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EDEXCEL A LEVEL MATHEMATICS: QUESTIONS BY TOPIC

Algebra and Functions (2)

A Level Mathematics Topic 2

30 questions · 223 marks · Mark schemes included

Pearson Edexcel A Level Mathematics (9MA0), June 2018 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

30 questions · 223 marks · 11 sub-topics



SUB-TOPIC 2.3

QUADRATIC FUNCTIONS

Sub-topic 2.3 · 1 question · 10 marks · lowest tariff first



5.

$$f(x) = 2x^2 + 4x + 9 \quad x \in \mathbb{R}$$

- (a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are integers to be found.

(3)

- (b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

- (c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \quad x \in \mathbb{R}$$

- (ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \quad x \in \mathbb{R}$$

(4)





Question 5 continued

(Total for Question 5 is 10 marks)



SUB-TOPIC 2.5

INEQUALITIES

Sub-topic 2.5 · 1 question · 5 marks · lowest tariff first







(Total for Question 7 is 5 marks)

SUB-TOPIC 2.6

POLYNOMIALS AND THE FACTOR THEOREM

Sub-topic 2.6 · 2 questions · 8 marks · lowest tariff first



4. Given that

- $f(x) = 2x^3 + 3x^2 - 16x + 16$
- $f(-4) = 0$

(a) write $f(x)$ in the form

$$(x + a) Q(x)$$

where a is a constant and $Q(x)$ is a quadratic expression.

(3)

(b) Hence prove that -4 is the only real root of the equation

$$f(x) = 0$$

(Solutions relying on calculator technology are not acceptable.)

(2)

Question 4 continued

(Total for Question 4 is 5 marks)



1.

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Factorise completely

$$2x^3 - 24x^2 + 40x$$

(3)



Question 1 continued

(Total for Question 1 is 3 marks)



SUB-TOPIC 2.7

GRAPHS OF FUNCTIONS

Sub-topic 2.7 · 4 questions · 35 marks · lowest tariff first



Question 13 continued

Lined area for writing the answer to Question 13 continued.

Question 13 continued

(Total for Question 13 is 11 marks)

7.

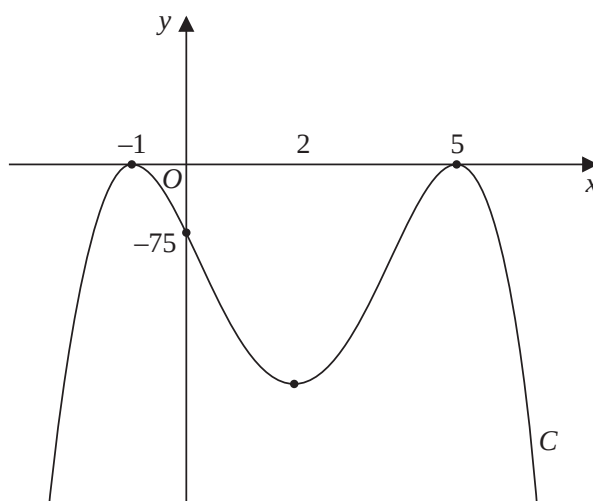


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$, where $f(x)$ is a quartic expression in x .

The curve

- has maximum turning points at $(-1, 0)$ and $(5, 0)$
- crosses the y -axis at $(0, -75)$
- has a minimum turning point at $x = 2$

(a) Find the set of values of x for which

$$f'(x) \geq 0$$

writing your answer in set notation.

(2)

(b) Find the equation of C . You may leave your answer in factorised form.

(3)

The curve C_1 has equation $y = f(x) + k$, where k is a constant.

Given that the graph of C_1 intersects the x -axis at exactly four places,

(c) find the range of possible values for k .

(2)

12.

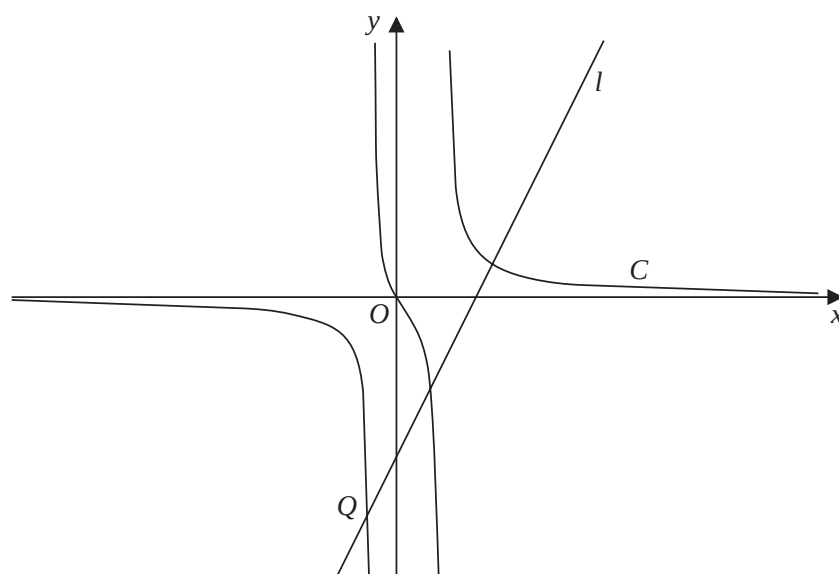
**Figure 3**

Figure 3 shows a sketch of the curve C with equation

$$y = \frac{15x}{(2x+3)(x-3)} \quad x \neq -\frac{3}{2} \quad x \neq 3$$

and the straight line l with equation

$$y = 2x - 10$$

(a) Verify that C and l intersect where $x = 6$

(2)

The curve and line also intersect at the point Q shown in Figure 3.

(b) Show that the x coordinate of Q is a solution of

$$4x^3 - 26x^2 - 3x + 90 = 0$$

(2)

(c) Using algebra and showing all stages of working, find the exact x coordinate of Q .

(4)

Question 12 continued

(Total for Question 12 is 8 marks)

8.

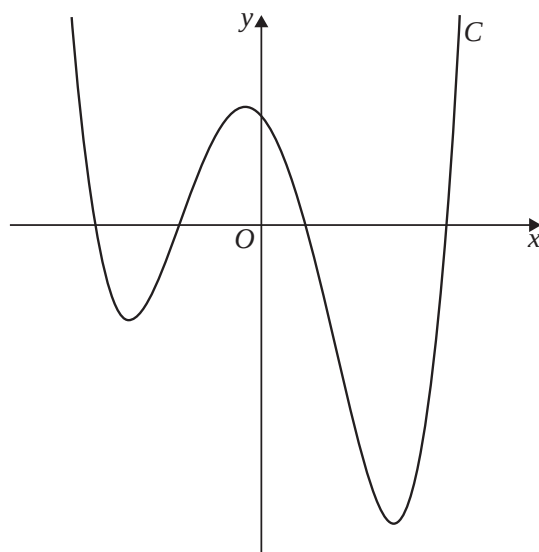


Figure 2

$$f(x) = x^4 + \frac{1}{3}x^3 - 8x^2 + ax + \frac{17}{3}$$

where a is a constant.

Figure 2 shows a sketch of the curve C with equation $y = f(x)$

Given that C has a local maximum at $x = -\frac{1}{4}$

(a) show that $a = -4$ (4)

(b) find the exact y coordinate of the local maximum. (1)

The equation $f(x) = k$, where k is a constant, has 4 distinct solutions.

(c) Using algebra and showing all stages of your working, find the range of values of k .
Give the answer using set notation.

(Solutions relying on calculator technology are not acceptable.)

(4)

SUB-TOPIC 2.8

COMPOSITE AND INVERSE FUNCTIONS

Sub-topic 2.8 · 1 question · 8 marks · lowest tariff first



12.

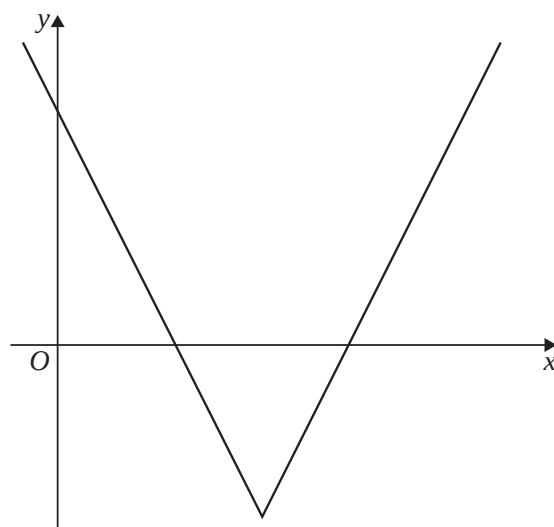


Figure 3

Figure 3 shows a sketch of the graph with equation $y = f(x)$ where

$$f(x) = 4|x - 3| - 5 \quad x \in \mathbb{R}$$

Given that a is a constant and $|a| = 1$

- (a) find the possible values of $f(a)$ (2)

The function g is defined by

$$g(x) = 2x + 17 \quad x \in \mathbb{R}$$

- (b) Find the range of $gf(x)$ (2)

The function h is defined by

$$h(x) = kx \quad x \in \mathbb{R}$$

where k is a constant.

Given that the equation $f(x) = h(x)$ has no solutions,

- (c) find the range of values of k . (4)





(Total for Question 12 is 6 marks)



SUB-TOPIC 2.9

TRANSFORMATIONS OF GRAPHS

Sub-topic 2.9 · 1 question · 4 marks · lowest tariff first





SUB-TOPIC 2.10

PARTIAL FRACTIONS

Sub-topic 2.10 · 5 questions · 44 marks · lowest tariff first







(Total for Question 10 is 7 marks)







(Total for Question 14 is 10 marks)









12. (a) Express $\frac{1}{V(25-V)}$ in partial fractions.

(2)

The volume, V microlitres, of a plant cell t hours after the plant is watered is modelled by the differential equation

$$\frac{dV}{dt} = \frac{1}{10}V(25 - V)$$





(Total for Question 12 is 12 marks)



(Total for Question 7 is 7 marks)



SUB-TOPIC 2.11

FUNCTIONS IN MODELLING

Sub-topic 2.11 · 5 questions · 41 marks · lowest tariff first







(Total for Question 8 is 7 marks)



9.

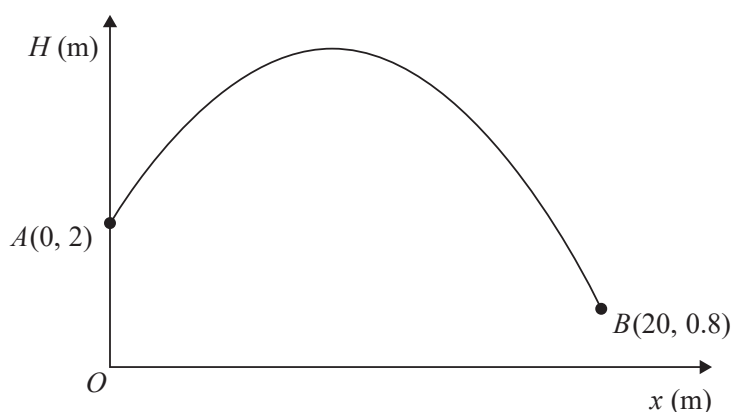


Figure 3

The graph in Figure 3 shows the path of a small ball.

The ball travels in a vertical plane above horizontal ground.

The ball is thrown from the point represented by A and caught at the point represented by B .

The height, H metres, of the ball above the ground has been plotted against the horizontal distance, x metres, measured from the point where the ball was thrown.

With respect to a fixed origin O , the point A has coordinates $(0, 2)$ and the point B has coordinates $(20, 0.8)$, as shown in Figure 3.

The ball reaches its maximum height when $x = 9$

A quadratic function, linking H with x , is used to model the path of the ball.

(a) Find H in terms of x .

(4)

(b) Give one limitation of the model.

(1)

Chandra is standing directly under the path of the ball at a point 16 m horizontally from O .

Chandra can catch the ball if the ball is less than 2.5 m above the ground.

(c) Use the model to determine if Chandra can catch the ball.

(2)





(Total for Question 9 is 7 marks)



12.

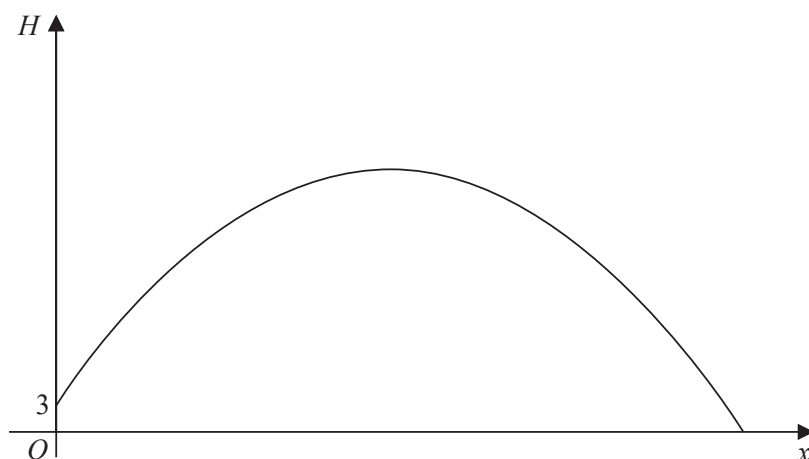


Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

- the maximum vertical height of the ball above the ground,
- the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)





(Total for Question 12 is 9 marks)







11. A company is trying to determine the most profitable selling price for a new toy.

Given that

- if the selling price of each toy is £30, the company expects to sell 1500 toys in one year
- if the selling price of each toy is £50, the company expects to sell 300 toys in one year

Using a **linear** model, with y being the expected number of toys sold in one year and x pounds being the selling price of the toy,

(a) find an equation for y in terms of x .

(3)

Given that

- the cost of making each toy is £10
 - the company has additional costs of £8 000 per year
- (b) show that, according to the model, the yearly profit, P , in **thousands** of pounds, is given by

$$P = -0.06x^2 + 3.9x - 41$$

(3)

Use the model given in part (b) to answer parts (c), (d) and (e).

Given that the company wishes to make a profit on sales of the toy,

(c) find the range of possible selling prices of the toy.

(2)

(d) Hence, or otherwise, deduce the selling price of the toy that maximises the profit.

(1)

In one particular year, the company sold the toy for £35 and made £21 750 profit.

(e) Use this information to evaluate the suitability of the model.

(2)







Question 11 continued

(Total for Question 11 is 11 marks)



SUB-TOPIC 3.2

CIRCLES

Sub-topic 3.2 · 2 questions · 16 marks · lowest tariff first





Question 14 continued



(Total for Question 7 is 9 marks)



SUB-TOPIC 4.1

BINOMIAL EXPANSION

Sub-topic 4.1 · 7 questions · 46 marks · lowest tariff first



4. In the binomial expansion of

$$(a + 2x)^7 \quad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a .

(3)

Question 4 continued

(Total for Question 4 is 3 marks)



$$(1 - 9x)^{\frac{1}{2}}$$

(3)

(1)



(Total for Question 2 is 4 marks)



$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

(Total for Question 1 is 5 marks)



(Total for Question 7 is 5 marks)



(Total for Question 4 is 6 marks)





Question 11 continued

(Total for Question 11 is 10 marks)



Question 13 continued



SUB-TOPIC 10.4

POSITION VECTORS AND DISTANCES

Sub-topic 10.4 · 1 question · 6 marks · lowest tariff first





Question 13 continued

Lined area for writing the answer to Question 13 continued.

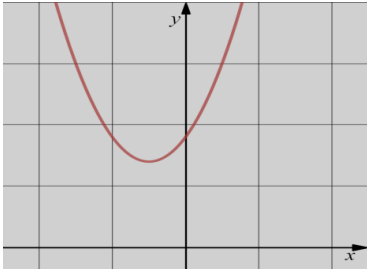
(Total for Question 13 is 6 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
5 (a)	$2x^2 + 4x + 9 = 2(x \pm k)^2 \pm \dots$ $a = 2$	B1	1.1b
	Full method $2x^2 + 4x + 9 = 2(x+1)^2 \pm \dots$ $a = 2$ & $b = 1$	M1	1.1b
	$2x^2 + 4x + 9 = 2(x+1)^2 + 7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0) y - intercept at (0,9) Minimum at (-1,7)	B1	1.2
		B1	1.1b
		B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	A1	2.2a
	(ii) $h(x) = \frac{21}{2(x+1)^2 + 7} \Rightarrow$ (maximum) value $\frac{21}{7} (= 3)$	M1	3.1a
	$0 < h(x) \leq 3$	A1ft	1.1b
		(4)	
(10 marks)			

(a)

B1: Achieves $2x^2 + 4x + 9 = 2(x \pm k)^2 \pm \dots$ or states that $a = 2$
M1: Deals correctly with first two terms of $2x^2 + 4x + 9$.

 Scored for $2x^2 + 4x + 9 = 2(x+1)^2 \pm \dots$ or stating that $a = 2$ and $b = 1$
A1: $2x^2 + 4x + 9 = 2(x+1)^2 + 7$

 Note that this may be done in a variety of ways including equating $2x^2 + 4x + 9$ with the expanded form of $a(x+b)^2 + c \equiv ax^2 + 2abx + ab^2 + c$

(b)

B1: For a U-shaped curve in any position not passing through $(0,0)$. Be tolerant of slips of the pen but do not allow if the curve bends back on itself

B1: A curve with a y -intercept on the +ve y axis of 9. The curve cannot just stop at $(0,9)$

Allow the intercept to be marked 9, $(0,9)$ but not $(9,0)$

B1ft: For a minimum at $(-1,7)$ in quadrant 2. This may be implied by -1 and 7 marked on the axes in the correct places. The curve must be a U shape and not a cubic say.

Follow through on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2 + c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $g(x) = f(x-2) - 4$ can score M1

For example, possible methods of arriving at this deduction are:

- $f(x) \rightarrow g(x)$ is $2x^2 + 4x + 9 \rightarrow 2(x-2)^2 + 4(x-2) + 5$ So $g(x) = f(x-2) - 4$
- $g(x) = 2(x-1)^2 + 3$ New curve has its minimum at $(1,3)$ so $(-1,7) \rightarrow (1,3)$
- Using a graphical calculator to sketch $y=g(x)$ and compares to the sketch of $y=f(x)$

In almost all cases you will not allow if the candidate gives two **different types of** transformations.

Eg, stretch and

A1: Requires both 'translate' and $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$, Allow 'shift' or move' instead of translate.

So condone " Move shift 2 (units) to the right and move 4 (units) down

However, for M1 A1, it is possible to reflect in $x=0$ and translate $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$, so please consider all responses.

SC: If the candidate writes translate $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ or " move 2 (units) to the left and 4 (units) up" score M1 A0

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

- Uses part (a) to write $h(x) = \frac{21}{2(x+1)^2 + 7}$ and attempts to find $\frac{21}{\text{their "7"}}$
- Attempts to differentiate, sets $4x+4=0 \rightarrow x=-1$ and substitutes into $h(x) = \frac{21}{2x^2 + 4x + 9}$
- Uses a graphical calculator to sketch $y=h(x)$ and establishes the 'maximum' value $(...,3)$

A1ft: $0 < h(x) \leq 3$ Allow for $0 < h \leq 3$ $[0,3]$ and $0 < y \leq 3$ but not $0 < x \leq 3$

Follow through on their $a(x+b)^2 + c$ so award for $0 < h(x) \leq \frac{21}{c}$



Question	Scheme	Marks	AOs
7	Attempts equation of line Eg Substitutes $(-2, 13)$ into $y = mx + 25$ and finds m	M1	1.1b
	Equation of l is $y = 6x + 25$	A1	1.1b
	Attempts equation of C Eg Attempts to use the intercept $(0, 25)$ within the equation $y = a(x \pm 2)^2 + 13$, in order to find a	M1	3.1a
	Equation of C is $y = 3(x+2)^2 + 13$ or $y = 3x^2 + 12x + 25$	A1	1.1b
	Region R is defined by $3(x+2)^2 + 13 < y < 6x + 25$ o.e.	B1ft	2.5
		(5)	
(5 marks)			
Notes:			

The first two marks are awarded for finding the equation of the line

M1: Uses the information in an attempt to find an equation for the line l .

E.g. Attempt using two points: Finds $m = \pm \frac{25-13}{2}$ and uses of one of the points in their $y = mx + c$ or equivalent to find c . Alternatively uses the intercept as shown in main scheme.

A1: $y = 6x + 25$ seen or implied. This alone scores the first two marks. Do not accept $l = 6x + 25$

It must be in the form $y = \dots$ but the correct equation can be implied from an inequality. E.g. $\dots < y < 6x + 25$

The next two marks are awarded for finding the equation of the curve

M1: A complete method to find the constant a in $y = a(x \pm 2)^2 + 13$ or the constants a, b in $y = ax^2 + bx + 25$.

An alternative to the main scheme is deducing equation is of the form $y = ax^2 + bx + 25$ and setting and solving a pair of simultaneous equations in a and b using the point $(-2, 13)$ the gradient being 0 at $x = -2$. Condone slips. Implied by $C = 3x^2 + 12x + 25$ or $3x^2 + 12x + 25$

FYI the correct equations are $13 = 4a - 2b + 25$ ($2a - b = -6$) and $-4a + b = 0$

A1: $y = 3(x+2)^2 + 13$ or equivalent such as $y = 3x^2 + 12x + 25$, $f(x) = 3(x+2)^2 + 13$.

Do not accept $C = 3x^2 + 12x + 25$ or just $3x^2 + 12x + 25$ for the A1 but may be implied from an inequality or from an attempt at the area, E.g. $\int 3x^2 + 12x + 25 \, dx$

B1ft: Fully defines the region R . Follow through on their equations for l and C .

Allow strict or non-strict inequalities as long as they are used consistently.

E.g. Allow for example " $3(x+2)^2 + 13 < y < 6x + 25$ " " $3(x+2)^2 + 13 \leq y \leq 6x + 25$ "

Allow the inequalities to be given separately, e.g. $y < 6x + 25, y > 3(x+2)^2 + 13$. Set notation may be used so

$\{(x, y) : y > 3(x+2)^2 + 13\} \cap \{(x, y) : y < 6x + 25\}$ is fine but condone with or without any of $(x, y) \leftrightarrow y \leftrightarrow x$

Incorrect examples include " $y < 6x + 25$ or $y > 3(x+2)^2 + 13$ ", $\{(x, y) : y > 3(x+2)^2 + 13\} \cup \{(x, y) : y < 6x + 25\}$

Values of x could be included but they must be correct. So $3(x+2)^2 + 13 < y < 6x + 25, x < 0$ is fine

If there are multiple solutions mark the final one.

