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EDEXCEL A LEVEL MATHEMATICS: QUESTIONS BY TOPIC

Coordinate Geometry in the (x, y) Plane (3)

A Level Mathematics Topic 3

31 questions · 213 marks · Mark schemes included

Pearson Edexcel A Level Mathematics (9MA0), June 2018 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

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QUESTIONS

31 questions · 213 marks · 8 sub-topics



SUB-TOPIC 2.5

INEQUALITIES

Sub-topic 2.5 · 1 question · 5 marks · lowest tariff first





Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 6 marks)



SUB-TOPIC 3.1

STRAIGHT LINES

Sub-topic 3.1 · 1 question · 11 marks · lowest tariff first



11. A company is trying to determine the most profitable selling price for a new toy.

Given that

- if the selling price of each toy is £30, the company expects to sell 1500 toys in one year
- if the selling price of each toy is £50, the company expects to sell 300 toys in one year

Using a **linear** model, with y being the expected number of toys sold in one year and x pounds being the selling price of the toy,

(a) find an equation for y in terms of x .

(3)

Given that

- the cost of making each toy is £10
 - the company has additional costs of £8 000 per year
- (b) show that, according to the model, the yearly profit, P , in **thousands** of pounds, is given by

$$P = -0.06x^2 + 3.9x - 41$$

(3)

Use the model given in part (b) to answer parts (c), (d) and (e).

Given that the company wishes to make a profit on sales of the toy,

(c) find the range of possible selling prices of the toy.

(2)

(d) Hence, or otherwise, deduce the selling price of the toy that maximises the profit.

(1)

In one particular year, the company sold the toy for £35 and made £21 750 profit.

(e) Use this information to evaluate the suitability of the model.

(2)



SUB-TOPIC 3.2

CIRCLES

Sub-topic 3.2 · 5 questions · 35 marks · lowest tariff first



$$x^2 + y^2 - 10x + 16y = 80$$

(ii) the radius of the circle.

(3)

(b) find the exact length OP

(2)









(Total for Question 14 is 8 marks)



6.

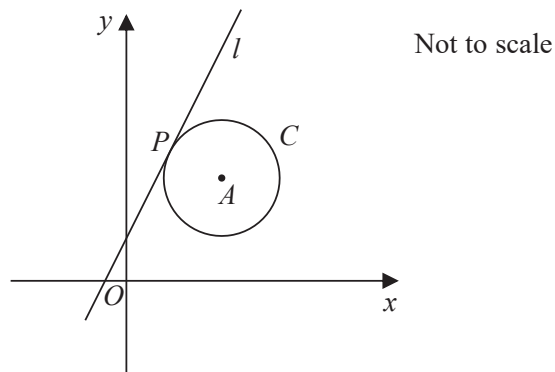


Figure 3

The circle C has centre A with coordinates $(7, 5)$.

The line l , with equation $y = 2x + 1$, is the tangent to C at the point P , as shown in Figure 3.

- (a) Show that an equation of the line PA is $2y + x = 17$ (3)

- (b) Find an equation for C . (4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C .

- (c) Find the value of the constant k . (3)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



SUB-TOPIC 3.3

PARAMETRIC EQUATIONS

Sub-topic 3.3 · 10 questions · 77 marks · lowest tariff first



13. A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \quad y = \frac{4t}{t^2 + 1} \quad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4$$

(3)





(Total for Question 4 is 6 marks)

(Total for Question 10 is 6 marks)

13. The curve C has parametric equations

$$x = \sin 2\theta \quad y = \operatorname{cosec}^3 \theta \quad 0 < \theta < \frac{\pi}{2}$$

- (a) Find an expression for $\frac{dy}{dx}$ in terms of θ

(3)

- (b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$

(3)

Question 13 continued

Lined area for writing the answer to Question 13.

(Total for Question 13 is 6 marks)





(Total for Question 9 is 7 marks)



Question 16 continued

Handwritten solution for Question 16 continued:

Let $x = 2\cos t$ and $y = 2\sin t$

Then $\frac{dy}{dx} = \frac{2\cos t}{-2\sin t} = -\cot t$

At $t = \frac{\pi}{3}$, $\frac{dy}{dx} = -\cot \frac{\pi}{3} = -\frac{1}{\sqrt{3}}$

Equation of the tangent line is $y - 2\sin \frac{\pi}{3} = -\frac{1}{\sqrt{3}}(x - 2\cos \frac{\pi}{3})$

$y - \sqrt{3} = -\frac{1}{\sqrt{3}}(x - 1)$

$y = -\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}} + \sqrt{3}$

$y = -\frac{1}{\sqrt{3}}x + \frac{4}{\sqrt{3}}$

Equation of the tangent line is $y = -\frac{1}{\sqrt{3}}x + \frac{4}{\sqrt{3}}$







(Total for Question 14 is 10 marks)

12.

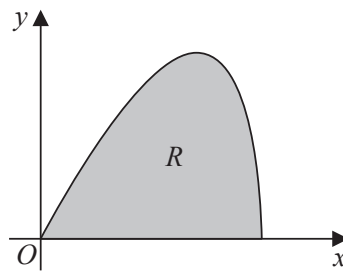


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6 \sin t \quad y = 5 \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

- (a) (i) Show that the area of R is given by $\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt$ (3)

- (ii) Hence show, by algebraic integration, that the area of R is exactly 20 (3)

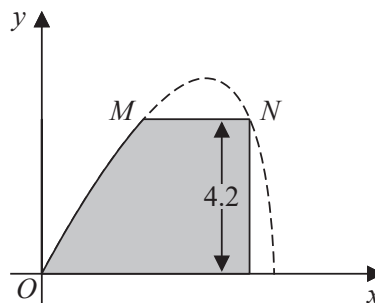


Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4. Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

- (b) calculate the width of the walkway. (5)



(Total for Question 12 is 11 marks)

16.

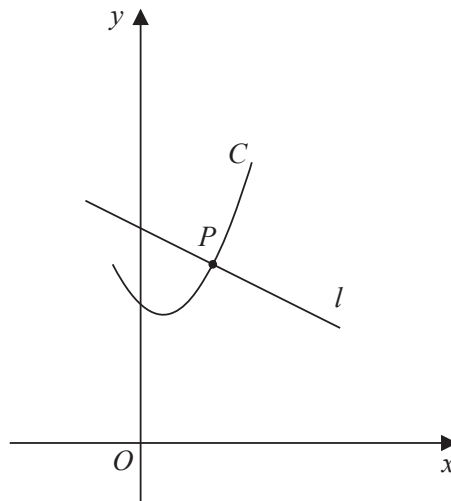


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 2 \tan t + 1 \quad y = 2 \sec^2 t + 3 \quad -\frac{\pi}{4} \leq t \leq \frac{\pi}{3}$$

The line l is the normal to C at the point P where $t = \frac{\pi}{4}$

(a) Using parametric differentiation, show that an equation for l is

$$y = -\frac{1}{2}x + \frac{17}{2} \quad (5)$$

(b) Show that all points on C satisfy the equation

$$y = \frac{1}{2}(x-1)^2 + 5 \quad (2)$$

The straight line with equation

$$y = -\frac{1}{2}x + k \quad \text{where } k \text{ is a constant}$$

intersects C at two distinct points.

(c) Find the range of possible values for k .

(5)









SUB-TOPIC 5.1

TRIGONOMETRIC RATIOS, RADIANS, ARCS AND SECTORS

Sub-topic 5.1 · 8 questions · 50 marks · lowest tariff first



Question 3 continued

(Total for Question 3 is 3 marks)



Question 3 continued

(Total for Question 3 is 4 marks)



Question 6 continued



(Total for Question 6 is 5 marks)



Question 8 continued



Question 8 continued



Question 8 continued

(Total for Question 8 is 10 marks)



15.

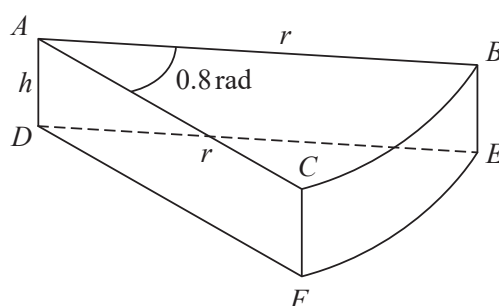


Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD , CF and BE are perpendicular to faces ABC and DEF
- edges AD , CF and BE have length h cm

Given that the volume of the toy is 240 cm^3

(a) show that the surface area of the toy, $S \text{ cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

Question 15 continued

Question 15 continued

Question 15 continued

(Total for Question 15 is 10 marks)



15.

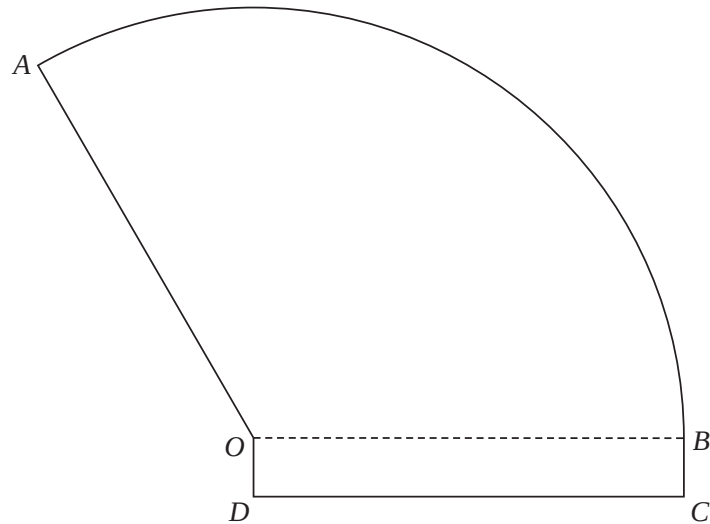


Figure 4

Figure 4 shows the plan view for the design of a stage.

The shape of this design consists of a sector of a circle AOB joined to a rectangle $OBCD$.

Given that

- the radius of the sector is r metres and angle AOB is θ radians
- the length and width of the rectangle are r metres and $\frac{1}{10}r$ metres respectively
- the total area of the stage is 240 m^2

(a) show that the perimeter of the stage, P metres, is given by

$$P = 2r + \frac{480}{r}$$

You must make your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which P has a stationary value.

(3)

(c) Prove, by further differentiation, that this value of r gives the minimum perimeter of the stage.

(2)

Question 15 continued



Question 15 continued



Question 15 continued

(Total for Question 15 is 9 marks)



Question 4 continued

(Total for Question 4 is 5 marks)



SUB-TOPIC 6.6

LOGARITHMIC GRAPHS AND PARAMETER ESTIMATION

Sub-topic 6.6 · 2 questions · 15 marks · lowest tariff first



10. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b \log_{10} l + \log_{10} a \quad (2)$$

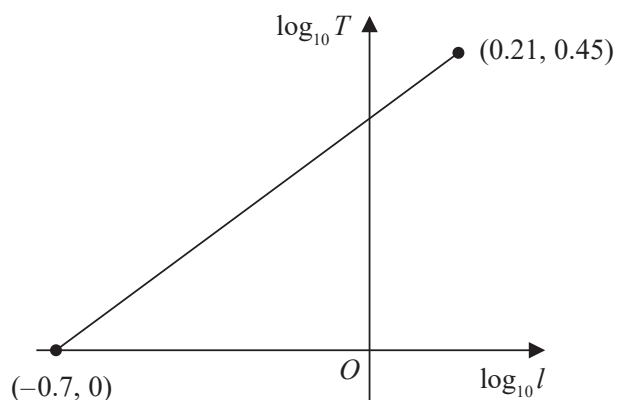


Figure 3

A student carried out an experiment to find the values of the constants a and b .

The student recorded the value of T for different values of l .

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data.

The straight line passes through the points $(-0.7, 0)$ and $(0.21, 0.45)$

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b , each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a .

(1)

Question 10 continued



Question 10 continued



Question 10 continued

(Total for Question 10 is 6 marks)



Question 13 continued



Question 13 continued

(Total for Question 13 is 9 marks)



SUB-TOPIC 10.4

POSITION VECTORS AND DISTANCES

Sub-topic 10.4 · 3 questions · 14 marks · lowest tariff first





Question 13 continued

Lined area for writing the answer to Question 13 continued.

Question 13 continued

(Total for Question 13 is 6 marks)



SUB-TOPIC 10.5

PROBLEM SOLVING WITH VECTORS

Sub-topic 10.5 · 1 question · 6 marks · lowest tariff first







(Total for Question 10 is 6 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
7	Attempts equation of line Eg Substitutes $(-2, 13)$ into $y = mx + 25$ and finds m	M1	1.1b
	Equation of l is $y = 6x + 25$	A1	1.1b
	Attempts equation of C Eg Attempts to use the intercept $(0, 25)$ within the equation $y = a(x \pm 2)^2 + 13$, in order to find a	M1	3.1a
	Equation of C is $y = 3(x+2)^2 + 13$ or $y = 3x^2 + 12x + 25$	A1	1.1b
	Region R is defined by $3(x+2)^2 + 13 < y < 6x + 25$ o.e.	B1ft	2.5
		(5)	
(5 marks)			
Notes:			

The first two marks are awarded for finding the equation of the line

M1: Uses the information in an attempt to find an equation for the line l .

E.g. Attempt using two points: Finds $m = \pm \frac{25-13}{2}$ and uses of one of the points in their $y = mx + c$ or equivalent to find c . Alternatively uses the intercept as shown in main scheme.

A1: $y = 6x + 25$ seen or implied. This alone scores the first two marks. Do not accept $l = 6x + 25$

It must be in the form $y = \dots$ but the correct equation can be implied from an inequality. E.g. $\dots < y < 6x + 25$

The next two marks are awarded for finding the equation of the curve

M1: A complete method to find the constant a in $y = a(x \pm 2)^2 + 13$ or the constants a, b in $y = ax^2 + bx + 25$.

An alternative to the main scheme is deducing equation is of the form $y = ax^2 + bx + 25$ and setting and solving a pair of simultaneous equations in a and b using the point $(-2, 13)$ the gradient being 0 at $x = -2$. Condone slips. Implied by $C = 3x^2 + 12x + 25$ or $3x^2 + 12x + 25$

FYI the correct equations are $13 = 4a - 2b + 25$ ($2a - b = -6$) and $-4a + b = 0$

A1: $y = 3(x+2)^2 + 13$ or equivalent such as $y = 3x^2 + 12x + 25$, $f(x) = 3(x+2)^2 + 13$.

Do not accept $C = 3x^2 + 12x + 25$ or just $3x^2 + 12x + 25$ for the A1 but may be implied from an inequality or from an attempt at the area, E.g. $\int 3x^2 + 12x + 25 \, dx$

B1ft: Fully defines the region R . Follow through on their equations for l and C .

Allow strict or non-strict inequalities as long as they are used consistently.

E.g. Allow for example " $3(x+2)^2 + 13 < y < 6x + 25$ " " $3(x+2)^2 + 13 \leq y \leq 6x + 25$ "

Allow the inequalities to be given separately, e.g. $y < 6x + 25, y > 3(x+2)^2 + 13$. Set notation may be used so

$\{(x, y) : y > 3(x+2)^2 + 13\} \cap \{(x, y) : y < 6x + 25\}$ is fine but condone with or without any of $(x, y) \leftrightarrow y \leftrightarrow x$

Incorrect examples include " $y < 6x + 25$ or $y > 3(x+2)^2 + 13$ ", $\{(x, y) : y > 3(x+2)^2 + 13\} \cup \{(x, y) : y < 6x + 25\}$

Values of x could be included but they must be correct. So $3(x+2)^2 + 13 < y < 6x + 25, x < 0$ is fine

If there are multiple solutions mark the final one.



Question	Scheme	Marks	AOs
11(a)	$\frac{300-1500}{50-30} = (-60)$	M1	3.1b
	$y-1500 = "-60"(x-30)$	dM1	1.1b
	$y = 3300 - 60x$	A1	3.3
	(3)		
(b)	Either $x("3300 - 60x")$ or $-10("3300 - 60x")$ seen	M1	1.1b
	$(P =) \frac{x("3300 - 60x") - 10("3300 - 60x") - 8000}{1000}$	dM1	3.3
	$P = 3.3x - 0.06x^2 - 33 + 0.6x - 8$ $P = -0.06x^2 + 3.9x - 41^*$	A1*	2.1
	(3)		
(c)	$\frac{-3.9 \pm \sqrt{3.9^2 - 4(-0.06)(-41)}}{2 \times -0.06}$	M1	3.1b
	$13.18 < x < 51.82$ (or $13.19 \leq x \leq 51.81$)	A1	3.2a
	(2)		
(d)	£32.50	B1ft	3.4
	(1)		
(e)	$(P =) -0.06(35)^2 + 3.9(35) - 41 = \dots$	M1	3.4
	£22 000 which is close to £21 750 so the model is suitable.	A1	3.5a
	(2)		
(11 marks)			

Notes

(a)

M1: Attempts the gradient of the straight line either way round.

Look for $\frac{300-1500}{50-30} = (-60)$ o.e. or $\frac{50-30}{300-1500} = \left(-\frac{1}{60}\right)$ o.e.Condone 1 sign/copying slip only if a correct formula is seen or implied e.g. $\frac{y_2 - y_1}{x_2 - x_1}$ Alternatively, sets up simultaneous equations $1500 = 30a + b$ and $300 = 50a + b$ (or $30 = 1500a + b$ and $50 = 300a + b$) and attempts to solve, leading to a value for a and/or b .
If using simultaneous equations, condone slips.dM1: Attempts the equation relating x and y with either coordinate (50, 300) or (30, 1500) usedcorrectly. Look for $y-1500 = "-60"(x-30)$ or $x-30 = "-\frac{1}{60}"(y-1500)$ or using (50, 300). Note that e.g. $y-30 = "-60"(x-1500)$ scores dM0 as the values have been used incorrectly. If using $y = mx + c$ they must proceed to $c = \dots$ If using simultaneous equations, they must proceed to values for a and b (Note for reference x in terms of y is $x = -\frac{1}{60}y + 55$)If they use e.g. $\frac{y-300}{1500-300} = \frac{x-50}{30-50}$ both M's can be scored together.

