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EDEXCEL A LEVEL MATHEMATICS: QUESTIONS BY TOPIC

Proof (1)

A Level Mathematics Topic 1

27 questions · 171 marks · Mark schemes included

Pearson Edexcel A Level Mathematics (9MA0), June 2018 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

27 questions · 171 marks · 4 sub-topics



SUB-TOPIC 1.1

PROOF BY DEDUCTION

Sub-topic 1.1 · 18 questions · 127 marks · lowest tariff first



(4)



Question 14 continued

(Total for Question 14 is 4 marks)



12.

$$y = \sin x$$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \rightarrow 0$, $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$

(5)



(Total for Question 12 is 5 marks)



Question 9 continued

(Total for Question 9 is 5 marks)



(Total for Question 9 is 6 marks)







Question 6 continued

(Total for Question 6 is 6 marks)







Question 10 continued

(Total for Question 10 is 6 marks)



Question 13 continued

Question 13 continued

Lined area for writing the answer to Question 13 continued.

15. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81} \quad (4)$$

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

Question 15 continued

Lined area for writing the answer to Question 15 continued.





(Total for Question 15 is 7 marks)



8.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1} \equiv 2 \tan \theta \sec \theta \quad \theta \neq (90n)^\circ, n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $0 < x < 90^\circ$, the equation

$$\frac{1}{\operatorname{cosec} 2x - 1} + \frac{1}{\operatorname{cosec} 2x + 1} = \cot 2x \sec 2x$$

Give each answer, in degrees, to one decimal place.

(4)













Question 11 continued

(Total for Question 11 is 7 marks)







Question 12 continued

(Total for Question 12 is 7 marks)



10.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan \theta \quad (4)$$

(b) Hence solve, for $0 < x < 180^\circ$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3 \sin 2x$$

giving your answers to one decimal place where appropriate. (4)



Question 10 continued

Lined area for writing the answer to Question 10 continued.



Question 15 continued

Lined area for writing the answer to Question 15.



(continued)





Question 12 continued

Lined area for writing the answer to Question 12 continued.

Question 12 continued

(Total for Question 12 is 9 marks)



15.

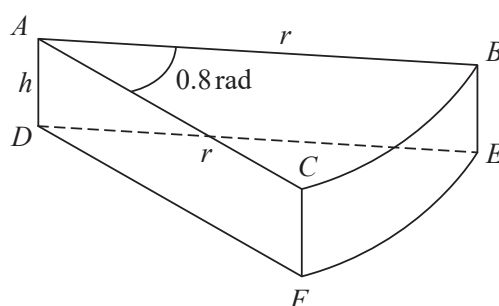


Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD , CF and BE are perpendicular to faces ABC and DEF
- edges AD , CF and BE have length h cm

Given that the volume of the toy is 240 cm^3

(a) show that the surface area of the toy, $S \text{ cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

Question 15 continued

Lined area for writing the answer to Question 15.





Question 15 continued

(Total for Question 15 is 10 marks)



Question 15 continued







15.

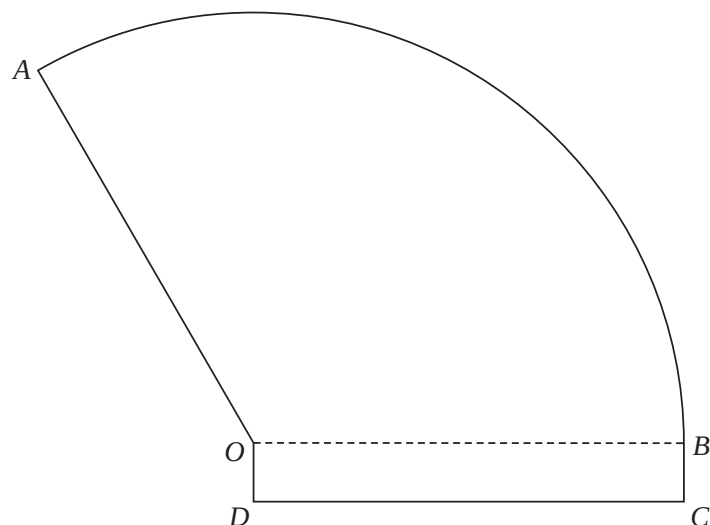


Figure 4

Figure 4 shows the plan view for the design of a stage.

The shape of this design consists of a sector of a circle AOB joined to a rectangle $OBCD$.

Given that

- the radius of the sector is r metres and angle AOB is θ radians
- the length and width of the rectangle are r metres and $\frac{1}{10}r$ metres respectively
- the total area of the stage is 240 m^2

(a) show that the perimeter of the stage, P metres, is given by

$$P = 2r + \frac{480}{r}$$

You must make your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which P has a stationary value.

(3)

(c) Prove, by further differentiation, that this value of r gives the minimum perimeter of the stage.

(2)

Question 15 continued







SUB-TOPIC 1.2

PROOF BY EXHAUSTION

Sub-topic 1.2 · 3 questions · 15 marks · lowest tariff first



16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)



10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) “Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$.”

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(Total for Question 10 is 6 marks)



8. A student was asked to prove that the square of any number can be expressed in the form $5n$ or $5n \pm 1$ where $n \in \mathbb{N}$

The start of the student's proof is shown in the box below.

Let $m = 5k$

Consider $m^2 = (5k)^2 = 25k^2 = 5 \times 5k^2 = 5n$ where $n = 5k^2$ ✓

Let $m = 5k + 1$

Consider $m^2 = (5k + 1)^2$

$$= 25k^2 + 10k + 1 = 5(5k^2 + 2k) + 1 = 5n + 1 \quad \text{where } n = 5k^2 + 2k \quad \checkmark$$

Let $m = 5k + 2$

Consider $m^2 = (5k + 2)^2$

$$= 25k^2 + 10k + 4 = 5(5k^2 + 2k + 1) - 1 = 5n - 1 \quad \text{where } n = 5k^2 + 2k + 1 \quad \checkmark$$

- (a) Identify and correct an algebraic error in the box above.

(1)

- (b) Show the calculations and statements that are required to complete the proof.

(4)

Question 8 continued

(Total for Question 8 is 5 marks)



SUB-TOPIC 1.3

DISPROOF BY COUNTER-EXAMPLE

Sub-topic 1.3 · 1 question · 5 marks · lowest tariff first



3. (a) “If m and n are irrational numbers, where $m \neq n$, then mn is also irrational.”

Disprove this statement by means of a counter example.

(2)

- (b) (i) Sketch the graph of $y = |x| + 3$

- (ii) Explain why $|x| + 3 \geq |x + 3|$ for all real values of x .

(3)



(Total for Question 3 is 5 marks)

SUB-TOPIC 1.4

PROOF BY CONTRADICTION

Sub-topic 1.4 · 5 questions · 24 marks · lowest tariff first







16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)



Question 16 continued

Lined area for writing the answer to Question 16 continued.







(Total for Question 7 is 5 marks)







Question 15 continued

Lined area for writing the answer to Question 15 continued.









MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
14	When n is even: $(2k+1)^3 - (2k)^3 = 8k^3 + 12k^2 + 6k + 1 - 8k^3 = 6k(2k+1) + 1$ $\Rightarrow \text{which is odd}$	M1	3.1a
	or		
	When n is odd: $(2k+2)^3 - (2k+1)^3 = 8(k^3 + 3k^2 + 3k + 1) - (8k^3 + 12k^2 + 6k + 1) = 6k(2k+3) + 7$ $\Rightarrow \text{which is odd}$	A1	2.2a
	When n is even: $(2k+1)^3 - (2k)^3 = 8k^3 + 12k^2 + 6k + 1 - 8k^3 = 6k(2k+1) + 1$ $\Rightarrow \text{which is odd}$	dM1	2.1
	and When n is odd: $(2k+2)^3 - (2k+1)^3 = 8(k^3 + 3k^2 + 3k + 1) - (8k^3 + 12k^2 + 6k + 1) = 6k(2k+3) + 7$ $\Rightarrow \text{which is odd}$ Hence odd for all $n (\in \mathbb{N})$ *	A1*	2.4
(4 marks)			
Notes			
General guidance It is likely that you will see a mixture of methods and approaches within some solutions. Mark the approach which scores the highest number of marks. There should be no errors in the algebra but allow e.g. invisible brackets to be “recovered”. Withhold the final mark if n is used instead of k or reference to $n \in \mathbb{R}$ but $n \in \mathbb{Z}^+$ is acceptable. Main scheme algebraic method using e.g. $n = 2k$ and $n = 2k \pm 1$ You will need to look at both cases and mark the one which is fully correct first. Allow a different variable to k and may be different letters for odd and even M1: For the key step attempting to find $(n+1)^3 - n^3$ when $n = 2k$ or $n = 2k \pm 1$ and attempting to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n = 2k + 2$ or $2n \pm 5$) Condone arithmetical slips and condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$ A1: Complete argument for $n = 2k$ or $n = 2k + 1$ (or e.g. $n = 2k - 1$) showing the result is odd. Requires: - Correct simplified quadratic expression e.g. $12k^2 + 6k + 1$ (when $n = 2k$), $12k^2 + 18k + 7$ (when $n = 2k + 1$), $12k^2 - 6k + 1$ (when $n = 2k - 1$) (may be factorised) - A reason why the expression is odd e.g. $2k(6k+3)+1$ or may use a divisibility argument e.g. $\frac{12k^2 + 6k + 1}{2} = 6k^2 + 3k + \frac{1}{2}$ - Concludes “odd” o.e. (may be within their final conclusion) There should be no errors in the algebra but allow e.g. invisible brackets if they are “recovered” Condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$ dM1: Attempts to find $(n+1)^3 - n^3$ when $n = 2k$ and $n = 2k \pm 1$ and attempts to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n = 2k + 2$, $2n \pm 5$) Condone arithmetical slips and condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$			

A1*: Complete argument for **both** $n = 2k$ **and** $n = 2k + 1$ (or e.g. $n = 2k - 1$) showing the result is odd for all $n (\in \mathbb{N})$

Requires for both cases:

- Correct simplified expressions for both odd and even (which may be factorised)
- A reason why both of the expressions are odd
- Minimal conclusion (may be within their final conclusion)

An overall conclusion is also required. “Hence odd for all $n (\in \mathbb{N})$ ” Accept “hence proven”, “statement proved”, “QED”

The conclusion for when $n = 2k$ and $n = 2k + 1$ may be within the final conclusion rather than separate which is acceptable e.g. “when $n = 2k$ and when $n = 2k + 1$ the expression is odd, hence proven” (following correct simplified expressions and reasons)

	$(n+1)^3$	n^3	$(n+1)^3 - n^3$
$n = 2k - 1$	$8k^3$	$8k^3 - 12k^2 + 6k - 1$	$12k^2 - 6k + 1$
$n = 2k$	$8k^3 + 12k^2 + 6k + 1$	$8k^3$	$12k^2 + 6k + 1$
$n = 2k + 1$	$8k^3 + 24k^2 + 24k + 8$	$8k^3 + 12k^2 + 6k + 1$	$12k^2 + 18k + 7$

Alternative methods:

Algebraic with logic example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic. Condone arithmetical slips.

A1: Correct quadratic expression $3n^2 + 3n + 1$

dM1: Attempts to factorise their quadratic such that $n^2 + n \rightarrow n(n+1)$ within their expression e.g. $3n(n+1) + 1$

A1*: Explains that e.g. $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even = even so $3n(n+1) + 1$ is odd hence odd for all $n (\in \mathbb{N})$

Proof by contradiction example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic.

A1: Correct quadratic expression $3n^2 + 3n + 1$

dM1: Sets $3n^2 + 3n + 1 = 2k$ (for some integer k) $\Rightarrow 3n(n+1) = 2k - 1$

A1*: Explains that $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even = even but $2k - 1$ is odd hence we have a contradiction so $(n+1)^3 - n^3$ is odd (for all $n (\in \mathbb{N})$). There must have been a correct opening statement setting up the contradiction e.g. “assume that there exists a value for n for which $(n+1)^3 - n^3$ is even”



Solutions via just logic (no algebraic manipulation)

e.g.

If n is odd, then $(n+1)^3 - n^3$ is $\text{even}^3 - \text{odd}^3 = \text{even} - \text{odd} = \text{odd}$ If n is even, then $(n+1)^3 - n^3$ is $\text{odd}^3 - \text{even}^3 = \text{odd} - \text{even} = \text{odd}$

Both cases must be considered to score any marks and scores SC 1010 if fully correct

Further Maths method (proof by induction)M1: Assumes true for $n = k$, substitutes $n = k + 1$ into $(n+1)^3 - n^3$, multiplies out the brackets and attempts to simplify to a three term quadratic e.g. $3k^2 + 9k + 7$ Condone arithmetical slipsA1: $(f(k+1) = 3k^2 + 3k + 1 + 6(k+1) = (k+1)^3 - k^3 + 6(k+1) = f(k) + 6(k+1)$
which is odd + even = odddM1: Attempts to substitute $n = 1 \Rightarrow (1+1)^3 - 1^3 = 7$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that

- it is true when $n = 1$
- if it is true for $n = k$ then it is true for $n = k + 1$
- therefore it is true for all $n (\in \mathbb{N})$



Question	Scheme	Marks	AOs
12	$\frac{\sin(x+h) - \sin x}{h}$	B1	2.1
	$\frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$	M1	1.1b
		A1	1.1b
	$(\text{As } h \rightarrow 0), \sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \rightarrow 0 \times \sin x + 1 \times \cos x$	dM1	2.1
	$\text{so } \frac{dy}{dx} = \cos x \quad *$	A1*	2.5

(5 marks)

Notes

Throughout the question allow the use of $h = \delta x$ if used consistently

There is no requirement to see "gradient of chord" written down.

B1: Gives the correct fraction such as $\frac{\sin(x+h) - \sin x}{x+h-x}$ or $\frac{\sin x - \sin(x+h)}{-h}$ or $\frac{\sin(x+h) - \sin(x-h)}{2h}$ or $\frac{\sin(x-h) - \sin x}{x-h-x}$. Condone invisible brackets. May be implied by $\frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$

M1: Uses the compound angle formula for $\sin(x \pm h)$ to give $\sin x \cos h \pm \cos x \sin h$

A1: Achieves $\frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$ or equivalent (may be implied by further work).
Allow invisible brackets to be recovered.

dM1: **It is dependent on both the B and the M marks being awarded.**

Complete attempt to apply the given limits to the gradient of their chord. They must isolate

$\left(\frac{\cos h - 1}{h} \right)$ and replace with 0 and isolate $\left(\frac{\sin h}{h} \right)$ and replace with 1.

e.g. $\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) = \sin x \times 0 + \cos x \times 1$

Accept as a minimum $\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) = \cos x$ (implying the application of the limits)

If they do not fully show $\left(\frac{\cos h - 1}{h} \right)$ and $\left(\frac{\sin h}{h} \right)$ being isolated but proceed from

e.g. $\frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$ to $0 \times \sin x + \cos x$ (or e.g. $0 + \cos x$) then this can be implied and

score dM1

$\frac{\sin x(\cos h - 1) + \cos x \sin h}{h} = \cos x$ is dM0

Condone if limit notation remains within their expression after the limits have been applied.

e.g. $\lim_{h \rightarrow 0} (\sin x \times 0 + \cos x \times 1)$

Alternatively, condone use of the small angle approximations such that

$$\frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \rightarrow \frac{-\frac{h^2}{2} \sin x + h \cos x}{h} = -\frac{h}{2} \sin x + \cos x$$
 and replaces $\frac{h}{2}$ with 0



A1*: Uses correct mathematical language of limiting arguments to show that $\frac{dy}{dx} = \cos x$ with no errors seen. (cso)

We need to see $h \rightarrow 0$ at some point in their solution and linking $\frac{dy}{dx}$ with $\cos x$ e.g.

- $\frac{dy}{dx} = \dots = \lim_{h \rightarrow 0} \left(\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right) = \cos x$
- $\frac{dy}{dx} = \dots = \lim_{h \rightarrow 0} \left(-\frac{h}{2} \sin x + \cos x \right) = 0 \times \sin x + \cos x = \cos x$ (using small angle approximations)
- $\frac{dy}{dx} = \dots = \frac{\sin x (\cos h - 1) + \cos x \sin h}{h} = \sin x \times 0 + 1 \times \cos x = \cos x$ as $h \rightarrow 0$

Condone $f'(x)$ or y' in place of $\frac{dy}{dx}$

Give final A0 for no evidence of limiting arguments:

e.g. when $h = 0$ $\frac{dy}{dx} = \dots = \sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) = \sin x \times 0 + \cos x \times 1 = \cos x$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h} = 1$ and $\frac{\cos h - 1}{h} = 0$ and attempting to apply these (without seeing e.g. $h \rightarrow 0$ at some point in their solution)

If they work in another variable (e.g. θ) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin x (\cos h - 1)$ e.g. $\frac{\sin x \cos h - 1 + \cos x \sin h}{h}$ but accept terms written as e.g. $\sin x \frac{\cos h - 1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.

