{\pi}{18}}^{\frac{5\pi}{18}}$	M1
	$V = \pi\left[\left(\dfrac{\sin 3\left(\frac{5\pi}{18}\right)}{3} - \dfrac{\sqrt{3}\cos 3\left(\frac{5\pi}{18}\right)}{3}\right) - \left(\dfrac{\sin 3\left(-\frac{\pi}{18}\right)}{3} - \dfrac{\sqrt{3}\cos 3\left(-\frac{\pi}{18}\right)}{3}\right)\right]$	M1
	$V = \dfrac{4\pi}{3}$	A1 [4]
		Total 7 marks

Part	Mark	Notes
(a)	M1	Sets the equation = 0 and obtains $\tan 3\theta = k$
	A1	Finds at least one correct value of 3θ $3\theta = -\dfrac{\pi}{6}$ **or** $\dfrac{5\pi}{6}$ Works in degrees Accept -30° **or** 150° for this mark. This mark is also implied by one correct solution for m or n in degrees or radians. **NB This is an M mark in Epen**
	A1	For $m = -\dfrac{\pi}{18} \quad n = \dfrac{5\pi}{18}$ which must be in radians. Accept embedded in coordinates. M and n do not need to be identified.

(b)	**Working in degrees**. Allow working in degrees up to the last M mark.	
	M1	For a correct statement for the volume of revolution with π and their limits. Allow $V=\pi\int_{-\frac{\pi}{18}}^{\frac{5\pi}{18}}\left(\left[\cos 3\theta+\sqrt{3}\sin 3\theta\right]^{\frac{1}{2}}\right)^2 \mathrm{d}\theta$ or $V=\pi\int_{-\frac{\pi}{18}}^{\frac{5\pi}{18}}\left(\cos 3\theta+\sqrt{3}\sin 3\theta\right)\mathrm{d}\theta$ Allow also: $V=\pi\int_{-\frac{\pi}{18}}^{\frac{5\pi}{18}}\left(\cos 3\theta+\sqrt{3}\sin 3\theta\right)\mathrm{d}x$ or even $V=\pi\int_{-\frac{\pi}{18}}^{\frac{5\pi}{18}}\left(\cos 3\theta+\sqrt{3}\sin 3\theta\right)$ **This mark can be implied by correct further working** Working in degrees $V=\pi\int_{-10^\circ}^{50^\circ}\left(\cos 3\theta+\sqrt{3}\sin 3\theta\right)\mathrm{d}\theta$
	M1	For an acceptable attempt at integration. Minimally acceptable integration is as follows. $\cos 3\theta \Rightarrow \pm\frac{\sin 3\theta}{3}$, $\sin 3\theta \Rightarrow \pm\frac{\cos 3\theta}{3}$ Ignore absence or incorrect limits and the absence of π for this mark.
	M1	For substitution of the correct limits into their integrated expression the correct way around. This must be a changed expression from the one given. If the integrated expression is correct with correct limits, allow a final volume of $\frac{4\pi}{3}$ seen without explicit substitution. If the final volume is incorrect without evidence of explicit substitution award M0. If the integrated expression is incorrect or the limits are incorrect, explicit substitution **must** be seen for the award of this mark. **Do not** allow use of degrees at this stage. Ignore absence of π for this mark.
	A1	For the correct volume $(V)=\frac{4\pi}{3}$

Question	Scheme	Marks
3(a)(i)	$v = 6t^2 - 16t + c$	M1
	$t = 0, \quad v = 12$	A1
	$v = \frac{6t^2}{2} - 16t + 12 = \left[3t^2 - 16t + 12\right]$	M1
(ii)	$s = \frac{3t^3}{3} - \frac{16t^2}{2} + 12t + k$	
	$\left[t = 0, \quad s = 0\right] \Rightarrow k = 0$	A1
	$s = t^3 - 8t^2 + 12t$	[4]
(b)	At the origin, $s = 0$	
	$t^3 - 4t^2 + 12t = 0$	M1
	$\Rightarrow t(t-2)(t-6) = 0$	dM1
	$\Rightarrow t = 2, 6, (0)$	
	P first returns to the origin when $t = 2$ seconds	A1 [3]
		Total 7 marks

Part	Mark	Notes
(a)(i)	M1	For integrating the given expression. (+c) not required for this mark See General Guidance for the definition of an attempt. No term to be differentiated.
	A1	Finds the value of c **and** the correct expression for v
(ii)	M1	For an acceptable attempt to integrate **their** v. See General Guidance for the definition of an attempt. (+k) not required for this mark. No term to be differentiated.
	A1	For the correct expression for the distance, which must have come from establishing **explicitly** that $k = 0$ from the integration. Without + k in their integration this mark is automatically lost.
(b)	M1	For setting their expression [which must be a cubic] for $s = 0$
	dM1	For solving the cubic Minimally acceptable attempt is $\Rightarrow t(t \pm A)(t \pm B) = 0$ where $\lvert AB \rvert = 12$ or also allow $(t \pm A)(t \pm B) = 0$ where $\lvert AB \rvert = 12$ as candidates are told that one value of $t = 0$ Any method to solve the quadratic is acceptable. Formula or completing the square. This is a dependent mark. <u>Use of a calculator</u> If there is no working seen and either their values of t are incorrect or their s is incorrect, only award this mark if a correct method is used/seen to solve the equation.
	A1	For the correct value of $t = 2$ explicitly stated.

Question	Scheme	Marks
6	$\frac{dV}{dt}=12$	B1
	$\frac{dV}{dh}=9h^2$	B1
	$1536=3h^3 \Rightarrow h=\sqrt[3]{\frac{1536}{3}}=8$	M1A1
	$\frac{dh}{dt}=\frac{dV}{dt}\times\frac{1}{\frac{dV}{dh}}=12\times\frac{1}{9h^2}=12\times\frac{1}{9\times8^2}=\frac{1}{48}$ (cm/s) oe	M1dM1A1
		[7]
		Total 7 marks

Mark	Notes
B1	For stating $\frac{dV}{dt}=12$ This must be clearly labelled $\frac{dV}{dt}$
B1	For differentiating the **given** expression for the volume. This must be clearly labelled $\frac{dV}{dh}$ **It must be correct for this mark.**
M1	For using the given formula, rearranged correctly to find the height of oil when the volume = 1536 That is, unless you see $h = 8$, you must see this expression. $h=\sqrt[3]{\frac{1536}{3}}$
A1	For the correct value of $h = 8$ Sight of $h = 8$ without working is M1A1
M1	For sight of a **correct** chain rule involving $\frac{dh}{dt}, \frac{dV}{dh}$ and $\frac{dV}{dt}$ only. Accept in any order. For example, accept $\frac{dh}{dt}\times\frac{dV}{dh}=\frac{dV}{dt}$ that is, $\frac{dh}{dt}$ does not need to be the subject. This mark can be implied by a correct next step.
dM1	For substituting in their values/expressions with their value of h into a correct chain rule. Note, this mark is dependent on the previous M mark.
A1	For the correct rate of increase. This question asks for an exact value. Do not accept a decimal estimate or accept $0.02083\dot{}$ unless you see a recurring sign.

	Accept any fraction that simplifies to $\frac{1}{48}$. You will see $\frac{12}{576}$ which is completely acceptable.

Question	Scheme	Marks
5	$\frac{dy}{dx} = 2e^{2x}\left(x^2-5x\right)+e^{2x}\left(2x-5\right) \Rightarrow \left[e^{2x}\left(2x-5\right) = \frac{dy}{dx} - 2y\right]$	M1A1A1
	$\frac{d^2y}{dx^2} = 4e^{2x}\left(x^2-5x\right)+2e^{2x}\left(2x-5\right)+2e^{2x}\left(2x-5\right)+2e^{2x}$	M1A1
	$\left[\text{OR } \frac{d^2y}{dx^2} = 2\frac{dy}{dx}+2e^{2x}\left(2x-5\right)+2e^{2x}\right]$	
	$\frac{d^2y}{dx^2} = 2\frac{dy}{dx}+2\left[\frac{dy}{dx}-2y\right]+2e^{2x}$	M1
	$\Rightarrow 2e^{2x} = \frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y \quad *$	A1 cso [7]
	ALT 1 LHS = RHS	
	$\frac{dy}{dx} = 2e^{2x}\left(x^2-5x\right)+e^{2x}\left(2x-5\right) = 2e^{2x}x^2-10xe^{2x}+2e^{2x}x-5e^{2x}$	
	$\left[=2e^{2x}x^2-8e^{2x}x-5e^{2x}\right]$	[M1A1A1
	$\frac{d^2y}{dx^2} = 4e^{2x}x^2+4e^{2x}x-8e^{2x}-16e^{2x}x-10e^{2x}$	
	$\left[=4e^{2x}x^2-12e^{2x}x-18e^{2x}\right]$	M1A1
	$\textbf{RHS} = 4e^{2x}x^2-12e^{2x}x-18e^{2x}-4\left(2e^{2x}x^2-8e^{2x}x-5e^{2x}\right)+4\left(e^{2x}x^2-5e^{2x}\right)$	
	$\textbf{RHS} = 4e^{2x}x^2-12e^{2x}x-18e^{2x}-8e^{2x}x^2+32e^{2x}x+20e^{2x}+4e^{2x}x^2-20e^{2x}x$	M1
	$\textbf{RHS} = 2e^{2x} = \textbf{LHS} \quad *$	A1]
	ALT 2 LHS = RHS	
	$\frac{dy}{dx} = 2e^{2x}\left(x^2-5x\right)+e^{2x}\left(2x-5\right) = \left[e^{2x}\left(2x^2-8x-5\right)\right]$	[M1A1A1
	$\frac{d^2y}{dx^2} = e^{2x}\left(4x-8\right)+2e^{2x}\left(2x^2-8x-5\right) = \left[e^{2x}\left(4x^2-12x-18\right)\right]$	M1A1
	$\textbf{RHS} = e^{2x}\left(4x^2-12x-18\right)-4\left(e^{2x}\left(2x^2-8x-5\right)\right)+4\left(e^{2x}\left(x^2-5x\right)\right)$	
	$\textbf{RHS} = 4e^{2x}x^2-12e^{2x}x-18e^{2x}-8e^{2x}x^2+32e^{2x}x+20e^{2x}+4e^{2x}x^2-20e^{2x}x$	M1
	$\textbf{RHS} = 2e^{2x} = \textbf{LHS} \quad *$	A1]
		Total 7 marks

Mark	Notes
General principles of marking this question. • You must read every line of working carefully. • The first 3 marks are for the correct application of Product Rule • The next 2 marks are for the correct reapplication of Product rule. • The final M mark is effectively for rearranging/substituting to the required form. • The final mark is only to be awarded if everything is correct**. This is a show question.** **Some candidates may use implicit differentiation, this is absolutely fine.**	
M1	For an attempt at product rule. • There must be an acceptable attempt to differentiate both terms. • The correct formula must be applied. • Minimally acceptable differentiation is as follows: $e^{2x} \Rightarrow \pm ke^{2x}$ and $x^2 - 5x \Rightarrow \pm(2x-5)$
A1	For one term correct within product rule
A1	For a fully correct derivative.
M1	For differentiating their first derivative a second time to find the second derivative. There are several different ways this can be completed. Please apply the same rules for the second derivative as for the first. There must be only 4 terms here out of which, award this mark for at **least two** terms correct.
A1	For a fully correct $\frac{d^2y}{dx^2}$ in any form
M1	For substituting $e^{2x}(2x-5) = \frac{dy}{dx} - 2y$
A1cso	For the correct final expression with no errors.
ALT 1 and 2	
M1	For an attempt at product rule. • There must be an acceptable attempt to differentiate both terms. • The correct formula must be applied. • Minimally acceptable differentiation is as follows: $e^{2x} \Rightarrow \pm ke^{2x}$ and $x^2 - 5x \Rightarrow \pm(2x-5)$
A1	For one term correct within product rule
A1	For both terms fully correct.
M1	For differentiating their first derivative a second time to find the second derivative. There are several different ways this can be completed. Please apply the same rules for the second derivative as for the first. Look for two out of four terms correct If their starting point is $\frac{dy}{dx} = e^{2x}(2x^2 - 8x - 5)$ or $2e^{2x}x^2 - 8e^{2x}x - 5e^{2x}$ check their differentiation for the M mark and apply 1 out of 2, or 2 out of 4 correct..
A1	For a fully correct $\frac{d^2y}{dx^2}$ in any form
M1	Collects up all terms on the RHS to obtain Ke^{2x} $K \neq 0$ **Please check their working carefully.**
A1cso	For the correct final expression with no errors.

Question number	Scheme	Marks
1 (a)	$b^2-4ac=(-9)^2-4\times3\times7=-3$ $-3<0$ so no (real) solutions for v	M1 A1 [2]
(b)	$\left\{\frac{dv}{dt}\right\}=6t-9\Rightarrow a=6\times4-9=15\left(\text{m/s}^2\right)$	M1A1 [2]
(c)	$s=\int_0^5(3t^2-9t+7)\,dt=\left[t^3-\frac{9}{2}t^2+7t\right]_0^5$ $=5^3-\frac{9}{2}\times5^2+7\times5=\frac{95}{2}(\text{m})$ oe	M1 M1A1 [3]
		Total 7 marks

Part	Mark	Additional Guidance
(a)	M1	Applies the discriminant, or any other valid method. e.g. Attempts to solve the 3TQ by quadratic formula, Completing square, achieves $3\left(t-\frac{3}{2}\right)^2+...$ Attempts to solve $v = 0$, finds complex roots
	A1	Verifies that the equation for v has no (real) solutions By completing square, it must be correct, $3\left(t-\frac{3}{2}\right)^2+\frac{1}{4}$, states the graph is above x axis, or > 0 Finds correct complex roots $\frac{9\pm\sqrt{3}i}{6}$, states imaginary numbers/roots **Note**: Just stating "error" is not enough
(b)	M1	Attempts to differentiate the expression for v. Power of t to decrease in at least one term and increase in none.
	A1	Substitutes $t = 4$ and obtains correct acceleration, with or without unit.
(c)	M1	Attempt to integrate the expression for v. Power of t to increase in at least one term and decrease in none. Ignore limits if shown.
	M1	Clear substitution of the **given** limits into their **integrated** expression for distance, no need to see substitution of zero, can be implied by the correct distance found from a **correct integrated** expression
	A1	$\frac{95}{2}$ (m) oe e.g. 47.5, ignore unit

Question number	Scheme	Marks
11	Intersection of the curve with the x-axis to find lower limit $\{y=\sqrt{4x-8}=0 \Rightarrow 4x-8=0\} \Rightarrow x=2$	B1
	Volume of rotation	
	$V=\pi\int_2^b\left(\sqrt{4x-8}\right)^2\,\mathrm{d}x=\left\{\pi\int_2^b(4x-8)\,\mathrm{d}x\right\}$	M1
	$V=\pi\left[\frac{4x^2}{2}-8x\right]_2^b$	M1
	$50\pi=\pi\left[\left(\frac{4\times b^2}{2}-8\times b\right)-\left(\frac{4\times 2^2}{2}-8\times 2\right)\right]$	M1
	$50=2b^2-8b+8 \Rightarrow 0=b^2-4b-21$ $0=(b+3)(b-7)$ $\Rightarrow b=7$	dddM1 ddddM1 A1 [7]
		Total 7 marks

Mark	Additional Guidance
B1	Finds the lower limit of the integral: For x = 2 or (2,0) seen/embedded in their integral/on the graph
M1	Sets up a correct expression for the volume with their lower limit (a numerical value) found earlier and the upper limit of b, must have π
M1	Correctly integrates $4x-8$, with or without π
M1	Substitutes their lower limit (a numerical value) and upper limit b correctly into **their integrated** expression (with or without π), and equates to 50π / 50
dddM1	Forms a 3TQ in b/x Dependent on previous three M marks
ddddM1	Solves the 3TQ with any correct method Can be implied by their correct b value from a correct 3TQ Dependent on all previous M marks
A1	For b = 7 only, from correct working, must be b=7, not x =7
Note: If attempted the volume of revolution about the y-axis, only B1 could be possibly awarded, no other marks could be scored.	

Question Number	Scheme	Marks
6(a)(i)	$\frac{dy}{dx} = 4e^{2x} + 2(4x-3)e^{2x}$	M1A1A1 (3)
(ii)	$(4x-3)\frac{dy}{dx} = (4x-3)(8x-2)e^{2x} = (8x-2)y$ *	M1A1cso (2)
(b)	$\frac{dy}{dx} = \frac{5\cos 5x \times (x-3)^2 - \sin 5x \times 2(x-3)}{(x-3)^4}$	M1A1A1 (3)
ALT	Using product rule:	
	$y = (x-3)^{-2}\sin 5x$	M1
	$\frac{dy}{dx} = -2(x-3)^{-3}\sin 5x + 5(x-3)^{-2}\cos 5x$	A1A1
		[8]

(a)(i)

M1 Use product rule to differentiate the given expression. Must have 2 terms added. One to be of the form ke^{2x} and the other of the form $k'(4x-3)e^{2x}$ where k' = 1 or 2

A1 Either term correct

A1 Second term correct

NB No simplification needed for these 3 marks

ALT $y = 4xe^{2x} - 3e^{2x} \Rightarrow \frac{dy}{dx} = 4e^{2x} + 8xe^{2x} - 6e^{2x}$

M1 Expand the given expression and differentiate using the product rule for $4xe^{2x}$

A1 Any 2 terms correct; A1 Third term correct.

NB No simplification needed for these 3 marks

(ii)

M1 Use their result from (i) to obtain an expression for $(4x-3)\frac{dy}{dx}$. No need to simplify.

A1cso Correct **given** result obtained with no errors in the working.
Can start with LHS and show equal to the RHS or vice versa or can start with each side and "meet in the middle"

(b)

M1 Attempt the quotient rule. The denominator must be $(x-3)^4$ and the numerator must be of the form $\left(k\cos 5x \times (x-3)^2 - \sin 5x \times l(x-3)\right)$ $k = \pm 5$ or ± 1, $l = 1$ or 2
(ie sine may have been differentiated to − cosine)

A1 One fully correct term in numerator.

A1 All fully correct.

ALT

M1 Rewrite without a quotient and apply the product rule obtaining 2 terms of the form shown

A1 Either term correct

A1 Second term correct
No need to simplify

Question number	Scheme	Marks
4 (a)	$v=\int(6t-12)\ \mathrm{d}t=3t^2-12t\ (+c)$ When $t=0$, $v=0$ so $c=0$ $v=3t^2-12t=3\times2^2-12\times2=-12\ (\text{m/s})$	M1A1 A1cso [3]
(b)	$v=0,\ 3t^2-12t=0\Rightarrow 3t(t-4)=0\Rightarrow T=4$	M1A1 [2]
(c)	$s=\int(3t^2-12t)\ \mathrm{d}t=t^3-6t^2\ (+k)$ $d=\left\lvert\left[t^3-6t^2\right]_0^4\right\rvert+\left[t^3-6t^2\right]_4^8=32+160=192\ \text{(m)}$	M1 M1A1 [3]
Alternative		
(c)	$s=\int(3t^2-12t)\ \mathrm{d}t=t^3-6t^2\ (+k)$ At time $t=0$, distance = 0, $k=0$ $t=4\quad s=4^3-6\times4^2=64-96=-32$ $t=8\quad s=8^3-6\times8^2=512-384=128$ Total distance travelled = 32 +32 +128 = 192 (m)	M1 M1 A1
		Total 8 marks

(a)	M1	Attempt to integrate $6t-12$, must see power of t increased by one in either term and no decrease in powers. Do not accept $v=t(6t-12)$.
	A1	Correct result of integration. Constant is not required.
	A1cso	Correct velocity. Negative sign is required. Must show or state constant = 0.
(b)	M1	$3t^2-12t=0$ and attempt to solve. ft their integration if M1 awarded in (a).
	A1	$T=4$ Allow $t=4$. Do not allow if $T=0$ is also given as an option.
(c)	M1	Attempt to integrate their velocity providing it includes at least one term in t^2 or higher powers. Must see power of t increased by one in at least one term and no decrease in powers. Do not accept $s=t\times$ their v.
	M1	Attempt to use limits of 0 to 4 and 4 to 8 and then add the magnitudes of the two values. ft their $T=4$ if M1 awarded in (b).
	A1	192
Alternative		
(c)	M1	Attempt to integrate their velocity providing it includes at least one term in t^2 or higher powers. Must see power of t increased by one in at least one term and no decrease in powers. Do not accept $v=t(6t-12)$.
	M1	Substitute and attempt to find distance when $t=4$ and when $t=8$. Working to confirm $k=0$ must be shown. ft their $T=4$ if M1 awarded in (b).
	A1	192

Question number	Scheme	Marks
8	$s=\int\left(3+5t-2t^2\right)\,\mathrm{d}t=3t+\frac{5t^2}{2}-\frac{2t^3}{3}+c$	M1
	when $t=0$ $s=5$	
	$5=0+0-0+c$	B1
	$s=5+3t+\frac{5t^2}{2}-\frac{2t^3}{3}$	A1
	When $s=x$	
	$\frac{\mathrm{d}x}{\mathrm{d}t}=3+5t-2t^2=0\Rightarrow(2t+1)(t-3)=0\Rightarrow t=3$	M1A1
	$\Rightarrow x=5+3\times3+\frac{5\times3^2}{2}-\frac{2\times3^3}{3}=\frac{37}{2}$ oe	A1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=5-4t$ when $t=3$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-7\Rightarrow$ max	M1A1
		[8]
		Total 8 marks

M1	Attempt to integrate
B1	$c=5$
A1	$s=5+3t+\frac{5t^2}{2}-\frac{2t^3}{3}$
M1	Solving $3+5t-2t^2=0$
A1	$t=3$ if shown must reject $t=-\frac{1}{2}$
A1	$x=\frac{37}{2}$ oe
M1	Differentiates to obtain $\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\right)5-4t$
A1	Establish that the maximum has been obtained and give a conclusion

Question Number	Scheme	Marks
6 (a)	When $x = \frac{3}{2}$ $y = 3 - \frac{9}{4} = \frac{3}{4}$ and $2y - \frac{3}{2} = 0$	M1
	So $\left(\frac{3}{2}, \frac{3}{4}\right)$ lies on both the line and the curve	A1cso (2)
(b)	$(\pi)\int_0^{\frac{3}{2}}(2x - x^2)^2 \mathrm{d}x$	M1
	$= (\pi)\int_0^{\frac{3}{2}}(4x^2 + x^4 - 4x^3)\mathrm{d}x$	A1
	$= (\pi)\left[\frac{4x^3}{3} + \frac{x^5}{5} - x^4\right]_0^{\frac{3}{2}}$	dM1
	$= \frac{153(\pi)}{160}$	A1
	$\frac{153\pi}{160} - \frac{1}{3}\pi\left(\frac{3}{4}\right)^2\left(\frac{3}{2}\right) = \frac{27\pi}{40}$	ddM1A1cao (6) **[8]**
ALT:	$\pi\int_0^{\frac{3}{2}}\left((2x - x^2)^2 - \left(\frac{1}{2}x\right)^2\right)\mathrm{d}x$	M1
	$\pi\int_0^{\frac{3}{2}}(\frac{15}{4}x^2 - 4x^3 + x^4)\,\mathrm{d}x$	A1
	$\pi\left[\frac{5x^3}{4} - x^4 + \frac{x^5}{5}\right]_0^{\frac{3}{2}}$	dM1A1
	$\pi\left[\frac{135}{32} - \frac{81}{16} + \frac{243}{160}\right] = \frac{27\pi}{40}$	ddM1A1cao

(a) M1	Attempt to show that $\left(\frac{3}{2}, \frac{3}{4}\right)$ lies on the curve and the line. Any valid method including solving the equations allowed.
A1cso	Appropriate conclusion following correct work. Verification, as shown, needs a conclusion.
	Solving the equations to obtain $\frac{x}{2} = 2x - x^2$ or $y = 4y - 4y^2$ and hence coordinates of A needs no conclusion. M1 for reaching coords, A1 for correct coords (decimals allowed)

Question Number	Scheme	Marks
(b)	**Algebraic integration must be seen – otherwise no marks.** The first 4 marks can be awarded with or without π provided the work is consistent. The first 3 marks can be awarded if no limits are shown.	
M1	Correct integral, with or without π. Limits may be missing – ignore any shown.	
A1	Square the bracket correctly.	
dM1	Attempt the integration of their integrand. The power of at least one term should increase and no power should decrease. Ignore limits.	
A1	Substitute the correct limits and obtain $\frac{153}{160}$ or $\frac{153\pi}{160}$ (0.95625(pi))	
ddM1	Subtract the volume of the cone from their previous answer. Both terms to include π	
A1cao	Correct final answer (0.675pi)	
ALT:	**See above for general instructions re integration**	
M1	Integral must be the difference of 2 squared terms	
A1	Correct integrand after squaring, need not be simplified	
dM1	Attempt the integration of their integrand. The power of at least one term should increase and no power should decrease.	
A1	Correct result	
ddM1	Substitute their limits	
A1cao	Correct final answer.	

Question Number	Scheme	Marks
8	$y=e^{3x}\sin 2x \quad \dfrac{dy}{dx}=2e^{3x}\cos 2x+3e^{3x}\sin 2x$	M1A1
	$\dfrac{d^2y}{dx^2}=\left(-4e^{3x}\sin 2x+6e^{3x}\cos 2x\right)+\left(6e^{3x}\cos 2x+9e^{3x}\sin 2x\right)$	M1A1A1
	$=12e^{3x}\cos 2x+5e^{3x}\sin 2x$	
	$\dfrac{d^2y}{dx^2}-6\dfrac{dy}{dx}+13y$	
	$=12e^{3x}\cos 2x+5e^{3x}\sin 2x-6\left(2e^{3x}\cos 2x+3e^{3x}\sin 2x\right)+13e^{3x}\sin 2x$	dM1
	$=12e^{3x}\cos 2x+5e^{3x}\sin 2x-12e^{3x}\cos 2x-18e^{3x}\sin 2x+13e^{3x}\sin 2x$	ddM1
	$=0$ *	A1cso **[8]**
ALT	$\dfrac{dy}{dx}=2e^{3x}\cos 2x+3e^{3x}\sin 2x$	M1A1
	$\dfrac{dy}{dx}=2e^{3x}\cos 2x+3y$	
	$\dfrac{d^2y}{dx^2}=\left(-4e^{3x}\sin 2x+6e^{3x}\cos 2x\right)+3\dfrac{dy}{dx}$	M1A1A1
	$\dfrac{d^2y}{dx^2}-6\dfrac{dy}{dx}+13y=\left(-4e^{3x}\sin 2x+6e^{3x}\cos 2x+3\dfrac{dy}{dx}\right)-6\dfrac{dy}{dx}+13y$	dM1
	$=-13y+6e^{3x}\cos 2x+9e^{3x}\sin 2x-3\dfrac{dy}{dx}+13y$	
	$=-13y+3\dfrac{dy}{dx}-3\dfrac{dy}{dx}+13y=0$ *	ddM1A1cso **[8]**

M1	Attempt the product rule. 2 terms of the form $\pm ke^{3x}\cos 2x$ and $\pm le^{3x}\sin 2x$ with $k=1$ or 2 and $l=1$ or 3
A1	Fully correct first derivative
M1	Attempt the second derivative using the product rule *correctly* on either term. Must have at least one of the terms in the first derivative fully correct.
A1 A1	A1 for each fully correct bracket
dM1	Substitute their derivatives and y in $\dfrac{d^2y}{dx^2}-6\dfrac{dy}{dx}+13y$ Depends on both previous M marks This and the following M mark may be awarded together.
ddM1	Remove the brackets
A1cso	Reach "0" from fully correct work.
ALT M1A1	As above
M1	Replace sin term with a y term and attempt the second derivative using the product rule on first term.
A1A1	A1 Correct bracket A1 Correct second term
dM1	As above
ddM1	Obtain an expression which is either all derivatives plus y terms or all trig terms
A1cso	Reach "0" from fully correct work

Question number	Scheme	Marks
9	$u = \mathrm{e}^{-t}$ $\quad v = \sin 2t$ $u' = -\mathrm{e}^{-t}$ $\quad v' = 2\cos 2t$ $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 2\mathrm{e}^{-t}\cos 2t - \mathrm{e}^{-t}\sin 2t = 2\mathrm{e}^{-t}\cos 2t - x$ $u = 2\mathrm{e}^{-t}$ $\quad v = \cos 2t$ $u' = -2\mathrm{e}^{-t}$ $\quad v' = -2\sin 2t$ $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4\mathrm{e}^{-t}\sin 2t - 2\mathrm{e}^{-t}\cos 2t - \dfrac{\mathrm{d}x}{\mathrm{d}t}$ $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4x - \left(\dfrac{\mathrm{d}x}{\mathrm{d}t} + x\right) - \dfrac{\mathrm{d}x}{\mathrm{d}t} = -5x - 2\dfrac{\mathrm{d}x}{\mathrm{d}t}$ $\therefore \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\dfrac{\mathrm{d}x}{\mathrm{d}t} + 5x = 0$ *	M1 A1 A1 M1 A1 dM1 dM1 A1 cso **[8]**

	Notes
M1	For an attempt to differentiate using the product rule. Must have 2 terms added together.
A1	For one correct term
A1	For two correct terms
M1	Attempts to differentiate $\dfrac{\mathrm{d}x}{\mathrm{d}t}$
A1	For $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4\mathrm{e}^{-t}\sin 2t - 2\mathrm{e}^{-t}\cos 2t - \dfrac{\mathrm{d}x}{\mathrm{d}t}$ oe
dM1	Dependant on previous M mark - for substitution of x and $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ into $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$ or for substitution of x, $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$ into the given equation
dM1	Dependant on previous M mark - for simplifying to $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -5x - 2\dfrac{\mathrm{d}x}{\mathrm{d}t}$ oe or All 5 correct terms seen and an attempt to simplify (5 correct terms may be implied by 7 correct terms)
A1 cso	Obtains the given equation or clear working to show that the equation = 0

Question number	Scheme	Marks
9 (a)	$\frac{dA}{dt} = 0.03$	B1
	$A = \frac{1}{2}x^2 \sin 60^\circ = \frac{\sqrt{3}}{4}x^2$	M1
	$\frac{dA}{dx} = \frac{\sqrt{3}}{2}x$	A1
	When $x = 2$ $\frac{dx}{dt} = \frac{1}{\sqrt{3}} \times 0.03 = 0.0173\,\text{cm/s}$	M1 A1 (5)
(b)	$V = \sqrt{3}x^3$ $\quad \frac{dV}{dx} = 3\sqrt{3}x^2$	M1
	When $x = 2$ $\frac{dV}{dt} = 12\sqrt{3} \times 0.0173 = 0.36$	M1 A1 (3)
		Total 8 marks

Part	Mark	Guidance
(a)	B1	For stating or using correctly in their Chain Rule $\frac{dA}{dt}=0.03$
	M1	For using the **correct** formula $\left(\frac{1}{2}ab\sin C\right)$ with the correct lengths and angle of 60° or $\frac{\pi}{3}$, for the cross-sectional area of the prism to obtain $A=\frac{1}{2}x^2\sin 60^{\circ}=\left(\frac{\sqrt{3}}{4}x^2\right)$ and differentiating their expression which must be as a minimum $A=px^2$ to obtain $\frac{dA}{dx}=qx$ [where p and q are constants]. [The height of the triangle is $\frac{\sqrt{3}}{2}x$ if they use $\frac{1}{2}\times\text{base}\times\text{height}$]
	A1	For the correct $\frac{dA}{dx}=\frac{\sqrt{3}}{2}x$
	M1	For applying a **correct** Chain rule using their $\frac{dA}{dx}$ and $x = 2$ to obtain $\frac{dx}{dt}=\left(\frac{1}{\frac{dA}{dx}}\times\frac{dA}{dt}\right)=\frac{dx}{dA}\times\frac{dA}{dt}=\text{'}\frac{2}{\sqrt{3}}\text{'}\times\frac{1}{2}\times 0.03$
	A1	$\frac{dx}{dt}=0.0173$
(b)	M1	For a correct expression for the volume using their A from part (a) to obtain $V=\frac{\sqrt{3}}{4}x^2\times 4x=\left(\sqrt{3}x^3\right)$ and differentiating their expression which must be as a minimum $V=mx^3$ to obtain as a minimum $\frac{dV}{dx}=nx^2$ [where m and n are constants] $\left(\frac{dV}{dx}=3\sqrt{3}x^2\right)$
	M1	For applying a correct Chain rule using their $\frac{dV}{dx}$ and $x = 2$ to obtain $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}=12\sqrt{3}\times 0.0173=\lceil 0.359...\rceil$ $\left(\text{ft their }\frac{dx}{dt}\right)$ **Note:** $\frac{dx}{dt}=0.0173$ or $\frac{\sqrt{3}}{100}$ $\left(\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}=12\sqrt{3}\times\frac{\sqrt{3}}{100}=\frac{9}{25}=0.36\right)$
	A1	For awrt 0.36

Question number	Scheme	Marks
11	$e^x = \frac{4}{e^x} \Rightarrow e^{2x} = 4$	M1
	$x = \frac{1}{2}\ln 4$ or $x = \ln 2$	A1
	$\pi\int_0^{\ln 2} e^{2x}dx + \pi\int_{\ln 2}^{a} 16e^{-2x}dx$	M1 M1
	$\pi\left[\frac{1}{2}e^{2x}\right]_0^{\ln 2} + \pi\left[-8e^{-2x}\right]_{\ln 2}^{a}$	M1
	$\pi\left(2-\frac{1}{2}\right)+\pi\left(-8e^{-2a}+2\right)=$	M1
	$\frac{7\pi}{2}-8\pi e^{-2a}$ $\quad a=2$ and $k=\frac{7\pi}{2}$	M1A1
		Total 8 marks

Mark	Guidance
M1	Sets $e^x = \frac{4}{e^x}$ and attempts to make x the subject A minimum of $e^{2x} = 4$ or $e^x = 2$ is required to be seen for this mark.
A1	For obtaining either $x = \frac{1}{2}\ln 4$ or $x = \ln 2$
M1	For stating $\pi\int_0^{\ln 2}\left(e^x\right)^2 dx = \pi\int_0^{\ln 2} e^{2x}dx$ using the limits of their ln 2 (which must be of the form ln k) and 0 correctly **Note:** Condone a missing π here if it is seen at the final M mark.
M1	For stating $\pi\int_{\ln 2}^{a}\left(4e^{-x}\right)^2 dx = \pi\int_{\ln 2}^{a} 16e^{-2x}dx$ using a and the limit of their ln 2 (which must be of the form ln k where k is consistent between the two integrals) correctly **Note:** Condone a missing π if it is seen at the final M mark.
M1	For an attempt to integrate **both** expressions obtaining: Either $\left[\frac{e^{2x}}{2}\right]$ or $\left[\frac{-16e^{-2x}}{2}\right]$ $\left(\text{condone } \left[\frac{16e^{-2x}}{2}\right]\right)$ For this mark ignore the absence of π or incorrect/absent limits [need not be simplified]
dM1	For substituting **their** limits correctly (where their ln 2 must be of the form ln k where k is consistent between the two integrals) into their integrated expression. For this mark ignore the absence of π $\pi\left(\frac{e^{2\ln 2}}{2} - \frac{e^0}{2}\right) + \pi\left(-16e^{-2a} - \frac{\left(-16e^{-2\ln 2}\right)}{2}\right) = \pi\left(2 - \frac{1}{2}\right) + \pi\left(-8e^{-2a} + 2\right)$ This mark is dependent on the previous M mark.
M1	For equating to the given expression for the volume and equating coefficients to find values for a and k. We must see π used here for the award of this mark. $\left(\pi\left(2 - \frac{1}{2}\right) + \pi\left(-8e^{-2a} + 2\right) = \frac{7\pi}{2} - 8\pi e^{-2a}\right)$ $\frac{7\pi}{2} - 8\pi e^{-2a} = k - 8\pi e^{-4} \Rightarrow k = \ldots,\ a = \ldots$
A1	For $a = 2$ and $k = \frac{7\pi}{2}$

Question number	Scheme	Marks
6 (a)	8 (m) (must be positive)	B1 [1]
(b)	$\frac{ds}{dt} = 3t^2 - 8t - 16$ $\frac{ds}{dt} = 0$ when P is stationary $\Rightarrow 3t^2 - 8t - 16 = 0$ $3t^2 - 8t - 16 = (3t + 4)(t - 4) = 0 \Rightarrow t = 4$	M1A1 M1A1 [4]
(c)	$\frac{d^2s}{dt^2} = 6t - 8$ $6t - 8 = 10 \Rightarrow t = 3$	B1ft M1A1 [3]
		Total 8 marks

Part	Mark	Notes
(a)	**B1**	For 8 (m) only
(b)	**M1**	For an attempt to differentiate the expression for s $\left[v \text{ or } \frac{ds}{dt}\right] = 3t^2 - 8t - 16$ See General guidance for the definition of an attempt with no terms integrated.
	A1	For the correct expression for v or $\frac{ds}{dt}$
	M1	For setting v or $\frac{ds}{dt} = 0$ and solving their 3TQ to find at least one value of t.
	A1	For $t = 4$ If two values of t are given $\left(t = -\frac{4}{3}\right)$ do not award this mark
(c)	**B1ft**	For the correct expression for the acceleration. Ft their $\frac{ds}{dt}$ The differentiation must be correct for this mark.
	M1	For setting their differentiated expression = 10 and solving for t
	A1	For $t = 3$

Question number	Scheme	Marks
8	When $(x=\pi \quad y=)2\pi^2$	B1
	$\frac{dy}{dx}=4x-\cos x$	M1
	When $x=\pi \quad \frac{dy}{dx}=4\pi-\cos\pi \quad =4\pi+1$	M1 A1
	Gradient of the normal $=-\frac{1}{"4\pi+1"}$	M1
	$y-2\pi^2=-\frac{1}{4\pi+1}(x-\pi)$ oe	M1
	$\left[(4\pi+1)(y-2\pi^2)=-x+\pi\right]$	
	$4\pi y+y-8\pi^3-2\pi^2-\pi+x=0$	dM1
	$x+(4\pi+1)y-\pi(8\pi^2+2\pi+1)=0$ *	A1 cso
		Total 8 marks

Mark	Notes
B1	For $(y=)2\pi^2$
M1	For differentiating *y* wrt *x*, to give an expression of the form $ax - \cos x$
M1	For correct substitution of $x=\pi$ into their derivative
A1	For $\frac{dy}{dx}=4\pi+1$
M1	For gradient of the normal $=-\frac{1}{"(4\pi+1)"}$
M1	For a fully correct method to find the equation of the line, using the correct *x* coordinate and their *y* coordinate and their gradient, which must be $=-\frac{1}{their(4\pi+1)}$ and must have come from differentiation. If $y = mx + c$ is the method used, *c* must be found and the line written as *y* =. If this method is used, this mark can be implied by the next method mark.
dM1	For expanding and rearranging their equation (allow one sign or algebraic error) to form an unsimplified equation = 0 Dependent on previous method mark.
A1cso	For obtaining the given expression

Question number	Scheme	Marks
5 (a)	$4e^{4x}(6x+2)^{\frac{3}{2}}+\frac{3}{2}.6.e^{4x}(6x+2)^{\frac{1}{2}}=\left[4e^{4x}(6x+2)^{\frac{3}{2}}+9e^{4x}(6x+2)^{\frac{1}{2}}\right]$ $e^{4x}\sqrt{6x+2}(24x+17)$	M1 A1 A1 dM1 A1 [5]
(b)	$\frac{3\cos 3x(2x-4)^3-\sin 3x.2.3.(2x-4)^2}{\left[(2x-4)^3\right]^2}=\left[\frac{3\cos 3x(2x-4)^3-6\sin 3x(2x-4)^2}{(2x-4)^6}\right]$ OR $\frac{3\cos 3x(2x-4)^3-\left(24x^2-96x+96\right)\sin 3x}{(2x-4)^6}$ ALT using product rule $\sin 3x(2x-4)^{-3}$ $3\cos 3x(2x-4)^{-3}+\sin 3x(-3)(2)(2x-4)^{-4}=\left[3\cos 3x(2x-4)^{-3}-6\sin 3x(2x-4)^{-4}\right]$	M1 A1 A1 M1 A1 A1 [3]
	Total	**8 marks**

Part	Mark	Additional Guidance
(a)	M1	Use of product rule to give an expression of the form $pe^{4x}(6x+2)^{\frac{3}{2}}+qe^{4x}(6x+2)^{\frac{1}{2}}$ where p = 1 or 4, q > 1. There must be a + sign between terms.
	A1	Either term correct. Note that simplification is not required for this mark.
	A1	Both terms correct. Note that simplification is not required for this mark.
	dM1	Obtains an expression of the form $e^{4x}\left(\sqrt{6x+2}\right)(Ax+B)$ where $A, B \neq 0$ You **must** check their working here that it follows from their attempt at product rule. There must be an intermediate step seen as follows for example: $4e^{4x}(6x+2)^{\frac{3}{2}}+9e^{4x}(6x+2)^{\frac{1}{2}}=e^{4x}(6x+2)^{\frac{1}{2}}\left(4\left(6x+2\right)+9\right)$ **Note: This mark is dependent on the first M Mark**
	A1	Correct A = 24 and correct B = 17
(b)	M1	Attempt at the quotient rule. Numerator must be the **difference** of two terms (either way round) of the form $k\cos 3x(2x-4)^3-l\sin 3x(2x-4)^2$, $k=\pm 3$ $l \geqslant 3$ OR accept: (terms wrong way around) $l\sin 3x(2x-4)^2-k\cos 3x(2x-4)^3$, $k=\pm 3$ $l \geqslant 3$ Denominator must be of the form $(2x-4)^6$ or accept $\left[(2x-4)^3\right]^2$
	A1	Either term correct, either way round.
	A1	Fully correct
	ALT	
	M1	Use of product rule to give an expression of the form $k\cos 3x(2x-4)^{-3}-l\sin 3x(2x-4)^{-4}$, $k=\pm 3$ $l \geqslant 3$ Accept for this mark $l\sin 3x(2x-4)^{-4}-k\cos 3x(2x-4)^{-3}$
	A1	Either term correct.
	A1	Fully correct
	NB: Ignore any subsequent simplification of their derivative.	

