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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Algorithms (D1.1)

International A Level Mathematics · D1: Decision Mathematics 1 (WDM11)

19 questions · 184 marks · Mark schemes included

Past papers June 2019 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Where a question is answered in a separate answer book, those answer book pages follow the question.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

19 questions · 184 marks · 2 sub-topics

SUB-TOPIC D1.1a

SORTING, SEARCHING & BIN PACKING

Sub-topic D1.1a · 16 questions · 167 marks · lowest tariff first



Write your answers in the D1 answer book for this paper.

1. Use the binary search algorithm to try to locate the word “Parallelogram” in the following alphabetical list. Clearly indicate how you choose your pivots and which part of the list you are rejecting at each stage.

Arc

Centre

Chord

Circle

Circumference

Diameter

Radius

Sector

Segment

Tangent

(Total 4 marks)



(Total 4 marks)

Write your answers in the D1 answer book for this paper.

1. The list of numbers shown below represents the masses, in kg, of nine items that are to be moved from one factory to another.

180 300 250 410 240 120 230 310 190

The items will all be moved, by van, at the same time.

Each van can carry at most 600 kg.

- (a) Calculate a lower bound for the number of vans required. (2)
- (b) Use the first-fit bin-packing algorithm to allocate the items to vans. (2)

(Total for Question 1 is 4 marks)



180 300 250 410 240 120 230 310 190

180 300 250 410 240 120 230 310 190

2. A list of eleven numbers is to be sorted into descending order.

After one pass, the quick sort algorithm produces the following list

17 33 14 25 23 28 21 13 9 6 10

- (a) State, with a reason, which number was used as a pivot for the first pass. (1)
- (b) Starting at the left-hand end of the above list, obtain the fully sorted list using a bubble sort. You need to write down only the list that results at the end of each pass. (3)
- (c) Apply the first-fit decreasing bin packing algorithm to the fully sorted list to pack the numbers into bins of size 85 (2)

(Total for Question 2 is 6 marks)





Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 6 marks)



Write your answers in the D1 answer book for this paper.

1. 175 135 210 105 100 150 60 20 70 125

The numbers in the list above represent the weights, in kilograms, of ten crates. The crates are to be transported in trucks that can each hold a maximum total crate weight of 300 kg.

- (a) Calculate a lower bound for the number of trucks that will be needed to transport the crates. (2)
- (b) Using the list provided, carry out a bubble sort to produce a list of the weights in descending order. You need only give the state of the list after each pass. (4)
- (c) Use the first-fit decreasing bin packing algorithm to allocate the crates to the trucks. (3)

(Total for Question 1 is 9 marks)



175 135 210 105 100 150 60 20 70 125

(Total for Question 1 is 9 marks)



6. The twelve numbers in the list below are to be packed into bins of size n , where n is a positive integer.

28 31 5 25 16 35 18 22 11 27 15 13

When the first-fit bin packing algorithm is applied to the list, the following allocation is obtained.

Bin 1: 28 31 5

Bin 2: 25 16 18 11

Bin 3: 35 22 15

Bin 4: 27 13

- (a) Based on the packing shown above, determine the possible values of n . You must give reasons for your answer.

(3)

- (b) The original list of twelve numbers is to be sorted into **ascending** order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

When the first-fit decreasing bin packing algorithm is applied to the list, the following allocation is obtained.

Bin 1: 35 31 5

Bin 2: 28 27 16

Bin 3: 25 22 18

Bin 4: 15 13 11

- (c) Determine the value of n . You must give a reason for your answer.

(2)

(Total for Question 6 is 9 marks)



28 31 5 25 16 35 18 22 11 27 15 13

28 31 5 25 16 35 18 22 11 27 15 13



Write your answers in the D1 answer book for this paper.

1. The following list of eleven numbers is to be packed into bins of size 55

26 19 31 10 14 30 33 18 3 28 5

- (a) Use the **first-fit** bin packing algorithm to pack the numbers into bins of size 55 (3)
- (b) Use **quick sort** to sort the list into descending order, showing the result after each pass. You must clearly identify the pivots used. (4)
- (c) Apply the **first-fit decreasing** bin packing algorithm to pack the numbers into bins of size 55 (2)

(Total for Question 1 is 9 marks)



1.

26 19 31 10 14 30 33 18 3 28 5

Lined area for writing the answer.

Question 1 continued

26 19 31 10 14 30 33 18 3 28 5

(Total for Question 1 is 9 marks)



3.

- (i) A list of twelve numbers is to be sorted into
- ascending**
- order.

28 18 50 27 48 8 25 12 56 31 33 42

- (a) Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

- (b) Show how the binary search algorithm is used to try to locate the number 40 in the list. You must clearly indicate your choice of pivots and which part of the list you are rejecting.

(3)

- (ii) A different list of nine distinct integers is to be sorted into
- descending**
- order.

23 20 x 32 15 8 19 6 22

The list is sorted using bubble sort. The first three passes are

23 20 32 x 15 19 8 22 6

23 32 20 x 19 15 22 8 6

32 23 20 19 x 22 15 8 6

Determine the range of possible values of x

(2)

(Total for Question 3 is 9 marks)



28 18 50 27 48 8 25 12 56 31 33 42



Question 3 continued

Lined area for writing the answer to Question 3.



Question 3 continued

23	20	x	32	15	8	19	6	22
23	20	32	x	15	19	8	22	6
23	32	20	x	19	15	22	8	6
32	23	20	19	x	22	15	8	6

(Total for Question 3 is 9 marks)



7. The numbers listed below are to be packed into bins of size n , where n is a positive integer.

14 20 23 17 15 22 19 25 13 28 32

A lower bound for the number of bins required is 4

- (a) Determine the range of possible values of n . You must make your method clear. (3)

- (b) Carry out a quick sort to produce a list of the numbers in descending order. You should show the result of each pass and identify your pivots clearly. (4)

When the first-fit bin packing algorithm is applied to the **original** list of numbers, the following allocation is achieved.

Bin 1: 14 20 23 15

Bin 2: 17 22 19 13

Bin 3: 25 28

Bin 4: 32

When the first-fit decreasing bin packing algorithm is applied to the sorted list of numbers, the following allocation is achieved.

Bin 1: 32 28

Bin 2: 25 23 22

Bin 3: 20 19 17 15

Bin 4: 14 13

- (c) Determine the value of n . You must explain your reasoning fully. (3)

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END



7.

14

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22

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25

13

28

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Question 7 continued

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Handwriting practice lines for Question 7 continued.

(Total 10 marks)

Q7

TOTAL FOR PAPER: 75 MARKS

END



Write your answers in the D1 answer book for this paper.

1.

17 9 15 8 20 13 28 4 12 5

The numbers in the list shown above are the weights, in kilograms, of ten boxes.
The boxes are to be transported in containers that will each hold a maximum weight of 40 kilograms.

- (a) Calculate a lower bound for the number of containers that will be needed to transport the boxes. You must show your working. (2)
- (b) Use the first-fit bin packing algorithm to allocate the boxes to the containers. (3)
- (c) Using the list provided, carry out a quick sort to produce a list of the weights in **ascending** order. You must make your pivots clear. (3)
- (d) Use the binary search algorithm to try to locate the weight of 9 in the sorted list. Clearly indicate how you choose your pivots and which part of the list is being rejected at each stage. (3)

(Total 11 marks)



17 9 15 8 20 13 28 4 12 5

Question 1 continued

Lined area for writing the answer to Question 1.



Question 1 continued

Lined area for writing the answer to Question 1.



(Total 11 marks)

Write your answers in the D1 answer book for this paper.

1.

16 23 18 9 4 20 35 5 17 13 6 11

The numbers in the list represent the weights, in kilograms, of twelve parcels. The parcels are to be transported in containers that will each hold a maximum weight of 45 kg.

- (a) Calculate a lower bound for the number of containers needed. You must make your method clear. (2)
- (b) Use the first-fit bin packing algorithm to allocate the parcels to the containers. (3)
- (c) Carry out a bubble sort, starting at the left-hand end of the list, to produce a list of the weights in descending order. You should only give the state of the list after each pass. (4)
- (d) Use the first-fit decreasing bin packing algorithm to allocate the parcels to the containers. (3)

(Total 12 marks)

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Question 1 continued

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Question 1 continued

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Handwriting practice area with 30 horizontal lines.

(Total 12 marks)

Q1
[] []

4.

35 17 10 7 28 23 41 15 20 29

- (a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 60 (3)
- (b) The list of numbers is to be sorted into descending order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly. (4)
- (c) Use the first-fit decreasing bin packing algorithm on your ordered list to pack the numbers into bins of size 60 (3)

The ten distinct numbers below are to be sorted into descending order.

20 24 17 26 8 15 x y 19 12

A bubble sort, starting at the left-hand end of the list, is to be used to obtain the sorted list.

After the second complete pass the list is

24 26 20 17 15 y 19 12 x 8

- (d) Find the constraints on the values of x and y . (3)

(Total 13 marks)

4.

35

17

10

7

28

23

41

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Question 4 continued

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Handwriting practice lines for the answer to Question 4.

(Total 13 marks)

Q4	
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2. (a) (i) Describe how to carry out the first pass of a bubble sort when it is used to sort a list of n numbers into ascending order.

(ii) Write down the circumstances under which a bubble sort stops.

(4)

A bubble sort, starting at the left-hand end of the list, is used to sort a list of ten numbers into ascending order. After a number of passes the list reads

0.9 1.2 1.5 0.5 1.4 1.1 0.7 1.7 2.2 3.2

- (b) Determine the maximum number of passes that could have taken place on this list. You must give a reason for your answer.

(2)

- (c) Complete the bubble sort to produce a list of the numbers in ascending order. You only need to give the state of the list after each complete pass.

(4)

- (d) Use the first-fit decreasing bin packing algorithm to determine how the ten numbers listed above can be packed into bins of size 4

(3)

(Total 13 marks)

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Question 2 continued

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Question 2 continued

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Question 2 continued

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(Total 13 marks)

Q2



3. 2.6 0.8 2.1 1.2 0.9 1.7 2.3 0.3 1.8 2.7

- (a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5

(3)

The list is to be sorted into **descending** order.

- (b) (i) Starting at the left-hand end of the above list, perform **two** passes through the list using a bubble sort. Write down the lists that result at the end of the first pass and the second pass.

- (ii) Write down, in the table in the answer book, the number of comparisons and the number of swaps performed during each of these two passes.

(4)

After a third pass using this bubble sort, the updated list is

2.6 2.1 1.7 2.3 1.2 1.8 2.7 0.9 0.8 0.3

- (c) Use a quick sort on this updated list to obtain the fully sorted list. You must make your pivots clear.

(3)

- (d) Apply the first-fit decreasing bin packing algorithm to the fully sorted list to pack the numbers into bins of size 5

(3)

(Total 13 marks)



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Question 3 continued

	Number of comparisons	Number of swaps
First pass		
Second pass		

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Question 3 continued

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Q3

(Total 13 marks)



3.

1.8 1.4 2.6 1.6 2.8 0.9 3.1 0.8 1.2 2.4 0.6

- (a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5

(3)

The list is to be sorted into descending order.

- (b) (i) Perform one pass of a bubble sort, starting at the left-hand end of the list. You must write down the list that results at the end of the first pass.
- (ii) Write down the number of comparisons and the number of swaps performed during the first pass.

(3)

After a second pass using this bubble sort, the updated list is

2.6 1.8 2.8 1.6 3.1 1.4 1.2 2.4 0.9 0.8 0.6

- (c) Use a quick sort on this updated list to obtain the fully sorted list in descending order. You must make your pivots clear.

(4)

- (d) Apply the first-fit decreasing bin packing algorithm to the fully sorted list to pack the numbers into bins of size 5

(3)

(Total for Question 3 is 13 marks)



1.8 1.4 2.6 1.6 2.8 0.9 3.1 0.8 1.2 2.4 0.6

Question 3 continued

Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

(Total for Question 3 is 13 marks)



Write your answers in the D1 answer book for this paper.

1.

5.2 4.7 6.5 4.5 3.1 5.1 1.8 2.9 3.4 3.8 1.2

- (a) Use the first-fit bin packing algorithm to determine how the eleven numbers listed above can be packed into bins of size 14
(3)
- (b) The list of numbers is to be sorted into **ascending** order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.
(4)
- (c) Apply the first-fit decreasing bin packing algorithm to the sorted list to pack the numbers into bins of size 14
(3)
- (d) Explain why the number of bins used in part (c) is optimal.
(1)
- (e) Use the binary search algorithm to **try** to locate 3.0 in the list of numbers. Clearly indicate how you choose your pivots and which part of the list is rejected at each stage.
(3)

(Total for Question 1 is 14 marks)



1.

5.2 4.7 6.5 4.5 3.1 5.1 1.8 2.9 3.4 3.8 1.2

Lined area for writing the answer.



Question 1 continued

Lined area for writing the answer to Question 1.





Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 14 marks)



3. Pupils from ten schools are visiting a museum on the same day. The museum needs to allocate each school to a tour group. The maximum size of each tour group is 42 pupils. A group may include pupils from more than one school. Pupils from each school must be kept in the same tour group. The numbers of pupils visiting from each school are given below.