Question	Scheme	Marks	AOs
9	$\frac{d}{d\theta}(\cos \theta) = -\sin \theta$; as $h \rightarrow 0$, $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$		
	$\frac{\cos(\theta + h) - \cos \theta}{h}$	B1	2.1
	$= \frac{\cos \theta \cos h - \sin \theta \sin h - \cos \theta}{h}$	M1	1.1b
		A1	1.1b
	$= -\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h}\right) \cos \theta$		
	As $h \rightarrow 0$, $-\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h}\right) \cos \theta \rightarrow -1 \sin \theta + 0 \cos \theta$	dM1	2.1
	so $\frac{d}{d\theta}(\cos \theta) = -\sin \theta$ *	A1*	2.5
		(5)	
(5 marks)			
Notes for Question 9			
B1:	Gives the correct fraction such as $\frac{\cos(\theta + h) - \cos \theta}{h}$ or $\frac{\cos(\theta + \delta\theta) - \cos \theta}{\delta\theta}$ Allow $\frac{\cos(\theta + h) - \cos \theta}{(\theta + h) - \theta}$ o.e. Note: $\cos(\theta + h)$ or $\cos(\theta + \delta\theta)$ may be expanded		
M1:	Uses the compound angle formula for $\cos(\theta + h)$ to give $\cos \theta \cos h \pm \sin \theta \sin h$		
A1:	Achieves $\frac{\cos \theta \cos h - \sin \theta \sin h - \cos \theta}{h}$ or equivalent		
dM1:	dependent on both the B and M marks being awarded Complete attempt to apply the given limits to the gradient of their chord		
Note:	They must isolate $\frac{\sin h}{h}$ and $\left(\frac{\cos h - 1}{h}\right)$, and replace $\frac{\sin h}{h}$ with 1 and replace $\left(\frac{\cos h - 1}{h}\right)$ with 0		
A1*:	cso. Uses correct mathematical language of limiting arguments to prove $\frac{d}{d\theta}(\cos \theta) = -\sin \theta$		
Note:	Acceptable responses for the final A mark include: $\frac{d}{d\theta}(\cos \theta) = \lim_{h \rightarrow 0} \left(-\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h}\right) \cos \theta \right) = -1 \sin \theta + 0 \cos \theta = -\sin \theta$ - Gradient of chord $= -\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h}\right) \cos \theta$. As $h \rightarrow 0$, gradient of chord tends to the gradient of the curve, so derivative is $-\sin \theta$ - Gradient of chord $= -\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h}\right) \cos \theta$. As $h \rightarrow 0$, gradient of curve is $-\sin \theta$		
Note:	Give final A0 for the following example which shows no limiting arguments : when $h = 0$, $\frac{d}{d\theta}(\cos \theta) = -\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h}\right) \cos \theta = -1 \sin \theta + 0 \cos \theta = -\sin \theta$		
Note:	Do not allow the final A1 for stating $\frac{\sin h}{h} = 1$ or $\left(\frac{\cos h - 1}{h}\right) = 0$ and attempting to apply these		
Note:	In this question $\delta\theta$ may be used in place of h		
Note:	Condone $f'(\theta)$ where $f(\theta) = \cos \theta$ or $\frac{dy}{d\theta}$ where $y = \cos \theta$ used in place of $\frac{d}{d\theta}(\cos \theta)$		

Notes for Question 9 Continued	
Note:	Condone x used in place of θ if this is done consistently
Note:	Give final A0 for $\frac{d}{d\theta}(\cos x) = \lim_{h \rightarrow 0} \left(-\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h} \right) \cos \theta \right) = -1 \sin \theta + 0 \cos \theta = -\sin \theta$ $\frac{d}{d\theta} = \dots$ - Defining $f(x) = \cos \theta$ and applying $f'(x) = \dots$ $\frac{d}{dx}(\cos \theta)$
Note:	Give final A1 for a correct limiting argument in x, followed by $\frac{d}{d\theta}(\cos \theta) = -\sin \theta$ e.g. $\frac{d}{d\theta}(\cos x) = \lim_{h \rightarrow 0} \left(-\frac{\sin h}{h} \sin x + \left(\frac{\cos h - 1}{h} \right) \cos x \right) = -1 \sin x + 0 \cos x = -\sin x$ $\Rightarrow \frac{d}{d\theta}(\cos \theta) = -\sin \theta$
Note:	Applying $h \rightarrow 0$, $\sin h \rightarrow h$, $\cos h \rightarrow 1$ to give e.g. $\lim_{h \rightarrow 0} \left(\frac{\cos \theta \cos h - \sin \theta \sin h - \cos \theta}{h} \right) = \left(\frac{\cos \theta(1) - \sin \theta(h) - \cos \theta}{h} \right) = \frac{-\sin \theta(h)}{h} = -\sin \theta$ is final M0 A0 for incorrect application of limits
Note:	$\lim_{h \rightarrow 0} \left(\frac{\cos \theta \cos h - \sin \theta \sin h - \cos \theta}{h} \right) = \lim_{h \rightarrow 0} \left(-\frac{\sin h}{h} \sin \theta + \left(\frac{\cos h - 1}{h} \right) \cos \theta \right)$ $= \lim_{h \rightarrow 0} (-1) \sin \theta + 0 \cos \theta = -\sin \theta. \text{ So for } \lim_{h \rightarrow 0} \text{ not removing } \lim_{h \rightarrow 0}$ when the limit was taken is final A0
Note:	Alternative Method: Considers $\frac{\cos(\theta + h) - \cos(\theta - h)}{(\theta + h) - (\theta - h)}$ which simplifies to $\frac{-2 \sin \theta \sin h}{2h}$

Question	Scheme	Marks	AOs
9(a)	$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \frac{3^{2(7-2k)}}{3^{4k-5}} = \frac{3^{2(k-1)}}{3^{2(7-2k)}}$	M1	3.1a
	$(3^{2(7-2k)})^2 = 3^{4k-5} \times 3^{2(k-1)}$ $\Rightarrow 28-8k = 6k-7 \Rightarrow k = \dots$	dM1	1.1b
	$k = \frac{5}{2}^*$	A1*	2.1
		(3)	
(b)	$a = 3^{4(2.5)-5}$ and $r = \frac{9^{7-2(2.5)}}{3^{4(2.5)-5}} \Rightarrow$ one of $a = 243$ or $r = \frac{1}{3}$	M1	2.2a
	$S_{\infty} = \frac{a}{1-r} = \frac{"243"}{1-"1/3"}$	M1	1.1b
	$S_{\infty} = \frac{729}{2} (364.5)$ cao	A1	1.1b
		(3)	
(6 marks)			
Notes			
(a) Special cases:			
SC 1: For those that verify rather than prove a SC 100 is awarded for substituting $k = \frac{5}{2}$ into all three terms to correctly obtain 243, 81 and 27 with a statement that this is geometric with $r = \frac{1}{3}$ (or equivalent reason). All statements must be correct.			
SC 2: Be aware that e.g. $\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow 81^{2(7-2k)} = 9^{6k-7}$ is an incorrect process (without some indication that they have intentionally squared both sides) that fortuitously leads to the correct answer and may score maximum SC 010.			
M1: Uses the 3 terms to set up an equation in k and either reaches a common base by replacing 9 with 3^2 or by replacing 3 with $9^{0.5}$ and uses the power law of indices correctly or uses the laws of indices correctly to reach $9^{14-4k} = 3^{6k-7}$ condoning slips in e.g. expanding brackets. Writing down e.g. $2(7-2k) - (4k-5) = 2(k-1) - 2(7-2k)$ is sufficient to imply the M1.			
dM1: Correct processing leading to a value for k .			
A1*: Correct value following correct working. Allow $k = 2.5$ in place of $k = \frac{5}{2}$ Condone missing/invisible brackets provided they are recovered correctly.			
Alt 1 Using Base 9: $\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \frac{9^{7-2k}}{9^{2k-2.5}} = \frac{9^{k-1}}{9^{7-2k}} \text{ o.e. scores M1}$ $\Rightarrow 9^{9.5-4k} = 9^{3k-8} \Rightarrow 9.5-4k = 3k-8 \Rightarrow 7k = 17.5 \Rightarrow k = 2.5$ Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.			

Alt 2 Finding r in terms of k and using e.g. $u_3 = ar^2$:

$$r = \frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(7-2k)}}{3^{4k-5}} = \{3^{19-8k}\} \text{ or } r = \frac{3^{2(k-1)}}{9^{7-2k}} = \frac{3^{2k-2}}{3^{2(7-2k)}} = \{3^{6k-16}\}$$

$$\Rightarrow 3^{4k-5} \times (3^{19-8k})^2 = 3^{2k-2} \text{ or } \Rightarrow 3^{4k-5} \times (3^{6k-16})^2 = 3^{2k-2} \text{ scores M1}$$

$$\Rightarrow 3^{4k-5} \times 3^{2(19-8k)} = 3^{2k-2} \Rightarrow 33-12k = 2k-2 \Rightarrow 14k = 35 \Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 3 Using Logs Way 1:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow (9^{7-2k})^2 = 3^{6k-7} \Rightarrow 9^{14-4k} = 3^{6k-7} \text{ scores M1}$$

$$\Rightarrow (14-4k)\log_3 9 = 6k-7$$

$$\Rightarrow 2(14-4k) = 6k-7$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 4 Using Logs Way 2:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}}$$

$$\Rightarrow (7-2k)\log_3 9 - (4k-5)\log_3 3 = (2k-2)\log_3 3 - (7-2k)\log_3 9 \text{ scores M1}$$

$$\Rightarrow 2(7-2k) - (4k-5) = 2k-2 - 2(7-2k)$$

$$\Rightarrow 19-8k = 6k-16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 5 Recognising that taking $\log_3$ forms an Arithmetic Sequence:

$$\{\log_3\}u_1 = 4k-5, \{\log_3\}u_2 = 2(7-2k), \{\log_3\}u_3 = 2(k-1)$$

$$\Rightarrow 2(7-2k) - (4k-5) = 2(k-1) - 2(7-2k) \text{ scores M1}$$

$$\Rightarrow 19-8k = 6k-16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.
There is no need to see any mention of log in this approach.

(b)

M1: Deduces expressions for the first term **and** the common ratio using $k = \frac{5}{2}$ in the correct formulae **and** finds at least one of $a = 243$ or $r = \frac{1}{3}$. Allow if seen in (a). May be implied by

correct values for a and r . For reference, $a = 3^{4(2.5)-5}$ and $r = \frac{9^{7-2(2.5)}}{3^{4(2.5)-5}} \left\{ \text{or } r = \frac{3^{2(2.5-1)}}{9^{7-2(2.5)}} \right\}$

M1: Recalls the sum to infinity formula and substitutes their values for a and r provided $|r| < 1$
Dependent on a correct attempt to find both a and r using $k = 2.5$ but allow if neither value is correct or if they are unprocessed e.g. $\frac{3^{4(2.5)-5}}{1 - \frac{9^{7-2(2.5)}}{3^{4(2.5)-5}}}$ scores this mark.

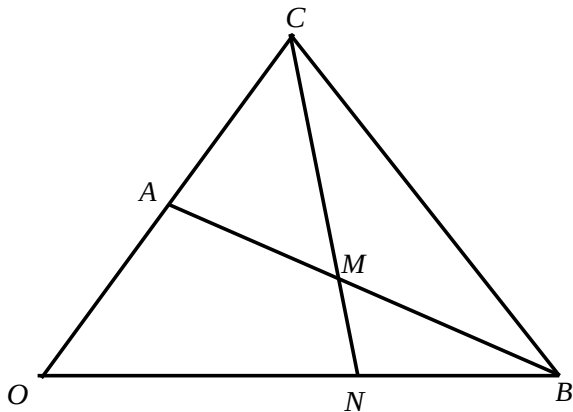
A1: cao. Correct sum to infinity. Answer only (with no working) scores full marks. Apply isw.



Question	Scheme	Marks	AOs
6	(a) $f(x) = -3x^3 + 8x^2 - 9x + 10, x \in \mathbb{R}$		
(a)	(i) $\{f(2) = -24 + 32 - 18 + 10 \Rightarrow f(2) = 0\}$	B1	1.1b
	(ii) $\{f(x) = (x-2)(-3x^2 + 2x - 5) \text{ or } (2-x)(3x^2 - 2x + 5)\}$	M1 A1	2.2a 1.1b
		(3)	
(b)	$-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$		
	Gives a partial explanation by - explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ - or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3\tan^3 \theta - 8\tan^2 \theta + 9\tan \theta - 10 = 0; 7\pi \leq \theta < 10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
(6 marks)			
Notes for Question 6			
(a)(i)			
B1:	$f(2) = 0$ or 0 stated by itself in part (a)(i)		
(a)(ii)			
M1:	Deduces that $(x-2)$ or $(2-x)$ is a factor and attempts to find the other quadratic factor by - using long division to obtain either $\pm 3x^2 \pm kx + \dots, k = \text{value} \neq 0$ or $\pm 3x^2 \pm \alpha x + \beta, \beta = \text{value} \neq 0, \alpha$ can be 0 - factorising to obtain their quadratic factor in the form $(\pm 3x^2 \pm kx \pm c), k = \text{value} \neq 0, c$ can be 0, or in the form $(\pm 3x^2 \pm \alpha x \pm \beta), \beta = \text{value} \neq 0, \alpha$ can be 0		
A1:	$(x-2)(-3x^2 + 2x - 5), (2-x)(3x^2 - 2x + 5)$ or $-(x-2)(3x^2 - 2x + 5)$ stated together as a product		
(b)			
M1:	See scheme		
A1:	See scheme. Proof must be correct with no errors , e.g. giving an incorrect discriminant value		
Note:	Correct calculation e.g. $(2)^2 - 4(-3)(-5), 4 - 60$ or -56 must be given for the first explanation		
Note:	Note that M1 can be allowed for - a correct follow through calculation for the discriminant of their “$-3y^4 + 2y^2 - 5$” which would lead to a value < 0 together with an explanation that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions - or for the omission of < 0		
Note:	< 0 must also been stated in a discriminant method for A1		
Note:	Do not allow A1 for incorrect working, e.g. $(2)^2 - 4(-3)(-5) = -54 < 0$		
Note:	$y^2 = 2 \Rightarrow y = \pm 2$, so 2 solutions is not allowed for A1, but can be condoned for M1		
Note:	Using the formula on $-3y^4 + 2y^2 - 5 = 0$ or $-3x^2 + 2x - 5 = 0$ gives y^2 or $x = \frac{-2 \pm \sqrt{-56}}{-6}$ or $\frac{-1 \pm \sqrt{-14}}{-3}$		



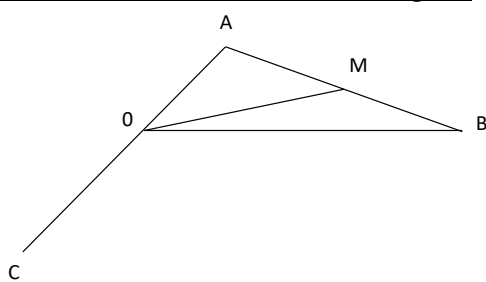
Notes for Question 6 Continued	
Note:	Completing the square on $-3x^2 + 2x - 5 = 0$ gives $x^2 - \frac{2}{3}x + \frac{5}{3} = 0 \Rightarrow \left(x - \frac{1}{3}\right)^2 - \frac{1}{9} + \frac{5}{3} = 0 \Rightarrow x = \frac{1}{3} \pm \sqrt{\frac{-14}{9}}$
Note:	Do not recover work for part (b) in part (c)
(c)	
B1:	See scheme
Note:	Give B0 for stating $\theta = \text{awrt } 23.1, \text{awrt } 26.2, \text{awrt } 29.4$ without reference to 3 solutions

Question	Scheme	Marks	AOs
10			
	$\overrightarrow{OA} = \mathbf{a}, \overrightarrow{OB} = \mathbf{b}$		
(a)	$\left\{ \overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2} \overrightarrow{AB} \Rightarrow \right\} \overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$	M1	3.1a
	$\left\{ \overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM} = \overrightarrow{CB} + \frac{1}{2} \overrightarrow{BA} \Rightarrow \right\} \overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$		
	$\Rightarrow \overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ (needs to be simplified and seen in (a) only)	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OC} + \lambda \overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda \left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \right) \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda \right) \mathbf{a} + \frac{1}{2}\lambda \mathbf{b} *$	A1*	2.1
		(2)	
(c) Way 1	$\left(2 - \frac{3}{2}\lambda \right) = 0 \Rightarrow \lambda = \dots$	M1	2.2a
	$\lambda = \frac{4}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON : NB = 2 : 1 *$	A1*	2.1
		(2)	
(c) Way 2	$\overrightarrow{ON} = \mu \mathbf{b} \Rightarrow \left(2 - \frac{3}{2}\lambda \right) \mathbf{a} + \frac{1}{2}\lambda \mathbf{b} = \mu \mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda \right) = 0 \Rightarrow \lambda = \dots \left\{ \mathbf{b}: \frac{1}{2}\lambda = \mu \text{ \& } \lambda = \frac{4}{3} \Rightarrow \mu = \frac{2}{3} \right\}$	M1	2.2a
	$\lambda = \frac{4}{3} \text{ or } \mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON : NB = 2 : 1 *$	A1*	2.1
		(2)	
(6 marks)			

Question	Scheme	Marks	AOs
10 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = \dots \quad \left\{ \mathbf{b}: 1 = \frac{1}{2}\lambda + K \quad \& \quad \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3} \right\}$	M1	2.2a
	$\lambda = \frac{4}{3} \text{ or } K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \text{ or } \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON:NB = 2:1 *$	A1	2.1
		(2)	
10 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b} \text{ \& } \overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3}, \quad \mathbf{b}: \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = \dots$	M1	2.2a
	$\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON:NB = 2:1 *$	A1	2.1
		(2)	

Notes for Question 10

(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors $\mathbf{a}$ and $\mathbf{b}$
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{ \overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow \right\} \overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ only o.e.
(b)	
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	Special Case Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \left\{ = \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b} \right\}$
Note:	Alternative 1: Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow \overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$
(c)	Way 1, Way 2 and Way 3
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ
A1*:	Correct proof
(c)	Way 4
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

Notes for Question 10 Continued	
Note:	Part (b) and part (c) can be marked together.
(a) Special Case	<u>Special Case where the point C is believed to be below the origin O</u>
	
	Give Special Case M1 A0 in part (a) for $\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow\} \overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$
	$\left\{ \text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \right\}$

Question	Scheme	Marks	AOs
13(a)	$k = e^2 \quad \text{or} \quad x \neq e^2$	B1	2.2a
		(1)	



(b)	$g'(x) = \frac{(\ln x - 2) \times \frac{3}{x} - (3 \ln x - 7) \times \frac{1}{x}}{(\ln x - 2)^2} = \frac{1}{x(\ln x - 2)^2}$ or $g'(x) = \frac{d}{dx} \left(3 - (\ln(x) - 2)^{-1} \right) = (\ln x - 2)^{-2} \times \frac{1}{x} = \frac{1}{x(\ln x - 2)^2}$ or $g'(x) = (\ln x - 2)^{-1} \times \frac{3}{x} - (3 \ln x - 7)(\ln x - 2)^{-2} \times \frac{1}{x} = \frac{1}{x(\ln x - 2)^2}$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) AND $\ln x - 2$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3 \ln x - 7 \dots 0$ or $\ln x - 2 \dots 0$ or $3 \ln a - 7 \dots 0$ or $\ln a - 2 \dots 0$ where $\dots$ is “=” or “>” to reach a value for x or a but may be seen as an inequality e.g. $x > \dots$ or $a > \dots$	M1	3.1a
	$0 < a < e^2, \quad a > e^{\frac{7}{3}}$	A1	2.2a
		(2)	
(6 marks)			

Notes:

(a)

B1: Deduces $k = e^2$ or $x \neq e^2$ Condone $k = \text{awrt } 7.39$ or $x \neq \text{awrt } 7.39$

(b)

M1: Attempts to differentiate via the quotient rule and with $\ln x \rightarrow \frac{1}{x}$ so allow for:

$$\frac{d}{dx}(g(x)) = \frac{(\ln x - 2) \times \frac{\alpha}{x} - (3 \ln x - 7) \times \frac{\beta}{x}}{(\ln x - 2)^2}, \quad \beta > 0$$

But a correct rule may be implied by their u, v, u', v' followed by applying $\frac{vu' - uv'}{v^2}$ etc.