Question	Scheme	Marks	AOs
4 (a)	States or uses $a = 4$	B1	1.1a
	Valid method to find $Q(x)$	M1	2.1
	$(x+4)(2x^2 - 5x + 4)$	A1	1.1b
		(3)	
(b)	Attempts to show that their $2x^2 - 5x + 4$ does not have any (real) roots	M1	3.1a
	Correct calculations, reason and conclusion	A1	2.1
		(2)	
(5 marks)			
Notes:			
(a) B1: States or uses $a = 4$ e.g. may be seen in their attempt at dividing algebraically by $x + 4$ M1: Attempts to divide $f(x)$ by $(x + 4)$ to find a three-term quadratic $Q(x)$. There are various methods or ways to present their solution so typically methods - by inspection look for $2x^3 + 3x^2 - 16x + 16 = (x + 4)(2x^2 + \dots x \pm 4)$ - by division look for a quadratic quotient of $2x^2 - 5x \pm \dots$ A1: $(x + 4)(2x^2 - 5x + 4)$ condone the missing trailing bracket i.e. $(x + 4)(2x^2 - 5x + 4$ isw if they attempt to factorise their quadratic factor. Allow to be scored if seen in (b). (b) M1: Attempts to show that their three-term quadratic “$2x^2 - 5x + 4$” does not have any roots: Attempts the discriminant e.g. $b^2 - 4ac = 25 - 4 \times 2 \times 4 (= -7)$ (may be embedded in the quadratic formula) Attempts to use the quadratic formula e.g. $\left(x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 4}}{4}\right)$ but do not allow directly from a calculator $\frac{5 \pm \sqrt{7}i}{4}$ Attempts to complete the square e.g. $2x^2 - 5x + 4 = 2\left(x^2 - \frac{5}{2}x\right) + 4 = 2\left(x - \frac{5}{4}\right)^2 + \dots \left(= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 4\right)$ Uses calculus to find the turning point e.g. $\frac{d(2x^2 - 5x + 4)}{dx} = 4x - 5 = 0 \Rightarrow x = \frac{5}{4} \Rightarrow y = \dots$ Note that any attempts using the discriminant or quadratic formula must have the values embedded in the correct places (may be partially evaluated) to score M1			

A1: **Dependent on a correct** $Q(x) = 2x^2 - 5x + 4$

Fully correct argument that requires:

- Fully correct work
- A justification depending on strategy and no incorrect reasoning seen
- A conclusion

Examples below – Note we must see working before they proceed to a correct root or minimum value – see M1 for guidance

Strategy	Correct work examples	Justification examples	Conclusion examples
Via discriminant	$b^2 - 4ac = -7$	$-7 < 0$ -7 so no (real) roots but NOT $-7 \neq 0$ so no roots	so “-4 is the only (real) root” / “only one (real) root”
Via using the quadratic formula	$x = \frac{5 \pm \sqrt{7}i}{4}$ or $x = \frac{5 \pm \sqrt{-7}}{4}$	$-7 < 0$ / which is not possible / complex roots o.e. / cannot square root a negative / no (real) roots	
Via completing the square	$2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8}$ or $2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8} = 0 \Rightarrow$ $2\left(x - \frac{5}{4}\right)^2 = -\frac{7}{8}$	which has a minimum value of $\frac{7}{8}$ / minimum (of the positive quadratic) is above the x-axis $-\frac{7}{8} < 0$ / cannot square root a negative / no (real) roots	
Via calculus	$x = \frac{5}{4} \Rightarrow y = \frac{7}{8}$	which has a minimum value of $\frac{7}{8}$ / minimum (of the positive quadratic) is above the x-axis	

Note that it is possible to justify and conclude in one step by using phrases e.g. “**no more** (real) roots” or “**no other** (real) roots”

e.g. $2x^2 - 5x + 4 \Rightarrow b^2 - 4ac = 25 - 32 = -7$ so no more roots which scores M1A1

Condone $25 - 32 < 0$ as a justification that the quadratic has no real roots

Condone the conclusion -4 is the only (real) solution (instead of (real) root).

Note if $(x + 4)$ is described as a root or -4 is described as a factor this scores A0



Question	Scheme	Marks	AOs
1	$2x(x^2 - 12x + 20)$ or $x(2x^2 - 24x + 40)$	B1	1.1b
	$x^2 - 12x + 20 = (x-2)(x-10)$ or e.g. $2x^2 - 24x + 40 = (2x-4)(x-10)$	M1	1.1b
	$2x(x-2)(x-10)$	A1	1.1b
		(3)	
(3 marks)			
Notes			
B1: Takes a factor of $2x$ or x out of the given cubic correctly. Dividing by 2 and then achieving $x(x^2 - 12x + 20)$ does not score this mark unless recovered. M1: Attempts to factorise their quadratic. Invisible brackets may be implied by later work. Score for $(x \pm b)(x \pm d)$ where $bd = 20$ coming from $x^2 \pm 12x \pm 20$ or for $(ax \pm b)(cx \pm d)$ where $ac = 2$ and $bd = 40$ coming from $2x^2 \pm 24x \pm 40$ May be scored if they have divided by x but not from e.g. $2x^3 - 24x^2 + 40x \rightarrow (2x-4)(x-10)$ without clear indication that they have divided by or taken out a factor of x or $2x$. There may be incorrect intermediate steps such as $(2x-20)(2x-4)$ which do not score the mark on their own but may be ignored if they return to e.g. $(2x-20)(x-2)$. A1: $2x(x-2)(x-10)$ or e.g. $2(x-10)(x-2)x$. Do not accept e.g. $x(2x-4)(x-10)$. Ignore = 0 ISW after a fully correct factorisation e.g. $2x(x-2)(x-10)$ that becomes $x(x-2)(x-10)$ and ignore any attempt to find roots (before or after factorisation). Allow $(2x)(x-2)(x-10)$ Alternative: B1: Takes a factor of $(x-2)$ or $(x-10)$ out correctly i.e. $(x-2)(2x^2 - 20x)$ or $(x-10)(2x^2 - 4x)$. Must be seen as a product of factors and not just in a division attempt but may be implied by later work. M1: Attempts to factorise their quadratic. Invisible brackets may be implied by later work. Score for $ax(bx \pm c)$ where $ab = "2"$ and $ac = \text{their "20" or "4"}$ coming from $Ax^2 + Bx$ A1: As main scheme. Note: Solutions that solve the cubic = 0 to achieve $x = 0, 2$ and 10 and then arrive at e.g. $x(x-2)(x-10)$ score no marks without a prior line of working such as $x(x^2 - 12x + 20)$ (would score M1) or $x(2x^2 - 24x + 40)$ (would score B1). Some examples: $2x^3 - 24x^2 + 40x \rightarrow x(x-2)(x-10)$ scores B0M0A0 $2x^3 - 24x^2 + 40x \rightarrow (2x-4)(x-10)$ scores B0M0A0 $x^2 - 12x + 20 \rightarrow (x+2)(x-10)$ scores B0M1A0 $x^3 - 12x^2 + 20x \rightarrow x(x-2)(x-10)$ scores B0M1A0 $x(2x^2 - 24x + 40) \rightarrow x(x-2)(x-10)$ scores B1M0A0 $x(2x^2 - 24x + 40) \rightarrow \{ x(2x+2)(2x+40) \rightarrow \} x(2x+2)(x+20)$ scores B1M1A0 $2x(x^2 - 12x + 20) \rightarrow 2x(x-1)(x+20)$ scores B1M1A0 $2x(x-2)(x-10)$ on its own scores B1M1A1			

Question	Scheme	Marks	AOs
13 (a)	(i) Explains $2x - q = 0$ when $x = 2$ oe Hence $q = 4$ *	B1*	2.4
	(ii) Substitutes $\left(3, \frac{1}{2}\right)$ into $y = \frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2} = \frac{p-9}{(2) \times (6)} \Rightarrow p-9=6 \Rightarrow p=15$ *	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x .	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)} = \frac{A}{(2x-4)} + \frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)} = \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)} - \frac{2.4}{(x+3)}$ oe	A1	1.1b
	$I = \int \frac{15-3x}{(2x-4)(x+3)} dx = m \ln(2x-4) + n \ln(x+3) + (c)$	M1	1.1b
	$I = \int \frac{15-3x}{(2x-4)(x+3)} dx = 0.9 \ln(2x-4) - 2.4 \ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5 \frac{15-3x}{(2x-4)(x+3)} dx$ Or $[.....]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6 = \ln 2 + \ln 3$ or $\ln 8 = 3 \ln 2$ $[0.9 \ln(6) - 2.4 \ln(8)] - [0.9 \ln(2) - 2.4 \ln(6)]$ $= 3.3 \ln 6 - 7.2 \ln 2 - 0.9 \ln 2$	dM1	2.1
	$= 3.3 \ln 3 - 4.8 \ln 2$	A1	1.1b
		(8)	
(11marks)			

(a)**B1*:** Is able to link $2x - q = 0$ and $x = 2$ to explain why $q = 4$ Eg "The asymptote $x = 2$ is where $2x - q = 0$ so $4 - q = 0 \Rightarrow q = 4$ ""The curve is not defined when $2 \times 2 - q = 0 \Rightarrow q = 4$ "There **must be some words** explaining why $q = 4$ and in most cases, you should see a reference to either "the asymptote $x = 2$ ", "the curve is not defined at $x = 2$ ", 'the denominator is 0 at $x = 2$ '**M1:** Substitutes $\left(3, \frac{1}{2}\right)$ into $y = \frac{p-3x}{(2x-4)(x+3)}$ and solvesAlternatively substitutes $\left(3, \frac{1}{2}\right)$ into $y = \frac{15-3x}{(2x-4)(x+3)}$ and shows $\frac{1}{2} = \frac{6}{(2) \times (6)}$ oe**A1*:** Full proof showing all necessary steps $\frac{1}{2} = \frac{p-9}{(2) \times (6)} \Rightarrow p-9=6 \Rightarrow p=15$ In the alternative there would have to be some recognition that these are equal eg ✓ hence $p = 15$ **(b)****M1:** Scored for an overall attempt at using PF's and integrating with lns seen with sight of limits 3 and another value of x .

Question	Scheme	Marks	AOs
7 (a)	Either $x \leq -1$ or $2 \leq x \leq 5$	M1	2.2a
	Both $\{x : x \in \mathbb{R}, x \leq -1\} \cup \{x : x \in \mathbb{R}, 2 \leq x \leq 5\}$ o.e.	A1	2.5
		(2)	
(b)	States $(y =) \alpha(x+1)^2(x-5)^2$ or $(f(x) =) \alpha(x+1)^2(x-5)^2$	M1	1.1b
	Substitutes $(0, -75)$ into $y = \alpha(x+1)^2(x-5)^2$ and attempts to find the value for α	dM1	3.1a
	$y = -3(x+1)^2(x-5)^2$ o.e.	A1	2.1
		(3)	
(c)	Substitutes $x = 2$ into their $y = -3(x+1)^2(x-5)^2 \Rightarrow y = (-243)$	M1	2.1
	$0 < k < 243$	A1ft	1.1b
		(2)	

(7 marks)

Notes:**(a)**

M1: Either

- $x \leq -1$ o.e. e.g. $-1 \geq x$
- $2 \leq x \leq 5$ o.e.

but condone use of strict inequalities anywhere for this mark.

e.g. $2 < x < 5$ or $2 < x \leq 5$ or $2 \leq x < 5$ May also write e.g. $x < 5$ and $x > 2$ which scores M1 but not " $x < 5$ or $x > 2$ "Allow interval notation such as e.g. $[2, 5]$ or $(-\infty, -1]$ or condone e.g. $(2, 5)$

Ignore incorrect inequality statements not related to the one which is valid.

e.g. " $2 \leq x < 5$ and $x > -1$ " which scores M1 for the first inequality.A1: Requires $\{ \}$ and $\cup$ $\{x : x \leq -1\} \cup \{x : 2 \leq x \leq 5\}$ or $\{x | x \leq -1\} \cup \{x | 2 \leq x \leq 5\}$ either way roundbut condone $\{x \leq -1\} \cup \{2 \leq x \leq 5\}$, $\{x \leq -1 \cup 2 \leq x \leq 5\}$.Allow e.g. $\{x : x \leq -1\} \cup \{x : 2 \leq x \cap x \leq 5\}$ Use of $\cap$ to join the two separate regions is A0It is acceptable (but not required) to mention $\mathbb{R}$ e.g. $\{x : x \in \mathbb{R}, x \leq -1\} \cup \{x : x \in \mathbb{R}, 2 \leq x \leq 5\}$ Condone use of a lower limit written as e.g. $\{x : -\infty \leq x \leq -1\} \cup \{x : 2 \leq x \leq 5\}$ 

(b) Note a correct equation written down scores all 3 marks.
A correct expression but missing e.g. $y = \dots$ or $f(x) = \dots$ scores M1dM1A0

M1: Forms the equation of the form $(y =) \alpha(x+1)^2(x-5)^2$. Condone $\alpha = 1$

Award for sight of $\alpha(x+1)^2(x-5)^2$ even with $\alpha = 1$ i.e. $(x+1)^2(x-5)^2$

dM1: Substitutes $(0, -75)$ into the form $y = \alpha(x+1)^2(x-5)^2$ and attempts to find the value for α . It is dependent on the previous method mark.

A1: $y = -3(x+1)^2(x-5)^2$ o.e. (e.g. $y = -3x^4 + 24x^3 - 18x^2 - 120x - 75$)

isw after a correct answer. Condone $f(x) = -3(x+1)^2(x-5)^2$ but not

$$C = -3(x+1)^2(x-5)^2$$

A correct equation scores all 3 marks. Allow if seen in (c)

isw if they attempt to multiply out.

Alternative I part (b):

Using the form $y = ax^4 + bx^3 + cx^2 + dx + e$, then setting up and solving simultaneous equations.

There are various versions of this but can be marked similarly.

M1: Sets e equal to -75 (may just be seen in their equation) and forms **three correct** different equations in a, b, c and d which may be unsimplified.

Note that the form $y = ax^4 + bx^3 + cx^2 + dx + e$ is M0 until e is set equal to -75

There are 5 equations that can be formed, only 3 are necessary for this mark.

Do not condone slips.

Using $(-1, 0) \quad \Rightarrow 0 = a - b + c - d - 75 \text{ o.e.}$

Using $(5, 0) \quad \Rightarrow 0 = 625a + 125b + 25c + 5d - 75 \text{ o.e.}$

Using $\frac{dy}{dx} = 0$ at $x = 2 \quad \Rightarrow 0 = 32a + 12b + 4c + d \text{ o.e.}$

Using $\frac{dy}{dx} = 0$ at $x = -1 \quad \Rightarrow 0 = -4a + 3b - 2c + d \text{ o.e.}$

Using $\frac{dy}{dx} = 0$ at $x = 5 \quad \Rightarrow 0 = 500a + 75b + 10c + d \text{ o.e.}$

dM1: Forms **four correct** different equations and solves to find values for a, b, c and d . You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y = -3x^4 + 24x^3 - 18x^2 - 120x - 75$ o.e. isw if they attempt to factorise but withhold this mark if they e.g. divide all terms by 3.

Condone $f(x) = \dots$ but not $C = \dots$

A correct equation scores all 3 marks. Allow if seen in (c)

Alternative II part (b): Uses the form $y = (x+1)(x-5)(ax^2 + bx + c)$

M1: Substitutes $x = 0, y = -75$ $-75 = -5c \Rightarrow c = 15$, multiplies out, differentiates

$$\Rightarrow \frac{dy}{dx} = (2x-4)(ax^2 + bx + 15) + (x^2 - 4x - 5)(2ax + b)$$

and forms a **correct equation** in a and b which may be unsimplified.

$$\text{Using } \frac{dy}{dx} = 0 \text{ at } x = 2 \quad \Rightarrow 0 = 4a + b \quad \text{o.e.}$$

$$\text{Using } \frac{dy}{dx} = 0 \text{ at } x = -1 \quad \Rightarrow 0 = a - b + 15 \quad \text{o.e.}$$

$$\text{Using } \frac{dy}{dx} = 0 \text{ at } x = 5 \quad \Rightarrow 0 = 5a + b + 3 = 0 \quad \text{o.e.}$$

dm1: Forms **two correct** different equations and solves to find values for a and b . You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y = (x+1)(x-5)(-3x^2 + 12x + 15)$ o.e. isw if they attempt to multiply out or factorise
Condone $f(x) = \dots$ but not $C = \dots$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

Alternative III part (b): Uses $\frac{dy}{dx} = \beta(x+1)(x-2)(x-5)$ (β may be 1) and integrates.

M1: Integrates $\left(\frac{dy}{dx} = \beta(x+1)(x-2)(x-5)\right)$ to $(y =) \beta\left(\frac{1}{4}x^4 - 2x^3 + \frac{3}{2}x^2 + 10x + k\right)$ and forms

one correct equation using either $(0, -75): -75 = \beta k$ (allow $-75 = k$)

$$(-1, 0): 0 = \beta\left(\frac{1}{4} + 2 + \frac{3}{2} - 10 + k\right) \quad (5, 0): 0 = \beta\left(\frac{625}{4} - 250 + \frac{75}{2} + 50 + k\right)$$

dm1: Forms a different equation using one of $(0, -75), (-1, 0), (5, 0)$ and solves to find values for β and k . You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y = -12\left(\frac{1}{4}x^4 - 2x^3 + \frac{3}{2}x^2 + 10x + \frac{25}{4}\right)$ o.e. isw if they attempt to multiply out or factorise

Condone $f(x) = \dots$ but not $C = \dots$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

(c)

M1: Substitutes $x = 2$ into their $y = -3(x+1)^2(x-5)^2$ (must be a quartic in any form) and proceeds to find a value for y . Sight of their $\pm y$ (or ± 243) scores M1.
You may need to check this on your calculator if only a value is seen.

A1ft: $0 < k < 243$ o.e. ft on their negative y value at $x = 2$.

Allow use of set notation, interval notation and allow e.g. $k < 243, k > 0$ but do not allow OR or $\cup$. Do not accept $0 \leq k \leq 243$ o.e.

If there are multiple attempts at describing the region, mark what appears to be their final answer.

This mark can only be scored if they have a negative quartic graph function

i.e. $\alpha < 0$ for their $y = \alpha(x+1)^2(x-5)^2$ or $a < 0$ for their $y = ax^4 + bx^3 + cx^2 + dx + e$

Question	Scheme	Marks	AOs
12 (a)	e.g. Substitutes $x = 6$ into both $(y =) \frac{15x}{(2x+3)(x-3)}$ and $(y =) 2x - 10$ and finds the y values for both	M1	1.1b
	e.g. Achieves 2 for both and makes a valid conclusion *	A1*	2.4
		(2)	
(b)	Sets $\frac{15x}{(2x+3)(x-3)} = 2x - 10$ and attempts to cross multiply	M1	1.1b
	$4x^3 - 26x^2 - 3x + 90 = 0$ *	A1*	2.1
		(2)	
(c)	Deduces that $(x - 6)$ is a factor and attempts to divide	M1	2.1
	$4x^3 - 26x^2 - 3x + 90 = (x - 6)(4x^2 - 2x - 15)$	A1	1.1b
	Solves their $4x^2 - 2x - 15$ using suitable method	M1	1.1b
	Deduces $x = \frac{1 - \sqrt{61}}{4}$ (see note)	A1	2.2a
		(4)	
(8 marks)			
Notes:			
(a) Must be seen in (a) to score M1: Examples to verify include: - substitutes $x = 6$ into both $(y =) \frac{15x}{(2x+3)(x-3)}$ and $(y =) 2x - 10$ and finds the y values for both - substitutes $x = 6$ into one of the two equations to find $y = 2$ and then substitutes this into the other equation and solves to find x - sets $\frac{15x}{(2x+3)(x-3)} = 2x - 10 \Rightarrow$ cubic equation ($=0$ implied) and either substitutes $x = 6$ into the expression, attempts $f(6)$ or else attempts to divide the cubic $= 0$ by $(x - 6)$. Condone without calculations for this mark only. i.e. $f(6) = 0$ Condone slips for this mark in any of the outlined approaches. There may be variations of these as well. A1*: Correct calculations must be seen with a minimal conclusion that they intersect e.g. $\frac{15 \times 6}{(2 \times 6 + 3)(6 - 3)} = 2$ and $12 - 10 = 2$ so they intersect (allow eg meet/cross/intersect) e.g. $\frac{15 \times 6}{(2 \times 6 + 3)(6 - 3)} = 2 \Rightarrow 2 = 2x - 10 \Rightarrow x = 6$ so they intersect Acceptable alternatives are: $f(x) = 4x^3 - 26x^2 - 3x + 90$, $f(6) = 4 \times 6^3 - 26 \times 6^2 - 3 \times 6 + 90 = 0 \Rightarrow$ so they intersect $f(x) = 4x^3 - 26x^2 - 3x + 90 \Rightarrow (x - 6)(4x^2 - 2x - 15)$ so $x = 6$ is a root so they intersect OR $(x - 6)$ is a factor hence they intersect Only accept verified, QED etc if there is a preamble mentioning intersection about how it will be shown.			

Special case: ($f(x) = 4x^3 - 26x^2 - 3x + 90 = 0$ o.e.) $\Rightarrow$ awrt -1.7, 6, awrt 2.2 Scores M1A0 with or without a conclusion. Requires all 3 roots.

(b) Work seen in (a) must be used in (b) to score

M1: Sets $\frac{15x}{(2x+3)(x-3)} = 2x-10$ and attempts to cross multiply to form a cubic equation.

Score for $15x = (2x-10)(2x+3)(x-3)$ or may be implied by an attempt to multiply two of the three brackets out e.g. $15x = (2x-10)(2x^2 - 3x - 9)$ or

$15x = (x-3)(4x^2 - 14x - 30)$ or $15x = (2x+3)(2x^2 - 16x + 30)$ (may be unsimplified)

Condone slips provided the intention is clear.

Condone invisible brackets to be implied by further work which is not the given answer.

