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EDEXCEL A LEVEL MATHEMATICS: QUESTIONS BY TOPIC

Numerical Methods (9)

A Level Mathematics Topic 9

14 questions · 109 marks · Mark schemes included

Pearson Edexcel A Level Mathematics (9MA0), June 2018 - June 2025

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HOW TO USE THIS BOOKLET

Questions are grouped by specification sub-topic and run from the lowest mark tariff to the highest within each one.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme.

Diagrams, graphs, tables and data are printed exactly as they appeared in the original papers.

QUESTION	SOURCE	MARKS	PAGE
9.2 ITERATION		7 Qs · 63 marks	3
1. Q4 <i>also Topic 6</i>	Paper 1: Pure Mathematics 1 June2018	4	4
2. Q6 <i>also Topic 6</i>	Paper 2: Pure Mathematics 2 June2024	7	6
3. Q4 <i>also Topic 6 & 7</i>	Paper 1: Pure Mathematics 1 November2021	9	10
4. Q7 <i>also Topic 6 & 7</i>	Paper 2: Pure Mathematics 2 October2020	10	14
5. Q11 <i>also Topic 6 & 7</i>	Paper 2: Pure Mathematics 2 June2019	11	18
6. Q15 <i>also Topic 6 & 7</i>	Paper 1: Pure Mathematics 1 June2023	13	22
7. Q9 <i>also Topic 6 & 7</i>	Paper 1: Pure Mathematics 1 June2025	9	27
9.3 NEWTON-RAPHSON METHOD		2 Qs · 13 marks	31
8. Q5	Paper 2: Pure Mathematics 2 June2018	6	32
9. Q6 <i>also Topic 5</i>	Paper 2: Pure Mathematics 2 June2022	7	34
9.4 THE TRAPEZIUM RULE		4 Qs · 25 marks	38
10. Q1 <i>also Topic 8</i>	Paper 2: Pure Mathematics 2 October2020	5	39
11. Q5 <i>also Topic 8</i>	Paper 1: Pure Mathematics 1 June2023	6	41
12. Q5 <i>also Topic 8</i>	Paper 2: Pure Mathematics 2 June2022	6	43
13. Q11 <i>also Topic 6 & 8</i>	Paper 1: Pure Mathematics 1 November2021	8	45
9.5 NUMERICAL METHODS IN CONTEXT		1 Qs · 8 marks	49
14. Q8 <i>also Topic 6</i>	Paper 1: Pure Mathematics 1 June2022	8	50



QUESTIONS

14 questions · 109 marks · 4 sub-topics



SUB-TOPIC 9.2

ITERATION

Sub-topic 9.2 · 7 questions · 63 marks · lowest tariff first



6.

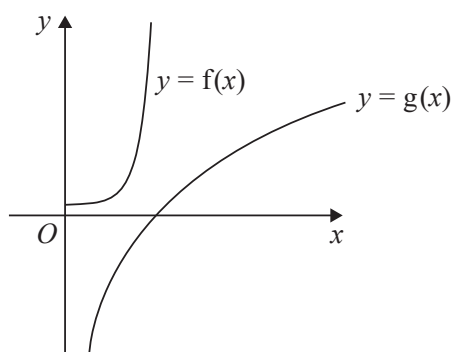


Figure 1

Figure 1 shows a sketch of the curves with equations $y = f(x)$ and $y = g(x)$ where

$$f(x) = e^{4x^2 - 1} \quad x > 0$$

$$g(x) = 8 \ln x \quad x > 0$$

(a) Find

(i) $f'(x)$

(ii) $g'(x)$

(2)

Given that $f'(x) = g'(x)$ at $x = \alpha$

(b) show that α satisfies the equation

$$4x^2 + 2 \ln x - 1 = 0$$

(2)

The iterative formula

$$x_{n+1} = \sqrt{\frac{1 - 2 \ln x_n}{4}}$$

is used with $x_1 = 0.6$ to find an approximate value for α

(c) Calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of α

(3)





(Total for Question 6 is 7 marks)







(Total for Question 4 is 9 marks)



7.

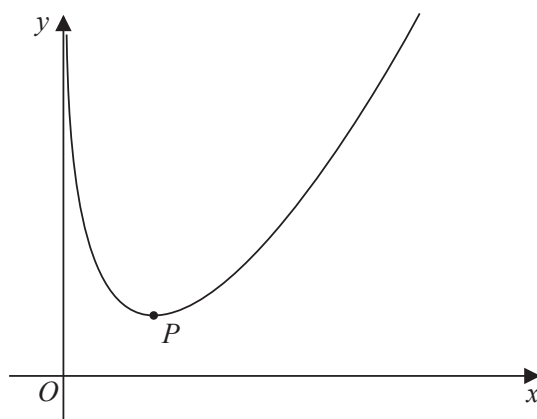


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}} \quad (4)$$

The point P , shown in Figure 1, is the minimum turning point on C .

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12} \right)^{\frac{2}{3}} \quad (3)$$

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12} \right)^{\frac{2}{3}} \quad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)







11.

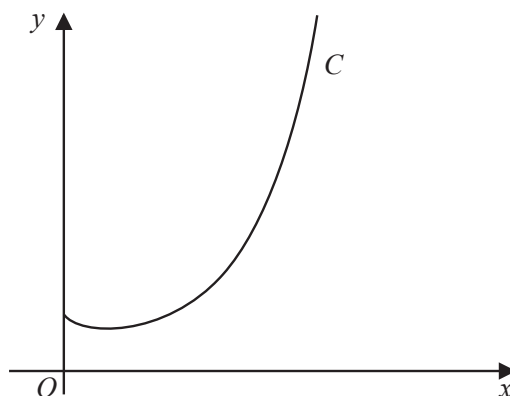


Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C .

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

The point $P(\alpha, 2)$ lies on C .

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n

(2)







15. A curve has equation $y = f(x)$, where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \quad x > \ln \sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of β

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)





Question 15 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

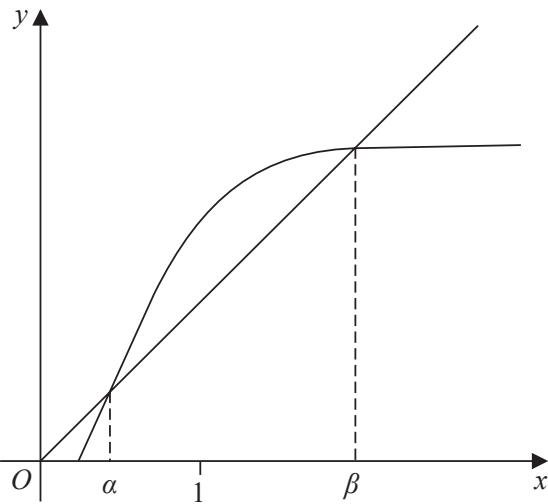
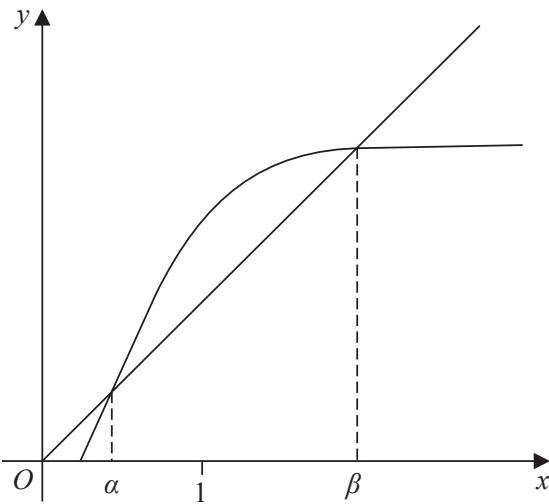


Diagram 1



copy of Diagram 1

Handwritten area for the answer, consisting of multiple horizontal lines.



(Total for Question 15 is 13 marks)



9.

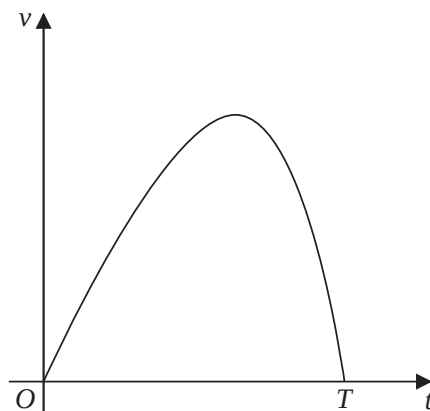


Figure 2

A racing car is driven along a straight road.

Figure 2 shows a graph of the speed of the car as it travels along the road.

The car starts from rest and is driven for T seconds before stopping.

The speed of the car is modelled by the equation

$$v = 15t - te^{0.2t} \quad 0 \leq t \leq T$$

where t seconds is the time after the car starts to move.