8 17 9 14 18 12 22 10 15 7

- (a) Calculate a lower bound for the number of tour groups required. You must make your method clear. (2)

- (b) Using the above list, apply the first-fit bin packing algorithm to allocate the pupils visiting from each school to tour groups. (2)

The above list of numbers is to be sorted into descending order.

- (c) Perform a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly. (4)

- (d) Using your sorted list from (c), apply the first-fit decreasing bin packing algorithm to obtain a second allocation of pupils to tour groups. (2)

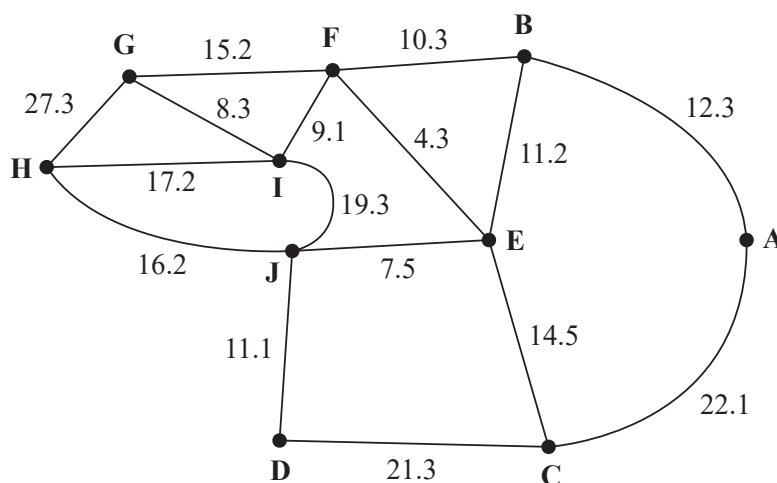


Figure 2

[The total weight of the network is 227.2]

Figure 2 represents the corridors in the museum. The number on each arc is the length, in metres, of the corresponding corridor. Sally is a tour guide in the museum and she must travel along each corridor at least once during each tour. Sally wishes to minimise the length of her route. She must start and finish at the museum's entrance at A.

- (e) Use an appropriate algorithm to find the corridors that Sally will need to traverse twice. You should make your method and working clear. (4)

- (f) Write down a possible shortest route, giving its length.

(2)

Sally is now allowed to start at H and finish her route at a different vertex. A route of minimum length that includes each corridor at least once needs to be found.

- (g) State the finishing vertex of Sally's new route and calculate the difference in length between this new route and the route found in (f).

(2)

(Total 18 marks)



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Question 3 continued

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8 17 9 14 18 12 22 10 15 7

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Question 3 continued

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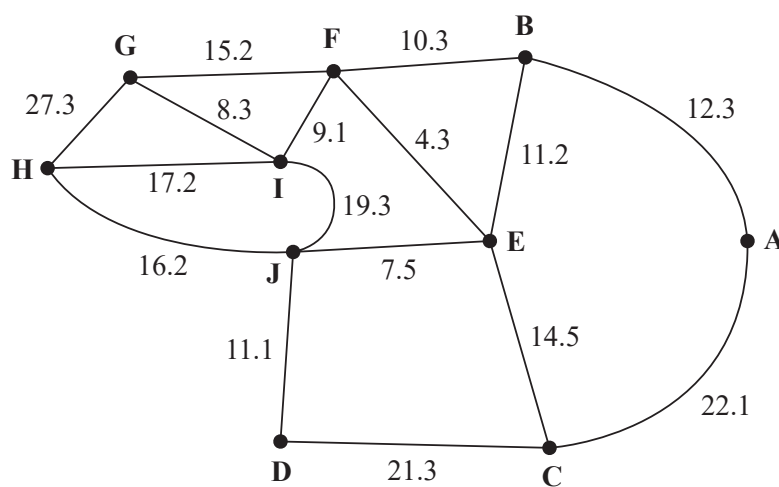


Figure 2

[The total weight of the network is 227.2]

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Question 3 continued

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(Total 18 marks)

Q3



SUB-TOPIC D1.1b

ALGORITHMS IN GENERAL (FLOW CHARTS & PSEUDOCODE)

Sub-topic D1.1b · 3 questions · 17 marks · lowest tariff first



3.

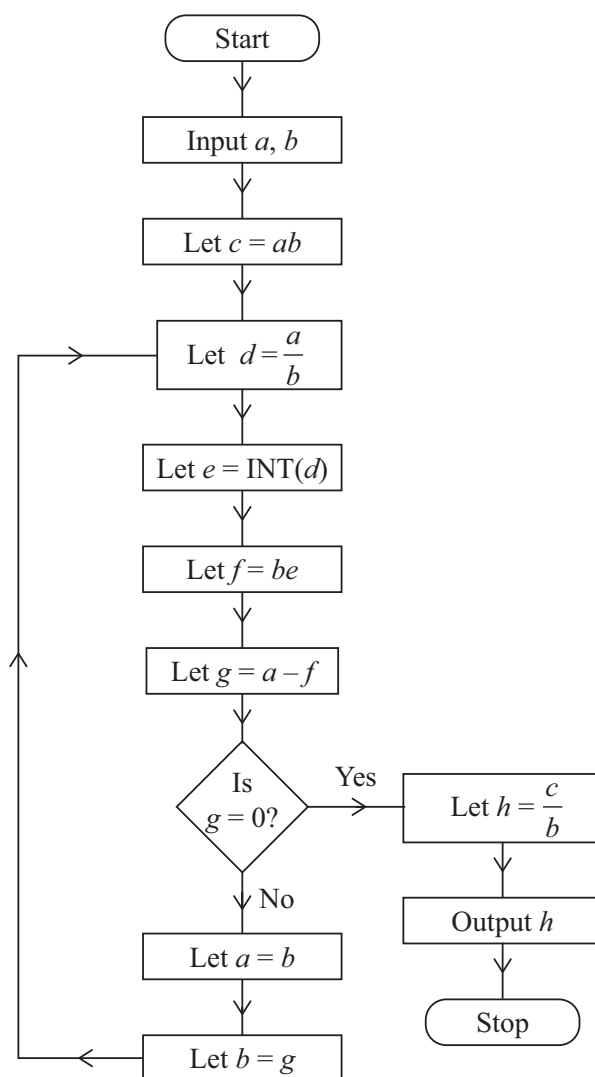


Figure 2

Figure 2 shows the flowchart for an algorithm.

The input for the algorithm is two positive integers.

The function $\text{INT}(X)$ is the largest integer less than or equal to X

- (a) Complete the table in the answer book, with $a = 1980$ and $b = 462$, showing the results at each step of the algorithm.

(4)

- (b) Explain how the output, h , relates to the inputs, a and b

(1)

(Total for Question 3 is 5 marks)



3.

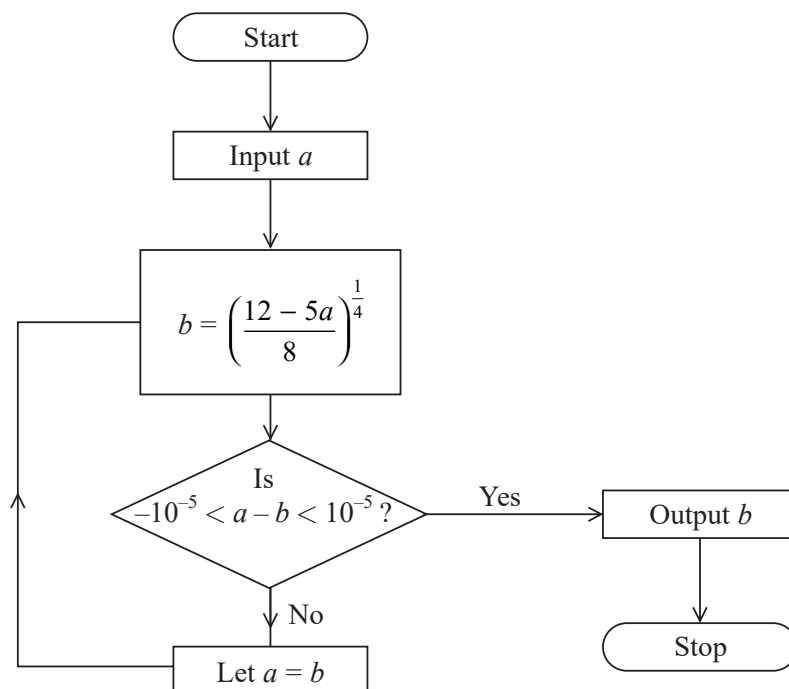


Figure 2

An algorithm for finding the positive real root of the equation $8x^4 + 5x - 12 = 0$ is described by the flow chart shown in Figure 2.

- (a) Use the flow chart, with $a = 1$, to complete the table in the answer book, stating values to at least 6 decimal places. Give the final output correct to 5 decimal places. (4)

Given that the value of the input a is a non-negative real number,

- (b) determine the set of values for a that **cannot** be used to find the positive real root of $8x^4 + 5x - 12 = 0$ using this flow chart. (2)

(Total 6 marks)

3.

You may not need to use all the rows in this table.

You should write your final output in the box provided at the bottom of this page.

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Final output



Question 3 continued

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Q3

(Total 6 marks)



3. In this question, the function $\text{INT}(X)$ is the largest integer less than or equal to X .

For example,

$$\text{INT}(5.7) = 5$$

$$\text{INT}(8) = 8$$

$$\text{INT}(-2.3) = -3$$

Consider the following algorithm.

- Step 1 Input N
- Step 2 Calculate $A = N \div 10$
- Step 3 Let $B = \text{INT}(A)$
- Step 4 Calculate $C = B \times 10$
- Step 5 Calculate $D = N - C$
- Step 6 Output D
- Step 7 Replace N by B
- Step 8 If $N = 0$ then STOP, otherwise go back to Step 2

- (a) Complete the table in the answer book, using $N = 4217$, to show the results obtained at each step of the algorithm.

(4)

- (b) Explain how the output values of the algorithm relate to the original input N , where N is any positive integer.

(2)

(Total for Question 3 is 6 marks)

3.

You may not need to use all the rows in this table.
It may not be necessary to complete all boxes in each row.

<i>N</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>

Outputs: _____

(Total for Question 5 is 6 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
1.	$\left[\frac{1+10}{2} \right] = 6$ Diameter – reject 1 – 6	M1
	$\left[\frac{7+10}{2} \right] = 9$ Segment – reject 9 – 10	A1
	$\left[\frac{7+8}{2} \right] = 8$ Sector – reject 8	A1
	$[7] = 7$ Radius – reject Parallelogram is <u>not</u> in list	A1 (4)
		4 marks
Notes for Question 1		
1M1: Choosing middle right pivot (choosing middle left ‘Circumference’ is M0) + an attempt at discarding/retaining half the list (condone if retaining the wrong half of the list or if only rejecting 1 – 5) 1A1: First pass correct i.e. 6th item and using 7 – 10 in the second pass (must not be using the 6th item in the second pass) – need not choose the 9th item or reject 9 – 10 for this mark 2A1: Second and third passes correct i.e. 9th (Segment) and 8th (Sector) items (no sticky pivots) – need not be rejecting the 8th item for this mark 3A1: CAO search complete (so rejecting 8th and 7th items) + ‘not found’ – must consider Radius by name or the ‘7th item’ after rejecting sector (or stating that the seventh item is not Parallelogram). Condone if Radius is rejected on the same line that Sector is rejected (but must be after Sector). Condone those candidates that correctly state that Radius is not Parallelogram or who do not explicitly reject the Radius e.g. ‘$[7] = 7$ Radius therefore Parallelogram is not in the list.’ Allow use of abbreviations provided clear and unambiguous. Also accept the new list of words being re-written after each pass (with or without the corresponding calculations). For reference: Arc Centre Chord Circle Circumference Diameter Radius Sector Segment Tangent		

Question	Scheme	Marks
1 (a)	$(180 + 300 + 250 + 410 + 240 + 120 + 230 + 310 + 190)/600$ = 3.72	M1
	Lower bound = 4	A1
		(2)
(b)	Van 1: 180 300 120	M1
	Van 2: 250 240	
	Van 3: 410 190	
	Van 4: 230 310	A1
		(2)
(4 marks)		
Notes		
a1M1	Calculates $(2230 \pm 410)/600$ a calculation using the sum of the activities (accept a total in the range 1820 – 2640) (accept 3.71 as correct truncated value). This can be awarded for sight of the calculation or awrt 3.7	
a1A1	CAO – must follow from a correct calculation or decimal (rounded or truncated), so we must see either 2230/600 o.e. or awrt 3.7 - an answer of 4 with no working scores M0	
b1M1	The correct first five items placed correctly (the bold values) and at least seven values placed in bins (allow repeated values). Condone cumulative totals or calculation of the remaining space for M1 only	
b2A1	CSO	