Do not allow this mark to be scored for proceeding straight from $\frac{15x}{(2x+3)(x-3)} = 2x-10$

to a simplified expression of $15x = 4x^3 - 26x^2 + 12x + 90$

A1*: Achieves the given answer with sufficient intermediate steps seen and **no errors including invisible brackets** in the main body of their solution (ignore “side workings”) Expect to see on the right hand side the product of a linear and a quadratic expression and an expression with all of the brackets multiplied out before simplifying to the given answer.

e.g.

$$\frac{15x}{(2x+3)(x-3)} = 2x-10 \Rightarrow 15x = (4x^2 - 14x - 30)(x-3) \Rightarrow 15x = 4x^3 - 26x^2 + 12x + 90 \\ \Rightarrow 4x^3 - 26x^2 - 3x + 90 = 0 \text{ * which is A1*}$$

(c) Work seen in (a) or (b) must be used in (c) to score

M1: For the key step in dividing the cubic by $(x-6)$. There are various methods or ways to present their solution so typically methods

- by inspection look for first and last terms e.g. $(x-6)(4x^2 \pm \dots x \pm 15)$
- by division look for a three term quadratic quotient of $4x^2 - 2x \pm \dots$

A1: $4x^3 - 26x^2 - 3x + 90 = (x-6)(4x^2 - 2x - 15)$ This may be implied by sight of $(4x^2 - 2x - 15)$ in their working.

M1: **Whilst not dependent on the previous method mark, it does require an attempt at dividing by $(x-6)$ however poor to achieve a three term quadratic factor**

Solves their $4x^2 - 2x - 15 = 0$ using either the quadratic formula or completing the square. Usual rules apply for solving a quadratic. **It cannot be scored for stating the roots directly via use of a calculator which is M0A0**

A1: $(x =) \frac{1-\sqrt{61}}{4}$ or exact equivalent e.g. $\frac{1}{4} - \sqrt{\frac{61}{16}}$

The root $(x =) \frac{1+\sqrt{61}}{4}$ must be rejected if found. Isw if they put their exact value in decimal form.



Question	Scheme	Marks	AOs
8(a)	$x^4 \rightarrow \dots x^3$ or $\dots x^3 \rightarrow \dots x^2$ or $\dots x^2 \rightarrow \dots x$ or $ax \rightarrow a$	M1	1.1b
	$(f'(x) =) 4x^3 + x^2 - 16x + a$	A1	1.1b
	$\left(f'\left(-\frac{1}{4}\right) =\right) 4\left(-\frac{1}{4}\right)^3 + \left(\pm\frac{1}{4}\right)^2 - 16\left(-\frac{1}{4}\right) + a = 0$ $\Rightarrow a = \dots$	dM1	3.1a
	$a = -4^*$	A1*	2.1
		(4)	
(b)	$\frac{4735}{768}$	B1	1.1b
		(1)	

Notes

(a) **The first two marks are the same for all approaches.**

M1: Reduces the power by one for any term in $f(x)$. Look for $x^n \rightarrow x^{n-1}$ and allow for $ax \rightarrow a$ including $ax^1 \rightarrow ax^0$ or $-4x \rightarrow -4$

A1: Correct differentiation $(f'(x) =) 4x^3 + x^2 - 16x + a$ or $4x^3 + x^2 - 16x - 4$

Ignore the absence of the LHS. May be in terms of a or have -4 substituted at this point as above.

Main Scheme:

dM1: Substitutes $x = -\frac{1}{4}$ into their $f'(x)$, sets $= 0$ ($= 0$ may be implied) and solves for a .

Condone $4 + a = 0$ as evidence of substitution for the dM1 only.

This mark is not available if their $f'(x)$ does not have a term with a in it using this approach.

A1*: cso Requires:

- a correct derivative
- correct substitution of $x = -\frac{1}{4}$ into $f'(x)$ and set $= 0$ (now the $= 0$ must be seen somewhere for this mark). Each term does not need to be evaluated.
- achieves $a = -4$

For the A1*, evidence of correct substitution might be $-\frac{1}{16} + \frac{1}{16} + 4 + a = 0$ but not $4 + a = 0$

Condone invisible brackets around $-\frac{1}{4}$ provided this is recovered before the given answer e.g.

$-\frac{1}{4}^2$ recovered to $+\frac{1}{16}$

Alt 1: Verification

dM1: Substitutes $x = -\frac{1}{4}$ and $a = -4$ into their $f'(x)$, and finds a value.

The $a = -4$ may already have been substituted into $f(x)$ as above.

A1*: cso Requires:

- a correct derivative



- correct substitution of $x = -\frac{1}{4}$ into $f'(x)$ and use of $a = -4$ (at some stage) to achieve $f'\left(-\frac{1}{4}\right) = \dots = 0$. The substitution must be embedded in $f'(x)$ **or** evaluated in an intermediate step e.g. $-\frac{1}{16} + \frac{1}{16} + 4 - 4$ before the $= 0$
- a minimal conclusion e.g. “hence proven” or “ $\therefore a = -4$ ” or e.g. a tick. but **not** e.g. “max = $-\frac{1}{4}$ ” or just “ $0 = 0$ ”

Alt 2: Algebraic division

dM1: Attempts to divide their $f'(x)$ by $(4x+1)$ $\left(\text{or } \left(x + \frac{1}{4}\right)\right)$, achieves a linear remainder in a only, sets $= 0$ and achieves a value for a . Condone slips in their calculations. As a minimum, expect to see $x^2 + \lambda x + \mu$ (or $4x^2 + \lambda x + \mu$) as their quotient (with $\mu \neq 0$) leading to a linear remainder in a only set $= 0$ leading to a value for a .

A1*: cso Requires:

- a correct derivative
- correct quotient $x^2 - 4$ (or $(4x^2 - 16)$) and correct remainder $a + 4$ set $= 0$
- achieves $a = -4$

(b)

B1: $\frac{4735}{768}$ cao Allow exact equivalents e.g. $6\frac{127}{768}$ or the recurring decimal $6.16536458\dot{3}$
There is no need to see a calculation. Decimal approximations e.g. $6.1653645833\dots$ score B0.



Question	Scheme	Marks	AOs
8(c)	$(f'(x) =) 4x^3 + x^2 - 16x - 4 = (4x+1)(x^2 + \dots)$	M1	3.1a
	$= (4x+1)(x^2 - 4)$ or e.g. $\frac{4x^3 + x^2 - 16x - 4}{4x+1} = x^2 - 4$	A1	1.1b
	$f(-2) = \dots(-5)$ $\Rightarrow -5 < k < \frac{4735}{768}$	M1	2.1
	$\left\{ k \in \mathbb{R} : -5 < k < \frac{4735}{768} \right\}$	A1ft	2.5
		(4)	
(9 marks)			
Notes			
(c) Note: Candidates who do not score the first M mark may score maximum M0A0M1A1ft via the special case at the end of the mark scheme for part (c). M1: For the key step in using the factor theorem to take $(4x+1)$ or $\left(x + \frac{1}{4}\right)$ out as a factor of their $f'(x)$, not $f(x)$. Look for $(4x+1)(x^2 + \lambda x - 4)$ or $(4x+1)(x^2 \pm \mu)$ or $\left(x + \frac{1}{4}\right)(4x^2 \pm \mu)$ or $\left(x + \frac{1}{4}\right)(4x^2 \pm \lambda x - 16)$ (where the λ or μ is a constant) either by inspection or division. If they attempt division in (a) then they must use their quotient in (c) to score the marks. They may “spot” that one of $f'(\pm 2) = 0$ and factor out $(x \pm 2)(4x^2 + \lambda x \pm 2)$ without evidence that $f'(\pm 2) = 0$. There must be some factorisation or algebraic division present – not just roots stated from a calculator. A1: Correct factorisation of the cubic to a linear and quadratic product, so $(4x+1)(x^2 - 4)$ or $\left(x + \frac{1}{4}\right)(4x^2 - 16)$ or $(x-2)(4x^2 + 9x + 2)$ or $(x+2)(4x^2 - 7x - 2)$ or for the correct quadratic seen e.g. $x^2 - 4$ or $4x^2 - 16$ (which may be the quotient in their algebraic division and may come from their work in (a) provided it is used in (c)). There is no need to complete the long division if the appropriate quotient is found. Note that proceeding directly to $(4x+1)(x-2)(x+2)$ does not score either mark without sight of the e.g. $(4x+1)(x^2 - 4)$ but they can score the final 2 marks via the special case. Similarly, proceeding directly to $\left(x + \frac{1}{4}\right)(2x-4)(2x+4)$ or $4\left(x + \frac{1}{4}\right)(x-2)(x+2)$ does not score either mark without sight of $\left(x + \frac{1}{4}\right)(4x^2 - 16)$ but they can score marks via the SC. M1: Solves their quadratic factor = 0, substitutes one of these solutions (not $x = -\frac{1}{4}$) into $f(x)$ to find a value and selects the inside region between this and their answer to (b). If their quadratic is of the form $Ax^2 - B$ then they can write down their solution for x but if it is a 3TQ then they must find their value of x using the usual non-calculator rules. Condone “incorrect” factorisation following the correct quadratic leading to $x = \pm 2$			

e.g. $\left(x + \frac{1}{4}\right)(4x^2 - 16) \rightarrow \left(x + \frac{1}{4}\right)(x - 2)(x + 2) = 0 \rightarrow x = \pm 2$

y (or k) = -5 (or $-\frac{47}{3}$) with nothing else can imply substitution of $x = -2$ (or 2)

Note that if they “spot” that $f'(\pm 2) = 0$ earlier then they can go straight to substitution.

Condone the use of y but not x in their region. Condone e.g. $k > -5, k < \frac{4735}{768}$ for this mark.

Condone the use of $\leq$ in place of $<$ for this mark. Note that $\left(2, -\frac{47}{3}\right)$ is the other minimum.

A1ft: $\left\{k : -5 < k < \frac{4735}{768}\right\}$ Requires correct use of set notation but condone absence of $\in \mathbb{R}$

Follow through on their upper limit from (b) which may be a decimal but must be positive. Must be strict inequalities i.e. not include the end points and must now be in terms of k .

Other acceptable notation includes: $k \in \left(-5, \frac{4735}{768}\right), \{k, k > -5\} \cap \left\{k, k < \frac{4735}{768}\right\}$

Condone $\left\{k : k > -5 \cap k < \frac{4735}{768}\right\}$ or $\left\{-5 < k < \frac{4735}{768}\right\}$

Do not allow e.g. $\{k : k > -5\} \cup \left\{k : k < \frac{4735}{768}\right\}$ or just $-5 < k < \frac{4735}{768}$

SC: Candidates that have not scored the first M1 may go on to score the final two marks by finding an acceptable range between -5 and $\frac{4735}{768}$.

Use of their “ -5 ” or “ $-\frac{47}{3}$ ”, coming from $x = \pm 2$, (or -5 or $-\frac{47}{3}$ directly from a calculator)

with $\frac{4735}{768}$ will score M1 if the inside region is selected. A correct $\left\{k : -5 < k < \frac{4735}{768}\right\}$ will score A1ft

There is no need to see any working to achieve the -5 (or $-\frac{47}{3}$).

All the relevant guidance regarding the region in the M1 and A1ft notes above continues to apply. e.g. for the M1 we will condone the use of y but not x in their region while for the A1ft we require use of k .

For example:

- $\left\{k : -5 < k < \frac{4735}{768}\right\}$ without scoring the first M scores M0A0M1A1ft
- “ -5 ” $< k < \frac{4735}{768}$ without scoring the first M scores M0A0M1A0ft
- $\left\{k : -\frac{47}{3} < k < \frac{4735}{768}\right\}$ without scoring the first M scores M0A0M1A0ft

Question	Scheme	Marks	AOs
12(a)	$4 1-3 -5 = \dots$ or $4 -1-3 -5 = \dots$	M1	1.1b
	3 and 11	A1	1.1b
		(2)	
(b)	$2(-5)+17 = \dots$	M1	1.1b
	$gf(x) \geq 7$	A1	2.2a
		(2)	
(c)	$k \dots -4$	B1	2.2a
	Minimum at $(3, -5)$ so $k \dots -\frac{5}{3}$	M1	3.1a
	$k < -\frac{5}{3}$	A1ft	1.1b
		A1	2.5
		(4)	
(8 marks)			
Notes			
(a) M1: Attempts $4 1-3 -5 = \dots$ or $4 -1-3 -5 = \dots$ proceeding to a value. Either correct value (3 or 11) that is clearly their answer can imply the M mark. Alternatively, attempts any of $-4(-1-3)-5$ or $-4(1-3)-5$ or $-4(-1)+7$ or $-4(1)+7$ i.e. without the modulus signs using the ‘left’ branch of the function. A1: Both 3 and 11 found and no other values that are clearly meant to be their answers (i.e. ignore reference to $a = 1$ or $a = -1$). Accept “3, 11” or “3 or 11” or “3 and 11” or $\{3, 11\}$ but not (3, 11). Condone $y = 3, 11$ but not $x = 3, 11$.			
(b) M1: Attempts $2(-5)+17$ which may be implied by sight of 7 used in their range, including in an incorrect range e.g. $f(x) > 7$ Alternatively, attempts $gf(x) = 2(4 x-3 -5)+17$ ($= 8 x-3 + 7$) and replaces $x-3$ with 0 or x with 3 or simply deduces the minimum value of their “7” in this case. Must be an attempt at gf and not fg. A1: Deduces the range $gf(x) \geq 7$ using correct notation. Condone $y \geq 7$ but not e.g. $x \geq 7$ or $g(x) \geq 7$ or $f(x) \geq 7$ or $fg(x) \geq 7$ Other acceptable notation includes: $gf(x) \in [7, \infty)$ Do not accept $gf(x) \in [7, \infty]$ or $gf(x) \in (7, \infty)$			
(c) B1: Deduces $k \dots -4$ allow any equality or inequality here. Condone $k \dots \pm 4$ (but not just $k \dots 4$) and condone e.g. “gradient $\dots -4$” May be seen coming from e.g. $kx = -4x + 7 \rightarrow (k+4)x = 7$ or $x = \frac{7}{k+4}$ which is acceptable but see the note below. Should not be x or y for this mark but may be m. Note: Finding the ‘discriminant’ of a linear equation e.g. $(k+4)x - 7 = 0$ in order to obtain $k \dots -4$ is invalid and cannot earn either the B1 or the final A1 mark unless there is an alternative valid reason given. M1: Attempts to use the vertex to find the (upper) limit for k. Look for $\frac{\pm 5}{\pm 3}$ but allow this mark to be scored for $\frac{\pm B}{\pm A}$ if there is clear evidence that they think the vertex is at (A, B)			



Allow use of m , x , y or another variable for this mark.

Alt 1 via squaring and the discriminant:

$$\begin{aligned} kx &= 4|x-3|-5 \Rightarrow kx+5 = 4|x-3| \\ \Rightarrow k^2x^2 + 10kx + 25 &= 16(x^2 - 6x + 9) \\ \Rightarrow (k^2 - 16)x^2 + (10k + 96)x - 119 &= 0 \\ \Rightarrow (10k + 96)^2 - 4(k^2 - 16)(-119) &= 0 \\ \Rightarrow 576k^2 + 1920k + 1600 &= 0 \Rightarrow k = -\frac{5}{3} \end{aligned}$$

Scores M1 for setting $kx = 4|x-3|-5$, isolating $|x-3|$ (or $4|x-3|$), squaring both sides, using $b^2 - 4ac \dots 0$ where ... is any equality or inequality, and solving the resulting 3TQ using the usual rules and may be by calculator, leading to a value for k .
Condone slips in expanding the brackets.

Alt 2 via solving simultaneous equations:

$$\begin{aligned} \text{e.g. } kx &= 4(x-3)-5 \Rightarrow x = \frac{-17}{k-4} \\ kx &= 4(3-x)-5 \Rightarrow (k+4)x = 7 \\ \Rightarrow (k+4)\left(\frac{-17}{k-4}\right) &= 7 \Rightarrow k = -\frac{5}{3} \end{aligned}$$

Scores M1 for setting $kx = 4(x-3)-5$ and $kx = 4(3-x)-5$, eliminating x , and solving for k

A1ft: Uses their value of k , found using one of the above methods, as the upper end of the range for k , i.e., $k < -\frac{5}{3}$ which may be seen as part of their range. Condone $k \leq -\frac{5}{3}$ for this mark.

Score once seen as an upper limit and do not withhold if they then incorrectly combine as
e.g. $-\frac{5}{3} < k < -4$

Condone the use of m for this mark, but not x or y .

A1: $-4 \leq k < -\frac{5}{3}$ o.e. and no other solutions seen. Their range must use k and not e.g. x , y or m .

Other acceptable notation includes: $k \in \left[-4, -\frac{5}{3}\right)$, " $k < -\frac{5}{3}$ and

$$k \geq -4$$
, $k < -\frac{5}{3} \cap k \geq -4$

Do not accept e.g. " $k < -\frac{5}{3}$, $k \geq -4$ " or " $k < -\frac{5}{3}$ or $k \geq -4$ " or
" $k < -\frac{5}{3} \cup k \geq -4$ "

Allow $-1.\dot{6}$ but **not** -1.6 or $-1.6\dots$ for $-\frac{5}{3}$

Question	Scheme	Marks	AOs
1 (a)	$(4, -4)$ or e.g. $x = 4, y = -4$ o.e.	B1	1.1b
		(1)	
(b)	$(-4, 6)$ or e.g. $x = -4, y = 6$ o.e.	B1	1.1b
		(1)	
(c)	$(6, \dots)$ or $(\dots, 5)$ or $x = 6$ or $y = 5$ o.e.	M1	1.1b
	$(6, 5)$ or $x = 6$ and $y = 5$	A1	1.1b
		(2)	
(4 marks)			

Notes:**General guidelines for all parts:**

Remember to check answers written against the questions.

If there is any contradiction, mark the answers given in the body of the scripts.

If there is no labelling, mark the responses in the order given.

The coordinates need to be values not just a calculation e.g. **not** $6 - 2$ for 4

Points can be written as a coordinate pair or separately as $x = \dots, y = \dots$

Do not allow coordinates written the wrong way round but isw if necessary

e.g. $x = 4, y = -4 \rightarrow (-4, 4)$ scores B1 and isw

Condone missing brackets (one or both) e.g. $x = 4, y = -4$ **or** $(4, -4$ **or** $4, -4$ for $(4, -4)$

Condone a missing comma e.g. $(4 -4)$ for $(4, -4)$

Condone use of a semi-colon e.g. $(4 ; -4)$ for $(4, -4)$

Condone vector notation e.g. $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$ for $(4, -4)$ and condone $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$

(a)

B1: $(4, -4)$ o.e. see above

(b)

B1: $(-4, 6)$ o.e. see above

(c)

M1: One correct coordinate. See above.

A1:

Both coordinates correct. See above.

Note that M0A1 is not a possible mark profile.

Note that in part (c), some candidates show their thinking by transforming the point piecewise e.g. $(6, -4) \rightarrow (6, 4) \rightarrow (6, 8) \rightarrow (6, 5)$

In such cases, mark their final **pair** of coordinates



Question	Scheme	Marks	AOs
10(a)	e.g. $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x+4} + \frac{B}{x-2} \Rightarrow 3kx-18 \equiv A(x-2) + B(x+4)$ or $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x-2} + \frac{B}{x+4} \Rightarrow 3kx-18 \equiv A(x+4) + B(x-2)$	B1	1.1b
	$6k-18=6B \Rightarrow B=\dots$ or $-12k-18=-6A \Rightarrow A=\dots$ or $3kx-18 \equiv (A+B)x + 4B - 2A \Rightarrow A+B=3k, -18=4B-2A$ $\Rightarrow A=\dots$ or $B=\dots$	M1	1.1b
	$\frac{2k+3}{x+4} + \frac{k-3}{x-2}$	A1	1.1b
		(3)	
(b)	$\int \left(\frac{"2k+3"}{x+4} + \frac{"k-3"}{x-2} \right) dx = \dots \ln(x+4) + \dots \ln(x-2)$	M1	1.2
	$("2k+3") \ln(x+4) + ("k-3") \ln(x-2)$	A1ft	1.1b
	$("2k+3") \ln(5) - ("k-3") \ln(5) \Rightarrow ("k+6") \ln 5 = 21 \Rightarrow k = \dots$	dM1	3.1a
	$(k =) \frac{21}{\ln 5} - 6$	A1	2.2a
		(4)	
(7 marks)			
Notes			
(a) B1: Correct form for the partial fractions and sets up the correct corresponding identity which may be implied by two equations in A and B if they are comparing coefficients. M1: Either - substitutes $x=2$ or $x=-4$ in an attempt to find A or B in terms of k - expands the rhs, collects terms and compares coefficients in an attempt to find A or B in terms of k Or may be implied by one correct fraction (numerator and denominator) You may see candidates substituting two other values of x and then solving simultaneous equations. A1: Achieves $\frac{2k+3}{x+4} + \frac{k-3}{x-2}$ with no errors. Must be the correct partial fractions not just for correct numerators. May be seen in (b). Correct answer implies B1M1A1. One correct fraction only B0M1A0			

(b)

M1: Attempts to find $\int \left(\frac{"2k+3"}{x+4} + \frac{"k-3"}{x-2} \right) dx$. Score for either $\frac{\cdots}{x+4} \rightarrow \dots \ln(x+4)$ or $\frac{\cdots}{x-2} \rightarrow \dots \ln(x-2)$

Allow the ... to be in terms of k or just constants but there must be no x terms.

Condone invisible brackets for this mark.

A1ft: $("2k+3") \ln|x+4| + ("k-3") \ln|x-2|$

but condone round brackets e.g. $("2k+3") \ln(x+4) + ("k-3") \ln(x-2)$ or equivalent e.g.

$("2k+3") \ln(x+4) + ("k-3") \ln(2-x)$

Follow through their partial fractions with numerators which must both be in terms of k .

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of k . To score this mark they must have attempted to integrate their partial fractions, substituted in the correct limits, subtracted either way round, set = 21 and attempted to solve to find k . Condone omission of the terms containing $\ln(1)$ or $\ln(-1)$.

Note that e.g. $\ln(-5)$ or $\ln(5)$ must be seen but may be disregarded **after** substitution and subtraction.

Do not be concerned with the processing as long as they proceed to $k = \dots$

Condone if they use x instead of k after limits have been used as long as the intention is clear.

A1: Deduces $(k =) \frac{21}{\ln 5} - 6$ or exact equivalent e.g. $\frac{21 - 6 \ln 5}{\ln 5}$, $\frac{21 - 3 \ln 25}{\ln 5}$.

