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EDEXCEL INTERNATIONAL GCSE FURTHER PURE MATHS: QUESTIONS BY TOPIC

Trigonometry (9)

International GCSE Further Pure Mathematics Topic 9

86 questions · 794 marks · Mark schemes included

Pearson Edexcel International GCSE (9–1) Further Pure Mathematics 4PM1, Papers 1, 1R, 2 & 2R, June 2019 - June 2025

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 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions run from the lowest mark tariff to the highest, so similar questions sit together.

Click any heading or question row below to jump straight to it.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

86 questions · 794 marks · lowest tariff first



3-MARK QUESTIONS

3 questions in this booklet



1

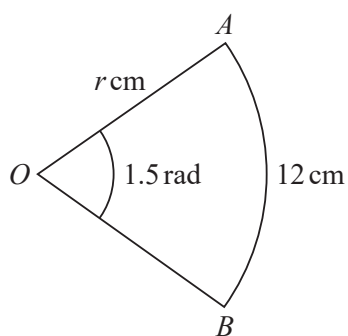
Diagram **NOT**
accurately drawn**Figure 1**

Figure 1 shows sector AOB of a circle with centre O and radius r cm. The angle AOB is 1.5 radians and the length of arc AB is 12 cm.

Calculate

(a) the value of r ,

(1)

(b) the area of the sector AOB .

(2)

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1

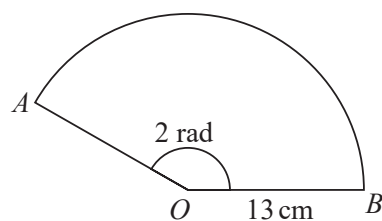
Diagram **NOT**
accurately drawn**Figure 1**

Figure 1 shows the sector AOB of a circle with centre O .
The radius of the circle is 13 cm and angle $AOB = 2$ radians.

(a) Find the length of the arc AB .

(1)

(b) Find the area of the sector AOB .

(2)

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2 In $\triangle ABC$,

$$\angle BAC = 50^\circ \quad AB = 10 \text{ cm} \quad BC = 9 \text{ cm}$$

Given that $\angle BCA = x^\circ$

find the two possible values, to one decimal place, of x

(3)



4-MARK QUESTIONS

3 questions in this booklet



For marks 11 to 14, show all the stages in your working.

1

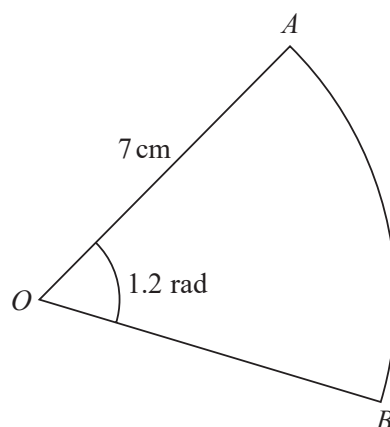


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows sector AOB of a circle with centre O and radius 7 cm .

The angle AOB is 1.2 radians.

Calculate

(a) the area of sector AOB (2)

(b) the perimeter of sector AOB (2)

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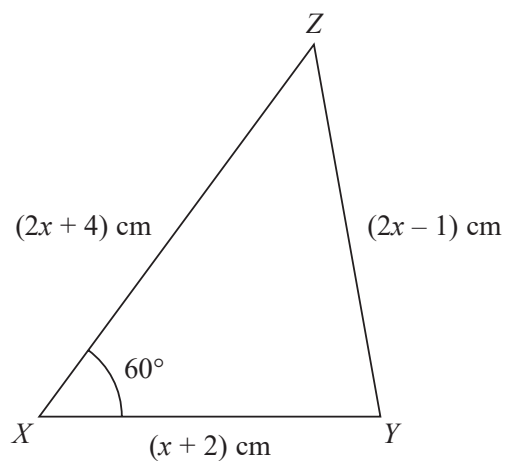
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2

Diagram **NOT**
accurately drawn**Figure 1**Figure 1 shows triangle XYZ in which

$$XY = (x + 2) \text{ cm} \quad XZ = (2x + 4) \text{ cm} \quad YZ = (2x - 1) \text{ cm} \quad \text{and} \quad \angle YXZ = 60^\circ$$

Find the value of x Give your answer in the form $p + q\sqrt{3}$ where p and q are integers to be found.

(4)

You must write down all the stages in your working.

- 1 In triangle ABC , $AB = 2x$ cm, $BC = 3x$ cm and $AC = 4x$ cm

The area of triangle ABC is 50 cm^2

Find, to 2 decimal places, the value of x

(4)



5-MARK QUESTIONS

7 questions in this booklet



- 4 In triangle ABC , $AB = 5x$ cm, $BC = (3x - 1)$ cm, $AC = (2x + 5)$ cm and angle $ABC = 60^\circ$

Find, to 3 significant figures, the value of x .

(5)

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- 3 In triangle ABC , $AB = 11$ cm and $BC = 12$ cm.

The area of triangle $ABC = 33$ cm²

Find, in cm to 3 significant figures, the two possible lengths of AC .

(5)



- 1 (a) Use the formula for $\cos(A + B)$ to show that $\cos 2A = 1 - 2 \sin^2 A$

(2)

Given that $\sin A = \frac{x+1}{2}$ and $\cos 2A = \frac{4-y}{3}$

- (b) show that $y = \frac{1}{2}(3x^2 + 6x + 5)$

(3)

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2 Solve the equation

$$\sin(2\theta - 20)^\circ - \sqrt{3} \cos(2\theta - 20)^\circ = 0 \quad \text{for } 0 \leq \theta \leq 180$$

(5)



3 Triangle ABC is such that

$$AC = 10\text{ cm} \quad BC = 7\text{ cm} \quad \text{angle } CAB = 25^\circ$$

Given that angle ABC is obtuse,

find, in cm to one decimal place, the length of AB

(5)



1

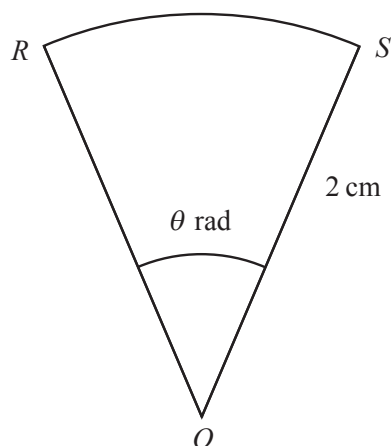
Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows sector ROS of a circle with centre O and radius 2 cm .
The size of angle ROS is θ radians.

The area of sector ROS is $\frac{\pi}{2}\text{ cm}^2$

(a) Find the exact value of θ

(2)

The perimeter of sector ROS is $P\text{ cm}$

(b) Find the exact value of P

(3)

1 Solve the equation

$$\sqrt{3} \tan A = 2 \sin A \quad \text{for } 0^\circ \leq A \leq 360^\circ$$

(5)



6-MARK QUESTIONS

14 questions in this booklet



3 In triangle ABC , $AC = 7$ cm, $BC = 10$ cm and angle $BAC = 65^\circ$

(a) Find, to the nearest 0.1° , the size of angle ABC .

(3)

(b) Find, in cm^2 to 3 significant figures, the area of triangle ABC .

(3)



4

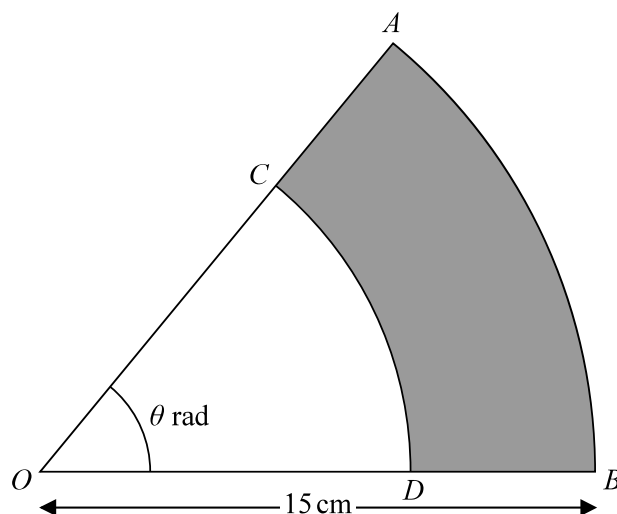
Diagram **NOT**
accurately drawn**Figure 1**

Figure 1 shows a sector OAB of a circle where angle $AOB = \theta$ radians. The circle has centre O and radius 15 cm . The point C divides OA in the ratio $2:1$ and the point D divides OB in the ratio $2:1$

The area of the region $ABDC$, shown shaded in Figure 1, is 100 cm^2

Find

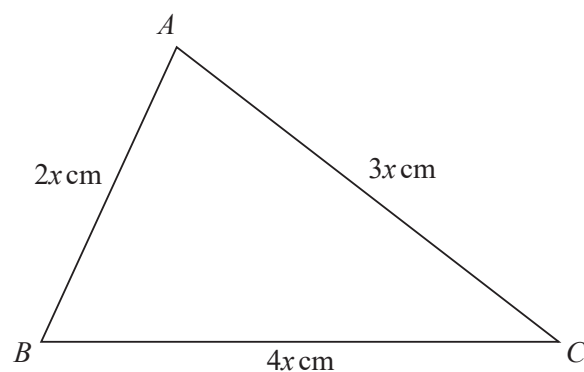
(a) the value of θ ,

(3)

(b) the perimeter of the region $ABDC$.

(3)

2

Diagram **NOT**
accurately drawn**Figure 2**Figure 2 shows triangle ABC in which

$$AB = 2x \text{ cm} \quad AC = 3x \text{ cm} \quad BC = 4x \text{ cm}$$

- (a) Show that $\sin ABC = \frac{3\sqrt{15}}{16}$ (4)

Given that the area of triangle ABC is $\frac{75\sqrt{15}}{64} \text{ cm}^2$

- (b) find the value of x . (2)

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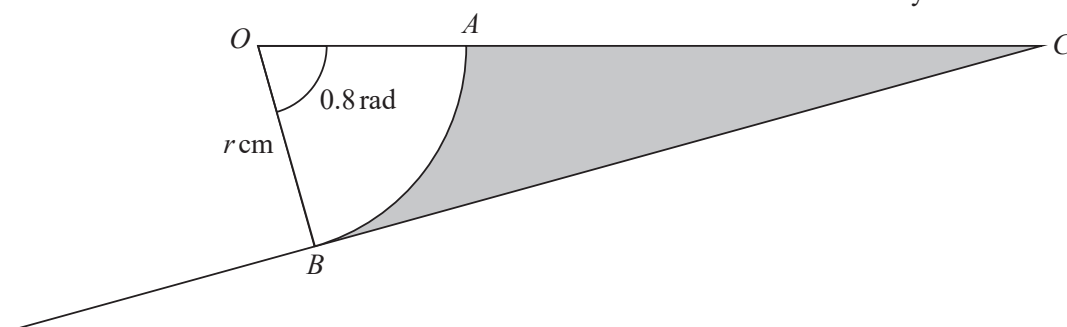
Diagram **NOT**
accurately drawn**Figure 1**

Figure 1 shows a sector AOB of a circle with centre O and radius r cm and a triangle BOC . The angle of sector AOB is 0.8 radians. The points O , A and C lie on a straight line so that CB is the tangent to the circle at B .

Given that the area of the shaded region in Figure 1 is 101 cm^2 , find the value of r .
Give your answer correct to 3 significant figures.

(6)

2 Angle α is acute such that $\cos \alpha = \frac{3}{5}$

Angle β is obtuse such that $\sin \beta = \frac{1}{2}$

(a) Find the exact value of

(i) $\tan \alpha$

(ii) $\tan \beta$

(3)

(b) Hence show that

$$\tan(\alpha + \beta) = \frac{m\sqrt{3} - n}{n\sqrt{3} + m}$$

where m and n are positive integers whose values are to be found.

(3)

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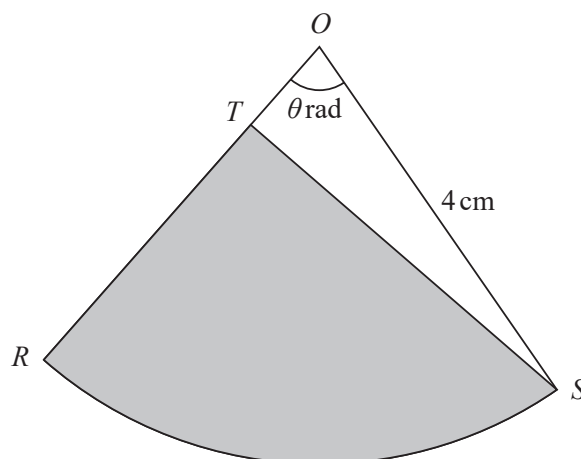
Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows sector ORS of a circle with centre O and radius 4 cm .
The size of angle ROS is θ radians.

The area of sector ORS is $2\pi\text{ cm}^2$

(a) Find the exact value of θ

(2)

(b) Find the perimeter, in cm to 3 significant figures, of the sector ORS .

(2)

The point T lies on OR such that $OT : TR = 1 : 3$

The region shown shaded in Figure 1 is bounded by the line TR , the line TS and the arc RS of the sector.

The area of this region is $A\text{ cm}^2$

(c) Find the exact value of A

(2)

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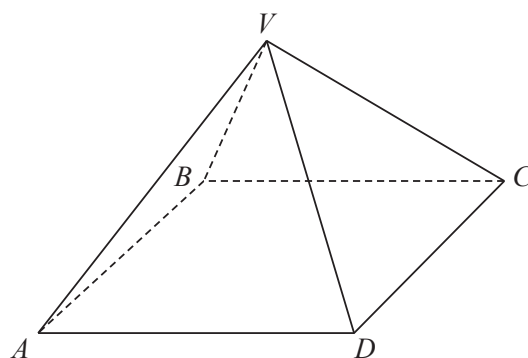


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows a right pyramid $VABCD$ with vertex V and square base $ABCD$.

Each of the edges of the pyramid has the same length.

Find the size, in degrees to one decimal place, of the angle between the plane CVD and the base $ABCD$.

(6)

3

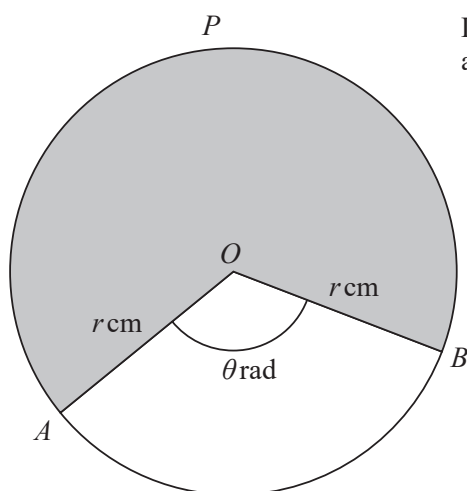
Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows a circle, centre O , with radius r cm.

The points A , P and B lie on the circle.

The obtuse angle $AOB = \theta$ radians.

The area of the sector $APBO$, shown shaded, is 372.4 cm^2 and the length of the arc APB is 53.2 cm.

Find, to 3 significant figures where appropriate, the value of

(i) r

(ii) θ

(6)

9

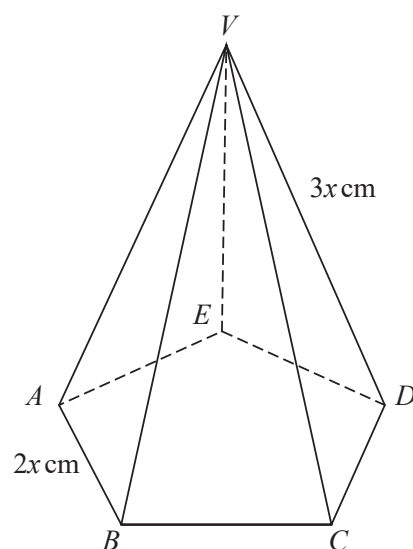


Diagram **NOT**
accurately drawn

Figure 4

Figure 4 shows a right pyramid with vertex V and base $ABCDE$ which is a regular pentagon.

$$AB = BC = CD = DE = EA = 2x \text{ cm}$$

$$VA = VB = VC = VD = VE = 3x \text{ cm}$$

Find, in degrees to one decimal place, the size of the angle between the plane VBC and the base $ABCDE$

(6)

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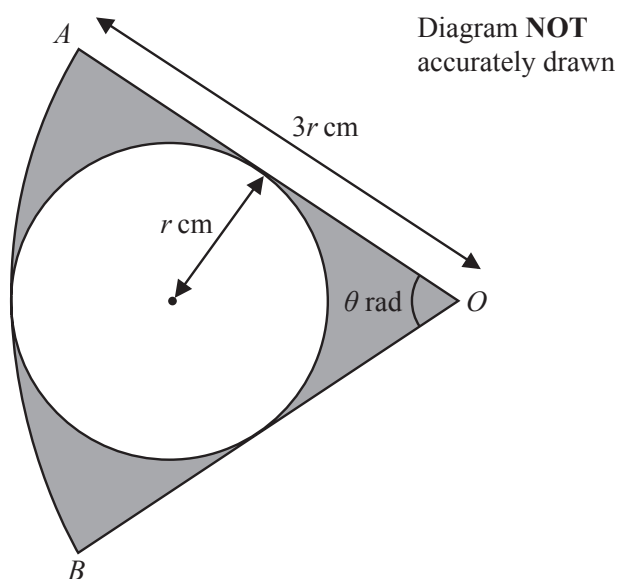
**Figure 1**

Figure 1 shows the sector AOB of a circle with centre O and radius $3r \text{ cm}$

A circle with radius $r \text{ cm}$ touches OA and OB and the arc AB

Angle AOB is θ radians, where $0 < \theta < \frac{\pi}{2}$

(a) Find the exact value of θ

(2)

The area of the region shown shaded in Figure 1 is $8\pi \text{ cm}^2$

(b) Find the value of r

(4)

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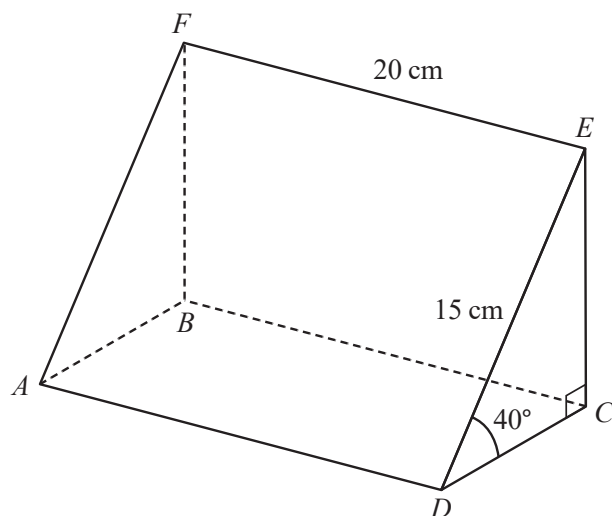


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows a right prism $ABCDEF$.
Triangle CDE is a cross section of the prism.

- $\angle DCE = \angle ABF = 90^\circ$
- $\angle EDC = \angle FAB = 40^\circ$
- $DE = AF = 15 \text{ cm}$
- $EF = DA = CB = 20 \text{ cm}$

Find, in degrees to one decimal place, the size of the angle between the line FD and the plane $DCBA$.

(6)

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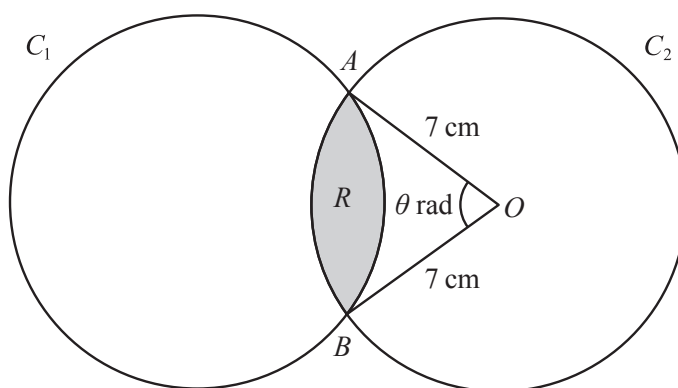
Diagram **NOT**
accurately drawn**Figure 1**

Figure 1 shows circle C_1 with radius 7 cm and circle C_2 with centre O and radius 7 cm

The circles C_1 and C_2 intersect at the point A and at the point B

The size of angle AOB is θ radians

The perimeter of the region R shown shaded in Figure 1 is $\frac{28\pi}{9}$ cm

(a) Find the exact value of θ

(2)

(b) Find the area, in cm^2 to 3 significant figures, of the region R

(4)

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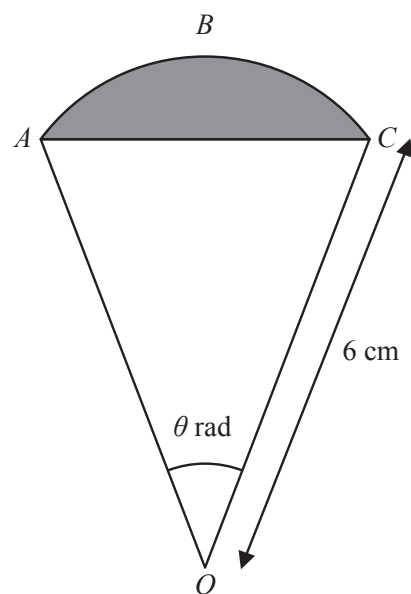


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows sector $OABC$ of a circle with centre O and radius 6 cm
The size of angle AOC is θ radians.

The perimeter of sector $OABC$ is $(12 + \pi)$ cm

The region shown shaded in Figure 1 is bounded by the arc ABC and the line AC
The area of this region is $S \text{ cm}^2$

Find the exact value of S

(6)

6

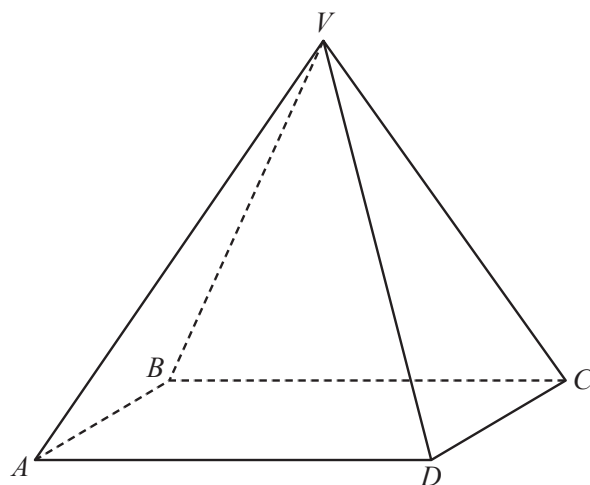


Diagram **NOT**
accurately drawn

Figure 2

Figure 2 shows a right pyramid $VABCD$ with vertex V and rectangular base $ABCD$

$$AD = VC = 3AB$$

Find in degrees, to one decimal place, the size of the angle between the plane CVD and the base $ABCD$

(6)

7-MARK QUESTIONS

7 questions in this booklet



1 Here is a formula

$$P = 3 + 2 \sin \left(\frac{3\pi t}{8} \right) \quad 0 \leq t \leq 12$$

(a) Find the exact value of P when $t = \frac{10}{3}$

(2)

(b) Find

(i) the largest value of P

(ii) the smallest value of P

(2)

(c) Find the least value of t for which $P = 4$

(3)

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6 (a) Show that

$$\sin(A + B) + \sin(A - B) = 2 \sin A \cos B \quad (2)$$

(b) Hence express $2 \sin 7x \cos x$ in the form $\sin mx + \sin nx$ where m and n are integers, giving the value of m and the value of n . (1)

(c) Use calculus to evaluate

$$\int_0^{\frac{\pi}{4}} (6 \sin 7x \cos x) dx \quad (4)$$

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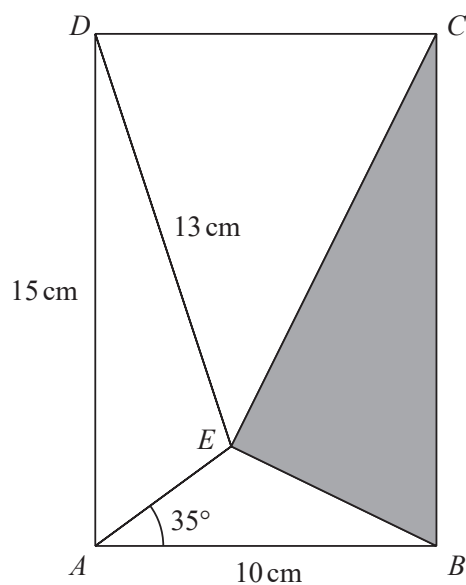
Diagram **NOT**
accurately drawn**Figure 2**

Figure 2 shows a rectangle $ABCD$ with $AB = 10$ cm and $AD = 15$ cm.
 E is the point inside the rectangle such that $DE = 13$ cm and angle $BAE = 35^\circ$

Given that angle AED is obtuse,

find the area, in cm^2 to one decimal place, of triangle BCE .

(7)

4

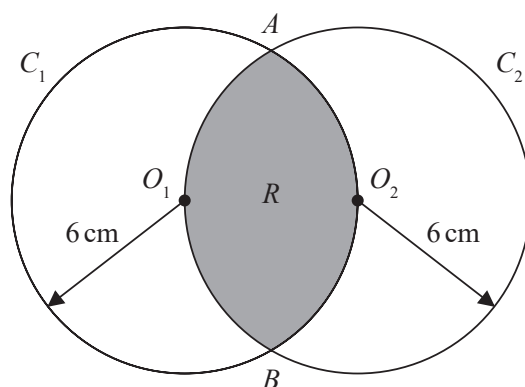
Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows two circles, C_1 and C_2 , each with a radius of 6 cm.

The centre of C_1 is O_1 such that O_1 lies on C_2

The centre of C_2 is O_2 such that O_2 lies on C_1

The circles intersect at the points A and B and enclose the region R , shown shaded in Figure 1

The area of region R is $P \text{ cm}^2$

Find the exact value of P , giving your answer in the form $a\pi - b\sqrt{c}$
where a , b and c are integers.

(7)

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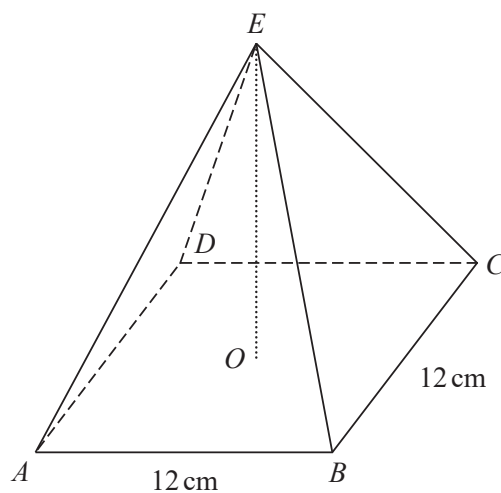
Diagram **NOT**
accurately drawn**Figure 2**

Figure 2 shows a right pyramid with a square base $ABCD$ and vertex E .

The base of the pyramid is horizontal with $AB = BC = 12$ cm.

The diagonals of the base intersect at the point O .

The vertex E of the pyramid is vertically above O and the angle between EA and the plane $ABCD$ is 30°

The height of the pyramid is h cm.

(a) Find the exact value of h

(3)

The point F lies on AD such that $AF:FD = 1:4$

(b) Calculate, to the nearest degree, the size of the angle EFO .

(4)

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2 In triangle ABC , $AB = 3x$ cm, $BC = 5x$ cm and $\angle ABC = 110^\circ$

(a) Find, in degrees to one decimal place, the size of $\angle BCA$

(4)

The area of triangle ABC is 24 cm^2

(b) Find, to 3 significant figures, the value of x

(3)

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4 Triangle ABC is such that

$$AB = 4.3 \text{ cm} \qquad BC = 5.9 \text{ cm} \qquad \text{angle } BCA = 29^\circ$$

(a) Find in degrees, to one decimal place, the two possible values for angle CAB

(4)

(b) Hence, find in cm, to 3 significant figures, the shortest possible length of AC

(3)

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8-MARK QUESTIONS

7 questions in this booklet



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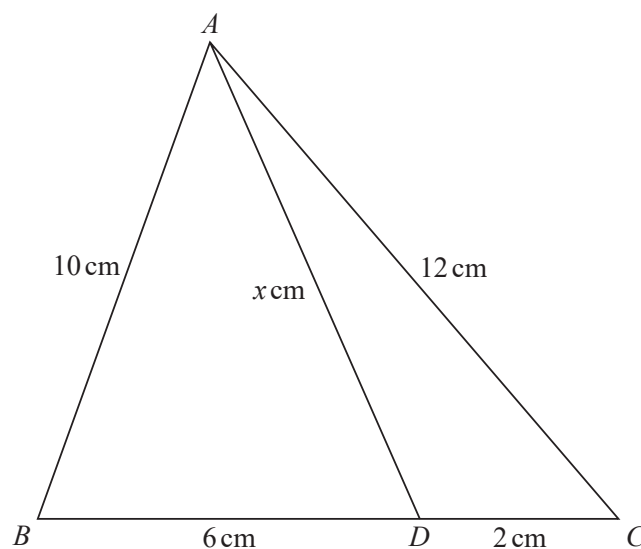
Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows the triangle ABC in which $AB = 10$ cm and $AC = 12$ cm.
The point D lies on BC such that $BD = 6$ cm, $DC = 2$ cm and $AD = x$ cm.

(a) Show that $x = 11$

(4)

(b) Find the area, in cm^2 to 3 significant figures, of triangle ADB .

(4)

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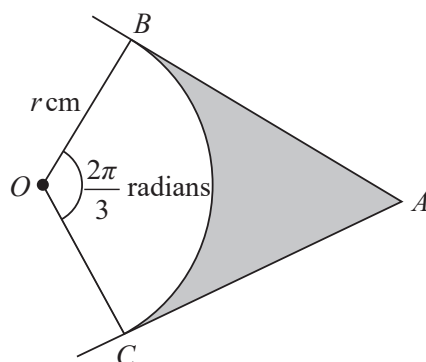


Diagram **NOT**
accurately drawn

Figure 2

In Figure 2, AB and AC are tangents to a circle with centre O and radius r cm.

The points B and C lie on the circle so that OBC is a sector of this circle and $\angle BOC = \frac{2\pi}{3}$ radians.

Given that the area of the shaded region is 10 cm^2 ,

find, to 3 significant figures, the value of r .

(8)

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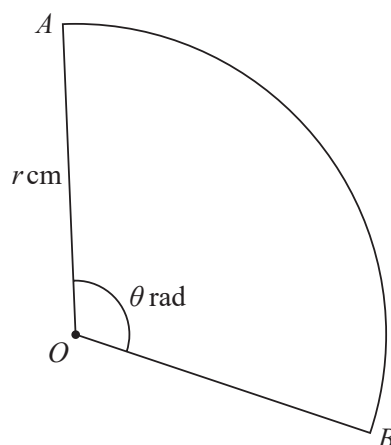
Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows the sector AOB of a circle with centre O and radius r cm, where r is an integer. The size of angle AOB is θ radians.

The sector has an area of 16.8 cm^2 and a perimeter of 16.4 cm .

Calculate

- (i) the value of r
- (ii) the value of θ

(8)

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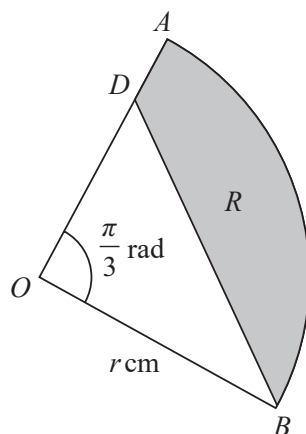
Diagram **NOT**
accurately drawn**Figure 1**

Figure 1 shows sector OAB of a circle with centre O and radius r cm.

The angle $AOB = \frac{\pi}{3}$ radians.

The point D divides OA in the ratio $3 : 1$

The area of the region R , shown shaded in Figure 1, is 79.5 cm^2

(a) Calculate the value, to 2 significant figures, of r

(4)

(b) Calculate the perimeter, in cm to 2 significant figures, of the region R

(4)

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7 Given that

$$\frac{3 + \sin^2 \theta}{\cos \theta - 2} = 3 \cos \theta$$

(a) show that $\cos \theta = -\frac{1}{2}$

(4)

(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3 \cos 3x \quad \text{for } 0^\circ \leq x < 180^\circ$$

(4)

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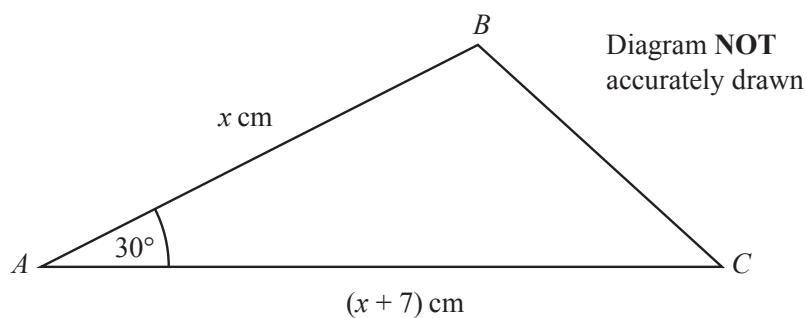
**Figure 1**

Figure 1 shows triangle ABC where

$$AB = x \text{ cm} \quad AC = (x + 7) \text{ cm} \quad \angle BAC = 30^\circ$$

The area of triangle $ABC = 36 \text{ cm}^2$

- (a) Show that $x = 9$ (3)
- (b) Find, in cm to 3 significant figures, the length of BC (2)
- (c) Find, in degrees to one decimal place, the size of
- (i) $\angle ABC$
- (ii) $\angle ACB$ (3)

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5 In triangle ABC , $AC = 12$ cm, $BC = 14$ cm, $AB = x$ cm and angle $ABC = 30^\circ$

(a) Show that $x = P\sqrt{3} \pm \sqrt{95}$ where P is a prime number.

(5)

(b) Hence or otherwise, find in cm^2 , the difference between the two possible areas of triangle ABC

Give your answer in the form $m\sqrt{n}$ where m is a prime number and n is an integer.

(3)

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9-MARK QUESTIONS

8 questions in this booklet



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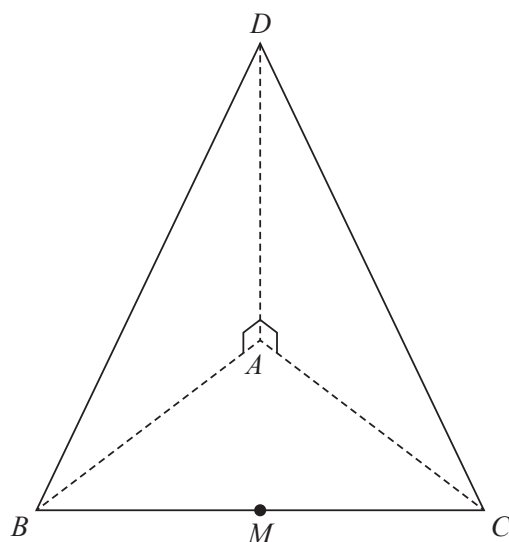
Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows a triangular pyramid $ABCD$.

The base, ABC , of the pyramid is a horizontal isosceles triangle with $AB = AC = 10$ cm and $BC = 16$ cm. The midpoint of BC is M .

The face BCD of the pyramid is an isosceles triangle with $BD = CD = 26$ cm and D is vertically above A .

$$\angle BAD = \angle CAD = 90^\circ$$

(a) Calculate the length, in cm, of AM .

(2)

Calculate, in degrees to the nearest degree,

(b) the size of $\angle BCD$,

(3)

(c) the size of the angle between the planes BCA and BCD .

(4)

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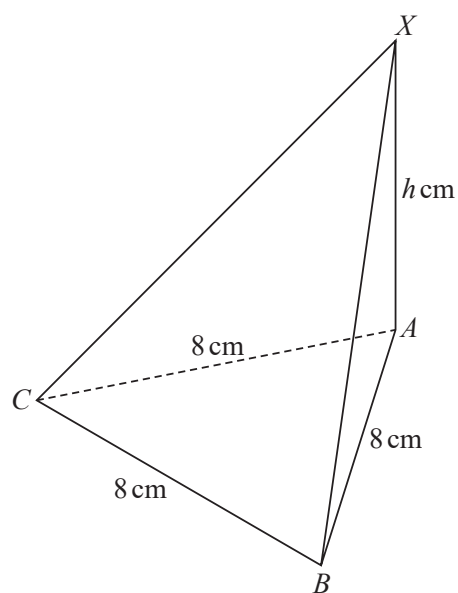
Diagram **NOT**
accurately drawn**Figure 3**

Figure 3 shows a triangular pyramid $ABCX$.

The base ABC of the pyramid is an equilateral triangle where $AB = BC = CA = 8$ cm.

The vertex X of the pyramid is such that AX is perpendicular to the base of the pyramid and $AX = h$ cm.

The volume of the pyramid is $48\sqrt{3}$ cm³

(a) Show that $h = 9$

(3)

(b) Find, in degrees to one decimal place, the size of angle BXC .

(3)

(c) Find, in degrees to one decimal place, the size of the angle between the plane BCX and the base ABC of the pyramid.

(3)

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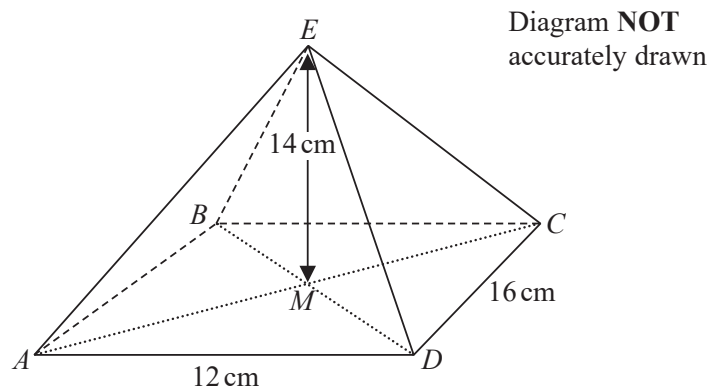


Figure 1

Figure 1 shows the right pyramid $ABCDE$. The base, $ABCD$, of the pyramid is a horizontal rectangle with $AD = 12$ cm and $CD = 16$ cm. The height ME of the pyramid is 14 cm where M is the point of intersection of the diagonals of the base.

The sloping edges, EA , EB , EC and ED of the pyramid are all of equal length.

- (a) Calculate, to 3 significant figures, the length of a sloping edge.

(3)

Calculate, in degrees to one decimal place, the size of

- (b) the angle between AE and the base,

(3)

- (c) the angle between the plane AED and the base.

(3)

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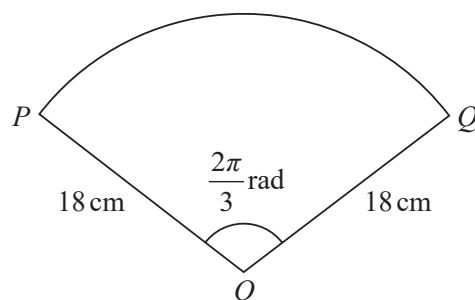
Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows a sector OPQ of a circle with centre O .

The radius of the circle is 18 cm and the angle POQ is $\frac{2\pi}{3}$ radians.

(a) Find the length of the arc PQ , giving your answer as a multiple of π

(2)

Figure 2 below shows the sector OPQ and the kite $OPTQ$.

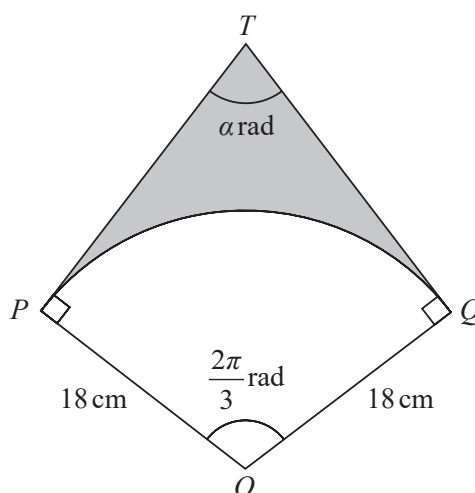
Diagram **NOT**
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Figure 2

PT is the tangent to the circle at P and QT is the tangent at Q , such that angle $PTQ = \alpha$ radians.

(b) (i) Find α in terms of π

(1)

(ii) Calculate, to 3 significant figures, the area of the region, shown shaded in Figure 2, which is bounded by the arc PQ and the tangents PT and QT .

(6)

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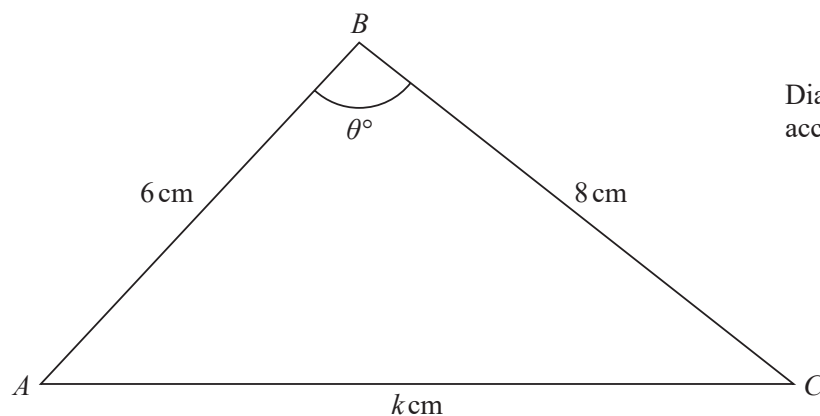


Figure 2

Figure 2 shows triangle ABC

$$AB = 6 \text{ cm} \quad BC = 8 \text{ cm} \quad AC = k \text{ cm} \quad \angle ABC = \theta^\circ$$

(a) Show that $\cos \theta^\circ = \frac{100 - k^2}{96}$ (2)

The area of triangle ABC is $\sqrt{455} \text{ cm}^2$

(b) Find the two possible values of k (7)

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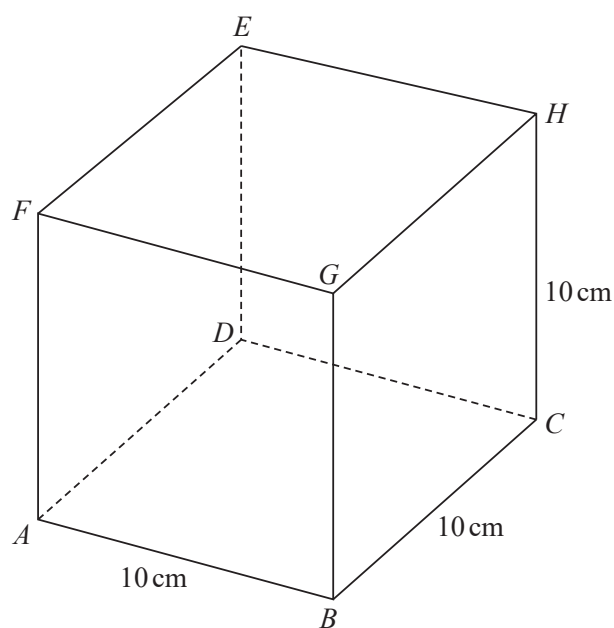
Diagram NOT
accurately drawn

Figure 2

Figure 2 shows a solid cube $ABCDEFGH$ with sides of length 10 cm.

- (a) Find, in cm, the exact length of FC (2)
- (b) Find, in degrees to one decimal place, the size of $\angle FCA$ (2)
- (c) Find, to the nearest degree, the size of the obtuse angle between the plane BFH and the plane BHC (5)

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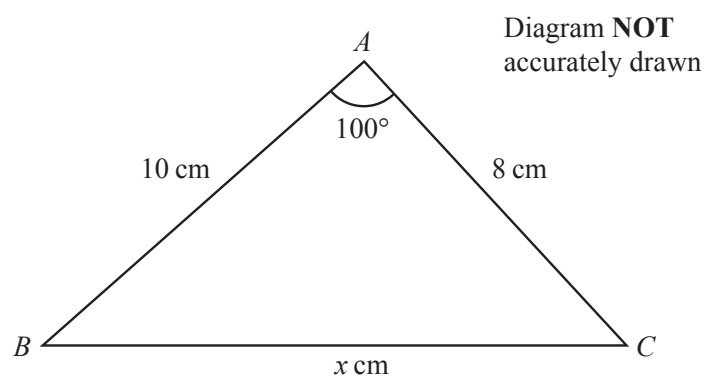
**Figure 2**

Figure 2 shows triangle ABC where

$$AB = 10 \text{ cm}, \quad AC = 8 \text{ cm}, \quad BC = x \text{ cm} \quad \text{and} \quad \angle BAC = 100^\circ$$

(a) Find, to 3 significant figures, the value of x (2)

(b) Find, in degrees to one decimal place, the size of

(i) angle ABC (ii) angle ACB (3)

The bisector of angle ABC meets AC at the point M

(c) Find the area, in cm^2 to 3 significant figures, of triangle BMC . (4)

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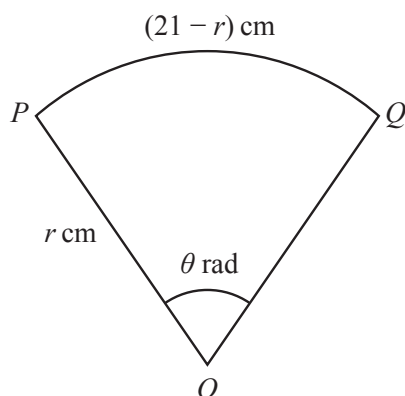
Diagram **NOT**
accurately drawn**Figure 2**

Figure 2 shows the sector OPQ of a circle with centre O and radius r cm.

$$OP = OQ = r \text{ cm} \quad \text{arc } PQ = (21 - r) \text{ cm} \quad \angle POQ = \theta \text{ radians}$$

The area of the sector is $A \text{ cm}^2$

(a) Show that $A = \frac{r}{2}(21 - r)$ (3)

The area of the sector must be greater than or equal to 27 cm^2

(b) Find the set of possible values of r (4)

(c) Hence write down the set of possible values of θ (2)

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10-MARK QUESTIONS

5 questions in this booklet



- 8 (a) Write down the value of k such that $\sin 2A = k \sin A \cos A$

(1)

$$g(A) = 2 + 3\cos A - \sin A - 3\sin 2A - 2\cos^2 A$$

Given that $g(A)$ can be written in the form $(p \cos A - \sin A)(q - r \sin A)$ where p , q and r are integers,

- (b) find the value of p , the value of q and the value of r .

(3)

- (c) Hence solve, in radians to 3 significant figures where appropriate, the equation

$$g(2\theta) = 0 \quad \text{for} \quad 0 \leq \theta < \pi$$

(6)

10 (a) Solve the equation

$$\tan x^\circ = -3 \quad \text{for } 0 \leq x < 360$$

Give your solutions to the nearest whole number.

(3)

Given that

$$7 \sin^2 \theta + \sin \theta \cos \theta = 6$$

(b) show that

$$\tan^2 \theta + \tan \theta - 6 = 0$$

(3)

(c) Hence solve the equation

$$7 \sin^2 y^\circ + \sin y^\circ \cos y^\circ = 6 \quad \text{for } 0 \leq y < 360$$

Give your solutions to the nearest whole number.

(4)

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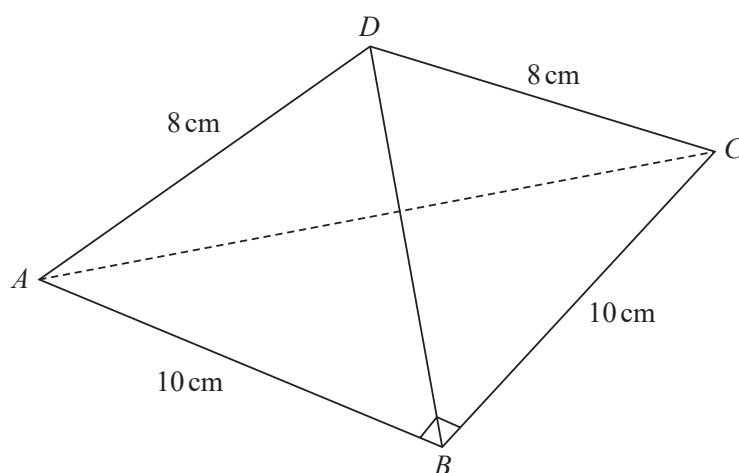


Figure 2

Figure 2 shows a triangular pyramid $ABCD$ with base ABC

$$AB = BC = 10 \text{ cm} \quad AD = CD = 8 \text{ cm} \quad \angle ABC = 90^\circ$$

- (a) Find the exact length of AC

Give your answer in the form $p\sqrt{q}$ cm where p is an integer and q is a prime number. (2)

The point M is the midpoint of AC

- (b) Find the exact length of BM

Give your answer in the form $m\sqrt{n}$ cm where both m and n are prime numbers. (2)

Given that $BD = 6$ cm,

- (c) find, in degrees to one decimal place, the size of the acute angle between the plane ACD and the plane ABC (4)

The base ABC of the pyramid is placed on a horizontal plane.

- (d) Find, in cm to 3 significant figures, the vertical height of D above the base. (2)

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6 In triangle ABC , $AC = x$ cm, $AB = (x + 3)$ cm and $\angle ABC = 30^\circ$

Given that $\angle ACB = \theta^\circ$ where $0 < \theta < 90$

(a) show that

$$(i) \sin \theta^\circ = \frac{x + 3}{2x}$$

$$(ii) \cos \theta^\circ = \frac{\sqrt{3x^2 - 6x - 9}}{2x}$$

(5)

Given that the size of $\angle BAC$: the size of $\angle ABC = 7:2$

(b) find the exact value of x

Give your answer in the form $a + a\sqrt{b}$ where a and b are prime numbers.

(5)

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- 9 Given that α is the acute angle such that $\tan \alpha = \frac{2}{3}$

(a) find the exact value of $\cos \alpha$

(1)

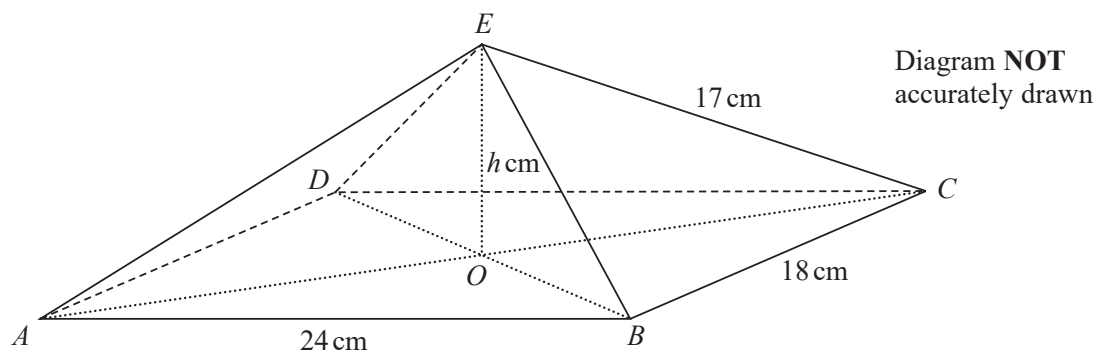


Figure 1

Figure 1 shows a right pyramid with a rectangular base $ABCD$ and vertex E

The rectangular base of the pyramid is horizontal with $AB = 24$ cm and $BC = 18$ cm.

The diagonals of the base intersect at the point O

The vertex E of the pyramid is vertically above O such that

$$AE = BE = CE = DE = 17 \text{ cm}$$

The height of the pyramid is h cm.

(b) Find the value of h

(3)

The size of the angle between the plane EBC and the plane $ABCD$ is θ°

(c) Show that $\tan \theta^\circ = \frac{2}{3}$

(2)

The point P is the midpoint of EB and the point Q is the midpoint of EC

(d) Find the size, in degrees to one decimal place, of the angle between the plane OPQ and the plane $BCQP$

(4)

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11-MARK QUESTIONS

6 questions in this booklet



- 8 (a) Solve, to the nearest integer, the equation

$$\sin(2x - 75)^\circ = -0.515 \quad \text{for } 0 \leq x < 180$$

(3)

- (b) Giving your solutions to one decimal place, where appropriate, solve the equation

$$2 \tan y^\circ + 5 \sin y^\circ = 0 \quad \text{for } 0 \leq y \leq 180$$

(4)

- (c) Explain mathematically why there are no values of θ that satisfy the equation

$$3 \cos^2 \theta^\circ - 3 \sin^2 \theta^\circ + \sin \theta^\circ + 12 = 0$$

(4)

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10 (a) Solve the equation

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \text{for } 0 \leq x \leq 2\pi$$

Give your solutions in terms of π , where appropriate.

(3)

(b) Solve the equation

$$3\sin\theta + 5\cos\theta = 0 \quad \text{for } -360^\circ \leq \theta \leq 360^\circ$$

Give your solutions to the nearest degree.