Question number	Scheme	Marks
6 a	$\cos 2A = \cos^2 A - \sin^2 A$ $= \left(1 - \sin^2 A\right) - \sin^2 A = 1 - 2\sin^2 A$ *	M1 A1 cso (2)
b	$\pi\int_0^{\frac{\pi}{4}} \left(3 + 2\sin x\right)^2 \mathrm{d}x$	M1
	$\pi\int_0^{\frac{\pi}{4}} \left(9 + 12\sin x + 4\sin^2 x\right) \mathrm{d}x$	M1
	$\pi\int_0^{\frac{\pi}{4}} \left(11 + 12\sin x - 2\cos 2x\right) \mathrm{d}x$	M1
	$\pi\left[11x - 12\cos x - \sin 2x\right]_0^{\frac{\pi}{4}}$	M1
	$\pi\left[\left(\frac{11\pi}{4} - 6\sqrt{2} - 1\right) - \left(0 - 12 - 0\right)\right] =$ awrt 35	M1 A1 (6)
		Total 8 marks

Part	Mark	Notes
(a)	**M1**	For the use of both $\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A = \left(1 - \sin^2 A\right)$ to attempt to form the required identity.
	A1	For $\cos 2A = 1 - 2\sin^2 A$ with no errors seen.
(b)	**M1**	For the correct use of use of $\pi\int_a^b y^2 \mathrm{d}x$ to obtain: $\pi\int_0^{\frac{\pi}{4}} (3 + 2\sin x)^2 \mathrm{d}x$ with the correct limits and π You may not see π and the limits at this stage. They may add these in at the end of their work. Check and if that is the case, award this mark.
	M1	For expanding the given quadratic correctly to achieve: This must be correct. $\pi\int_0^{\frac{\pi}{4}} \left(9 + 12\sin x + 4\sin^2 x\right) \mathrm{d}x$ or accept $\pi\int_0^{\frac{\pi}{4}} \left(9 + 6\sin x + 6\sin x + 4\sin^2 x\right) \mathrm{d}x$ Ignore limits and π for this mark. **This is an A mark in Epen**
	M1	For using $\sin^2 A = \frac{1 - \cos 2A}{2}$ and substituting this identity into their expansion to attempt to obtain: $\pi\int_0^{\frac{\pi}{4}} \left(11 + 12\sin x - 2\cos 2x\right) \mathrm{d}x$ As a minimum we need to see: $A + B\sin x \pm C\cos 2x$ Ignore limits and π for this mark.
	M1	For an attempt to integrate their expression which must be of the form: $A + B\sin x \pm C\cos 2x$ **or** $A + B\sin x \pm C\sin^2 x$ where there must be an acceptable attempt to integrate as a minimum at least **both** A **and** $B\sin x$ to obtain $A \to Ax$ and $B\sin x \to -B\cos x$ Ignore limits and π for this mark.
	M1	For the correct substitution of the correct given limits the correct way around into an integrated/changed expression. $\pi\left[\left(\frac{11\pi}{4} - 6\sqrt{2} - 1\right) - (0 - 12 - 0)\right] = 35.041....$ This mark can be implied by the correct final answer **ONLY** if the correct integration is seen explicitly in the response. If their integrated expression is incorrect, and no substitution is seen award M0 here.
	A1	For awrt 35

Question	Scheme	Marks
3	$\frac{dy}{dx} = 3e^{3x}\sin 2x + 2e^{3x}\cos 2x$	M1A1A1
	Method A $\frac{d^2y}{dx^2} = 3\left(3e^{3x}\sin 2x + 2e^{3x}\cos 2x\right) + 6e^{3x}\cos 2x - 4e^{3x}\sin 2x$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 6e^{3x}\cos 2x - 4y$ $2e^{3x}\cos 2x = \frac{dy}{dx} - 3y \Rightarrow 6e^{3x}\cos 2x = 3\frac{dy}{dx} - 9y$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 3\frac{dy}{dx} - 9y - 4y \Rightarrow 13y + \frac{d^2y}{dx^2} = 6\frac{dy}{dx}$	M1 M1 M1 M1A1 [8]
	Method B $\frac{dy}{dx} = 3y + 2e^{3x}\cos 2x$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 6e^{3x}\cos 2x - 4e^{3x}\sin 2x,\ \ \frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 6e^{3x}\cos 2x - 4y$ $2e^{3x}\cos 2x = \frac{dy}{dx} - 3y \Rightarrow 6e^{3x}\cos 2x = 3\frac{dy}{dx} - 9y$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 3\frac{dy}{dx} - 9y - 4y \Rightarrow 13y + \frac{d^2y}{dx^2} = 6\frac{dy}{dx}$	[M1,M1 M1 M1A1]
	Method C $\frac{d^2y}{dx^2} = 3\left(3e^{3x}\sin 2x + 2e^{3x}\cos 2x\right) + 6e^{3x}\cos 2x - 4e^{3x}\sin 2x$ $13y + \frac{d^2y}{dx^2} = 13e^{3x}\sin 2x + (5e^{3x}\sin 2x + 12e^{3x}\cos 2x)$ $6\frac{dy}{dx} = 6(3e^{3x}\sin 2x + 2e^{3x}\cos 2x)$ $(= 18e^{3x}\sin 2x + 12e^{3x}\cos 2x)$ $LHS = 13e^{3x}\sin 2x + (5e^{3x}\sin 2x + 12e^{3x}\cos 2x)$ $= 18e^{3x}\sin 2x + 12e^{3x}\cos 2x$ $RHS = 6(3e^{3x}\sin 2x + 2e^{3x}\cos 2x)$ $= 18e^{3x}\sin 2x + 12e^{3x}\cos 2x$ $LHS = RHS$	[M1 M1 M1 M1A1]
		Total 8 marks

Question	Notes	Marks
3	$y = e^{3x}\sin 2x$	
	For an attempt to differentiate the given expression.	

	• There needs to be an acceptable attempt to differentiate both terms. $e^{3x}\sin 2x \rightarrow ke^{3x}\sin 2x + le^{3x}\cos 2x$ with $k, l \neq 0$ • There need to be two terms added. $\frac{dy}{dx} = 3e^{3x}\sin 2x + 2e^{3x}\cos 2x$	M1
	One term completely correct. $\frac{dy}{dx} = 3e^{3x}\sin 2x + '2e^{3x}\cos 2x'$ or $\frac{dy}{dx} = '3e^{3x}\sin 2x' + 2e^{3x}\cos 2x$	A1
	Fully correct differentiated expression. $\frac{dy}{dx} = 3e^{3x}\sin 2x + 2e^{3x}\cos 2x$	A1
	Method A	
	For an attempt to find $\frac{d^2y}{dx^2}$ Minimally acceptable attempt is $ke^{3x}\sin 2x + le^{3x}\cos 2x \rightarrow k(me^{3x}\sin 2x + ne^{3x}\cos 2x) + l(pe^{3x}\cos 2x + qe^{3x}\sin 2x)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\frac{d^2y}{dx^2} = 3\left(3e^{3x}\sin 2x + 2e^{3x}\cos 2x\right) + 6e^{3x}\cos 2x - 4e^{3x}\sin 2x$	M1
	For substituting y and $\frac{dy}{dx}$ into their $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 6e^{3x}\cos 2x - 4y$	M1
	For preparing to eliminate cos2x by rearranging their $\frac{dy}{dx}$ $2e^{3x}\cos 2x = \frac{dy}{dx} - 3y \Rightarrow 6e^{3x}\cos 2x = 3\frac{dy}{dx} - 9y$ Allow errors in arithmetic but not mathematically incorrect process.	M1
	For an unsimplified expression only in terms of y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 3\frac{dy}{dx} - 9y - 4y$	M1
	For the required simplified expression with fully correct working. $13y + \frac{d^2y}{dx^2} = 6\frac{dy}{dx}$	A1 [8]
	Method B	
	For an attempt to find $\frac{d^2y}{dx^2}$ using $\frac{dy}{dx} = 3y + 2e^{3x}\cos 2x$	

	Minimally acceptable attempt is $\frac{dy}{dx} = ky + le^{3x}\cos 2x \rightarrow \frac{d^2y}{dx^2} = k\frac{dy}{dx} + l(pe^{3x}\cos 2x + qe^{3x}\sin 2x)$ k, l as in their first derivative, $p, q \neq 0$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 6e^{3x}\cos 2x - 4e^{3x}\sin 2x$, $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 6e^{3x}\cos 2x - 4y$	M1,M1
	For preparing to eliminate cos2x by rearranging their $\frac{dy}{dx}$ $2e^{3x}\cos 2x = \frac{dy}{dx} - 3y \Rightarrow 6e^{3x}\cos 2x = 3\frac{dy}{dx} - 9y$ Allow errors in arithmetic but not mathematically incorrect process.	M1
	For an unsimplified expression only in terms of y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 3\frac{dy}{dx} - 9y - 4y$	M1
	For the required simplified expression with fully correct working. $13y + \frac{d^2y}{dx^2} = 6\frac{dy}{dx}$	A1
	Method C	
	For an attempt to find $\frac{d^2y}{dx^2}$ Minimally acceptable attempt is $ke^{3x}\sin 2x + le^{3x}\cos 2x \rightarrow k(me^{3x}\sin 2x + ne^{3x}\cos 2x) + l(pe^{3x}\cos 2x + qe^{3x}\sin 2x)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\frac{d^2y}{dx^2} = 3\left(3e^{3x}\sin 2x + 2e^{3x}\cos 2x\right) + 6e^{3x}\cos 2x - 4e^{3x}\sin 2x$	M1
	For substituting y and $\frac{d^2y}{dx^2}$ into the LHS of the result.	M1
	For substituting $\frac{dy}{dx}$ into the RHS of the result.	For simplifying the LHS of the result and writing as $k(3e^{3x}\sin 2x + 2e^{3x}\cos 2x)$ — M1
	For unsimplified expressions for the LHS and RHS of the result in terms of $e^{3x}\sin 2x$ and $e^{3x}\cos 2x$ only and method to simplify both sides	M1
	For correctly simplifying the LHS and RHS and showing equal.	A1
		Total 8 marks

Question	Scheme	Marks
7	$x=0,\ y=0^2+2=2$ $\left[y=x^2+2\Rightarrow x^2=y-2\right]$	B1
	$V=\pi\int_2^a x^2\ \mathrm{d}y\Rightarrow 18\pi=\pi\int_2^a (y-2)\ \mathrm{d}y,=\pi\left[\frac{y^2}{2}-2y\right]_2^a$	M1,M1
	$18\pi=\pi\left[\left(\frac{a^2}{2}-2a\right)-\left(\frac{2^2}{2}-2\times 2\right)\right]$	M1
	$0=a^2-4a-32$	M1A1
	$a^2-4a-32=(a+4)(a-8)=0\Rightarrow a=8$	M1A1 [8]
	ALT $x=0,\ y=0^2+2=2$ $\left[y=x^2+2\Rightarrow x^2=y-2\right]$	[B1
	$18\pi=\pi\int_2^a (y-2)\ \mathrm{d}y,\ \ 18\pi=\pi\left[\frac{(y-2)^2}{2}\right]_2^a$	M1,M1
	$18\pi=\pi\left[\left(\frac{a-2}{2}\right)^2-\left(\frac{2-2}{2}\right)^2\right]$	M1
	$18=\left(\frac{a-2}{2}\right)^2\Rightarrow 36=(a-2)^2$	M1A1
	$a=2\pm\sqrt{36}\Rightarrow a=8$	M1A1]
		Total 8 marks

Question	Notes	Marks
7	The intersection of S with the y-axis is at the point with coordinates (0, 2). May be given as when $x = 0, y = 2$	B1
	For a correct statement for the volume, condone missing or incorrect limits and missing π $\left[y = x^2 + 2 \Rightarrow x^2 = y - 2\right]$ $V = \pi\int_2^a x^2 \ \mathrm{d}y \Rightarrow 18\pi = \pi\int_2^a (y-2)\ \mathrm{d}y$	M1
	METHOD A For an attempt to integrate the expression which must be in the minimally acceptable form $y \pm 2$ $18\pi = \pi\left[\frac{y^2}{2} - 2y\right]_2^a$ [Ignore limits and π for this mark]	M1
	For substituting the limits into their integrated expression to form an equation in a. No simplification is required for this mark. $18\pi = \pi\left[\left(\frac{a^2}{2} - 2a\right) - \left(\frac{2^2}{2} - 2\times 2\right)\right]$	M1
	For forming a 3TQ in terms of a $\quad 0 = a^2 - 4a - 32$	M1
	For the correct 3TQ $0 = a^2 - 4a - 32$	A1
	For an attempt to solve their 3TQ $a^2 - 4a - 32 = (a+4)(a-8) = 0 \Rightarrow a = ...$	M1
	For the correct value of $a = 8$ If $a = -4$ is also stated then it must be rejected.	A1 [8]
	METHOD B For an attempt to integrate the expression which must be in the minimally acceptable form $y \pm 2$ $18\pi = \pi\left[\frac{(y-2)^2}{2}\right]_2^a$ [Ignore limits and π for this mark]	M1
	For substituting the limits into their integrated expression to form an equation in a. No simplification is required for this mark. $18\pi = \pi\left[\left(\frac{a-2}{2}\right)^2 - \left(\frac{2-2}{2}\right)^2\right]$	M1
	For forming an equation in a $\quad 18 = \left(\frac{a-2}{2}\right)^2 \Rightarrow 36 = (a-2)^2$	M1A1
	For an attempt to solve their equation. $a = 2 \pm \sqrt{36} \Rightarrow a = 8$	M1A1

Total 8 marks

Question	Scheme	Marks
7(a)	$\frac{dy}{dx} = 2mx + \frac{1}{2} \times 64x^{-\frac{1}{2}}$	M1A1
	$32x^{-\frac{1}{2}} + 2mx = 0 \Rightarrow 32 \times 4^{-\frac{1}{2}} + 2m \times 4 = 0 \Rightarrow 16 + 8m = 0 \Rightarrow m = -2$	dM1A1
	$n = 39 + 64\sqrt{4} - 2 \times 4^2 = 135$	dM1A1 [6]
(b)	$\frac{d^2y}{dx^2} = -\frac{1}{2} \times 32x^{-\frac{3}{2}} - 4 = -16x^{-\frac{3}{2}} - 4$	M1
	$\frac{d^2y}{dx^2} = -6 < 0 \Rightarrow$ maximum	A1FT [2]
		Total 8 marks

Part	Mark	Notes
(a)	M1	For an attempt to differentiate the given expression. Minimum required is **two** of: • $mx^2 \rightarrow kmx, \quad k \neq 0$ • $64\sqrt{x} \rightarrow nx^{-\frac{1}{2}}, \quad n \neq 0$ • $39 \rightarrow 0$
	A1	For a correct derivative in terms of m
	dM1	For setting their $\frac{dy}{dx} = 0$, substitute $x = 4$ and attempting to solve to find the value of m Dep on 1st M mark.
	A1	For $m = -2$
	dM1	For using their value of m to find a value for n $n = 4^2 \times m + 167$ Dep on 1st M mark.
	A1	For the correct value of n
(b)	M1	For finding the second derivative. Minimum required is $\frac{d^2y}{dx^2} = km + px^{-\frac{3}{2}}$ with k, p not 0. Allow for substituting x values on either side of $x = 4$ into *their* $\frac{dy}{dx}$ provided $\frac{dy}{dx} = kmx + nx^{-\frac{1}{2}}$ with k, n not 0.
	A1FT	For substituting their value of m and forming a correct conclusion. Minimum required is substitution of their value of m together with $x = 4$ into their second derivative, evaluation and correct conclusion. Allow for evaluating of $\frac{dy}{dx}$ on either side of $x = 4$ and forming a correct conclusion. $x = 3.9, \frac{dy}{dx} = 0.6038 \ldots \quad x = 4.1, \frac{dy}{dx} = -0.596 \ldots$

Question	Scheme	Marks
4(a)	**Mark parts (i) and (ii) together.** $2y-x-4=0 \Rightarrow y=\dfrac{x+4}{2}$ oe $\dfrac{x^2}{4}+2=\dfrac{x+4}{2}$ oe $\Rightarrow \dfrac{x^2+8}{4}=\dfrac{x+4}{2}$ oe $\Rightarrow x^2+8=2x+8$ oe $\Rightarrow x^2-2x=0 \Rightarrow x(x-2)=0$ oe $\Rightarrow x=(0,)\ 2$ $\Rightarrow y=2,\ 3$ So coordinates at point *A* are (0, 2) * and at point *B* (2, 3)	M1 M1 A1* cso A1 [4]
	NB SC1 – for correct and complete substitution of $x = 0$ into the equation of curve *S* or line *l* to show that $y = 2$, where there is no other work, other work is incorrect or no marks would otherwise be gained. Award this as 1st M mark. THERE ARE NO OTHER MARKS AVAILABLE FOR PART (a) IF THIS IS ALL STUDENTS DO	
(b)	$V=\pi\int_2^{"3"}(4y-8)\,dy-\pi\int_2^{"3"}(2y-4)^2\,dy$ $\left[\Rightarrow \pi\int_2^{"3"}(-4y^2+20y-24)\,dy \quad \text{or} \quad \pi\int_2^{"3"}(4y-8)\,dy-\pi\int_2^{"3"}(4y^2-16y+16)\,dy\right]$ $=(\pi)\left[-\dfrac{4y^3}{3}+\dfrac{20y^2}{2}-24y\right]_{("2")}^{("3")}$ or $(\pi)\left[\dfrac{4y^2}{2}-8y\right]_{("2")}^{("3")}-(\pi)\left[\dfrac{4y^3}{3}-\dfrac{16y^2}{2}+16y\right]_{("2")}^{("3")}$ $=(\pi)\left[\left(\dfrac{4\times"3"^3}{3}+\dfrac{20\times"3"^2}{2}-24\times"3"\right)-\left(\dfrac{4\times"2"^3}{3}+\dfrac{20\times"2"^2}{2}-24\times"2"\right)\right]$ $=\dfrac{2}{3}\pi$ **ALT** $V=\pi\int_2^{"3"}(4y-8)\,dy-\pi\times\dfrac{1}{3}\times2^2\times1$ $=\pi\left[\dfrac{4y^2}{2}-8y\right]_{"2"}^{"3"}\left(-\dfrac{4}{3}\pi\right)$ $=\pi\left[\left(2\times"3"^2-8\times"3"\right)-\left(2\times"2"^2-8\times"2"\right)\right]\left(-\dfrac{4}{3}\pi\right)$ $=\dfrac{2}{3}\pi$	M1 M1 M1 A1 [4] M1 M1 M1 A1
		Total 8 marks

Part	Mark	Notes
(a)	M1	Correctly equates the equation of S with the equation of l
	M1	Forms a quadratic and a minimally acceptable attempt to solve their quadratic. For a 2TQ the **solution** of their quadratic for must be correct, **but any zero solution doesn't need to be shown as it's obvious from the given diagram.** If they achieve a 3TQ and attempt to solve – see general guidance.
	A1* cso	For correct substitution of $x = 0$ to show coordinates of A (0, 2) or $x = 0$, $y = 2$. Allow coordinates without brackets. SC1 (see MS) if this is the only work done.
	A1	For the coordinates of B (2, 3) $x = 2$, $y = 3$ Allow coordinates without brackets.
Note on labelling: **If the labels (i) and (ii) are not present, the marks can be awarded if** • **The coordinates appear in the correct order or** • **They are labelled with *A* and *B*** **If there is ambiguity award A1 A0 if both correct and A0 A0 if only one is correct**		
(b)	M1	For a correct statement for the volume of rotation with 2 and their upper limit used correctly and correctly including π. The lower limit **must** be 2.
	M1	For a minimally acceptable attempt to integrate (see general guidance) **and** no power of y must decrease. π and limits do not need to be present to gain this mark. There must be a minimum of 2 terms to integrate.
	M1	For substituting their limits into their changed expression the correct way round. There must be at least one clear substitution of each limit. π does not need to be present to gain this mark.
	A1	For the correct final volume $= \frac{2}{3}\pi$ accept $0.\dot{6}\pi, 0.67\pi$ or better
	ALT	
	M1	For a correct statement for the volume of rotation of the curve with 2 and their upper limit used correctly including π minus the correct formula used for the volume of a cone. The lower limit **must** be 2. **This mark may also be awarded if the integral is subtracted the wrong way round.**
	M1	For a minimally acceptable attempt to integrate (see general guidance) **and** no power of y must decrease. π, limits and $-\frac{4}{3}\pi$ do not need to be present to gain this mark. There must be a minimum of 2 terms to integrate.
	M1	For substituting their limits into their changed expression. There must be at least one clear explicit substitution of each limit. This mark can be implied by a correct final answer. π and $-\frac{4}{3}\pi$ do not need to be present to gain this mark.
	A1	For the correct final volume $= \frac{2}{3}\pi$ accept $0.\dot{6}\pi, 0.67\pi$ or better **If a negative value is found and changed at the end to a positive value, this final A mark cannot be awarded.**
	Note: any candidate incorrectly rotating around *x* axis, 2nd & 3rd M marks are available for integrating an expression & substituting in. Max mark M0 M1 M1 A0.	

Question	Scheme	Marks
5	$\sqrt{2x+6}=\frac{x}{3}+1\Rightarrow 2x+6=\frac{x^2}{9}+\frac{2x}{3}+1\Rightarrow \frac{x^2}{9}-\frac{4x}{3}-5=0$	M1
	$\Rightarrow x^2-12x-45=0\Rightarrow (x+3)(x-15)=0\Rightarrow x=-3,15$	M1A1
	[When $x=15,\quad y=6$]	
	$(V=)\pi\int_{'-3'}^{'15'}(2x+6)\,dx-\frac{1}{3}\times\pi\times'6'^2\times('15'-'-3')$	M1M1
	$(V=)(\pi)\left[\frac{2x^2}{2}+6x\right]_{'-3'}^{'15'}-216\pi$	M1
	$(V=)\pi\left[\left(\frac{2\times'15'^2}{2}+6\times'15'\right)-\left(\frac{2\times('-3')^2}{2}+6\times('-3')'\right)\right]-216\pi(=324\pi-216\pi)=108\pi$	M1A1
ALT last 5 marks	OR	[8]
	$(V=)\pi\int_{'-3'}^{'15'}(2x+6)\,dx-\pi\int_{'-3'}^{'15'}\left(\frac{x^2}{9}+\frac{2x}{3}+1\right)dx$ or $\pi\int_{'-3'}^{'15'}(2x+6)\,dx-\pi\int_{'-3'}^{'15'}\left(\frac{x}{3}+1\right)^2dx$	M1M1
	$(V=)(\pi)\left[\frac{2x^2}{2}+6x\right]_{'-3'}^{'15'}-\pi\left[\frac{x^3}{9\times3}+\frac{2x^2}{3\times2}+x\right]_{'-3'}^{'15'}$	M1
	$V=\pi\left[\left(\frac{2\times'15'^2}{2}+6\times'15'\right)-\left(\frac{2\times('-3')^2}{2}+6\times('-3')\right)\right]$	
	$-\pi\left[\left(\frac{'15'^3}{9\times3}+\frac{2\times'15'^2}{3\times2}+15\right)-\left(\frac{('-3')^3}{9\times3}+\frac{2\times('-3')^2}{3\times2}-'3'\right)\right]=108\pi$	M1A1
		Total 8 marks

Mark	Notes
M1	Sets the equation of the curve = equation of line and attempts to form a 3TQ, which must include an attempt to expand $\left(\frac{1}{3}x+1\right)\left(\frac{1}{3}x+1\right)$. Allow only 1 error in processing overall. Accept as a minimum for this mark: $\frac{x^2}{9}\pm Jx-K=0$ or $x^2\pm Lx-M=0$ $J,K,L,M>0$
M1	Solves their 3TQ using a full and minimally acceptable method (see general guidance) to find two values. They must show their method if using an incorrect 3TQ. Note this is not a dependent mark, so the candidate can solve a 3TQ not of the form required for the first M mark.
A1	$x = -3, 15$ M1 M1 A1 may be awarded for both correct values given without working.
M1	For the correct expression for the volume of rotation under the curve including π and ft their $x = -3, 15$.
M1	For the correct expression for the volume of the cone. Ft their $x = -3, 15$ and their value of y only (must be clear they are using a value of y. '6' cannot be 15 or -3). $\frac{1}{3}\times\pi\times'6'^2\times('15'-'-3')$
M1	For an attempt to integrate their expression for the curve. Minimally acceptable attempt (see general guidance) – no power of x to decrease. π and limits don't need to be present.
M1	For substitution of their limits into any changed expression representing their volume of the curve and evaluation of the final volume using their volume of the cone. Not a dependent mark but substitution **must** be into their changed expression. This mark may be awarded for the work being done in parts and subtracted in any order at any point, but **there must be a subtraction.** This M mark may be implied by a correct final answer.
A1	For $V = 108\pi$ **This mark may not be awarded if the subtraction is the incorrect way around and the sign of the answer is simply changed at the end.**
ALT last 5 marks	
M1	For the correct expression for the volume of rotation under the curve including π and ft their $x = -3, 15$
M1	For the correct expression for the volume of rotation under the straight line including π and ft their $x = -3, 15$
Where candidates have combined the 2 expressions, M1 M1 can be awarded for $\pm\pi\int_{-3}^{15}\left(\frac{x^2}{9}-\frac{4x}{3}-5\right)\mathrm{d}x$ **or** for a clear attempt to combine the correct two expressions, even if the following simplification is incorrect. [Note for final A mark correct integral is $\pi\int_{-3}^{15}\left(-\frac{x^2}{9}+\frac{4x}{3}+5\right)\mathrm{d}x$	
M1	For an attempt to integrate the expression for **either** the curve **or** the line **or** what they intend to be **<u>their</u>** combined expression. Minimally acceptable attempt (see general guidance) – no power of x to decrease. π and limits do not need to be present for this mark.
M1	For substitution of their limits into any changed expressions representing their volume of the line and the curve and evaluation of the final volume or their combined expression. There must be at least one correct substitution of both limits into each expression. Not a dependent mark but substitution **must** be into changed expressions. This mark may be awarded for the work being done in parts and subtracted in any order at any point, but **there must be a subtraction.** This M mark may be implied by a correct final answer.
A1	For $V = 108\pi$ **This mark may not be awarded if the subtraction is the incorrect way around and the sign of the answer is simply changed at the end.**

Question	Scheme	Marks
PLEASE REMEMBER TO CHECK DIAGRAM FOR RELEVANT LENGTHS AND WORKING **Throughout the question where lengths are in brackets eg (*AC* =), this means you don't have to see *AC* =.** **But the mark cannot be awarded for a clearly incorrect method eg *VO* =** $\sqrt{x^2+x^2}$ **for the**		

Question	Scheme	Marks
8(a)	$\left(\frac{dy}{dx}=\right)\frac{5x^2\times2\times3\times e^{3x+1}-10x\times2e^{3x+1}}{(5x^2)^2}$ oe	M1A1A1
	$\left(\frac{dy}{dx}=\right)\frac{xe^{3x+1}(30x-20)}{25x^4}$ or $\frac{10xe^{3x+1}(3x-2)}{25x^4}$	dM1
	$=\left[\frac{10e^{3x+1}(3x-2)}{25x^3}\right]=\frac{2e^{3x+1}(3x-2)}{5x^3}$	A1 [5]
ALT – use of product rule	$\left(\frac{dy}{dx}=\right)2e^{3x+1}(-1)\times2\times5x\times(5x^2)^{-2}+2\times3e^{3x+1}(5x^2)^{-1}$ oe	M1A1A1
	$=x\times(5x^2)^{-2}e^{3x+1}(-20+30x)$ or $10x\times(5x^2)^{-2}e^{3x+1}(-2+3x)$	dM1
	$=\left[\frac{10e^{3x+1}(3x-2)}{25x^3}\right]=\frac{2e^{3x+1}(3x-2)}{5x^3}$	A1 [5]
(b)	$\left(\frac{\delta y}{y}\times100=\right)"\frac{2xe^{3x+1}(3x-2)}{5x^4}"\times\delta x\times\frac{100}{y}$ or $"\frac{2e^{3x+1}(3x-2)}{5x^3}"\times\delta x\times\frac{100}{y}$ oe	M1
	or $(\delta y=)"\frac{2xe^{3x+1}(3x-2)}{5x^4}"\times\delta x$ or $"\frac{2e^{3x+1}(3x-2)}{5x^3}"\times\delta x$ or $y\times\frac{(3x-2)}{5x}\times\delta x$ δx may also be written as $0.02x$	
	$\left(\frac{\delta y}{y}\times100=\right)\frac{2e^{3x+1}(3x-2)}{5x^3}\times0.02x\times\frac{100}{\frac{2e^{3x+1}}{5x^2}}$ oe	dM1
	or $y\times\frac{(3x-2)}{x}\times0.02x\times\frac{100}{y}$ $(\%\text{ change in } y=)6x-4$	A1 [3]
		Total 8 marks

Part	Mark	Notes
(a)	M1	For attempting to use the quotient rule. The definition of an attempt is as follows: • The denominator must be correct. • The terms in the numerator subtracted either way around The expression **must** be of the form: $\frac{K\times x^2\times e^{3x+1}-L\times x\times e^{3x+1}}{\left(5x^2\right)^2}$ $K\neq 10$ or 15 and $L\neq 10$ and $K,L>0$ Note K and L may not be simplified and will often need to be checked.
	A1	Following M1 (general principle of marking), at least one fully correct simplified or unsimplified term on the numerator.
	A1	Fully correct **unsimplified** derivative
	dM1	For correctly taking out common factors of x and e^{3x+1} and attempting to simplify. A numerical factor does not need to be taken out at this point. Dependent on the 1st method mark.
	A1	For the correct derivative in the required form.
ALT use of product rule	M1	For attempting to use the product rule to give an expression of the form $Me^{3x+1}x\times(5x^2)^{-2}+Ne^{3x+1}(5x^2)^{-1}$ $M<0$ and $\neq 4,10$ $N>0$ and $\neq 2,3$
	A1	Following M1 (general principle of marking), at least one term is fully correct.
	A1	Fully correct **unsimplified** derivative
	M1	For correctly taking out common factors of x and $(5x^2)^{-2}$ and e^{3x+1} and attempting to simplify. A numerical factor does not need to be taken out at this point. Dependent on the 1st method mark.
	A1	For the correct derivative in the required form.
For the final M mark in particular – any work felt to be of credit not fitting the mark scheme – send to Review.		
(b)	M1	For any of the statements shown, using their answer from part (a), which must be of the form: $\frac{Ae^{3x+1}(Bx-C)}{Dx^3}$ Note, we'll allow candidates to use an expression here that is not quite of the form given in part (a). This mark may be implied by further correct working.
	dM1	For substituting in the expression for y **and** a correct attempt to simplify their expression to the required form. You must check their working here, do not allow an answer that has come from incorrect working (general principle anyway, but extra care needed for this question). Condone 100 written as 100% if the candidate is correctly multiplying by 100. Dependent on previous method mark.
	A1	For the correct expression as shown.

Question	Scheme	Marks
4	$\frac{dA}{dt}=50\pi$ $\quad\frac{dr}{dt}=\frac{5}{12}$	B1,B1
	$\left(A=4\pi r^2 \Rightarrow \frac{dA}{dr}=\right)8\pi r$	B1
	$\frac{dr}{dt}=\frac{1}{\frac{dA}{dr}}\times\frac{dA}{dt}$ **oe** $\quad\Rightarrow\frac{5}{12}=\frac{1}{8\pi r}\times 50\pi \Rightarrow r=15$	M1dM1A1
	$V=\frac{4}{3}\times\pi\times 15^3=4500\pi\left[\text{cm}^3\right]$	M1A1
		[8]
		Total 8 marks

Mark	Notes
B1	For either the correct $\frac{dA}{dt}=50\pi$ or $\frac{dr}{dt}=\frac{5}{12}$ May be seen explicitly or used implicitly in the candidate's work.
B1	For both correct $\frac{dA}{dt}=50\pi$ and $\frac{dr}{dt}=\frac{5}{12}$ May be seen explicitly or implicitly in the candidate's working.
B1	For $8\pi r$ May be seen explicitly or implicitly in the candidate's working.
M1	For a correct chain rule, relevant to the question. Look for equivalences. This may be explicitly stated or can be awarded for use of the appropriate values or expressions implicitly.
dM1	For correctly substituting their values and rearranging their equation to find a value for *r*. Allow errors in rearrangement. Dependent on the previous method mark, though M1 dM1 can be awarded if the implicit use of their values in a correct chain rule is correct, without the chain rule being stated.
A1	For r = 15 (cm)
M1	For using the formula for the volume of a sphere with their *r* **Note – although this has not been made a dependent mark, this mark can only be awarded if 'their *r*' has come from some attempt at calculus.**
A1	For the correct volume of a sphere given exactly.

Question	Scheme	Marks
9	**Area under the curve** $A_c = \int_{-1}^{0}\left(-2e^{3x}+4\right)dx, = \left[\frac{-2e^{3x}}{3}+4x\right]_{-1}^{0}$ $\Rightarrow \left(\frac{-2e^{3\times 0}}{3}+4\times 0\right)-\left(\frac{-2e^{3\times -1}}{3}+4\times -1\right), = \frac{10}{3}+\frac{2e^{-3}}{3}$ **Area under trapezium** Method 1 [uses formula for the area of a trapezium] $y=2 \quad y=-2e^{-3}+4$ $A_T = \frac{1}{2}\times 1\times\left('2'+'-2e^{-3}+4'\right)=\left[3-e^{-3}\right]$ Method 2 [finds equation of the line and integrates] Equation of the line: $y=\left(2e^{-3}-2\right)x+2$ (accept $y=-1.9x+2$) $A_T = \int_{-1}^{0}'\left(\left(2e^{-3}-2\right)x+2\right)'dx = \left[\frac{'\left(2e^{-3}-2\right)'x^2}{2}+\frac{'2'x}{2}\right]_{-1}^{0} = \left[3-e^{-3}\right]$ **Shaded Area** $A_{shaded} = \left(\frac{10}{3}+\frac{2e^{-3}}{3}\right)-\left(3-e^{-3}\right)=\frac{1+5e^{-3}}{3}$	M1M1 M1A1 B1 M1 M1A1 [8]
		Total 8 marks

Mark	Notes
M1	For a correct statement for the area under the curve with **correct limits**. This mark can be implied/embedded by later correct work. **Condone missing d*x***
M1	Attempts to integrate the given curve to a form of $Ae^{3x} + Bx$ **Ignore limits for this mark**.
M1	For substituting in both of their limits into their integrated expression in the form $Ae^{3x} + Bx$ and subtracts. (A, B are nonzero constants) Correct exact answer $\frac{10}{3} + \frac{2e^{-3}}{3}$ implies the correct substitution. If their answer is not correct and exact, explicit substitution needs to be seen.
A1	For the correct **exact** area under the curve. If not seen explicitly, this can be implied by correct exact final answer.
Method 1 - trapezium	
B1	For both correct **exact** values of y
M1	For the correct method of finding area of the trapezium using their values for y Look for $\frac{1}{2} \times 1 \times (sum\ of\ their\ y\ values)$
Method 2 – equation of line	
B1	For the correct equation of the line L, must reach $y = \ldots$ $y = \left(2e^{-3} - 2\right)x + 2$ $\left(\text{accept } y = -1.9x + 2\right)$
M1	For the correct area under the line L using their line. Check for correct integration of their line and full substitution of – 1 if their area is incorrect. (If the terms are equal to zero, substitution of 0 does not need to be seen.) Allow also $A_T = \int_{-1}^{0} '\left(-1.9x + 2\right)' \, dx = \left[\frac{'-1.9'x^2}{2} + \frac{'2'x}{2}\right]_{-1}^{0}$
Combines the areas	
M1	Correct strategy to find the value of shaded area The integrated area of the curve - the area of trapezium
A1	For the correct **exact** area of the shaded region in the required form
Some candidates are combining the curve – line in one calculation – mark this as follows. • Wherever you see the **correct** equation of the line, score the 5th Mark [B mark] • Mark the work for the curve (which may be embedded in a combined integral) using the first 4 marks. • Mark the area under the line using the 6th mark [M mark – using their equation in the form $y = mx + c$] • When you see them subtracting the areas – award the penultimate M mark If they add the areas, this is M0 • Final A mark is for the correct answer only	

Question number	Scheme	Marks
11	$\frac{3}{r}=\frac{4}{h}$ oe $\rightarrow$ $r=\frac{3h}{4}$ oe	M1 A1
	$(V=)\frac{1}{3}\pi\left(\frac{3h}{4}\right)^2 h$ $\left(V=\frac{3\pi h^3}{16}\right)$ oe	M1
	$\left(\frac{dV}{dh}=\right)\frac{9\pi h^2}{16}$ oe	M1 A1
	$\frac{dh}{dt}=\frac{dV}{dt}\cdot\frac{dh}{dV}$ oe	M1
	$\left(\frac{dh}{dt}=\right)9\pi\times\frac{16}{9\pi(2)^2}$ oe	dddM1
	$\left(\frac{dh}{dt}=\right)4$	A1 [8]
	OR	
	$\frac{3}{r}=\frac{4}{h}$ oe $\Rightarrow$ $h=\frac{4r}{3}$ oe	M1A1
	$(V=)\frac{1}{3}\pi r^2 h$ $\left(V=\frac{4\pi r^3}{9}\right)$ oe	M1
	$\left(\frac{dV}{dr}=\right)\frac{4\pi r^2}{3}$ AND $\frac{dh}{dr}=\frac{4}{3}$	M1A1
	$\frac{dh}{dt}=\frac{dV}{dt}\cdot\frac{dr}{dV}\cdot\frac{dh}{dr}$ oe	M1
	$\left(\frac{dh}{dt}=\right)9\pi\times\frac{1}{\frac{4\pi\left(\frac{3}{2}\right)^2}{3}}\times\frac{4}{3}$ oe $\left[h=2,\quad r=\frac{3}{2}\right]$	M1
	$\left(\frac{dh}{dt}=\right)4$	A1 [8]
		Total 8 marks

Mark	Additional Guidance
M1	For any correct equation using similar triangles to link together r and h
A1	For $r=\frac{3h}{4}$. This may be implied in working.
M1	For using their expression for r in terms of h to get an expression for V of the form $a\pi h^3$
M1	For a minimally acceptable attempt to differentiate their expression for V to get an equation of the form $b\pi h^2$
A1	For $\left(\frac{\mathrm{d}V}{\mathrm{d}h}=\right)\frac{9\pi h^2}{16}$ oe
M1	For a correct, relevant chain rule. This may be implied in working.
dddM1	Correct substitution into chain rule, dependent on the previous 3 method marks.
A1	Correct value.
	ALT
M1	For any correct equation using similar triangles to link together r and h
A1	For $h=\frac{4r}{3}$
M1	For using their expression for r in terms of h to get an expression for V of the form $a\pi r^3$
M1	For a minimally acceptable attempt to differentiate their expression for V to get an equation of the form $b\pi r^2$
A1	For $\left(\frac{\mathrm{d}V}{\mathrm{d}r}=\right)\frac{4\pi r^2}{3}$ AND $\frac{\mathrm{d}h}{\mathrm{d}r}=\frac{4}{3}$
M1	For a correct, relevant chain rule. This may be implied in working
dddM1	Correct substitution into chain rule, dep on previous three marks
A1	Correct value

Question number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = -4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t \Rightarrow 2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4y$ *	M1A1A1 Cso [3]
(b)	**Method A** $\frac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$ $\frac{d^2y}{dt^2} = -4\frac{dy}{dt} + 8e^{-4t}\sin 2t - 4y$ $2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4e^{-4t}\cos 2t \Rightarrow 8e^{-4t}\sin 2t = -4\frac{dy}{dt} - 16y$ $\frac{d^2y}{dt^2} = -4\frac{dy}{dt} + 8e^{-4t}\sin 2t - 4y \Rightarrow \frac{d^2y}{dt^2} = -4\frac{dy}{dt} + \left\{-4\frac{dy}{dt} - 16y\right\} - 4y$ $\Rightarrow \frac{d^2y}{dt^2} = -8\frac{dy}{dt} - 20y \Rightarrow \frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 20y = 0$ $\frac{d^2y}{dt^2} + M\frac{dy}{dt} + Ny = 0 \Rightarrow M = 8, N = 20$	M1 M1 M1 M1 A1 [5]
	Method B $\frac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$ $\left(= 16e^{-4t}\sin 2t + 12e^{-4t}\cos 2t\right)$ $\frac{d^2y}{dt^2} + M\frac{dy}{dt} + Ny$ $= 16e^{-4t}\sin 2t + 12e^{-4t}\cos 2t + M(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + Ne^{-4t}\cos 2t$ $= (16 - 2M)e^{-4t}\sin 2t + (12 - 4M + N)e^{-4t}\cos 2t$ $16 - 2M = 0$ and $12 - 4M + N = 0$ $M = 8,\ N = 20$	[M1 M1 M1 M1 A1]
	Method C By inspection – please see additional guidance.	
		Total 8 marks

Part	Mark	Additional Guidance

(a)	M1	For an attempt to differentiate the given expression. • There needs to be an acceptable attempt to differentiate both terms. $e^{-4t}\cos 2t \rightarrow ke^{-4t}\sin 2t + le^{-4t}\cos 2t$ with $k, l \neq 0$ • There need to be two terms added. $\frac{dy}{dt} = -4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t$
	A1	One term completely correct. $\frac{dy}{dt} = -4e^{-4t}\cos 2t + '-2e^{-4t}\sin 2t$ OR $\frac{dy}{dt} = '-4e^{-4t}\cos 2t' - 2e^{-4t}\sin 2t$
	A1	Fully correct differentiated expression and rearranged into the required form $\frac{dy}{dt} = -4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t \Rightarrow 2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4y$ Accept also, For example: $-\left(-4e^{-4t}\cos 2t' - 2e^{-4t}\sin 2t\right) - 4\left(e^{-4t}\sin 2t\right) = 2e^{-4t}\sin 2t$ With any unambiguous indication that a conclusion has been reached, e.g., QED, hash sign, double underline, RHS = LHS etc.
		Method A
(b)	M1	For an attempt to find $\frac{d^2y}{dt^2}$ Minimally acceptable attempt is $ke^{-4x}\sin 2t + le^{-4t}\cos 2t \rightarrow k(me^{-4t}\sin 2t + ne^{-4t}\cos 2t) + l(pe^{-4t}\cos 2t + qe^{-4t}\sin 2t)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\frac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$
	M1	For substituting y and $\frac{dy}{dt}$ into their $\frac{d^2y}{dt^2}$ $\frac{d^2y}{dt^2} = -4\frac{dy}{dt} + 8e^{-4t}\sin 2t - 4y$
	M1	For preparing to eliminate cos2t by rearranging their $\frac{dy}{dt}$ $2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4e^{-4t}\cos 2t \Rightarrow 8e^{-4t}\sin 2t = -4\frac{dy}{dt} - 16y$ Allow errors in arithmetic but not mathematically incorrect process.
	M1	For an unsimplified expression only in terms of y, $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ $\frac{d^2y}{dt^2} = -4\frac{dy}{dt} + \left\{-4\frac{dy}{dt} - 16y\right\} - 4y$
	A1 [8]	For finding the value of M and the value of N. $M = 8,\ N = 20$
		Method B

	M1	For an attempt to find $\frac{d^2y}{dt^2}$ Minimally acceptable attempt is $ke^{-4t}\sin 2t + le^{-4t}\cos 2t \rightarrow k(me^{-4t}\sin 2t + ne^{-4t}\cos 2t) + l(pe^{-4t}\cos 2t + qe^{-4t}\sin 2t)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\frac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$
	M1	For substituting y, $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ into the equation. $\frac{d^2y}{dt^2} + M\frac{dy}{dt} + Ny$ $= 16e^{-4t}\sin 2t + 12e^{-4t}\cos 2t + M(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + Ne^{-4t}\cos 2t$
	M1	For collecting terms $\frac{d^2y}{dt^2} + M\frac{dy}{dt} + Ny = ('16' - 2M)e^{-4t}\sin 2t + ('12' - 4M + N)e^{-4t}\cos 2t$
	M1	For identifying two equations from their working and a method to solve them simultaneously For example, $'16' - 2M = 0$ and $'12' - 4M + N = 0$
	A1	For finding the value of M and the value of N. $M = 8, N = 20$
		Method C (By inspection)
	M1	For an attempt to find $\frac{d^2y}{dt^2}$ Minimally acceptable attempt is $ke^{-4t}\sin 2t + le^{-4t}\cos 2t \rightarrow k(me^{-4t}\sin 2t + ne^{-4t}\cos 2t) + l(pe^{-4t}\cos 2t + qe^{-4t}\sin 2t)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\frac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$
	M1	For simplifying $\frac{d^2y}{dt^2} = e^{-4t}16\sin 2t + e^{-4t}12\cos 2t = \left[e^{-4t}(16\sin 2t + 12\cos 2t)\right]$
	M1	For comparing $\frac{d^2y}{dt^2}$ with $\frac{dy}{dt}[\sin 2t]$ and deducing that $'16' + (\underline{8} \times -2) = 0 \Rightarrow M = 8$
	M1	For comparing $\frac{d^2y}{dt^2}$ with $\frac{dy}{dt}$ and y and deducing that $'12' - ('8' \times 4) + \underline{20} = 0 \Rightarrow N = 20$
	A1	For both $M = 8$ and $N = 20$
		Total 8 marks

Question number	Scheme	Marks
9	Intersections of curve and line: $x^2-2=-2x+13$ $\Rightarrow x^2+2x-15=0$ $\Rightarrow (x+5)(x-3)=0$ or $y=\left(\frac{y-13}{2}\right)^2-2$ $\Rightarrow y^2-30y+161=0$ $\Rightarrow x=(-5),\ 3$ $\Rightarrow (y-7)(y-23)=0$ when $x=3, y=7$ $\Rightarrow y=7,\ (23)$	M1 M1 A1
	<u>Volume of cone</u> Using formula: Volume of cone $=\frac{1}{3}\times\pi\times3^2\times(13-7)=18\pi$ or using integration: Volume of cone $=\pi\int_7^{13}\left(\frac{13}{2}-\frac{y}{2}\right)^2\,dy=\frac{\pi}{4}\left[169y-13y^2+\frac{y^3}{3}\right]_7^{13}=18\pi$	B1
	<u>Volume of solid generated by rotating the curve:</u> When $x=0, y=-2$ Volume $=\pi\int_{-2}^{7}(y+2)\,dy=\pi\left[\frac{y^2}{2}+2y\right]_{-2}^{7}$ $=\pi\left\{\left(\frac{49}{2}+14\right)-\left(\frac{(-2)^2}{2}+(-4)\right)\right\}=\frac{81}{2}\pi$ Total volume = $18\pi+\frac{81}{2}\pi=\frac{117}{2}\pi$ oe	M1M1 ddM1 A1 A1 [9]
		Total 9 marks

	M1	Attempt to solve $y+2x=13$ with $y=x^2-2$ and obtain a three term quadratic in x or y. ($x^2+2x-15=0$ or $y^2-30y+161=0$) (Condone = 0 missing.)
	M1	Attempt to solve (see general principles for marking quadratics).
	A1	$y = 7$
	B1	18π correct volume of cone, in terms of π.
	M1	Correct integral for volume generated by rotating curve. Volume $=\pm\pi\int_{(-2)}^{(7)}(y+2)$ (dy) Ignore limits and condone dy omitted or with the wrong variable.
	M1	Evidence of completing integration (see general principles for integration). Ignore limits. π may be missing. If $y + 2$ is combined with another expression in an attempt to add or subtract two volumes, you may accept evidence of integration for the combined expression.
	dd M1	Dep on previous two M marks. Substitute both correct limits $y=-2$ and $y=7$ into the result of their integration. The full substitution should be shown unless the answer is $\frac{81}{2}\pi$ oe.
	A1	$\frac{81}{2}\pi$ oe, given in terms of π. A correct answer from the correct integration implies M1 for a correct substitution of limits.
	A1	$\frac{117}{2}\pi$ oe, correct total volume, in terms of π.