Allow recovery from expressions that contain e.g. $\ln(-5)$ as long as it is dealt with subsequently.

Also allow recovery from invisible brackets. Condone $x = \frac{21}{\ln 5} - 6$

Some candidates may use substitution in part (b) e.g.

$$\int \left(\frac{"2k+3"}{x+4} + \frac{"k-3"}{x-2} \right) dx = \int \left(\frac{"2k+3"}{x+4} \right) dx + \int \left(\frac{"k-3"}{x-2} \right) dx$$

$$u = x+4 \Rightarrow \int \left(\frac{"2k+3"}{x+4} \right) dx = \int \left(\frac{"2k+3"}{u} \right) du = \dots \ln u$$

$$u = x-2 \Rightarrow \int \left(\frac{"k-3"}{x-2} \right) dx = \int \left(\frac{"k-3"}{u} \right) du = \dots \ln u$$

Score **M1** for integrating at least once to an appropriate form as in the main scheme e.g. $\dots \ln u$

A1ft: For $("2k+3") \ln|u| + ("k-3") \ln|u|$

but condone $("2k+3") \ln u + ("k-3") \ln u$ which may be seen separately

Follow through their "A" and "B" in terms of k .

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of k . To score this mark they must have attempted to integrate their partial fractions using substitution, substituted in the correct changed limits and subtracts either way

round, set = 21 and attempted to solve to find k . Do not be concerned with processing as long as they proceed to $k = \dots$. Condone omission of terms which contain e.g. $\ln(1)$ or $\ln(-1)$.

Note that e.g. $\ln(-5)$ or $\ln(5)$ must be seen but may be disregarded **after** substitution and subtraction.

$$[(2k+3)\ln u]_1^5 + [(k-3)\ln u]_{-5}^{-1} = 21 \Rightarrow (2k+3)\ln 5 - (2k+3)\ln 1 + (k-3)\ln 1 - (k-3)\ln 5 = 21$$

$$\Rightarrow (2k+3)\ln 5 - (k-3)\ln 5 = 21 \Rightarrow (k+6)\ln 5 = 21 \Rightarrow k = \dots$$

A1: $k = \frac{21}{\ln 5} - 6$ or exact equivalent e.g. $\frac{21-6\ln 5}{\ln 5}$, $\frac{21-3\ln 25}{\ln 5}$, $21\log_5 e - 6$.

Allow recovery from expressions that contain e.g. $\ln(-5)$ as long as it is dealt with subsequently.

Also allow recovery from invisible brackets.



Question	Scheme	Marks	AOs
14(a)	$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow A = \dots, B = \dots$	M1	1.1b
	Either $A = 2$ or $B = -1$	A1	1.1b
	$\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$	A1	1.1b
		(3)	
(b)	$\int \frac{1}{V} dV = \int \frac{3}{(2t-1)(t+1)} dt$	B1	1.1a
	$\int \frac{2}{2t-1} - \frac{1}{t+1} dt = \dots \ln(2t-1) - \dots \ln(t+1) (+c)$	M1	3.1a
	$\ln V = \ln(2t-1) - \ln(t+1) (+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = (\ln 3)$	M1	3.4
	$\ln V = \ln(2t-1) - \ln(t+1) + \ln 3$ $V = \frac{3(2t-1)}{(t+1)} *$	A1*	2.1
		(5)	
	(b) Alternative separation of variables:		
	$\int \frac{1}{3V} dV = \int \frac{1}{(2t-1)(t+1)} dt$	B1	1.1a
	$\frac{1}{3} \int \frac{2}{2t-1} - \frac{1}{t+1} dt = \dots \ln(2t-1) - \dots \ln(t+1) (+c)$	M1	3.1a
	$\frac{1}{3} \ln 3V = \frac{1}{3} \ln(2t-1) - \frac{1}{3} \ln(t+1) (+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = \left(\frac{1}{3} \ln 3\right)$	M1	3.4
	$\frac{1}{3} \ln V = \frac{1}{3} \ln(2t-1) - \frac{1}{3} \ln(t+1) + \frac{1}{3} \ln 3$ $V = \frac{3(2t-1)}{(t+1)} *$	A1*	2.1
		(5)	
(c)	(i) 30 (minutes)	B1	3.2a
	(ii) 6 (m ³)	B1	3.4
		(2)	
(10 marks)			
Notes:			

(a)

M1: Correct method of partial fractions leading to values for their A and B

 E.g. substitution:
$$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(x+1) + B(2x-1) \Rightarrow A = \dots, B = \dots$$

 Or compare coefficients
$$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = x(A+2B) + A-B \Rightarrow A = \dots, B = \dots$$

 Note that
$$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(2x-1) + B(x+1) \Rightarrow A = \dots, B = \dots$$
 scores M0


A1: Correct value for “A” or “B”

A1: Correct partial fractions not just values for “A” and “B”. $\frac{2}{2x-1} - \frac{1}{x+1}$ or e.g. $\frac{2}{2x-1} + \frac{-1}{x+1}$

Must be seen as **fractions** but if not stated here, allow if the correct fractions appear later.

(b)

B1: Separates variables $\int \frac{1}{V} dV = \int \frac{3}{(2t-1)(t+1)} dt$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $\dots \ln(2t-1) + \dots \ln(t+1)$ where $\dots$ are constants.

Condone missing brackets around the $(2t-1)$ and/or the $(t+1)$ for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for $+c$ here.

The brackets around the $(2t-1)$ and/or the $(t+1)$ must be seen or implied for this mark

M1: Attempts to find “c” or e.g. “ $\ln k$ ” using $t=2$, $V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2$, $V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the $\ln$ ’s have been combined.

A1*: Correct processing leading to the given answer $V = \frac{3(2t-1)}{(t+1)}$

Alternative:

B1: Separates variables $\int \frac{1}{3V} dV = \int \frac{1}{(2t-1)(t+1)} dt$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $\dots \ln(2t-1) + \dots \ln(t+1)$ where $\dots$ are constants.

Condone missing brackets around the $(2t-1)$ and/or the $(t+1)$ for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for $+c$ here.

The brackets around the $(2t-1)$ and/or the $(t+1)$ must be seen or implied for this mark

M1: Attempts to find “c” or e.g. “ $\ln k$ ” using $t=2$, $V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2$, $V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the $\ln$ ’s have been combined.

A1*: Correct processing leading to the given answer $V = \frac{3(2t-1)}{(t+1)}$

(Note the working may look like this:

$$\frac{1}{3} \ln 3V = \frac{1}{3} \ln(2t-1) - \frac{1}{3} \ln(t+1) + c, \quad \frac{1}{3} \ln 9 = \frac{1}{3} \ln(3) - \frac{1}{3} \ln 3 + c, \quad c = \frac{1}{3} \ln 9$$

$$\ln 3V = \ln \frac{9(2t-1)}{(t+1)} \Rightarrow 3V = \frac{9(2t-1)}{(t+1)} \Rightarrow V = \frac{3(2t-1)}{(t+1)} \text{ *)}$$

Note that B0M1A1M1A1 is not possible in (b) as the B1 must be implied if all the other marks have been awarded.

Note also that some candidates may use different variables in (b) e.g.

$$\frac{dy}{dx} = \frac{3y}{(2x-1)(x+1)} \Rightarrow \int \frac{1}{y} dy = \int \frac{3}{(2x-1)(x+1)} dx \text{ etc. In such cases you should award marks for}$$

equivalent work but they must revert to the given variables at the end to score the final mark.

Also if e.g. a “t” becomes an “x” within their working but is recovered allow full marks.



(c)

B1: Deduces 30 minutes. Units not required so just look for 30 but allow equivalents e.g. $\frac{1}{2}$ an hour.

If units are given they must be correct so do not allow e.g. 30 hours.

B1: Deduces 6 m^3 . Units not required so just look for 6. Condone $V < 6$ or $V \leq 6$

If units are given they must be correct so do not allow e.g. 6 m.



Question	Scheme	Marks	AOs
9(a)(i)	$50x^2 + 38x + 9 \equiv A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$ $\Rightarrow B = \dots$ or $C = \dots$	M1	1.1b
	$B = 1$ and $C = 2$	A1	1.1b
(a)(ii)	E.g. $x = 0 \Rightarrow 9 = 2A + B + 4C$ $\Rightarrow 9 = 2A + 1 + 8 \Rightarrow A = \dots$	M1	2.1
	$A = 0^*$	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2} = (5x+2)^{-2} = 2^{-2} \left(1 + \frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2} = 2^{-2} + \dots$	M1	3.1a
	$\left(1 + \frac{5}{2}x\right)^{-2} = 1 - 2\left(\frac{5}{2}x\right) + \frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2 + \dots$	M1	1.1b
	$2^{-2} \left(1 + \frac{5}{2}x\right)^{-2} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots$	A1	1.1b
	$\frac{1}{(1-2x)} = (1-2x)^{-1} = 1 + 2x + \frac{-1(-1-1)}{2!}(2x)^2 + \dots$	M1	1.1b
	$\frac{1}{(5x+2)^2} + \frac{2}{1-2x} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots + 2 + 4x + 8x^2 + \dots$	dM1	2.1
	$= \frac{9}{4} + \frac{11}{4}x + \frac{203}{16}x^2 + \dots$	A1	1.1b
(b)(ii)	$ x < \frac{2}{5}$	B1	2.2a
		(7)	
(11 marks)			
Notes			

(a)(i)

M1: Uses a correct identity and makes progress using an appropriate strategy (e.g. sub $x = \frac{1}{2}$) to find a value for B or C . May be implied by one correct value (cover up rule).

A1: Both values correct

(a)(ii)

M1: Uses an appropriate method to establish an equation connecting A with B and/or C and uses their values of B and/or C to find a suitable equation in A .

Amongst many different methods are:

Compare terms in $x^2 \Rightarrow 50 = -10A + 25C$ which would be implied by $50 = -10A + 25 \times 2$

Compare constant terms or substitute $x = 0 \Rightarrow 9 = 2A + B + 4C$ implied by $9 = 2A + 1 + 4 \times 2$

A1*: Fully correct proof with no errors.

Note: The second part is a proof so it is important that a suitable proof/show that is seen.

Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0



(b)(i)

M1: Applies the key steps of writing $\frac{1}{(5x+2)^2}$ as $(5x+2)^{-2}$ and takes out a factor of 2^{-2} to form an

expression of the form $(5x+2)^{-2} = 2^{-2}(1+*x)^{-2}$ where * is not 1 or 5

Alternatively uses direct expansion to obtain $2^{-2} + \dots$

M1: Correct attempt at the binomial expansion of $(1+*x)^{-2}$ up to the term in x^2

Look for $1 + (-2)*x + \frac{(-2)(-3)}{2}*x^2$ where * is not 5 or 1.

Condone sign slips and lack of $*^2$ on term 3.

Alt Look for correct structure for 2^{nd} and 3^{rd} terms by direct expansion. See below

A1: For a fully correct expansion of $(2+5x)^{-2}$ which may be unsimplified. This may have been combined with their 'B'

A direct expansion would look like $(2+5x)^{-2} = 2^{-2} + (-2)2^{-3} \times 5x + \frac{(-2)(-3)}{2}2^{-4} \times (5x)^2$

M1: Correct attempt at the binomial expansion of $(1-2x)^{-1}$

Look for $1 + (-1)*x + \frac{(-1)(-2)}{2}*x^2$ where * is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.

There must be some attempt to use their values of B and C

A1: Correct expression or correct values for p, q and r.

(b)(ii)

B1: Correct range. Allow also other forms, for example $-\frac{2}{5} < x < \frac{2}{5}$ or $x \in \left(-\frac{2}{5}, \frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered



Question	Scheme	Marks	AOs
12(a)	$\frac{1}{V(25-V)} = \frac{P}{V} + \frac{Q}{25-V}$ e.g. $1 = P(25-V) + QV$ $V = 0$ or $V = 25$ leading to $P = \dots$ or $Q = \dots$	M1	1.1b
	$\frac{1}{V(25-V)} = \frac{1}{25V} + \frac{1}{25(25-V)}$	A1	1.1b
		(2)	
Notes			
(a) M1: Sets $\frac{1}{V(25-V)} = \frac{P}{V} + \frac{Q}{25-V}$ and uses a correct method to identify the value of at least one constant. Do not condone incorrect work e.g. $\frac{1}{V(25-V)} = \frac{P}{V} + \frac{Q}{25-V} \Rightarrow 1 = PV + Q(25-V)$ etc. this scores M0 A1: Correct partial fractions in any form e.g. $\frac{1}{25V} + \frac{1}{25(25-V)}, \frac{1}{25V} + \frac{1}{625-25V}, \frac{1/25}{V} + \frac{1/25}{(25-V)}, \frac{1}{25V} - \frac{1}{25(V-25)}, \frac{1}{25} \left(\frac{1}{V} + \frac{1}{25-V} \right)$ etc. Note that this mark is not just for the correct constants, it is for the correctly written fractions either seen in part (a) or used in part (b). Allow 0.04 for $\frac{1}{25}$. Correct partial fractions only scores both marks. If the correct fractions are obtained following incorrect work score M0A0 but allow full recovery in the rest of the question.			

(b) Way 1	$\int \frac{1}{V(25-V)} dV = \int \frac{1}{25V} + \frac{1}{25(25-V)} dV = \frac{1}{25} \ln V - \frac{1}{25} \ln(25-V)$	M1	3.1a
	$\frac{1}{25} \ln V - \frac{1}{25} \ln(25-V) = \frac{1}{10} t (+c)$	A1ft	1.1b
	$t = 0, V = 20 \Rightarrow \frac{1}{25} \ln 20 - \frac{1}{25} \ln(25-20) = c \left(\Rightarrow c = \frac{1}{25} \ln 4 \right)$	M1	3.4
	$V = 24 \Rightarrow t = \frac{2}{5} \ln 24 - \frac{2}{5} \ln(25-24) - \frac{2}{5} \ln 4$	dM1	3.1b
	$= 43 \text{ (or exact } 24 \ln 6)$	A1	3.2a
		(5)	
Alternative for the final 3 marks:			
	$\left[\frac{1}{25} \ln V - \frac{1}{25} \ln(25-V) \right]_{20}^{24} = \left[\frac{1}{10} t \right]_0^T \Rightarrow \frac{1}{25} \ln 24 - \frac{1}{25} \ln 4 = \frac{1}{10} T$	M1	3.4
	$T = \frac{10}{25} \ln 24 - \frac{10}{25} \ln 4 = \dots$	dM1	3.1b
	$= 43 \text{ (or exact } 24 \ln 6)$	A1	3.2a
(c)	$\frac{1}{25} \ln V - \frac{1}{25} \ln(25-V) = \frac{1}{10} t + \frac{1}{25} \ln 4$ $\ln V - \ln(25-V) = 2.5t + \ln 4$ $\ln \frac{V}{4(25-V)} = 2.5t \Rightarrow \frac{V}{4(25-V)} = e^{2.5t}$	M1	2.1
	$\Rightarrow V = 4e^{2.5t} (25-V) \Rightarrow V + 4Ve^{2.5t} = 100e^{2.5t} \Rightarrow V = \dots$	M1	2.1
	$\Rightarrow V = \frac{100e^{2.5t}}{1 + 4e^{2.5t}} = \frac{100}{e^{-2.5t} + 4}$	A1	1.1b
		(3)	
(d)	25 (microlitres)	B1	2.2a
	Since e.g. As $t \rightarrow \infty$, $e^{-2.5t} \rightarrow 0$	B1	2.4
		(2)	
(12 marks)			
Notes			
(b) Mark (b) and (c) together			
M1: Realises that $\int \frac{\dots}{V(25-V)} (dV)$ is required and reaches the form $p \ln \alpha V \pm q \ln \beta(25-V)$ (or e.g. $p \ln \alpha V \pm q \ln \beta(V-25)$) or equivalent for this integration e.g. $p \ln 25V - q \ln(625-25V)$ with p and q non-zero. But note that $\int \frac{\dots}{V(25-V)} dV = \ln V(25-V)$ does not score this mark unless we see an attempt to integrate the partial fractions first. Condone missing brackets e.g. around the $V-25$ for this mark Note that the rhs may be incorrect or missing for this mark. A1ft: Fully correct <u>equation</u> following through their P and Q. The “+ c” is not required here. You may need to check carefully when awarding this mark as there will be various alternative correct (or correct ft) forms e.g. these are correct (for correct PF's):			

In general look for an equation of the form $P \ln \alpha V - Q \ln \beta(25 - V) = \frac{1}{10}t(+c)$ or a multiple of this equation. Do **not** condone missing brackets unless they are implied by later work e.g. $\ln 25 - V$ for $\ln(25 - V)$

Allow () or | around the arguments of the ln's and condone "log" for ln.

M1: States or uses $t = 0$ and $V = 20$ consistently leading to a constant of integration which may be simplified or unsimplified. May be implied by their constant so may need to be checked.

This mark is not formally dependent but depends on having made some attempt to integrate both sides, however poor.

dM1: States or uses $V = 24$ and proceeds to find a value for t (even if $t < 0$). You do not need to check the processing provided they reach a value for t . **Depends on the previous method mark** and depends on an attempt to integrate both sides however poor.

May be implied by their value for t so may need to be checked.

A1: Correct value of 43 or awrt 43.0 or exact value of $24 \ln 6$.

Units are not required but if any are given it must be minutes or condone "m".

Note that in hours the time is 0.7167037877... and scores A0

Alternative for final 3 marks:

M1:
$$\left[\frac{1}{25} \ln V - \frac{1}{25} \ln(25 - V) \right]_{20}^{24} = \left[\frac{1}{10} t \right]_0^T \Rightarrow \frac{1}{25} \ln 24 - \frac{1}{25} \ln 4 = \frac{1}{10} T$$

Applies the limits 20 and 24 to lhs and 0 to "T" or e.g. "t" on rhs

This mark is not formally dependent but depends on having made some attempt to integrate both sides, however poor.

dM1: $T = \frac{10}{25} \ln 24 - \frac{10}{25} \ln 4 = \dots$ Solves to find "T". You do not need to check the

processing provided they reach a value for t/T . **Depends on the previous method mark** and depends on having made some attempt to integrate both sides however poor.

A1: Correct value of 43 or awrt 43.0 or exact value of $24 \ln 6$.

Units are not required but if any are given it must be minutes or condone "m".

Note that in hours the time is 0.7167037877... and scores A0

Note

$$\int \frac{1}{25V} + \frac{1}{25(25-V)} dV = \int \frac{1}{25V} - \frac{1}{25V-625} dV = \frac{1}{25} \ln V - \frac{1}{25} \ln(25V-625) = \frac{1}{10} t(+c)$$

is also correct integration and scores M1A1

The subsequent marks are also available as described above and could lead to the correct answer if the limits and constant of integration are dealt with correctly.

Use review for any examples like these if you are unsure but generally apply the MS as above.

(c) The marks in (c) depend on having integrated their partial fractions to obtain an equation of the form $\pm \dots \ln \dots V \pm \dots \ln \dots (25 - V) = \pm kt \pm c$, $k, c \neq 0$ and ... are non-zero constants or equivalent if they have already attempted to eliminate the ln's in (b)

e.g. $\frac{\dots V}{\dots(25-V)} = e^{\pm kt \pm c}$ oe

M1: Uses fully correct log work, having obtained a constant of integration, to eliminate all the $\ln$'s including from e.g. $e^{\ln 4}$. We condone sign or coefficient slips only.

M1: Proceeds from an **equation of the form** $\frac{\dots V}{\dots(25-V)} = \dots e^{\dots t}$ oe using correct algebra to

$$V = \dots \text{ e.g. } \frac{\dots V}{\dots(25-V)} = \dots e^{\dots t} \Rightarrow \dots V = \dots(25-V) \dots e^{\dots t} \Rightarrow (\dots \pm \dots)V = \dots e^{\dots t} \Rightarrow V = \dots$$

Condone sign/coefficient slips only.

A1: Correct expression not just values for the constants.

(d)

B1: Correct value of 25 seen

Allow e.g. < 25 or $,, 25$

Condone " > 25 " but the following mark is then not available

B1: Depends on a correct final equation in any form in (c) e.g. $V = \frac{100e^{2.5t}}{1 + 4e^{2.5t}}$ oe **and one of:**

- Considers the behaviour as $t \rightarrow \infty$ e.g. states that as $t \rightarrow \infty$, $e^{-2.5t} \rightarrow 0$ (condone " $= 0$ ") oe
- $V < 25$ as $\ln(25 - V)$ is not possible when $V \dots 25$
- Verifies the 25 using a value of t , $t \dots 9$

Using the differential equation:

B1: Correct value of 25 seen

Allow e.g. " < 25 " or $,, 25$

Condone " > 25 " but the following mark is then not available

B1: E.g. when $V = 25$, $\frac{dV}{dt} = 0$ **or** $\frac{dV}{dt} < 0$ if $V > 25$

Question	Scheme	Marks	AOs
7	Any one of $A = 3, B = -2, C = 4, D = -5$ correct from correct work	B1	2.2a
	Finds the value of B, C or D using a valid method e.g. $3x^3 - 8x^2 - 6x - 11 = (Ax + B)(x + 1)(x - 3) + C(x - 3) + D(x + 1)$ and substitutes $x = -1 \Rightarrow \dots = \dots C \Rightarrow C = \dots$ or substitutes $x = 3 \Rightarrow \dots = \dots D \Rightarrow D = \dots$	M1	1.1b
	Valid method for finding all four constants. e.g. Deduces $A = 3$, substitutes $x = 3$ leading to $D = \dots$ and substitutes $x = -1$ leading to $C = \dots$ and substitutes e.g. $x = 0$ leading to $B = \dots$	M1 (A1 on EPEN)	1.1b
	$A = 3, B = -2, C = 4$ and $D = -5$	A1	2.1
		(4)	
(4 marks)			
Notes			
Note: Condone different uses of 'A', 'B' etc for the M marks provided the intention is clear but values must be correctly attributed to each letter or implied (if embedded in a final expression) for the A mark. B1: Any one of $A = 3, B = -2, C = 4, D = -5$ correct (usually $A = 3$ is deduced) from correct work. May be implied by $3x$ seen. Condone $A = 3x$ for this mark. M1: This mark is for finding the value of B, C or D, using a valid method, which may be in one of the following ways: - Attempts algebraic division of $(3x^3 - 8x^2 - 6x - 11)$ by their expanded $(x + 1)(x - 3) = (x^2 - 2x - 3)$ or separately by both $(x + 1)$ and $(x - 3)$ as far as $3x \pm 2$. Ignore any remainder for this mark. For reference, the correct remainder is $-x - 17$. - Algebraic division might be seen as $3x(x^2 - 2x - 3) \pm 2(x^2 - 2x - 3) + "9x - 4x - 6 - 6x - 11"$ o.e. and scores the B1 for the $3x$ in the correct place and M1 for both the $3x$ and the ± 2 in the correct places. In the next two bullets, they should achieve $3x^3 - 8x^2 - 6x - 11 = (Ax + B)(x + 1)(x - 3) + C(x - 3) + D(x + 1)$ o.e. but condone small slips e.g. sign errors or a slip on a constant. The bracketed expressions should be in the correct places and Ax and B should both be multiplied by $(x + 1)(x - 3)$. Note that $3x^3 - 8x^2 - 6x - 11 = C(x - 3) + D(x + 1)$ without sight of the identity above (condoning the previously mentioned slips) is an incorrect method and scores M0. - Attempts to multiply up by $(x + 1)(x - 3)$ and substitutes one of $x = -1$ or $x = 3$ leading to $C = \dots$ or $D = \dots$ (you do not need to check the accuracy of their substitution) or attempts to multiply up by $(x + 1)(x - 3)$, expands and compares coefficients leading to a value for B, C or D. Do not be concerned about how they expand the brackets. - Attempts at the cover up method may be seen. If unsure then send to review. M1: For finding values of all four constants via a valid method. Not dependent on the previous method mark so B1M0M1A0 is possible.			