According to the model,

(a) find the value of T , giving your answer to one decimal place,

(2)

(b) show that the maximum speed of the car occurs when

$$t = 5 \ln \left(\frac{75}{t + 5} \right) \quad (4)$$

Using the iteration formula

$$t_{n+1} = 5 \ln \left(\frac{75}{t_n + 5} \right) \quad \text{with } t_1 = 8$$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)





SUB-TOPIC 9.3

NEWTON-RAPHSON METHOD

Sub-topic 9.3 · 2 questions · 13 marks · lowest tariff first



(Total for Question 5 is 6 marks)



6.

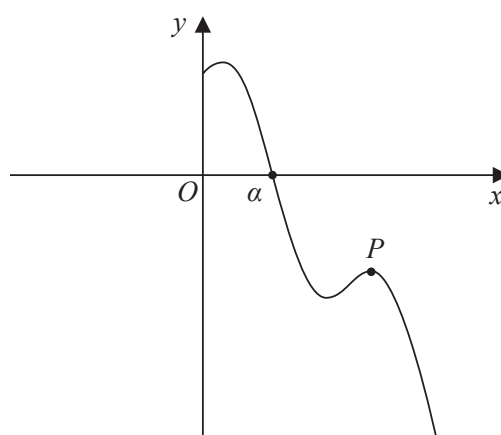


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8 \sin\left(\frac{1}{2}x\right) - 3x + 9 \quad x > 0$$

and x is measured in radians.

The point P , shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P , giving your answer to 3 significant figures.

(4)

The curve crosses the x -axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $f(4) = 4.274$ and $f(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(1)

(c) Taking $x_0 = 5$ as a first approximation to α , apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to α .

Show your method and give your answer to 3 significant figures.

(2)





(Total for Question 6 is 7 marks)



SUB-TOPIC 9.4

THE TRAPEZIUM RULE

Sub-topic 9.4 · 4 questions · 25 marks · lowest tariff first



(Total for Question 5 is 6 marks)



(Total for Question 5 is 6 marks)



11.

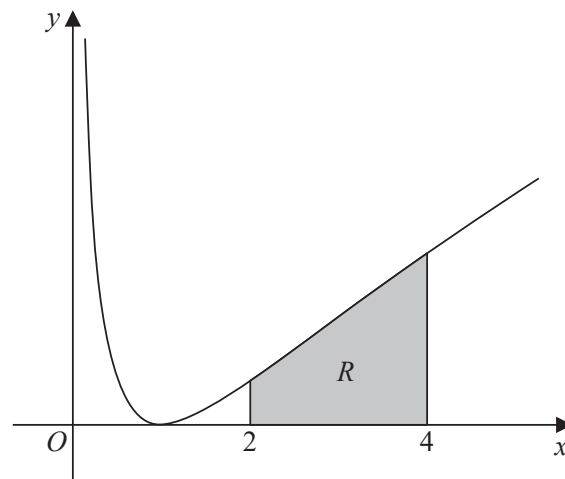


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \quad x > 0$$

The finite region R , shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x -axis and the line with equation $x = 4$

The table below shows corresponding values of x and y , with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

- (a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R , giving your answer to 3 significant figures.

(3)

- (b) Use algebraic integration to find the exact area of R , giving your answer in the form

$$y = a(\ln 2)^2 + b \ln 2 + c$$

where a , b and c are integers to be found.

(5)

Question 11 continued

Lined area for writing the answer to Question 11 continued.





Question 11 continued

Lined area for writing the answer to Question 11.

(Total for Question 11 is 8 marks)

SUB-TOPIC 9.5

NUMERICAL METHODS IN CONTEXT

Sub-topic 9.5 · 1 question · 8 marks · lowest tariff first



8.

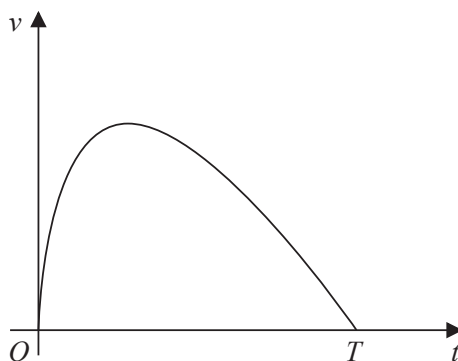


Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v \text{ ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t) \ln(t + 1) \quad 0 \leq t \leq T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T (1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1 \quad (4)$$

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,
 (ii) find, by repeated iteration, the time taken for the car to reach maximum speed. (3)





(Total for Question 6 is 6 marks)



MARK SCHEMES

official mark schemes



Question	Scheme	Marks	AOs
4 (a)	Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x) - x)$	M1	2.1
	$f(3) = (2\ln(5) - x) = (+)0.22$ and $f(4) = (2\ln(4) - 4) = -1.23$ <u>Change of sign</u> and function <u>continuous</u> in interval $[3, 4] \Rightarrow$ <u>Root</u> *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1 = 4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula can be used to find an approximation for α because the cobweb spirals inwards for the cobweb diagram	A1	2.2a
		(2)	
(4 marks)			

Notes:
(a)

M1: Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x) - x)$ or alternatively **compares** $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

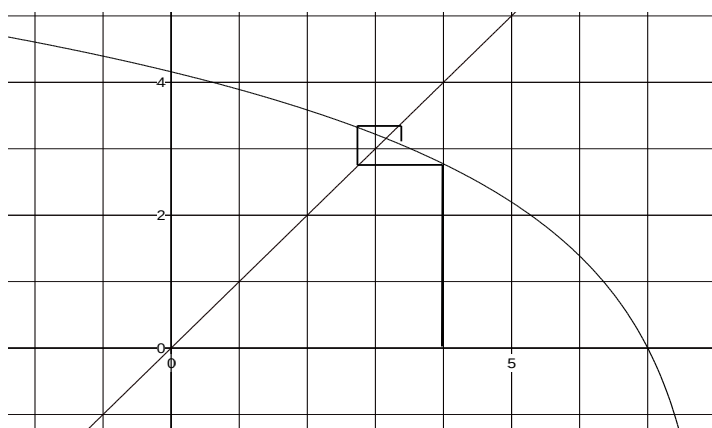
A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)

When comparing terms, allow reasons to be $2\ln 8 = 3.21 > 3$, $2\ln 4 = 2.77 < 4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**

A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must statement that it can be used with a suitable reason. Suitable reasons could be "it spirals inwards", it gets closer to the root", it converges "



Question	Scheme	Marks	AOs
6(a)(i)	$(f'(x) =) 8xe^{4x^2-1}$ or e.g. $\frac{8xe^{4x^2}}{e}$ oe	B1	1.1b
(ii)	$(g'(x) =) \frac{8}{x}$ or e.g. $8x^{-1}$ oe	B1	1.2
		(2)	
(b)	$8xe^{4x^2-1} = \frac{8}{x} \Rightarrow e^{4x^2-1} = \frac{1}{x^2} \Rightarrow 4x^2 - 1 = \ln \frac{1}{x^2}$	M1	1.1b
	$4x^2 - 1 = \ln \frac{1}{x^2} \Rightarrow 4x^2 - 1 = -2 \ln x$ $\Rightarrow 4x^2 + 2 \ln x - 1 = 0^*$	A1*	2.1
		(2)	
(c)(i)	$x_1 = 0.6 \Rightarrow x_2 = \sqrt{\frac{1-2 \ln 0.6}{4}}$	M1	1.1b
	$(x_2 =) 0.7109$	A1	1.1b
(ii)	$(\alpha =) 0.6706$	B1 (A1 In ePEN)	1.1b
		(3)	
(7 marks)			
(a)(i) B1: Correct derivative in any form. "$f'(x) =$" is not required. Apply isw if necessary. (ii) B1: Correct derivative in any form. "$g'(x) =$" is not required. Apply isw if necessary. (b) M1: Eliminates e by setting their $f'(x) =$ their $g'(x)$ where $f'(x) = Axe^{4x^2-1}$ oe and $g'(x) = \frac{B}{x}$ oe with $A \times B > 0$ and proceeds via $e^{4x^2-1} = \frac{\dots}{x^2}$ or equivalent work (see below) to obtain $4x^2 - 1 = \ln \frac{\dots}{x^2}$ oe e.g. $\ln x + 4x^2 - 1 = \ln \frac{1}{x}$ Allow if they use α for x. Note that there are various alternatives for this mark but the derivatives must be of the form defined above and the processing must be correct with coefficient/sign slips only. <u>Examples of equivalent work:</u> $8xe^{4x^2-1} = \frac{8}{x} \Rightarrow x^2 e^{4x^2-1} = 1 \Rightarrow \ln x^2 + \ln e^{4x^2-1} = 0 \Rightarrow \ln e^{4x^2-1} = -\ln x^2 \Rightarrow 4x^2 - 1 = -2 \ln x$ $\frac{8xe^{4x^2}}{e} = \frac{8}{x} \Rightarrow \frac{1}{e} e^{4x^2} = \frac{1}{x^2} \Rightarrow e^{4x^2} = \frac{e}{x^2} \Rightarrow \ln e^{4x^2} = \ln \frac{e}{x^2} \Rightarrow 4x^2 = \ln \frac{e}{x^2} = 1 - 2 \ln x$ A1*: Obtains the printed answer with sufficient working and no errors. Sufficient work would require the "e" eliminated before the given answer. Must follow correct derivatives in part (a). Condone $4x^2 + 2 \ln x - 1 = 0$ and condone $4\alpha^2 + 2 \ln \alpha - 1 = 0$ or $4\alpha^2 + 2 \ln \alpha - 1 = 0$			