Question Number	Scheme	Marks
10	$\frac{dV}{dt} = 40\ (\text{cm}^3/\text{s})$	B1
	$A = 4\pi r^2 \quad \frac{dA}{dr} = 8\pi r$	M1A1
	$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dr} = 4\pi r^2$	M1A1
	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}, = 8\pi r \times \frac{1}{4\pi r^2} \times 40 \quad (= \frac{80}{r})$	M1,A1ft
	$r = 4$ so $\frac{80}{4} = 20\ (\text{cm}^2/\text{s})$	dM1A1cao **[9]**

	Any letters can be used for volume and area, inc *SA* for area, but their choice must be used consistently.
B1	State or use $\frac{dV}{dt} = 40\ (\text{cm}^3/\text{s})$ (units not needed)
M1	Attempt to differentiate $4\pi r^2$ with respect to r (Formula for area of sphere is given on formula page)
A1	Correct derivative = d*A*/d*t*
M1	Attempt to differentiate $\frac{4}{3}\pi r^3$ with respect to r (Formula for volume of sphere is given on formula page)
A1	Correct derivative = d*A*/d*t*
M1	Show (or use) a useful chain rule. Terms can be in any order as long as it is possible to obtain d*A*/d*t* from it. OR Use chain rule twice to obtain an expression from which d*A*/d*t* could be obtained.
A1ft	Substitute their expressions for the 3 derivatives in their chain rule. Need not be simplified.
dM1	Use the resulting expression(s) with $r = 4$ to obtain a value for d*A*/d*t* All previous M marks needed.
A1cao +cso	Correct value, units may be missing. Solution must be correct.

Question number	Scheme	Marks
7 (a)	$(V=)x^3 \quad (\frac{dV}{dx}=)3x^2 \quad (\text{at } x=2 \quad \frac{dV}{dx}=12)$	M1
	$\frac{dx}{dt}=0.1$	B1 (A1 on ePen)
	$\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}="12"\times 0.1$ oe	M1
	1.2 m^3/s cao oe	A1cao (4)
(b)	(Surface Area =) $6x^2 \quad (\frac{dA}{dx})=12x \quad$ at $x=6$	M1
	$\frac{dA}{dx}=72$	A1
	$\frac{dV}{dx}=108 \quad \frac{dA}{dt}=0.05$	B1
	$\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dA}\times\frac{dA}{dt}="108"\times"\frac{1}{72}"\times 0.05$	M1
	0.075 m^3/s	A1 (5)
	<u>ALT</u>	
	A = $6x^2$ leading to an expression in A for $V \quad V=\left(\frac{A}{6}\right)^{\frac{3}{2}}$	M1
	$\frac{dV}{dA}=\frac{1}{4}\left(\frac{A}{6}\right)^{\frac{1}{2}}$ oe	A1
	$\frac{dV}{dA}=\frac{3}{2}$ and $\frac{dA}{dt}=0.05$ oe	B1
	$\frac{dV}{dt}=\frac{dV}{dA}\times\frac{dA}{dt}="\frac{3}{2}"\times 0.05$	M1
	1.2 m^3/s cao oe	A1cao [5] **[9]**

Part	Mark	Additional Guidance
(a)	M1	Correct expression for Volume, attempt at differentiation to ax^2, a is an integer, $a > 1$.
	B1	$\frac{dx}{dt} = 0.1$, can be explicit or implicitly used in a chain rule.
	M1	For **any correct chain rule**, that would lead to a value for $\frac{dV}{dt}$ and substitution of 0.1 and their value for $\frac{dV}{dx}$. They must show an attempt to find $\frac{dV}{dx}$, need not be a correct attempt, this isn't a dependent mark.
	A1	cao oe
(b)	M1	Correct expression for Surface Area, attempt at differentiation to bx, b is an integer, $b > 1$.
	A1	$\frac{dA}{dx} = 72$
	B1	$\frac{dV}{dx} = 108$ & $\frac{dA}{dt} = 0.05$ clearly stated, implicitly in chain rule or explicitly
	M1	For **any correct chain rule**, that would lead to a value for $\frac{dV}{dt}$ and substitution of 0.05 and their values for $\frac{dA}{dx}$ and $\frac{dV}{dx}$ They must show an attempt to find $\frac{dV}{dx}$, need not be a correct attempt, this isn't a dependent mark.
	A1	cao oe
(b) ALT	M1	Correct expression for area, attempt to rearrange, expression for V in terms of A
	A1	oe
	B1	Both derivatives clearly stated, implicitly in a chain rule or explicitly
	M1	For **any correct chain rule**, that would lead to a value for $\frac{dV}{dt}$ and substitution of 0.05 and their value for $\frac{dV}{dA}$. $\frac{dV}{dA}$ doesn't need to come from correct working, but there must have been some attempt to find an expression for V in terms of A and $\frac{dV}{dA}$ presented somewhere in the working.
	A1	cao oe

Question	Scheme	Marks
11 (a)	$\frac{e^{4x}}{32}$ **and** $8x^2-4x+1$	M1
	$\frac{dy}{dx}=\frac{e^{4x}}{32}(16x-4)+\frac{e^{4x}}{8}(8x^2-4x+1)$	A1A1
	$\frac{dy}{dx}=\frac{16xe^{4x}}{32}-\frac{4e^{4x}}{32}+\frac{8x^2e^{4x}}{8}-\frac{4xe^{4x}}{8}+\frac{e^{4x}}{8}$	dM1
	$\frac{dy}{dx}=x^2e^{4x}$ *	A1 cso [5]
(b)	Volume $=\pi\int_{-2}^{0}\left(3xe^{2x}\right)^2\,dx$	M1
	$V=\pi\int_{-2}^{0}9x^2e^{4x}\,dx=\pi\left[\frac{9e^{4x}}{32}(8x^2-4x+1)\right]_{-2}^{0}$	M1
	$\pi\int_{-2}^{0}9x^2e^{4x}\,dx=\pi\left[\frac{9e^{4\times0}}{32}(8\times0^2-4\times0+1)\right]-\pi\left[\frac{9e^{4\times-2}}{32}(8\times-2^2-4\times-2+1)\right]$	dM1
	$V=0.87142.....\approx0.87$ (2sf)	A1 [4]
	SC - attempts integration by parts.	
	Volume $=\pi\int_{-2}^{0}\left(3xe^{2x}\right)^2\,dx$	M1
	$\int x^2e^{4x}\,dx=\frac{x^2e^{4x}}{'4'}-\int2x\frac{e^{4x}}{'4'}\,dx$ $\int x^2e^{4x}\,dx=\frac{x^2e^{4x}}{'4'}-\left[\frac{2xe^{4x}}{'16'}-\int\frac{2e^{4x}}{'16'}\right]=\frac{x^2e^{4x}}{'4'}\pm\frac{2xe^{4x}}{'16'}\pm\frac{e^{4x}}{'32'}$	M1
	$\pi\int_{-2}^{0}9x^2e^{4x}\,dx=9\pi\left[\frac{x^2e^{4x}}{'4'}-\frac{2xe^{4x}}{'16'}+\frac{e^{4x}}{'32'}\right]_{-2}^{0}$ $=9\pi\left[\left(0-0+\frac{1}{32}\right)-\left(\frac{(-2)^2e^{4(-2)}}{'4'}-\frac{2(-2)e^{4(-2)}}{'16'}+\frac{e^{4(-2)}}{'32'}\right)\right]$	dM1
	For the correct volume of $0.87142.....\approx0.87$ rounded correctly to 2sf	A1 [4]
		Total 9 marks

Part	Mark	Notes
(a)	**M1**	For using product rule correctly with an attempt to differentiate both $\frac{e^{4x}}{32}$ **and** $8x^2-4x+1$ Correct application of $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$ is required For attempt to differentiate: $8x^2-4x+1\rightarrow 16x-4$ $\frac{e^{4x}}{32}\rightarrow ke^{4x}$ where $k\neq 1$ or $\frac{1}{32}$ or $\frac{1}{128}$
	A1	For either $\frac{e^{4x}}{32}(16x-4)$ OR $\frac{e^{4x}}{8}(8x^2-4x+1)$ o.e. in both cases
	A1	For $\frac{dy}{dx}=\frac{e^{4x}}{32}(16x-4)+\frac{e^{4x}}{8}(8x^2-4x+1)$ fully correct Accept $\frac{dy}{dx}=\frac{e^{4x}}{32}(16x-4)+\frac{4e^{4x}}{32}(8x^2-4x+1)$
	dM1	For multiplying out both sets of brackets, or factorising, their $\frac{dy}{dx}$ provided the earlier M mark has been achieved e.g., $\frac{dy}{dx}=\frac{16xe^{4x}}{32}-\frac{4e^{4x}}{32}+\frac{8x^2e^{4x}}{8}-\frac{4xe^{4x}}{8}+\frac{e^{4x}}{8}$ Some working to collect terms between product rule and final expression.
	A1 cso	For the correct expression only $\frac{dy}{dx}=x^2e^{4x}*$
(b)	**M1**	This is a strategy mark for using the correct expression with the correct limits for the volume of rotation. Volume = $\pi\int_{-2}^{0}\left(3xe^{2x}\right)^2\,dx$
	M1	For using the **given** result from part (a) to integrate $V=\pi\int_{-2}^{0}9x^2e^{4x}\,dx=\pi\left[\frac{9e^{4x}}{32}(8x^2-4x+1)\right]_{-2}^{0}$ Ignore π and limits for this mark even if they are missing or incorrect.
	dM1	For applying the correct limits in an attempt to evaluate the integral $\pi\int_{-2}^{0}9x^2e^{4x}\,dx=\pi\left[\frac{9e^{4\times0}}{32}(8\times0^2-4\times0+1)\right]-\pi\left[\frac{9e^{4\times-2}}{32}(8\times-2^2-4\times-2+1)\right]$ Dependent on previous method mark. Condone omission of π for this mark.
	A1	For the correct volume of $0.87142\ldots\approx 0.87$ rounded correctly to 2sf
SC -attempts integration by parts.		
	M1	This is a strategy mark for using the correct expression with the correct limits for the volume of rotation.

		Volume = $\pi\int_{-2}^{0}\left(3xe^{2x}\right)^2\ dx$
	M1	For an attempt to integrate by parts. • They must use the correct formula • $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$ **TWICE** in the correct direction First integration – ignore π and 3^2 for this mark $\int x^2e^{4x}\ dx = \frac{x^2e^{4x}}{'4'} - \int 2x\frac{e^{4x}}{'4'}\,dx$ Second integration $\int x^2e^{4x}\ dx = \frac{x^2e^{4x}}{'4'} - \left[\frac{2xe^{4x}}{'16'} - \int\frac{2e^{4x}}{'16'}\right] = \frac{x^2e^{4x}}{'4'} \pm \frac{2xe^{4x}}{'16'} \pm \frac{e^{4x}}{'32'}$
	dM1	For applying the correct limits in an attempt to evaluate the integral $\pi\int_{-2}^{0}9x^2e^{4x}\ dx = 9\pi\left[\frac{x^2e^{4x}}{'4'} - \frac{2xe^{4x}}{'16'} + \frac{e^{4x}}{'32'}\right]_{-2}^{0}$ $= 9\pi\left[\left(0-0+\frac{1}{32}\right) - \left(\frac{(-2)^2e^{4(-2)}}{'4'} - \frac{2(-2)e^{4(-2)}}{'16'} + \frac{e^{4(-2)}}{'32'}\right)\right]$ Dependent on previous method mark. Condone omission of π for this mark.
	A1	For the correct volume of $0.87142..... \approx 0.87$ rounded correctly to 2sf

Question number	Scheme	Marks
11 (a)(i)	$(a=)-8$	B1 [1]
(a)(ii)	$e^{2x}=9\Rightarrow 2x=\ln 9$	M1
	$x=\frac{1}{2}\ln 9=\ln 3$ *	A1cso [2]
ALT	$e^{2x}=9\Rightarrow\left[\left(e^{x}\right)^{2}=3^{2}\right]\Rightarrow e^{x}=3$	M1
	$x=\ln 3$ *	A1cso
(b)	$\pi\int_{0}^{\ln 3}\left(e^{2x}-9\right)^{2}dx$	M1
	$\pi\int_{0}^{\ln 3}\left(e^{4x}-18e^{2x}+81\right)dx$	A1
	$(\pi)\left[\frac{1}{4}e^{4x}-9e^{2x}+81x\right]_{(0)}^{("\ln 3")}$ oe	M1 A1ft
	$(\pi)\left[\left(\frac{81}{4}-81+81\ln 3\right)-\left(\frac{1}{4}-9\right)\right]$	M1
	$\pi(81\ln 3-52)$	A1 [6]
		Total 9 marks

	A1*cso	For a fully correct solution.

Part	Mark	Notes
(a)(i)	**B1**	For $(a=)-8$
(a)(ii)	**M1**	For substituting y = 0 and rearranging to $2x=\ln 9$ or $x=\frac{1}{2}\ln 9$
	A1*cso	For a fully correct solution with no errors – must see $x=\frac{1}{2}\ln 9$ *or* $2x=2\ln 3$
ALT	**M1**	For substituting y = 0 and manipulating to give $e^{x}=3$
	A1*cso	For a fully correct solution with no errors
(b)	**M1**	For $\pi\int_{0}^{\ln 3}\left(e^{2x}-9\right)^{2}dx$
	A1	For $\pi\int_{0}^{\ln 3}\left(e^{4x}-18e^{2x}+81\right)dx$ oe. If π seen for first M1, can be omitted.
	M1	For minimally acceptable attempt to integrate two terms of their expression (at least one term integrated correctly). π and limits can be omitted.
	A1ft	For $\frac{1}{4}e^{4x}-9e^{2x}+81x$ Ft their expression of the form $e^{4x}+ce^{2x}+d$ π and limits can be omitted
	M1	For correct substitution of 0 and their *b* into a changed expression. The substitution must be fully shown for this mark. π can be omitted. This mark can be implied by a correct final answer.
	A1	For obtaining the correct expression

π must only be present for the final A1 mark and the first M1 mark (can be implied by first or final A1)

Question	Scheme	Mark
5(a)	$3t^2-23t+30=0\Rightarrow(3t-5)(t-6)=0\Rightarrow t=6,\ \frac{5}{3}$	M1A1A1 [3]
(b)	$a=\frac{dv}{dt}=6t-23$ $(6t-23>0)\Rightarrow t>\frac{23}{6}$	M1 A1 [2]
(c)	$s=\int 3t^2-23t+30\ dt=\frac{3t^3}{3}-\frac{23t^2}{2}+30t+c$ $26=\frac{3(8)^3}{3}-\frac{23(8)^2}{2}+30(8)+c\Rightarrow c='10'$ $d=0+0+0+'10'\Rightarrow d=10$	M1A1 M1 A1 [4]
	Total	**9 marks**

Part	Mark	Notes
5 (a)	M1	For setting the given expression for $v = 0$ and attempting to solve the 3TQ See general guidance for the definition of an attempt $3t^2-23t+30=0\Rightarrow(3t-5)(t-6)=0\Rightarrow t=...,...$
	A1	For either $t=6$ or $t=\frac{5}{3}$
	A1	For both $t=6$ and $t=\frac{5}{3}$
(b)	M1	For differentiating the given expression for v which must be correct for this mark. $a=\frac{dv}{dt}=6t-23$
	A1	$(6t-23>0)\Rightarrow t>\frac{23}{6}$ Accept equivalent exact values including $t>3.8\dot{3}$
(c)	M1	For attempting to integrate the given expression for v See General Guidance for the definition of an attempt with no terms differentiated. $+c$ is not required for the award of this mark. $s=\int 3t^2-23t+30\ dt=\frac{3t^3}{3}-\frac{23t^2}{2}+30t+c$
	A1	For the correct integrated expression, which must include a constant term e.g.+ c $s=\frac{3t^3}{3}-\frac{23t^2}{2}+30t+c$ Simplification is not required for this mark
	M1	For substituting the given values of $t = 8$ when $s = 26$ into their **integrated** expression to find c $26=\frac{3(8)^3}{3}-\frac{23(8)^2}{2}+30(8)+c\Rightarrow c='10'$ $[d=0+0+0+'10'\Rightarrow d=...]$
	A1	For the correct value of $d = 10$

Question number	Scheme	Marks
10 a	$(y-1)+y^2=11\Rightarrow y^2+y-12=0$ or $x^2+\left(x^2+1\right)^2=11\Rightarrow x^4+3x^2-10=0$	M1
	$(y-3)(y+4)=0$ or $x^2+5 \quad x^2-2 \quad =0$	M1
	$(y=3 \ y=-4) \qquad \Rightarrow x^2=2 \quad x^2=-5$	M1
	For stating the x coordinate of A is $-\sqrt{2}$ **and** the x coordinate of B is $\sqrt{2}$	A1 (4)
b	$\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2)-(x^2+1)^2\ \mathrm{d}x$ or $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^2+1)^2-(11-x^2)\mathrm{d}x$	M1
	$\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(-x^4-3x^2+10)\ \mathrm{d}x"$	A1ft
	$(\pi)\left[-\frac{x^5}{5}-x^3+10x\right]_{(-\sqrt{2})}^{(\sqrt{2})}$	M1
	$(\pi)\left[\left(-\frac{("\sqrt{2}")^5}{5}-("\sqrt{2}")^3+10"\sqrt{2}"\right)-\left(-\frac{("-\sqrt{2}")^5}{5}-("-\sqrt{2}")^3-10"\sqrt{2}"\right)\right]=63.98$	M1 A1 (5)
ALT		
	$\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2)\ \mathrm{d}x-\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^2+1)^2\ \mathrm{d}x$ or $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^2+1)^2\ \mathrm{d}x-\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2)\ \mathrm{d}x$	{M1}
	$\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2)\ \mathrm{d}x-\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^4+2x^2+1)\ \mathrm{d}x$	{A1ft}
	$(\pi)\left[11x-\frac{x^3}{3}\right]_{(-\sqrt{2})}^{(\sqrt{2})}-(\pi)\left[\frac{x^5}{5}+\frac{2x^3}{3}+x\right]_{(-\sqrt{2})}^{(\sqrt{2})}$	{M1}
	$(\pi)\left[\left(11("\sqrt{2}")-\frac{("\sqrt{2}")^3}{3}\right)-\left(11("-\sqrt{2}")-\frac{("-\sqrt{2}")^3}{3}\right)\right]$	
	$-(\pi)\left[\left(\frac{("\sqrt{2}")^5}{5}+\frac{2("\sqrt{2}")^3}{3}+"\sqrt{2}"\right)-\left(\frac{("-\sqrt{2}")^5}{5}+\frac{2("-\sqrt{2}")^3}{3}"-\sqrt{2}"\right)\right]$	{M1} {A1}
	$=63.98$	(5)
		Total 9 marks

Note: the candidate may also set up the integral as

$2\times\pi\int_0^{\sqrt{2}}(11-x^2)-(x^2+1)^2\ \mathrm{d}x$ or $2\times\pi\int_{-\sqrt{2}}^{0}(11-x^2)-(x^2+1)^2\ \mathrm{d}x$ or

$2\times\left(\pi\int_0^{\sqrt{2}}(11-x^2)\ \mathrm{d}x-\pi\int_0^{\sqrt{2}}(x^2+1)^2\ \mathrm{d}x\right)$ *or* $2\times\left(\pi\int_{-\sqrt{2}}^{0}(11-x^2)\ \mathrm{d}x-\pi\int_{-\sqrt{2}}^{0}(x^2+1)^2\ \mathrm{d}x\right)$

If it is clear that the integral(s) being used at any point needs to be multiplied by 2, all marks can be awarded, follow the same principles in the main or ALT schemes if the multiply by 2 is seen anywhere in the solution.

If multiply by 2 is nowhere present or implied by a final answer, the maximum mark is M0 A0 M1 M1 A0

Part	Mark	Additional Guidance
(a)	M1	For correctly rearranging to get $x^2 = y - 1$ and correctly substituting to get an equation of the form $ay^2 + by + c = 0 \quad a,b,c \neq 0$ **or** For correctly substituting $y = x^2 + 1$, an attempt to expand $(x^2+1)^2$ and attaining an equation of the form $dx^4 + ex^2 + f = 0 \quad d,e,f \neq 0$
	M1	For a complete attempt to solve either $ay^2 + by + c$ or $dx^4 + ex^2 + f$ by any valid method – see general guidance for definition of minimally acceptable attempt.
	M1	For correctly solving their quadratic equation in y or quartic equation in x **and** for obtaining a value(s) for x^2 or x. Award this mark if the candidate does not state or dismisses a solution where x^2 is negative.
	A1	For stating the x coordinate of A is $-\sqrt{2}$ **and** the x coordinate of B is $\sqrt{2}$ Decimal equivalent allowed.
(b)	M1	For $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"} (11 - x^2) - (x^2+1)^2 \, dx$ or $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"} (x^2+1)^2 - (11 - x^2) dx$ Allow use of their limits from part a.
	A1ft	For expanding the bracket and simplifying to give $\pi\int_{-\sqrt{2}}^{\sqrt{2}} (-x^4 - 3x^2 + 10) \, dx$ The ft is only for use of their limits from part a.
	M1	For correct integration of their expressions. There must be at least 3 terms overall and there must be a term in x^4 or y^4. It is not necessary for π or limits to be present for this mark.
	M1	For correct substitution of their limits seen into their changed expression. Each of the limits needs to be substituted into the expression correctly at least once. Allow a fully correct final answer to imply this mark. Brackets must be correct but can be recovered later.
	A1	For awrt 63.98 (Calc: 63.9775…) If a negative value is found and changed at the end to a positive value, this final A mark cannot be awarded.
ALT	M1	For $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"} (11 - x^2) \, dx - \pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"} (x^2+1)^2 \, dx$ or $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"} (x^2+1)^2 \, dx - \pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"} (11 - x^2) \, dx$ Allow use of their limits from part a.
	A1ft	For expanding the bracket and simplifying $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"} (11 - x^2) \, dx - \pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"} (x^4 + 2x^2 + 1) \, dx$ The ft is only for use of their limits from part a.
	M1	For correct integration of their expressions. There must be at least 3 terms overall and there must be a term in x^4 or y^4. It is not necessary for π or limits to be present for this mark.
	M1	For correct substitution of their limits seen into their changed expression. Each of the limits needs to be substituted into each expression correctly at least once. Allow a fully correct final answer, with correct integration shown, to imply this mark. Brackets must be correct but can be recovered later.
	A1	For awrt 63.98 (Calc: 63.9775…) If a negative value is found and changed at the end to a positive value, this final A mark cannot be awarded.

Question number	Scheme	Marks
7 a	$\frac{dy}{dx} = 2e^{2x}\cos 2x - 2e^{2x}\sin 2x$	M1 A1 A1
	$\frac{dy}{dx} = 2y - 2e^{2x}\sin 2x$ *	A1 cso (4)
b	$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 4e^{2x}\sin 2x - 4e^{2x}\cos 2x$	M1 A1
	$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} + 2\left(\frac{dy}{dx} - 2y\right) - 4y$	dM1ddM1
	$\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y$ *	A1 cso (5)
		Total 9 marks

Part	Mark	Notes
(a)	M1	For an attempt to use the product rule. The definition of an attempt is as follows: • There must be a correct attempt to differentiate both terms. $\cos 2x \Rightarrow \pm 2\sin 2x \quad e^{2x} \Rightarrow 2e^{2x}$ where $x \neq 0$ • The terms must be added The correct formula must be used.
	A1	For one term correct
	A1	For both terms correct
	A1 cso	For obtaining the given result with no errors seen.
(b)	**Method 1**	
	M1	For attempting to differentiate $\frac{dy}{dx}$ to obtain $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 4e^{2x}\cos 2x - 4e^{2x}\sin 2x - 4e^{2x}\sin 2x - 4e^{2x}\cos 2x$ For this mark accept either: $\left(4e^{2x}\cos 2x - 4e^{2x}\sin 2x\right)$ **OR** $-4e^{2x}\sin 2x - 4e^{2x}\cos 2x$
	A1	For the correct $\frac{d^2y}{dx^2}$ (This is an M mark in Epen)
	dM1	For simplifying $\frac{d^2y}{dx^2}$ to obtain $\frac{d^2y}{dx^2} = -8e^{2x}\sin 2x$ (A mark in Epen) **This mark is dependent on the first M mark**

	ddM1	For using the substitution $-2e^{2x}\sin 2x = \frac{dy}{dx} - 2y \Rightarrow -8e^{2x}\sin 2x = 4\frac{dy}{dx} - 8y$ **This mark is dependent on BOTH previous M marks.**
	A1cso	For obtaining the given result with no errors seen. $\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y$
	Method 2	
	M1	For attempting to differentiate $\frac{dy}{dx}$ to obtain $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 4e^{2x}\sin 2x - 4e^{2x}\cos 2x$ For this mark accept either: $2\frac{dy}{dx}$ or $-4e^{2x}\sin 2x - 4e^{2x}\cos 2x$
	A1	For the correct $\frac{d^2y}{dx^2}$ **This is an M mark in Epen**
	dM1	For substituting $-4e^{2x}\cos 2x \Rightarrow -2y$ **This is an A mark in Epen**
	ddM1	For using the substitution $-4e^{2x}\sin 2x = 2\frac{dy}{dx} - 4y$
	A1	For obtaining the given result with no errors seen. $\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y$
	Method 3 – Works form LHS and RHS together.	
	M1	For attempting to differentiate $\frac{dy}{dx}$ to obtain $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 4e^{2x}\cos 2x - 4e^{2x}\sin 2x - 4e^{2x}\sin 2x - 4e^{2x}\cos 2x$ For this mark accept either: $\left(4e^{2x}\cos 2x - 4e^{2x}\sin 2x\right)$ **OR** $-4e^{2x}\sin 2x - 4e^{2x}\cos 2x$
	A1	For the correct $\frac{d^2y}{dx^2}$ **This is an M mark in Epen**
	dM1	For simplifying $\frac{d^2y}{dx^2}$ to obtain $\frac{d^2y}{dx^2} = -8e^{2x}\sin 2x$ (A mark in Epen) **This mark is dependent on the first M mark**
	ddM1	Multiplies out the given expression in (b) $\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y = 4\left(2e^{2x}\cos 2x - 2e^{2x}\sin 2x\right) - 8\left(e^{2x}\cos 2x\right)$ $= 8e^{2x}\cos 2x - 8e^{2x}\sin 2x - 8e^{2x}\cos 2x$ $= -8e^{2x}\sin 2x$

		This mark is dependent on BOTH previous M marks.
	A1	For a conclusion. A simple # sign, 'shown', 'QED' or underlining is sufficient.

Question	Scheme	Marks
7(a)	$S = 5x^3 + (3x-4)^2$ $\Rightarrow S = 5x^3 + 9x^2 - 24x + 16$ *	M1 A1 cso [2]
(b)	$\frac{dS}{dx} = 15x^2 + 18x - 24 = 0$ $\Rightarrow (5x-4)(x+2) = 0 \Rightarrow x = \frac{4}{5}, -2$ $\frac{d^2S}{dx^2} = 30x + 18 = 30\left(\frac{4}{5}\right) + 18 \Rightarrow$ +ve hence minimum	M1 M1A1 M1A1 [5]
(c)	$S = 5\left(\frac{4}{5}\right)^3 + 9\left(\frac{4}{5}\right)^2 - 24\left(\frac{4}{5}\right) + 16 = \frac{128}{25}$ or 5.12	M1A1 [2]
		Total 9 marks

Part	Mark	Notes
(a)	M1	For substituting y into the given S Substituting x will not yield the required expression.
	A1 cso	For obtaining the given expression with no errors. You must check every line of their working.
(b)	M1	For an attempt to differentiate **the given** expression for S wrt x, Accept **at least** two terms fully correct with no power of x to increase.
	M1	Sets their differentiated expression = 0 and attempts to solve, **provided** it is a quadratic. See General Guidance for the definition of an attempt to solve a QE
	A1	For the correct **two** values of x.
	M1	Attempts to differentiate again. Minimally acceptable attempt is $\left(\frac{d^2S}{dx^2}\right) = Ax + B$
	A1	Conclusion: Concludes that the positive value of x $\left(\frac{4}{5}\right)$ will give a positive $\frac{d^2S}{dx^2}$ hence will be a minimum. For example, positive + positive = positive hence minimum. OR Substitutes either value of x, with the appropriate conclusion and correctly concludes that $x = \frac{4}{5}$ gives a minimum.

		NOTE: If they evaluate $\frac{d^2S}{dx^2}$ it must be correct [= 42] An incorrect value is A0.
(c)	M1	Uses **their** minimum value of x in the **given** expression, even if they identify $x = -2$ as the value giving a minimum S. If their minimum is not explicitly identified, then award M0. If they substitute both values without identifying which is which, award M0.
	A1	For the correct value of S

Question	Scheme	Marks
5(a)	$\left(v=\int(3t-4)\,\mathrm{d}t=\right)\frac{3t^2}{2}-4t+c$ (When $t=0$, $v=0 \Rightarrow c=0$) $\left(v=\right)\frac{3\times4^2}{2}-4\times4=8$ [m/s]	M1 M1A1 [3]
(b)	$0=\frac{3t^2}{2}-4t \Rightarrow t\left(\frac{3t}{2}-4\right)=0 \Rightarrow \frac{3t}{2}=4 \Rightarrow t=\frac{8}{3}$	M1A1 [2]
(c)	$\left(x=\int\left(\frac{3t^2}{2}-4t\right)\mathrm{d}t=\right)\frac{3t^3}{2\times3}-\frac{4t^2}{2}(+c)=\left[\frac{t^3}{2}-2t^2+c\right]$ When $t=0$ P is at the point with coordinates $(-10, 0)$ $\Rightarrow c=-10$ $\Rightarrow \left(x=\right)\frac{3^3}{2}-2\times3^2-10=-\frac{29}{2}$ (m)	M1 M1 dM1A1 [4]
		Total 9 marks

Part	Mark	Notes
(a)	M1	For a minimally acceptable attempt (see general guidance) to integrate the given expression. **At least one term correctly integrated.** Terms do not need to be simplified to attain this mark. No power of t to decrease.
	M1	For substituting the value of $t = 4$ into their changed expression (general rule of marking unless precluded, this method mark may be implied from a correct answer, unless from incorrect working).
	A1	For $(v =)8$
We do not need to see the calculation of $c = 0$ Note: substitution of $t = 4$ into the expression for a also gives an answer of 8 – please watch out for this and do not mark this correct.		
(b)	M1	For setting their changed expression = 0 and a **fully correct** attempt to solve, leading to a value for t. Note a correct vale of t can imply this mark. If the quadratic is not the correct quadratic, method must be shown.
	A1	For the value of $t=\frac{8}{3}$, ignore $t = 0$ is given. Accept answers which round to 2.7 or clear indication of 2.6 recurring.
(c)	M1	For a **minimally acceptable attempt** (see general guidance) to integrate their expression for v, which must be a **minimum 2 term** expression. Terms don't need to be simplified at this point. c does not need to be present. No power of t to decrease.
	M1	For correct substitution, into their changed expression, of $t = 0$ and $P = -10$ to find the value of c. -10, if seen at any point will usually imply this mark. The mark is for substitution of the correct values, to find c, if they rearrange incorrectly, the mark can still be awarded.
	dM1	For substituting the value of $t = 3$ into their changed expression, with their c. Dependent on the previous method mark.
	A1	For the displacement of $-\frac{29}{2}$ If distance of $\frac{29}{2}$ is given A0, no isw here.

Question number	Scheme	Marks
9 (a)	$f'(x) = 4\times 2x(1+x^2)^3 = 8x(1+x^2)^3$ *	M1 A1* cso [2]
(b)	Find the gradient of the tangent to C at A:	
	$\frac{dy}{dx} = \frac{3\cos 3x(1+x^2)^4 - 8x(1+x^2)^3\sin 3x}{(1+x^2)^8}$	M1A1A1
	$= \frac{(1+x^2)^3[3(1+x^2)\cos 3x - 8x\sin 3x]}{(1+x^2)^8}$	
	$= \frac{3\cos 3x + 3x^2\cos 3x - 8x\sin 3x}{(1+x^2)^5}$	
	$\frac{3\cos\left(3\times\frac{2\pi}{3}\right)+3\left(\frac{2\pi}{3}\right)^2\cos\left(3\times\frac{2\pi}{3}\right)-8\times\frac{2\pi}{3}\sin\left(3\times\frac{2\pi}{3}\right)}{\left(1+\left(\frac{2\pi}{3}\right)^2\right)^5}$	M1
	$= \frac{3\left(1+\left(\frac{2\pi}{3}\right)^2\right)}{\left(1+\left(\frac{2\pi}{3}\right)^2\right)^5} = \frac{3}{\left(1+\left(\frac{2\pi}{3}\right)^2\right)^4}$	A1
	Find the negative reciprocal:	
	$= -\frac{1}{3}\left(1+\left(\frac{2\pi}{3}\right)^2\right)^4$	M1A1* cso [7]
ALT (b)	Find the gradient of the tangent to C at A:	
	Using product rule from	M1
	$y = \sin 3x\left(1+x^2\right)^{-4}$	A1A1
	$\frac{dy}{dx} = \sin 3x\left(-8x\left(1+x^2\right)^{-5}\right)+3\cos 3x\left(1+x^2\right)^{-4}$	
	$\frac{dy}{dx}\left(\frac{2\pi}{3}\right) = \sin\left(3\times\frac{2\pi}{3}\right)\left(-8\times\frac{2\pi}{3}\left(1+\left(\frac{2\pi}{3}\right)^2\right)^{-5}\right)+3\times\cos\left(3\times\frac{2\pi}{3}\right)\left(1+\left(\frac{2\pi}{3}\right)^2\right)^{-4}$	M1
	$= \frac{3}{\left(1+\left(\frac{2\pi}{3}\right)^2\right)^4}$ oe	A1
	Find the negative reciprocal:	
	$= -\frac{1}{3}\left(1+\left(\frac{2\pi}{3}\right)^2\right)^4$	M1A1* cso [7]
		Total 9 marks

Part	Mark	Additional Guidance
9 (a)	M1	For showing an intermediate equation in which are shown multiplication by the original index, multiplication by the derivative of the bracketed term and a reduction in the index of the bracketed term. Should be in the form $ax(1+x^2)^3$
	A1*cso	For a fully correct solution, minimum steps shown, no omissions or error. Accept "$\frac{dy}{dx}=$" or "$\frac{d}{dx}=$"
(b)	M1	For a minimally acceptable attempt to differentiate using the quotient rule • $\sin 3x \Rightarrow k\cos 3x \quad k \neq 0,1$ where k is an integer • $(1+x^2)^4$ as given in (a) • Allow + /- in the numerator • Denominator must be correct
	A1	For both terms in the numerator, in either order
	A1	For the correct expression shown oe **NB: This mark appears as an M mark in ePen**
	M1	For substitution of $x=\frac{2\pi}{3}$ into any changed expression representing the gradient of the tangent. **NB: This mark appears as an A mark in ePen**
	A1	For the correct expression shown. This may be implied by further correct work. **NB: This mark appears as an M mark in ePen**
	M1	For attempting to find the negative reciprocal of their expression.
	A1cso	For the correct expression shown
ALT (b)	M1	For a minimally acceptable attempt to differentiate using the product rule • $\sin 3x \Rightarrow k\cos 3x \quad k \neq 0,1$ where k is an integer • $(1+x^2)^4$ as given in (a) • Allow + /- between the terms
	A1	For one term correct
	A1	For the correct expression shown oe **NB: This mark appears as an M mark in ePen**
	M1	For substitution of $x=\frac{2\pi}{3}$ into any changed expression representing the gradient of the tangent. **NB: This mark appears as an A mark in ePen**
	A1	For the correct expression shown This may be implied by further correct work **NB: This mark appears as an M mark in ePen**
	M1	For attempting to find the negative reciprocal of their expression.
	A1cso	For the correct expression shown

Question number	Scheme	Marks
4 (a)	$f'(x) = 3e^{3x}(1+2x)^{\frac{1}{2}} + e^{3x} \times \frac{1}{2} \times 2(1+2x)^{-\frac{1}{2}}$	M1A1
	$\Rightarrow f'(x) = \frac{3e^{3x}(1+2x)+e^{3x}}{\sqrt{1+2x}} \Rightarrow f'(x) = \frac{2e^{3x}(2+3x)}{\sqrt{1+2x}}$*	M1A1 [4]
(b)	When $x = 0$	
	$f'(0) = \frac{2e^0(2+0)}{\sqrt{1+0}} = 4$ Gradient of Normal $= -\frac{1}{4}$	B1B1
	$f(0) = e^0\sqrt{1+2\times 0} = 1$	B1
	Equation of Normal to curve $y = f(x)$ when $x = 0$	
	$y - 1 = -\frac{1}{4}(x-0)$	M1A1
	$\Rightarrow x + 4y - 4 = 0$	A1 [6]
		Total 10 marks

(a)	
M1	Use of the product rule. Sum of two terms (either way round) with $x^n \rightarrow x^{n-1}$ (Condone e^{3x} instead of $3e^{3x}$)
A1	Both terms correct
M1	Simplifying their product rule expression to a single expression with denominator $a\sqrt{1+2x}$ where a is a constant
A1	Obtains the **given** expression
(b)	
B1	$f'(0) = 4$
B1	Gradient of Normal $= -\frac{1}{4}$
B1	$f(0) = 1$
M1	Substitution of (0, '1') and 'gradient of normal' (but not 4) into the formula for a line
A1	$y - 1 = -\frac{1}{4}(x-0)$ oe
A1	$x + 4y - 4 = 0$

Question number	Scheme	Marks
9 (a)	$540 = 3x^2h \Rightarrow h = \dfrac{180}{x^2}$ $S = 2\left(3x^2 + 3xh + xh\right) = 6x^2 + 8xh$ $\Rightarrow S = 6x^2 + 8x \times \dfrac{180}{x^2} = 6x^2 + \dfrac{1440}{x}$ *	M1 M1 depM1A1 [4]
(b)	$S = 6x^2 + 1440x^{-1}$ $\dfrac{dS}{dx} = 12x - 1440x^{-2}$ At min/max $\dfrac{dS}{dx} = 0$ $12x - 1440x^{-2} = 0 \Rightarrow x^3 = 120 \Rightarrow x = 4.93242....$ $x \approx 4.93$ (3sf) $\dfrac{d^2S}{dx^2} = 12 + \dfrac{2880}{x^3} \Rightarrow$ Always positive for positive values of x, hence minimum	 M1 M1A1 M1A1ft [5]
(c)	$S = 6 \times 4.93242^2 + \dfrac{1440}{4.93242} = 437.9185 \approx 438$	B1 [1]
		Total 10 marks