Condone 1 sign/copying slip only if a correct formula is seen or implied.

A1: cao Either $y = 3300 - 60x$ or $y = -60x + 3300$ or equivalent with y in terms of x

The correct equation with no working scores full marks.



(b)

M1: Attempts either $\pm x(3300 - 60x)$ or $\pm 10(3300 - 60x)$ with or without dividing by 1000.
May see both in one step, i.e., $\pm(x \pm 10)(3300 - 60x)$

Alternatively, allow $(P =) \pm xy \pm 10y \pm 8000$ with or without dividing by 1000.

Allow their model for y to be any expression in x (i.e. it does not need to be linear) for M1.

dM1: Full attempt to find P in terms of x only. There is no need to see a LHS for this mark.

$$(P =) \frac{x(3300 - 60x) - 10(3300 - 60x) - 8000}{1000} \text{ or } (P =) \frac{(x - 10)(3300 - 60x) - 8000}{1000}$$

Condone the consistent absence of division by 1000 e.g.

$$(P =) x(3300 - 60x) - 10(3300 - 60x) - 8000 \text{ or } (P =) (x - 10)(3300 - 60x) - 8000$$

Condone invisible brackets if recovered. Dependent on the previous method mark.

Their model for y must now be a linear expression in x for dM1.

A1*: Achieves the given answer through rigorous argument with all elements pulled together clearly. Requires $P =$ (not just “profit =”) at some point which may be on a previous line. Any brackets seen must have been expanded in an intermediate line before the given answer. Allow otherwise correct work leading to $P = -60x^2 + 3900x - 41000$ followed by $P = -0.06x^2 + 3.9x - 41$ (without justifying the division by 1000). Allow recovery if they write e.g. 800 but it is recovered before the given answer.

Note: Attempts by verification are unlikely to score any marks. If unsure, send to review.
e.g. $x = 10 \rightarrow P = -0.06(10)^2 + 3.9(10) - 41 = -8$ without an accompanying argument involving the (x, y) values (50, 300) and (30, 1500) scores M0dM0A0*.

(c)

M1: Attempts to find either end point of the interval which may come from a calculator.

The exact values are $\frac{195 \pm 5\sqrt{537}}{6}$ but may just be seen in the quadratic formula, e.g.,

$$\frac{-3.9 \pm \sqrt{3.9^2 - 4(-0.06)(-41)}}{2 \times -0.06} \text{ or using completing the square – see general marking principles.}$$

Accept awrt 13.2 or awrt 51.8 for this mark.

A1: Either $13.18 < x < 51.82$ or $13.19 \leq x \leq 51.81$ No units are required but 2d.p. are required. Correct answer scores both marks. Allow e.g. $13.18 < x \leq 51.81$ or $13.19 \leq x < 51.82$ Allow e.g. “ $x > 13.18$ and $x < 51.82$ ” or “ $x > 13.18 \cap x < 51.82$ ” but not use of exact values and not “ $x > 13.18, x < 51.82$ ” or “ $x > 13.18 \cup x < 51.82$ ” or “ $x > 13.18$ or $x < 51.82$ ” Must be x or e.g. “selling price” for the range, not y or P .

(d)

B1ft: £32.50. Units and 2dp required. Not £32.50p. Ignore attempts to find maximum profit. Allow FT on the mean of their end points from (c) (their end points must be > 0 , their answer requires £ and 2dp).

$$\text{May come directly from the given answer to (b) using } x = -\frac{b}{2a} = -\frac{3.9}{2 \times -0.06} = £32.50$$

or from $\frac{dP}{dx} = -0.12x + 3.9 = 0$ or directly from a calculator. No working required.

Note: Following use of 13.2 and/or 51.8 (not e.g. 51.80) with either type of inequality in part (c), £32.5 will score B1ft so that the missing decimal place(s) is only penalised once.



(e)

M1: Attempts to substitute 35 (and not any other value) into the given equation for P to find a value for P .

May be implied by 22 or 22000. Condone slips.

Alternatively, substitutes $P = 21.75$ into the model, and solves via any acceptable method, including by calculator, to find a value for x . If using 21750 it must be substituted into their $(P =) -60x^2 + 3900x - 41000$. Note $x = 35.73$ or 29.27

A1: Requires:

- A correct value. Usually (£)22 000 but accept, e.g., 22 thousand (pound). If using 22 they must compare with 21.75. The alternative requires $x = \text{awrt } 35.7$ to be compared to 35.
- A correct comparison. Some examples below:
 - “Close to (£)21 750” o.e. e.g. “the model predicts 22000 and they make close to this amount”
 - “Agree to 2sf”. Do not condone “it rounds to 22 000” without mention of the degree of accuracy.
 - 1.1% error (accept approx. 1%) Do not be concerned about the mechanics of any percentage error calculation seen.
 - “The difference is small”
 - Condone e.g. $22 \approx 21.75$ o.e.
 - **“Only £250 off”**. However, just stating “£250 off” without suggesting this is a (relatively) small difference is not sufficient for this component. Similarly, “£350 off is a small difference” is incorrect (requires correct £250).
- A conclusion. e.g. Concludes it (the model) is suitable / reliable / good / fairly accurate / accurate.



Question	Scheme	Marks	AOs
3 (a)	(i) $x^2 + y^2 - 10x + 16y = 80 \Rightarrow (x-5)^2 + (y+8)^2 = \dots$	M1	1.1b
	Centre $(5, -8)$	A1	1.1b
	(ii) Radius 13	A1	1.1b
		(3)	
(b)	Attempts $\sqrt{5^2 + 8^2} + 13$	M1	3.1a
	$13 + \sqrt{89}$ but ft on their centre and radius	A1ft	1.1b
		(2)	
(5 marks)			
Notes:			

(a)(i)

M1: Attempts to complete the square on **both** x and y terms.

Accept $(x \pm 5)^2 + (y \pm 8)^2 = \dots$ or imply this mark for a centre of $(\pm 5, \pm 8)$

Condone $(x \pm 5)^2 \dots (y \pm 8)^2 = \dots$ where the first $\dots$ could be $,$ or even $-$

A1: Correct centre $(5, -8)$.

Accept without brackets. May be written $x = 5, y = -8$

(a)(ii)

A1: 13. The M mark must have been awarded, so it can be scored following a centre of $(\pm 5, \pm 8)$.

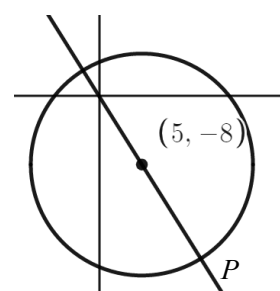
Do not allow for $\sqrt{169}$ or ± 13

(b)

M1: Attempts $\sqrt{5^2 + 8^2} + 13$ for their centre $(5, -8)$ and their radius 13.

Award when this is given as a decimal, e.g. 22.4 for correct centre and radius. Look for $\sqrt{a^2 + b^2} + r$ where centre is $(\pm a, \pm b)$ and radius is r

A1ft: $13 + \sqrt{89}$ Follow through on their $(5, -8)$ and their 13 leading to an exact answer. ISW for example if they write $13 + \sqrt{89} = 22.4$



There are more complicated attempts which could involve finding P by solving $y = -\frac{8}{5}x$ and

$x^2 + y^2 - 10x + 16y = 80$ simultaneously and choosing the coordinate with the greatest modulus. The method is only scored when the distance of the largest coordinate from O is attempted. Such methods are unlikely to result in an exact value but can score 1 mark for the method. Condone slips

FYI. Solving $y = -\frac{8}{5}x$ and $x^2 + y^2 - 10x + 16y = 80 \Rightarrow 89x^2 - 890x - 2000 = 0 \Rightarrow P = (11.89, -19.02)$

Hence $OP = \sqrt{11.89^2 + 19.02^2} (= 22.43)$ scores M1 A0 but $OP = \sqrt{258 + 26\sqrt{89}}$ is M1 A1

Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2 + (y-r)^2 = r^2$ or $x^2 + y^2 - 2rx - 2ry + r^2 = 0$	B1	2.2a
	$y = 12 - 2x$, $x^2 + y^2 - 2rx - 2ry + r^2 = 0$ $\Rightarrow x^2 + (12 - 2x)^2 - 2rx - 2r(12 - 2x) + r^2 = 0$	M1	1.1b

Question	Scheme	Marks	AOs
14(a)(i) (ii)	Centre is (3, -7)	B1	1.1b
	$(x-3)^2 + (y+7)^2 = 49 + 9 - 33 \Rightarrow r^2 = \dots (25)$	M1	1.1b
	$r = 5$	A1	1.1b
		(3)	
(b)	Distance between centres = $\sqrt{(3+6)^2 + (-7+8)^2} = \sqrt{82}$	M1 A1ft	3.1a 1.1b
	" $\sqrt{82}$ " - "5" or " $\sqrt{82}$ " + "5"	dM1	3.1a
	$\sqrt{82} - 5$ and $\sqrt{82} + 5$	A1	2.2a
	$\{k : \sqrt{82} - 5 < k\} \cap \{k : k < \sqrt{82} + 5\}$ or e.g. $\{k : \sqrt{82} - 5 < k < \sqrt{82} + 5\}$	A1	2.5
		(5)	
(8 marks)			
Notes			
(a)(i) B1: Correct centre. Allow as a coordinate pair or written separately e.g. $x = 3, y = -7$ or as a column vector $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$ Condone missing brackets e.g. 3, -7 but do not allow coordinates the wrong way round. (ii) M1: Uses a correct strategy to find the radius or radius² Requires an attempt at: $(x \pm 3)^2 + (y \pm 7)^2 - 3^2 - 7^2 \pm 33 = 0 \Rightarrow (x \pm 3)^2 + (y \pm 7)^2 = \alpha, \alpha > 0$ Award for $(x \pm 3)^2 + (y \pm 7)^2 - a^2 - b^2 \pm 33 = 0 \Rightarrow (x \pm 3)^2 + (y \pm 7)^2 = \alpha, \alpha > 0$ with at least one of $a = 3$ or $b = 7$ (or 9 or 49) You may see an attempt at "$f^2 + g^2 - c$" or "$\sqrt{f^2 + g^2 - c}$" e.g. "$3^2 + 7^2 \pm 33$" or "$\sqrt{3^2 + 7^2 \pm 33}$" A1: Correct radius of 5. Do not allow ± 5 or $\sqrt{25}$. May be scored following $(x \pm 3)^2 + (y \pm 7)^2 = 25$ Correct answers only in (a) scores B1M1A1 (b) M1: Uses Pythagoras correctly on their centre from part (a) and the given centre to find the distance between the centres. Look for $\sqrt{(-6 - (\text{their } x))^2 + (-8 - (\text{their } y))^2}$ or e.g. $\sqrt{((\text{their } x) - (-6))^2 + ((\text{their } y) - (-8))^2}$ but condone one sign slip with their coordinates if the intention is clear. A1ft: Correct distance or follow through their centre from part (a). This may be implied by their value. Condone the use of decimals so allow 3sf accuracy e.g. awrt 9.06 for $\sqrt{82}$ or you may need to check their value following an incorrect centre in (a)(i). Not e.g. $\pm\sqrt{82}$ unless the positive root is subsequently used. dM1: Correct strategy for one of the limits. E.g. adds or subtracts <u>their 5</u> to their distance between centres.			

A1: Correct limits. There is no follow through but allow decimals to 3sf e.g. awrt 4.06 and awrt 14.1

A1: Correct answer with **exact values** using set notation.

Allow as shown in the main scheme but also allow equivalent set notation e.g.

$$\{k : k \in \mathbb{R}, \sqrt{82} - 5 < k < \sqrt{82} + 5\}, \{k : \sqrt{82} - 5 < k < \sqrt{82} + 5\}, k \in (\sqrt{82} - 5, \sqrt{82} + 5)$$

and allow “|” for “:” and allow the “k:” or “k ∈” to be missing

e.g. $\{\sqrt{82} - 5 < k < \sqrt{82} + 5\}$ and $(\sqrt{82} - 5, \sqrt{82} + 5)$ are both acceptable.

But $\{k : k < \sqrt{82} + 5\} \cup \{k : k > \sqrt{82} - 5\}$ or $\{k : k < \sqrt{82} - 5, k < \sqrt{82} + 5\}$ score A0

Do **not** allow solutions not in set notation such as $\sqrt{82} - 5 < k < \sqrt{82} + 5$

Correct answers with no working should be sent to review.

Scenario for part (b) for reference:



Algebraic approach for part (b):

$$\begin{aligned}
 x^2 + y^2 - 6x + 14y + 33 &= x^2 + 12x + 36 + y^2 + 16y + 64 - k^2 \\
 \Rightarrow 18x + 2y + 67 - k^2 &= 0 \Rightarrow y = \frac{k^2 - 67}{2} - 9x \\
 (x - 3)^2 + (y + 7)^2 &= 25 \Rightarrow x^2 - 6x + 9 + \left(\frac{k^2 - 67}{2} - 9x + 7 \right)^2 = 25 \\
 \Rightarrow 82x^2 + 471x - 9k^2x + \frac{k^4 - 106k^2 + 2745}{4} &= 0 \\
 \text{When circles touch } b^2 - 4ac &= 0 \\
 \Rightarrow (471 - 9k^2)^2 - 4 \times 82 \left(\frac{k^4 - 106k^2 + 2745}{4} \right) &= 0 \\
 \Rightarrow k^4 - 214k^2 + 3249 &= 0 \\
 \Rightarrow (k^2 - 10k - 57)(k^2 + 10k - 57) &= 0 \\
 \Rightarrow k = 5 + \sqrt{82}, 5 - \sqrt{82}, -5 + \sqrt{82}, -5 - \sqrt{82} \\
 k = \underline{5 + \sqrt{82}, -5 + \sqrt{82}}
 \end{aligned}$$

We will mark this as follows:

M1: This requires a valid strategy that:

- solves the 2 circle equations simultaneously to find y in terms of x and k , or x in terms of y and k
- substitutes for y or x into one of the circle equations to obtain an equation in x and k only, or y and k only,
- attempts $b^2 - 4ac = 0$ or e.g. $b^2 - 4ac > 0$ or equivalent to obtain an equation in k only. You do not need to look at the details of their algebra.

A1: Correct simplified 3TQ in k^2

dM1: Solves their 3TQ in k^2 by any correct method including a calculator to find k .

A1: Both correct values for k (exact or decimals as in the main scheme) (they may have extras which can be ignored)

A1: As main scheme (exact and in set notation)

Note that work such as

$$(x + 6)^2 + (y + 8)^2 = k^2 \Rightarrow x + 6 + y + 8 = k$$

**is not a valid strategy as it greatly simplifies the problem
and would generally score no marks.**

Implicit differentiation approach for part (b):

$$x^2 + y^2 - 6x + 14y + 33 = 0 \Rightarrow 2x + 2y \frac{dy}{dx} - 6 + 14 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{6 - 2x}{2y + 14}$$

$$\frac{3 - x}{y + 7} = -\frac{x + 6}{y + 8} \Rightarrow (3 - x)(y + 8) = -(x + 6)(y + 7) \Rightarrow x = 9y + 66$$

$$(9y + 66)^2 + y^2 - 6(9y + 66) + 14y + 33 = 0 \Rightarrow 82y^2 + 1148y + 3993 = 0$$

$$(\text{or } 82x^2 - 492x - 1287 = 0)$$

$$y = \frac{-574 \pm 5\sqrt{82}}{82} \Rightarrow x = \frac{246 \pm 45\sqrt{82}}{82}$$

$$\left(\frac{246 + 45\sqrt{82}}{82}, \frac{-574 + 5\sqrt{82}}{82} \right) \rightarrow \left(\frac{246 + 45\sqrt{82}}{82} + 6 \right)^2 + \left(\frac{-574 + 5\sqrt{82}}{82} + 8 \right)^2 = k^2$$

$$\Rightarrow k^2 = 107 + 10\sqrt{82} \Rightarrow k = 5 + \sqrt{82}$$

$$\text{Then the same for } \left(\frac{246 - 45\sqrt{82}}{82}, \frac{-574 - 5\sqrt{82}}{82} \right) \rightarrow k = -5 + \sqrt{82}$$

We will mark this as follows:

M1: This requires a valid strategy that:

- differentiates the equations of both circles implicitly and equates the derivatives to obtain an equation connecting y and x . (Note that the equation connecting y and x is the common equation through the centres which can also be found from using the coordinates of the centres)
- substitutes for x or y into the equation for C_1 to obtain an equation in one variable

A1: Correct 3TQ in y or x

dM1: This requires:

- solves their 3TQ in y or x by any correct means including a calculator and finds at least one point of intersection
- substitutes this point into C_2 and proceeds to a value for k

A1: Correct values for k (exact or decimals as in the main scheme)

A1: As main scheme (exact and in set notation)

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient " $-\frac{1}{2}$ " and point $(7,5)$ $y - 5 = -\frac{1}{2}(x - 7)$	M1	1.1b
	Completes proof $2y + x = 17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y + x = 17$ and $y = 2x + 1$ simultaneously	M1	2.1
	$P = (3, 7)$	A1	1.1b
	Length $PA = \sqrt{(3-7)^2 + (7-5)^2} = (\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2 + (y-5)^2 = 20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y = 2x + k$ meets C using $\vec{OA} + \vec{PA}$	M1	3.1a
	Substitutes their $(11, 3)$ in $y = 2x + k$ to find k	M1	2.1
	$k = -19$	A1	1.1b
		(3)	
(10 marks)			
(c)	Attempts to find where $y = 2x + k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2 + (4k - 34)x + k^2 - 10k + 54 = 0$	M1	3.1a
	Uses $b^2 - 4ac = 0$ oe and proceeds to $k = \dots$	M1	2.1
	$k = -19$	A1	1.1b
		(3)	

Notes:
(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y = -\frac{1}{2}x + c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7, 5)$

So sight of $y - 5 = \frac{1}{2}(x - 7)$ would score this mark

If the form $y = mx + c$ is used expect the candidates to proceed as far as $c = \dots$ to score this mark.