(3)

(c) Solve the equation

$$1 + \sin 2y = 2\cos^2 2y \quad \text{for } -180^\circ \leq y \leq 0^\circ$$

(5)

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- 7 (i) (a) Using a formula from page 2, show that

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \quad (2)$$

Given that $\tan 2\alpha = 1$

- (b) show that $\tan \alpha = a \pm \sqrt{b}$ where a and b are integers whose values need to be found. (3)

- (ii) (a) Using formulae from page 2, show that $\cos(x - 30)^\circ = \sin(x + 30)^\circ$ can be written as $\tan x^\circ = 1$ (4)

- (b) Hence, or otherwise, solve

$$\cos(2y - 30)^\circ = \sin(2y + 30)^\circ \quad \text{for } -90 < y \leq 90 \quad (2)$$

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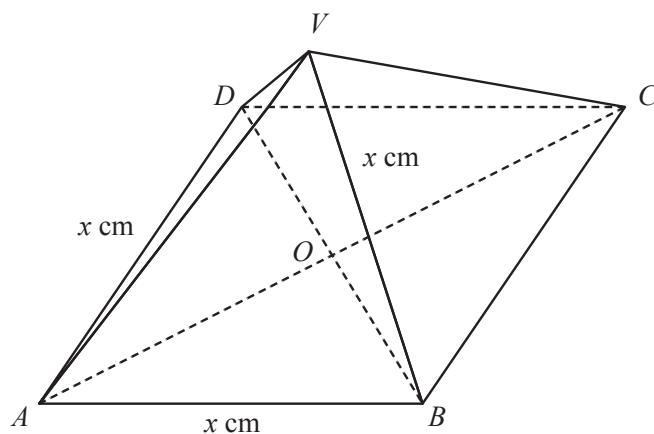


Diagram **NOT**
accurately drawn

Figure 3

Figure 3 shows a right pyramid with a horizontal square base.

$$AB = BC = CD = DA = x \text{ cm}$$

$$AV = BV = CV = DV = x \text{ cm}$$

O is the point of intersection of the diagonals of the base.

The vertex V of the pyramid is vertically above O

(a) Show that $VO = \frac{\sqrt{2}}{2}x \text{ cm}$ (3)

(b) Find, in degrees, the size of the angle AVC (2)

(c) Find, in degrees to one decimal place, the size of the angle between the plane VAB and the plane VDC (3)

The volume of the pyramid is 200 cm^3

Given that the volume of a pyramid $= \frac{1}{3} \times \text{base area} \times \text{height}$

(d) Find to 3 significant figures, the value of x (3)

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11 $f(\theta) = (2\cos\theta - \sin\theta)(2\sin\theta + \cos\theta)$

(a) Show that $f(\theta) = \frac{3}{2}\sin 2\theta + 2\cos 2\theta$

(3)

Diagram **NOT**
accurately drawn

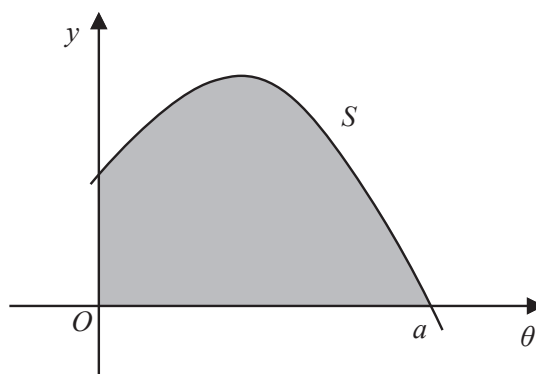


Figure 4

Figure 4 shows part of the curve S with equation $y = f(\theta) + 2$

Given that S intersects with the θ -axis at the point with coordinates $(a, 0)$

(b) using $\sin^2\theta + \cos^2\theta = 1$, or otherwise, show that $a = \frac{\pi}{2}$

(5)

(c) Using algebraic integration, find the exact area bounded by S , the positive θ -axis and the positive y -axis shown shaded in Figure 4

(3)

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- 8 (i) (a) Using a formula given on page 2, show that

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \quad (2)$$

- (b) Hence, solve the equation

$$\tan A^\circ - \tan 2A^\circ = 0 \quad \text{for } 0 \leq A \leq 180 \quad (5)$$

- (ii) Using a formula given on page 2, solve, giving your solutions as exact values

$$\cos\left(x - \frac{\pi}{6}\right) = \sin x \quad \text{for } -\pi \leq x \leq 2\pi \quad (4)$$

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12-MARK QUESTIONS

6 questions in this booklet



- 7 (a) Solve, in degrees to one decimal place,

$$(3 \cos \theta + 5)(5 \sin \theta - 3) = 0 \quad \text{for } 0 \leq \theta < 180^\circ \quad (2)$$

- (b) Show that the equation

$$8 \sin(x - \alpha) = 3 \sin(x + \alpha)$$

can be written in the form

$$5 \tan x = 11 \tan \alpha \quad (5)$$

- (c) Hence solve, to one decimal place,

$$8 \sin(2y - 30^\circ) = 3 \sin(2y + 30^\circ) \quad \text{for } 0 \leq y < 180^\circ \quad (5)$$

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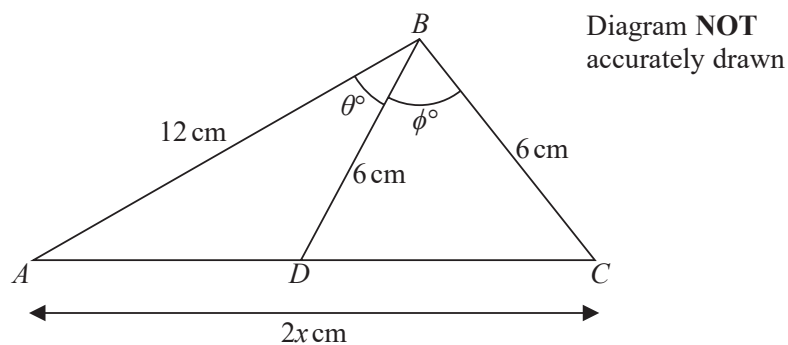


Figure 3

Figure 3 shows triangle ABC with $AB = 12$ cm, $BC = 6$ cm and $AC = 2x$ cm.

The point D is the midpoint of AC and $BD = 6$ cm.

$\angle ABD = \theta^\circ$ and $\angle DBC = \phi^\circ$ where $\theta \neq 0$ and $\phi \neq 0$

(a) Show that $\cos ADB = \frac{x^2 - 108}{12x}$ (2)

(b) Hence, or otherwise, show that $AC = 6\sqrt{6}$ cm. (4)

(c) Show that $\sin(\theta^\circ + \phi^\circ) = \sin \phi^\circ$ (4)

(d) Hence show that $\theta = 180 - 2\phi$ (2)

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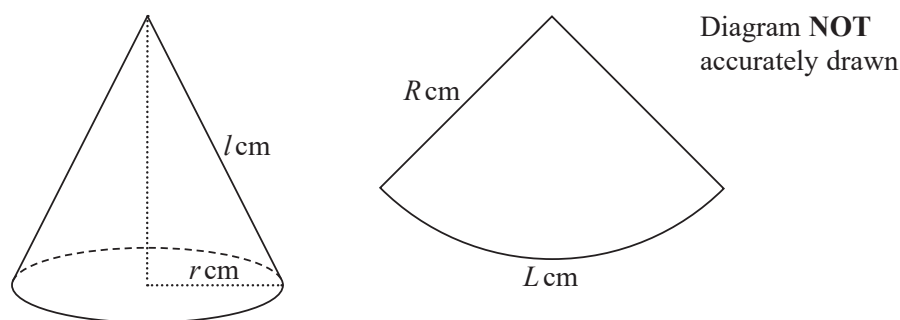


Figure 4

Figure 4 shows a right circular cone with base radius r cm and slant height l cm.
Figure 4 also shows a sector of a circle with radius R cm and arc length L cm.

The area of the curved surface of the cone is A cm²

By considering how the sector of the circle can be folded to exactly form the curved surface of the cone with R and L suitably chosen,

- (a) prove that $A = \pi rl$

(4)

Sand is poured onto a horizontal surface at a constant rate of 1.5 cm³/s.

The sand forms a pile in the shape of a right circular cone with its base on the surface.

The curved surface area of the cone, A cm², increases in such a way that the height of the cone is always three times the radius of the base of the cone.

Given that $\frac{dA}{dr} = k\pi r$, where k is a constant,

- (b) find the exact value of k .

(3)

- (c) Calculate the rate, in cm²/s, to 3 significant figures, at which the curved surface area of the pile is increasing when the height of the pile is 24 cm.

(5)

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7 (a) Solve the equation

$$\cos(3x - 15)^\circ = \frac{\sqrt{3}}{2} \quad \text{for } 0 \leq x < 180$$

(4)

(b) Solve, giving your solutions to one decimal place where appropriate,

$$3 \tan y^\circ + 4 \sin y^\circ = 0 \quad \text{for } -180 \leq y < 180$$

(4)

(c) Solve, giving your solutions to one decimal place where appropriate,

$$\cos \theta^\circ = 3 \sin^2 \theta^\circ - 1 \quad \text{for } -180 \leq \theta < 180$$

(4)

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11 (a) Use a formula on page 2 to show that $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ (3)

(b) Show that $\sin^4 x + \cos^4 x = \frac{3 + \cos 4x}{4}$ (5)

(c) Hence solve, in degrees to one decimal place, the equation

$$8 \sin^4 \left(\frac{\theta}{2} \right) + 8 \cos^4 \left(\frac{\theta}{2} \right) = 5 \sin(2\theta) + 6 \quad \text{for } 0^\circ \leq \theta < 180^\circ \quad (4)$$

11 (a) Using a formula on page 2 show that $\cos 2A = 2\cos^2 A - 1$

(2)

(b) Hence show that $(2\cos^2 A - 1)^2 = \frac{\cos 4A + 1}{2}$

(3)

The curve with equation $y = \frac{\sin 2x}{2} + \frac{(2\cos^2 x - 1)^2}{2} + \frac{1}{8}$ has a stationary point P in the range $0 \leq x \leq \frac{\pi}{6}$

(c) Find the exact coordinates of P

(7)

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13-MARK QUESTIONS

8 questions in this booklet



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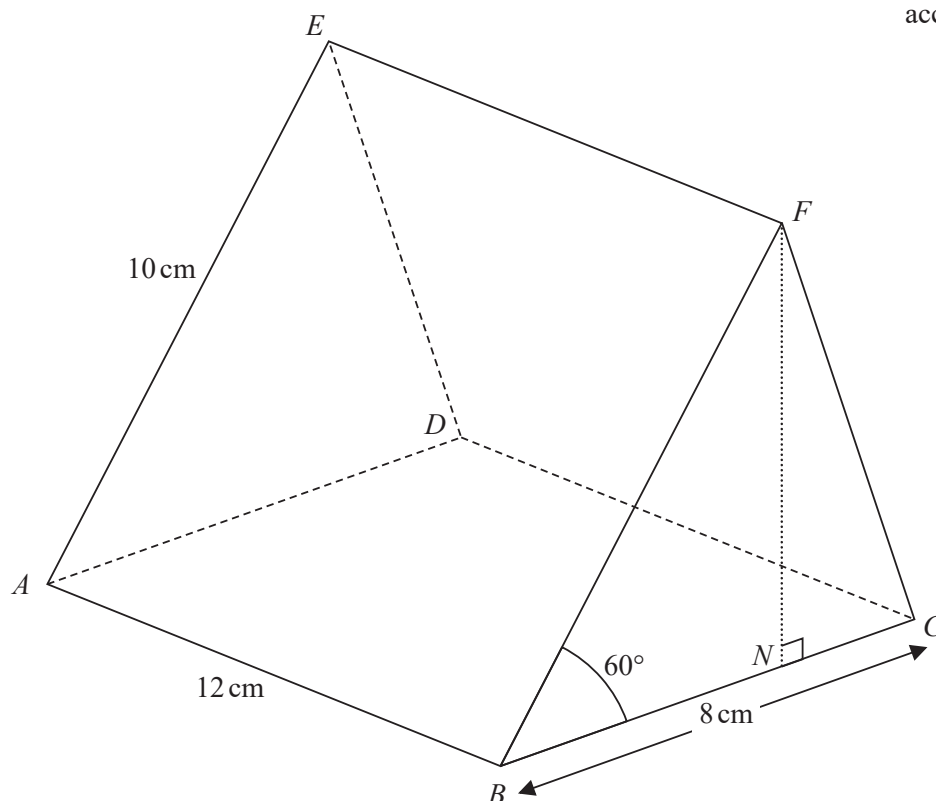
Diagram NOT
accurately drawn

Figure 3

Figure 3 shows a right prism $ABCDEF$. The cross section BCF of the prism is a triangle.

$$AB = DC = 12 \text{ cm} \quad BC = AD = 8 \text{ cm} \quad BF = AE = 10 \text{ cm} \quad \angle FBC = \angle EAD = 60^\circ$$

The point N lies on BC such that FN is perpendicular to BC .

(a) Show that $BN = 5 \text{ cm}$. (2)

(b) Find, in cm to 3 significant figures, the length of EN . (3)

The midpoint of BF is X and the midpoint of FC is Y .

(c) Find, in degrees to one decimal place, the size of the angle between the plane $ABCD$ and the plane $AXYD$. (2)

(d) Find, in degrees to one decimal place, the size of the angle AYE . (6)

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11 (a) Express the equation

$$3 \sin(A - B) = \sin(A + B)$$

in the form $\tan A = k \tan B$, giving the value of the integer k .

(4)

(b) Given that $\theta \neq \frac{(2n+1)\pi}{2}$ where $n \in \mathbb{Z}$,

show that $\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} = 1 - \tan^2 \theta$

(3)

(c) Using the exact values of $\sin x^\circ$, $\cos x^\circ$ and $\tan x^\circ$ for $x = 30, 45, 60$

show that

(i) $\cos 15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$

(2)

(ii) $\tan 255^\circ = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$

(4)

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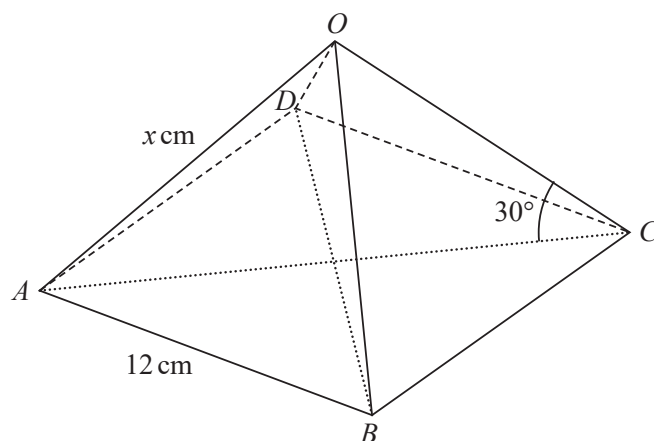
Diagram **NOT**
accurately drawn**Figure 2**

Figure 2 shows the right pyramid $OABCD$ with a square base $ABCD$ of side 12 cm.

$$OA = OB = OC = OD = x \text{ cm} \quad \text{and} \quad \angle OAC = \angle ODB = \angle OCA = \angle OBD = 30^\circ$$

- (a) Find the exact length of AC (2)
- (b) Show that $x = 4\sqrt{6}$ (2)
- (c) Find the total surface area, to the nearest cm^2 , of the pyramid. (5)
- (d) Find the size of the obtuse angle, to the nearest degree, between the plane OAB and the plane OBC (4)

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10 Using formulae given on page 2

(a) show that

(i) $\sin 2\theta = 2\sin \theta \cos \theta$

(ii) $\cos 2\theta = 2\cos^2 \theta - 1$

(5)

Given that $\theta \neq (90^\circ + 180^\circ n)$ where $n \in \mathbb{Z}$

(b) use the results from part (a) to show that $\sin 2\theta - \tan \theta$ can be written as $\tan \theta \cos 2\theta$

(4)

(c) Solve for $0 < x < 360$

$$\sin 2x^\circ - \tan x^\circ = 0$$

(4)

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9 (a) Using the formulae on page 2, show that

$$(i) \cos^2 A = \frac{\cos 2A + 1}{2}$$

$$(ii) \sin^2 A = \frac{1 - \cos 2A}{2}$$

(4)

(b) Show that

$$(2 \sin x - \cos x)(\sin x - 3 \cos x) = \frac{1}{2}(\cos 2x - 7 \sin 2x + 5)$$

(5)

$$y = (2 \sin x - \cos x)(\sin x - 3 \cos x)$$

(c) Solve, for $0^\circ \leq x \leq 180^\circ$ the equation, $\frac{dy}{dx} = 0$

Give your answers to the nearest whole number.

(4)

9

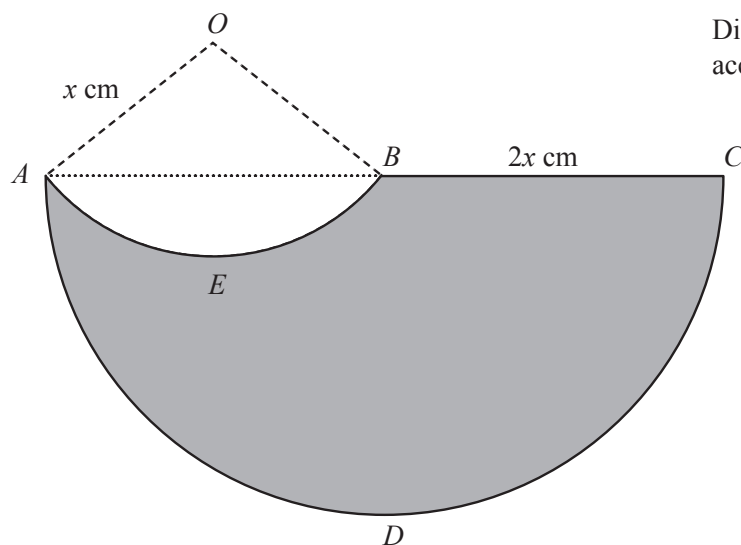


Diagram **NOT**
accurately drawn

Figure 2

A logo, $AEBCD$, is shown shaded in Figure 2.

The straight line ABC is the diameter of the semicircle ADC

AEB is an arc of a circle with centre O

All angles are measured in radians.

- $BC = 2x$ cm
- $OA = OB = x$ cm
- length of arc $AEB = 1.8x$ cm

The perimeter of the logo is P

(a) Show that $P = ax(\pi + \pi \sin 0.9 + b)$ where a and b are constants to be found.

(7)

Given that $x = 10$ cm,

(b) find, in cm^2 to 3 significant figures, the area of the logo.

(6)

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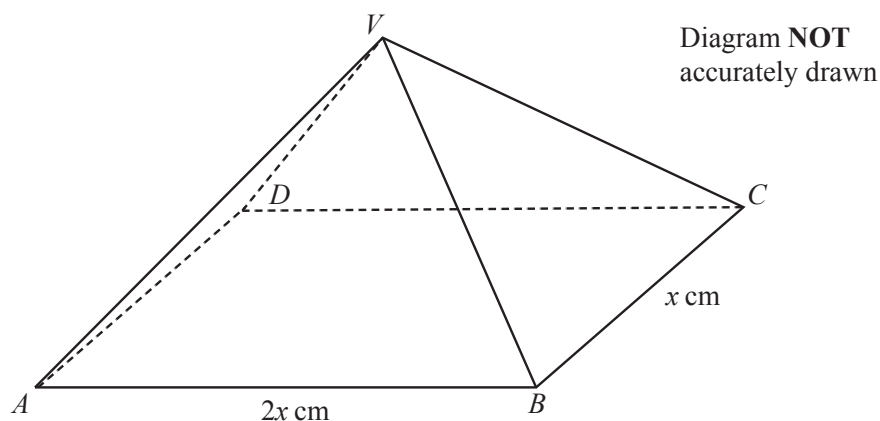


Figure 4

Figure 4 shows a right pyramid $ABCDV$

The base of the pyramid is a rectangle where,

$$AB = DC = 2x \text{ cm} \quad AD = BC = x \text{ cm}$$

The edges VA, VB, VC and VD are all of equal length.

The angle between VA and $ABCD$ is 45°

(a) Show that $VA = \frac{\sqrt{10}}{2}x \text{ cm}$ (3)

(b) Find in cm, the exact height of the pyramid in terms of x (2)

Find, in degrees to one decimal place,

(c) the size of angle VBA (2)

(d) the size of the obtuse angle between the plane AVC and the plane BVD (4)

Given that the volume of the pyramid is $9\sqrt{5} \text{ cm}^3$

(e) find the value of x (2)

8

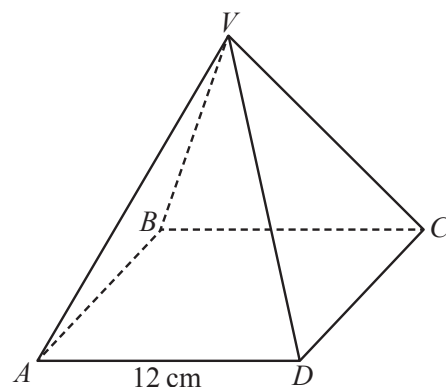


Diagram **NOT**
accurately drawn

Figure 3

Figure 3 shows a right pyramid with vertex V and square base, $ABCD$, of side 12 cm

The size of the angle between the plane VAB and the base $ABCD$ is 30°

- (a) Show that the height of the pyramid is $2\sqrt{3}\text{ cm}$ (2)
- (b) Find, in cm , the exact length of VA (3)
- (c) Find, in cm^2 , the exact area of triangle VAD (3)
- (d) Find, in cm , the exact length of the perpendicular from D to VA (2)
- (e) Find, in degrees to one decimal place, the size of the obtuse angle between the plane VAB and the plane VAD (3)

14-MARK QUESTIONS

1 question in this booklet



- 9 (a) Using a formula given on page 2, show that

$$\cos 2\theta = 2\cos^2 \theta - 1 \quad (2)$$

- (b) Hence show that

$$\int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} (2\cos^2 \theta - 1) \, d\theta = -\frac{a + \sqrt{b}}{c}$$

where a , b and c are integers to be found.

(4)

Diagram **NOT**
accurately drawn

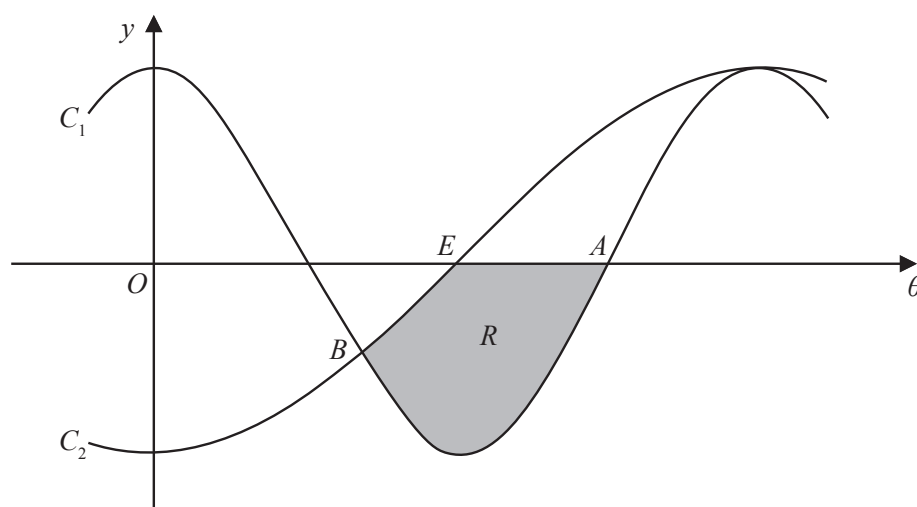


Figure 3

Figure 3 shows part of the curve C_1 with equation $y = 2\cos^2 \theta - 1$ and part of the curve C_2 with equation $y = -\cos \theta$

Point B is the intersection of C_1 and C_2 as shown in Figure 3

Point $A \left(\frac{3\pi}{4}, 0 \right)$ is the intersection of C_1 with the θ -axis as shown in Figure 3

Point $E \left(\frac{\pi}{2}, 0 \right)$ is the intersection of C_2 with the θ -axis as shown in Figure 3

The finite region R , shown shaded in Figure 3, is bounded by the θ -axis, C_1 and C_2

- (c) Use calculus to find, in its simplest form, the exact area of R (8)

15-MARK QUESTIONS

6 questions in this booklet



10 (a) Use the formula for $\cos(A + B)$ to show that $\cos 2A = 2\cos^2 A - 1$ (2)

(b) Show that $\cos 4A = 8\cos^4 A - 8\cos^2 A + 1$ (4)

(c) Solve the equation $\cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right)\left[\cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right) - 1\right] = -\frac{1}{16}$ $0 \leq \theta < 2\pi$

Give your answers in terms of π . (5)

$$f(A) = 4\cos^4 A - 4\cos^2 A + 1$$

(d) Using calculus, find the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f(A) dA$

Give your answer in the form $a\pi - b\sqrt{c}$ where a and b are fractions in their lowest terms and c is a prime number. (4)

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- 11 (a) Using a formula from page 2, show that $\cos 2x = 1 - 2 \sin^2 x$

(3)

Diagram **NOT**
accurately drawn

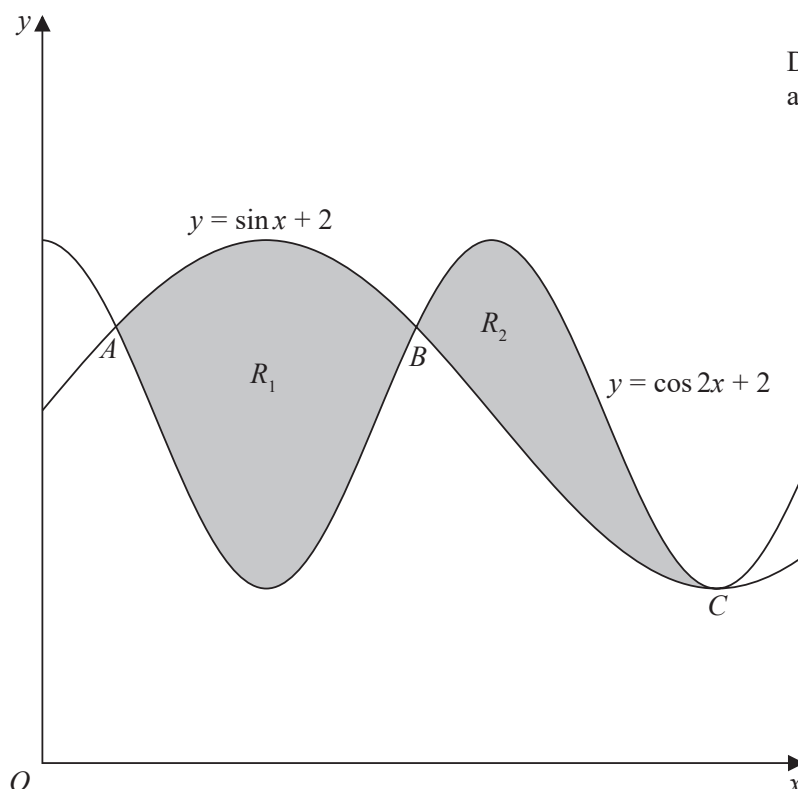


Figure 2

Figure 2 shows a sketch of part of the curves with equations $y = \sin x + 2$ and $y = \cos 2x + 2$

The points A , B and C , shown in Figure 2, are three points that are common to both curves.

- (b) Find the coordinates of each of these points.

(4)

R_1 and R_2 , shown shaded in Figure 2, are two regions enclosed by the two curves.

- (c) Use calculus to find, in its simplest form, the ratio

area of R_1 : area of R_2

(8)

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- 10 (a) Use the factor theorem to show that $(4x - 3)$ is a factor of

$$16x^3 + 11x - 15 \quad (2)$$

- (b) Using formulae given on page 2, show that

(i) $\sin 2\theta = 2 \sin \theta \cos \theta$

(ii) $\cos 2\theta = 2 \cos^2 \theta - 1$ (5)

- (c) Show that the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

becomes the equation

$$16x^3 + 11x - 15 = 0$$

by using the substitution $x = \cos \theta$

(4)

- (d) Hence show that any solution of the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

is given by $\cos \theta = \frac{3}{4}$

(4)

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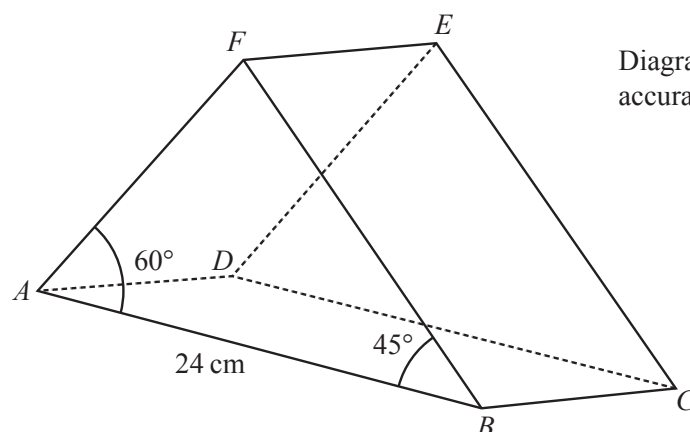
Diagram **NOT**
accurately drawn**Figure 5**

Figure 5 shows a right triangular prism $ABCDEF$ where $ABCD$ is a rectangle.

$$AF = DE \quad BF = CE \quad AD = FE = BC \quad AB = DC = 24 \text{ cm}$$

$$\angle ABF = \angle DCE = 45^\circ \quad \angle BAF = \angle CDE = 60^\circ$$

Using a formula from page 2,

(a) show that $\sin AFB = \frac{\sqrt{2} + \sqrt{6}}{4}$ (3)

Without using a calculator,

(b) show that $BF = 12(3\sqrt{2} - \sqrt{6}) \text{ cm}$ (5)

The angle between the plane AEB and the plane $ABCD$ is 65°

(c) Find, in cm to 2 significant figures, the length of EF (3)

(d) Find, in degrees to one decimal place, the size of the angle between the line CF and the plane $ABCD$ (4)

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10 (a) Using a formula from page 2, show that

(i) $\sin^2 A = \frac{1 - \cos 2A}{2}$

(ii) $\cos^2 A = \frac{\cos 2A + 1}{2}$

(5)

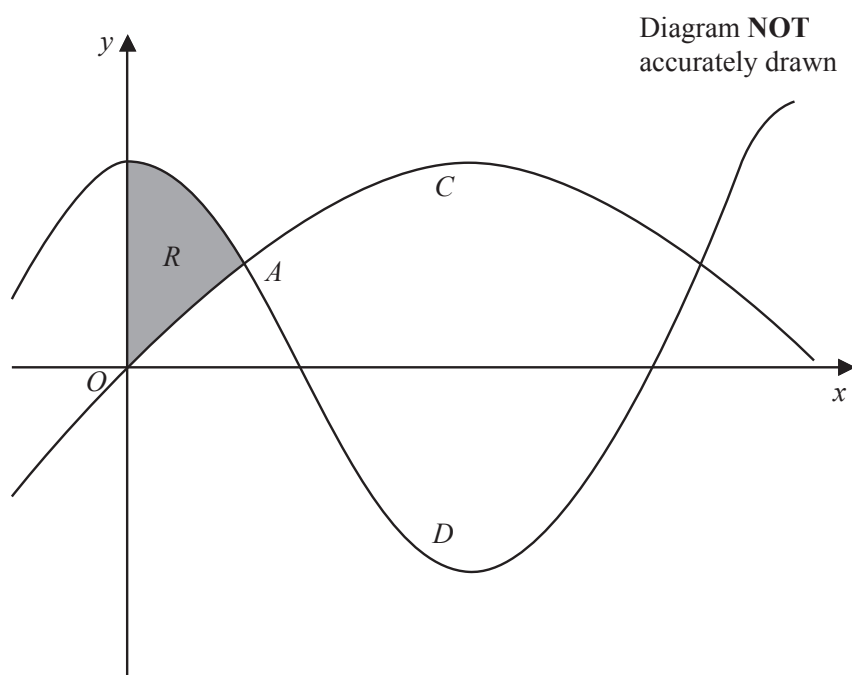


Figure 2

Figure 2 shows part of the curve C with equation $y = \sin x$ and part of the curve D with equation $y = \cos 2x$.
Curve C and curve D intersect at the point A .

(b) Use algebra to find the exact coordinates of A

(5)

The shaded region R is rotated through 360° about the x -axis.

(c) Use algebraic integration to find the exact volume of the solid generated.

Give your answer in the form $\frac{a\sqrt{a}}{b}\pi$

where a is a prime number and b is an integer.

(5)

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11 (a) Show that $\cos^4 \theta - \sin^4 \theta = \cos 2\theta$

(4)

(b) Hence, or otherwise, solve the equation

$$8 \cos^2 \left(2\theta + \frac{\pi}{4} \right) - 3 = 2 \cos^4 \left(\theta + \frac{\pi}{8} \right) - 2 \sin^4 \left(\theta + \frac{\pi}{8} \right) \quad \text{for } 0 \leq \theta < \pi$$

Give your solutions to 2 decimal places.

(7)

(c) Using calculus, find the exact value of $\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} (\cos^4 2x - \sin^4 2x - 8 \sin 4x) \, dx$

Give your answer in the form $a - b\sqrt{2}$ where a and b are rational numbers.

(4)

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16-MARK QUESTIONS

4 questions in this booklet



11

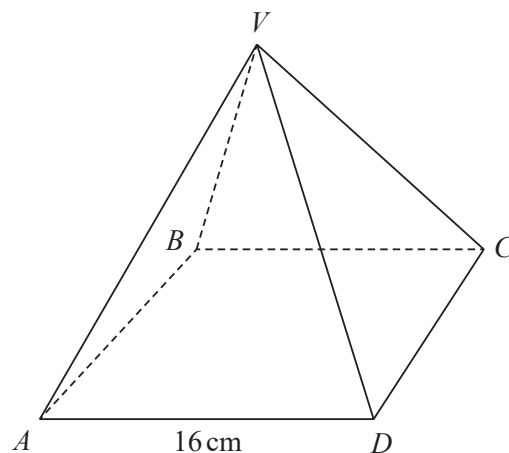
Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows a right pyramid with vertex V and square base, $ABCD$, of side 16 cm.

The size of angle AVC is 90°

(a) Show that the height of the pyramid is $8\sqrt{2}$ cm. (4)

(b) Find, in cm, the length of VA . (3)

(c) Find, in cm, the exact length of the perpendicular from D onto VA . (3)

Find, in degrees to one decimal place, the size of

(d) the angle between the plane VAB and the base $ABCD$, (3)

(e) the obtuse angle between the plane VAB and the plane VAD . (3)

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10 (a) Show that

$$\cos(A + B) + \cos(A - B) = 2 \cos A \cos B \quad (2)$$

(b) Hence show that

$$\cos P + \cos Q = 2 \cos \frac{P + Q}{2} \cos \frac{P - Q}{2} \quad (3)$$

(c) Solve, for $0 \leq \theta \leq \frac{\pi}{2}$, the equation

$$\cos 5\theta + \cos 7\theta = 0$$

Give each solution in terms of π

(4)

(d) Show that

$$\cos 8x + 2 \cos 6x + \cos 4x = 4 \cos 6x \cos^2 x \quad (3)$$

(e) Use calculus to find the exact value of

$$\int_0^{\frac{\pi}{3}} \cos 6x \cos^2 x \, dx \quad (4)$$

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10 Using formulae from page 2

(a) show that $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$ (2)

(b) Hence show that $\cos 5\theta - \cos 9\theta = 2 \sin 7\theta \sin 2\theta$ (1)

(c) Solve the equation

$$\cos 5\theta - \cos 9\theta = \sqrt{3} \sin 7\theta \quad \text{for } 0 < \theta \leq \frac{1}{3}\pi$$

Give your solutions in terms of π (7)

(d) Using calculus and showing your working, evaluate

$$\int_0^{\frac{\pi}{7}} 8 \sin 7x \cos 2x \tan 2x \, dx$$

Give your answer to 3 decimal places. (6)

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11 Using formulae from page 2, show that

$$(a) \text{ (i) } \cos 2A = 2\cos^2 A - 1 \quad (3)$$

$$\text{ (ii) } \sin 2A = 2\sin A \cos A \quad (1)$$

$$(b) \text{ Show that } \cos^3 A = \frac{\cos 3A + 3\cos A}{4} \quad (4)$$

Hence, or otherwise,

(c) solve, giving exact values in terms of π

$$8\cos^3\left(\frac{\theta}{2}\right) - 6\cos\left(\frac{\theta}{2}\right) - 1 = 0 \quad \text{for } 0 \leq \theta \leq 2\pi \quad (4)$$

(d) use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{6}} (4\cos^3 \theta - \sin 2\theta) \, d\theta \quad (4)$$

17-MARK QUESTIONS

1 question in this booklet



10 (a) Using formulae on page 2, show that

(i) $\sin 2A = 2 \sin A \cos A$

(ii) $\cos 2A = 2 \cos^2 A - 1$

(3)

$$f(\theta) = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

(b) Show that $f(\theta) = \sin 2\theta$

(4)

(c) Solve, in radians to 3 significant figures, for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, the equation

$$5 \tan \left(x + \frac{\pi}{6} \right) = \left[1 + \tan^2 \left(x + \frac{\pi}{6} \right) \right] \left[1 - 2 \cos^2 \left(x + \frac{\pi}{6} \right) \right]$$

(6)

(d) Using calculus, find the exact value of

$$\int_0^{\frac{\pi}{2}} \left(\frac{4 \tan \theta}{1 + \tan^2 \theta} - \cos 5\theta + 2 \right) d\theta$$

(4)

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MARK SCHEMES

official Pearson Edexcel mark schemes



Question number	Scheme	Marks
1 (a)	$l = r\theta \Rightarrow r = \frac{12}{1.5} = 8$	B1 [1]
(b)	$A = \frac{1.5}{2} \times 8^2 = 48 \text{ (cm}^2\text{)}$ ALT 1 $A = \frac{l^2}{2\theta} = \frac{12^2}{2 \times 1.5} = 48 \text{ (cm}^2\text{)}$ ALT 2 $A = \frac{1}{2}rl = \frac{1}{2} \times 8 \times 12 = 48 \text{ (cm}^2\text{)}$	M1A1 [2] {M1A1} [2] {M1A1} [2]
Total 3 marks		
(a) B1 (b) M1 A1 ALT 1: M1 A1 ALT 2: M1 A1	$r = 8$ $A = 48 \text{ (cm}^2\text{)}$ units not required Use of $A = \frac{1}{2}r^2\theta$ $A = 48 \text{ (cm}^2\text{)}$ units not required Use of $A = \frac{l^2}{2\theta}$ $A = 48 \text{ (cm}^2\text{)}$ units not required Use of $A = \frac{1}{2}rl$ $A = 48 \text{ (cm}^2\text{)}$ units not required	

Question number	Scheme	Marks
1 (a)	$l = r\theta \Rightarrow l = 13 \times 2 = 26 \text{ (cm)}$	B1 [1]
(b)	$A = \frac{\theta}{2} r^2 = \frac{2}{2} \times 13^2 = 169 \text{ (cm}^2\text{)}$	M1A1 [2]
Total 3 marks		
(a) B1 (b) M1 A1	$l = 26$ Use of $A = \frac{\theta}{2} r^2$ or $A = \frac{1}{2} rl$ or $A = \frac{l^2}{2\theta}$ $A = 169$	

Question	Scheme	Marks
2	$\frac{\sin \angle BCA}{10} = \frac{\sin 50}{9} \Rightarrow \angle BCA = 58.3381...^\circ \Rightarrow 58.3^\circ, 121.7^\circ$	M1A1A1
Total 3 marks		

Question	Notes	Marks
2	Uses sine rule or any other appropriate trigonometry in triangle ABC $\frac{\sin \angle BCA}{10} = \frac{\sin 50}{9}$ Note: the perpendicular height of the triangle from B to AC is 7.66044 cm. Their method must be complete for the award of this mark.	M1
	$\angle BCA = 58.3381...^\circ$	A1
	One possible value is awrt 58.3° and the other possible value is awrt 121.7°	A1 [3]
Total 3 marks		

Question	Scheme	Mark
1(a)	$A = \frac{1.2}{2} \times 7^2 = 29\frac{2}{5}$	M1A1 [2]
(b)	$P = 7 + 7 + 7 \times 1.2 = 22\frac{2}{5}$ (cm)	M1A1 [2]
Total		4 marks

Part	Mark	Notes
(a)	M1	For using the correct formula for the area of a sector with correct substitution of the given values. $A = \frac{1.2}{2} \times 7^2 = \dots$
	A1 [2]	For $A = 29\frac{2}{5}$ o.e. (cm ²)
(b)	M1	For a complete method to find the perimeter of the sector with correct substitution of the given values $P = 7 + 7 + 7 \times 1.2 = \dots$
	A1 [2]	$P = 22\frac{2}{5}$ o.e. (cm)

Question number	Scheme	Marks
2	$(2x-1)^2 = (2x+4)^2 + (x+2)^2 - 2(2x+4)(x+2)\cos 60^\circ$	M1
	$= x^2 - 16x - 11 = 0$	M1
	$x = \frac{16 \pm \sqrt{16^2 + 4 \times 11}}{2} = 8 \pm 5\sqrt{3}$	M1
	[If $x = 8 - 5\sqrt{3}$ then $2x - 1$ is negative, so] $x = 8 + 5\sqrt{3}$	A1 (4)
Total 4 marks		

Mark	Notes
M1	For the correct use of a correct cosine rule. Either $(2x-1)^2 = (2x+4)^2 + (x+2)^2 - 2(2x+4)(x+2)\cos 60^\circ$ or $\cos 60^\circ = \frac{(2x+4)^2 + (x+2)^2 - (2x-1)^2}{2 \times (2x+4) \times (x+2)}$
M1	For simplification of their expression to a 3TQ They must reach as a minimum; $Px^2 \pm Qx \pm R = 0$ where P , Q and R are non-zero constants Accept the terms in any order. Accept even for example $x^2 - 16x = 11$
M1	For an attempt to solve their 3TQ using any method. See General Guidance for the definition of an attempt. NOTE: If their 3TQ is incorrect and they do NOT show us a valid method to solve it, and two roots just appear, this is M0 .
A1	For $x = 8 + 5\sqrt{3}$ only (must reject $x = 8 - 5\sqrt{3}$)



Question	Scheme	Marks
1	Angle $(4x)^2 = (2x)^2 + (3x)^2 - 2 \times 2x \times 3x \times \cos ABC \text{ or}$ $(\cos ABC =) \frac{(2x)^2 + (3x)^2 - (4x)^2}{2 \times 2x \times 3x} \Rightarrow \angle ABC = 104.47751...^\circ (1.823476...^\circ)$ OR $(3x)^2 = (2x)^2 + (4x)^2 - 2 \times 2x \times 4x \times \cos BAC \text{ or}$ $(\cos BAC =) \frac{(2x)^2 + (4x)^2 - (3x)^2}{2 \times 2x \times 4x} \Rightarrow \angle BAC = 46.56746...^\circ (0.812755...^\circ)$ OR $(2x)^2 = (3x)^2 + (4x)^2 - 2 \times 3x \times 4x \times \cos ACB \text{ or}$ $(\cos ACB =) \frac{(3x)^2 + (4x)^2 - (2x)^2}{2 \times 3x \times 4x} \Rightarrow \angle ACB = 28.95502...^\circ (0.505360...^\circ)$ Area $50 = \frac{1}{2} \times 2x \times 3x \times \sin 104.47751^\circ \Rightarrow x^2 (=17.213259...)$ $\Rightarrow x = 4.15$ OR $50 = \frac{1}{2} \times 2x \times 4x \times \sin 46.56746^\circ \Rightarrow x^2 (=17.213259...)$ $\Rightarrow x = 4.15$ OR $50 = \frac{1}{2} \times 3x \times 4x \times \sin 28.95502^\circ \Rightarrow x^2 (=17.213259...)$ $\Rightarrow x = 4.15$	M1A1 [M1A1] [M1A1] dM1 A1 [dM1 A1] [4]
ALT1	Uses Heron's formula $s = \frac{2x + 3x + 4x}{2} = \frac{9x}{2} \quad \text{oe}$ $50 = \sqrt{\left(\frac{9x}{2}\right)\left(\frac{9x}{2} - 2x\right)\left(\frac{9x}{2} - 3x\right)\left(\frac{9x}{2} - 4x\right)} = \sqrt{\frac{135x^4}{16}}$ $\Rightarrow x^2 = 17.213259... \Rightarrow x = 4.15$	M1A1 dM1 A1 [4]

ALT 2	$(4x)^2 = (2x)^2 + (3x)^2 - 2 \times 2x \times 3x \times \cos ABC \text{ or}$ $(\cos ABC =) \frac{(2x)^2 + (3x)^2 - (4x)^2}{2 \times 2x \times 3x} \left(= -\frac{1}{4} \right)$ $\Rightarrow (\sin ABC =) \sqrt{\frac{15}{16}} \left(= \frac{\sqrt{15}}{4} = 0.9682458... \right) \text{ oe}$ OR $(3x)^2 = (2x)^2 + (4x)^2 - 2 \times 2x \times 4x \times \cos BAC \text{ or}$ $(\cos BAC =) \frac{(2x)^2 + (4x)^2 - (3x)^2}{2 \times 2x \times 4x} \left(= \frac{11}{16} \right)$ $\Rightarrow (\sin ABC =) \sqrt{\frac{135}{256}} \left(= 3 \frac{\sqrt{15}}{16} = 0.7261843... \right) \text{ oe}$ OR $(2x)^2 = (3x)^2 + (4x)^2 - 2 \times 3x \times 4x \times \cos ACB \text{ or}$ $(\cos ACB =) \frac{(3x)^2 + (4x)^2 - (2x)^2}{2 \times 3x \times 4x} \left(= \frac{7}{8} \right)$ $\Rightarrow (\sin ABC =) \sqrt{\frac{15}{64}} \left(= \frac{\sqrt{15}}{8} = 0.4841229... \right) \text{ oe}$ $50 = \frac{1}{2} \times 2x \times 3x \times \sqrt{\frac{15}{16}} \Rightarrow x^2 (=17.213259...) \text{ or } x =$ $\Rightarrow x = 4.15$ OR $50 = \frac{1}{2} \times 2x \times 4x \times \sqrt{\frac{135}{256}} \Rightarrow x^2 (=17.213259...) \text{ or } x =$ $\Rightarrow x = 4.15$ OR $50 = \frac{1}{2} \times 3x \times 4x \times \sqrt{\frac{15}{64}} \Rightarrow x^2 (=17.213259...) \text{ or } x =$ $\Rightarrow x = 4.15$	M1 A1 {M1 A1} {M1 A1} dM1 A1 {dM1 A1} {dM1 A1} [4]
Total 4 marks		

Mark	Notes
M1	For a fully correct substitution into the cosine formula as shown. ie including an equals sign. OR for a fully correct expression for $\cos ABC$ or $\cos BAC$ or $\cos ACB$ Allow students to use just B , A and C for angles or any labelling.
A1	For one of the correct angles in the triangle.
Note for M1 A1 In this question, we will override the general principle of marking for multiple attempts, if the attempts are finding other angles. Mark the attempt that is correct. The x can be consistently omitted or the x can be recovered (also indicated by a correct angle) or values in the correct proportions can be used as an alternative (also indicated by a correct angle). M1 may also be awarded if the values in the expression or equation lead to the correct angle for those values, regardless of the labelling of the angle. This may also lead to A1	

4	$(2x+5)^2 = (3x-1)^2 + (5x)^2 - 2 \times (3x-1) \times 5x \cos 60^\circ$ $15x^2 - 21x - 24 (= 0) \quad (5x^2 - 7x - 8 = 0)$ $x = \frac{21 \pm \sqrt{21^2 + 4 \times 15 \times 24}}{30}$ $x = 2.1456..... \text{ (or } -0.7456...)$ $\therefore x = 2.15$	M1A1 A1 M1 A1	[5]
M1 A1 A1 M1 Alcao	Use the cosine rule in either form. Rule to be correct either by quoting and using the general formula or by implication from a correct substitution. Correct substitution in their cosine rule. Simplify to obtain a 3TQ. Terms in any order. $= 0$ may be missing Solve their 3TQ by formula (correct general formula or correct substitution for their equation) or completing the square. Reach a positive value for x. Negative need not be seen. Calculator solutions: Correct answer from correct equation scores M1A1, otherwise M0A0 Correct value for x. Must be 3 sf Negative value (if shown) must be eliminated or positive clearly identified as the required value..		

Question number	Scheme	Marks
3	$33 = \frac{1}{2} \times 11 \times 12 \times \sin \angle ABC \Rightarrow \sin \angle ABC = \frac{1}{2} \quad [30^\circ \text{ or } 150^\circ]$ $AC = \sqrt{11^2 + 12^2 - 2 \times 11 \times 12 \cos 30^\circ} = 6.0306... \approx 6.03 \text{ (cm)}$ $AC = \sqrt{11^2 + 12^2 - 2 \times 11 \times 12 \cos 150^\circ} = 22.2178... \approx 22.2 \text{ (cm)}$	M1A1 M1A1 A1 [5]
Total 5 marks		
M1	Use of $\frac{1}{2}ab \sin C = 33$ (must be set = 33)	
A1	30° and 150° (Accept $\frac{\pi}{6}$ and $\frac{5\pi}{6}$)	
M1	Use of $c^2 = a^2 + b^2 - 2ab \cos C$ (square root not required for M1)	
A1	6.03	
A1	22.2	
	NB If both answers are not given to 3sf but are correct then award A1A0	

Question number	Scheme	Marks
1 (a)	$\left[\cos 2A = \cos^2 A - \sin^2 A \text{ and } \cos^2 A + \sin^2 A = 1 \right]$ $\Rightarrow [\cos 2A] = (1 - \sin^2 A) - \sin^2 A$ $\Rightarrow [\cos 2A] = 1 - 2\sin^2 A^*$	M1 A1 cso [2]
(b)	$\frac{4-y}{3} = 1 - 2\left(\frac{x+1}{2}\right)^2$ $\Rightarrow \frac{4-y}{3} = 1 - 2\left(\frac{x^2 + 2x + 1}{4}\right) \Rightarrow y = \dots$ $\Rightarrow y = \frac{1}{2}(3x^2 + 6x + 5)^*$	M1 M1 A1* cso [3]
Total 5 marks		

Part	Mark	Notes
(a)	M1	Substitutes the identity $\cos^2 A + \sin^2 A = 1$ into the simplified summation formula for cosine to achieve the given identity [for $\cos 2A$]
	A1*	For the correct identity with no errors seen.
(b)	M1	For writing $\frac{4-y}{3} = 1 - 2\left(\frac{x+1}{2}\right)^2$ or equivalent at any stage in their working. Allow equivalence to include any attempted expansion of $\left(\frac{x+1}{2}\right)^2$
	M1	For expanding the given expression for $\sin A$ to achieve an expression for y in terms of x . Their working must reach $y = \dots$ for the award of this mark. Note: The expansion of $\left(\frac{x+1}{2}\right)^2$ must be correct. Allow one error only in re-arrangement.
	A1*	For the correct expression. Note: this is a given answer, there must be no errors in their working for the award of this mark.