Alternatively attempts to write $g(x) = \frac{3 \ln(x) - 7}{\ln(x) - 2} = 3 - (\ln(x) - 2)^{-1}$ and attempts the chain rule so allow for:

$$3 - (\ln(x) - 2)^{-1} \rightarrow (\ln(x) - 2)^{-2} \times \frac{\alpha}{x}$$

Alternatively writes $g(x) = (3 \ln(x) - 7)(\ln(x) - 2)^{-1}$ and attempts the product rule so allow for:

$$g'(x) = (\ln x - 2)^{-1} \times \frac{\alpha}{x} - (3 \ln x - 7)(\ln x - 2)^{-2} \times \frac{\beta}{x}$$

In general condone missing brackets for the M mark. E.g. if they quote $u = 3 \ln x - 7$ and $v = \ln x - 2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: $\frac{1}{x(\ln x - 2)^2}$ Allow $\frac{\frac{1}{x}}{(\ln x - 2)^2}$ i.e. we need to see the numerator simplified to $1/x$

Note that some candidates establish the correct numerator and correct denominator independently and provided they obtain the correct expressions, this mark can be awarded.

But allow a correctly expanded denominator.

A1cso: States that as $x > 0$ **AND** $\ln x - 2$ is squared so $g'(x) > 0$



Question	Scheme	Marks	AOs
15 (a)	$x^2 \tan y = 9 \Rightarrow 2x \tan y + x^2 \sec^2 y \frac{dy}{dx} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\frac{dy}{dx}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + f(x)$	M1	1.1b
	$\frac{dy}{dx} = \frac{-2x \times \frac{9}{x^2}}{x^2 \left(1 + \frac{81}{x^4}\right)} = \frac{-18x}{x^4 + 81} \quad *$	A1*	2.1
		(4)	
(b)	$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{-18 \times (x^4 + 81) - (-18x)(4x^3)}{(x^4 + 81)^2} = \frac{54(x^4 - 27)}{(x^4 + 81)^2} \text{ o.e.}$	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \frac{d^2y}{dx^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \frac{d^2y}{dx^2} = 0$ AND when $x > \sqrt[4]{27} \Rightarrow \frac{d^2y}{dx^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
(7 marks)			
Notes:			

(a)

M1: Attempts to differentiate $\tan y$ implicitly. Eg. $\tan y \rightarrow \sec^2 y \frac{dy}{dx}$ or $\cot y \rightarrow -\operatorname{cosec}^2 y \frac{dy}{dx}$

You may well see an attempt $\tan y = \frac{9}{x^2} \Rightarrow \sec^2 y \frac{dy}{dx} = \dots$

When a candidate writes $x^2 \tan y = 9 \Rightarrow x = 3 \tan^{\frac{1}{2}} y$ the mark is scored for $\tan^{\frac{1}{2}} y \rightarrow \dots \tan^{\frac{3}{2}} y \sec^2 y$

A1: Correct differentiation $2x \tan y + x^2 \sec^2 y \frac{dy}{dx} = 0$

Allow also $\sec^2 y \frac{dy}{dx} = -\frac{18}{x^3}$ or $2x = -9 \operatorname{cosec}^2 y \frac{dy}{dx}$ amongst others

M1: Full method to get $\frac{dy}{dx}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + f(x)$

A1*: Proceeds correctly to the given answer of $\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$



(b)

M1: Attempts to differentiate the given expression using the product or quotient rule.

For example look for a correct attempt at $\frac{vu' - uv'}{v^2}$ with $u = -18x, v = x^4 + 81, u' = \pm 18, v' = \dots x^3$

If no method is seen or implied award for $\frac{\pm 18 \times (x^4 + 81) \pm 18x(ax^3)}{(x^4 + 81)^2}$

Using the product rule award for $\pm 18(x^4 + 81)^{-1} \pm 18x(x^4 + 81)^{-2} \times cx^3$

A1: Correct **simplified** $\frac{d^2y}{dx^2} = \frac{54(x^4 - 27)}{(x^4 + 81)^2}$ o.e. such as $\frac{d^2y}{dx^2} = \frac{54x^4 - 1458}{(x^4 + 81)^2}$

Alternatively score for showing that when a correct (unsimplified) $\frac{d^2y}{dx^2} = 0 \Rightarrow x^4 = 27 \Rightarrow x = \sqrt[4]{27}$

Or for substituting $x = \sqrt[4]{27}$ into an unsimplified but correct $\frac{d^2y}{dx^2}$ and showing that it is 0

A1: Correct explanation with a minimal conclusion and correct second derivative.

See scheme.

It can be also be argued from $x^4 < 27$, $x^4 = 27$ and $x^4 > 27$ provided the conclusion states that the point of inflection is at $x = \sqrt[4]{27}$

Alternatively substitutes values of x either side of $\sqrt[4]{27}$ and at $\sqrt[4]{27}$, into $\frac{d^2y}{dx^2}$, finds all three values and makes a minimal conclusion.

A different method involves finding $\frac{d^3y}{dx^3}$ and showing that $\frac{d^3y}{dx^3} \neq 0$ and $\frac{d^2y}{dx^2} = 0$ when $x = \sqrt[4]{27}$

$$\text{FYI } \frac{d^3y}{dx^3} = \frac{23328x^3}{(x^4 + 81)^3} = 0.219 \text{ when } x = \sqrt[4]{27}$$

Alternative part (a) using arctan

M1: Sets $y = \arctan \frac{9}{x^2} \rightarrow \frac{dy}{dx} = \frac{1}{1 + \left(\frac{9}{x^2}\right)^2} \times \dots$ where $\dots$ could be 1

A2: $y = \arctan \frac{9}{x^2} \rightarrow \frac{dy}{dx} = \frac{1}{1 + \left(\frac{9}{x^2}\right)^2} \times -\frac{18}{x^3}$

A1*: $\frac{dy}{dx} = \frac{1}{1 + \frac{81}{x^4}} \times -\frac{18}{x^3} = \frac{-18x}{x^4 + 1}$ showing correct intermediate step and no errors.

Question	Scheme	Marks	AOs
8(a) Way 1	$\left(\frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1} \right) \frac{\operatorname{cosec} \theta + 1 + \operatorname{cosec} \theta - 1}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)}$	B1	1.1b
	$\equiv \frac{2 \operatorname{cosec} \theta}{\operatorname{cosec}^2 \theta - 1} \equiv \frac{2 \operatorname{cosec} \theta}{\cot^2 \theta} \text{ or e.g. } \equiv \frac{2 \sin \theta}{1 - \sin^2 \theta} = \frac{2 \sin \theta}{\cos^2 \theta}$	M1	1.1b
	$\frac{2 \operatorname{cosec} \theta}{\cot^2 \theta} \equiv \frac{2}{\sin \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta} \equiv 2 \tan \theta \sec \theta^*$ or $\frac{2 \sin \theta}{\cos^2 \theta} \equiv 2 \tan \theta \sec \theta^*$	A1*	2.1
		(3)	
(a) Way 2 “Meets in the middle”			
	$\left(\text{LHS} = \frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1} \right) \frac{\operatorname{cosec} \theta + 1 + \operatorname{cosec} \theta - 1}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)}$	B1	1.1b
	$\equiv \frac{2 \operatorname{cosec} \theta}{\operatorname{cosec}^2 \theta - 1} \equiv \frac{2 \operatorname{cosec} \theta}{\cot^2 \theta}$	M1	1.1b
	$\text{RHS} = 2 \tan \theta \sec \theta \equiv \frac{2 \sin \theta}{\cos^2 \theta} \equiv \frac{2 \sin^2 \theta}{\sin \theta \cos^2 \theta}$ $\equiv \frac{2}{\sin \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta} \equiv \frac{2 \operatorname{cosec} \theta}{\cot^2 \theta} = \text{LHS or e.g. QED or e.g. Proven}$	A1*	2.1
		(3)	
Part (a) Notes			
(a) Condone a complete proof <u>entirely</u> in x (or another variable) instead of θ Condone “=” for “$\equiv$” Note that we are marking this as B1M1A1 not M1M1A1 B1: Adds the fractions to obtain a correct single fraction (not fractions over fractions) in any form. Condone missing brackets when they combine their fractions as long as they are recovered to give a correct fraction. This can be done in a variety of ways but when combined, the fraction must be correct e.g. $\frac{\operatorname{cosec} \theta + 1 + \operatorname{cosec} \theta - 1}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)} \text{ or } \frac{2 \operatorname{cosec} \theta}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)} \text{ or } \frac{2 \operatorname{cosec} \theta}{\operatorname{cosec}^2 \theta - 1}$ or e.g. $\left(\frac{1}{\frac{1}{\sin \theta} - 1} + \frac{1}{\frac{1}{\sin \theta} + 1} = \frac{\sin \theta}{1 - \sin \theta} + \frac{\sin \theta}{1 + \sin \theta} \right) = \frac{2 \sin \theta}{1 - \sin^2 \theta} \text{ etc.}$ M1: Uses a correct Pythagorean identity anywhere in their attempt e.g. $\operatorname{cosec}^2 \theta - 1 = \cot^2 \theta$, $\sin^2 \theta + \cos^2 \theta = 1$ etc. or equivalent A1*: Correct work with all necessary steps shown leading to the given answer. See scheme for the necessary steps. They need to proceed via sine and cosine to the given answer. There should be no notational or bracketing errors and no mixed or missing variables. E.g. we would consider $\cos^2 \theta$ written as $\cos \theta^2$ a notational error. Condone reaching $2 \sec \theta \tan \theta^*$			
Way 2 (Meet in the middle)			

B1: See Way 1

M1: See Way 1

A1*: Correct work on the RHS with all necessary steps shown leading to showing the equivalence with the LHS. See scheme for the necessary steps. There should be no notational or bracketing errors and no mixed or missing variables.
For this approach there must be a (minimal) conclusion e.g. “= LHS”, “QED”, “Hence proven” etc.

It is possible to start with the rhs e.g.:

$$\begin{aligned}
 2 \tan \theta \sec \theta &= 2 \frac{\sin \theta}{\cos \theta} \times \frac{1}{\cos \theta} \\
 &= \frac{2 \sin \theta}{\cos^2 \theta} = \frac{2 \operatorname{cosec} \theta}{\cot^2 \theta} \\
 &= \frac{2 \operatorname{cosec} \theta}{\operatorname{cosec}^2 \theta - 1} = \frac{2 \operatorname{cosec} \theta}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)} \\
 &= \frac{\operatorname{cosec} \theta + 1 + \operatorname{cosec} \theta - 1}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)} \\
 &= \frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1}
 \end{aligned}$$

B1: Correctly reaches $2 \tan \theta \sec \theta = \frac{2 \operatorname{cosec} \theta}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)}$

M1: Uses a **correct** Pythagorean identity anywhere in their attempt e.g.
 $\operatorname{cosec}^2 \theta - 1 = \cot^2 \theta$, $\sin^2 \theta + \cos^2 \theta = 1$ etc. or equivalent

A1*: Correct work with all necessary steps shown leading to the lhs. See scheme for the necessary steps. There should be no notational or bracketing errors and no mixed or missing variables.

8(b)	$2 \tan 2x \sec 2x = \cot 2x \sec 2x$	B1	2.2a
	$2 \tan 2x \sec 2x - \cot 2x \sec 2x = 0$ $\Rightarrow \sec 2x(2 \tan 2x - \cot 2x) = 0$ $2 \tan 2x - \cot 2x = 0 \Rightarrow 2 \tan 2x = \cot 2x \Rightarrow \tan^2 2x = \frac{1}{2}$ or $2 \tan 2x - \cot 2x = 0 \Rightarrow 2 \frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\sin 2x} \Rightarrow 2 \sin^2 2x = \cos^2 2x$ $\Rightarrow 2 \sin^2 2x = 1 - \sin^2 2x \Rightarrow \sin^2 2x = \frac{1}{3}$ or $2(1 - \cos^2 2x) = \cos^2 2x \Rightarrow \cos^2 2x = \frac{2}{3}$ or $2 \tan 2x = \cot 2x \Rightarrow \frac{4 \tan x}{1 - \tan^2 x} = \frac{1 - \tan^2 x}{2 \tan x}$ $\Rightarrow \tan^4 x - 10 \tan^2 x + 1 = 0$ or $2 \tan 2x - \cot 2x = 0 \Rightarrow 2 \frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\sin 2x} \Rightarrow 2 \sin^2 2x = \cos^2 2x$ $2 \sin^2 2x = \cos^2 2x \Rightarrow 1 - \cos 4x = \frac{1}{2}(\cos 4x + 1) \Rightarrow \cos 4x = \frac{1}{3}$	M1	2.1
	$2x = \tan^{-1} \frac{1}{\sqrt{2}} = K \Rightarrow x = \frac{K}{2}$ or $2x = \sin^{-1} \frac{1}{\sqrt{3}} = K \Rightarrow x = \frac{K}{2}$ or $2x = \cos^{-1} \sqrt{\frac{2}{3}} = K \Rightarrow x = \frac{K}{2}$ or $\tan^2 x = 5 \pm 2\sqrt{6} \Rightarrow x = \tan^{-1}(\sqrt{5 \pm 2\sqrt{6}})$ or $\cos 4x = \frac{1}{3} \Rightarrow 4x = \cos^{-1}\left(\frac{1}{3}\right) = K \Rightarrow x = \frac{1}{4}K$	M1	1.1b
	$x = 17.6^\circ, 72.4^\circ$	A1	1.1b
		(4)	
(7 marks)			
(b) Notes			



- (b) Note that attempts solve an equation of the form:
 $2 \tan x \sec x = \cot 2x \sec 2x$ or e.g. $2 \tan \theta \sec \theta = \cot 2\theta \sec 2\theta$ or e.g. $2 \tan \theta \sec \theta = \cot 2x \sec 2x$
 Will generally score no marks in part (b)

Condone the use of θ instead of x here.

- B1:** Deduces the correct equation using the result from part (a)
M1: Factors out or cancels the $\sec 2x$ to obtain $\dots \tan 2x \pm \dots \cot 2x = 0$ or e.g.
 $\dots \tan 2x = \pm \dots \cot 2x$ leading to an equation of the form:
 $\tan^2 2x = \alpha$ or e.g. $\cot^2 2x = \frac{1}{\alpha}$ where $\alpha > 0$
Or uses $\tan 2x = \frac{\sin 2x}{\cos 2x}$ and $\cot 2x = \frac{\cos 2x}{\sin 2x}$ and then $\sin^2 2x = \pm 1 \pm \cos^2 2x$ or
 $\cos^2 2x = \pm 1 \pm \sin^2 2x$ to obtain an equation of the form $\sin^2 2x = \beta$ or $\cos^2 2x = \gamma$ or
 e.g. $\operatorname{cosec}^2 2x = \frac{1}{\beta}$ or $\sec^2 2x = \frac{1}{\gamma}$ where $0 < \beta < 1$ or $0 < \gamma < 1$
Or uses $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ and $\cot 2x = \frac{1 - \tan^2 x}{2 \tan x}$ to obtain a 3TQ in $\tan^2 x$ (or possibly
 in $\sec^2 x$)
Or uses $\tan 2x = \frac{\sin 2x}{\cos 2x}$ and $\cot 2x = \frac{\cos 2x}{\sin 2x}$ and then $2 \sin^2 2x = \pm 1 \pm \cos 4x$ and
 $2 \cos^2 2x = \pm 1 \pm \cos 4x$ to obtain an equation of the form $\cos 4x = k$, $0 < k < 1$
M1: Correct order of operations from $\tan^2 2x = \alpha$ or $\sin^2 2x = \beta$ or $\cos^2 2x = \gamma$ or $\cos 4x = k$
 or equivalents e.g. $\operatorname{cosec}^2 2x = \frac{1}{\beta}$ where $\alpha > 0$ or $0 < \beta < 1$ or $0 < \gamma < 1$ or $0 < k < 1$
 leading to at least one value for x e.g. square roots, finds inverse tan/sin/cos/cosec and
 divides by 2 or inverse cos and divides by 4
or from $\tan^2 x = k$, $k > 0$ which follows their equation (may need to check) and then
 finds $x = \tan^{-1} \sqrt{k}$
 You may need to check their value(s) (in degrees or radians) to see if the correct order of
 operations has been used. May be implied by e.g. 17.6° or 17.7° provided no incorrect
 work is seen.
A1: Correct values. Allow awrt 17.6° and awrt 72.4° . The degrees symbol is not required.
 Ignore any values outside the range (correct or incorrect) but if there are extra angles in
 range score A0. Answers in radians score A0.

Note that some candidates may convert to $\sin x$ or $\cos x$ and then solve:

E.g.

$$\tan^2 2x = \frac{1}{2} \Rightarrow \frac{\sin^2 2x}{\cos^2 2x} = \frac{1}{2} \Rightarrow \frac{4 \sin^2 x \cos^2 x}{(2 \cos^2 x - 1)^2} \rightarrow 12 \cos^4 x - 12 \cos^2 x + 1 = 0$$

$$\text{or } \frac{4 \sin^2 x \cos^2 x}{(1 - 2 \sin^2 x)^2} \rightarrow 12 \sin^4 x - 12 \sin^2 x + 1 = 0$$

$$\Rightarrow \cos^2 x / \sin^2 x = \frac{3 \pm \sqrt{6}}{6} \Rightarrow \cos x / \sin x = \pm \sqrt{\frac{3 \pm \sqrt{6}}{6}} \Rightarrow x = 17.6^\circ, 72.4^\circ$$

These can be marked in a similar way.