A1*: Completes proof with no errors or omissions $2y + x = 17$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point P , ie for an attempt at solving $2y + x = 17$ and $y = 2x + 1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17 - x = 2x + 1$ as they have set $2y = y$ but condone bracketing errors, eg $2 \times 2x + 1 + x = 17$

A1: $P = (3, 7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P = (3, 7)$ and $(7, 5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x - 7)^2 + (y - 5)^2 = 20$. Do not accept $(x - 7)^2 + (y - 5)^2 = (\sqrt{20})^2$

(c)

M1: Attempts to find where $y = 2x + k$ meets C .

Awarded for using $\overrightarrow{OA} + \overrightarrow{PA}$. $(11, 3)$ or one correct coordinate of $(11, 3)$ is evidence of this award.

M1: For a full method leading to k . Scored for either substituting their $(11, 3)$ in $y = 2x + k$

or, **in the alternative**, for solving their $(4k - 34)^2 - 4 \times 5 \times (k^2 - 10k + 54) = 0 \Rightarrow k = \dots$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by $\pm$ correct roots

A1: $k = -19$ only

Alternative I

M1: For solving $y = 2x + k$ with their $(x - 7)^2 + (y - 5)^2 = 20$ and creating a quadratic eqn of the form

$ax^2 + bx + c = 0$ **where both b and c are dependent upon k** . The terms in x^2 and x must be collected together or implied to have been collected by their correct use in " $b^2 - 4ac$ "

FYI the correct quadratic is $5x^2 + (4k - 34)x + k^2 - 10k + 54 = 0$

M1: For using the discriminant condition $b^2 - 4ac = 0$ to find k . It is not dependent upon the previous M and may be awarded from only one term in k .

$(4k - 34)^2 - 4 \times 5 \times (k^2 - 10k + 54) = 0 \Rightarrow k = \dots$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by $\pm$ correct roots

A1: $k = -19$ only

Alternative II

M1: For solving $2y + x = 17$ with their $(x - 7)^2 + (y - 5)^2 = 20$, creating a 3TQ and solving.

M1: For substituting their $(11, 3)$ into $y = 2x + k$ and finding k

A1: $k = -19$ only

Other method are possible using trigonometry.

Question	Scheme	Marks	AOs
2 (a)	(i) Centre $(-3, 4)$	B1	1.1b
	(ii) States or implies that $r^2 = 24$ or $r = \sqrt{24}$	M1	1.1b
	$2\sqrt{6}$	A1	1.1b
		(3)	
(b)	Attempts a valid method e.g. Finds distance of centre from origin, Sets $y = 0$ and finds values of x	M1	3.1a
	Correct calculations, reason and conclusion (see notes)	A1	2.4
		(2)	
(5 marks)			
Notes:			
Mark (i) and (ii) together (a)(i) B1: Centre $(-3, 4)$ Accept without brackets. May be written e.g. $x = -3$, $y = 4$ (a)(ii) M1: States or implies that $r^2 = 24$ or $r = \sqrt{24}$. A final answer of $\sqrt{24}$ or $2\sqrt{6}$ implies the radius May multiply out the brackets, collect terms ($x^2 + y^2 + 6x - 8y + 1 = 0$) and states the radius is $r^2 = \frac{6^2}{4} + \frac{(-8)^2}{4} - 1$ o.e. Do not condone slips for this mark. A1: $2\sqrt{6}$ isw if they proceed to write as a decimal (b) Note that if their radius is incorrect in (a) then maximum score is M1A0 unless they restart in (b) M1: Attempts a valid method. For example - Finds the distance (or distance ²) of the centre from the origin. They must be attempting $(d =) \sqrt{(\pm 3)^2 + (\pm 4)^2} = \dots$ or $(d^2 =) (\pm 3 - 0)^2 + (\pm 4 - 0)^2 = \dots$ and proceed to a value. May be seen as substituting the coordinates of the origin into the equation for C proceeding to a value for the left hand side e.g. 25 (to be able to compare with 24) - Sets $y = 0$ and attempts to solve $(x + 3)^2 + (-4)^2 = 24 \Rightarrow x = \dots$ (at least one value) - Sets $x = 0$ and attempts to solve $(3)^2 + (y - 4)^2 = 24 \Rightarrow y = \dots$ (at least one value) In each method the starting expression or equation must be correct but do not be concerned by slips when evaluating or processing in finding the distance, the x coordinate or y coordinate.			

A1: Correct calculation(s), reason and conclusion examples:

Calculation examples	Reason examples	Conclusion examples
e.g. $(d^2 =) 25$ or e.g. $(d =) 5$	$25 > 24$ o.e. or $5 > \sqrt{24}$ o.e. (allow 4.9 or better)	e.g. origin does NOT lie within circle / origin lies outside circle / not in circle o.e.
e.g. $(x+3)^2 + (-4)^2 = 24$ $\Rightarrow (x =) -3 \pm \sqrt{8}$ (allow decimals awrt -0.2 and awrt -5.8)	roots are both negative (same signs) o.e.	
e.g. $(3)^2 + (y-4)^2 = 24$ $\Rightarrow (y =) 4 \pm \sqrt{15}$ (allow decimals awrt 0.1 and awrt 7.9)	roots are both positive (same signs) o.e.	

Note if their reasoning is incorrect e.g. referring to the radius as 24 instead of $\sqrt{24}$ then A0 but allow referencing to “the radius of C” provided their radius in (a) was correct.

Note if they give a reason that the origin does not lie inside the circle C because e.g. $25 \neq 24$ this scores M1A0 (M1 for 25 but A0 incorrect reasoning)

Question	Scheme	Marks	AOs
13	$(x-3)^2 + y^2 = \left(\frac{t^2+5}{t^2+1} - 3\right)^2 + \left(\frac{4t}{t^2+1}\right)^2$	M1	3.1a
	$= \frac{(2-2t^2)^2 + 16t^2}{(t^2+1)^2} = \frac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\frac{4(t^4+2t^2+1)}{(t^2+1)^2} = \frac{4(t^2+1)^2}{(t^2+1)^2} = 4^*$	A1*	2.1
		(3)	

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the

Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.

dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.

A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
Alt	$x = \frac{t^2+5}{t^2+1} \Rightarrow xt^2 + x = t^2 + 5 \Rightarrow t^2 = \frac{5-x}{x-1}$ $y = \frac{4t}{t^2+1} \Rightarrow y^2 = \frac{16t^2}{(t^2+1)^2} = \frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1} + 1\right)^2}$	M1	3.1a
	$y^2 = \frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1} + 1\right)^2} = 16\left(\frac{5-x}{x-1}\right) \times \left(\frac{(x-1)}{5-x+x-1}\right)^2 \Rightarrow y^2 = (5-x)(x-1)$	dM1	1.1b
	$y^2 = (5-x)(x-1) \Rightarrow y^2 = 6x - x^2 - 5$ $\Rightarrow y^2 = 4 - (x-3)^2 \text{ or other intermediate step}$ $\Rightarrow (x-3)^2 + y^2 = 4^*$	A1*	2.1
		(3)	
(3 marks)			
Notes			

M1: Adopts a correct strategy for eliminating t to obtain an equation in terms of x and y only. See scheme.

Other methods exist which also lead to an appropriate equation. E.g using $t = \frac{y}{x-1}$

dM1: Uses correct processing to eliminate the fractions and start to simplify

A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
4	$C_1: x=10\cos t, y=4\sqrt{2}\sin t, 0 \leq t < 2\pi; C_2: x^2 + y^2 = 66$		
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$100(1 - \sin^2 t) + 32\sin^2 t = 66$	M1	2.1
	$100\cos^2 t + 32(1 - \cos^2 t) = 66$	A1	1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$ $\Rightarrow \sin t = \dots$	dM1	1.1b
	$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$ $\Rightarrow \cos t = \dots$		
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. Note: These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2}, y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
	(6)		
Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1 \Rightarrow 32x^2 + 100y^2 = 3200\}$	M1	3.1a
	$\frac{x^2}{100} + \frac{66 - x^2}{32} = 1$	M1	2.1
	$\frac{66 - y^2}{100} + \frac{y^2}{32} = 1$	A1	1.1b
	$32x^2 + 6600 - 100x^2 = 3200$ $x^2 = 50 \Rightarrow x = \dots$	dM1	1.1b
	$2112 - 32y^2 + 100y^2 = 3200$ $y^2 = 16 \Rightarrow y = \dots$		
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. Note: These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2}, y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
	(6)		
Way 3	$\{C_2: x^2 + y^2 = 66 \Rightarrow x = \sqrt{66}\cos\alpha, y = \sqrt{66}\sin\alpha$ $\{C_1 = C_2 \Rightarrow 10\cos t = \sqrt{66}\cos\alpha, 4\sqrt{2}\sin t = \sqrt{66}\sin\alpha$ $\{\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \left(\frac{10\cos t}{\sqrt{66}}\right)^2 + \left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2 = 1$	M1	3.1a
	then continue with applying the mark scheme for Way 1		
Way 4	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$100\left(\frac{1 + \cos 2t}{2}\right) + 32\left(\frac{1 - \cos 2t}{2}\right) = 66$	M1	2.1
	$50 + 50\cos 2t + 16 - 16\cos 2t = 66 \Rightarrow 34\cos 2t + 66 = 66$ $\Rightarrow \cos 2t = \dots$	A1	1.1b
	$50 + 50\cos 2t + 16 - 16\cos 2t = 66 \Rightarrow 34\cos 2t + 66 = 66$ $\Rightarrow \cos 2t = \dots$	dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the x-coordinate and value of the y-coordinate. Note: These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2}, y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
	(6)		
	Note: Give final A0 for writing $x = 5\sqrt{2}, y = -4$ followed by $S = (-4, 5\sqrt{2})$		
(6 marks)			
Notes for Question 4			

4	$C_1: x=10\cos t, y=4\sqrt{2}\sin t, 0 \leq t < 2\pi; C_2: x^2 + y^2 = 66$		
Way 5	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66(\sin^2 t + \cos^2 t)$	M1	2.1
		A1	1.1b
	$100\cos^2 t + 32\sin^2 t = 66\sin^2 t + 66\cos^2 t \Rightarrow 34\cos^2 t = 34\sin^2 t$ $\Rightarrow \tan t = \dots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. Note: These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2}, y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	
	Way 5		
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.		
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only with no constant term		
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term		
dM1:	dependent on both the previous M marks Rearranges to make $\tan t = \dots$		
M1:	See scheme		
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$		

Question	Scheme	Marks	AOs
10(a)	$x = 4, y = 2 \Rightarrow t = -1$	B1	2.2a
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -3t^2 \times \frac{1}{2(t+3)}$	M1	1.1b
	$\frac{dy}{dx} = -3(-1)^2 \times \frac{1}{2(-1+3)} = -\frac{3}{4}$	M1	1.1b
	$\Rightarrow y - 2 = -\frac{3}{4}(x - 4)$ or $\Rightarrow y = -\frac{3}{4}x + c \rightarrow 2 = -\frac{3}{4} \times 4 + c \Rightarrow c = \dots$	ddM1	2.1
	$y - 2 = -\frac{3}{4}(x - 4) \Rightarrow 4y - 8 = -3x + 12$ or $c = 5 \Rightarrow y = -\frac{3}{4}x + 5$ $\Rightarrow 3x + 4y = 20^*$	A1*	1.1b
		(5)	
(b)	Maximum height is 9m	B1	3.4
		(1)	
(6 marks)			
Notes			
(a) If parametric differentiation is not used in part (a) (e.g. uses Cartesian form) then only the B mark is available but see alternative below.			
B1: Uses the given Cartesian coordinates to deduce the correct value for t . If more than one value for t e.g. $t = -5$ is given and $t = -1$ is not “selected” score B0 but if just $t = -1$ is used subsequently allow recovery and score B1			
M1: Attempts to use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ or equivalent with their differentiated equations. There must be an attempt to differentiate both parameters, however poor, and divide or multiply correctly so using $\frac{dy}{dx} = \frac{y}{x}$ scores M0. Both parameters must be “changed”. Condone confusion with the variables e.g. referring to $\frac{dy}{dt}$ as $\frac{dy}{dx}$ if the intention is clear. This may be implied by e.g. $\frac{dy}{dt} = -3t^2$, $\frac{dx}{dt} = 2(t+3)$, $t = -1$, $\frac{dy}{dt} = -3$, $\frac{dx}{dt} = 4 \Rightarrow \frac{dy}{dx} = -\frac{3}{4}$			
M1: Uses their numerical value of t (not 4) in their $\frac{dy}{dx}$ to obtain a value. Condone attempts with different values of t e.g. $t = -1$ and $t = -5$			
ddM1: Applies a correct straight line method with their value of $\frac{dy}{dx}$ which has come from an attempt to use parametric differentiation with their value of t (not 4) and with $x = 4$ and $y = 2$ correctly placed. An attempt at the equation of the normal is M0. If using $y = mx + c$ they must reach as far as $c = \dots$ Depends on both previous M marks.			
A1*: Correct equation as printed with no errors but condone $4y + 3x = 20^*$ Allow equivalents e.g. $20 = 4y + 3x^*$ or $3x + 4y = 20^*$ This is a printed answer so there must be at least one intermediate step as shown in the main scheme.			

Alternative for (a) using parametric differentiation but avoids the need for a value for t :

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{dt} \times \frac{dt}{dx} = -3t^2 \times \frac{1}{2(t+3)} \\ -3t^2 \times \frac{1}{2(t+3)} &= -3(1-y)^{\frac{2}{3}} \times \frac{1}{2\sqrt{x}} = -3(1-2)^{\frac{2}{3}} \times \frac{1}{2\sqrt{4}} = -\frac{3}{4} \\ \text{or} \\ -3t^2 \times \frac{1}{2(t+3)} &= -3(\sqrt{x}-3)^2 \times \frac{1}{2\sqrt{x}} = -3(2-3)^2 \times \frac{1}{2\sqrt{4}} = -\frac{3}{4} \\ \text{or} \\ -3t^2 \times \frac{1}{2(t+3)} &= -3(1-y)^{\frac{2}{3}} \times \frac{1}{2\left((1-y)^{\frac{1}{3}}+3\right)} = -3(1-2)^2 \times \frac{1}{2 \times 2} = -\frac{3}{4} \\ \Rightarrow y-2 &= -\frac{3}{4}(x-4) \Rightarrow 3x+4y=20^*\end{aligned}$$

B1: **Either** a correct expression for $\frac{dy}{dx}$ in terms of x and/or y following a correct $\frac{dy}{dx}$ in terms of t **or** for $t = -1$ seen anywhere.

M1: Attempts to use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ with their differentiated equations.

There must be an attempt to differentiate both parameters, however poor, and divide or multiply correctly so using $\frac{dy}{dx} = \frac{y}{x}$ scores M0

Condone confusion with the variables e.g. referring to $\frac{dy}{dt}$ as $\frac{dy}{dx}$ if the intention is clear.

This may be implied by e.g. $\frac{dy}{dt} = -3t^2$, $\frac{dx}{dt} = 2(t+3)$, $t = -1$, $\frac{dy}{dt} = -3$, $\frac{dx}{dt} = 4 \Rightarrow \frac{dy}{dx} = -\frac{3}{4}$

M1: Attempts to express their $\frac{dy}{dx}$ which is in terms of t , in terms of x and/or y **and** uses $x = 4$ and $y = 2$ correctly placed in an attempt to find the gradient of the tangent.

ddM1: Applies a correct straight line method with their value of $\frac{dy}{dx}$ which has come from an attempt to use parametric differentiation with their gradient and with $x = 4$ and $y = 2$ correctly placed.

If using $y = mx + c$ they must reach as far as $c = \dots$

Depends on both previous M marks.

A1*: Correct equation as printed with no errors.

This is a printed answer so there must be at least one intermediate step as shown in the main scheme.

(b)

B1: 9m or equivalent **including correct units**. Accept e.g. 9 metres, 900cm etc.