(a)	
M1	Rearrange the equation for volume to make h the subject
M1	Obtains an expression for S in terms of x and h.
depM1	Dependent on previous M1. Use the equation to eliminate h to give an expression for S in terms of x only.
A1	Obtains the **given** expression for S.
(b)	
M1	Attempts to differentiates S wrt x with $x^n \rightarrow x^{n-1}$
M1	Equate their derivative to zero and solve for x
A1	Correct value of x, min 3 sf (Do not accept $\sqrt[3]{120}$)
M1	Obtains a correct second derivative from their first derivative. (If signs of $\dfrac{dS}{dx}$ on either side of their x are considered, numerical calculations must be shown.)
A1 ft	Establish that the minimum has been obtained and give a conclusion. No need to calculate the value of the second derivative. Follow through their x provided $x > 0$ and the second derivative is algebraically correct or if signs of $\dfrac{dS}{dx}$ on either side of their x were considered these need to be calculated and correct
(c)	
B1	Correct value of S. Must be 3 sf

Question Number	Scheme	Marks
8(a)	$2xy+5y=e^x \quad y=\dfrac{e^x}{(2x+5)}$	
	$\dfrac{dy}{dx}=\dfrac{e^x(2x+5)-2e^x}{(2x+5)^2}$	M1A1A1
	$\dfrac{dy}{dx}=\dfrac{e^x}{(2x+5)}\times\dfrac{(2x+5-2)}{(2x+5)}=\dfrac{y(2x+3)}{(2x+5)}$ *	M1A1 (5)
(b)	$x=0\Rightarrow \dfrac{dy}{dx}=\dfrac{5-2}{5^2}=\dfrac{3}{25}$	M1A1 (2)
ALT	$x=0\Rightarrow y=\dfrac{1}{5},\ \dfrac{dy}{dx}=\dfrac{1}{5}\times\dfrac{3}{5}=\dfrac{3}{25}$	
(c)	$x=0\Rightarrow y=\dfrac{e^0}{(2\times0+5)}=\dfrac{1}{5}$	M1(Award if seen in (b) and **used** in (c))
	$y-\dfrac{1}{5}=-\dfrac{25}{3}x$	M1
	$125x+15y-3=0$	A1 (3) [10]

Part	Mark	Notes
(a)		$2xy+5y=e^x \Rightarrow y=\dfrac{e^x}{(2x+5)}$
	M1	For attempting Quotient Rule • Both terms must be differentiated correctly $e^x\Rightarrow e^x \quad 2x+5\Rightarrow 2$ • There must be two terms subtracted in the numerator either way around • The denominator must the denominator squared. $\dfrac{dy}{dx}=\dfrac{e^x(2x+5)-2e^x}{(2x+5)^2}$
	A1	For $e^x(2x+5)$ or $2e^x$
	A1	For a fully correct differentiated expression. $\dfrac{dy}{dx}=\dfrac{e^x(2x+5)-2e^x}{(2x+5)^2}$

	M1	Subs in $y=\dfrac{e^x}{(2x+5)}$ as a common factor $\dfrac{dy}{dx}=\dfrac{e^x}{(2x+5)}\times\dfrac{(2x+5-2)}{(2x+5)}=\dfrac{y(2x+5-2)}{(2x+5)}$	Subs in $e^x=y(2x+5)$ and factorises $\dfrac{dy}{dx}=\dfrac{y(2x+5)(2x+5)-2y(2x+5)}{(2x+5)^2}$
	A1	For the correct answer with no errors. **Note this is a given answer.** $\dfrac{dy}{dx}=\dfrac{y(2x+3)}{(2x+5)}$	
	ALT – uses implicit differentiation on $2xy+5y=e^x$		
	M1	$2\left(y+x\dfrac{dy}{dx}\right)+5\dfrac{dy}{dx}=e^x$	
	A1	Takes out $\dfrac{dy}{dx}$ as a common factor $\dfrac{dy}{dx}(2x+5)=e^x-2y$	
	A1	For a fully correct differentiated expression as below. $\dfrac{dy}{dx}=\dfrac{e^x-2y}{(2x+5)}$	
	M1	For separating the fraction, taking out y as a common factor and attempting to form a single fraction $\dfrac{dy}{dx}=\dfrac{e^x}{(2x+5)}-\dfrac{2y}{(2x+5)}=\dfrac{y(2x+5)}{(2x+5)}-\dfrac{2y}{(2x+5)}=\dfrac{y(2x+5-2)}{2x+5}$	
	A1	For the correct answer with no errors. $\dfrac{dy}{dx}=\dfrac{y(2x+3)}{(2x+5)}$	
(b)	M1	For substituting $x=0$ into $\dfrac{dy}{dx}=\dfrac{e^x(2x+5)-2e^x}{(2x+5)^2}=\dfrac{e^0(2\times0+5)-2e^0}{(2\times0+5)^2}=...$	
	A1	For the correct value of $\dfrac{dy}{dx}=\dfrac{3}{25}$	
	ALT		
	M1	When $x=0\Rightarrow y=\dfrac{1}{5}$, $\dfrac{dy}{dx}=\dfrac{1}{5}\times\dfrac{3}{5}=...$	
	A1	For the correct value of $\dfrac{dy}{dx}=\dfrac{3}{25}$	
(c)	M1	$x=0\Rightarrow y=\dfrac{e^0}{(2\times0+5)}=\dfrac{1}{5}$ $x=0\Rightarrow y=\dfrac{1}{5}$ Award if seen in (b) and **used** in (c) **This is a B mark in Epen.**	
	M1	Inverts the gradient found in (b) and forms equation of the normal. ft their value of y $y-\dfrac{1}{5}=-\dfrac{25}{3}x$	
	A1	Equation of line is given in the required form. $125x+15y-3=0$	

Question	Scheme	Marks
8 (a)	$90\pi = \pi r^2 h \Rightarrow h = \frac{90}{r^2}$ $S = 2\pi r^2 + 2\pi rh \Rightarrow S = 2\pi r^2 + 2\pi r \times \frac{90}{r^2}$ $S = 2\pi r^2 + \frac{2\times 90\pi}{r} = 2\pi r^2 + \frac{180\pi}{r}$ *	B1 M1 A1cso [3]
(b)	$\frac{dS}{dr} = 4\pi r - \frac{180\pi}{r^2}$ $\frac{dS}{dr} = 0 \Rightarrow 4\pi r - \frac{180\pi}{r^2} = 0 \Rightarrow 4\pi r = \frac{180\pi}{r^2} \Rightarrow r^3 = 45 \Rightarrow r = ...$ $r = 3.55689... \Rightarrow r \approx 3.56$ $\frac{d^2S}{dr^2} = 4\pi + \frac{360\pi}{r^3}$ $\frac{d^2S}{dr^2} = 4\pi + \frac{360\pi}{r^3} \Rightarrow \left(\frac{d^2S}{dr^2} = 37.699...\right)$ $37.699 > 0 \Rightarrow$ hence minimum	M1 M1 A1 M1 A1ft [5]
(c)	$S = 2\pi \times 3.556...^2 + \frac{2\times 90\pi}{3.556..} = ...$ $S = 238.4769... \Rightarrow S = 238\ (\text{cm}^2)$	M1 A1 [2]
		Total 10 marks

Part	Marks	Scheme
(a)	**B1**	For finding an expression for h in terms of r $90\pi = \pi r^2 h \Rightarrow h = \frac{90}{r^2}$ Award for finding an expression for hr in terms of r $90\pi = \pi r^2 h \Rightarrow hr = \frac{90}{r}$
	M1	For substituting their expression for h into a **correct** formula for the closed surface area of a cylinder $S = 2\pi r^2 + 2\pi rh \Rightarrow S = 2\pi r^2 + 2\pi r \times \frac{'90'}{r^2}$ Or for substitution of their expression for hr into a **correct** formula for the closed surface area of a cylinder $S = 2\pi r^2 + 2\pi rh \Rightarrow S = 2\pi r^2 + 2\pi \times \frac{'90'}{r}$
	A1 cso	For the correct expression for the area as shown

		$S = 2\pi r^2 + \frac{2\times 90\pi}{r} = 2\pi r^2 + \frac{180\pi}{r}$ Must have the $S =$ for this mark.
(b)	**M1**	For attempting to differentiate the given expression for S at least one power to decrease and neither power to increase. $\frac{dS}{dr} = 4\pi r - \frac{180\pi}{r^2}$
	M1	Sets their $\frac{dS}{dr} = 0$ and attempts to solve for r $4\pi r - \frac{180\pi}{r^2} = 0 \Rightarrow 4\pi r = \frac{180\pi}{r^2} \Rightarrow r^3 = 45 \Rightarrow r = ...$
	A1	For the correct value of $r = 3.55689... \Rightarrow r \approx 3.56$ Accept awrt 3.56
	M1	For attempting to differentiate their expression for $\frac{dS}{dr}$ at least one power to decrease and neither power to increase. $\frac{d^2S}{dr^2} = 4\pi + \frac{360\pi}{r^3}$
	A1ft	For correct work throughout $\frac{d^2S}{dr^2} = 4\pi + \frac{180\pi}{r^3} \Rightarrow \left(\frac{d^2S}{dr^2} = 37.699...\right)$ $37.699 > 0 \Rightarrow$ hence minimum Evaluation not required as both terms positive so $\frac{d^2S}{dr^2} > 0$ hence minimum Indication of positive or >0 required. If $\frac{d^2S}{dr^2}$ evaluated incorrectly then do not award. If evaluated then accept awrt 38
(c)	**M1**	For substituting their value of r into the **given** expression for S $S = 2\pi \times '3.556'...^2 + \frac{2\times 90\pi}{'3.556'..} = ...$ Their value of r >0
	A1	$S = 238.4769... \Rightarrow S = 238\ (\text{cm}^2)$ Accept awrt 238

Question	Scheme	Marks
6(a)	$\frac{dy}{dx} = \frac{\left(x^2+1\right)2xe^{\left(x^2+1\right)} - 2xe^{\left(x^2+1\right)}}{\left(x^2+1\right)^2}$	M1A1A1
	$\frac{dy}{dx} = \frac{2xe^{\left(x^2+1\right)}\left(x^2+1-1\right)}{\left(x^2+1\right)^2} = \frac{2x^3e^{\left(x^2+1\right)}}{\left(x^2+1\right)^2}$	M1A1 cso [5]
(b)	When $x = -1$ $\frac{dy}{dx} = \frac{-e^2}{2}$, $y = \frac{e^2}{2}$	B1ft, B1
	$y - \frac{e^2}{2} = -\frac{e^2}{2}(x+1), \Rightarrow y = -\frac{e^2x}{2}$ oe	M1A1ft, A1 [5]
		Total 10 marks

Question	Notes	Marks
6(a)	$y = \frac{e^{\left(x^2+1\right)}}{x^2+1}$	
	Using Quotient Rule $\frac{dy}{dx} = \frac{\left(x^2+1\right)2xe^{\left(x^2+1\right)} - 2xe^{\left(x^2+1\right)}}{\left(x^2+1\right)^2}$ • For an attempt to differentiate both $e^{\left(x^2+1\right)}$ and x^2+1 Award for either $e^{x^2+1} \Rightarrow 2xe^{x^2+1}$ or $x^2+1 \Rightarrow 2x$ but both must be **changed** expressions. • Numerator is to have two terms in either order **subtracted.** • Denominator must be $\left(x^2+1\right)^2$	M1
	At least one term fully correct in the numerator	A1
	Fully correct unsimplified.	A1
	For an attempt to take out a common factor of either $2x$ or $e^{\left(x^2+1\right)}$ $\frac{dy}{dx} = \frac{2xe^{\left(x^2+1\right)}\left(x^2+1-1\right)}{\left(x^2+1\right)^2}$ OR Multiplies out the first term in the numerator $\frac{2x^3e^{\left(x^2+1\right)} + 2xe^{\left(x^2+1\right)} - 2xe^{\left(x^2+1\right)}}{\left(x^2+1\right)^2}$	M1 [M1]

	Using Product Rule	
	$\frac{dy}{dx} = \left(x^2+1\right)^{-1} \times 2xe^{x^2+1} + (-2x)e^{x^2+1} \times \left(x^2+1\right)^{-2}$ • For an attempt to differentiate both $e^{\left(x^2+1\right)}$ and $\left(x^2+1\right)^{-1}$ Award for either $e^{x^2+1} \Rightarrow 2xe^{x^2+1}$ or $\left(x^2+1\right)^{-1} \Rightarrow (-2x)\left(x^2+1\right)^{-2}$ but both must be **changed** expressions. • Numerator is to have two terms added	M1
	At least one term correct	A1
	Fully correct unsimplified or simplified	A1
	For an attempt to take out a common factor of either $2x$ or $e^{\left(x^2+1\right)}$ and set a common denominator of $\left(x^2+1\right)^2$	M1
	For the fully correct expression for the derivative, $\frac{dy}{dx} = \frac{2x^3e^{\left(x^2+1\right)}}{\left(x^2+1\right)^2}$ NB: Any further simplification [e.g., cancelling $\left(x^2+1\right)$] following a correct answer seen, is A0.	A1 [5]
(b)	When $x = -1$, $\frac{dy}{dx} = \frac{-e^2}{2}$ ft their K in their $\frac{dy}{dx}$ Allow awrt $\frac{dy}{dx} = -3.7$ Even allow $\frac{dy}{dx} = \frac{-Ke^2}{4}$	B1ft
	When $x = -1$, $y = \frac{e^2}{2}$ Allow awrt $y = 3.7$	B1
	For a correctly used method for the equation of the tangent using their values for $\frac{dy}{dx}$ and y $y - '\frac{e^2}{2}' = '-\frac{e^2}{2}'(x+1)$ Also allow: $y - 3.7 = -3.7(x+1)$ If they use $y = mx + c$ they must obtain a value for c for the award of this mark.	M1
	For the correct equation in any form	A1ft
	For a correct simplified equation in any form but this must be in exact form. $y = -\frac{e^2x}{2}$ or $2y + e^2x = 0$	A1 [5]
		Total 10 marks

Question	Scheme	Marks
11	$S = \pi r^2 + 2\pi rh$	B1
	$\pi r^2 + 2\pi rh = 625\pi \Rightarrow 2rh = 625 - r^2 \Rightarrow h = \dfrac{625 - r^2}{2r}$ *	M1A1 [cso] [3]
(b)	$V = \pi r^2\left(\dfrac{625 - r^2}{2r}\right)$ or $\left(V = \dfrac{625\pi}{2}r - \dfrac{\pi r^3}{2}\right)$	M1
	$\dfrac{dV}{dr} = \dfrac{625\pi}{2} - \dfrac{3\pi r^2}{2}$	M1
	$\dfrac{625\pi}{2} - \dfrac{3\pi r^2}{2} = 0 \Rightarrow 625 = 3r^2 \Rightarrow r = 14.4337... \Rightarrow r \approx 14.4$	M1A1
	For differentiating their $\dfrac{dV}{dr}$ $\dfrac{d^2V}{dr^2} = -3\pi r \ \ (= -136.03\ ...)$	M1
	For concluding that as $\dfrac{d^2V}{dr^2}$ is negative then the volume is a maximum. (Explicit substitution of r is not required).	A1 [6]
(c)	$h = \dfrac{625 - 14.43375^2}{2 \times 14.43375} = 14.4337... \approx 14.4$	B1ft [1]
		Total 10 marks

Question	Scheme	Marks
11	For a correct expression for the surface area of the cup. $S = \pi r^2 + 2\pi rh$	B1
	For equating their expression for the surface area to 625π and attempting to make h the subject. $\pi r^2 + 2\pi rh = 625\pi \Rightarrow 2rh = 625 - r^2 \Rightarrow h = \dfrac{625 - r^2}{2r}$ Allow errors in arithmetic but not mathematically incorrect process.	M1
	For the correct expression for h as shown with no errors. $h = \dfrac{625 - r^2}{2r}$ *	A1 cso [3]

(b)	For the correct volume of the container in terms of r only. $V = \pi r^2\left(\frac{625-r^2}{2r}\right)$ or $\left(V = \frac{625\pi}{2}r - \frac{\pi r^3}{2}\right)$	M1
	Minimally acceptable attempt at differentiation, see general guidance, no power to increase. $\frac{dV}{dr} = \frac{625\pi}{2} - \frac{3\pi r^2}{2}$	M1
	Places their derivative = 0 and attempts to rearrange to find r. Minimally acceptable derivative is of the form $a \pm b\pi r^2$ $\frac{625\pi}{2} - \frac{3\pi r^2}{2} = 0 \Rightarrow 625 = 3r^2 \Rightarrow r = ...$	M1
	For the correct value of r $r = 14.4337... \Rightarrow r \approx 14.4$ (cm) Must reject negative value if found, award A0 if not rejected. May be implied from subsequent working.	A1
	For differentiating their $\frac{dV}{dr}$ For minimally acceptable attempt to differentiate their first derivative, see general guidance, no power to increase. Or testing gradients or a sketch. $\frac{d^2V}{dr^2} = -3\pi r$	M1
	For concluding that as $\frac{d^2V}{dr^2}$ is negative then the volume is a maximum. (Explicit substitution of r is not required).	A1 [6]
(c)	For the value of h using their value of r $h = \frac{625-14.43375^2}{2\times 14.43375} = 14.4337... \approx 14.4$ Allow for 14.5 if substituting $r = 14.4$	B1ft [1]
		Total 10 marks

Question	Scheme	Marks
4(a)	$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = 4t - 16 \quad [t = 5] \quad a = 4\times 5 - 16 = 4$	M1A1 [2]
(b)	$v = 0 \Rightarrow 2t^2 - 16t + 30 = 0 \Rightarrow (t-5)(2t-6) = 0$ $\Rightarrow t_1 = 3, \ \ t_2 = 5$ Displacement $= \int_3^5 (2t^2 - 16t + 30)\,\mathrm{d}t = \left[\dfrac{2t^3}{3} - \dfrac{16t^2}{2} + 30t\right]_3^5$ $= \left[\dfrac{2\times 5^3}{3} - 8\times 25 + 30\times 5\right] - \left[\dfrac{2\times 3^3}{3} - 8\times 9 + 30\times 3\right] = \dfrac{100}{3} - 36 = -\dfrac{8}{3}$ Distance $= \dfrac{8}{3}$ (m)	M1 A1A1 M1A1 M1A1 A1ft [8]
		Total 10 marks

Part	Mark	Notes
(a)	**M1**	For an attempt to differentiate the given expression and substitute $t = 5$ See General Guidance for the definition of an attempt. Do not accept an expression with any terms integrated.
	A1	Obtains the value 4
(b)	**M1**	Sets $v = 0$ and attempts to solve the equation using a correct method. Please check their work carefully. See General Guidance for acceptable ways to solve a quadratic. If they use a calculator and only $t = 3, 5$ are seen with no working – award M1A1A1
	A1	For either $t_1 = 3$ or $t_2 = 5$
	A1	For both $t_1 = 3$ and $t_2 = 5$
	No working seen to find the distance.	
	SC: If a final answer of $\frac{8}{3}$ is seen with no evidence of algebraic integration, **award the final A mark only.**	
	M1	For an attempt to integrate the given expression for v with or without limits. Ignore poor or absent notation. [ie. No integral sign] See General Guidance for the definition of an attempt. Note: Do not accept an integrated expression with any **algebraic** terms differentiated.
	A1	For the correct integrated expression unsimplified or simplified. isw incorrect simplification following a correctly integrated expression. Accept for this mark the inclusion of $+ c$ Ignore limits for this mark.
	M1	Substitutes their values of $t_1 = 3$ or $t_2 = 5$ into their integrated expression and attempts to evaluate. Allow the subtraction either way around. Their integral must be a changed expression. Accept even if they have differentiated it. If the final answer is not $\pm\frac{8}{3}$ and there is no explicit substitution seen, award M0.
	A1	For the correct value of $-\frac{8}{3}$ You must see the negative value for this mark. Accept awrt -2.67 for this mark.
	A1ft	For the correct distance of $\frac{8}{3}$ (m) or exact equivalent. Do not accept 2.67 (m) but accept $2.\dot{6}$ **Note:** This is a ft mark. We will only award this mark if their value for the displacement is **negative** and the final distance is positive.

Question	Scheme	Marks
7(a)	$y=\sqrt{\dfrac{e^{4x}}{2x-3}}=e^{2x}(2x-3)^{-\frac{1}{2}}$	B1
	$y=\dfrac{e^{2x}}{\sqrt{2x-3}}\Rightarrow\dfrac{dy}{dx}=\dfrac{2(2x-3)^{\frac{1}{2}}e^{2x}-2\times\frac{1}{2}\times e^{2x}(2x-3)^{-\frac{1}{2}}}{\left[(2x-3)^{\frac{1}{2}}\right]^2}$	M1A1A1
	$\dfrac{dy}{dx}=\dfrac{\frac{2(2x-3)e^{2x}-2\times\frac{1}{2}\times e^{2x}}{(2x-3)^{\frac{1}{2}}}}{2x-3}=\dfrac{2(2x-3)e^{2x}-2\times\frac{1}{2}\times e^{2x}}{(2x-3)^{\frac{3}{2}}}$	M1 A1
	$=\dfrac{e^{2x}(4x-7)}{(2x-3)^{\frac{3}{2}}}$	A1cso
	$\Rightarrow\delta y\approx\dfrac{e^{2x}(4x-7)}{(2x-3)^{\frac{3}{2}}}\delta x*$	[7]
(b)	When $x=2.5,\ \delta x=\dfrac{0.2}{100}\times 2.5=0.005$	B1
	$\delta y\approx\dfrac{e^{2\times2.5}(4\times2.5-7)}{(2\times2.5-3)^{\frac{3}{2}}}\times0.005$	M1
	$\Rightarrow\delta y\approx0.79$	A1 [3]
		Total 10 marks

Part	Mark	Notes
(a)	**If any candidate attempts part (a) using Chain Rule – please send to Review.**	
	B1	Simplifies the equation into a form which can be differentiated. For example, Award this mark for correct subsequent use in differentiation. $y=\sqrt{\dfrac{e^{4x}}{2x-3}}=e^{2x}(2x-3)^{-\frac{1}{2}}$ or $\dfrac{e^{2x}}{(2x-3)^{\frac{1}{2}}}$ or even $\dfrac{\left(e^{4x}\right)^{\frac{1}{2}}}{(2x-3)^{\frac{1}{2}}}$ or $\dfrac{\left(e^{4x}\right)^{\frac{1}{2}}}{\sqrt{(2x-3)}}$
	Uses Quotient rule. – **NB – This is a 'show' question. Check every line of working.**	
	M1	• The denominator must be correct and squared. • There must be an attempt to differentiate both terms • The two terms in the numerator must be subtracted either way around. Minimally acceptable differentiation is as follows: $e^{2x}\to 2e^{2x},\quad (2x-3)^{\frac{1}{2}}\to k(2x-3)^{-\frac{1}{2}}$ Allow $\left(e^{4x}\right)^{\frac{1}{2}}\to 4e^{4x}\times\frac{1}{2}\times\left(e^{4x}\right)^{-\frac{1}{2}}$

	A1	One term must be fully correct Either $2(2x-3)^{\frac{1}{2}}e^{2x}$ or $-2\times\frac{1}{2}\times e^{2x}(2x-3)^{-\frac{1}{2}}$
	A1	$\frac{dy}{dx}$ fully correct. Ignore poor notation and erroneous subsequent simplification. $'\frac{dy}{dx}'=\frac{2(2x-3)^{\frac{1}{2}}e^{2x}-2\times\frac{1}{2}\times e^{2x}(2x-3)^{-\frac{1}{2}}}{2x-3}$ **OR** $'\frac{dy}{dx}'=\frac{(2x-3)^{\frac{1}{2}}\times 4e^{4x}\times\frac{1}{2}\times(e^{4x})^{-\frac{1}{2}}-2\times\frac{1}{2}\times(e^{4x})^{\frac{1}{2}}(2x-3)^{-\frac{1}{2}}}{2x-3}$
	Uses Product Rule **NB – This is a 'show' question. Check every line of working.**	
	M1	The correct formula must be used. • $\frac{dy}{dx}=uv'+vu'$ • There must be an attempt to differentiate both terms $e^{2x}\to 2e^{2x}$, $(2x-3)^{-\frac{1}{2}}\to k(2x-3)^{-\frac{3}{2}}$ Allow $(e^{4x})^{\frac{1}{2}}\to 4e^{4x}\times\frac{1}{2}\times(e^{4x})^{-\frac{1}{2}}$
	A1	One term must be fully correct Either $(2x-3)^{-\frac{1}{2}}\times 2e^{2x}$ or $\left(-\frac{1}{2}\times 2\times(2x-3)^{-\frac{3}{2}}\right)e^{2x}$
	A1	$\frac{dy}{dx}$ fully correct. Ignore poor notation and erroneous subsequent simplification. $'\frac{dy}{dx}'=(2x-3)^{-\frac{1}{2}}\times 2e^{2x}+\left(-\frac{1}{2}\times 2\times(2x-3)^{-\frac{3}{2}}\right)e^{2x}$ **OR** $'\frac{dy}{dx}'=(2x-3)^{-\frac{1}{2}}\times 4e^{4x}\times\frac{1}{2}\times(e^{4x})^{-\frac{1}{2}}+\left(-\frac{1}{2}\times 2\times(2x-3)^{-\frac{3}{2}}\right)(e^{4x})^{\frac{1}{2}}$
	Simplification – Check their work carefully here.	
	M1	Quotient Rule A correct attempt to simplify the numerator by forming a fraction over $(2x-3)^{\frac{1}{2}}$ Product Rule A correct attempt to simplify by forming a fraction over $(2x-3)^{\frac{3}{2}}$
	A1	For $'\frac{dy}{dx}'=\frac{e^{2x}(4x-7)}{(2x-3)^{\frac{3}{2}}}$
	A1 cso	For the expression exactly as given $\delta y\approx\frac{e^{2x}(4x-7)}{(2x-3)^{\frac{3}{2}}}\delta x$
(b)	**B1**	For finding the change in x
	M1	For using the given expression to substitute the values and evaluate the expression.

		Do not accept a substitution of 0.2 or 0.2% for δx
	A1	For $'\delta y' \approx 0.79$ accept awrt 0.79 Do not penalise poor notation here. Allow dy = 0.79

	ALT 2 Uses Chain Rule	
	B1	This mark is scored when they divide through later by e^{2x} and $(2x-3)^{\frac{1}{2}}$
	M1	The correct form must be used. $y = u^{\frac{1}{2}} \quad u' = \dfrac{4(2x-3)e^{4x} - 2e^{4x}}{(2x-3)^2} = \left[\dfrac{e^{4x}(8x-14)}{(2x-3)^2}\right] \quad y' = \dfrac{1}{2}\left(\dfrac{e^{4x}}{2x-3}\right)^{-\frac{1}{2}}$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{e^{4x}}{2x-3}\right)^{-\frac{1}{2}} \times \left(\dfrac{4(2x-3)e^{4x} - 2e^{4x}}{(2x-3)^2}\right)$ Both terms must be differentiated correctly $e^{4x} \to 4e^{4x}$, $(2x-3) \to k$
	A2	For the correct derivative in unsimplified form. We must see this in full to determine if they simplify further correctly as it is a show question. Please award **both** A marks for the correct derivative seen.
	B1	This mark is awarded at this point. Simplifies $\left(\dfrac{e^{4x}}{2x-3}\right)^{-\frac{1}{2}}$ to e^{2x} and $(2x-3)^{\frac{1}{2}}$ accept inverted or not.
	M1	Simplifies e^{2x} and $(2x-3)^{\frac{1}{2}}$ to obtain a denominator of $(2x-3)^{\frac{3}{2}}$ $\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{(2x-3)^{\frac{1}{2}}}{e^{2x}}\right) \times \left(\dfrac{4(2x-3)e^{4x} - 2e^{4x}}{(2x-3)^2}\right) = \dfrac{1}{2}\left(\dfrac{4(2x-3)e^{2x} - 2e^{2x}}{(2x-3)^{\frac{3}{2}}}\right)$
	A1	For the correct expression with no errors.

Question	Scheme	Marks
3(a)	$675 = \frac{\theta r^2}{2} \Rightarrow \theta = \frac{675\times 2}{r^2} = \frac{1350}{r^2}$ $P = 2r + r\theta \Rightarrow P = 2r + r\left(\frac{1350}{r^2}\right) \Rightarrow P = 2r + \frac{1350}{r}$	M1 M1A1 cso [3]
(b)	$\frac{dP}{dr} = 2 - \frac{1350}{r^2}$ $2 - \frac{1350}{r^2} = 0 \Rightarrow r = 15\sqrt{3}$ or $\sqrt{675}$ $P = 2\times 15\sqrt{3} + \frac{1350}{15\sqrt{3}} = 60\sqrt{3}$	M1 M1A1 M1A1 [5]
(c)	$\frac{d^2P}{dr^2} = \frac{2700}{r^3}$ $r>0 \Rightarrow \frac{d^2P}{dr^2} > 0 \Rightarrow$ minimum	M1A1ft [2]
		Total 10 marks

Part	Mark	Notes
(a)	**Note:** Accept any variable for P in this part of the question for the first two M marks **only** , even no variable as long as it is clear it is the perimeter.	
	M1	Applies the **correct** formula for the area of a sector with 675 cm^2 and attempts to rearrange to find an expression for θ, r or $r\theta$ Minimally acceptable expression: $\theta = \frac{k}{r^2}, r = \frac{k}{\theta r}$ or $\theta r = \frac{k}{r}$
	M1	Applies the **correct** formula for the perimeter of a sector and substitutes their expression for θ provided it is as a minimum of the form $\theta = \frac{k}{r^2}$ Minimally acceptable expression: $P = 2r + \frac{k}{r}$ where k is an integer
	A1	For a fully correct expression for P with no errors seen. You must see $P =$ …….
	ALT – uses the formula $A = \frac{1}{2}rS$ where S is the arc length. S is very popular!	
	M1	Applies the **correct** formula for the area of a sector and rearranges to find an expression for S. $675 = \frac{rS}{2} \Rightarrow S = \frac{1350}{r}$ minimally acceptable $S = \frac{k}{r}$
	M1	Applies the correct formula for the perimeter of a sector. $P = 2r + S \Rightarrow P = 2r + \frac{1350}{r}$ minimally acceptable $P = 2r + \frac{k}{r}$
	A1	For a fully correct expression for P with no errors seen. You must see $P =$ …….

	ALT 2 – works in degrees	
	M1	Applies a **correct** formula for the area of a sector in degrees and attempts to find an expression for θ $675=\frac{\theta}{360}\times\pi r^2 \Rightarrow \theta=\frac{675\times360}{\pi r^2}=\frac{243000}{\pi r^2}$ Min: $\theta=\frac{K}{\pi r^2}$
	M1	Applies a **correct** formula for the perimeter of a sector and substitutes their expression for θ $P=2r+\frac{\theta}{360}\times2\pi r\left(\frac{243000}{\pi r^2}\right)\Rightarrow P=2r+\frac{1350}{r}$ Min: $P=2r+\frac{K}{r}$ **Do not accept solution with a mix of degrees and radians.**
	A1	For a fully correct expression for P with no errors seen. You must see P = …….
(b)	M1	**NB:** Allow poor notation here, even $\frac{dy}{dx}$ or nothing at all. For attempting to differentiate the **given** expression for P The minimally acceptable expression for the derivate is $\frac{dP}{dr}=2-\frac{Q}{r^2}$ where Q is a **positive** integer.
	M1	For setting their differentiated expression = 0 finding a value for r This is a simple equation to solve. Go through their working checking that it is correct. Do not award this mark for incorrect processing.
	A1	For $r=15\sqrt{3}$ oe [An approximate value is $r=25.98\ldots$.]
	NB: Award the next 2 marks if they appear in part (b) only	
	M1	For substituting their value for r into the given expression for P Only allow this mark if: • They use the correct r and obtain the correct perimeter. • They use an incorrect r provided it is a positive value and **show explicit substitution**
	A1	For the correct final answer in exact form. There is no follow through here.
(c)	**NB: Award the next two marks if they appear in part (c) only.**	
	M1	Finds the second derivative. The minimally acceptable expression for the second derivative is $\frac{d^2P}{dr^2}=\frac{X}{r^3}$ where x is an integer. (The value for $\frac{d^2P}{dr^2}$ is awrt 0.15) If they test $\frac{dP}{dr}$ around the minimum point – send to Review. If they test P either side – score M0A0
	A1ft	For a correct conclusion. FT their 2nd derivative provided it is of the form $\frac{d^2P}{dr^2}=\frac{X}{r^3}$ r must be positive and if they find a value for $\frac{d^2P}{dr^2}$ then substitution **must** be seen unless they use $15\sqrt{3}$ and obtain awrt 0.15

Question	Scheme	Marks
6(a)	$2x^2 = \frac{1}{4x} \Rightarrow x^3 = \frac{1}{8} \Rightarrow x = \frac{1}{2}$ $y = 2\times\left(\frac{1}{2}\right)^2 = \frac{1}{2} \Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right)$	M1A1 B1 [3]
(b)	$y = 2x^2 \Rightarrow x^2 = \frac{y}{2}$, $y = \frac{1}{4x} \Rightarrow x^2 = \frac{y^{-2}}{16}$ or $\left(\frac{1}{4y}\right)^2$ o.e. $V = \pi\int_{0.5}^{4}\left(\frac{y}{2}\right)\mathrm{d}y - \pi\int_{0.5}^{4}\left(\frac{y^{-2}}{16}\right)\mathrm{d}y$ $V = \pi\left[\frac{y^2}{4}\right]_{0.5}^{4} - \pi\left[\frac{y^{-1}}{-16}\right]_{0.5}^{4}$ $V = \frac{\pi}{4}\left(4^2 - \left(\frac{1}{2}\right)^2\right) + \frac{\pi}{16}\left(\frac{1}{4} - \frac{1}{0.5}\right) = \frac{245\pi}{64}$ [The decimal equivalent of the area is 12.0264….]	B1,B1 M1 M1A1 M1A1 [7]
		Total 10 marks

Part	Mark	Notes
(a)	M1	For setting the two equations together and attempting to find a value for x A minimally acceptable attempt is reaching at least. $x^3 = \frac{1}{8}$
	A1	For $x = \frac{1}{2}$
	B1	For the correct y coordinate $y = \frac{1}{2}$
(b)	B1	For rearranging the equation for S to $x^2 = \frac{y}{2}$ seen explicitly or embedded
	B1	For rearranging the equation for C to $x^2 = \frac{y^{-2}}{16}$ or $\frac{1}{16y^2}$ or $\left(\frac{1}{4y}\right)^2$ Seen explicitly or embedded
	M1	For a correct expression for the volume with the correct limits [ft their y coord of $\frac{1}{2}$] and π Accept the **[correct only]** expressions either way around. You may see π added in at the end. That is fine and please award this mark if that is the case. Ignore poor notation as long as the intention is clear. For example, ignore missing dy
	M1	For an attempt to integrate at least one of their two only expressions. Ignore limits and π for this mark. See General Guidance.
	A1	For a fully correct integrated expression for the volume.

		Ignore limits and π
	M1	For substituting their limits the correct way around into their integrated expression. You must see this if their integrated expression is incorrect, or their limits are incorrect. If the integration and limits are all correct, the correct volume seen (either in exact or in decimal form i.e. 12.0….) scores this mark. Accept partly processed, for example $\frac{\pi}{4}\left(16-\frac{1}{4}\right)+\frac{\pi}{16}\left(\frac{1}{4}-\frac{1}{0.5}\right)$ as long as you can see **four** calculations/terms as above. Ignore π for this mark.
	A1	For $\frac{245\pi}{64}$
	SC rotates around the *x* axis. Maximum score is B0B0M1M1A0M1A0	
	B0B0	Not available
	M1	For a correct expression for the volume with the correct calculated limits for x $\left(\sqrt{2}\right.$ and $\left.\frac{1}{16}\right)$ and π Accept the [**correct only]** expressions either way around. You may see π added in at the end. That is fine and please award this mark if that is the case. $V=\pi\int_{\frac{1}{16}}^{\sqrt{2}}\left(\frac{x^{-1}}{4}\right)^2\,\mathrm{d}x-\pi\int_{\frac{1}{16}}^{\sqrt{2}}\left(2x^2\right)^2\,\mathrm{d}x$
	M1	For an attempt to integrate one of their two only expressions. $V=\pi\left[-\frac{x^{-1}}{16}-\frac{4x^5}{5}\right]_{\frac{1}{16}}^{\sqrt{2}}$ Ignore π and limits for this mark
	A0	Not available
	M1	For substituting their values into the integrated expression correctly the correct way around. $V=\pi\left[\left(-\frac{\left(\sqrt{2}\right)^{-1}}{16}-\frac{4\left(\sqrt{2}\right)^5}{5}\right)-\left(-\frac{\left(\frac{1}{16}\right)^{-1}}{16}-\frac{4\left(\frac{1}{16}\right)^5}{5}\right)\right]$
	A0	Not available

8(a)	Finds the length DE $x^2 = (DE)^2 + (DE)^2 \Rightarrow DE = \frac{x}{\sqrt{2}}$ or $DE^2 = \frac{x^2}{2}$ Volume of prism $3.6 = \left(\frac{x}{\sqrt{2}}\right)^2 \times \frac{1}{2} \times y \Rightarrow \left[y = \frac{72}{5x^2}\right]$ Surface area of prism $S = 2 \times \frac{x^2}{4} + 2 \times \frac{x}{\sqrt{2}} \times \frac{72}{5x^2} + x \times \frac{72}{5x^2} = \left[\frac{x^2}{2} + \sqrt{2} \times \frac{72}{5x} + \frac{72}{5x}\right]$ $\Rightarrow S = \frac{x^2}{2} + \frac{72(\sqrt{2}+1)}{5x}$ *	B1 M1 M1 A1 cso [4]
(b)	$\frac{dS}{dx} = x - \frac{72(\sqrt{2}+1)}{5x^2} = 0$ $\Rightarrow \frac{72(\sqrt{2}+1)}{5x^2} = x$ $\Rightarrow x = \sqrt[3]{\frac{72(\sqrt{2}+1)}{5}} = 3.26371... \approx 3.26$ (cm) $\frac{d^2S}{dr^2} = 1 + \frac{144(\sqrt{2}+1)}{5x^3}$ $\Rightarrow$ positive + positive = positive $\Rightarrow$ is always greater than 0, hence minimum	M1 M1 A1 B1ft [4]
(c)	$S_{(\min)} = \frac{3.26371^2}{2} + \frac{72(\sqrt{2}+1)}{5 \times 3.26371} = 15.97779... \approx 16 \ (\text{cm}^2)$	M1A1 [2]
		Total 10 marks

Part	Mark	Notes

(a)	B1	For using Pythagoras theorem or any appropriate trigonometry to find the length *DE* (or any equivalent side)
	M1	For using the correct formula to find the volume of a prism and rearranging to find an expression for *y* minimally of the form $y = \frac{A}{Bx^2}$ This need not be simplified.
	M1	For forming an expression for *S* using their *y* The basic expression for the surface area must be correct, with correct substitution of their values. $S = 2\times'DE'+2\times'DE'\times y+xy \Rightarrow S = 2\times'\frac{x^2}{4}'+2\times'\frac{x}{\sqrt{2}}'\times\left('\frac{72}{5x^2}'\right)+x\times\left('\frac{72}{5x^2}'\right)$
	A1 cso	For $S = \frac{x^2}{2}+\frac{72\left(\sqrt{2}+1\right)}{5x}$ as written
(b)	M1	For differentiating the **given** expression for *S* Minimally acceptable differentiated expression is as follows: $\frac{dS}{dx} = x \pm Kx^{-2}$ oe. where *K* is a constant
	M1	For setting their $\frac{dS}{dx} = 0$ and attempting to solve to obtain a value for *x*
	A1	For the correct value of *x* Accept awrt 3.26 (cm)
	B1ft	For differentiating their $\frac{dS}{dr}$ to minimally obtain $\frac{d^2S}{dr^2} = 1 \pm Lx^{-3}$ and coming to an appropriate conclusion. The justification has to be correct and complete. If they refer/use a substitution it must be seen and it must be correct.. That is, $\frac{d^2S}{dr^2} = 1+\frac{144\left(\sqrt{2}+1\right)}{5\times3.26^3} = 3.00...$ $3>0$ so positive, hence minimum Alternatively: *x* is positive so; positive + $\frac{\text{positive}}{\text{positive}} \Rightarrow$ positive hence minimum
(c)	M1	For substituting their value of *x* (provided it is positive) and attempting to find a value of *S*
	A1	For awrt *S* = 16

Question number	Scheme	Marks
5 (a)	$3t^2-16t+5=0 \rightarrow (3t-1)(t-5) \rightarrow t =$ $t=\frac{1}{3}$ oe $t=5$	M1 M1 A1 [3]
(b)	$(a=)6t-16>0$ $t>\frac{8}{3}$ oe	M1 A1 [2]
(c)	$\int_{(1)}^{(4)}(3t^2-16t+5)\,dt$ $\left[t^3-8t^2+5t\right]_{(1)}^{(4)}$ $\left(4^3-8(4)^2+5(4)\right)-\left(1^3-8(1)^2+5(1)\right)$ -42 (distance) = 42	M1 A1 M1 A1 A1 [5]
		Total 10 marks

Part	Mark	Additional Guidance
(a)	M1	For setting $v = 0$ and attempting to solve, see general guidance for definition of a minimum attempt to solve leading to $t =$
	M1	For a correct value of t. Any correct value will imply M2.
	A1	For both correct values of t Both correct values will imply M2 A1
(b)	M1	For an attempt to differentiate and setting their expression > 0 (allow placing = 0 as a concession) See general guidance for definition of a valid attempt. Additionally for this mark, no power of t to increase, at least one term differentiated correctly.
	A1	For $t>\frac{8}{3}$ oe Accept rounding to 1 dp or better eg 2.7, 2.67, 2.667 etc Or indication of recurring e.g. $2.\dot{6}$ or 2.6^r or 2.6... minimum 3 dots
(c)	M1	For an attempt to integrate the given expression for v, see general guidance for the definition of a valid attempt. Additionally, no power of t to decrease. Limits do not need to be present.
	A1	For a correct integration. Limits do not need to be present.
	M1	For correct substitution of limits into **their** changed expression. Can be any changed expression with a minimum of 2 terms. This mark may be implied by sight of – 42 or 42 or – 44 – – 2 or 44 – 2. If the final answer is incorrect, must see substitution of both limits correctly at least once. For this mark, students may assume an arbitrary constant (typically $C = 0$) or leave an arbitrary constant in their working and have the equivalent $\left(4^3-8(4)^2+5(4)+C\right)-\left(1^3-8(1)^2+5(1)+C\right)$ Candidates cannot attain this mark for just – 44 and – 2 without a subtraction.
	A1	For – 42 or 42.
	A1	Correct positive distance given.
Any candidate not showing the step of integration – 0 marks for this part of the question.		