Note that if both derivatives in (a) **are correct** we will allow fully correct work using the equation in (b) to work backwards to verify that $pf'(x) = qg'(x)$ for M1 then obtains

$f'(x) = g'(x)$ with a minimal conclusion for A1

If either derivative in (a) is incorrect or missing, candidates who work backwards score no marks in (b).

(c)(i)/(ii)

M1: Attempts to use the iterative formula with $x_1 = 0.6$

Award this mark for e.g. $(x_2 =) \sqrt{\frac{1 - 2 \ln 0.6}{4}}$ or may be implied by awrt 0.71 provided no incorrect working is seen.

Candidates sometimes find x_3 (or possibly subsequent terms) rather than x_2 in which case the M1 can be implied. (See table below for first few iterations)

A1: ($x_2 =$) awrt 0.7109

Sight of ($x_2 =$) awrt 0.7109 scores M1A1

B1(A1 on ePEN): ($\alpha =$) 0.6706 (4dp)

Must be this value and **not** awrt 0.6706

For reference:

x_1	0.6
x_2	0.7109239143
x_3	0.6485329086
x_4	0.6830236199
x_5	0.6637868021
x_6	0.6744606223
.	.
.	.
.	.
α	0.6706416243



Question	Scheme	Marks	AOs
4(a)	$f'(x) = 2x + \frac{4x - 4}{2x^2 - 4x + 5}$	M1 A1	1.1b 1.1b
	$2x + \frac{4x - 4}{2x^2 - 4x + 5} = 0 \Rightarrow 2x(2x^2 - 4x + 5) + 4x - 4 = 0$	dM1	1.1b
	$2x^3 - 4x^2 + 7x - 2 = 0^*$	A1*	2.1
		(4)	
(b)	(i) $x_2 = \frac{1}{7}(2 + 4(0.3)^2 - 2(0.3)^3)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$h(x) = 2x^3 - 4x^2 + 7x - 2$ $h(0.3415) = 0.00366... \quad h(0.3405) = -0.00130...$	M1	3.1a
	States: • there is a change of sign • $f'(x)$ is continuous • $\alpha = 0.341$ to 3dp	A1	2.4
		(2)	
(9 marks)			
Notes			

(a)

M1: Differentiates $\ln(2x^2 - 4x + 5)$ to obtain $\frac{g(x)}{2x^2 - 4x + 5}$ where $g(x)$ could be 1

A1: For $f'(x) = 2x + \frac{4x - 4}{2x^2 - 4x + 5}$

dM1: Sets their $f'(x) = ax + \frac{g(x)}{2x^2 - 4x + 5} = 0$ and uses "correct" algebra, condoning slips, to obtain a

cubic equation. E.g Look for $ax(2x^2 - 4x + 5) \pm g(x) = 0$ o.e. , condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3 - 4x^2 + 7x - 2 = 0$ with no errors. (The dM1 mark must have been awarded)

(b)(i)

M1: Attempts to use the iterative formula with $x_1 = 0.3$. If no method is shown award for $x_2 = \text{awrt } 0.33$

A1: $x_2 = \text{awrt } 0.3294$ Note that $\frac{1153}{3500}$ is correct

Condone an incorrect suffix if it is clear that a correct value has been found

(b)(ii)

A1: $x_4 = \text{awrt } 0.3398$ Condone an incorrect suffix if it is clear that a correct value has been found

(c)

M1: Attempts to substitute $x = 0.3415$ and $x = 0.3405$ into a suitable function and gets one value correct (rounded or truncated to 1 sf). It is allowable to use a tighter interval that contains the root 0.340762654

Examples of suitable functions are $2x^3 - 4x^2 + 7x - 2$, $x - \frac{1}{7}(4x^2 - 2x^3 + 2)$ and $f'(x)$ as this has been

found in part (a) with $f'(0.3405) = -0.00067\dots$, $f'(0.3415) = (+) 0.0018$

There must be sufficient evidence for the function, which would be for example, a statement such as $h(x) = 2x^3 - 4x^2 + 7x - 2$ or sight of embedded values that imply the function, not just a value or values

even if both are correct. Condone $h(x)$ being mislabelled as f

$h(0.3415) = 2 \times 0.3415^3 - 4 \times 0.3415^2 + 7 \times 0.3415 - 2$

A1: Requires

- both calculations correct (rounded or truncated to 1sf)
- a statement that there is a change in sign and that the function is continuous
- a minimal conclusion e.g. ✓, proven, $\alpha = 0.341$, root

$$\text{or } {}^7C_4 a^3 2^4, \frac{7!}{4!3!} a^3 2^4, \binom{7}{4} a^3 2^4, 35a^3 2^4, 560a^3 \text{ etc.}$$

$$\text{or } {}^7C_3 a^3 (2x)^4, \frac{7!}{4!3!} a^3 (2x)^4, \binom{7}{3} a^3 (2x)^4, 35a^3 (2x)^4, 560a^3 x^4, \binom{7}{3} a^3 16x^4 \text{ etc.}$$

$$\text{or } {}^7C_3 a^3 2^4, \frac{7!}{4!3!} a^3 2^4, \binom{7}{3} a^3 2^4, 35a^3 2^4, 560a^3$$

You can condone missing brackets around the “2x” so allow e.g. $\frac{7!}{4!3!} a^3 2x^4$

An alternative is to attempt to expand $a^7 \left(1 + \frac{2x}{a}\right)^7$ to give $a^7 \left(\dots \frac{7 \times 6 \times 5 \times 4}{4!} \left(\frac{2x}{a}\right)^4 \dots \right)$

Allow M1 for e.g. $a^7 \left(\dots \frac{7 \times 6 \times 5 \times 4}{4!} \left(\frac{2x}{a}\right)^4 \dots \right)$, $a^7 \left(\dots \binom{7}{4} \left(\frac{2x}{a}\right)^4 \dots \right)$, $a^7 \left(\dots 35 \left(\frac{2x}{a}\right)^4 \dots \right)$ etc.

but condone missing brackets around the $\frac{2x}{a}$

Note that ${}^7C_3, \binom{7}{3}$ etc. are equivalent to ${}^7C_4, \binom{7}{4}$ etc. and are equally acceptable.

If the candidate attempts $(a + 2x)(a + 2x)(a + 2x) \dots$ etc. then it must be a complete method to reach the required term. Send to review if necessary.

dM1: For “560” $a^3 = 15120 \Rightarrow a = \dots$ Condone slips on copying the 15120 but their “560” must be an attempt at

${}^7C_4 \times 2$ or ${}^7C_4 \times 2^4$ and must be attempting the cube root of $\frac{15120}{560}$. **Depends on the first mark.**

A1: $a = 3$ and no other values i.e. ± 3 scores A0

Note that this is fairly common:

$${}^7C_4 a^3 2x^4 = 70a^3 x^4 \Rightarrow 70a^3 = 15120 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

and scores M1 dM1 A0



7(a)	$\ln x \rightarrow \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2 + x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2 \times \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{dy}{dx} = 3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2 + x - 16\sqrt{x} = 0 \Rightarrow 12x^{\frac{3}{2}} + x^{\frac{1}{2}} - 16 = 0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}} = 16 - \sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}} = \frac{4}{3} - \frac{\sqrt{x}}{12} \Rightarrow x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2 = \sqrt[3]{\left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	$x_2 = \text{awrt } 1.13894$	A1	1.1b
	$x = 1.15650$	A1	2.2a
		(3)	
(10 marks)			

Notes:

(a)

B1: Differentiates $\ln x \rightarrow \frac{1}{x}$ seen or implied

M1: Correct method to differentiate $\frac{4x^2 + x}{2\sqrt{x}}$:

Look for $\frac{4x^2 + x}{2\sqrt{x}} \rightarrow \dots x^{\frac{3}{2}} + \dots x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}} + \dots$ **or** $\dots + Qx^{-\frac{1}{2}}$

Alternatively uses the quotient rule on $\frac{4x^2 + x}{2\sqrt{x}}$.