Question Number	Scheme	Marks
2.(a)	The 13 was used as a pivot in the first pass as all numbers greater than 13 are on the left (of the 13) and all numbers less than 13 are on the right (of the 13) (and this is not the case for any other value)	B1 (1)
(b)	17 33 14 25 23 28 21 13 9 6 10 33 17 25 23 28 21 14 13 9 10 6 33 25 23 28 21 17 14 13 10 9 6 33 25 28 23 21 17 14 13 10 9 6 33 28 25 23 21 17 14 13 10 9 6 33 28 25 23 21 17 14 13 10 9 6	M1 A1 A1 (3)
(c)	Bin 1: 33 28 23 Bin 2: 25 21 17 14 6 Bin 3: 13 10 9	M1 A1 (2)
		6 marks

Notes for Question 2

a1B1: pivot value of 13 correctly stated + correct reasoning (give bod provided there is a clear intention of 'values greater than the pivot, pivot, values less than the pivot' e.g. 'after the first pass all the values greater than 13 are to the left and all values smaller are on the right'. Allow for **B1** a statement such as, 'values on one side are bigger than 13 and on the other are smaller', however just a statement like 'all the values greater than 13 are to the left' is **B0** – must have an indication of values on both sides of the 13)

PLEASE NOTE NO MISREADS IN PARTS (b) AND (c) – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE IN (b)

b1M1: Bubble sort. Consistent direction, end number (6) in place and the list beginning with the correct first four numbers (33 17 25 23) after the first pass. Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate's numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 6 appears at the end of the list is the end of their first pass. Their first pass must be of the form 33 17 25 23 x 6 where x is either 5, 6 or 7 numbers

b1A1: The first, second **and** third passes correct

b2A1: Fourth and fifth passes correct – **must include a fifth pass** (ISW after correctly completing the fifth pass). If after the fourth pass they state that the list is in order and simply re-write the list then **A0** (but give bod if it could be interpreted as a fifth pass)

SC in (b): Ascending sort: First two passes correct scores **M1** only in **(b)**

17 14 25 23 28 21 13 9 6 10 33 followed by 14 17 23 25 21 13 9 6 10 28 33

c1M1: The **correct** first five values placed correctly (so must be the 33 28 25 23 and the 21) and at least eight values placed in bins - condone cumulative totals for **M1** only (the bold values)

c1A1: CSO – no additional or repeated values



Question Number	Scheme	Marks																														
3.(a)	<table><tr><th>N</th><th>A</th><th>B</th><th>C</th><th>D</th></tr><tr><td>4217</td><td>421.7</td><td>421</td><td>4210</td><td>7</td></tr><tr><td>421</td><td>42.1</td><td>42</td><td>420</td><td>1</td></tr><tr><td>42</td><td>4.2</td><td>4</td><td>40</td><td>2</td></tr><tr><td>4</td><td>0.4</td><td>0</td><td>0</td><td>4</td></tr><tr><td>0</td><td></td><td></td><td></td><td></td></tr></table>	N	A	B	C	D	4217	421.7	421	4210	7	421	42.1	42	420	1	42	4.2	4	40	2	4	0.4	0	0	4	0					M1
	N	A	B	C	D																											
	4217	421.7	421	4210	7																											
	421	42.1	42	420	1																											
	42	4.2	4	40	2																											
	4	0.4	0	0	4																											
	0																															
Output values: 7, 1, 2, 4	A1																															
	A1																															
	A1																															
	A1																															
	(4)																															
(b)	The first value is the units digit of N , the second value is the tens digit, the third is the hundreds digit, and so on	B2, 1, 0																														
		(2)																														
		6 marks																														
Notes for Question 3																																
a1M1: At least three rows of cells completed with a correct first row – condone repeated values in all columns or a single value in each row a1A1: CAO – the values in the second and third row correct a2A1: CAO – fourth row correct and a zero only in the fifth N row a3A1: Correct outputs (7, 1, 2, 4) – dependent on the first four rows being correct – the output must either be stated on the given answer line or ‘output 7, 1, 2, 4’ must be clearly written somewhere near the table (do not bod column D being circled, etc.). Condone the output being stated as 7124 b1B1: Indication that the outputs are the digits of N b2B1: Indication that the digits are in the reverse order Examples of B1 B1: • The output is N (or the original input or 4217) in reverse order • The output is N backwards • The output is (the digits of) N written right to left • The first value is the unit digit of N, the second value is the tens digit, and so on Examples for B1 B0: • The output is each number of N • The output is N • Output values are the values that make up the original input Examples for B0 B0: • The output comes from/is derived from the original input • The algorithm removes the last digit step by step • The output values are the last digits of N • The output is the last digit of N • The output is N + contradictory statement (e.g. ‘output is N or a number that is smaller than N’) Example for B0 B1 (not common): • The output is in reverse/right to left/backwards (so no mention of N or original input or 4217)																																

Question Number	Scheme	Marks																																																																						
1.(a)	$\frac{1150}{300} = 3.83\ldots$ so lower bound is 4	M1 A1 (2)																																																																						
(b)	<table><tr><td>175</td><td>135</td><td>210</td><td>105</td><td>100</td><td>150</td><td>60</td><td>20</td><td>70</td><td>125</td></tr><tr><td>175</td><td>210</td><td>135</td><td>105</td><td>150</td><td>100</td><td>60</td><td>70</td><td>125</td><td>20</td></tr><tr><td>210</td><td>175</td><td>135</td><td>150</td><td>105</td><td>100</td><td>70</td><td>125</td><td>60</td><td>20</td></tr><tr><td>210</td><td>175</td><td>150</td><td>135</td><td>105</td><td>100</td><td>125</td><td>70</td><td>60</td><td>20</td></tr><tr><td>210</td><td>175</td><td>150</td><td>135</td><td>105</td><td>125</td><td>100</td><td>70</td><td>60</td><td>20</td></tr><tr><td>210</td><td>175</td><td>150</td><td>135</td><td>125</td><td>105</td><td>100</td><td>70</td><td>60</td><td>20</td></tr><tr><td>210</td><td>175</td><td>150</td><td>135</td><td>125</td><td>105</td><td>100</td><td>70</td><td>60</td><td>20</td></tr></table>	175	135	210	105	100	150	60	20	70	125	175	210	135	105	150	100	60	70	125	20	210	175	135	150	105	100	70	125	60	20	210	175	150	135	105	100	125	70	60	20	210	175	150	135	105	125	100	70	60	20	210	175	150	135	125	105	100	70	60	20	210	175	150	135	125	105	100	70	60	20	M1 A1 A1ft A1cso (4)
	175	135	210	105	100	150	60	20	70	125																																																														
	175	210	135	105	150	100	60	70	125	20																																																														
	210	175	135	150	105	100	70	125	60	20																																																														
	210	175	150	135	105	100	125	70	60	20																																																														
	210	175	150	135	105	125	100	70	60	20																																																														
	210	175	150	135	125	105	100	70	60	20																																																														
210	175	150	135	125	105	100	70	60	20																																																															
(c)	Truck 1: 210 <u>70</u> 20 Truck 2: 175 <u>125</u> Truck 3: 150 135 Truck 4: <u>105</u> <u>100</u> 60	M1 <u>A1</u> A1 (3)																																																																						
		9 marks																																																																						

Notes for Question 1

a1M1: Attempt to find the lower bound $(1150 \pm 210) / 300$ (a value of 3.83 (or better) seen with no working can imply this mark)

a1A1: Correct calculation seen **or** 3.83 (or better) followed by a lower bound of 4. An answer of 4 with no working scores M0A0. **Only** seeing 3.8 followed by 4 scores M1A0

b1M1: Bubble sort. Consistent direction, end number (20) in place and the list beginning with the correct first five numbers (175 210 135 105 150). Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate's numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 20 appears at the end of the list is the end of their first pass

b1A1: The first, second **and** third passes correct – so end three numbers in place

b2A1ft: Fourth and fifth passes correct following through from the candidate's third pass – so end five numbers in place

b3A1: cso (correct solution only – so previous three marks must have been awarded in this part). Must show a 6th pass showing no swaps/changes (give bod if the passes are not labelled but do not award this mark if it is clear that after the 5th pass the list is simply being written out again (rather than a genuine 6th pass taking place)). Condone if the sort continues until a 9th pass has been completed (but there should be no changes in the 9th pass)

SC in (b) if list is sorted into ascending order (regardless of reversing at the end of the sort) award

M1 for 135 175 105 100 150 60 20 70 125 210 and then

A1 for 135 105 100 150 60 20 70 125 175 210 **and** 105 100 135 60 20 70 125 135 175 210 so 2 marks max.

c1M1: **Their** first four items placed correctly and at least eight values placed in trucks (if correct this will be the bold items but must check **their** packing if any of **their** first four values are incorrect – note that the maximum weight is 300). Condone cumulative totals for M1 only. First-fit increasing scores no marks in this part. If no sort seen in (b) then mark (c) assuming the correct ordered list in descending order

c1A1: First eight items placed correctly (the underlined and bold values). No additional or repeated values. No follow through or misreads for the A marks in this part. Must be using the correct ten values (so **any** wrong values regardless of where they appear in the trucks is A0)

c2A1: cso. No additional or repeated values



6.(a)	Bin 3 has the largest sum of the four bins with $72 \Rightarrow n \geq 72$	B1																																																																																				
	After the first three numbers were placed in Bin 1 (with a total of 64) the next smallest value (the 11) could not fit in Bin 1 $\Rightarrow n < 64 + 11$	B1																																																																																				
	$72 \leq n \leq 74$ oe e.g. $n = 72, 73$ or 74	B1 (3)																																																																																				
(b)	middle right <table><tr><td>28</td><td>31</td><td>5</td><td>25</td><td>16</td><td>35</td><td>18</td><td>22</td><td>11</td><td>27</td><td>15</td><td>13</td><td>Pivots</td><td></td></tr><tr><td>5</td><td>16</td><td>11</td><td>15</td><td>13</td><td>18</td><td>28</td><td>31</td><td>25</td><td>35</td><td>22</td><td>27</td><td>18</td><td>M1</td></tr><tr><td>5</td><td>11</td><td>16</td><td>15</td><td>13</td><td>18</td><td>28</td><td>31</td><td>25</td><td>22</td><td>27</td><td>35</td><td>11, 35</td><td>A1</td></tr><tr><td>5</td><td>11</td><td>13</td><td>15</td><td>16</td><td>18</td><td>22</td><td>25</td><td>28</td><td>31</td><td>27</td><td>35</td><td>15, 25</td><td>A1ft</td></tr><tr><td>5</td><td>11</td><td>13</td><td>15</td><td>16</td><td>18</td><td>22</td><td>25</td><td>28</td><td>27</td><td>31</td><td>35</td><td>31</td><td>A1</td></tr><tr><td>5</td><td>11</td><td>13</td><td>15</td><td>16</td><td>18</td><td>22</td><td>25</td><td>27</td><td>28</td><td>31</td><td>35</td><td>27</td><td>(4)</td></tr></table>	28	31	5	25	16	35	18	22	11	27	15	13	Pivots		5	16	11	15	13	18	28	31	25	35	22	27	18	M1	5	11	16	15	13	18	28	31	25	22	27	35	11, 35	A1	5	11	13	15	16	18	22	25	28	31	27	35	15, 25	A1ft	5	11	13	15	16	18	22	25	28	27	31	35	31	A1	5	11	13	15	16	18	22	25	27	28	31	35	27	(4)	
28	31	5	25	16	35	18	22	11	27	15	13	Pivots																																																																										
5	16	11	15	13	18	28	31	25	35	22	27	18	M1																																																																									
5	11	16	15	13	18	28	31	25	22	27	35	11, 35	A1																																																																									
5	11	13	15	16	18	22	25	28	31	27	35	15, 25	A1ft																																																																									
5	11	13	15	16	18	22	25	28	27	31	35	31	A1																																																																									
5	11	13	15	16	18	22	25	27	28	31	35	27	(4)																																																																									
	middle left <table><tr><td>28</td><td>31</td><td>5</td><td>25</td><td>16</td><td>35</td><td>18</td><td>22</td><td>11</td><td>27</td><td>15</td><td>13</td><td>Pivots</td><td></td></tr><tr><td>28</td><td>31</td><td>5</td><td>25</td><td>16</td><td>18</td><td>22</td><td>11</td><td>27</td><td>15</td><td>13</td><td>35</td><td>35</td><td></td></tr><tr><td>5</td><td>16</td><td>11</td><td>15</td><td>13</td><td>18</td><td>28</td><td>31</td><td>25</td><td>22</td><td>27</td><td>35</td><td>11, 25</td><td></td></tr><tr><td>5</td><td>11</td><td>16</td><td>15</td><td>13</td><td>18</td><td>22</td><td>25</td><td>28</td><td>31</td><td>27</td><td>35</td><td>15, 31</td><td></td></tr><tr><td>5</td><td>11</td><td>13</td><td>15</td><td>16</td><td>18</td><td>22</td><td>25</td><td>28</td><td>27</td><td>31</td><td>35</td><td>28</td><td></td></tr><tr><td>5</td><td>11</td><td>13</td><td>15</td><td>16</td><td>18</td><td>22</td><td>25</td><td>27</td><td>28</td><td>31</td><td>35</td><td></td><td></td></tr></table>	28	31	5	25	16	35	18	22	11	27	15	13	Pivots		28	31	5	25	16	18	22	11	27	15	13	35	35		5	16	11	15	13	18	28	31	25	22	27	35	11, 25		5	11	16	15	13	18	22	25	28	31	27	35	15, 31		5	11	13	15	16	18	22	25	28	27	31	35	28		5	11	13	15	16	18	22	25	27	28	31	35			
28	31	5	25	16	35	18	22	11	27	15	13	Pivots																																																																										
28	31	5	25	16	18	22	11	27	15	13	35	35																																																																										
5	16	11	15	13	18	28	31	25	22	27	35	11, 25																																																																										
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5	11	13	15	16	18	22	25	28	27	31	35	28																																																																										
5	11	13	15	16	18	22	25	27	28	31	35																																																																											
(c)	After the 27 has been placed in Bin 2 (giving a total of 55) the 18 was placed in Bin 3 not Bin 2 indicating that $n < 55 + 18$	B1																																																																																				
	$n = 72$	dB1 (2)																																																																																				
		9 marks																																																																																				
	Notes for Question 6																																																																																					
a1B1	Correct reasoning why n is at least 72 States or shows a bin (3) has total 72 and concludes that this is the least value																																																																																					
a2B1	Correct reasoning regarding the upper bound for n – that is as the second smallest value (the 11) was not put in Bin 1 then $n < \text{the sum of Bin 1} + 11$																																																																																					
a3B1	$n = 72, 73$ or 74																																																																																					
b1M1	Quick sort – pivots, p , selected and first pass gives $<p, p, >p$. If only choosing 1 pivot per iteration M1 only. If sorting into descending order then mark as a misread																																																																																					
b1A1	First pass correct and next pivots chosen correctly/consistently for second pass																																																																																					
b2A1ft	Second and third passes correct (ft from their first pass and choice of pivots)																																																																																					
b3A1	CSO (including a fifth pass)																																																																																					
c1B1	Correct reasoning regarding the placement of the 18 in Bin 3 rather than Bin 2 (must explicitly mention 18)																																																																																					
c2B1	Dependent on previous B mark - CAO ($n = 72$)																																																																																					