Question	Scheme	Mark
2	$\sin(2\theta - 20)^\circ = \sqrt{3} \cos(2\theta - 20)^\circ \Rightarrow \tan(2\theta - 20)^\circ = \sqrt{3}$	M1
	$(2\theta - 20)^\circ = 60^\circ, 240^\circ, 420^\circ, \dots$	M1A1
	$\theta = \frac{'60' + 20}{2} = 40^\circ, \quad \theta = \frac{'240' + 20}{2} = 130^\circ$	M1A1 [5]
Total		5 marks

Mark	Notes
M1	For using the identity $\frac{\sin A}{\cos A} = \tan A$ to reach $\tan(2\theta - 20)^\circ = k$ where k is a numerical value $\sin(2\theta - 20)^\circ = \sqrt{3} \cos(2\theta - 20)^\circ \Rightarrow \tan(2\theta - 20)^\circ = \sqrt{3}$
M1	For finding at least one correct angle for $(2\theta - 20)^\circ$ $(2\theta - 20)^\circ = 60^\circ, 240^\circ, 420^\circ, \dots$ Allow even for eg., -120° This mark can be implied by correct final answers.
A1	For both of the angles 60° and 240° Ignore any extra values, even if within range. $0 \leq (2\theta - 20) \leq 360$ This mark can be implied by correct final answers.
M1	For correct processing of their values for $(2\theta - 20)^\circ$ $\theta = \frac{'60' + 20}{2} = \dots$ or $\theta = \frac{'240' + 20}{2} = \dots$
A1 [5]	For both correct values of $\theta = 40$ and 130 Ignore other angles out of range, penalise extra angles within the range by the loss of this A mark

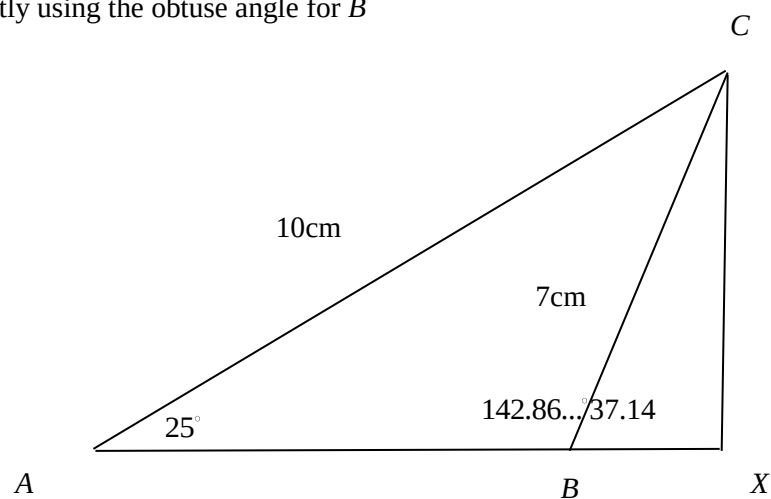
Q	SCHEME	MARKS
3	$\frac{\sin 25}{7} = \frac{\sin B}{10} \text{ oe } (B =) \sin^{-1}\left(\frac{10 \sin 25}{7}\right) \text{ oe } (B = 37.138(254541) \text{ or } 142.861(7455))$ $(C = 180 - [(180 - 37.138(25454)) + 25])$ $(C =) 12.138(25451)$ $(c^2 =) 7^2 + 10^2 - 2(7)(10) \cos "12.138(25451)" \text{ or }$ $\frac{c}{\sin "12.138(25451)" } = \frac{7}{\sin 25} \text{ oe eg } (c =) \frac{7}{\sin 25} \times \sin "12.138(25451)" \text{ or }$ $\frac{c}{\sin "12.138(25451)" } = \frac{10}{\sin "142.861(7455)" } \text{ oe }$ $\text{eg } (c =) \frac{10}{\sin "142.861(7455)" } \times \sin "12.138(25451)"$ **allow students to follow through using their acute angle for B $(c^2 =) 7^2 + 10^2 - 2(7)(10) \cos "117.861(7455)" \text{ or }$ $\frac{c}{\sin "117.861(7455)" } = \frac{7}{\sin 25} \text{ oe eg } (c =) \frac{7}{\sin 25} \times \sin "117.861(7455)" \text{ or }$ $\frac{c}{\sin "117.861(7455)" } = \frac{10}{\sin "37.138(254541)" } \text{ oe }$ $\text{eg } (c =) \frac{10}{\sin "37.138(254541)" } \times \sin "117.861(7455)"$ $(c =) 3.5(\text{cm})$	M1dM1 A1 (M1 on ePen) ddM1 A1 [5]
ALT1	$7^2 = 10^2 + (AB)^2 - 2(10)(AB) \cos 25$ $0 = (AB)^2 - 20 \cos 25(AB) + 100 - 49 \quad (20 \cos 25 = 18.12615574)$ $0 = (AB)^2 - 20 \cos 25(AB) + 51$ $(AB =) \frac{-(-20 \cos 25) \pm \sqrt{(-20 \cos 25)^2 - 4(1)(51)}}{2 \times 1} (= 3.48, 14.64)$ $(c =) 3.5(\text{cm})$	M1 dM1 A1 (M1 on ePen) ddM1 A1 [5]
ALT2	M1 M1 as main scheme (then students are working in right angled triangles). $(AX =) 10 \cos 25 \text{ or } 9.(06307787)$ $(BX =) 7 \sin ("37.14.....") \Rightarrow 10 \cos 25 - 7 \sin ("37.14.....")$ or if working with acute angle $(BX =) 7 \sin ("37.14.....") \Rightarrow 10 \cos 25 + 7 \sin ("37.14.....")$ $= 3.5((\text{cm}))$	B1 (M1 on ePen) ddM1 A1 [5]
Total 5 marks		

Part	Mark	Additional Guidance
	M1	For a fully correct substitution into the sine rule.
	dM1	For a correct rearrangement to find angle B , implied by sight of the correct acute or obtuse angle. Dependent on previous method mark.
	A1 (M1 on ePen)	For angle C , accept angles which round to 12.
	ddM1	Correct substitution into the cosine rule or sine rule using their angles B and/or C . Allow angle B to be an acute angle and angle C to be the angle that comes from this acute angle using the sum of angles in a triangle. This mark is dependent on the first 2 method marks and is for correct use of the cosine or sine rule, given they have found an angle for B and used this to find angle C .
	A1	For awrt 3.5 (cm)
ALT1	M1	For a fully correct substitution into the cosine rule. Allow AB to be denoted by any letter
	dM1	For a fully correct rearrangement = 0, dependent on the previous method mark.
	A1 (M1 on ePen)	For the correct 3 term quadratic equation = 0.
	ddM1	For a minimally acceptable attempt to solve their quadratic – see general guidance. Dependent on both previous method marks. Sight of 3.48..... or 14.64 may imply this mark
	A1	For awrt 3.5 (cm) Must reject any second solution.
ALT2	M2	As first 2 marks in main scheme
	B1 (M1 on ePen)	For AX either $10\cos 25$ or answers which round to 9
	ddM1	For finding BX correctly and a correct method to find AB , allowing use of acute B
	A1	For awrt 3.5 (cm)

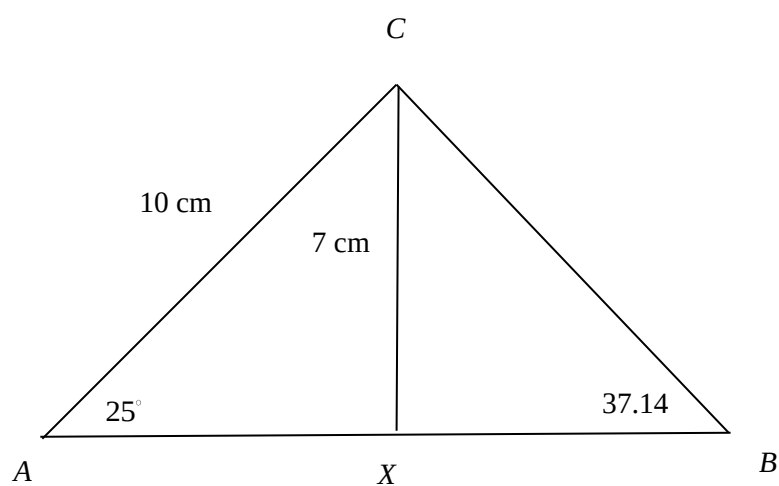


Particularly to help with ALT2:

If correctly using the obtuse angle for B



Using the acute angle for B



Question number	Scheme	Marks
1 (a)	$\frac{1}{2}2^2\theta = \frac{\pi}{2} \Rightarrow \theta =$ $\theta = \frac{\pi}{4}$	M1 A1 [2]
(b)	$2 \times \frac{\pi}{4} \left(= \frac{\pi}{2} \right)$ $\frac{\pi}{2} + 2 + 2 \quad \text{or} \quad \frac{\pi}{2} + 2 \times 2 \quad \text{or} \quad \frac{\pi}{2} + 4$ $4 + \frac{\pi}{2} \quad \text{oe}$	M1 M1 A1 [3]
Total 5 marks		

Part	Mark	Additional Guidance
(a)	M1	Correct use of formula and rearrangement to give θ . Allow one error in rearrangement. Allow work in degrees for this mark.
	A1	Correct exact value in radians. isw
(b)	M1	Correct use of formula for length of arc with their value for θ
	M1	Their arc length + 2 + 2
	A1	Correct exact value – allow work in degrees throughout. Do not isw Note: 5.6 is the correct estimated value, which implies M1M1A0

Question number	Scheme	Marks
1	$\sqrt{3} \tan A = 2 \sin A \Rightarrow \frac{\sqrt{3} \sin A}{\cos A} = 2 \sin A$	M1
	$\Rightarrow \sqrt{3} \sin A - 2 \sin A \cos A = 0 \Rightarrow \sin A (\sqrt{3} - 2 \cos A) = 0$	M1
	$\Rightarrow \sin A = 0 \quad \text{or} \quad \cos A = \frac{\sqrt{3}}{2}$	M1
	$\Rightarrow A = \sin^{-1} 0 = 0^\circ, 180^\circ, 360^\circ, \quad \cos A = \frac{\sqrt{3}}{2} \Rightarrow A = 30^\circ, 330^\circ$	A1A1 [5]
Total 5 marks		

Mark	Additional Guidance
M1	Uses identity $\tan = \frac{\sin}{\cos}$
M1	Rearranges to obtain $\sqrt{3} \sin A - 2 \sin A \cos A = 0$ or it's equivalent AND factorises to obtain $\sin A (\sqrt{3} - 2 \cos A) = 0$ This step can be implied by further correct work. If they cancel through by $\sin A$ this is M0
M1	Finds any one value for either $\sin A$ or $\cos A$
A1	For at least any two correct angles obtained from either $\cos A$ or $\sin A$
A1	For all five correct angles Ignore any values outside of the range. Withhold this mark for any extra angles within range.

Question	Scheme	Marks
3 (a)	$\frac{\sin B}{7} = \frac{\sin 65^\circ}{10}$ $B = 39.37\dots = 39.4^\circ$	M1A1 A1 [3]
(b)	$\text{Area} = \frac{1}{2} \times 10 \times 7 \sin(180 - 65 - "39.37")$ $= 33.90\dots = 33.9 \text{ (cm}^2\text{)}$	M1A1 A1 [3]
Total 6 marks		

Additional Guidance		
(a)	M1	For the correct use of a correct Sine rule $\frac{\sin B}{7} = \frac{\sin 65^\circ}{10}$ seen only is sufficient for the award of this mark.
	A1	For reaching $\sin B = \frac{7 \times \sin 65}{10}$
	A1	For $B = 39.37\dots^\circ \approx 39.4^\circ$ correctly rounded answer only.
(b)	M1	For writing a correct expression for the area reaching a value for the area. $\text{Area} = \frac{1}{2} \times 10 \times 7 \sin(180 - 65 - "39.37")$
	A1	For using the correct value for angle B ($39.37\dots^\circ$ or 39.4°)
	A1	For an area of $33.9 \text{ (cm}^2\text{)}$ rounded correctly.
Rounding		In this question penalise rounding separately for part (a) and part (b)

Question	Scheme	Marks
4 (a)	$\frac{1}{2} \times 15^2 \theta - \frac{1}{2} \times 10^2 \theta = 100$ $\theta = \frac{200}{125} = \frac{8}{5} \quad (\text{accept } 1.6)$	M1M1 A1 [3]
(b)	$\text{Perimeter} = 10 \times \frac{8}{5} + 15 \times \frac{8}{5} + 2 \times 5 = 50 \text{ (cm)}$	M1M1A1 [3]
Total 6 marks		

Additional Guidance		
(a)	M1	For the correct length of OC (and OD) $OC = OD = 10$ (cm)
	M1	For using the correct formula for the area of a sector and evaluating to reach a value for θ $\frac{1}{2} \times 15^2 \theta - \frac{1}{2} \times 10^2 \theta = 100 \Rightarrow \theta = \dots$ (using their value for OC or OD)
	A1	For the correct value of $\theta = \frac{200}{125}$ o.e
(b)	M1	For using the correct arc formula for the length of both arcs using their values for OC and θ $10 \times \frac{8}{5} \text{ and } 15 \times \frac{8}{5}$
	M1	For a complete expression for the perimeter for the region $ABDC$ and for evaluating their expression. $\text{Perimeter} = 10 \times \frac{8}{5} + 15 \times \frac{8}{5} + 2 \times 5 = \dots \text{ (cm)}$
	A1	For 50 (cm)

Question number	Scheme	Marks
4	Area of sector = $0.4r^2$	B1
	$BC = r \tan 0.8$	B1
	Area of triangle = $\frac{1}{2}r^2 \tan 0.8$	B1 ft
	Shaded region = $\frac{1}{2}r^2 \tan 0.8 - 0.4r^2 = 101$	M1 M1
	$r^2 = \frac{101}{\frac{1}{2} \tan 0.8 - 0.4}$	A1
	$r = 29.7$	[6]

Mark	Additional Guidance
	Accept angle converted to degrees $0.8^\circ = 45.84^\circ$ throughout $\tan(0.8) = \tan(45.8\dots)^\circ = 1.0296$
B1	For the correct area of the sector = $\frac{0.8}{2}r^2$ oe (need not be simplified)
B1	For $BC = r \tan 0.8$ oe e.g. accept $\tan\left(\frac{4}{5}\right) = \frac{BC}{r}$ This may be embedded in ' $\frac{r \times r \tan 0.8}{2}$ ', ' $\frac{0.8}{2}r^2$ ' = 101 Award when seen.
B1ft	$A = \frac{r \times r \tan 0.8}{2}$
M1	Shaded region = ' $\frac{r \times r \tan 0.8}{2}$ ', ' $\frac{0.8}{2}r^2$ ' = 101 Ft their expressions for the areas of the sector and triangle provided they are as a minimum $kr^2 \tan 0.8$ and lr^2 where k and l are constants
dM1	This mark is dependent on the previous M mark For attempting to solve their equation $r = \sqrt{\frac{101}{\frac{1}{2} \tan 0.8 - 0.4}} = (29.658\dots)$ This is an A mark in Epen
A1	$r = 29.7$ only

Question number	Scheme	Marks
2 (a)(i)	$(\tan \alpha =) \frac{4}{3}$	B1
(a)(ii)	$(\tan \beta =) -\frac{1}{\sqrt{3}} \left(= -\frac{\sqrt{3}}{3} \right)$	B1 B1 (3)
(b)	$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ $= \frac{\frac{4}{3} + \left(-\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{4}{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}$ $= \frac{4\sqrt{3} \pm 3}{3\sqrt{3} \pm 4} \quad \text{or} \quad \frac{12 + (-3\sqrt{3})}{9 + 4\sqrt{3}}$ $= \frac{4\sqrt{3} - 3}{3\sqrt{3} + 4}$	M1 dM1 A1 cso (3)
Total 6 marks		

Part	Mark	Notes
(a)(i)	B1	For $(\tan \alpha =) \frac{4}{3}$ accept $1.\dot{3}$ or 1.3^r or 1.3 recurring
(a)(ii)	B1	For $(\tan \beta =) \pm \frac{1}{\sqrt{3}}$ (i.e. 1st B mark allow + or – sign with the exact value shown oe)
	B2	$(\tan \beta =) -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$ oe (i.e. 2 B marks correct sign with correct exact value)
B marks are awarded independent of method, so award for the exact values stated oe		
(b)	M1	For using the correct formula for $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ Note: The formula is given on page 2 of this paper and there must be correct substitution for their exact values obtained in part (a) $\tan(\alpha + \beta) = \frac{\frac{4}{3} + \left(-\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{4}{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}$
	dM1	For attempting to simplify their expression for $\tan(\alpha + \beta)$ as far as $\frac{a \pm b\sqrt{c}}{d\sqrt{c} \pm e}$ where a, b, c, d and e are integers. For the correct substitution of their values and check the common denominators are correct for their values in both the numerator and denominator of $\tan(\alpha + \beta)$ The numerator and denominator must be of the form $p \pm q$ where q contains a surd. If they use $\tan \beta = \pm \frac{1}{\sqrt{3}}$ they will get to: $\tan(\alpha + \beta) = \frac{\frac{4}{3} + \left(\pm \frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{4}{3}\right)\left(\pm \frac{1}{\sqrt{3}}\right)} = \frac{\frac{4\sqrt{3} \pm 3}{3\sqrt{3}}}{\frac{3\sqrt{3} \mp 4}{3\sqrt{3}}} = \frac{4\sqrt{3} \pm 3}{3\sqrt{3} \mp 4}$ If they use $\tan \beta = -\frac{\sqrt{3}}{3}$ they will get to: $\tan(\alpha + \beta) = \frac{\frac{4}{3} + \left(\pm \frac{\sqrt{3}}{3}\right)}{1 - \left(\frac{4}{3}\right)\left(\pm \frac{\sqrt{3}}{3}\right)} = \frac{\frac{4 + (\pm\sqrt{3})}{3}}{\frac{9 \mp 4\sqrt{3}}{9}} \text{ or } \frac{\frac{12 + (\pm 3\sqrt{3})}{9}}{\frac{9 \mp 4\sqrt{3}}{9}} = \frac{12 + (\pm 3\sqrt{3})}{9 \mp 4\sqrt{3}}$ Note: This mark is dependent on the previous M mark.
	A1	For simplifying to the correct final answer with no errors seen. $\tan(\alpha + \beta) = \frac{4\sqrt{3} - 3}{3\sqrt{3} + 4} \text{ o.e. for example } \frac{12\sqrt{3} - 9}{9\sqrt{3} + 12}$ Must be in the form $\frac{m\sqrt{3} - n}{n\sqrt{3} + m}$

Question number	Scheme	Marks
3 (a)	$\left(\frac{1}{2} \times 4^2 \times \theta = 2\pi\right)$ $\frac{1}{2} \times "16" \times \theta = 2\pi \Rightarrow \theta = \dots$ $\theta = \frac{2\pi}{8} = \frac{\pi}{4} \quad \text{oe}$	M1 A1 [2]
(b)	$(\text{arc } SR =) 4 \times " \frac{\pi}{4} " (= \pi)$ $(4 + 4 + \pi) = 11.1 \text{ (cm)} \quad \text{Accept awrt 11.1 (cm)}$	M1 A1ft [2]
(c)	$\frac{1}{2} \times (1) \times 4 \times \sin " \frac{\pi}{4} " (= \sqrt{2})$ $2\pi - \sqrt{2} \quad \text{oe}$	M1 A1 [2]
Total 6 marks		

Part	Mark	Additional Guidance
(a)	M1	Use of the correct formula for the area of sector, with correct substitution of 4 and 2π , an attempt to square r and to rearrange to find a value for θ . The rearrangement has to be correct for this mark.
	A1	Accept any valid exact equivalent. Do not accept answers in degrees or answers not given exactly for A1. However, 45° or $0.785(3981\dots)$ stated with no method - award implied M1.
(b)	M1	Use of correct formula for length of an arc with the correct radius and “their θ ” (allow working in degrees for this mark) from part (a). Do not allow $\theta = \frac{\pi}{2}$ or 90°
	A1ft	Allow ft of their angle from part (a), if complete method shown and if their answer is correct to 3sf. Note this is ft mark. Check working carefully.
(c)	M1	Use of correct formula for area of a triangle with correct sides substituted and “their θ ” (allow working in degrees) from part (a). Do not allow $\theta = \frac{\pi}{2}$ or 90°
	A1	For the correct answer shown.

Question number	Scheme	Marks
6	Correctly identifies the angle VXO where X is the midpoint of CD and O is the foot of the perpendicular from V There are other valid triangles that can be used. $VC = a$ and $CX = \frac{a}{2}$ note could also be $2a$ and a or could use values where VC is twice the length of CX $((VX)^2) = a^2 - \left(\frac{a}{2}\right)^2$ $VX = \frac{\sqrt{3}a}{2} \text{ oe}$ $\cos \theta = \frac{\frac{a}{2}}{\frac{\sqrt{3}a}{2}} \text{ oe}$ $\left[\cos \theta = \frac{1}{\sqrt{3}} \text{ oe} \right]$ leading to $\theta =$ 54.7 $((VO)^2) = \sqrt{a^2 - \left(\frac{\sqrt{2}}{2}a\right)^2}$ $VO = \frac{\sqrt{2}}{2}a \text{ oe}$ $\tan \theta = \frac{\frac{\sqrt{2}a}{2}}{\frac{a}{2}} \text{ oe}$ $\left[\tan \theta = \sqrt{2} \right]$ leading to $\theta =$ 54.7	B1 M1 dM1 A1 M1 A1
Total 6 marks		

Part	Mark	Additional Guidance
	B1	Angle identified in written work or on diagram. Allow labelling to be any letters.
	M1	Denotes any side of the pyramid with a and any appropriate length on the base $\frac{a}{2}$. This can be in written work or on the diagram. The two sides can be any two sides (including values) which will form a triangle with the required angle and must be used in the work that follows (even if incorrectly). Allow if the candidate denotes any side of the pyramid with a and identifies AO as $\frac{\sqrt{2}a}{2}$ oe
	dM1	Uses Pythagoras with a minus sign in a correct triangle with correctly labelled sides.
	A1	Correct expression for their correct choice of sides oe.
	M1	Working in triangle VXO (or other valid triangle) with their values from previous working, using any appropriate trigonometry.
	A1	Awrt 54.7

Question	Scheme	Marks
3	Mark parts (i) and (ii) together. (Let the obtuse angle $AOB = \alpha$ - students do not need to state this, allow use of any symbol or letter, including θ) $53.2 = r\alpha$ $372.4 = \frac{r^2\alpha}{2}$ $\alpha = \frac{53.2}{r}, \quad 372.4 = \frac{r^2}{2} \times \frac{53.2}{r} \text{ oe } \Rightarrow r = 14$ $\alpha = \frac{53.2}{14} = 3.8$ $\theta = (2\pi - 3.8) = 2.48$	B1 B1 M1A1 M1 A1 [6]
ALT1 final 4 marks	First 2 marks as ALT2 $r = \frac{53.2}{\alpha}, \quad 372.4 = \frac{\left(\frac{53.2}{\alpha}\right)^2}{2} \times \alpha \quad \text{oe}$ $\Rightarrow \alpha = \frac{372.4}{1415.12} = 3.8$ $r \left(= \frac{53.2}{3.8} \right) = 14$ $\theta = (2\pi - 3.8) = 2.48$	B1B1 M1 M1(A1 on ePen) A1(M1 on ePen) A1 [6]
ALT2	$2\pi r - r\theta = 53.2 \quad \text{oe}$ $\pi r^2 - \frac{1}{2}r^2\theta = 372.4 \quad \text{oe}$ $\theta = 2\pi - \frac{53.2}{r} \Rightarrow \pi r^2 - \frac{1}{2}\left(2\pi - \frac{53.2}{r}\right)r^2 = 372.4 \quad \text{oe}$ $26.6r = 372.4 \quad \text{oe}$ $r = 14$ $\theta = 2.48$	B1 B1 M1 M1(A1 on ePen) A1(M1 on ePen) A1 [6]

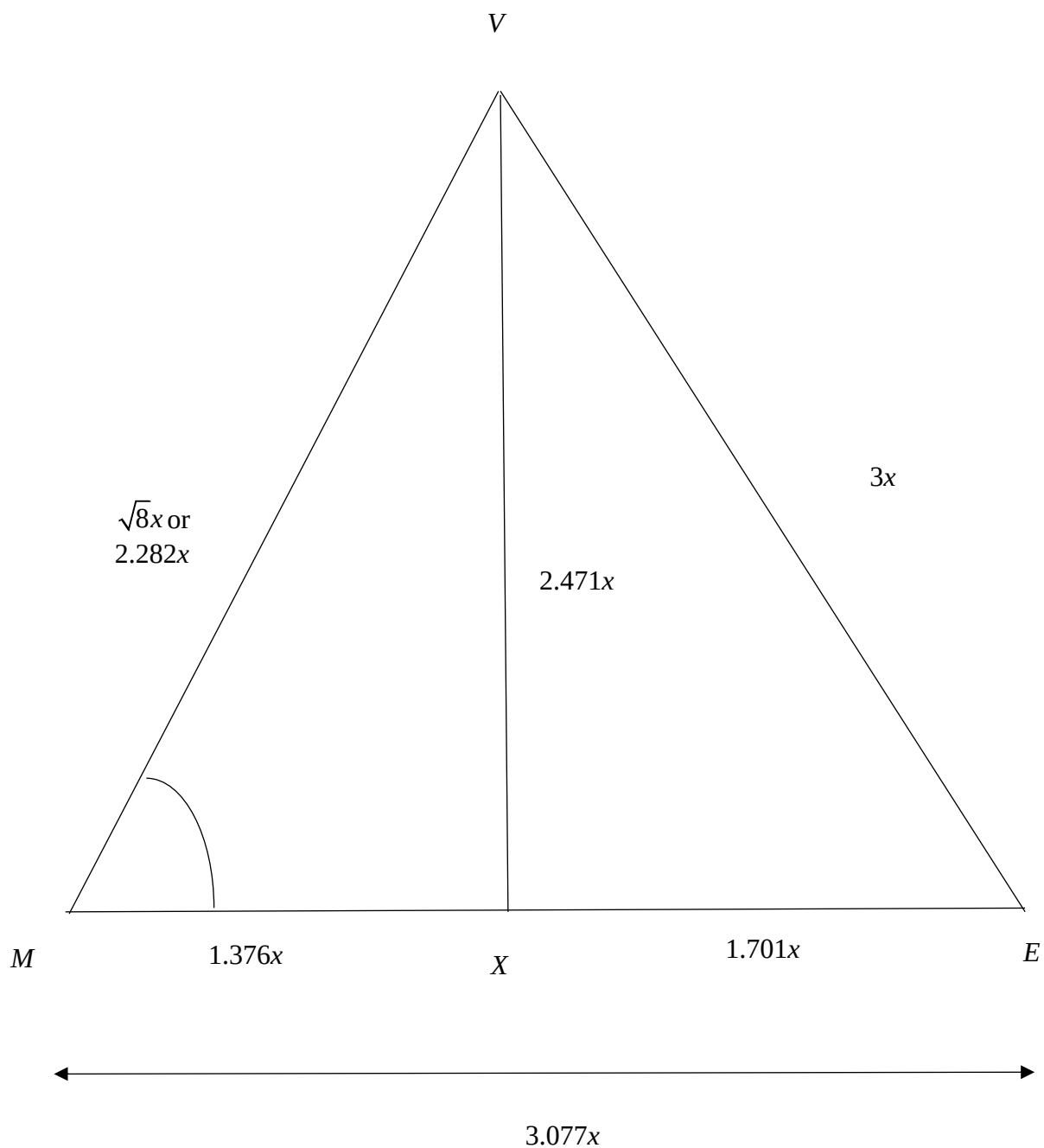
ALT3	First 2 marks as ALT2 $r = \frac{53.2}{2\pi - \theta}$ $r^2 \left(\pi - \frac{\theta}{2} \right) = 372.4 \Rightarrow \left(\frac{53.2}{2\pi - \theta} \right)^2 = \frac{372.4}{\pi - \frac{\theta}{2}} \text{ oe eg } \left(\frac{53.2}{2\pi - \theta} \right)^2 = \frac{744.8}{2\pi - \theta}$ $\frac{53.2^2}{744.8} = 2\pi - \theta \left(\Rightarrow \theta = 2\pi - \frac{53.2^2}{744.8} \right)$ $\theta = 2.48$ $r = \text{awrt } 14.0$	B1B1 M1 M1(A1 on ePen) A1(M1 on ePen) A1 [6]
Total 6 marks		

Mark	Notes
B1	Correctly uses the formula for length of an arc $53.2 = r \times \text{angle in radians}$ or correctly uses the formula for length of an arc $53.2 = 2\pi r \times \frac{\text{angle in degrees}}{360}$
B1	Correctly uses the formula for area of a sector $372.4 = \frac{r^2}{2} \times \text{angle in radians}$ or correctly uses the formula for area of a sector $372.4 = \pi r^2 \times \frac{\text{angle in degrees}}{360}$
M1	Eliminates α from both equations, allow up to one error. If they have worked in degrees, they must reach an equation where π must not be present (ie it has cancelled).
A1	For $r = 14$
M1	For using their r in a correct equation to find a value for α If they are working in degrees, there must follow a correct attempt to convert their angle into radians.
A1	For the correct value of $\theta = 2.48$
ALT1 B1B1	As main scheme
M1	Eliminates r from both equations and reaches an equation in α , allow one error
M1 (A1 on ePen)	Solves their equation (allow one error) to find a value for α
A1(M1 on ePen)	For the correct value for r
A1	For the correct value of $\theta = 2.48$
ALT2 B1	Correctly uses the formula for length of an arc
B1	Correctly uses the formula for area of a sector
M1	Rearranges for θ (allow one error) and substitutes correctly into the other equation to reach an equation for r
M1 (A1 on ePen)	For an equation of the form $ar = b$ $ab > 0$
A1(M1 on ePen)	$r = 14$
A1	$\theta = 2.48$
ALT3 B1B1	As ALT2
M1	Rearranges for r (allow one error) and substitutes correctly into the other equation to reach an equation for θ
M1 (A1 on ePen)	For reaching an equation of the form $d = 2\pi - \theta$ $d > 0$
A1(M1 on ePen)	$\theta = 2.48$
A1	$r = \text{awrt } 14.0$

Question	Scheme	Marks
9	Be careful to look on the sketch to award marks for the question and for any equivalent calculations or alternative methods. IF IN DOUBT SEND TO REVIEW X is the point directly below V positioned on the base $ABCDE$ Perpendicular from the mid-point of BC (M) to point X $(\angle BXC =) 72^\circ$ or $(\angle ABC =) 108^\circ$ Length of BX eg $(BX =) \frac{x}{\sin 36^\circ} (= 1.701x)$ or $\frac{\sin 54}{\sin 72} \times 2x$ or $\frac{x}{\cos 54}$ oe $(VX =) \sqrt{(3x)^2 - (1.701x)^2} = 2.471x$ or $(VM =) \sqrt{(3x)^2 - (x)^2} = \sqrt{8}x (= 2.828x)$ $(MX =) \sqrt{(1.701x)^2 - x^2} = 1.376x$ or $\frac{x}{\tan 36}$ oe or $x \tan 54$ Required angle is $\angle VMX$ or $\angle VME$ $(\tan \angle VMX =) \frac{2.471x}{1.376x} \Rightarrow (\angle VMX = 60.888...^\circ)$ oe $(\cos \angle VME =) \frac{(\sqrt{8}x)^2 + (1.701x + 1.376x)^2 - (3x)^2}{2 \times \sqrt{8}x \times (1.701x + 1.376x)}$ oe $\Rightarrow$ awrt 60.9° or better	B1 M1 M1 M1 dddM1 A1 [6]
ALT1	May only be applied if they attempt to find MX in one step and attempt to find VM and use triangle VMX. If they find MX any other way, apply one of the other schemes $(\angle BXC =) 72^\circ$ or $(\angle ABC =) 108^\circ$ $(MX =) \frac{x}{\tan 36}$ oe or $x \tan 54$ $(VM =) \sqrt{(3x)^2 - (x)^2} = \sqrt{8}x (= 2.828x)$ $(\cos \angle VMX =) \frac{x \tan 54}{2\sqrt{2}x} \Rightarrow (\angle VMX = 60.888...^\circ)$ $\Rightarrow$ awrt 60.9° or better	B1 M2 M1 dddM1 A1 [6]

ALT2	$(\angle BXC =) 72^\circ$ or $(\angle ABC =) 108^\circ$	B1
	$(EB =) \sqrt{(2x)^2 + (2x)^2 - 2(2x)(2x)\cos 108} (= 3.236x)$	M1
	or $(EB =) \frac{2x}{\sin 36} \times \sin 108 (= 3.236x)$	
	$(VM =) \sqrt{(3x)^2 - (x)^2} = \sqrt{8}x (= 2.828x)$	M1
	$(EM =) \sqrt{("3.236x")^2 - x^2} (= 3.077x)$	M1
	$(\cos \angle VME =) \frac{(" \sqrt{8}x ")^2 + ("1.701" x + "1.376" x)^2 - (3x)^2}{2 \times " \sqrt{8}x " \times ("1.701" x + "1.376" x)}$	dddM1
	$\Rightarrow$ awrt 60.9° or better	A1
Total 6 marks		

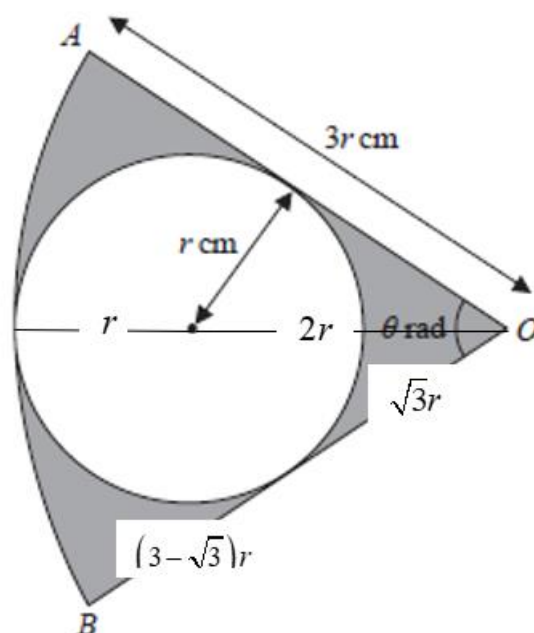
Mark	Notes
B1	For writing down or finding the angle $BXC = 72^\circ$ or $\angle ABC = 108^\circ$, any notation. Clear identification or implicit use of this angle in later working is acceptable. Note use of 36 degrees or 54 degrees implied this mark.
M1	Ignore missing x 's throughout their solution. For any correct, suitable trigonometry to find the length BX
M1	Correct use of Pythagoras theorem (with a – sign) or suitable trigonometry to find the height VX of the pyramid or the length VM
M1	Correct use of Pythagoras theorem (with a – sign) or suitable trigonometry to find the length of the midpoint of BC to point X
dddM1	For any suitable trigonometry to find the size of the required angle. Dependent on all 3 previous method marks.
A1	For the correct angle, awrt 60.9°
ALT1	For writing down or finding the angle $BXC = 72^\circ$ or $\angle ABC = 108^\circ$, any notation. Clear identification or implicit use of this angle in later working is acceptable. Note use of 36 degrees or 54 degrees implied this mark.
B1	
M2	Ignore missing x 's throughout their solution. For any correct method to find the length MX
M1	Correct method to find the length VM
dddM1	For any suitable trigonometry to find the size of the required angle. Dependent on all 3 previous method marks.
A1	For the correct angle, awrt 60.9°
ALT2	For writing down or finding the angle $BXC = 72^\circ$ or $\angle ABC = 108^\circ$, any notation. Clear identification or implicit use of this angle in later working is acceptable. Note use of 36 degrees or 54 degrees implied this mark.
B1	
M1	Ignore missing x 's throughout their solution. For any correct method to find the length EB
M1	For any correct method to find VM
M1	For any correct method to find EM
dddM1	For any suitable trigonometry to find the size of the required angle. Dependent on all 3 previous method marks.
A1	For the correct angle, awrt 60.9°
Where students are allowed to use their values, indicated by inverted commas, it MUST be clear from working or their sketch this is the correct use of the relevant side. Beware $\sqrt{8}x$ coming from incorrect working of $\sqrt{(2x)^2 + (2x)^2}$	



Question	Scheme	Marks
3(a)	$\sin\left(\frac{\theta}{2}\right) = \frac{r}{2r} \Rightarrow \frac{\theta}{2} = \frac{\pi}{6}$ $\Rightarrow \theta = \frac{\pi}{3}$ ALT works in degrees $\sin\left(\frac{\theta}{2}\right) = \frac{r}{2r} \Rightarrow 30^\circ$ $\Rightarrow \theta = \frac{\pi}{3}$	M1 A1 [2] [M1 A1]
(b)	Area of circle = πr^2 Area of sector = $\frac{\frac{\pi}{3}}{2\pi} \times \pi (3r)^2 = \frac{3}{2} \pi r^2$ Shaded area: $8\pi = \frac{3}{2} \pi r^2 - \pi r^2 = \frac{\pi r^2}{2} \Rightarrow r^2 = 16 \Rightarrow r = 4 \text{ (cm)}$ ALT works in degrees Area of circle = πr^2 Area of sector = $\frac{60^\circ}{360^\circ} \times \pi \times (3r)^2 = \frac{3}{2} \pi r^2$ Shaded area: $8\pi = \frac{3}{2} \pi r^2 - \pi r^2 = \frac{\pi r^2}{2} \Rightarrow r^2 = 16 \Rightarrow r = 4 \text{ (cm)}$	B1 M1 dM1A1 [4] [B1 M1 dM1A1]
Total 6 marks		

Part	Mark	Notes
(a)	M1	For the correct use of sine with r and $2r$ to obtain $\frac{\theta}{2} = \frac{\pi}{6}$ Allow working in degrees for the M mark only. i.e $\frac{\theta}{2} = 30^\circ$ Any responses not based on trigonometry are M0A0 If you are not sure, please send to Review
	A1	For $\theta = \frac{\pi}{3}$ ONLY
(b)	B1	For the correct area of the circle seen anywhere in part (b)
	M1	For using the correct formula for the area of a sector with their θ provided θ is within the specified range. $\left[0 < \theta < \frac{\pi}{2}, \quad 0 < \theta < 1.57.. \right]$ Allow working in degrees for the sector provided it is correct. Do not allow use of the radian formula for a sector with degrees.
	dM1	For subtracting the area of the circle from the area of the sector and finding a value for r $r = \sqrt{\frac{8\pi}{\left(\frac{1}{2} \times (3)^2 \times \frac{\pi}{3} - \pi\right)}}$ This mark is dependent on the previous M mark
	A1	For $r = 4$

USEFUL SKETCH

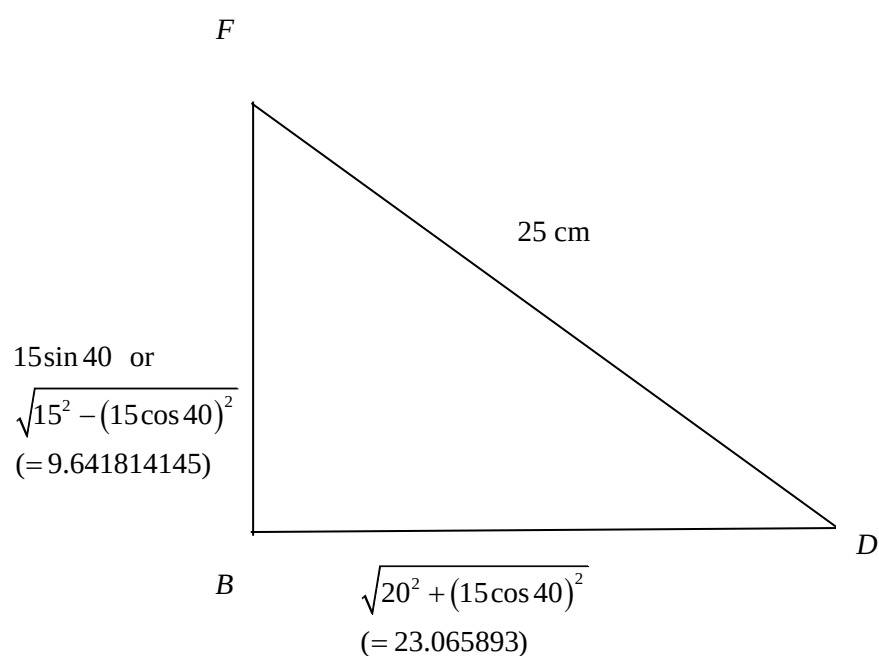
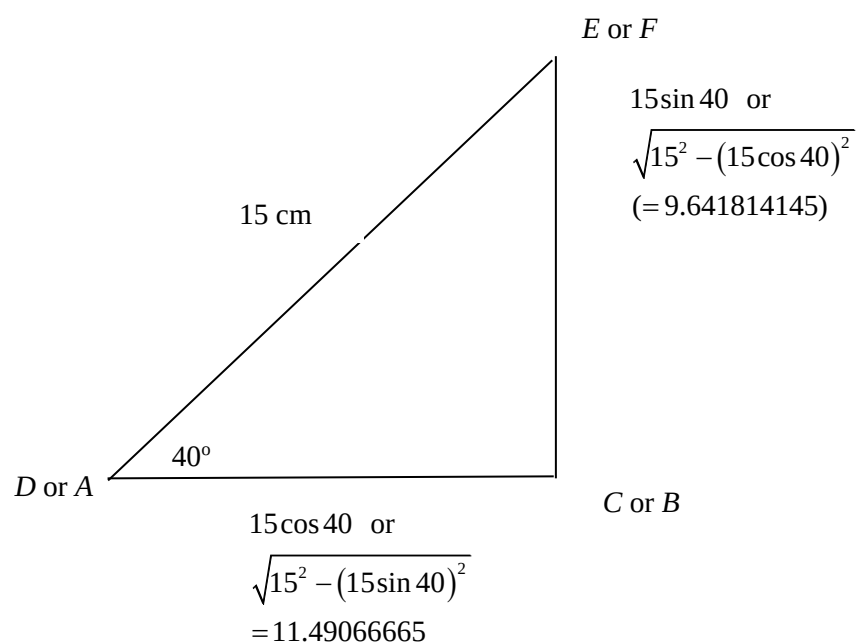


Question number	Scheme	Marks
Remember to look for work on the diagram.		
4	$(FD) = \sqrt{15^2 + 20^2} \quad (FD =) 25$ $(BF =) 15 \sin 40 \quad \text{oe} \quad (\Rightarrow (BF) = 9.641814145)$ $\sin BDF = \frac{"15 \sin 40"}{"\sqrt{15^2 + 20^2}"} (= 0.3856725658) \quad \text{or} \quad \sin^{-1} \left(\frac{"9.641814145"}{"25"} \right)$ $(BDF) = 22.7^\circ$	M1 A1 M1 M1 dM1 A1 [6]
Example of an ALT	$(DC =) 15 \times \cos 40 \quad \text{and} \quad (BD =) \sqrt{20^2 + ("15 \cos 40")^2} \quad (BD =) \text{awrt } 23.1$ $(DC =) 15 \times \cos 40 \quad \text{and} \quad (FB =) \sqrt{15^2 - (15 \cos 40)^2} \quad \text{oe} \quad (\Rightarrow (FB) = 9.641814145)$ $\tan BDF = \frac{"\sqrt{15^2 - (15 \cos 40)^2}"}{"\sqrt{20^2 + ("15 \cos 40")^2}"} (= 0.4180117433) \quad \text{or} \quad \tan^{-1} \left(\frac{"9.641814145"}{"23.1"} \right)$ $(BDF) = 22.7^\circ$	M1 A1 M1 dM1 M1 A1 [6]
Total 6 marks		

Part	Mark	Additional Guidance
$(BF =) 15 \sin 40 \quad \text{oe} \quad (\Rightarrow (BF) = 9.641814145)$ means does not need to be labelled BF but there mustn't be anything to contradict this. (General principle of marking) It is fine to label with equivalent sides eg $(EC = FB)$ There are a number of ways candidates are completing this question. Please mark to the following general principles – send anything to review that can't be marked using these.		
	M1	For a full and correct method to find FD or BD
	A1	For $FD = 25$ or $BD = \text{awrt } 23.1$
	M1	For a full and correct method to find a second side in triangle BDF
	M1	For correctly identifying the correct angle to be found. Dependent on previous method mark. Look for any clear identification that this is the angle they're finding, including on the diagram or in written work.
	dM1	For the use of their values for two sides in triangle BDF in a correct trig equation or the correct expression to find the angle with the relevant inverse trig function. There must be a full, complete and correct method which leads to an equation or expression to find angle BDF , part methods should not be awarded this mark. Dependent on the previous method mark only ie the correct angle identified, which may be implied in a correct equation or expression. If it is clear candidates are using 2 of their values for BF , FD and BD , this mark can be awarded, even if the work to calculate either is incorrect. $\sin BDF = \left(\frac{"BF"}{"DF"} \right) \quad \text{or} \quad \cos BDF = \left(\frac{"BD"}{"DF"} \right) \quad \text{or} \quad \tan BDF = \left(\frac{"BF"}{"BD"} \right)$ $\text{or} \quad \sin^{-1} \left(\frac{"BF"}{"DF"} \right) \quad \text{or} \quad \cos^{-1} \left(\frac{"BD"}{"DF"} \right) \quad \text{or} \quad \tan^{-1} \left(\frac{"BF"}{"BD"} \right)$ If incorrect lengths are found, you may allow $\left(\frac{"BF"}{"DF"} \right) \quad \text{or} \quad \left(\frac{"BD"}{"DF"} \right) > 1$ for this mark Note: if students choose to use cosine rule, they must additionally use a full and correct method to find a third side of triangle BDF and have either of these $\cos BDF = \frac{DF^2 + BD^2 - BF^2}{2 \times DF \times BD} \quad \text{or} \quad \cos^{-1} \left(\frac{DF^2 + BD^2 - BF^2}{2 \times DF \times BD} \right)$
	A1	Any answer which rounds to (awrt) 22.7°



Useful triangles and sides.



Question number	Scheme	Marks
3 (a)	$2 \times 7 \times \theta = \frac{28\pi}{9}$ oe or $2 \times \pi \times 7 \times \frac{\text{angle}}{360} = \frac{14\pi}{9}$ oe and attempts to change back to radians $\theta = \frac{2\pi}{9}$	M1 A1 [2]
(b)	<u>Area of sector</u> $A = \frac{1}{2} \times 7^2 \times \frac{2\pi}{9}$, <u>Area of triangle</u> $A = \frac{1}{2} \times 7^2 \times \sin \frac{2\pi}{9}$, <u>Area of segment</u> $[A =] [2 \times] \left(\frac{1}{2} \times 7^2 \times \frac{2\pi}{9} - \frac{1}{2} \times 7^2 \times \sin \frac{2\pi}{9} \right) = 2.71$ (awrt)	M1 M1 ddM1A1 [4]
Total marks 6		

Part	Mark	Additional Guidance
(a)	M1	For forming a correct equation using arc length and $r = 7$ If attempted by degrees, must see attempts to change degrees to radians
	A1	For $\theta = \frac{2\pi}{9}$ oee (or exact equivalent)
(b)	M1	For a correct method finding area of the sector using their θ $A = \frac{1}{2} \times 7^2 \times \frac{2\pi}{9}$ or $A = \pi \times 7^2 \times \frac{40}{360}$ (Note: $\frac{2\pi}{9} = 40^\circ$)
	M1	For a fully correct method finding the area of a triangle using their θ , can be implied by the correct area of triangle 15.7 awrt
	ddM1	For finding the difference between the area of the sector and the area of the triangle. The subtraction must be shown, or it could be implied by correct final answer. Dependent on both previous M marks
	A1	For 2.71 (awrt) ignore unit



Question number	Scheme	Mark
3	$(l = r\theta)6\theta$ or $(l = \frac{\theta}{360} \times 2\pi r =) \frac{\theta}{360} \times 2\pi \times 6$ oe $6 + 6 + "6\theta" = 12 + \pi$ or $6 + 6 + " \frac{\theta}{360} \times 2\pi \times 6" = 12 + \pi$ oe $(6\theta = \pi) \Rightarrow \theta = \frac{\pi}{6}$ or $\theta = 30^\circ$ $\left(A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta = \right) \frac{1}{2}6^2 " \frac{\pi}{6} " - \frac{1}{2}6^2 \sin " \frac{\pi}{6} "$ or $\frac{"30"}{360} \times \pi \times 6^2 - \frac{1}{2}6^2 \sin "30"$ $= 3\pi - 9$ oe	M1 M1 A1 M1 M1 A1 [6]
Total 6 marks		

Part	Mark	Additional Guidance
Students may work in radians or degrees. If using both, marks may be given for accurate use of formulas.		
	M1	Correct use of formula for length of an arc.
	M1	Allow use of their length of arc, ie for following through their length of arc and equating an equation of the form $6 + 6 + a\theta = 12 + \pi$ or $6 + 6 + \frac{\theta}{360} \times 2\pi \times a = 12 + \pi$ a doesn't have to be an integer, can be irrational. This may be done in stages. Take care to look out for equivalences, where stages have been missed, this is not a show that question. For example, " 6θ " = π would imply both method marks as the candidate could have realised the 12 cancels from each side. As can often be the case, a correct angle will imply method marks (unless incorrect method shown). General marking principle.
	A1	$\theta = \frac{\pi}{6}$
	M1	Correct use and substitution into the formula for area of sector using their θ
	M1	Correct use and substitution into the of formula for area of triangle, using their θ
	A1	$3\pi - 9$ oe (correct decimal is 0.4247.... and is likely to imply correct method)



Question number	Scheme	Marks
6	Correctly identifies the angle VXO where X is the midpoint of CD and O is the foot of the perpendicular from V oe	B1
	If y is the length of AD . $VC = 3y$ $CX = \frac{y}{2}$ $OX = \frac{3y}{2}$	M1
	$(VX) = \sqrt{(3y)^2 - \left(\frac{y}{2}\right)^2}$ $VX = \frac{\sqrt{35}y}{2}$ oe	dM1
	$(OC) = \sqrt{\left(\frac{3y}{2}\right)^2 + \left(\frac{y}{2}\right)^2} = \left(\frac{\sqrt{10}}{2}y\right)$ and $VO = \sqrt{(3y)^2 - \left(\frac{\sqrt{10}}{2}y\right)^2} = \left(\frac{\sqrt{26}}{2}y = \sqrt{\frac{13}{2}}y\right)$	A1
	$\cos \theta = \frac{\frac{3y}{2}}{\frac{\sqrt{35}y}{2}} \left(= \frac{OX}{VC} = 0.507062..... \right)$ oe eg $\tan \theta = \frac{\frac{\sqrt{26}}{2}y}{\frac{3y}{2}} \left(= \frac{OV}{OX} = 1.699673.... \right)$ $\sin \theta = \frac{\frac{\sqrt{26}}{2}y}{\frac{\sqrt{35}y}{2}} \left(= \frac{OV}{VC} = 0.861891..... \right)$ $(\text{Cosine rule}) \cos \theta = \frac{\left(\frac{\sqrt{35}y}{2}\right)^2 + (3y)^2 - \left(\frac{\sqrt{35}y}{2}\right)^2}{2 \times \frac{\sqrt{35}}{2}y \times 3y}$ oe $\cos \theta = \frac{\left(\frac{\sqrt{35}y}{2}\right)^2 + \left(\frac{3y}{2}\right)^2 - \left(\frac{\sqrt{26}y}{2}\right)^2}{2 \times \frac{\sqrt{35}}{2}y \times 3y}$ $\left[\cos \theta = \frac{3}{\sqrt{35}} \right]$ oe leading to $\theta =$ 59.5	M1 A1 [6]
Total 6 marks		

Part	Mark	Additional Guidance
	B1	Angle identified in written work or on diagram . Allow labelling to be any letters. This mark should be permitted for any work where it is clear the candidate is identifying or using the correct angles. If ambiguous or unclear, cannot be awarded.
	M1	Denotes correct and appropriate letters for any two relevant lengths on the pyramid to make the next stage of progress. The sides are shown in the MS, but we will accept sides with different labels elsewhere in the triangle. Eg $DX = \frac{y}{2}$ is the same as CX This can be implied by written work or on the diagram. The two sides can be any values or expressions in the correct ratio which are necessary to form part of the working. Watch out for equivalences. Eg $AD = VC = y$ $DC = \frac{y}{3}$ $CX = \frac{y}{6}$ or $AD = VC = 9$ $DC = \frac{9}{3}$ $CX = \frac{9}{6}$ May be implied by correct working. For this mark, the sides used must have the correct algebraic expressions, (including multiples of those shown) or numbers in the correct proportion.
	dM1	Correct full method to find VX or VO. Correct full method means they can't use a side from an incorrect method in another method for VX or VO (so for example if they use an incorrect method to find VX and then use this to find VO they can't have the mark for finding VO) Dependent on the previous mark.
	A1	Correct expression or values for VO or VX ie all sides that they will be using.
	M1	Working in triangle VXO (or other valid triangle) with their values or expressions from previous working, using any appropriate trigonometry. The candidate can use sides which are labelled with the correct lengths for the trig ratio used, even if calculated incorrectly in previous work.
	A1	awrt 59.5 (59.52964053) Note: 60 is likely to have come from an incorrect angle identified And it is possible to get an answer close without a correct method. The method must always be checked through (in any question)

Question number	Scheme	Marks
1 (a)	$t = \frac{10}{3}$ $P = 3 + 2 \sin \frac{5\pi}{4}$ $= 3 - \sqrt{2}$ oe (e.g. $3 - \frac{2}{\sqrt{2}}$)	M1 A1 (2)
(b) (i)	5	B1
(ii)	1	B1 (2)
(c)	$4 = 3 + 2 \sin \left(\frac{3\pi t}{8} \right)$ $\frac{1}{2} = \sin \left(\frac{3\pi t}{8} \right)$ $\frac{\pi}{6} = \left(\frac{3\pi t}{8} \right)$ $t = \frac{4}{9}$ oe	M1 M1 A1 (3)
		[7]

Part	Mark	Additional Guidance
(a)	M1	Correct substitution of $t = \frac{10}{3}$, leading to a value for P, simplification not required.
	A1	Answer stated, oe exact value.
(c)	M1	Correctly substitutes the value of $P = 4$ and rearranges to give an expression of the form $a = \sin \left(\frac{3\pi t}{8} \right)$ Do not allow this mark if $a > 1$ or $a < -1$
	M1	Correctly uses the inverse sin function to arrive at $b = \left(\frac{3\pi t}{8} \right)$ and solves to find a value of $\frac{3\pi t}{8}$, allow this value to be in degrees If the inverse sin function is not shown, then the value of the angle obtained must be correct for their b .
	A1	oe

Question number	Scheme	Marks
6 (a)	$(\sin A \cos B + \cos A \sin B) + (\sin A \cos B - \cos A \sin B)$ $= 2 \sin A \cos B *$	M1 A1 cso (2)
(b)	$\sin 8x + \sin 6x$	B1 (1)
(c)	$3 \int_0^{\frac{\pi}{4}} (\sin 8x + \sin 6x) dx$ $= (3) \left[-\frac{1}{8} \cos 8x - \frac{1}{6} \cos 6x \right]_0^{\frac{\pi}{4}}$ $= (3) \left[\left(-\frac{1}{8} - 0 \right) - \left(-\frac{1}{8} - \frac{1}{6} \right) \right]$ $= \frac{1}{2} \text{ cao oe}$	M1 A1 M1 A1 cao (4) [7]

Part	Mark	Additional Guidance
(a)	M1	Correct expression show.
	A1	cso
(b)	B1	For the expression shown
(c)	M1	$k \int_0^{\frac{\pi}{4}} (\sin 8x + \sin 6x) dx$ $k \neq 0$ or 1 k must be an integer. This mark can be awarded if the limits aren't seen on the integral.
	A1	Correctly integrated, the 3 and the limits need not be present for this mark
	M1	Correctly shown substitution of limits, with a subtract sign between. The 3 need not be present for this mark, need not be simplified. <i>This mark can be implied if first M1 A1 awarded and final correct answer.</i> Allow a correct substitution of limits into any changed expression.
	A1	cao oe