Question number	Scheme	Marks
9	$\left(x=y^2+1 \qquad x=4-2y\right) \quad \left(y=\sqrt{x-1} \quad y=\frac{4-x}{2}\right)$	
	$4-2y=y^2+1$ or $\sqrt{x-1}=\frac{4-x}{2}$	M1
	$0=y^2+2y-3 \qquad 0=x^2-12x+20$	A1
	$0=(y+3)(y-1) \qquad 0=(x-2)(x-10)$	M1
	$(y=1,)\,x=2 \qquad x=2$	A1
	$\left(y^2=\left(2-\frac{x}{2}\right)^2=4-2x+\frac{x^2}{4}\right)$	
	$(0=x-1)\Rightarrow x=1 \qquad (2(0)+x-4=0)\Rightarrow x=4$	B1B1
	$\pi\int_{"1"}^{"2"}(x-1)\,dx+\pi\int_{"2"}^{"4"}\left(2-\frac{x}{2}\right)^2 dx$ or $\pi\int_{"1"}^{"2"}(x-1)\,dx+\frac{1}{3}\pi\times"1"^2\times("4"-"2")^2$	ddM1
	$\left[\pi\int_1^2(x-1)\,dx+\pi\int_2^4\left(4-2x+\frac{x^2}{4}\right)dx \text{ or } \frac{1}{3}\pi\times1^2\times2\right]$	
	$(\pi)\left[\frac{x^2}{2}-x\right]_{(1)}^{(2)}+(\pi)\left[\left(4x-\frac{2x^2}{2}+\frac{x^3}{12}\right)\right]_{(2)}^{(4)}$	M1
	$(\pi)\left[\left(\frac{"2"^2}{2}-"2"\right)-\left(\frac{"1"^2}{2}-"1"\right)\right]+(\pi)\left[\left(4("4")-\frac{2("4")^2}{2}+\frac{"4"^3}{12}\right)-\left(4("2")-\frac{2("2")^2}{2}+\frac{"2"^3}{12}\right)\right]$	M1
	$\left(\frac{1}{2}\pi+\frac{2}{3}\pi\right)$	
	$=\frac{7}{6}\pi$ oe	A1 [10]
		Total 10 marks

Part	Mark	Additional Guidance
	M1	For correctly placing the 2 equations equal to each other and any attempt to form a quadratic = 0 in x or y This attempt doesn't need to be correct, but they must reach an unsimplified quadratic
	A1	For the correct 3TQ = 0
	M1	For any method to solve their quadratic – see general guidance for minimally acceptable attempt. $x = 2$ will imply this mark.
	A1	For $x = 2$
	B1	For $x = 1$ or $x = 4$
	B1	For $x = 1$ and $x = 4$
	ddM1	For $\pi\int_{"1"}^{"2"}(x-1)\mathrm{d}x + \pi\int_{"2"}^{"4"}\left(2-\frac{x}{2}\right)^2 \mathrm{d}x$ or $\pi\int_{"1"}^{"2"}(x-1)\mathrm{d}x + \frac{1}{3}\pi\times"1"^2\times("4"-"2")^2$ Dependent on both previous method marks. Also dependent on having attempted to find the intersection of the curve and the line with the x-axis. Allow their "1" and their "4" limits as long as it's clear the candidate has attempted to find the intersections of the **line and curve** with the x axis
	M1	For integrating (minimum attempt – see general guidance and also, no power of x to decrease) their terms in their volume of revolution. There must be a minimum of 2 terms. This is not a dependent mark, the mark is given for integration, π and limits **do not need to be present** for this mark to be awarded. If students have written an incorrect formula for the volume of the cone, this mark may still be awarded for integrating. If expressions have been combined incorrectly, this mark for integration of a minimum of 2 terms may be given.
	M1	For correct substitution of limits into any changed expression. Also not dependent, any substitution of what candidates consider to be their limits into their changed expression for the volume of revolution may be awarded this mark. If students have written an incorrect formula for the volume of the cone, this mark may still be awarded for integrating. If expressions have been combined incorrectly, this mark is for substitution of limits into any changed expression(s), minimum 2 terms, with each limit substituted correctly at least once and may be given.
	A1	For $\frac{7}{6}\pi$ oe

Question number	Scheme	Marks
3 (a)	$\left(v = 2t^2 - 19t + 35\right)$ $\left(\frac{dv}{dt} =\right) 4t - 19$ $4t - 19 = 0$ $t = \frac{19}{4}$	 M1 M1 A1 [3]
(b)	$\left(v = 0 = 2t^2 - 19t + 35\right)$ $0 = (2t - 5)(t - 7)$ $T_1 = \frac{5}{2},\ T_2 = 7$	 M1 A1, A1 [3]
(c)	$\left(\int\left(2t^2 - 19t + 35\right)dt\right) = \frac{2}{3}t^3 - \frac{19}{2}t^2 + 35t(+C)$ oe **OR** $\int_{'2.5'}^{'7'}\left(2t^2 - 19t + 35\right)\,dt = \left[\frac{2}{3}t^3 - \frac{19}{2}t^2 + 35t\right]_{'2.5'}^{'7'}$ $\left(\frac{2}{3}(7)^3 - \frac{19}{2}(7)^2 + 35(7)\right) - \left(\frac{2}{3}\left(\frac{5}{2}\right)^3 - \frac{19}{2}\left(\frac{5}{2}\right)^2 + 35\left(\frac{5}{2}\right)\right) = -\frac{243}{8}$ $d = \frac{243}{8}$	M1 A1 M1 A1 [4]
		Total 10 marks

Part	Mark	Additional Guidance
3 (a)	M1	For a minimally acceptable attempt to differentiate, see general guidance.
	M1	For placing their changed expression for $a = 0$ and an attempt to solve, allow 1 processing error.
	A1	For $t = \frac{19}{4}$
(b)	M1	For setting $v = 0$ and attempting to solve, see general guidance for definition of a valid attempt to solve.
	A1	For one correct value of either T_1 or T_2
	A1	For both correct values
(c)	**Note:** **The question states 'Show your working clearly' No integration seen – award M0A0M0A0**	
	M1	For an attempt to integrate, see general guidance.
	A1	For a fully correct integration (constant of integration need not be shown).
	M1	For correct substitution of **their** T_1 and T_2 **into a changed expression AND subtracts their values correctly.** If these are not the correct values, then must see each limit correctly substituted into their expression at least once, If the correct values are used, but their final answer is not correct, explicit substitution must be seen for the award of this mark. **Note:** Mark may be implied by a correct answer given **following correct integration**
	A1	Correct answer **which must be a positive value.**

Question number	Scheme	Marks
6 (a)	$700\pi = 2\pi r^2 + 2\pi rh$	B1
	$h = \dfrac{350 - r^2}{r}$ or $rh = 350 - r^2$	M1
	$V = \pi r^2 h = \pi r^2\left(\dfrac{350 - r^2}{r}\right) = \left\{\pi r\left(350 - r^2\right)\right\} = \pi r\left(350 - r^2\right)$*	M1A1* cso [4]
(b)	$V = 350\pi r - \pi r^3$	
	$\dfrac{\mathrm{d}V}{\mathrm{d}r} = 350\pi - 3\pi r^2 = 0$	M1
	$\Rightarrow 350\pi = 3\pi r^2 \Rightarrow r = \sqrt{\dfrac{350}{3}} = 10.8$ (3sf)	M1A1
	$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} = -6\pi r$	M1
	$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} < 0 \therefore$ max	A1 [5]
(c)	$h = \dfrac{350 - r^2}{r} = \dfrac{350 - \dfrac{350}{3}}{\sqrt{\dfrac{350}{3}}} = 21.6$ (3 s.f.)	B1 [1]
		Total 10 marks

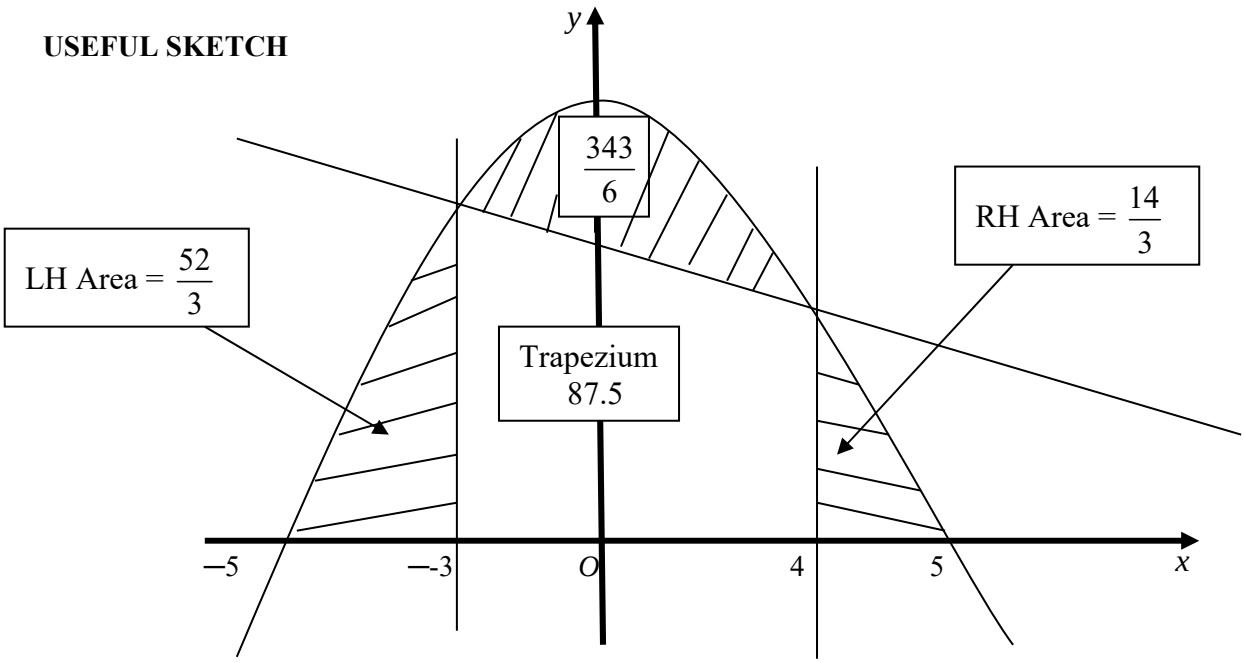
Part	Mark	Additional Guidance
(a)	B1	For a **correct** equation for the surface area in terms of the radius and height
	M1	For rearranging to find h in terms of r which must be as a minimum $h = \frac{k - lr^2}{mr}$ where k, l and m are integers OR $rh = k - lr^2$ where k and l are integers
	M1	For a correct expression for volume using *their* h in terms of r
	A1* cso	For rearranging and obtaining the given result with no errors or omissions and must include $V = \ldots$ [Allow Volume = …. or Vol = …..]
(b)	M1	For an attempt to differentiate the **given** expression for V which must be of the form $\frac{dV}{dr} = 350\pi - K\pi r^2$ where K is a constant Using Product Rule: $\frac{dV}{dr} = \pi r(-2r) + \pi(350 - r^2) \Rightarrow \frac{dV}{dr} = \left[350\pi - K\pi r^2\right]$
	M1	For setting their $\frac{dV}{dr} = 0$ **and** attempting to find a value for r.
	A1	$r = \sqrt{\frac{350}{3}}$ $[= 10.801...]$ accept awrt 10.8
	M1	For finding $\frac{d^2V}{dr^2}$ Accept only $\frac{d^2V}{dr^2} = \pm k\pi r$ where k is a constant See general guidance for what constitutes an attempt to differentiate.
	A1	For the correct conclusion derived from the correct working throughout.
(c)	B1	For obtaining the value of h correct to 3 s.f. Accept awrt 21.6

Question	Scheme	Marks
6 (a)	$2y+2\pi x=90$ $S=2yx+\pi x^2,=(90-2\pi x)x+\pi x^2=90x-\pi x^2 \quad k=90$	B1 M1,M1A1 [4]
(b)	$\frac{dS}{dx}="90"-2\pi x$ $\frac{dS}{dx}=0 \Rightarrow x=\frac{"90"}{2\pi}=14.32$ $\frac{d^2S}{dx^2}=-2\pi, <0 \therefore \max$ **ALT** Use graph of $S=90x-\pi x^2$ to justify max or values of ds/dx either side of their x	M1 M1A1 M1A1 [5]
(c)	$S_{max}=90\times\frac{90}{2\pi}-\pi\left(\frac{90}{2\pi}\right)^2=644.57....=645$ (Allow 644)	M1,A1 [2]

Additional Guidance		
(a)	B1	For the correct expression for the perimeter $2y+2\pi x=90$
	M1	For the correct expression for the area. $S=2yx+\pi x^2$ (That is all that is required for this mark)
	M1	For substituting their rearranged expression $\left(2y=90-2\pi x\right)$ into their expression for the area $\left(S=2yx+\pi x^2\right)$
	A1	For simplifying their expression to achieve $S=90x-\pi x^2$ **AND** the value of k must be stated; $k=90$
(b)	M1	For an attempt to differentiate their expression for S. Minimally acceptable attempt $S='90'x-\pi x^2\Rightarrow\frac{\mathrm{d}S}{\mathrm{d}x}='90'-p\pi x$ where p is a constant
	M1	For setting their $\frac{\mathrm{d}S}{\mathrm{d}x}=0$ **and** attempting to find a value for x $\Rightarrow x=\frac{'90'}{p\pi}$ and obtaining a numerical value for x.
	A1	$x=14.32$ cao
	Verifying the value of x is a maximum. **Using 2nd derivative**	
	M1	For finding $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}$ of their $\frac{\mathrm{d}S}{\mathrm{d}x}$. Minimally acceptable attempt $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}=\pm q$ where q is a constant
	A1	For the correct $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}=-2\pi$ with a correct conclusion.
	ALT	
	M1	(i) Uses a graph (must be evidence of this) to draw the conclusion. Eg General graph of S is  so value is a maximum (ii) Uses value of x around their value for the maximum. For example, tests x = 14 and x = 15 in their $\frac{\mathrm{d}S}{\mathrm{d}x}$
	A1	For the correct conclusion derived from correct working throughout. For example is x = 14 and x = 15 then $\frac{\mathrm{d}S}{\mathrm{d}x}=2.035$ and $\frac{\mathrm{d}S}{\mathrm{d}x}=-4.25$ respectively so the gradient moves from positive to negative and therefore the turning point is a maximum.
(c)	M1	For substituting their values of k and x into the given expression for S and obtaining a value. $S_{\max}=90\times'x'-\pi'x'^2=....$
	A1	$S_{\max}=645$ or 644

Question number	Scheme	Marks
5 (a)	$25 - x^2 = 13 - x$	M1
	$x^2 - x - 12 = 0$	
	$(x-4)(x+3) = 0$	M1
	$A = (-3, 16)$	A1
	$B = (4, 9)$	A1
		(4)
(b)	$\int_{-5}^{5}(25-x^2)\mathrm{d}x - \left[\int_{-3}^{4}(25-x^2)\,\mathrm{d}x - \frac{1}{2}(16+9)\times 7\right]$	M1 A1
	$\left[25x - \frac{x^3}{3}\right]_{-5}^{5} - \left\{\left[25x - \frac{x^3}{3}\right]_{-3}^{4} - 87.5\right\}$	M1 A1 B1
	$\left(\frac{250}{3} + \frac{250}{3}\right) - \left[\left(100 - \frac{64}{3}\right) - (-75+9) - 87.5\right]$	M1
	$\frac{219}{2} = (109.5)$	A1
		(7)
		[11]
	Alternative (b)	
	$\int_{-5}^{5}(25-x^2)\mathrm{d}x - \int_{-3}^{4}(12-x^2+x)\,\mathrm{d}x$	M1 A1
	$\left[25x - \frac{x^3}{3}\right]_{-5}^{5} - \left[12x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-3}^{4}$	M1 A1
		A1
	$\left(\frac{250}{3} + \frac{250}{3}\right) - \left(\frac{104}{3} + \frac{45}{2}\right)$	M1
	$\frac{219}{2} = (109.5)$	A1
		(7)

USEFUL SKETCH



Part	Mark	Additional Guidance
(a)	M1	For setting the given equation of the curve = given equation of the line $25-x^2=13-x$ and attempting to form a 3TQ $x^2-x-k=0$ (k is an integer) Ignore the absence of = 0 if further work shows that they are attempting to solve a 3TQ = 0
	M1	For attempting to solve their 3TQ See general guidance for the definition of an attempt.
	A1	For **either** $(-3,\ 16)$ or $(4,\ 9)$
	A1	For **both** $(-3,\ 16)$ and $(4,\ 9)$
(b)		There are two ways to calculate this area. In each case; The first M mark is for a correct strategy (allow ft from (a) in their limits) The first A mark (M mark in Epen) is a fully correct strategy with correct limits The second M mark is for an attempt to integrate The second A mark is for a fully correct integration – ignore limits for this mark. The B mark (and A mark in Epen) is for the area of the trapezium of 87.5 seen anywhere. The third M mark is for substituting in their limits The final A mark is the correct answer only.
		Method 1 – Trapezium + two sides
	M1	For an attempt at the correct strategy to find the area. Allow for this mark a correct statement with using their limits correctly. This may well be seen at the end when they combine individual areas. $(A=)\frac{1}{2}('16'+'9')\times'7'+\int_{'4'}^{'5'}\left(25-x^2\right)\,\mathrm{d}x+\int_{'-5'}^{'-3'}\left(25-x^2\right)\,\mathrm{d}x$ OR $(A=)\int_{-3}^{4}(13-x)\,\mathrm{d}x+\int_{'4'}^{'5'}\left(25-x^2\right)\,\mathrm{d}x+\int_{'-5'}^{'-3'}\left(25-x^2\right)\,\mathrm{d}x$
	A1	Fully correct expression with correct limits. $(A=)\frac{1}{2}(16+9)\times7+\int_{4}^{5}\left(25-x^2\right)\,\mathrm{d}x+\int_{-5}^{-3}\left(25-x^2\right)\,\mathrm{d}x$ OR $(A=)\int_{-3}^{4}(13-x)\,\mathrm{d}x+\int_{'4'}^{'5'}\left(25-x^2\right)\,\mathrm{d}x+\int_{'-5'}^{'-3'}\left(25-x^2\right)\,\mathrm{d}x$
	M1	For an attempt to integrate their expression for area. (Follow General Guidance for the definition of an attempt) Ignore limits for this mark
	A1	For a fully correct integrated expression for the Area with a correct expression for the trapezium (**Ignore limits for this mark**.) $\left[13x-\frac{x^2}{2}\right],\ \left[25x-\frac{x^3}{3}\right],\ \left[25x-\frac{x^3}{3}\right]$ OR $\frac{1}{2}(16+9)\times7,\ \left[25x-\frac{x^3}{3}\right],\ \left[25x-\frac{x^3}{3}\right]$
	B1	For the correct area of the trapezium of 87.5. Award wherever seen. $\frac{343}{6}$ seen implies B1 If not seen explicitly, this can be implied from a correct final answer. **This is an A mark in Epen**

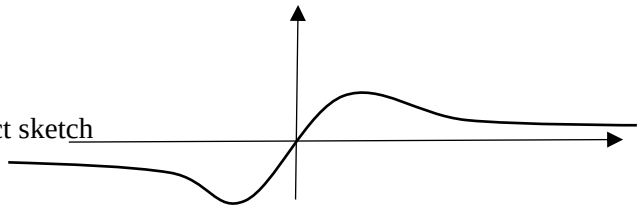
	M1	For an attempt to substitute **their** limits into **their integrated** expression.
	A1	For the correct final area of $A=\frac{219}{2}$ oe
	Method 2 – Using the area under the whole curve between −5 and 5; minus the area of the curve between −4 and 3; plus the area of the trapezium	
	M1	For an attempt at the correct strategy to find the area Allow for this mark, the correct strategy with **their** limits This may well be seen at the end when they combine individual areas. $(A=)\int_{-5}^{5}\left(25-x^2\right)\,\mathrm{d}x-\int_{'-3'}^{'4'}\left(25-x^2\right)\,\mathrm{d}x+\frac{1}{2}('16'+'9')\times'7'$ OR $(A=)\int_{-5}^{5}\left(25-x^2\right)\,\mathrm{d}x-\int_{'-3'}^{'4'}\left(25-x^2\right)\,\mathrm{d}x+\int_{'-3'}^{'4'}(13-x)\,\mathrm{d}x$ OR $(A=)\int_{-5}^{5}\left(25-x^2\right)\,\mathrm{d}x-\int_{'-3'}^{'4'}\left(12+x-x^2\right)\,\mathrm{d}x$
	A1	For the correct expression with correct limits $(A=)\int_{-5}^{5}\left(25-x^2\right)\,\mathrm{d}x-\int_{'-3'}^{'4'}\left(25-x^2\right)\,\mathrm{d}x+\frac{1}{2}('16'+'9')\times'7'$ OR $(A=)\int_{-5}^{5}\left(25-x^2\right)\,\mathrm{d}x-\int_{'-3'}^{'4'}\left(25-x^2\right)\,\mathrm{d}x+\int_{'-3'}^{'4'}(13-x)\,\mathrm{d}x$ OR $(A=)\int_{-5}^{5}\left(25-x^2\right)\,\mathrm{d}x-\int_{'-3'}^{'4'}\left(12+x-x^2\right)\,\mathrm{d}x$
	M1	For an attempt to integrate their expression for area. (Follow General Guidance for the definition of an attempt) Ignore limits for this mark
	A1	For a fully correct integrated expression for the Area with a correct expression for the trapezium. Accept this seen as individual parts Ignore limits for this mark. $\left[25x-\frac{x^3}{3}\right],\quad\left[25x-\frac{x^3}{3}\right],\quad\left[13x-\frac{x^2}{2}\right]$ OR $\left[25x-\frac{x^3}{3}\right],\quad\left[25x-\frac{x^3}{3}\right],\quad\frac{1}{2}(16+9)\times7$ OR $\left[25x-\frac{x^3}{3}\right],\quad\left[12x+\frac{x^2}{2}-\frac{x^3}{3}\right]$
	B1	For the correct area of the trapezium of 87.5 $\frac{343}{6}$ seen implies B1 Award wherever seen. If not seen explicitly, this can be implied from a correct final answer. **This is an A mark in Epen**
	M1	For an attempt to substitute **their** limits into **their integrated** expression or individual parts..
	A1	For the correct final area of $A=\frac{219}{2}$ oe

Question number	Scheme	Marks
11 (a)	at P $\quad b=\sqrt{a-2} \Rightarrow b^2=a-2$*	M1A1cso [2]
(b)	At P $y=b \Rightarrow y^2=b^2 \Rightarrow y^2=a-2$	B1
	$V=\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,\mathrm{d}x-\pi\int_a^{16}(a-2)\,\mathrm{d}x=\pi\int_a^{16}(x-2)\,\mathrm{d}x-\pi\int_a^{16}(a-2)\,\mathrm{d}x$	M1
	$\Rightarrow \pi\int_a^{16}(x-a)\,\mathrm{d}x$ or $\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,\mathrm{d}x-\pi(a-2)(16-a)$	
	$50\pi=\pi\left[\frac{x^2}{2}-ax\right]_a^{16}$ or $50\pi=\pi\left[\frac{x^2}{2}-2x\right]_a^{16}-\pi(a-2)(16-a)$	depM1A1
	$50\pi=\pi\left[\left(\frac{256}{2}-16a\right)-\left(\frac{a^2}{2}-a^2\right)\right]$	depM1
	or $50\pi=\pi\left[\left(96-\frac{a^2}{2}+2a\right)-\left(18a-a^2-32\right)\right]$	
	$\Rightarrow a^2-32a+156=0$	M1
	$\Rightarrow (a-6)(a-26)=0 \Rightarrow a<16$ so $a=6$	
	$b^2=a-2 \Rightarrow b^2=4 \Rightarrow b=2$	M1A1 A1 [9]
		Total 11 marks

(a)	
M1	$b=\sqrt{a-2}$
A1 cso	Obtains the **given** answer with no errors in the working
(b)	
B1	$y^2=a-2$
M1	Use of $V=\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,\mathrm{d}x-\pi\int_a^{16}(a-2)\,\mathrm{d}x$ or $V=\pi\int_a^{16}(x-a)\,\mathrm{d}x$ or $\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,\mathrm{d}x-\pi(a-2)(16-a)$ Ignore limits
depM1	Attempts to integrate. Ignore limits (Dependent on previous M1)
A1	Correct integration. Ignore limits
depM1	Correct substitution of limits (Dependent on previous M1)
M1	Obtaining the 3TQ
M1	Solving the 3TQ
A1	$a=6$
A1	$b=2$

Question number	Scheme	Marks
7 (a)	$\frac{dy}{dx} = \frac{(x^2+4)-x(2x)}{(x^2+4)^2}$	M1
	$= \frac{4-x^2}{(x^2+4)^2}$	A1
	$\frac{dy}{dx} = 0 \Rightarrow 4-x^2 = 0$	M1
	So $\left(2, \frac{1}{4}\right)$ and $\left(-2, -\frac{1}{4}\right)$	A1 A1 (5)
(b)	$\frac{d^2y}{dx^2} = \frac{-2x(x^2+4)^2 - 4x(4-x^2)(x^2+4)}{(x^2+4)^4}$	M1
	$\frac{d^2y}{dx^2} = \frac{-2x^3-8x-16x+4x^3}{(x^2+4)^3}$	M1
	$\frac{d^2y}{dx^2} = \frac{2x^3-24x}{(x^2+4)^3}$	M1
	$\frac{d^2y}{dx^2} = \frac{2x(x^2-12)}{(x^2+4)^3}$ *	A1 cso (4)
(c)	When $x = 2$ $\left[\frac{d^2y}{dx^2} = -\frac{1}{16}\right]$ When $x = -2$ $\left[\frac{d^2y}{dx^2} = \frac{1}{16}\right]$	
	$\frac{d^2y}{dx^2} < 0$ so maximum $\frac{d^2y}{dx^2} > 0$ so minimum	M1 A1ft (2)
		Total 11 marks

Part	Mark	Notes
(a)	**M1**	For an attempt at Quotient rule. The definition of an attempt is that there must be a minimal attempt to differentiate both terms and the denominator must be $(x^2+4)^2$. Allow the terms in the numerator to be the wrong way around, but the terms must be subtracted. [See General Guidance for an attempt at differentiation]. $\frac{dy}{dx} = \frac{(x^2+4)-x(2x)}{(x^2+4)^2}$
	A1	For the correct $\frac{dy}{dx}$ simplified or unsimplified. Award the mark for a correct $\frac{dy}{dx}$ seen even if there are later errors in simplification.
	M1	For setting their $\frac{dy}{dx} = 0$ which must be a quadratic equation solving to find x: $4-x^2 = 0 \Rightarrow x = \pm 2$

	A1	For the correct coordinates of either $\left(2,\frac{1}{4}\right)$ or $\left(-2,-\frac{1}{4}\right)$
	A1	For both correct coordinates $\left(2,\frac{1}{4}\right)$ and $\left(-2,-\frac{1}{4}\right)$
(b)	**M1**	For an attempt at Quotient rule on their $\frac{dy}{dx}=\frac{(x^2+4)-x(2x)}{(x^2+4)^2}$ which must be as a minimum: $\frac{ax^2+bx+c}{(x^2+4)^2}$ where a, b and c are constants and $a,c\neq 0$ The definition of an attempt is that there must be a minimal attempt to differentiate the numerator and denominator and the correct formula applied. $(x^2+4)^2$ must differentiate to $ax(x^2+4)$, the denominator must be $(x^2+4)^4$ Allow the terms in the numerator to be the wrong way around, but the terms must be subtracted. Apply General Guidance for an attempt at differentiation on ax^2+bx+c. $\frac{d^2y}{dx^2}=\frac{-2x(x^2+4)^2-4x(4-x^2)(x^2+4)}{(x^2+4)^4}$
	M1	For cancelling through by (x^2+4) $\frac{d^2y}{dx^2}=\frac{-2x(x^2+4)-4x(4-x^2)}{(x^2+4)^3}$
	M1	For simplifying the numerator to achieve as a minimum $\frac{d^2y}{dx^2}=\frac{ax^3+bx}{(x^2+4)^3}$ where a and b are constants
	A1 cso	For obtaining the answer as given with no errors. $\frac{d^2y}{dx^2}=\frac{2x(x^2-12)}{(x^2+4)^3}$ *
(c)	**M1**	Substitutes either their 2 or their – 2 into their $\frac{d^2y}{dx^2}$ Note: When $x = 2$ $\left[\frac{d^2y}{dx^2}=-\frac{1}{16}\right]$ and when $x = -2$ $\left[\frac{d^2y}{dx^2}=\frac{1}{16}\right]$
	A1ft	For the conclusion $\frac{d^2y}{dx^2}<0$ so maximum $\frac{d^2y}{dx^2}>0$ so minimum
	ALT – tests gradient or sight of a sketch	
	M1	Tests gradient on either side of one the turning points (their 2 or their – 2) using their $\frac{dy}{dx}$ or a correct sketch 
	A1ft	For the correct conclusion

Question number	Scheme	Marks
4 (a)	$f(x)=2(x-3)^2-5$ $\Rightarrow f(x)=2(x^2-6x+9)-5=2x^2-12x+13$ $p=-12,*\ \ q=13$ **ALT** $\frac{dy}{dx}=4x+p=0$ at $(3,-5)$ $\Rightarrow 4\times 3+p=0\Rightarrow p=-12*$ $-5=2\times 3^2-12\times 3+q\Rightarrow q=13$	M1M1 M1A1 cso [4] M1 M1 M1A1 cso [4]
(b)	Minimum since coefficient of x^2 is positive	B1 [1]
(c)	$\frac{dy}{dx}=4x-12\Rightarrow$ when $x=1$ $\frac{dy}{dx}=-8$ So gradient of normal = $\frac{1}{8}$ When $x=1$ $y=3$ $y-'3'=\frac{1}{'8'}(x-1)$ Eg. $y-3=\frac{1}{8}(x-1)$, $x-8y+23=0$ oe	B1 B1ft M1 M1 A1, A1 [6]
		Total 11 marks

Part	Mark	Notes
(a)	**M1**	For using the coordinate of the stationary point and forming the expression: $f(x)=2(x-3)^2-5$ This must be correct for this mark
	M1	Expands their $2(x-3)^2-5$ to form a 3TQ
	M1	For equating coefficients with the given equation to find the value of p and q
	A1	For $p=-12,*\ \ q=13$ **NB:** The value of p is given in the question.
	ALT – Uses differentiation	
	M1	For a correct differentiation of the expression for y This must be correct for this mark
	M1	For equating their $\frac{dy}{dx}=0,$ substituting in $x = 3$ and attempting to find a value for p
	M1	For substituting the given value of p into $y=2x^2+px+q$ using the coordinates $(3,-5)$ to find the value of q. Explicit substitution of the value of p into the equation must be seen if p has not been found using differentiation. [M0M0M1A0 can be scored]
	A1	For $p=-12,*\ \ q=13$ **NB:** The value of p is given in the question.
(b)	**B1**	For the correct statement: Minimum since coefficient of x^2 is positive
	ALT	
	B1	For $\frac{d^2y}{dx^2}=4\Rightarrow$ positive, hence minimum. A correct conclusion is required.
(c)	**B1**	For the value of the gradient when $x = 1$ $[m=-8]$
	B1ft	For the gradient of the normal which must have come from differentiation. Ft their -8
	M1	For the value of y when $x = 1$ using their value of q
	M1	For a complete equation of the line in any form using their normal and their value of y Allow this mark for an equation of the line with $x = 1$ and $y =$ '3' with their normal, provided there is evidence that the normal comes from the negative reciprocal of their value of the tangent. For this mark the gradient of the tangent need not have come from differentiation.
	A1	For the correct equation in any form.
	A1	For the correct equation in the required form or any multiples of the coefficients.

Question number	Scheme	Marks
7 (a)	(Surface area =) $12x^2 + 4xy + 3xy + 5xy = 144$ oe	M1
	$12x^2 + 12xy = 144$ oe leading to $\frac{144 - 12x^2}{12x}$ oe	A1
	$V = \frac{1}{2} \times 3x \times 4x \times y \ (= 6x^2y)$	M1
	$= 6x^2 \times \frac{144 - 12x^2}{12x} = 6x(12 - x^2)$	M1
	$= 72x - 6x^3$ *	A1 cso [5]
(b)	$\left(\frac{dV}{dx}\right) = 72 - 18x^2$	M1
	$\left(\frac{dV}{dx} = \right)"72 - 18x^2" = 0$ $\Rightarrow x =$	M1 (A1 on ePen)
	$x = 2$	A1 (M1 on ePen)
	$\left(\frac{d^2V}{dx^2}\right) = -36x$	B1 (A1 on ePen)
	$x = 2 \ \frac{d^2V}{dx^2} < 0$ Therefore maximum	[4]
(c)	$V = 72 \times 2 - 6 \times 2^3 = 96$	M1 A1 [2]
		Total 11 marks

Part	Mark	Notes
(a)	M1	For surface area as an equation in x and y (correct and unsimplified)
	A1	For $12x^2+12xy=144$ oe and correctly making y the subject $\frac{144-12x^2}{12x}$ (oe)
	M1	$V=\frac{1}{2}\times 3x\times 4x\times y$ $(=6x^2y)$
	M1	For correct substitution of their expression for y, which must be in terms of x only, into their formula for V.
	A1*cso	Obtains the given expression
(b)	M1	For attempting to differentiate V wrt x At least one term must be fully correct, the other – follow general guidance
	M1 (A1 on ePen)	For setting their derivative (which must involve a changed expression) = 0 and a complete solution to find the value of x
	A1 (M1 on ePen)	For $x = 2$ only
	B1 (A1 on ePen)	For a correct second derivative and correct justification that this is a maximum, with some form of conclusion. All work to be correct for this mark. i.e it is acceptable, as this is a simple substitution to state if $x = 2$, $\frac{d^2V}{dx^2}<0$, but if a substitution of $x = 2$ is made, the value for the second derivative must be given as – 72.
ALT final B1	B1 (A1 on ePen)	Testing and substituting into the correct second derivative with appropriate values either side of $x = 2$. For correct justification that this is a maximum, with some form of conclusion. All work to be correct for this mark
(c)	M1	For correct substitution of their x into V
	A1	96

Question number	Scheme	Marks
6 (a)	$\frac{dy}{dx}=\frac{2}{\sqrt{x}}$	M1
	When $x=9$ $\frac{dy}{dx}=\frac{2}{3}$	A1
	$y-12=\frac{2}{3}(x-9)$	M1
	When $y=0$ $-12=\frac{2}{3}(x-9)$	M1
	$x=-9$ So $(-9,0)$	A1
		(5)
(b)	Gradient of Normal: $-\frac{3}{2}$	M1
	$y-12=-\frac{3}{2}(x-9)$	M1
	When $y=0$ $-12=-\frac{3}{2}(x-9)$	M1
	$x=17$ So $(17,0)$	A1
		(4)
(c)	$\frac{1}{2}\times 12\times 26=156$	M1 A1
		(2)
		Total 11 marks

Part	Mark	Notes
(a)	**M1**	For an attempt to differentiate y wrt x. Accept $\frac{dy}{dx} = 2x^{-\frac{1}{2}}$ Allow as a minimum $\frac{dy}{dx} = kx^{-\frac{1}{2}}$ where k is a constant $\neq 0$
	A1	For $\frac{dy}{dx} = \frac{2}{3}$
	M1	For a complete method to find the equation of a straight line. If they use $y = mx + c$ then they must reach a value for c for this mark. Accept the equation of the line with their value of $\frac{dy}{dx}$ using the given coordinates.
	M1	For substitution of $y = 0$ to find a value for x
	A1	For $(-9, 0)$
(b)	**M1**	For gradient of Normal: $-\frac{3}{2}$ Which is the negative reciprocal of their gradient obtained in (a)
	M1	For a complete method to find the equation of a straight line with a gradient of $-\frac{3}{2}$ (ft their gradient of normal) If they use $y = mx + c$ then they must reach a value for c for this mark.
	M1	For substitution of $y = 0$ to find a value for x
	A1	For $(17, 0)$
(c)	**M1**	For any correct method for finding the area of a triangle. e.g., $A = \frac{1}{2} \times 12 \times ('17' - '-9') = 156$ OR $A = \frac{1}{2}\begin{bmatrix} 9 & -9 & 17 & 9 \\ 12 & 0 & 0 & 12 \end{bmatrix} = \frac{1}{2}\left[(0+0+204)-(0+0-108)\right] = 156$
	A1	For 156

Question number	Scheme	Marks
10 (a)	$\sin(A+B)+\sin(A-B)=\sin A\cos B+\sin B\cos A+\sin A\cos B-\sin B\cos A$ $=2\sin A\cos B$	M1
	$\left[\sin(A+B)+\sin(A-B)=\sin 5x+\sin 3x\right]$ $Ax+Bx=5x$ $Ax-Bx=3x$	M1
	$\Rightarrow A=4x,\quad B=x$	
	$\Rightarrow \sin(4x+x)+\sin(4x-x)=\sin 5x+\sin 3x=2\sin 4x\cos x$	A1 cso [3]
	ALT	
	$\sin(A+B)-\sin(A-B)=\sin A\cos B+\sin B\cos A-(\sin A\cos B-\sin B\cos A)$ $=2\sin B\cos A$	[M1
	$\left[\sin(A+B)+\sin(A-B)=\sin(A+B)-\sin\left[-(A-B)\right]=\sin 5x+\sin 3x\right]$ $\Rightarrow A=x,\quad B=4x$	M1
	$\Rightarrow \sin(x+4x)-\sin(x-4x)=\sin 5x-\sin(-3x)=\sin 5x+\sin 3x=2\sin 4x\cos x$ $\left[-\sin(-3x)=+\sin 3x\right]$	A1cso]
(b)	**Intersections on x-axis** $6\sin x\cos x=0\Rightarrow \sin 4x=0,\quad \cos x=0$	M1
	from $\sin 4x=0\Rightarrow 4x=0,\pi,2\pi,3\pi,4\pi\rightarrow x=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{2},2\pi$ **and**	M1
	from $\cos x=0\Rightarrow x=0,\frac{\pi}{2},\frac{3\pi}{2}$ $\Rightarrow x=0,\frac{\pi}{4},\frac{\pi}{2}$	A1
	Integration $(3)\int(\sin 5x+\sin 3x)\,dx=(3)\left(-\frac{\cos 5x}{5}-\frac{\cos 3x}{3}\right)$	M1 A1 ft
	Limits $\int_0^{\frac{\pi}{4}}(\sin 5x+\sin 3x)\,dx\pm\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin 5x+\sin 3x)\,dx$	M1
	Substitution of values	

	$3\left\{\left[\left(-\frac{\cos 5\left(\frac{\pi}{4}\right)}{5}-\frac{\cos 3\left(\frac{\pi}{4}\right)}{3}\right)-\left(-\frac{\cos 5(0)}{5}-\frac{\cos 3(0)}{3}\right)\right]-\left[\left(-\frac{\cos 5\left(\frac{\pi}{2}\right)}{5}-\frac{\cos 3\left(\frac{\pi}{2}\right)}{3}\right)-\left(-\frac{\cos 5\left(\frac{\pi}{4}\right)}{5}-\frac{\cos 3\left(\frac{\pi}{4}\right)}{3}\right)\right]\right\}$	ddM1
	$= 2.73137... + \lvert -1.313170... \rvert = 3.8267... \approx 3.86$	A1 cao [8]
		Total 11 marks

Part	Mark	Additional Guidance
(a)	M1	Use of both correct trig identities [either adding or subtracting], allow any two distinct letters or symbols and simplify to reach either $2\sin A\cos B$ or $2\sin B\cos A$ Allow use of $4x$ and x in place of A and B
	M1	Finds or deduces that $A = 4x, \quad B = x$ Or if they use the **ALT** scheme then $A = x, \quad B = 4x$ This may be implied by the next step or just written down.
	A1cso	For a final correct statement with no errors seen.
(b)	M1	(Placing the equation = 0) and arriving at one of $\sin 4x = 0$ ***or*** $\cos x = 0$
	M1	At least any **one** correct value for x from $\sin 4x = 0$ **and** at least any **one** correct value for x from $\cos x = 0$
	A1	Correctly identifies the required intersections of the curve with the x-axis either explicitly here or used in limits in following work, 0 need not be explicitly stated.
	Note:	Award for these values seen without working, and allow working here in degrees.
	M1	**Integration** An attempt at integration of both terms from their part (a) to an expression of the form $\pm\frac{\cos 5x}{5}\pm\frac{\cos 3x}{3}$ Ignore: • the presence or absence of the multiplier of 3 for this mark • any limits • incorrect notation
	A1	Fully correct integration. Again, ignore the presence/absence of the multiplier of 3. $(3)\left(-\frac{\cos 5x}{5}-\frac{\cos 3x}{3}\right)$
	M1	**Limits** Candidate clearly shows the integration of the curve needs to be done in two parts, where their limits are explicitly shown and come from the limits identified from solving the equation $6\sin x\cos x = 0$.