Question	Scheme	Marks	AOs
13(a)	$y = \operatorname{cosec}^3 \theta \Rightarrow \frac{dy}{d\theta} = -3 \operatorname{cosec}^2 \theta \operatorname{cosec} \theta \cot \theta$	B1	1.1b
	$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	1.1b
	$\frac{dy}{dx} = \frac{-3 \operatorname{cosec}^3 \theta \cot \theta}{2 \cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y = 8 \Rightarrow \operatorname{cosec}^3 \theta = 8 \Rightarrow \sin^3 \theta = \frac{1}{8} \Rightarrow \sin \theta = \frac{1}{2}$	M1	3.1a
	$\theta = \frac{\pi}{6} \Rightarrow \frac{dy}{dx} = \frac{-3 \operatorname{cosec}^3 \left(\frac{\pi}{6} \right) \cot \left(\frac{\pi}{6} \right)}{2 \cos \left(\frac{2\pi}{6} \right)} = \dots$ or $\sin \theta = \frac{1}{2} \Rightarrow \frac{dy}{dx} = \frac{\frac{-3}{\sin^3 \theta} \times \frac{\cos \theta}{\sin \theta}}{2(1 - 2 \sin^2 \theta)} = \frac{-3 \times 8 \times \frac{\sqrt{3}/2}{1/2}}{2 \left(1 - 2 \times \frac{1}{4} \right)}$	M1	2.1
	$= -24\sqrt{3}$	A1	2.2a
		(3)	
(6 marks)			
Notes			
(a) B1: Correct expression for $\frac{dy}{d\theta}$ seen or implied in any form e.g. $\frac{-3 \cos \theta}{\sin^4 \theta}$ M1: Obtains $\frac{dx}{d\theta} = k \cos 2\theta$ or $\alpha \cos^2 \theta + \beta \sin^2 \theta$ (from product rule on $\sin \theta \cos \theta$) and attempts $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ A1: Correct expression in any form. May see e.g. $\frac{-3 \cos \theta}{2 \sin^4 \theta \cos 2\theta}, -\frac{3}{4 \sin^4 \theta \cos \theta - 2 \sin^3 \theta \tan \theta}$ (b) M1: Recognises the need to find the value of $\sin \theta$ or θ when $y = 8$ and uses the y parameter to establish its value. This should be correct work leading to $\sin \theta = \frac{1}{2}$ or e.g. $\theta = \frac{\pi}{6}$ or 30°. M1: Uses their value of $\sin \theta$ or θ in their $\frac{dy}{dx}$ from part (a) (working in exact form) in an attempt to obtain an exact value for $\frac{dy}{dx}$. May be implied by a correct exact answer. If no working is shown but an exact answer is given you may need to check that this follows their $\frac{dy}{dx}$. A1: Deduces the correct gradient			

Question	Scheme	Marks	AOs
9(a) Way 1	$x = (t + 3)^2 - 25$	M1	1.1b
	$\Rightarrow x + 25 = (t + 3)^2 \Rightarrow (x + 25)^{\frac{1}{2}} = (t + 3) \Rightarrow y = \dots$	M1	2.1
	$y = 6 \ln(x + 25)^{\frac{1}{2}} \Rightarrow y = 3 \ln(x + 25)$	A1cso	1.1b
		(3)	
	(a) Way 2		
	$y = 6 \ln(t + 3) = 3 \ln(t + 3)^2$	M1	1.1b
	$y = 3 \ln(t + 3)^2 = 3 \ln(t^2 + 6t + 9) = 3 \ln(x + 16 + 9)$	M1	2.1
	$y = 3 \ln(x + 25)$	A1cso	1.1b
	(a) Way 3		
	$y = 6 \ln(t + 3) \Rightarrow \frac{y}{6} = \ln(t + 3) \Rightarrow t + 3 = e^{\frac{y}{6}} \Rightarrow t = e^{\frac{y}{6}} - 3$	M1	1.1b
	$x = \left(e^{\frac{y}{6}} - 3\right)^2 + 6\left(e^{\frac{y}{6}} - 3\right) - 16 \Rightarrow y = \dots$ or $x = \left(e^{\frac{y}{6}} - 3 + 8\right)\left(e^{\frac{y}{6}} - 3 - 2\right) \Rightarrow y = \dots$	M1	2.1
	$y = 3 \ln(x + 25)$	A1cso	1.1b
	(a) Way 4		
	$x = (t + 3)^2 - 25$	M1	1.1b
	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{3}{(t+3)^2} = \frac{3}{x+25} \Rightarrow y = 3 \ln(x+25)(+c)$	M1	2.1
	e.g. $t = 0 \Rightarrow x = -16, y = 6 \ln 3 \Rightarrow 6 \ln 3 = 3 \ln(9) \Rightarrow c = 0$ $y = 3 \ln(x + 25)$	A1cso	1.1b
(b)	$x = 0, y = 3 \ln 25$ oe e.g. $6 \ln 5$	B1ft	2.2a
	$\frac{dy}{dx} = \frac{3}{x + "25"} \Rightarrow \frac{dy}{dx} = \frac{3}{0 + "25"} \quad \left(= \frac{3}{25} \right)$ or $\frac{dy}{dx} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{6}{2+3} \quad \left(= \frac{6}{50} = \frac{3}{25} \right)$	M1	2.1
	$y - "3 \ln 25" = "\frac{3}{25}"(x \{-0\})$	dM1	3.1a
	$25y - 3x = 150 \ln 5$	A1	2.2a
		(4)	
	(7 marks)		
Notes			
Choose the mark scheme that best matches their chosen method.			



(a)

Way 1

M1: Attempts to complete the square. Award for sight of $x = (t + 3)^2 \pm \dots$ where $\dots \neq 0$

M1: Rearranges their $x = (t + 3)^2 - 25$ to either $(t + 3) = \dots$ or $(t + 3)^2 = \dots$ and then substitutes correctly their expression into the parametric equation for y . So e.g., $t = \sqrt{x + 25} - 3 \rightarrow y = 6 \ln(\sqrt{x + 25} - 3)$ is M0.

A1cso: $y = 3 \ln(x + 25)$ including brackets with all stages of working shown.

The “ $y =$ ” must appear at some point.

Way 2

M1: Attempts to use the power rule for logarithms $y = 6 \ln(t + 3) = \dots \ln(t + 3)^2$ where $\dots \neq 6$

M1: Writes $y = 6 \ln(t + 3)$ as $3 \ln(t + 3)^2$ and then multiplies out and substitutes correctly in for t to obtain a Cartesian equation for C

A1cso: $y = 3 \ln(x + 25)$ including brackets with all stages of working shown.

The “ $y =$ ” must appear at some point.

Way 3

M1: Attempts to make t the subject for $y = 6 \ln(t + 3)$ to obtain $t = e^{\frac{y}{6}} \pm \dots$ where $\dots \neq 0$

M1: Substitutes $t = e^{\frac{y}{6}} \pm \dots$ correctly into $x = t^2 + 6t - 16$ and rearranges to make y the subject.

A1cso: $y = 3 \ln(x + 25)$ including brackets with all stages of working shown.

The “ $y =$ ” must appear at some point.

Way 4

M1: Attempts to complete the square. Award for sight of $x = (t + 3)^2 \pm \dots$ where $\dots \neq 0$

M1: Attempts to find $\frac{dy}{dx}$ where $\frac{dy}{dx} = \frac{\left(\frac{\dots}{t+3}\right)}{at+b}$, $a, b \neq 0$ and uses the completed square form to find $\frac{dy}{dx}$ in terms of x and then integrates to obtain a Cartesian equation for C

A1cso: A complete method using any correct point on the curve to show that $c = 0$ and obtain $y = 3 \ln(x + 25)$ with all stages of working shown. The “ $y =$ ” must appear at some point.

Note that a common incorrect approach in (a) is:

$$x = t^2 + 6t - 16 = (t - 2)(t + 8) \Rightarrow x = t - 2 \Rightarrow t = x + 2 \Rightarrow y = 6 \ln(x + 5)$$

which scores no marks.



(b)

B1ft: Deduces $y = 3 \ln 25$ or e.g. $y = 6 \ln 5$ but allow follow through on their Cartesian equation with $x = 0$ and apply isw after a correct value or ft value for y

M1: Attempts to find $\frac{dy}{dx}$ when $x = 0$ so score for obtaining $\frac{dy}{dx} = \frac{\dots}{x + "25"}$ and substituting in $x = 0$

Allow this mark if they use the letters A and B e.g. $\frac{dy}{dx} = \frac{\dots}{x + B} = \frac{\dots}{0 + B}$ or allow a “made up” A and B .

or

Attempts to find $\frac{dy}{dx}$ when $t = 2$ by finding $\frac{dy}{dx} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{\frac{6}{5}}{2 \times 2 + 6} \left(= \frac{6}{50} = \frac{3}{25} \right)$

For the derivative look for $\frac{dy}{dx} = \frac{\left(\frac{\dots}{t+3}\right)}{at+b}$ or e.g. $\left(\frac{\dots}{t+3}\right) \times \frac{1}{at+b}$ $a, b \neq 0$

NOTE if candidates find $\frac{dy}{dx} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} = \frac{6(2t+6)}{t+3} = 12$ we will give BOD that $t = 2$ has been used unless

there is clear evidence that $t = 2$ has not been used.

dM1: Attempts to find the equation of the tangent. Score for sight of $y - "3 \ln 25" = \frac{3}{25} (x - \{-0\})$ or if they use

$y = mx + c$ they must proceed as far as $c = \dots$ **It is dependent on the previous method mark.**

Must have numeric A and B now.

A1: $25y - 3x = 150 \ln 5$ or any integer multiple of this equation in the form $ax + by = c \ln 5$



Question	Scheme	Marks	AOs
16 (a)	Attempts $y \cdot \frac{dx}{dt} = (2 \sin 2t + 3 \sin t) \times 16 \sin t \cos t$ and uses $\sin 2t = 2 \sin t \cos t$	M1	2.1
	Correct expanded integrand. Usually for one of $(R) = \int \underline{48 \sin^2 t \cos t + 16 \sin^2 2t} \, dt$ $(R) = \int \underline{48 \sin^2 t \cos t + 64 \sin^2 t \cos^2 t} \, dt$ $(R) = \int \underline{24 \sin 2t \sin t + 16 \sin^2 2t} \, dt$	A1	1.1b
	Attempts to use $\cos 4t = 1 - 2 \sin^2 2t = (1 - 8 \sin^2 t \cos^2 t)$	M1	1.1b
	$R = \int_0^a 8 - 8 \cos 4t + 48 \sin^2 t \cos t \, dt$ *	A1*	2.1
	Deduces $a = \frac{\pi}{4}$	B1	2.2a
		(5)	
(b)	$\int 8 - 8 \cos 4t + 48 \sin^2 t \cos t \, dt = 8t - 2 \sin 4t + 16 \sin^3 t$	M1 A1	2.1 1.1b
	$\left[8t - 2 \sin 4t + 16 \sin^3 t \right]_0^{\frac{\pi}{4}} = 2\pi + 4\sqrt{2}$	M1 A1	2.1 1.1b
		(4)	
(9 marks)			
Notes:			

(a) Condone work in another variable, say $\theta \leftrightarrow t$ if used consistently for the first 3 marks

M1: For the key step in attempting $y \cdot \frac{dx}{dt} = (2 \sin 2t + 3 \sin t) \times 16 \sin t \cos t$ with an attempt to use

$\sin 2t = 2 \sin t \cos t$ Condone slips in finding $\frac{dx}{dt}$ but it must be of the form $k \sin t \cos t$

E.g. I $y \cdot \frac{dx}{dt} = (2 \sin 2t + 3 \sin t) \times k \sin t \cos t = (4 \sin t \cos t + 3 \sin t) \times k \sin t \cos t$

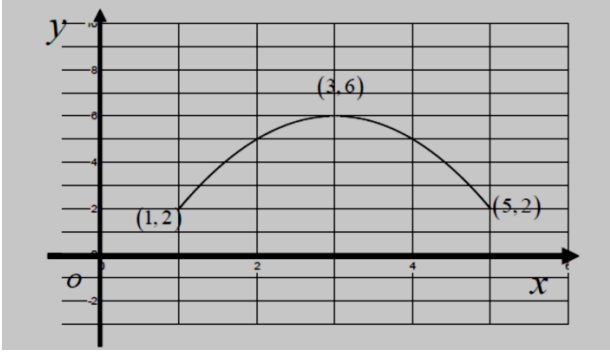
E.g. II $y \cdot \frac{dx}{dt} = (2 \sin 2t + 3 \sin t) \times k \sin t \cos t = (2 \sin 2t + 3 \sin t) \times \frac{k}{2} \sin 2t$

A1: A correct (expanded) integrand in t . Don't be concerned by the absence of $\int$ or dt or limits

$$(R) = \int \underline{48 \sin^2 t \cos t + 16 \sin^2 2t} \, dt \quad \text{or} \quad (R) = \int \underline{48 \sin^2 t \cos t + 64 \sin^2 t \cos^2 t} \, dt$$

but watch for other correct versions such as $(R) = \int \underline{24 \sin 2t \sin t + 16 \sin^2 2t} \, dt$



Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{y-4}{2} = 1 - 2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \frac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 shaped parabola Fully correct with 'ends' at (1,2) & (5,2) Suitable reason : Eg states as $x = 3 + 2\sin t$, $1 \leq x \leq 5$	M1	1.1b
		A1	1.1b
		B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ $\Rightarrow k - x = 6 - (x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $x^2 - 7x + (k+3) = 0$ Or y $y^2 + (7-2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow 49 - 4 \times 1 \times (k+3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$ or $(7-2k)^2 - 4 \times 1 \times (k^2 - 6k + 3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leq k < \frac{37}{4}\right\}$	A1	2.5
		(5)	
(10 marks)			
(a)			

A1*: Proceeds to $y = 6 - (x - 3)^2$ without any errors

Allow a proof where they start with $y = 6 - (x - 3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1 - 2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar.

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1, 2)$ and $(5, 2)$

B1: Any suitable explanation as to why C does not include all points of $y = 6 - (x - 3)^2$, $x \in \mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on x or y**. For example

‘As $-1 \leq \sin t \leq 1$ then $1 \leq x \leq 5$ ’ Condone in words ‘x lies between 1 and 5’ and strict inequalities

‘As $\sin t \leq 1$ then $x \leq 5$ ’ Condone in words ‘x is less than 5’

‘As $-1 \leq \cos(2t) \leq 1$ then $2 \leq y \leq 6$ ’ Condone in words ‘y lies between 2 and 6’

Withhold if the statement is incorrect Eg "because the domain is $2 \leq x \leq 5$ "

Do not allow a statement on the top limit of y as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k = 7$ This can be found by substituting into $(5, 2)$
 $x + y = k \Rightarrow k = 7$ or substituting $x = 5$ into $x^2 - 7x + (k + 3) = 0 \Rightarrow 25 - 35 + k + 3 = 0$
 $\Rightarrow k = 7$
- or deduces that $k < \frac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for k.

Finds where $x + y = k$ meets $y = 6 - (x - 3)^2$ once by using an appropriate method.

Eg. Sets $k - x = 6 - (x - 3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2 - 7x + (k + 3) = 0$ The $= 0$ may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2 = 4ac$ or $b^2 \dots 4ac$ where ... is any inequality

leading to a critical value for k. Eg. one root $\Rightarrow 49 - 4 \times 1 \times (k + 3) = 0 \Rightarrow k = \frac{37}{4}$

A1: Range of values for $k = \left\{ k : 7 \leq k < \frac{37}{4} \right\}$ Accept $k \in \left[7, \frac{37}{4} \right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x - 3)^2$ once by using an appropriate method. Eg. Sets gradient of $y = 6 - (x - 3)^2$ equal to -1	M1	3.1a
	$-2x + 6 = -1 \Rightarrow x = 3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of k. $y = 6 - (3.5 - 3)^2 = 5.75$ Hence using $k = 3.5 + 5.75 = 9.25$	M1	2.1
	Range of values for $k = \{k : 7 \leq k < 9.25\}$	A1	2.5



Question	Scheme	Marks	AOs
12(a)(i)	$y \times \frac{dx}{dt} = 5 \sin 2t \times 6 \cos t \quad \text{or} \quad 5 \times 2 \sin t \cos t \times 6 \cos t$	M1	1.2



	$(\text{Area} =) \int 5 \sin 2t \times 6 \cos t \, dt = \int 5 \times 2 \sin t \cos t \times 6 \cos t \, dt$ or $\int 5 \sin 2t \times 6 \cos t \, dt = \int 60 \sin t \cos^2 t \, dt$	dM1	1.1b
	$(\text{Area} =) \int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt *$	A1*	2.1*
		(3)	
(a)(ii)	$\int 60 \sin t \cos^2 t \, dt = -20 \cos^3 t$	M1 A1	1.1b 1.1b
	$\text{Area} = \left[-20 \cos^3 t \right]_0^{\frac{\pi}{2}} = 0 - (-20) = 20 *$	A1*	2.1
		(3)	
(b)	$5 \sin 2t = 4.2 \Rightarrow \sin 2t = \frac{4.2}{5}$	M1	3.4
	$t = 0.4986..., 1.072...$	A1	1.1b
	Attempts to find the x values at both t values	dM1	3.4
	$t = 0.4986... \Rightarrow x = 2.869...$ $t = 1.072 \Rightarrow x = 5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
(11 marks)			

Notes:

(a)(i)

M1: Attempts to multiply y by $\frac{dx}{dt}$ to obtain $A \sin 2t \cos t$ but may apply $\sin 2t = 2 \sin t \cos t$ here

dM1: Attempts to use $\sin 2t = 2 \sin t \cos t$ within an integral which may be implied by

e.g. $A \int \sin 2t \times \cos t \, dt = \int k \sin t \cos^2 t \, dt$

A1*: Fully correct work leading to the given answer.

This must include $\sin 2t = 2 \sin t \cos t$ or e.g. $5 \sin 2t = 10 \sin t \cos t$ seen explicitly in their proof and a correct intermediate line that includes an integral sign and the “dt”

Allow the limits to just “appear” in the final answer e.g. working need not be shown for the limits.