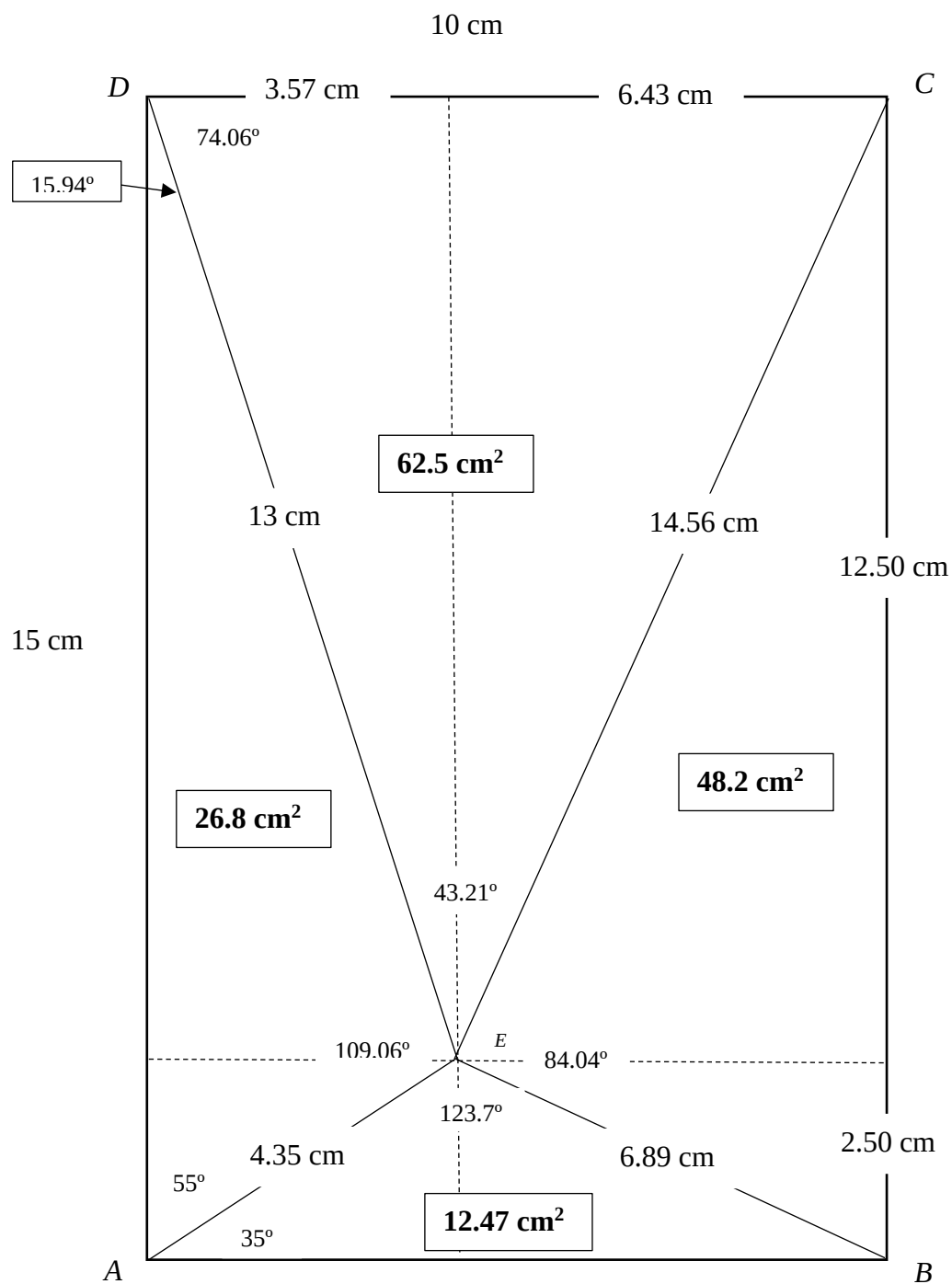
Question number	Scheme	Marks
3	Working in triangle AED – main scheme, ALT1 and ALT2 – first 3 marks: $\frac{\sin E}{15} = \frac{\sin 55}{13}$ $E = 70.9...^\circ$ or $109...^\circ$ (awrt) $ADE = 180 - 55 - "109.1" (=15.9...^\circ)$ or $180 - 55 - "70.9" (=54.0...^\circ)$	M1 A1 M1 (B1 on ePen)
Main Scheme	(Let h be the perpendicular height of triangle ADE) $\sin "15.9...^\circ" = \frac{h}{13} \quad \text{Allow use of "54.0"}$ $h = 3.57... \text{ or } 10.5.....$ (awrt) Area = $\frac{1}{2} \times 15 \times (10 - '3.57...')$ Do not allow use of h that has come from "54.0" $= 48.2 \text{ (cm}^2\text{)}$ (awrt)	M1 A1 ddM1 A1 (7)
ALT1	First 3 marks as main scheme $((AE)^2 = 13^2 + 15^2 - 2 \times 13 \times 15 \cos("15.9....")^\circ \quad \text{allow use of "54.0...." for "15.9...."}$ or $(AE) = \sqrt{13^2 + 15^2 - 2 \times 13 \times 15 \cos("15.9....")^\circ} \quad \text{allow use of "54.0...." for "15.9...."}$ $\frac{AE}{\sin "15.9"} = \frac{13}{\sin 55} \quad \text{allow use of "54.0...." for "15.9...."}$ $(AE) =$ awrt 4.35 or 4.36 or awrt 12.8 or 12.9 if using using 54.0....rather than 15.9.... FOR THE NEXT PART DO NOT ALLOW USE OF 54.0 or 12.8..... $\frac{1}{2} \times "4.35...." \times 10 \times \sin 35$ (Area AEB = 12.4....) $\frac{1}{2} \times "4.35...." \times 15 \times \sin 55$ or $\frac{1}{2} \times 13 \times 15 \times \sin "15.9...."$ (Area AED = 26.7....) $\frac{1}{2} \times 13 \times 10 \times \sin "90 - 15.9"$ (Area EDC = 62.5....) Area triangle BCE = $(10 \times 15) - (\text{Area AEB} + \text{Area AED} + \text{Area EDC})$ awrt 48.2 (cm²)	M1 A1 ddM1 A1 (7)
ALT2	First 5 marks as ALT1 DO NOT ALLOW USE OF 54.0 or 12.8.....for this 2 marks $((EB)^2 = 10^2 + "4.36"{}^2 - 2 \times 10 \times "4.36" \cos(35)^\circ$ or $(EB) = \sqrt{10^2 + "4.36"{}^2 - 2 \times 10 \times "4.36" \cos(35)^\circ}$ $\frac{\sin EBA}{4.36....} = \frac{\sin 35}{6.96....} \Rightarrow \text{Angle EBA} = 21.2..... \Rightarrow \text{Angle EBC} = 90 - 21.2..... = 68.7.....$ Area = $0.5 \times 6.96 \times 15 \times \sin(68.7.....)$ Area = awrt 48.2	ddM1 A1 (7)

ALT3	Working in triangle ABE or denoting side AE as y : $13^2 = 15^2 + y^2 - 2 \times y \times 15 \cos 55$ $(\Rightarrow 0 = y^2 - (17.2 \dots) y + 56)$ $(y =) \text{awrt } 4.35 \text{ or } 4.36$ or $(y =) \text{awrt } 12.8$	M1
	$c^2 = "4.36"^2 + 10^2 - 2 \times "4.36" \times 10 \cos 35$ 12.8 could also be used for 4.36 $(c = \text{awrt } 6.89 \text{ or } 7.38)$	A1
	$\frac{\sin 35}{"6.89" \text{ or } "7.83"} = \frac{\sin ABE}{"4.36" \text{ or } "12.8"}$	M1
	angle $ABE = \text{awrt } 21.2 \text{ or } 85.9$ and $c = \text{awrt } 6.89 \text{ or } 7.38$	M1
	Area $= \frac{1}{2} \times 15 \times "6.89" \times \sin(90 - "21.2")$	A1
	awrt 48.2	ddM1
Total 7 marks		
If the final answer given rounds to 48.2 and working is present, students may be given full marks if their intermediate values are not given to the degree of accuracy demanded in the mark scheme. Look carefully for answers written on the diagram and award marks.		

Mark	Notes for ALT3
M1	For a correct substitution into the cosine rule to find AE as shown
A1	For either $y = \text{awrt } 4.35/4.36$ or $y = \text{awrt } 12.8$
M1	Correct substitution into the cosine rule to find BE using their 4.36 or 12.8
M1	Correct substitution into the sine rule to find angle ABE
ddM1	For this mark, they must have used $y = 4.35/4.36$ throughout Correct substitution into the area of a triangle formula as shown
A1	awrt 48.2

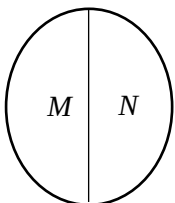
Marks	Notes
Working in triangle AED – main scheme, ALT1 and ALT2 – first 3 marks:	
M1	For correct use of the sine rule to find angle AED
A1	For either $AED = 70.9...^\circ$ or $109...^\circ$ awrt
M1 (B1 on ePen)	For $ADE = (180 - 55 - \text{their "109".1...})^\circ$ or $180 - 55 - \text{"70.9"} (=54.0...^\circ)$ It must be clear this is their angle AED if the angle is incorrect or for finding "74.1" – the angle which h makes with DE using "109.1" or "70.9"
Main scheme next 4 marks:	
M1	For $\sin "15.9"...^\circ = \frac{h}{13}$ Use of their 15.9. Allow use of their 54.0..... Note, using the same triangle, this could also be presented as: $\cos "74.1"...^\circ = \frac{h}{13}$
A1	For $h = 3.57... \text{ or } 10.5$ (awrt)
ddM1	For area = $\frac{1}{2} \times 15 \times (10 - "3.57...")$ Dependent on both previous method marks – for this mark, candidates must have used angle $ADE = (180 - 55 - \text{their "109".1...} = "15.9")^\circ$ ie they must have used the obtuse angle found for angle AED
A1	For awrt 48.2 (cm^2) (Units not required)
For all solutions: if the final answer given rounds to 48.2 and working is present, students may be given full marks if their intermediate values are not given to the degree of accuracy demanded in the mark scheme.	
ALT1	
M1	For correct substitution into cosine rule $13^2 + 15^2 - 2 \times 13 \times 15 \cos("15.9...")$ or $\sqrt{13^2 + 15^2 - 2 \times 13 \times 15 \cos("15.9...")}$ Use of their 15.9..... Allow use of their 54.0.....
A1	For awrt 4.36
ddM1	For a complete method to find the area of triangle BCE as shown in the scheme – use of their values. Dependent on both previous method marks – for this mark, candidates must have used angle $ADE = (180 - 55 - \text{their "109".1...} = "15.9")^\circ$ ie they must have used the obtuse angle found for angle AED
A1	For awrt 48.2 (cm^2) (Units not required)
ALT2	For a complete method to find area of triangle BCE
Final ddM1	Finds EB, finds angle EBC Dependent on both previous method marks – for this mark, candidates must have used angle $ADE = (180 - 55 - \text{their "109".1...} = "15.9")^\circ$ ie they must have used the obtuse angle found for angle AED
A1	For awrt 48.2 (cm^2) (Units not required)
General principles for marking this question with any other method seen: The first 5 marks will be allocated as the main scheme and/or the ALTS The final ddM1 mark will be awarded for a complete method, using their values but MUST use the obtuse angle as stated in the question. Final A1 for awrt 48.2	

USEFUL SKETCH

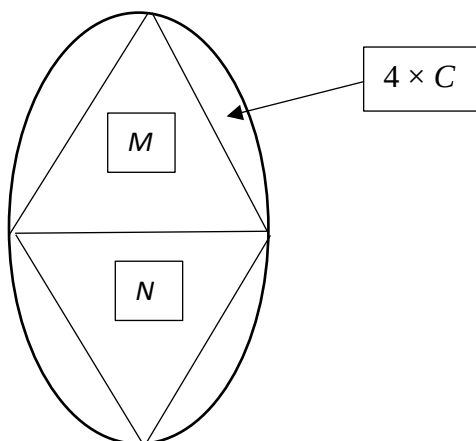


Question number	Scheme	Marks
4	$O_1A = O_2A = O_1O_2 = 6$ so triangle AO_1O_2 is equilateral	B1
	So angle $AO_1O_2 = \frac{\pi}{3}$	M1
	Area of sector $AO_1BO_2 = \frac{1}{2} \times \frac{2\pi}{3} \times 6^2 = (12\pi)$	M1
	Area of triangle $AO_1B = \frac{1}{2} \times 6^2 \times \sin\left(\frac{2\pi}{3}\right) = 9\sqrt{3}$	M1
	Area of segment $AO_1B = 12\pi - 9\sqrt{3}$	M1
	So $P = 2 \times (12\pi - 9\sqrt{3}) = 24\pi - 18\sqrt{3} \text{ (cm}^2\text{)}$	M1 A1
	Decimal value for reference is 44.221... (cm ²)	(7)
Total 7 marks		



Mark	Notes
B1	For recognising that triangle AO_1O_2 is equilateral. This can be implied by further work. Look at the diagram. The angles are frequently marked on there. This mark is common to all three methods of finding the area. Note: Correct calculations for either the area of the sector or area of triangle implies the first two marks.
Finds area of segment AO_1B and doubles. (Area M + Area N)	
	
M1	For angle $AO_1O_2 = \frac{\pi}{3}$ or 60° NB This is often marked on the diagram. Please check that carefully.
M1	Correct method to find area of sector $AO_1BO_2 = \frac{1}{2} \times \frac{2\pi}{3} \times 6^2 = (12\pi)$ or $\frac{1}{3} \times \pi \times 6^2 = (12\pi)$
M1	Correct method to find area of triangle $AO_1B = \frac{1}{2} \times 6^2 \times \sin\left(\frac{2\pi}{3}\right) = 9\sqrt{3}$
M1	So area of segment $AO_1B = 12\pi - 9\sqrt{3}$
dM1	For area of $R = 2 \times (12\pi - 9\sqrt{3})$ Dependent on all 4 previous M marks
A1	For $24\pi - 18\sqrt{3}$ (cm^2) (Units not required)
ALT	
M1	For angle $AO_1O_2 = \frac{\pi}{3}$ or 60° NB This is often marked on the diagram. Please check that carefully.
M1	Correct method to find area of sector $AO_1O_2 = \frac{1}{2} \times \frac{\pi}{3} \times 6^2 = (6\pi)$ or $\frac{1}{6} \times \pi \times 6^2$
M1	Correct method to find area of triangle $AO_1O_2 = \frac{1}{2} \times 6^2 \times \sin\left(\frac{\pi}{3}\right) = 9\sqrt{3}$ OR height of triangle $AO_1O_2 = 3\sqrt{3} \Rightarrow \text{Area } AO_1O_2 = \frac{1}{2} \times 3\sqrt{3} \times 6 = 9\sqrt{3}$
M1	So area of segment C (on diagram) = $'6\pi' - '9\sqrt{3}'$

Method 1 - 2 triangles + 4 segments = $M + N + 4 \times C$



dM1

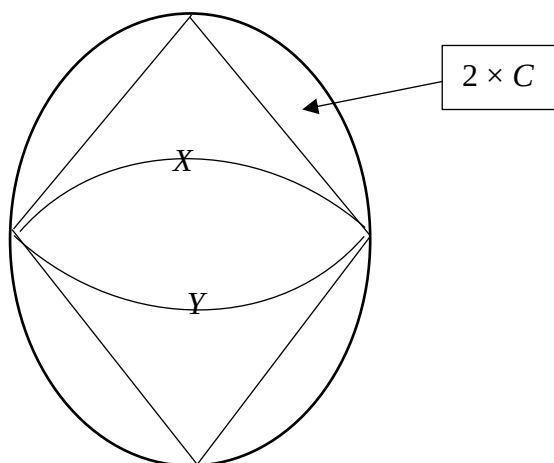
$$\text{Area of } R = 2 \times \Delta AO_1O_2 + 4 \times \text{segment } AO_1O_2 = 2 \times ('9\sqrt{3} ') + 4 ('6\pi ' - '9\sqrt{3} ')$$

Dependent on all 4 previous M marks

A1

$$\text{For } 24\pi - 18\sqrt{3} \text{ (cm}^2\text{)} \text{ (Units not required)}$$

Method 2 - 2 sectors + 2 segments = $X + Y + 2C$



dM1

$$\text{Area of } R = 2 \times \text{Sector } AO_1O_2 + 2 \times \text{segment } AO_1O_2 = 2 \times '6\pi ' + 2 ('6\pi ' - '9\sqrt{3} ')$$

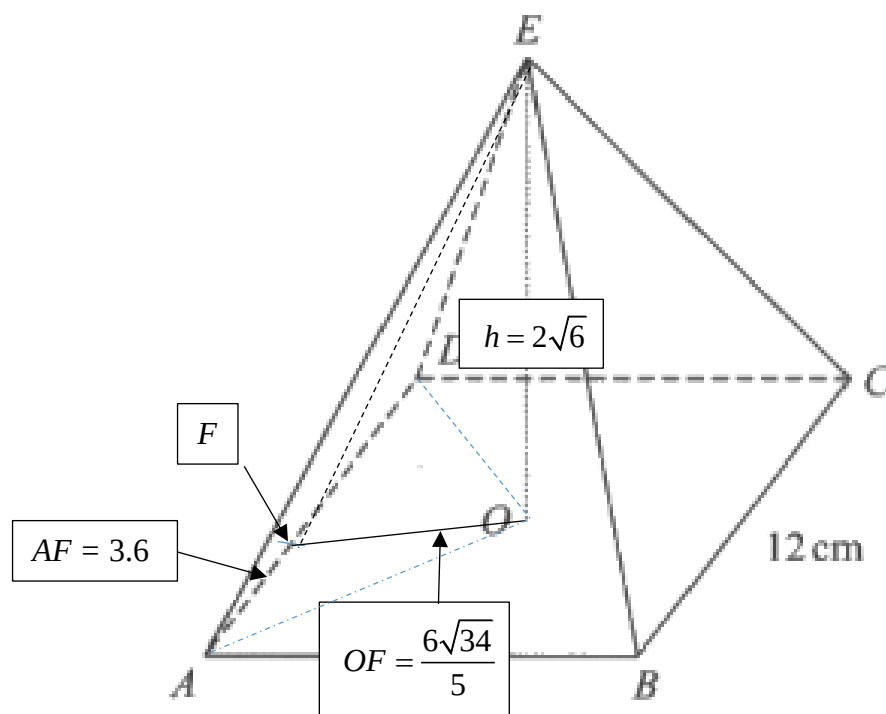
Dependent on all 4 previous M marks

A1

$$\text{For } 24\pi - 18\sqrt{3} \text{ (cm}^2\text{)} \text{ (Units not required)}$$

Question number	Scheme	Marks
5 a	$OC = \frac{1}{2}\sqrt{12^2 + 12^2} = [6\sqrt{2}] \quad \text{or} \quad AC = \sqrt{12^2 + 12^2} = [12\sqrt{2}]$ $h = EO = 6\sqrt{2} \times \frac{\sqrt{3}}{3} = 2\sqrt{6} \quad \text{oe}$	M1 M1 A1 (3)
b	Midpoint of AD to $F = \left(6 - \frac{12}{1+4}\right) [= 3.6]$ $OF = \sqrt{6^2 + 3.6^2} = \frac{6\sqrt{34}}{5}$ $\tan \theta = \frac{2\sqrt{6}}{\frac{6\sqrt{34}}{5}} = 35^\circ$	M1 M1 M1 A1 (4)
Total 7 marks		

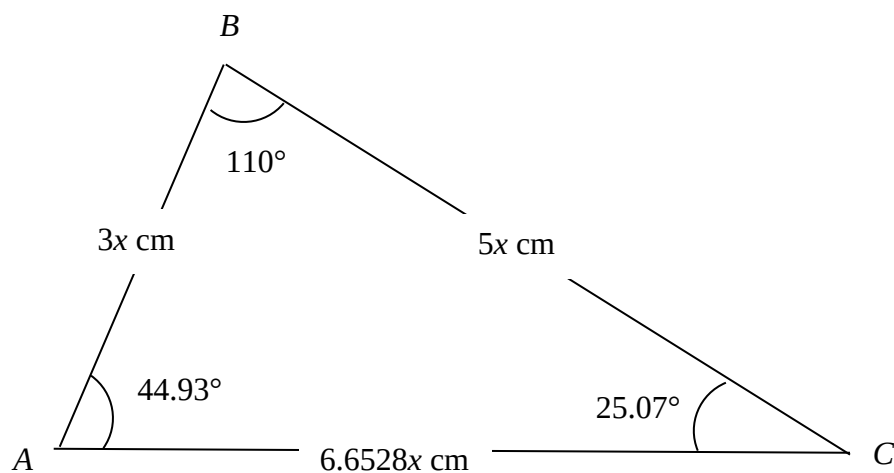
USEFUL SKETCH



Part	Mark	Notes
(a)	M1	For using a correct Pythagoras theorem to find either OC , OA ($6\sqrt{2}$) or AC ($12\sqrt{2}$)
	M1	For using the correct trigonometry (tan ratio): $h = '6\sqrt{2}' \tan 30^\circ = [2\sqrt{6}]$ or equivalent. Eg $\tan 30 = \frac{h}{6\sqrt{2}} \Rightarrow h = \dots$
	A1	For $2\sqrt{6}$ oe
(b)	M1	For correct expression for the midpoint of AD to F $\left(6 - \frac{12}{1+4}\right)$ or 3.6 oe seen
	M1	For the correct use of Pythagoras' to find OF $\frac{6\sqrt{34}}{5}$ oe [6.997..] ft their 3.6
	ALT	
	M1	Finds the length OF using cosine rule. $OF = \sqrt{2.4^2 + (6\sqrt{2})^2 - 2 \times 2.4 \times 6\sqrt{2} \times \cos 45} = \left[\frac{6\sqrt{34}}{5}\right]$ OR $OF = \sqrt{9.6^2 + (6\sqrt{2})^2 - 2 \times 9.6 \times 6\sqrt{2} \times \cos 45} = \left[\frac{6\sqrt{34}}{5}\right]$ OR $\cos 45^\circ = \frac{9.6^2 + (6\sqrt{2})^2 - OF^2}{2 \times 9.6 \times 6\sqrt{2}} \Rightarrow OF = \dots$ oe [6.997..]
	M1	For the correct evaluation of their cosine rule. $\left(\frac{6\sqrt{34}}{5}\right)$ oe Accept awrt 7.00 [6.997142...]
	dM1	For the correct use of $\tan \theta = \frac{'EO'}{'OF'}$ This mark is dependent on both previous M marks. They must have a valid method to find OF for the award of this mark.
	A1	For 35(°) or better (Calculator value is 34.997...°.)
Note: There are other methods – if unsure, send to Review.		

Question	Scheme	Marks
2(a)	$AC = \sqrt{(3x)^2 + (5x)^2 - 2 \times 3x \times 5x \times \cos 110^\circ} = 6.6528...x$	M1A1
	$\sin \angle BCA = \frac{3x \sin 110^\circ}{6.6528'x} = 0.4237... \Rightarrow \angle BCA = 25.07...^\circ$	M1
	Accept awrt 25.1°	A1 [4]
	ALT (for 2 nd M1 A1) $\cos \angle BCA = \frac{(5x)^2 + (6.6528'x)^2 - (3x)^2}{2 \times 5x \times 6.6528'x} = 0.9057... \Rightarrow \angle BCA = 25.07^\circ$	[M1
	Accept awrt 25.1°	A1]
(b)	$24 = \frac{1}{2} \times 3x \times 5x \times \sin 110^\circ$	M1
	$\Rightarrow x = \sqrt{\frac{24 \times 2}{3 \times 5 \times \sin 110^\circ}} = (1.84536...)$	M1
	Accept awrt 1.85	A1 [3]
	Total 7 marks	

Useful sketch

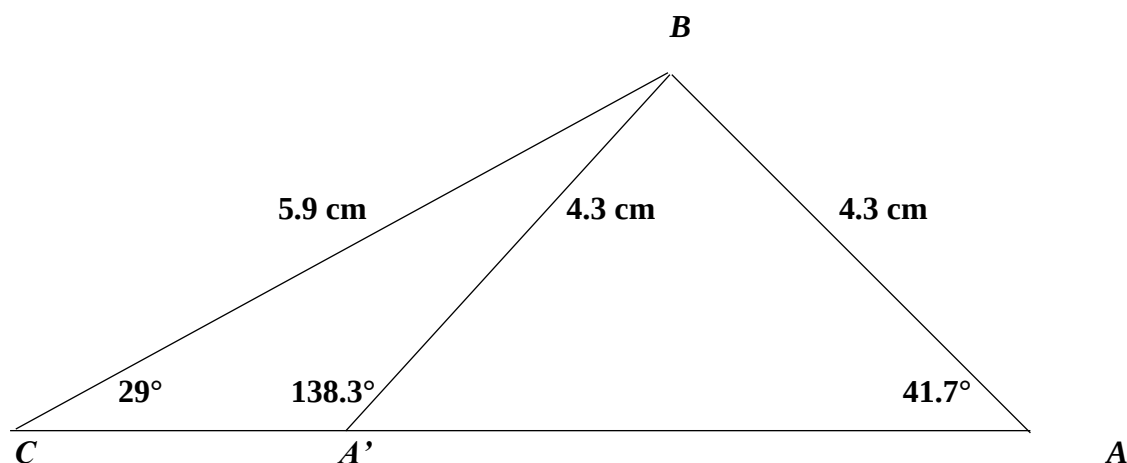


Part	Mark	Notes
(a)	M1	For using a correct cosine rule for AC or AC^2 . Note, this is given in the formula sheet so must be correct for this mark.
	A1	For finding a length of $AC = 6.6528...x$ or $AC = \sqrt{44.26...x^2}$ accept awrt $6.7x$ or $\sqrt{44.3x^2}$ Note: Allow the missing x here and throughout their working This mark can be implied by sight of $AC = \sqrt{34x^2 - 30x^2 \cos 110}$ if they carry it through to find the angle in the next step. For example: $\sin \angle BCA = \frac{3x \sin 110^\circ}{\sqrt{34x^2 - 30x^2 \cos 110}}$
	M1	For using any appropriate trigonometry to find the size of angle BCA . For example; Sine Rule $\sin \angle BCA = \frac{3x \sin 110^\circ}{6.6528'x} = 0.4237... \Rightarrow \angle BCA = 25.07...^\circ$ Cosine Rule $\cos \angle BCA = \frac{(5x)^2 + (6.6529x)^2 - (3x)^2}{2 \times 5x \times 6.6529x} \Rightarrow \angle BCA = 25.07$ Allow a missing x from their working provided it is consistent. Do not allow for example $\sin \angle BCA = \frac{3x \sin 110^\circ}{6.6528'}$
	A1	For awrt 25.1°
	ALT – Uses sine rule	
	M1	Use of sine rule: $\frac{\sin(70 - \theta)}{5x} = \frac{\sin \theta}{3x} \Rightarrow \left[\frac{\sin(70 - \theta)}{\sin \theta} = \frac{5}{3} \right]$
	A1	Expands $\sin(70 - \theta) = \sin 70 \cos \theta - \cos 70 \sin \theta$
	M1	Uses the tan identity and rearranges to make $\tan \theta$ the subject. $\frac{\sin 70 \cos \theta - \cos 70 \sin \theta}{\sin \theta} = \frac{5}{3}$ $\Rightarrow \frac{\sin 70}{\tan \theta} - \cos 70 = \frac{5}{3} \Rightarrow \tan \theta = \frac{\sin 70}{\cos 70 + \frac{5}{3}}$
	A1	For awrt 25.1°
	M1	For using the correct formula for the area of a triangle with the correct angle and side lengths. e.g., Can use AB and AC with $\angle ABC = 110^\circ$ or BC and AC with their $\angle BCA$ $[25.1^\circ]$ or AB and AC with their $(180^\circ - 110^\circ - 25.07^\circ = 44.93^\circ)$
	M1	For finding a value of x using their value for AC and BCA or BAC The processing must be correct for this mark and they must be finding the square root of x for this mark.
	A1	For awrt 1.85

Question number	Scheme	Marks
4 (a)	$\frac{\sin A}{5.9} = \frac{\sin 29}{4.3} \quad \text{oe}$ $(A =) \sin^{-1} \left(\frac{\sin 29}{4.3} \times 5.9 \right) \quad \text{or} \quad \sin^{-1} ("0.66520.....") \quad \text{oe}$ $A = 41.7$ $A = 180 - "41.69796....." = 138.3$	M1 dM1 A1 A1 [4]
(b)	$180 - (29 + "138.30203") \quad (= 12.69796.....)$ $(AC)^2 = 5.9^2 + 4.3^2 - 2 \times 5.9 \times 4.3 \cos(180 - (29 + "138.30203")) \quad \text{or}$ $AC = \sqrt{5.9^2 + 4.3^2 - 2 \times 5.9 \times 4.3 \cos(180 - (29 + "138.30203"))}$ $= 1.95(\text{cm})$	M1 M1 A1 [3]
ALT (b)	$180 - (29 + "138.30203") \quad (= 12.69796.....)$ $\frac{\sin 12.7}{AC} = \frac{\sin 29}{4.3}$ $AC = 0.2198 \times \frac{4.3}{0.4848}$ $= 1.95 (\text{cm})$	[M1] [M1] [A1] [3]
Total 7 marks		

Part	Mark	Additional Guidance
4 (a)	M1	For correct substitution into the sine rule, any form.
	dM1	For rearrangement to give $A =$, allow one processing error
	A1	For awrt 41.7
	A1	For awrt 138.3
(b)	M1	For calculating angle using $180 - (29 + "138.30203")$ They must use their 'larger' angle. If they have only one angle to use then this mark is M0
	M1	For correct substitution into the cosine rule, any form, allow any angle calculated from their acute or obtuse value from part (a) applied in $180 - (29 + \text{"their angle"})$
	A1	For awrt 1.95 (cm)
ALT (b)	M1	For calculating angle using $180 - (29 + "138.30203")$ They must use their 'larger' angle. If they have only one angle to use then this mark is M0
	M1	For correct substitution into the sine rule, any form, allow any angle calculated from their acute or obtuse value from part (a) applied in $180 - (29 + \text{"their angle"})$
	A1	For awrt 1.95 (cm)

Useful Sketch



Question number	Scheme	Marks
3 (a)	$\cos(\angle B) = \frac{10^2 + 8^2 - 12^2}{2 \times 10 \times 8} = \frac{10^2 + 6^2 - x^2}{2 \times 10 \times 6}$ $\frac{20}{160} = \frac{136 - x^2}{120}$ $x^2 = 121$ $x = 11 \quad *$ Alternative Method $\cos(ADB) = \frac{x^2 + 6^2 - 10^2}{2 \times 6 \times x} = \frac{x^2 - 64}{12x}$ $\cos(ADC) = \frac{x^2 + 2^2 - 12^2}{2 \times 2 \times x} = \frac{x^2 - 140}{4x}$ $\frac{x^2 - 64}{12x} + \frac{x^2 - 140}{4x} = 0$ $4x^2 = 484$ $x = 11 \quad *$	M1 M1 dM1 A1 cso (4) M1 M1 dM1 A1 cso (4)
(b)	Angle $ABC / ABD = \cos^{-1}\left(\frac{10^2 + 8^2 - 12^2}{2 \times 10 \times 8}\right) = 82.8^\circ$ $\frac{1}{2} \times 10 \times 6 \times \sin 82.8 = 29.8$ Alternative Method 1 Angle $ADB = \cos^{-1}\left(\frac{11^2 - 64}{12 \times 11}\right) = 64.42^\circ$ $\frac{1}{2} \times 6 \times 11 \times \sin 64.42 = 29.8$ Alternative Method 2 $\cos ABC = \frac{10^2 + 8^2 - 12^2}{2 \times 10 \times 8} = \frac{1}{8}$ $\sin ABC / ABD = \sqrt{1 - \left(\frac{1}{8}\right)^2} = \sqrt{\frac{63}{64}} = \frac{3}{8}\sqrt{7}$ $\frac{1}{2} \times 10 \times 6 \times \frac{3}{8}\sqrt{7} = 29.8$	B1 M1 M1 A1 (4) [8] B1 M1 M1 A1 (4) B1 M1 M1 A1 (4)

	Notes
(a)	
M1	Use of cosine rule to obtain a correct expression for $\cos(\angle B)$. The correct formula in either form may be used.
M1	Use of cosine rule to obtain a second correct expression for $\cos(\angle B)$. The correct formula in either form may be used.
dM1	Dependant on previous M mark - for solving leading to $x^2 = \dots$
A1 cso	For obtaining the given result
	Alternative
M1	Use of cosine rule to obtain a correct expression for $\cos(\angle ADB)$. The correct formula in either form may be used.
M1	Use of cosine rule to obtain a correct expression for $\cos(\angle ADC)$. The correct formula in either form may be used.
dM1	Dependant on previous M mark - for solving leading to $4x^2 = \dots$
A1	For obtaining the given result
(b)	
B1	For use of the cosine rule to find angle B
M1	For $\cos^{-1}\left(\frac{10^2 + 8^2 - 12^2}{2 \times 10 \times 8}\right)$ oe
M1	Use of $\frac{1}{2}ab \sin C$ (correct for their angle)
A1	29.8
	Alternative 1
B1	For use of the cosine rule to find angle ADB
M1	For $\cos^{-1}\left(\frac{11^2 - 64}{12 \times 11}\right)$ oe
M1	Use of $\frac{1}{2}ab \sin C$ (correct for their angle)
A1	29.8
	Alternative 2
B1	For use of the cosine rule to find $\cos B$
M1	Use of $\sin^2 A + \cos^2 A = 1$
M1	Use of $\frac{1}{2}ab \sin C$ (correct for their angle)
A1	29.8



Question number	Scheme	Marks
5	Triangle OBA: $\tan\left(\frac{\pi}{3}\right) = \frac{AB}{r}$ $AB = r \tan\left(\frac{\pi}{3}\right)$ Area of triangle $OBA = \frac{1}{2}r^2 \tan\left(\frac{\pi}{3}\right)$ Area of quadrilateral = $\sqrt{3}r^2$ Area of sector = $\frac{1}{2}r^2 \times \frac{2}{3}\pi = \frac{\pi}{3}r^2$ Area of shaded region = $\sqrt{3}r^2 - \frac{\pi}{3}r^2 = 10$ oe $r^2 = \frac{10}{\sqrt{3} - \frac{\pi}{3}}$ $r = 3.82$	M1 A1 M1 A1 M1 dM1 M1 A1 [8]
Notes		
Accept angles converted to degrees throughout		
M1	For $\tan\left(\frac{\pi}{3}\right) = \frac{AB}{r}$ or $\tan\left(\frac{\pi}{3}\right) = \frac{AC}{r}$	
A1	For $r \tan\left(\frac{\pi}{3}\right)$ (= AB or AC)	
M1	For finding the area of either triangle OBA or $OCA = \frac{1}{2}r^2 \tan\left(\frac{\pi}{3}\right)$	
A1	For finding the area of the quadrilateral = $\sqrt{3}r^2$	
M1	For finding the area of the sector = $\frac{1}{2}r^2 \times \frac{2}{3}\pi$	
dM1	For area of quadrilateral – area of sector = 10 dependant on a reasonable attempt to find both the area of the quadrilateral and the area of the sector	
M1	For solving the equation as far as $r^2 = \dots$	
A1	For 3.82	

Question number	Scheme	Marks
3	Mark parts (i) and (ii) together $\frac{r^2\theta}{2} = 16.8 \Rightarrow \left(\theta = \frac{33.6}{r^2} \right)$ $r\theta + 2r = 16.4$ $r \left(\frac{33.6}{r^2} \right) + 2r = 16.4 \Rightarrow r^2 - 8.2r + 16.8 = 0$ $r = \frac{8.2 \pm \sqrt{(8.2)^2 - 4 \times 1 \times 16.8}}{2 \times 1} = 4, (4.2)$ $\theta = \frac{33.6}{4^2} = 2.1$ ALT $r \left(\frac{33.6}{r^2} \right) + 2r = 16.4 \Rightarrow 5r^2 - 41r + 84 = 0 \Rightarrow (5r - 21)(r - 4) = 0$ $\Rightarrow r = 4, \left(\frac{21}{5} \right)$ $\theta = \frac{33.6}{4^2} = 2.1$ ALT- elimination of r by substitution $\frac{r^2\theta}{2} = 16.8$ $r\theta + 2r = 16.4 \quad \left(\Rightarrow r = \frac{16.4}{(\theta+2)} \right)$ $\frac{\left(\frac{16.4}{(\theta+2)} \right)^2}{2} \theta = 16.8$ $33.6\theta^2 - 134.56\theta + 134.4 = 0$ $\theta = 2.1, (1.90 \dots)$ $r = \sqrt{\frac{33.6}{2.1}} \text{ or } r = \sqrt{\frac{33.6}{1.90\dots}} \Rightarrow r = 4, \left(\frac{21}{5} \right)$ ALT $\frac{r^2\theta}{2} = 16.8 \Rightarrow \left(r = \sqrt{\frac{33.6}{\theta}} \right)$ $r\theta + 2r = 16.4$ $16.4 = 2\sqrt{\frac{33.6}{\theta}} + \theta\sqrt{\frac{33.6}{\theta}} \Rightarrow 16.4 = (2 + \theta)\sqrt{\frac{33.6}{\theta}}$ $33.6\theta^2 - 134.56\theta + 134.4 = 0$	B1 B1 M1M1 dM1A1 dM1A1 [8] [M1M1 dM1A1 dM1A1] [B1 B1 M1 M1 dM1A1 dM1A1 B1 B1 M1 M1 dM1A1]

$\theta = 2.1, (1.90 \dots)$	$r = \sqrt{\frac{33.6}{2.1}} \text{ or } r = \sqrt{\frac{33.6}{1.90\dots}} \Rightarrow r = 4, \left(\frac{21}{5}\right)$	dM1A1
Total 8 marks		

Mark	Notes
B1	Uses the correct formula for the area of a sector to give $\frac{r^2\theta}{2} = 16.8$
B1	Uses the correct formula for the length of an arc to give $r\theta + 2r = 16.4$
M1	For attempting to eliminate θ by substitution: $r\left(\frac{33.6}{r^2}\right) + 2r = 16.4$ An attempt involves rearrangement of their linear equation to $\theta = \dots$ followed by substitution into the area of a sector equation or rearrangement of their area of a sector equation to $\theta = \dots$ followed by substitution into the linear equation. Allow if $2r$ omitted in their perimeter equation.
M1	For forming a 3TQ in r using only their expressions. $r^2 - 8.2r + 16.8 = 0$ or $5r^2 - 41r + 84 = 0$
dM1	For an attempt to solve their 3TQ to give at least one value of r See General Guidance for the definition of an attempt. This mark is dependent on the first M mark being awarded.
A1	For the correct value of r : $r = 4, (4.2)$ Reject $r = 4.2$ if given.
M1	For substituting <i>their</i> r into one of the two equations and rearranging to obtain θ
A1	For the correct value of θ . $\theta = 2.1$ Condone the value of θ that corresponds to 4.2 being included.
ALT – elimination of r by substitution	
M1	For attempting to eliminate r by substitution: $\frac{\left(\frac{16.4}{(\theta+2)}\right)^2}{2} \theta = 16.8$ or $16.4 = 2\sqrt{\frac{33.6}{\theta}} + \theta\sqrt{\frac{33.6}{\theta}}$ An attempt involves rearrangement of their linear equation to $r = \dots$ followed by substitution into the area of a sector equation or rearrangement of their area of a sector equation to $r = \dots$ followed by substitution into the linear equation. Allow if $2r$ omitted in their perimeter equation.
M1	For forming a 3TQ in θ using only their expressions. $33.6\theta^2 - 134.56\theta + 134.4 = 0$
dM1	For an attempt to solve their 3TQ to give at least one value of θ See General Guidance for the definition of an attempt. This mark is dependent on the first M mark being awarded.
A1	For the correct value of θ . $\theta = 2.1$ Condone the value of θ that corresponds to 4.2 being included.
M1	For substituting <i>their</i> θ into one of the two equations and rearranging to obtain r
A1	For the correct value of r : $r = 4, (4.2)$ Reject $r = 4.2$ if given.
	Note: In open, award first A for r , second A for θ



Question number	Scheme	Marks
4 (a)	$\frac{1}{2} \cdot \frac{\pi}{3} r^2 - \frac{1}{2} \cdot r \cdot \frac{3r}{4} \sin\left(\frac{\pi}{3}\right) (= 79.5)$	M1 M1
	$\frac{\pi}{6} r^2 - \frac{3\sqrt{3}}{16} r^2 = 79.5$ leading to $r =$	ddM1
	$r = \sqrt{\frac{79.5}{\left(\frac{\pi}{6} - \frac{3\sqrt{3}}{16}\right)}} = 20$	A1 [4]
(b)	$(BD^2) = "20"'^2 + \left(\frac{3}{4} \times "20"\right)^2 - 2 \times 20 \times \left(\frac{3}{4} \times "20"\right) \cos \frac{\pi}{3}$	M1
	$= 5\sqrt{13}$ or awrt 18.02	M1
	Perimeter = $("20" - 15) + \frac{"20" \pi}{3} + "5\sqrt{13}" = 44$	M1 A1 [4]
Total 8 marks		

Part	Marks	Notes
(a)	M1	For $\frac{1}{2} \cdot \frac{\pi}{3} r^2$
	M1	For $\frac{1}{2} \cdot r \cdot \frac{3r}{4} \sin\left(\frac{\pi}{3}\right)$
	ddM1	For the correct unsimplified equation and attempting to rearrange (allow two slips) to make r the subject (dep on both previous M marks)
	A1	For 20
(b)	M1	For correct use of the cosine rule, as shown (ft their r)
	M1 (A1 on ePen)	For $5\sqrt{13}$ or awrt 18.02 – allow the candidate show they are finding the square root of their BD^2 to give an answer for BD
	M1	For correctly finding the perimeter of ABD with correct use of arc length $= r\theta$ (ft their r and their BD)
	A1	For 44

Question number	Scheme	Marks
7 (a)	$\frac{3 + (1 - \cos^2 \theta)}{\cos \theta - 2} = 3 \cos \theta$ $4 - \cos^2 \theta = 3 \cos^2 \theta - 6 \cos \theta$ $(2 \cos \theta + 1)(2 \cos \theta - 4) = 0$ $(\cos \theta = 2 \text{ does not exist so}) \cos \theta = -\frac{1}{2} \quad *$	M1 M1 M1 A1 cso (4)
(b)	$\cos 3x = -\frac{1}{2}$ $3x = 120^\circ, 240^\circ, 480^\circ$ $x = 40^\circ, 80^\circ, 160^\circ$	M1 A1 A1 A1 (4)
Total 8 marks		

Part	Mark	Notes
(a)	M1	For the correct use of $\sin^2 \theta + \cos^2 \theta = 1$
	M1	For multiplying both sides by $\cos \theta - 2$ and expanding brackets
	M1	For solving their 3TQ using any method. $4 \cos^2 \theta - 6 \cos \theta - 4 = 0 \Rightarrow 2(2 \cos \theta - 1)(\cos \theta - 2) = 0$ See General Guidance.
	A1cso	For obtaining the given equation: $\cos \theta = -\frac{1}{2}$ Must reject $\cos \theta = 2$
	ALT	
	M1	For the correct use of $\sin^2 \theta + \cos^2 \theta = 1$
	M1	Factorises LHS $\frac{4 - \cos^2 \theta}{\cos \theta - 2} = 3 \cos \theta \Rightarrow \frac{(2 - \cos \theta)(2 + \cos \theta)}{\cos \theta - 2} = 3 \cos \theta$
	M1	Cancels through by $\cos \theta - 2$ and solves their linear equation in terms of $\cos \theta$ $-(2 + \cos \theta) = 3 \cos \theta \Rightarrow \cos \theta = \dots$
	A1cso	For obtaining the given equation: $\cos \theta = -\frac{1}{2}$
(b)	M1	For $\cos 3x = -\frac{1}{2}$
	A1	For $3x = 120^\circ$ or any other correct angle, e.g. even $3x = -120^\circ$ Allow an angle in radians for this mark. E.g. $3x = \frac{2\pi}{3}$
	A1	For one from $x = 40^\circ, 80^\circ, 160^\circ$
	A1	For all angles correct with no additional angles within range. $x = 40^\circ, 80^\circ, 160^\circ$ Ignore any angles out of range.

	Penalise any extra angles within range by deducting the final A mark.
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Question	Scheme	Marks
3(a)	$36 = \frac{1}{2} \times x \times (x+7) \times \sin 30^\circ \Rightarrow 36 = \frac{x(x+7)}{4} \Rightarrow x^2 + 7x - 144 = 0$ [Accept also e.g., $x^2 + 7x = 144$] $(x-9)(x+16) = 0 \Rightarrow x = 9, (-16) \text{ (cm) }^*$	M1 M1A1 cso [3]
(b)	$BC = \sqrt{9^2 + 16^2 - 2 \times 9 \times 16 \cos 30^\circ} = 9.3586... \approx 9.36 \text{ (cm)}$	M1A1 [2]
(c)(i)	$\frac{\sin ABC}{16} = \frac{\sin 30^\circ}{9.3586} \Rightarrow \angle ABC = 180^\circ - 58.7408... \approx 121.3^\circ$	M1A1
(ii)	$\angle BCA = 180^\circ - 30^\circ - 121.259^\circ = 28.740... \approx 28.7^\circ$	B1ft [3]
Total 8 marks		

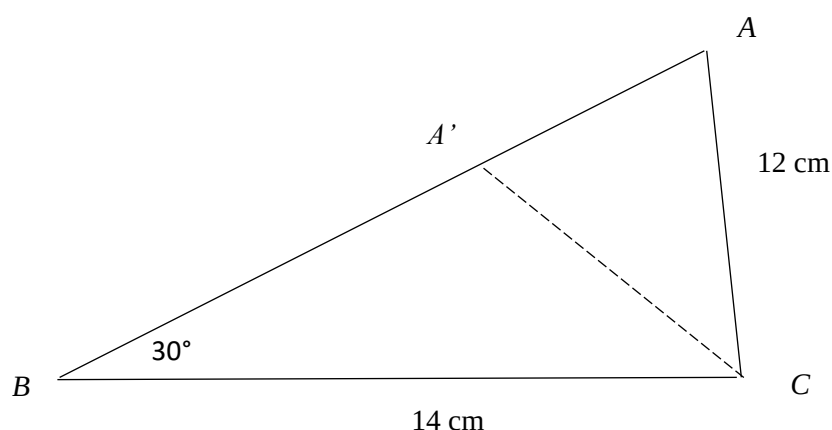
Part	Mark	Notes
(a)	M1	Applies $\frac{1}{2}ab \sin C$ correctly with the given lengths to form a 3TQ in terms of x
	M1	Solves their 3TQ by any correct method to find the value of x This is a show question – this step MUST be seen
	A1 cso	For $x = 9$ Sight of $x = -16$ not rejected is A0
(b)	M1	Applies cosine rule correctly to find the length BC
	A1	For the correct length awrt 9.36 (cm)
	ALT	
	M1	Finds the height of the triangle from vertex B [4.5 cm] and applies trigonometry or Pythagoras theorem to find the length of the point from the base of the perpendicular to C $AX^2 = 9^2 - 4.5^2 = \frac{243}{4} \Rightarrow XC = 16 - \sqrt{\frac{243}{4}}$ $BC = \sqrt{4.5^2 + \left(16 - \sqrt{\frac{243}{4}}\right)^2} = \dots$
	A1	For the correct length awrt 9.36 (cm)

(c)	M1	Uses any appropriate trigonometry to find angle ABC Award this mark for awrt $58.7...^\circ$ or an acute angle if their AC is incorrect.
	A1	For $\angle ABC = 180^\circ - 58.7408... \approx 121.3^\circ$ Accept awrt 121.3° or 121.2°
	B1ft	For awrt 28.7° or 28.8° Allow a ft here from their angle BCA NB: this is an A mark in Epen
	ALT 1 Uses trigonometry again for the final B mark	
	B1ft	Sine rule $\frac{\sin \angle ACB}{9} = \frac{\sin 30^\circ}{9.3586} \Rightarrow [\angle BCA \approx 28.7^\circ]$ or $\frac{\sin \angle ACB}{9} = \frac{\sin 121.3^\circ}{16} \Rightarrow [\angle BCA \approx 28.7^\circ]$ cosine rule $\angle ACB = \cos^{-1} \left(\frac{9.36^2 + 16^2 - 9^2}{2 \times 9.36 \times 16} \right) = [28.7^\circ]$ For awrt 28.7° allow 28.8°
	ALT 2 Uses the area of a triangle	
	M1	For using the given area of the triangle with their BC For example; $36 = \frac{1}{2} \times 9 \times 9.36 \times \sin ABC \Rightarrow \sin ABC = 0.85470... = (58.726...^\circ)$
	A1	$180^\circ - 58.726...^\circ \approx 121.3^\circ$ or 121.2°
	B1ft	As above or restarts using the area of the triangle. $36 = \frac{1}{2} \times 16 \times 9.36 \times \sin ACB \Rightarrow (\angle ACB = 28.7^\circ)$

Question number	Scheme	Marks
5(a)	$12^2 = 14^2 + x^2 - 2 \times 14x \cos 30$ $x^2 - 14\sqrt{3}x + 52 = 0$ $x = \frac{14\sqrt{3} \pm \sqrt{(14\sqrt{3})^2 - 4(1)(52)}}{2(1)}$ $x = 7\sqrt{3} \pm \sqrt{95}$	M1 M1 A1 M1 A1cso [5]
(b)	Area = $\frac{1}{2} \times 14 \times (7\sqrt{3} \pm \sqrt{95}) \times \sin 30$ or uses Area = $\frac{1}{2} \times 12 \times (7\sqrt{3} \pm \sqrt{95}) \times \frac{7}{12}$ Difference in areas $D = \frac{1}{2} \times 14 \times (7\sqrt{3} + \sqrt{95}) \times \sin 30 - \frac{1}{2} \times 14 \times (7\sqrt{3} - \sqrt{95}) \times \sin 30$ $D = 7\sqrt{95}$	M1 dM1 A1 [3]
ALT Part (a) – sine rule		
	$\frac{12}{\sin 30} = \frac{14}{\sin A} \Rightarrow \sin A = \frac{7}{12}$ $\left(\frac{7}{12}\right)^2 + \cos^2 A \equiv 1 \Rightarrow \cos A = \sqrt{1 - \left(\frac{7}{12}\right)^2}$ $\sin A = \frac{7}{12}, \cos A = \pm \frac{\sqrt{95}}{12}$ $x = 12 \cos A + 14 \cos 30 \Rightarrow x = 12 \times \frac{\pm\sqrt{95}}{12} + 14 \times \frac{\sqrt{3}}{2}$ $x = 7\sqrt{3} \pm \sqrt{95} *$	{M1 M1 A1 M1 A1 Cso [5]}
Total 8 marks		



USEFUL SKETCH



Part	Mark	Additional Guidance
(a)	M1	Use the cosine rule to form an equation. Rule to be correct but may be in any rearranged form.
	M1	Rearrange to obtain a 3TQ. Terms in any order. = 0 may be missing.
	A1	Correct 3TQ Allow poor notation – missing = 0
	M1	Attempt to solve their 3TQ by use of the formula or completing the square. See general guidance for what constitutes an attempt to solve. Use of calculators: If the 3TQ is correct, accept correct solutions without working.
	A1	Correct values for x $x = 7\sqrt{3} \pm \sqrt{95}$
(b)	M1	For use of area of triangle $= \frac{1}{2}ac \sin B$ with $a = 14$, $B = 30^\circ$ and their x May use either value found for x . Or uses $= \frac{1}{2}bc \sin A$ with $c = 12$ and $\sin A = \frac{7}{12}$
	dM1	For finding the difference between their two areas provided both areas are positive This M mark is dependent on the previous M mark
	A1	Correct difference in areas (must be exact value)

ALT Part (a) – Uses sine rule		
	M1	Use of the sine rule to find $\sin A$
	M1	Use of $\sin^2 A + \cos^2 A \equiv 1$ to find an exact value for $\cos A$
	A1	Correct exact values for $\sin A$ and $\cos A$
	M1	Method to find x $x = 12 \cos A + 14 \cos 30$ Allow their $\cos A$
	A1	Correct values for x $x = 7\sqrt{3} \pm \sqrt{95}$

Question Number	Scheme	Marks
3(a)	$AM = \sqrt{10^2 - 8^2} = 6$	M1,A1 (2)
(b)	$\cos C = \frac{26^2 + 16^2 - 26^2}{2 \times 16 \times 26} = \frac{256}{832} \left(= \frac{4}{13} \text{ oe} \right)$	M1A1
	$\angle BCD = 72^\circ$	A1cao (3)
(c)	$AD = \sqrt{26^2 - 10^2} = 24 \text{ or } DM = \sqrt{26^2 - 8^2} = 6\sqrt{17} \text{ oe } (24.73\dots)$	M1A1
	$\tan(\angle DMA) = \frac{24}{6} \text{ or } \cos(\angle DMA) = \frac{6}{6\sqrt{17}} \text{ or } \sin(\angle DMA) = \frac{24}{6\sqrt{17}}$	M1
	$\angle DMA = 76^\circ$	A1cao (4)
[9]		
(a) M1 A1 NB	Use Pythagoras, with a minus sign, to obtain the length of AM . Correct length obtained. Answers w/o working get both marks (use of (3,4,5) triangle)	
(b) M1	Use the cosine rule in $\triangle BCD$ to obtain a numerical expression for $\cos C$. Correct formula in either form may be quoted and substitution attempted or correct formula can be implied by the correct substitution. Complete method needed, so if another angle found first do not award this mark until a value for angle BCD is obtained.	
A1	Correct value for the cosine obtained (Decimal to be awrt 0.308 (0.30769...)) Award by implication if final answer is awrt 72° .	
A1cao	For 72° (from correct working)	
ALT	Use the isosceles triangle	
M1A1	$\cos C = \frac{8}{26} \text{ oe } (Any \text{ trig function allowed provided work completed to a value for angle } BCD)$	
A1cao	For 72° (from correct working)	
(c) M1 A1 M1	Attempt the length of AD or DM using Pythagoras with a minus sign. Correct value for their choice of line, Use an appropriate trig function. The length of AD or DM must have been attempted with a + or a - sign.	
A1cao	Correct answer.	
Penalise once only in (b) and (c) for failing to round as instructed.		