- If attempting algebraic division, they must have a quotient of $3x \pm k$ for constant k ($3x \pm 2$ is **not** required for this mark) **and** they must go on to deal with their remainder using a correct method, i.e., " $-x-17$ " = $C(x-3) + D(x+1)$ followed by either comparing coefficients or substitution of appropriate values. You do not need to check the accuracy of their substitution.
- For those attempting repeated algebraic division, the values of C and D must come from a correct method and so cannot be equal to e.g. the remainders from successive division by $(x+1)$ and $(x-3)$ or vice versa. If, e.g., they divide by $(x+1)$ (to get remainder R_1) and then $(x-3)$ (to get remainder R_2) they should get $3x \pm k + \frac{R_1}{(x+1)(x-3)} + \frac{R_2}{(x-3)}$. They can then either combine the two fraction terms before using partial fractions or they can use partial fractions on $\frac{R_1}{(x+1)(x-3)}$ before combining their two terms in $\frac{1}{(x-3)}$.
- If using $3x^3 - 8x^2 - 6x - 11 = (Ax+B)(x+1)(x-3) + C(x-3) + D(x+1)$ o.e. they may use a combination of substitution and comparing coefficients, but the method must be valid. Do not be concerned about how they expand the brackets, and you do not need to check the accuracy of their substitution.
- If using substitution only they are likely to use $x = -1, 3$ and 0 but the final value for x could be anything (and in fact they could use three/four different values for x). You do not need to check the accuracy of their substitution.
- If attempting to compare coefficients, do not be concerned about how they expand the brackets.

A1: Requires:

- all of $A = 3$, $B = -2$, $C = 4$ and $D = -5$ which may be stated separately in their work ($A = 3x$ that is not recovered would score final A0).
- **or** $3x - 2 + \frac{4}{x+1} - \frac{5}{x-3}$ seen. Condone $3x + -2 + \frac{4}{x+1} + \frac{-5}{x-3}$

There is no requirement to write out the form given in the question once correct values for A , B , C and D have been seen.

ISW after the four correct values (e.g. if they swap C and D in the fractions so

$A = 3$, $B = -2$, $C = 4$, $D = -5$ followed by $3x - 2 - \frac{5}{x+1} + \frac{4}{x-3}$ scores full marks).

Repeated Algebraic Division

An example for repeated algebraic division. Note that the repeated division could be carried out in the other order – i.e. dividing by $(x - 3)$ first followed by $(x + 1)$.

$$\begin{array}{r} 3x^2 - 11x + 5 \\ x + 1 \overline{) 3x^3 - 8x^2 - 6x - 11} \\ \underline{3x^3 + 3x^2} \\ -11x^2 - 6x \\ \underline{-11x^2 - 11x} \\ 5x - 11 \\ \underline{5x + 5} \\ -16 \end{array}$$

Followed by

$$\begin{array}{r} 3x - 2 \\ x - 3 \overline{) 3x^2 - 11x + 5} \\ \underline{3x^2 - 9x} \\ -2x + 5 \\ \underline{-2x + 6} \\ -1 \end{array}$$

B1 Scored for either '3' or '-2' embedded in the quotient.

1st M1 Scored for achieving $3x \pm 2$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{-16}{(x + 1)(x - 3)} + \frac{-1}{(x - 3)}$$

Correctly deals with both remainders from the repeated division.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{-16}{(x + 1)(x - 3)} + \frac{-(x + 1)}{(x + 1)(x - 3)}$$

Sets up an identity in x and C and D using partial fractions.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{-x - 17}{(x + 1)(x - 3)}$$

2nd M1 Finds the values of all four constants via a correct method (in this case by substitution of appropriate values).

$$-x - 17 = C(x - 3) + D(x + 1)$$

$$x = 3 \Rightarrow D = -5 \text{ and } x = -1 \Rightarrow C = 4$$

$$A = 3, B = -2, C = 4, D = -5$$

A1 $A = 3, B = -2, C = 4$ and $D = -5$

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{F}{(x + 1)} + \frac{G}{(x - 3)} + \frac{-1}{(x - 3)}$$

Alternatively, splits the first fraction using partial fractions and sets this equal to their remainder divided by $(x + 1)(x - 3)$ and multiplies up correctly.

$$-16 = G(x + 1) + F(x - 3)$$

$$x = -1 \Rightarrow F = 4, x = 3 \Rightarrow G = -4$$

2nd M1 Finds the values of all four constants via a valid method.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{4}{(x + 1)} + \frac{-4}{(x - 3)} + \frac{-1}{(x - 3)}$$

A1 $A = 3, B = -2, C = 4$ and $D = -5$ seen embedded in a final expression.

$$\frac{3x^3 - 8x^2 - 6x - 11}{(x + 1)(x - 3)} = 3x - 2 + \frac{4}{(x + 1)} - \frac{5}{(x - 3)}$$

Question	Scheme	Marks	AOs
8 (a) Way 1	$H = Ax(40 - x)$ {or $H = Ax(x - 40)$ }	M1	3.3
	$x = 20, H = 12 \Rightarrow 12 = A(20)(40 - 20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40 - x)$ or $H = -\frac{3}{100}x(x - 40)$	A1	1.1b
	(3)		
(a) Way 2	$H = 12 - \lambda(x - 20)^2$ {or $H = 12 + \lambda(x - 20)^2$ }	M1	3.3
	$x = 40, H = 0 \Rightarrow 0 = 12 - \lambda(40 - 20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12 - \frac{3}{100}(x - 20)^2$	A1	1.1b
	(3)		
(a) Way 3	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) Both $x = 0, H = 0 \Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$ and either $x = 40, H = 0 \Rightarrow 0 = 1600a + 40b$ or $x = 20, H = 12 \Rightarrow 12 = 400a + 20b$ or $\frac{-b}{2a} = 20 \Rightarrow b = -40a$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
	(3)		
(b)	$\{H = 3 \Rightarrow\} 3 = \frac{3}{100}x(40 - x) \Rightarrow x^2 - 40x + 100 = 0$ or $\{H = 3 \Rightarrow\} 3 = 12 - \frac{3}{100}(x - 20)^2 \Rightarrow (x - 20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	{chooses $20 + \sqrt{300} \Rightarrow$ } greatest distance = awrt 37.3 m	A1	3.2a
	(3)		
(c)	Gives a limitation of the model. Accept e.g. - the ground is horizontal - the ball needs to be kicked from the ground - the ball is modelled as a particle - the horizontal bar needs to be modelled as a line - there is no wind or air resistance on the ball - there is no spin on the ball - no obstacles in the trajectory (or path) of the ball - the trajectory of the ball is a perfect parabola	B1	3.5b
	(1)		
(7 marks)			

Notes for Question 8	
(a)	
M1:	Translates the situation given into a suitable equation for the model. E.g. Way 1: {Uses $(0, 0)$ and $(40, 0)$ to write} $H = Ax(40 - x)$ o.e. {or $H = Ax(x - 40)$ } Way 2: {Uses $(20, 12)$ to write} $H = 12 - \lambda(x - 20)^2$ or $H = 12 + \lambda(x - 20)^2$ Way 3: Writes $H = ax^2 + bx + c$, and uses $(0, 0)$ to deduce $c = 0$ and an attempt at using either $(40, 0)$ or $(20, 12)$ Special Case: Allow SC M1dM0A0 for not deducing $c = 0$ but attempting to apply both $(40, 0)$ and $(20, 12)$
dM1:	Applies a complete strategy with appropriate constraints to find all constants in their model. Way 1: Uses $(20, 12)$ on their model and finds $A = \dots$ Way 2: Uses either $(40, 0)$ or $(0, 0)$ on their model to find $\lambda = \dots$ Way 3: Uses $(40, 0)$ and $(20, 12)$ on their model to find $a = \dots$ and $b = \dots$
A1:	Finds a correct equation linking H to x E.g. $H = \frac{3}{100}x(40 - x)$, $H = 12 - \frac{3}{100}(x - 20)^2$ or $H = -0.03x^2 + 1.2x$
Note:	Condone writing y in place of H for the M1 and dM1 marks.
Note:	Give final A0 for $y = -0.03x^2 + 1.2x$
Note:	Give special case M1dM0A0 for writing down any of $H = 12 - (x - 20)^2$ or $H = x(40 - x)$ or $H = x(x - 40)$
Note:	Give M1 dM1 for finding $-0.03x^2 + 1.2x$ or $a = -0.03$, $b = 1.2$, $c = 0$ in an implied $ax^2 + bx + c$ (with no indication of $H = \dots$)
(b)	
M1:	Substitutes $H = 3$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x \pm \alpha)^2 = \beta$; $\alpha, \beta \neq 0$
Note:	E.g. $1.2x - 0.03x^2 = 3$ or $40x - x^2 = 100$ are acceptable for the 1 st M mark
Note:	Give M0 dM0 A0 for $(\text{their } A)x^2 = 3 \Rightarrow x = \dots$ or their $(\text{their } A)x^2 + (\text{their } k) = 3 \Rightarrow x = \dots$
dM1:	Correct method of solving their quadratic equation to give at least one solution
A1:	Interprets their solution in the original context by selecting the larger correct value and states correct units for their value . E.g. Accept awrt 37.3 m or $(20 + \sqrt{300})$ m or $(20 + 10\sqrt{3})$ m
Note:	Condone the use of inequalities for the method marks in part (b)
(c):	
B1:	See scheme
Note:	Give no credit for the following reasons H (or the height of ball) is negative when $x > 40$ - Bounce of the ball should be considered after hitting the ground - Model will not be true for a different rugby ball - Ball may not be kicked in the same way each time

Question	Scheme	Marks	AOs
9(a) Way 1	$H = \pm ax^2 \pm bx \pm c$ $x = 0, H = 2 \Rightarrow c = 2$ and either $x = 20, H = 0.8 \Rightarrow 0.8 = 400a + 20b + 2$ or $H = ax^2 + bx + c \Rightarrow \frac{dH}{dx} = 2ax + b$ $x = 9, \frac{dH}{dx} = 0 \Rightarrow 18a + b = 0$	M1	3.3
	$H = \pm ax^2 \pm bx \pm c$ $x = 0, H = 2 \Rightarrow c = 2$ and $x = 20, H = 0.8 \Rightarrow 0.8 = 20^2 a + 20b + 2$ and $H = ax^2 + bx + c \Rightarrow \frac{dH}{dx} = 2ax + b$ $x = 9, \frac{dH}{dx} = 0 \Rightarrow 18a + b = 0$	dM1	3.1b
	$0.8 = 400a + 20b + 2, 18a + b = 0 \Rightarrow a = \dots, b = \dots$	ddM1	1.1b
	$H = -0.03x^2 + 0.54x + 2$	A1	2.2a
		(4)	
(a) Way 1 Notes			
Condone use of y for H for the <u>method</u> marks. A model of the form $H = x^2 + ax + b$ or $H = -x^2 + ax + b$ will score no marks. Note that it is possible to identify (by symmetry) that the points $(-2, 0.8)$ and $(18, 2)$ also lie on the parabola so you may see valid use of these points. M1: Uses the equation $H = \pm ax^2 \pm bx \pm c$ to model the path and uses $x = 0$ and $H = 2$ correctly placed to establish the value of the constant term and uses $x = 20$ and $H = 0.8$ or $x = 9, \frac{dH}{dx} = 0$ to give an equation in ‘a’ and ‘b’ with $\frac{dH}{dx}$ of the form $\dots ax + b$ An alternative is to recognise that the maximum occurs when $x = -\frac{b}{2a} = 9$ or equivalent e.g. maximum when $x = 9 \Rightarrow H = a(x - 9)^2 + \dots = ax^2 - 18ax + \dots \Rightarrow b = -18a$ Award for $\pm \frac{b}{2a} = 9$ or equivalent. They may also use e.g. $(-2, 0.8)$ or $(18, 2)$ to give an equation in a and b. dM1: This mark requires: - uses the equation $H = \pm ax^2 \pm bx \pm c$ to model the path and uses $x = 0$ and $H = 2$ correctly placed to establish the value of the constant term - uses $x = 20$ and $H = 0.8$ correctly placed and $x = 9, \frac{dH}{dx} = 0$ to give 2 equations in ‘a’ and ‘b’ with $\frac{dH}{dx}$ of the form $\dots ax + b$ or as above using $\pm \frac{b}{2a} = 9$ They may also use e.g. $(-2, 0.8)$ or $(18, 2)$ to give an equation in a and b. ddM1: Solves their 2 equations in ‘a’ and ‘b’ to find their ‘a’ and ‘b’. This may be done on a calculator. You do not need to check their method for solving.			

A1: Correct equation . Must be $H = f(x)$.			
(a)	$x = 9 \text{ at max} \Rightarrow H = A \pm B(x-9)^2$		
Way 2	and either $x = 0, H = 2 \Rightarrow 2 = A + 81B$ or $x = 20, H = 0.8 \Rightarrow 0.8 = A + 121B$	M1	3.3
	$x = 9 \text{ at max} \Rightarrow H = A + B(x-9)^2$ and $x = 0, H = 2 \Rightarrow 2 = A + 81B$ and $x = 20, H = 0.8 \Rightarrow 0.8 = A + 121B$	dM1	3.1b
	$2 = A + 81B, 0.8 = A + 121B \Rightarrow A = 4.43, B = -0.03$	ddM1	1.1b
	$H = 4.43 - 0.03(x-9)^2$	A1	2.2a
		(4)	
(a) Way 2 Notes			
Condone use of y for H for the <u>method</u> marks. A model of the form $H = A \pm (x-9)^2$ will score no marks. M1: Uses the equation $H = A \pm B(x-9)^2$ or $H = A \pm B(9-x)^2$ to model the path and uses one of the ‘end points’ correctly placed to give an equation in ‘A’ and ‘B’ They may also use e.g. $(-2, 0.8)$ or $(18, 2)$ to give an equation in A and B. dM1: Uses the equation $H = A + B(x-9)^2$ or $H = A + B(9-x)^2$ to model the path and uses both ‘end points’ correctly placed to give 2 equations in ‘A’ and ‘B’ They may also use e.g. $(-2, 0.8)$ or $(18, 2)$ to give an equation in A and B. ddM1: Solves their 2 equations in “A” and “B” to find their ‘A’ and ‘B’. This may be done on a calculator. You do not need to check their method for solving. A1: Correct equation. Must be $H = f(x)$. Note that using $H = A + B(x-9)^2$ followed by the incorrect assumption that $A = 2$ is unlikely to score any marks as they will subsequently not be able to produce 2 equations in “A” and “B” Possible alternative 3: $H = A((x-9)^2 - 81) + B$ $x = 0, H = 2 \Rightarrow B = 2$ $x = 20, H = 0.8 \Rightarrow 0.8 = 40A + B$ $B = 2 \Rightarrow A = -0.03$ $H = 2 - 0.03((x-9)^2 - 81)$ M1: Uses the equation $H = A((x-9)^2 - 81) + B$ to model the path and uses $H = 2$ when $x = 0$ correctly placed to find “B” dM1: Uses the equation $H = A((x-9)^2 - 81) + B$ to model the path and uses $H = 0.8$ when $x = 20$ correctly placed. May also use e.g. $(-2, 0.8)$ or $(18, 2)$ ddM1: Substitutes their value for “B” to find a value for “A” A1: Correct equation. Must be $H = f(x)$.			

(b)	Examples must focus on why the model may not be appropriate or give situations where the model would break down e.g.: • H is unlikely to be a quadratic function in x • The path is unlikely to be parabolic • Wind may affect the path of the ball • Wind may affect the distance the ball travels • Air resistance has not been considered • The ball is unlikely to travel in a vertical plane (as it may spin) • The ball is not a particle so has dimensions/size • The ground is unlikely to be horizontal • There may be trees (or other hazards) that would affect the path of the ball • The shape of the ball may affect the motion Condone statements (where the link to the model is not completely made) such as • The ball will spin • Ground is not flat • The ball is not a particle Do not accept statements that refer to the situation outside the range of the throw e.g. • The model is not valid for all values of x • H will become negative Do not accept statements that do not refer to the given model or single word vague answers e.g. • The distances may have been measured incorrectly • The ball is not modelled as a particle • “Friction”, “Spin”, “Force”, “air resistance” • It does not take into account the weight of the ball • It depends how good the thrower is • You cannot throw the ball the same way every time	B1	3.5b
		(1)	
(c)	$x = 16 \Rightarrow H = -0.03(16)^2 + 0.54(16) + 2 = \dots$	M1	3.4
	$H = 2.96$ So Chandra would not be able to catch the ball	A1	3.2a
		(2)	
(7 marks)			
Notes for (b) and (c)			
(b) B1: Gives a suitable limitation – see scheme If more than one limitation is given and one is acceptable then award this mark as long as none of the other statements are contradictory (they may be incorrect/inappropriate)			

(c)

M1: Substitutes $x = 16$ into their equation modelling the path **to obtain a value** for H . This may be seen explicitly as above or may be implied by their value (you may need to check). **Must have a quadratic function in x .**

A1: Depends on

- A correct equation
- $H = 2.96$
- Correct conclusion that she cannot catch the ball or equivalent

A minimum for M1A1 could be e.g. $x = 16 \Rightarrow H = 2.96$ “so no”

(c) Alternative:

$$\text{e.g. } 2.5 = 4.43 - 0.03(x - 9)^2 \Rightarrow x = 9 + \frac{\sqrt{579}}{3} = 17.02\dots$$

So Chandra would not be able to catch the ball

M1: Substitutes $H = 2.5$ into their **quadratic** equation modelling the path **to obtain a value** for x . This may be seen explicitly as above or may be implied by their value (you may need to check). **Must have a quadratic function in x .**

A1: Depends on

- A correct equation
- $x = \text{awrt } 17$
- Correct conclusion that she cannot catch the ball or equivalent.

A minimum for M1A1 could be e.g. $H = 2.5 \Rightarrow x = 17$ “so no”



Question	Scheme	Marks	AOs
12(a)	$H = ax^2 + bx + c$ and $x=0, H=3 \Rightarrow H = ax^2 + bx + 3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x=120, H=27 \Rightarrow 27 = 14400a + 120b + 3$	M1	3.1b
	or $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	A1	1.1b
	$H = ax^2 + bx + 3$ and $x=120, H=27 \Rightarrow 27 = 14400a + 120b + 3$ and $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = \dots, b = \dots$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
	(5)		
(b)(i)	$x = 90 \Rightarrow H \left(-\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3 \right) = 30 \text{ m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = \dots$	M1	3.4
	$x = (-4.868\dots) 184.868\dots$ $\Rightarrow x = 185 \text{ (m)}$	A1	3.2a
	(3)		
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. - The ground is unlikely to be horizontal - The ball is not a particle so has dimensions/size - The ball is unlikely to travel in a vertical plane (as it will spin) H is not likely to be a quadratic function in x	B1	3.5b
	(1)		
(9 marks)			
Notes			

(a)

M1: Translates the problem into a suitable model and uses $H = 3$ when $x = 0$ to establish $c = 3$ Condone with $a = \pm 1$ so $H = x^2 + bx + 3$ will score M1 but little elseM1: For a correct attempt at **using one of the two other pieces** of information within a quadratic model**Either** uses $H = 27$ when $x = 120$ (with $c = 3$) to produce a linear equation connecting a and b forthe model **Or** differentiates and uses $\frac{dH}{dx} = 0$ when $x = 90$. Alternatives exist here, using thesymmetrical nature of the curve, so they could use $x = -\frac{b}{2a}$ at vertex or use point (60, 27) or (180, 3).A1: At least one correct equation connecting a and b . Remember " a " could have been set as negative so an equation such as $27 = -14400a + 120b + 3$ would be correct in these circumstances.dM1: Fully correct strategy that uses $H = ax^2 + bx + 3$ with the two other pieces of information in order to establish the values of **both a and b** for the modelA1: Correct equation, not just the correct values of a , b and c . Award if seen in part (b)

(b)(i)

B1: Correct height including the units. CAO

(b)(ii)

M1: Uses $H = 0$ and attempts to solve for x . Usual rules for quadratics.

A1: Discards the negative solution (may not be seen) and identifies awrt 185 m. Condone lack of units



(c)

B1: Candidate should either refer to an issue with one of the four aspects of how the situation has been modelled or give a situation where the model breaks down

- the ball has been modelled as a particle
- there may be trees (or other hazards) in the way that would affect the motion

Condone answers (where the link to the model is not completely made) such as

- the ball will spin
- ground is not flat

Do not accept answers which refer to the situation after it hits the ground (this isn't what was modelled)

- the ball will bounce after hitting the ground
- it gives a negative height for some values for x

Do not accept answers that do not refer to the model in question, or else give single word vague answers

- the height of tee may have been measured incorrectly
- "friction", "spin", "force" etc
- it does not take into account the weight of the ball
- it depends on how good the golfer is
- the shape of the ball will affect the motion
- you cannot hit a ball the same distance each time you hit it

The method using an alternative form of the equation can be scored in a very similar way.