(a)(ii)

M1: Obtains $\int 60 \sin t \cos^2 t \, dt = k \cos^3 t$. This may be attempted via a substitution of $u = \cos t$ to obtain

$$\int 60 \sin t \cos^2 t \, dt = k u^3$$

A1: Correct integration $-20 \cos^3 t$ or equivalent e.g. $-20u^3$

A1*: Rigorous proof with all aspects correct including the correct limits and the $0 - (-20)$ and

not just: $-20 \cos^3 \frac{\pi}{2} - (-20 \cos^3 0) = 20$

(b)

M1: Uses the given model and attempts to find value(s) of t when $\sin 2t = \frac{4.2}{5}$. Look for $2t = \sin^{-1} \frac{4.2}{5} \Rightarrow t = \dots$

A1: At least one correct value for t , correct to 2 dp. FYI $t = 0.4986..., 1.072...$ or in degrees $t = 28.57..., 61.42...$



Question	Scheme	Marks	AOs
16(a)	Attempts $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4\sec^2 t \tan t}{2\sec^2 t} (= 2 \tan t)$	M1 A1	1.1b 1.1b
	At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = 2, x = 3, y = 7$	M1	2.1
	Attempts equation of normal $y - 7 = -\frac{1}{2}(x - 3)$	M1	1.1b
	$y = -\frac{1}{2}x + \frac{17}{2}$ *	A1*	2.1
		(5)	
(b)	Attempts to use $\sec^2 t = 1 + \tan^2 t \Rightarrow \frac{y-3}{2} = 1 + \left(\frac{x-1}{2}\right)^2$	M1	3.1a
	$\Rightarrow y - 3 = 2 + \frac{(x-1)^2}{2} \Rightarrow y = \frac{1}{2}(x-1)^2 + 5$ *	A1*	2.1
		(2)	
(b) Alternative 1:			
	$y = \frac{1}{2}(x-1)^2 + 5 = \frac{1}{2}(2 \tan t + 1 - 1)^2 + 5$ $= \frac{1}{2}4 \tan^2 t + 5 = 2(\sec^2 t - 1) + 5$	M1	3.1a
	$= 2\sec^2 t + 3 = y$ *	A1	2.1
(b) Alternative 2:			
	$x = 2 \tan t + 1 \Rightarrow t = \tan^{-1}\left(\frac{x-1}{2}\right) \Rightarrow y = 2\sec^2\left(\tan^{-1}\left(\frac{x-1}{2}\right)\right) + 3$ $\Rightarrow y = 2\left(1 + \tan^2\left(\tan^{-1}\left(\frac{x-1}{2}\right)\right)\right) + 3$	M1	3.1a
	$\Rightarrow y = 2\left(1 + \left(\frac{x-1}{2}\right)^2\right) + 3 = \frac{1}{2}(x-1)^2 + 5$ *	A1	2.1
(b) Alternative 3:			
	$\frac{dy}{dx} = 2 \tan t = x - 1 \Rightarrow y = \int (x-1) dx = \frac{x^2}{2} - x + c$ $(3, 7) \rightarrow 7 = \frac{3^2}{2} - 3 + c \Rightarrow c = \frac{11}{2}$	M1	3.1a
	$\frac{x^2}{2} - x + \frac{11}{2} = \frac{1}{2}(x^2 - 2x) + \frac{11}{2} = \frac{1}{2}(x-1)^2 - \frac{1}{2} + \frac{11}{2} = \frac{1}{2}(x-1)^2 + 5$ *	A1	2.1



(c)	Attempts the lower limit for k: $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + (11 - 2k) = 0$ $b^2 - 4ac = 1 - 4(11 - 2k) = 0 \Rightarrow k = \dots$	M1	2.1
	$(k =) \frac{43}{8}$	A1	1.1b
	Attempts the upper limit for k: $(x, y)_{t=-\frac{\pi}{4}} : t = -\frac{\pi}{4} \Rightarrow x = 2 \tan\left(-\frac{\pi}{4}\right) + 1 = -1, y = 2 \sec^2\left(-\frac{\pi}{4}\right) + 3 = 7$ $(-1, 7), y = -\frac{1}{2}x + k \Rightarrow 7 = \frac{1}{2} + k \Rightarrow k = \dots$	M1	2.1
	$(k =) \frac{13}{2}$	A1	1.1b
	$\frac{43}{8} < k \leq \frac{13}{2}$	A1	2.2a
		(5)	
(12 marks)			
Notes:			

(a) **Must use parametric differentiation to score any marks in this part and not e.g. Cartesian form**

M1: For the key step of attempting $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. There must be some attempt to differentiate both

parameters however poor and divide the right way round so using $\frac{dy}{dx} = \frac{y}{x}$ scores M0.

This may be implied by e.g. $\frac{dx}{dt} = 2 \sec^2 t$, $\frac{dy}{dt} = 4 \sec^2 t \tan t$, $t = \frac{\pi}{4} \Rightarrow \frac{dx}{dt} = 4$, $\frac{dy}{dt} = 8 \Rightarrow \frac{dy}{dx} = 2$

A1: $\frac{dy}{dx} = \frac{4 \sec^2 t \tan t}{2 \sec^2 t}$. Correct expression in any form. May be implied as above.

Condone the confusion with variables as long as the intention is clear e.g.

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4 \sec^2 x \tan x}{2 \sec^2 x} (= 2 \tan x)$ and allow subsequent marks if this is interpreted correctly

M1: For attempting to find the values of x, y and the gradient at $t = \frac{\pi}{4}$ **AND** getting at least two correct.

Follow through on their $\frac{dy}{dx}$ so allow for any two of $x = 3$, $y = 7$, $\frac{dy}{dx} = 2$ (or their $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$)

Note that the $x = 3$, $y = 7$ may be seen as e.g. (3, 7) on the diagram. There must be a non-trivial $\frac{dy}{dx}$ for this mark e.g. they must have a $\frac{dy}{dx}$ to substitute into.

M1: For a correct attempt at the normal equation using their x and y at $t = \frac{\pi}{4}$ with the negative

reciprocal of their $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ having made some attempt at $\frac{dy}{dx}$ and all correctly placed.

For attempts using $y = mx + c$ they must reach as far as a value for c using their x and y at $t = \frac{\pi}{4}$

with the negative reciprocal of their $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ all correctly placed.

(b)

M1: Attempts to use $\sec^2 t = 1 + \tan^2 t$ oe to obtain an equation involving y and $(x-1)^2$ E.g. as above or e.g. $y = 2\sec^2 t + 3 = 2(1 + \tan^2 t) + 3 = 2\left(1 + \left(\frac{x-1}{2}\right)^2\right) + 3$ for M1 and then

$$y = \frac{1}{2}(x-1)^2 + 5^* \text{ for A1}$$

A1*: Proceeds with a clear argument to the given answer with no errors**Alternative 1:****M1:** Uses the given result, substitutes for x and attempts to use $\sec^2 t = 1 + \tan^2 t$ oe**A1:** Proceeds with a clear argument to the y parameter and makes a (minimal) conclusion e.g. “ $= y$ ”
QED, hence proven etc.**Alternative 2:****M1:** Uses the x parameter to obtain t in terms of arctan, substitutes into y and attempts to use
 $\sec^2 t = 1 + \tan^2 t$ oe**A1:** Proceeds with a clear argument to the given answer with no errors**Alternative 3:****M1:** Uses $\frac{dy}{dx}$ from part (a) to express $\frac{dy}{dx}$ in terms of x , integrates and uses (3, 7) to find “ c ” to reach
a Cartesian equation.**A1:** Proceeds with a clear argument to the given answer with no errors**Allow the marks for (b) to score anywhere in their solution e.g. if they find the Cartesian equation in part (a)**

(c)

M1: A full attempt to find the **lower** limit for k .

$$\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + (11 - 2k) = 0 \Rightarrow b^2 - 4ac = 1 - 4(11 - 2k) = 0 \Rightarrow k = \dots$$

Score **M1** for setting $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k$, rearranging to 3TQ form and attempts $b^2 - 4ac \dots 0$ e.g. $b^2 - 4ac > 0$ or e.g. $b^2 - 4ac < 0$ correctly to find a value for k .**A1:** $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$ **An alternative method using calculus for lower limit:**

$$y = \frac{1}{2}(x-1)^2 + 5 \Rightarrow \frac{dy}{dx} = x-1, x-1 = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$$

$$x = \frac{1}{2} \Rightarrow y = \frac{1}{2}\left(\frac{1}{2}-1\right)^2 + 5 = \frac{41}{8}$$

$$y = -\frac{1}{2}x + k \Rightarrow \frac{41}{8} = -\frac{1}{4} + k \Rightarrow k = \dots$$

Score **M1** for $\frac{dy}{dx} =$ “a linear expression in x ”, sets $= -\frac{1}{2}$, solves a linear equation to find x andthen substitutes into the given result in (b) to find y and then uses $y = -\frac{1}{2}x + k$ to find a valuefor k . **A1:** $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$ 

An alternative method using parameters for lower limit:

$$\begin{aligned}
 y &= -\frac{1}{2}x + k \Rightarrow 2\sec^2 t + 3 = -\frac{1}{2}(2\tan t + 1) + k \\
 \Rightarrow 2(1 + \tan^2 t) + 3 &= -\frac{1}{2}(2\tan t + 1) + k \Rightarrow 2\tan^2 t + \tan t + 5.5 - k = 0 \\
 b^2 - 4ac &= 0 \Rightarrow 1 - 4 \times 2(5.5 - k) = 0 \Rightarrow k = \frac{43}{8}
 \end{aligned}$$

Score **M1** for substituting parametric form of x and y into $y = -\frac{1}{2}x + k$, uses $\sec^2 t = 1 + \tan^2 t$ rearranges to 3TQ form and attempts $b^2 - 4ac \dots 0$ or e.g. $b^2 - 4ac > 0$ or $b^2 - 4ac < 0$ correctly to find a value for k .

A1: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

M1: A full attempt to find the **upper** limit for k . This requires an attempt to find the value of x and the value of y using $t = -\frac{\pi}{4}$, the substitution of these values into $y = -\frac{1}{2}x + k$ and solves for k .

A1: $k = \frac{13}{2}$. Look for this value e.g. may appear in an inequality.

A1: Deduces the correct range for k : $\frac{43}{8} < k \leq \frac{13}{2}$

Allow equivalent notation e.g. $\left(k \leq \frac{13}{2} \text{ and } k > \frac{43}{8}\right)$, $\left(k \leq \frac{13}{2} \cap k > \frac{43}{8}\right)$, $\left(\frac{43}{8}, \frac{13}{2}\right]$

But not e.g. $\left(k \leq \frac{13}{2}, k > \frac{43}{8}\right)$, $\left(k \leq \frac{13}{2} \cup k > \frac{43}{8}\right)$, $\left(k \leq \frac{13}{2} \text{ or } k > \frac{43}{8}\right)$ and do not allow if in terms of x .

Allow equivalent exact values for $\frac{43}{8}$, $\frac{13}{2}$

There may be other methods for finding the upper limit which are valid. If you are in any doubt if a method deserves credit then use Review.

Question	Scheme	Marks	AOs
5(a)	$-3 = \frac{t-1}{2} \Rightarrow t = -5 \Rightarrow y = 5(-5+2)^4$	M1	1.1b
	$(y =) 405$	A1	1.1b
		(2)	
(b)	$x = \frac{t-1}{2} \Rightarrow t = 2x+1 \Rightarrow y = 5(2x+1+2)^4$	M1	1.1b
	$y = 5(2x+3)^4$	A1	1.1b
		(2)	
(c)	$(2x+3)^n \rightarrow \dots (2x+3)^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right) 40(2(-3)+3)^3$	dM1	1.1b
	$\left(\frac{dy}{dx}\right) -1080$	A1	1.1b
		(3)	
(7 marks)			
Notes			
(a) Mark parts (a) and (b) together.			
M1: Substitutes $x = -3$ into $x = \frac{t-1}{2}$, attempts to find t , and substitutes their t into $y = 5(t+2)^4$ Condone slips. May be implied by substitution of $t = -5$ into y or by 405 or by $y = 5(-3)^4$ Alternatively, they may substitute $x = -3$ into their $y = f(x)$			
A1: cao Answer only scores full marks and ISW after seeing 405 e.g. $(-5, 405)$ Accept $(-3, 405)$ or e.g. $P = 405$			
(b) Mark parts (a) and (b) together.			
M1: Attempts to make t the subject of $x = \frac{t-1}{2}$ using the correct order of operations and substitutes into $y = 5(t+2)^4$ Condone slips e.g. dividing by 2 first: $t = \frac{x}{2} + 1$ or missing the +2 in $y = 5(t+2)^4$			
Alt 1: Writes $t = \left(\frac{y}{5}\right)^{\frac{1}{4}} - 2$, substitutes into $x = \frac{t-1}{2}$ and attempts to make y the subject using			
the correct order of operations, i.e., $2x = \left(\frac{y}{5}\right)^{\frac{1}{4}} - 3 \Rightarrow (2x+3)^4 = \frac{y}{5} \Rightarrow y = 5(2x+3)^4$			
Alt 2: Makes t the subject and then substitutes into an expanded $5(t+2)^4$ Do not be too concerned about their expansion, but it must contain t^4 and a constant term.			
A1: cao and ISW after a correct answer seen. May be scored for $y = 5(2x+1+2)^4$ Allow e.g. $y = 80x^4 + 480x^3 + 1080x^2 + 1080x + 405$ or e.g. $y = 5(2x+1)^4 + 40(2x+1)^3 + 120(2x+1)^2 + 160(2x+1) + 80$ Their RHS may be unsimplified but do not allow if e.g. binomial coefficients are still present. Do not accept $f(x) = \dots$ It must be $y = \dots$			

(c) Note: Differentiation seen in (a) or (b) can score marks if used in (c)

M1: Reduces the power of their $(2x+3)^n$ by one to $Q(2x+3)^{n-1}$ where Q is a constant and could be 1. There should be no other terms using this method.

Alternatively, attempts to expand their $(2x+3)^n$ (may have been expanded in (b)) and reduces the power of x by one in at least one term.

They may use parametric differentiation, i.e., $\frac{dy}{dt} = a(t+2)^3$ and $\frac{dx}{dt} = b$ where a and b are

constants, leading to $\frac{dy}{dx} = \frac{a(t+2)^3}{b}$ i.e. they must divide the correct way round.

Condone attempts at the chain rule that reach e.g. $y' = \dots u^3$ but make a slip when substituting

back in for x or t , e.g., $\frac{dy}{dx} = (5x+3)^3$ or $\frac{dy}{dt} = (t+2)^2$, provided the intention is clear.

dM1: Substitutes $x = -3$ into their $\frac{dy}{dx}$ in terms of x which may be implied by their answer.

Using parametric differentiation, substitutes their t (found from an attempt at substituting $x = -3$ into $x = \frac{t-1}{2}$ which may have been seen in (a)) into their $\frac{dy}{dx}$ in terms of t .

Note for reference, if correct, the parametric differentiation is $\frac{dy}{dt} = 20(t+2)^3$ and $\frac{dx}{dt} = \frac{1}{2}$

leading to $\frac{dy}{dx} = 40(t+2)^3$

They may substitute their value of t into $\frac{dy}{dt}$ first before using the chain rule to reach $\frac{dy}{dx}$ which is acceptable and implies the first M mark.

A1: $\left(\frac{dy}{dx} = \right) -1080$

Correct answer only scores full marks.

May be seen labelled as m or something else.

Question	Scheme	Marks	AOs
3 (a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2 \left(\frac{\theta}{360} \right)$ {where θ is in degrees} • correct formula is $\pi r^2 \left(\frac{40}{360} \right)$	B1	2.3
		(1)	
(b) Way 1	$\{ \text{Area of sector} = \} \quad \frac{1}{2} (5^2) \left(\frac{2\pi}{9} \right)$	M1	1.1b
	$= \frac{25}{9} \pi \text{ {cm}^2} \quad \text{or} \quad \text{awrt } 8.73 \text{ {cm}^2}$	A1	1.1b
		(2)	
(b) Way 2	$\{ \text{Area of sector} = \} \quad \pi (5^2) \left(\frac{40}{360} \right)$	M1	1.1b
	$= \frac{25}{9} \pi \text{ {cm}^2} \quad \text{or} \quad \text{awrt } 8.73 \text{ {cm}^2}$	A1	1.1b
		(2)	
(3 marks)			
Notes for Question 3			
(a)			
B1:	Explains that the formula use is only valid when angle AOB is applied in radians. See scheme for examples of suitable explanations.		
(b)	Way 1		
M1:	Correct application of the sector formula using a correct value for θ in radians		
Note:	Allow exact equivalents for θ e.g. $\theta = \frac{40\pi}{180}$ or θ in the range $[0.68, 0.71]$		
A1*:	Accept $\frac{25}{9}\pi$ or awrt 8.73 Note: Ignore the units		
(b)	Way 2		
M1:	Correct application of the sector formula in degrees		
A1:	Accept $\frac{25}{9}\pi$ or awrt 8.73 Note: Ignore the units.		
Note:	Allow exact equivalents such as $\frac{50}{18}\pi$		
Note:	Allow M1 A1 for $500 \left(\frac{\pi}{180} \right) = \frac{25}{9}\pi \text{ {cm}^2} \quad \text{or} \quad \text{awrt } 8.73 \text{ {cm}^2}$		

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = \dots$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]

(4 marks)

Notes:

B1: States or uses $\frac{1}{2}r^2\theta = 11$ This may be implied with an embedded found value for θ

B1: States or uses $2r + r\theta = 4r\theta$ or equivalent

M1: Full method to find $r = \dots$ This involves combining the equations to eliminate θ or find θ
The initial equations must be of the same "form" (see **) but condone slips when attempting to solve.

It cannot be scored from impossible values for θ Hence only score if $0 < \theta < 2\pi$ FYI $\theta = \frac{2}{3}$ radians

Allow this to be scored from equations such as $\dots r^2\theta = 11$ and ones that simplify to $\dots r = \dots r\theta$ **

Allow their $2r + r\theta = 4r\theta \Rightarrow \theta = \dots$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $2r + r\theta = 4r\theta \Rightarrow r\theta = \dots$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $\frac{1}{2}r^2\theta = 11 \Rightarrow \theta = \frac{\dots}{r^2}$ then substitute into their $2r + r\theta = 4r\theta \Rightarrow r = \dots$

A1: $r = \sqrt{33}$ only but isw after a correct answer.

.....
The whole question can be attempted using θ in degrees.