Condone slips but if rule is not quoted expect $\left(\frac{dy}{dx}\right) = \frac{2\sqrt{x}(Ax + B) - (4x^2 + x)Cx^{-\frac{1}{2}}}{(2\sqrt{x})^2} (A, B, C > 0)$

But a correct rule may be implied by their u, v, u', v' followed by applying $\frac{vu' - uv'}{v^2}$ etc.

Alternatively uses the product rule on $(4x^2 + x)(2\sqrt{x})^{-1}$

Condone slips but expect $\left(\frac{dy}{dx}\right) = Ax^{-\frac{1}{2}}(Bx + C) + D(4x^2 + x)x^{-\frac{3}{2}} (A, B, C > 0)$

In general condone missing brackets for the M mark. If they quote $u = 4x^2 + x$ and $v = 2\sqrt{x}$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: Correct differentiation of $\frac{4x^2 + x}{2\sqrt{x}}$ although may not be simplified.



Question	Scheme	Marks	AOs	
11 (a) Way 1	$\{y = x^x \Rightarrow\} \ln y = x \ln x$	B1	1.1a	
	$\frac{1}{y} \frac{dy}{dx} = 1 + \ln x$	M1	1.1b	
		A1	2.1	
	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \frac{x}{x} + \ln x = 0 \text{ or } 1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \dots$	M1	1.1b	
	$x = e^{-1} \text{ or awrt } 0.368$	A1	1.1b	
	Note: $k \neq 0$	(5)		
(a) Way 2	$\{y = x^x \Rightarrow\} y = e^{x \ln x}$	B1	1.1a	
	$\frac{dy}{dx} = \left(\frac{x}{x} + \ln x \right) e^{x \ln x}$	M1	1.1b	
		A1	2.1	
	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \frac{x}{x} + \ln x = 0 \text{ or } 1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \dots$	M1	1.1b	
	$x = e^{-1} \text{ or awrt } 0.368$	A1	1.1b	
	Note: $k \neq 0$	(5)		
(b) Way 1	Attempts both $1.5^{1.5} = 1.8\dots$ and $1.6^{1.6} = 2.1\dots$ and at least one result is correct to awrt 1 dp	M1	1.1b	
	$1.8\dots < 2$ and $2.1\dots > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1	
		(2)		
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$	M1	1.1b	
	Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63			
	$\{x_4 = 1.67313\dots \Rightarrow\} x_4 = 1.673$ (3 dp) cao	A1	1.1b	
		(2)		
(d)	Give 1 st B1 for any of - oscillates - periodic - non-convergent - divergent - fluctuates - goes up and down 1, 2, 1, 2, 1, 2 - alternates (condone)	Give B1 B1 for any of - periodic {sequence} with period 2 - oscillates between 1 and 2	B1	2.5
		Condone B1 B1 for any of - fluctuates between 1 and 2 - keep getting 1, 2 - alternates between 1 and 2 - goes up and down between 1 and 2 1, 2, 1, 2, 1, 2, ...	B1	2.5
			(2)	
(11 marks)				
Note	A common solution			
	A maximum of 3 marks (i.e. B1 1 st M1 and 2 nd M1) can be given for the solution			
	$\log y = x \log x \Rightarrow \frac{1}{y} \frac{dy}{dx} = 1 + \log x$			
	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} 1 + \log x = 0 \Rightarrow x = 10^{-1}$			
	1st B1 for $\log y = x \log x$ 1st M1 for $\log y \rightarrow \lambda \frac{1}{y} \frac{dy}{dx}; \lambda \neq 0 \text{ or } x \log x \rightarrow 1 + \log x \text{ or } \frac{x}{x} + \log x$ 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = \dots; k \neq 0$			

Question	Scheme	Marks	AOs
11 (b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 3	For $\ln y = x \ln x$, attempts both $1.5 \ln 1.5 = 0.608...$ and $1.6 \ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 4	For $\log y = x \log x$, attempts both $1.5 \log 1.5 = 0.264...$ and $1.6 \log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
Notes for Question 11			
(a)	Way 1		
B1:	$\ln y = x \ln x$. Condone $\log_x y = x \log_x x$ or $\log_x y = x$		
M1:	For either $\ln y \rightarrow \frac{1}{y} \frac{dy}{dx}$ or $x \ln x \rightarrow 1 + \ln x$ or $\frac{x}{x} + \ln x$		
A1:	Correct differentiated equation. i.e. $\frac{1}{y} \frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y} \frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1 + \ln x)$ or $\frac{dy}{dx} = x^x(1 + \ln x)$		
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$		
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)		
Note:	Give no marks for no working leading to 0.368		
Note:	Give M0 A0 M0 A0 for $\ln y = x \ln x \rightarrow x = 0.368$ with no intermediate working		
(a)	Way 2		
B1:	$y = e^{x \ln x}$		
M1:	For either $y = e^{x \ln x} \Rightarrow \frac{dy}{dx} = f(\ln x) e^{x \ln x}$ or $x \ln x \rightarrow 1 + \ln x$ or $\frac{x}{x} + \ln x$		
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right) e^{x \ln x}$ or $\frac{dy}{dx} = (1 + \ln x) e^{x \ln x}$ or $\frac{dy}{dx} = x^x(1 + \ln x)$		
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$		
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)		
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow \ln y = x \ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = e^{-1} \text{ or awrt 0.368}$		



Notes for Question 11 Continued	
(b)	Way 1
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} = \text{awrt } 1.8...$ and $1.6^{1.6} = \text{awrt } 2.1...$, reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	Way 2
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	Way 3
M1:	Attempts both $1.5 \ln 1.5 = 0.608...$ and $1.6 \ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5 \ln 1.5 = 0.608...$ and $1.6 \ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	Way 4
M1:	Attempts both $1.5 \log 1.5 = 0.264...$ and $1.6 \log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5 \log 1.5 = 0.264...$ and $1.6 \log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ cao (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 = \text{awrt } 1.673$
Note:	$x_2 = 1.63299...$, $x_3 = 1.46626...$, $x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for “Converges in a cob-web pattern” or “Converges up and down to α ”

Question	Scheme	Marks	AOs
15(a)	$\dots xe^x + \dots e^x$	M1	1.1b
	$k(xe^x + e^x)$	A1	1.1b
	$\frac{d}{dx}(\sqrt{e^{3x}-2}) = \frac{1}{2} \times 3e^{3x}(e^{3x}-2)^{-\frac{1}{2}}$	B1	1.1b
	$(f'(x) =) \frac{(e^{3x}-2)^{\frac{1}{2}}(7xe^x + 7e^x) - \frac{3}{2}e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times 7xe^x}{e^{3x}-2}$	dM1	2.1
	$f'(x) = \frac{7e^x(e^{3x}(2-x) - 4x - 4)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
		(5)	
(b)	$e^{3x}(2-x) - 4x - 4 = 0 \Rightarrow x(\dots e^{3x} \pm \dots) = \dots e^{3x} \pm \dots$	M1	1.1b
	$\Rightarrow x = \frac{2e^{3x}-4}{e^{3x}+4} *$	A1*	2.1
		(2)	
(c)	Draws a vertical line $x=1$ up to the curve then across to the line $y=x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
		(1)	
(d)(i)	$x_2 = \frac{2e^3-4}{e^3+4} = 1.5017756\dots$	M1	1.1b
	$x_2 = \text{awrt } 1.502$	A1	1.1b
(ii)	$\beta = 1.968$	dB1	2.2b
		(3)	
(e)	$h(x) = \frac{2e^{3x}-4}{e^{3x}+4} - x$ $h(0.4315) = -0.000297\dots \quad h(0.4325) = 0.000947\dots$	M1	3.1a
	Both calculations correct and e.g. states: - There is a change of sign - e.g $f'(x)$ is continuous $\alpha = 0.432$ (to 3dp)	A1cao	2.4
		(2)	
(13 marks)			
Notes			
(a)			
M1:	Attempts the product rule on xe^x (or may be $7xe^x$) achieving an expression of the form $\dots xe^x \pm \dots e^x$. If it is clear that the quotient rule has been applied instead which may be quoted then M0.		
A1:	$k(xe^x + e^x)$ (e.g. $7(xe^x + e^x)$) or equivalent which may be unsimplified (may be implied by further work)		
B1:	$\left(\frac{d}{dx}(\sqrt{e^{3x}-2})\right) = \frac{1}{2} \times 3e^{3x}(e^{3x}-2)^{-\frac{1}{2}}$ (simplified or unsimplified)		

dM1: Attempts to use the quotient rule. It is dependent on the previous method mark.