The first M is for the completed square form of the quadratic showing a maximum at $x = 90$

So award M1 for $H = \pm a(x-90)^2 + c$ or $H = \pm a(90-x)^2 + c$. Condone for this mark an equation with

$a = 1 \Rightarrow H = (x-90)^2 + c$ or $c = 3 \Rightarrow H = a(x-90)^2 + 3$ but will score little else

Alt (a)	$H = a(x+b)^2 + c$ and $x=90$ at $H_{\max} \Rightarrow H = a(x-90)^2 + c$	M1	3.3
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ or $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$	M1 A1	3.1b 1.1b
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ and $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$ $\Rightarrow a = \dots, c = \dots$	dM1	3.1b
	$H = -\frac{1}{300}(x-90)^2 + 30$ o.e	A1	1.1b
		(5)	
(b)	$x = 90 \Rightarrow H = 0^2 + 30 = 30$ m	B1	3.4
		(1)	
	$H = 0 \Rightarrow 0 = -\frac{1}{300}(x-90)^2 + 30 \Rightarrow x = \dots$	M1	3.4
	$\Rightarrow x = 185$ (m)	A1	3.2a
		(2)	

Note that $H = -\frac{1}{300}(x-90)^2 + 30$ is equivalent to $H = -\frac{1}{300}(90-x)^2 + 30$

Other versions using symmetry are also correct so please look carefully at all responses

E.g. Using a starting equation of $H = a(x-60)(x-120) + b$ leads to $H = -\frac{1}{300}(x-60)(x-120) + 27$



Question	Scheme	Marks	AOs
6 (a)	Attempts to use $h = A - B t^{1.5}$ to form one correct equation with either $17.6 = A - B \times 4^{1.5}$ or $11.9 = A - B \times 9^{1.5}$	M1	3.1b
	Correct equations $A - 8B = 17.6$ $A - 27B = 11.9$	A1	1.1b
	Solves simultaneously to find values for A and B	dM1	1.1b
	$h = 20 - 0.3 t^{1.5}$	A1	3.3
		(4)	
(b)	20 metres	B1ft	3.2a
		(1)	
(c)	$0 = 20 - 0.3 T^{1.5} \left(\Rightarrow T^{1.5} = \frac{200}{3} \right) \Rightarrow T = \dots$	M1	3.4
	$(T =) 16.4$	A1	1.1b
		(2)	
(7 marks)			
Notes:			
(a) M1: Forms one correct equation either $17.6 = A - 8B$ or $11.9 = A - 27B$ May be unsimplified e.g. $17.6 = A - B \times 4^{1.5}$ or $11.9 = A - B \times 9^{1.5}$ A1: $17.6 = A - 8B$ and $11.9 = A - 27B$ which may be unsimplified dM1: Solves simultaneously to find values for A and B. It is dependent on the previous method mark so at least one equation must be correct. Do not be concerned with the process as calculators may be used. Score if values for A and B are reached from a pair of simultaneous equations. A1: $h = 20 - 0.3 t^{1.5}$ o.e. e.g. $t = \left(\frac{10}{3} (20 - h) \right)^{\frac{2}{3}}$ isw once a correct equation is found. Requires the complete equation including $h = \dots$ Just stating the values for A and B is A0 but allow A1 to be scored if the correct equation is seen in (b) or (c) (b) B1ft: 20 metres but ft on their A metres (provided $A > 0$). Condone if they have a value for A which is given to a greater degree of accuracy than 3sf but they round this to 3sf or better in (b). Requires units as well (allow m for metres)			

(c) **Do not be concerned with the use of t or T . If awrt 16.4 is seen in (a) it must be seen or used in (c) to score.**

M1: Sets " 20 " – " 0.3 " $T^{1.5} = 0$ and proceeds to a value or expression for T e.g. $\left(\frac{"20"}{"0.3"}\right)^{\frac{2}{3}}$

Do not be concerned by the use of any inequalities instead of " $=$ "

You do not need to be concerned by the mechanics of the rearrangement, they just need to achieve a value or expression.

If no equation is seen then may be implied by a correct value for T (to the nearest integer e.g. awrt " 16 ") or a correct expression for T for their A and B . You may need to check

$\left(\frac{"20"}{"0.3"}\right)^{\frac{2}{3}}$ on your calculator. They may also achieve this by trial and improvement.

A1: ($T =$) awrt 16.4 including $\left(\frac{200}{3}\right)^{\frac{2}{3}}$ or exact equivalent e.g. $\sqrt[3]{\frac{200}{3}}$ ignore any units for time. If an exact value is given condone the radical symbol not fully covering the fraction

provided it is not clearly e.g. $\frac{\sqrt[3]{200}}{3}$

Condone $(0 \leq) T \leq$ awrt 16.4 o.e. but not $(0 \leq) T <$ awrt 16.4

Do not accept any greater than (or equal) inequalities e.g. $T \geq$ awrt 16.4

isw if there is a written response to (c) once a correct value or valid expression for T is seen.



Question	Scheme	Marks	AOs
11(a)	$\frac{300-1500}{50-30} = (-60)$	M1	3.1b
	$y-1500 = "-60"(x-30)$	dM1	1.1b
	$y = 3300 - 60x$	A1	3.3
	(3)		
(b)	Either $x("3300 - 60x")$ or $-10("3300 - 60x")$ seen	M1	1.1b
	$(P =) \frac{x("3300 - 60x") - 10("3300 - 60x") - 8000}{1000}$	dM1	3.3
	$P = 3.3x - 0.06x^2 - 33 + 0.6x - 8$ $P = -0.06x^2 + 3.9x - 41^*$	A1*	2.1
	(3)		
(c)	$\frac{-3.9 \pm \sqrt{3.9^2 - 4(-0.06)(-41)}}{2 \times -0.06}$	M1	3.1b
	$13.18 < x < 51.82$ (or $13.19 \leq x \leq 51.81$)	A1	3.2a
	(2)		
(d)	£32.50	B1ft	3.4
	(1)		
(e)	$(P =) -0.06(35)^2 + 3.9(35) - 41 = \dots$	M1	3.4
	£22 000 which is close to £21 750 so the model is suitable.	A1	3.5a
	(2)		
(11 marks)			

Notes

(a)

M1: Attempts the gradient of the straight line either way round.

Look for $\frac{300-1500}{50-30} = (-60)$ o.e. or $\frac{50-30}{300-1500} = \left(-\frac{1}{60}\right)$ o.e.Condone 1 sign/copying slip only if a correct formula is seen or implied e.g. $\frac{y_2 - y_1}{x_2 - x_1}$ Alternatively, sets up simultaneous equations $1500 = 30a + b$ and $300 = 50a + b$ (or $30 = 1500a + b$ and $50 = 300a + b$) and attempts to solve, leading to a value for a and/or b .
If using simultaneous equations, condone slips.dM1: Attempts the equation relating x and y with either coordinate (50, 300) or (30, 1500) usedcorrectly. Look for $y-1500 = "-60"(x-30)$ or $x-30 = "-\frac{1}{60}"(y-1500)$ or using (50, 300). Note that e.g. $y-30 = "-60"(x-1500)$ scores dM0 as the values have been used incorrectly. If using $y = mx + c$ they must proceed to $c = \dots$ If using simultaneous equations, they must proceed to values for **a and b** (Note for reference x in terms of y is $x = -\frac{1}{60}y + 55$)If they use e.g. $\frac{y-300}{1500-300} = \frac{x-50}{30-50}$ both M's can be scored together.

Condone 1 sign/copying slip only if a correct formula is seen or implied.

A1: cao Either $y = 3300 - 60x$ or $y = -60x + 3300$ or equivalent with y in terms of x

The correct equation with no working scores full marks.



(b)

M1: Attempts either $\pm x("3300 - 60x")$ or $\pm 10("3300 - 60x")$ with or without dividing by 1000.
May see both in one step, i.e., $\pm(x \pm 10)("3300 - 60x")$

Alternatively, allow $(P =) \pm xy \pm 10y \pm 8000$ with or without dividing by 1000.

Allow their model for y to be any expression in x (i.e. it does not need to be linear) for M1.

dM1: Full attempt to find P in terms of x only. There is no need to see a LHS for this mark.

$$(P =) \frac{x("3300 - 60x") - 10("3300 - 60x") - 8000}{1000} \text{ or } (P =) \frac{(x - 10)("3300 - 60x") - 8000}{1000}$$

Condone the consistent absence of division by 1000 e.g.

$$(P =) x("3300 - 60x") - 10("3300 - 60x") - 8000 \text{ or } (P =) (x - 10)("3300 - 60x") - 8000$$

Condone invisible brackets if recovered. Dependent on the previous method mark.

Their model for y must now be a linear expression in x for dM1.

A1*: Achieves the given answer through rigorous argument with all elements pulled together clearly. Requires $P =$ (not just "profit = ") at some point which may be on a previous line. Any brackets seen must have been expanded in an intermediate line before the given answer. Allow otherwise correct work leading to $P = -60x^2 + 3900x - 41000$ followed by $P = -0.06x^2 + 3.9x - 41$ (without justifying the division by 1000). Allow recovery if they write e.g. 800 but it is recovered before the given answer.

Note: Attempts by verification are unlikely to score any marks. If unsure, send to review.
e.g. $x = 10 \rightarrow P = -0.06(10)^2 + 3.9(10) - 41 = -8$ without an accompanying argument involving the (x, y) values (50, 300) and (30, 1500) scores M0dM0A0*.

(c)

M1: Attempts to find either end point of the interval which may come from a calculator.

The exact values are $\frac{195 \pm 5\sqrt{537}}{6}$ but may just be seen in the quadratic formula, e.g.,

$$\frac{-3.9 \pm \sqrt{3.9^2 - 4(-0.06)(-41)}}{2 \times -0.06} \text{ or using completing the square – see general marking principles.}$$

Accept awrt 13.2 or awrt 51.8 for this mark.

A1: Either $13.18 < x < 51.82$ or $13.19 \leq x \leq 51.81$ No units are required but 2d.p. are required. Correct answer scores both marks. Allow e.g. $13.18 < x \leq 51.81$ or $13.19 \leq x < 51.82$ Allow e.g. " $x > 13.18$ and $x < 51.82$ " or " $x > 13.18 \cap x < 51.82$ " but not use of exact values and not " $x > 13.18, x < 51.82$ " or " $x > 13.18 \cup x < 51.82$ " or " $x > 13.18$ or $x < 51.82$ " Must be x or e.g. "selling price" for the range, not y or P .

(d)

B1ft: £32.50. Units and 2dp required. Not £32.50p. Ignore attempts to find maximum profit. Allow FT on the mean of their end points from (c) (their end points must be > 0 , their answer requires £ and 2dp).

$$\text{May come directly from the given answer to (b) using } x = -\frac{b}{2a} = -\frac{3.9}{2 \times -0.06} = £32.50$$

or from $\frac{dP}{dx} = -0.12x + 3.9 = 0$ or directly from a calculator. No working required.

Note: Following use of 13.2 and/or 51.8 (not e.g. 51.80) with either type of inequality in part (c), £32.5 will score B1ft so that the missing decimal place(s) is only penalised once.



(e)

M1: Attempts to substitute 35 (and not any other value) into the given equation for P to find a value for P .

May be implied by 22 or 22000. Condone slips.

Alternatively, substitutes $P = 21.75$ into the model, and solves via any acceptable method, including by calculator, to find a value for x . If using 21750 it must be substituted into their $(P =) -60x^2 + 3900x - 41000$. Note $x = 35.73$ or 29.27

A1: Requires:

- A correct value. Usually (£)22 000 but accept, e.g., 22 thousand (pound). If using 22 they must compare with 21.75. The alternative requires $x = \text{awrt } 35.7$ to be compared to 35.
- A correct comparison. Some examples below:
 - “Close to (£)21 750” o.e. e.g. “the model predicts 22000 and they make close to this amount”
 - “Agree to 2sf”. Do not condone “it rounds to 22 000” without mention of the degree of accuracy.
 - 1.1% error (accept approx. 1%) Do not be concerned about the mechanics of any percentage error calculation seen.
 - “The difference is small”
 - Condone e.g. $22 \approx 21.75$ o.e.
 - **“Only £250 off”**. However, just stating “£250 off” without suggesting this is a (relatively) small difference is not sufficient for this component. Similarly, “£350 off is a small difference” is incorrect (requires correct £250).
- A conclusion. e.g. Concludes it (the model) is suitable / reliable / good / fairly accurate / accurate.



Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2 + (y-r)^2 = r^2$ or $x^2 + y^2 - 2rx - 2ry + r^2 = 0$	B1	2.2a
	$y = 12 - 2x$, $x^2 + y^2 - 2rx - 2ry + r^2 = 0$ $\Rightarrow x^2 + (12 - 2x)^2 - 2rx - 2r(12 - 2x) + r^2 = 0$	M1	1.1b

Question	Scheme	Marks	AOs
7(a)(i)	$(x-5)^2 + (y+2)^2 = \dots$	M1	1.1b
	$(5, -2)$	A1	1.1b
(ii)	$r = \sqrt{5^2 + 2^2 - 11}$	M1	1.1b
	$r = 3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y = 3x + k \Rightarrow x^2 + (3x+k)^2 - 10x + 4(3x+k) + 11 = 0$ $\Rightarrow x^2 + 9x^2 + 6kx + k^2 - 10x + 12x + 4k + 11 = 0$	M1	2.1
	$\Rightarrow 10x^2 + (6k+2)x + k^2 + 4k + 11 = 0$	A1	1.1b
	$b^2 - 4ac = 0 \Rightarrow (6k+2)^2 - 4 \times 10 \times (k^2 + 4k + 11) = 0$	M1	3.1a
	$\Rightarrow 4k^2 + 136k + 436 = 0 \Rightarrow k = \dots$	M1	1.1b
	$k = -17 \pm 6\sqrt{5}$	A1	2.2a
		(5)	
(9 marks)			
Notes			

(a)(i)

 M1: Attempts to complete the square on by halving both x and y terms.

 Award for sight of $(x \pm 5)^2, (y \pm 2)^2 = \dots$ This mark can be implied by a centre of $(\pm 5, \pm 2)$.

 A1: Correct coordinates. (Allow $x = 5, y = -2$)

(a)(ii)

 M1: Correct strategy for the radius or radius². For example award for $r = \sqrt{5^2 + 2^2 - 11}$

 or an attempt such as $(x-a)^2 - a^2 + (y-b)^2 - b^2 + 11 = 0 \Rightarrow (x-a)^2 + (y-b)^2 = k \Rightarrow r^2 = k$

 A1: $r = 3\sqrt{2}$. Do not accept for the A1 either $r = \pm 3\sqrt{2}$ or $\sqrt{18}$

 The A1 can be awarded following sign slips on $(5, -2)$ so following $r^2 = 5^2 + 2^2 - 11$

(b) Main method seen

 M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of $= 0$

A1: Correct 3 term quadratic equation.

 The terms must be collected but this can be implied by correct a, b and c

 M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both b and c are expressions in k . It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation

 M1: Solves 3TQ in k . See General Principles.

 The 3TQ in k must have been found as a result of attempt at $b^2 - 4ac \dots 0$

A1: Correct simplified values

Look carefully at the method used. It is possible to attempt this using gradients

(b) Alt 1	$x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow 2x + 2y \frac{dy}{dx} - 10 + 4 \frac{dy}{dx} = 0$	M1	2.1
		A1	1.1b
	Sets $\frac{dy}{dx} = 3 \Rightarrow x + 3y + 1 = 0$ and combines with equation for C $\Rightarrow 5x^2 - 50x + 44 = 0$ or $5y^2 + 20y + 11 = 0$ $\Rightarrow x = \dots$ or $y = \dots$	M1	3.1a
	$x = \frac{25 \pm 9\sqrt{5}}{5}, y = \frac{-10 \pm 3\sqrt{5}}{5}, k = y - 3x \Rightarrow k = \dots$ $k = -17 \pm 6\sqrt{5}$	M1	1.1b
		A1	2.2a



M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms

A1: Correct differentiation.

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y .

M1: Uses at least one pair of coordinates and l to find at least one value for k . It is dependent upon having attempted both M's

A1: Correct simplified values

(b) Alt 2	$x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow 2x + 2y \frac{dy}{dx} - 10 + 4 \frac{dy}{dx} = 0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx} = 3 \Rightarrow x + 3y + 1 = 0$ and combines with equation for l $y = 3x + k, x + 3y = 1$ $\Rightarrow x = \dots$ and $y = \dots$ in terms of k	M1	3.1a
	$x = \frac{-3k-1}{10}, y = \frac{k-3}{10}, x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow k = \dots$	M1	1.1b
	$k = -17 \pm 6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3

M1 A1: Correct differentiation (See alt 1)

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k

M1: Substitutes for x and y into C and solves resulting 3TQ in k

A1: Correct simplified values

(b) Alt 3	$y = 3x + k \Rightarrow m = 3 \Rightarrow m_r = -\frac{1}{3}$	M1	
	$y + 2 = -\frac{1}{3}(x - 5)$	A1	
	$(x - 5)^2 + (y + 2)^2 = 18, y + 2 = -\frac{1}{3}(x - 5)$ $\Rightarrow \frac{10}{9}(x - 5)^2 = 18 \Rightarrow x = \dots$ or $\Rightarrow 10(y + 2)^2 = 18 \Rightarrow y = \dots$	M1	
	$x = \frac{25 \pm 9\sqrt{5}}{5}, y = \frac{-10 \pm 3\sqrt{5}}{5}, k = y - 3x \Rightarrow k = \dots$	M1	
	$k = -17 \pm 6\sqrt{5}$	A1	

M1: Applies negative reciprocal rule to obtain gradient of radius

A1: Correct equation of radial line passing through the centre of C

M1: Solves simultaneously to find x or y

Alternatively solves " $y = -\frac{1}{3}x - \frac{1}{3}$ " and $y = 3x + k$ to get x in terms of k which they substitute in

$x^2 + (3x + k)^2 - 10x + 4(3x + k) + 11 = 0$ to form an equation in k .

M1: Applies $k = y - 3x$ with at least one pair of values to find k

A1: Correct simplified values



$$\frac{1}{4}(0.5774 + 0.7071) + \frac{1}{4}(0.7071 + 0.7746) + \frac{1}{4}(0.7746 + 0.8165) + \frac{1}{4}(0.8165 + 0.8452)$$

May be awarded for using the function e.g. $\frac{1}{2}h \times \left\{ \sqrt{\frac{0.5}{1+0.5}} + \sqrt{\frac{2.5}{1+2.5}} + 2 \left(\sqrt{\frac{1}{1+1}} + \sqrt{\frac{1.5}{1+1.5}} + \sqrt{\frac{2}{1+2}} \right) \right\}$

A1: Awrt 1.50 (Apply isw if necessary)

Correct answers with no working – send to review

(b)

B1ft: See main scheme. Must be considering $3 \times$ (a) and not e.g. attempting trapezium rule again.

(c)

B1: See scheme



Question Number	Scheme	Marks	AO's
2	Attempts any one of $(\pm \overrightarrow{PQ} =) \pm (\mathbf{q} - \mathbf{p}), (\pm \overrightarrow{PR} =) \pm (\mathbf{r} - \mathbf{p}), (\pm \overrightarrow{QR} =) \pm (\mathbf{r} - \mathbf{q})$ Or e.g. $(\pm \overrightarrow{PQ} =) \pm (\overrightarrow{OQ} - \overrightarrow{OP}), (\pm \overrightarrow{PR} =) \pm (\overrightarrow{OR} - \overrightarrow{OP}), (\pm \overrightarrow{QR} =) \pm (\overrightarrow{OR} - \overrightarrow{OQ})$	M1	1.1b
	Attempts e.g. $\mathbf{r} - \mathbf{q} = 2(\mathbf{q} - \mathbf{p})$ $\mathbf{r} - \mathbf{p} = 3(\mathbf{q} - \mathbf{p})$ $\frac{2}{3}(\mathbf{q} - \mathbf{p}) = \frac{1}{3}(\mathbf{r} - \mathbf{q})$ $\mathbf{q} = \mathbf{p} + \frac{1}{3}(\mathbf{r} - \mathbf{p})$ $\mathbf{q} = \mathbf{r} + \frac{2}{3}(\mathbf{p} - \mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r} - \mathbf{q} = 2\mathbf{q} - 2\mathbf{p} \Rightarrow 2\mathbf{p} + \mathbf{r} = 3\mathbf{q} \Rightarrow \mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})^*$	A1*	2.1
		(3)	
(3 marks)			

Notes:

M1: Attempts any of the relevant vectors by subtracting either way around. This may be implied by sight of any one of $\pm(\mathbf{q} - \mathbf{p}), \pm(\mathbf{r} - \mathbf{p}), \pm(\mathbf{r} - \mathbf{q})$ ignoring how they are labelled

dM1: Uses the given information and writes it correctly in vector form that if rearranged would give the printed answer

A1*: Fully correct work leading to the given answer. Allow $\text{OQ} = \dots$ as long as OQ has been defined as $\mathbf{q}$ earlier.

In the working allow use of P instead of $\mathbf{p}$ and Q instead of $\mathbf{q}$ as long as the intention is clear.

Question	Scheme	Marks	AOs
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3(a)	$2\log(4-x) = \log(4-x)^2$	B1	1.2
	$2\log(4-x) = \log(x+8) \Rightarrow \log(4-x)^2 = \log(x+8)$ $(4-x)^2 = (x+8)$ or $2\log(4-x) = \log(x+8) \Rightarrow \log(4-x)^2 - \log(x+8) = 0$ $\frac{(4-x)^2}{(x+8)} = 1$	M1	1.1b
	$16 - 8x + x^2 = x + 8 \Rightarrow x^2 - 9x + 8 = 0 *$	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2 - 9x + 8 = 0 \Rightarrow (4-x)^2 - x - 8 = 0$	B1	1.2
	$\Rightarrow (4-x)^2 = x + 8$ $\Rightarrow \log(4-x)^2 = \log(x+8)$	M1	1.1b
	$\Rightarrow 2\log(4-x) = \log(x+8) *$ Hence proved.	A1	2.1
(b)	(i) $(x =) 1, 8$	B1	1.1b
	(ii) 8 is not a solution as $\log(4-8)$ cannot be found	B1	2.3
		(2)	
(5 marks)			

Notes:

(a)

B1: States or uses $2\log(4-x) = \log(4-x)^2$

M1: Correct attempt at eliminating the logs to form a quadratic equation in x .