		As a minimum we need to see two integrals of the function (this can either be written in the original form or the changed form). Their limits must be clearly shown for this mark (following through their solutions) and the upper limit on the first integral must be equal to the lower limit on the second. The candidate does not need to show that the modulus of the second integral must be taken for this mark. Ignore the presence/absence of the multiplier of 3.
	ddM1	Substitution of all limits –dependent on the previous two method marks, allow slips in simplification. The substitution must be correct. If you see any of the decimal answers shown in the working column or any of $\frac{8+4\sqrt{2}}{5}$ (1st integral) or $\frac{4\sqrt{2}}{5}$ (2nd integral) or $\frac{8+8\sqrt{2}}{5}$ (sum of integrals) award this mark. **NOTE: They must consider the modulus of the second integral, and the multiplier of 3 must also be taken into account for this mark.**
	A1cao	For awrt 3.86

Question number	Scheme	Marks
8 (a)(i)	$e^{3x} - 1 = 9 - 9e^{-3x}$	M1
	$(e^{3x})^2 - e^{3x} = 9e^{3x} - 9$	M1
	$\left(e^{3x}\right)^2 - 10e^{3x} + 9 = 0$ *	A1cso*
(ii)	$(e^{3x} - 1)(e^{3x} - 9)$ [= 0] leading to $e^{3x} =$	M1
	$e^{3x} = 9 \rightarrow x = \frac{1}{3}\ln 9$ *	A1cso* [5]
Note: subscripts on marks in (b) indicate which mark is being awarded on epen (1=1st etc)		
(b)	$\left(\int_0^{\frac{1}{3}\ln 9}(9 - 9e^{-3x})\,dx =\right)$ $9x + 3e^{-3x}$ $\left(\int_0^{\frac{1}{3}\ln 9}(e^{3x} - 1)\,dx =\right)$ $\frac{e^{3x}}{3} - x$	$M1_1$ $A1_2A1_3$
	"$9x + 3e^{-3x}$" − "$\frac{e^{3x}}{3} - x$" $M1_4$ $\left(10 \times \frac{1}{3}\ln 9 - \frac{9}{-3}e^{-3\times\frac{1}{3}\ln 9} - \frac{e^{3\times\frac{1}{3}\ln 9}}{3}\right) - \left(10 \times 0 - \frac{9}{-3}e^{-3\times 0} - \frac{e^{3\times 0}}{3}\right)$ $dM1_5$	$\left(9 \times \frac{1}{3}\ln 9 + 3e^{-3\times\frac{1}{3}\ln 9}\right) - (9 \times 0 + 3e^0)$ **and** $\left(\frac{e^{3\times\frac{1}{3}\ln 9}}{3} - \frac{1}{3}\ln 9\right) - \left(\frac{e^0}{3} - 0\right)$ $dM1_5$ "$3\ln 9 - \frac{8}{3}$" − "$\frac{8}{3} - \frac{1}{3}\ln 9$" $M1_4$
	$\frac{10}{3}\ln 9 - \frac{16}{3}$	$A1_6$
	ALT $\int(9 - 9e^{-3x}) - (e^{3x} - 1)dx$	$M1_4$
	$(\int(10 - 9e^{-3x} - e^{3x})\,dx =)\ 10x - \frac{9}{-3}e^{-3x} - \frac{e^{3x}}{3}$	$M1_1$ $A1_2$ $A1_3$
	$\left(10\times\frac{1}{3}\ln 9 - \frac{9}{-3}e^{-3\times\frac{1}{3}\ln 9} - \frac{e^{3\times\frac{1}{3}\ln 9}}{3}\right) - \left(10\times 0 - \frac{9}{-3}e^{-3\times 0} - \frac{e^{3\times 0}}{3}\right)$	$dM1_5$
	$\frac{10}{3}\ln 9 - \frac{16}{3}$ oe	$A1_6$ [6]
		Total 11 marks

Part	Mark	Additional Guidance
Mark parts (i) and (ii) together.		
(a)(i)	M1	For equating the two equations.
	M1	For multiplying through by e^{3x}, minimum of 2 out of 4 correct terms. (presence of $\pm 10e^{3x}$ indicates 2 correct terms).
	A1*cso	Correct solution only, no errors or omissions.
(ii)	M1	Minimally acceptable attempt at solving the equation leading to $e^{3x} =$ See general guidance, if the formula is quoted allow up to two slips in substitution, otherwise the substitution must be correct.
	A1*cso	Correct solution only, no errors or omissions. If 0 also included then this should be rejected.
(b)	$M1_1$	For attempt to integrate one of: $9 - 9e^{-3x}$ or $e^{3x} - 1$ or $\pm[(9 - 9e^{-3x}) - (e^{3x} - 1)]$ Limits may not be present. At least one term correct. Ignore +c if included.
	$A1_2$	For correct integration of one of the exponential terms $\pm 9e^{-3x} \rightarrow \mp\frac{9}{3}e^{-3x}$ **or** $\pm e^{3x} \rightarrow \pm\frac{1}{3}e^{3x}$ Limits need not be present. Ignore +c if included.
	$A1_3$	For correct integration of both curves $9x + 3e^{-3x}$ **and** $\frac{e^{3x}}{3} - x$ or for a **fully correct** integration where the difference between two expressions is found $\pm\left(10x - \frac{9}{-3}e^{-3x} - \frac{e^{3x}}{3}\right)$ or $\pm\left(9x - \frac{9}{-3}e^{-3x} - \frac{e^{3x}}{3} + x\right)$ **Note: this is an M mark in epen**
	$M1_4$	For the difference between the two expressions either before or after integration. Allow subtraction either way around. **Note: this is an A mark in epen**
	$dM1_5$	Substitution of correct limits into their integrated expressions (limits subtracted the correct way around). Dependent on first M scored. If substituting before difference found then must substitute into both integrated expressions. May be implied by awrt 1.99 If integration is not correct then substitution must be shown.
	$A1_6$	For the correct answer oe. Must be exact value.

Question	Scheme	Marks
7(a)	$v = 1^2 - 10\times1 + 28 = 19$ [m/s]	B1 [1]
(b)	$s = \int\left(t^2 - 10t + 28\right)\,\mathrm{d}t = \frac{t^3}{3} - \frac{10t^2}{2} + 28t\,(+c)$ $24 = \frac{3^3}{3} - \frac{10\times3^2}{2} + 28\times3 + c \Rightarrow c = -24 \Rightarrow \left[s = \frac{t^3}{3} - \frac{10t^2}{2} + 28t - 24\right]$ $t = 5, \quad s = \frac{5^3}{3} - \frac{10\times5^2}{2} + 28\times5 - 24 = \frac{98}{3}$ [m]	M1 M1A1 M1A1 [5]
(c)	$\frac{\mathrm{d}v}{\mathrm{d}t} = 2t - 10$ when t = 9, acceleration = 8 [m/s ^{2}]	M1 A1 [2]
(d)	(i) $v = (t-5)^2 + 3$ Irrespective of the value of t $v \geqslant 3$ so the particle never comes to rest. **ALT** $b^2 - 4ac < 0 \Rightarrow (-10)^2 - 4\times1\times28 = -12$ No real solutions so the particle never comes to rest. (ii) Least value of v is 3 [m/s]	M1 A1 [M1 A1] B1 [3]
		Total 11 marks

Question	Notes	Marks
7(a)	$v = t^2 - 10t + 28$	
	$v = 1^2 - 10\times1 + 28 = 19$ [m/s]	B1 [1]
(b)	For an attempt to integrate the given expression for v [See general guidance for the definition of an attempt] $s = \int\left(t^2 - 10t + 28\right)\,\mathrm{d}t = \frac{t^3}{3} - \frac{10t^2}{2} + 28t\,(+c)$	M1
	For finding the value of c. They cannot score this mark without $+c$ $24 = \frac{3^3}{3} - \frac{10\times3^2}{2} + 28\times3 + c \Rightarrow c = ...$	M1
	For the correct expression for s. This need not be explicitly stated. $s = \frac{t^3}{3} - \frac{10t^2}{2} + 28t - 24$	A1

	Award for the correct value of c seen -24 [m]	
	For using their integrated expression for s to find a value of the displacement when $t = 5$ $s = \frac{5^3}{3} - \frac{10\times5^2}{2} + 28\times5 - 24 = ...$	M1
	For the correct value of $s = \frac{98}{3}$ [m] Accept $s = 32.7$ [or better]	A1 [5]
(c)	For an attempt to differentiate the given v and substituting $t = 9$ into their differentiated expression. $\frac{dv}{dt} = 2t - 10 \Rightarrow \frac{dv}{dt} = 2\times9 - 10 = ...$	M1
	For acceleration = 8 [m/s ^{2}]	A1 [2]
(d)(i)	**Method A** Completes the square to give $v = (t-5)^2 + 3$	M1
	Concludes that at the minimum velocity [3 m/s] $t = 5$ so P never comes to rest	A1
	Method B Finds the value of the discriminant $b^2 - 4ac < 0 \Rightarrow (-10)^2 - 4\times1\times28 = -12$	M1
	Concludes that as there are no real solutions, **so *P* does not come to rest.**	A1
	Method C Solves the 3TQ to give the following two [non-real] values of t: $5+\sqrt{3}i$ and $5-\sqrt{3}i$ or $\left(t-\left[5+\sqrt{3}i\right]\right)\left(t-\left[5+\sqrt{3}i\right]\right)=0$	M1
	Concludes that as there are no real solutions, **so *P* does not come to rest.**	A1
	Method D Uses their result from (c) $\frac{dv}{dt} = 2t - 10 = 0 \Rightarrow t = 5$	M1
	Concludes that at the minimum velocity [3 m/s] $t = 5$ so P never comes to rest	A1
(ii)	For the correct value of $v = 3$ [m/s}	B1 [3]
		Total 11 marks

Question	Scheme	Marks
5(a)	$V = x \times 4x \times h \Rightarrow 4x^2h = 75 \Rightarrow h = \dfrac{75}{4x^2}$	B1
	$S = 2\left(4x^2 + xh + 4xh\right) = \left[8x^2 + 2xh + 8xh\right] \Rightarrow S = 8x^2 + 10xh$	M1
	$\Rightarrow S = 8x^2 + 10x \times \dfrac{75}{4x^2} \Rightarrow S = 8x^2 + \dfrac{375}{2x}$ *	M1A1 cso
		[4]
(b)(i)	$\dfrac{\mathrm{d}S}{\mathrm{d}x} = 16x - \dfrac{375}{2x^2} = 0$	M1
	$\Rightarrow 16x = \dfrac{375}{2x^2} \Rightarrow x^3 = \dfrac{375}{32} \Rightarrow x = 2.2714... \approx 2.27 \ (\text{cm})$	M1A1
(ii)	$\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2} = 16 + 375x^{-3} \approx 48$	M1
	Conclusion: (As x is always positive) $\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2} > 0 \Rightarrow$ minimum	A1ft [5]
(c)	$S = 8 \times '2.27'^2 + \dfrac{375}{2 \times '2.27'} = 123.822... \approx 124 \ (\text{cm}^2)$	M1A1 [2]
		Total 11 marks

Part	Mark	Notes
(a)	**B1**	For finding the correct expression for h in terms of x Accept this mark seen anywhere in part (a) Accept also for example, $xh = \dfrac{75}{4x}$ or any equivalent that can be substituted into an expression for S.
	M1	For finding an expression for S is terms of x and h. Accept as a minimum $Ax^2 + Bxh$ where A and B are constants. Accept other letters for S at this stage. E.g SA etc. or even no letter at all.
	M1	For substituting their h into their S
	A1	For the correct expression for S as shown in the question. You must see $S = 8x^2 + \dfrac{375}{2x}$ including S although S seen at the top of a column of working is also fine.
(b)	**M1**	For an attempt to differentiate the given S which as a minimum must be of the form

		$\frac{dS}{dx} = 16x - \frac{k}{x^2}$ where k is a constant.
	M1	For setting their $\frac{dS}{dx} = 0$ and attempting to solve the equation and reaching a value of x. It must be a clearly differentiated expression even if they do not score the first M mark in (b). That is, they must reduce the power of at least one term by 1.
	A1	For awrt $x = 2.27$
	M1	For an attempt to differentiate their $\frac{dS}{dx}$ to find $\frac{d^2S}{dx^2}$ which must be as a minimum $\frac{d^2S}{dx^2} = 16 + lx^{-3}$ where l is a constant **Other methods** Please send to Review any examples of candidates who test the gradient on either side of the minimum, or who draw a sketch.
	A1ft	Concludes that as $x > 0$ so $\frac{d^2S}{dx^2} > 0$ and therefore S is a minimum when $x = 2.27$ Dependent on correct method seen. Follow through their value of x provided it is positive. **NB:** They do not need to evaluate $\frac{d^2S}{dx^2}$ to score this mark. [See above] However, if they use the correct [2.27] or an incorrect positive value of x, provided substitution is seen award the mark even if the final evaluation is incorrect. If no substitution is seen and either the value of x is incorrect or they do not obtain approximately 48, withhold this mark.
(c)	**M1**	Substitutes their $x = 2.27$ into the given S Follow through their value of x provided it is a positive value. Use of a negative value of x is M0. If the final value of S is incorrect following an incorrect value of x award this mark only for explicit substitution seen.
	A1	For awrt $S = 124$ (cm^2)

Question	Scheme	Marks
6(a)	$\tan 30^\circ = \frac{r}{h} \Rightarrow r = \frac{h}{\sqrt{3}}$	B1
	$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h \Rightarrow V = \frac{1}{9}\pi h^3$ *	M1A1 cso [3]
(b)	$\frac{dV}{dt} = -4$	B1
	$A = \pi r^2 \Rightarrow A = \pi\left(\frac{h}{\sqrt{3}}\right)^2 = \frac{\pi h^2}{3}$	B1
	$\frac{dA}{dh} = \frac{2\pi h}{3}$ $\quad \frac{dV}{dh} = \frac{3\pi h^2}{9} = \frac{\pi h^2}{3}$	M1M1A1
	$\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dV}{dt} \times \frac{1}{\frac{dV}{dh}}$	M1
	$\frac{dA}{dt} = \frac{2\pi h}{3} \times (-4) \times \frac{3}{\pi h^2} = -\frac{8}{h} \Rightarrow \frac{dA}{dt} = -\frac{8}{24} = -\frac{1}{3}$ $\left(\text{cm}^2/\text{s}\right)$	dM1A1 [8]
	ALT – working in terms of *r* rather than *h*	
	$\frac{dV}{dt} = -4$	[B1
	$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi r^2\left(\sqrt{3}r\right) = \frac{\sqrt{3}}{3}\pi r^3$	B1
	$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$ $\quad V = \frac{\sqrt{3}}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = \sqrt{3}\pi r^2$	M1M1A1
	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{1}{\frac{dV}{dr}} \times \frac{dV}{dt}$	M1
	$\frac{dA}{dt} = 2\pi r \times \frac{1}{\sqrt{3}\pi r^2} \times -4 = -\frac{8}{\sqrt{3}r} \Rightarrow \frac{dA}{dt} = -\frac{8}{\sqrt{3}\times\frac{24}{\sqrt{3}}} = -\frac{1}{3}$	dM1A1]
		Total 11 marks

Part	Mark	Notes
(a)	B1	For $r = \frac{h}{\sqrt{3}}$ or $r^2 = \frac{1}{3}h^2$ $r = \frac{h}{\sqrt{3}}$ may be implied by substitution into the correct formula for volume of a cone.
	M1	For using the correct volume of a cone and substituting in their expression for r or r^2 to find an expression for the volume in terms of h
	A1	For the correct expression with no errors seen. Must have $V =$
(b)	B1	For stating $\frac{dV}{dt} = -4$ (allow +) May be embedded in working.
	B1	For finding the correct expression for the area of a circle in terms of h Note: if differentiation is attempted before writing in terms of h then award 1st M1 for $\frac{dA}{dh} = \frac{dA}{dr} \times \frac{dr}{dh}$ with attempts at $\frac{dA}{dr}, \frac{dr}{dh}$ found and substituted in and attempt to substitute for r in terms of h. B1 once $\frac{dA}{dh}$ found completely in terms of h
	M1	For attempting to differentiate their expression for A in terms of h See general guidance for what constitutes an attempt to differentiate. A must have the form $A = kh^2$ (dimensionally correct for area).
	M1	For attempting to differentiate the given V See general guidance for what constitutes an attempt to differentiate.
	A1	Both derivatives correct. $\frac{dA}{dh}$ and $\frac{dV}{dh}$
	M1	For a correct expression of chain rule to find $\frac{dA}{dt}$
	dM1	For substituting their derivatives into a correct chain rule. Dep on previous three M marks.
	A1	For the correct $\frac{dA}{dt} = -\frac{1}{3}$ $\left(\text{cm}^2/\text{s}\right)$ Allow $-0.\dot{3}$ but not -0.3. ISW once exact answer found.
	ALT	
	B1	For stating $\frac{dV}{dt} = -4$ (allow +) May be embedded in working.
	B1	For finding the correct expression for the volume in terms of r Must be working with correct volume expression as given in (a).
	M1	For attempting to differentiate expression for A in terms of r See general guidance for what constitutes an attempt to differentiate. A must have the form $A = kr^2$ (dimensionally correct for area).
	M1	For attempting to differentiate expression for V in terms of r See general guidance for what constitutes an attempt to differentiate.

	A1	Both derivatives correct. $\frac{dA}{dr}$ and $\frac{dV}{dr}$
	M1	For a correct expression of chain rule to find $\frac{dA}{dt}$
	dM1	For substituting their derivatives into a correct chain rule. Dep on previous three M marks.
	A1	For the correct $\frac{dA}{dt} = -\frac{1}{3}$ $\left(\text{cm}^2/\text{s}\right)$ Allow $-0.\dot{3}$ but not -0.3. ISW once exact answer found.

Question number	Scheme	Marks
7 (a)	$\frac{1}{4}\pi r^2 + 2xr = 100 \quad \left(100 - \frac{1}{4}\pi r^2 = 2xr\right) \Rightarrow x = \frac{50}{r} - \frac{1}{8}\pi r$ oe for example $\frac{100 - \frac{\pi r^2}{4}}{2r}$ $\frac{400 - \pi r^2}{8r}$	M1
	$(P =)\frac{1}{4}(2\pi r) + 4x + 2r$ oe for example $(P =)\frac{1}{2}\pi r + 2x + 2x + 2r$	M1
	$(P =)\frac{1}{2}\pi r + 4\left(\frac{50}{r} - \frac{1}{8}\pi r\right) + 2r \left(= \frac{1}{2}\pi r + \frac{200}{r} - \frac{1}{2}\pi r + 2r\right)$ oe	M1
	$P = \frac{200}{r} + 2r$ *	A1*cso [4]
(b)	$\left(\frac{dP}{dr} =\right) -\frac{200}{r^2} + 2$	M1
	$0 = "-\frac{200}{r^2} + 2" \Rightarrow r =$	M1
	$(r =)10$	A1
	$\left(\frac{d^2P}{dr^2} =\right)\frac{400}{r^3}$	M1
	$r = 10, \frac{d^2P}{dr^2} > 0$ therefore minimum or $\frac{d^2P}{dr^2} = \frac{400}{10^3}$ or $\frac{2}{5} > 0$ therefore minimum	A1 [5]
(c)	$P = \frac{200}{("10")} + 2("10") = 40$	M1 A1 [2]
		Total 11 marks

Part	Mark	Additional Guidance
(a)	M1	Writes the correct (unsimplified) equation for the area and rearranges to give $x =$ The rearrangement may contain up to 2 errors and does not have to be simplified.
	M1	Writes the correct formula or expression for the perimeter. It is not necessary to see $P =$ at this point and the expression does not need to be simplified.
	M1	Correctly substitutes their expression for x into their formula for the perimeter Not a dependent mark **but** ***x* must be of the form** $\frac{a}{r}+b\pi r \quad a,b\neq 0$ ***P* must be of the form** $d\pi r+ex+fr \quad d,e,f\neq 0$ Not necessary to see $P =$ for this mark.
	A1*cso	For the given result, minimum steps as shown, with no errors seen. It is not necessary to see the steps in brackets. They are present to help examiners check intermediate work as there shouldn't be incorrect work for this mark. It is necessary to see $P =$ at some point for this mark to be awarded.
(b)	M1	For attempting to differentiate P wrt r At least one term must be fully correct, the other – follow general guidance.
	M1	For setting their derivative (must involve a changed expression and have 2 terms) = 0 and a completely correct rearrangement to find the value of r
	A1	For ($r =$) 10, ignore $r = -10$
	M1	For an attempt to find the second derivative to give an expression of the form $\pm\frac{a}{r^3}$
	A1	For a correct second derivative and correct justification that this is a minimum, with some form of conclusion. All work to be correct for this mark. i.e it is acceptable, as this is a simple substitution to state (if $r = 10$) $\frac{\mathrm{d}^2P}{\mathrm{d}r^2}>0$, but if a substitution of $r = 10$ is made, the value for the second derivative must be given as 0.4 Work with $r = -10$ to find a maximum may be ignored.
ALT final A1	A1	Testing and substituting into the correct first derivative with appropriate values either side of $r = 10$. For correct justification that this is a minimum, with some form of conclusion. All work to be correct for this mark.
(c)	M1	For correct substitution of their r into the given formula for P.
	A1	40

Question number	Scheme	Marks
7 (a)	$2x^2y$	M1
	$72=2x^2y \Rightarrow y=\frac{36}{x^2}$ oe or $xy=\frac{36}{x}$ oe	M1
	$(A=)2\times 2xy+2\times xy+2\times 2x^2\left(=6xy+4x^2\right)$	
	$=6x\left(\frac{36}{x^2}\right)+4x^2$ or $=6\left(\frac{36}{x}\right)+4x^2$	dM1
	$A=\frac{216}{x}+4x^2$ *	A1*cso [4]
(b)	$\left(\frac{dA}{dx}=\right)-\frac{216}{x^2}+8x$	M1
	$0=-\frac{216}{x^2}+8x\rightarrow x=$ or $x^3=$	dM1
	$x=3$	A1
	$(A=)\frac{216}{3}+4\times 3^2$	dM1
	$=108$	A1
	$\left(\frac{d^2A}{dx^2}=\right)\frac{432}{x^3}+8$	M1
	$=\frac{432}{3^3}+8(=24)>0$ therefore minimum	A1 [7]
		Total 11 marks

Part	Mark	Additional Guidance
(a)	M1	Correct expression for volume.
	M1	Equates their expression of the form ax^2y to 72 and rearranges correctly for y or xy Simplification is not required to be awarded this mark.
	dM1	Substitutes their expression for y or xy into a correct (unsimplified) formula or expression for area.. Dep on previous M mark
	A1*	Correct solution only, minimum steps as shown. A = must be present at least once.
(b)	M1	Differentiates to get an expression of the form $\pm\frac{a}{x^2}+8x \quad a\neq 0$
	dM1	Places their expression of the correct form = 0 and rearranges to give x or x^3 = Allow errors in rearrangement. Dependent on previous method mark. So long as differentiation shown, this mark can be implied by correct value.
	A1	$x = 3$ dependent on having gained 1st method mark – the question directs the candidates to use calculus.
	dM1	Substitutes their value for x into the **correct** expression for A. Dependent on 1st method mark. So long as differentiation shown, this mark can be implied by a correct value.
	A1	For 108 dependent on having gained 1st method mark – the question directs the candidates to use calculus.
	M1	Differentiates their expression for $\frac{\mathrm{d}A}{\mathrm{d}x}$ to give an expression of the form $\pm\frac{b}{x^3}+8 \; b\neq 0$
	A1	Correctly substitutes $x = 3$ into the correct second derivative or argues the correct second derivative is > 0 and draws a conclusion. It is not necessary to see an evaluation, but the evaluation must be correct if carried out. Second derivative **must be stated to be > 0 or positive.** As always with A1, must have gained all previous marks. **Note: it is highly unusual to have a final A1, when any of the preceding marks have been 0. The structure of this question means, this is possible, if they either don't find the minimum area or calculate it incorrectly.** **So for the final 4 marks dM1 A0 M1 A1 and dM0 A0 M1 A1**

Question number	Scheme	Marks
10 (a)	$\int\left(3x^2-4x-p\right)\,\mathrm{d}x=\frac{3x^3}{3}-\frac{4x^2}{2}-px+c\quad\left[=x^3-2x^2-px+c\right]$	M1A1
	$y=x^3-2x^2-px+c$ At $(2,0)\quad 0=8-8-2p+c\Rightarrow c=2p$ At $(-1,9)\quad 9=-1-2+p+c\Rightarrow c=12-p$ $\Rightarrow p=4,c=8\Rightarrow y=x^3-2x^2-4x+8$*	M1 M1 A1A1cso [6]
(b)	$x^3-2x^2-4x+8=8-4x\Rightarrow x^3-2x^2=0\Rightarrow x^2\left(x-2\right)=0$ $x=0,x=2$	M1 A1
	Area $=\int_0^2\left(8-4x\right)\,\mathrm{d}x-\int_0^2\left(x^3-2x^2-4x+8\right)\,\mathrm{d}x$ Area $=\int_0^2\left(8-4x\right)\,\mathrm{d}x-\int_0^2\left(x^3-2x^2-4x+8\right)\,\mathrm{d}x=\int_0^2\left(-x^3+2x^2\right)\,\mathrm{d}x$ Area $=\left[-\frac{x^4}{4}+\frac{2x^3}{3}\right]_0^2=\left(-4+\frac{16}{3}\right)-\left(0\right)=\frac{4}{3}$	M1 M1M1A1 [6]
		Total 12 marks

(a)

M1 Attempts to integrate

A1 Correct integration including + *c*

M1 Substitution of (2, 0) (Does not have to be simplified)

M1 Substitution of (–1, 9 (Does not have to be simplified)

A1 $p=4,c=8$

A1 cso Obtains the **given** answer with no errors in the working

(b)

M1 Equating *C* and *l*

A1 $x=0$ and $x=2$

NB If correct limits are seen then M1A1 is awarded

M1 Use of $\int_a^b\left(\mathrm{f}(x)-\mathrm{g}(x)\right)\mathrm{d}x$ or $\int_a^b\mathrm{f}(x)\mathrm{d}x-\int_a^b\mathrm{g}(x)\mathrm{d}x$ or $\int_a^b\mathrm{f}(x)\mathrm{d}x-\frac{1}{2}\times2\times8$

Ignore limits (These can be either way round)

M1 Attempt the integration. Limits not needed.

M1 Substitute the correct limits. $\left(\text{May be implied by } \pm\frac{4}{3}\right)$

A1 $\frac{4}{3}$

NB If no integration is seen then M0M0A0 is awarded for the last 3 marks for an answer of $\frac{4}{3}$

Question number	Scheme	Marks
10 (a)	$\sin x+1=\cos x+1 \Rightarrow \tan x=1$	M1
	$x=\frac{\pi}{4},\frac{5\pi}{4}$	A1 A1 (3)
(b)	Area R_1 :	
	$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x+1)-(\cos x+1)\,dx=\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x-\cos x)\,dx$	M1
	$\left[-\cos x-\sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$	M1
	$\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)-\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)=2\sqrt{2}$	M1 A1
	Area R_2 :	
	$\int_{\pi}^{\frac{5\pi}{4}}(\cos x+1)\,dx+\int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}(\sin x+1)\,dx$	M1
	$\left[\sin x+x\right]_{\pi}^{\frac{5\pi}{4}}+\left[-\cos x+x\right]_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}$	A1
	$\left[\left(-\frac{\sqrt{2}}{2}+\frac{5\pi}{4}\right)-\pi\right]+\left[\frac{3\pi}{2}-\left(\frac{\sqrt{2}}{2}+\frac{5\pi}{4}\right)\right]=-\sqrt{2}+\frac{1}{2}\pi$	M1 A1
	area of R_1 : area of R_2 $=2:\left(\frac{\pi\sqrt{2}}{4}-1\right)$ oe	A1 (9)
		Total 12 marks

Part	Mark	Notes
(a)	M1	For $\tan x = 1$
	A1	For $x = \frac{\pi}{4}$ [Allow 45°]
	A1	For $x = \frac{5\pi}{4}$ [Allow 225°]
(b)	M1	For stating $A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x + 1) - (\cos x + 1)\,dx$
	M1	For attempting to integrate their expression for the area. This must be correct for this mark. $\int (\sin x - \cos x)\,dx = -\cos x - \sin x$
	M1	For substitution of correct limits
	A1	For $2\sqrt{2}$
	M1	For attempting $\int f(x)dx + \int g(x)dx$ with the correct limits. $\int_{\pi}^{\frac{5\pi}{4}} (\cos x + 1)\,dx + \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}} (\sin x + 1)\,dx$
	M1	For attempting to integrate, which must be correct for this mark.
	M1	For substitution of their limits correctly.
	A1	For $-\sqrt{2} + \frac{1}{2}\pi$
	A1	For the simplified ratio $2 : \left(\frac{\pi\sqrt{2}}{4} - 1\right)$

Question number	Scheme	Marks
8 a	Height of the waste paper basket = $\sqrt{(5x)^2-(3x)^2}=4x$ (cm)	M1
	$V=\frac{1}{2}(2x+8x)\times 4x\times h=20x^2h=2250\Rightarrow h=\frac{2250}{20x^2}$ oe	M1
	$S=2\times 20x^2+2xh+2(5xh)$	M1
	$S=40x^2+12x\left(\frac{2250}{20x^2}\right)$ oe	M1
	$S=40x^2+\frac{1350}{x}$ *	A1 cso (5)
b	$\frac{dS}{dx}=80x-\frac{1350}{x^2}$ oe	M1
	$\frac{dS}{dx}=80x-\frac{1350}{x^2}=0$ so $x^3=\frac{135}{8}\Rightarrow x=...$	M1
	$x=2.56$ awrt	A1
	$\frac{d^2S}{dx^2}=80+\frac{2700}{x^3}>0$ for all positive values of $x\therefore$ minimum	M1 A1ft (5)
c	When $x=2.56$ $S=40(2.56)^2+\frac{1350}{2.56}=789$ awrt	M1 A1 (2)
		Total 12 marks

Part	Mark	Notes
(a)	**M1**	For finding the height of the waste-paper basket using Pythagoras theorem. $\sqrt{(5x)^2-(3x)^2}=4x$ (cm)
	M1	For finding the volume which must come from; V = correct area of trapezium (using their height) × h $V=\frac{1}{2}(2x+8x)\times'4x'\times h=2250\Rightarrow('20'x^2h=2250)$ **and** for obtaining an expression for the height h: $h=\frac{2250}{'20'x^2}$ or $xh=\frac{2250}{'20'x}$ Please check their algebra carefully, as some may even substitute $hx^2=...$ **NB: This is an A mark in Epen.**
	M1	For writing the surface area in terms of 2 unknowns [x and h] which need not be simplified. This must be **correct** using their expression in terms of x for the height of the prism. e.g. $S=2\times\frac{(8x+2x)\times'4x'}{2}+2xh+2(5xh)=\left[40x^2+12xh\right]$
	M1	For eliminating h from their expression for S S must be in the form Ax^2+Bxh and h must be of the form $\frac{C}{x^2}$ where A, B and C are constants.
	A1 cso	For the given result exactly as written. $S=40x^2+\frac{1350}{x}$
(b)	**M1**	For an acceptable attempt to differentiate the **given** expression for S. See General Guidance.
	M1	For setting their $\frac{dS}{dx}=0$ and attempting to find a value for x The minimum acceptable expression is $\frac{dS}{dx}=Px\pm\frac{Q}{x^2}$ where P and Q are constants.
	A1	For awrt $x=2.56$
	M1	For differentiating their $\frac{dS}{dx}$, which must be as a minimum $\frac{dS}{dx}=Px\pm\frac{Q}{x^2}$ to find the second derivative to achieve as a minimum $\frac{d^2S}{dx^2}=\pm M\pm\frac{N}{x^3}$
	A1ft	Concludes that as $\frac{d^2S}{dx^2}$ will always be positive, [either by substitution, or by inference] so the value of x obtained will be a minimum. $\left[\frac{d^2S}{dx^2}=80+\frac{2700}{2.56^3}=240.9...\right]$ with a conclusion. **NOTE:** The ft only applies to their value of x . Do not ft an incorrect $\frac{d^2S}{dx^2}$
(c)	**M1**	Substitutes their value of x, obtained using a correct method) into the **given** expression for S [provided it is a positive value, do **NOT** allow negative values of x]
	A1	For awrt 789

Question number	Scheme	Marks
11 (a)	$(600 =)2\pi r^2 + 2\pi rh$ oe eg $(300 =)\pi r^2 + \pi rh$	M1
	$h = \dfrac{300 - \pi r^2}{\pi r}$ oe | $\pi rh = 300 - \pi r^2$ oe	A1 cao
	$(V =)\ \pi r^2 "\left(\dfrac{300 - \pi r^2}{\pi r}\right)"$ oe | $V = "(300 - \pi r^2)"r$ oe	M1
	$V = 300r - \pi r^3$ *	A1* cso [4]
(b) (i)	$\dfrac{dV}{dr} = 300 - 3\pi r^2$	M1
	$0 = 300 - 3\pi r^2 \quad \rightarrow \quad r =$	M1
	$r = \sqrt{\dfrac{100}{\pi}}$ * cso	A1* cso
(ii)	$\dfrac{d^2V}{dr^2} = -6\pi r$	M1
	$\rightarrow \dfrac{d^2V}{dr^2} = -6\pi\sqrt{\dfrac{100}{\pi}}$ or $\dfrac{d^2V}{dr^2} = -6\pi \times 5.6418958.....$	
	When r is positive, $-6\pi r$ is negative (-106.347231…..) and therefore this value of r gives a maximum	A1 [5]
(c)	$(V =)300"\sqrt{\frac{100}{\pi}}" - \pi\left("\sqrt{\frac{100}{\pi}}"\right)^3 (= 1128.(379167))$	M1
	$p^3 = \dfrac{300"\sqrt{\frac{100}{\pi}}" - \pi\left("\sqrt{\frac{100}{\pi}}"\right)^3}{\frac{4}{3}\pi}$ (=269(.3806…))	dM1
	p = 6.5 cm	A1 [3]
		Total 12 marks

Part	Mark	Additional Guidance
(a)	M1	Correct expression for the surface area of the cylinder and an attempt to rearrange to h = or $\pi rh =$ Allow errors in arithmetic but not mathematically incorrect process. πrh may be embedded, e.g. $300 = \pi r^2 + \pi rh$ becoming $300 = \pi r^2 + V$ would score M1A1M1 and may score full marks if correct final result obtained.
	A1	cao
	M1	Substitutes their expression for height or their expression for πrh into a correct expression for the volume.
	A1	cso no errors or omissions, must state V =
	Mark parts (i) and (ii) together.	
(b) (i)	M1	Minimally acceptable attempt at differentiation, see general guidance, no power to increase.
	M1	Places their derivative = 0 and attempts to rearrange to find r. Minimally acceptable derivative is of the form $a \pm b\pi r^2$
	A1	Correct value for r, exact value only. Must reject negative value if found, award A0 if not rejected.
(ii)	M1	Minimally acceptable attempt to differentiate their first derivative, see general guidance, no power to increase. Or testing gradients or a sketch.
	A1	Correct evaluation of second derivative or explanation of why second derivative is negative. Conclusion this value of r gives a maximum. No incorrect work.
(c)	M1	Correct substitution of their r into the expression for V.
	dM1	Attempts rearrangement using the formula for volume of a sphere to make p^3 the subject Correct order of operations applied to right hand side. Accept arithmetic slips.
	A1	p = 6.5 Accept awrt 6.5

Question number	Scheme	Marks
11 (a)	$(p=)3e^{\frac{2}{3}}$ oe	B1 [1]
(b)	$\left(\frac{dy}{dx}=\right)e^{\frac{x}{3}} = e^{\frac{2}{3}}$ oe	M1 A1
	$(m_{norm}=)-\frac{1}{"e^{\frac{2}{3}}"}=\left(-e^{-\frac{2}{3}}\right)$ oe	M1
	$y-"3e^{\frac{2}{3}}"="-\frac{1}{"e^{\frac{2}{3}}"}"(x-2)$ oe $\left(y=-e^{-\frac{2}{3}}x+2e^{-\frac{2}{3}}+3e^{\frac{2}{3}}\right)$	M1
	$0-3e^{\frac{2}{3}}=-e^{-\frac{2}{3}}(x-2)$ oe	dM1
	$x=2+3e^{\frac{4}{3}}$	A1
	$(\text{Area of triangle}=)\ \frac{1}{2}\left("3e^{\frac{2}{3}}"\right)\left("2+3e^{\frac{4}{3}}"-2\right)\left(=\frac{9}{2}e^2\right)$	ddM1
	$\int_0^2 3e^{\frac{x}{3}}dx=\left[\frac{3e^{\frac{x}{3}}}{\frac{1}{3}}\right]_0^2$	B1 M1
	$\left(\frac{3e^{\frac{2}{3}}}{\frac{1}{3}}\right)-\frac{3e^0}{\frac{1}{3}}$ $\left(=9e^{\frac{2}{3}}-9\right)$	dM1
	$=\frac{9}{2}e^2+9e^{\frac{2}{3}}-9$	A1cso
ALT for ddM1	$\int_2^{"2+3e^{\frac{4}{3}}"}(-e^{-\frac{2}{3}}x+2e^{-\frac{2}{3}}+3e^{\frac{2}{3}})dx=-e^{-\frac{2}{3}}\frac{x^2}{2}+2e^{-\frac{2}{3}}x+3e^{\frac{2}{3}}x$	
	$=\left[-e^{-\frac{2}{3}}\left(\frac{"2+3e^{\frac{4}{3}}"^2}{2}\right)+2e^{-\frac{2}{3}}\left("2+3e^{\frac{4}{3}}"\right)+3e^{\frac{2}{3}}\left("2+3e^{\frac{4}{3}}"\right)\right]$	ddM1
	$-\left[-e^{-\frac{2}{3}}\left(\frac{"2^2}{2}\right)+2e^{-\frac{2}{3}}(2)+3e^{\frac{2}{3}}(2)\right]$	
		[11]
		Total 12 marks

Part	Mark	Additional Guidance
(a)	B1	For the correct value of *p*.
(b)	M1	Differentiates to get an expression of the form $ae^{\frac{x}{3}}$ $a \neq 3$
	A1	For $e^{\frac{2}{3}}$ This must be an exact value, but isw any following decimals.
	M1	Finds the negative reciprocal of their gradient. This is not a completely dependent mark but must come from any attempt at differentiation.
	M1	For a complete unsimplified attempt (if using $y - y_1 = m(x - x_1)$ to find the equation of *l*, using their value of *p* and their gradient, which can be any changed gradient after finding an expression and value for $\frac{dy}{dx}$ If $y = mx + c$ is used, a complete attempt must involve correctly rearranging (using their values for *p* and their gradient) to find *c*. **Different to question 2, if the candidate is not working with correct values and the rearrangement to find c with exact values is not shown, this mark cannot be awarded**
	dM1	Sets their equation = 0 and attempts to find a value for *x*. Allow errors in rearrangement. Dependent on previous method mark. This must be an exact value, but isw any following decimals.
	A1	Finds the correct value of *x*
	ddM1	For correct substitution into the formula for the area of a triangle, using (their *x*-intercept – 2) and their answer for *p* Does not need to be simplified. Dependent on previous 2 method marks. Or for stating the integral, with their *p* and their *x* intercept as limits and correct integration of at least 2 terms. **We don't need to see substitution of limits for this mark.** Their integral must be of the minimum form $\int_2^{"f+ge^{\frac{h}{3}}"} \left(ae^{-\frac{c}{3}}x + be^{-\frac{d}{3}} \right) dx \quad a,b,c,d,f,g,h \neq 0$ See notes for next *B* mark, can be awarded if limits are correct, within a combined integral.
	B1	For stating $\int_0^2 3e^{\frac{x}{3}}$ Note: some candidates are listing this integral by adding or subtracting it from the equation of the line. ie $\int_a^{"b"} \left(-e^{-\frac{2}{3}}x + 2e^{-\frac{2}{3}} + 3e^{\frac{2}{3}} \pm 3e^{\frac{x}{3}} \right) dx$ - mark B1 if limits are correct and B0 if not and potentially next M1 dM1 for anything that is obviously their line $\pm 3e^{\frac{x}{3}}$
	M1	Integrates to get an expression of the form $ae^{\frac{x}{3}}$ $a \neq 3$
	dM1	Substitution of correct limits into any changed expression, must see substitution of $x = 2$ and 0 (unless the 0 limit gives an evaluation of 0) Allow a correct final answer or a correct evaluation of this integral $9e^{\frac{2}{3}} - 9$ to imply this mark, if no errors in working. Examiners are not expected to evaluate to check incorrect expressions integrated. Dependent on previous method mark.
	A1cso	For a full and correct worked solution with no errors.