Score for achieving an expression of the form

$$(f'(x) =) \frac{(e^{3x} - 2)^{\frac{1}{2}} ("7"xe^x + "7"e^x) - \frac{3}{2}e^{3x}(e^{3x} - 2)^{-\frac{1}{2}} \times "7"xe^x}{e^{3x} - 2} \text{ or equivalent (do not be}$$

concerned by the constants for their “7” or their “ $\frac{3}{2}$ ” which may be both 1)

If it is clear that the quotient rule has been applied the wrong way round then score M0.

Alternatively, applies the product rule. Score for achieving an expression of the form

$$(f'(x) =) (e^{3x} - 2)^{\frac{1}{2}} ("7"xe^x + "7"e^x) - \frac{3}{2}e^{3x}(e^{3x} - 2)^{\frac{3}{2}} \times "7"xe^x \text{ or equivalent (do not be}$$

concerned by the constants for their “7” or their “ $\frac{3}{2}$ ” which may be both 1)

Do not condone invisible brackets.

A1: $(f'(x) =) \frac{7e^x(e^{3x}(2-x) - 4x - 4)}{2(e^{3x} - 2)^{\frac{3}{2}}}$ following a fully correct differentiated expression.

You may need to check to see if (a) is continued after other parts for evidence of this.

Condone the lack of $f'(x) =$ on the left hand side or allow the use of $\frac{dy}{dx}$ or y' instead.

Alternative (a) attempt using the triple product rule

$$\begin{aligned} \text{e.g. } \frac{d}{dx} \left(7xe^x (e^{3x} - 2)^{-\frac{1}{2}} \right) &= 7e^x (e^{3x} - 2)^{-\frac{1}{2}} + 7xe^x (e^{3x} - 2)^{-\frac{1}{2}} + 7xe^x \times \left(-\frac{1}{2} \right) \times 3e^{3x} (e^{3x} - 2)^{-\frac{3}{2}} \\ &\Rightarrow \frac{(7e^x + 7xe^x)(e^{3x} - 2) + 7xe^x \times \left(-\frac{1}{2} \right) \times 3e^{3x}}{(e^{3x} - 2)^{\frac{3}{2}}} = \frac{7e^x \left(e^{3x} - 2 + xe^{3x} - 2x - \frac{3}{2}xe^{3x} \right)}{(e^{3x} - 2)^{\frac{3}{2}}} \Rightarrow \frac{7e^x (e^{3x}(2-x) - 4x - 4)}{2(e^{3x} - 2)^{\frac{3}{2}}} \end{aligned}$$

M1: Attempts the product rule on $xe^x \rightarrow \dots xe^x \pm \dots e^x$ which may be seen within the expression $\dots e^x (e^{3x} - 2)^{-\frac{1}{2}} \pm \dots xe^x (e^{3x} - 2)^{-\frac{1}{2}} + \dots$ simplified or unsimplified.

A1: $k(xe^x + e^x)$ which may be seen within the expression $k \left(e^x (e^{3x} - 2)^{-\frac{1}{2}} + xe^x (e^{3x} - 2)^{-\frac{1}{2}} \right) + \dots$ simplified or unsimplified.

B1: $\left(-\frac{1}{2} \right) \times 3e^{3x} (e^{3x} - 2)^{-\frac{3}{2}}$ which may be seen within the expression $\dots + k \left(xe^x \times \left(-\frac{1}{2} \right) \times 3e^{3x} (e^{3x} - 2)^{-\frac{3}{2}} \right)$ simplified or unsimplified.

dM1: A complete method using all three products (which may appear all on one line). Do not condone invisible brackets.

A1: As above in main scheme notes.

(b) Note that if they do not have values $A = -4$, $B = -4$ in (a) (which may be seen later) then maximum score is M1A0*

M1: Sets their $e^{3x}(2-x) - 4x - 4$ equal to zero, collects terms in x on one side of the equation and non x terms on the other and attempts to factorise the side with x as a common factor. Condone sign slips only for this mark. Allow A and B to be used instead of “-4” and “-4”

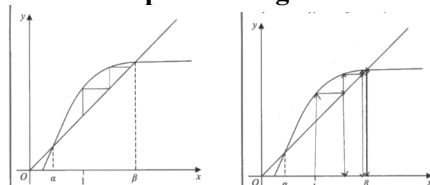


A1*: Achieves the given answer with no errors including invisible brackets. If they do not reach the printed answer then it is A0. If they subsequently write $x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$ then isw

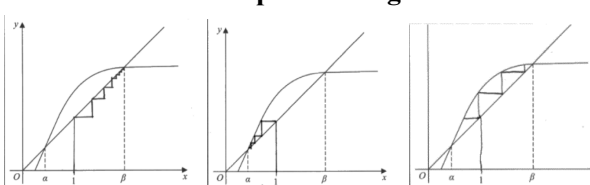
(c)

B1: Starting at $x_1 = 1$ look for at least 2 sets of vertical and horizontal lines drawn (may be dashes) tending to β . Condone a lack of arrows on the lines but the sequence of lines should finish at the point of intersection where the root is. Condone the initial vertical line not starting from the x -axis. Mark the intention to draw horizontal and vertical lines. If they have any lines to the left of $x = 1$ this is B0. If they use both diagrams and do not indicate which one they want marking, then the “copy of Diagram 1” should be marked.

Examples scoring B1:



Examples scoring B0:



(d)(i)

M1: Substitutes 1 into the iterative formula. The values embedded in the formula is sufficient for this mark. May be implied by awrt 1.50

A1: awrt 1.502 isw

(d)(ii)

dB1: 1.968 cao (which can only be scored if M1 is scored in (d)(i))

SC: If (d)(i) is rounded to 1.50 then allow 1.97 in (d)(ii) to score M1A0dB1 for (d)

(e)

M1: Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct (rounded or truncated to 1sf). It is allowable to use a tighter interval that contains the root $0.4317388728...$

If no function is stated then may be implied by their answers to e.g. $f'(0.4315)$, $f'(0.4325)$

You will need to check their calculation is correct.

Other possible functions include:

- $h(x) = x - \frac{2e^{3x} - 4}{e^{3x} + 4}$ (other way round to MS) $h(0.4315) = 0.0002974...$, $h(0.4325) = -0.0009479...$

- their $f'(x) = \pm \left(\frac{7e^x (e^{3x} (2-x) - 4x - 4)}{2(e^{3x} - 2)^{\frac{3}{2}}} \right)$

(If correct A and B then $f'(0.4315) = \mp 0.005789...$, $f'(0.4325) = \pm 0.01831...$)

- their $g(x) = \pm (e^{3x} (2-x) - 4x - 4)$

(If correct A and B then $g(0.4315) = \mp 0.002275...$, $g(0.4325) = \pm 0.007261...$)

A1: Requires

- Both calculations correct (rounded or truncated to 1sf)
- A statement that there is a change in sign and that their **function** is continuous (must refer to the function used for the substitution (which is not $f(x)$))
Accept equivalent statements for $f'(0.4315) < 0$, $f'(0.4325) > 0$ e.g.
 $f'(0.4315) \times f'(0.4325) < 0$, “one negative one positive”. A minimum is “change of sign and continuous” but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. “because x is continuous” or “because the interval is continuous”
- A minimal conclusion e.g. “hence $\alpha = 0.432$ ”, “so rounds to 0.432”. Do not allow “hence root”

Question	Scheme	Marks	AOs
9 (a)	$0 = 15T - T e^{0.2T} \Rightarrow e^{0.2T} = 15$	M1	3.4
	$(T =)$ awrt 13.5	A1	1.1b
		(2)	
(b)	Attempts to differentiate using the product rule $\left(\frac{d}{dt}(t e^{0.2t})\right) = 0.2t e^{0.2t} + e^{0.2t}$	M1	1.1b
	$v = 15t - t e^{0.2t} \Rightarrow \left(\frac{dv}{dt} =\right) 15 - (0.2t e^{0.2t} + e^{0.2t})$	A1	1.1b
	Sets their $\frac{dv}{dt} = 0 \Rightarrow e^{0.2t} (0.2t + 1) = 15 \Rightarrow e^{0.2t} = \frac{15}{0.2t + 1}$	dM1	3.1b
	$\Rightarrow t = 5 \ln\left(\frac{75}{t+5}\right) *$	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2 = 5 \ln\left(\frac{75}{8+5}\right)$	M1	1.1b
	awrt 8.478	A1	1.1b
	(ii) awrt 8.55 seconds (including units)	A1	3.2a
		(3)	
(9 marks)			
Notes:			
(a) May use t or another variable which is acceptable. M1: Uses the model with $v = 0$ and proceeds to $e^{0.2T} = 15$ Do not be concerned by the use of an inequality sign instead of an equals. May be implied by awrt 13.5 A1: $(T =)$ awrt 13.5 units are not required but if given they must be seconds (or e.g. secs or s)			

(b) If no attempt is seen for (b) then allow differentiation seen in (a) to score in (b)

M1: Attempts to use the product rule to differentiate $t e^{0.2t}$ achieving the form $Ae^{0.2t} + Bte^{0.2t}$ (A and B both non zero but may be 1) which may be unsimplified. It is likely to be part of an expression.