Alternative not using part (a):

$$\frac{1}{\operatorname{cosec} 2x - 1} + \frac{1}{\operatorname{cosec} 2x + 1} = \cot 2x \sec 2x$$

$$\Rightarrow \frac{2 \operatorname{cosec} 2x}{\operatorname{cosec}^2 2x - 1} = \cot 2x \sec 2x$$

$$\Rightarrow \frac{2 \operatorname{cosec} 2x}{\cot^2 2x} = \frac{\cos 2x}{\sin 2x} \times \frac{1}{\cos 2x} = \operatorname{cosec} 2x$$

$$\Rightarrow 2 \tan^2 2x = 1 \Rightarrow \tan^2 2x = \frac{1}{2}$$

Score as:

M1: For correct work leading to one of the forms in the main scheme e.g.

$$\tan^2 2x = \alpha \text{ or e.g. } \cot^2 2x = \frac{1}{\alpha} \text{ where } \alpha > 0$$

or

$$\sin^2 2x = \beta \text{ or } \cos^2 2x = \gamma \text{ or e.g. } \operatorname{cosec}^2 2x = \frac{1}{\beta} \text{ or } \sec^2 2x = \frac{1}{\gamma}$$

where $0 < \beta < 1$ or $0 < \gamma < 1$

B1: Any correct equation e.g. $\tan^2 2x = \frac{1}{2}$, $\cot^2 2x = 2$, $\sin^2 2x = \frac{1}{3}$ etc.

Then **M1A1** as main scheme



Question	Scheme	Marks	AOs
11	$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x) + C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x) + C(x-3)$	A1	1.1b
		(4)	
(b)	$f(x) = 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}$ { $= 3 + 4(x-3)^{-1} - 2(1-2x)^{-1}$ } ; $x > 3$		
	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2} \left\{ = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2} \right\}$	M1	2.1
	Correct $f'(x)$ and as $(x-3)^2 > 0$ and $(1-2x)^2 > 0$, then $f'(x) = -(+ve) - (+ve) < 0$, so $f(x)$ is a decreasing function	A1	2.4
		(3)	
(7 marks)			
Notes for Question 11			
(a)			
M1:	Way 1: Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3)$ in a complete method to find values for B and C . Note: Allow one slip in copying $1+11x-6x^2$ Way 2: Uses a correct identity $-10x+10 \equiv B(1-2x) + C(x-3)$ (which has been found from long division) in a complete method to find values for B and C		
B1:	$A=3$		
M1:	Attempts to find the value of either B or C from their identity This can be achieved by <i>either</i> substituting values into their identity <i>or</i> by comparing coefficients and solving the resulting equations simultaneously		
A1:	See scheme		
Note:	Way 1: Comparing terms: x^2 : $-6 = -2A$; x : $11 = 7A - 2B + C$; constant: $1 = -3A + B - 3C$ Way 1: Substituting: $x=3$: $-20 = -5B \Rightarrow B=4$; $x=\frac{1}{2}$: $5 = -\frac{5}{2}C \Rightarrow C=-2$		
Note:	Way 2: Comparing terms: x : $-10 = -2B + C$; constant: $10 = B - 3C$ Way 2: Substituting: $x=3$: $-20 = -5B \Rightarrow B=4$; $x=\frac{1}{2}$: $5 = -\frac{5}{2}C \Rightarrow C=-2$		

Note:	$A=3, B=4, C=-2$ from no working scores M1B1M1A1
Note:	The final A1 mark is effectively dependent upon both M marks

Notes for Question 11 Continued	
(a) ctd	
Note:	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4, C=-2$ will get 1 st M0, 2 nd M1, 1 st A0
Note:	Way 1: You can imply a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ from seeing $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
Note:	Way 2: You can imply a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ from seeing $\frac{-10x+10}{(x-3)(1-2x)} \equiv \frac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
(b)	
M1:	Differentiates to give $\{f'(x) = \} \pm \lambda(x-3)^{-2} \pm \mu(1-2x)^{-2}; \lambda, \mu \neq 0$
A1ft:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$, which can be simplified or un-simplified
Note:	Allow A1ft for $f'(x) = -(their\ B)(x-3)^{-2} + (2)(their\ C)(1-2x)^{-2}; (their\ B), (their\ C) \neq 0$
A1:	$f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ or $f'(x) = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2}$ and a correct explanation e.g. $f'(x) = -(+ve) - (+ve) < 0$, so $f(x)$ is a decreasing {function}
Note:	The final A mark can be scored in part (b) from an incorrect $A=...$ or from $A=0$ or no value of A found in part (a)

Notes for Question 11 Continued - Alternatives						
(a)						
Note:	Be aware of the following alternative solutions, by initially dividing by " $(x-3)$ " or " $(1-2x)$ "					
	$\bullet \frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{-6x-7}{(1-2x)} - \frac{20}{(x-3)(1-2x)} \equiv 3 - \frac{10}{(1-2x)} - \frac{20}{(x-3)(1-2x)}$ $\frac{20}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x) + E(x-3) \Rightarrow D = -4, E = -8$ $\Rightarrow 3 - \frac{10}{(1-2x)} - \left(\frac{-4}{(x-3)} + \frac{-8}{(1-2x)} \right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}; A=3, B=4, C=-2$ $\bullet \frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{3x-4}{(x-3)} + \frac{5}{(x-3)(1-2x)} \equiv 3 + \frac{5}{(x-3)} + \frac{5}{(x-3)(1-2x)}$ $\frac{5}{(x-3)(1-2x)} \equiv \frac{D}{(x-3)} + \frac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x) + E(x-3) \Rightarrow D = -1, E = -2$ $\Rightarrow 3 + \frac{5}{(x-3)} + \left(\frac{-1}{(x-3)} + \frac{-2}{(1-2x)} \right) \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}; A=3, B=4, C=-2$					
(b)						
	Alternative Method 1:					
	$f(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)}, x > 3 \Rightarrow f(x) = \frac{1+11x-6x^2}{-2x^2+7x-3}; \begin{cases} u=1+11x-6x^2 & v=-2x^2+7x-3 \\ u'=11-12x & v'=-4x+7 \end{cases}$					
	$f'(x) = \frac{(-2x^2+7x-3)(11-12x) - (1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$	<table><tr><td>Uses quotient rule to find $f'(x)$</td><td>M1</td></tr><tr><td>Correct differentiation</td><td>A1</td></tr></table>	Uses quotient rule to find $f'(x)$	M1	Correct differentiation	A1
Uses quotient rule to find $f'(x)$	M1					
Correct differentiation	A1					
	$f'(x) = \frac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2}$ and a correct explanation, e.g. $f'(x) = -\frac{(+ve)}{(+ve)} < 0$, so $f(x)$ is a decreasing {function}	A1				
	Alternative Method 2:					
	Allow M1A1A1 for the following solution: Given $f(x) = 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)} = 3 + \frac{4}{(x-3)} + \frac{2}{(2x-1)}$ as $\frac{4}{(x-3)}$ decreases when $x > 3$ and $\frac{2}{(2x-1)}$ decreases when $x > 3$ then $f(x)$ is a decreasing {function}					



Question	Scheme	Marks	AOs
12	$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2 \cot 2\theta$		
(a) Way 1	$\{\text{LHS} = \} \frac{\cos 3\theta \cos \theta + \sin 3\theta \sin \theta}{\sin \theta \cos \theta}$	M1	3.1a
	$= \frac{\cos(3\theta - \theta)}{\sin \theta \cos \theta} \left\{ = \frac{\cos 2\theta}{\sin \theta \cos \theta} \right\}$	A1	2.1
	$= \frac{\cos 2\theta}{\frac{1}{2} \sin 2\theta} = 2 \cot 2\theta *$	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	$\{\text{LHS} = \} \frac{\cos 2\theta \cos \theta - \sin 2\theta \sin \theta}{\sin \theta} + \frac{\sin 2\theta \cos \theta + \cos 2\theta \sin \theta}{\cos \theta}$		
	$= \frac{\cos 2\theta \cos^2 \theta - \sin 2\theta \sin \theta \cos \theta + \sin 2\theta \cos \theta \sin \theta + \cos 2\theta \sin^2 \theta}{\sin \theta \cos \theta}$	M1	3.1a
	$= \frac{\cos 2\theta (\cos^2 \theta + \sin^2 \theta)}{\sin \theta \cos \theta} \left\{ = \frac{\cos 2\theta}{\sin \theta \cos \theta} \right\}$	A1	2.1
	$= \frac{\cos 2\theta}{\frac{1}{2} \sin 2\theta} = 2 \cot 2\theta *$	dM1	1.1b
		A1 *	2.1
(a) Way 3	$\{\text{RHS} = \} \frac{2 \cos 2\theta}{\sin 2\theta} = \frac{2 \cos(3\theta - \theta)}{\sin 2\theta} = \frac{2(\cos 3\theta \cos \theta + \sin 3\theta \sin \theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$= \frac{2(\cos 3\theta \cos \theta + \sin 3\theta \sin \theta)}{2 \sin \theta \cos \theta}$	dM1	1.1b
	$= \frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} *$	A1 *	2.1
		(4)	
(b) Way 1	$\left\{ \frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4 \Rightarrow \right\} 2 \cot 2\theta = 4 \Rightarrow 2 \left(\frac{1}{\tan 2\theta} \right) = 4$	M1	1.1b
	Rearranges to give $\tan 2\theta = k; k \neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{ 90^\circ < \theta < 180^\circ, \tan 2\theta = \frac{1}{2} \Rightarrow \right\}$		
	Only one solution of $\theta = 103.3^\circ$ (1 dp) or awrt 103.3°	A1	2.2a
		(3)	
(b) Way 2	$\left\{ \frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4 \Rightarrow \right\} 2 \cot 2\theta = 4 \Rightarrow \frac{2}{\tan 2\theta} = 4$	M1	1.1b
	$\frac{2}{\left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right)} = 4 \Rightarrow 2(1 - \tan^2 \theta) = 8 \tan \theta$		
	$\Rightarrow \tan^2 \theta + 4 \tan \theta - 1 = 0 \Rightarrow \tan \theta = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$	dM1	1.1b
	$\{\Rightarrow \tan \theta = -2 \pm \sqrt{5}\} \Rightarrow \tan \theta = k; k \neq 0 \Rightarrow \text{applies } \arctan k$		
	$\{90^\circ < \theta < 180^\circ, \tan \theta = -2 - \sqrt{5} \Rightarrow \}$		
	Only one solution of $\theta = 103.3^\circ$ (1 dp) or awrt 103.3°	A1	2.2a
(7 marks)			

Notes for Question 12	
(a)	Way 1 and Way 2
M1:	Correct valid method forming a common denominator of $\sin \theta \cos \theta$ i.e. correct process of $\frac{(\dots)\cos \theta + (\dots)\sin \theta}{\cos \theta \sin \theta}$
A1:	Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta - \theta)$ or $\cos 2\theta$
dM1:	dependent on the previous M mark Applies a correct $\sin 2\theta \equiv 2\sin \theta \cos \theta$ to the common denominator $\sin \theta \cos \theta$
A1*:	Correct proof
Note:	Writing $\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = \frac{\cos 3\theta \cos \theta}{\sin \theta \cos \theta} + \frac{\sin 3\theta \sin \theta}{\sin \theta \cos \theta}$ is considered a correct valid method of forming a common denominator of $\sin \theta \cos \theta$ for the 1 st M1 mark
Note:	Give 1 st M0 e.g. for $\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = \frac{\cos 4\theta + \sin 4\theta}{\sin \theta \cos \theta}$ but allow 1 st M1 for $\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = \frac{\cos 3\theta \cos \theta + \sin 3\theta \sin \theta}{\sin \theta \cos \theta} = \frac{\cos 4\theta + \sin 4\theta}{\sin \theta \cos \theta}$
Note:	Give 1 st M0 e.g. for $\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = \frac{\cos^2 3\theta + \sin^2 3\theta}{\sin \theta \cos \theta}$ but allow 1 st M1 for $\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = \frac{\cos 3\theta \cos \theta + \sin 3\theta \sin \theta}{\sin \theta \cos \theta} = \frac{\cos^2 3\theta + \sin^2 3\theta}{\sin \theta \cos \theta}$
Note:	Allow 2 nd M1 for stating a correct $\sin 2\theta = 2\sin \theta \cos \theta$ and for attempting to apply it to the common denominator $\sin \theta \cos \theta$
(a)	Way 3
M1:	Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta \cos \theta \pm \sin 3\theta \sin \theta$
A1:	Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta \cos \theta + \sin 3\theta \sin \theta$
dM1:	dependent on the previous M mark Applies $\sin 2\theta \equiv 2\sin \theta \cos \theta$ to their denominator
A1*:	Correct proof
Note:	Allow 1 st M1 1 st A1 (together) for any of $\text{LHS} \rightarrow \frac{\cos 2\theta}{\sin \theta \cos \theta}$ or $\text{LHS} \rightarrow \frac{\cos 2\theta(\cos^2 \theta + \sin^2 \theta)}{\sin \theta \cos \theta}$ or $\text{LHS} \rightarrow \cos 2\theta(\cot \theta + \tan \theta)$ or $\text{LHS} \rightarrow \cos 2\theta \left(\frac{1 + \tan^2 \theta}{\tan \theta} \right)$ (i.e. where $\cos 2\theta$ has been factorised out)
Note:	Allow 1 st M1 1 st A1 for progressing as far as $\text{LHS} = \dots = \cot x - \tan x$
Note:	The following is a correct alternative solution $\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = \frac{\cos 3\theta \cos \theta + \sin 3\theta \sin \theta}{\sin \theta \cos \theta} = \frac{\frac{1}{2}(\cos 4\theta + \cos 2\theta) - \frac{1}{2}(\cos 4\theta - \cos 2\theta)}{\sin \theta \cos \theta}$ $= \frac{\cos 2\theta}{\sin \theta \cos \theta} = \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2 \cot 2\theta *$
Note:	E.g. going from $\frac{\cos 2\theta \cos^2 \theta - \sin 2\theta \sin \theta \cos \theta + \sin 2\theta \cos \theta \sin \theta + \cos 2\theta \sin^2 \theta}{\sin \theta \cos \theta}$ to $\frac{\cos 2\theta}{\sin \theta \cos \theta}$ with no intermediate working is 1 st A0

Notes for Question 12 Continued	
(b)	Way 1
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	dependent on the previous M mark Rearranges to give $\tan 2\theta = k$, $k \neq 0$, and applies $\arctan k$
A1:	Uses $90^\circ < \theta < 180^\circ$ to deduce the only solution $\theta = \text{awrt } 103.3^\circ$
Note:	Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
Note:	1 st M1 can be implied by seeing $\tan 2\theta = \frac{1}{2}$
Note:	Condone 2 nd M1 for applying $\frac{1}{2} \arctan\left(\frac{1}{2}\right) \{= 13.28...\}$
(b)	Way 2
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	dependent on the previous M mark Applies $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$, forms and uses a correct method for solving a 3TQ to give $\tan \theta = k$, $k \neq 0$, and applies $\arctan k$
A1:	Uses $90^\circ < \theta < 180^\circ$ to deduce the only solution $\theta = \text{awrt } 103.3^\circ$
Note:	Give M1 dM1 A1 for no working leading to $\theta = \text{awrt } 103.3^\circ$ and no other solutions
Note:	Give M1 dM1 A0 for no working leading to $\theta = \text{awrt } 103.3^\circ$ and other solutions which can be either outside or inside the range $90^\circ < \theta < 180^\circ$

Question	Scheme	Marks	AOs
10(a)	$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \frac{1 - (1 - 2\sin^2 \theta) + 2\sin \theta \cos \theta}{1 + \cos 2\theta + \sin 2\theta}$ or $\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \frac{1 - \cos 2\theta + \sin 2\theta}{1 + (2\cos^2 \theta - 1) + 2\sin \theta \cos \theta}$	M1	2.1
	$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \frac{1 - (1 - 2\sin^2 \theta) + 2\sin \theta \cos \theta}{1 + (2\cos^2 \theta - 1) + 2\sin \theta \cos \theta}$	A1	1.1b
	$= \frac{2\sin^2 \theta + 2\sin \theta \cos \theta}{2\cos^2 \theta + 2\sin \theta \cos \theta} = \frac{2\sin \theta (\sin \theta + \cos \theta)}{2\cos \theta (\cos \theta + \sin \theta)}$	dM1	2.1
	$= \frac{\sin \theta}{\cos \theta} = \tan \theta^*$	A1*	1.1b
		(4)	
(b)	$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3\sin 2x \Rightarrow \tan 2x = 3\sin 2x \quad \text{o.e.}$	M1	3.1a
	$\Rightarrow \sin 2x - 3\sin 2x \cos 2x = 0$ $\Rightarrow \sin 2x(1 - 3\cos 2x) = 0$ $\Rightarrow (\sin 2x = 0,) \cos 2x = \frac{1}{3}$	A1	1.1b
	$x = 90^\circ, \text{ awrt } 35.3^\circ, \text{ awrt } 144.7^\circ$	A1 A1	1.1b 2.1
		(4)	
(8 marks)			
Notes			



(a)

 M1: Attempts to use a correct double angle formulae for both $\sin 2\theta$ and $\cos 2\theta$ (seen once).

 The application of the formula for $\cos 2\theta$ must be the one that cancels out the "1"

 So look for $\cos 2\theta = 1 - 2\sin^2 \theta$ in the numerator or $\cos 2\theta = 2\cos^2 \theta - 1$ in the denominator

 Note that $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ may be used as well as using $\cos^2 \theta + \sin^2 \theta = 1$

$$\text{A1: } \frac{1 - (1 - 2\sin^2 \theta) + 2\sin \theta \cos \theta}{1 + (2\cos^2 \theta - 1) + 2\sin \theta \cos \theta} \text{ or } \frac{2\sin^2 \theta + 2\sin \theta \cos \theta}{2\cos^2 \theta + 2\sin \theta \cos \theta}$$

 dM1: Factorises numerator and denominator in order to demonstrate cancelling of $(\sin \theta + \cos \theta)$

A1*: Fully correct proof with no errors.

You must see an intermediate line of $\frac{2\sin \theta (\cancel{\sin \theta + \cos \theta})}{2\cos \theta (\cancel{\cos \theta + \sin \theta})}$ or $\frac{\sin \theta}{\cos \theta}$ or even $\frac{2\sin \theta}{2\cos \theta}$

Withhold this mark if you see, within the body of the proof,

- notational errors. E.g. $\cos 2\theta = 1 - 2\sin^2$ or $\cos \theta^2$ for $\cos^2 \theta$
- mixed variables. E.g. $\cos 2\theta = 2\cos^2 x - 1$

(b)

 M1: Makes the connection with part (a) and writes the lhs as $\tan 2x$. Condone $x \leftrightarrow \theta$ $\tan 2\theta = 3\sin 2\theta$

 A1: Obtains $\cos 2x = \frac{1}{3}$ o.e. with $x \leftrightarrow \theta$. You may see $\sin^2 x = \frac{1}{3}$ or $\cos^2 x = \frac{2}{3}$ after use of double angle formulae.