Note that this may be implied by e.g. $\log \frac{(4-x)^2}{(x+8)} = 0 \Rightarrow (4-x)^2 = x+8$

A1*: Proceeds to the given answer with at least one line where the $(4-x)^2$ has been multiplied out.

There must be no errors or omissions but condone invisible brackets around the arguments of the logs e.g. allow $\log 16 - 8x + x^2$ for $\log(16 - 8x + x^2)$ and $\log x + 8$ for $\log(x + 8)$

Note we will allow a start of $(4-x)^2 = x+8$ with no previous work for full marks.

Some examples of how to mark (a) in particular cases:



Question	Scheme	Marks	AOs
4	${}^7C_4a^3(2x)^4$	M1	1.1b



Question	Scheme	Marks	AOs
2(a)	$\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm 9x)^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-9x)^3$	M1	1.1b
	$(1-9x)^{\frac{1}{2}} = 1 + \frac{1}{2}(-9x) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm 9x)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-9x)^3$	A1	1.1b
	$(1-9x)^{\frac{1}{2}} = 1 - \frac{9}{2}x - \frac{81}{8}x^2 - \frac{729}{16}x^3$	A1	1.1b
		(3)	
(b)	Expansion is valid for $ x < \frac{1}{9}$ and $x = -\frac{2}{9}$ is outside this range.	B1	2.4
		(1)	

(4 marks)

Notes

(a)

M1: For an attempt at the binomial expansion with $n = \frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4. Award for the correct coefficient with the correct power of x .

e.g. $\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ **or** $\frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(\lambda x)^3$ where $\lambda \neq 1$

Condone missing or incorrect brackets around the x terms but the binomial coefficients must be correct. Allow $2!$ and/or $3!$ or 2 and/or 6 .

Do not allow notation such as $\begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}$ unless these are interpreted correctly.

A1: Correct unsimplified expression as shown but the bracketing must be correct unless any missing brackets are implied by subsequent work.
May be implied by a correct simplified expression.

OR allow this mark for at least 2 correct simplified terms from $-\frac{9}{2}x, -\frac{81}{8}x^2$ and $-\frac{729}{16}x^3$

A1: $1 - \frac{9}{2}x - \frac{81}{8}x^2 - \frac{729}{16}x^3$ or simplified equivalent. Correct answer with no working can score full marks. Ignore any extra terms and allow the terms to be listed or in a different order. Apply isw once a correct expansion is seen. Condone $+-$ (equivalent to listing). Allow recovery if applicable e.g. if an “ x ” is lost then “reappears”.

Allow decimal equivalents $1 - 4.5x - 10.125x^2 - 45.5625x^3$ provided they are exact.

Allow mixed numbers $1 - 4\frac{1}{2}x - 10\frac{1}{8}x^2 - 45\frac{9}{16}x^3$



Note: You may see attempts via direct expansion, but these will be scored using the main scheme, ignoring absence of powers on the 1s. The below attempts both score first M1A1.

If you are unsure, send to review.

$$(1-9x)^{\frac{1}{2}} = 1^{\frac{1}{2}} + \frac{1}{2} \left(1^{-\frac{1}{2}} \right) (-9x) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \left(1^{-\frac{3}{2}} \right) (-9x)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \left(1^{-\frac{5}{2}} \right) (-9x)^3$$

$$9^{\frac{1}{2}} \left(\frac{1}{9} - x \right)^{\frac{1}{2}} = 3 \left[\left(\frac{1}{9} \right)^{\frac{1}{2}} + \frac{1}{2} \left(\left(\frac{1}{9} \right)^{-\frac{1}{2}} \right) (-x) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \left(\left(\frac{1}{9} \right)^{-\frac{3}{2}} \right) (-x)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \left(\left(\frac{1}{9} \right)^{-\frac{5}{2}} \right) (-x)^3 \right]$$

(b)

B1: Expansion is valid for $|x| < \frac{1}{9}$ or $|x|, \frac{1}{9}$ and $x = -\frac{2}{9}$ is outside this range.

Requires:

- an acceptable range of validity given
- an acceptable comparison of $-\frac{2}{9}$ or $\frac{2}{9}$ with their range leading to e.g. "not valid".

Examples of acceptable alternatives include:

- (Valid for) $|9x| < 1$ or $|9x|, 1$ and as $9x = -2$ (the expansion is) not valid.
- (Valid for) $|x| < \frac{1}{9}$ or $|x|, \frac{1}{9}$ and as $\frac{2}{9} > \frac{1}{9}$ (or $-\frac{2}{9} < -\frac{1}{9}$) the expansion is not valid.
- (Valid for) $-\frac{1}{9} < x < \frac{1}{9}$ and $x = -\frac{2}{9}$ is too small/big (condoned as minimally acceptable).
- (Series converges for) $|-9x|, 1$ and as $-9x = 2$ the series will diverge.
- (Valid for) $|x| < \frac{1}{9}$ but $\left| -\frac{2}{9} \right| > \frac{1}{9}$ so $-\frac{2}{9}$ cannot be used.

Do not accept vague statements such as "it is too big", "it is outside the range" without any mention of what the range is. $-\frac{2}{9} < -\frac{1}{9}$ alone is insufficient evidence (without any mention of what the range is) and scores B0.

An attempt to evaluate the expansion and compare with $\sqrt{3}$ is not acceptable on its own.

Question	Scheme	Marks	AOs
1 (a)	$(1+8x)^{\frac{1}{2}} = 1 + \frac{1}{2} \times 8x + \frac{\frac{1}{2} \times -\frac{1}{2}}{2!} \times (8x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!} \times (8x)^3$	M1 A1	1.1b 1.1b
	$= 1 + 4x - 8x^2 + 32x^3 + \dots$	A1	1.1b
		(3)	
(b)	Substitutes $x = \frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x = \frac{1}{32}$ is substituted into $1 + 4x - 8x^2 + 32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
(5 marks)			
Notes:			

(a)

M1: Attempts the binomial expansion with $n = \frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4.

Award for the correct coefficient with the correct power of x . Do not accept ${}^nC_r$ notation for coefficients.

For example look for term 3 in the form $\frac{\frac{1}{2} \times -\frac{1}{2}}{2!} \times (*x)^2$ or $\frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!} \times (*x)^3$

A1: Correct (unsimplified) expression. May be implied by correct simplified expression

A1: $1 + 4x - 8x^2 + 32x^3$

Award if there are extra terms (even if incorrect).

Award if the terms are listed $1, 4x, -8x^2, 32x^3$

(b)

M1: Score for substituting $x = \frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to obtain $\frac{\sqrt{5}}{2}$ or equivalent such as $\sqrt{\frac{5}{4}}$

Alternatively award for substituting $x = \frac{1}{32}$ into **both sides** and making a connection between the two sides by use of an $=$ or $\approx$.

E.g. $\left(1 + \frac{8}{32}\right)^{\frac{1}{2}} = 1 + 4 \times \frac{1}{32} - 8 \times \left(\frac{1}{32}\right)^2 + 32 \times \left(\frac{1}{32}\right)^3$ following through on their expansion

Also implied by $\frac{\sqrt{5}}{2} = \frac{1145}{1024}$ for a correct expansion

It is not enough to state substitute $x = \frac{1}{32}$ into "the expansion" or just the rhs " $1 + 4x - 8x^2 + 32x^3$ "

A1ft: Requires a full (and correct) **explanation** as to how the expansion can be used to estimate $\sqrt{5}$

E.g. Calculates $1 + 4 \times \frac{1}{32} - 8 \times \left(\frac{1}{32}\right)^2 + 32 \times \left(\frac{1}{32}\right)^3$ and multiplies by 2.

This can be scored from an incorrect binomial expansion or a binomial expansion with more terms.

The explanation could be mathematical. So $\frac{\sqrt{5}}{2} = \frac{1145}{1024} \rightarrow \sqrt{5} = \frac{1145}{512}$ is acceptable.

SC: For 1 mark, M1, A0 score for a statement such as "substitute $x = \frac{1}{32}$ into both sides of part (a) and make $\sqrt{5}$ the subject"



Question	Scheme	Marks	AOs
7(a)	$\sqrt{4-9x} = 2(1 \pm \dots)^{\frac{1}{2}}$	B1	1.1b
	$\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}} = \dots + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \left(-\frac{9x}{4}\right)^2}{2!} \text{ or } \dots + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right) \left(-\frac{9x}{4}\right)^3}{3!}$	M1	1.1b
	$1 + \frac{1}{2} \times \left(-\frac{9x}{4}\right) + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \left(-\frac{9x}{4}\right)^2}{2!} + \frac{\frac{1}{2} \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right) \left(-\frac{9x}{4}\right)^3}{3!}$	A1	1.1b
	$\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$	A1	1.1b
		(4)	
(b)	States that the approximation will be an overestimate since all terms (after the first one) in the expansion are negative (since $x > 0$)	B1	3.2b
		(1)	
(5 marks)			
Notes:			

(a)

B1: Takes out a factor of 4 and writes $\sqrt{4-9x} = 2(1 \pm \dots)^{\frac{1}{2}}$ or $\sqrt{4}(1 \pm \dots)^{\frac{1}{2}}$ or $4^{\frac{1}{2}}(1 \pm \dots)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion of $(1+ax)^{\frac{1}{2}}$ $a \neq 1$ to form term 3 or term 4 with the correct structure. Look for the correct binomial coefficient multiplied by the corresponding power of x e.g.

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(\dots x)^2 \text{ or } \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(\dots x)^3 \text{ where } \dots \neq 1$$

Condone missing or incorrect brackets around the x terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expression for the expansion of $\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}}$ e.g.

$$1 + \frac{1}{2} \times \left(-\frac{9x}{4}\right) + \frac{\frac{1}{2} \times \left(\frac{1}{2}-1\right) \left(-\frac{9x}{4}\right)^2}{2!} + \frac{\frac{1}{2} \times \left(\frac{1}{2}-1\right) \times \left(\frac{1}{2}-2\right) \left(-\frac{9x}{4}\right)^3}{3!}$$

which may be left unsimplified as shown but the bracketing must be correct unless any missing brackets are implied by subsequent work. If the 2 outside this expansion is only partially applied to this expansion then score A0 but if it is applied to all terms this A1 can be implied.

OR at least 2 correct simplified terms for the final expansion from, $-\frac{9x}{4}, -\frac{81x^2}{64}, -\frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$$

Ignore any extra terms if found. Allow terms to be “listed” and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an “x” is lost then “reappears”.

Direct expansion in (a) can be marked in a similar way:

$$\sqrt{4-9x} = (4-9x)^{\frac{1}{2}} = 4^{\frac{1}{2}} + \left(\frac{1}{2}\right)4^{-\frac{1}{2}} \times (-9x)^1 + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)4^{-\frac{3}{2}} \times \frac{(-9x)^2}{2!} + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)4^{-\frac{5}{2}} \times \frac{(-9x)^3}{3!}$$

B1: For 2 or $\sqrt{4}$ or $4^{\frac{1}{2}}$ as the constant term in the expansion.

M1: Correct form for term 3 or term 4.

E.g. $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) \times \frac{(-9x)^2}{2!}$ or $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) \times \frac{(-9x)^3}{3!}$ where $\dots \neq 1$

Condone missing brackets around the x terms but the binomial coefficients must be correct.

Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expansion (unsimplified as above)

OR at least 2 correct simplified terms from, $-\frac{9x}{4}, -\frac{81x^2}{64}, -\frac{729x^3}{512}$

A1: $\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$ oe and condone e.g. $2 + \frac{-9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$$

Ignore any extra terms if found. Allow terms to be “listed” and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an “x” is lost then “reappears”.

(b)

B1: States that the approximation will be an **overestimate** due to the fact that all terms (after the first one) in the expansion are negative or equivalent statements e.g.

- Overestimate because the terms are negative
- Overestimate as the terms are being taken away (from 2)

Condone “overestimate as every term is negative”

If you think a response is worthy of credit but are unsure then use Review.

This mark depends on having obtained an expansion in (a) of the form

$k - px - qx^2 - rx^3$ $k, p, q, r > 0$ but note that if e.g. one of the algebraic terms is zero or was “lost” or there are extra negative terms this mark is still available.

Question	Scheme	Marks	AOs
8	$y = \frac{(x-2)(x-4)}{4\sqrt{x}} = \frac{x^2 - 6x + 8}{4\sqrt{x}} = \frac{1}{4}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$\int \frac{1}{4}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} dx = \frac{1}{10}x^{\frac{5}{2}} - x^{\frac{3}{2}} + 4x^{\frac{1}{2}} (+c)$	dM1 A1	3.1a 1.1b
	Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{10}x^{\frac{5}{2}} - x^{\frac{3}{2}} + 4x^{\frac{1}{2}}$	M1	2.2a
	$\left(\frac{32}{10} - 8 + 8\right) - \left(\frac{2}{5}\sqrt{2} - 2\sqrt{2} + 4\sqrt{2}\right) = \frac{16}{5} - \frac{12}{5}\sqrt{2}$ Area R = $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ (or $\frac{16}{5} - \frac{12}{5}\sqrt{2}$)	A1	2.1
		(6)	
(6 marks)			
Notes:			

M1: Correct attempt to write $\frac{(x-2)(x-4)}{4\sqrt{x}}$ as a sum of terms with **indices**.

Look for at least two different terms with the correct index e.g. two of $x^{\frac{3}{2}}$, $x^{\frac{1}{2}}$, $x^{-\frac{1}{2}}$ which have come from the correct places.

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\frac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

A1: $\frac{1}{4}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$ which can be left unsimplified e.g. $\frac{1}{4}x^{2-\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}} - x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$
or as e.g. $\frac{1}{4}\left(x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 8x^{-\frac{1}{2}}\right)$

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\frac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

dM1: Integrates $x^n \rightarrow x^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $x^{\frac{3}{2}} \rightarrow x^{\frac{5}{2}}$, $x^{\frac{1}{2}} \rightarrow x^{\frac{3}{2}}$, $x^{-\frac{1}{2}} \rightarrow x^{\frac{1}{2}}$

It is dependent upon the first M so at least two terms must have had a correct index.

A1: $\frac{1}{10}x^{\frac{5}{2}} - x^{\frac{3}{2}} + 4x^{\frac{1}{2}} (+c)$. Allow unsimplified e.g. $\frac{1}{4} \times \frac{2}{5}x^{\frac{3}{2}+1} - \frac{1}{2} \times \frac{2}{3}x^{\frac{1}{2}+1} - \frac{2}{3}x^{\frac{1}{2}+1} + 2 \times 2x^{\frac{1}{2}}$
or as e.g. $\frac{1}{4}\left(\frac{2}{5}x^{\frac{5}{2}} - 4x^{\frac{3}{2}} + 16x^{\frac{1}{2}}\right)(+c)$.

M1: Substitutes the limits 4 and 2 to their $\frac{1}{10}x^{\frac{5}{2}} - x^{\frac{3}{2}} + 4x^{\frac{1}{2}}$ and subtracts either way round.

There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{10} \times 4^{\frac{5}{2}} - 4^{\frac{3}{2}} + 4 \times 4^{\frac{1}{2}} - \frac{1}{10} \times 2^{\frac{5}{2}} - 2^{\frac{3}{2}} + 4 \times 2^{\frac{1}{2}}$

This is an independent mark but the limits must be applied to an expression that is not y so they may even have differentiated.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ but also allow $\frac{16}{5} - \frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw

See overleaf for integration by parts and integration by substitution.



Integration by parts:

$\int \frac{(x-2)(x-4)}{4\sqrt{x}} dx = \int \frac{1}{4}(x-2)(x-4)x^{-\frac{1}{2}} dx = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}} dx$	M1 A1	1.1b 1.1b
$\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}} dx = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int x^{\frac{3}{2}} - 3x^{\frac{1}{2}} dx$ $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ Or e.g. $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	dM1 A1	3.1a 1.1b
Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	M1	2.2a
$0 - \frac{16}{3} + \frac{128}{15} - \left(0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}\right)$ $\text{Area } R = \frac{12}{5}\sqrt{2} - \frac{16}{5} \left(\text{or } \frac{16}{5} - \frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies integration by parts and reaches the form $\alpha(x-2)(x-4)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}} dx$ $\alpha, p \neq 0$

oe e.g. $\alpha(x^2 - 6x + 8)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}} dx$ $\alpha, p \neq 0$

A1: Correct first application of parts in any form

dM1: Attempts their $\int (px+q)x^{\frac{1}{2}} dx$ by expanding and integrating or may attempt parts again.

E.g. $\int (2x-6)x^{\frac{1}{2}} dx = \int \left(2x^{\frac{3}{2}} - 6x^{\frac{1}{2}}\right) dx = \dots$ or e.g. $\int (2x-6)x^{\frac{1}{2}} dx = \frac{2}{3}x^{\frac{3}{2}}(2x-6) - \frac{4}{3}\int x^{\frac{3}{2}} dx$

If they expand then at least one term requires $x^n \rightarrow x^{n+1}$ or if parts is attempted again, the structure must be correct.

A1: Fully correct integration in any form

M1: Substitutes the limits 4 and 2 to their $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ and subtracts

either way round. There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $0 - \frac{16}{3} + \frac{128}{15} - 0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}$

This is an independent mark but the limits must be applied to a “changed” function.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ but also allow $\frac{16}{5} - \frac{12}{5}\sqrt{2}$ or exact equivalents



Integration by substitution example:

$u = \sqrt{x} \left(x = u^2 \right) \Rightarrow \int \frac{(x-2)(x-4)}{4\sqrt{x}} dx = \int \frac{(u^2-2)(u^2-4)}{4u} \frac{dx}{du} du$ $= \int \frac{(u^2-2)(u^2-4)}{4u} 2u du$	M1 A1	1.1b 1.1b
$= \frac{1}{2} \int (u^4 - 6u^2 + 8) du = \frac{1}{2} \left(\frac{u^5}{5} - \frac{6u^3}{3} + 8u \right) (+c)$	dM1 A1	3.1a 1.1b
Deduces limits of integral are $\sqrt{2}$ and 2 and applies to their $\frac{1}{2} \left(\frac{u^5}{5} - \frac{6u^3}{3} + 8u \right)$	M1	2.2a
$\frac{1}{2} \left(\frac{32}{5} - 16 + 16 - \left(\frac{4\sqrt{2}}{5} - 4\sqrt{2} + 8\sqrt{2} \right) \right)$ $\text{Area } R = \frac{12}{5}\sqrt{2} - \frac{16}{5} \left(\text{or } \frac{16}{5} - \frac{12}{5}\sqrt{2} \right)$	A1	2.1
	(6)	

Notes:

M1: Applies the substitution e.g. $u = \sqrt{x}$ and attempts $k \int \frac{(u^2-2)(u^2-4)}{u} \frac{dx}{du} du$

A1: Fully correct integral in terms of u in any form e.g. $\frac{1}{2} \int (u^2-2)(u^2-4) du$

dM1: Expands the bracket and integrates $u^n \rightarrow u^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $u^4 \rightarrow u^5$, $u^2 \rightarrow u^3$, $k \rightarrow ku$

A1: $\frac{1}{2} \left(\frac{u^5}{5} - \frac{6u^3}{3} + 8u \right) (+c)$. Allow unsimplified.

M1: Substitutes the limits 2 and $\sqrt{2}$ to their $\frac{1}{2} \left(\frac{u^5}{5} - \frac{6u^3}{3} + 8u \right)$ and subtracts either way round.

There is no requirement to evaluate but 2 and $\sqrt{2}$ must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{2} \left(\frac{32}{5} - 16 + 16 - \frac{4\sqrt{2}}{5} - 4\sqrt{2} + 8\sqrt{2} \right)$

Alternatively reverses the substitution and applies the limits 4 and 2 with the same conditions.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ but also allow $\frac{16}{5} - \frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw.

There may be other substitutions seen and the same marking principles apply.



4 (a)	$\frac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} \times (1 \pm \dots)$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n = 1 + nax + \frac{n(n-1)}{2} a^2 x^2 +$	M1	1.1b
	$\left(1 - \frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x = -14$ and gives a valid reason. Eg explains that the expansion is not valid for $ x > 4$	B1	2.4
		(1)	
(b)(ii)	States $x = -\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
(6 marks)			

(a)

M1: For the strategy of expanding $\frac{1}{\sqrt{4-x}}$ using the binomial expansion.

You must see $4^{-\frac{1}{2}}$ oe and an expansion which may or may not be combined.

M1: Uses a correct binomial expansion for their $(1 \pm ax)^n = 1 \pm nax \pm \frac{n(n-1)}{2} a^2 x^2 +$

Condone sign slips and the "a" not being squared in term 3. Condone $a = \pm 1$

Look for an attempt at the correct binomial coefficient for their n , being combined with the correct power of ax

A1: $\left(1 - \frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \left(-\frac{x}{4}\right)^2$ unsimplified

FYI the simplified form is $1 + \frac{x}{8} + \frac{3x^2}{128}$ Accept the terms with commas between.

A1: $\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$ Ignore subsequent terms. Allow with commas between.

Note: Alternatively $(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}4^{-\frac{5}{2}}(-x)^2 + \dots$

M1: For $4^{-\frac{1}{2}} + \dots$ **M1:** As above but allow slips on the sign of x and the value of n **A1:** Correct unsimplified (as above) **A1:** As main scheme

(b) Any evaluations of the expansions are irrelevant.

Look for a suitable value and a suitable reason for both parts.

(b)(i)



B1: Requires $x = -14$ with a suitable reason.

Eg. $x = -14$ as the expansion is only valid for $|x| < 4$ or equivalent.

Eg. ' $x = -14$ as $|-14| > 4$ ' or 'I cannot use $x = -14$ as $\left|\frac{-14}{4}\right| > 1$ '

Eg. ' $x = -14$ as is outside the range $|x| < 4$ '

Do not allow ' -14 is too big' or ' $x = -14, |x| < 4$ ' either way around without some reference to the validity of the expansion.

(b)(ii)

B1: Requires $x = -\frac{1}{2}$ with a suitable reason.

Eg. $x = -\frac{1}{2}$ as it is 'the smallest/smaller value' or ' $x = -\frac{1}{2}$ as the value closest to zero' (that will give the more accurate approximation). The bracketed statement is not required.