	MISREAD – sorting into descending order													
6.(b)	middle right													
													Pivots	
	28	31	5	25	16	35	18	22	11	27	15	13	18	M1
	28	31	25	35	22	27	<u>18</u>	5	16	11	15	13	11, 35	A1
	<u>35</u>	28	31	25	22	27	<u>18</u>	16	15	13	<u>11</u>	5	15, 25	
	<u>35</u>	28	31	27	<u>25</u>	22	<u>18</u>	16	<u>15</u>	13	<u>11</u>	<u>5</u>	31	A1ft
	<u>35</u>	<u>31</u>	28	27	<u>25</u>	<u>22</u>	<u>18</u>	<u>16</u>	<u>15</u>	<u>13</u>	<u>11</u>	<u>5</u>	27	A1
	<u>35</u>	<u>31</u>	28	<u>27</u>	<u>25</u>	<u>22</u>	<u>18</u>	<u>16</u>	<u>15</u>	<u>13</u>	<u>11</u>	<u>5</u>		(4)
	middle left													
													Pivots	
	28	31	5	25	16	35	18	22	11	27	15	13	35	
	<u>35</u>	28	31	5	25	16	18	22	11	27	15	13	18	
	<u>35</u>	28	31	25	22	27	<u>18</u>	5	16	11	15	13	11, 25	
	<u>35</u>	28	31	27	<u>25</u>	22	<u>18</u>	16	15	13	<u>11</u>	5	15, 31	
	<u>35</u>	<u>31</u>	28	27	<u>25</u>	<u>22</u>	<u>18</u>	16	<u>15</u>	13	<u>11</u>	<u>5</u>	28	
	<u>35</u>	<u>31</u>	<u>28</u>	27	<u>25</u>	<u>22</u>	<u>18</u>	<u>16</u>	<u>15</u>	<u>13</u>	<u>11</u>	<u>5</u>		
	Notes for Question 6													
b1M1	Quick sort – pivots, p, selected and first pass gives <p, p, >p. If only choosing 1 pivot per iteration M1 only. If sorting into descending order then mark as a misread													
b1A1	First pass correct and next pivots chosen correctly/consistently for second pass													
b2A1ft	Second and third passes correct (ft from their first pass and choice of pivots)													
b3A1	CSO (including a fifth pass)													
	REMOVE final two A marks earned in this section so max 2/4													

Question Number	Scheme	Marks
1.(a)	Bin 1: 26 19 10 Bin 2: 31 14 3 5 Bin 3: <u>30</u> <u>18</u> Bin 4: <u>33</u> Bin 5: 28	M1 <u>A1</u> A1(3)
(b)	middle right 26 19 31 10 14 30 33 18 3 28 5 Pivot 30 31 33 <u>30</u> 26 19 10 14 18 3 28 5 33, 18 <u>33</u> 31 <u>30</u> 26 19 28 <u>18</u> 10 14 3 5 (31) 19, 3 <u>33</u> <u>31</u> <u>30</u> 26 28 <u>19</u> <u>18</u> 10 14 5 <u>3</u> 28, 14 <u>33</u> <u>31</u> <u>30</u> 28 26 <u>19</u> <u>18</u> <u>14</u> 10 5 <u>3</u> (26) 5 <u>33</u> <u>31</u> <u>30</u> <u>28</u> <u>26</u> <u>19</u> <u>18</u> <u>14</u> 10 <u>5</u> <u>3</u> (10)	M1 A1 A1ft A1 (4)
	middle left 26 19 31 10 14 30 33 18 3 28 5 Pivot 30 31 33 <u>30</u> 26 19 10 14 18 3 28 5 31, 14 33 <u>31</u> <u>30</u> 26 19 18 28 <u>14</u> 10 3 5 (33), 19, 3 <u>33</u> <u>31</u> <u>30</u> 26 28 <u>19</u> 18 <u>14</u> 10 5 <u>3</u> 26, (18) 10 <u>33</u> <u>31</u> <u>30</u> 28 <u>26</u> <u>19</u> 18 <u>14</u> <u>10</u> 5 <u>3</u> (28), (5)	
(c)	Bin 1: 33 19 3 Bin 2: 31 18 5 Bin 3: 30 14 10 Bin 4: 28 26	M1 A1(2)
		9 marks
	Notes for Question 1	
a1M1	The correct first five items placed correctly (the bold values) and at least eight values placed in bins (allow repeated values). Condone cumulative totals or calculation of the remaining space for M1 only	
a1A1	First eight values placed correctly (the bold and underlined values) with all eleven correct values only placed in bins. This mark cannot be awarded if any repeated values or incorrect values are seen (even if the first eight values have been placed correctly)	
a2A1	CSO – no additional or repeated values (dependent on both previous marks)	

b1M1	Quick sort using all 11 numbers (condone one item error or omission), pivot, p, chosen (must be choosing middle item – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). If only choosing one pivot per iteration then M1. Bubble sort is M0.	
b1A1	First and second passes correct but not choice of pivots for 3 rd pass	
b2A1ft	Third and fourth passes correct following through from their second pass and choice of pivots for the third pass. The pivots for the third and fourth pass must be consistent with their second pass (MR or ML) If they choose one MR and one ML pivot for the second pass, they can score this mark if they then consistently choose either MR or ML pivots for the rest of their sort.	
b3A1	CSO (correct solution only – all previous marks in this part must have been awarded) including a fifth pass for middle right and choosing either 5 (MR) or 10 (ML) as pivots	
	Special Case for (b) If the candidate sorts into ascending order they can score M1 as per the main scheme (but with the values either side of the pivot reversed), A1 for a fully correct sort then A0 A0 even if the list is reversed at the end (so 2 marks max.). See below	
c1M1	Their seven largest items placed correctly and at least nine values placed in bins (if correct this will be the bold items but must check their packing if any of their seven largest values are incorrect – note that the maximum weight of a bin is 55). Condone cumulative totals or calculation of remaining space for M1 only. First-fit increasing scores no marks in this part. If no sort seen in (b) then mark (c) assuming the correct ordered list is being used	
c1A1	CAO	
	Ascending Sort Middle right 26 19 31 10 14 30 33 18 3 28 5 Pivot 30 26 19 10 14 18 3 28 5 <u>30</u> 31 33 18, 33 10 14 3 5 <u>18</u> 26 19 28 <u>30</u> 31 <u>33</u> 3, 19, (31) <u>3</u> 10 14 5 <u>18</u> <u>19</u> 26 28 <u>30</u> <u>31</u> <u>33</u> 14, 28 <u>3</u> 10 5 <u>14</u> <u>18</u> <u>19</u> 26 <u>28</u> <u>30</u> <u>31</u> <u>33</u> 5 (26) <u>3</u> <u>5</u> 10 <u>14</u> <u>18</u> <u>19</u> <u>26</u> <u>28</u> <u>30</u> <u>31</u> <u>33</u> (10)	

	Middle left												
	26	19	31	10	14	30	33	18	3	28	5	Pivot 30	
	26	19	10	14	18	3	28	5	<u>30</u>	31	33	14, 31	
	10	3	5	<u>14</u>	26	19	18	28	<u>30</u>	<u>31</u>	33	3, 19, (31)	
	<u>3</u>	10	5	<u>14</u>	18	<u>19</u>	26	28	<u>30</u>	<u>31</u>	<u>33</u>	10, (18) 26	
	<u>3</u>	5	<u>10</u>	<u>14</u>	<u>18</u>	<u>19</u>	<u>26</u>	28	<u>30</u>	<u>31</u>	<u>33</u>	(28), (5)	



Question Number	Scheme	Marks
2.(a) (i)		M1 A1 (ADBCE) A1 (HF) A1ft (GJ)
	Shortest path from A to J is ABFGJ	A1
(a) (ii)	Shortest length is 357 (metres)	A1ft (6)
(b)	Consider all pairings of A, C, E and J	M1
	$AC + EJ \quad 147 + 210 = 357$ $AE + CJ \quad 161 + 364 = 525$ $AJ + CE \quad 357 + 168 = 525$	A1ft A1ft
	Length of route = $1\,960 + 357 = 2\,317$ (metres)	A1 ft (4)
(c)(i)	Consider arcs between odd nodes not using A – shortest CE so repeat. Finish at J	M1 A1
(c)(ii)	Difference in length their “357” – $168 = 189$ metres or their “2317” – $2128 = 189$ (metres)	A1ft (3)
		13 Marks

Question	Scheme	Marks
3. (i)(a)	Middle Right	
	Pivot	M1
	28 18 50 27 48 8 <u>25</u> 12 56 31 33 42 25	
	18 <u>8</u> 12 25 28 50 27 48 <u>56</u> 31 33 42 8, 56	A1
	8 18 <u>12</u> 25 28 50 27 <u>48</u> 31 33 42 56 12, 48	A1ft
	8 12 18 25 28 27 <u>31</u> 33 42 48 50 56 (18), 31, (50)	A1
	8 12 18 25 28 <u>27</u> 31 33 <u>42</u> 48 50 56 27, 42	
	8 12 18 25 27 28 31 33 42 48 50 56	
	Middle Left	
	Pivot	
	28 18 50 27 48 <u>8</u> 25 12 56 31 33 42 8	
	8 28 18 50 27 48 <u>25</u> 12 56 31 33 42 25	
	8 <u>18</u> 12 25 28 50 27 <u>48</u> 56 31 33 42 18, 48	
	8 12 18 25 28 27 <u>31</u> 33 42 48 <u>50</u> 56 (12), 31, 50	
	8 12 18 25 <u>28</u> 27 31 <u>33</u> 42 48 50 56 28, 33, (56)	
	8 12 18 25 27 28 31 33 42 48 50 56	
		(4)



(i)(b)	8 12 18 25 27 28 31 33 42 48 50 56 Pivot 31 31 < 40 so discard 31 and below 33 42 48 50 56 Pivot 48 48 > 40 so discard 48 and above 33 42 Pivot 42 42 > 40 so discard 42 33 33 ≠ 40 so 40 is not in the list	M1 A1 A1
	Alternatively 56 50 48 42 33 31 28 27 25 18 12 8 Pivot 28 28 < 40 so discard 28 and below 56 50 48 42 33 31 Pivot 42 42 > 40 so discard 42 and above 33 31 Pivot 31 31 < 40 so discard 31 33 33 ≠ 40 so 40 is not in the list	
(ii)	$x > 15$ or $x < 19$ o.e. $x > 15$ and $x < 19$ leading to $15 < x < 19$ or $16 \leq x \leq 18$ o.e.	B1 B1
		(2)
(9 marks)		