B1: States or uses $\frac{\theta}{360} \times \pi r^2 = 11$

B1: States or uses $2r + \frac{\theta}{360} \times 2\pi r = 4 \times \frac{\theta}{360} \times 2\pi r$



Question	Scheme	Marks	AOs
11	Identifies angle $BAO = \frac{\pi}{3}$ or angle $BOC = \frac{2\pi}{3}$	B1	2.2a
	$\text{Area}(\text{segment}) = \frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} \left\{ = \frac{25\pi}{6} - \frac{25\sqrt{3}}{4} \right\}$ or $\text{Area}(AOB) = 2 \times \frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} \left\{ = \frac{25\pi}{3} - \frac{25\sqrt{3}}{4} \right\}$	M1	2.1
	$\text{Area of } R = \frac{1}{2} \times 5^2 \times \frac{2\pi}{3} - \left(\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} \right)$ or $\text{Area of } R = \frac{1}{2} \times \pi \times 5^2 - \left(\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} + \frac{1}{2} \times 5^2 \times \frac{\pi}{3} \right)$	dM1	3.1a
	$= \frac{25}{4} \sqrt{3} + \frac{25}{6} \pi \text{ (cm}^2\text{)}$	A1	1.1b
		(4)	
(4 marks)			
Notes			
Note: Use of degrees, if used in formulae in degrees, can score full marks. B1: Deduces angle BAO or angle BOA is $\frac{\pi}{3}$ radians or 60° or deduces that angle BOC is $\frac{2\pi}{3}$ radians or 120°. May be seen on the diagram or their own sketches or embedded in a formula. May be implied if they find e.g. $\frac{1}{6} \times \pi \times 5^2$ for the area of the minor sector. M1: Uses a correct process and an angle of $\frac{\pi}{3}$ radians or 60° to find - the area of the segment bounded by the arc OB and straight line OB - or the area of the segment bounded by the arc AB and straight line AB - or the area of unshaded region AOB. Allow decimal values to imply the method (require 3sf rounded or truncated). Use of 60° must be in a correct formula in degrees. $0.5 \times 5^2 \times 60$ scores M0. For reference these are the areas of the regions: - segment $= \frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \times \sin \frac{\pi}{3} \left\{ = \frac{25\pi}{6} - \frac{25\sqrt{3}}{4} = 2.26 \right\}$ (3 s.f.) scores M1 $AOB = \frac{1}{2} \times 5^2 \times \frac{\pi}{3} + \frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \times \sin \frac{\pi}{3} \left\{ = \frac{25\pi}{3} - \frac{25\sqrt{3}}{4} = 15.35 \right\}$ (2 d.p.) scores M1 - Minor sector $= \frac{1}{2} \times 5^2 \times \frac{\pi}{3} \left\{ = \frac{25\pi}{6} = 13.09 \right\}$ (2 d.p.) scores M0 on its own - Major sector $= \frac{1}{2} \times 5^2 \times \frac{2\pi}{3} \left\{ = \frac{25\pi}{3} = 26.18 \right\}$ (2 d.p.) scores M0 on its own			

- Triangle $AOB = \frac{1}{2} \times 5^2 \times \sin \frac{\pi}{3} \left\{ = \frac{25\sqrt{3}}{4} = 10.8 \right\}$ (3 s.f.) scores M0 on its own.

This mark may be implied if seen as part of a **correct** strategy for the area of R .

Note that the area of triangle AOB may also be found using Pythagoras and $\frac{1}{2}bh$ but their method must be correct.

dM1: A fully correct strategy for finding the area of R .

Follow through on incorrectly simplified areas but the method must be correct.

Allow decimal values to imply the method (require 3sf rounded or truncated).

For reference, the area is approximately 23.92 (2d.p.) and is likely to imply B1M1dM1 but their work should be checked.

Either:

- major sector – segment $= \frac{1}{2} \times 5^2 \times \frac{2\pi}{3} - \left(\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} \right)$

or

- semicircle – $AOB = \frac{1}{2} \times \pi \times 5^2 - \left(\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} + \frac{1}{2} \times 5^2 \times \frac{\pi}{3} \right)$

or

- semicircle – $2 \times$ sector AOB + triangle $AOB = \frac{1}{2} \times \pi \times 5^2 - 2 \times \left(\frac{1}{2} \times 5^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5^2 \sin \frac{\pi}{3} \right) + \frac{1}{2} \times 5^2 \sin \frac{\pi}{3}$

or

- sector AOB + triangle $AOB = \frac{1}{2} \times 5^2 \times \frac{\pi}{3} + \frac{1}{2} \times 5^2 \sin \frac{\pi}{3}$ (this also implies the first M1)

If they go straight to one of these expressions then we would imply the first M1 (and the B1).

A1: Correct expression $\frac{25}{4}\sqrt{3} + \frac{25}{6}\pi$ o.e. in the correct form e.g. $\frac{50}{8}\sqrt{3} + \frac{75}{18}\pi$

Ignore any reference to (or absence of) units.

Do not apply isw if they go on to add or subtract additional areas.

However, we can condone poor simplification e.g. $\frac{25}{12}(\sqrt{3} + \pi)$ following a correct answer.

Note: Attempts via integration for the area underneath a circle or polar coordinates are unlikely but if seen then use Review. The schemes for these approaches follow the main scheme but the first M1 may also be scored for finding half of the area of AOB .

You might find the following diagram helpful:

Question	Scheme	Marks	AOs
6(a)	Angle $AOB = \frac{\pi - \theta}{2}$	B1	2.2a
		(1)	
(b)	Area = $2 \times \frac{1}{2} r^2 \left(\frac{\pi - \theta}{2} \right) + \frac{1}{2} (2r)^2 \theta$	M1	2.1
	$= \frac{1}{2} r^2 \pi - \frac{1}{2} r^2 \theta + 2r^2 \theta = \frac{3}{2} r^2 \theta + \frac{1}{2} r^2 \pi = \frac{1}{2} r^2 (3\theta + \pi)^*$	A1*	1.1b
		(2)	
(c)	Perimeter = $4r + 2r \left(\frac{\pi - \theta}{2} \right) + 2r\theta$	M1	3.1a
	$= 4r + r\pi + r\theta$ or e.g. $r(4 + \pi + \theta)$	A1	1.1b
		(2)	
(5 marks)			
Notes			
(a) B1: Deduces the correct expression for angle AOB Note that $\frac{180 - \theta}{2}$ scores B0 (b) M1: Fully correct strategy for the area using their angle from (a) appropriately. Need to see $2 \times \frac{1}{2} r^2 \alpha$ or just $r^2 \alpha$ where α is their angle in terms of θ from part (a) + $\frac{1}{2} (2r)^2 \theta$ with or without the brackets. A1*: Correct proof. For this mark you can condone the omission of the brackets in $\frac{1}{2} (2r)^2 \theta$ as long as they are recovered in subsequent work e.g. when this term becomes $2r^2 \theta$ The first term must be seen expanded as e.g. $\frac{1}{2} r^2 \pi - \frac{1}{2} r^2 \theta$ or equivalent (c) M1: Fully correct strategy for the perimeter using their angle from (a) appropriately Need to see $4r + 2r\alpha + 2r\theta$ where α is their angle from part (a) in terms of θ A1: Correct simplified expression Note that some candidates may change the angle to degrees at the start and all marks are available e.g. (a) $\frac{180 - \frac{180\theta}{\pi}}{2}$ (b) $2 \left(\frac{180 - \frac{180\theta}{\pi}}{2} \right) \times \frac{1}{360} \times \pi r^2 + \frac{\theta}{360} \times \frac{180}{\pi} \times \pi (2r)^2 = \frac{1}{2} \pi r^2 - \frac{1}{2} r^2 \theta + 2r^2 \theta = \frac{1}{2} r^2 (3\theta + \pi)$ (c) $4r + 2 \left(\frac{180 - \frac{180\theta}{\pi}}{2} \right) \times \frac{1}{360} \times 2\pi r + \frac{180\theta}{\pi} \times \frac{1}{360} \times 2\pi (2r) = 4r + \pi r + r\theta$			

Question	Scheme	Marks	AOs
8(a)	$OC \times 2.3 = 27.6$	M1	1.1b
	e.g. $OC = \frac{27.6}{2.3} = 12 \text{ m}^*$	A1*	2.1
		(2)	
(b)	e.g. $(2AOB =) \pi - 2.3$	M1	1.1b
	$\frac{\pi - 2.3}{2} \Rightarrow 0.421 \text{ rad}^*$	A1*	2.1
		(2)	
(c)	Area $OCDE = \frac{1}{2} \times 12^2 \times 2.3$	M1	1.1b
	$= 165.6 \text{ (m}^2\text{)} \text{ (accept awrt 166)}$	A1	1.1b
	$(OB =) \frac{35 - 27.6}{2} + 12 = 15.7 \text{ m}$	B1	2.1
	Area of OAB (or OFG) $= \frac{1}{2} \times "15.7" \times 7.5 \times \sin 0.421 \text{ (= 24.0...m}^2\text{)}$	M1	1.1b
	Total area $= "165.6" + 2 \times "24.1"$	dM1	3.1a
	$= \text{awrt } 214 \text{ (m}^2\text{)}$	A1	1.1b
		(6)	
(10 marks)			
Notes			
(a)			
M1:	Uses $l = r\theta$ with $l = 27.6$ and $\theta = 2.3$ correctly substituted in (may be labelled differently in their equation). Values just need to be embedded in an equation or accept an expression for OC e.g. $\frac{27.6}{2.3}$. May work in degrees which is acceptable. Condone an alternative letter being used to denote OC such as r Alternatively, they use $l = r\theta$ with $r = 12$ and $\theta = 2.3$ and verify that $l = 27.6 \text{ m}$		
A1*:	Achieves an expression for OC before proceeding to $OC = 12 \text{ (m)}$ with no errors seen (condone lack of units) They must show at least $\frac{27.6}{2.3} \Rightarrow OC = 12 \text{ (m)}$ which can score M1A1* $r = \frac{27.6}{2.3} = 12$ is M1A1* (condone alternative letters for OC) BUT e.g. $\frac{27.6}{2.3} = 12 \text{ (m)}$ on its own is M1A0* e.g. $OC \times 2.3 = 27.6 \Rightarrow OC = 12 \text{ (m)}$ is M1A0* In the alternative method they verify $l = 27.6$ and conclude that $OC = 12 \text{ m}$ We must see the calculation $12 \times 2.3 = 27.6$ and conclude that $OC = 12 \text{ (m)}$ e.g. $\text{arc} = 12 \times 2.3 = 27.6$ so $OC = 12 \text{ (m)}$ is M1A1* whereas $12 \times 2.3 = 27.6$ is M1A0* Also allow e.g. if $OC = 12 \text{ (m)}$ then $12 \times 2.3 = 27.6 \checkmark$ is M1A1* If they work in degrees and use rounded values this scores A0* (If they work with e.g. $\frac{414}{\pi}$ to keep the angle exact then A1* can still be scored)		



(b)

M1: Attempts to subtract 2.3 from π (which may be implied by an expression for AOB which is not the given answer)

e.g. $\frac{1}{2}(\pi - 2.3)$ or $\frac{\pi}{2} - 1.15$ score M1

May work in degrees e.g. $180 - \text{awrt } 132$ is M1

Condone invisible brackets e.g. $\pi - 2.3 \div 2$ can still score M1.

A1*: Achieves 0.421 (rad) with no errors seen (ignore any side working which is not part of their main solution). Look for a correct expression which is awrt 0.421 before proceeding to the answer.

Alternatively, they may write

e.g. $2AOB = \pi - 2.3 (= 0.8415...) \Rightarrow AOB = 0.421$

Condone if they do not round their answer at the end to 0.421.

Condone lack of units. Condone poor labelling of other angles and it does not require $AOB =$ to score this mark, but do not accept e.g. $ABO =$

If they work in degrees then withhold this mark if they do not show the conversion back to radians.

e.g. $\frac{\pi - 2.3}{2} = 0.421 \text{ (rad)}$ is M1A1*

e.g. $\frac{180 - \text{awrt } 131.8}{2} \div \frac{180}{\pi} = 0.421 \text{ (rad)}$ is M1A1* (conversion from degrees to radians seen)

e.g. $\pi - 2.3 \div 2 = 0.421 \text{ (rad)}$ is M1A0* (invisible/lack of brackets)

e.g. $\pi - 2.3 = \frac{0.842...}{2} = 0.421$ M1A0* (incorrect joined statement)

(c)

M1: Attempts to use $A = \frac{1}{2}r^2\theta$ with $r = 12$ and $\theta = 2.3$ The values embedded in the formula is sufficient for this mark. May be implied by a correct answer or further work. Look out for alternative more complex ways to find the area of the sector. e.g. area of semicircle – area of two sectors with $r = 12$ and $\theta = 0.421$

A1: awrt 166 (may be implied by later work)

B1: A correct expression or value for the length OB or OF which may be a part of a calculation (may see 15.7 in the equation to find the area of AOB). May be implied by a correct value for the area of a congruent triangle (or both)

M1: Attempts to find the area of at least one of the two congruent triangles using their OB found from $\frac{35 - 27.6}{2} + 12 (= 15.7)$, $OA = 7.5$ and $\theta = 0.421$ in $\frac{1}{2} \times OA \times OB \times \sin C$ (may work in degrees)

Be aware that omitting sine in the formula may give a value close to the area of the triangle which would be M0. **Condone use of $\theta = 0.4$ or $\theta = 0.42$** if they have rounded angle AOB .

The values embedded in the expression is sufficient to score the mark or may be implied by the value.

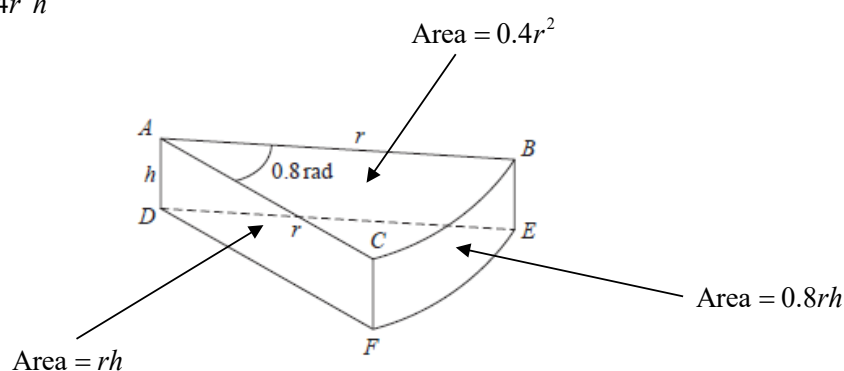
Look out for more complex methods to find the area of one or both of the two congruent triangles

e.g. they may split the congruent triangle into two right angled triangles and add the separate areas.



Question	Scheme	Marks	AOs
15 (a)	Sets up an allowable equation using volume = 240 E.g. $\frac{1}{2}r^2 \times 0.8h = 240 \Rightarrow h = \frac{600}{r^2}$ o.e.	M1 A1	3.4 1.1b
	Attempts to substitute their $h = \frac{600}{r^2}$ into $(S =) \frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + 2rh + 0.8rh$	dM1	3.4
	$S = 0.8r^2 + 2.8rh = 0.8r^2 + 2.8 \times \frac{600}{r} = 0.8r^2 + \frac{1680}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$	M1 A1	3.1a 1.1b
	Sets $\frac{dS}{dr} = 0 \Rightarrow r^3 = 1050$ $r = \text{awrt } 10.2$	dM1 A1	2.1 1.1b
		(4)	
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right) = 1.6 + \frac{3360}{r^3}$ and considers its value or sign	M1	1.1b
	E.g. Correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$ with $\frac{d^2S}{dr^2} \bigg _{r=10.2} = 5 > 0$ proving a minimum value of S	A1	1.1b
		(2)	
(10 marks)			
Notes:			

$$\text{Volume} = 0.4r^2h$$



$$\text{Total surface area} = 2rh + 0.8r^2 + 0.8rh$$

(a)



M1: Attempts to use the fact that the volume of the toy is 240 cm^3

Sight of $\frac{1}{2}r^2 \times 0.8 \times h = 240$ leading to $h = \dots$ or $rh = \dots$ scores this mark

But condone an equation of the correct form so allow for $kr^2h = 240 \Rightarrow h = \dots$ or $rh = \dots$

A1: A correct expression for $h = \frac{600}{r^2}$ or $rh = \frac{600}{r}$ which may be left unsimplified.

This may be implied when you see an expression for S or part of S E.g. $2rh = 2r \times \frac{600}{r^2}$

dM1: Attempts to substitute their $h = \frac{a}{r^2}$ o.e. such as $hr = \frac{a}{r}$ into a **correct** expression for S

Sight of $\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + rh + rh + 0.8rh$ with an appropriate substitution

Simplified versions such as $0.8r^2 + 2rh + 0.8rh$ used with an appropriate substitution is fine.

A1*: Correct work leading to the given result.

$S =$, $SA =$ or surface area = must be seen at least once in the correct place

The method must be made clear so expect to see evidence. For example

$S = 0.8r^2 + 2rh + 0.8rh \Rightarrow S = 0.8r^2 + 2r \times \frac{600}{r^2} + 0.8r \times \frac{600}{r^2} \Rightarrow S = 0.8r^2 + \frac{1680}{r}$ would be fine.

(b) There is no requirement to see $\frac{dS}{dr}$ in part (b). It may even be called $\frac{dy}{dx}$.

M1: Achieves a derivative of the form $pr \pm \frac{q}{r^2}$ where p and q are non-zero constants

A1: Achieves $\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$

dM1: Sets or implies that their $\frac{dS}{dr} = 0$ and proceeds to $mr^3 = n$, $m \times n > 0$. It is dependent upon a

correct attempt at differentiation. This mark may be implied by a correct answer to their $pr - \frac{q}{r^2} = 0$

A1: $r = \text{awrt } 10.2$ or $\sqrt[3]{1050}$

(c)

M1: Attempts to substitute their positive r (found in (b)) into $\left(\frac{d^2S}{dr^2}\right) = e \pm \frac{f}{r^3}$ where e and f are non zero

and finds its value or sign.

Alternatively considers the sign of $\left(\frac{d^2S}{dr^2}\right) = e \pm \frac{f}{r^3}$ (at their positive r found in (b))

Condone the $\frac{d^2S}{dr^2}$ to be $\frac{d^2y}{dx^2}$ or being absent, but only for this mark.