Question	Scheme	Marks	AOs
11 (a)	$\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{0.5 \times -0.5}{2} \times (4x)^2$	M1	1.1b
	$(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5) \times (-1.5)}{2} (-x)^2$	M1	1.1b
	$(1+4x)^{0.5} = 1 + 2x - 2x^2$ and $(1-x)^{-0.5} = 1 + 0.5x + 0.375x^2$ oe	A1	1.1b
	$(1+4x)^{0.5} \times (1-x)^{-0.5} = (1+2x-2x^2 \dots) \times \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 \dots\right)$ $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + 2x + x^2 - 2x^2 + \dots$ $= A + Bx + Cx^2$	dM1	2.1
	$= 1 + \frac{5}{2}x - \frac{5}{8}x^2 \dots *$	A1*	1.1b
		(6)	
(b)	Expression is valid $ x < \frac{1}{4}$ Should not use $x = \frac{1}{2}$ as $\frac{1}{2} > \frac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x = \frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}} = \frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	
(10 marks)			

(a)

B1: Scored for key step in setting up the process so that it can be attempted using binomial expansionsThis could be achieved by $\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$ See end for other alternatives

It may be implied by later work.

M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{(0.5) \times (-0.5)}{2} \times (4x)^2$ There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1 + 2x \pm 0.5x^2$ **M1:** Award for an attempt at the binomial expansion $(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5) \times (-1.5)}{2} (-x)^2$ There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1 \pm 0.5x \pm 0.375x^2$ **A1:** Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end**dM1:** In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step

the multiplication on one term only. It is dependent upon having scored the first B and one of the other two M's

In the alternative it is for multiplying $\left(1 + \frac{5}{2}x - \frac{5}{8}x^2\right)(1-x)^{0.5}$ and comparing it to $(1+4x)^{0.5}$

It is for the key step in adding 'six' terms to produce the quadratic expression.

A1*: Completes proof with no errors or omissions. In the alternative there must be some reference to the fact that both sides are equal.

(b)

B1: States that the expansion may not / is not valid when $|x| > \frac{1}{4}$

This may be implied by a statement such as $\frac{1}{2} > \frac{1}{4}$ or stating that the expansion is only valid when $|x| < \frac{1}{4}$

Condone, for this mark a candidate who substitutes $x = \frac{1}{2}$ into the $4x$ **and** states it is not valid as $2 > 1$ oe

Don't award for candidates who state that $\frac{1}{2}$ is too big without any reference to the validity of the expansion.

As a rule you should see some reference to $\frac{1}{4}$ or $4x$

(c)(i)

M1: Substitutes $x = \frac{1}{11}$ into BOTH sides $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ and attempts to find at least one side.

As the left hand side is $\frac{\sqrt{6}}{2}$ they may multiply by 2 first which is acceptable

A1: Finds both sides leading to a correct equation/statement $\sqrt{\frac{15}{10}} = \frac{1183}{968}$ oe $\sqrt{6} = 2 \times \frac{1183}{968}$

A1: $\sqrt{6} = \frac{1183}{484}$ or $\sqrt{6} = \frac{2904}{1183}$ $\sqrt{6} = 2 \times \frac{1183}{968} = \frac{1183}{484}$ would imply all 3 marks

Watch for other equally valid alternatives for 11(a) including

B1: $(1+4x)^{0.5} \approx \left(1 + \frac{5}{2}x - \frac{5}{8}x^2\right)(1-x)^{0.5}$ then the M's are for $(1+4x)^{0.5}$ and $(1-x)^{0.5}$

M1: $(1-x)^{0.5} = 1 + (0.5)(-x) + \frac{(0.5) \times (-0.5)}{2}(-x)^2$

Or

B1: $\sqrt{\frac{1+4x}{1-x}} = \sqrt{1 + \frac{5x}{1-x}} = \left(1 + 5x(1-x)^{-1}\right)^{\frac{1}{2}}$ then the first M1 for one application of binomial and the second would be for both $(1-x)^{-1}$ and $(1-x)^{-2}$

Or

B1: $\sqrt{\frac{1+4x}{1-x}} \times \frac{\sqrt{1-x}}{\sqrt{1-x}} = \sqrt{(1+3x-4x^2)} \times (1-x)^{-1} = \left(1 + (3x-4x^2)\right)^{\frac{1}{2}} \times (1-x)^{-1}$

Question	Scheme	Marks	AOs
13(a)	$3^{-2} \left(1 + \frac{x}{3}\right)^{-2} = 3^{-2}(1 + \dots x + \dots x^2)$	M1	1.1b
	$(-2)\left(\frac{x}{3}\right)$ or $\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	M1	1.1b
	$\left(1 + \frac{x}{3}\right)^{-2} = 1 + (-2)\left(\frac{x}{3}\right) + \frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	A1	1.1b
	$3^{-2} \left(1 + \frac{x}{3}\right)^{-2} = \frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}$	A1	2.1
		(4)	

(a)

M1: Attempts a binomial expansion by taking out a factor of 3^{-2} or $\frac{1}{3^2}$ or $\frac{1}{9}$ and achieves at least the first 3 terms in their expansion. May be seen separately e.g. $\frac{1}{9}$ and $(1 + \dots x + \dots x^2)$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)(kx)$ or $\frac{(-2)(-2-1)}{2!}(kx)^2$ where $k \neq 1$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $\left(1 + \frac{x}{3}\right)^{-2}$ e.g. $= 1 + (-2)\left(\frac{x}{3}\right) + \frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2 - \dots$ or $1 - \frac{2x}{3} + \frac{x^2}{3} - \dots$ Do not condone missing brackets unless they are implied by subsequent work.

Condone $\left(-\frac{x}{3}\right)^2$ for $\left(\frac{x}{3}\right)^2$

Also allow this mark for 2 correct simplified terms from $\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

Direct expansion, if seen, should be marked as follows:

$$\left((3+x)^{-2} = 3^{-2} - 2 \times 3^{-3} \times x + \frac{-2(-2-1)}{2!} \times 3^{-4} \times x^2\right)$$

$$\mathbf{M1:} \text{ For } (3+x)^{-2} = 3^{-2} + 3^{-3} \times \alpha x + 3^{-4} \times \beta x^2$$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2) \times 3^{-3} x$ or $\frac{(-2)(-2-1)}{2!} \times 3^{-4} x^2$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $(3+x)^{-2}$ e.g. $3^{-2} - 2 \times 3^{-3} \times x + \frac{-2(-2-1)}{2!} \times 3^{-4} \times x^2$

Also award for at least 2 correct simplified terms from $\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

Note that M0M1A1A0 is a possible mark trait in either method



Note regarding a possible misread in parts (b) and (c)

Some candidates are misreading $\int \frac{6x}{(3+x)^2} dx$ in parts (b) and (c) as $\int \frac{6}{(3+x)^2} dx$

If parts (b) and (c) are consistently attempted with $\int \frac{6}{(3+x)^2} dx$ then we will allow the M

marks in (b) **only**. **M1** for $x^n \rightarrow x^{n+1}$ applied to their expansion in part (a) or $6 \times$ (their expansion in part (a)) and **dM1** for substituting in 0.4 and 0.2 and subtracting either way round (may be implied).

No marks are available in part (c)

MARK PARTS (b) and (c) TOGETHER

(b)	$\int 6x \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27} \right) dx = \int \left(\frac{2x}{3} - \frac{4x^2}{9} + \frac{2x^3}{9} \right) dx = \dots$	M1	1.1b
	$\int \left(\frac{2x}{3} - \frac{4x^2}{9} + \frac{2x^3}{9} \right) dx = \frac{x^2}{3} - \frac{4x^3}{27} + \frac{x^4}{18} \text{ oe}$	A1	1.1b
	$\left[\frac{x^2}{3} - \frac{4x^3}{27} + \frac{x^4}{18} \right]_{0.2}^{0.4} = \left(\frac{(0.4)^2}{3} - \frac{4(0.4)^3}{27} + \frac{(0.4)^4}{18} \right) - \left(\frac{(0.2)^2}{3} - \frac{4(0.2)^3}{27} + \frac{(0.2)^4}{18} \right)$	dM1	3.1a
	$= \text{awrt } 0.03304 \text{ or } \frac{223}{6750}$	A1	1.1b
		(4)	

(b)

M1: Attempts to multiply their expansion from part (a) by $6x$ or just x and attempts to integrate. Condone copying slips and slips in expanding. Look for $x^n \rightarrow x^{n+1}$ at least once having multiplied by $6x$ or x . Ignore e.g. spurious integral signs.

A1: Correct integration, simplified or unsimplified.

$$\frac{x^2}{3} - \frac{4x^3}{27} + \frac{x^4}{18} \text{ oe e.g. } \frac{1}{9} \left(3x^2 - \frac{4x^3}{3} + \frac{x^4}{2} \right), 6 \left(\frac{x^2}{18} - \frac{2x^3}{81} + \frac{x^4}{108} \right)$$

If they have extra terms they can be ignored.

Ignore e.g. spurious integral signs.

dM1: An overall problem-solving mark for

- using part (a) by integrating $6x \times$ their binomial expansion and
- substituting in 0.4 and 0.2 and subtracting either way round (may be implied)

For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the limits 0.4 and 0.2.

This could be e.g. $\left[f(x) \right]_{0.2}^{0.4} = \dots$ provided the first M was scored. If the integration was correct, evidence can be taken from answer of awrt 0.0330 if limits are not seen elsewhere. **Depends on the first M mark.**

A1: awrt 0.03304 (NB allow the exact value which is $\frac{223}{6750} = 0.033037037\dots$).

Is w following a correct answer.

Note answers which use additional terms in the expansion to give a different approximation score A0

Also note that the actual value is 0.032865...



Some may use integration by parts in (b) and the following scheme should be applied.

Integration by parts in (b):

Either by taking $u = 6x$ and $\frac{dv}{dx} = \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)$

$$\begin{aligned}\int 6x \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) dx &= 6x \times \left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - 6 \int \left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) dx \\ &= 6x \left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - \left(\frac{1}{3}x^2 - \frac{2x^3}{27} + \frac{6x^4}{324}\right)\end{aligned}$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) dx = kx \times f(x) - k \int f(x) dx = kx \times f(x) - kg(x)$$

Where $f(x)$ is an attempt to integrate their expansion from (a) with $x^n \rightarrow x^{n+1}$ at least once

and $g(x)$ is an attempt to integrate their $f(x)$ with $x^n \rightarrow x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

Or by taking $u = \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)$ and $\frac{dv}{dx} = 6x$

$$\begin{aligned}\int 6x \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) dx &= 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int 3x^2 \times \left(-\frac{2}{27} + \frac{2x}{27}\right) dx \\ &= 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int \left(\frac{6x^3}{27} - \frac{6x^2}{27}\right) dx = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \left(\frac{6x^4}{108} - \frac{6x^3}{81}\right)\end{aligned}$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) dx = kx^2 \times f(x) - k \int x^2 g(x) dx = kx^2 \times f(x) - kh(x)$$

Where $f(x)$ is their expansion from (a) and $g(x)$ is an attempt to differentiate their $f(x)$ with

$x^n \rightarrow x^{n-1}$ at least once **and** $h(x)$ is an attempt to integrate their $x^2 g(x)$ with $x^n \rightarrow x^{n+1}$ at

least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme



(c)	Overall problem-solving mark (see notes)	M1	3.1a
	$u = 3 + x \Rightarrow \int_{3.2}^{3.4} f(u) du \Rightarrow \int_{3.2}^{3.4} \frac{6(u-3)}{u^2} du = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2} du \Rightarrow \dots \ln u + \dots u^{-1}$	M1	1.1b
	$\int_{3.2}^{3.4} \frac{6(u-3)}{u^2} du = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2} du \Rightarrow 6 \ln u + 18u^{-1}$	A1	1.1b
	$\left[6 \ln u + 18u^{-1} \right]_{3.2}^{3.4} = \left(6 \ln 3.4 + \frac{18}{3.4} \right) - \left(6 \ln 3.2 + \frac{18}{3.2} \right) = \dots$	ddM1	1.1b
	$6 \ln \left(\frac{17}{16} \right) - \frac{45}{136} \text{ oe}$	A1	2.1
		(5)	
(c) Alt 1	Overall problem-solving mark (see notes)	M1	3.1a
	$\int 6x(3+x)^{-2} dx = \frac{\dots x}{3+x} \pm \dots \int (3+x)^{-1} dx = \frac{\dots x}{3+x} \pm \dots \ln(3+x) \text{ oe}$	M1	1.1b
	$= 6 \ln(3+x) - \frac{6x}{3+x} \text{ oe}$	A1	1.1b
	$\left(6 \ln(3+0.4) - \frac{6(0.4)}{3+0.4} \right) - \left(6 \ln(3+0.2) - \frac{6(0.2)}{3+0.2} \right) = \dots$	ddM1	1.1b
	$6 \ln \left(\frac{17}{16} \right) - \frac{45}{136} \text{ oe}$	A1	2.1

(c) Alt 2	Overall problem-solving mark (see notes)	M1	3.1a
	$\int 6x(3+x)^{-2} dx = \int \left(\frac{\dots}{(3+x)} + \frac{\dots}{(3+x)^2} \right) dx = \dots \ln(3+x) + \frac{\dots}{3+x} \text{ oe}$	M1	1.1b
	$= 6 \ln(3+x) + \frac{18}{3+x} \text{ oe}$	A1	1.1b
	$\left(6 \ln(3+0.4) + \frac{18}{3+0.4} \right) - \left(6 \ln(3+0.2) + \frac{18}{3+0.2} \right) = \dots$	ddM1	1.1b
	$6 \ln \left(\frac{17}{16} \right) - \frac{45}{136} \text{ oe}$	A1	2.1

(13 marks)

Notes

(c) There are various methods which can be used

M1: An overall problem-solving mark for all of

- using an appropriate integration technique e.g. substitution, by parts or partial fractions – note that this may not be correct but mark positively if they have tried one of these approaches
- integrates one of their terms to a natural logarithm, e.g., $\frac{a}{3+x} \rightarrow b \ln(3+x)$ or $\frac{a}{u} \rightarrow b \ln u$
- substitutes in correct limits and subtracts either way round

M1: Integrates to achieve an expression of the required form for their chosen method

- substitution: $u = x + 3 \rightarrow \pm \frac{a}{u} \pm b \ln u$ or e.g. $u = (x+3)^2 \rightarrow \pm \frac{a}{\sqrt{u}} \pm b \ln u$
- parts: $\pm a \ln(3+x) \pm \frac{bx}{3+x}$ condone missing brackets e.g. $\dots \ln x + 3$ for $\dots \ln(3+x)$
- partial fractions: $\pm a \ln(3+x) \pm \frac{b}{3+x}$ condone missing brackets e.g. $\dots \ln 3 + x$ for $\dots \ln(3+x)$



A1: Correct integration for their method e.g.

- substitution: $u = x + 3 \rightarrow 6 \ln u + 18u^{-1}$ or e.g. $u = (x + 3)^2 \rightarrow 3 \ln u + \frac{18}{\sqrt{u}}$
- parts: $6 \ln(3 + x) - \frac{6x}{3 + x}$
- partial fractions: $6 \ln(3 + x) + \frac{18}{3 + x}$ or e.g. $3 \ln(9 + 6x + x^2) + \frac{18}{3 + x}$

Note that the above terms may appear “separated” but must be correct with the correct signs.
(ignore any reference to a constant of integration)

Do not condone missing brackets e.g. $6 \ln x + 3$ for $6 \ln(3 + x)$ unless they are implied by later work.

ddM1: Substitutes in the correct limits for their integral and subtracts either way round to find a value

Depends on both previous method marks.

For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the appropriate limits.

This could be e.g. $[f(x)]_{0.2}^{0.4} = \dots$ provided both previous M marks were scored.

Note that for substitution they may revert back to $3 + x$ and so should be using 0.4 and 0.2

A1: A full and rigorous argument leading to $6 \ln\left(\frac{17}{16}\right) - \frac{45}{136}$ or exact equivalent e.g. $3 \ln\left(\frac{289}{256}\right) - \frac{45}{136}$ or

e.g. $-6 \ln\left(\frac{16}{17}\right) - \frac{45}{136}$

The brackets are not required around the $\frac{17}{16}$ and allow exact equivalents e.g. allow 1.0625 or $1\frac{1}{16}$

but not e.g. $\frac{3.4}{3.2}$. The $\frac{45}{136}$ must be exact or an exact equivalent. Also allow e.g. $6 \ln\left|\frac{17}{16}\right| - \frac{45}{136}$

Ignore spurious integral signs that may appear as part of their solution.

Question	Scheme	Marks	AOs
13(a)	Attempts two of the relevant vectors $\pm \overrightarrow{AB} = \pm(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$ $\pm \overrightarrow{AC} = \pm(-20\mathbf{i} + (p+3)\mathbf{j} + 5\mathbf{k})$ $\pm \overrightarrow{BC} = \pm(-16\mathbf{i} + (p-4)\mathbf{j} + 4\mathbf{k})$	M1	3.1a
	Uses two of the three vectors in such a way as to find the value of p . E.g. $p+3 = 5 \times 7$	dM1	2.1
	$p = 32$	A1	1.1b
		(3)	
	(a) Alternative:		
	$r_{AB} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$	M1	3.1a
	$4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow \lambda = 5$ $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow p = 35 - 3$	dM1	2.1
	$p = 32$	A1	1.1b
(b)	Deduces that $\overrightarrow{OD} = \lambda \overrightarrow{OB} = 4\lambda \mathbf{j} + 6\lambda \mathbf{k}$ and attempts $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - 32)\mathbf{j} + (6\lambda - 10)\mathbf{k}$	M1	3.1a
	Correct attempt at λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$ $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - 32)\mathbf{j} + (6\lambda - 10)\mathbf{k}$ $\overrightarrow{OA} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ $4\lambda - 32 = -12 \Rightarrow \lambda = \dots \quad \text{OR} \quad 6\lambda - 10 = 20 \Rightarrow \lambda = \dots$	dM1	1.1b
	$ \overrightarrow{OD} = 5 \times \sqrt{4^2 + 6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
	(b) Alternative:		
	Deduces that $\overrightarrow{OD} = \lambda \overrightarrow{OB} = 4\lambda \mathbf{j} + 6\lambda \mathbf{k}$ and attempts $\overrightarrow{OD} = \overrightarrow{OC} + \mu \overrightarrow{OA} = -16\mathbf{i} + 32\mathbf{j} + 10\mathbf{k} + \mu(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	M1	3.1a
	Correct attempt at λ or μ using the fact that $\lambda \overrightarrow{OB} = \overrightarrow{OC} + \mu \overrightarrow{OA}$ E.g. $-16 + 4\mu = 0 \Rightarrow \mu = 4$	dM1	1.1b
	$ \overrightarrow{OD} = 5 \times \sqrt{4^2 + 6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
(6 marks)			
Notes:			

(a)

M1: Attempts two of the three relevant vectors by **subtracting** either way around. See scheme.Allow equivalent work e.g. $\pm \overrightarrow{AB} = \pm(\overrightarrow{OB} + \overrightarrow{AO})$ 

dM1: For the key step in using the fact that if the vectors are parallel, they will be multiples of each other (where the multiple is something other than 1) to find p .

$$\text{E.g. } p+3=5\times 7, \quad p-4=\frac{4}{5}(p+3), \quad p-4=4\times 7$$

A1: $p=32$ (Condone 32j)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB}$, $\overrightarrow{AC}=5\overrightarrow{AB}$, $\overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find p and if the 3rd component has errors but is not used, full marks can be awarded.

Alternative:

M1: Forms the vector equation using A or B as position and $\pm\overrightarrow{AB}$ as the direction

dM1: For the key step in using the fact that C lies on the line to find p

A1: $p=32$ (Condone 32j)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB}$, $\overrightarrow{AC}=5\overrightarrow{AB}$, $\overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find p and if the 3rd component has errors but is not used, full marks can be awarded.

There will be other approaches e.g. using “gradients” and “ratios” and the method marks can be implied – if you are unsure if such attempts deserve credit use Review

(b) Vector approach

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-32)\mathbf{j}+(6\lambda-10)\mathbf{k}$

dM1: Correct attempt at finding λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

$$\text{E.g. } 16\mathbf{i}+(4\lambda-32)\mathbf{j}+(6\lambda-10)\mathbf{k}=4\alpha\mathbf{i}-3\alpha\mathbf{j}+5\alpha\mathbf{k} \Rightarrow \alpha=4 \Rightarrow 4\lambda-32=-3\times 4 \Rightarrow \lambda=...$$

A1: $|\overrightarrow{OD}|=10\sqrt{13}$

Alternative:

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts

$$\overrightarrow{OD}=\overrightarrow{OC}+\mu\overrightarrow{OA}=-16\mathbf{i}+32\mathbf{j}+10\mathbf{k}+\mu(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$$

dM1: Correct attempt at finding λ or μ using the fact that $\lambda\overrightarrow{OB}=\overrightarrow{OC}+\mu\overrightarrow{OA}$

$$\text{E.g. } (-16+4\mu)\mathbf{i}+(32-3\mu)\mathbf{j}+(10+5\mu)\mathbf{k}=4\lambda\mathbf{j}+6\lambda\mathbf{k} \Rightarrow -16+4\mu=0 \Rightarrow \mu=...$$

May also solve simultaneously using y and z components to find λ or μ

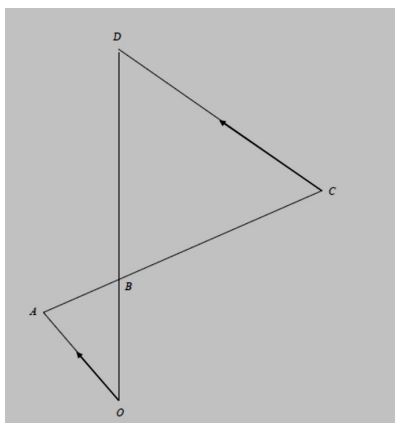
A1: $|\overrightarrow{OD}|=10\sqrt{13}$

Note that the correct vector is $20\mathbf{j}+30\mathbf{k}$

PTO for similar triangle approach



(b) Similar triangle approach



M1: For the key step in recognising that triangle BCD and triangle BAO are similar with a ratio of lengths of 4:1

dM1: States or uses the fact that $|\overrightarrow{OD}| = 5 \times |\overrightarrow{OB}|$

Stating this will score M1 dM1 provided there is no evidence of incorrect work

Note that they may establish this result using the work from (a) but must be used here to score.

A1: $|\overrightarrow{OD}| = 10\sqrt{13}$