A1: $\left(\frac{dv}{dt}\right) = 15 - (0.2t e^{0.2t} + e^{0.2t})$ o.e. which may be unsimplified. (Condone a missing trailing bracket)

Do not allow recovery of signs to score this mark if they initially write

e.g. $15 - 0.2t e^{0.2t} + e^{0.2t} = 0$ and on a later line correct this. e.g. $15 - 0.2t e^{0.2t} - e^{0.2t} = 0$

dM1: Sets $15 \pm Ae^{0.2t} \pm Bte^{0.2t} = 0$ (the $=0$ may be implied), attempts to make $e^{\pm 0.2t}$ (or $Ce^{\pm 0.2t}$) the subject **and** proceeds to the form

$$Ce^{0.2t} = \frac{D}{E + Ft} \text{ or } Ce^{-0.2t} = \frac{E + Ft}{D} \text{ (where } C \text{ can be 1 and } A, B, C, D, E, F \neq 0 \text{)}$$

It is dependent on the previous method mark.

May see $15 - "0.2t e^{0.2t} - e^{0.2t}" = 0 \Rightarrow e^{0.2t} = \frac{15}{0.2t + 1}$ which scores dM1

They must take out a factor of $e^{\pm 0.2t}$ (or $Ce^{\pm 0.2t}$) and divide by their bracket. Condone sign slips in their rearrangement, however, if they take logs of both sides first, the rearrangement must be correct (with no sign slips).

Allow invisible brackets to be implied by further work which is not the given answer.

A1*: Achieves the given answer with **no errors seen including use of invisible brackets** (but condone a missing trailing bracket) All previous marks in (b) must have been scored. The $= 0$ must have been seen somewhere in their solution. Do not allow this mark to be scored for proceeding directly from

$$e^{0.2t} (0.2t + 1) = 15 \Rightarrow t = 5 \ln \left(\frac{75}{t + 5} \right) \text{ which is A0*}$$

We must see either $e^{0.2t} = \frac{15}{0.2t + 1}$ o.e. or an unsimplified expression for t

e.g. $t = 5 \ln \left(\frac{15}{0.2t + 1} \right)$ before achieving the given answer. Condone $t = 5 \ln \frac{75}{t + 5}$

(c) (i) Check by the question. If there is a contradiction between answers, the answer in the main body of the script takes precedence.

M1: Attempts to use the iteration formula at least once. $t_2 = 5 \ln \left(\frac{75}{8 + 5} \right)$. May be implied by awrt 8.76 or awrt 8.48 or awrt 8.58. It is not implied by awrt 8.55

A1: awrt 8.478 (on its own can score M1A1)

(c)(ii) This mark can only be scored provided in (c)(i) M1 has been scored so M0A0A1 is not a possible mark profile.

A1: awrt 8.55 **seconds** (e.g. s or secs) **Requires units.**

If the candidate lists their iterations but does not select an answer then take the final value, which still requires units to be stated (which in many cases is likely to be omitted)

Note that awrt 8.55 does not imply M1.



Question	Scheme	Marks	AOs
5	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\{f(x) = 2x^3 + x^2 - 1 \Rightarrow f'(x) = 6x^2 + 2x\}$	B1	1.1b
	$\left\{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow \right\} \{x_{n+1}\} = x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$	M1	1.1b
	$= \frac{x_n(6x_n^2 + 2x_n) - (2x_n^3 + x_n^2 - 1)}{6x_n^2 + 2x_n} \Rightarrow x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n} *$	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow\} x_2 = \frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)} \text{ or } x_2 = 1 - \frac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	M1	1.1b
	$\Rightarrow x_2 = \frac{3}{4}, x_3 = \frac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson method cannot be used with $x_1 = 0$ which refer or allude to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the x-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	B1	2.3
		(1)	
(6 marks)			
Notes for Question 5			
(a)			
B1:	States that $f'(x) = 6x^2 + 2x$ or states that $f'(x_n) = 6x_n^2 + 2x_n$ (Condone $\frac{dy}{dx} = 6x^2 + 2x$)		
M1:	Substitutes $f(x_n) = 2x_n^3 + x_n^2 - 1$ and their $f'(x_n)$ into $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$		
A1*:	A correct intermediate step of making a common denominator which leads to the given answer		
Note:	Allow B1 if $f'(x) = 6x^2 + 2x$ is applied as $f'(x_n)$ (or $f'(x)$) in the NR formula $\{x_{n+1}\} = x_n - \frac{f(x_n)}{f'(x_n)}$		
Note:	Allow M1A1 for • $x_{n+1} = x - \frac{2x^3 + x^2 - 1}{6x^2 + 2x} = \frac{x(6x^2 + 2x) - (2x^3 + x^2 - 1)}{6x^2 + 2x} \Rightarrow x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$		
Note	Condone $x = x - \frac{2x^3 + x^2 - 1}{6x^2 + 2x}$ for M1		
Note	Condone $x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$ or $x - \frac{2x^3 + x^2 - 1}{6x^2 + 2x}$ (i.e. no $x_{n+1} = \dots$) for M1		
Note:	Give M0 for $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ followed by $x_{n+1} = 2x_n^3 + x_n^2 - 1 - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$		
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*		

Notes for Question 5 Continued	
(b)	
M1:	An attempt to use the given or their formula once. Can be implied by $\frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or 0.75 o.e.
Note:	Allow one slip in substituting $x_1 = 1$
A1:	$x_2 = \frac{3}{4}$ and $x_3 = \frac{2}{3}$
Note:	Condone $x_2 = \frac{3}{4}$ and $x_3 = \text{awrt } 0.667$ for A1
Note:	Condone $\frac{3}{4}, \frac{2}{3}$ listed in a correct order ignoring subscripts
(c)	
B1:	See scheme
Note:	Give B0 for the following isolated reasons: e.g. • You cannot divide by 0 • The fraction (or the NR formula) is undefined at $x = 0$ • At $x = 0$, $f'(x_1) = 0$ • x_1 cannot be 0 • $6x^2 + 2x$ cannot be 0 • the denominator is 0 which cannot happen • if $x_1 = 0$, $6x^2 + 2x = 0$

Question	Scheme	Marks	AOs
6(a)	$(f'(x) =) 4 \cos\left(\frac{1}{2}x\right) - 3$	M1 A1	1.1b 1.1b
	Sets $f'(x) = 4 \cos\left(\frac{1}{2}x\right) - 3 = 0 \Rightarrow x =$	dM1	3.1a
	$x = 14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $f(4) > 0$, $f(5) < 0$ and the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1 = 5 - \frac{8 \sin 2.5 - 15 + 9}{"4 \cos 2.5 - 3"}$ (NB $f(5) = -1.212\dots$ and $f'(5) = -6.204\dots$)	M1	1.1b
	$x_1 =$ awrt 4.80	A1	1.1b
		(2)	
(7 marks)			
Notes:			

(a)

M1: Differentiates to obtain $k \cos\left(\frac{1}{2}x\right) \pm \alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

A1: Correct derivative $f'(x) = 4 \cos\left(\frac{1}{2}x\right) - 3$. Allow unsimplified e.g. $f'(x) = \frac{1}{2} \times 8 \cos\left(\frac{1}{2}x\right) - 3x^0$

There is no need for $f'(x) = \dots$ or $\frac{dy}{dx} = \dots$ just look for the expression and the brackets are not required.

dM1: For the complete strategy of proceeding to a value for x .

Look for

- $f'(x) = a \cos\left(\frac{1}{2}x\right) + b = 0$, $a, b \neq 0$
- Correct method of finding a valid solution to $a \cos\left(\frac{1}{2}x\right) + b = 0$

Allow for $a \cos\left(\frac{1}{2}x\right) + b = 0 \Rightarrow \cos\left(\frac{1}{2}x\right) = \pm k \Rightarrow x = 2 \cos^{-1}(\pm k)$ where $|k| < 1$

If this working is not shown then you may need to check their value(s).