Question number	Scheme	Marks
11 (a)	Let x = the length of the side of the triangle and h = the length of the prism	
	$\frac{1}{2}x^2 \sin 60\, h = 72$ or $\frac{1}{2}\left(\sqrt{\left(x^2 - \left(\frac{1}{2}x\right)^2\right)}\right)xh = 72$	M1
	$\frac{\sqrt{3}x^2h}{4} = 72$ $h = \frac{288}{\sqrt{3}x^2}$	M1 A1
	$S = 2 \times \frac{1}{2}x^2 \sin 60 + 3xh$ or $2 \times \frac{1}{2}\left(\sqrt{\left(x^2 - \left(\frac{1}{2}x\right)^2\right)}\right)x + 3xh$	M1
	$S = \frac{\sqrt{3}x^2}{2} + 3x\left(\frac{288}{\sqrt{3}x^2}\right)$ $S = \frac{\sqrt{3}x^2}{2} + \frac{288\sqrt{3}}{x}$ *	M1 A1 cso (6)
(b)	$\frac{dS}{dx} = \sqrt{3}x - \frac{288\sqrt{3}}{x^2} (= 0)$	M1
	$x^3 = 288$ $x = \sqrt[3]{288} = 6.604$	dM1 A1
	$\frac{d^2S}{dx^2} = \sqrt{3} + \frac{576\sqrt{3}}{x^3}$ $\frac{d^2S}{dx^2} > 0$ (when x =6.6) ∴ value is a minimum	ddM1 A1 (5)
(c)	Substitutes their x into $S = \frac{\sqrt{3}x^2}{2} + \frac{288\sqrt{3}}{x}$ $S = 113$	M1 A1 (2)
		[13]

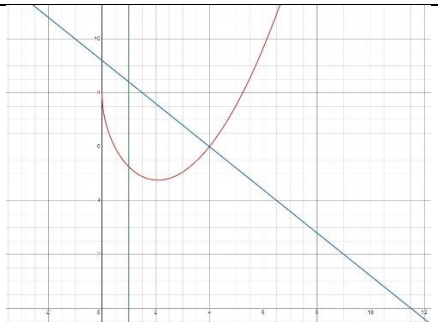
Part	Mark	Additional Guidance
(a)	M1	For the correct expression for the volume of the prism in terms of x and h (or other letter for the length, e.g. l) Simplification not required for this mark $72 = \left(\frac{1}{2} \times x \times x \times \sin 60^\circ\right) \times h$ or $72 = \left(\frac{1}{2} \times x \times \sqrt{x^2 - \frac{x^2}{4}}\right) \times h$ or $72 = \left(\frac{\sqrt{3}}{4}x^2\right) \times h$
	M1	For an attempt to find an expression for h in terms of x Accept as a minimum $h = \frac{k}{x^2}$ where k is a positive integer
	A1	For $h = \frac{288}{\left(\sqrt{3}\right)x^2}$ or $h = \frac{96\sqrt{3}}{x^2}$

	M1	For an expression for S in terms of x and h (ft their area of the triangle) $S = 2\left(\frac{1}{2}\times x\times x\times \sin 60^\circ\right)+3xh$ or $S = 2\left(\frac{1}{2}\times x\times\sqrt{x^2-\frac{x^2}{4}}\right)+3xh$ $\left(S=\frac{\sqrt{3}}{2}x+3xh\right)$
	M1	For substituting their h into their S $S = 2\left(\frac{1}{2}\times x\times x\times \sin 60^\circ\right)+\left(3x\times\frac{288}{\left(\sqrt{3}\right)x^2}\right)$ or $S = 2\left(\frac{1}{2}\times x\times\sqrt{x^2-\frac{x^2}{4}}\right)+\left(3x\times\frac{288}{\left(\sqrt{3}\right)x^2}\right)$
	A1	For the correct expression for S as given. **The expression must be set equal to S.** $S=\frac{\sqrt{3}x^2}{2}+\frac{288\sqrt{3}}{x}$ exactly as seen here. * This is a given result so full working must be seen.
(b)	M1	For an attempt to differentiate the **given** expression for S $\frac{dS}{dx}=\sqrt{3}x-\frac{288\sqrt{3}}{x^2}$ or $\frac{dS}{dx}=\sqrt{3}x-288\sqrt{3}x^{-2}$ (See General Guidance for the definition of an attempt)
	dM1	For setting their differentiated expression = 0 and attempting to solve for x $\sqrt{3}x-\frac{288\sqrt{3}}{x^2}=0\Rightarrow x^3=288\Rightarrow x=\left(6.604\right)$ (rounded correctly) This mark is dependent on the first M mark in (b)
	A1	For $x = 6.604$ rounded correctly
	dM1	For attempting the second derivative (usual definition of an attempt) $\frac{d^2S}{dx^2}=\sqrt{3}+\frac{576\sqrt{3}}{x^3}$ This mark is dependent on first M mark in (b)
	A1ft	Concludes either that $\frac{d^2S}{dx^2}>0$ for all **positive** values of x **or** substitutes in their value of x to show that $\frac{d^2S}{dx^2}=5.19.....$ hence positive so must be a minimum. Only ft if the final conclusion is a minimum provided their $\frac{d^2S}{dx^2}$ is algebraically correct
	ALT – for justifying the minimum using their derivative	
	dM1	Chooses a value either side of their value of x and substituting them into their $\frac{dS}{dx}$ e.g. $x = 6$ and 7 $\frac{dS}{dx}=\sqrt{3}\times 6-\frac{288\sqrt{3}}{6^2}=-3.46...$ and $\frac{dS}{dx}=\sqrt{3}\times 7-\frac{288\sqrt{3}}{7^2}=1.944...$
	A1ft	Concludes that the gradient function moves from negative to positive hence must be a minimum.
(c)	M1	Substitutes their value of x into the given expression for S $S=\frac{\sqrt{3}\text{'6.604'}^2}{2}+\frac{288\sqrt{3}}{\text{'6.604'}}=...$
	A1	For $S = 113$ rounded correctly

Question number	Scheme	Marks
8 a	(Area) = $2xy+\frac{1}{2}x^2\left(\frac{1}{2}\right)\left(=2xy+\frac{1}{4}x^2=50\right)$	M1
	So $y=\frac{50-\frac{1}{4}x^2}{2x}\left(=\frac{25}{x}-\frac{1}{8}x\right)$	dM1
	$P=2x+4y+\frac{1}{2}x=\frac{5}{2}x+4y$	M1
	$=\frac{5}{2}x+4\left(\frac{25}{x}-\frac{1}{8}x\right)\Rightarrow P=2x+\frac{100}{x}$ *	M1 A1 cso (5)
b	$\left(\frac{dP}{dx}=\right)2-\frac{100}{x^2}$	M1 A1
	$\left(\frac{dP}{dx}=0\Rightarrow\right)2-\frac{100}{x^2}=0\Rightarrow x^2=50\Rightarrow x=5\sqrt{2}$ oe	M1 A1
	$\frac{d^2P}{dx^2}=\frac{200}{x^3}$ When $x=5\sqrt{2}$ $\frac{d^2P}{dx^2}=\frac{2}{5}\sqrt{2}(=0.565.....)>0$ therefore minimum	M1 A1 (6)
c	$P=2\left(5\sqrt{2}\right)+\frac{100}{5\sqrt{2}}=20\sqrt{2}$	M1 A1 (2)
		Total 13 marks

Part	Mark	Additional Guidance
(a)	M1	For writing an expression (may not be simplified) for the area in terms of x and y. Allow any expression of the form $xy + xy + ax^2$ oe where $a > 0$ This mark can be implied by an equation for A with the required $xy + xy + ax^2$ oe where $a > 0$
	dM1	For placing their expression for area = 50 and correctly rearranging to make y the subject. The expression for y does not need to be simplified.
	M1	For correctly writing an equation for perimeter in terms of x and y (may not be simplified). There must be a $P =$, though this may be seen later in their work. As long as $P =$ appears by the end of the work, this mark can be awarded. Perimeter = instead of $P =$ is acceptable for this mark
	M1	For correct substitution of their expression for y into their expression for **P or the Perimeter to reach an expression for P which must be in terms of x only. Simplification not required.**
	A1*cso	Obtains the given result with no errors. Perimeter = instead of $P =$ is **NOT** acceptable for this mark
For this question, if we are confident any second solution provided is a re-start and the work the candidate wishes us to consider, this is the work can be marked.		
(b)	M1	For an attempt to differentiate the given expression for P wrt x It is not necessary to see $\frac{dP}{dx} =$ Award this mark for any expression of the form $2 \pm \frac{b}{x^2}$, $b \neq 0$ oe
	A1	For $2 - \frac{100}{x^2}$ oe
	M1	For placing their expression for $\frac{dP}{dx} = 0$ which must have a term in x^2, a correct rearrangement to solve the equation formed and an attempt to find a value for x
	A1	For $x = 5\sqrt{2}$ oe ignore $x = -5\sqrt{2}$
	M1	For $\frac{d^2P}{dx^2} = \pm\frac{c}{x^3}$ $c \neq 0$
	A1	For substituting in a correct value for x, evaluating correctly, stating > 0 and drawing a conclusion. This can be as simple as # or shown. Students may also argue that the second derivative has to be positive as x is positive. This must be a convincing argument that since x is positive the derivative is positive and then a valid minimal conclusion. Do not award this mark if a negative value of x is used and it is not discounted at some point.
	ALT M1	Final 2 marks For substitution of valid values for x either side of their value of x found for the minimum value into $\frac{dP}{dx} = 2 \pm \frac{b}{x^2}$, $b \neq 0$
	A1	For fully correct substitution of their values for x into a correct expression for a first derivative, correctly evaluating or arguing the sign of the derivative and a conclusion drawn. Do not award this mark if a negative value of x is used and it is not discounted at some point.
(c)	M1	For substitution of their value for $x = 5\sqrt{2}$ oe into the given expression for P
	A1	For $20\sqrt{2}$

Question	Scheme	Marks
7(a)	$(a=)\dfrac{4^2}{4}-3\sqrt{4}+8=6$	B1 cso [1]
(b)	$\left(\dfrac{dy}{dx}=\right)\dfrac{2x}{4}-3\times\dfrac{1}{2}\times x^{-\frac{1}{2}}$ oe	M1
	$\left(x=4\Rightarrow\dfrac{dy}{dx}=\right)\dfrac{2\times4}{4}-3\times\dfrac{1}{2}\times4^{-\frac{1}{2}}=\dfrac{5}{4}\Rightarrow$	dM1A1
	$M_n=-\dfrac{1}{"\frac{5}{4}"}$	M1 (now matches ePen)
	$y-6="-\dfrac{4}{5}"(x-4)\Rightarrow 5y+4x-46=0$ *	M1A1 cso [6]
(c)	Area under C	
	$\left(A_C=\int_1^4\left(\dfrac{x^2}{4}-3\sqrt{x}+8\right)\,dx\right)$	
	$=\left[\dfrac{x^3}{3\times4}-\dfrac{3\times x^{\frac{3}{2}}}{\frac{3}{2}}+8x\right]_1^4$	M1
	$=\left[\dfrac{4^3}{3\times4}-\dfrac{3\times4^{\frac{3}{2}}}{\frac{3}{2}}+8\times4\right]-\left[\dfrac{1^3}{3\times4}-\dfrac{3\times1^{\frac{3}{2}}}{\frac{3}{2}}+8\times1\right]$	M1
	$=\dfrac{61}{4}$	A1
	Area under the line	
	$5\times0+4x-46=0\Rightarrow x(=11.5)$ $\quad A=\dfrac{1}{2}\times("11.5"-4)\times6=\dfrac{45}{2}$	
	ALT $\int_4^{11.5}\left(-\dfrac{4}{5}x+\dfrac{46}{5}\right)dx=\left[-\dfrac{4}{5\times2}x^2+\dfrac{46}{5}x\right]_4^{11.5}$	M1A1
	$=\left(-\dfrac{4}{5\times2}\times11.5^2+\dfrac{46}{5}\times11.5\right)-\left(-\dfrac{4}{5\times2}\times4^2+\dfrac{46}{5}\times4\right)$	
	Required area $=\left(\dfrac{61}{4}+\dfrac{45}{2}=\right)\dfrac{151}{4}(=37.75)$ oe	A1 [6]

ALT	Line - curve $\int_1^4\left(\left(-\frac{4}{5}x+\frac{46}{5}\right)-\left(\frac{x^2}{4}-3\sqrt{x}+8\right)\right)\mathrm{d}x$ or $\int_1^4\left(-\frac{x^2}{4}+3\sqrt{x}-\frac{4}{5}x+\frac{6}{5}\right)\mathrm{d}x$ $\left[-\frac{4}{5\times2}x^2+\frac{46}{5}x-\frac{x^3}{4\times3}+\frac{3\times x^{\frac{3}{2}}}{\frac{3}{2}}-8x\right]_1^4$ or $\left[-\frac{x^3}{4\times3}+\frac{3\times x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{4}{5\times2}x^2+\frac{6}{5}x\right]_1^4$ $\left(-\frac{4}{5\times2}\times4^2+\frac{46}{5}\times4-\frac{4^3}{4\times3}+\frac{3\times4^{\frac{3}{2}}}{\frac{3}{2}}-8\times4\right)-$ $\left(-\frac{4}{5\times2}\times1^2+\frac{46}{5}\times1-\frac{1^3}{4\times3}+\frac{3\times1^{\frac{3}{2}}}{\frac{3}{2}}-8\times1\right)$ or $\left(-\frac{4^3}{4\times3}+\frac{3\times4^{\frac{3}{2}}}{\frac{3}{2}}-\frac{4}{5\times2}\times4^2+\frac{6}{5}\times4\right)-\left(-\frac{1^3}{4\times3}+\frac{3\times1^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{5\times2}\times4^2+\frac{6}{5}\times1\right)$ $=\frac{127}{20}$ Area under the line $5\times0+4x-46=0\Rightarrow x\left(=11.5\right)$ $5\times y+4\times1-46=0\Rightarrow x\left(=\frac{42}{5}\right)$ $A=\frac{1}{2}\times\left("11.5"-1\right)\times"\frac{42}{5}"=\frac{45}{2}$ $A=\frac{1}{2}\times\left("11.5"-1\right)\times"\frac{42}{5}"=\frac{441}{10}$ **ALT** $\int_1^{"11.5"}\left(-\frac{4}{5}x+\frac{46}{5}\right)\mathrm{d}x=\left[-\frac{4}{5\times2}x^2+\frac{46}{5}x\right]_1^{"11.5"}$ $=\left(-\frac{4}{5\times2}\times"11.5"^2+\frac{46}{5}\times"11.5"\right)-\left(-\frac{4}{5\times2}\times1^2+\frac{46}{5}\times1\right)=\frac{441}{10}$ $\left(\frac{441}{10}-\frac{127}{20}=\right)\frac{151}{4}$	M1 M1 A1 M1 A1 A1
Useful diagram 	
	Total 13 marks

Part	Mark	Notes
(a)	B1	Correct substitution, no errors and shows $a = 6$
(b)	M1	For an attempt to differentiate the given function. Simplification not required. See General Guidance for the definition of an attempt. No power of x to increase.
	dM1	For substituting the value of $x = 4$ into their derivative. Can be implied by sight of $\frac{5}{4}$ Dependent upon the previous method mark.
	A1	For the correct gradient of $\frac{5}{4}$
	M1	For finding the negative reciprocal of their gradient.
	M1	For forming the equation of the normal, using the given value for a and their **changed** gradient (ie not the gradient of the tangent). This is not a dependent mark, but the candidate may not use the gradient of the tangent and the gradient used must come from some differentiation work. If the candidate uses $y = mx + c$, a fully correct rearrangement to find c must be shown (this can be implied by a correct c for their equation) and concluded with the equation of the line written.
	A1 cso	For the correct equation of the line, minimum steps shown, no errors or omissions.
(c)	M1	For an attempt to integrate the given function. See General Guidance for the definition of an attempt. In this question at least one term correctly integrated and no power of t to decrease. Terms do not need to be simplified. Limits don't need to be present or correct.
	M1	For substituting the correct coordinates into their **changed** expression, the correct way around and subtracting. At least one correct substitution in one term of each limit fully shown. Their limits must be correct.
	A1	For the correct area under the curve. A correct answer here will imply M1 M1 A1 if the integration step has been shown and first M1 awarded. Solutions where the integration step has not been shown will score M0 M0 A0.
	M1	For a correct method to find the area of the triangle using their value for the intersection of line $L$with the x-axis . or Fully correct integration for line and substitution of limits of their 11.5 (following any attempt to find where L crosses the x-axis) and 4. May be implied by correct area. Terms do not need to be simplified.
	A1	For the correct area of the triangle
	A1	For the correct area of $\frac{151}{4}$

ALT	M1	For an attempt to integrate either of the functions as given – there **must** be a subtraction sign if listed separately. Also award this mark for integrating $\int_1^4\left(-\frac{x^2}{4}+3\sqrt{x}-\frac{4}{5}x+E\right)\mathrm{d}x \quad E\neq 0$ (Combines the constant terms incorrectly) See General Guidance for the definition of an attempt. In this question at least one term correctly integrated and no power of t to decrease. Terms do not need to be simplified. Limits don't need to be present or correct.
	M1	For substituting the correct coordinates into their **changed** expression, the correct way around and subtracting. At least one correct substitution in one term of each limit fully shown. Their limits must be correct.
	A1	For $\frac{127}{20}$
	M1	For a correct method to find the area of the large triangle using their value for the intersection of line $L$with the x-axis . or Fully correct integration for line and substitution of limits of their 11.5 (following any attempt to find where L crosses the x-axis) and 4. May be implied by correct area. Terms do not need to be simplified.
	A1	For $\frac{441}{10}$
	A1	For the correct area of $\frac{151}{4}$

Where students have combined expression incorrectly as follows:

$\int_1^4\left(\left(\frac{x^2}{4}-3\sqrt{x}+8\right)\pm\left(-\frac{4}{5}x+\frac{46}{5}\right)\right)\mathrm{d}x$ If the constant term is unsimplified, the highest mark will be up to M1 M1 A1 M0 A0 A0 if the work is seen for the first 3 marks as described above.

If they have simplified the constant term to give eg

$\pm\left(\frac{x^2}{4}-3\sqrt{x}+\frac{86}{5}\right)\pm\frac{4}{5}x$ or $\pm\left(\frac{x^2}{4}-3\sqrt{x}+\frac{86}{5}\right)\pm\frac{86}{5}x$

The highest mark will be M1 M0 A0 M0 A0 A0 if the first two terms in the curve and any constant term are integrated as per conditions above.

Question number	Scheme	Marks
9(a)	$f'(x)=\lambda(x-2)(3x+1)$	M1
	$f(x)=\int\lambda(x-2)(3x+1)\,dx=\left[\int\lambda\left(3x^2-5x-2\right)\right]dx=\lambda\left(\frac{3x^3}{3}-\frac{5x^2}{2}-2x+k\right)$	M1A1
	At $(2,-9)$	
	$-9=2(2^3)-5(2^2)-4(2)+c\Rightarrow c=3$	M1
	$a=-5,b=-4,c=3\Rightarrow\left\{f(x)=2x^3-5x^2-4x+3\right\}$ *	A1*cso
	ALT	[5]
	$f(x)=2x^3+ax^2+bx+c$	
	$f'(x)=6x^2+2ax+b$	[M1
	Both $\begin{cases}0=6(2^2)+2(2)a+b\\0=6\times\left(-\frac{1}{3}\right)^2+2\times\left(-\frac{1}{3}\right)\times a+b\end{cases}$	M1A1
	$\begin{cases}72+12a+3b=0 \text{ and } 12-12a+18b=0\Rightarrow a=...,b=...\\\text{At } (2,9)\quad -9=2(2^3)-5(2^2)-4(2)+c\Rightarrow c=...\end{cases}$	M1
	$a=-5,b=-4,c=3\Rightarrow\left\{f(x)=2x^3-5x^2-4x+3\right\}$ *	A1*cso]
(b)(i)	$f(-1)=2(-1)^3-5(-1)^2-4(-1)+3=0$ oe	B1 [1]
(b)(ii)	$x+1\overline{)2x^3-5x^2-4x+3}$ $\quad 2x^2-7x+3$	M1
	$2x^2-7x+3=(2x-1)(x-3)\Rightarrow(x+1)(x-3)(2x-1)$	M1A1 [3]
(c)	$A=\int_{-1}^{\frac{1}{2}}(2x^3-5x^2-4x+3)\,dx+\left\lvert\int_{\frac{1}{2}}^{3}(2x^3-5x^2-4x+3)\,dx\right\rvert$	M1
	$A=\left[\frac{1}{2}x^4-\frac{5}{3}x^3-2x^2+3x\right]_{-1}^{\frac{1}{2}}+\left\lvert\left[\frac{1}{2}x^4-\frac{5}{3}x^3-2x^2+3x\right]_{\frac{1}{2}}^{3}\right\rvert$	M1
	$A=\left\{\left[\frac{1}{2}.\left(\frac{1}{2}\right)^4-\frac{5}{3}.\left(\frac{1}{2}\right)^3-2.\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)\right]-\left[\frac{1}{2}.(-1)^4-\frac{5}{3}.(-1)^3-2.(-1)^2+3.(-1)\right]\right\}$	M1
	$+\left\lvert\left\{\left[\frac{1}{2}.(3)^4-\frac{5}{3}.(3)^3-2.(3)^2+3(3)\right]-\left[\frac{1}{2}.\left(\frac{1}{2}\right)^4-\frac{5}{3}.\left(\frac{1}{2}\right)^3-2.\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)\right]\right\}\right\rvert$	
	$A=\left\{\frac{117}{32}+\left\lvert-\frac{1375}{96}\right\rvert\right\}=\frac{863}{48}$	A1 [4]
		Total 13 marks

Part	Mark	Additional Guidance
(a)	M1	For forming an expression for $f'(x)$ using the two turning points given. Students may be working with $\lambda = 1$ at this stage and recognise the need to multiply through later.
	M1	Attempts to integrate, see General Guidance for the definition of an attempt Students may be working with $\lambda = 1$ at this stage and recognise the need to multiply through later.
	A1	Correct integration including +c, Students may be working with $\lambda = 1$ at this stage and recognise the need to multiply through later.
	M1	$\lambda = 2$ is stated or implied, $\lambda k = c$ Substitution of $(2,-9)$ to their $f(x)$ (Does not have to be simplified) or substitution of $\left(-\frac{1}{3},\frac{100}{27}\right)$ (Does not have to be simplified)
	A1*cso	Obtains the **given** answer with no errors in the working
	ALT - starts with $f(x) = 2x^3 + ax^2 + bx + c$	
	M1	Attempts to differentiate, see General Guidance for the definition of an attempt
	M1	Equates differentiated expression to 0 and substitutes $x = 2$ **and** equates differentiated expression to 0 and substitutes $x = -\frac{1}{3}$
	A1	Two correct equations, simplified or unsimplified
	M1	Attempts to solve two **correct** equations simultaneously (including using calculators) to find a, b **and** substitution of their $a, b, (2,-9)$ to $f(x)$ to find c or substitution of their $a, b, \left(-\frac{1}{3},\frac{100}{27}\right)$ to $f(x)$ to find c
	A1*cso	Obtains the **given** answer with no errors in the working
(b)(i)	B1	Demonstrating that $x+1$ is a factor $f(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3 = 0$ (factor theorem) If shown via polynomial division, the division must be fully correct and obtains 0 as the remainder.
(b)(ii)	M1	Dividing by $(x+1)$ to obtain a three-term quadratic quotient: $2x^2 - 7x + ...$
	M1	Attempt to factorise their 3TQ, see General Guidance. Or attempt to solve their 3TQ using the quadratic formula
	A1	Correct three linear factors written in one line : e.g. $(x+1)(x-3)(2x-1)$
	ALT I – equates coefficients	
	M1	For stating $2x^3 - 5x^2 - 4x + 3 = (x+1)(Ax^2 + Bx + C) \Rightarrow$ $2x^3 - 5x^2 - 4x + 3 = Ax^3 + (A+B)x^2 + (B+C)x + C \Rightarrow$ $A = 2, B = -7, C = ...$
	M1	Attempt to factorise their 3TQ, see General Guidance. Or attempt to solve their 3TQ using the quadratic formula
	A1	Correct three linear factors written in one line: e.g. $(x+1)(x-3)(2x-1)$
	ALT II - inspection	
	M1	For finding the quadratic factor. Minimum required is $(x+1)(2x^2 - 7x + C)$
	M1	Attempt to factorise their 3TQ, see General Guidance. Or attempt to solve their 3TQ using the quadratic formula
	A1	Correct three linear factors written in one line: e.g. $(x+1)(x-3)(2x-1)$

(c)	M1	For identifying the correct limits, uses a correct strategy to find the area above the x-axis and below the x-axis: $\int_{-1}^{\frac{1}{2}}(2x^3-5x^2-4x+3)\,\mathrm{d}x+\left\lvert\int_{\frac{1}{2}}^{3}(2x^3-5x^2-4x+3)\,\mathrm{d}x\right\rvert$ or $\int_{-1}^{\frac{1}{2}}(2x^3-5x^2-4x+3)\,\mathrm{d}x-\int_{\frac{1}{2}}^{3}(2x^3-5x^2-4x+3)\,\mathrm{d}x$ or implied by their working
	M1	For an attempt to integrate $2x^3-5x^2-4x+3$ See General Guidance for the definition of an attempt.
	M1	For clear substitution of **correct** limits into any **one** of their integrated expressions (must be a changed expression). Can be implied by **one** of the **correct exact** areas found from a **correctly** integrated expression
	A1	For correctly obtaining $\frac{863}{48}$ or $17\frac{47}{48}$

Question Number	Scheme	Marks
9(a)	$f(x)=\frac{1}{4}x^4-\frac{1}{3}x^3-2x^2+4x\ (+c)$	M1A1
	$x=-2\ \ y=-\frac{28}{3}\ \Rightarrow c=0$	M1
	$\left(f(x)=\frac{1}{4}x^4-\frac{1}{3}x^3-2x^2+4x\right)\ \therefore C$ passes through O	A1 cso (4)
(b)(i)	$x=2\ \ f'(x)=8-4-8+4=0$	M1
	$\frac{d^2y}{dx^2}=3x^2-2x-4$	M1
	$x=2\quad \frac{d^2y}{dx^2}=12-4-4>0\quad \therefore$ min at $x=2$	A1cso
	$x=1\ \ f'(x)=1-1-4+4=0$	M1
	$x=1\quad \frac{d^2y}{dx^2}=3-2-4<0\quad \therefore$ max at $x=1$	A1 cso
ALT	$f'(x)=(x-2)(x-1)(x+2)\ (=0)$ factorise	M1
	$x=2,1,(-2)$ solve (solutions to be 2,1 (and another))	M1
	OR: $f'(x)\ (=0)$ solved by calculator.	
	All 3 solutions needed (and correct) $=0$ not needed	M2
	$\frac{d^2y}{dx^2}=3x^2-2x-4$ differentiate	M1
	$x=2\quad \frac{d^2y}{dx^2}=12-4-4>0\quad \therefore$ min at $x=2$	A1cso
	$x=1\quad \frac{d^2y}{dx^2}=3-2-4<0\quad \therefore$ max at $x=1$	A1cso
(ii)	$x=1\Rightarrow y=1\frac{11}{12}\qquad x=2\Rightarrow y=1\frac{1}{3}$	B1B1 (7)
(c)	$y'=(x-1)(x-2)(x+2)$	M1
(i)	$x=-2,\ y=-\frac{28}{3}$ or $\left(-2,-\frac{28}{3}\right)$	A1
(ii)	$x=-2\quad \frac{d^2y}{dx^2}=12+4-4>0\quad \therefore$ min point	A1cso (3)
		[14]

Question Number	Scheme	Marks
(a)M1	Attempt to integrate f'(x). The power of at least one x term must increase and none should decrease. c not needed	
A1	Correct integration, c not needed	
M1	Substitute the given coordinates to show $c = 0$. If c is not included (or assumed to be 0), then showing that substitution of $x = -2$ gives $y = -28/3$ is acceptable. Substitutions must be shown.	
A1cso	Correct conclusion from fully correct work. Accept eg $f(0) = 0 \therefore$ shown	
(b)	Ignore labels (i) and (ii) when marking (b)	
(i)M1	Substitute $x = 2$ in the expression for f'(x) to show f'(x) = 0. Substitution must be shown	
M1	Differentiate the expression for f'(x). At least one power must decrease and none increase.	
A1cso	Show second derivative is > 0 at $x = 2$ and give the conclusion. No errors or omissions in the working.	
M1	Substitute $x = 1$ in the expression for f'(x) to show f'(x) = 0. Substitution must be shown	
A1cso	Show second derivative is < 0 at $x = 1$ and give the conclusion. No errors or omissions in the working.	
(ii)B1	For either y coordinate correct (and x coordinate correctly indicated; substitution shown indicates this)	
B1	For the second y coordinate correct	
(c)	(May have been seen in (b))	
M1	Factorise f'(x) completely – any valid method OR use the factor theorem to find $x = -2$	
(i)A1	Extract the x coordinate of the third turning point and obtain the corresponding y coordinate. May quote y coordinate from the question	
(ii)A1cso	Test the sign of the second derivative at this point and make the conclusion. All work in (c) and $\frac{d^2y}{dx^2}$ (from (b)) must be completely correct for this mark to be awarded.	
	Alternative ways to determine the nature of the turning points:	
1.	If the change of sign of f'(x) is used then values of f'(x) either side of 1 and 2 must be calculated to provide evidence.	
2.	The continuity of a cubic function can be used to establish the nature of the turning points. If in doubt send to review.	

Question number	Scheme	Marks
10 (a)	$f(1) = 32(1^3) - 33(1) + 1 = 0$ *	B1 cso (1)
(b)	$(x-1)(32x^2+32x-1)=0$	M1
	A correct method shown to solve a quadratic e.g. $\frac{-32 \pm \sqrt{32^2+4\times32}}{64}$	M1
	$(x = 1)$ or $\frac{-4+3\sqrt{2}}{8}$ or $\frac{-4-3\sqrt{2}}{8}$ Accept decimals correct to 3 sf e.g. = 0.0303… or = –1.03 …	A1 A1 (4)
(c)	$\sqrt{x} = \frac{1}{8x}$	M1
	$p = \frac{1}{4}$	A1 (2)

Notes

(a)

B1 cso For substitution of 1 into $f(x)$ to obtain the given result

(b)

M1 For $(x-1)(32x^2+32x-1)=0$

M1 A correct method shown to solve a quadratic. If an algebraic method is not shown then M0A0A0 is awarded.

A1 For $\frac{-4+3\sqrt{2}}{8}$ Allow 0.0303 or better

A1 For $\frac{-4-3\sqrt{2}}{8}$ Allow -1.03 or better

(c)

M1 For equating the two equations

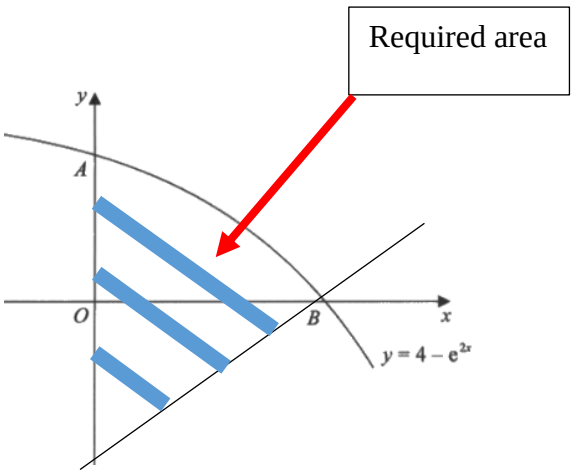
A1 For $p = \frac{1}{4}$

(d)	$\pi\int_{\frac{1}{4}}^{a} x\ \mathrm{d}x = \pi\left[\frac{x^2}{2}\right]_{\frac{1}{4}}^{a}$	M1
	$= \pi\left(\frac{a^2}{2} - \frac{1}{32}\right)$	A1
	$\pi\int_{\frac{1}{4}}^{a} \frac{1}{64x^2}\ \mathrm{d}x = \pi\left[-\frac{1}{64x}\right]_{\frac{1}{4}}^{a}$	M1
	$= \pi\left(\frac{1}{16} - \frac{1}{64a}\right)$	A1
	$= \pi\left(\frac{a^2}{2} - \frac{1}{32}\right) - \pi\left(\frac{1}{16} - \frac{1}{64a}\right) = \frac{27\pi}{64}$	dM1
	$32a^3 - 33a + 1 = 0$	A1
	So $a = 1$	A1 (7)
	Alternative Method	
	$\pi\int_{\frac{1}{4}}^{a}\left(x - \frac{1}{64x^2}\right)\mathrm{d}x = \frac{27\pi}{64}$	M1
	$\int_{\frac{1}{4}}^{a}\left(64x - \frac{1}{x^2}\right)\mathrm{d}x = 27$	A1
	$\left[32x^2 + \frac{1}{x}\right]_{\frac{1}{4}}^{a} = 27$	M1 A1
	$\left(32a^2 + \frac{1}{a}\right) - (2 + 4) = 27$	dM1
	$32a^3 - 33a + 1 = 0$	A1
	So $a = 1$	A1 (7) **[14]**

	Notes
(d)	
M1	For an attempt to integrate $\pi\int_{\frac{1}{4}}^{a} x\,\mathrm{d}x$ Ignore limits
A1	For $\pi\left(\frac{a^2}{2}-\frac{1}{32}\right)$
M1	For an attempt to integrate $\pi\int_{\frac{1}{4}}^{a}\frac{1}{64x^2}\,\mathrm{d}x$ Ignore limits
A1	For $=\pi\left(\frac{1}{16}-\frac{1}{64a}\right)$
M1	Dependant on at least one previous M mark being awarded - for subtraction of the two integrals.
A1	For $32a^3-33a+1=0$ oe
A1	For $a = 1$
	Alternative
M1	For $\pi\int_{\frac{1}{4}}^{a}\left(x-\frac{1}{64x^2}\right)\mathrm{d}x=\frac{27\pi}{64}$ Ignore limits
A1	For $\int_{\frac{1}{4}}^{a}\left(64x-\frac{1}{x^2}\right)\mathrm{d}x=27$ Ignore limits
M1	For an attempt to integrate $\int_{\frac{1}{4}}^{a}\left(64x-\frac{1}{x^2}\right)\mathrm{d}x$ Ignore limits
A1	$\left[32x^2+\frac{1}{x}\right]_{\frac{1}{4}}^{a}=27$
M1	Dependant on at least one previous M mark being awarded - for correct substitution of the limits
A1	For $32a^3-33a+1=0$ oe
A1	For $a = 1$

Question number	Scheme	Marks
11 a	When $x=0 \quad y=3$	B1
	When $y=0 \quad 0=4-e^{2x} \Rightarrow \ln 4=2x \Rightarrow x=\ln 2$	M1 A1 (3)
b	$\frac{dy}{dx}=-2e^{2x}$	M1
	When $x=\ln 2 \quad \frac{dy}{dx}=-8$	M1
	$y-0=\frac{1}{8}(x-\ln 2) \Rightarrow y=\frac{1}{8}x-\frac{1}{8}\ln 2$	M1 A1 (4)
c	$\int_0^{\ln 2}\left(4-e^{2x}\right)dx=\left[4x-\frac{1}{2}e^{2x}\right]_0^{\ln 2}=\left(4\ln 2-\frac{1}{2}e^{2\ln 2}\right)-\left(0-\frac{1}{2}e^{0}\right)$	M1M1M1
	$\frac{1}{2}(\ln 2)\left(\frac{1}{8}\ln 2\right)=\frac{1}{16}(\ln 2)^2$	M1M1
	$\frac{1}{8}\int_0^{\ln 2}\lvert x-\ln 2\rvert\,dx=\frac{1}{8}\left\lvert\frac{x^2}{2}-x\ln 2\right\rvert_0^{\ln 2}=\left\lvert\frac{(\ln 2)^2}{16}-\frac{(\ln 2)^2}{8}\right\rvert=\frac{(\ln 2)^2}{16}$	[M1M1]
	$4\ln 2-\frac{3}{2}+\frac{1}{16}(\ln 2)^2=1.3$	M1 A1 (7)
		Total 14 marks

USEFUL SKETCH



Part	Mark	Notes

(a)(i) (ii)	B1	For $y = 3$
	M1	For $\ln 4 = 2x$ or $\sqrt{4} = \sqrt{e^{2x}}$ seen explicitly
	A1	For $x = \ln 2$
(b)	M1	For differentiating the given expression. This must be correct for this mark. $\frac{dy}{dx} = -2e^{2x}$
	M1	For substituting ln2 into their $\frac{dy}{dx} = -2e^{2\ln 2} = [-8]$
	M1	For a correct method for finding the equation of a straight line using their numerical perpendicular gradient and $y = 0$ and $x = \ln 2$ Award when they substitute into a correct formula, or if they use $y = mx + c$ award when c is obtained. (accept a decimal value for c for this mark awrt $c = -0.087$)
	A1	For $y = \frac{1}{8}x - \frac{1}{8}\ln 2$ or $y = \frac{x}{8} - \frac{\ln 2}{8}$ in exact form only.
(c)	**Area under curve**	
	M1	For a correct statement for the area under the curve with correct limits. Accept the limits either way around. Ignore poor notation. This mark can be implied by later correct work. $\int_0^{\ln 2}\left(4 - e^{2x}\right)dx$
	M1	For a minimally acceptable attempt to integrate as follows. $\int_0^{\ln 2}\left(4 - e^{2x}\right)dx = \left[4x \pm \frac{e^{2x}}{2}\right]$
	M1	For substitution of both limits into their integral of the curve. $A = \left(4\ln 2 - \frac{1}{2}e^{2\ln 2}\right) - \left(0 - \frac{1}{2}e^{0}\right) = \left[4\ln 2 - \frac{3}{2}\right] = [1.27258....]$
	Area of the triangle	
	M1	**Method 1** For a statement of the area. $A = \frac{1}{2}(\ln 2)\left('\frac{1}{8}\ln 2'\right)$ **Method 2** For a statement of the area, with limits either way around. $A = \frac{1}{8}\int_0^{\ln 2}(x - \ln 2)\,dx$ ft their equation of the line.
	M1	For a correct method to evaluate the area of the triangle. **Method 1** $A = \frac{1}{2}(\ln 2)\left('\frac{1}{8}\ln 2'\right) = \frac{1}{16}(\ln 2)^2 = [0.03002.....]$ **Method 2** The integration and substitution must be correct for this mark $A = \frac{1}{8}\int_0^{\ln 2}\lvert x - \ln 2\rvert\,dx = \frac{1}{8}\left\lvert\frac{x^2}{2} - x\ln 2\right\rvert_0^{\ln 2} = \left\lvert\frac{(\ln 2)^2}{16} - \frac{(\ln 2)^2}{8}\right\rvert = \frac{(\ln 2)^2}{16}$

	Combined area	
	M1	For the total final area using their values $4\ln 2-\frac{3}{2}+\frac{1}{16}(\ln 2)^2$ **This is an A mark in Epen.**
	A1	For awrt 1.3 Also accept the exact Area = $4\ln 2-\frac{3}{2}+\frac{1}{16}(\ln 2)^2$

8(a)

$5e^{-2x}+4=e^{2x}$ $\quad 5e^{-2x}+4-e^{2x}=0$ OR $y=\frac{5}{y}+4\Rightarrow y^2-4y-5=0$ M1

$\left(5e^{-x}-e^{x}\right)\left(e^{-x}+e^{x}\right)=0$ $\quad \left(y-5\right)\left(y+1\right)=0$ M1

$5e^{-x}=e^{x}$ $\quad e^{2x}=5$ $\quad x=\frac{1}{2}\ln 5$ $\left(\text{oe eg } \ln\sqrt{5}\right)$ $\quad y=5$ A1

$\left(e^{-x}=-e^{x} \text{ not possible}\right)$ $\quad e^{2x}=5$ $\quad x=\frac{1}{2}\ln 5$

A is $\left(\frac{1}{2}\ln 5, 5\right)$ A1 (4)

(b)

$y=5e^{-2x}+4\Rightarrow \frac{dy}{dx}=-10e^{-2x}$ M1

At A $\quad \frac{dy}{dx}=-10e^{-2x}=-10\times\frac{1}{5}=-2$ A1ft

Eqn tgt: $\quad y-5=-2\left(x-\frac{1}{2}\ln 5\right)$ dM1A1

$y=0\Rightarrow x=\frac{1}{2}\left(5+\ln 5\right)$ (= x coordinate of B)* A1cso (5)

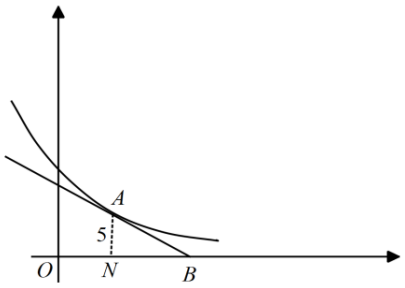
ALT

For last 3 marks:

Hence $\frac{5}{NB}=2\Rightarrow NB=\frac{5}{2}$ dM1A1

$ON=\frac{1}{2}\ln 5$

$OB=\frac{1}{2}\ln 5+\frac{5}{2}=\frac{1}{2}\left(5+\ln 5\right)$ * A1cso



(c)

$C_2: \frac{dy}{dx}=2e^{2x}\Rightarrow$ grad tgt at A is $2\times 5=10$ B1ft

Eqn tgt: $\quad y-5=10\left(x-\frac{1}{2}\ln 5\right)$ M1

At D: $\quad x=\frac{1}{2}\left(-1+\ln 5\right)$ A1

Area $\Delta ABD=\frac{1}{2}\left(\frac{1}{2}\left(5+\ln 5\right)-\frac{1}{2}\left(-1+\ln 5\right)\right)\times 5$ M1A1

$=\frac{15}{2}$ or $7\frac{1}{2}$ $\left(\text{units}^2\right)$ A1 (6)

See notes for area by "determinant" method

ALT	For second and third marks: $\frac{5}{ND}=10\Rightarrow ND=\frac{1}{2}$ $OD=\frac{1}{2}\ln 5-\frac{1}{2}$	 M1 A1 [15]

(a)	
M1	Equate the 2 curve equations. No need to simplify
M1	Factorise their equation
A1	Obtain the one possible value for x (other root need not be seen; if seen it must be rejected) **Must be exact**
A1	Obtain the corresponding value for y. **Must be exact.** Need not be shown in coordinate brackets. Use of $e^{2x} = 5$ leads to $y = 5$ without use of a value of x, so M1M1A0A1 can be scored. There must only be one correct y shown. Accept $y=e^{\ln 5}$
(b)	
M1	Differentiate the equation of C_1 $5e^{-2x}\rightarrow ke^{-2x}$ where $k=\pm 5$ or ± 10 and no integration seen
A1ft	Grad at A = -2 follow through their x coordinate
dM1	Obtain the equation of the tangent at A using their gradient and their coordinates of A. Can be in any form but if $y=mx+c$ is used a value for c must be found. Gradient of the tangent must be numerical.
A1	Correct equation in any form
A1cso	Correct x coordinate of B obtained from correct working.
ALT	For last 3 marks
dM1	Use their y coordinate of A and their (numerical) gradient of the tangent to find the length NB (where N is the foot of the perpendicular from A to the x-axis)
A1	Correct length of NB
A1cso	Add the x coordinate of A to obtain the x coordinate of B
(c)	
B1ft	Correct gradient of tangent to C_2 at A follow through their x coordinate
M1	Obtain an equation for the tangent using their gradient and their coordinates of A Gradient of the tangent must be numerical.
A1	Correct x coordinate of D (exact or minimum 3 sf)
M1	Use a correct formula for the area of a triangle with their y coordinate of A, their x coordinate of D and the **given** x coordinate of B
A1	Correct, unsimplified area Allow use of correct but non-exact coordinates
A1	Correct area Accept only $7\frac{1}{2}, \frac{15}{2}$ or 7.5
	Heron's formula: Nos which may be seen: $AB=\frac{5\sqrt{5}}{2}, AD=\frac{\sqrt{101}}{2}, BD=3, s=\frac{1}{2}(a+b+c)=6.8$
ALT	For second and third marks:
M1	Use their y coordinate of A and their gradient of the tangent to find the length ND
A1	Use the x coordinate of A to obtain the x coordinate of D

ALT	Area by "determinant" method:
M1	Eg Area $= \frac{1}{2}\begin{vmatrix} \frac{1}{2}\ln 5 & \frac{1}{2}(5+\ln 5) & \frac{1}{2}(\ln 5-1) & \frac{1}{2}\ln 5 \\ 5 & 0 & 0 & 5 \end{vmatrix}$ *y* coordinates of *B* and *D* must be 0 Must include the ½ and have 4 sets of coordinates with first and last the same.
A1	$= \frac{1}{2}\left(\frac{1}{2}(\ln 5-1)\times 5 - \frac{1}{2}(\ln 5+1)\right)$ Allow use of correct but non-exact coordinates
A1	Correct area Accept only $7\frac{1}{2}$, $\frac{15}{2}$ or 7.5 **Must be positive**

Question number	Scheme	Marks
6 (a)	$x^2 - 5x + 4 = (x-4)(x-1)$	M1
	$A(1, 0)$ $B(4, 0)$	A1 A1
		(3)
	Notes	
(a)		
M1	For solving the quadratic	
A1	For $A(1, 0)$	
A1	For $B(4, 0)$	
(b)	$\frac{dy}{dx} = 2x - 5$	M1
	When $x = 1$ $\frac{dy}{dx} = -3$	A1
	Tangents meet on the axis of symmetry of curve C so $x = \frac{1+4}{2} = \frac{5}{2}$	M1 A1
	When $x = \frac{5}{2}$ $y = -3\left(\frac{5}{2} - 1\right) = -\frac{9}{2}$	M1
	$\left(\frac{5}{2}, -\frac{9}{2}\right)$	A1
		(6)
	Notes	
(b)		
M1	For $\frac{dy}{dx} = 2x - 5$ (Allow if seen in part(c))	
A1	$\frac{dy}{dx} = -3$ when $x = 1$	
M1	For use of $\frac{x_1 + x_2}{2}$ or $\frac{-b}{2a}$ or $\frac{dy}{dx} = 0$	
A1	$x = \frac{5}{2}$	
M1	For substitution of $x = \frac{5}{2}$ into $y - y_1 = m(x - x_1)$ oe	
A1	For $\left(\frac{5}{2}, -\frac{9}{2}\right)$	