A1: States that $\frac{d^2S}{dr^2}$ or $S'' = 1.6 + \frac{3360}{r^3} = \text{awrt } 5 > 0$ proving a minimum value of S

This is dependent upon having achieved $r = \text{awrt } 10$ and a correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$

It can be argued without finding the value of $\frac{d^2S}{dr^2}$. E.g. $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3} > 0$ as $r > 0$, so

minimum value of S. For consistency it is also dependent upon having achieved $r = \text{awrt } 10$



Question	Scheme	Marks	AOs
15 (a)	$\frac{1}{2}r^2\theta + \frac{1}{10}r^2 = 240 \Rightarrow r\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r} \text{ or } \theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r^2}$	M1 A1	3.4 1.1b
	Substitutes into the expression for P $r\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r} \text{ into } (P =) r\theta + 2r + \frac{1}{5}r$	dM1	3.4
	$P = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r} + 2r + \frac{1}{5}r = \frac{480}{r} - \frac{1}{5}r + 2r + \frac{1}{5}r = 2r + \frac{480}{r} *$	A1*	2.1
		(4)	
(b)	$\left(\frac{dP}{dr}\right) = 2 - \frac{480}{r^2}$	M1	1.1b
	Sets $\frac{dP}{dr} = 0 \Rightarrow r^2 = 240$ $r = \text{awrt } 15.5$	dM1 A1	2.1 1.1b
		(3)	
(c)	$\left(\frac{d^2P}{dr^2}\right) = \frac{960}{r^3}$	M1	1.1b
	$\left(\frac{d^2P}{dr^2}\right) = \text{awrt } 0.26 > 0 \text{ proving a minimum value of } P$	A1	1.1b
		(2)	
(9 marks)			
Notes:			
(a) Note that just finding a correct equation for the area and/or a correct equation for the perimeter (before any substitution) is insufficient to score any marks. M1: Uses area formulae to form an equation of the form $\alpha r^2\theta + \beta r^2 = 240$ o.e. ($\alpha, \beta \neq 0$) and rearranges to make $r\theta$, θ or $r\theta + \frac{1}{5}r$ the subject. Look for: $r\theta = \frac{M \pm Nr^2}{r} \left(= \frac{M}{r} \pm Nr \right) \text{ o.e. or } \theta = \frac{M \pm Nr^2}{r^2} \left(= \frac{M}{r^2} \pm N \right) \text{ o.e. where } M, N \neq 0$ or $r\theta + \frac{1}{5}r = \frac{L}{r}$ $L \neq 0$ o.e. May work in degrees. A1: A correct rearrangement for θ or $r\theta$ or $r\theta + \frac{1}{5}r$ which may be unsimplified (may be in degrees) $r\theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r} \text{ o.e. e.g. } r\theta = \frac{2400 - r^2}{5r} \text{ or } r\theta = \frac{480 - 0.2r^2}{r}$ or $r\theta + \frac{1}{5}r = \frac{480}{r}$ o.e.			

$$\text{or } \theta = \frac{240 - \frac{1}{10}r^2}{\frac{1}{2}r^2} \text{ o.e. e.g. } \theta = \frac{2400 - r^2}{5r^2} \text{ or } \theta = \frac{480}{r^2} - \frac{1}{5} \text{ or } \theta = 480r^{-2} - 0.2$$

dM1: Substitutes their $r\theta = \frac{M \pm Nr^2}{r}$ o.e. or $\theta = \frac{M \pm Nr^2}{r^2}$ o.e. or $r\theta + \frac{1}{5}r = \frac{L}{r}$ into an

expression of the form $(P =) r\theta + Qr$, $Q \neq 0$ (typically $P = r\theta + \frac{11}{5}r$) which may be unsimplified or in degrees. It is dependent on the previous method mark. It is acceptable for their valid expression for θ , $r\theta$ or $r\theta + \frac{1}{5}r$ to be substituted into the perimeter expression directly (without first seeing them in the perimeter expression).

A1*: $P = 2r + \frac{480}{r}$ following a correct method (condone slips to be recovered) and all previous marks scored. Condone invisible brackets to be recovered.
 $P =$, Perimeter = must be seen at least once in their solution in the correct place.

(b) Mark (b) and (c) together. There is no requirement to see the notation $\frac{dP}{dr}$ in part

(b). It may even be called $\frac{dy}{dx}$. Allow use of e.g. P' or e.g. y'

M1: $\left(\frac{dP}{dr} =\right) p \pm \frac{q}{r^2}$ where p and q are non-zero constants

dM1: Sets or implies that their $\frac{dP}{dr} = 0$ and proceeds to $mr^{\pm 2} = n$, $m \times n > 0$. It is dependent on the previous method mark. Do not be concerned by the mechanics of the rearrangement. This mark may be implied by a correct answer to their $p - \frac{q}{r^2} = 0$. You may need to check this on your calculator.

A1: $r = \text{awrt } 15.5$ or $\sqrt{240}$ ($= 4\sqrt{15}$) Do not accept $\pm$ (ignore any units if given)

(c) Condone other letters used instead of P and r for $\frac{d^2P}{dr^2}$ e.g. $\frac{d^2y}{dx^2}$ for M1 only.

Just using $\frac{dP}{dr}$ and considering a sign change is M0A0

M1: Differentiates and finds $\left(\frac{d^2P}{dr^2} =\right) \pm \frac{f}{r^3}$ (do not be concerned about the sign)

A1: **Note if they score A0 in (b) then this mark cannot be scored.**
 Requires

- a correct a correct expression for $\frac{d^2P}{dr^2}$
- a correct value for $\left(\frac{d^2P}{dr^2} =\right) \frac{960}{r^3} = \text{awrt } 0.26$ using awrt 15.5 (but allow 0.23(43..) if using 16)
- a correct comparison with 0 **and** a conclusion e.g. minimum

The expression for the second derivative does not need to be labelled but if it is then it must be $\frac{d^2P}{dr^2}$ o.e. or accept e.g. P'' BUT $\frac{d^2y}{dx^2}$ used in their conclusion is A0



Question	Scheme	Marks	AOs
4(a)	e.g. $6.4^2 = 8^2 + 13^2 - 2(8)(13)\cos ABC$	M1	1.1b
	$(\text{angle } ABC =) \cos^{-1}\left(\frac{8^2 + 13^2 - 6.4^2}{2 \times 8 \times 13}\right) = 0.394 \text{ (radians)*}$	A1*	1.1b
		(2)	
(b)	e.g. $(\text{Area}(ABC) =) \frac{1}{2}(8)^2(0.394) (= \text{awrt } 12.6)$ or $(\text{Area}(ABD) =) \frac{1}{2}(8)(13)\sin 0.394 (= \text{awrt } 20.0)$	M1	1.1b
	$(\text{Area}(R) =) \frac{1}{2}(8)(13)\sin 0.394 - \frac{1}{2}(8)^2(0.394) = \dots$	dM1	3.1a
	$(\text{Area}(R) =) \text{awrt } 7.34 \text{ to } 7.37$	A1	1.1b
		(3)	
(5 marks)			
Notes			
It is acceptable in this question to work in degrees and convert if necessary. NB Angle ABC in degrees is 22.591...° (a) M1: Attempts to use the cosine rule with values correctly placed. May be implied by, e.g., $\cos \theta = \frac{8^2 + 13^2 - 6.4^2}{2 \times 8 \times 13}$ or $\cos^{-1}\left(\frac{6.4^2 - 8^2 - 13^2}{-2 \times 8 \times 13}\right)$ Condone slips in substitution if a correct cosine formula is seen. The placement of the values should be correct. So, condone e.g. a missing 2 once a correct cosine formula is seen. Alt 1: attempts the cosine rule and finds angle ADB (= awrt 0.5 or 29°) or angle BAD (= awrt 2.2 or 2.3 or 129°) and uses the sine rule correctly with angle ABC involved. Alt 2: attempts the cosine rule and finds angle ADB (= awrt 0.5 or 29°) and angle BAD (= awrt 2.2 or 2.3 or 129°) and sums angles ADB, BAD, and ABC to π (or 180°) Alt 3: using Pythagoras and simultaneous equations to find the length of AN or BN (see diagram on the next page) and uses e.g. $\cos ABC = \frac{BN}{8}$ or $\sin ABC = \frac{AN}{8}$ A1*: cso Correct proof. If starting with $6.4^2 = 8^2 + 13^2 - 2(8)(13)\cos ABC$ then there must be an intermediate line such as $\cos ABC = \frac{8^2 + 13^2 - 6.4^2}{2 \times 8 \times 13}$ or $\text{angle } ABC = \cos^{-1}\left(\frac{8^2 + 13^2 - 6.4^2}{2 \times 8 \times 13}\right)$ $\cos ABC = \text{awrt } 0.92$ or $\frac{4801}{5200}$ $\text{angle } ABC = \text{awrt } 0.3943$ The minimum required is e.g. $\cos^{-1}\left(\frac{8^2 + 13^2 - 6.4^2}{2 \times 8 \times 13}\right)$ or $\cos \theta = \frac{8^2 + 13^2 - 6.4^2}{2 \times 8 \times 13} \rightarrow \theta = 0.394$ both of which score M1A1*. The final value must be 0.394 and not e.g. 0.3943. There should be no obvious incorrect statements in the proof e.g. $\cos(0.92) = 0.394$ and no obvious incorrect work. Allow the use of, e.g., θ, x, or even e.g. A throughout. Mention of radians is not required. Those working in degrees and achieve 22.6 can convert to 0.394 without calculation for M1A1			



Attempts via verification e.g. $AD^2 = 8^2 + 13^2 - 2(8)(13)\cos 0.394 \rightarrow AD = \text{awrt } 6.4$ score maximum M1A0*.

(b) **Note: you may need to check the diagram for working.**

M1: Attempts the area of the sector **or** the area of the triangle ABD via a **correct** method. May be implied by a full attempt at the area of R .

The sector area may be implied by $\frac{1576}{125}$ or by $\frac{0.394}{2\pi} \times \pi \times 8^2$ or $\frac{22.6}{360} \times \pi \times 8^2$

The angle ABC is given in the question as 0.394 and should be used (or a more accurate value). Do not allow use of a different value for angle ABC (other than the angle in degrees).

There are many acceptable alternative approaches to find the area of triangle ABD , e.g. using their angle ADB **or** their angle BAD **or** using right-angled triangles

e.g. $\frac{1}{2}(13)(\text{"3.07"})$ **or** $\frac{1}{2}(\text{"7.39"})(\text{"3.07"}) + \frac{1}{2}(\text{"5.614"})(\text{"3.07"})$ and could be implied by e.g. $11.3 + 8.6$

Alternatively, attempts the area of the triangle ACD $\left(\text{e.g. } \frac{1}{2}(5)(6.4)\sin \text{"0.501"} = \text{awrt } 7.68 \right)$

or the area of the segment (in sector ABC) $\left(\frac{1}{2}(8^2)(0.394) - \frac{1}{2}(8^2)\sin 0.394 = \text{awrt } 0.324 \right)$

via a **correct** method. The area of the triangle ABC alone is insufficient for this mark.

Note that $\sin 0.394$ might be seen as $\sqrt{1 - \left(\frac{4801}{5200}\right)^2}$

For values, see the diagram below.

dM1: Complete and correct method for the area of R . Requires correct attempts at both the area of the sector **and** the area of the triangle ABD **and** subtraction of the two values (or expressions). Allow sector – triangle ABD if this is then recovered by making the area positive.

Alternatively, correct attempts at both the area of the triangle ACD **and** the area of the segment (in sector ABC) **and** subtraction of the two values (or expressions).

Condone slips in substitution provided a correct formula is seen.

May be implied by a correct answer.

A1: awrt 7.34 to 7.37 no units required but penalise incorrect units.

ISW after an acceptable answer is seen.

Helpful Diagram:



Question	Scheme	Marks	AOs
10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b \log_{10} l^*$ or $\Rightarrow \log_{10} T = b \log_{10} l + \log_{10} a^*$	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \frac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
(6 marks)			
Notes			
(a) M1: Takes logs of both sides and shows the addition law. Implied by $T = al^b \Rightarrow \log_{10} a + \log_{10} l^b$ A1*: Uses the power law to obtain the given equation with no errors. Allow the bases to be missing in the working but they must be present in the final answer. Also allow t rather than T and A rather than a. Allow working backwards e.g. $\log_{10} T = b \log_{10} l + \log_{10} a \Rightarrow \log_{10} T = \log_{10} l^b + \log_{10} a$ $\Rightarrow \log_{10} T = \log_{10} al^b \Rightarrow T = al^b^*$ M1: Uses the given answer and uses the power law and addition law correctly A1: Reaches the given equation with no errors as above (b) B1: Deduces the correct value for b (Allow awrt 0.495 or $\frac{45}{91}$) M1: Correct strategy to find the value of a. E.g. substitutes one of the given points and their value for b into $\log_{10} T = \log_{10} a + b \log_{10} l$ and uses correct log work to identify the value of a. Allow slips in rearranging their equation but must be correct log work to find a. Alternatively finds the equation of the straight line and equates the constant to $\log_{10} a$ and uses correct log work to identify the value of a. E.g. $y - 0.45 = "0.495"(x - 0.21) \Rightarrow y = "0.495"x + 0.346 \Rightarrow a = 10^{0.346} = \dots$ A1: Complete equation $T = 2.22l^{0.495}$ or $T = 2.22l^{\frac{45}{91}}$ (Allow awrt 2.22 and awrt 0.495 or $\frac{45}{91}$) Must see the <u>equation</u> not just correct values as it is a requirement of the question. (c) B1: Correct interpretation			

Question	Scheme	Marks	AOs
13(a)	$\log_{10} b = 0.0054 \Rightarrow b = 10^{0.0054}$ or $\log_{10} a = 0.81 \Rightarrow a = 10^{0.81}$	M1	3.1a
	$b = 1.01$ or $a = 6.46$	A1	1.1b
	$\log_{10} b = 0.0054 \Rightarrow b = 10^{0.0054}$ and $\log_{10} a = 0.81 \Rightarrow a = 10^{0.81}$	M1	2.1
	$b = 1.013$ and $a = 6.457$	A1	1.1b
		(4)	
(b)(i)	e.g. The world population in billions in 2004	B1ft	3.2a
(ii)	$b = 1.013$ represents the scale factor of the <u>yearly increase</u> in the world population	B1ft	3.2a
		(2)	
(c)	$P = 6.457 \dots (1.013 \dots)^{26}$ or e.g. $\log P = 0.81 + 26 \times 0.0054 \Rightarrow P = \dots$	M1	3.4
	awrt 9 billion	A1	2.2b
		(2)	
(d)	Not reliable since the data used for the model covered the years 2004 – 2007 and it would not be sensible to assume that the model still holds in 2030	B1	3.2b
		(1)	
(9 marks)			
Notes			
(a) Must be using base 10 in (a). Ignore any units associated with a and b in part (a). M1: Correct strategy to get a numerical expression or value for a or b e.g. $a = 10^{0.81}$ or $b = 10^{0.0054}$. This may be implied by $a = \text{awrt } 6.46$ or $b = \text{awrt } 1.01$ if no incorrect work is seen. A1: Correct value for a or b. Allow 3 sf for this mark so allow $a = \text{awrt } 6.46$ or $b = \text{awrt } 1.01$. May be seen embedded in their formula. M1: Correct strategy to get a numerical expression or value for a and b e.g. $a = 10^{0.81}$ and $b = 10^{0.0054}$. This may be implied by $a = \text{awrt } 6.46$ and $b = \text{awrt } 1.01$ if no incorrect work is seen. A1: Correct values. This requires $a = \text{awrt } 6.457$ and $b = \text{awrt } 1.013$ for this mark. May be seen embedded in their formula. Is w once correct answers are seen. Special case: Constants the wrong way round: $a = 1.013$ and $b = 6.457$ with or without working scores M1A1M1A0 unless the equation is formed correctly in which case the final A mark can be recovered. Note that having found the value of a, it is possible to find b by substituting e.g. $t = 1$ as follows: $a = 10^{0.81} = 6.457 \quad t = 1 \Rightarrow P = ab \Rightarrow b = \frac{P}{a}$ $t = 1 \Rightarrow \log_{10} P = 0.0054 + 0.81 = 0.8154 \Rightarrow P = 10^{0.8154} \Rightarrow b = \frac{P}{a} = \frac{10^{0.8154}}{6.457} = 1.013$			



Note that a misread of 0.0054 as 0.054 is quite common and may score 1110 as it does not simplify the question.

(b)(i) Follow through their a .

B1ft: Correct interpretation for a but must reference “billions”.

Allow equivalent alternatives e.g.

- The original/initial population **in billions**
- The population in 2004 was “6.46” **billion**

(b)(ii) Follow through their b .

B1ft: Correct interpretation for b but must reference “each year” or e.g. “yearly” or

Allow equivalent alternatives e.g.

- The proportional increase/change in each year.
- The population will rise by “1.3%” each year. Must follow their value for b .
- The rate/factor at which the population is rising/increasing/changing per annum.
- “1.013” is the multiplier representing the year on year increase.

Do **not** accept

- The amount it is rising
- How much it is rising
- The rate the population increases
- The percentage increase each year
- The rate of increase in billions annually

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way round as long as clearly labelled “ a is.....” and “ b is”

(c)

M1: Substitutes $t = 25$ or 26 or 27 into their model to find a value for P

Must be using **their** a and b correctly in $P = ab^t$

May be implied by sight of “9” or 9 billion if no incorrect working is seen.

A1: Correct value including units (allow awrt 9 billion) **from a correct model but condone incorrect/premature rounding or truncating in an otherwise correct model that leads to the correct value of awrt 9 billion.**

Allow e.g. awrt 9 000 000 000 or e.g. awrt 9×10^9

Just awrt 9 without the “billions” is A0

(d)

B1: The response must refer to the fact that the answer is unreliable together with a reference to the fact that the data used for the model is a long way from 2030

Examples:

- Not good as 2030 is a long way from 2004 – 2007
- Unreliable as based on old data
- Questionable as it has been extrapolated over a long time
- Not reliable due to how far out we have extrapolated
- By the time 2030 arrives it will be unreliable

But not e.g.

- Unreliable, extrapolation
- Not good as outside the range
- Not good as the population rises 101.3% each year
- Disease may happen
- Reliable as based on old data

LOOK FOR $\frac{1}{2}h \times \{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$ condensing steps on the terms but must use all y values with no repeats.

There must be a clear attempt at $\frac{1}{2}h \times (\text{first } y + \text{last } y + 2 \times \text{"sum of the rest"})$

Give M0 for $\frac{1}{2} \times \frac{1}{2} 0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)$ unless the missing brackets are implied.

NB this incorrect method gives 5.85...