For example $4 \cos\left(\frac{1}{2}x\right) - 3 = 0 \Rightarrow x = 1.4\dots$ or $11.1\dots$ (or $82.8\dots$ or $637\dots$ or 803 in degrees) would indicate this method.

A1: Selects the correct turning point $x = 14.0$ and not just 14 or unrounded e.g. $14.011\dots$

Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the y coordinate.

Correct answer with no working scores no marks.

(b)

B1: See scheme. Must be a full reason, (e.g. change of sign and continuous)

Accept equivalent statements for $f(4) > 0$, $f(5) < 0$ e.g. $f(4) \times f(5) < 0$, "there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is



(c)

M1: Attempts $x_1 = 5 - \frac{f(5)}{f'(5)}$ to obtain a value following through on their $f'(x)$ as long as it is a “changed” function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees $f(5) = -5.65\dots$ and $f'(5) = 0.996\dots$ and gives $x_1 = 10.67\dots$

There must be clear evidence that $5 - \frac{f(5)}{f'(5)}$ is being attempted.

$$\text{so e.g. } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = 4.80 \text{ scores M0 as does e.g. } x_1 = x - \frac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$$

BUT evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is 4.804624337 so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

A1: $x_1 = \text{awrt } 4.80$ not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$$5 - \frac{f(5)}{f'(5)} = \text{awrt } 4.80 \text{ following a correct derivative scores M1A1}$$

$$5 - \frac{f(5)}{f'(5)} \neq \text{awrt } 4.80 \text{ with no evidence that } 5 - \frac{f(5)}{f'(5)} \text{ was attempted scores M0}$$



Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2} \{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: The integration on a calculator gives 1.511549071 The full accuracy for y values gives 1.504726147 The accuracy from the table gives 1.50475		
		(3)	
(b)	$3 \times \text{their (a)}$ If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working. Only allow 4.5 if (a) is correct and working is shown e.g. 3×1.5 If (a) is incorrect allow $3 \times \text{their (a)}$ given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))	B1ft	2.2a
	For reference the integration on a calculator gives 4.534647213		
		(1)	
(c)	<u>This mark depends on the B1 having been awarded in part (b) with awrt 4.5</u> Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as $4.535 \approx 4.50$ • Very accurate as 4.535 to 2 sf is 4.5 • $4.51425 < 4.535$ so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • $4.535 - 4.5 = 0.035$ so not far out But not just “it is an underestimate” or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left \frac{4.535 - 4.50}{4.535} \right \times 100 = 0.77\% \quad \text{or} \quad \left \frac{4.535 - 4.51}{4.535} \right \times 100 = 0.55\% \quad \text{or}$ $\left \frac{4.535 - 4.51425}{4.535} \right \times 100 = 0.46\% \quad \text{or} \quad \left \frac{4.50}{4.535} \right \times 100 = 99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. This mark should be withheld if there are any contradictory statements	B1	3.2b
		(1)	
(5 marks)			

Notes:

(a)



B1: States or uses $h = 0.5$. May be implied by $\frac{1}{4} \times \{ \dots \}$ below.

M1: Correct attempt at the trapezium rule.

Look for $\frac{1}{2}h \times (0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165))$ condoning slips on the terms but must use all y values



Question	Scheme	Marks	AOs
5(a)	$h = 0.2$	B1	1.1b
	$\frac{1}{2} \times 0.2 \times \{a + 13.5 + 2(16.8 + b + 20.2 + 18.7)\} = 17.59$	M1	1.1b
	e.g. $\Rightarrow a + 13.5 + 2b + 111.4 = 175.9 \Rightarrow a + 2b = 51^*$	A1*	2.1
		(3)	
(b)	$a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2 \Rightarrow a + b = 28 \Rightarrow a = \dots$ (or $b = \dots$)	M1	3.1a
	$a = 5$ or $b = 23$	A1	1.1b
	$a = 5$ and $b = 23$	A1	1.1b
		(3)	
(6 marks)			
Notes			
(a)			
B1: States or uses $h = 0.2$ o.e.			
M1: Forms the equation $\frac{1}{2} \times 0.2 \times \{a + 13.5 + 2(16.8 + b + 20.2 + 18.7)\} = 17.59$ o.e. but condone copying slips. They may have added some of the y values together so as a minimum accept e.g. $0.1 \times \{a + 13.5 + 2(55.7 + b)\} = 17.59$ Condone invisible brackets as long as they are recovered or implied in further work before achieving the given answer. Condone the use of $\approx$ for this mark. Allow this mark if they add the areas of individual trapezia e.g. $\frac{\text{their } h}{2} \{a + 16.8\} + \frac{\text{their } h}{2} \{16.8 + b\} + \frac{\text{their } h}{2} \{b + 20.2\} + \frac{\text{their } h}{2} \{20.2 + 18.7\} + \frac{\text{their } h}{2} \{18.7 + 13.5\}$ Condone copying slips but it must be a complete method using all the trapezia. h must be numerical but condone $h = 1$			
A1*: A rigorous argument leading to $a + 2b = 51$ from correct working and no errors seen including brackets, although do not penalise a missing trailing bracket at the end e.g. $\frac{1}{2} \times 0.2 \times \{a + 13.5 + 2(16.8 + b + 20.2 + 18.7)\} = 17.59 \Rightarrow \dots \Rightarrow a + 2b = 51$ could score B1M1A1but $\frac{1}{2} \times 0.2 \times a + 13.5 + 2(16.8 + b + 20.2 + 18.7) = 17.59 \Rightarrow \dots \Rightarrow a + 2b = 51$ could score max B1M1A0 provided later work implied correct brackets. Both sets of brackets must be dealt with correctly before proceeding to the final answer such that e.g. $\dots \Rightarrow a + 2b + 124.9 = 175.9 \Rightarrow a + 2b = 51$ is M1A1* $\dots \Rightarrow a + 13.5 + 33.6 + 2b + 40.4 + 37.4 = 175.9 \Rightarrow a + 2b = 51$ is M1A1* $\dots \Rightarrow 0.1a + 1.35 + 3.36 + 0.2b + 4.04 + 3.74 = 17.59 \Rightarrow a + 2b = 51$ is M1A1* Note that $a + 2b \approx 51$ as the final answer is A0*			
(b)			
M1: Attempts to form the equation $a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2$, condoning copying errors, (may just be stated as e.g. $a + b = 28$ o.e.) and attempts to solve their equation simultaneously with the given equation (or condone their equation from part (a)). Do not be too concerned with the process here as calculators may be used. Score if values for a or b are reached from a pair of simultaneous equations.			
A1: for $a = 5$ or $b = 23$			
A1: for both $a = 5$ and $b = 23$			

Question	Scheme	Marks	AOs
5(a)	States or uses $h = 1.5$	B1	1.1a
	Full attempt at the trapezium rule $= \frac{h}{2} \{1.63 + 2.63 + 2 \times (2 + 2.26 + 2.46)\}$	M1	1.1b
	= awrt 13.3 or $\frac{531}{40}$	A1	1.1b
		(3)	
(b)(i)	$\int_3^9 \log_3(2x)^{10} dx = 10 \times "13.3" = \text{awrt } 133 \text{ or e.g. } \frac{531}{4}$	B1ft	2.2a
(ii)	$\int_3^9 \log_3 18x dx = \int_3^9 \log_3(9 \times 2x) dx = \int_3^9 2 + \log_3 2x dx$ $= [2x]_3^9 + \int_3^9 \log_3 2x dx = 18 - 6 + \int_3^9 \log_3 2x dx = \dots$	M1	3.1a
	Awrt 25.3 or $\frac{1011}{40}$	A1ft	1.1b
		(3)	
(6 marks)			
Notes:			

(a)

B1: States or uses $h = 1.5$

M1: A full attempt at the trapezium rule.

Look for $\frac{\text{their } h}{2} \{1.63 + 2.63 + 2 \times (2 + 2.26 + 2.46)\}$ but condone copying slips

Note that $\frac{\text{their } h}{2} \{1.63 + 2.63 + 2 \times (2 + 2.26 + 2.46)\}$ scores M0 unless the missing brackets are recovered or implied by their answer. You may need to check.

Allow this mark if they add the areas of individual trapezia e.g.

$$\frac{\text{their } h}{2} \{1.63 + 2\} + \frac{\text{their } h}{2} \{2 + 2.26\} + \frac{\text{their } h}{2} \{2.26 + 2.46\} + \frac{\text{their } h}{2} \{2.46 + 2.63\}$$

Condone copying slips but must be a complete method using all the trapezia.

A1: awrt 13.3 (Note full accuracy is 13.275) or exact equivalent.

Note that the calculator answer is 13.324 so you must see correct working to award awrt 13.3

Use of $h = -1.5$ leading to a negative area can score B1M1A0 but allow full marks if then stated as positive.