Notes	
ia1M1	Quick sort using all 12 numbers (condone one item error or omission), pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values less than the pivot), pivot, (values greater than the pivot). If only choosing one pivot per iteration then M1 only
ia1A1	First and second passes correct, but not choice of pivots for 3 rd pass
ia2A1ft	Third and fourth passes correct following through from their second pass and choice of pivots for the third pass. The pivots for the third, and fourth passes must be consistent (so all MR or all ML)
ia3A1	CSO (correct solution only – all previous marks in this part must have been awarded) including a fifth pass (with both 27 and 42 chosen as pivots MR or both 28 and 33 chosen as pivots ML)
	Special Case for (a) If the candidate sorts into descending order they can score M1 as per the main scheme (but with the values either side of the pivot reversed), A1 for a fully correct sort then A0 A0 even if the list is reversed at the end (so 2 marks max.). See below
ib1M1	Correct choice of pivot (must be using MR) (31 ascending or 28 descending) and discards 8 to 31 or 28 (must be using the sorted list, but could be sorted into either ascending or descending order) It must be clear which values are being discarded or the reduced list must be shown. If pivots are retained, this will score M1 only. Candidates may either compare with the actual values or the positions of items in a correctly sorted list. The actual rejections may be shown as values from the list or the positions in the list. (If they do not have the correct 12 numbers in their sorted list in (a) they can score M1 only for using their sorted list here)
ib1A1	2 nd and 3 rd pass correct choice of pivots and discards (if pivot not discarded A0)
ib2A1	CSO - Final pass correct, compares with 33 and concludes 40 is not in list
ii1B1	One inequality correct – if a list of inequalities is seen, they must clearly select from this list otherwise B0
ii2B1	Both inequalities correct and combined to give a range of values (accept either form or list of values 16, 17, 18) (SC if just 16, 17, 18 seen accept this for both marks) (accept interval notation either (15, 19) or [16, 18])

	Special Case Descending Max M1 A1 A0 A0												
	Middle Right												
													Pivot
	28	18	50	27	48	8	<u>25</u>	12	56	31	33	42	25
	28	50	27	48	<u>56</u>	31	33	42	25	18	<u>8</u>	12	56, 8
	56	28	50	27	<u>48</u>	31	33	42	25	18	<u>12</u>	8	48, 12
	56	<u>50</u>	48	28	27	<u>31</u>	33	42	25	<u>18</u>	12	8	(50), 31, (18)
	56	50	48	33	<u>42</u>	31	28	<u>27</u>	25	18	12	8	42, 27
	56	50	48	42	33	31	28	27	25	18	12	8	
	Middle Left												
													Pivot
	28	18	50	27	48	<u>8</u>	25	12	56	31	33	42	8
	28	18	50	27	48	<u>25</u>	12	56	31	33	42	8	25
	28	50	27	<u>48</u>	56	31	33	42	25	<u>18</u>	12	8	48, 18
	<u>50</u>	56	48	28	27	<u>31</u>	33	42	25	18	<u>12</u>	8	50, 31, (12)
	56	50	48	<u>33</u>	42	31	<u>28</u>	27	25	18	12	8	33, 28
	56	50	48	42	33	31	28	27	25	18	12	8	

Question Number	Scheme	Marks																																																							
7. (a)	$3 < \frac{228}{n} \leq 4$	M1																																																							
	Critical value of 57 and 76 (or 57 and 75)	A1																																																							
	$57 \leq n < 76$ (or $57 \leq n \leq 75$)	A1 (3)																																																							
(b)	e.g. middle right	M1 A1 A1ft A1 (4)																																																							
	<table><tr><td>14</td><td>20</td><td>23</td><td>17</td><td>15</td><td>22</td><td>19</td><td>25</td><td>13</td><td>28</td><td>32</td></tr><tr><td>23</td><td>25</td><td>28</td><td>32</td><td>22</td><td>14</td><td>20</td><td>17</td><td>15</td><td>19</td><td>13</td></tr><tr><td>32</td><td>28</td><td>23</td><td>25</td><td>22</td><td>20</td><td>17</td><td>19</td><td>15</td><td>14</td><td>13</td></tr><tr><td>32</td><td>28</td><td>25</td><td>23</td><td>22</td><td>20</td><td>19</td><td>17</td><td>15</td><td>14</td><td>13</td></tr><tr><td>32</td><td>28</td><td>25</td><td>23</td><td>22</td><td>20</td><td>19</td><td>17</td><td>15</td><td>14</td><td>13</td></tr></table>		14	20	23	17	15	22	19	25	13	28	32	23	25	28	32	22	14	20	17	15	19	13	32	28	23	25	22	20	17	19	15	14	13	32	28	25	23	22	20	19	17	15	14	13	32	28	25	23	22	20	19	17	15	14	13
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	14		20	23	17	15	22	19	25	13	28	32																																													
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32	28	25	23	22	20	19	17	15	14	13																																															
(c)	From first-fit Bin 1 could not fit the 17 so $n < 74$ (or $n \leq 73$) but could fit the 15 so n is either 72 (as the largest total is 72 in Bin 1 from first-fit) or 73	B1																																																							
	From first-fit decreasing the 13 could not fit in Bin 1	B1																																																							
	So $n = 72$	ddB1 (3)																																																							
		10 marks																																																							

Question Number	Scheme	Marks
	Notes for Question 7	
a1M1: An equation or inequality linking the expression $\frac{228}{n}$ with either 3 or 4 a1A1: Correct critical values of 57 and 76 (or 57 and 75) a2A1: $57 \leq n < 76$ or $57 \leq n \leq 75$ b1M1: Quick sort – pivots, p, selected and first pass gives $>p$, p, $<p$. If only choosing 1 pivot per iteration M1 only. If sorting into ascending order then mark as a misread b1A1: First pass correct and next pivots chosen correctly/consistently for second pass b2A1ft: Second and third passes correct (ft from their first pass and choice of pivots) b3A1: cso (including a fourth pass with 19 used as a pivot if middle right or 14 if middle left) c1B1: Correct deduction from first-fit that n is at least 72 or at most 73 (oe e.g. less than 74). For example, may see $(14 + 20 + 23 + 15 =) 72$ stated therefore $n \geq 72$ or $14 + 20 + 23 + 17 = 74$ followed by $n < 74$ (so realising that the 17 did not fit in Bin 1). As a minimum accept the statement that $n < 74$ or $n \leq 73$ or $n \geq 72$ c2B1: Correct deduction from first-fit decreasing that the 13 was not placed in Bin 1. For example, may see $32 + 28 + 13 = 73$ so therefore $n < 73$ or $n \neq 72$. As a minimum accept the statement that $n < 73$ or $n \neq 72$ or simply stating that '13 did not fit in Bin 1' (give bod here if not clear which Bin 1 they are considering) c3ddb1: cao (dependent on both previous B marks) – must state that the <u>largest</u> total in any bin is <u>72</u> (or they need to say or show there exists a bin with 72) and that the <u>13 did not fit in Bin 1 in first-fit decreasing</u> so n cannot be <u>73</u> and therefore $n = 72$ (not just '13 does not fit in Bin 1' – must be clear that they are talking about first-fit decreasing) No marks in (c) if $n = 72$ stated with no working or if all the candidate does is to sum the numbers in each bin Note that the first B mark in (c) can be implied if the candidate considers the first-fit decreasing packing first or argues with first-fit decreasing before considering first-fit, e.g., 'The 13 does not fit in Bin 1 in the first-fit decreasing packing therefore n is at most 72 and the total of Bin 1 in first-fit is 72' would imply the first two B marks in this part. Stating that therefore n must be 72 would then score all three marks		

Question Number	Scheme	Marks
1.(a)	$\frac{131}{40} = 3.275$ so lower bound is 4	M1 A1 (2)
(b)	Container 1: <u>17</u> <u>9</u> <u>8</u> 4 Container 2: <u>15</u> <u>20</u> 5 Container 3: <u>13</u> 12 Container 4: <u>28</u>	<u>M1</u> <u>A1</u> A1 (3)
(c)	middle right 17 9 15 8 20 13 28 4 12 5 9 8 4 12 5 <u>13</u> 17 15 20 28 <u>4</u> 9 8 12 5 <u>13</u> 17 15 <u>20</u> 28 <u>4</u> 9 8 5 <u>12</u> <u>13</u> <u>15</u> 17 <u>20</u> 28 <u>4</u> 5 <u>8</u> 9 <u>12</u> <u>13</u> <u>15</u> 17 <u>20</u> 28 Pivots 13 4, 20 12, 15, (28) 8, (17) Sort complete	M1 A1 A1 (3)
	middle left 17 9 15 8 20 13 28 4 12 5 17 9 15 8 13 4 12 5 <u>20</u> 28 4 5 <u>8</u> 17 9 15 13 12 <u>20</u> 28 <u>4</u> 5 <u>8</u> 9 13 12 <u>15</u> 17 <u>20</u> 28 <u>4</u> 5 <u>8</u> 9 12 <u>13</u> <u>15</u> 17 <u>20</u> 28 <u>4</u> 5 <u>8</u> <u>9</u> 12 <u>13</u> <u>15</u> 17 <u>20</u> 28 Pivots 20 8, (28) 4, 15 (5), 13, (17) 9 (Sort Complete)	
(d)	$\left\lceil \frac{1+10}{2} \right\rceil = 6$ 13 – reject 13 – 28 $\left\lceil \frac{1+5}{2} \right\rceil = 3$ 8 – reject 4 – 8 $\left\lceil \frac{4+5}{2} \right\rceil = 5$ 12 – reject 12 $\lceil 4 \rceil = 4$ 9 – found	M1 A1 A1 (3)
		11 marks
Notes for Question 1		
a1M1: Attempt to find the lower bound $(131 \pm 28) / 40$. A value of 3.275 seen with no corresponding calculation can imply this mark. Allow this mark for a clear intention of adding all ten values and dividing by 40, for example, $\frac{17 + 9 + 15 + 8 + 20 + 13 + 28 + 4 + 12 + 5}{40} \text{ scores M1}$ a1A1: cao - correct calculation seen followed by 4 or the correct value of 3.275 followed by 4. However, an answer of 4 with no working scores no marks in (a) b1M1: First four values placed correctly and at least eight values placed in containers. The first four values are those that are boxed. Condone cumulative totals for M1 only (e.g. for Container 1: 17 26 34 38)		

Question Number	Scheme	Marks																																																																																																
1. (a)	$\frac{177}{45} = 3.933\dots$ so lower bound is 4	M1 A1 (2)																																																																																																
(b)	Container 1: 16 23 4 Container 2: 18 9 <u>5</u> 13 Container 3: <u>20</u> 17 6 Container 4: <u>35</u> Container 5: 11	M1 <u>A1</u> A1 (3)																																																																																																
(c)	<table><tr><td>16</td><td>23</td><td>18</td><td>9</td><td>4</td><td>20</td><td>35</td><td>5</td><td>17</td><td>13</td><td>6</td><td>11</td></tr><tr><td>23</td><td>18</td><td>16</td><td>9</td><td>20</td><td>35</td><td>5</td><td>17</td><td>13</td><td>6</td><td>11</td><td>4</td></tr><tr><td>23</td><td>18</td><td>16</td><td>20</td><td>35</td><td>9</td><td>17</td><td>13</td><td>6</td><td>11</td><td>5</td><td>4</td></tr><tr><td>23</td><td>18</td><td>20</td><td>35</td><td>16</td><td>17</td><td>13</td><td>9</td><td>11</td><td>6</td><td>5</td><td>4</td></tr><tr><td>23</td><td>20</td><td>35</td><td>18</td><td>17</td><td>16</td><td>13</td><td>11</td><td>9</td><td>6</td><td>5</td><td>4</td></tr><tr><td>23</td><td>35</td><td>20</td><td>18</td><td>17</td><td>16</td><td>13</td><td>11</td><td>9</td><td>6</td><td>5</td><td>4</td></tr><tr><td>35</td><td>23</td><td>20</td><td>18</td><td>17</td><td>16</td><td>13</td><td>11</td><td>9</td><td>6</td><td>5</td><td>4</td></tr><tr><td>35</td><td>23</td><td>20</td><td>18</td><td>17</td><td>16</td><td>13</td><td>11</td><td>9</td><td>6</td><td>5</td><td>4</td></tr></table>	16	23	18	9	4	20	35	5	17	13	6	11	23	18	16	9	20	35	5	17	13	6	11	4	23	18	16	20	35	9	17	13	6	11	5	4	23	18	20	35	16	17	13	9	11	6	5	4	23	20	35	18	17	16	13	11	9	6	5	4	23	35	20	18	17	16	13	11	9	6	5	4	35	23	20	18	17	16	13	11	9	6	5	4	35	23	20	18	17	16	13	11	9	6	5	4	M1 A1 A1ft A1cso (4)
16	23	18	9	4	20	35	5	17	13	6	11																																																																																							
23	18	16	9	20	35	5	17	13	6	11	4																																																																																							
23	18	16	20	35	9	17	13	6	11	5	4																																																																																							
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35	23	20	18	17	16	13	11	9	6	5	4																																																																																							
(d)	Container 1: 35 <u>9</u> Container 2: 23 20 Container 3: 18 17 6 4 Container 4: 16 <u>13</u> <u>11</u> 5	M1 <u>A1</u> A1 (3)																																																																																																
		12 marks																																																																																																