 A1: Two "correct" values. Condone accuracy of awrt 90° , 35° , 145°

Also condone radian values here. Look for 2 of awrt 0.62, 1.57, 2.53

 A1: All correct (allow awrt) and no other values in range. Condone $x \leftrightarrow \theta$ if used consistently

 Answers without working in (b): Just answers and no working score 0 marks.

 If the first line is written out, i.e. $\tan 2x = 3\sin 2x$ followed by all three correct answers score 1100.


Question	Scheme	Marks	AOs
15(a)	$S_n = a + ar + ar^2 + \dots + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + \dots + ar^n \Rightarrow S_n - rS_n = \dots$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b



	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \frac{a(1-r^n)}{(1-r)}^*$	A1*	2.1
		(4)	
(b)	$\frac{a(1-r^{10})}{1-r} = 4 \times \frac{a(1-r^5)}{1-r} \text{ or } 4 \times \frac{a(1-r^{10})}{1-r} = \frac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1-r$)	M1	3.1a
	$1-r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5 - 1)(r^5 - 3) = 0 \Rightarrow r^5 = \dots$ or e.g. $1-r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = \dots$	dM1	2.1
	$r = \sqrt[5]{3} \text{ oe only}$	A1	1.1b
		(4)	
(8 marks)			

Notes:

(a)

B1: Writes out the sum or lists terms. Condone the omission of S .

The sum must include the first and last terms and (at least) two other correct terms and no incorrect terms e.g. ar^n

Note that the sum may be seen embedded within their working.

M1: For the key step in attempting to multiply the first series by r and subtracting.

A1: $S_n - rS_n = a - ar^n$ either way around but condone one side being prematurely factorised (but not both)

following correct work but this could follow B0 if insufficient terms were shown.

A1*: Depends on all previous marks. Proceeds to given result showing all steps including seeing both sides unfactorised at some point in their working.

Note: If terms are listed rather than added then allow the first 3 marks if otherwise correct but withhold the final mark.

(b)

M1: For the correct strategy of producing an equation in just r^{10} and r^5 (and possibly $(1-r)$) with the “4” on either side using the result from part (a) and makes progress to at least cancel through by a

Some candidates retain the “ $1-r$ ” and start multiplying out e.g. $(1-r)(1-r^{10})$ and this mark is still available as long as there is progress in cancelling the “ a ”.

A1: Correct equation with the a and the $1-r$ cancelled. Allow any correct equation in just r^5 and r^{10}

dM1: Depends on the first M. Solves as far as $r^5 = \dots$ by solving a 3 term quadratic in r^5 by a valid method – see general guidance or by difference of 2 squares – see above

A1: $r = \sqrt[5]{3}$ oe only. The solution $r = 1$ if found must be rejected here.

(b) Note: For candidates who use $S_5 = 4S_{10}$ expect to see:

$$4 \times \frac{a(1-r^{10})}{1-r} = \frac{a(1-r^5)}{1-r} \Rightarrow 4(1-r^{10}) = (1-r^5) \text{ M1A0}$$

Example for (a): $4r^{10} - 3 = 0 \Rightarrow (4r^5 + 3)(r^5 - 1) = 0 \Rightarrow r^5 = \dots$ or $4(1-r^5)(1+r^5) = (1-r^5) \Rightarrow r^5 = \dots$ dM1A0

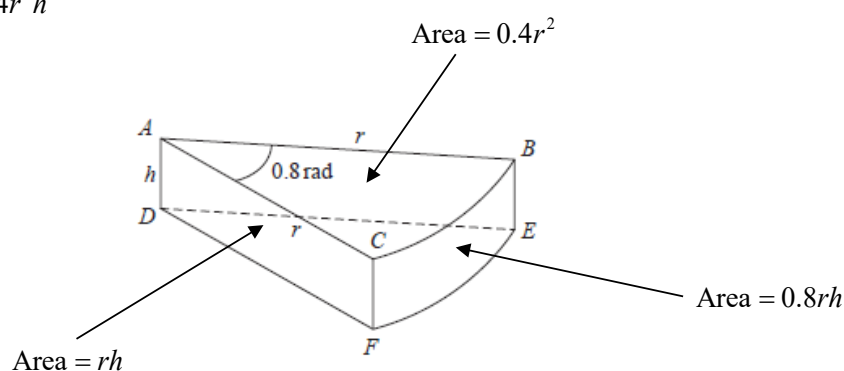


Question	Scheme	Marks	AOs
12	$1 - \cos 2\theta \equiv \tan \theta \sin 2\theta, \theta \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$		
(a) Way 1	$\tan \theta \sin 2\theta = \left(\frac{\sin \theta}{\cos \theta}\right)(2 \sin \theta \cos \theta)$	M1	1.1b
	$= \left(\frac{\sin \theta}{\cancel{\cos \theta}}\right)(2 \sin \theta \cancel{\cos \theta}) = 2 \sin^2 \theta = 1 - \cos 2\theta *$	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1 - \cos 2\theta = 1 - (1 - 2 \sin^2 \theta) = 2 \sin^2 \theta$	M1	1.1b
	$= \left(\frac{\sin \theta}{\cos \theta}\right)(2 \sin \theta \cos \theta) = \tan \theta \sin 2\theta *$	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x - 5)(1 - \cos 2x) = 3 \tan^2 x \sin 2x, -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x - 5) \tan x \sin 2x = 3 \tan^2 x \sin 2x$ or $(\sec^2 x - 5)(1 - \cos 2x) = 3 \tan x(1 - \cos 2x)$		
	Deduces $x = 0$	B1	2.2a
	Uses $\sec^2 x = 1 + \tan^2 x$ and cancels/factorises out $\tan x$ or $(1 - \cos 2x)$ e.g. $(1 + \tan^2 x - 3 \tan x - 5) \tan x = 0$ or $(1 + \tan^2 x - 3 \tan x - 5)(1 - \cos 2x) = 0$ or $1 + \tan^2 x - 5 = 3 \tan x$	M1	2.1
	$\tan^2 x - 3 \tan x - 4 = 0$	A1	1.1b
	$(\tan x - 4)(\tan x + 1) = 0 \Rightarrow \tan x = \dots$	M1	1.1b
	$x = -\frac{\pi}{4}, 1.326$	A1	1.1b
		A1	1.1b
		(6)	
(9 marks)			
Notes for Question 12			
(a)	Way 1		
M1:	Applies $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin 2\theta = 2 \sin \theta \cos \theta$ to $\tan \theta \sin 2\theta$		
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1 - 2 \sin^2 \theta$		
A1*:	For a correct proof showing all steps of the argument		
(a)	Way 2		
M1:	For using $\cos 2\theta = 1 - 2 \sin^2 \theta$		
Note:	If the form $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ or $\cos 2\theta = 2 \cos^2 \theta - 1$ is used, the mark cannot be awarded until $\cos^2 \theta$ has been replaced by $1 - \sin^2 \theta$		
M1:	Attempts to write their $2 \sin^2 \theta$ in terms of $\tan \theta$ and $\sin 2\theta$ using $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin 2\theta = 2 \sin \theta \cos \theta$ within the given expression		
A1*:	For a correct proof showing all steps of the argument		
Note:	If a proof meets in the middle; e.g. they show $\text{LHS} = 2 \sin^2 \theta$ and $\text{RHS} = 2 \sin^2 \theta$; then some indication must be given that the proof is complete. E.g. $1 - \cos 2\theta \equiv \tan \theta \sin 2\theta$, QED, box		

Notes for Question 12 Continued			
(b)			
B1:	Deduces that the given equation yields a solution $x = 0$		
M1:	For using the key step of $\sec^2 x = 1 + \tan^2 x$ and cancels/factorises out $\tan x$ or $(1 - \cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$		
Note:	Allow the use of $\pm \sec^2 x = \pm 1 \pm \tan^2 x$ for M1		
A1:	Correct 3TQ in $\tan x$. E.g. $\tan^2 x - 3\tan x - 4 = 0$		
Note:	E.g. $\tan^2 x - 4 = 3\tan x$ or $\tan^2 x - 3\tan x = 4$ are acceptable for A1		
M1:	For a correct method of solving their 3TQ in $\tan x$		
A1:	Any one of $-\frac{\pi}{4}$, awrt -0.785 , awrt 1.326 , -45° , awrt 75.964°		
A1:	Only $x = -\frac{\pi}{4}, 1.326$ cao stated in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$		
Note:	Alternative Method (Alt 1)		
	$(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$ or $(\sec^2 x - 5)(1 - \cos 2x) = 3\tan x(1 - \cos 2x)$		
	Deduces $x = 0$		B1 2.2a
	$\sec^2 x - 5 = 3\tan x \Rightarrow \frac{1}{\cos^2 x} - 5 = 3\left(\frac{\sin x}{\cos x}\right)$ $1 - 5\cos^2 x = 3\sin x \cos x$ $1 - 5\left(\frac{1 + \cos 2x}{2}\right) = \frac{3}{2}\sin 2x$ $-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ $\{3\sin 2x + 5\cos 2x = -3\}$		M1 2.1
	Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A \pm B \cos 2x = \pm C \sin 2x$ $-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ o.e.		A1 1.1b
	$\sqrt{34}\sin(2x + 1.03) = -3$		M1 1.1b
	Expresses their answer in the form $R\sin(2x + \alpha) = k$; $k \neq 0$ with values for R and α		
	$\sin(2x + 1.03) = -\frac{3}{\sqrt{34}}$		
	$x = -\frac{\pi}{4}, 1.326$		A1 1.1b A1 1.1b

Question	Scheme	Marks	AOs
15 (a)	Sets up an allowable equation using volume = 240 E.g. $\frac{1}{2}r^2 \times 0.8h = 240 \Rightarrow h = \frac{600}{r^2}$ o.e.	M1 A1	3.4 1.1b
	Attempts to substitute their $h = \frac{600}{r^2}$ into $(S =) \frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + 2rh + 0.8rh$	dM1	3.4
	$S = 0.8r^2 + 2.8rh = 0.8r^2 + 2.8 \times \frac{600}{r} = 0.8r^2 + \frac{1680}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$	M1 A1	3.1a 1.1b
	Sets $\frac{dS}{dr} = 0 \Rightarrow r^3 = 1050$ $r = \text{awrt } 10.2$	dM1 A1	2.1 1.1b
		(4)	
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right) = 1.6 + \frac{3360}{r^3}$ and considers its value or sign	M1	1.1b
	E.g. Correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$ with $\frac{d^2S}{dr^2}_{r=10.2} = 5 > 0$ proving a minimum value of S	A1	1.1b
		(2)	
(10 marks)			
Notes:			

$$\text{Volume} = 0.4r^2h$$



$$\text{Total surface area} = 2rh + 0.8r^2 + 0.8rh$$

(a)

M1: Attempts to use the fact that the volume of the toy is 240 cm^3

Sight of $\frac{1}{2}r^2 \times 0.8 \times h = 240$ leading to $h = \dots$ or $rh = \dots$ scores this mark

But condone an equation of the correct form so allow for $kr^2h = 240 \Rightarrow h = \dots$ or $rh = \dots$

A1: A correct expression for $h = \frac{600}{r^2}$ or $rh = \frac{600}{r}$ which may be left unsimplified.

This may be implied when you see an expression for S or part of S E.g. $2rh = 2r \times \frac{600}{r^2}$

dM1: Attempts to substitute their $h = \frac{a}{r^2}$ o.e. such as $hr = \frac{a}{r}$ into a **correct** expression for S

Sight of $\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + rh + rh + 0.8rh$ with an appropriate substitution

Simplified versions such as $0.8r^2 + 2rh + 0.8rh$ used with an appropriate substitution is fine.

A1*: Correct work leading to the given result.

$S =$, $SA =$ or surface area = must be seen at least once in the correct place

The method must be made clear so expect to see evidence. For example

$S = 0.8r^2 + 2rh + 0.8rh \Rightarrow S = 0.8r^2 + 2r \times \frac{600}{r^2} + 0.8r \times \frac{600}{r^2} \Rightarrow S = 0.8r^2 + \frac{1680}{r}$ would be fine.

(b) There is no requirement to see $\frac{dS}{dr}$ in part (b). It may even be called $\frac{dy}{dx}$.

M1: Achieves a derivative of the form $pr \pm \frac{q}{r^2}$ where p and q are non-zero constants

A1: Achieves $\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$

dM1: Sets or implies that their $\frac{dS}{dr} = 0$ and proceeds to $mr^3 = n$, $m \times n > 0$. It is dependent upon a correct attempt at differentiation. This mark may be implied by a correct answer to their $pr - \frac{q}{r^2} = 0$

A1: $r = \text{awrt } 10.2$ or $\sqrt[3]{1050}$

(c)

M1: Attempts to substitute their positive r (found in (b)) into $\left(\frac{d^2S}{dr^2} = \right) e \pm \frac{f}{r^3}$ where e and f are non zero and finds its value or sign.

Alternatively considers the sign of $\left(\frac{d^2S}{dr^2} = \right) e \pm \frac{f}{r^3}$ (at their positive r found in (b))

Condone the $\frac{d^2S}{dr^2}$ to be $\frac{d^2y}{dx^2}$ or being absent, but only for this mark.

A1: States that $\frac{d^2S}{dr^2}$ or $S'' = 1.6 + \frac{3360}{r^3} = \text{awrt } 5 > 0$ proving a minimum value of S

This is dependent upon having achieved $r = \text{awrt } 10$ and a correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$

It can be argued without finding the value of $\frac{d^2S}{dr^2}$. E.g. $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3} > 0$ as $r > 0$, so

minimum value of S. For consistency it is also dependent upon having achieved $r = \text{awrt } 10$



Question	Scheme	Marks	AOs
15(a)	$3(x+y)^2 \left(1 + \frac{dy}{dx}\right) = 6x - 3 \frac{dy}{dx}$	M1 A1 A1	3.1a 1.1b 1.1b
	$(3(x+y)^2 + 3) \frac{dy}{dx} = 6x - 3(x+y)^2 \Rightarrow \frac{dy}{dx} = \dots$	M1	2.1
	$\frac{dy}{dx} = \frac{6x - 3(x+y)^2}{3(x+y)^2 + 3} \left(\text{oe e.g. } \frac{2x - (x+y)^2}{(x+y)^2 + 1} \right)$	A1	1.1b
		(5)	
	Alternative – expands $(x+y)^3$ before differentiating $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$		
	$\Rightarrow 3x^2 + 3x^2 \frac{dy}{dx} + 6xy + 6xy \frac{dy}{dx} + 3y^2 + 3y^2 \frac{dy}{dx} = 6x - 3 \frac{dy}{dx}$	M1 A1 A1	3.1a 1.1b 1.1b
	$(3x^2 + 6xy + 3y^2 + 3) \frac{dy}{dx} = 6x - 3x^2 - 6xy - 3y^2 \Rightarrow \frac{dy}{dx} = \dots$	M1	2.1
	$(3x^2 + 6xy + 3y^2 + 3) \frac{dy}{dx} = 6x - 3x^2 - 6xy - 3y^2$ $\Rightarrow \frac{dy}{dx} = \frac{6x - 3x^2 - 6xy - 3y^2}{3x^2 + 6xy + 3y^2 + 3} \left(\text{oe e.g. } \frac{2x - x^2 - 2xy - y^2}{x^2 + 2xy + y^2 + 1} \right)$	A1	1.1b

(a) Notes

- (a) Some candidates have a spurious " $\frac{dy}{dx} =$ " appearing as their intention to differentiate e.g.

$$\left(\frac{dy}{dx} = \right) 3(x+y)^2 \left(1 + \frac{dy}{dx} \right) = 6x - 3 \frac{dy}{dx}$$

This can be condoned for the first 3 marks in both versions.

Allow equivalent notation for the $\frac{dy}{dx}$ e.g. y'

M1: Award this mark for one of:

- $(x+y)^3 \rightarrow k(x+y)^2 \left(\lambda + \frac{dy}{dx} \right)$ where λ is 1, x or 0 but condone missing brackets e.g.

$$3(x+y)^2 1 + \frac{dy}{dx}$$

- $3x^2 - 3y - 2 \rightarrow \dots x + \dots \frac{dy}{dx}$ but condone $3x^2 - 3y - 2 \rightarrow \dots x + \dots \frac{dy}{dx} - 2$

A1: **Either** $3(x+y)^2 \left(1 + \frac{dy}{dx} \right)$ **or** $6x - 3 \frac{dy}{dx}$ **oe**

May be implied if e.g. they collect terms to one side initially.

Do not condone missing brackets unless they are implied by subsequent work.

A1: $3(x+y)^2 \left(1 + \frac{dy}{dx} \right)$ **and** $6x - 3 \frac{dy}{dx}$ (seen separately or equated)

If they collect terms to one side initially then the signs must be correct.



M1: A valid attempt to make $\frac{dy}{dx}$ the subject with exactly 2 **different** terms in $\frac{dy}{dx}$, one coming from the differentiation of $(x+y)^3$ and the other coming from the differentiation of “ $-3y$ ”

Note that here, 2 **different** terms means terms such as $3\frac{dy}{dx}$ and $3(x+y)^2\frac{dy}{dx}$ and not

e.g. $3\frac{dy}{dx}$ and $-8\frac{dy}{dx}$

Look for $(\dots\pm\dots)\frac{dy}{dx} = \dots \Rightarrow \frac{dy}{dx} = \dots$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\frac{dy}{dx} = \dots$ at the start, they may incorporate this in

their rearrangement in which case they will have 3 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

Note that from $3(x+y)^2\left(1+\frac{dy}{dx}\right) = 6x - 3\frac{dy}{dx}$, candidates may expand the brackets before

rearranging, in which case they would need 4 **different** $\frac{dy}{dx}$ terms coming from the appropriate places.

Note that the different $\frac{dy}{dx}$ terms do not have to be correct as long as the above conditions are satisfied.

A1: Fully correct expression for $\frac{dy}{dx}$. Allow any equivalent correct forms.

Apply isw as soon as a correct expression is seen.

(a) alternative by expanding:

M1: Award this mark for one of:

- $3x^2 - 3y - 2 \rightarrow \dots x + \dots \frac{dy}{dx}$ but condone $3x^2 - 3y - 2 \rightarrow \dots x + \dots \frac{dy}{dx} - 2$
- Expanding $(x+y)^3$ to obtain either an x^2y term or an xy^2 term and then uses the product rule to obtain $\dots x^2y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$ or $\dots xy^2 \rightarrow \dots xy \frac{dy}{dx} + \dots y^2$

A1: **Either** $3x^2 + 3x^2 \frac{dy}{dx} + 6xy + 6xy \frac{dy}{dx} + 3y^2 + 3y^2 \frac{dy}{dx}$ **or** $6x - 3\frac{dy}{dx}$.