(b)(i)

B1ft: Deduces that $\int_3^9 \log_3(2x)^{10} dx = 10 \times "13.3" = \text{awrt } 133$

FT on their 13.3 look for 3sf accuracy but follow through on e.g. their rounded answer to part (a) so if 13 was their answer to part (a) then allow 130 here **following a correct method**.

A correct method must be seen here but a minimum is e.g. $10 \times "13.3" = "133"$

Note that $\int_3^9 \log_3(2x)^{10} dx = 133.2414316\dots$ so a correct method must be seen to award marks.

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the answer to part (a).



(b)(ii)

M1: Shows correct log work to relate the given question to part (a)

Must reach as far as e.g. $[2x]_3^9 + \int_3^9 \log_3 2x \, dx = \dots$ with correct use of limits on $[2x]_3^9$ which

may be implied or equivalent work e.g. finds the area of the rectangle as 2×6

A1ft: **Correct working** followed by awrt 25.3 but ft on their 13.3 so allow for $12 +$ their answer to part (a) **following correct work** as shown.

Note that $\int_3^9 \log_3 18x \, dx = 25.32414\dots$ so a correct method must be seen to award marks.

Some examples of an acceptable method are:

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9 \times 2x) \, dx = \int_3^9 2 + \log_3 2x \, dx = 6 \times 2 + "13.3" = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9 \times 2x) \, dx = \int_3^9 2 + \log_3 2x \, dx = 12 + "13.3" = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9 \times 2x) \, dx = \int_3^9 2 + \log_3 2x \, dx = [2x]_3^9 + \int_3^9 \log_3 2x \, dx = 25.3$$

BUT just $12 + "13.3" = 25.3$ scores M0

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the answer to part (a).



Question	Scheme	Marks	AOs
11(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2} \times \frac{1}{2} \{0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int (\ln x)^2 \, dx = x(\ln x)^2 - \int x \times \frac{2 \ln x}{x} \, dx$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2 \int \ln x \, dx = x(\ln x)^2 - 2 \left(x \ln x - \int dx \right)$ $= x(\ln x)^2 - 2 \int \ln x \, dx = x(\ln x)^2 - 2x \ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2 \, dx = \left[x(\ln x)^2 - 2x \ln x + 2x \right]_2^4$ $= 4(\ln 4)^2 - 2 \times 4 \ln 4 + 2 \times 4 - \left(2(\ln 2)^2 - 2 \times 2 \ln 2 + 2 \times 2 \right)$ $= 4(2 \ln 2)^2 - 16 \ln 2 + 8 - 2(\ln 2)^2 + 4 \ln 2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12 \ln 2 + 4$	A1	1.1b
		(5)	
(8 marks)			
Notes			

(a)

B1: Correct strip width. May be implied by $\frac{1}{2} \times \frac{1}{2} \{ \dots \}$ or $\frac{1}{4} \times \{ \dots \}$

M1: Correct application of the trapezium rule.

Look for $\frac{1}{2} \times "h" \{0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)\}$ condoning slips in the digits.

The bracketing must be correct but it is implied by awrt 2.41

A1: 2.41 only. This is not awrt

(b)

M1: Attempts parts the correct way round to achieve $\alpha x(\ln x)^2 - \beta \int \ln x dx$ o.e.

May be unsimplified (see scheme). Watch for candidates who know or learn $\int \ln x dx = x \ln x - x$

who may write $\int (\ln x)^2 dx = \int (\ln x)(\ln x) dx = \ln x(x \ln x - x) - \int \frac{x \ln x - x}{x} dx$

A1: Correct expression which may be unsimplified

dM1: Attempts parts again to (only condone coefficient errors) to achieve $\alpha x(\ln x)^2 - \beta x \ln x \pm \gamma x$ o.e.

ddM1: Applies the limits 4 and 2 to an expression of the form $\pm \alpha x(\ln x)^2 \pm \beta x \ln x \pm \gamma x$, subtracts and applies $\ln 4 = 2 \ln 2$ at least once. Both M's must have been awarded

A1: Correct answer

.....
It is possible to do $\int (\ln x)^2 dx$ via a substitution $u = \ln x$ but it is very similar.

M1 A1, dM1: $\int u^2 e^u du = u^2 e^u - \int 2u e^u du = u^2 e^u - 2ue^u \pm 2e^u$

ddM1: Applies appropriate limits and uses $\ln 4 = 2 \ln 2$ at least once to an expression of the form $u^2 e^u - \beta u e^u \pm \gamma e^u$ Both M's must have been awarded

.....

Question	Scheme	Marks	AOs
8 (a)	25	B1	3.4
		(1)	
(b)	Attempts to differentiate using the product rule $\frac{dv}{dt} = \ln(t+1) \times -0.4 + \frac{(10-0.4t)}{t+1}$	M1 A1	3.1b 1.1b
	Sets their $\frac{dv}{dt} = 0 \Rightarrow \frac{(10-0.4t)}{(t+1)} = 0.4 \ln(t+1)$ and then makes progress towards making "t" the subject (See notes for this)	dM1	1.1b
	$t = \frac{25 - \ln(t+1)}{1 + \ln(t+1)}$ $t = \frac{26}{1 + \ln(t+1)} - 1 \quad *$	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2 = \frac{26}{1 + \ln 8} - 1$	M1	1.1b
	awrt 7.298	A1	1.1b
	(ii) awrt 7.33 seconds	A1	3.2a
		(3)	
(8 marks)			
Notes:			

(a)

B1: 25 but condone 25 seconds. If another value is given (apart from 0) it is B0

(b)

M1: Attempts to use the product rule in an attempt to differentiate $v = (10 - 0.4t) \ln(t+1)$ Look for $(10 - 0.4t) \times \frac{1}{(t+1)} \pm k \ln(t+1)$, where k is a constant, condoning slips.If you see direct evidence of an incorrect rule used e.g. $vu' - uv'$ it is M0You will see attempts from $v = 10 \ln(t+1) - 0.4t \ln(t+1)$ which can be similarly marked.In this case look for $\frac{a}{t+1} \pm \frac{bt}{t+1} \pm c \ln(t+1)$ A1: Correct differentiation. Condone a missing left hand or it seen as v' , $\frac{dy}{dx}$ or even $= 0$

$$\left(\frac{dv}{dt}\right) = \ln(t+1) \times -0.4 + \frac{(10-0.4t)}{t+1} \text{ or equivalent such as } \left(\frac{dv}{dt}\right) = \frac{10}{t+1} - \frac{0.4t}{(t+1)} - 0.4 \ln(t+1)$$



dM1: Score for setting their $dV/dt = 0$ (which must be in an appropriate form) and proceeding to an equation where the variable t occurs only once – ignoring $\ln(t+1)$.

See two examples of how this can be achieved below. It is dependent upon the previous M.

Look for the following steps

- An allowable derivative set (or implied) = 0 E.g. $\ln(t+1) \times 0.4 = \frac{(10-0.4t)}{t+1}$
- Cross multiplication (or division) and rearrangement to form an equation where the variable t only occurs once.

E.g.1.

$$\begin{aligned}\ln(t+1) \times 0.4 &= \frac{(10-0.4t)}{t+1} \\ \Rightarrow \ln(t+1) &= \frac{25-t}{t+1} \\ \Rightarrow \ln(t+1) &= -1 + \frac{26}{t+1}\end{aligned}$$

E.g.2

$$\begin{aligned}\ln(t+1) \times 0.4 &= \frac{(10-0.4t)}{t+1} \\ \Rightarrow 0.4t \ln(t+1) + 0.4 \ln(t+1) &= 10 - 0.4t \\ \Rightarrow 0.4t(1 + \ln(t+1)) &= 10 - 0.4 \ln(t+1)\end{aligned}$$

A1*: Correctly proceeds to the given answer of $t = \frac{26}{1 + \ln(t+1)} - 1$ showing all key steps.

The key steps must include

- use of $\frac{dv}{dt}$ or v' which must be correct
- a correct line preceding the given answer, usually $t = \frac{25 - \ln(t+1)}{1 + \ln(t+1)}$ or $\frac{26}{t+1} - 1 = \ln(t+1)$

(c) (i)

M1: Attempts to use the iteration formula at least once.

Usually to find $t_2 = \frac{26}{1 + \ln 8} - 1$ which may be implied by awrt 7.44

A1: awrt 7.298. This alone will score both marks as iteration is implied. ISW after sight of this value. As t_3 is the only value that rounds to 7.298 just score the rhs, it does not need to be labelled t_3

(c) (ii)

A1: Uses repeated iteration until value established as awrt 7.33 **seconds**. Allow awrt 7.33 s

Requires units. It also requires some evidence of iteration which will be usually be awarded from the award of the M