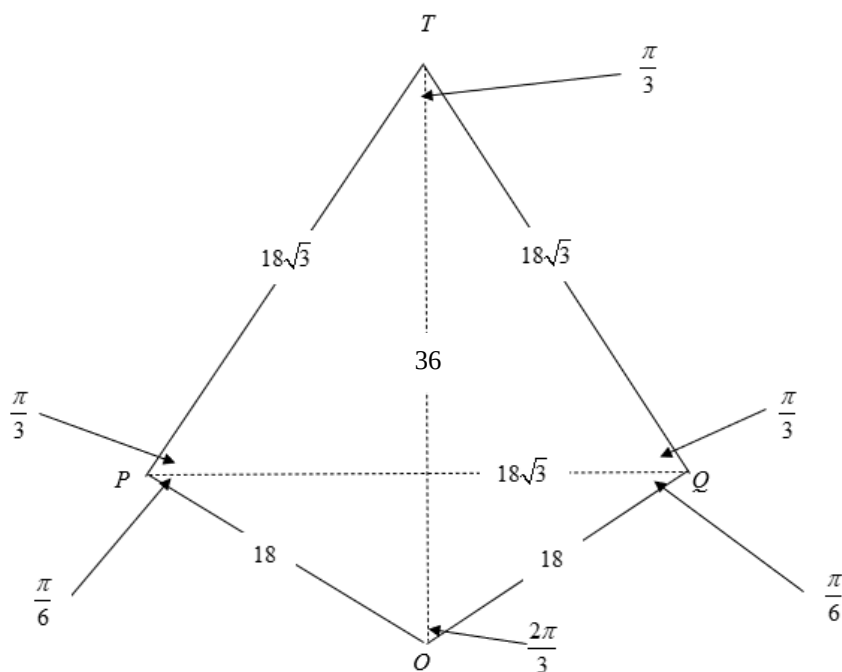
Question number	Scheme	Marks
4 (a)	$A = \frac{1}{2} \times 8 \times 8 \times \sin 60^\circ = (16\sqrt{3})$ $48\sqrt{3} = \frac{1}{3} \times 16\sqrt{3} \times h \Rightarrow h = 9 *$	M1 M1 A1cso [3]
(b)	$BX = \sqrt{9^2 + 8^2} = \sqrt{145}$ $\angle BXC = \frac{'145' + '145' - 8^2}{2 \times ' \sqrt{145} ' \times ' \sqrt{145} '} = 38.8025...^\circ \approx 38.8^\circ$	M1 M1A1 [3]
(c)	Let midpoint of BC be M $AM = \sqrt{8^2 - 4^2} = (4\sqrt{3}) \text{ or } MX = \sqrt{145 - 4^2} = (\sqrt{129})$ $\angle XMA = \tan^{-1}\left(\frac{9}{'4\sqrt{3}'}\right) \text{ or } \angle XMA = \sin^{-1}\left(\frac{9}{'\sqrt{129}'}\right) \text{ or } \angle XMA = \cos^{-1}\left(\frac{'4\sqrt{3}'}{'\sqrt{129}'}\right)$ $= 52.4109...^\circ \approx 52.4^\circ$	M1 M1 A1 [3]
	ALT 1	
	Let midpoint of BC be M $(XA)^2 = (AM)^2 + (XM)^2 - 2(AM)(XM)\cos\theta$ $9^2 = (8^2 - 4^2) + (8^2 + 9^2 - 4^2) - 2\sqrt{8^2 - 4^2}\sqrt{8^2 + 9^2 - 4^2}\cos\theta$ $\theta = 52.4109...^\circ \approx 52.4^\circ$	{M1} {M1} {A1} [3]
Total 9 marks		
(a)	Use of $\frac{1}{2}ab\sin C$ (may be implied by $(16\sqrt{3})$) Use of $\frac{1}{3} \times \text{'Area of base'} \times h$ Obtains the given answer with no errors in the working Use of $\sqrt{(\text{part } a)^2 + 8^2}$ (may be implied by $\sqrt{145}$) Use the cosine rule, either form. If not for angle BXC there must be a complete method shown for obtaining BXC (follow through their BX) awrt 38.8° Use of Pythagoras' to find the length of AM or MX (may be implied by $4\sqrt{3}$ or $\sqrt{129}$) $\tan^{-1}\left(\frac{9}{'4\sqrt{3}'}\right) \text{ or } \sin^{-1}\left(\frac{9}{'\sqrt{129}'}\right) \text{ or } \cos^{-1}\left(\frac{'4\sqrt{3}'}{'\sqrt{129}'}\right)$ awrt 52.4°	M1
M1		M1
A1 cso		A1 cso
(b)		M1
M1		M1
A1		A1
(c)		M1
M1		M1
A1		A1

ALT	
M1	Use of cosine rule using AX , AM and XM e.g. $(XA)^2 = (AM)^2 + (XM)^2 - 2(AM)(XM)\cos\theta$
M1	Correct values substituted into the cosine rule in any form e.g. $9^2 = (8^2 - 4^2) + (8^2 + 9^2 - 4^2) - 2\sqrt{8^2 - 4^2}\sqrt{8^2 + 9^2 - 4^2}\cos\theta$
A1	awrt 52.4°

Question Number	Scheme	Marks
3(a)	$AM = \sqrt{6^2 + 8^2} = 10$ $AE = \sqrt{14^2 + 10^2} = \sqrt{296} = 17.20... = 17.2 \text{ cm}$	M1 M1A1 (3)
(b)	$\tan \phi = \frac{EM}{MA} = \frac{14}{10}, \phi = 54.46... = 54.5^\circ$ or using another trig function	M1A1ft,A1(3)
(c)	$\tan \theta = \frac{EM}{\frac{1}{2}CD} = \frac{14}{8}, \theta = 60.255...^\circ = 60.3^\circ$	M1A1ft,A1 (3) [9]

Part	Mark	Notes
(a)	M1	Applies Pythagoras theorem to find the length of AM $AM = \sqrt{6^2 + 8^2} = 10$ or $AM = \frac{\sqrt{12^2 + 16^2}}{2} = 10$
	M1	Applies Pythagoras to find the length of one of the sloping edges $AE = \sqrt{14^2 + 10^2} = \sqrt{296} = ...$
	A1	For the correct length of either AE , DE , CE or BE $AE = 17.2 \text{ cm}$ rounded correctly
	ALT	
	M1M1	Applies Pythagoras in 3D $AE = \sqrt{14^2 + 6^2 + 8^2} = \sqrt{296} = ...$
(b)	M1	For applying any acceptable trigonometry to find the required angle. $\tan \phi = \frac{EM}{MA} = \frac{14}{10}$, or $\sin \phi = \frac{14}{\sqrt{296}}$, or $\cos \phi = \frac{10}{\sqrt{296}} \Rightarrow \phi = ...$
	A1ft	For the correct trigonometry if they use sine or cosine following through their $\sqrt{296}$
	A1	Required angle = 54.5° Rounded correctly
(c)	M1	For applying trigonometry to find the required angle. $\tan \theta = \frac{EM}{\frac{1}{2}CD} = \frac{14}{8} \Rightarrow \theta = ...$ OR The length of the perpendicular from E to the mid-point of AD is $\sqrt{260}$ $\sin \theta = \left(\frac{14}{\sqrt{260}} \right)$, or $\cos \left(\frac{8}{\sqrt{260}} \right) \Rightarrow \theta = ...$
	A1ft	Ft their $\sqrt{260}$
	A1	$\theta = 60.3^\circ$
		Rounding: Penalise rounding only the first time it occurs in either (b) or (c)

Question number	Scheme	Marks
3 (a)	$PQ = 18 \times \frac{2}{3} \pi = 12\pi$	M1 A1 (2)
(b)(i)	$\alpha = \frac{1}{3} \pi$	B1
(b)(ii)	$PT = 18 \tan \frac{\pi}{3} = 18\sqrt{3}$	M1 A1
	Area of $OPTQ = 2 \times \frac{1}{2} \times 18 \times 18\sqrt{3}$	M1
	Area of Sector $OPQ = \frac{1}{2} \times 18^2 \times \frac{2\pi}{3}$	M1
	Shaded Area = $2 \times \frac{1}{2} \times 18 \times 18\sqrt{3} - \frac{1}{2} \times 18^2 \times \frac{2\pi}{3} = 222 \text{ cm}^2$	M1 A1 (7)
Total 9 marks		



Area of triangle $OPQ = 81\sqrt{3}$ or $140.29... \text{ cm}^2$

Area of triangle $PQT = 243\sqrt{3}$ or $420.88... \text{ cm}^2$

Area of quadrilateral $OTPQ = 324\sqrt{3}$ or $561.18... \text{ cm}^2$

Part	Mark	Guidance
(a)	M1	Uses the correct formula for the length of arc to give $PQ = 18 \times \frac{2}{3} \pi = ...$
	A1	For $PQ = 12\pi$

ALT – works in degrees (but the angle must be correct at 120°)	
M1	Uses the correct formula for length of arc to give $PQ = \frac{120}{360} \times 2\pi \times 18 = \dots$
A1	For $PQ = 12\pi$
(b)	(i) B1
	For stating $\alpha = \frac{\pi}{3}$ (Please check the diagram as it may written on there)
	(ii) Method 1 - Allow use of degrees throughout provided the angles are correct. $(\angle POQ = 120^\circ, \angle PTQ = 60^\circ)$
	Finds length of PT
	M1
	For finding length PT : e.g., $\tan\left(\frac{\pi}{3}\right) = \frac{PT}{18} \Rightarrow PT = 18 \tan\left(\frac{\pi}{3}\right) = \dots$ The given values must be used correctly
	A1
	For $PT = 18\sqrt{3}$
	M1
	For the area of $OPTQ = 2 \times \frac{1}{2} \times 18 \times '18\sqrt{3}' = (561.18\dots)$ Their $18\sqrt{3}$ must come from an attempt at using trigonometry.
	Method 2
	Finds lengths PQ and TO
	M1
	For finding the lengths PQ and TO using any acceptable correct trigonometry. e.g., $PQ = \sqrt{18^2 + 18^2 - 2 \times 18 \times 18 \cos\left(\frac{2\pi}{3}\right)} = \dots$ and $\cos\left(\frac{\pi}{3}\right)$ $TO = \frac{\dots}{18} = \dots$ The given values must be used correctly
	A1
	For both correct lengths: $PQ = 18\sqrt{3}$ and $TO = 36$
	M1
	For the area of $OPTQ = \frac{1}{2} \times '18\sqrt{3}' \times '36' = (561.18\dots \text{ or } 324\sqrt{3})$ Their $18\sqrt{3}$ and 36 must come from an attempt at using trigonometry
	M1
	For the area of sector $OPQ = \frac{1}{2} \times 18^2 \times \frac{2\pi}{3} = (339.29\dots \text{ or } 108\pi)$ or $\frac{120^\circ}{360^\circ} \times \pi \times 18^2 = (339.29\dots)$
	M1
	For area of $OPTQ$ – area of Sector OPQ $561.18 - 339.29 = 221.887\dots$
	A1
	For 222 cm^2 (must be 3sf) (Units are not required)

Question number	Scheme	Marks
7 (a)	Throughout this question condone missing degree signs $\cos \theta^\circ = \frac{6^2 + 8^2 - k^2}{2 \times 6 \times 8} = \frac{100 - k^2}{96} *$	M1A1cso [2]
(b)	$\sqrt{455} = \frac{1}{2} \times 6 \times 8 \times \sin \theta^\circ$ $\Rightarrow \sin \theta^\circ = \frac{\sqrt{455}}{24} \Rightarrow \left(\sin^2 \theta^\circ = \frac{455}{576} \right)$ $\cos^2 \theta^\circ = 1 - \sin^2 \theta^\circ \Rightarrow \cos^2 \theta^\circ = 1 - \frac{455}{576} = \frac{121}{576}$ $\Rightarrow \cos \theta^\circ = \pm \frac{11}{24} \quad \text{both values required}$ $\cos \theta^\circ = \frac{11}{24} = \frac{100 - k^2}{96} \Rightarrow k^2 = 56 \Rightarrow k = \sqrt{56} = (2\sqrt{14})$ $\cos \theta^\circ = -\frac{11}{24} = \frac{100 - k^2}{96} \Rightarrow k^2 = 144 \Rightarrow k = 12$ ALT $\sqrt{455} = \frac{1}{2} \times 6 \times 8 \times \sin \theta^\circ$ $\Rightarrow \sin \theta^\circ = \frac{\sqrt{455}}{24}$ $\theta^\circ = \sin^{-1} \left(\frac{\sqrt{455}}{24} \right) (= 62.72 \dots^\circ)$ $\theta^\circ = 62.72 \dots, 117.27 \dots \quad \text{both values required}$ $\cos \theta^\circ = \frac{100 - k^2}{96} \Rightarrow k^2 = 100 - 96 \cos \theta^\circ$ $\Rightarrow k^2 = 100 - 96 \cos 62.72 \dots \Rightarrow k^2 = 56 \Rightarrow k = \sqrt{56}$ $\cos \theta^\circ = \frac{100 - k^2}{96} \Rightarrow k^2 = 100 - 96 \cos \theta^\circ$ $\Rightarrow k^2 = 100 - 96 \cos 117.27 \dots \Rightarrow k^2 = 144 \Rightarrow k = 12$	M1 A1 M1 A1 M1A1 A1 [7] [M1 A1 M1 A1 M1A1 A1]
Total 9 marks		

Part	Mark	Notes
(a)	M1	For correct substitution into the cosine rule and attempt to rearrange to find an expression for $\cos \theta^\circ$
	A1 cso	For obtaining the given expression for $\cos \theta$ $\cos \theta^\circ = \frac{100 - k^2}{96}$ Note: This is a show question. There must be no errors seen.

(b)	M1	For using the correct formula for area of a triangle and substitution of the given values to obtain $\sqrt{455} = \frac{1}{2} \times 6 \times 8 \times \sin \theta^\circ$ and attempt to rearrange to obtain $\sin \theta^\circ = \dots$
	A1	$\sin \theta^\circ = \frac{\sqrt{455}}{24}$ Allow for $\sin \theta = \frac{\sqrt{455}}{0.5 \times 8 \times 6}$
	M1	For use of $\sin^2 \theta + \cos^2 \theta = 1$ to obtain a value for $\cos \theta^\circ$. $\cos^2 \theta^\circ = 1 - \frac{'455'}{576} \Rightarrow \cos \theta^\circ = \pm \sqrt{1 - \frac{'455'}{576}}$ Allow use of their $\sin \theta^\circ$ provided $-1 \leq \sin \theta^\circ \leq 1$ Allow if only one value of $\cos \theta^\circ$ obtained.
	A1	$\cos \theta^\circ = \pm \frac{11}{24}$
	M1	For forming an equation for k using their $\cos \theta^\circ$ and attempt to solve for k . $\cos \theta^\circ = \frac{'11'}{24} = \frac{100-k^2}{96} \Rightarrow k^2 = 56 \Rightarrow k = \sqrt{56}$ $\cos \theta^\circ = -\frac{11'}{24} = \frac{100-k^2}{96} \Rightarrow k^2 = 144 \Rightarrow k = 12$
	A1	For one correct value of k $\sqrt{56}$ or awrt 7.48 or awrt 12
	A1	For both correct values of k $\sqrt{56}$ or awrt 7.48 and awrt 12
ALT – working with angles		
	M1	For using the correct formula for area of a triangle and substitution of the given values to obtain $\sqrt{455} = \frac{1}{2} \times 6 \times 8 \times \sin \theta^\circ$ and attempt to rearrange to obtain $\sin \theta^\circ = \dots$
	A1	$\sin \theta^\circ = \frac{\sqrt{455}}{24}$ Allow for $\sin \theta = \frac{\sqrt{455}}{0.5 \times 8 \times 6}$
	M1	For use of the inverse trigonometric function to obtain a value for θ° $\theta^\circ = 62.72 \dots, 117.27 \dots$ If working not shown then award for angle correct to a minimum of 1 d.p. Allow if only one value of θ° found. Condone working in radians awrt 1.09, awrt 2.05
	A1	$\theta^\circ = 62.72 \dots, 117.27 \dots$ Allow awrt 62.7° , 117.3° Both angles found. Condone working in radians awrt 1.09, awrt 2.05
	M1	For forming an equation in k using their θ and an attempt to solve for k . $\cos \theta^\circ = \frac{100-k^2}{96} \Rightarrow k^2 = 100 - 96 \cos \theta^\circ$ $\Rightarrow k^2 = 100 - 96 \cos 62.72 \dots \Rightarrow k^2 = 56 \Rightarrow k = \sqrt{56}$ $\cos \theta^\circ = \frac{100-k^2}{96} \Rightarrow k^2 = 100 - 96 \cos \theta^\circ$ $\Rightarrow k^2 = 100 - 96 \cos 117.27 \dots \Rightarrow k^2 = 144 \Rightarrow k = 12$
	A1	For one correct value of k $\sqrt{56}$ or awrt 7.48 or awrt 12
	A1	For both correct values of k and no others. $\sqrt{56}$ or awrt 7.48 and awrt 12

Question	Scheme	Mark
4(a)	$FC = \sqrt{10^2 + 10^2 + 10^2} = \sqrt{300}$	M1A1 [2]
(b)	$\cos FCA = \frac{\sqrt{200}}{\sqrt{300}} \Rightarrow \angle FCA = 35.3^\circ$	M1A1 [2]
(c)	$CX = \frac{\sqrt{200}}{2} = \sqrt{50}, FX = \sqrt{50+100} = \sqrt{150}$ $\cos \angle FXC = \frac{'150' + '50' - '300'}{2 \times \sqrt{'150'} \times \sqrt{'50'}} \Rightarrow \angle FXC = 125.2643...^\circ \Rightarrow \text{awrt } 125^\circ$	M1,M1A1 M1A1 [5]
Total		9 marks

Part	Mark	Notes
(a)	M1	For using Pythagoras theorem or any appropriate trigonometry to find FC . $FC = \sqrt{10^2 + 10^2 + 10^2} = \dots$ OR $AC = \sqrt{10^2 + 10^2} = \sqrt{200}$ $FC = \sqrt{10^2 + 200} = \sqrt{300}$
	A1	For the correct exact length of $FC = \sqrt{300} (=10\sqrt{3})$
(b)	M1	For using any appropriate trigonometry to find the required angle. They must find a value for the award of this mark. $\cos \angle FCA = \frac{\sqrt{200}}{\sqrt{300}} \Rightarrow \angle FCA = (35.2643^\circ), \sin \angle FCA = \frac{10}{\sqrt{300}} \Rightarrow \angle FCA = (35.2643^\circ),$ $\tan \angle FCA = \frac{10}{\sqrt{200}} \Rightarrow \angle FCA = (35.2643^\circ)$ Allow awrt 14.1 for $\sqrt{200}$ and awrt 17.3 for $\sqrt{300}$
	A1	$\angle FCA = 35.2643...^\circ \approx 35.3^\circ$ (awrt) Accept answers which round to 35.3° , but they must round to this value.

(c)	M1	Let X be the midpoint of BH . For any method (Pythagoras or trigonometry) to find the length CX $CX = \frac{\sqrt{200}}{2} = (\sqrt{50} \text{ or } 5\sqrt{2})$
	M1	For any method (Pythagoras or trigonometry) to find the length FX $FX = \sqrt{50+100} = (\sqrt{150} \text{ or } 5\sqrt{6})$ or $FX = \sqrt{200-50} = (\sqrt{150} \text{ or } 5\sqrt{6})$
	A1	For both correct lengths $\sqrt{50}$ and $\sqrt{150}$
	M1	For the correct cosine rule to find angle FXC using their lengths $\cos \angle FXC = \frac{'150' + '50' - '300'}{2 \times \sqrt{'150'} \times \sqrt{'50'}} \Rightarrow \angle FXC = (125.2643...^\circ)$ NB: Check that the cosine rule they are using is to find the required angle.
	A1	For the correct angle with awrt. $\angle FXC = 125^\circ$ Accept answers which round to 125° , but they must round to this value.
	ALT – Uses the right-angled triangle FGX	
	M1	Let X be the midpoint of BH . For any method (Pythagoras or trigonometry) to find the length GX $GX = \frac{\sqrt{200}}{2} = (\sqrt{50} \text{ or } 5\sqrt{2} \text{ or awrt } 7.07)$ If they use the sin ratio to find the angle, identify $FG = 10$ (cm)
	M1	For any method (Pythagoras or trigonometry) to find the length FX $FX = \sqrt{50+100} = (\sqrt{150} \text{ or } 5\sqrt{6} \text{ awrt } 12.2)$ or $FX = \sqrt{200-50} = (\sqrt{150} \text{ or } 5\sqrt{6} \text{ awrt } 12.2)$ OR If they use the tan ratio to find the angle, identify $FG = 10$ (cm)
	A1	For both correct lengths $\sqrt{50}$ and 10 for tan or $\sqrt{50}$ and $\sqrt{150}$ for cos or $\sqrt{150}$ and 10 for sin Allow decimal equivalents, i.e., 7.07 and 12.2
	M1	$\cos \angle FXG = \frac{\sqrt{50}}{\sqrt{150}} \Rightarrow \angle FXG = (54.7356...^\circ)$, $\sin \angle FXG = \frac{10}{\sqrt{150}} \Rightarrow \angle FXG = (54.7356...^\circ)$, $\tan \angle FXG = \frac{10}{\sqrt{50}} \Rightarrow \angle FXG = (54.7356...^\circ)$ So the required angle is $\angle FXC = 180 - '54.7356...'^\circ = (125.264...^\circ)$ We must see this calculation for the award of this mark. Do not award just for $54.7356...^\circ$
	A1	For the correct angle with awrt. $\angle FXC = 125^\circ$ Accept answers which round to 125° , but they must round to this value.



Question	Scheme	Marks
3(a)	$BC^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \times \cos 100^\circ$ $BC = 13.848... \approx 13.8$	M1 A1 [2]
(b)	(i) $\sin ABC = \frac{8 \sin 100^\circ}{13.848...} \Rightarrow ABC = 34.6752...^\circ \approx 34.7^\circ$ (ii) $\angle ACB = 180^\circ - 100^\circ - 34.6752...^\circ = 45.324...^\circ \approx 45.3^\circ$	M1A1 B1FT [3]
	(i)(ii) $\cos ACB = \frac{8^2 + 13.848...^2 - 10^2}{2 \times 8 \times 13.848...} \Rightarrow ACB = 45.330 ...$ $ABC = 180^\circ - 100^\circ - 45.330 ... = 34.6699 ...$	[M1A1 B1FT]
(c)	$\angle MBC = 34.675 \div 2 = 17.337...^\circ$ and $\angle BMC = 180^\circ - 17.337^\circ = 117.33...^\circ$ $MC = \frac{13.848 \sin 17.337^\circ}{\sin 117.33^\circ} = 4.6293...$ Area of $BMC =$ $\frac{1}{2} \times 4.629 \times 13.848 \times \sin 45.324^\circ = 22.79... \approx 22.8 \text{ (cm}^2\text{)}$	B1FT M1 M1A1 [4]
	ALT $ABM = 34.675 \div 2 = 17.337 ...^\circ$ and $AMB = 180 - 100 - 17.33 ... = 62.66 ...$ $BM = \frac{10 \sin 100}{\sin 62.66...} = 11.086 ...$ Area of $BMC =$ $\frac{1}{2} \times 13.848 ... \times 11.086 ... \times \sin 17.337 ... = 22.87 ...$ $\approx 22.9 \text{ (cm}^2\text{)}$	[B1FT M1 M1A1]
Total 9 marks		

Part	Mark	Notes
(a)	M1	For applying the cosine rule correctly to obtain BC^2
	A1	For awrt 13.8 (cm)
(b)(i)	M1	For applying the sine rule correctly to find angle ABC Accept any appropriate trigonometry. $\frac{\sin ABC}{8} = \frac{\sin 100}{13.8}$ leading to $ABC = 34.8$ is M1A0
	A1	For awrt 34.7°
(ii)	B1FT	For awrt 45.3° FT $180 - 100 - \text{their } 34.3752 ...$ $0 < \text{their } 34.3752 ... < 80$

ALT		
(b)(i)(ii)	M1	For applying cosine rule correctly to find angle ACB Accept any appropriate trigonometry.
	A1	For awrt 45.3° (angle ACB)
	B1FT	FT $180 - 100 - \text{their } 45.330 \dots$ (angle ABC)
(c)	B1ft	For both angles MBC and BMC
	M1	For applying the sine rule using their angles and BC to find length MC Allow alternative correct methods to find MC .
	M1	For using Area = $\frac{1}{2}ab \sin C$ using their length and angles. $\text{Their } MC \neq 4$
	A1	Awrt 22.8 or awrt 22.9
ALT		
(c)	B1ft	For both angles ABM and AMB
	M1	For applying sine rule using their angles to find length BM . Allow alternative correct methods to find BM .
	M1	For using Area = $\frac{1}{2}ab \sin C$ using their length and angles. $\text{Their } MC \neq 4$ if used.
	A1	Awrt 22.8 or awrt 22.9

Question	Scheme	Marks
4a)	$\theta = \frac{21-r}{r}$ $A = \frac{r^2}{2} \theta \Rightarrow A = \frac{r^2}{2} \left(\frac{21-r}{r} \right) \Rightarrow A = \frac{r}{2} (21-r)^*$ ALT OR $\theta r = 21-r$ or $l = 21-r$ $A = \frac{r}{2} r \theta \text{ or } A = \frac{1}{2} r l \Rightarrow A = \frac{r}{2} (21-r)^*$	B1 M1A1 cso [3] [B1 M1A1]
(b)	$27 \leq \frac{r}{2} (21-r) \Rightarrow 54 \leq 21r - r^2 \Rightarrow r^2 - 21r + 54 \leq 0$ Finds c.v's $r^2 - 21r + 54 = 0 \Rightarrow (r-18)(r-3) = 0 \Rightarrow r = 18, 3$ $3 \leq r \leq 18$	B1 M1 M1A1 [4]
(c)	$\theta = \frac{21-r}{r} \Rightarrow \theta = \frac{21-18}{18} = \frac{1}{6} \quad \theta = \frac{21-3}{3} = 6$ $\Rightarrow \frac{1}{6} \leq \theta \leq 6$	B1ft B1 [2]
Total 9 marks		

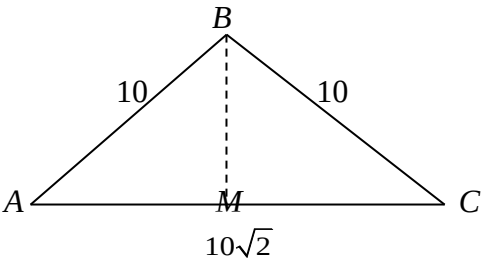
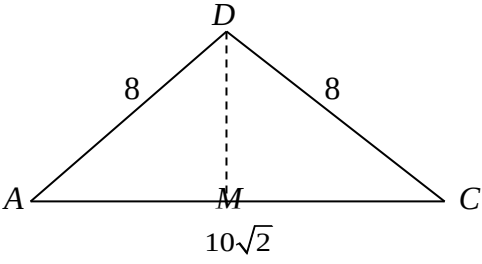
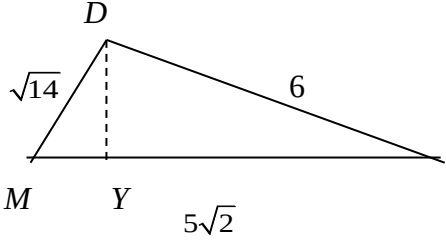
Part	Mark	Notes
(a)	B1	States or uses $\theta = \frac{21-r}{r}$
	M1	Uses the correct formula for the area of a sector and substitutes their expression for θ
	A1 cs0	For the correct expression for A exactly as written with no errors. That is $A = \frac{r}{2}(21-r)$ including $A = ..$ However, if they have a string of working, then allow $A =$ at the top For example $., A = ...$ $=$ $= = \frac{r}{2}(21-r)$ is acceptable
	ALT	
	B1	States $l = 21-r$
	M1	Uses the alternative formula $A = \frac{1}{2}rl$ and substitutes in their l
	A1 cs0	For the correct expression for A exactly as written with no errors.
(b)	B1	Forms a correct inequality and a correct 3TQ (accept in any form) using the given expression for A and the value of 27
	M1	Solves their 3TQ by any method to obtain two values for r If they use a calculator to solve an incorrect 3TQ, award only if a method is seen. If the 3TQ is correct, and they use a calculator, award for correct roots seen.
	M1	Forms an inequality for the inside region using their values of r Allow any critical values which were obtained from their quadratic equation. Accept any acceptable form that defines a continuous inside region For example: $3 \leq r$ and $r \leq 18$ or $3 \leq r \cap r \leq 18$ Accept even $3 \leq r$ or $r \leq 18$
	A1	For the correct inequality only in this form. $3 \leq r \leq 18$
(c)	B1ft	For both $\theta = \frac{1}{6}$ and $\theta = 6$ You must follow through their values for r for this mark.
	B1	For the correct inequality only in this form.. $\frac{1}{6} \leq \theta \leq 6$

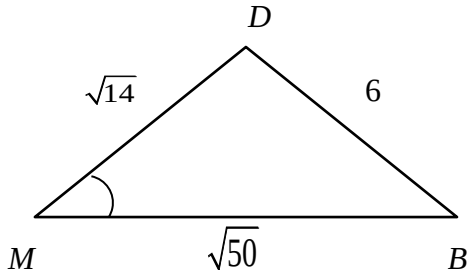
Question number	Scheme	Marks
8 (a)	$k = 2$	B1 [1]
(b)	$2 + 3 \cos A - \sin A - 3 \sin 2A - 2 \cos^2 A = 3 \cos A - \sin A - 6 \sin A \cos A + 2 \sin^2 A$ $3 \cos A - \sin A - 6 \sin A \cos A + 2 \sin^2 A = 3 \cos A(1 - 2 \sin A) - \sin A(1 - 2 \sin A)$ $3 \cos A(1 - 2 \sin A) - \sin A(1 - 2 \sin A) = (1 - 2 \sin A)(3 \cos A - \sin A)$ $\Rightarrow p = 3, q = 1, r = 2$	B1 M1 A1 [3]
	ALT	
	$2 + 3 \cos A - \sin A - 3 \sin 2A - 2 \cos^2 A = 3 \cos A - \sin A - 6 \sin A \cos A + 2 \sin^2 A$ $(p \cos A - \sin A)(q - r \sin A) = pq \cos A - q \sin A - pr \cos A \sin A + r \sin^2 A$ $pq \cos A - q \sin A - pr \cos A \sin A + r \sin^2 A$ $\Rightarrow p = 3, q = 1, r = 2$	{B1} {M1} {A1} [3]
(c)	$(1 - 2 \sin 2\theta)(3 \cos 2\theta - \sin 2\theta) = 0$ $\Rightarrow 3 \cos 2\theta - \sin 2\theta = 0 \Rightarrow \tan 2\theta = 3$ $\Rightarrow 1 - 2 \sin 2\theta = 0 \Rightarrow \sin 2\theta = \frac{1}{2}$ $\tan 2\theta = 3 \Rightarrow 2\theta = 1.1071..., 4.7124... \Rightarrow \theta = 0.5536, 2.3562$ $\sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = 0.5235..., 2.6179... \Rightarrow \theta = 0.2618, 1.3090$	B1 B1 M1A1 M1A1 [6]
Total 10 marks		
(a) B1 (b) B1 M1 A1 ALT B1 M1 A1	$k = 2$ Substituting $k = 2$ and use of $\sin^2 A + \cos^2 A = 1$ to obtain $3 \cos A - \sin A - 6 \sin A \cos A + 2 \sin^2 A$ Factorising to obtain $(1 - 2 \sin A)(3 \cos A - \sin A)$ $p = 3, q = 1, r = 2$ If p, q and r are stated then they must be correct (may be implied by a correct factorisation if p, q and r are not stated) Substituting $k = 2$ and use of $\sin^2 A + \cos^2 A = 1$ to obtain $3 \cos A - \sin A - 6 \sin A \cos A + 2 \sin^2 A$ Expanding $(p \cos A - \sin A)(q - r \sin A)$ to obtain $pq \cos A - q \sin A - pr \cos A \sin A + r \sin^2 A$ $p = 3, q = 1, r = 2$	

(c)	
B1	$\tan 2\theta = 3$
B1	$\sin 2\theta = \frac{1}{2}$
M1	$2\theta = 1.24(90\dots), 4.39(06\dots)$
A1	$\theta = 0.625, 2.2(0)$
M1	$2\theta = 0.5235\dots, 2.61(79\dots)$ allow $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$
A1	$\theta = 0.262, 1.31$ allow $\left[\frac{\pi}{12}, \frac{5\pi}{12}\right]$

Question number	Scheme	Marks
10 (a)	$x = \tan^{-1}(-3) = -72$ $x = 108 \quad x = 288$	M1 A1 A1 (3)
(b)	$7\sin^2 \theta + \sin \theta \cos \theta = 6(\sin^2 \theta + \cos^2 \theta)$ $\sin^2 \theta + \sin \theta \cos \theta - 6\cos^2 \theta = 0$ $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\sin \theta}{\cos \theta} - 6 = 0$ $\tan^2 \theta + \tan \theta - 6 = 0$	M1 A1 cso (3)
(c)	$(\tan y + 3)(\tan y - 2) = 0$ $\tan y = -3 \quad \tan y = 2$ $y = 108, 288 \quad y = 63, 243$	M1 A1 A1ft A1 (4)
Total 10 marks		

Part	Mark	Guidance
(a)	M1	For using inverse tan to obtain any correct angle $\tan^{-1}(-3) \Rightarrow x = -71.565...^{\circ}$ Accept awrt -72°
	A1	For either 108 or 288
	A1	For both 108 and 288
(b)	M1	Uses $\sin^2 \theta + \cos^2 \theta = 1$ on the given equation to obtain $7\sin^2 \theta + \sin \theta \cos \theta = 6(\sin^2 \theta + \cos^2 \theta)$
	M1	For rearranging and dividing through by $\cos^2 \theta$ with the $\frac{\sin \theta}{\cos \theta} = \tan \theta$ identity to obtain a 3TQ: $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\sin \theta}{\cos \theta} - 6 = 0 \Rightarrow (\tan^2 \theta + \tan \theta - 6 = 0)$
	ALT 1	
	M1	Divides the given equation through by $\cos^2 \theta$ with the $\frac{\sin \theta}{\cos \theta} = \tan \theta$ identity to obtain $7\tan^2 \theta + \tan \theta = \frac{6}{\cos^2 \theta}$
	M1	Uses $\sin^2 \theta + \cos^2 \theta = 1$ to obtain $\tan^2 \theta + 1 = \frac{1}{\cos^2 \theta}$ and uses this result on the given equation and rearranges to achieve a 3TQ to obtain $7\tan^2 \theta + \tan \theta = 6(1 + \tan^2 \theta) \Rightarrow (\tan^2 \theta + \tan \theta - 6 = 0)$
	A1	For obtaining the given expression $\tan^2 \theta + \tan \theta - 6 = 0$ * in full. Note: This is a show question, there must be no errors in the solution.
	ALT 2	
	M1	Uses the identity $\frac{\sin \theta}{\cos \theta} = \tan \theta$ with $\tan^2 \theta + \tan \theta - 6 = 0$ to achieve $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\sin \theta}{\cos \theta} - 6 = 0$ and multiplies through by $\cos^2 \theta$ to obtain $\sin^2 \theta + \sin \theta \cos \theta - 6\cos^2 \theta = 0$
	M1	Uses $\sin^2 \theta + \cos^2 \theta = 1$ to obtain $\sin^2 \theta + \sin \theta \cos \theta - 6(1 - \sin^2 \theta) = 0$ and rearranges to obtain $7\sin^2 \theta + \sin \theta \cos \theta = 6$
	A1 CSO	For obtaining the given expression $7\sin^2 \theta + \sin \theta \cos \theta = 6$ in full. Note: This is a show question, there must be no errors in the solution.
(c)	M1	For changing $7\sin^2 y + \sin y \cos y = 6$ to $\tan^2 y + \tan y - 6 = 0$ [this step must be correct] and then attempting to solve the 3TQ by any method.
	A1	For $\tan y = -3$ and $\tan y = 2$
	A1ft	For both $y = 108$ and 288 (ft from (a)) Do not ft angles out of range
	A1	For both $y = 63$ and 243
Rounding errors: Penalise rounding only once in this question when first seen provided angles round to 108, 288, 63 or 243		
Extra angles: Deduct one A mark for any extra angles within range. Ignore angles outside of range.		

Question number	Scheme	Marks
9 (a)	$AC = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$	M1A1 [2]
(b)	 $BM = \sqrt{10^2 - (5\sqrt{2})^2} = 5\sqrt{2}$	M1A1 [2]
(c)	 $DM = \sqrt{8^2 - (5\sqrt{2})^2} = \sqrt{14}$ $\angle DMB = \cos^{-1} \left(\frac{14 + 50 - 36}{2 \times \sqrt{14} \times 5\sqrt{2}} \right) = 58.0519...^\circ \approx 58.1^\circ$	M1A1 M1A1 [4]
(d)	Let perpendicular from point D to BM be the point Y  Vertical height of shape $ABCD$ In triangle DMY (above) Height = $DY = \sqrt{14} \sin 58.0519^\circ = 3.1749... \approx 3.17$ (cm)	M1A1 [2]
Total 10 marks		

Part	Mark	Notes
(a)	M1	For using Pythagoras theorem correctly on triangle ABC to find AC
	A1	For $AC = 10\sqrt{2}$
(b)	M1	For using Pythagoras theorem or any appropriate trigonometry correctly on triangle ABC to find BM
	A1	For the correct length $BM = 5\sqrt{2}$ NB: Allow for the answer just seen without any working as the triangle ABC is isosceles.
(c)	M1	For using Pythagoras theorem to find the length DM
	A1	For $DM = \sqrt{14}$ or accept awrt 3.74
	M1	For using cosine rule to find the required angle DMB 
	A1	For awrt 58.1°
(d)	M1	For using any appropriate trigonometry to find the required length. Ft their angle DMB
	A1	For awrt 3.17 (cm) Accept 3.18 (cm)

Question number	Scheme	Marks
6(a)(i)	$\frac{\sin 30^\circ}{x} = \frac{\sin ACB}{x+3}$ $\sin \theta^\circ = \frac{x+3}{2x} *$	M1 A1 CSO
(ii)	$\cos^2 \theta^\circ = 1 - \left(\frac{x+3}{2x} \right)^2$ $\cos^2 \theta^\circ = \frac{(2x)^2 - (x+3)^2}{(2x)^2}$ $\cos \theta^\circ = \frac{\sqrt{3x^2 - 6x - 9}}{2x} *$	M1 M1 A1 [5]
Alt – use of right-angled triangle with Pythagoras' theorem		
	$\text{Adjacent} = \sqrt{(2x)^2 - (x+3)^2}$ $\cos \theta = \frac{\sqrt{(2x)^2 - (x+3)^2}}{2x}$ $\cos \theta^\circ = \frac{\sqrt{3x^2 - 6x - 9}}{2x} *$	[M1 M1 A1]
(b)	$\frac{\angle BAC}{30^\circ} = \frac{7}{2} \Rightarrow \angle BAC = 105^\circ$ $\theta = 180 - 30 - 105 = 45$ $\cos 45^\circ = \frac{\sqrt{2}}{2} = \frac{\sqrt{3x^2 - 6x - 9}}{2x} \Rightarrow 2x^2 = 3x^2 - 6x - 9 \Rightarrow x^2 - 6x - 9 = 0$ $x^2 - 6x - 9 = 0 \Rightarrow (x-3)^2 - 18 = 0 \Rightarrow x = \dots$ $x = 3 + 3\sqrt{2}$	B1 B1 M1 M1 A1 [5]
Alt – last three marks		
	$\sin \theta^\circ = \frac{x+3}{2x} = \frac{\sqrt{2}}{2} \Rightarrow x+3 = \sqrt{2}x$ $x+3 = \sqrt{2}x \Rightarrow x(\sqrt{2}-1) = 3 \Rightarrow x = \frac{3}{\sqrt{2}-1}$ $x = 3 + 3\sqrt{2}$	[M1 M1 A1]
Total 10 marks		

Part	Marks	Scheme
(a) (i)	M1	For using a correct sine rule to give, $\frac{\sin 30^\circ}{x} = \frac{\sin ACB}{x+3}$
	A1 cao	For correctly obtaining the expression for $\sin \theta$ $\sin \theta^\circ = \frac{x+3}{2x}$ *
(ii)	M1	For using the Pythagorean identity $\cos^2 \theta^\circ = 1 - \left(\frac{x+3}{2x}\right)^2$
	M1	For simplifying to form a single fraction $\cos^2 \theta^\circ = \frac{(2x)^2 - (x+3)^2}{(2x)^2}$
	A1 cao	For simplifying to achieve the given expression, $\cos \theta^\circ = \frac{\sqrt{3x^2 - 6x - 9}}{2x}$ * Note this is a show question
Alt – use of right-angled triangle with Pythagoras' theorem		
	M1	For use of a right-angled triangle with Pythagoras' theorem to determine the adjacent $\text{Adjacent} = \sqrt{(2x)^2 - (x+3)^2}$
	M1	For use of cosine ratio $\cos \theta = \frac{\sqrt{(2x)^2 - (x+3)^2}}{2x}$
	A1 cao	For simplifying to achieve the given expression, $\cos \theta^\circ = \frac{\sqrt{3x^2 - 6x - 9}}{2x}$ * Note this is a show question
(b)	B1	For finding the size of $\angle BAC$ $\frac{\angle BAC}{30^\circ} = \frac{7}{2} \Rightarrow \angle BAC = 105^\circ$
	B1	For finding the value of $\theta = 180 - 30 - 105 = 45$
	M1	For substituting the value of $\angle ABC$ into the given expression for $\cos \theta$ and forming a 3TQ, condone arithmetic errors in rearrangement. $\cos 45^\circ = \frac{\sqrt{2}}{2} = \frac{\sqrt{3x^2 - 6x - 9}}{2x} \Rightarrow 2x^2 = 3x^2 - 6x - 9 \Rightarrow x^2 - 6x - 9 = 0$ Allow for use of their 45° but this must come from an attempt at working with the ratio. Do not allow if their 45° is 30° .
	M1	For an attempt to solve their 3TQ by any valid method (see general guidance) $x^2 - 6x - 9 = 0 \Rightarrow (x-3)^2 - 18 = 0 \Rightarrow x = \dots$
	A1 cao	For the correct value of x in the correct form $x = 3 + 3\sqrt{2}$ Allow $a = 3, b = 2$
Alternative method		
	M1	For substituting the value of $\angle ABC$ into the given expression for $\sin \theta$ and forming a linear equation $\sin \theta^\circ = \frac{x+3}{2x} = \frac{\sqrt{2}}{2} \Rightarrow x+3 = \sqrt{2}x$

	M1	For an attempt to solve their linear equation $x + 3 = \sqrt{2}x \Rightarrow x(\sqrt{2} - 1) = 3 \Rightarrow x = \frac{3}{\sqrt{2} - 1}$
	A1 cao	For the correct value of x in the correct form $x = 3 + 3\sqrt{2}$ Allow $a = 3, b = 2$

Question number	Scheme	Marks
9 (a)	$\cos \alpha = \frac{3}{\sqrt{13}}$	B1 (1)
(b)	$h = \sqrt{17^2 - (9^2 + 12^2)} = 8$	M1 M1 A1 (3)
(c)	Let M be the midpoint of BC $\tan \theta = \frac{h}{OM} = \frac{8}{12} = \frac{2}{3}$ *	M1 A1 (2)
(d)	Let N be the midpoint of EM $EM = \sqrt{8^2 + 12^2} = 4\sqrt{13}$ $NO = \sqrt{12^2 + (2\sqrt{13})^2 - 2(12)(2\sqrt{13})\left(\frac{3}{\sqrt{13}}\right)} \Rightarrow NO = 2\sqrt{13}$ Hence triangle ONM is isosceles $180 - 2 \times 33.7 = 112.6^\circ$	M1 M1 M1 A1 (4)
Total 10 marks		

Part	Mark	Notes
(a)	B1	For $\cos \alpha = \frac{3}{\sqrt{13}}$
(b)	M1	For use of Pythagoras to OC find e.g. $\sqrt{(9^2 + 12^2)}$ oe
	M1	For use of Pythagoras to find h e.g. $\sqrt{17^2 - (9^2 + 12^2)}$
	A1	For 8
(c)	M1	For $\tan \theta = \frac{h}{OM} = \frac{8}{12}$ (condone the omission of the degree sign)
	A1 cs0	For obtaining the given result
(d)	M1	For use of Pythagoras to find EM e.g. $\sqrt{8^2 + 12^2}$
	M1	For use of the cosine rule to find NO e.g. $\sqrt{12^2 + (2\sqrt{13})^2 - 2(12)(2\sqrt{13})\left(\frac{3}{\sqrt{13}}\right)}$
	M1	For $180 - 2 \times 33.7$
	A1	For 112.6°

Question number	Scheme	Marks
8 (a)	$2x - 75 = -31, 211$ $x = 22, 143$	M1A1 A1 (3)
(b)	$2 \frac{\sin y^\circ}{\cos y^\circ} + 5 \sin y^\circ = 0$ $\sin y^\circ \left(\frac{2}{\cos y^\circ} + 5 \right) = 0$ $\cos y^\circ = -\frac{2}{5} \quad (\sin y^\circ = 0)$ $y = 113.6^\circ$ $y = 0^\circ, 180^\circ$	M1 M1 A1 B1 (4)
(c)	$3(1 - \sin^2 \theta) - 3\sin^2 \theta + \sin \theta + 12 = 0$ $6\sin^2 \theta - \sin \theta - 15 = 0$ $(2\sin \theta + 3)(3\sin \theta - 5) = 0$ $\sin \theta = -\frac{3}{2} \quad \sin \theta = \frac{5}{3}$ As $-1 \leq \sin \theta \leq 1$ no such values for θ exist	M1 M1 A1 B1 (4)
		[11]

Part	Mark	Additional Guidance
(a)	M1	For finding at least one correct value of $(2x - 75) = -31^\circ$ or 211° and attempting to find one value of $x \Rightarrow x = \frac{-31+75}{2}$ or $x = \frac{211+75}{2}$
	A1	For $x = 22$ or 143
	A1	For $x = 22$ and 143 Extra values within range – A0 Extra values outside of the range - ignore
(b)	M1	For using the identity $\tan y^\circ = \frac{\sin y^\circ}{\cos y^\circ}$
	M1	For factorising their expression and finding values for $\sin y^\circ$ and $\cos y^\circ$ $\sin y^\circ \left(\frac{2}{\cos y^\circ} + 5 \right) = 0 \Rightarrow \sin y^\circ = 0, \cos y^\circ = -\frac{2}{5} \Rightarrow y = \dots$
	A1	For $y = 113.6$ if there are extra values within range – A0
	B1	For both $y = 0$ and 180 Both required
	ALT	
	M1	For multiplying $\sin y \times \frac{\cos y}{\cos y} \Rightarrow \tan y \cos y \Rightarrow (2 \tan y + 5 \tan y \cos y = 0)$
	M1	For factorising the above expression and finding values for $\tan y^\circ$ and $\cos y^\circ$ $2 \tan y + 5 \tan y \cos y = 0 \Rightarrow \tan y(2 + 5 \cos y) = 0$ $\Rightarrow \tan y = 0, \cos y = -\frac{2}{5} \Rightarrow y = \dots$
	A1	For $y = 113.6$ Extra values within range – A0 Extra values outside of the range - ignore
	B1	For both $y = 0$ and 180 Both required
(c)	SC	$2 \frac{\sin y}{\cos y} = -5 \sin y \Rightarrow \cos y = -\frac{2}{5} \Rightarrow y = 113.6$ no evidence of factorising – award M1M0A1B0 only (unless there is later recovery)
	M1	For using the identity $\cos^2 \theta + \sin^2 \theta = 1 \Rightarrow 3(1 - \sin^2 \theta) - 3 \sin^2 \theta + \sin \theta + 12 = 0$ to form a 3TQ in terms of $\sin \theta$ Minimally acceptable attempt is $6 \sin^2 \theta \pm \sin \theta \pm 15 = 0$
	M1	For an attempt to solve their 3TQ (see general guidance for the definition of an attempt) $6 \sin^2 \theta - \sin \theta - 15 = 0 \Rightarrow (2 \sin \theta + 3)(3 \sin \theta - 5) = 0 \Rightarrow \sin \theta = \dots, \dots$
	A1	$\sin \theta = -\frac{3}{2}, \frac{5}{3}$
	B1	For the conclusion; $ \sin \theta > 1$ therefore no values exist for $\sin \theta$ Do not accept ‘undefined’ without an explanation that $ \sin \theta > 1$
Penalise rounding only once in this question		

Question number	Scheme	Marks
10 (a)	$(x + \frac{\pi}{3}) = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ or $\frac{7\pi}{3}$	M1
	$(x + \frac{\pi}{3}) = \frac{\pi}{3}$ and $\frac{2\pi}{3}$ and $\frac{7\pi}{3}$	A1
	$x = 0, \frac{\pi}{3}, 2\pi$	A1
		(3)
(b)	$\tan \theta = -\frac{5}{3}$	M1
	$\theta = -59^\circ, -239^\circ, 121^\circ, 301^\circ$	M1 A1 (3)
(c)	$1 + \sin 2y - 2(1 - \sin^2 2y) = 0$	M1
	$2\sin^2 2y + \sin 2y - 1 = 0$	A1
	$(\sin 2y + 1)(2\sin 2y - 1) = 0$	
	$\sin 2y = -1$ or $\sin 2y = \frac{1}{2}$	dM1
	$2y = -90^\circ, (30^\circ), (150^\circ), -330^\circ, -210^\circ$	A1
	$y = -45^\circ, -105^\circ, -165^\circ$	A1
		(5)
		[11]

Part	Mark	Additional Guidance
(a)	M1	Any one of the three indicated angles, radians only, ignore any other angles.
	A1	For all three indicated angles, ignore other angles out of the range $\frac{\pi}{3} \leq x + \frac{\pi}{3} \leq \frac{7\pi}{3}$
	A1	For all three angles, ignore angles out of range, A0 if additional angles in range.
(b)	M1	For $\tan \theta = k$. $k \neq 0, k \neq \pm 1$
	M1	Any one correct value, does not need to be to the nearest degree. Allow one correct value to imply the first M1. Ignore any other angles.
	A1	For all four angles, ignore angles out of range, A0 if additional angles in range. All four angles must be given to the nearest degree.
(c)	M1	For the correct use of $1 - \sin^2 2y$ in the equation on the left or right side, equation doesn't need to be = 0.
	A1	Correct 3TQ, must be = 0 and a valid attempt to solve leading to $\sin 2y =$
	dM1	$\sin 2y = -1$ or $\sin 2y = \frac{1}{2}$ (allow $\sin 2y = a$ and b from a valid attempt to solve their 3TQ). Allow $\sin 2y$ to be x or any other variable.
	A1	For minimum of 3 of the 5 values shown, (including the ones in brackets), ignore other angles outside the range $-360^\circ \leq 2y \leq 360^\circ$. Allow sight of 270 if -90° present.
	A1	For all 3 values shown. Ignore extras out of range. Rounding answers (where accuracy is specified in the question) Penalise only once per question for failing to round as instructed - ie giving more digits in the answers.