	Alternative Method $\frac{dy}{dx} = 2x - 5$ When $x = 1$ $\frac{dy}{dx} = -3$ So $y = -3x + 3$ When $x = 4$ $\frac{dy}{dx} = 3$ So $y = 3x - 12$ Meet when $-3x + 3 = 3x - 12$ $\left(\frac{5}{2}, -\frac{9}{2}\right)$	M1 A1 M1 A1 M1 A1 (6)
	Notes	
	Alternative	
M1	For an attempt to find the equation of the tangent of C at A	
A1	For $y = -3x + 3$	
M1	For an attempt to find the equation of the tangent of C at B	
A1	For $y = 3x - 12$	
M1	For equating the two equations	
A1	For $\left(\frac{5}{2}, -\frac{9}{2}\right)$	

(c)	Gradient of normal at (0, 1) = $\frac{1}{3}$	M1
	When $x=\frac{5}{2}$ $y=\frac{1}{3}\left(\frac{5}{2}-1\right)=\frac{1}{2}$	M1
	$\left(\frac{5}{2},\frac{1}{2}\right)$	A1 (3)
	Alternative Method $y=\frac{1}{3}x-\frac{1}{3}$ and $y=-\frac{1}{3}x+\frac{4}{3}$	M1
	Meet when $\frac{1}{3}x-\frac{1}{3}=-\frac{1}{3}x+\frac{4}{3}$	M1
	$\left(\frac{5}{2},\frac{1}{2}\right)$	A1 (3)
	Notes	
(c)		
M1	For Gradient of normal at (0, 1) = $\frac{1}{3}$ ft the gradient found in part (b)	
M1	For substitution of $x=\frac{5}{2}$ into $y-y_1=m\left(x-x_1\right)$ oe	
A1	For $\left(\frac{5}{2},\frac{1}{2}\right)$	
	Alternative	
M1	For $y=\frac{1}{3}x-\frac{1}{3}$ and $y=-\frac{1}{3}x+\frac{4}{3}$	
M1	For equating the two equations	
A1	For $\left(\frac{5}{2},\frac{1}{2}\right)$	

(d)	Area = $\frac{1}{2}(4-1)\times\left(\frac{1}{2}+\frac{9}{2}\right)=\frac{15}{2}$	M1 M1 A1 (3)
	Alternative Method 1 Area = $\frac{1}{2}\times 3\times\frac{1}{2}+\frac{1}{2}\times 3\times\frac{9}{2}=\frac{15}{2}$	M1 M1 A1 (3)
	Alternative Method 2 $AN=\sqrt{1.5^2+0.5^2}=\frac{\sqrt{10}}{2}$ $AT=\sqrt{1.5^2+4.5^2}=\frac{3\sqrt{10}}{2}$	M1
	Area = $2\times\frac{1}{2}\times\frac{\sqrt{10}}{2}\times\frac{3\sqrt{10}}{2}=\frac{30}{4}=\frac{15}{2}$	M1 A1 (3)
	Alternative Method 3 Area = $\frac{1}{2}\begin{vmatrix}1 & \frac{5}{2} & 4 & \frac{5}{2} & 1\\ 0 & -\frac{9}{2} & 0 & \frac{1}{2} & 0\end{vmatrix}\Rightarrow\frac{1}{2}\left(-\frac{9}{2}+2+18-\frac{1}{2}\right)=\frac{15}{2}$	M1 M1 A1 (3)
		[15]

	Notes
(d)	
M1	For $\frac{1}{2}\times AB\times NT$
M1	For $\frac{1}{2}(4-1)\times\left(\frac{1}{2}+\frac{9}{2}\right)$
A1	For $\frac{15}{2}$
	Alternative 1
M1	For area of triangle *ANB* + area of triangle *ATB*
M1	For $\frac{1}{2}\times 3\times\frac{1}{2}+\frac{1}{2}\times 3\times\frac{9}{2}$
A1	$\frac{15}{2}$
	Alternative 2
M1	For finding *AN* and *AT*
M1	For $2\times\frac{1}{2}\times AN\times AT$
A1	$\frac{15}{2}$

	Alternative 3
M1	Use of area = $\frac{1}{2}\begin{vmatrix}1 & a & 4 & c & 1\\ 0 & b & 0 & d & 0\end{vmatrix}$ oe where (a, b) and (c, d) are the coordinates of T and N
M1	$\frac{1}{2}\left(-\frac{9}{2}+2+18-\frac{1}{2}\right)$
A1	$\frac{15}{2}$

Question number	Scheme	Marks
10 (a)	$\frac{1}{2}+\sin 3x=0 \Rightarrow \sin 3x=-\frac{1}{2} \Rightarrow 3x=-\frac{\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6} \Rightarrow x=\frac{7\pi}{18},\frac{11\pi}{18}$	M1
	Coordinates of M are $\left(\frac{7\pi}{18},\ 0\right)$*	A1
	Coordinates of N are $\left(\frac{11\pi}{18},\ 0\right)$	A1 [3]
	ALT	
	$\frac{1}{2}+\sin\left(3\times\frac{7\pi}{18}\right)=0$	[M1
	Coordinates of M are $\left(\frac{7\pi}{18},0\right)$ *	A1
	Coordinates of N are $\left(\frac{11\pi}{18},0\right)$	A1]
(b)	$\frac{dy}{dx}=3\cos 3x=0 \Rightarrow 3x=\frac{\pi}{2} \Rightarrow x=\frac{\pi}{6}$	M1A1
	$y=\frac{1}{2}+\sin 3\left('\frac{\pi}{6}'\right)=1.5$	dM1A1 [4]
	$\left[\text{Coordinates of point } A \text{ are } \left(\frac{\pi}{6},\ 1.5\right)\right]$	
	ALT	
	Max of sine curve is 1 so that $\frac{1}{2}+1=\frac{3}{2} \Rightarrow y=\frac{3}{2}$	[M1A1
	$\sin 3\theta=1 \ \Rightarrow\ 3\theta=\frac{\pi}{2} \Rightarrow\ \theta=\frac{\pi}{6}$	dM1A1]
	$\left[\text{Coordinates of point } A \text{ are } \left(\frac{\pi}{6},\ \frac{3}{2}\right)\right]$	
(c)	Uses the given x coordinate for point M	
	At point M $\quad \frac{dy}{dx}=3\cos\left(3\times\frac{7\pi}{18}\right)=-\frac{3\sqrt{3}}{2}$	B1
	$y-0=-\frac{3\sqrt{3}}{2}\left(x-\frac{7\pi}{18}\right) \Rightarrow 12y+18\sqrt{3}x-7\sqrt{3}\pi=0$ o.e.	M1A1A1 [4]
(d)	$A=\int_0^{\frac{7\pi}{18}}\left(\frac{1}{2}+sin\,3x\right)\,dx+\left\|\left[\int_{\frac{7\pi}{18}}^{\frac{11\pi}{18}}\left(\frac{1}{2}+sin\,3x\right)\,dx\right]\right\|$	M1
	$A=\left[\frac{1}{2}x-\frac{cos\,3x}{3}\right]_0^{\frac{7\pi}{18}}+\left\|\left[\frac{1}{2}x-\frac{cos\,3x}{3}\right]\right\|_{\frac{7\pi}{18}}^{\frac{11\pi}{18}}$	M1
	$A=\left[\left(\frac{1}{2}\times\frac{7\pi}{18}-\frac{cos\,3\left(\frac{7\pi}{18}\right)}{3}\right)-\left(0-\frac{cos\,3\times 0}{3}\right)\right]$	M1

	$+\left\|\left[\left(\frac{1}{2}\times\frac{11\pi}{18}-\frac{cos\,3\left('\frac{11\pi}{18}'\right)}{3}\right)-\left(\frac{1}{2}\times\frac{7\pi}{18}-\frac{cos\,3\left(\frac{7\pi}{18}\right)}{3}\right)\right]\right\|$ $A=[(1.23287)+(0.22828)]=1.46115\ldots\approx 1.46$	A1 [4]
		Total 15 marks

Part	Mark	Notes
(a)	**M1**	Sets the equation equal to 0, solves using inverse sin and obtains a correct angle $\frac{1}{2}+\sin 3x=0 \Rightarrow \sin 3x=-\frac{1}{2} \Rightarrow 3x=-\frac{\pi}{6}$ Condone working in degrees for M mark.
	A1 cso	For correctly obtaining $\left(\frac{7\pi}{18},0\right)$ * with no errors. Award for finding $\frac{7\pi}{18}$ and $\frac{11\pi}{18}$ but not shown as coordinates.
	A1	For correct coordinates of N: $\left(\frac{11\pi}{18},0\right)$
Alternative method		
	M1	For correct substitution of $\frac{7\pi}{18}$ into $\frac{1}{2}+\sin 3x$
	A1 cso	For correctly showing that $\frac{1}{2}+\sin\left(3\times\frac{7\pi}{18}\right)=0$ and stating $\left(\frac{7\pi}{18},0\right)$ * Award for showing $\frac{7\pi}{18}$ and finding $\frac{11\pi}{18}$ but not shown as coordinates
	A1	For correct coordinates of N: $\left(\frac{11\pi}{18},0\right)$
(b)	**M1**	For attempt to differentiate $y=\frac{1}{2}+\sin 3x$, set equal to 0 and solve for x. $\frac{dy}{dx}=3\cos 3x=0 \Rightarrow 3x=\frac{\pi}{2} \Rightarrow x=\frac{\pi}{6}$ Condone finding x in degrees for this mark. Attempt at differentiation of two terms: $\frac{1}{2}\rightarrow 0$ $\sin(3x)\rightarrow k\cos(3x)$
	A1	For correctly obtaining $x=\frac{\pi}{6}$ **Note: Do not award this mark if $\frac{dy}{dx}$ is incorrect.**
	dM1	Substitutes $x=\frac{\pi}{6}$ to find a value for y. $y=\frac{1}{2}+\sin 3\left('\frac{\pi}{6}'\right)=1.5$ Condone working with x in degrees for this mark.
	A1	For correct coordinates of A. $\left(\frac{\pi}{6},\frac{3}{2}\right)$ Allow values equivalent to $\frac{3}{2}$
Alternative method		
	M1	For stating that the maximum of a sine curve is 1 and adding $\frac{1}{2}$
	A1	For correctly obtaining $y=\frac{3}{2}$
	dM1	For setting $\sin 3x$ equal to 1 and attempt to solve for x $\sin 3x=1 \Rightarrow 3x=\frac{\pi}{2} \Rightarrow x=\frac{\pi}{6}$ Condone working with x in degrees for this mark.

	A1	For correctly obtaining $x = \frac{\pi}{6}$
(c)	**B1**	For correct gradient at point M $\left(\frac{dy}{dx}\right) = -\frac{3\sqrt{3}}{2}$
	M1	For a fully correct method of finding the equation of the tangent to C at M $y - 0 = {}' -\frac{3\sqrt{3}}{2}{}' \left(x - \frac{7\pi}{18}\right)$ Allow use of their $-\frac{3\sqrt{3}}{2}$ obtained from substitution of $x = \frac{7\pi}{18}$ into their $\frac{dy}{dx}$.
	A1	For a correct equation of the tangent in any form: $y - 0 = -\frac{3\sqrt{3}}{2}\left(x - \frac{7\pi}{18}\right)$
	A1	For correct equation of the tangent in the required form: $12y + 18\sqrt{3}x - 7\sqrt{3}\pi = 0$ Accept integer multiples of this equation.
(d)	**M1**	For identifying the correct limits to find the area above the curve and the area below the curve: $\int_0^{\frac{7\pi}{18}}\left(\frac{1}{2} + \sin 3x\right) + \left|\int_{\frac{7\pi}{18}}^{\frac{11\pi}{18}}\left(\frac{1}{2} + \sin 3x\right)\right|$ or $\int_0^{\frac{7\pi}{18}}\left(\frac{1}{2} + \sin 3x\right) - \int_{\frac{7\pi}{18}}^{\frac{11\pi}{18}}\left(\frac{1}{2} + \sin 3x\right)$ Allow use of their $\frac{11\pi}{18}$. Needs to correctly indicate dealing with areas above and below axis.
	M1	For an attempt to integrate $\frac{1}{2} + \sin 3x$ obtaining: $\frac{1}{2}x - k\cos 3x$ where $k \neq -3$ For this mark ignore incorrect / absent limits.
	M1	For substituting limits correctly into their integrated expression (must be a changed expression). $A = \left[\left(\frac{1}{2} \times \frac{7\pi}{18} - \frac{cos\,3\left(\frac{7\pi}{18}\right)}{3}\right) - \left(0 - \frac{cos\,3 \times 0}{3}\right)\right]$ $+ \left|\left[\left(\frac{1}{2} \times \frac{11\pi}{18} - \frac{cos\,3\left(\frac{'11\pi'}{18}\right)}{3}\right) - \left(\frac{1}{2} \times \frac{7\pi}{18} - \frac{cos\,3\left(\frac{7\pi}{18}\right)}{3}\right)\right]\right|$ Allow use of their $\frac{11\pi}{18}$. Must show substitution or M0.
	A1	For correctly obtaining awrt 1.46

Question number	Scheme	Marks
7 a (i)	$f\ x = \int (4x^3 - 12x^2 - 19x + 12)\,dx = \frac{4}{4}x^4 - \frac{12}{3}x^3 - \frac{19}{2}x^2 + 12x + D$ oe For the point $4, -104$ it follows that $-104 = (4)^4 - 4(4)^3 - \frac{19}{2}(4)^2 + 12(4) + D$ * $-104 = 256 - 256 - 152 + 48 + D \Leftrightarrow D = 0$*	M1 A1 M1 A1* cso (4)
a (ii)	$x = 0.5$ $f'(x) = 4(0.5)^3 - 12(0.5)^2 - 19(0.5) + 12 = 0$ $f''(x) = 12x^2 - 24x - 19$ $x = 0.5$ $f''(x) = 12(0.5)^2 - 24(0.5) - 19 (= -28) < 0$ Therefore maximum *	M1 M1 A1 cso (3)
b (i)	$f'(x) = (2x-1)(2x^2 - 5x - 12)$ $f'(x) = (2x-1)(2x+3)(x-4) \rightarrow x =$ $x = -\frac{3}{2}$ or $x = 4$, $\left[x = \frac{1}{2}\right]$ $A\left(-\frac{3}{2}, -\frac{333}{16}\right)$ $B(4, -104)$	M1 M1 A1 A1 A1 (5)
b (ii)	$f''(x) = 12(-1.5)^2 - 24(-1.5) - 19 (= 44) > 0$ Therefore minimum $x = 4$ $f''(x) = 12(4)^2 - 24(4) - 19 (= 77) > 0$ Therefore minimum (Read notes carefully for allocation of these marks) **Alternative** As we have a maximum at $x = 0.5$ and C is a continuous curve $x = -\frac{3}{2}$ is a minimum and $x = 4$ is a minimum	ddM1 A1 A1 (3) {ddM1} {A1}{A1} (3)
		Total 15 marks

Part	Marks	Notes
(a) (i)	**M1**	Attempt to integrate – see general guidance for minimally acceptable attempt – at least one term must be fully correct in this integration (unsimplified).
	A1	Fully correct integration – does not need to include a constant of integration, need not be simplified.
	M1	Substitution of $x = 4$ and $y = -104$ into their expression, which must include a constant of integration.
	A1 cso	Obtains the given result, making it clear the constant of integration is 0 or listing fully the function at the end after fully correct working. There must be no incorrect working for this final mark to be awarded.
Note: candidates may assume $D = 0$ & show, by substituting, (4, -104) lies on the curve **final M1 A1**		
(a) (ii)	**M1**	Substitution of $x = 0.5$ into $f'(x)$ and shows $f'(x) = 0$. A candidate may also solve $f'(x) = 0$ and solve to get $x = 0.5$.
	M1	For finding $f''(x)$ - see general guidance for minimally acceptable attempt at differentiation. At least one term must be fully correct for this differentiation.
	A1 cso*	Substitutes $x = 0.5$ into a fully correct second derivative, shows that the result is negative and draws a conclusion. This can be as simple as "shown" or #. It is not necessary to work out the actual value of $f''(0.5)$ if clear substitution is shown, but if calculated, it must be correct. No incorrect work for this mark to be awarded. Note: as always, this A mark must follow 2 M1 marks.

(b)	Part (i) and (ii) may be marked together	
(i)	**M1**	A clear attempt to divide by $(2x-1)$ or compare coefficients. The student must arrive at a quadratic factor of the form $(2x^2+Ax-12)$ if dividing It is also possible to divide by $\left(x-\frac{1}{2}\right)$ and arrive at a quadratic factor of the form $4x^2+Bx-24$
	M1	Uses a complete method to solve their 3TQ, see general guidance for a minimally acceptable attempt. Must progress to $x =$.
	A1	$x=-\frac{3}{2}$ or $x=4$ Award M1 M1 A1 if **both** correct values are given without working.
	A1	$A\left(-\frac{3}{2},-\frac{333}{16}\right)$ or $B(4,-104)$ Allow x = and y = (clearly paired) for **both** final A marks.
	A1	$A\left(-\frac{3}{2},-\frac{333}{16}\right)$ and $B(4,-104)$ accept 20.8 or better – answer is 20.8125
	Factorising in (a)(ii) and then using in this part can be awarded the marks above.	
(b) (ii)	**ddM1**	Correctly substitutes $x=-\frac{3}{2}$ or $x=4$ into their $f''(x)$. Their values Dependent on both previous M marks in part (b). A fully correct evaluation of the second derivative can imply this mark ie sight of 77 and 44. Part (i) and (ii) may be marked together
	A1	Substitutes $x = -1.5$ **or** $x = 4$ into a fully correct second derivative, shows that the result is positive and draws the conclusion this is a minimum. It is not necessary to work out the actual value of $f''(x)$ if clear substitution is shown, but if calculated, it must be correct.
	A1	Substitutes $x = -1.5$ **and** $x = 4$ into a fully correct second derivative, shows that the result is positive and draws the conclusion that both values of x give a minimum. It is not necessary to work out the actual value of $f''(x)$ if clear substitution is shown, but if calculated, it must be correct.
ALT	**ddM1**	For stating the curve is continuous and there is a maximum at $x = 0.5$ Dep both previous M
	A1	For stating "as we have a maximum at $x = 0.5$" and progressing to state either A or B must be a minimum.
	A1	Fully correct conclusion, stating A and B must both be minimum points.
There **MUST** be an appropriate justification for these marks to be awarded, such as that given in M1		

Question	Scheme	Marks
8(a)	$\int 17+2x-3x^2\ \mathrm{d}x = 17x+\frac{2x^2}{2}-\frac{3x^3}{3}+k$	M1A1
	$0=\left(17\times(-1)+\frac{2\times(-1)^2}{2}-\frac{3\times(-1)^3}{3}\right)+k \Rightarrow k=15$	M1
	$y=15+17x+x^2-x^3$	A1 [4]
(b)	$\frac{(15+17x+x^2-x^3)}{(x+1)}=-x^2+2x+15$	M1A1
	$-x^2+2x+15=(x+3)(5-x)$	M1A1
	$a=-3,\ [-1]$ and $b=5$	A1
	When $x=0$ $y=15$ so $c=15$	B1 [6]
(c)	$\int_0^5(15+17x+x^2-x^3)\ \mathrm{d}x-\frac{1}{2}\times5\times15$	M1
	OR $\int_0^5(15+17x+x^2-x^3)\ \mathrm{d}x-\int_0^5(15-3x)\ \mathrm{d}x$	
	$\int_0^5(15+17x+x^2-x^3)\ \mathrm{d}x=\left[15x+\frac{17x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^5$	M1
	$\left(15\times5+\frac{17\times5^2}{2}+\frac{5^3}{3}-\frac{5^4}{4}\right)-(0)=\left(\frac{2075}{12}\right)$	M1
	Area of triangle Area $=\frac{1}{2}\times5\times15=37.5$	B1
	OR Area $=\int_0^5(15-3x)\ \mathrm{d}x=\left[15x-\frac{3x^2}{2}\right]_0^5=37.5$	[B1]
	For the correct area of $R=\frac{2075}{12}-37\frac{1}{2}=\frac{1625}{12}=135\frac{5}{12}$	A1 [5]
		Total 15 marks

Question	Notes	Marks
8(a)	$f'(x)=17+2x-3x^2$	
	For an attempt to integrate $f'(x)$ $y=17x+\frac{2x^2}{2}-\frac{3x^3}{3}+(k)$	M1
	For the correct integral **including** a constant of integration $y=17x+\frac{2x^2}{2}-\frac{3x^3}{3}+k$	A1
	For substituting $(-1\ ,\ 0)$ into their integrated expression, which must include a constant of integration. $0=\left(17\times(-1)+\frac{2\times(-1)^2}{2}-\frac{3\times(-1)^3}{3}\right)+k\Rightarrow(k=15)$	M1
	For writing the equation in the required form $y=15+17x+x^2-x^3*$ **This is a given equation. Every step above must be seen for the award of full marks.**	A1 cso [4]
(b)	Divides $(15+17x+x^2-x^3)$ by $(x+1)$ $x+1\overline{)15+17x+x^2-x^3}\quad Q+2x+x^2$ **OR** Equates coefficients $(15+17x+x^2-x^3)=(x+1)(Ax^2+Bx+c)\Rightarrow(x+1)(-x^2+2x+Q)$ Where Q is a constant **Minimal working here is sight of the quadratic factor.**	M1
	For obtaining the correct 3TQ $-x^2+2x+15$	A1
	For factorising their 3TQ [or otherwise solving] $-x^2+2x+15=(x+3)(5-x)$	M1A1
	For $a=-3,\ [-1]$ and $b=5$ identified clearly.	A1
	SC No working seen [use of a root finder on a calculator]	
	For $(x+1)(x-5)(x+3)$ seen leading to $a=-3$ and $b=5$ with no working award: M0A0MA1A1	
	For $a=-3$ and $b=5$ seen with no other working seen award: M0A0M0A0A1	
	For the value of $c=15$	B1
(c)	For writing a correct expression for the area of R with the correct limits $\int_0^5(15+17x+x^2-x^3)\,dx-\frac{1}{2}\times5\times15$ OR	M1

	$\int_0^5 \left(15+17x+x^2-x^3\right)\,\mathrm{d}x-\int_0^5 (15-3x)\,\mathrm{d}x$	
	For an attempt to integrate the expression for the curve. [Ignore limits for this mark] Area = $\int_0^5 \left(15+17x+x^2-x^3\right)\,\mathrm{d}x=\left[15x+\frac{17x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^5$	M1
	For evaluating their integral using their limits. Area = $\left(15\times5+\frac{17\times5^2}{2}+\frac{5^3}{3}-\frac{5^4}{4}\right)-(0)=\left(\frac{2075}{12}\right)$	M1
	For the area of the triangle Area $=\frac{1}{2}\times5\times15=37.5$ **ALT** Integrates the line Area = $\int_0^5 (15-3x)\,\mathrm{d}x=\left[15x-\frac{3x^2}{2}\right]_0^5=37.5$	B1 [B1]
	For the correct area of R $\frac{2075}{12}-37\frac{1}{2}=\frac{1625}{12}=135\frac{5}{12}$	A1 [5]
	ALT	
	For writing an expression for the area of R with the correct limits. $\int_0^5 \left(15+17x+x^2-x^3\right)\,\mathrm{d}x-\int_0^5 (15-3x)\,\mathrm{d}x=\left[\int_0^5 \left(20x+x^2-x^3\right)\,\mathrm{d}x\right]$ Award the B mark for a correct expression for the combined area.	M1 B1
	For an attempt to integrate the expression for the combined area or just the curve. [Ignore limits for this mark] Area = $\int_0^5 \left(20x+x^2-x^3\right)\,\mathrm{d}x=\left[\frac{20x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^5$	M1
	For evaluating their integral using their limits, but the lower limit must be 0. Area = $\left(\frac{20\times5^2}{2}+\frac{5^3}{3}-\frac{5^4}{4}\right)=\left(\frac{1625}{12}\right)$	M1
	For the correct final area.	A1 [5]
		Total 15 marks

Question	Scheme	Marks
5(a)	$y = Q(x-6)(x+2)$ where Q is a constant Using the coordinates $(4,-6)$	M1
	$-6 = Q(4-6)(4+2) \Rightarrow Q = \dfrac{-6}{-12} = \dfrac{1}{2}$	M1
	$y = \dfrac{1}{2}(x-6)(x+2) \Rightarrow y = \dfrac{x^2}{2} - 2x - 6$ *	A1 cso [3]
(b)	$\dfrac{dy}{dx} = \dfrac{2x}{2} - 2$	M1
	$x = 4, \quad \dfrac{dy}{dx} = 4 - 2 = 2$	M1
	Gradient of normal is $-\dfrac{1}{2}$	B1ft
	Equation of l : $y-(-6) = -\dfrac{1}{2}(x-4)$	M1
	$\Rightarrow y = -\dfrac{x}{2} - 4 \Rightarrow 2y + x + 8 = 0$ *	A1 cso [5]
(c)	$\dfrac{x^2}{2} - 2x - 6 = -\dfrac{x}{2} - 4 \Rightarrow \dfrac{x^2}{2} - \dfrac{3}{2}x - 2 = 0$	M1
	$\Rightarrow (x-4)(x+1) = 0 \Rightarrow x = -1, 4$	M1 A1
	Area $= \displaystyle\int_{-1}^{4}\left(\frac{x^2}{2} - 2x - 6\right) dx - \int_{-1}^{4}\left(-\frac{x}{2} - 4\right) dx$	M1
	$= \left[\dfrac{x^3}{6} - \dfrac{3}{4}x^2 - 2x\right]_{-1}^{4}$	M1
	$= \left(\dfrac{4^3}{6} - \dfrac{3}{4}\times 4^2 - 2\times 4\right) - \left(\dfrac{(-1)^3}{6} - \dfrac{3}{4}\times(-1)^2 - 2\times(-1)\right)$	M1
	$= \left(\dfrac{4^3}{6} - \dfrac{3}{4}\times 4^2 - 2\times 4\right) - \left(\dfrac{(-1)^3}{6} - \dfrac{3}{4}\times(-1)^2 - 2\times(-1)\right)$	
	$= -\dfrac{125}{12} \Rightarrow$ Area $= \dfrac{125}{12}$ (units^2) oe	A1 [7]
		Total 15 marks

Part	Mark	Notes
(a)	M1	Uses the intersections with the x-axes to form a quadratic equation of the form $y = Q(x \pm 6)(x \pm 2)$

	M1	Uses their Quadratic with the coordinates (4, – 6) to find the value of Q Allow just one processing error here.
	A1 cso	For the correct equation in the required form. Note this equation is given to candidates. Both above steps must be complete and correct for the award of this mark.
	ALT – Uses simultaneous equations	
	M1	Sets up all three equations with the given coordinates. These must be correct. $y = px^2 + qx + r$ $0 = 4p - 2q + r$ **1** $0 = 36p + 6q + r$ **2** $-6 = 16p + 4q + r$ **3**
	M1	Attempts to solve their three simultaneous equations to find the values of p, q and r At least one correct value is evidence of correct method. **2 - 1** $0 = 32p + 8q$ **4** **3 - 2** $6 = 20p + 2q$ **5** **5** $\times 4$ $24 = 80p + 8q$ **6** **6 - 4** $24 = 48p \Rightarrow p = \frac{1}{2}$, $24 = 40 + 8q \Rightarrow q = -2$ $0 = 2 + 4 + r \Rightarrow r = -6$
	A1 cso	For the correct equation in the required form. Note this equation is given to candidates. All of the above steps must be complete and correct for the award of this mark.
(b)	M1	For differentiating the given expression. This must be correct, simplified or unsimplified for this mark.
	M1	For substituting $x = 4$ into their $\frac{dy}{dx}$ to find the gradient of the tangent.
	B1ft	For finding the gradient of the normal. Ft their gradient.
	M1	For forming an equation for l using the equation of the normal which must have come from use of calculus. If they use $y = mx + c$ then they must find c and form an equation for the award of this mark. For example; $c = -4 \Rightarrow y = -\frac{x}{2} - 4$
	A1 cso	For the correct equation in the required form. Accept the terms in any order. For example: even $0 = -8 - x - 2y$
(c)	M1	For equating the equation of S to their l and forming a 3TQ
	M1	For attempting to solve the 3TQ [see General Guidance for the definition of an attempt] to find two points of intersection.
	A1	For **both** correct values of x
	M1	For a correct statement for the area using **their two points of intersection** correctly. **Do not accept limits of $x = -2$ and 4 or 6** They may complete these two areas separately and combine at the end. Check to the end of their work before you score this mark. Accept either $\int_{-1}^{4} \text{Curve} - \int_{-1}^{4} \text{Line}$ or $\int_{-1}^{4} \text{Line} - \int_{-1}^{4} \text{Curve}$ for this mark.
	M1	For an attempt to integrate either the expression for the line or the curve.

		See General guidance – but no power of x is to decrease.
	M1	For substituting in their values correctly **AND** subtract the two integrals. Explicit substitution must be seen if the final area is incorrect, or the limits are incorrect. A final correct area which follows correct integration is adequate evidence.
	A1	For the correct area

Question number	Scheme	Marks
8 (a)	$\left(\frac{dy}{dx}=\right)4x-4$	M1
	Gradient of $l=8$	B1
	Gradient of normal = $-\frac{1}{8}$	B1
	$4x-4=-\frac{1}{8}\Rightarrow x=\frac{31}{32}$	M1A1 [5]
(b) (i)	$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3=\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)$	B1*cso [1]
(ii)	$\alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2=\left((\alpha+\beta)^2-2\alpha\beta\right)^2-2(\alpha\beta)^2$	B1*cso [1]
(c)	$\alpha+\beta=2\quad\alpha\beta=\frac{9}{2}$ oe	B1
	$\left(\alpha^3-\beta+\beta^3-\alpha=\alpha^3+\beta^3-(\alpha+\beta)\right)$	
	$=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)-(\alpha+\beta)$	M1
	(sum of roots =)$2^3-3\left(\frac{9}{2}\right)(2)-2(=-21)$	A1
	$\left[\left(\alpha^3-\beta\right)\left(\beta^3-\alpha\right)=\right]\alpha^3\beta^3-\alpha^4-\beta^4+\alpha\beta$	M1
	$=(\alpha\beta)^3-\left(\alpha^4+\beta^4\right)+\alpha\beta$	
	Now, $\left(\alpha^4+\beta^4\right)=\left((2)^2-2\times\frac{9}{2}\right)^2-2\times\left(\frac{9}{2}\right)^2=\left(-\frac{31}{2}\right)$	A1ft
	(product of roots =) $=\left(\frac{9}{2}\right)^3-\left(-\frac{31}{2}\right)+\frac{9}{2}\left(=\frac{889}{8}\right)$	A1
	$x^2-"-21"x+"\frac{889}{8}"$	M1
	$8x^2+168x+889=0$	A1 [8]
		Total 15 marks

Part	Mark	Additional Guidance
8 (a)	M1	For a correct differentiation.
	B1	For 8, identified as the gradient or used to calculate gradient of normal
	B1	For $-\frac{1}{8}$ stated as gradient of normal or used in subsequent work
	M1	For correctly solving their $ax+b=-\frac{1}{8}$
	A1	For $\frac{31}{32}$
(b) (i)	B1*cso	Complete and full algebra to show the given identity
(ii)	B1*cso	Complete and full algebra to show the given identity
(c)	B1	Correct values for $\alpha+\beta$ **and** $\alpha\beta$. Award if values not seen explicitly but embedded in the sum/product calculations for the new equation.
	M1	**Sum:** Using the result from part (b) and correctly manipulating to prepare for substitution of $\alpha+\beta$ **and** $\alpha\beta$
	A1	For -21
	M1	**Product:** For the correct algebraic expansion shown.
	A1ft	Correct substitution of their values into $\alpha^4+\beta^4$ within the product of roots
	A1	For $\frac{889}{8}$
	M1	Use $x^2-(\text{their sum of their roots})x+\text{their product of roots}$ "= 0" may be missing.
	A1	Correct equation oe with integer coefficients.

Question number	Scheme	Marks
9 a	$\left(ax^4 = 3x^4\right) \Rightarrow a = 3$	B1
	$\left(4c = 64\right) \Rightarrow c = 16$	B1
	$\left(bx^3 - 4ax^3 = 4x^3\right) \Rightarrow b - 4a = 4$ or $\left(4ax^2 - 4bx^2 + cx^2 = -36x^2\right)$ $\Rightarrow 4a - 4b + c = -36$ *or* $\left(4bx - 64x = 0x\right) \Rightarrow 4b - 64 = 0$	M1
	$b = 16$	A1 (4)
b	$\int_0^2 (x^3 + x^2 - 6x)\mathrm{d}x = \left[\frac{x^4}{4} + \frac{x^3}{3} - \frac{6}{2}x^2\right]_0^2$	B1 (M1 on ePen) M1
	$= \left("\frac{2^4}{4} + \frac{2^3}{3} - 3(2)^2"\right) - (0)$	M1 (A1 on ePen)
	$= \pm\frac{16}{3}$ oe	A1 (M1 on ePen)
	$\left(\left["\frac{x^4}{4} + \frac{x^3}{3} - 3x^2"\right]_x^0 = \pm\frac{16}{3} \Rightarrow\right)(0) - \left("\frac{x^4}{4} + \frac{x^3}{3} - 3x^2"\right) = \pm\frac{16}{3}$ oe	M1
	$3x^4 + 4x^3 - 36x^2 + 64 = 0$	A1
	$(x-2)^2(3x^2 + 16x + 16) = 0$ *	A1* cso (7)
ALT	$\int_x^2 (x^3 + x^2 - 6x)\mathrm{d}x = \left["\frac{x^4}{4} + \frac{x^3}{3} - 3x^2"\right]_x^2$	B1 (M1 on ePen) M1
	$= \left("\frac{2^4}{4} + \frac{2^3}{3} - 3(2)^2"\right) - \left("\frac{x^4}{4} + \frac{x^3}{3} - 3(x)^2"\right)$	M1 (A1 on ePen)
	$\pm\left(-\frac{16}{3} - \frac{x^4}{4} - \frac{x^3}{3} + 3x^2\right) = 0$	A1 (M1 on ePen)
	$3x^4 + 4x^3 - 36x^2 + 64 = 0$	M1 A1
	$(x-2)^2(3x^2 + 16x + 16) = 0$ *	A1
c	(When $(x-2) = 0$ $x = 2$ and when $3x^2 + 16x + 16 = 0$)	
	$(3x+4)(x+4) = 0$ so $x = -\frac{4}{3}$ and $x = -4$	M1 A1
	($x \neq -4$ as it is to the left of the point at $x = -3$) $x = -\frac{4}{3}$	M1
	When $"x = -\frac{4}{3}"$ $y = \left("-\frac{4}{3}"\right)^3 + \left("-\frac{4}{3}"\right)^2 - 6\left("-\frac{4}{3}"\right) = \frac{200}{27}$	dM1
	So $A = \left(-\frac{4}{3}, \frac{200}{27}\right)$	A1 (5)
		Total 16 marks

Part	Mark	Notes
(a)	B1	$a = 3$
	B1	$c = 16$
	M1	For a clear process to equate coefficients to find either of the equations $b-4a=4$ or $4a-4b+c=-36$ or $4b - 64 = 0$. Allow one error. For algebraic division: there must be a complete attempt to divide by $x^2 + px + q$ and reach an expression of the form $3x^2+rx+16,\ \ p,q,r\neq 0$
	A1	$b=16$
Full marks may be awarded for all of *a*, *b* and *c* appearing correctly with no method shown. Embedded *a*, *b* and *c* should be awarded full marks.		
(b)	B1 (M1 on ePen)	Correct limits of $x = 0$ and $x = 2$ – or used later in working.
	M1	For a minimally acceptable attempt to integrate any 3 term cubic of the form $x^3 + fx^2 + gx$. Limits do not need to be present. See general guidance for the definition of a minimally acceptable attempt.
	M1 (A1 on ePen)	Substitution of their limits into any changed expression, the correct way round minimum 3 terms. Sub of 0 not needed. Allow this mark to be implied by a fully correct value.
	A1 (M1 on ePen)	Either for a fully correct substitution, or for $\pm\frac{16}{3}$
	M1	For $(0)-\left("\frac{x^4}{4}+\frac{x^3}{3}-3x^2"\right)=\pm\frac{16}{3}$ oe. Allow use of their integrated expression and their $\pm\frac{16}{3}$ which can be in any equivalent form, including an unevaluated expression. Allow use of *n* rather than *x*. The limit of 0 need not be seen.
	A1	For $3x^4+4x^3-36x^2+64=0$ Allow use of *n* rather than *x*. $\frac{16}{3}$ must have been evaluated.
	A1* cso	For $(x-2)^2(3x^2+16x+16)=0$ .No errors in working. Candidates can use *n* throughout.
ALT	B1 (M1 on ePen)	Correct limits of $x = x$ and $x = 2$ – seen on the integral or used later in working. These can be either way round.
	M1	For a minimally acceptable attempt to integrate any 3 term cubic of the form $x^3 + fx^2 + gx$. Limits do not need to be present. See general guidance for the definition of a minimally acceptable attempt.
	M1 (A1 on ePen)	For substitution of both of their limits into any changed expression. Minimum 3 terms
	A1 (M1 on ePen)	For $\pm\left(-\frac{16}{3}-\frac{x^4}{4}-\frac{x^3}{3}+3x^2\right)=0$
	M1	For a valid attempt to multiply their equation throughout to arrive at integer coefficients.
	A1 A1	As main scheme. Candidates can use *n* throughout.
(c)	M1	For a complete and minimally acceptable attempt to solve $3x^2+16x+16=0$ to give $x =$ See general guidance for definition of minimally acceptable. Some candidates may have factorised in part b), if used in part c), marks can be awarded
	A1	For $x=-\frac{4}{3}$ and $x=-4$
	M1	Chooses $"x=-\frac{4}{3}"$, can be stated explicitly or used later implicitly. Allow choice of their *x* coordinate for $-3<x<0$
	dM1	For substituting $x="-\frac{4}{3}"$ into $y=x(x+3)(x-2)$ Allow substitution of their *x*, dependent on previous method mark
	A1	$A=\left(-\frac{4}{3},\frac{200}{27}\right)$ students may list $x=-\frac{4}{3}, y=\frac{200}{27}$

Question	Scheme	Marks
11 (a)	$x=4 \Rightarrow y=6$	B1
	$\frac{dy}{dx}=\frac{2}{3}x$	M1
	Grad tgt $=\frac{8}{3}$	A1
	$y-6=\frac{8}{3}(x-4)$	M1A1
	$3y=8x-14$	A1 [6]
(b)	$y-6=-\frac{3}{8}(x-4)$	M1
	$8y=-3x+60$	A1 [2]
(c)	Base $=\left(20-\frac{7}{4}\right) \quad \left(=\frac{73}{4}\right)$	B1
	Area $=\frac{1}{2}\times\frac{73}{4}\times 6, =54\frac{3}{4}$ or 54.75	M1,A1 [3]
	ALT	
	Vertices $\left(\frac{7}{4},0\right)$ and $(20,0)$, Area $=\frac{1}{2}\begin{vmatrix}4 & \frac{7}{4} & 20 & 4\\ 6 & 0 & 0 & 6\end{vmatrix}=54\frac{3}{4}$	B1,M1A1 [3]
(d)	Vol $=\pi\int_0^4\left(\frac{1}{3}(x^2+2)\right)^2 dx$ + cone, base 6 and ht 16	M1
	$(\pi)\int_0^4\left(\frac{1}{3}(x^2+2)\right)^2 dx=(\pi)\frac{1}{9}\int_0^4(x^4+4x^2+4)dx=(\pi)\frac{1}{9}\left[\frac{x^5}{5}+\frac{4x^3}{3}+4x\right]_0^4$	M1A1
	Cone $=\frac{\pi}{3}\times 6^2\times 16 \quad (=192\pi)$ (or equation of normal squared and integrated correctly)	B1
	Vol $=\frac{\pi}{9}\left(\frac{4^5}{5}+\frac{4^4}{3}+4^2\right)+192\pi=710$	M1A1 [6]
		Total 17 marks

Additional Guidance		
(a)	B1	Finds the value of y when $x = 6$. $x = 4 \Rightarrow y = 6$
	M1	Differentiates the given expression to give $\frac{dy}{dx} = \frac{2}{3}x$ This must be correct.
	A1	For the gradient of the tangent = $\frac{8}{3}$
	M1	For the equation of the line k. $y - '6' = '\frac{8}{3}'(x-4)$ Follow through their y and their gradient.
	A1	For the correct equation in any form.
	A1	For the correct equation in the required form. $3y = 8x - 14$
(b)	M1	For the equation of the normal to the curve. Allow their y and the negative inverse of their gradient of the tangent. $y - '6' = '-\frac{3}{8}'(x-4)$ The gradient of the normal **must be** the negative inverse of their numerical value
	A1	For the correct equation in the required form. $8y = -3x + 60$
(c)	B1	For the length of the base $\left(\frac{73}{4}\right)$
	M1	For the area of the triangle Area $= \frac{1}{2} \times '\frac{73}{4}' \times '6'$
	A1	For the correct area of the triangle $54\frac{3}{4}$ or 54.75
	ALT	
	B1	For the correct coordinates of the vertices Vertices $\left(\frac{7}{4}, 0\right)$ and $(20, 0)$
	M1	For the correct method to find the area using their vertices $\frac{1}{2}\begin{vmatrix} 4 & '\frac{7}{4}' & '20' & 4 \\ '6' & 0 & 0 & '6' \end{vmatrix} = ...$
	A1	For the correct area of the triangle $54\frac{3}{4}$ or 54.75
(d)	M1	For a correct statement of the volume of the solid. $\pi\int_0^4 \left(\frac{1}{3}(x^2+2)\right)^2 dx$ + cone, with a radius of the base of their value of y (6) and the height derived from their equation of line l (normal) Alternatively, they can use Volume of cone $= \pi\int_4^{'20'} \left('-\frac{3}{8}'x + '\frac{60}{8}'\right)^2 dx$ The limits must be correct for this mark.

	M1	For squaring the equation of the **curve** only and attempting to integrate. (See general guidance for the definition of an attempt) Ignore the limits and π for this mark.
	A1	For a fully correct integrated expression for the cone. Ignore the limits and π for this mark.
	B1	For finding the volume of the cone. Volume of cone = $=\frac{\pi}{3}\times 6^2\times 16 \quad (=192\pi)$ **ALT** Volume of cone $=\pi\int_4^{'20'}\left(-\frac{3}{8}x+\frac{15}{2}\right)^2 \mathrm{d}x=\pi\int_4^{'20'}\left(\frac{9}{64}x^2-\frac{45}{8}x+\frac{225}{4}\right)\mathrm{d}x$ $=\pi\left[\frac{3}{64}x^3-\frac{45}{16}x^2+\frac{225}{4}x\right]_4^{'20'}=192\pi$ If candidates chose to use the **ALT** method – it must be fully correct.
	M1	Evaluating their integrated expression which must have as a minimum a term in x^4, a term in x^3, and a term in x, **AND** adding their volume of the cone.
	A1	For 710 (cm^3)