May be implied if e.g. they collect terms to one side initially.

A1: $3x^2 + 3x^2 \frac{dy}{dx} + 6xy + 6xy \frac{dy}{dx} + 3y^2 + 3y^2 \frac{dy}{dx}$ **and** $6x - 3\frac{dy}{dx}$ oe. (seen separately or equated) If they collect terms to one side initially then the signs must be correct.

M1: A valid attempt to make $\frac{dy}{dx}$ the subject with exactly 4 **different** terms in $\frac{dy}{dx}$, 3 coming from the differentiation of $(x+y)^3$ and the other coming from the differentiation of “ $-3y$ ”

Note that here, 4 **different** terms means terms such as $x^2 \frac{dy}{dx}$ and $6xy \frac{dy}{dx}$ and not e.g.



$$3 \frac{dy}{dx} \text{ and } -8 \frac{dy}{dx}$$

Look for $(\dots \pm \dots \pm \dots \pm \dots) \frac{dy}{dx} = \dots \Rightarrow \frac{dy}{dx} = \dots$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\frac{dy}{dx} = \dots$ at the start, they may incorporate this in

their rearrangement in which case they will have 5 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

Note that the different $\frac{dy}{dx}$ terms do not have to be correct as long as the above conditions

are satisfied. E.g. if they have an incorrect term such as $6x \frac{dy}{dx}$, this mark is still available.

A1: Fully correct expression for $\frac{dy}{dx}$. Allow any equivalent correct forms.

Condone e.g. $3x2y$ for $6xy$.

Apply isw as soon as a correct expression is seen.

Alternative making y the subject in (a):

$$(x + y)^3 = 3x^2 - 3y - 2$$

$$x + y = (3x^2 - 3y - 2)^{\frac{1}{3}} \Rightarrow y = (3x^2 - 3y - 2)^{\frac{1}{3}} - x$$

$$\frac{dy}{dx} = \frac{1}{3}(3x^2 - 3y - 2)^{-\frac{2}{3}} \left(6x - 3 \frac{dy}{dx} \right) - 1$$

$$\frac{dy}{dx} \left(1 + (3x^2 - 3y - 2)^{-\frac{2}{3}} \right) = 2x(3x^2 - 3y - 2)^{-\frac{2}{3}} - 1$$

$$\frac{dy}{dx} = \frac{2x(3x^2 - 3y - 2)^{-\frac{2}{3}} - 1}{1 + (3x^2 - 3y - 2)^{-\frac{2}{3}}}$$

Score as follows:

M1: Cube roots both sides and makes $x + y$ or y the subject then award for

- $(3x^2 - 3y - 2)^{\frac{1}{3}} \rightarrow \dots (3x^2 - 3y - 2)^{-\frac{2}{3}}$ or
- $3x^2 - 3y - 2 \rightarrow \dots x + \dots \frac{dy}{dx}$ but condone $3x^2 - 3y - 2 \rightarrow \dots x + \dots \frac{dy}{dx} - 2$

A1: For the $\frac{1}{3}(3x^2 - 3y - 2)^{-\frac{2}{3}}$ **or** $6x - 3 \frac{dy}{dx}$

A1: Fully correct

M1: A valid attempt to make $\frac{dy}{dx}$ the subject with exactly 2 **different** terms in $\frac{dy}{dx}$

A1: Correct expression

Using partial derivatives in (a):



$$(x+y)^3 = 3x^2 - 3y - 2 \rightarrow f(x, y) = (x+y)^3 - 3x^2 + 3y + 2$$

$$\frac{\partial f}{\partial x} = 3(x+y)^2 - 6x \quad \frac{\partial f}{\partial y} = 3(x+y)^2 + 3$$

$$\frac{dy}{dx} = -\frac{\partial f}{\partial x} \div \frac{\partial f}{\partial y} = \frac{6x - 3(x+y)^2}{3(x+y)^2 + 3}$$

or

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = 3x^2 - 3y - 2$$

$$f(x, y) = x^3 + 3x^2y + 3xy^2 + y^3 - 3x^2 + 3y + 2$$

$$\frac{\partial f}{\partial x} = 3x^2 + 6xy + 3y^2 - 6x \quad \frac{\partial f}{\partial y} = 3x^2 + 6xy + 3y^2 + 3$$

$$\frac{dy}{dx} = -\frac{\partial f}{\partial x} \div \frac{\partial f}{\partial y} = \frac{-3x^2 - 6xy - 3y^2 + 6x}{3x^2 + 6xy + 3y^2 + 3}$$

Score as follows:

M1: Correct structure for either partial derivative:

$$\text{Doesn't expand: } \frac{\partial f}{\partial x} = \dots(x+y)^2 + \dots x \quad \text{or} \quad \frac{\partial f}{\partial y} = \dots(x+y)^2 + \dots$$

or

$$\text{Expands: } \frac{\partial f}{\partial x} = \dots x^2 + \dots xy + \dots y^2 + \dots x \quad \text{or} \quad \frac{\partial f}{\partial y} = \dots x^2 + \dots xy + \dots y^2 + \dots$$

Where “...” are non-zero constants

A1: Correct $\frac{\partial f}{\partial x}$ **or** correct $\frac{\partial f}{\partial y}$

A1: Correct $\frac{\partial f}{\partial x}$ **and** correct $\frac{\partial f}{\partial y}$

M1: Attempts $\frac{dy}{dx} = -\frac{\partial f}{\partial x} \div \frac{\partial f}{\partial y}$

A1: Correct expression



(b)	$\frac{dy}{dx} = \frac{6(1) - 3(0+1)^2}{3(0+1)^2 + 3} = \frac{1}{2}$ or e.g. $\frac{dy}{dx} = \frac{6(1) - 3(1)^2 - 6(1)(0) - 3(0)^2}{3(1)^2 + 6(1)(0) + 3(0)^2 + 3} = \frac{1}{2}$ $\Rightarrow y - 0 = -2(x - 1)$ or $\Rightarrow y = -2x + c \Rightarrow 0 = -2 + c \Rightarrow c = \dots$	M1	2.1
	$y = -2x + 2^*$		
		(2)	
(b) Notes			
(b) Note that the gradient of $\frac{1}{2}$ could have been deduced from the given equation so you will need to check their solution carefully. M1: Substitutes $x = 1$ and $y = 0$ into their $\frac{dy}{dx}$ to obtain the tangent gradient and then uses the negative reciprocal and $x = 1$ and $y = 0$ in a correct straight line method to obtain the normal equation with $x = 1$ and $y = 0$ correctly placed. Note that when finding the normal gradient, they may find the negative reciprocal of their expression from part (a) and then substitute $x = 1$ and $y = 0$ which is fine. If using $y = mx + c$ they must proceed as far as finding a value for c. If no substitution of $x = 1$ and $y = 0$ into their $\frac{dy}{dx}$ is seen you will need to check their value. If they just state a value for $\frac{dy}{dx}$ then it must follow their $\frac{dy}{dx}$ with $x = 1$ and $y = 0$ A1*: Correct equation with no errors following a correct $\frac{dy}{dx}$ from part (a) (unless they start again which is unlikely) Be aware that some incorrect expressions for $\frac{dy}{dx}$ from part (a) may fortuitously give $\frac{dy}{dx} = \frac{1}{2}$ and would generally score A0 In general A1* must follow the final A1 in (a) or correct differentiation in (a)			

(c)	$y = -2x + 2 \Rightarrow (x - 2x + 2)^3 = 3x^2 - 3(-2x + 2) - 2$ or $x = \frac{2-y}{2} \Rightarrow \left(\frac{2-y}{2} + y\right)^3 = 3\left(\frac{2-y}{2}\right)^2 - 3y - 2$	M1	1.1b
	$x^3 - 3x^2 + 18x - 16 = 0$ or $y^3 + 60y = 0$	A1	1.1b
	$\Rightarrow (x-1)(x^2 - 2x + 16) = 0$ (x = 1 is known) or $\Rightarrow y(y^2 + 60) = 0$ (y = 0 is known)	dM1	2.1
	For $x^2 - 2x + 16 = 0$, $b^2 - 4ac = 4 - 4 \times 1 \times 16$ or For $y^2 + 60 = 0$, $y^2 \neq -60$	ddM1	2.1
	As $b^2 - 4ac < 0$ or as $y^2 \neq -60$ there are no other real roots and so the normal does not meet C again.	A1	2.4
		(5)	
(c) Notes			
(c) M1: Uses the equation from part (a) and substitutes $y = \pm 2x \pm 2$ or $x = \frac{\pm 2 \pm y}{2}$ to obtain an equation in one variable (usually x) (not necessarily a cubic equation). Allow slips in rearranging to obtain x in terms of y (or y in terms of x) as long as the intention is clear. A1: Correct cubic equation with terms collected and “= 0” seen or implied. Note that both $-x^3 + 3x^2 - 18x + 16 = 0$ and $-y^3 - 60y = 0$ are correct equations. To access any of the following marks, candidates must attempt to use either the factor of (x – 1) with their cubic in x or the factor of y in their cubic in y to obtain a quadratic expression in x or y. Attempts that just use a calculator to solve the cubic equation score no more marks in this part. dM1: Uses the fact that (x – 1) or y is a factor in an attempt to establish the quadratic factor. For the cubic in x, it must be of the form $ax^3 + bx^2 + cx + d = 0$ $a, b, c, d \neq 0$ For the cubic in y, it must be of the form $ay^3 + by = 0$ $a, b \neq 0$ For the cubic in x, the attempt at the quadratic factor using (x – 1) may be via inspection or e.g. long division to obtain a 3 term quadratic expression. There may or may not be a remainder but they must obtain 3 terms. For the cubic in y, they would need to take out a factor of y (or divide through by y) to obtain a factor of the form $k(y^2 + \alpha)$			

ddM1: This mark requires:

- a correct cubic equation in x or y
- the correct quadratic factor or a multiple of it e.g. $k(x^2 - 2x + 16)$ or $k(y^2 + 60)$
- an attempt to show that the quadratic factor has no real roots

For the quadratic in x this could be:

Attempts discriminant: e.g. $b^2 - 4ac = 4 - 4 \times 1 \times 16$ (may be embedded in the quadratic formula)

Attempts to complete the square: e.g. $x^2 - 2x + 16 = (x-1)^2 - 1 + 16$

Uses calculus to find the turning point: e.g. $\frac{d(x^2 - 2x + 16)}{dx} = 2x - 2 = 0 \Rightarrow x = 1 \Rightarrow y = \dots$

Attempts to solve: e.g. $x^2 - 2x + 16 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 4 \times 16}}{2}$ or from a calculator $x = 1 \pm \sqrt{15}i$

For the quadratic in y this is likely to be:

Attempts to solve: e.g. $y^2 + 60 = 0 \Rightarrow y^2 = -60 \Rightarrow \dots$

A1: Fully correct argument that requires:

- fully correct work
- a justification depending on their strategy
- a conclusion depending on their strategy

Via discriminant: $4 - 4 \times 1 \times 16 < 0$ so no real roots so they do not meet again

Via completing the square: $\rightarrow (x-1)^2 + 15$ which has a minimum value of 15 so no real roots so they do not meet again.

Via calculus: $x = 1 \Rightarrow y = 15$ is the minimum so no real roots so they do not meet again.

Via solving: $x = 1 \pm \sqrt{15}i$ or e.g. $x = 1 \pm 3.87i$ or e.g. $x = 1 \pm \sqrt{-15}$ so math error so they do not meet again.

For y it is likely to be more straightforward e.g. $y^2 \neq -60$ which cannot be solved so they do not meet again.

Allow equivalent statements for “they do not meet again” e.g. so they only meet once.

(But do **not** condone incorrect statements such as “therefore P does not meet C again”)

A minimum justification could be:

$$x^2 - 2x + 16 = 0 \rightarrow b^2 - 4ac = (-2)^2 - 4 \times 1 \times 16 \quad \text{ddM1}$$

$$4 - 4 \times 1 \times 16 < 0 \quad \text{so no more roots so no more intersections} \quad \text{A1}$$

Do not allow e.g.

“ $x^2 - 2x + 16 = 0$ gives a math error so they do not meet again”

as there has been no attempt to show why the “math error” occurs – this scores M0A0



Alternative to (c) by showing the cubic is strictly increasing (or decreasing):

M1A1: As in the main scheme then

$$f(x) = x^3 - 3x^2 + 18x - 16 \Rightarrow f'(x) = 3x^2 - 6x + 18$$

$$3x^2 - 6x + 18 = 3(x^2 - 2x + 6) = 3(x-1)^2 + 15$$

$$3(x-1)^2 + 15 > 0 \text{ so } f(x) \text{ is an increasing function}$$

Hence there can only be one intersection (at $x = 1$) so the normal and curve do not intersect again.

dM1: Differentiates their cubic of the form $ax^3 + bx^2 + cx + d = 0$ $a, b, c, d \neq 0$ to obtain a 3 term quadratic expression with only coefficient errors on the non-constant terms.

ddM1: This mark requires:

- a correct cubic equation in x
- the correct derivative or a multiple of it
- an attempt to show that the quadratic expression is always positive (or negative)

A1: Fully correct concluding argument e.g. that as the derivative is always positive (or always negative) the function is strictly increasing (or decreasing) and therefore there can only be one intersection (at $x = 1$) so the normal and curve do not meet again.

(12 marks)



Question	Scheme	Marks	AOs
4 (a)	States or uses $a = 4$	B1	1.1a
	Valid method to find $Q(x)$	M1	2.1
	$(x+4)(2x^2 - 5x + 4)$	A1	1.1b
		(3)	
(b)	Attempts to show that their $2x^2 - 5x + 4$ does not have any (real) roots	M1	3.1a
	Correct calculations, reason and conclusion	A1	2.1
		(2)	
(5 marks)			
Notes:			
(a) B1: States or uses $a = 4$ e.g. may be seen in their attempt at dividing algebraically by $x + 4$ M1: Attempts to divide $f(x)$ by $(x + 4)$ to find a three-term quadratic $Q(x)$. There are various methods or ways to present their solution so typically methods - by inspection look for $2x^3 + 3x^2 - 16x + 16 = (x + 4)(2x^2 + \dots x \pm 4)$ - by division look for a quadratic quotient of $2x^2 - 5x \pm \dots$ A1: $(x + 4)(2x^2 - 5x + 4)$ condone the missing trailing bracket i.e. $(x + 4)(2x^2 - 5x + 4$ isw if they attempt to factorise their quadratic factor. Allow to be scored if seen in (b). (b) M1: Attempts to show that their three-term quadratic “$2x^2 - 5x + 4$” does not have any roots: Attempts the discriminant e.g. $b^2 - 4ac = 25 - 4 \times 2 \times 4 (= -7)$ (may be embedded in the quadratic formula) Attempts to use the quadratic formula e.g. $\left(x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 4}}{4}\right)$ but do not allow directly from a calculator $\frac{5 \pm \sqrt{7}i}{4}$ Attempts to complete the square e.g. $2x^2 - 5x + 4 = 2\left(x^2 - \frac{5}{2}x\right) + 4 = 2\left(x - \frac{5}{4}\right)^2 + \dots \left(= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 4\right)$ Uses calculus to find the turning point e.g. $\frac{d(2x^2 - 5x + 4)}{dx} = 4x - 5 = 0 \Rightarrow x = \frac{5}{4} \Rightarrow y = \dots$ Note that any attempts using the discriminant or quadratic formula must have the values embedded in the correct places (may be partially evaluated) to score M1			

A1: **Dependent on a correct** $Q(x) = 2x^2 - 5x + 4$

Fully correct argument that requires:

- Fully correct work
- A justification depending on strategy and no incorrect reasoning seen
- A conclusion

Examples below – Note we must see working before they proceed to a correct root or minimum value – see M1 for guidance

Strategy	Correct work examples	Justification examples	Conclusion examples
Via discriminant	$b^2 - 4ac = -7$	$-7 < 0$ -7 so no (real) roots but NOT $-7 \neq 0$ so no roots	so “-4 is the only (real) root” / “only one (real) root”
Via using the quadratic formula	$x = \frac{5 \pm \sqrt{7}i}{4}$ or $x = \frac{5 \pm \sqrt{-7}}{4}$	$-7 < 0$ / which is not possible / complex roots o.e. / cannot square root a negative / no (real) roots	
Via completing the square	$2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8}$ or $2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8} = 0 \Rightarrow$ $2\left(x - \frac{5}{4}\right)^2 = -\frac{7}{8}$	which has a minimum value of $\frac{7}{8}$ / minimum (of the positive quadratic) is above the x-axis $-\frac{7}{8} < 0$ / cannot square root a negative / no (real) roots	
Via calculus	$x = \frac{5}{4} \Rightarrow y = \frac{7}{8}$	which has a minimum value of $\frac{7}{8}$ / minimum (of the positive quadratic) is above the x-axis	

Note that it is possible to justify and conclude in one step by using phrases e.g. “**no more** (real) roots” or “**no other** (real) roots”

e.g. $2x^2 - 5x + 4 \Rightarrow b^2 - 4ac = 25 - 32 = -7$ so no more roots which scores M1A1

Condone $25 - 32 < 0$ as a justification that the quadratic has no real roots

Condone the conclusion -4 is the only (real) solution (instead of (real) root).