Question number	Scheme	Marks
7(i) (a)	$\tan 2\theta = \frac{\tan \theta + \tan \theta}{1 - \tan \theta \tan \theta}$ $= \frac{2 \tan \theta}{1 - \tan^2 \theta} *$	M1 A1 cso (2)
(b)	$1 = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$ $\tan^2 \alpha + 2 \tan \alpha - 1 = 0$ $\tan \alpha = -1 \pm \sqrt{2}$	M1 M1 A1 (3)
(ii) (a)	$\cos(x - 30) = \sin(x + 30)$ $\Rightarrow \cos x \cos 30^\circ + \sin x \sin 30^\circ = \sin x \cos 30^\circ + \cos x \sin 30^\circ$ $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) \cos x = \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) \sin x$ $\frac{\sin x}{\cos x} = \tan x = \frac{\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)}$ $\tan x = 1 *$	M1 M1 M1 A1 cso (4)
(b)	$\tan 2y = 1 \Rightarrow \tan y = -1 \pm \sqrt{2}$ $y = -67.5^\circ, 22.5^\circ$	M1 A1 (2)
Total 11 marks		

Part	Mark	Notes
(i)(a)	M1	For use of $\tan(A + B) \Rightarrow \tan 2\theta = \frac{\tan \theta + \tan \theta}{1 - \tan \theta \tan \theta}$ and simplifying to obtain the given result.
	A1	For the given result with no errors seen
(i)(b)	M1	For setting $1 = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$ and for rearranging to obtain a 3TQ with terms in any order
	M1	For a correct attempt using a valid method to solve the 3TQ with two solutions for $\tan \alpha$ Accept use of θ for example in place of α
	A1	For $\tan \alpha = -1 \pm \sqrt{2}$
(ii)(a)		General principle. (The second and third M marks can be carried out in either order) First M1 – For expanding the two addition formulae correctly and equating. Second M1 – For collecting up like terms to obtain $\sin x(\dots) = \cos x(\dots)$ allow sign errors. Third M1 – For use of the $\tan = \frac{\sin}{\cos}$ identity Final A1 – For the given answer with no errors

They can work in terms of $\sin 30^\circ$, $\cos 30^\circ$ or $\tan 30^\circ$ or alternatively $\frac{1}{2}$, $\frac{\sqrt{3}}{2}$, $\frac{1}{\sqrt{3}}$ respectively	
M1	For the correct use of $\cos(A - B)$ and $\sin(A + B)$ to obtain $\cos x \cos 30^\circ + \sin x \sin 30^\circ = \sin x \cos 30^\circ + \cos x \sin 30^\circ$
M1	For collecting up like terms to obtain: $\sin x (\sin 30^\circ - \cos 30^\circ) = \cos x (\sin 30^\circ - \cos 30^\circ)$ NB: This is an A mark in Epen
M1	For use of $\tan = \frac{\sin}{\cos}$ at any stage in their working.
A1	For $\tan x = 1$
ALT	
M1	For the correct use of $\cos(A - B)$ and $\sin(A + B)$ to obtain $\cos x \cos 30^\circ + \sin x \sin 30^\circ = \sin x \cos 30^\circ + \cos x \sin 30^\circ$
M1	For dividing through by $\cos x$ and collecting up like terms to obtain: $\frac{\cos x \cos 30^\circ + \sin x \sin 30^\circ}{\cos x} = \frac{\sin x \cos 30^\circ + \cos x \sin 30^\circ}{\cos x}$ $\Rightarrow \frac{\sin x}{\cos x} (\sin 30^\circ - \cos 30^\circ) = \sin 30^\circ - \cos 30^\circ$ OR For dividing through by $\cos x$ and $\cos 30^\circ$ and also collecting up like terms to obtain: $\frac{\cos x \cos 30^\circ + \sin x \sin 30^\circ}{\cos x \cos 30^\circ} = \frac{\sin x \cos 30^\circ + \cos x \sin 30^\circ}{\cos x \cos 30^\circ}$ $\Rightarrow 1 + \frac{\sin x \sin 30^\circ}{\cos x \cos 30^\circ} = \frac{\sin x}{\cos x} + \frac{\sin 30^\circ}{\cos 30^\circ} \Rightarrow \frac{\sin x}{\cos x} \left(\frac{\sin 30^\circ}{\cos 30^\circ} - 1 \right) = \frac{\sin 30^\circ}{\cos 30^\circ} - 1$
M1	For use of $\tan = \frac{\sin}{\cos}$ at any stage in their working.
A1	For $\tan x = 1$
(ii)(b)	For solving the equation $\tan 2y = 1 \Rightarrow \tan y = -1 \pm \sqrt{2} \Rightarrow y = \dots, \dots$ M1 (leading to two values for y) NB: This is an B mark in Epen
	$y = -67.5^\circ, 22.5^\circ$ A1 Both values required. NB: This is an B mark in Epen
	ALT
	For solving the equation $\tan 2y = 1$ to achieve at least one correct value for $2y$ $2y = \tan^{-1} 1 = 45^\circ, -135^\circ$ [Accept even for example 225° for this mark] M1
	$y = -67.5^\circ, 22.5^\circ$ A1 Both values required. For extra angles in range withhold this mark. Extra angles outside of the range – ignore. NB: This is an B mark in Epen



6(a)	$(AC =) \sqrt{x^2 + x^2} (= \sqrt{2}x) \text{ or } (AC =) \frac{x}{\sin 45} \text{ or } (AC =) \frac{x}{\cos 45}$ $(AO =) \sqrt{\left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^2} \left(= \sqrt{\frac{x^2}{2}} = \frac{x}{\sqrt{2}} \right) \text{ or } (AO =) \frac{\frac{x}{2}}{\sin 45} \text{ or } (AO =) \frac{\frac{x}{2}}{\cos 45}$ $(VO =) \sqrt{x^2 - \left(\frac{\sqrt{2}x}{2}\right)^2} \text{ or } \sqrt{x^2 - \left(\frac{1}{2}\sqrt{x^2 + x^2}\right)^2} \text{ or } \sqrt{x^2 - \frac{x^2}{2}} \left(= \sqrt{\frac{1}{2}x^2} \right)$ $\text{or } (VO =) \frac{\sqrt{2}}{2} \tan 45$ $= \frac{\sqrt{2}}{2} x \text{ (cm)*}$	M1 M1 A1*cs [3]
ALT	Any indication the candidate realises triangle AVC is right angled and isosceles. $\sin 45 = \frac{VO}{x} \text{ or } x \sin 45 = VO$ $VO = \frac{\sqrt{2}}{2} x *$	M1 M1 A1*cs o [3]
(b)	$(\angle AVC =) \cos^{-1} \left(\frac{x^2 + x^2 - (\sqrt{2}x)^2}{2 \times x \times x} \right) \text{ or } (\angle AVC =) 2 \times \sin^{-1} \left(\frac{\frac{\sqrt{2}}{2}x}{x} \right)$ $\text{or } (\angle AVC =) 2 \times \cos^{-1} \left(\frac{\frac{\sqrt{2}}{2}x}{x} \right) \text{ or } (\angle AVC =) 2 \times \tan^{-1} \left(\frac{\frac{\sqrt{2}}{2}x}{\frac{\sqrt{2}}{2}x} \right)$ $= 90^\circ$	M1 A1 [2]

(c)	Let M be the midpoint of AB and let N be the midpoint of DC $(VM = VN =) \sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \left(\frac{\sqrt{3}}{2}x\right)$ $\cos MVN = \frac{\left(\frac{\sqrt{3}}{2}x\right)^2 + \left(\frac{\sqrt{3}}{2}x\right)^2 - x^2}{2 \times \left(\frac{\sqrt{3}}{2}x\right) \times \left(\frac{\sqrt{3}}{2}x\right)} \text{ or}$ $(\angle MVN =) \cos^{-1} \left(\frac{\left(\frac{\sqrt{3}}{2}x\right)^2 + \left(\frac{\sqrt{3}}{2}x\right)^2 - x^2}{2 \times \left(\frac{\sqrt{3}}{2}x\right) \times \left(\frac{\sqrt{3}}{2}x\right)} \right) \text{ or}$ $(\angle MVN =) 2 \times \cos^{-1} \left(\frac{\frac{\sqrt{2}}{2}x}{\frac{\sqrt{3}}{2}x} \right) \text{ or } (\angle MVN =) 2 \times \sin^{-1} \left(\frac{\frac{1}{2}x}{\frac{\sqrt{3}}{2}x} \right)$ or $(\angle MVN =) 2 \times (\angle MVO =) \text{ and } \cos MVO = \frac{\frac{\sqrt{2}}{2}x}{\frac{\sqrt{3}}{2}x}$ $70.528...^\circ \approx 70.5^\circ$ ALT $OM = \frac{x}{2}$ $(\angle MVN =) 2 \tan^{-1} \left[\frac{\frac{x}{2}}{\frac{\sqrt{2}x}{2}} \right] = \left(2 \tan^{-1} \left[\frac{1}{\sqrt{2}} \right] \right) = 70.528...^\circ \approx 70.5^\circ$	M1 M1 A1 [3] M1 M1A1 [3]
(d)	Volume = $\frac{1}{3} \times \left(\frac{\sqrt{2}}{2}x\right) \times x^2 = \frac{\sqrt{2}}{6}x^3$ oe $200 = \frac{1}{3} \times \left(\frac{\sqrt{2}}{2}x\right) \times x^2 \Rightarrow x =$ $x = 9.4672... \approx 9.47(\text{cm})$	M1 dM1 A1 [3]
Total 11 marks		

Part	Mark	Notes
For part (a), we must see the algebraic proof, including x throughout.		
(a)	M1	For correct use of Pythagoras theorem or any correct appropriate trigonometry and a complete method to find the length AC or AO ie candidates must rearrange correctly – minimum steps as shown in mark scheme and no errors must be present.
	M1	For a full, correct and complete method to find VO, using their AC/AO.
	A1 cso	For $\frac{\sqrt{2}}{2}x$ (cm) – correct solution only, no errors or omissions, minimum steps as shown.
ALT	M1	Any indication the candidate realises triangle AVC is right angled and isosceles. This mark may be implied by correct working for the next method mark or present on the diagram.
	M1	For the correct trig minimum steps as shown and no errors.
	A1 cso	For $\frac{\sqrt{2}}{2}x$ (cm) – correct solution only, no errors or omissions.
(b)	M1	For any correct appropriate trigonometry to find angle AVC. ($\angle AVC =$) If finding $\angle AVO$, must be doubled for this mark. AC must be $\sqrt{2}x$ oe, AO must be $\frac{\sqrt{2}}{2}x$ oe.
	A1	For 90°
	Allow M1A1 for candidates writing down the correct answer by inference.	
Consistent omission of x is permissible for all marks in part (b).		
(c)	M1	For correct Pythagoras theorem to find the lengths of VM or VN
	M1	For any correct trigonometry to find angle MVN, such as the statements shown in the mark scheme. The candidate can state, for example, $\cos(MVN) =$ fully correct expression (using their VN and VM) or, for example, $\cos^{-1}(\text{fully correct expression})$, where they do not have to write angle MVN = i.e – this can involve any fully correct method to reach an equation which would allow them to use inverse trig to find MVN or if they use inverse trig, a fully correct expression must be seen with the correct inverse trig function. Send any items where allocation of marks is unclear (potentially using other methods to the main and ALT) to Review. or fully correct method to find angle MVO which must then be doubled for this mark.
	A1	For any answer stated which rounds to 70.5° (ie more decimal places is condoned).
Consistent omission of x is permissible for all marks in part (c).		
ALT	M1	States or uses $OM = \frac{x}{2}$
	M1	For fully correct method to find angle MVO which must then be doubled for this mark.
	A1	For any answer stated which rounds to 70.5° (ie more decimal places is condoned).
(d)	M1	For a correct unsimplified expression for the volume of the pyramid as shown.
	dM1	For placing their expression = 200, which must involve a term in x^3 , and a fully correct method/rearrangement to solve. Note this mark is dependent on the first method mark, so they must have got the correct unsimplified expression even if then simplified incorrectly.
	A1	For any final answer stated which rounds to 9.47 (ie more decimal places is condoned).

Question	Scheme	Marks
11(a)	$f(\theta) = (2 \cos \theta - \sin \theta)(2 \sin \theta + \cos \theta)$ $= 4 \sin \theta \cos \theta + 2 \cos^2 \theta - 2 \sin^2 \theta - \sin \theta \cos \theta$ $= 3 \sin \theta \cos \theta + 2(\cos^2 \theta - \sin^2 \theta) \Rightarrow \frac{3}{2} \sin 2\theta + 2 \cos 2\theta$ $= \frac{3}{2} \sin 2\theta + 2 \cos 2\theta \quad *$	M1 M1A1 CSO [3]
(b)	$\frac{3}{2} \sin 2\theta + 2 \cos 2\theta + 2 = 0 \Rightarrow 3 \sin 2\theta + 4 \cos 2\theta + 4 = 0$ $\Rightarrow 3 \sin 2\theta + 4(\cos^2 \theta - \sin^2 \theta) + 4(\sin^2 \theta + \cos^2 \theta) = 0$ $\Rightarrow 6 \sin \theta \cos \theta + 8 \cos^2 \theta = 0$ $\Rightarrow \cos \theta (6 \sin \theta + 8 \cos \theta) = 0$ $\Rightarrow \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2} \quad *$ $\left[\Rightarrow \tan \theta = -\frac{4}{3} \right]$	M1M1 M1 M1 A1 CSO [5]
(c)	$\text{Area} = \int_0^{\frac{\pi}{2}} \left(\frac{3}{2} \sin 2\theta + 2 \cos 2\theta + 2 \right) d\theta = \left[-\frac{3}{4} \cos 2\theta + \frac{2 \sin 2\theta}{2} + 2\theta \right]_0^{\frac{\pi}{2}}$ $= \left(-\frac{3}{4} \cos 2\left(\frac{\pi}{2}\right) + \frac{2 \sin 2\left(\frac{\pi}{2}\right)}{2} + 2\left(\frac{\pi}{2}\right) \right) - \left(-\frac{3}{4} \cos 0 + \frac{2 \sin 0}{2} + 2 \times 0 \right)$ $= \left(\frac{3}{4} + \pi \right) - \left(-\frac{3}{4} \right) = \frac{3}{2} + \pi$	M1 M1 A1 [3]
Total 11 marks		

Part	Mark	Notes
(a)	M1	For multiplying out the brackets correctly and simplifying to $k \sin \theta \cos \theta$, $l(\cos^2 \theta - \sin^2 \theta)$ where k and l are integers. Condone invisible brackets if the working is clear.
	M1	For using the identity for $\cos 2\theta$ on their $2(\cos^2 \theta - \sin^2 \theta)$ OR For using the identity for $\sin 2\theta$ on their $3 \sin \theta \cos \theta$
	A1 CSO	For the correct identity as shown with no errors. You must check every line of working carefully – this is a given answer.
	Solutions based on RHS = LHS are acceptable. Apply the above marks – if you are not sure, please send to REVIEW	

(b)	M1	For correctly using the identity $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
	M1	For correctly using the identity $\sin^2 \theta + \cos^2 \theta = 1$
	M1	For correctly using the identity $\sin 2\theta = 2 \sin \theta \cos \theta$
	M1	For factorising the resulting expression. You must see this step
	A1 cso	For $\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$
	ALT	
	M1	For correctly using $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $\frac{3}{2} \sin 2\theta + 2 \cos 2\theta + 2 = \frac{3}{2} \sin 2\theta + 2(\cos^2 \theta - \sin^2 \theta) = 0$
	M1	For correctly using the identity $\sin^2 \theta + \cos^2 \theta = 1$ $\frac{3}{2} \sin 2\theta + 2 \cos 2\theta + 2 = \frac{3}{2} \sin 2\theta + 2(\cos^2 \theta - 1 + \cos^2 \theta) + 2 = 0$ $\Rightarrow \left[\frac{3}{2} \sin 2\theta + 4 \cos^2 \theta = 0 \right]$
	M1	For correctly using the identity $\sin 2\theta = 2 \sin \theta \cos \theta$ $\frac{3}{2} \sin 2\theta + 4 \cos^2 \theta = 3 \sin \theta \cos \theta + 4 \cos^2 \theta = 0$
	M1	For factorising the resulting expression. You must see this step $3 \sin \theta \cos \theta + 4 \cos^2 \theta = \cos \theta (3 \sin \theta + 4 \cos \theta) = 0$
	A1	$\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2} *$
(c)	M1	For integrating the given expression. Minimally acceptable integration as shown below: $\frac{3}{2} \sin 2\theta \Rightarrow \pm \frac{3}{4} \cos 2\theta$ $2\theta \cos 2\theta \Rightarrow \pm \left(\frac{2}{2} \right) \sin 2\theta$ $2 \Rightarrow 2\theta$
	M1	For substituting BOTH of the given limits the correct way around into their changed expression and subtracting.. There must be a minimum of two terms to substitute limits into. This must be explicitly seen
	A1	For the correct exact area as shown. [Approximate area = 4.641....]

Question number	Scheme	Marks
8 (i) (a)	$(\tan 2A = \tan(A + A) =) \frac{\tan A + \tan A}{1 - \tan A \tan A}$ $= \frac{2 \tan A}{1 - \tan^2 A} \quad *$	M1 A1cso* [2]
(b)	$\tan A - \frac{2 \tan A}{1 - \tan^2 A} = 0$ $\tan A(1 - \tan^2 A) - 2 \tan A = 0$ $\tan A - \tan^3 A - 2 \tan A = 0$ $\tan^3 A + \tan A = 0$ $\tan A(\tan^2 A + 1) = 0$ $\tan A = 0 \quad A = 0, 180$ $(\tan^2 A = -1 \quad \text{no solutions})$	M1 M1 M1 A1, A1 [5]
ALT	$\tan A - \frac{2 \tan A}{1 - \tan^2 A} = 0$ $\tan A \left(1 - \frac{2}{1 - \tan^2 A} \right) = 0 \Rightarrow 1 - \tan^2 A - 2 = 0 \quad \text{or} \quad 2 = 1 - \tan^2 A$ $\tan^2 A = -1$ $\tan A = 0 \quad A = 0, 180$ $(\tan^2 A = -1 \quad \text{no solutions})$	
(ii)	$\cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6} (= \sin x)$ $\sin x + \sqrt{3} \cos x = 2 \sin x \Rightarrow \sqrt{3} = \tan x$ $x = -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}$	M1 dM1 A1 A1 [4]
Total 11 marks		

Part	Mark	Additional Guidance
(i) (a)	M1	For $\frac{\tan A + \tan A}{1 - \tan A \tan A}$
	A1cso*	For the given result shown with minimum steps as shown in the MS, no errors or omissions seen. Extra steps must be correct and checked.
(b)	M1	For the correct substitution of the given result from part (a).
	M1	For correctly multiplying by $1 - \tan^2 A$ or for sight of a correct single fraction (which could possibly afterwards be simplified incorrectly) and elimination of $1 - \tan^2 A$ as the denominator.
	M1	For a correct rearrangement of their equation, factorising correctly, leading to a quadratic factor and a linear factor = 0
	A1	For $A = 0$ or 180 must follow M3
	A1	For $A = 0$ & 180 with no other values in range. Ignore values outside range. Must follow M3 Note if you see any solutions based on trial and improvement, for this question, this will score 0 marks.
	eg	$\frac{\tan A - \tan^3 A - 2 \tan A}{1 - \tan^2 A} = 0$ is M1 M1 M0 $3 \tan A - \tan^3 A = 0$
ALT	M1	For the correct substitution of the given result from part (a).
	M1	For a correct factorisation and correctly multiplying by $1 - \tan^2 A$
	M1	For a correct rearrangement of their equation leading to $\tan^2 A = P$ $P \neq 0$
	A1	For $A = 0$ or 180 must follow M3
	A1	For $A = 0$ & 180 with no other values in range. Ignore values outside range. Must follow M3 Note if you see any solutions based on trial and improvement, for this question, this will score 0 marks.
(ii)	M1	For correctly using the formula for $\cos(A - B)$ as shown.
	dM1	For simplifying and rearranging to $\tan x = a$, allow errors in rearrangement, a must be an exact value. Dependent on previous method mark.
	A1 A1	A1 for one value or for any correct value(s) with extra value in range. A2 for all three values with no extra values in range. Ignore extra values out of range correct or otherwise.

Question	Scheme	Marks
7 (a)	$\left(\cos \theta = -\frac{5}{3} \text{ not possible} \right)$ $\sin \theta = \frac{3}{5} \quad \theta = 36.9^\circ \text{ or } 143.1^\circ$	M1A1 [2]
(b)	$8(\sin x \cos \alpha - \cos x \sin \alpha) = 3(\sin x \cos \alpha + \cos x \sin \alpha)$ $5 \sin x \cos \alpha = 11 \cos x \sin \alpha$ $5 \frac{\sin x}{\cos x} = 11 \frac{\sin \alpha}{\cos \alpha}$ $5 \tan x = 11 \tan \alpha^*$	M1 A1 M1 M1A1 [5]
	$5 \tan 2y = 11 \tan 30^\circ$ $\tan 2y = \frac{11}{5} \tan 30^\circ$ $2y = 51.78\dots, 231.78\dots$ $y = 25.89\dots, 115.89\dots$ $y = 25.9^\circ, 115.9^\circ$	M1 A1 M1 A1A1 [5]
Total 12 marks		

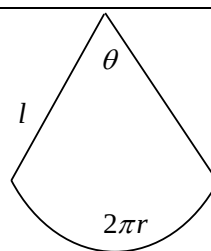
Additional Guidance		
(a)	M1	For rejecting a value for $\cos \theta$ and finding a value for $\sin \theta$ $\sin \theta = \frac{3}{5} \Rightarrow \theta = 36.9^\circ$ or any other valid value (143.1°) Accept even for example -216.9° Working in radians is not acceptable unless a correct conversion to degrees is later applied.
	A1	For both correct values of $\theta = 36.9^\circ, 143.1^\circ$ rounded correctly.
(b)	M1	For the correct expansion of $\sin(x + \alpha)$ and $\sin(x - \alpha)$ Allow $8(\sin x \cos \alpha + \cos x \sin[-\alpha]) = 3(\sin x \cos \alpha + \cos x \sin \alpha)$
	A1	For rearranging and obtaining $5 \sin x \cos \alpha = 11 \cos x \sin \alpha$
	M1	For rearranging their expression to obtain ' $5, \frac{\sin x}{\cos x} = 11, \frac{\sin \alpha}{\cos \alpha}$ '
	M1	For using the identity $\tan A = \frac{\sin A}{\cos A}$ on their ' $5, \frac{\sin x}{\cos x} = 11, \frac{\sin \alpha}{\cos \alpha}$ ' to obtain ' $5 \tan x = 11 \tan \alpha$ '
	A1	For a fully correct method to achieve $5 \tan x = 11 \tan \alpha$ Note: This is a show question.
(c)	M1	$[x = 2y \text{ and } \alpha = 30^\circ]$ For using the given identity $5 \tan x = 11 \tan \alpha$ with the correct substitution for x and α to give $5 \tan 2y = 11 \tan 30^\circ$
	A1	For achieving $\tan 2y = \frac{11}{5} \tan 30^\circ$ or $\tan 2y = 2.2 \tan 30^\circ$
	M1	For achieving any valid value for $2y$ i.e. 51.78° or 231.78° Accept values out of range for this mark, even for example -128.22°
	A1	For either 25.9° or 115.9°
	A1	For both 25.9° and 115.9°
Penalise rounding only once in this question. However, the values given must round to 25.9° and 115.9° otherwise it is the incorrect value. Values given of 25.90° and/or 115.90° would score A0A1		

Question Number	Scheme	Marks
9(a)	$\cos ADB = \frac{6^2 + x^2 - 12^2}{2 \times 6x} = \frac{x^2 - 108}{12x}$	M1A1 (2)
(b)	$\cos BDC = \frac{6^2 + x^2 - 6^2}{2 \times 6x} = \frac{x^2}{12x}$ $ADB = 180 - BDC \Rightarrow -\frac{x^2}{12x} = \frac{x^2 - 108}{12x}$ $2x^2 = 108 \Rightarrow x = 3\sqrt{6}$ $AC = 6\sqrt{6}$	B1 M1A1 A1 (4)
(c)	$\frac{\sin(\theta^\circ + \phi^\circ)}{2x} = \frac{\sin BCD}{12} \Rightarrow \frac{\sin(\theta^\circ + \phi^\circ)}{x} = \frac{\sin BCD}{6}$ $\frac{\sin \phi^\circ}{x} = \frac{\sin BCD}{6}$ $\therefore \sin \phi^\circ = \sin(\theta^\circ + \phi^\circ)$	M1A1 M1 A1 (4)
(d)	$\sin(\theta^\circ + \phi^\circ) = \sin \phi^\circ \Rightarrow (\theta + \phi) = 180 - \phi$ (or ϕ or $360 + \phi$ (not possible)) $\therefore \theta = 180 - 2\phi$	M1 A1 (2) [12]

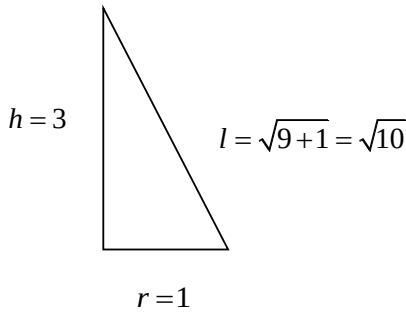
Part	Mark	Notes
(a)	M1	For using a correct cosine rule $\cos ADB = \frac{6^2 + x^2 - 12^2}{2 \times 6x} = \frac{x^2 - 108}{12x}$ or $12^2 = x^2 + 6^2 - 2 \times 6 \times x \cos ADB$
	A1	Simplifies to $\cos ADB = \frac{x^2 - 108}{12x}$
(b)	B1	For the correct expression $\cos BDC = \frac{6^2 + x^2 - 6^2}{2 \times 6x} = \frac{x^2}{12x}$
	M1	$\cos BDC = -\cos ADB$ and $\cos BDC = -\frac{x^2}{12x}$ so $-\frac{x^2}{12x} = \frac{x^2 - 108}{12x}$ and attempts to solve
	ALT 1 – uses triangles BAD and BAC	
	B1	For both of the following correct expressions for $\cos BAD$ and $\cos BAC$: $\cos BAD = \frac{12^2 + x^2 - 6^2}{2 \times 12 \times x}$ and $\cos BAC = \frac{12^2 + (2x)^2 - 6^2}{2 \times 12 \times 2x}$
	M1	$\angle BAC = \angle BAD$ so equates their two expressions $\frac{12^2 + x^2 - 6^2}{2 \times 12 \times x} = \frac{12^2 + (2x)^2 - 6^2}{2 \times 12 \times 2x} \Rightarrow \frac{108 + x^2}{24x} = \frac{108 + 4x^2}{48x}$ and attempts to solve
	ALT 2 – uses triangles BCD and BCA	

	B1	For both of the following correct expressions for $\cos BAD$ and $\cos BAC$: $\cos BCD = \frac{6^2 + x^2 - 6^2}{2 \times 6 \times x} \quad \text{and} \quad \cos BCA = \frac{6^2 + (2x)^2 - 12^2}{2 \times 6 \times 2x}$
	M1	$\frac{6^2 + x^2 - 6^2}{2 \times 6 \times x} = \frac{6^2 + (2x)^2 - 12^2}{2 \times 6 \times 2x} \Rightarrow x^2 = 2x^2 - 54$ and attempts to solve
	Final A marks for all three methods	
	A1	For the correct value of $x = 3\sqrt{6}$
	A1	For $AC = 6\sqrt{6}$
	(c)	
	M1	Uses sine rule on triangle ABC : $\frac{\sin(\theta^\circ + \phi^\circ)}{2x} = \frac{\sin BCD}{12} \Rightarrow \frac{\sin(\theta^\circ + \phi^\circ)}{x} = \frac{\sin BCD}{6}$
	A1	Achieves the correct expression for $\sin(\theta^\circ + \phi^\circ) = \frac{x \sin BCD}{6}$
	M1	Uses sine rule on triangle BDC : $\frac{\sin \phi^\circ}{x} = \frac{\sin BCD}{6} \Rightarrow \left(\sin \phi^\circ = \frac{x \sin BCD}{6} \right)$
	A1	Shows that $\sin \phi^\circ = \sin(\theta^\circ + \phi^\circ)$ with no errors
	ALT 1 – Uses exact values for the trigonometric ratios and the expansion for $\sin(A + B)$	
	M1	Finds $\cos \theta = \frac{7}{8} \Rightarrow \sin \theta = \frac{\sqrt{15}}{8}$ or $\cos \phi = \frac{1}{4} \Rightarrow \sin \phi = \frac{\sqrt{15}}{4}$ Accept $\theta = 28.95...^\circ$ or $\phi = 75.52...^\circ \Rightarrow \sin \phi = 0.968...$
	A1	Finds $\cos \theta = \frac{7}{8} \Rightarrow \sin \theta = \frac{\sqrt{15}}{8}$ and $\cos \phi = \frac{1}{4} \Rightarrow \sin \phi = \frac{\sqrt{15}}{4}$ Accept $\theta = 28.95...^\circ$ and $\phi = 75.52...^\circ \Rightarrow \sin \phi = 0.968...$
	M1	Expands $\sin(\theta + \phi) = \frac{\sqrt{15}}{8} \times \frac{1}{4} + \frac{\sqrt{15}}{4} \times \frac{7}{8} = \left(\frac{\sqrt{15}}{4} \right)$ Or $\sin(\theta + \phi)^\circ = \sin(28.95^\circ) \cos(75.52^\circ) + \sin(75.52^\circ) \cos(28.95^\circ) = 0.968... = \sin \phi^\circ$
	A1	Shows that $\sin(\theta + \phi) = \frac{\sqrt{15}}{4}$ and $\sin \phi = \frac{\sqrt{15}}{4}$ so $\sin(\theta + \phi) = \sin \phi$ with no errors. If they use approximate values for $\sin \theta$ and ϕ withhold this final mark so A0
	ALT 2 – Uses $\angle BCD = 52.2...^\circ$	
	M1	Finds $\angle BCD = 52.2...^\circ$ using cosine rule and applies sine rule on triangle ABC $\frac{\sin(\theta^\circ + \phi^\circ)}{6\sqrt{6}} = \frac{\sin 52.2^\circ}{12}$
	A1	Shows that $\sin(\theta^\circ + \phi^\circ) = 0.968...$
	M1	Uses sine rule on triangle BD : $\frac{\sin \phi^\circ}{3\sqrt{6}} = \frac{\sin 52.2^\circ}{6} \Rightarrow \sin \phi^\circ = 0.968... = \sin(\theta^\circ + \phi^\circ)$
	A1	If they use an approximate value for angle BCD withhold this final mark so A0
(d)	M1	For writing $\sin \phi^\circ = \sin(180 - \phi)^\circ \Rightarrow \theta^\circ + \phi^\circ = 180^\circ - \phi^\circ$
	A1	For rearranging $\theta^\circ + \phi^\circ = 180^\circ - \phi^\circ$ to achieve $\therefore \theta = 180 - 2\phi$ This is a show question and there must be no errors here.

Question Number	Scheme	Marks
10(a)	Circumference of base = $2\pi r$	B1
	$l\theta = 2\pi r \Rightarrow \theta = \frac{2\pi r}{l}$	B1
	$A = \frac{1}{2}l^2\theta = \frac{1}{2}l^2 \frac{2\pi r}{l} = \pi rl$	M1A1 (4)
(b)	$A = \pi rl$	
	$l = r\sqrt{10} \Rightarrow A = \pi r^2\sqrt{10}$	B1
	$\frac{dA}{dr} = 2\pi r\sqrt{10} \Rightarrow k = 2\sqrt{10}$	M1A1 (3)
(c)	$\frac{dV}{dt} = 1.5 \text{ (cm}^3/\text{s)}$	B1
	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$	M1
	$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times 3r = \pi r^3$	B1
	$\frac{dV}{dr} = 3\pi r^2$	B1ft
	$\therefore \frac{dA}{dt} = 2\pi \times 8\sqrt{10} \times 1.5 \times \frac{1}{3 \times 64\pi} = 0.3952... = 0.395 \text{ cm}^2/\text{s}$	A1 (5)
		[12]



Part	Mark	Notes	
(a)	B1	For the circ of the base $L = 2\pi r$	
	B1	$R = l$ and $l = r\theta$ Therefore $l\theta = 2\pi r \Rightarrow \theta = \frac{2\pi r}{l}$	$R = l$
	M1	$A = \frac{1}{2}l^2\theta$ and substituting their expression for θ to give $A = \frac{1}{2}l^2\theta = \frac{1}{2}l^2 \frac{2\pi r}{l}$	Uses the formula $A = \frac{1}{2}RL$ $A = \frac{1}{2} \times l \times 2\pi r \Rightarrow (A = \pi rl)$
	A1	For the required expression for A, $A = \pi rl$ with no errors.	

(b)	B1	For finding that the slant height is $\sqrt{10}$ times the radius of the cone  So $l = r\sqrt{10}$
	M1	Substitutes $l = r\sqrt{10}$ into the given expression $A = \pi rl$ and differentiates their resulting expression to find $\frac{dA}{dr}$ $A = \pi r^2 \sqrt{10}$ so $\frac{dA}{dr} = 2\pi r \sqrt{10}$
	A1	Therefore $k = 2\sqrt{10}$
(c)	B1	States $\frac{dV}{dt} = 1.5 \text{ (cm}^3/\text{s)}$ Award if it seen explicitly in (b) and used in (c)
	M1	States (or uses) a correct chain rule $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$
	B1	For finding the volume of a cone in terms of r only $V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^2 \times 3r = \pi r^3$
	B1ft	Differentiates their expression for the volume of a cone provided it is in terms of V and r only. $\frac{dV}{dr} = 3\pi r^2$
	A1	For combining all required terms into their chain rule and evaluating to 3 significant figures, $\frac{dA}{dt} = 2\pi \times 8\sqrt{10} \times 1.5 \times \frac{1}{3 \times 64\pi} = 0.3952... = 0.395 \text{ cm}^2/\text{s}$
	ALT – in terms of h	
	B1	States $\frac{dV}{dt} = 1.5 \text{ (cm}^3/\text{s)}$ Award if it seen explicitly in (b) and used in (c)
	M1	States (or uses) a correct chain rule $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$ and $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$
	B1	For finding the area and volume of a cone in terms of h $r = \frac{h}{3}, A = \pi r^2 \sqrt{10} \Rightarrow A = \frac{\sqrt{10}}{9} \pi h^2$ and $V = \frac{1}{3} \pi r^2 h \Rightarrow V = \frac{1}{27} \pi h^3$
	B1ft	Differentiates their expressions for the area and volume of a cone provided they are both in terms of h only. $\frac{dA}{dh} = \frac{2\sqrt{10}}{9} \pi h$ and $\frac{dV}{dh} = \frac{3}{27} \pi h^2$
	A1	Combines the required terms into their chain rules and evaluating to 3 significant figures $\frac{dA}{dt} = 0.395 \text{ 102}$

Question number	Scheme	Marks
7 (a)	$(3x - 15 =) 30 \text{ or } 330 \text{ or } 390$ $x = 15 \quad x = 115 \quad x = 135$	M1 M1 A1 A1 [4]
(b)	$3 \frac{\sin y}{\cos y} + 4 \sin y = 0 \rightarrow \sin y \left(\frac{3}{\cos y} + 4 \right) = 0$ $\sin y = 0 \quad \text{and} \quad \cos y = -\frac{3}{4} \quad \rightarrow \quad y =$ $y = -180, 0$ $y = 138.6, -138.6$	M1 A1ft A1 A1
	ALT $3 \tan y + 4 \tan y \cos y = 0 \rightarrow \tan y (3 + 4 \cos y) = 0$ $\tan y = 0 \quad \text{and} \quad \cos y = -\frac{3}{4} \quad \rightarrow \quad y =$ $y = -180, 0$ $y = 138.6, -138.6$	M1 A1ft A1 A1 [4]
(c)	$\cos \theta = 3(1 - \cos^2 \theta) - 1 \rightarrow 3\cos^2 \theta + \cos \theta - 2 = 0$ $(3\cos \theta - 2)(\cos \theta + 1)$ $\theta = -180$ $\theta = 48.2, -48.2$	M1 M1 A1 A1 [4]
Total 12 marks		

Part	Mark	Additional Guidance
(a)	M1	For any of 30 or 330 or 390 May be implied by correct answers.
	M1	Solves a linear equation coming from attempt at use of inverse trigonometric function to obtain one value. e.g. solves $3x - 15 = "30"$
	A1 A1	First A1 for any correct value, second A1 for all 3 correct values and no others in the range. Ignore values outside the range.
(b)	M1	Correctly replaces the identity for $\tan y$ and attempts to deal with $\sin y$. Allow for factorising. Condone dividing through by $\sin y$. Minimally acceptable attempt for factorisation is $A \sin y \left(\frac{B}{\cos y} + C \right)$
	A1ft	$\sin y = 0$ and $\cos y = -\frac{B}{C}$, follow through their B and C only .
	A1	From sine: Both values, ignore extra values out of range, A0 for extra values in range.
	A1	From cosine: Both values, ignore extra values out of range, A0 for extra values in range. $y = \text{awrt } 138.6, \text{ awrt } -138.6$
	ALT	
	M1	Correctly replaces the identity for $\tan y$ and attempts to deal with $\tan y$. Allow for factorising. Condone dividing through by $\tan y$. Minimally acceptable attempt for factorisation is $A \tan y (B + C \cos y)$
	A1ft	$\tan y = 0$ and $\cos y = -\frac{B}{C}$, follow through their B and C only .
	A1	From tangent: Both values, ignore extra values out of range, A0 for extra values in range.
	A1	From cosine: Both values, ignore extra values out of range, A0 for extra values in range. $y = \text{awrt } 138.6, \text{ awrt } -138.6$
(c)	M1	Correctly uses the identity for $\cos^2 \theta$ and rearranges to get a 3TQ Minimally acceptable attempt is $\pm 3\cos^2 \theta \pm \cos \theta \pm 2 = 0$
	M1	Solves a 3TQ to arrive at 2 distinct values for $\cos \theta$. See general guidance.
	A1	For -180 , ignore extra values out of range, A0 for extra values in range.
	A1	Both values, ignore extra values out of range, A0 for extra values in range. $\theta = \text{awrt } 48.2, \text{ awrt } -48.2$

Question	Scheme	Marks
11(a)	$\cos 2A = \cos^2 A - \sin^2 A \Rightarrow \cos 2A = (1 - \sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$ $2\sin^2 A = 1 - \cos 2A \Rightarrow \sin^2 A = \frac{1}{2}(1 - \cos 2A) \quad *$	M1M1 A1cso [3]
(b)	$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ $\sin^4 x + \cos^4 x = \left(\left(\frac{1 - \cos 2x}{2} \right)^2 + \left(\frac{1 + \cos 2x}{2} \right)^2 \right)$ $= \frac{1}{4}((1 - \cos 2x)^2 + (1 + \cos 2x)^2)$ $= \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x + 1 + 2\cos 2x + \cos^2 2x)$ $= \frac{1}{4}(2 + 2\cos^2 2x) = \left\{ \frac{1}{2}(1 + \cos^2 2x) \right\}$ $\left\{ \cos^2 2x = \frac{1 + \cos 4x}{2} \right\}$ $\sin^4 x + \cos^4 x = \frac{1}{2} \left(1 + \frac{1 + \cos 4x}{2} \right) = \frac{3 + \cos 4x}{4} \quad *$	B1 M1 A1 M1A1 cso [5]
(c)	$5\sin 2\theta + 6 = 8 \left(\frac{3 + \cos 2\theta}{4} \right) \Rightarrow 5\sin 2\theta + 6 = 6 + 2\cos 2\theta$ $\Rightarrow 5\sin 2\theta = 2\cos 2\theta \Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{2}{5} \Rightarrow \tan 2\theta = \frac{2}{5}$ $2\theta = 21.801...^\circ, 201.801...^\circ, 381.801...^\circ$ $\Rightarrow \theta = 10.9^\circ, 100.9^\circ$ Penalise extra angles in range by withholding the final A mark. Extra angles out of range – ignore.	M1 M1 A1 A1 [4]
Total 12 marks		

Part	Mark	Notes
(a)	M1	For using the summation formula for $\cos 2A$ They must start with either $\cos 2A = \cos^2 A - \sin^2 A$ or $\cos(A + A) = \cos A \cos A - \sin A \sin A$
	M1	For eliminating $\cos^2 A$ using $\cos^2 A + \sin^2 A = 1$ and attempting to rearrange to the required result. This is not dependent on the first M mark, so if they start with another identity for $\cos 2A$ they can still get this mark.
	A1 cso	For the correct identity with no errors seen. This is a given result.
	NB Some candidates work backwards – that is fine, please follow their working.	

Working with a different variable.

If they work in this part with a different variable (eg A) then award all the marks as appropriate up to the last mark.

If they leave their final answer in terms of another variable, withhold the final A mark only.

If they however, change to x on the final line award all the marks [provided everything is correct]

(b)	Main method	
	B1	For use of the correct identity for $\cos^2 x$ $\left[\cos^2 x = \frac{1}{2}(1 + \cos 2x) \right]$ or if they convert $\cos^2 x$ to $\sin^2 x$ use the Pythagorean identity $\cos^2 x = 1 - \sin^2 x$ and apply the given identity.
	M1	For squaring both identities. This must be a correct expansion. For their identity for $\left[\cos^2 x = \frac{1}{2}(1 + \cos 2x) \right]$
	A1	For collecting like terms and obtaining $\frac{1}{2}(1 + \cos^2 2x)$ oe. For example, $\frac{1}{4}(2 + 2\cos^2 2x)$
	M1	For applying the identity: $\cos^2 A = \frac{1}{2}(1 + \cos 2A) \Rightarrow \left[\cos^2 2A = \frac{1}{2}(1 + \cos 4A) \right]$ on $\cos^2 2x$ only again to achieve an expression in $\cos 4x$ only
	A1 cso	For the correct identity with no errors seen. This is a given result. You must check every line of their work carefully.
ALT 1		
	B1	For use of the correct identity for $\sin 2A = 2 \sin A \cos A$ [seen later in their working].
	M1	For using the expansion of $\sin^2 x + \cos^2 x$ as follows $\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$ This must be correct
	A1	For obtaining $\sin^4 x + \cos^4 x = 1 - \frac{1}{2}\sin^2 2x$ oe. For example; $\frac{1}{2}(2 - \sin^2 2x)$
	M1	For applying the given identity on $\sin^2 2x$ only to achieve an expression in $\cos 4x$ only $\sin^2 A = \frac{1}{2}(1 - \cos 2A) \Rightarrow \left[\sin^2 2A = \frac{1}{2}(1 - \cos 4A) \right]$ $\sin^4 x + \cos^4 x = 1 - \frac{1}{2}\left(\frac{1 - \cos 4x}{2}\right) \Rightarrow \left(\sin^4 x + \cos^4 x = \frac{3 + \cos 4x}{4} \right)$
	A1 cso	For the correct identity with no errors seen. This is a given result. You must check every line of their work carefully.

ALT 2 – Works backwards from the given result	
B1	For use of the correct identity for $\sin 2A = 2 \sin A \cos A$ [seen later in their working].
M1	Applies the $\cos 2A$ identity and converts 3 into $3(\sin^2 2x + \cos^2 2x)$ $\frac{3 + \cos 4x}{4} = \frac{3\sin^2 2x + 3\cos^2 2x + (\cos^2 2x - \sin^2 2x)}{4}$
A1	Obtains $\frac{3 + \cos 4x}{4} = \frac{4\cos^2 2x + 2\sin^2 2x}{4}$
M1	Applies the $\cos 2A$ identity and expands the bracket. $\frac{3 + \cos 4x}{4} = \frac{4(\cos^2 x - \sin^2 x)^2 + 8\sin^2 x \cos^2 x}{4}$ $= \frac{4\cos^4 x - 8\sin^2 x \cos^2 x + 4\sin^4 x + 8\sin^2 x \cos^2 x}{4}$
A1 cso	Simplifies to the required result with no errors seen $\frac{3 + \cos 4x}{4} = \sin^4 x + \cos^4 x \quad *$

(c)	M1	For obtaining the correct equation in terms of $\sin 2\theta$ and $\cos 2\theta$ $6 + 2 \cos 2\theta = 5 \sin 2\theta + 6$ Accept unsimplified, accept even: $8\left(\frac{3 + \cos 2\theta}{4}\right) = 5 \sin 2\theta + 6$
	M1	For using the $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$ identity correctly on their expression following an expression in the form $A \cos 2\theta = B \sin 2\theta$ and must be in terms of 2θ
	A1	For achieving at least one correct angle for 2θ NB This is an M mark in Epen
	A1	For awrt both 10.9° and 100.9° Penalise extra angles in range by withholding the final A mark. Extra angles out of range – ignore.

Question	Scheme	Marks
11(a)	$\cos(A + A) = \cos A \cos A - \sin A \sin A$ $\Rightarrow \cos 2A = \cos^2 A - (1 - \cos^2 A)$ $\cos 2A = 2 \cos^2 A - 1 *$	M1 A1 cso [2]
(b)	$(2\cos^2 A - 1)^2 = (\cos 2A)^2$ $\cos 4A = 2\cos^2 2A - 1 \Rightarrow \cos^2 2A = \frac{\cos 4A + 1}{2}$ $(2\cos^2 2A - 1)^2 = \frac{\cos 4A + 1}{2} *$	M1 M1 A1 cso [3]
(b) ALT	$\cos 4A = 2 \cos^2 2A - 1$ $= 2(2 \cos^2 A - 1)^2 - 1$ $(2 \cos^2 A - 1)^2 = \frac{\cos 4A + 1}{2} *$	M1 M1 A1 cso [3]
(c)	$y = \frac{\sin 2x}{2} + \frac{(2 \cos^2 x - 1)^2}{2} + \frac{1}{8} \Rightarrow y = \frac{\sin 2x}{2} + \frac{\cos 4x}{4} + \frac{3}{8}$ $\frac{dy}{dx} = \cos 2x - \sin 4x$ $\frac{dy}{dx} = \cos 2x - 2 \sin 2x \cos 2x \{= 0\}$ $\{\cos 2x(1 - 2 \sin 2x) = 0\} \Rightarrow \sin 2x = \frac{1}{2}$ $\Rightarrow x = \frac{\sin^{-1}\left(\frac{1}{2}\right)}{2} = \dots$ $x = \frac{\pi}{12}$ $y = \frac{\sin 2\left(\frac{\pi}{12}\right)}{2} + \frac{\cos 4\left(\frac{\pi}{12}\right)}{4} + \frac{3}{8} = \frac{3}{4} \Rightarrow \left(\frac{\pi}{12}, \frac{3}{4}\right\}$	M1 dM1 B1 ddM A1 M1A1 [7]
Total 12 marks		

Part	Mark	Notes
(a)	M1	For correct use of $\cos(A + B) = \cos A \cos B - \sin A \sin B \Rightarrow \cos(A + A) = \cos A \cos A - \sin A \sin A$ OR $\cos 2A = \cos^2 A - \sin^2 A$ with the Pythagorean identity to eliminate sine from the identity.
	A1 cso	Achieves printed answer with no errors seen. Left hand side must be $\cos 2A$ not $\cos(A+A)$
(b)	M1	Uses the given result from (a), replaces $(2\cos^2 A - 1)^2$ with $(\cos 2A)^2$
	M1	Uses the given result from (a) or a correct identity to replace $\cos^2 2A$ with $\frac{\cos 4A + 1}{2}$
	A1 cso	Fully correct proof showing all necessary steps. (If they meet in the middle, they must have a conclusion, accept e.g. shown, proved, L=R, etc.)
(b) ALT	M1	Uses the given result from (a) on $\cos 4A$
	M1	Uses the given result from (a) or a correct identity on $\cos 2A$
	A1 cso	Fully correct proof showing all necessary steps. (If they meet in the middle, they must have a conclusion, accept e.g. shown, proved, L=R, etc.)
(c)	M1	Rearranges the given y into the form $\frac{\sin 2x}{2} + \frac{\cos 4x + 1}{2} + l$, oe, where k and l are nonzero constants Or $\frac{\sin 2x}{2} + \frac{\cos 4x}{P} + Q$, oe, where P and Q are nonzero constants (Allow mixed variables x and A for this method mark only)
	dm1	For an attempt to differentiate their y into the form $P\cos(2x) \pm Q\sin(4x)$ oe, where P, Q are nonzero constants Depends on the previous method mark.
	B1	Uses $\sin 4x = 2\sin 2x \cos 2x$ or $\sin 4x = \sqrt{1 - \cos^2 4x}$ oe in their dy/dx (may be implied by correct working)
	ddm1	Sets their differentiated expression = 0 (can be implied) and attempts a correct method to solve for $x = \dots$ Depends on both previous method marks.
	A1	For $x = \frac{\pi}{12}$, ignore extra x values are outside of given range. Withhold A mark if extra solution seen in the given range.
	M1	For attempting to find the value of y using their x , found by solving their $\frac{dy}{dx} = 0$, provided x is in the range $0 \leq x \leq \frac{\pi}{6}$ Substitutes into the given y or their y , correct answer implies this mark, but for incorrect answer, must see explicit substitution
	A1	For the correct exact coordinates of P, $\left(\frac{\pi}{12}, \frac{3}{4}\right)$ or $\left(\frac{\pi}{12}, 0.75\right)$, accept $x = \frac{\pi}{12}$, $y = \frac{3}{4}$ or $x = \frac{\pi}{12}$, $y = 0.75$

Question number	Scheme	Marks
8 (a)	$BN = 10 \cos 60^\circ$ $= 5 \text{ (cm)}^*$	M1 A1cso [2]
(b)	$FN^2 = 10^2 - 5^2 = 75$ or $FN = 10 \sin 60 = 5\sqrt{3}$ or $FN = 5 \tan 60$ $NE = \sqrt{12^2 + 75} = 14.7986... \approx 14.8 \text{ (cm)}$	M1 M1A1 [3]
(c)	Let R be the midpoint of FN and P be the point on AD that corresponds to point N on BC . $\angle RPN = \tan^{-1} \left(\frac{\frac{\sqrt{75}}{2}}{12} \right) = 19.84167...^\circ \approx 19.8^\circ$	M1A1 [2]
(d)	Let the perpendicular from Y to line BC meet BC at Z $AY = \sqrt{AZ^2 + ZY^2} = \sqrt{12^2 + 6.5^2 + \left(\frac{\sqrt{75}}{2} \right)^2} = \sqrt{205}$ $FC = \sqrt{FN^2 + NC^2} = \sqrt{75 + 3^2} = \sqrt{84} \Rightarrow FY = \sqrt{21}$ So $EY = \sqrt{12^2 + 21} = \sqrt{165}$ $\angle AYE = \cos^{-1} \left(\frac{205 + 165 - 100}{2 \times \sqrt{205} \times \sqrt{165}} \right) = 42.7745...^\circ \approx 42.8^\circ$	M1A1 M1 M1 M1A1 [6]
Total 13 marks		

(a)	M1	Use cosine or sine in triangle BNF , or any complete method. $BN = 10 \cos 60^\circ$ or $\cos 60^\circ = \frac{BN}{10}$ or $\frac{BN}{\sin 30} = \frac{10}{\sin 90}$ or $BN = 10 \sin 30$ or $FN = 10 \sin 60 = 5\sqrt{3}$ and $BN^2 = 10^2 - (5\sqrt{3})^2$ (may use any single letter instead of BN)
	A1 cso	$BN = 5 \text{ (cm)}$ Correct answer from correct working. (Answer given.) Must show a calculation in the form $BN = \dots$ e.g. $BN = 10 \cos 60$
(b)	M1	Use Pythag or trig to find a numerical expression for FN or FN^2 eg $FN^2 = 10^2 - 5^2 (= 75)$ or $FN = 10 \sin 60 (= 5\sqrt{3} = 8.66...)$ or $FN = 5 \tan 60$
	M1	Use Pythag with their FN to find a numerical expression for NE , eg $NE = \sqrt{12^2 + (5\sqrt{3})^2} (= \sqrt{219})$
	A1	$NE = 14.8 \text{ (cm)}$ given to 3SF.
(c)	M1	Use tangent with their FN , eg $\tan RPN = \left(\frac{\frac{1}{2}\sqrt{75}}{12} \right)$ or any complete method.
	A1	19.8° given to 1DP.

(d)	M1	Complete method to find AY eg $AY = \sqrt{(12^2 + 6.5^2) + \left(\frac{\sqrt{75}}{2}\right)^2} (= \sqrt{205} = 14.31\dots)$ (allow their FN)
	A1	$AY = \sqrt{205} (= 14.31\dots)$ (may be implied by subsequent working)
	M1	Complete method to find FY eg $FC = \sqrt{75 + 3^2} (= \sqrt{84}) \Rightarrow FY = \frac{1}{2}\sqrt{84} (= \sqrt{21})$ (allow their FN) eg $FC = \sqrt{8^2 + 10^2 - 2 \times 8 \times 10 \cos 60} (= \sqrt{84}) \Rightarrow FY = \frac{1}{2}\sqrt{84} (= \sqrt{21} = 4.58\dots)$
	M1	Method to find EY using their FY (correct or from a correct method) eg $EY = \sqrt{12^2 + 21} (= \sqrt{165} = 12.84\dots)$
	M1	Substitute their values for AY and EY into the cosine rule used in triangle AYE . $\cos AYE = \left(\frac{205 + 165 - 100}{2 \times \sqrt{205} \times \sqrt{165}}\right)$ oe
	A1	$\angle AYE = 42.8^\circ$ given to 1DP. Allow more than 1DP if 1DP was penalised in part (c).