Notes for Question 1

a1M1: Attempt to find the lower bound $(177 \pm 35)/45$. A value of 3.9 or better (for example, 3.93, 3.933, 3.9333..., $\frac{59}{15}$, etc.) seen with no corresponding calculation can imply this mark. Allow this mark for a clear intention of adding **all twelve** values and dividing by 45, for example,

$$\frac{16 + 23 + 18 + 9 + 4 + 20 + 35 + 5 + 17 + 13 + 6 + 11}{45}$$
 scores M1

a1A1: CAO (correct answer only) - correct calculation seen followed by 4 **or** the correct value of 3.9 or better stated (see note above for mark **a1M1**) followed by 4. However, an answer of 4 with no working scores no marks in this part

b1M1: First five items placed correctly and at least eight values placed in containers. The first five values are those in bold. Condone cumulative totals for M1 only (e.g. for Crate 1: 16 39 43)

b1A1: First eight items placed correctly (the underlined **and** bold values). This mark cannot be awarded if you see **any** repeated values or more than twelve items placed in crates (even if the first eight items have been placed correctly)

b2A1: CSO (correct solution only). No additional or repeated values

c1M1: Bubble sort. Consistent direction, end number (4) in place and the list beginning with the correct first four numbers (23 18 16 9). Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate's numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 4 appears at the end of the list is the end of their first pass

c1A1: The first, second **and** third passes correct – so end three numbers in place

Question Number	Scheme	Marks																																																																																																																								
c2A1ft: Fourth and fifth passes correct following through from the candidate's third pass – so end five numbers in place c3A1: CSO (correct solution only – so previous three marks must have been awarded in this part). Must show a 7th pass showing no swaps/changes. Condone if the sort continues until an 11th pass has been completed (but there should be no changes in the 8th to 11th passes) or if the pass stops between an 8th and 11th pass (provided there are no changes in the values in any pass after the 6th pass) d1M1: Must be using correct sorted list in descending order (so no follow through or misreads from an incorrect list from (c)). First six items placed correctly and at least eight values placed in containers. Note that the first six items are the bold values. Condone cumulative totals for M1 only. First-fit increasing scores no marks in this part d1A1: First nine items placed correctly (the underlined and bold values). No additional or repeated values d2A1: CSO. So no additional or repeated values Sorting into ascending order in (c) • If the candidate sorts the list into ascending order and reverses the list they can score full marks • If the list is not reversed in (c) then mark as a misread (so mark according to the main scheme and then subtract the final two A marks earned). If the candidate says that the list needs reversing but doesn't actually show the reversed list then remove the final A mark earned Misreads for part (c) only – if there is a 'misread' of a single number (this could take the form of an extra number, a number missing, or a number changed, for example, 42 rather than 23) before starting the sort in (c) then mark as a misread. If they 'misread' more than one number then M0. If they miscopy one of their own numbers during the sort then this is an accuracy error and loses the corresponding A mark(s) according to the scheme. No misreads permitted in (a), (b) and (d) (so mark to the main scheme) For reference the sort in ascending order is: <table><tr><td>16</td><td>23</td><td>18</td><td>9</td><td>4</td><td>20</td><td>35</td><td>5</td><td>17</td><td>13</td><td>6</td><td>11</td></tr><tr><td>16</td><td>18</td><td>9</td><td>4</td><td>20</td><td>23</td><td>5</td><td>17</td><td>13</td><td>6</td><td>11</td><td>35</td></tr><tr><td>16</td><td>9</td><td>4</td><td>18</td><td>20</td><td>5</td><td>17</td><td>13</td><td>6</td><td>11</td><td>23</td><td>35</td></tr><tr><td>9</td><td>4</td><td>16</td><td>18</td><td>5</td><td>17</td><td>13</td><td>6</td><td>11</td><td>20</td><td>23</td><td>35</td></tr><tr><td>4</td><td>9</td><td>16</td><td>5</td><td>17</td><td>13</td><td>6</td><td>11</td><td>18</td><td>20</td><td>23</td><td>35</td></tr><tr><td>4</td><td>9</td><td>5</td><td>16</td><td>13</td><td>6</td><td>11</td><td>17</td><td>18</td><td>20</td><td>23</td><td>35</td></tr><tr><td>4</td><td>5</td><td>9</td><td>13</td><td>6</td><td>11</td><td>16</td><td>17</td><td>18</td><td>20</td><td>23</td><td>35</td></tr><tr><td>4</td><td>5</td><td>9</td><td>6</td><td>11</td><td>13</td><td>16</td><td>17</td><td>18</td><td>20</td><td>23</td><td>35</td></tr><tr><td>4</td><td>5</td><td>6</td><td>9</td><td>11</td><td>13</td><td>16</td><td>17</td><td>18</td><td>20</td><td>23</td><td>35</td></tr><tr><td>4</td><td>5</td><td>6</td><td>9</td><td>11</td><td>13</td><td>16</td><td>17</td><td>18</td><td>20</td><td>23</td><td>35</td></tr></table>			16	23	18	9	4	20	35	5	17	13	6	11	16	18	9	4	20	23	5	17	13	6	11	35	16	9	4	18	20	5	17	13	6	11	23	35	9	4	16	18	5	17	13	6	11	20	23	35	4	9	16	5	17	13	6	11	18	20	23	35	4	9	5	16	13	6	11	17	18	20	23	35	4	5	9	13	6	11	16	17	18	20	23	35	4	5	9	6	11	13	16	17	18	20	23	35	4	5	6	9	11	13	16	17	18	20	23	35	4	5	6	9	11	13	16	17	18	20	23	35
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Question Number	Scheme	Marks
2.(a)		M1 A1 M1 A1 (4)
(b)	e.g.	M1 A1 A1 A1 (4)
(c)	Lower bound is 4 workers e.g. activities K, M, N and Q together with $20 < \text{time} < 21$	M1 A1 (2)
		10 marks

Question Number	Scheme	Marks
4. (a)	Bin 1: 35 17 7 Bin 2: 10 <u>28</u> <u>15</u> Bin 3: <u>23</u> 20 Bin 4: <u>41</u> Bin 5: 29	M1 <u>A1</u> A1 (3)
(b)	e.g. middle right Pivot(s)	
	35 17 10 7 28 <u>23</u> 41 15 20 29 23 35 28 <u>41</u> 29 23 <u>17</u> 10 <u>7</u> 15 20 41, 7 41 35 <u>28</u> 29 23 17 10 <u>15</u> 20 7 28, 15 41 35 <u>29</u> 28 23 17 <u>20</u> 15 10 7 29, 20, (10) 41 35 29 28 23 20 17 15 10 7 Sort complete	M1 A1 A1ft A1 (4)
(c)	Bin 1: 41 <u>17</u> Bin 2: 35 23 Bin 3: 29 28 Bin 4: <u>20</u> 15 10 7	M1 <u>A1</u> A1 (3)
(d)	$8 < x < 12, y > x, (y > 8)$	B3, 2, 1, 0 (3)
		13 marks

Notes for Question 4

PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASES IN PART (b) and the guidance for the M mark in (c)

a1M1: First four items placed correctly (the values in bold) and at least seven values placed in bins.

Condone cumulative totals for M1 only – if one of the bold values appears in more than one bin then M0

a1A1: First eight items placed correctly (the underlined and bold values) – if one of the underlined values appears in two different bins then this is A0

a2A1: CSO (correct solution only – so no additional/repeated values)

b1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then M1 only**

b1A1: First two passes correct

b2A1ft: Third pass correct (follow through from their second pass and choice of pivots for the third pass (these pivots for the third pass must be either middle left or middle right))

b3A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including a ‘sort complete’ - this could be shown by the final list being re-written or ‘sorted’ statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice)

middle left

```

35 17 10 7 28 23 41 15 20 29 28
35 41 29 28 17 10 7 23 15 20 41, 7
41 35 29 28 17 10 23 15 20 7 35, 23
41 35 29 28 23 17 10 15 20 7 (29), 10
41 35 29 28 23 17 15 20 10 7 15
41 35 29 28 23 17 20 15 10 7 17
41 35 29 28 23 20 17 15 10 7 Sort complete

```

Question Number	Scheme	Marks
SC for (b): If using an incorrect list from the start of (b) with only one error (an error is either one missing number, one extra number, two numbers transposed or one incorrect number) then the most they can score is M1A0A1ftA0 Sorting list into ascending order in (b) • If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in (b) • If the list is not reversed in (b) then remove the last two A marks earned in (b). If the list is reversed at the start of (c) but not in (b) then still remove the last two A marks earned in (b). If the list is in ascending order in (b) award no marks for first-fit increasing in (c). If the candidate says that the list needs reversing in (b) but does not actually show the reversed list in (b) then remove the last A mark earned c1M1: Must be using a list that is in strictly descending order. If it is clear that their list is not in descending order then M0. First five items placed correctly (the bold values) and at least eight values placed in bins – condone cumulative totals for M1 only – if one of the bold values appears in more than one bin then M0. If it is clear that their list is not correct (41 35 29 28 23 20 17 15 10 7) then M1 only and for this M mark allow them to be using their final list from (b) which can contain one error and in this case an error is either one missing number or one extra number or one incorrect number (e.g. 24 for 23) but their list must be in descending order. c1A1: First seven items placed correctly (the underlined and bold values) – if one of the underlined values appears in two different bins then this is A0 c2A1: CSO (so no additional/repeated values) d1B1: $x > 8$ d2B1: $x < 12$ If B0 B0 then award B1 B0 for $8 \leq x \leq 12$ (oe) d3B1: $x < y$ but not $y > 8$ only For full marks in (d) no additional incorrect constraints, for example, if B1B1B1 initially awarded but an additional incorrect constraint seen (e.g. $y > 15$) then award B1B1B0 but do not penalise any additional incorrect constraints unless all three other correct constraints seen		

Question Number	Scheme	Marks
5. (a)		M1 A1 A1 A1 A1 (5)
(b)	Activities E, I, J and L cannot be critical	M1 A1 (2)
		7 marks

Notes for Question 5

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'F dealt with correctly' requires the correct precedences for this activity, i.e. A, B and C labelled correctly and leading into the same node and F starting from that node but do not consider the end event for F. The table below is very useful in checking this point. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be) '

Ignore incorrect or lack of arrows on the activities for the first four marks only

a1M1: Seven activities (labelled on arc), one start and at least two dummies placed

a1A1: Activities A, B, C, 1st two dummies (including correct arrows on these two dummies), D and E dealt with correctly. The first two dummies are those at the end of activities A and C

a2A1: A dummy linking D, F and G into an event node and a dummy linking D, F, G, I and E into an event node and F, G and I dealt with correctly. The dummies must have arrows

a3A1: Activities H, J, K and L dealt with correctly

a4A1: CSO – all arrows correctly placed for each activity with one finish and at most four dummies. Note that this is not a unique solution due to for example the 3rd and 4th dummies so please check these carefully.

Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only.

Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

b1M1: At least two correct activities with no more than 5 activities stated

b1A1: All four activities (E, I, J and L) and no others

A	B	C	D	E	F	G	H	I	J	K	L
-	-	-	A	C	A, B, C	A, B, C	D, F, G	A, B, C	D, F, G	H	D, E, F, G, I

Question Number	Scheme	Marks																																																																						
2.(a)(i)	In the first pass of a bubble sort we compare the first value with the second and swap if the first is larger than the second. We then compare the value that is second with the third value and swap if the second is larger than the third. Continue like this until the end of the list.	M1 A1																																																																						
(a)(ii)	Bubble sort stops when we either have a list of length 1 to sort or we have a pass in which no swaps were made.	B1 B1 (4)																																																																						
(b)	Maximum number of passes is 3 as only the three largest values are in the correct position.	B1 B1dep (2)																																																																						
(c)	<table><tr><td>0.9</td><td>1.2</td><td>1.5</td><td>0.5</td><td>1.4</td><td>1.1</td><td>0.7</td><td>1.7</td><td>2.2</td><td>3.2</td></tr><tr><td>0.9</td><td>1.2</td><td>0.5</td><td>1.4</td><td>1.1</td><td>0.7</td><td>1.5</td><td>1.7</td><td>2.2</td><td>3.2</td></tr><tr><td>0.9</td><td>0.5</td><td>1.2</td><td>1.1</td><td>0.7</td><td>1.4</td><td>1.5</td><td>1.7</td><td>2.2</td><td>3.2</td></tr><tr><td>0.5</td><td>0.9</td><td>1.1</td><td>0.7</td><td>1.2</td><td>1.4</td><td>1.5</td><td>1.7</td><td>2.2</td><td>3.2</td></tr><tr><td>0.5</td><td>0.9</td><td>0.7</td><td>1.1</td><td>1.2</td><td>1.4</td><td>1.5</td><td>1.7</td><td>2.2</td><td>3.2</td></tr><tr><td>0.5</td><td>0.7</td><td>0.9</td><td>1.1</td><td>1.2</td><td>1.4</td><td>1.5</td><td>1.7</td><td>2.2</td><td>3.2</td></tr><tr><td>0.5</td><td>0.7</td><td>0.9</td><td>1.1</td><td>1.2</td><td>1.4</td><td>1.5</td><td>1.7</td><td>2.2</td><td>3.2</td></tr></table>	0.9	1.2	1.5	0.5	1.4	1.1	0.7	1.7	2.2	3.2	0.9	1.2	0.5	1.4	1.1	0.7	1.5	1.7	2.2	3.2	0.9	0.5	1.2	1.1	0.7	1.4	1.5	1.7	2.2	3.2	0.5	0.9	1.1	0.7	1.2	1.4	1.5	1.7	2.2	3.2	0.5	0.9	0.7	1.1	1.2	1.4	1.5	1.7	2.2	3.2	0.5	0.7	0.9	1.1	1.2	1.4	1.5	1.7	2.2	3.2	0.5	0.7	0.9	1.1	1.2	1.4	1.5	1.7	2.2	3.2	M1 A1 A1ft A1cso (4)
0.9	1.2	1.5	0.5	1.4	1.1	0.7	1.7	2.2	3.2																																																															
0.9	1.2	0.5	1.4	1.1	0.7	1.5	1.7	2.2	3.2																																																															
0.9	0.5	1.2	1.1	0.7	1.4	1.5	1.7	2.2	3.2																																																															
0.5	0.9	1.1	0.7	1.2	1.4	1.5	1.7	2.2	3.2																																																															
0.5	0.9	0.7	1.1	1.2	1.4	1.5	1.7	2.2	3.2																																																															
0.5	0.7	0.9	1.1	1.2	1.4	1.5	1.7	2.2	3.2																																																															
0.5	0.7	0.9	1.1	1.2	1.4	1.5	1.7	2.2	3.2																																																															
(d)	Bin 1: 3.2 0.7 Bin 2: 2.2 1.7 Bin 3: 1.5 1.4 1.1 Bin 4: 1.2 0.9 0.5	M1 A1 A1(3)																																																																						
		13 marks																																																																						