May be awarded for separate trapezia e.g.

$$\frac{1}{2}(0.5774 + 0.7071) + \frac{1}{2}(0.7071 + 0.7746) + \frac{1}{2}(0.7746 + 0.8165) + \frac{1}{2}(0.8165 + 0.8452)$$



Question	Scheme	Marks	AOs
7(a)	$(\overrightarrow{AB}) = 3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}$	B1	1.1b
		(1)	
Notes for (a)			
B1: Correct <u>vector</u>. Allow $3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix}$ but not $\begin{pmatrix} 3\mathbf{i} \\ 9\mathbf{j} \\ 3\mathbf{k} \end{pmatrix}$ and not $(3, 9, 3)$ Condone 9 for $\begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix}$ Do not apply isw here but award for e.g. $3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k} = \begin{pmatrix} 3\mathbf{i} \\ 9\mathbf{j} \\ 3\mathbf{k} \end{pmatrix}$ E.g. if they obtain $\overrightarrow{AB} = 3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}$ and then say $\overrightarrow{AB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ then award B0 If part (a) is not attempted and the correct $\overrightarrow{AB}$ is seen in part (b) the B1 can be awarded there.			

General Guidance for part (b):

As with most vector questions we will see a variety of approaches (correct and incorrect).

In general, the marks are awarded as follows:

- M1 for a correct complete strategy to find at least one position for P (May be implied by at least 2 correct components)
- A1 for one correct position for P
- dM1 for a correct complete strategy to find both positions for P (May be implied by at least 2 correct components for both positions)
- A1 both correct positions for P and no others

Various examples are shown below.

Other methods will be seen but the above marking principles should be applied.

You can condone slips in their algebra/processing as long as the intention is clear.

The examples given below give the detail to look for depending on the approach.

If you see a response and you are not sure if it deserves credit use Review.

Note that adding vectors when they should be subtracting will generally score M0 but use review if necessary.



(b)	Examples: $\overrightarrow{OP} = \overrightarrow{OA} + 2\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + 2(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{AB} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} + (3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \frac{2}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} - \frac{1}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$	M1	3.1a
	$8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$ or $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	A1	1.1b
	Examples: $\overrightarrow{OP} = \overrightarrow{OA} + 2\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + 2(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{AB} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} + (3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ and $\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \frac{2}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$ or $\overrightarrow{OP} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA} = 5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k} - \frac{1}{3}(3\mathbf{i} + 9\mathbf{j} + 3\mathbf{k}) = \dots$	dM1	3.1a
	$8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	A1	2.2a
		(4)	
(5 marks)			
Notes for (b)			
Note that sight of at least one correct position for P implies M1A1 M1: Attempts at least one correct strategy for finding P A1: One correct position vector or allow coordinates for <u>this</u> mark e.g. (8, 15, 11) or (4, 3, 7) or $x = \dots, y = \dots, z = \dots$ If given as a vector, allow e.g. $8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$, $\begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$ but not $\begin{pmatrix} 8\mathbf{i} \\ 15\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$ dM1: Attempts two correct strategies for finding P A1: Both correct position vectors Must both be vectors so e.g. $8\mathbf{i} + 15\mathbf{j} + 11\mathbf{k}$, $\begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$ and $4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$, $\begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix}$ but not e.g. $\begin{pmatrix} 8\mathbf{i} \\ 15\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$ Condone e.g. 15 for $\begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$			

Alternative 1 using vector equation of l :

$$\mathbf{r} = \overrightarrow{OA} + \lambda \overrightarrow{AB} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix} \quad \left(\text{or e.g. } \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \right)$$

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}| \Rightarrow \begin{vmatrix} 3\lambda \\ 9\lambda \\ 3\lambda \end{vmatrix} = 2 \begin{vmatrix} 3\lambda + 2 - 5 \\ 9\lambda - 3 - 6 \\ 3\lambda + 5 - 8 \end{vmatrix} \Rightarrow 9\lambda^2 + 81\lambda^2 + 9\lambda^2 = 4[(3\lambda - 3)^2 + (9\lambda - 9)^2 + (3\lambda - 3)^2]$$

$$\Rightarrow \lambda = 2, \frac{2}{3} \Rightarrow \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 3 \\ 9 \\ 3 \end{pmatrix}$$

M1: Forms the vector equation of line l using their $\overrightarrow{AB}$ from part (a) or by starting again, forms the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ then uses $|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$ and Pythagoras to produce a quadratic equation in " λ " which they then solve to find " λ " and use correctly to find at least one position for P .

Note if they use $\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ for the direction, they should get $\lambda = 6, 2$

If all other work is correct, condone not squaring the "2" when applying Pythagoras

A1: See main scheme

dM1: As the first M and finds both positions for P

A1: See main scheme

Alternative 2 using P as $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and that $\overrightarrow{AP}$ and $\overrightarrow{BP}$ are parallel:

$$\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix}, \quad \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix}$$

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}| \Rightarrow \begin{vmatrix} x-2 \\ y+3 \\ z-5 \end{vmatrix} = 2 \begin{vmatrix} x-5 \\ y-6 \\ z-8 \end{vmatrix} \Rightarrow \begin{aligned} (x-2)^2 &= 4(x-5)^2 \\ (y+3)^2 &= 4(y-6)^2 \\ (z-5)^2 &= 4(z-8)^2 \end{aligned}$$

$$\begin{aligned} (x-2)^2 &= 4(x-5)^2 \Rightarrow x = 4, 8 \\ (y+3)^2 &= 4(y-6)^2 \Rightarrow y = 3, 15 \\ (z-5)^2 &= 4(z-8)^2 \Rightarrow z = 7, 11 \end{aligned} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$$

M1: Sets P as a general point, forms $\overrightarrow{AP}$ and $\overrightarrow{BP}$ (either way round) then uses $|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$ then squares components and equates to produce quadratic equations in x and y and z which they then solve to find at least one position for P . It is not just for finding values which are not then used to form a point (or vector).

If all other work is correct, condone not squaring the "2" when squaring.

A1: See main scheme

dM1: As the first M and finds both positions for P .

A1: See main scheme



Note that if the modulus is not used, this method can lead to one correct position for P e.g.

$$\overrightarrow{AP} = 2\overrightarrow{BP} \Rightarrow \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = 2 \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix} \Rightarrow \begin{matrix} x=8 \\ y=15 \\ z=11 \end{matrix} \text{ and scores M1 A1}$$

But it is possible to find the other position without squaring e.g.

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}| \Rightarrow \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = 2 \begin{pmatrix} 5-x \\ 6-y \\ 8-z \end{pmatrix} \Rightarrow \begin{matrix} x=4 \\ y=3 \\ z=7 \end{matrix} \text{ and scores dM1 then A1 as main scheme.}$$

This requires at least 2 correct equations for x , y or z for the dM1

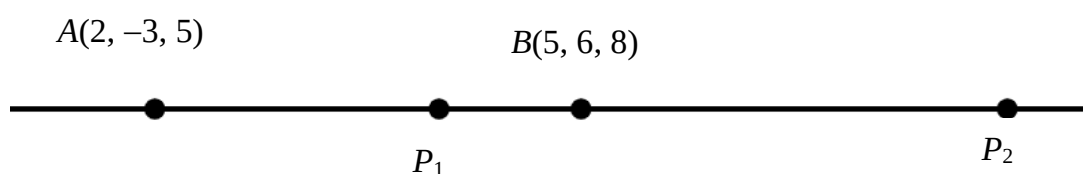
e.g. **Alternative 3 using P as $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and eliminating 2 of the variables:**

$$\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix}, \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix}$$

$$\overrightarrow{AP} = \begin{pmatrix} x-2 \\ y+3 \\ z-5 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x-2 \\ 3x-6 \\ x-2 \end{pmatrix}, \overrightarrow{BP} = \begin{pmatrix} x-5 \\ y-6 \\ z-8 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \Rightarrow \overrightarrow{BP} = \begin{pmatrix} x-5 \\ 3x-15 \\ x-5 \end{pmatrix}$$

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}| \Rightarrow (x-2)^2 + (3x-6)^2 + (x-2)^2 = 4[(x-5)^2 + (3x-15)^2 + (x-5)^2] \Rightarrow x = 4, 8$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} \text{ (or } \overrightarrow{OB} + \overrightarrow{BP}) = \begin{pmatrix} x \\ 3x-9 \\ x+3 \end{pmatrix} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix} \text{ or } \begin{pmatrix} 8 \\ 15 \\ 11 \end{pmatrix}$$



Question	Scheme	Marks	AOs
13(a)	Attempts two of the relevant vectors $\pm \overrightarrow{AB} = \pm(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$ $\pm \overrightarrow{AC} = \pm(-20\mathbf{i} + (p+3)\mathbf{j} + 5\mathbf{k})$ $\pm \overrightarrow{BC} = \pm(-16\mathbf{i} + (p-4)\mathbf{j} + 4\mathbf{k})$	M1	3.1a
	Uses two of the three vectors in such a way as to find the value of p . E.g. $p+3 = 5 \times 7$	dM1	2.1
	$p = 32$	A1	1.1b
		(3)	
(a) Alternative:			
	$r_{AB} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$	M1	3.1a
	$4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow \lambda = 5$ $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow p = 35 - 3$	dM1	2.1
	$p = 32$	A1	1.1b
(b)	Deduces that $\overrightarrow{OD} = \lambda \overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - 32)\mathbf{j} + (6\lambda - 10)\mathbf{k}$	M1	3.1a
	Correct attempt at λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$ $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - 32)\mathbf{j} + (6\lambda - 10)\mathbf{k}$ $\overrightarrow{OA} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ $4\lambda - 32 = -12 \Rightarrow \lambda = \dots \quad \text{OR} \quad 6\lambda - 10 = 20 \Rightarrow \lambda = \dots$	dM1	1.1b
	$ \overrightarrow{OD} = 5 \times \sqrt{4^2 + 6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
(b) Alternative:			
	Deduces that $\overrightarrow{OD} = \lambda \overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts $\overrightarrow{OD} = \overrightarrow{OC} + \mu \overrightarrow{OA} = -16\mathbf{i} + 32\mathbf{j} + 10\mathbf{k} + \mu(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	M1	3.1a
	Correct attempt at λ or μ using the fact that $\lambda \overrightarrow{OB} = \overrightarrow{OC} + \mu \overrightarrow{OA}$ E.g. $-16 + 4\mu = 0 \Rightarrow \mu = 4$	dM1	1.1b
	$ \overrightarrow{OD} = 5 \times \sqrt{4^2 + 6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
(6 marks)			
Notes:			

(a)

M1: Attempts two of the three relevant vectors by **subtracting** either way around. See scheme.

 Allow equivalent work e.g. $\pm \overrightarrow{AB} = \pm(\overrightarrow{OB} + \overrightarrow{AO})$


dM1: For the key step in using the fact that if the vectors are parallel, they will be multiples of each other (where the multiple is something other than 1) to find p .

E.g. $p+3=5\times 7$, $p-4=\frac{4}{5}(p+3)$, $p-4=4\times 7$

A1: $p=32$ (Condone 32j)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB}$, $\overrightarrow{AC}=5\overrightarrow{AB}$, $\overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find p and if the 3rd component has errors but is not used, full marks can be awarded.

Alternative:

M1: Forms the vector equation using A or B as position and $\pm\overrightarrow{AB}$ as the direction

dM1: For the key step in using the fact that C lies on the line to find p

A1: $p=32$ (Condone 32j)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB}$, $\overrightarrow{AC}=5\overrightarrow{AB}$, $\overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC}$, $\overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find p and if the 3rd component has errors but is not used, full marks can be awarded.

There will be other approaches e.g. using “gradients” and “ratios” and the method marks can be implied – if you are unsure if such attempts deserve credit use Review

(b) Vector approach

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-32)\mathbf{j}+(6\lambda-10)\mathbf{k}$

dM1: Correct attempt at finding λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

E.g. $16\mathbf{i}+(4\lambda-32)\mathbf{j}+(6\lambda-10)\mathbf{k}=4\alpha\mathbf{i}-3\alpha\mathbf{j}+5\alpha\mathbf{k}\Rightarrow\alpha=4\Rightarrow 4\lambda-32=-3\times 4\Rightarrow\lambda=...$

A1: $|\overrightarrow{OD}|=10\sqrt{13}$

Alternative:

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts

$$\overrightarrow{OD}=\overrightarrow{OC}+\mu\overrightarrow{OA}=-16\mathbf{i}+32\mathbf{j}+10\mathbf{k}+\mu(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$$

dM1: Correct attempt at finding λ or μ using the fact that $\lambda\overrightarrow{OB}=\overrightarrow{OC}+\mu\overrightarrow{OA}$

E.g. $(-16+4\mu)\mathbf{i}+(32-3\mu)\mathbf{j}+(10+5\mu)\mathbf{k}=4\lambda\mathbf{j}+6\lambda\mathbf{k}\Rightarrow -16+4\mu=0\Rightarrow\mu=...$

May also solve simultaneously using y and z components to find λ or μ

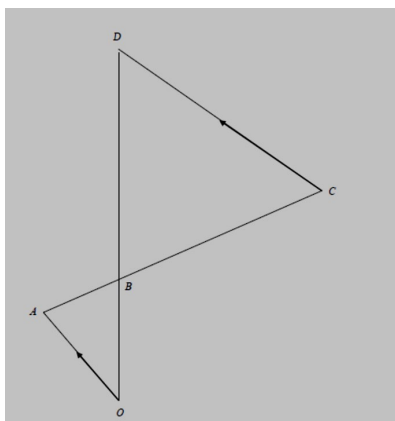
A1: $|\overrightarrow{OD}|=10\sqrt{13}$

Note that the correct vector is $20\mathbf{j}+30\mathbf{k}$

PTO for similar triangle approach



(b) Similar triangle approach



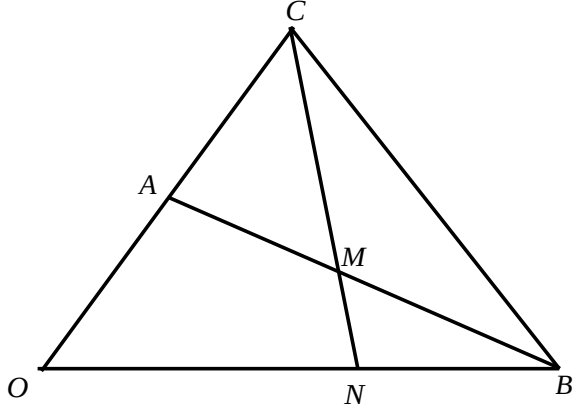
M1: For the key step in recognising that triangle BCD and triangle BAO are similar with a ratio of lengths of 4:1

dM1: States or uses the fact that $|\overrightarrow{OD}| = 5 \times |\overrightarrow{OB}|$

Stating this will score M1 dM1 provided there is no evidence of incorrect work

Note that they may establish this result using the work from (a) but must be used here to score.

A1: $|\overrightarrow{OD}| = 10\sqrt{13}$

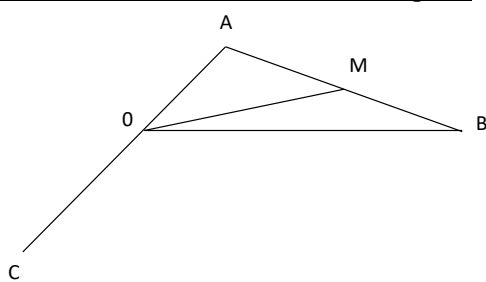
Question	Scheme	Marks	AOs
10			
	$\overrightarrow{OA} = \mathbf{a}, \overrightarrow{OB} = \mathbf{b}$		
(a)	$\left\{ \overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2} \overrightarrow{AB} \Rightarrow \overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) \right\}$	M1	3.1a
	$\left\{ \overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM} = \overrightarrow{CB} + \frac{1}{2} \overrightarrow{BA} \Rightarrow \overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b}) \right\}$		
	$\Rightarrow \overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ (needs to be simplified and seen in (a) only)	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OC} + \lambda \overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda \left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \right) \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda \right) \mathbf{a} + \frac{1}{2}\lambda \mathbf{b} *$	A1*	2.1
		(2)	
(c) Way 1	$\left(2 - \frac{3}{2}\lambda \right) = 0 \Rightarrow \lambda = \dots$	M1	2.2a
	$\lambda = \frac{4}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON : NB = 2 : 1 *$	A1*	2.1
		(2)	
(c) Way 2	$\overrightarrow{ON} = \mu \mathbf{b} \Rightarrow \left(2 - \frac{3}{2}\lambda \right) \mathbf{a} + \frac{1}{2}\lambda \mathbf{b} = \mu \mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda \right) = 0 \Rightarrow \lambda = \dots \left\{ \mathbf{b}: \frac{1}{2}\lambda = \mu \text{ \& } \lambda = \frac{4}{3} \Rightarrow \mu = \frac{2}{3} \right\}$	M1	2.2a
	$\lambda = \frac{4}{3} \text{ or } \mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON : NB = 2 : 1 *$	A1*	2.1
		(2)	
(6 marks)			



Question	Scheme	Marks	AOs
10 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = \dots \quad \left\{ \mathbf{b}: 1 = \frac{1}{2}\lambda + K \quad \& \quad \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3} \right\}$	M1	2.2a
	$\lambda = \frac{4}{3} \text{ or } K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \text{ or } \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON:NB = 2:1 *$	A1	2.1
		(2)	
10 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b} \text{ \& } \overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3}, \quad \mathbf{b}: \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = \dots$	M1	2.2a
	$\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON:NB = 2:1 *$	A1	2.1
		(2)	

Notes for Question 10

(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors $\mathbf{a}$ and $\mathbf{b}$
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{ \overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow \right\} \overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ only o.e.
(b)	
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	Special Case Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \left\{ = \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b} \right\}$
Note:	Alternative 1: Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow \overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$
(c)	Way 1, Way 2 and Way 3
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ
A1*:	Correct proof
(c)	Way 4
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

Notes for Question 10 Continued	
Note:	Part (b) and part (c) can be marked together.
(a) Special Case	<u>Special Case where the point C is believed to be below the origin O</u>
	
	Give Special Case M1 A0 in part (a) for $\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow\} \overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$
	$\left\{ \text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \right\}$