Question Number	Scheme	Marks
11(a)	$(3 \sin A \cos B - 3 \cos A \sin B) = (\sin A \cos B + \cos A \sin B)$ $2 \sin A \cos B = 4 \cos A \sin B$ $\rightarrow \frac{\sin A}{\cos A} = 2 \frac{\sin B}{\cos B}$ $\tan A = 2 \tan B \quad k = 2$	M1 M1 M1 A1 (4)
(b)	$\frac{(\cos^4 \theta - \sin^4 \theta)}{\cos^2 \theta} = \frac{(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}{\cos^2 \theta}$ $= \frac{(\cos^2 \theta - \sin^2 \theta)}{\cos^2 \theta}$ $= 1 - \tan^2 \theta \quad *$	M1 M1 A1 cso (3)
ALT 1	$1 - \tan^2 \theta = 1 - \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}$ $= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta} \times (\cos^2 \theta + \sin^2 \theta)$ $= \frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta}$	M1 M1 A1
ALT 2	$\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} = \cos^2 \theta - \frac{\sin^4 \theta}{\cos^2 \theta} = \cos^2 \theta - \tan^2 \theta \sin^2 \theta$ $= \cos^2 \theta - \tan^2 \theta (1 - \cos^2 \theta)$ $= \cos^2 \theta - \tan^2 \theta + \sin^2 \theta = 1 - \tan^2 \theta$	M1 Eliminate 4 th powers M1 Eliminate sin ² A1
(c)(i)	$\cos(45 - 30) \text{ or } \cos(60 - 45) = \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$ $= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4} \quad *$	M1 A1cso (2)
ALT	By using double angle formula: $\cos^2 15^\circ = \frac{1}{2} (1 + \cos 30^\circ) = \frac{1}{2} \left(1 + \frac{\sqrt{3}}{2} \right)$ Leading to the <i>given</i> answer. $\cos 15^\circ = \sqrt{\left(\frac{2 + \sqrt{3}}{4} \right)}$ or $\frac{\sqrt{2 + \sqrt{3}}}{2}$ must be seen.	M1 A1

Question Number	Scheme	Marks
(ii)	$\tan 255 = \tan 75$ $= \tan(30 + 45) = \frac{\tan 30 + \tan 45}{1 - \tan 30 \tan 45}$ $= \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}}$ $\frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} *$	B1 M1 dM1 A1cso (4) [13]
(a) M1 M1 M1 A1 (b) M1 M1 A1cso ALT 1 M1 M1 A1cso (c)(i) M1 A1cso (ii) B1 M1 dM1 A1cso	Expand both sides of the equation using correct formulae Collect like terms from their expansions. (Not dependent) Divide through by $\cos A \cos B$ Replace each fraction with the appropriate tangent and show $k = 2$ (value need not be shown explicitly) Factorise the numerator using the difference of 2 squares. Replace $\sin^2 \theta + \cos^2 \theta$ with 1 Divide both terms by $\cos^2 \theta$ and obtain the <i>given</i> answer with no errors seen. Use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and obtain a single fraction with no tan Indicate multiplication by $\sin^2 \theta + \cos^2 \theta$ Multiply and obtain the <i>given</i> answer with no errors seen. Express 15 as the difference of 2 suitable numbers, expand using a correct formula and substitute the correct exact values for the trig functions (substitution must be shown). Simplify and combine the fractions to obtain the <i>given</i> answer with no errors seen. $\tan 255 = \tan 75$ seen explicitly or used. OR eg $\tan(210 + 45)$ – give B1 for $\tan 210 = \tan 30$ used Express 75 as $30 + 45$ and expand $\tan(30 + 45)$ using the correct formula (given on the formula page) OR expand $\tan(210 + 45)$ If $75 = 15 + 60$ is used $\tan 15$ can be obtained from a calculator but must be in exact form.. Substitute the correct exact values for the trig functions Simplify the fractions to obtain the <i>given</i> answer with full working and no errors seen.	

Question	Scheme	Marks
9(a)	$AC = \sqrt{12^2 + 12^2} = \sqrt{288} = 12\sqrt{2}$	M1A1 [2]
(b)	$x = \frac{12\sqrt{2}}{\cos 30^\circ} = \sqrt{96} = (4\sqrt{6})^*$	M1A1 cso [2]
(c)	$\cos \angle AOB = \frac{96 + 96 - 12^2}{2 \times \sqrt{96} \times \sqrt{96}} \Rightarrow \angle AOB = 75.522...^\circ$ $\text{Area } \triangle OAB = \frac{1}{2} \times \sqrt{96} \times \sqrt{96} \times \sin 75.522^\circ = (46.4758...)$ $4 \times 46.4758 + 12^2 = 329.90... \Rightarrow \text{Area} = 330 \text{ [m}^2\text{]}$	M1A1 M1 M1A1 [5]
(d)	$\left[\angle OBC = \frac{180 - 75.522}{2} = 52.239^\circ \right]$ Let Y on OB be the foot of the perpendicular from A to OB $AY = 12 \sin 52.239^\circ = (9.4868...)$ $\cos \angle AYC = \frac{'9.4868'^2 + '9.4868'^2 - 288}{2 \times '9.4868' \times '9.4868'} \Rightarrow \angle AYC = 126.87... \approx 127^{[o]}$	M1 A1 M1A1 [4]
Total 13 marks		

Question	Notes	Marks
9(a)	For using Pythagoras theorem on triangle ABC or triangle ADC $AC = \sqrt{12^2 + 12^2} = ...$	M1
	For the correct value of AC $AC = \sqrt{288} = 12\sqrt{2}$	A1 [2]
(b)	Let the intersection of AC and BD be X Uses any appropriate trigonometry on triangle OAX $\frac{12\sqrt{2}}{\cos 30^\circ} = ...$ For example; $x = \frac{12\sqrt{2}}{\cos 30^\circ} = ...$ OR $\frac{\sin 120^\circ}{12\sqrt{2}} = \frac{\sin 30^\circ}{OA} \Rightarrow OA = ...$ OR $(12\sqrt{2})^2 = x^2 + x^2 - 2 \times x \times x \cos 120 \Rightarrow x = ...$	M1
	For the correct value of x	A1 cso

	$x = \sqrt{96} = (4\sqrt{6})^*$	[2]
(c)	For using trigonometry to find angle $\angle AOB$ $\cos \angle AOB = \frac{96+96-12^2}{2 \times \sqrt{96} \times \sqrt{96}} = \dots$	M1
	$\angle AOB = 75.522\dots^\circ$	A1
	Area of triangle OAB $\text{Area} = \frac{1}{2} \times \sqrt{96} \times \sqrt{96} \times \sin 75.522^\circ = (46.4758\dots)$	M1
	Total area of the pyramid = $4 \times 46.4758 + 12^2 = (329.90\dots)$	M1
	For the correct final area = awrt 330 (cm ²)	A1 [5]
	ALT	
	Height of the triangle of one of the triangular faces: $h = \sqrt{(4\sqrt{6})^2 - 6^2} = \dots$	M1
	$h = 2\sqrt{15}$	A1
	Area of triangle $OAB = \frac{1}{2} \times 2\sqrt{15} \times 12 = 12\sqrt{15} = (46.4758\dots)$	M1
	Total area of the pyramid = $4 \times 12\sqrt{15} + 144 = (329.903\dots)$	M1
(d)	$\cos \angle AOB = \frac{96+96-12^2}{2 \times \sqrt{96} \times \sqrt{96}} = 75.522^\circ$ or $\angle OBC = \frac{180^\circ - 75.522^\circ}{2} = 52.239^\circ$	
	Let Y on OB be the foot of the perpendicular from A to OB Length $AY = 12 \sin 52.239^\circ = (9.4868\dots)$ OR Length $AY = 4\sqrt{6} \sin 75.522^\circ = (9.4868\dots)$	M1A1 [M1A1]
	For the appropriate trigonometry on triangle AYC to find angle AYC $\cos \angle AYC = \frac{9.4868^2 + 9.4868^2 - 288}{2 \times 9.4868 \times 9.4868} \Rightarrow (\angle AYC = 126.87)$	M1
	Angle between plane AOB and plane OBC = awrt 127[°]	A1 [4]
	Total 13 marks	

Question number	Scheme	Marks
10 a (i)	$\sin 2\theta = \sin \theta \cos \theta + \cos \theta \sin \theta = 2 \sin \theta \cos \theta$ *	M1 A1 cso (2)
a (ii)	$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \cos^2 \theta - (1 - \cos^2 \theta)$ $= 2 \cos^2 \theta - 1$ *	M1 M1 A1 cso (3)
b	$\sin 2\theta - \tan \theta = 2 \sin \theta \cos \theta - \frac{\sin \theta}{\cos \theta}$ $\sin \theta \left(2 \cos \theta - \frac{1}{\cos \theta} \right)$ $\sin \theta \left(\frac{2 \cos^2 \theta - 1}{\cos \theta} \right) = \tan \theta \cos 2\theta$ *	M1 M1 dM1 A1cso (4)
c	$\tan x = 0$ so $x = 180$ $\cos 2x = 0$ so $2x = 90, 270, 450, 630$ $x = 45, 135, 225, 315$	B1 M1 A1 A1 (4)
Total 13 marks		

Part	Mark	Notes
(a)(i)	M1	For the correct use of: $\sin(A + B) = \sin A \cos B + \cos A \sin B \Rightarrow \sin 2\theta = \sin \theta \cos \theta + \sin \theta \cos \theta$ Allow any letter to be used for this mark.
	A1 Cso	For obtaining the given result $\sin 2\theta = 2 \sin \theta \cos \theta$ or allow $\sin(\theta + \theta) = 2 \sin \theta \cos \theta$
(a)(ii)	M1	For the correct use of: $\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\Rightarrow \cos 2A = \cos^2 A - \sin^2 A$ or $\cos(A + A) = \cos A \cos A - \sin A \sin A$ Allow any letter to be used for this mark.
	M1	For use of $\sin^2 A + \cos^2 A = 1$ and substituting Allow any letter to be used for this mark.
	A1 cso	For obtaining the given result $\cos 2\theta = 2 \cos^2 \theta - 1$ or allow $\cos(\theta + \theta) = 2 \cos^2 \theta - 1$
(b)	M1	For correct use of $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\tan \theta = \frac{\sin \theta}{\cos \theta}$

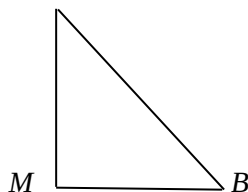
	M1	For factorising the resulting expression to give $\sin \theta \left(2 \cos \theta - \frac{1}{\cos \theta} \right)$ and using a common denominator of $\cos \theta$ correctly to obtain $\sin \theta \left(\frac{2 \cos^2 \theta - 1}{\cos \theta} \right)$ oe e.g. $\frac{\sin \theta}{\cos \theta} (2 \cos^2 \theta - 1)$
	dM1	For use of $2 \cos^2 \theta - 1 = \cos 2\theta$ Note: This mark is dependent on both previous M marks
	A1 CSO	For obtaining the given result
	ALT - Allow candidates to work from both sides. For example.	
	M1	For correct use of $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\sin 2\theta - \tan \theta = \tan \theta \cos 2\theta$ $\Rightarrow 2 \sin \theta \cos \theta - \frac{\sin \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta} (2 \cos^2 \theta - 1)$
	M1	For using a common denominator on the LHS to give: $\frac{2 \sin \theta \cos^2 \theta - \sin \theta}{\cos \theta} = \frac{\sin \theta (2 \cos^2 \theta - 1)}{\cos \theta}$
	dM1	Multiplies or factorises one side to make LHS = RHS For example; $\frac{2 \sin \theta (\cos^2 \theta - 1)}{\cos \theta} = \frac{2 \sin \theta (\cos^2 \theta - 1)}{\cos \theta}$
	A1 CSO	Concludes RHS = LHS therefore shown. Note there must be no errors seen. If unsure, send to Review.
(c)	Some candidates are starting again without using the given expression in (b) and are getting their angles from $\cos x$ rather than $\cos 2x$ Allow the use of θ or any other variable throughout this part of the question. As a general principle of marking this part: • B1 – For only 180° seen. If 0° is included, then B0. • M1 – For either $2x = 90, (270, 450, 630)$ OR $x = 45^\circ$ or 135° or 225° or 315° or any other correct angle in or out of range. • A1 – For any two correct angles apart from 180° • A1 – For all four correct angles, apart from 180°	
	B1	For $x = 180$ If $x = 0$ included then B0
	M1	For obtaining $2x = 90, (270, 450, 630)$
	A1	For any two correct angles from $45^\circ, 135^\circ, 225^\circ$ or 315°
	A1	For all four angles from $\cos x$ or $\cos 2x$
	For extra angles within range, penalise the last A mark only. For example; If they have [coming from a correct method] $x = 45^\circ, 67.5^\circ, 135^\circ, 225^\circ, 315^\circ$ score M1A1A0 If they have $x = 45^\circ, 67.5^\circ, 135^\circ$ then award M1A1A0	

Question	Scheme	Marks
9(a)	$\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$ OR $\cos 2A = \cos^2 A - \sin^2 A = (1 - \sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$ (i) $\cos^2 A = \frac{\cos 2A + 1}{2}$, (ii) $\sin^2 A = \frac{1 - \cos 2A}{2}$ *	M1M1 A1A1cso [4]
(b)	$(2\sin x - \cos x)(\sin x - 3\cos x)$ $= 2\sin^2 x - 7\sin x \cos x + 3\cos^2 x$ $= 2\left(\frac{1 - \cos 2x}{2}\right), \dots, + 3\left(\frac{\cos 2x + 1}{2}\right)$ $= \dots - \frac{7\sin 2x}{2} + \dots$ $= \frac{1}{2}(\cos 2x - 7\sin 2x + 5)$ *	M1 M1, M1 M1 A1 cso [5]
(c)	$y = \frac{1}{2}(\cos 2x - 7\sin 2x + 5) \Rightarrow \frac{dy}{dx} = -\sin 2x - 7\cos 2x = 0$ $\Rightarrow -\sin 2x = 7\cos 2x \Rightarrow \tan 2x = -7$ $2x = -81.869, 98.130, 278.130$ $\Rightarrow x = 49, 139$	M1 M1 A1 A1 [4]
Total 13 marks		

Part	Mark	Notes
NB: Parts (a) and (b) are proofs with given answers. You must check every line of their working.		
(a)	M1	Uses the formula for $\cos(A+B) = \cos A \cos B - \sin A \sin B$ correctly.
	M1	Uses the identity $\cos^2 A = 1 - \sin^2 A$ or $\sin^2 A = 1 - \cos^2 A$
(i)	A1 cso	For the correct identity for either $\sin^2 A$ or $\cos^2 A$ with no errors seen
(ii)	A1 cso	For the correct identity for $\sin^2 A$ and $\cos^2 A$ with no errors seen
SC	If a candidate does not use $\cos(A+B) = \cos A \cos B - \sin A \sin B$ to find the required identities for $\sin^2 A$ or $\cos^2 A$ but uses them to show $\sin^2 A = \frac{1 - \cos 2A}{2}$ or $\cos^2 A = \frac{\cos 2A + 1}{2}$ apply the following scheme.	
	First M mark – M0	
	2nd M1	Uses the identity $\cos^2 A = 1 - \sin^2 A$ or $\sin^2 A = 1 - \cos^2 A$ For example:

		$\cos^2 A = \frac{\cos 2A + 1}{2} = 1 - \sin^2 A$ $\sin^2 A = 1 - \frac{\cos 2A + 1}{2} = \frac{2 - (\cos 2A + 1)}{2} = \frac{1 - \cos 2A}{2}$
(i)/(ii)	A1 cso	For the correct identity for $\sin^2 A$ or $\cos^2 A$ with no errors seen
	A0	Not available
(b)	M1	Multiplies out the brackets with no errors.
	M1	Substitutes at least one of the given identities for $\cos 2x$ into their expansion [either for $\sin^2 x$ or $\cos^2 x$]. This is not a dependent mark, allow for substitution into an erroneous expression.
	M1	Substitutes both of the given identities for $\cos 2x$ into their expansion [for both $\sin^2 x$ and $\cos^2 x$]. This is not a dependent mark, allow for substitution into an erroneous expression. NB: This is a B mark in Epen
	M1	Uses the identity $\sin 2A = 2\sin A \cos A$ seen applied explicitly before the final answer. Look for, $k \sin x \cos x \rightarrow \frac{k \sin 2x}{2}$
	A1 cso	For the correct identity with no errors seen.
	ALT – Works backwards – these steps must contain no errors (to maintain compatibility with the main method. Starting from $\frac{1}{2}(\cos 2x - 7\sin 2x + 5)$ award as follows:	
	M1	Uses: $\cos 2x = \cos^2 x - \sin^2 x$
	M1	Uses: $5 = 5\cos^2 x + 5\sin^2 x$
	M1	Uses: $-7\sin 2x = 14\sin x \cos x$
	M1	Collects like terms and divides by 2 to give: $3\cos^2 x - 7\sin x \cos x + 2\sin^2 x$
	A1	Factorises their expression to give: $(2\sin x - \cos x)(\sin x - 3\cos x)$
(c)	M1	For an attempt to differentiate the given equation. A minimally acceptable is defined as follows: $\frac{dy}{dx} = \pm k \sin 2x \pm l \cos 2x$ where k and l are integers with no other terms .
	M1	Uses the identity $\tan 2x = \frac{\sin 2x}{\cos 2x}$ This is an independent mark, but it must be used correctly following correct division of $\cos 2x$ throughout.
	A1	Find at least one correct value for $2x$ e.g., -81.869 , 98.130 , 278.130 etc NB: This is an M mark in Epen. [Accept -81 , 98 , 278]
	A1	For $x = 49$ and 139 rounded to the nearest whole number or better.

ALT - Differentiates using product rule	
M1	$y = (2 \sin x - \cos x)(\sin x - 3 \cos x)$ $\frac{dy}{dx} = (2 \sin x - \cos x)(\cos x + 3 \sin x) + (\sin x - 3 \cos x)(2 \cos x + \sin x)$ $\frac{dy}{dx} = 2 \sin x \cos x + 6 \sin^2 x - \cos^2 x - 3 \sin x \cos x +$ $+ 2 \sin x \cos x + \sin^2 x - 6 \cos^2 x - 3 \sin x \cos x$ $\frac{dy}{dx} = -2 \sin x \cos x + 6 \sin^2 x - 7 \cos^2 x$ Allow $\frac{dy}{dx} = \pm p \sin x \cos x \pm q \sin^2 x \pm r \cos^2 x$ NB: This will not factorise
M1	Divides through by $\cos^2 x$ provided they have an expression of the form $\frac{dy}{dx} = \pm p \sin x \cos x \pm q \sin^2 x \pm r \cos^2 x$ This gives : $\frac{dy}{dx} = -2 \sin x \cos x + 6 \sin^2 x - 7 \cos^2 x = 0 \rightarrow \div \cos^2 x$ $-7 - 2 \tan x + 7 \tan^2 x = 0$ AND solves to give a value for $\tan x = \frac{1 \mp 5\sqrt{2}}{7}$ or allow $\tan x = \frac{1 \pm 5\sqrt{2}}{7}$
A1	At least one correct value for x of for example, awrt $49.1 \dots$ awrt -40.9
A1	For both 49° and 139°

Question	Scheme	Marks
9(a)	$\angle AOB = \frac{1.8x}{x} = 1.8$  $MB = x \sin 0.9 \Rightarrow AB = 2x \sin 0.9$ Hence diameter $AC = 2x \sin 0.9 + 2x$ [or Radius = $x \sin 0.9 + x$] Arc length of semicircle: $\frac{\pi D}{2} = \frac{\pi(2x \sin 0.9 + 2x)}{2} = \pi x(1 + \sin 0.9)$ Perimeter of logo: $P = \pi x(1 + \sin 0.9) + 2x + 1.8x \Rightarrow P = x(\pi + \pi \sin 0.9 + 3.8)$ <a>$a=1, b=3.8$	B1 M1 dM1A1 M1 M1A1 [7]
(b)	$D = 2 \times 10 \times \sin 0.9 + 2 \times 10 = 35.6665...$ $\text{Area of semicircle} = \frac{\frac{\pi \times 35.6665...^2}{4}}{2} = 499.553...$ $\text{Area of sector} = 1.8 \times \frac{10^2}{2} = 90$ $\text{Area of segment} = 90 - \frac{1}{2} \times 10 \times 10 \times \sin 1.8 = 41.307...$ OR Area of triangle = $\frac{1}{2} \times 10 \times 10 \times \sin 1.8 = 48.692$ $\text{Area of logo} = 499.553 - 41.307 = 458.245... \approx \text{awrt } 458 \text{ (cm}^2\text{)}$	M1 M1 B1 B1 M1A1 [6]
Total 13 marks		

Part	Mark	Notes
(a)	B1	For finding angle $AOB = 1.8$ in radians only seen anywhere.
	M1	For find the length AB or $\frac{1}{2} AB$ using their angle.

		$AB = 2x \sin 0.9 \quad \frac{1}{2} AB = x \sin 0.9$ [Allow $AB = 1.57x$] Please check their diagram for their labelling/notation.
	dM1	For attempting to find the diameter (or radius) of the semi-circle ADC Allow $2x + 1.57x$ or $3.57x$ [Dependent on the previous M mark]
	A1	For the correct expression for the diameter $[2x \sin 0.9 + 2x]$ or the radius $[x \sin 0.9 + x]$ of the semi-circle.
	M1	For finding the arc length of the semi-circle. This can be simplified or unsimplified. [Allow also $1.79\pi x$]
	M1	For finding the perimeter of the logo using their values in terms of x and $\sin 0.9$ only That is: $P = 1.8x + 2x +$ their arc length of ADC
	A1	For the correct expression exactly as written.
	ALT – Uses cosine rule	
	B1	For finding angle $AOB = 1.8$
	M1	Finds the length AB or $\frac{1}{2} AB$ using their angle. [They must sqrt to find AB] $AB = \sqrt{x^2 + x^2 - 2 \times x \times x \cos 1.8}$
	dM1	For attempting to find the diameter (or radius) of the semi-circle ADC $AC = 2x + 'x\sqrt{2 - 2 \cos 1.8}'$
	A1	For the correct diameter or radius of the circle.
	M1	For finding the arc length of the semi-circle. $P = \frac{\pi(x\sqrt{2 - 2 \cos 1.8})}{2}$
	M1	For finding the perimeter of the logo using their values. That is: $P = 1.8x + 2x + '\pi x(1 + \sin 0.9)'$ They must change cosine to sine for this mark. $\frac{1}{2}(\sqrt{2 - 2 \cos 1.8}) = \sin 0.9$ must be explicitly seen NOTE: $\left[\sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}} \right]$
	A1	For the correct expression exactly as written.
(b)	Allow work in degrees in this part as it makes no difference.	
	M1	For attempting to find the diameter or radius of the semicircle. Accept: $D = 2 \times 10 \times \sin \angle \text{their } AOB + 2 \times 10 = (35.6665...)$ $r = \frac{2 \times 10 \times \sin \angle \text{their } AOB + 2 \times 10}{2} = (17.833...)$
	M1	For using their D , [where $D \neq 10$] to find the area of the semicircle . $\text{Area}_{\text{semicircle}} = \frac{\pi \times ('35.6665...')^2}{4} \quad \text{OR} \quad \frac{\pi \times 17.8333...^2}{2} = (499.553...)$ Note: This is an A mark in Epen
	B1	For finding the area of the sector If they work in degrees, accept awrt 90. This is not a ft mark.



	B1	For finding the area of the segment, awrt 41 OR for finding the area of the triangle $OAB = \frac{1}{2} \times 10^2 \times \sin 1.8 = [48.692...]$ This is not a ft mark.
	M1	For finding the area of the logo Area = Their area of semicircle – their area of the segment. Or Area = Area of whole shape – area of sector
	A1	For the correct value of the area of the logo.



Question	Scheme	Marks
10(a)	$AC = \sqrt{(2x)^2 + x^2} = (\sqrt{5}x)$ $VA = '\sqrt{5}x' \sin 45^\circ = \frac{\sqrt{10}}{2}x *$	B1 M1A1 CSO [3]
(b)	$\text{Height} = \sqrt{\left(\frac{\sqrt{10}}{2}x\right)^2 - \left(\frac{\sqrt{5}}{2}x\right)^2} = \frac{\sqrt{5}}{2}x$	M1A1 [2]
(c)	$\cos \angle VBA = \frac{1}{\frac{\sqrt{10}}{2}}$ $\Rightarrow \angle VBA = 50.768\dots^\circ \approx 50.8^\circ \text{ awrt}$	M1 A1 [2]
(d)	Let M be the point at which the diagonals AC and BD meet. The required angle is either AMB or DMC $AM = MB = \frac{AC}{2} = \frac{\sqrt{5}x}{2}$ or $DM=MC=\frac{AC}{2} = \frac{\sqrt{5}x}{2}$ $\cos \angle AMB = \frac{\left(\frac{\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2 - 2^2}{2 \times \frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2}} \text{ oe}$ $(\cos \angle AMB = -\frac{3}{5} \Rightarrow \angle AMB = 126.869\dots^\circ) \approx 126.9^\circ \text{ awrt}$	B1 B1 M1 A1 [4]
(e)	$9\sqrt{5} = \frac{1}{3} \times 2x \times x \times \frac{\sqrt{5}}{2}x \Rightarrow x = \dots$ $x = 3$	M1 A1 [2]
Total 13 marks		

Part	Mark	Notes
(a)	B1	For using correct Pythagoras theorem to find length AC or $\frac{1}{2}$ AC, explicit substitution must be seen.
	M1	For correct trigonometry using their AC in triangle VAC or half of their AC in triangle VAM or correct Pythagoras theorem (let M be the centre of the base at which the diagonals AC and BD meet) to find length VA. e.g. $VA = \sqrt{5}x \cos(45^\circ) = \frac{\sqrt{10}}{2}x \text{ or }$ $VA = \frac{\sqrt{5}}{2}x \div \cos(45^\circ) = \frac{\sqrt{10}}{2}x \text{ or } VA = \frac{\sqrt{5}}{2}x \div \sin(45^\circ) = \frac{\sqrt{10}}{2}x$
	A1 cso	Achieves printed answer with no errors. Condone AV instead of VA Note: $\frac{\sqrt{10}}{2}x$ written as $\frac{\sqrt{10}x}{2}$ is A0, $\frac{\sqrt{10}}{2}x$ written as $\frac{\sqrt{10}x}{2}$ is A1
(b)	M1	Uses the correct Pythagoras theorem or correct trigonometry with the given length $VA = \frac{\sqrt{10}}{2}x$ to find the height of the pyramid.
	A1	For $h = \frac{\sqrt{5}}{2}x$ Correct exact answer only scores M1A1. This can be written down as the triangle is isosceles.
(c)	M1	Ignore missing x consistently throughout their solution. For using any appropriate trigonometry to find the angle VBA If attempts cosine rule, look for correct application of the cosine rule for the required angle: $\left(\frac{\sqrt{10}}{2}x\right)^2 = \left(\frac{\sqrt{10}}{2}x\right)^2 + (2x)^2 - 2 \times \frac{\sqrt{10}}{2}x \times 2x \cos(VBA) \text{ oe}$
	A1	For the correct angle. awrt 50.8° (with or without degree sign)
(d)	B1	Identifies the angle required.(can be implied/embedded in their working)
	B1	For the correct lengths AM and MB or DM and MC or BE and ME Let E to be mid point of AB, correct length $BE=x$, $ME=0.5x$ scores B1, this mark can be implied/embedded in their working)
	M1	Ignore missing x consistently throughout their solution. For using any appropriate trigonometry to find the required angle or half of the required angle.
	A1	For the correct angle. awrt 126.9° (with or without degree sign)
(e)	M1	For using the correct formula for the volume of a pyramid with their height of the pyramid and attempts to find a value of x.
	A1	For $x = 3$ with or without unit, ignore incorrect unit



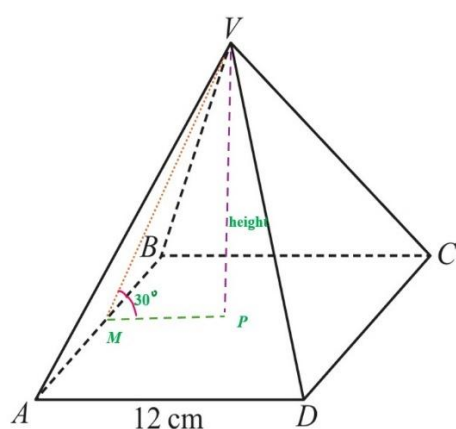
Question number	Scheme	Marks
8(a)	$\tan 30^\circ = \frac{VP}{6}$ where P is the centre of the base $VP = 6 \tan 30^\circ = 2\sqrt{3} *$	M1 A1*cso [2]
(b)	$AC = \sqrt{12^2 + 12^2} = 12\sqrt{2}$ or $AC = \frac{12}{\sin 45}$ or $AC = \frac{12}{\cos 45}$ $VA^2 = (2\sqrt{3})^2 + (6\sqrt{2})^2 (=84)$ $VA = 2\sqrt{21}$ (cm) oee	M1 oe M1 oe A1 [3]
(c)	$VX = \sqrt{(2\sqrt{21})^2 - 6^2} (=4\sqrt{3})$ or $VX = \frac{6}{\cos 30}$ or $VX = \sqrt{6^2 + (2\sqrt{3})^2}$ where X is the midpoint of the side AD $\text{Area} = \frac{1}{2} \times 4\sqrt{3} \times 12 = 24\sqrt{3}$ (cm ²) oee	M1 M1A1 [3]
(d)	$'24\sqrt{3}' = \frac{1}{2} \times '2\sqrt{21}' \times DY$ where Y is the foot of the perpendicular from D to VA $DY = \frac{24\sqrt{7}}{7}$ oee	M1 A1 [2]
(e)	$\cos \theta = \frac{\left(\frac{24\sqrt{7}}{7}\right)^2 + \left(\frac{24\sqrt{7}}{7}\right)^2 - (12\sqrt{2})^2}{2 \times \frac{24\sqrt{7}}{7} \times \frac{24\sqrt{7}}{7}} = -\frac{3}{4}$ $\theta = 138.6^\circ$	M1A1 A1 [3]
Total is 13 marks		

Part	Mark	Additional Guidance
(a)	M1	For forming a correct equation using trigonometry. e.g. $\tan 60^\circ = \frac{6}{VP}$ or $\frac{h}{\sin 30^\circ} = \frac{6}{\sin 60^\circ}$ (If attempted by ratio, send to review)
	A1*cso	For rearranging to show the required result. Minimum need to see one intermediate step from their equation to the answer, if the height is not the subject of their equation: e.g. $\tan 30^\circ = \frac{VP}{6} \Rightarrow VP = 6 \tan 30^\circ = 2\sqrt{3}$ is M1A1 $VP = 6 \tan 30^\circ = 2\sqrt{3}$ is M1A1 $\tan 30^\circ = \frac{VP}{6} \Rightarrow \frac{\sqrt{3}}{3} = \frac{VP}{6} \Rightarrow VP = 2\sqrt{3}$ is M1A1 $\tan 30^\circ = \frac{VP}{6} \Rightarrow VP = 2\sqrt{3}$ is M1A0
(b)	M1oe	Correct use of Pythagoras' theorem or trigonometry to obtain the length/half length of the diagonal of the base AC or $\frac{1}{2} AC$ Or other valid method to find VM , (Let M to be the midpoint of AB) e.g.: $VM = \frac{6}{\cos 30^\circ} (= 4\sqrt{3})$ or $VM = \sqrt{6^2 + (2\sqrt{3})^2} (= 4\sqrt{3})$
	M1oe	Use Pythagoras' theorem to obtain a numerical value for VA/VA^2 Any correct method. e.g. $VA^2 = (4\sqrt{3})^2 + (6)^2$
	A1	Correct exact length $2\sqrt{21}$ or $\sqrt{84}$ oee (or exact equivalent)
(c)	M1	Correct use of Pythagoras' theorem or trigonometry to obtain the length of VX , which is the same as VM , could be found in part b Or correct use of cosine rule (ft their VA) to find the value of cosine value of any angle in triangle VAD e.g. $AD^2 = VA^2 + VA^2 - 2VA \times VA \times \cos \angle AVD$ $12^2 = ('2\sqrt{21}')^2 + ('2\sqrt{21}')^2 - 2 \times '2\sqrt{21}' \times '2\sqrt{21}' \times \cos \angle AVD$ $\left\{ \Rightarrow \cos \angle AVD = \frac{1}{7} \right\}$
	M1	Correct method to find the area of triangle VAD (Do not have to use exact values) Accept any correct valid method. e.g. $\left\{ \cos \angle AVD = \frac{1}{7} \Rightarrow \sin \angle AVD = \frac{4\sqrt{3}}{7} \right\} \Rightarrow A = \frac{1}{2} \times '2\sqrt{21}' \times '2\sqrt{21}' \times \frac{4\sqrt{3}}{7} = 24\sqrt{3}$
	A1	Correct exact value for area
(d)	M1	Use of area of triangle with VA as the base to form an equation for DY Or other valid method: e.g. using trigonometry ratio: $\sin \angle VAD = \frac{'4\sqrt{3}'}{'2\sqrt{21}}} = \frac{DY}{12}$

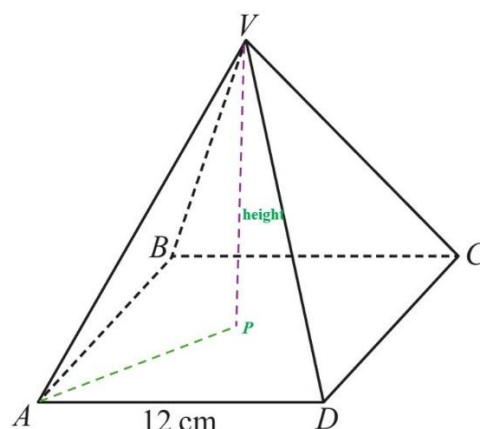
	A1	Correct exact length $DY = \frac{24\sqrt{7}}{7}$ oee, e.g. $DY = \frac{8\sqrt{63}}{7}$ or $24\sqrt{\frac{1}{7}}$
(e)	M1	Correct use of cosine rule: $\cos \theta = \frac{{}^{\text{"DY"}}^2 + {}^{\text{"BY"}}^2 - {}^{\text{"BD"}}^2}{2 \times {}^{\text{"DY"}} \times {}^{\text{"BY"}}$ with their values Or other valid method, e.g. $\sin \frac{\theta}{2} = \frac{{}^{\text{"BD"}}}{2} = \frac{{}^{\text{"6}\sqrt{2}}}{\text{"24}\sqrt{7}}$
	A1	Obtains $\cos \theta = -\frac{3}{4}$ or $\sin \frac{\theta}{2} = \frac{\sqrt{14}}{4}$
	A1	Correct angle found

Useful diagrams:

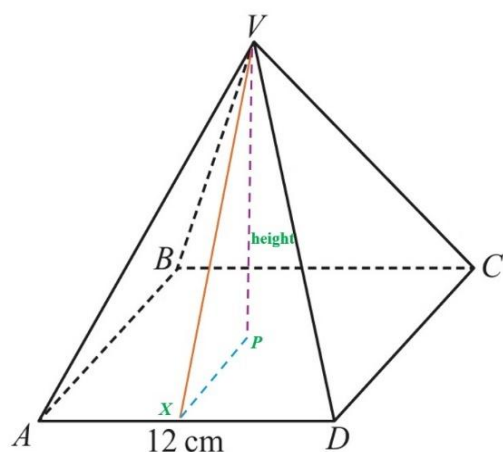
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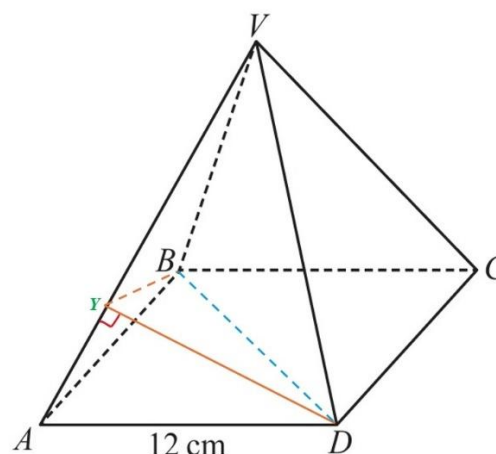
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3



4



Question number	Scheme	Marks
9 (a)	$\cos 2\theta = \{\cos \theta \cos \theta - \sin \theta \sin \theta\} = \cos^2 \theta - \sin^2 \theta = \cos^2 \theta - (1 - \cos^2 \theta)$ $= 2\cos^2 \theta - 1 \quad *$	M1 A1*cs0 [2]
(b)	$\int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} \cos 2\theta \{d\theta\}$ $\left[\frac{\sin 2\theta}{2} \right]_{\frac{\pi}{3}}^{\frac{3\pi}{4}} = \frac{\sin 2\left(\frac{3\pi}{4}\right)}{2} - \frac{\sin 2\left(\frac{\pi}{3}\right)}{2} = -\frac{2 + \sqrt{3}}{4}$	M1 M1M1A1 [4]
(c)	$2\cos^2 \theta - 1 = -\cos \theta$ $2\cos^2 \theta + \cos \theta - 1 = 0 \Rightarrow (2\cos \theta - 1)(\cos \theta + 1) = 0 \Rightarrow \cos \theta = \dots$ $\theta = \frac{\pi}{3}$ $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} -\cos \theta \, d\theta$ $= \left[-\sin \theta \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ $= -\sin \frac{\pi}{2} - \left(-\sin \frac{\pi}{3} \right) \left\{ = \frac{-2 + \sqrt{3}}{2} \right\}$ $\frac{2 + \sqrt{3}}{4} - \frac{2 - \sqrt{3}}{2}$ $\frac{-2 + 3\sqrt{3}}{4}$ ALT for last 5 marks $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (-\cos \theta - \cos 2\theta) \{d\theta\} \{+\} \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (-\cos 2\theta) \{d\theta\}$ $= \left[-\sin \theta - \frac{\sin 2\theta}{2} \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \{+\} \left[-\frac{\sin 2\theta}{2} \right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}$ $= \left(-\sin \frac{\pi}{2} - \frac{\sin 2 \times \frac{\pi}{2}}{2} \right) - \left(-\sin \frac{\pi}{3} - \frac{\sin 2 \times \frac{\pi}{3}}{2} \right) \{+\} \left(-\frac{\sin 2 \times \frac{3\pi}{4}}{2} \right) - \left(-\frac{\sin 2 \times \frac{\pi}{2}}{2} \right)$ $= \frac{-4 + 3\sqrt{3}}{4} + \frac{1}{2}$ $= \frac{-2 + 3\sqrt{3}}{4}$	M1 M1 A1 M1 M1 M1 ddM1 A1 [8] M1 M1 M1 ddM1 A1
Total 14 marks		

Part	Mark	Additional Guidance
(a)	M1	For $\cos^2 \theta - \sin^2 \theta = \cos^2 \theta - (1 - \cos^2 \theta)$ or $\cos^2 \theta - \sin^2 \theta = \cos^2 \theta + \cos^2 \theta - 1$
	A1*cs0	Obtains the correct result with no errors seen. Must have $\cos 2\theta$ on the lhs in their working, does not have to be in the final line provided there is a chain of reasoning.
(b)	M1	For replacement of $2\cos^2 \theta - 1$ with $\cos 2\theta$, ignore limits
	M1	For an attempt to integrate to an expression of the form $\pm \frac{\sin 2\theta}{2}$
	M1	Correct substitution of the given limits to a changed expression in terms of $\sin 2\theta$ and subtracts the correct way around. A correct value from a correctly integrated expression scores this mark, no need to see explicit substitution
	A1	Correct answer. isw
	M1	Correctly equates the two curves.
(c)	M1	Rearranges their $3TQ = 0$ and attempts to solve (see general guidance). Can be implied by their correct $\cos \theta$ values, $\frac{1}{2}$ and -1 from a correct $3TQ$, if their $3TQ$ is not correct, a clear method must be shown.
	A1	Correct angle found. Ignore extra angles.
	M1	Main: For stating the correct integral with “correct” limits of their $\frac{\pi}{3}$ and $\frac{\pi}{2}$ $\pm \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} -\cos \theta \, d\theta$ and $\pm \int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} (2\cos^2 \theta - 1) d\theta$ (the later integral can be implied by later work) (their $\frac{\pi}{3}$ must be positive and $< \frac{\pi}{2}$) ALT: Correct integrals $\pm \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (-\cos \theta - \cos 2\theta) \{d\theta\}$ and $\pm \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (-\cos 2\theta) \{d\theta\}$ oe
	M1	For a correct integration of all their integrals in $\cos 2\theta$ and/or $\cos \theta$ only.
	A1 on Epen	Ignore limits
	M1	For correct explicit substitution of their limits into either changed expression, subtracts the correct way around – substitution of 0 must be seen unless substitution into their changed function gives 0 Can be implied by $\pm \left(\frac{-2 + 3\sqrt{3}}{4} \right)$ from correct working
	ddM1	Correct strategy finding the required area: Main: For subtracting the modulus of their 2 integrals. oe. Either way around. ALT: Add the values of two positive area “ $\frac{-4 + 3\sqrt{3}}{4}$ ” and “ $\frac{1}{2}$ ” oe Dependent on previous three method marks
	A1	Correct answer, simplest form and exact area of R

Question number	Scheme	Marks
10 (a)	$\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1^*$	M1A1cso [2]
(b)	$\cos 4A = 2\cos^2 2A - 1$ $= 2(2\cos^2 A - 1)^2 - 1$ $= 2(4\cos^4 A - 4\cos^2 A + 1) - 1$ $= 8\cos^4 A - 8\cos^2 A + 1^*$	M1 M1 M1 A1cso [4]
(c)	Let $\frac{\theta}{4} + \frac{\pi}{24} = x$ $\cos^2 x (\cos^2 x - 1) = \cos^4 x - \cos^2 x$ $\Rightarrow \cos^4 x - \cos^2 x = \frac{\cos 4x - 1}{8}$ $4\left(\frac{\theta}{4} + \frac{\pi}{24}\right) = \theta + \frac{\pi}{6}$ $\frac{\cos\left(\theta + \frac{\pi}{6}\right) - 1}{8} = -\frac{1}{16} \Rightarrow \cos\left(\theta + \frac{\pi}{6}\right) = \frac{1}{2}$ $\Rightarrow \theta + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3} \Rightarrow \theta = \frac{\pi}{6}, \frac{3\pi}{2}$	M1 M1 M1 M1A1 [5]
Alternative (with x defined as above)		
(c)	$\cos^4 x - \cos^2 x = -\frac{1}{16} \Rightarrow 8\cos^4 x - 8\cos^2 x + 1 = 1 - \frac{8}{16}$ $\cos 4x = \frac{1}{2} \Rightarrow 4x = \theta + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \dots \Rightarrow \theta = \frac{\pi}{6}, \frac{3\pi}{2}$	M1 M1M1 M1A1
Alternative (with x defined as above)		
(c)	$16(\cos^2 x)^2 - 16\cos^2 x + 1 = 0$ $\cos^2 x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4 \times 16 \times 1}}{2 \times 16} = \frac{2 \pm \sqrt{3}}{4}$ $\cos x = \sqrt{\frac{2 \pm \sqrt{3}}{4}} (= \pm 0.9659... \text{ or } \pm 0.2588...)$ $x = 0.2617..., 1.3089..., 1.8325..., 2.8787... (= \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12} ...)$ $\theta = 4\left(x - \frac{\pi}{24}\right) = \frac{\pi}{6}, \frac{3\pi}{2}$	M1 M1 M1 M1A1
(d)	$f(A) = \frac{\cos 4A}{2} + \frac{1}{2}$ $\left[\frac{\sin 4A}{8} + \frac{A}{2}\right]^{\frac{\pi}{6}} = \left(\frac{\sin 2\pi}{8} + \frac{\pi}{4}\right) - \left(\frac{\sin \frac{2\pi}{3}}{8} + \frac{\pi}{12}\right) = \frac{\pi}{4} - \frac{\sqrt{3}}{16} - \frac{\pi}{12} = \frac{\pi}{6} - \frac{\sqrt{3}}{16}$	M1 dM1 ddM1 A1 [4]
Total 15 marks		

(a)	M1	Use $\cos(A + A) = \cos A \cos A - \sin A \sin A$ (may go straight to $\cos 2A = \cos^2 A - \sin^2 A$) and then try to replace $\sin^2 A$ by $1 - \cos^2 A$. Allow with any other single variable.
	A1 cso	Obtain $\cos 2A = 2 \cos^2 A - 1$ with no errors in the working. Must be stated with A as the variable.
(b)		Allow with any other single variable for all M marks. Allow one sign error removing brackets and ft for all M marks.
	M1	$\cos 4A = 2 \cos^2 2A - 1$ or $\cos 4A = \cos^2 2A - \sin^2 2A$ Obtain an expression in terms of $\cos 2A$ and/or $\sin 2A$.
	M1	eg $\cos 4A = 2(2 \cos^2 A - 1)^2 - 1$, $\cos 4A = (2 \cos^2 A - 1)^2 - 4(1 - \cos^2 A) \cos^2 A$ Obtain a correct expression in $\cos A$. Eliminate terms in $\sin A$ and $\sin 2A$.
	M1	Expand brackets. Numerical factors may remain. e.g. $\cos 4A = 2(4 \cos^4 A - 2 \cos^2 A - 2 \cos^2 A + 1) - 1$ e.g. $\cos 4A = 4 \cos^4 A - 4 \cos^2 A + 1 - 4 \cos^2 A + 4 \cos^4 A$
	A1 cso	Obtain the given result with no errors in the working. Must be stated with A as the variable.
(c)	M1	Expand brackets and use the result from part (b) to express the LHS of the equation in terms of $4x$, $\frac{\cos 4x - 1}{8}$, where $x = \frac{\theta}{4} + \frac{\pi}{24}$ Alternative Arrange the equation in the form $8 \cos^4 x - 8 \cos^2 x + 1 = 1 - \frac{8}{16}$ Allow a single numerical or sign error in the constants for both methods.
	M1	Rearrange the equation to give $\cos\left(\theta + \frac{\pi}{6}\right) = \frac{1}{2}$ or oe Note that this mark is shown as B1 on ePEN.
	M1	Find any correct positive value for $4\left(\frac{\theta}{4} + \frac{\pi}{24}\right)$, given in radians. eg $\frac{\pi}{3}$
	M1	Find any correct positive value for θ , given in radians. eg $\frac{\pi}{6}$
	A1	Both correct answers with no extras, given in radians as multiples of π . Ignore answers outside the given range.

Alternative		
(c)	M1	Write the equation as a three term quadratic. e.g. $16(\cos^2 x)^2 - 16\cos^2 x + 1 = 0, \left(x = \frac{\theta}{4} + \frac{\pi}{24}\right),$ $A^2 - A + \frac{1}{16} = 0, \left(A = \cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right)\right)$
	M1	Any valid method to solve their quadratic equation. Note that this mark is shown as B1 on ePEN.
	M1	Any correct value for $x = \frac{\theta}{4} + \frac{\pi}{24}$ in the range $0 \leq x \leq \pi$, given in radians, either as a decimal or as a multiple of π .
	M1	Either value of θ given in radians, $\theta = \frac{\pi}{6}$ or $\theta = \frac{3\pi}{2}$ or $\theta = 0.523\dots$ or $\theta = 4.712\dots$
	A1	Both $\theta = \frac{\pi}{6}$ and $\theta = \frac{3\pi}{2}$ given as multiples of π .
(d)	M1	Use result from part (b) to write $f(A) = k(\cos 4A \pm 1)$.
	dM1	Integrate their $f(A)$ correctly, dep on M1 awarded. Ignore limits.
	ddM1	Substitute both given limits, dep on both M marks awarded.
	A1	$\frac{\pi}{6} - \frac{\sqrt{3}}{16}$ or $\frac{1}{6}\pi - \frac{1}{16}\sqrt{3}$ simplified in this format. Do not isw.

Question number	Scheme	Marks
11 (a)	$\cos 2x = \cos(x+x)$ $= \cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x$ $= 1 - 2\sin^2 x$ *	M1 M1 A1 cso (3) M1dM1
(b)	$\sin x + 2\sin^2 x - 1 = 0 \Rightarrow (2\sin x - 1)(\sin x + 1) = 0$ When $\sin x = \frac{1}{2}$ When $\sin x = -1$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{3\pi}{2}$ $\left(\frac{\pi}{6}, \frac{5}{2}\right), \left(\frac{5\pi}{6}, \frac{5}{2}\right), \left(\frac{3\pi}{2}, 1\right)$	A1 A1 (4)
(c)	$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin x + 2) dx - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\cos 2x + 2) dx$ $\left[-\cos x + 2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - \left[\frac{\sin 2x}{2} + 2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$ $\left[\left(-\cos \frac{5\pi}{6} + 2\left(\frac{5\pi}{6}\right)\right) - \left(-\cos \frac{\pi}{6} + 2\left(\frac{\pi}{6}\right)\right)\right]$ $-\left[\left(\frac{\sin 2\left(\frac{5\pi}{6}\right)}{2} + 2\left(\frac{5\pi}{6}\right)\right) - \left(\frac{\sin 2\left(\frac{\pi}{6}\right)}{2} + 2\left(\frac{\pi}{6}\right)\right)\right]$ $= \frac{3\sqrt{3}}{2}$ $\int_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} (\cos 2x + 2) dx - \int_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} (\sin x + 2) dx$ $\left[\frac{\sin 2x}{2} + 2x\right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} - \left[-\cos x + 2x\right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}$ $\left[\left(\frac{\sin 2\left(\frac{3\pi}{2}\right)}{2} + 2\left(\frac{3\pi}{2}\right)\right) - \left(\frac{\sin 2\left(\frac{5\pi}{6}\right)}{2} + 2\left(\frac{5\pi}{6}\right)\right)\right]$ $-\left[\left(-\cos \left(\frac{3\pi}{2}\right) + 2\left(\frac{3\pi}{2}\right)\right) - \left(-\cos \left(\frac{5\pi}{6}\right) + 2\left(\frac{5\pi}{6}\right)\right)\right]$ $\frac{3\sqrt{3}}{4}$ $R_1:R_2 = 2:1$ See Special Case at end of notes	M1 A1 M1 M1 A1 B1ft (8)
Total 15 marks		

Part	Mark	Notes
(a)	M1	For using the addition formula on page 2 and reaching the result $\cos 2x = \cos^2 x - \sin^2 x$ This must be correct for this mark.
	M1	For using and applying $\sin^2 x + \cos^2 x = 1$ in their $\cos 2x$ $\cos 2x = 1 - \sin^2 x - \sin^2 x$
	A1 CSO	For the correct identity as shown in full. $\cos 2x = 1 - 2\sin^2 x$ * Note: This is a show question, there must be no errors seen.
(b)	M1	Equates the two given equations together. $\sin x + 2 = \cos 2x + 2$ and uses the given identity in part (a) to attempt to form a 3TQ in $\sin x$: $\sin x + 2\sin^2 x - 1 = 0$ or $2\sin^2 x + \sin x - 1 = 0$ At least either $2\sin^2 x$ or $\sin x$ must be correct.
	dM1	Solves their 3TQ by any method to reach two values for $\sin x$ -see General Guidance $(2\sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = \left(-1, \frac{1}{2}\right)$ This mark is dependent on the previous M mark.
	A1	For finding all three values of x : $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ Allow $x = 30^\circ, 150^\circ, 270^\circ$ for this mark – condone missing degree signs
	A1	For all three sets of correct coordinates: $\left(\frac{\pi}{6}, \frac{5}{2}\right), \left(\frac{5\pi}{6}, \frac{5}{2}\right), \left(\frac{3\pi}{2}, 1\right)$ Allow $\left(30^\circ, \frac{5}{2}\right), \left(150^\circ, \frac{5}{2}\right), (270^\circ, 1)$ – condone missing degree signs
(c)	Area R_1	
	M1	For stating a correct method to find the area R_1 using their limits from part (b) of $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ applied the correct way. Condone angles in degrees. $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin x + 2) dx - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\cos 2x + 2) dx$ (The $+2 - 2$ may have been simplified).
	M1	For an attempt to integrate their expression for R_1 which must involve $\sin x$ and $\cos 2x$ A minimally acceptable integral of $\sin x \rightarrow -\cos x$ or $\cos 2x \rightarrow \pm \frac{\sin 2x}{2}$ and $2 \rightarrow 2x$ (The $2 \rightarrow 2x$ may not be present if they've simplified first). At least one trig term must be acceptable with $2 \rightarrow 2x$ (if present) correct. $\left[-\cos x + 2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - \left[\frac{\sin 2x}{2} + 2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$ Ignore incorrect, angles in degrees or even absent limits for this mark.