Notes for Question 2

ai1M1: Compare first value with second value (allow ‘compare first and second’) and swap if first is larger (oe) - allow ‘wrong order’ for M1 only – must be clear that the first value is being compared with the second value

ai1A1: Compare second with third (not just ‘next two’) and so on until the end of the list (oe e.g. ‘all’, ‘last two’, ‘last’) – must be clear that **all** the list of numbers has been considered

aii1B1: CAO – one of ‘until only one item left’ (oe e.g. ‘stops after $n-1$ passes’ but not just a statement along the lines of ‘until all the required passes have been done’) or ‘until no swaps’ (oe e.g. ‘one pass gives the same result as the next pass’) – no marks though if they only say ‘when the list is in order’ (oe)

aii2B1: CAO – both reasons stated correctly

b1B1: CAO (3)

b2B1dep: Correct reasoning – dependent on previous B mark – must mention that the three largest numbers are in the correct position (and not just that 1.7, 2.2 and 3.2 are in the correct position)

c1M1: Bubble sort. Consistent direction throughout sort, and first pass correct – do check these carefully as some candidates show the result of each comparison and swap in their first pass. No marks for quick sort or descending order

c1A1: Second and third passes correct – so end six numbers in place after third pass

c2A1ft: Their fourth and fifth passes correct following through from the candidate’s third pass – so end eight numbers in place after fifth pass

c3A1: CSO (correct solution only) – with sixth pass showing no swaps (not just a statement that the list is in order after a fifth pass)

d1M1: First four items placed correctly (the boxed values). Condone cumulative totals for M1 only

d1A1: First eight items placed correctly (the boxed and underlined values) – any additional/repeated values scores M1 only

d2A1: CSO

Question Number	Scheme	Marks									
3. (a)	Bin 1: 2.6 0.8 1.2 0.3 Bin 2: 2.1 <u>0.9</u> <u>1.7</u> Bin 3: <u>2.3</u> 1.8 Bin 4: 2.7	M1 <u>A1</u> A1 (3)									
(b)(i)	First pass: 2.6 2.1 1.2 0.9 1.7 2.3 0.8 1.8 2.7 0.3 Second pass: 2.6 2.1 1.2 1.7 2.3 0.9 1.8 2.7 0.8 0.3	B1 B1									
(b)(ii)	<table border="1"> <thead> <tr> <th></th><th>Comparisons</th><th>Swaps</th></tr> </thead> <tbody> <tr> <td>First pass</td><td>9</td><td>7</td></tr> <tr> <td>Second pass</td><td>8</td><td>4</td></tr> </tbody> </table>		Comparisons	Swaps	First pass	9	7	Second pass	8	4	B1 B1 (4)
	Comparisons	Swaps									
First pass	9	7									
Second pass	8	4									
(c)	e.g. middle right Pivot(s) 2.6 2.1 1.7 2.3 1.2 <u>1.8</u> 2.7 0.9 0.8 0.3 1.8 2.6 2.1 <u>2.3</u> 2.7 1.8 1.7 1.2 <u>0.9</u> 0.8 0.3 2.3, 0.9 2.6 <u>2.7</u> 2.3 2.1 1.8 1.7 <u>1.2</u> 0.9 0.8 <u>0.3</u> 2.7, (2.1), 1.2, 0.3 2.7 2.6 2.3 2.1 1.8 1.7 1.2 0.9 0.8 0.3 Sort complete	M1 A1 A1 (3)									
(d)	Bin 1: 2.7 2.3 Bin 2: 2.6 2.1 0.3 Bin 3: <u>1.8</u> <u>1.7</u> <u>1.2</u> Bin 4: 0.9 0.8	M1 <u>A1</u> A1 (3)									
		13 marks									

Notes for Question 3

a1M1: First four items placed correctly and at least seven values placed in bins. Condone cumulative totals for M1 only (the values in bold)

a1A1: First seven items placed correctly (the underlined and bold values) – any repeated/additional values then A0

a2A1: CSO (correct solution only – so no additional/repeated values)

bi1B1: CAO (first pass) – some candidates may show each comparison/swap within the first pass so take the first pass to be the list when the 0.3 is in the correct position

bi2B1: CAO (second pass) – some candidates may show each comparison/swap within the second pass so take the second pass to be the list when the 0.8 and the 0.3 are in the correct positions

ISW if completing more than two passes

bii1B1: Two correct values in the given table

bii2B1: Fully correct table completed

Mark table on page 2 of the AB (and ignore any other answers given elsewhere) but if table blank then check answer space carefully and mark this attempt instead



c1M1: Quick sort – pivot, p, chosen (must be choosing middle left or middle right – choosing first/last item as a pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then M1 only. If sorting into ascending order then M0**

c1A1: First two passes correct (second pass pivot consistent with choice of pivot in first pass) – but need not be choosing pivots for the third pass

c2A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including a ‘sort complete’ - this could be shown by the final list being re-written or ‘sorted’ statement (e.g. ‘done’, ‘complete’, etc.) or each item being used as a pivot (which would therefore mean that the final list would have been written twice)

middle left:

2.6	2.1	1.7	2.3	<u>1.2</u>	1.8	2.7	0.9	0.8	0.3		1.2
2.6	2.1	<u>1.7</u>	2.3	1.8	2.7	1.2	0.9	<u>0.8</u>	0.3		1.7, 0.8
2.6	2.1	<u>2.3</u>	1.8	2.7	1.7	1.2	0.9	0.8	0.3		2.3, (0.9), (0.3)
<u>2.6</u>	2.7	2.3	<u>2.1</u>	1.8	1.7	1.2	0.9	0.8	0.3		2.6, 2.1
2.7	2.6	2.3	2.1	1.8	1.7	1.2	0.9	0.8	0.3		Sort Complete

Two Special Cases for (c): Case I: Those that perform a quick sort on the original list can score M1 only. Case II: Those that perform a quick sort on 2.6 2.1 1.7 2.3 1.2 1.8 2.7 (so not including the last three numbers in the list) can score M1A1 only

No misreads in (d) – mark according to scheme in all cases

d1M1: First four items placed correctly and at least seven values placed in bins – condone cumulative totals for M1 only (the bold values)

d1A1: First seven items placed correctly (the underlined and bold values) – any repeated/additional values is A0

d2A1: CSO (so no additional/repeated values)

Question number	Scheme	Marks
3.(a)	Bin 1: 1.8 1.4 1.6 Bin 2: 2.6 <u>0.9</u> <u>0.8</u> 0.6 Bin 3: <u>2.8</u> 1.2 Bin 4: <u>3.1</u> Bin 5: 2.4	M1 A1 A1 (3)
(b)(i)	1.8 2.6 1.6 2.8 1.4 3.1 0.9 1.2 2.4 0.8 0.6	B1
(ii)	Comparisons: 10 Swaps: 6	B1 B1 (3)
(c)	e.g. using middle right 2.6 1.8 2.8 1.6 3.1 <u>1.4</u> 1.2 2.4 0.9 0.8 0.6 pivot 1.4 2.6 1.8 2.8 <u>1.6</u> 3.1 2.4 <u>1.4</u> 1.2 0.9 <u>0.8</u> 0.6 pivots 1.6 0.8 2.6 1.8 <u>2.8</u> 3.1 2.4 <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivots 2.8 0.9 (0.6) 3.1 <u>2.8</u> 2.6 <u>1.8</u> 2.4 <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivot(s) (3.1) 1.8 (1.2) 3.1 <u>2.8</u> 2.6 <u>2.4</u> <u>1.8</u> <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivot 2.4 3.1 <u>2.8</u> 2.6 <u>2.4</u> <u>1.8</u> <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 (sort complete) e.g. using middle left 2.6 1.8 2.8 1.6 3.1 <u>1.4</u> 1.2 2.4 0.9 0.8 0.6 pivot 1.4 2.6 1.8 <u>2.8</u> 1.6 3.1 2.4 <u>1.4</u> 1.2 <u>0.9</u> 0.8 0.6 pivots 2.8 0.9 3.1 <u>2.8</u> 2.6 <u>1.8</u> 1.6 2.4 <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivots (3.1) 1.8 (1.2) 0.8 3.1 <u>2.8</u> <u>2.6</u> 2.4 <u>1.8</u> 1.6 <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivot(s) 2.6 (1.6) (0.6) 3.1 <u>2.8</u> <u>2.6</u> 2.4 <u>1.8</u> <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 (sort complete)	M1 A1 A1ft A1
(d)	Bin 1: 3.1 1.8 Bin 2: 2.8 <u>1.6</u> 0.6 Bin 3: 2.6 2.4 Bin 4: <u>1.4</u> <u>1.2</u> 0.9 0.8	M1 A1 A1 (3)
		(13 marks)

Notes for Question 3

PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE IN PART (c) AND THE GUIDANCE FOR THE M MARK IN (d)

a1M1: The **correct** first four items placed correctly (the bold values) and at least eight values placed in bins (allow repeated values). Condone cumulative totals for M1 only

a1A1: First eight values placed correctly (the bold **and** underlined values) with all eleven correct values only placed in bins. This mark cannot be awarded if **any** repeated values or incorrect values are seen (even if the first eight values have been placed correctly)

a2A1: CSO – no additional or repeated values (dependent on both previous marks)

bi1B1: CAO – isw after one complete pass. Please check these carefully as some candidates show all the swaps and comparisons in the first pass and some show more than one complete pass. As a guide consider the placement of the 0.8 (when this is the second value from the right-hand side this will indicate the completion of the first pass)



bii1B1: Comparisons correct (10)

bii2B1: Swaps correct (6)

If the comparisons and swaps are not labelled then assume that the first number seen is the comparisons and the second number is the swaps (so seeing after the 1st pass of bubble sort the numbers 10, 6 then award both the second and third B marks in this part). If all they state is 6 then 10 then give **SC B1 B0** for the final two marks in this part

c1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then M1 only**

c1A1: First pass correct and choosing consistent pivots for the second pass for this mark

c2A1ft: Second and third pass correct following through from their first pass and choice of pivots for the second pass these pivots for the second pass must be consistent (either both middle left or both middle right)

c3A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including a fifth pass shown (not just saying ‘sort complete’ after the fourth pass) for ‘middle right’ or a fourth pass shown for ‘middle left’

SC for (c): If using the original list or an incorrect list from the start of (c), or after the first pass, with only one error (an error is either one missing number, one extra number, two numbers transposed or one incorrect number) then they can score at most M1A0A1ftA0. If the candidate sorts into ascending order they can score M1 as per the main scheme (but with the values either side of the pivot reversed), A1 for a fully correct sort then A0 A0 even if the list is reversed at the end (so 2 marks max.).

d1M1: **Their** five largest items placed correctly and at least eight values placed in bins (if correct this will be the bold items but must check **their** packing if any of **their** five largest values are incorrect – note that the maximum weight of a bin is 5). Condone cumulative totals for M1 only. First-fit increasing scores no marks in this part. If no sort seen in (c) then mark (d) assuming the correct ordered list is being used

d1A1: First eight values placed correctly (the bold **and** underlined values) with all eleven correct values only placed in bins. This mark cannot be awarded if **any** repeated values or incorrect values are seen (even if the first eight values have been placed correctly)

d2A1: CSO – no additional or repeated values (dependent on both previous marks)