Note if $(x + 4)$ is described as a root or -4 is described as a factor this scores A0

Question	Scheme	Marks	AOs
15 (a)	$\frac{1}{2}r^2\theta + \frac{1}{10}r^2 = 240 \Rightarrow r\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r}$ or $\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r^2}$	M1 A1	3.4 1.1b
	Substitutes into the expression for P $r\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r}$ into $(P =) r\theta + 2r + \frac{1}{5}r$	dM1	3.4
	$P = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r} + 2r + \frac{1}{5}r = \frac{480}{r} - \frac{1}{5}r + 2r + \frac{1}{5}r = 2r + \frac{480}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dP}{dr} =\right) 2 - \frac{480}{r^2}$	M1	1.1b
	Sets $\frac{dP}{dr} = 0 \Rightarrow r^2 = 240$ $r = \text{awrt } 15.5$	dM1 A1	2.1 1.1b
		(3)	
(c)	$\left(\frac{d^2P}{dr^2} =\right) \frac{960}{r^3}$	M1	1.1b
	$\left(\frac{d^2P}{dr^2} =\right) \text{awrt } 0.26 > 0$ proving a minimum value of P	A1	1.1b
		(2)	
(9 marks)			
Notes:			
(a) Note that just finding a correct equation for the area and/or a correct equation for the perimeter (before any substitution) is insufficient to score any marks. M1: Uses area formulae to form an equation of the form $\alpha r^2\theta + \beta r^2 = 240$ o.e. ($\alpha, \beta \neq 0$) and rearranges to make $r\theta$, θ or $r\theta + \frac{1}{5}r$ the subject. Look for: $r\theta = \frac{M \pm Nr^2}{r} \left(= \frac{M}{r} \pm Nr \right) \text{ o.e. or } \theta = \frac{M \pm Nr^2}{r^2} \left(= \frac{M}{r^2} \pm N \right) \text{ o.e. where } M, N \neq 0$ or $r\theta + \frac{1}{5}r = \frac{L}{r}$ $L \neq 0$ o.e. May work in degrees. A1: A correct rearrangement for θ or $r\theta$ or $r\theta + \frac{1}{5}r$ which may be unsimplified (may be in degrees) $r\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r} \text{ o.e. e.g. } r\theta = \frac{2400 - r^2}{5r} \text{ or } r\theta = \frac{480 - 0.2r^2}{r}$ or $r\theta + \frac{1}{5}r = \frac{480}{r}$ o.e.			

$$\text{or } \theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r^2} \text{ o.e. e.g. } \theta = \frac{2400 - r^2}{5r^2} \text{ or } \theta = \frac{480}{r^2} - \frac{1}{5} \text{ or } \theta = 480r^{-2} - 0.2$$

dM1: Substitutes their $r\theta = \frac{M \pm Nr^2}{r}$ o.e. or $\theta = \frac{M \pm Nr^2}{r^2}$ o.e. or $r\theta + \frac{1}{5}r = \frac{L}{r}$ into an

expression of the form $(P =) r\theta + Qr$, $Q \neq 0$ (typically $P = r\theta + \frac{11}{5}r$) which may be unsimplified or in degrees. It is dependent on the previous method mark. It is acceptable for their valid expression for θ , $r\theta$ or $r\theta + \frac{1}{5}r$ to be substituted into the perimeter expression directly (without first seeing them in the perimeter expression).

A1*: $P = 2r + \frac{480}{r}$ following a correct method (condone slips to be recovered) and all previous marks scored. Condone invisible brackets to be recovered.
 $P =$, Perimeter = must be seen at least once in their solution in the correct place.

(b) Mark (b) and (c) together. There is no requirement to see the notation $\frac{dP}{dr}$ in part

(b). It may even be called $\frac{dy}{dx}$. Allow use of e.g. P' or e.g. y'

M1: $\left(\frac{dP}{dr} =\right) p \pm \frac{q}{r^2}$ where p and q are non-zero constants

dM1: Sets or implies that their $\frac{dP}{dr} = 0$ and proceeds to $mr^{\pm 2} = n$, $m \times n > 0$. It is dependent on the previous method mark. Do not be concerned by the mechanics of the rearrangement. This mark may be implied by a correct answer to their $p - \frac{q}{r^2} = 0$. You may need to check this on your calculator.

A1: $r = \text{awrt } 15.5$ or $\sqrt{240}$ ($= 4\sqrt{15}$) Do not accept $\pm$ (ignore any units if given)

(c) Condone other letters used instead of P and r for $\frac{d^2P}{dr^2}$ e.g. $\frac{d^2y}{dx^2}$ for M1 only.

Just using $\frac{dP}{dr}$ and considering a sign change is M0A0

M1: Differentiates and finds $\left(\frac{d^2P}{dr^2} =\right) \pm \frac{f}{r^3}$ (do not be concerned about the sign)

A1: **Note if they score A0 in (b) then this mark cannot be scored.**
 Requires

- a correct a correct expression for $\frac{d^2P}{dr^2}$
- a correct value for $\left(\frac{d^2P}{dr^2} =\right) \frac{960}{r^3} = \text{awrt } 0.26$ using awrt 15.5 (but allow 0.23(43..) if using 16)
- a correct comparison with 0 **and** a conclusion e.g. minimum

The expression for the second derivative does not need to be labelled but if it is then it must be $\frac{d^2P}{dr^2}$ o.e. or accept e.g. P'' BUT $\frac{d^2y}{dx^2}$ used in their conclusion is A0

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k, 3k + 1, 3k + 2$ or equivalent e.g. $3k - 1, 3k, 3k + 1$		
	Attempts to square any two distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for any two of the possible three cases: E.g.	A1 M1 on	1.1b
	$(3k + 1)^2 = 9k^2 + 6k + 1 = 3 \times (3k^2 + 2k) + 1$ is one more than a multiple of 3 $(3k + 2)^2 = 9k^2 + 12k + 4 = 3 \times (3k^2 + 4k + 1) + 1$ (or $(3k - 1)^2 = 9k^2 - 6k + 1 = 3 \times (3k^2 - 2k) + 1$) is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k, 3k + 1, 3k + 2$ or e.g. $3k - 1, 3k, 3k + 1$	M1 A1 on EPEN	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
(4 marks)			

Notes:

M1: Makes the key step of attempting to write the natural numbers in any 2 of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square these expressions.

A1(M1 on EPEN): Successfully shows for 2 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra. This must be made explicit e.g. reaches $3 \times (3k^2 + 2k) + 1$ and makes a statement that this is one more than a multiple of 3 but also allow other rigorous arguments that reason why $9k^2 + 6k + 1$ is one more than a multiple of 3 e.g. “ $9k^2$ is a multiple of 3 and $6k$ is a multiple of 3 so $9k^2 + 6k + 1$ is one more than a multiple of 3”

M1(A1 on EPEN): Recognises that all natural numbers can be written in one of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square in all 3 cases.

A1: Successfully shows for all 3 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra and makes a conclusion

Question 10 (i)	Scheme	Marks	AOs
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Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up (i)

M1: Awarded for setting up the proof for either the even or odd numbers.

A1: Concludes correctly with a reason why $n^2 + 2$ cannot be divisible by 4 for either n odd or even.

dM1: Awarded for setting up the proof for both even and odd numbers

A1: Fully correct proof with valid explanation and conclusion for all n

Example of an algebraic proof

For $n = 2m$, $n^2 + 2 = 4m^2 + 2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n = 2m + 1$, $n^2 + 2 = (2m + 1)^2 + 2 = \dots$ FYI $(4m^2 + 4m + 3)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4(m^2 + m) + 3$ AND states hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m$, $\frac{4m^2 + 2}{4} = m^2 + \frac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\frac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m + 1$, $\frac{n^2 + 2}{4} = \frac{4m^2 + 4m + 3}{4} = m^2 + m + \frac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\frac{3}{4}$...AND states hence for all n , $n^2 + 2$ is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When n is odd, "odd $\times$ odd" = odd	M1	2.1
so $n^2 + 2$ is odd, so (when n is odd) $n^2 + 2$ cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$ even" is a multiple of 4	dM1	2.1
Concludes that when n is even $n^2 + 2$ cannot be divisible by 4 because n^2 is divisible by 4.....AND STATES true for all n .	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction ‘Assume that $n^2 + 2$ is divisible by 4 $\Rightarrow n^2 + 2 = 4k$ ’	M1	2.1
$\Rightarrow n^2 = 4k - 2 = 2(2k - 1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then $n^2 + 2$ cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n .	A1*	2.4
	(4)	

A similar proof exists via contradiction where

A1: $n^2 = 2(2k - 1) \Rightarrow n = \sqrt{2} \times \sqrt{2k - 1}$

dM1: States that $2k - 1$ is odd, so does not have a factor of 2, meaning that n is irrational



Question 10 (ii)	Scheme	Marks	AOs
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(ii)

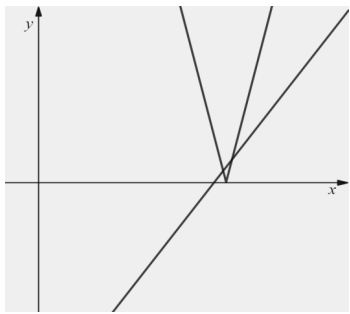
M1: States or implies ‘sometimes true’ or ‘not always true’ and gives an example where it is not true.

A1: and gives an example where it is true,

Proof using numerical values

SOMETIMES TRUE and chooses any number $x : 9.25 < x < 9.5$ and shows false Eg $x = 9.4$ $ 3x - 28 = 0.2$ and $x - 9 = 0.4$ ✗	M1	2.3
Then chooses a number where it is true Eg $x = 12$ $ 3x - 28 = 8$ $x - 9 = 3$ ✓	A1	2.4
	(2)	

Graphical Proof

 States or implies “sometimes true” Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x-axis Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $ 3x - 28 < x - 9$ and points where $ 3x - 28 > x - 9$ or in words like ‘above’ and ‘below’ or ‘dips below at one point’	A1	2.4
	(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x - 28 < x - 9$ and $-3x + 28 < x - 9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true Solves both $3x - 28 \geq x - 9$ and $-3x + 28 \geq x - 9$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Question	Scheme	Marks	AOs
8 (a)	e.g. The last line should start $25k^2 + 20k + 4$	B1	2.3
		(1)	
(b)	Considers one of the missing calculations $m = 5k + 3$ and attempts $m^2 = (5k + 3)^2 = \dots$ or $m = 5k + 4$ and attempts $m^2 = (5k + 4)^2 = \dots$	M1	2.1
	Achieves one correct statement $m^2 = (5k + 3)^2 = 25k^2 + 30k + 9 = 5(5k^2 + 6k + 2) - 1$ or $m^2 = (5k + 4)^2 = 25k^2 + 40k + 16 = 5(5k^2 + 8k + 3) + 1$	A1	1.1b
	Considers both of the missing calculations	dM1	1.1b
	Achieves both correct statements with final concluding remark (see notes)	A1	2.1
		(4)	
	(5 marks)		
Notes:			
(a) B1: Corrects the error for the case when $m = 5k + 2$. The correction may be on the proof in the box or may be described in the main body of the text. May just see the $10k$ crossed out and replaced with $20k$ in the box or described in the main body of the work. Sight of the quadratic $(25k^2) + 20k (+4)$ scores B1 and isw e.g. if they make other errors in the expansion. If B1 is not scored in (a) then allow to score if seen correct in (b) e.g. they may attempt the case $m = 5k + 2$ as part of their proof in (b).			
(b) Main scheme method uses $m = 5k + 3$ and $m = 5k + 4$ You will need to look at both cases and mark the one which is fully correct first. Allow a different variable to k and may be different letters for the two cases. If a candidate attempts repeated cases e.g. $m = 5k + 4$ and $m = 5k - 1$ then mark both and award the higher mark of the two. Condone use of m as a variable for the first three marks. There should be no errors in the algebra for the A marks including invisible brackets but do not be concerned with any re-attempt at doing the case $m = 5k + 2$			

Note that there are other allowable valid pairs of combinations covering the final two distinct cases e.g. $m = 5k - 2$ and $m = 5k + 4$, or $m = 5k - 1$ and $m = 5k + 3$ but NOT e.g. $m = 5k + 3$ and $m = 5k - 2$

Note that we are not expecting candidates to state what set of numbers k belongs to but we will condone pairs such as $m = -5k - 1$ and $m = -5k - 2$

Typically candidates will show the algebraic steps as in the main scheme but for this particular pair they may justify equivalence using the results in the box for $m = 5k + 1$ and $m = 5k + 2$ without requiring calculations which is acceptable.

M1: Considers one valid case e.g. $m = 5k + 3$ and attempts $m^2 = (5k + 3)^2$ or $m = 5k + 4$ and attempts $m^2 = (5k + 4)^2$

Look for expanding out the brackets and simplifying to a 3TQ. Condone slips.

A1: Achieves one correct statement which includes the case, the quadratic multiplied out and written in the required form

e.g. $m^2 = (5k + 3)^2 = 25k^2 + 30k + 9 = 5(5k^2 + 6k + 2) - 1$ or

e.g. $m^2 = (5k + 4)^2 = 25k^2 + 40k + 16 = 5(5k^2 + 8k + 3) + 1$

dM1: Considers **both cases** for a valid pair (see first M1 for guidance). It is dependent on the previous method mark. Condone slips.

A1: Full proof with correct statements for both cases for a valid pair. Each must include the case, the quadratic multiplied out and written in the form which is not in terms of m (we do not need the "where $n = \dots$ " at the end of the statements - you can ignore these)

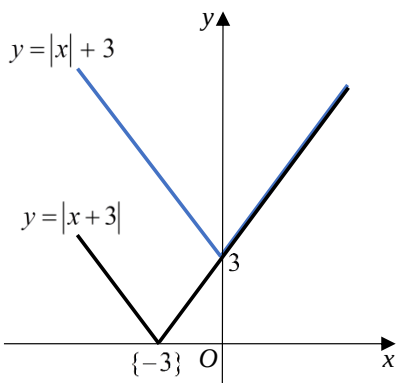
Requires a minimal overall conclusion eg. Proven, QED, tick

Condone recovery of interchanging of variables.

m	m^2	$5n \pm 1$
$5k + 3$	$25k^2 + 30k + 9$	$5(5k^2 + 6k + 2) - 1$
$5k + 4$	$25k^2 + 40k + 16$	$5(5k^2 + 8k + 3) + 1$
$5k - 1$	$25k^2 - 10k + 1$	$5(5k^2 - 2k) + 1$
$5k - 2$	$25k^2 - 20k + 4$	$5(5k^2 - 4k + 1) - 1$

**Ignore any additional cases that are not required to complete the proof
(and ignore replications of the ones given in the box in the question)**



Question	Scheme		Marks	AOs
3	Statement: “If m and n are irrational numbers, where $m \neq n$, then mn is also irrational.”			
(a)	E.g. $m = \sqrt{3}$, $n = \sqrt{12}$		M1	1.1b
	$\{mn = \} \quad (\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue or 6 is not irrational or 6 is rational		A1	2.4
			(2)	
(b)(i), (ii) Way 1		V shaped graph {reasonably} symmetrical about the y-axis with vertical intercept (0, 3) or 3 stated or marked on the positive y-axis	B1	1.1b
		Superimposes the graph of $y = x + 3 $ on top of the graph of $y = x + 3$	M1	3.1a
	the graph of $y = x + 3$ is either the same or above the graph of $y = x + 3 $ {for corresponding values of x } or when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = x + 3$ is above the graph of $y = x + 3 $		A1	2.4
			(3)	
(b)(ii) Way 2	<u>Reason 1</u> When $x \geq 0$, $ x + 3 = x + 3 $	Any one of Reason 1 or Reason 2	M1	3.1a
	<u>Reason 2</u> When $x < 0$, $ x + 3 > x + 3 $	Both Reason 1 and Reason 2	A1	2.4

(5 marks)

Notes for Question 3

(a)	
M1:	States or uses any pair of different numbers that will disprove the statement. E.g. $\sqrt{3}$, $\sqrt{12}$; $\sqrt{2}$, $\sqrt{8}$; $\sqrt{5}$, $-\sqrt{5}$; $\frac{1}{\pi}$, 2π ; $3e$, $\frac{4}{5e}$;
A1:	Uses correct reasoning to disprove the given statement, with a correct conclusion
Note:	Writing $(3e)\left(\frac{4}{5e}\right) = \frac{12}{5} \Rightarrow$ untrue is sufficient for M1A1
(b)(i)	
B1:	See scheme
(b)(ii)	
M1:	For constructing a method of comparing $ x + 3$ with $ x + 3 $. See scheme.
A1:	Explains fully why $ x + 3 \geq x + 3 $. See scheme.
Note:	Do not allow either $x > 0$, $ x + 3 \geq x + 3 $ or $x \geq 0$, $ x + 3 \geq x + 3 $ as a valid reason
Note:	$x = 0$ (or where necessary $x = -3$) need to be considered in their solutions for A1
Note:	Do not allow an incorrect statement such as $x \leq 0$, $ x + 3 > x + 3 $ for A1



Notes for Question 3 Continued			
(b)(ii)			
Note:	Allow M1A1 for $x > 0$, $ x + 3 = x + 3 $ and for $x \leq 0$, $ x + 3 \geq x + 3 $		
Note:	Allow M1 for any of x is positive, $x + 3 = x + 3$ x is negative, $x + 3 > x + 3$ $x > 0$, $x + 3 = x + 3$ $x \leq 0$, $x + 3 \geq x + 3$ $x > 0$, $x + 3$ and $x + 3$ are equal $x \geq 0$, $x + 3$ and $x + 3$ are equal - when $x \geq 0$, both graphs are equal - for positive values $x + 3$ and $x + 3$ are the same Condone for M1 $x \leq 0$, $x + 3 > x + 3$ $x < 0$, $x + 3 \geq x + 3$		
(b)(ii) Way 3	- For $x > 0$, $x + 3 = x + 3$ - For $-3 < x < 0$, as $x + 3 > 3$ and $\{0 < x + 3 < 3\}$, then $x + 3 > x + 3$ - For $x \leq -3$, as $x + 3 = -x + 3$ and $x + 3 = -x - 3$, then $x + 3 > x + 3$	M1	3.1a
		A1	2.4

Question	Scheme	Marks	AOs
15	$(\sin x - \cos x)^2 < 1 \Rightarrow \sin^2 x - 2 \sin x \cos x + \cos^2 x < 1$ o.e.	M1	1.1b
	Examples: $1 - 2 \sin x \cos x < 1$, $1 - \sin 2x < 1$, $-2 \sin x \cos x < 0$, $-\sin 2x < 0$	A1	2.2a
	As x is obtuse then $-2 \sin x \cos x$ is positive because $\sin x > 0$ and $\cos x < 0$ so we have a contradiction. Therefore $\sin x - \cos x \geq 1$ *	A1*	2.4
(3 marks)			
Notes			
Condone poor notation e.g. $\sin x^2$ or e.g. $-2 \sin \theta \cos x < 1$ for the first two marks only. M1: Expands $(\sin x - \cos x)^2$ to obtain $\sin^2 x \pm k \sin x \cos x + \cos^2 x$ where $k = 1$ or 2 o.e. May be implied. A1: Uses a correct identity $\sin^2 x + \cos^2 x = 1$ or e.g. $-\sin^2 x - \cos^2 x = -1$ to obtain a correct inequality in any form that does not include the $\sin^2 x$ and $\cos^2 x$ terms. Condone e.g. $-2 \sin \cos x < 0$ A1*: Fully correct work which includes - a convincing argument that explains why their inequality is not true - a statement that indicates there is a contradiction - a conclusion that $\sin x - \cos x \geq 1$ (there is no need to repeat “when x is obtuse”) - no contradictory statements - no mixed/missed variables, e.g., $-2 \sin \theta \cos x < 1$ or $1 - \sin 2 < 1$ Examples: From $-2 \sin x \cos x < 0$: ----- In the second quadrant $-2 \sin x \cos x$ is $- \times + \times - = +$ “(this is a contradiction)” or equivalent (therefore) $\sin x - \cos x \geq 1$ or As x is obtuse, $\sin x > 0$, $\cos x < 0$ so $-2 \sin x \cos x > 0$ “(this is a contradiction)” or equivalent (therefore) $\sin x - \cos x \geq 1$ From $-\sin 2x < 0$: ----- As x is obtuse, $2x$ is reflex o.e. (i.e. $\pi < 2x < 2\pi$) so $-\sin 2x > 0$ “(this is) wrong” or equivalent (therefore) $\sin x - \cos x \geq 1$ From $1 - \sin 2x < 1$: ----- As x is obtuse, $2x$ is reflex o.e. (i.e. $180 < 2x < 360$) so $\sin 2x < 0$ so $1 - \sin 2x > 1$ “(this is a contradiction)” or equivalent (therefore) $\sin x - \cos x \geq 1$ From $\sin 2x > 0$: ----- As x is obtuse, $2x$ is reflex o.e. (i.e. $180 < 2x < 360$) so $\sin 2x < 0$ “(this is) incorrect” or equivalent (therefore) $\sin x - \cos x \geq 1$ Note that you may condone the absence of a statement referring to the fact that $(\sin x - \cos x)^2 < 1$ is only valid since $\sin x - \cos x > 0$ when x is obtuse.			

Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p + q)(2p - q) = 25$	M1	2.1
	If true then $2p + q = 25$ or $2p + q = 5$ $2p - q = 1$ or $2p - q = 5$ Award for deducing either of the above statements	M1	2.2a
	Solutions are $p = 6.5, q = 12$ or $p = 2.5, q = 0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	
	(4 marks)		
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either $2p+q=25$ or $2p+q=5$ must be true.
 $2p-q=1$ or $2p-q=5$

Award for deducing either of the above statements.

You can ignore any reference to $2p+q=1$ or $2p-q=25$ as this could not occur for positive p and q .

A1: For correctly solving one of the given statements,

For $2p+q=25$ candidates only really need to proceed as far as $p=6.5$ to show the contradiction.
 $2p-q=1$

For $2p+q=5$ candidates only really need to find either p or q to show the contradiction.
 $2p-q=5$

Alt for $2p+q=5$ candidates could state that $2p+q \neq 2p-q$ if p, q are positive integers.
 $2p-q=5$

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
16 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*) , or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow q^2 = 4p^2 - 25$ with * or ** Or There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow 4p^2 = q^2 + 25$ with * or **	M1	2.1
	Sets $q = 2n \pm 1$ and expands $(2n \pm 1)^2 = 4p^2 - 25$	M1	2.2a
	Proceeds to an expression such as $4p^2 = 4n^2 + 4n + 26 = 4\left(n^2 + n + 6\right) + 2$ $4p^2 = 4n^2 + 4n + 26 = 4\left(n^2 + n\right) + \frac{13}{2}$ $p^2 = n^2 + n + \frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2 - q^2 = 4 \times (2m)^2 - (2n+1)^2 = 16m^2 - 4n^2 - 4n - 1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2 - q^2 = 4 \times (2m+1)^2 - (2n+1)^2 = 16m^2 + 16m - 4n^2 - 4n + 3$	Three more than a multiple of 4 so cannot equal 25

Question	Scheme	Marks	AOs
7 (i)	For setting up the contradiction: There exists integers p and q such that pq is even and both p and q are odd	B1	2.5
	For example, sets $p = 2m + 1$ and $q = 2n + 1$ and then attempts $pq = (2m + 1)(2n + 1) = \dots$	M1	1.1b
	Obtains $pq = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1$ $= 2(2mn + m + n) + 1$ States that this is odd, giving a contradiction so "if pq is even, then at least one of p and q is even" *	A1*	2.1
		(3)	
(ii)			
	$(x + y)^2 < 9x^2 + y^2 \Rightarrow 2xy < 8x^2$	M1	2.2a
	States that as $x < 0 \Rightarrow 2y > 8x$ $\Rightarrow y > 4x$ *	A1*	2.1
		(2)	
(5 marks)			
Notes:			

(i)

B1: For using the "correct"/ allowable language in setting up the contradiction.

Expect to see a minimum of

- "assume" or "let" or "there is " or other similar words
- " pq is even" and " p and q are (both) odd"

M1: Uses a correct algebraic form for p and q and attempting to multiply.Allow any correct form so $p = 2n + 1$ and $q = 2m + 3$ would be fine to use**Different variables must be used** for p and q , so $p = 2n + 1$ and $q = 2n - 1$ would be M0

A1*: Full argument .

This requires (1) a correct calculation for their pq

(2) a correct reason and conclusion that it is odd

$$\text{E.g. } (2m+1)(2n+1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1 = \text{odd}$$

$$\text{E.g. } (2m-1)(2n+1) = 4mn + 2m - 2n - 1 = \text{even} + \text{even} - \text{even} - 1 = \text{odd}$$

and (3) a minimal statement implying that they have proven what was required which could be QED, proven etc

Note that B0 M1 A1 is possible

(ii)

M1: For multiplying out and cancelling terms before proceeding to a correct intermediate line such as

$$2xy < 8x^2 \text{ o.e. such as } 2x(4x - y) > 0$$

A1*: Full and rigorous proof with reason shown as to why inequality reverses. The point at which it reverses must be correct and a correct reason given

See scheme

$$\text{Alt: } 2xy < 8x^2 \Rightarrow xy - 4x^2 < 0 \Rightarrow x(y - 4x) < 0$$

$$\text{as } x < 0, (y - 4x) > 0 \Rightarrow y > 4x \text{ scores M1 A1}$$

So, the following should be scored M1 A0 as line 3 is incorrect

$$2xy - 8x^2 < 0$$

$$\Rightarrow 2xy < 8x^2$$

$$\Rightarrow y < 4x$$

$$\Rightarrow y > 4x \text{ as } x < 0$$

There should be no incorrect lines in their proof



Question	Scheme	Marks	AOs
15(i)	$k^2 - 4k + 5 = (k - 2)^2 + \dots$	M1	2.1
	$k^2 - 4k + 5 = (k - 2)^2 + 1$ so $k^2 - 4k + 5 \dots 1$ so $k^2 - 4k + 5$ is always positive*	A1*	2.4
		(2)	
Notes			
(i) Note: Using e.g. x throughout is acceptable for both marks. M1: Starts the process of showing the given expression is positive. Completing the square requires $(k - 2)^2 \pm \dots$ Differentiation requires them to differentiate to a linear expression in k, set $= 0$ (which may be implied), solve for k and substitute their k into $k^2 - 4k + 5$ to reach a value. Ignore e.g. $\frac{dy}{dx}$ Discriminant requires a calculation of $b^2 - 4ac = (\pm 4)^2 - 4 \times 1 \times 5 \{ = -4 \}$ and might be seen embedded in the quadratic formula. Sketches on their own are insufficient without working to find the minimum point algebraically. Stating the minimum is $(2, 1)$ without any evidence is M0. A1*: Completes the proof with no errors and correct reasoning. Accept e.g. “hence proved” in place of “so $k^2 - 4k + 5$ is always positive” as long as there is sufficient justification for it being “proved”. As a minimum expect to see e.g.: $k^2 - 4k + 5 = (k - 2)^2 + 1$ which is always positive as $(k - 2)^2 \dots 0$ (or as squares are always positive or zero). Must be a correct statement. Do not condone e.g. $(k - 2)^2 > 0$ $k^2 - 4k + 5 = (k - 2)^2 + 1$ so $(2, 1)$ is the minimum (point) so $k^2 - 4k + 5$ is always positive. $k^2 - 4k + 5 = (k - 2)^2 + 1$ so $k = 2 \pm \sqrt{-1}$ hence $k^2 - 4k + 5$ has no real roots and as k^2 has a positive coefficient (condone e.g. positive k^2 or positive quadratic or $a > 0$), hence proved. $2k - 4 = 0 \Rightarrow k = 2 \Rightarrow k^2 - 4k + 5 = 1$ is the minimum value so $k^2 - 4k + 5$ is always positive. $b^2 - 4ac = 16 - 20 \leq 0$ so $k^2 - 4k + 5 = 0$ has no real roots and as k^2 has a positive coefficient (condone e.g. positive k^2 or positive quadratic or $a > 0$), hence proved. Note that stating $k^2 - 4k + 5 = (k - 2)^2 + 1 > 0$ alone is not sufficient for the A1* Similarly, stating $k^2 - 4k + 5 = (k - 2)^2 + 1 > 0$ because $(k - 2)^2 > 0$ (or $(k - 2)^2$ is positive) is incorrect and scores A0. SC: Attempts at completing the square for $k = 2n$ and/or $k = 2n + 1$ (or $k = 2n - 1$) can score M1 provided at least one is attempted as far as $(\dots)^2 + \dots$ e.g. to $(2n - 2)^2 + \dots$ or e.g. $4(n - 1)^2 + \dots$ Candidates are unlikely to complete the argument to score A1 (it is not sufficient to just consider odd/even) but if you think they might deserve the A1 then send to Review. $k = 2n \rightarrow k^2 - 4k + 5 = 4n^2 - 8n + 5$ leading to $(2n - 2)^2 + 1$ or $4(n - 1)^2 + 1$ $k = 2n + 1 \rightarrow k^2 - 4k + 5 = 4n^2 - 4n + 2$ leading to $(2n - 1)^2 + 1$ or $4\left(n - \frac{1}{2}\right)^2 + 1$ $k = 2n - 1 \rightarrow k^2 - 4k + 5 = 4n^2 - 12n + 10$ leading to $(2n - 3)^2 + 1$ or $4\left(n - \frac{3}{2}\right)^2 + 1$			

Question	Scheme	Marks	AOs
15(ii)	Attempts to solve any 1 pair of the relevant 11 sim. equations. e.g. one of $\left. \begin{array}{l} 3x + 2y = 28 \\ 2x - 5y = 1 \end{array} \right\} \Rightarrow x = \dots, (y = \dots)$ or ... (see notes for all 11)	M1	2.1
	Attempts to solve any 2 pairs of the relevant 11 sim. equations with at least one correct and correctly rejected. e.g. both $\left. \begin{array}{l} 3x + 2y = 28 \\ 2x - 5y = 1 \end{array} \right\} \Rightarrow x = \frac{142}{19}, \left(y = \frac{53}{19} \right) \text{ Not integers}$ and $\left. \begin{array}{l} 3x + 2y = 7 \\ 2x - 5y = 4 \end{array} \right\} \Rightarrow x = \dots, (y = \dots)$ or ... (see notes for all 11 options)	dM1 (A1 on EPEN)	2.2a
	Attempts to solve all 5 pairs of the relevant sim. equations with positive RHS or e.g. $\left. \begin{array}{l} 3x + 2y = 28 \\ 2x - 5y = 1 \end{array} \right\} \Rightarrow x = \frac{142}{19}, \left(y = \frac{53}{19} \right) \text{ Not integers}$ and $\left. \begin{array}{l} 3x + 2y = 7 \\ 2x - 5y = 4 \end{array} \right\} \Rightarrow x = \dots, (y = \dots)$ and all other cases are not possible as $3x + 2y \geq 5$	ddM1	2.1
	Requires: - All cases considered, with correct values and rejected - Correct reasons given in each case e.g. “not integers” - Concluding statement e.g. “hence proven”	A1	2.4
		(4)	

(6 marks)

Notes

15(ii)

General Note:

Throughout this question we are condoning if candidates do not reference the following two points which are deemed fairly trivial and acceptable to be omitted at A-level:

- As x and y are integers then both $3x + 2y$ and $2x - 5y$ are integers.
- As x and y are positive then $3x + 2y > 0$

As such, we only *require* candidates to prove that there are no positive integer solutions x and y to simultaneous equations that have a positive RHS.

Note: Any attempt to solve the given simultaneous equations does not score any marks on its own. Any attempts that use substitutions such as $x = 2n$ etc. are unlikely to score any marks. There are other methods to eliminate some pairs of simultaneous equations e.g. by showing that the only solution to $3x + 2y = 7$ is $(1, 2)$ which is not on $2x - 5y = 4$. Use Review in such cases.



M1: Attempt to solve any one of the other 11 possible cases (labelled A-K) below to find a value for x or a value for y . The attempt may be implied by a value for x or y which need not be correct.

dM1: Attempts to solve any two of the other 11 possible cases below to find a value for x or a value for y **with** at least one correct **and** correctly rejected. It is not necessary to find both values of x and y unless the correct value found does not cause a contradiction (see cases D and G).

ddM1: Attempts to solve all 5 pairs (A-E) of the relevant simultaneous equations with positive RHS **or** attempts to solve cases A and B and justifies that these are the only cases that need checking using either:

- $3x + 2y \geq 5$ (because x and y are positive) **or**
- $3x + 2y > 2x - 5y$ (because x and y are positive) [cases F, G and H do not need checking using this approach because of the general note]

Their values for x and y do not need to be correct for this mark as long as the dM1 is scored.

A1: cso Shows that all necessary cases are impossible with correct values, correct reasons, and a minimal conclusion e.g. “hence proved”. There is no need to say “contradiction” or restate the objective (you can also ignore any inaccurate attempt to restate the objective).

All 5 pairs of simultaneous may be rejected in one go if the rejection is sufficiently clear.

The mechanics of solving the simultaneous equations does not need to be shown.

$$A: \begin{cases} 3x + 2y = 28 \\ 2x - 5y = 1 \end{cases} \Rightarrow x = \frac{142}{19}, \left(y = \frac{53}{19} \right) \text{ Not integers}$$

$$B: \begin{cases} 3x + 2y = 7 \\ 2x - 5y = 4 \end{cases} \Rightarrow x = \frac{43}{19}, \left(y = \frac{2}{19} \right) \text{ Not integers}$$

$$C: \begin{cases} 3x + 2y = 4 \\ 2x - 5y = 7 \end{cases} \Rightarrow x = \frac{34}{19}, \left(y = -\frac{13}{19} \right) \text{ Not integers/not positive}$$

$$D: \begin{cases} 3x + 2y = 2 \\ 2x - 5y = 14 \end{cases} \Rightarrow (x = 2), y = -2 \text{ Not positive}$$

$$E: \begin{cases} 3x + 2y = 1 \\ 2x - 5y = 28 \end{cases} \Rightarrow x = \frac{61}{19}, \left(y = -\frac{82}{19} \right) \text{ Not integers/not positive}$$

$$F: \begin{cases} 3x + 2y = -1 \\ 2x - 5y = -28 \end{cases} \Rightarrow x = -\frac{61}{19}, \left(y = \frac{82}{19} \right) \text{ Not integers/not positive}$$

$$G: \begin{cases} 3x + 2y = -2 \\ 2x - 5y = -14 \end{cases} \Rightarrow x = -2, (y = 2) \text{ Not positive}$$

$$H: \begin{cases} 3x + 2y = -4 \\ 2x - 5y = -7 \end{cases} \Rightarrow x = -\frac{34}{19}, \left(y = \frac{13}{19} \right) \text{ Not integers/not positive}$$

$$I: \begin{cases} 3x + 2y = -7 \\ 2x - 5y = -4 \end{cases} \Rightarrow x = -\frac{43}{19}, \left(y = -\frac{2}{19} \right) \text{ Not integers/not positive}$$

$$J: \begin{cases} 3x + 2y = -14 \\ 2x - 5y = -2 \end{cases} \Rightarrow x = -\frac{74}{19}, \left(y = -\frac{22}{19} \right) \text{ Not integers/not positive}$$

$$K: \begin{cases} 3x + 2y = -28 \\ 2x - 5y = -1 \end{cases} \Rightarrow x = -\frac{142}{19}, \left(y = -\frac{53}{19} \right) \text{ Not integers/not positive}$$



Question	Scheme	Marks	AOs
15(i)	$n = 1, 2^3 = 8, 3^1 = 3, (8 > 3)$ $n = 2, 3^3 = 27, 3^2 = 9, (27 > 9)$ $n = 3, 4^3 = 64, 3^3 = 27, (64 > 27)$ $n = 4, 5^3 = 125, 3^4 = 81, (125 > 81)$	M1	2.1
	So if $n \leq 4, n \in \mathbb{N}$ then $(n + 1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m = (2p \pm 1)$ and attempt $m^3 + 5 = (2p \pm 1)^3 + 5 = \dots$	M1	2.1
	$= 8p^3 + 12p^2 + 6p + 6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion - reason for $8p^3 + 12p^2 + 6p + 6$ being even - acceptable statement such as "this is a contradiction so if $m^3 + 5$ is odd then m must be even"	A1	2.4
		(4)	
(6 marks)			
Notes			

(i)

M1: A full and rigorous argument that uses all of $n = 1, 2, 3$ and 4 in an attempt to prove the given result. Award for attempts at both $(n + 1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27 > 9$

Extra values, say $n = 0$, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for $n = 1, 2, 3$ and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept ✓ or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m = 2p \pm 1$ and attempting to expand $(2p \pm 1)^3 + 5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p + 1)^3 + 5 = 8p^3 + 12p^2 + 6p + 6$ and **states** even.

Alternatively reaches $(2p - 1)^3 + 5 = 8p^3 - 12p^2 + 6p + 4$ and **states** even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) **A reason** why the expression $8p^3 + 12p^2 + 6p + 6$ or $8p^3 - 12p^2 + 6p + 4$ is even

Acceptable reasons are

- all terms are even
- sight of a factorised expression E.g. $8p^3 - 12p^2 + 6p + 4 = 2(4p^3 - 6p^2 + 3p + 2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if $m^3 + 5$ is odd then m is even"
- "this is contradiction, so proven."
- "So if $m^3 + 5$ is odd then m is even"

S.C If the candidate misinterprets the demand and does not use proof by contradiction but states a

counter example to the statement "if $m^3 + 5$ is odd then m must be even" such as when $m = \sqrt[3]{2}$ then they can score special case mark B1