	M1	Substitutes their limits correctly into their integrated expression. Must involve a changed expression. $\left[\left(-\cos \frac{5\pi}{6} + 2 \left(\frac{5\pi}{6} \right) \right) - \left(-\cos \frac{\pi}{6} + 2 \left(\frac{\pi}{6} \right) \right) \right] -$ $\left[\left(\frac{\sin 2 \left(\frac{5\pi}{6} \right)}{2} + 2 \left(\frac{5\pi}{6} \right) \right) - \left(\frac{\sin 2 \left(\frac{\pi}{6} \right)}{2} + 2 \left(\frac{\pi}{6} \right) \right) \right]$ Do not accept a substitution in degrees – it must be in radians. If the work for the first two method marks to find the area of R_1 is fully correct, this mark can be implied by $\frac{3\sqrt{3}}{2}$. If the student does not get $\left[-\cos x + 2x \right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - \left[\frac{\sin 2x}{2} + 2x \right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$ from the first two methods marks, they must show the clear substitution of their limits to be awarded this mark.
	A1	For the correct area.
Area R_2		
	M1	For an attempt to integrate their expression for R_2 which must involve $\int (\cos 2x + 2) dx - \int (\sin x + 2) dx$ or $\int \cos 2x - \sin x dx$ if they have simplified the $+2 - 2$. Ignore incorrect, angles in degrees or even absent limits for this mark. A minimally acceptable integral of $\sin x \rightarrow -\cos x$ or $\cos 2x \rightarrow \pm \frac{\sin 2x}{2}$ and $2 \rightarrow 2x$ (The $2 \rightarrow 2x$ may not be present if they've simplified first). At least one trig term must be acceptable with $2 \rightarrow 2x$ (if present) correct. $\left[\frac{\sin 2x}{2} + 2x \right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} - \left[-\cos x + 2x \right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}$
	M1	For substituting their limits correctly into their integrated expression. Must involve a changed expression. $\left[\left(\frac{\sin 2 \left(\frac{3\pi}{2} \right)}{2} + 2 \left(\frac{3\pi}{2} \right) \right) - \left(\frac{\sin 2 \left(\frac{5\pi}{6} \right)}{2} + 2 \left(\frac{5\pi}{6} \right) \right) \right]$ $- \left[\left(-\cos \left(\frac{3\pi}{2} \right) + 2 \left(\frac{3\pi}{2} \right) \right) - \left(-\cos \left(\frac{5\pi}{6} \right) + 2 \left(\frac{5\pi}{6} \right) \right) \right]$ Do not accept a substitution in degrees – it must be in radians. If the student's work for the first method mark to find the area of R_2 is fully correct, this M mark can be implied by $\frac{3\sqrt{3}}{4}$. If the student does not get

		$\left[\frac{\sin 2x}{2} + 2x \right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} - \left[-\cos x + 2x \right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}$ from the first method mark, they must show the clear substitution of their limits to be awarded this mark.
	A1	For the correct area.
Ratio area of R_1: area of R_2		
	B1ft	For a ratio given in its simplest form $a : b$ where a, b are positive integers, ft their values for the area of R_1 and R_2 Correct ratio is $R_1 : R_2 = 2 : 1$
	SC	Special case: For the use of degrees in part (c). <u>If the area of R_1 and R_2 are both correct when limits in degrees have been substituted</u> (possible, for example, when the candidate has simplified before integrating): Penalise the first substitution in degrees for the area R_1 by awarding M1 M1 M0 A0. For the area R_2 allow M1 M1 A1 if the substitution (in degrees) and the area of R_2 is correct. B1ft for the correct ratio.

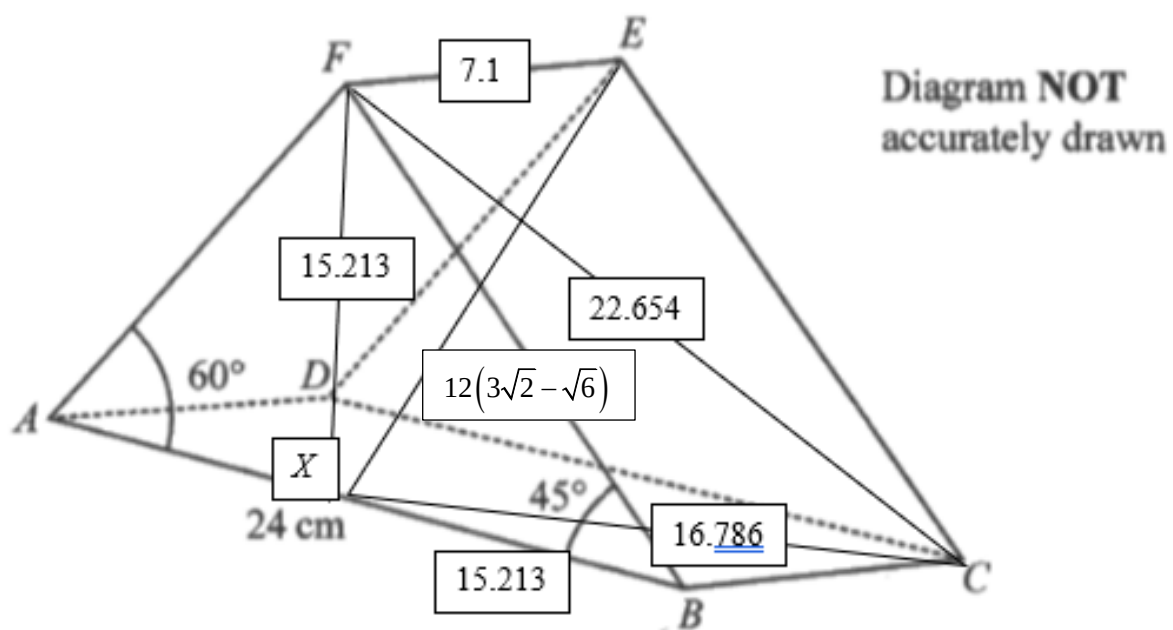
Question number	Scheme	Marks
10 (a)	$f\left(\frac{3}{4}\right) = 16\left(\frac{3}{4}\right)^3 + 11\left(\frac{3}{4}\right) - 15$ $\left(\frac{27}{4} + \frac{33}{4} - 15\right) = 0$	M1 A1cso [2]
(b)(i)	$[\sin 2\theta = \sin(\theta + \theta)] = \sin \theta \cos \theta + \sin \theta \cos \theta$	M1
	$(\sin 2\theta =) 2 \sin \theta \cos \theta$	A1cso [2]
(b)(ii)	$[\cos 2\theta = \cos(\theta + \theta)] = \cos \theta \cos \theta - \sin \theta \sin \theta = \cos^2 \theta - \sin^2 \theta$	M1
	$= \cos^2 \theta - (1 - \cos^2 \theta) = 2 \cos^2 \theta - 1$	M1 A1cso [3]
(c)	$27 \cos \theta (2 \cos^2 \theta - 1) + 19 \sin \theta (2 \sin \theta \cos \theta) - 15 (= 0)$	M1
	$54 \cos^3 \theta - 27 \cos \theta + 38(1 - \cos^2 \theta) \cos \theta - 15 (= 0)$	M1
	$54 \cos^3 \theta - 27 \cos \theta + 38 \cos \theta - 38 \cos^3 \theta - 15 (= 0)$ $(16 \cos^3 \theta + 11 \cos \theta - 15 = 0) \text{ oe}$	A1
	(When $x = \cos \theta$) $16x^3 + 11x - 15 = 0$	A1cso [4]
(d)	$(16x^3 + 11x - 15 = 0 \Rightarrow (4x - 3)(4x^2 + 3x + 5) = 0) \rightarrow 4x^2 + 3x + 5$	M1
	$b^2 - 4ac = 9 - 80 < 0$ (So no real roots)	M1 A1
	(Only solution is) $4x - 3 = 0$ or $4 \cos \theta - 3 = 0$ so $\cos \theta = \frac{3}{4}$	A1cso [4]
Total 15 marks		

Part	Marks	Notes
(a)	M1	For $f\left(\frac{3}{4}\right) = 16\left(\frac{3}{4}\right)^3 + 11\left(\frac{3}{4}\right) - 15$
	A1*cso	For a fully correct solution, with $= 0$ stated, with no errors seen
(b)(i)	M1	For sight of $\sin \theta \cos \theta + \sin \theta \cos \theta$
	A1*cso	For obtaining the given expression, with no errors seen
(b)(ii)	M1	For $\cos \theta \cos \theta - \sin \theta \sin \theta = \cos^2 \theta - \sin^2 \theta$
	M1	For correctly using $\sin^2 \theta = 1 - \cos^2 \theta$
	A1*cso	For obtaining the given expression with no errors seen
(c)	M1	For correctly substituting $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\cos 2\theta = 2 \cos^2 \theta - 1$
	M1	For expanding the first set of brackets (allow one error only in expansion) and correct use of $\sin^2 \theta = 1 - \cos^2 \theta$,
	A1	$54 \cos^3 \theta - 27 \cos \theta + 38 \cos \theta - 38 \cos^3 \theta - 15 (= 0)$ oe all brackets expanded
	A1*cso	For obtaining the given equation
(d)	M1	For any valid, complete method to find the quadratic factor which must be of the form $4x^2 + ax \pm 5$
	M1	For use of the discriminant from their quadratic factor
	A1	For $b^2 - 4ac = 9 - 80 < 0$

Question	Scheme	Marks
9(a)	$\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ$ $= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} *$ ALT 1 $\sin 75^\circ = \sin(120^\circ - 45^\circ) = \sin 120^\circ \cos 45^\circ - \sin 45^\circ \cos 120^\circ$ $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \times \left(-\frac{1}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4} *$ ALT 2 $\sin 75^\circ = \sin(60^\circ + 15^\circ) = \sin 60^\circ \cos 15^\circ + \sin 15^\circ \cos 60^\circ$ $= \frac{\sqrt{3}}{2} \times \left(\frac{\sqrt{6} + \sqrt{2}}{4}\right) + \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) \times \frac{1}{2}$ $= \frac{3\sqrt{2} + \sqrt{6}}{8} + \frac{\sqrt{6} - \sqrt{2}}{8} = \frac{\sqrt{6} + \sqrt{2}}{4}$	M1 M1A1 cso [3] [M1 M1A1] [M1 M1 A1]
(b)	$[\angle AFC = 180^\circ - 60^\circ - 45^\circ = 75^\circ]$ $\frac{24}{\sin 75^\circ} = \frac{BF}{\sin 60^\circ}$ $\Rightarrow \frac{24}{\sin 75^\circ} = \frac{BF}{\sin 60^\circ} \Rightarrow \frac{24}{\frac{\sqrt{6} + \sqrt{2}}{4}} = \frac{BF}{\frac{\sqrt{3}}{2}}, \Rightarrow BF = \frac{48\sqrt{3}}{\sqrt{6} + \sqrt{2}}$ $BF = \frac{48\sqrt{3}}{(\sqrt{6} + \sqrt{2})} \times \frac{(\sqrt{6} - \sqrt{2})}{(\sqrt{6} - \sqrt{2})} = \frac{48\sqrt{3}\sqrt{6} - 48\sqrt{6}}{6 - 2} = 12(3\sqrt{2} - \sqrt{6}) *$	M1, M1A1 M1A1 cso [5]
(c)	Let the vertical point from F to the line AB be X and the corresponding point on CD be Y. The required angle is EXY $FX = 12(3\sqrt{2} - \sqrt{6}) \sin 45^\circ = 36 - 12\sqrt{3} = (15.2153...)$ OR Area $ABC = \frac{1}{2} \times 24 \times FX = \frac{1}{2} \times 24 \times 12(3\sqrt{2} - \sqrt{6}) \sin 45^\circ$ $\Rightarrow FX = 12(3\sqrt{2} - \sqrt{6}) \sin 45^\circ = 36 - 12\sqrt{3} = (15.2153...)$ $EF = \frac{36 - 12\sqrt{3}}{\tan 65^\circ} = 7.09505... \approx 7.1 \text{ (cm)}$	M1 M1A1 [3]
(d)	Identifies the required angle FCX $FC = \sqrt{[12(3\sqrt{2} - \sqrt{6})]^2 + '7.09505'^2} = 22.65735....$ $\angle FCX = \sin^{-1}\left(\frac{15.2153}{'22.6573'}\right) = 42.186...^\circ \approx 42.2^\circ$	B1 M1 M1A1 [4]
Total 15 marks		

Part	Mark	Notes
(a)	M1	For the correct use of the addition formula for sine with $\sin(45^\circ + 30^\circ)$ Accept also $\sin(120^\circ - 45^\circ)$ and $\sin(60^\circ + 15^\circ)$ but not $\sin(90^\circ - 15^\circ)$ or $\sin(180^\circ - 105^\circ)$
	M1	For applying the exact values of these sines and cosines and for evaluating the expression.. This step must be explicitly seen.
	A1 cso	For the correct final answer with no errors seen
(b)	M1	For the correct use of sine rule using the exact value for $\sin 75^\circ$ and $\sin 60^\circ$
	M1	For rearranging to obtain $BF = \frac{k\sqrt{3}}{\sqrt{6} + \sqrt{2}}$ or $\frac{l\sqrt{3}}{2\sqrt{6} + 2\sqrt{2}}$ Where k and l are integers
	A1	For the correct exact BF which need not be simplified. $BF = \frac{48\sqrt{3}}{\sqrt{6} + \sqrt{2}}$ or $\frac{96\sqrt{3}}{2\sqrt{6} + 2\sqrt{2}}$ or even $\frac{12\sqrt{3} \times 4}{\sqrt{6} + \sqrt{2}}$ $\frac{12\sqrt{3}}{\sqrt{6} + \sqrt{2}}$ but not for example, $\frac{\sqrt{6} + \sqrt{2}}{4}$ unless recovery is seen later
	M1	For multiplying numerator and denominator by their conjugate which must of the form e.g $\sqrt{A} \pm \sqrt{B}$ AND simplifying to the required form. We need to see at least one intermediate line of working before the final answer here.
	A1 cso	The answer as shown with no errors or omissions seen. This is a given result.
(c)	M1	For finding the length FX using any appropriate trigonometry.
	M1	For using any appropriate trigonometry to find length EF [or XY]
	A1	For awrt 7.1 (cm)
(d)	B1	For identifying the correct angle required.
	M1	For using Pythagoras theorem to find FC $FC = \sqrt{21.51781^2 + 7.09505^2} = 22.65735.....$ Or For using Pythagoras theorem to find CX $FC = \sqrt{15.213^2 + 7.095^2} = 16.786...$ [Note: $BX = 15.213...$ and $CX = 16.786...$]
	M1	For using any appropriate trigonometry to find angle FCX . For example; $\angle FCX = \tan^{-1}\left(\frac{15.2153}{16.786}\right) = 42.186...^\circ \approx 42.2^\circ$
	A1	For awrt 42.2°

USEFUL SKETCH

**NB:**

$$AC = 25.03 \text{ cm}$$

$$AF = 17.57 \text{ cm}$$

$$CF = 22.65 \text{ cm}$$

$$\text{Angle } FCA = 42.9^\circ$$

Question number	Scheme	Marks
10 (a)(i)	$\cos(A + A) = \cos A \cos A - \sin A \sin A \rightarrow \cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = 1 - \sin^2 A - \sin^2 A \Rightarrow \cos 2A = 1 - 2\sin^2 A$ * $\cos 2A - 1 = -2\sin^2 A$ or $1 - \cos 2A = 2\sin^2 A$ $\sin^2 A = \frac{1 - \cos 2A}{2}$ *	M1 M1 A1*cso [3]
/ALT (a)(i)	Starting from r.h.s of equation: $\cos(A + A) = \cos A \cos A - \sin A \sin A \rightarrow \cos 2A = \cos^2 A - \sin^2 A$ Gives $\frac{1 - \cos 2A}{2} = \frac{1 - (\cos^2 A - \sin^2 A)}{2}$ $= \frac{\sin^2 A + \sin^2 A}{2} = \sin^2 A$ as required.	M1 M1 A1*cso [3]
(ii)	$(\cos 2A = \cos^2 A - \sin^2 A$ or $\cos 2A = 1 - 2\sin^2 A)$ $\cos 2A = 1 - \sin^2 A - \sin^2 A$ or $\cos 2A = 1 - 2(1 - \cos^2 A)$ $\rightarrow \cos 2A = 2\cos^2 A - 1$ $\cos 2A + 1 = 2\cos^2 A$ $\cos^2 A = \frac{\cos 2A + 1}{2}$ *	M1 A1*cso [2]
ALT (ii)	Starting from r.h.s of equation: $\cos 2A = \cos^2 A - \sin^2 A$ Gives $\frac{\cos 2A + 1}{2} = \frac{\cos^2 A - \sin^2 A + 1}{2}$ $= \frac{\cos^2 A + \cos^2 A}{2} = \cos^2 A$ as required.	M1 A1*cso [2]
(b)	$\cos 2x = \sin x$ $1 - 2\sin^2 x = \sin x \rightarrow 0 = 2\sin^2 x + \sin x - 1$ $0 = (2\sin x - 1)(\sin x + 1)$ $\sin x = \frac{1}{2}, (-1) \rightarrow x =$ $\left(\frac{\pi}{6}, \frac{1}{2}\right)$	M1 M1 M1 A1 A1 [5]

(c)	$\pi \int_0^{\frac{\pi}{6}} (\cos^2 2x - \sin^2 x) dx$	B1ft
	$(\pi) \int_0^{\frac{\pi}{6}} \left(\frac{\cos 4x + 1}{2} \right) - \left(\frac{1 - \cos 2x}{2} \right) dx$	M1
	$\left[(\pi) \int_0^{\frac{\pi}{6}} \left(\frac{\cos 4x}{2} + \frac{\cos 2x}{2} \right) dx \right]$	
	$(\pi) \left[\frac{\sin 4x}{8} + \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{6}}$	M1
	$(\pi) \left(\frac{\sin 4\left(\frac{\pi}{6}\right)}{8} + \frac{\sin 2\left(\frac{\pi}{6}\right)}{4} \right)$	M1
	$\frac{3\sqrt{3}}{16} \pi$	A1 [5]
Total 15 marks		

Part	Mark	Additional Guidance
10 (a)(i)	M1	Correct use of the addition formula as shown leading to $\cos 2A = \cos^2 A - \sin^2 A$
	M1	Correct substitution of $1 - \sin^2 A$ to get $\cos 2A = 1 - 2\sin^2 A$
	A1*cso	Fully correct solution with the minimum steps shown, no errors.
ALT (a)(i)	M1	Correct use of the addition formula as shown leading to $\cos 2A = \cos^2 A - \sin^2 A$
	M1	Correct substitution into the right hand side of the equation.
	A1*cso	Rearrangement of resultant expression to reach left hand side of original equation., without errors..
(ii)	M1	Replacement of $\sin^2 A$ with $1 - \cos^2 A$ in an expression from their part (i)
	A1*cso	Rearrangement of a correct equation to attain the given result, minimum steps shown, no errors.
ALT (ii)	M1	Replacement pf $\cos 2A$ with expression from (i)
	A1*cso	Rearrangement of resultant expression to reach left hand side of original equation, without errors.
(b)	M1	Correctly equates the 2 expressions
	M1	Correctly uses the identity for $\cos 2x$ and rearranges to an equation of the form $0 = a \sin^2 x + b \sin x + c$ $a, b, c \neq 0$
	M1	Uses a full method to solve their equation of the form $0 = a \sin^2 x + b \sin x + c$ $a, b, c \neq 0$ to find the inverse sine to arrive at a value for x . See general guidance for the solution of a 3TQ.
	A1	For the correct value of x
	A1	For correct coordinates, allow omission of brackets or $x =$ and $y =$. SC: Allow A1 where candidate works in degrees and gets $x = 30$ degrees
(c)	B1ft	For the integral shown, ft their limits only.
	M1	An attempt to replace $\cos^2 2x$ and $\sin^2 x$ with an identity from part (a). At least one replacement must be fully correct.
	M1	At least one term correctly integrated.
	M1	Substitution of their limits clearly seen into a changed expression. This can be implied by a correct answer. Where the expression is not correct, or their limits are not correct, the substitution must be seen into at least half of their terms.
	A1	For the correct answer. Withold this mark if the candidate works in degrees rather than radians

Question number	Scheme	Marks
11 (a)	$\cos^4 \theta - \sin^4 \theta = (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)$ $= (\cos^2 \theta - \sin^2 \theta) \times 1 = \cos 2\theta^*$ ALT example $\cos^4 \theta - \sin^4 \theta = \cos^4 \theta - \sin^2 \theta \sin^2 \theta$ $= \cos^4 \theta - (1 - \cos^2 \theta)(1 - \cos^2 \theta)$ $= \cos^4 \theta - (1 - 2\cos^2 \theta + \cos^4 \theta)$ $= (1 - 2\cos^2 \theta)$ $= \cos 2\theta$	M1 M1M1A1 cso [4 2nd M1 1st M1 3rd M1 A1
(b)	$8\cos^2\left(2\theta + \frac{\pi}{4}\right) - 3 = 2\left[\cos^4\left(\theta + \frac{\pi}{8}\right) - \sin^4\left(\theta + \frac{\pi}{8}\right)\right]$ $= 2\cos\left(2\theta + \frac{\pi}{4}\right)$ $8\cos^2\left(2\theta + \frac{\pi}{4}\right) - 2\cos\left(2\theta + \frac{\pi}{4}\right) - 3 = 0$ $(4\cos\left(2\theta + \frac{\pi}{4}\right) - 3)(2\cos\left(2\theta + \frac{\pi}{4}\right) + 1) [= 0]$ $\cos\left(2\theta + \frac{\pi}{4}\right) = \frac{3}{4} \quad \cos\left(2\theta + \frac{\pi}{4}\right) = -\frac{1}{2}$ $2\theta + \frac{\pi}{4} = 0.72273... \quad 2\theta + \frac{\pi}{4} = \frac{2\pi}{3}$ $\theta = 2.39, 3.11, 0.65, 1.70 \text{ accept } \theta = \frac{5\pi}{24}, \frac{13\pi}{24}$	M1 A1 M1 A1 dM1 A1A1 [7]
(c)	$\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} (\cos^4 2x - \sin^4 2x - 8\sin 4x) dx = \int_{\frac{\pi}{16}}^{\frac{\pi}{8}} (\cos 4x - 8\sin 4x) dx$ $= \left[\frac{1}{4} \sin 4x + \frac{8}{4} \cos 4x \right]_{\frac{\pi}{16}}^{\frac{\pi}{8}}$ $= \frac{1}{4} \left[\sin\left(4 \times \frac{\pi}{8}\right) - \sin\left(4 \times \frac{\pi}{16}\right) \right] + 2 \left[\cos\left(4 \times \frac{\pi}{8}\right) - \cos\left(4 \times \frac{\pi}{16}\right) \right]$ $= \frac{1}{4} \left[1 - \frac{\sqrt{2}}{2} \right] + 2 \left[0 - \frac{\sqrt{2}}{2} \right] = \frac{1}{4} - \frac{9\sqrt{2}}{8}$	M1 M1 M1 A1 [4]
Total 15 marks		

Part	Mark	Additional Guidance
General Principles for marking part (a)		
- First M mark is for either applying the difference of two squares or multiplying out their expression and collecting terms. - Second M mark is for applying $\cos^2 \theta + \sin^2 \theta = 1$ in any form. This can be implied from correct work. However, do not award for just $\cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$ seen. - Third M mark is for applying any identity for $\cos 2\theta$ - A1 For obtaining the required result with no errors.		
(a)	M1	For factorising as the difference of two squares $(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)$
	M1	For use of $\cos^2 \theta + \sin^2 \theta = 1$
	M1	For use of $\cos^2 \theta - \sin^2 \theta = \cos(2\theta)$
	A1 CSO	Obtain the given result with no errors in the working.
(b)	M1	For using the given result to obtain a quadratic in $\left(2\theta + \frac{\pi}{4}\right)$ Note: another variable may be used in place of $\left(2\theta + \frac{\pi}{4}\right)$ for this mark
	A1	For rearranging to the correct quadratic
	M1	For a valid attempt to solve their 3TQ . See General Guidance for what constitutes an attempt to factorise. If candidates use a calculator [so no working shown] then award this mark only if the 3TQ is correct with the correct solutions.
	A1	For obtaining correct values for $\cos\left(2\theta + \frac{\pi}{4}\right)$ Note: another variable may be used in place of $\left(2\theta + \frac{\pi}{4}\right)$ for this mark
	dM1	For achieving any valid value in radians for their $2\theta + \frac{\pi}{4}$ i.e. $0.7227\dots$ or $\frac{2}{3}\pi$ Accept values out of range for this mark. This is dependent on the previous M mark.
	A1	For two correct solutions in range
	A1	For all four correct solutions in range and no others. Extra solutions within range scores A0 Ignore solutions outside of range. Exact solutions when given must be given as multiples of π .
(c)	M1	For use of the given result to obtain $\int_{\frac{\pi}{16}}^{\frac{\pi}{8}} \cos 4x \, dx$
	M1	For an attempt to integrate $\cos kx \rightarrow l \sin kx$ where $k, l \neq 1$
	M1	For substituting limits into a changed expression
	A1	For the correct answer in the required form.

Question Number	Scheme	Marks
11(a) ALT:	$AC = \sqrt{(16^2 + 16^2)} = 16\sqrt{2}$ $AP^2 + PD^2 = 16^2 \Rightarrow AP = 8\sqrt{2}$ $VP = 8\sqrt{2} \tan 45 = 8\sqrt{2}$ (where P is the centre of the base) *	M1A1 M1A1cso (4)
(b)	$VA^2 = (8\sqrt{2})^2 + (8\sqrt{2})^2 (= 256)$ or $VA = \frac{8\sqrt{2}}{\sin 45^\circ}$ $VA = 16$ cm	M1A1 A1 (3)
(c)	$DX^2 = 16^2 - 8^2$ where X is the foot of the perpendicular from D to VA $DX = 8\sqrt{3}$	M1A1 A1 (3)
(d)	$\tan \theta = \frac{8\sqrt{2}}{8}, \sin \theta = \frac{8\sqrt{2}}{8\sqrt{3}}, \cos \theta = \frac{8}{8\sqrt{3}}$ (or unsimplified if cosine or sine rule used) $\theta = 54.7^\circ$	M1A1 A1 (3)
(e)	$\cos \phi = \frac{(8\sqrt{3})^2 + (8\sqrt{3})^2 - (16\sqrt{2})^2}{2 \times 8\sqrt{3} \times 8\sqrt{3}} \left(= -\frac{1}{3} \right)$ $\phi = 109.5^\circ$	M1A1 A1 (3) [16]
(a) M1 A1 M1 A1cso	Use Pythagoras (with a + sign) to obtain the length of the diagonal of the base. Or use Pythagoras with correct sign to obtain the half diagonal Correct length for the diagonal or half diagonal Use tan in $\triangle APV$, their AP and angle of 45° to obtain the height Correct answer with no errors in the working OR: State $\triangle APV$ is isosceles, or shown the 2 correct angles – can be on a diagram $VP (= AP) = 8\sqrt{2}$ OR use any other complete valid method	M1 A1 M1 A1
(b) M1 A1 A1	Use Pythagoras or trigonometry in $\triangle APV$ or $\triangle AVC$ (or any other complete, valid method) to obtain a numerical expression for VA . Correct numbers in their choice of method. Correct length obtained. $AV = 16$ w/o working scores	M1A1A1

Question Number	Scheme	Marks
(c) M1	Use Pythagoras with a minus sign (seen or implied) or trigonometry in $\triangle ADX$ OR any other complete valid method NB: triangle $\triangle ADV$ is equilateral.	
A1	Correct numbers in their choice of method.	
A1	Correct exact length for the perpendicular.	
(d) M1	Identify the correct triangle needed with the required angle marked (may be on Figure 1). This may be shown explicitly or implied by their work that follows.	
A1	Reach one of $\tan \theta = \frac{8\sqrt{2}}{8}$, $\sin \theta = \frac{8\sqrt{2}}{8\sqrt{3}}$, $\cos \theta = \frac{8}{8\sqrt{3}}$ oe	
A1	Correct answer, must be 1 dp.	
(e) M1	Use cosine rule: $\cos \phi = \frac{DX^2 + XB^2 - BD^2}{2 \times DX \times XB}$ (their values)	
A1	Correct numbers substituted, follow through their previous answers	
A1	Correct answer, must be 1 dp unless already penalised in (d)	
	Any other routes should be marked:	
	M1 Correct, complete method (ie it must be possible to reach a value for the required angle)	
	A1 Correct numbers substituted	
	A1 Correct answer, must be 1 dp unless already penalised in (d)	

Question Number	Scheme	Marks
10(a)	$\cos(A+B) + \cos(A-B) = \cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B$ $= 2 \cos A \cos B$ *	M1 A1 cso (2)
(b)	Let $A+B=P$, $A-B=Q \Rightarrow A=\frac{1}{2}(P+Q)$, $B=\frac{1}{2}(P-Q)$ As $\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$ $\cos P + \cos Q = 2 \cos \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$ *	M1 M1A1cso (3)
ALT	Let $A=\frac{1}{2}(P+Q)$, $B=\frac{1}{2}(P-Q) \Rightarrow A+B=P$, $A-B=Q$ As $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$ $2 \cos \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q) = \cos P + \cos Q$ *	M1 M1A1cso
(c)	$\cos 5\theta + \cos 7\theta = 2 \cos 6\theta \cos \theta = 0$ $\cos 6\theta = 0 \Rightarrow 6\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \Rightarrow \theta = \frac{\pi}{12}, \frac{\pi}{4} \left(\text{or } \frac{3\pi}{12} \right), \frac{5\pi}{12}$ $\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$	M1 A1A1 A1 (4)
(d)	$\cos 8x + 2 \cos 6x + \cos 4x = (\cos 8x + \cos 6x) + (\cos 6x + \cos 4x)$ $= 2 \cos 7x \cos x + 2 \cos 5x \cos x$ $= 2 \cos x (\cos 7x + \cos 5x) = 2 \cos x \times 2 (\cos 6x \cos x) = 4 \cos 6x \cos^2 x$ *	M1 dM1,A1cso (3)
ALT 1	$\cos 8x + 2 \cos 6x + \cos 4x = (\cos 8x + \cos 4x) + 2 \cos 6x$ $= 2 \cos 6x \cos 2x + 2 \cos 6x$ $= 2 \cos 6x (\cos 2x + 1) = 2 \cos 6x (2 \cos^2 x - 1 + 1) = 4 \cos 6x \cos^2 x$ *	M1 dM1A1cso (3)
ALT 2	Working from right to left $4 \cos 6x \cos^2 x = 4 \cos 6x \times \frac{1}{2} (\cos 2x + 1) = 2 \cos 6x \cos 2x + 2 \cos 6x$ $= \cos 8x + \cos 4x + 2 \cos 6x$ *	M1 dM1A1cso (3)

(e) ALT	$\int_0^{\frac{\pi}{3}} \cos 6x \cos^2 x \, dx = \frac{1}{4} \int_0^{\frac{\pi}{3}} (\cos 8x + 2 \cos 6x + \cos 4x) \, dx$ $= \frac{1}{4} \left[\frac{1}{8} \sin 8x + \frac{1}{3} \sin 6x + \frac{1}{4} \sin 4x \right]_0^{\frac{\pi}{3}}$ $= \frac{1}{4} \left[\frac{1}{8} \sin \frac{8\pi}{3} + \frac{1}{3} \sin 2\pi + \frac{1}{4} \sin \frac{4\pi}{3} - 0 \right] = -\frac{\sqrt{3}}{64} \quad (-0.02706... \text{ scores M1A0})$ For the first M mark Award M1 only when the integrand has been changed (by a valid method) to a function which can be integrated $\int_0^{\frac{\pi}{3}} \cos 6x \cos^2 x \, dx = \int_0^{\frac{\pi}{3}} \cos 6x \times \frac{1}{2} (\cos 2x + 1) \, dx$ $= \frac{1}{2} \int_0^{\frac{\pi}{3}} (\cos 6x \times \cos 2x + \cos 6x) \, dx = \frac{1}{2} \int_0^{\frac{\pi}{3}} \frac{1}{2} (\cos 8x + \cos 4x + 2 \cos 6x) \, dx$ Rest as above.	M1 A1 dM1A1 (4) [16] M1
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(a) M1 A1cso (b) M1 M1 A1cso ALT (c) M1 A1 A1 A1 NB (d) M1 dM1 A1cso	Use the standard formulae. This is a “show that” question so these formulae must be written out in full. Both sides of the result must be seen although the working may appear between them as seen in the scheme. Final given result. Do not award if the expansions shown are not in the correct order (this suggests incorrect signs in the formulae). Use $A + B = P$, $A - B = Q$ to obtain A and B in terms of P and Q Substitute in the result from (a) to obtain an identity in P and Q only Correct result reached with no errors seen Working right to left: Notes similar to above Use the result from (b) to show that $(2) \cos 6\theta \cos(\pm\theta) = 0$ Obtain one correct solution of $\cos 6\theta = 0$. Allow if in decimal form but must be radians. Two further correct solutions and no more within the range. If any of the 3 solutions of $\cos 6\theta = 0$ is not exact do not award this mark. State the solution of $\cos(\pm\theta) = 0$. Must be exact unless this is penalised above. If answers are given in degrees deduct 2A marks from any that would otherwise have been given. (Answers in degrees but then changed to radians are acceptable – mark the radian answers.) Ignore extra answers outside the stated range – any within are incorrect. Use the result from (a) or (b) once. Allow if one or both “2”s are missing Use the result from (a) or (b) again. Must include both “2”s this time. Depends on previous M mark. Reach the given result with no errors seen
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ALT 1	
M1	Use the result from (a) or (b). Allow if the “2” is missing.
dM1	Factorise and use the <i>correct</i> double angle formula on $\cos 2x$ Depends on the previous M mark
A1cso	Reach the given result with no errors seen
ALT 2	
M1	Use the <i>correct</i> half angle formula on $\cos^2 x$
dM1A1	M1: Use the result from (a) or (b)
cso	A1: reach the given result with no errors seen
(e)M1	Obtain a function which can be integrated either by using the given result from (d) OR deriving the same result. Allow if $1/4$ is missing. (Integration by Parts – send to review)
A1	Correct integration (must have included the $1/4$) Limits not needed for these 2 marks.
dM1	Correct use of the given limits. All sines are 0 at the lower limit so these need not be shown
A1	Correct final answer which must be exact and stated as a single fraction .

Question	Scheme	Marks
10(a)	$\cos(A - B) = \cos A \cos B + \sin A \sin B$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\cos(A - B) - \cos(A + B) = \sin A \sin B - (-\sin A \sin B) = 2 \sin A \sin B$ *	M1A1cso [2]
(b)	$[A - B = 5\theta, A + B = 9\theta \Rightarrow A = 7\theta, B = 2\theta]$ $\cos 5\theta - \cos 9\theta = 2 \sin 7\theta \sin 2\theta$ *	B1 cso [1]
(c)	$\sqrt{3} \sin 7\theta = 2 \sin 7\theta \sin 2\theta \Rightarrow 0 = 2 \sin 7\theta \sin 2\theta - \sqrt{3} \sin 7\theta$ $0 = \sin 7\theta (2 \sin 2\theta - \sqrt{3}) \Rightarrow \sin 7\theta = 0, 2 \sin 2\theta - \sqrt{3} = 0$ $\sin 7\theta = 0 \Rightarrow 7\theta = 0, \pi, 2\pi \Rightarrow \theta = \frac{\pi}{7}, \frac{2\pi}{7}$ $2 \sin 2\theta - \sqrt{3} = 0 \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} \Rightarrow 2\theta = \frac{\pi}{3}, \frac{2\pi}{3} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{3}$	M1A1 M1 M1A1 M1A1 [7]
(d)	$\tan 2x = \frac{\sin 2x}{\cos 2x} \Rightarrow \tan 2x \cos 2x = \sin 2x$ $\int_0^{\frac{\pi}{7}} 8 \sin 7x \sin 2x \, dx = \left[\int_0^{\frac{\pi}{7}} 4 \times (2 \sin 7x \sin 2x) \, dx \right]$ $\int_0^{\frac{\pi}{7}} 4 (\cos 5x - \cos 9x) \, dx = 4 \left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9} \right]_0^{\frac{\pi}{7}}$ $4 \left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9} \right]_0^{\frac{\pi}{7}} = 4 \left[\left(\frac{\sin 5 \times \frac{\pi}{7}}{5} - \frac{\sin 9 \times \frac{\pi}{7}}{9} \right) - \left(\frac{\sin 0}{5} - \frac{\sin 0}{9} \right) \right] \#$ $= 0.9729... \approx 0.973$	M1 M1 M1M1 M1A1 [6]
Total 16 marks		

Question	Notes	Marks
10(a)	From the formula sheet $\cos(A - B) = \cos A \cos B + \sin A \sin B$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$ Subtracts the two equations to give: $\cos(A - B) - \cos(A + B) = \sin A \sin B - (-\sin A \sin B)$	M1
	For the correct identity as shown with no errors, $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$ *	A1 cso [2]
(b)	For finding the value of A and the value of B $A - B = 5\theta, \quad A + B = 9\theta$ $\Rightarrow A = 7\theta, B = 2\theta$ Or as a minimum: $\cos(7\theta - 2\theta) - \cos(7\theta + 2\theta) = 2 \sin 7\theta \sin 2\theta$ *	B1cso [1]
(c)	Sets $\sqrt{3} \sin 7\theta = 2 \sin 7\theta \sin 2\theta$	M1
	Achieves the correct equation allow the terms in any order. $0 = 2 \sin 7\theta \sin 2\theta - \sqrt{3} \sin 7\theta$	A1
	Factorises their equation $0 = \sin 7\theta (2 \sin 2\theta - \sqrt{3}) \Rightarrow \sin 7\theta = 0, 2 \sin 2\theta - \sqrt{3} = 0$	M1
	For finding at least one correct value for θ using $\sin 7\theta = 0$ $\sin 7\theta = 0 \Rightarrow 7\theta = 0, \pi, 2\pi \Rightarrow \theta = \frac{\pi}{7}, \frac{2\pi}{7}$	M1
	For both correct values $\theta = \frac{\pi}{7}, \frac{2\pi}{7}$	A1
	For finding one correct value of θ using $2 \sin 2\theta - \sqrt{3} = 0$ $2 \sin 2\theta - \sqrt{3} = 0 \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} \Rightarrow 2\theta = \frac{\pi}{3}, \frac{2\pi}{3} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{3}$	M1
	For both correct values $\theta = \frac{\pi}{6}, \frac{\pi}{3}$	A1 [7]
(d)	Uses the identity for $\tan 2x = \frac{\sin 2x}{\cos 2x} \Rightarrow \tan 2x \cos 2x = \sin 2x$	M1
	Substitutes the above into $8 \sin 7x \cos 2x \tan 2x$ to give $\int_0^{\frac{\pi}{7}} 8 \sin 7x \sin 2x \, dx = \left[\int_0^{\frac{\pi}{7}} 4 \times (2 \sin 7x \sin 2x) \, dx \right]$	M1
	Ignore integral sign and limits for this mark.	
	For substituting $\cos 5x - \cos 9x$ for $2 \sin 7x \sin 2x$ to give $\int_0^{\frac{\pi}{7}} 4 (\cos 5x - \cos 9x) \, dx$ Ignore integral sign and limits for this mark.	M1

	Integrates $\int_0^{\frac{\pi}{7}} 4(\cos 5x - \cos 9x) \, dx = 4 \left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9} \right]_0^{\frac{\pi}{7}}$ Ignore limits for this mark. As a minimum they must obtain: $\left(\pm \frac{\sin 5x}{5} \pm \frac{\sin 9x}{9} \right)$ for the integration.	M1
	Substitutes the limits the correct way around $4 \left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9} \right]_0^{\frac{\pi}{7}} = 4 \left[\left(\frac{\sin 5 \times \frac{\pi}{7}}{5} - \frac{\sin 9 \times \frac{\pi}{7}}{9} \right) - \left(\frac{\sin 0}{5} - \frac{\sin 0}{9} \right) \right]$ $= (0.9729...)$	M1
	For the correct value of $\int_0^{\frac{\pi}{7}} 8 \sin 7x \cos 2x \tan 2x \, dx = 0.973$	A1 [6]
Total 16 marks		

Question	Scheme	Marks
11(a)(i)	$\cos 2A = \cos^2 A - \sin^2 A, = \cos^2 A - (1 - \cos^2 A)$ $\cos 2A = 2\cos^2 A - 1$ *	M1, M1 A1 CSO [3]
(ii)	$\sin 2A = \sin A \cos A + \sin A \cos A = 2 \sin A \cos A$ *	B1 CSO [1]
(b)	$\cos 3A = \cos(2A + A)$ $= \cos 2A \cos A - \sin A \sin 2A$ $= (2\cos^2 A - 1)\cos A - 2\sin A \sin A \cos A$ $= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$ $= 2\cos^3 A - \cos A + 2\cos^3 A - 2\cos A$ $= 4\cos^3 A - 3\cos A$ $\Rightarrow \cos^3 A = \frac{\cos 3A + 3\cos A}{4}$ * ALT $\cos^3 A = \frac{\cos 3A + 3\cos A}{4}$ $= \frac{\cos 2A \cos A - \sin 2A \sin A + 3\cos A}{4}$ $= \frac{(2\cos^2 A - 1)\cos A - 2\sin A \sin A \cos A + 3\cos A}{4}$ $= \frac{2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A + 3\cos A}{4}$ $= \frac{2\cos^3 A - \cos A + 2\cos^3 A - 2\cos A + 3\cos A}{4}$ $= \frac{4\cos^3 A}{4} = \cos^3 A$ [LHS = RHS]	M1 M1 M1 A1 Cso [4] [M1 M1 M1 A1]
(c)	$[\cos 3A = 4\cos^3 A - 3\cos A \Rightarrow 2\cos 3A = 8\cos^3 A - 6\cos A]$ $8\cos^3\left(\frac{\theta}{2}\right) - 6\cos\left(\frac{\theta}{2}\right) - 1 = 0 \Rightarrow 2\cos 3\left(\frac{\theta}{2}\right) - 1 = 0$ $\cos 3\left(\frac{\theta}{2}\right) = \frac{1}{2} \Rightarrow 3\left(\frac{\theta}{2}\right) = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \left(\frac{11\pi}{3}\right)$ $\theta = \frac{2\pi}{9}, \frac{10\pi}{9}, \frac{14\pi}{9}$	M1 M1 M1A1 [4]

(d)	$I = \int_0^{\frac{\pi}{6}} (4\cos^3 \theta - \sin 2\theta) d\theta = \int_0^{\frac{\pi}{6}} (\cos 3\theta + 3\cos \theta - \sin 2\theta) d\theta$	M1
	$I = \left[\frac{\sin 3\theta}{3} + 3\sin \theta + \frac{\cos 2\theta}{2} \right]_0^{\frac{\pi}{6}}$	M1
	$= \left[\frac{\sin 3\left(\frac{\pi}{6}\right)}{3} + 3\sin\left(\frac{\pi}{6}\right) + \frac{\cos 2\left(\frac{\pi}{6}\right)}{2} \right] - \left[\frac{\sin 3(0)}{3} + 3\sin(0) + \frac{\cos 2(0)}{2} \right]$	M1
	$= \left[\frac{1}{3} + \frac{3}{2} + \frac{1}{4} \right] - \left[0 - 0 + \frac{1}{2} \right] = \frac{19}{12}$	A1 [4]
Total 16 marks		

Part	Mark	Notes
(a)(i)	M1	For the correct use of the addition formula for $\cos 2A$
	M1	For using the Pythagorean identity correctly
	A1 CSO	For the correct identity with no errors.
(a)(ii)	B1 CSO	$[\sin(A + A)] = \sin 2A = \sin A \cos A + \cos A \sin A = 2 \sin A \cos A$
(b)	Method 1 – Starting with $\cos 3A$	
	M1	For using the correct addition formula for $\cos 3A$ to obtain the correct expansion.
	M1	For applying at least TWO of the following seen anywhere in this part of the question: $\cos 2A$ identity from (a) correctly - the $\sin 2A$ identity from (a) applied correctly - the identity $\sin^2 A = 1 - \cos^2 A$ applied correctly
	M1	For simplifying the expression to be in terms of $\cos^3 A$, $\cos 3A$ and $\cos A$ only
	A1 CSO	For the correct identity with no errors
	ALT – Method 2 – starting with the RHS to show = LHS	
	M1	For using the correct addition formula for $\cos 3A$ to obtain the correct expansion. You can ignore everything else for this and the next mark.
	M1	For applying at least TWO of the following seen anywhere in this part of the question: [again, ignore anything other than the $\cos 3A$ here] $\cos 2A$ identity from (a) correctly - the $\sin 2A$ identity from (a) applied correctly - the identity $\sin^2 A = 1 - \cos^2 A$ applied correctly
	M1	Collects up like terms and simplifies to obtain $\frac{4\cos^3 A}{4}$
	A1	For showing LHS = RHS with no errors

(c)	M1	Substitutes $2 \cos 3\left(\frac{\theta}{2}\right)$ in place of $8 \cos^3\left(\frac{\theta}{2}\right) - 6 \cos\left(\frac{\theta}{2}\right)$ correctly to obtain at least $k \cos 3\left(\frac{\theta}{2}\right) - 1 = 0$ where k is a constant
	M1	Rearranges to obtain $\cos 3\left(\frac{\theta}{2}\right) = \frac{1}{k}$ and takes the inverse cosine to find at least any one correct value for $3\left(\frac{\theta}{2}\right)$
	M1	Finds at least one correct value for θ
	A1	Finds all three values with no extra values within range. Ignore any values given outside of the range.
(d)	M1	For a correct substitution as shown. $4 \cos^3 \theta = \cos 3\theta + 3 \cos \theta$
	M1	For an acceptable attempt to integrate either $\cos 3\theta \rightarrow \pm \frac{\sin 3\theta}{3}$ or $-\sin 2\theta \rightarrow \pm \frac{\cos 2\theta}{2}$
	M1	For substituting both given values the correct way around. This mark can be awarded for a complete substitution SEEN into any changed expression. Award this mark for correct integration and a value of $\frac{19}{12}$ seen without explicit substitution.
	A1	For $\frac{19}{12}$

Question	Scheme	Marks
10(a)(i) (ii)	$\sin(A + A) = \sin A \cos A + \cos A \sin A = 2 \sin A \cos A^*$ $\cos(A + A) = \cos A \cos A - \sin A \sin A$ $\Rightarrow \cos 2A = \cos^2 A - \sin^2 A$ $= \cos^2 A - (1 - \cos^2 A)$ $= 2 \cos^2 A - 1^*$	B1 CSO M1 A1*CSO [3]
(b)	$\left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = \frac{2 \frac{\sin \theta}{\cos \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}$ $= \frac{2 \frac{\sin \theta}{\cos \theta}}{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}} \quad \text{or} \quad \frac{2 \frac{\sin \theta}{\cos \theta}}{1 + \frac{1}{\cos^2 \theta} - \frac{\cos^2 \theta}{\cos^2 \theta}} \quad \text{or} \quad \frac{2 \frac{\sin \theta}{\cos \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} \times \frac{\cos^2 \theta}{\cos^2 \theta} \quad \text{or} \quad \frac{2 \sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta}$ $= \frac{2 \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos^2 \theta}} \left(= 2 \frac{\sin \theta}{\cos \theta} \times \cos^2 \theta \right) \quad \text{or} \quad \left(\frac{2 \sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta} \right) = 2 \sin \theta \cos \theta$ $= \sin 2\theta^*$	M1 dM1 ddM1 A1*CSO [4]
ALT	$\left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = \frac{2 \frac{\sin \theta}{\cos \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}$ $= \frac{2 \frac{\sin \theta}{\cos \theta}}{1 + \tan^2 \theta}$ $\cos^2 \theta + \sin^2 \theta = 1 \Rightarrow \tan^2 \theta + 1 = \frac{1}{\cos^2 \theta} \Rightarrow \frac{2 \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos^2 \theta}} \left(= 2 \frac{\sin \theta}{\cos \theta} \times \cos^2 \theta \right) = 2 \sin \theta \cos \theta$ $= \sin 2\theta^*$	M1 dM1 ddM1 A1*CSO [4]
(c)	$\frac{5 \tan \left(x + \frac{\pi}{6} \right)}{1 + \tan^2 \left(x + \frac{\pi}{6} \right)} = \left(1 - 2 \cos^2 \left(x + \frac{\pi}{6} \right) \right) \Rightarrow \frac{5}{2} \sin \left(2 \left(x + \frac{\pi}{6} \right) \right) = \left(1 - 2 \cos^2 \left(x + \frac{\pi}{6} \right) \right)$ $1 - 2 \cos^2 \left(x + \frac{\pi}{6} \right) = -\cos \left(2x + \frac{\pi}{3} \right)$ $\left(\frac{5}{2} \sin \left(2x + \frac{\pi}{3} \right) = -\cos \left(2x + \frac{\pi}{3} \right) \Rightarrow \tan \left(2x + \frac{\pi}{3} \right) = -\frac{2}{5} \right.$ $\left(\Rightarrow 2x + \frac{\pi}{3} = \right) -0.38050, \quad 2.76108$ $(\Rightarrow x =) -0.714, \quad 0.857$	M1 M1 M1 M1 A1A1 [6]

(d)	$I = \int_0^{\pi} (2 \sin 2\theta - \cos 5\theta + 2) \{d\theta\}$	M1
	$= \left[\frac{-2 \cos 2\theta}{2} - \frac{\sin 5\theta}{5} + 2\theta \right]_0^{\pi}$	M1
	$= \left(-\cos 2\left(\frac{\pi}{2}\right) - \frac{\sin 5\left(\frac{\pi}{2}\right)}{5} + 2 \times \left(\frac{\pi}{2}\right) \right) - \left(-\cos 2(0) - \frac{\sin 5(0)}{5} + 2 \times 0 \right)$	M1
	$\left(= \left(1 - \frac{1}{5} + \pi \right) - (-1 - 0 + 0) \right) = \frac{9}{5} + \pi$	A1 [4]
Total 17 marks		

Part	Mark	Notes
(a)	B1 cso	For $\sin(A + A)$ or $\sin 2A = \sin A \cos A + \cos A \sin A = 2 \sin A \cos A$ No steps missing or incorrect work. Condone $\sin A + A$ with brackets missing if all other work correct.
	M1	For $\cos A \cos A - \sin A \sin A \Rightarrow \cos^2 A - \sin^2 A$
	A1* cso	For $\cos(A + A)$ or $\cos 2A = \cos A \cos A - \sin A \sin A \Rightarrow \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A)$ $= 2 \cos^2 A - 1$ No steps missing, no incorrect work. Condone $\cos A + A$ with brackets missing if all other work correct.
Students who miss 'A' in their working, (eg coscos – sinsin) may be awarded M mark, but not A or B marks.		
(b)	M1	Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$ in the given identity. Allow the 2 to be missing for this mark but not the final A mark.
	dM1	For $1 + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}$ or multiplying by $\frac{\cos^2 \theta}{\cos^2 \theta}$ to give $\left(\frac{2 \frac{\sin \theta}{\cos \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} \right) = \frac{2 \sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta}$ The $\times \frac{\cos^2 \theta}{\cos^2 \theta}$ does not need to be explicitly written. Dependent on previous method mark.
	ddM1	For minimum steps (and dependent on previous method marks): $\frac{2 \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos^2 \theta}}$ or $\frac{2 \sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta} = 2 \sin \theta \cos \theta$
	A1*cso	Achieves the required result – minimum steps as shown, no errors. Steps in brackets do not need to be shown.
	ALT	
ALT	M1	Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$ in the given identity.
	dM1	For replacing $1 + \frac{\sin^2 \theta}{\cos^2 \theta}$ with $1 + \tan^2 \theta$ Dependent on previous method mark.
	ddM1	For replacing $\tan^2 \theta + 1 = \frac{1}{\cos^2 \theta} \Rightarrow \frac{2 \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos^2 \theta}} = 2 \sin \theta \cos \theta$ Dependent on both previous method marks.
	A1*cso	Achieves the required result – minimum steps as shown, no errors. Steps in brackets do not need to be shown.
Students who miss θ in their working, may be awarded M marks, but not the A mark.		

Allow work throughout where candidates have let, for example $\theta = x + \frac{\pi}{6}$ and worked in θ and 2θ throughout in (c)

(c)	M1	Rearranges the given equation into $\frac{5 \tan\left(x + \frac{\pi}{6}\right)}{1 + \tan^2\left(x + \frac{\pi}{6}\right)} = \left(1 - 2 \cos^2\left(x + \frac{\pi}{6}\right)\right)$ and changes the left hand side into $P \sin\left(2\left(x + \frac{\pi}{6}\right)\right)$ where P is any constant.
	M1	Correctly states or uses $1 - 2 \cos^2\left(x + \frac{\pi}{6}\right) = -\cos\left(2x + \frac{\pi}{3}\right)$ (condone being replaced with $-\cos 2A$)
	M1	For $\tan\left(2x + \frac{\pi}{3}\right) = Q$ where Q is any constant.
	M1	For -0.38050 or 2.76108 This may be awarded for either value being seen, ignore any extra values seen. Accept any values with at least 3 significant figures.
	A1	For $(x =) -0.714$ or $(x =) 0.857$ This may be awarded for either value being seen, ignore any extra values seen. Accept any value with more than 3 significant rounding to those given.
	A1	For $(x =) -0.714$ and $(x =) 0.857$ Ignore any extra values out of range . Extra values in range will mean A0. Accept any value with more than 3 significant rounding to those given.

Students must use θ or $x + \frac{\pi}{6}$ and 2θ or $2\left(x + \frac{\pi}{6}\right)$

(d)	M1	For changing the integral into $(I =) \int_0^{\frac{\pi}{2}} (2 \sin 2\theta - \cos 5\theta + 2) \{d\theta\}$ $d\theta$ may be omitted. If the integral sign and limits are not present, these can be implied by a clear attempt in later work to integrate all the terms and a substitution of limits. Students must use 2θ and 5θ in their working.
	M1	For a minimum of 2 out of the 3 terms shown above correctly integrated. Students must use 2θ and 5θ in their working.
	M1	For substituting in the given limits the correct way around into their changed function with at least two terms, there must be at least one explicit substitution of each limit. If the candidate has a constant term as part of the function, they can omit this from this step (as the terms cancel out anyway). This mark may be implied by a correct answer. If the candidate is working with an incorrect function and substitution of 0 clearly gives 0, this substitution does not need to be shown.
	A1	For $\frac{9}{5} + \pi$ The approximate decimal value $4.941\dots$ will usually indicate correct working (as a guide), but a substitution of limits will be needed to get the 3 rd method mark.

In this question, students are directed to use calculus, so must show a correct integration. The penultimate method mark may be implied from a fully correct answer, but if the answer is not correct, we must see substitution of limits as stated.