Question number	Scheme	Marks																																										
4.(a)	<table><tr><th>Activity</th><th>Immediately preceded by</th><th>Activity</th><th>Immediately preceded by</th><th>Activity</th><th>Immediately preceded by</th></tr><tr><td>A</td><td>-</td><td>G</td><td>A, B, E</td><td>M</td><td>D, G</td></tr><tr><td>B</td><td>-</td><td>H</td><td>A, B, E</td><td>N</td><td>H, K</td></tr><tr><td>C</td><td>-</td><td>I</td><td>A, B, E</td><td>P</td><td>H, K</td></tr><tr><td>D</td><td>A</td><td>J</td><td>A, B, E, F</td><td>Q</td><td>H, I, J, K</td></tr><tr><td>E</td><td>C</td><td>K</td><td>D, G</td><td>R</td><td>P, Q</td></tr><tr><td>F</td><td>C</td><td>L</td><td>D, G</td><td></td><td></td></tr></table>	Activity	Immediately preceded by	Activity	Immediately preceded by	Activity	Immediately preceded by	A	-	G	A, B, E	M	D, G	B	-	H	A, B, E	N	H, K	C	-	I	A, B, E	P	H, K	D	A	J	A, B, E, F	Q	H, I, J, K	E	C	K	D, G	R	P, Q	F	C	L	D, G			B2, 1, 0 (2)
	Activity	Immediately preceded by	Activity	Immediately preceded by	Activity	Immediately preceded by																																						
	A	-	G	A, B, E	M	D, G																																						
	B	-	H	A, B, E	N	H, K																																						
	C	-	I	A, B, E	P	H, K																																						
	D	A	J	A, B, E, F	Q	H, I, J, K																																						
	E	C	K	D, G	R	P, Q																																						
F	C	L	D, G																																									
(b)		M1 A1 M1 A1 (4)																																										
(c)	Critical activities: C, E, G, K and N	B1 (1)																																										
(d)	Float on J = 35 – 22 – 5 = 8	B1ft (1)																																										
(e)	Lower bound is $\frac{133}{43} = 3.0930... = 4$	B1 (1)																																										
(f)	e.g. 	M1 A1 A1 A1 (4)																																										
		(13 marks)																																										

Notes for Question 4

a1B1: Any four (of the eight blank) rows correct

a2B1: All eight rows correct

b1M1: All top boxes complete, values generally increasing in the direction of the arrows (so generally going from 'left to right' across the network), condone one 'rogue' value (if values do not increase in the



Question Number	Scheme	Marks
1.(a)	Bin 1: 5.2 4.7 3.1 Bin 2: 6.5 4.5 <u>1.8</u> 1.2 Bin 3: <u>5.1</u> <u>2.9</u> 3.4 Bin 4: 3.8	M1 <u>A1</u> A1(3)
(b)	e.g. middle right 5.2 4.7 6.5 4.5 3.1 5.1 1.8 2.9 3.4 3.8 1.2 4.7 4.5 3.1 1.8 2.9 3.4 3.8 1.2 <u>5.1</u> 5.2 6.5 1.8 1.2 <u>2.9</u> 4.7 4.5 3.1 3.4 3.8 <u>5.1</u> 5.2 <u>6.5</u> <u>1.2</u> 1.8 <u>2.9</u> <u>3.1</u> 4.7 4.5 3.4 3.8 <u>5.1</u> 5.2 <u>6.5</u> <u>1.2</u> 1.8 <u>2.9</u> <u>3.1</u> <u>3.4</u> 4.7 4.5 3.8 <u>5.1</u> 5.2 <u>6.5</u> <u>1.2</u> 1.8 <u>2.9</u> <u>3.1</u> <u>3.4</u> 3.8 <u>4.5</u> 4.7 <u>5.1</u> 5.2 <u>6.5</u> Pivots 5.1 2.9, 6.5 1.2, 3.1, (5.2) (1.8), 3.4 4.5 (Sort Complete)	M1 A1 A1ft A1 (4)
	e.g. middle left 5.2 4.7 6.5 4.5 3.1 5.1 1.8 2.9 3.4 3.8 1.2 4.7 4.5 3.1 1.8 2.9 3.4 3.8 1.2 <u>5.1</u> 5.2 6.5 1.2 <u>1.8</u> 4.7 4.5 3.1 2.9 3.4 3.8 <u>5.1</u> <u>5.2</u> 6.5 1.2 <u>1.8</u> 2.9 <u>3.1</u> 4.7 4.5 3.4 3.8 <u>5.1</u> <u>5.2</u> 6.5 1.2 <u>1.8</u> 2.9 <u>3.1</u> 3.4 3.8 <u>4.5</u> 4.7 <u>5.1</u> <u>5.2</u> 6.5 1.2 <u>1.8</u> 2.9 <u>3.1</u> <u>3.4</u> 3.8 <u>4.5</u> 4.7 <u>5.1</u> <u>5.2</u> 6.5 Pivots 5.1 1.8, 5.2 (1.2), 3.1, (6.5) (2.9), 4.5 3.4, (4.7) (Sort Complete)	
(c)	Bin 1: 6.5 5.2 1.8 Bin 2: 5.1 4.7 3.8 Bin 3: 4.5 <u>3.4</u> <u>3.1</u> <u>2.9</u> Bin 4: 1.2	M1 <u>A1ft</u> A1(3)
(d)	$\frac{1.2+1.8+\cdots+6.5}{14} = \frac{42.2}{14} = 3.01\dots$ so a minimum of 4 bins is required and (c) is optimal	B1 (1)
(e)	$\left\lceil \frac{1+11}{2} \right\rceil = 6$ 3.8 > 3.0 so reject 3.8 to 6.5 (or reject 6 th to 11 th items) $\left\lceil \frac{1+5}{2} \right\rceil = 3$ 2.9 < 3.0 so reject 1.2 to 2.9 (or reject 1 st to 3 rd item) $\left\lceil \frac{4+5}{2} \right\rceil = 5$ 3.4 > 3.0 so reject 3.4 (or reject 5 th item) [4] = 4 3.1 is not 3.0 so 3.0 not found	M1 A1ft A1 (3)
		14 marks

	Notes for Question 1
a1M1	The correct first five items placed correctly (the bold values) and at least eight values placed in bins (allow repeated values). Condone cumulative totals for M1 only
a1A1	First eight values placed correctly (the bold and underlined values) with all eleven correct values only placed in bins. This mark cannot be awarded if any repeated values or incorrect values are seen (even if the first eight values have been placed correctly)
a2A1	CSO – no additional or repeated values (dependent on both previous marks)
b1M1	Quick sort using all 11 numbers (condone one slip), pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values less than the pivot), pivot, (values greater than the pivot). If only choosing one pivot per iteration then M1 only. Bubble sort is M0.
b1A1	First and second passes correct and pivots correctly chosen for third pass
b2A1ft	Third and fourth passes correct following through from their second pass and choice of pivots for the third pass. The pivots for the third pass must be consistent (if appropriate either both middle left or both middle right)
b3A1	CSO (correct solution only – all previous marks in this part must have been awarded) including a fifth pass (we do not need to see “sort complete” oe)
	Special Case for (b) If the candidate sorts into descending order they can score M1 as per the main scheme (but with the values either side of the pivot reversed), A1 for a fully correct sort then A0 A0 even if the list is reversed at the end (so 2 marks max.). See below
c1M1	Their six largest items placed correctly and at least eight values placed in bins (if correct this will be the bold items but must check their packing if any of their six largest values are incorrect – note that the maximum weight of a bin is 14). Condone cumulative totals for M1 only. First-fit increasing scores no marks in this part. If no sort seen in (b) then mark (c) assuming the correct ordered list is being used
c1A1ft	Their first nine values placed correctly (the bold and underlined values) with all their eleven values only placed in bins. This mark cannot be awarded if any repeated values are seen (even if their first nine values have been placed correctly)
c2A1	CSO – must be the correct values with no additional or repeated values (dependent on both previous marks)
d1B1	CAO using either a correct lower bound calculation (accept awrt 3.01 without seeing the full calculation) or spare capacity in the first 3 bins (e.g. total spare in first three bins is $0.5 + 0.4 + 0.1 = 1 < 1.2$ so not possible to pack into 3 bins oe e.g. $42.2 > 42$ hence cannot fit in 3 bins) and conclusion of optimal or states using the minimum number of bins (dependent on scoring at least M1 in (c))
e1M1	Must be using their sorted list. Choosing their middle pivot (3.8) and an attempt at discarding/retaining half the list (condone if retaining the wrong half of the list) (If pivot is retained at any stage M1 only). The actual rejections may be shown as values from the list or the positions in the list.
e1A1ft	First and second passes correct for their list i.e. selecting the 6 th item in the first pass and using 1 st to 5 th items in the second pass (must not be using the 6 th item for the second pass) and then correctly selecting the 3 rd item (the 2.9) in the second pass and rejecting the 1 st to 3 rd items. Candidates may either compare 3.0 with the actual values or the positions of items in a correctly sorted list. Comparisons may be implied by the choice of the correct item and corresponding rejection. If a candidate rennumbers the list after a pass check the position numbers and values carefully.
e2A1	CSO – must be the correct values with the search completed correctly (so rejecting the 3.4 in the third pass) together with ‘not found’.

In (e) if candidates use the list sorted into descending order, they can score full marks. The scheme above applies in the same way. **Note there are only 3 comparisons needed.**

6.5 5.2 5.1 4.7 4.5 3.8 3.4 3.1 2.9 1.8 1.2

$\left[\frac{1+11}{2} \right] = 6$ 3.8 > 3.0 so reject 6.5 to 3.8
 $\left[\frac{7+11}{2} \right] = 9$ 2.9 < 3.0 so reject 2.9 to 1.2
 $\left[\frac{7+8}{2} \right] = 8$ 3.1 > 3.0 so reject 3.4 to 3.1
 3.0 is not in the list

For reference – descending sort

Middle right

5.2	4.7	6.5	4.5	3.1	5.1	1.8	2.9	3.4	3.8	1.2	Pivots
5.2	6.5	<u>5.1</u>	4.7	4.5	3.1	1.8	2.9	3.4	3.8	1.2	5.1
<u>6.5</u>	5.2	<u>5.1</u>	4.7	4.5	3.1	3.4	3.8	<u>2.9</u>	1.8	1.2	6.5, 2.9
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	4.5	3.4	3.8	<u>3.1</u>	<u>2.9</u>	1.8	<u>1.2</u>	(5.2), 3.1, 1.2
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	4.5	3.8	<u>3.4</u>	<u>3.1</u>	<u>2.9</u>	<u>1.8</u>	<u>1.2</u>	3.4 (1.8)
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	<u>4.5</u>	3.8	<u>3.4</u>	<u>3.1</u>	<u>2.9</u>	<u>1.8</u>	<u>1.2</u>	4.5
											(Sort Complete)

Middle left

5.2	4.7	6.5	4.5	3.1	5.1	1.8	2.9	3.4	3.8	1.2	Pivots
5.2	6.5	<u>5.1</u>	4.7	4.5	3.1	1.8	2.9	3.4	3.8	1.2	5.1
6.5	<u>5.2</u>	<u>5.1</u>	4.7	4.5	3.1	2.9	3.4	3.8	<u>1.8</u>	1.2	5.2, 1.8
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	4.5	3.4	3.8	<u>3.1</u>	2.9	<u>1.8</u>	<u>1.2</u>	(6.5), 3.1, (1.2)
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	<u>4.5</u>	3.4	3.8	<u>3.1</u>	<u>2.9</u>	<u>1.8</u>	<u>1.2</u>	4.5, (2.9)
<u>6.5</u>	<u>5.2</u>	<u>5.1</u>	4.7	<u>4.5</u>	3.8	<u>3.4</u>	<u>3.1</u>	<u>2.9</u>	<u>1.8</u>	<u>1.2</u>	(4.7), 3.4
											(Sort Complete)

Question Number	Scheme	Marks
2. (a)(i)	The diagram shows a project network with the following activities and their durations: A(5), B(4), C(7), D(4), E(3), F(3), G(2), H(4), I(6), J(7), K(7), L(3), M(2), N(2). The earliest start and finish times are indicated in the boxes. The network starts at activity A and ends at activity M. The total duration of the project is 20 days.	M1 A1 M1 A1
(a)(ii)	Minimum completion time is 20 (days)	A1ft (5)
(b)	Lower bound = $\frac{59}{20} = 2.95 = 3$ workers	B1 (1)
(c)	The Gantt chart shows the scheduling of activities A through N over a 22-day period. The chart is divided into three rows, each representing a different worker. Activity A is scheduled from day 0 to 5, B from 0 to 4, and C from 4 to 7. Activity D is scheduled from day 5 to 8, E from 7 to 10, and F from 10 to 12. Activity G is scheduled from day 15 to 17, H from 17 to 19, and I from 12 to 15. Activity J is scheduled from day 7 to 14, K from 10 to 17, and L from 17 to 20. Activity M is scheduled from day 19 to 20, and N from 19 to 20. The chart shows that the project can be completed within 22 days using 3 workers.	M1 A1 A1 A1 (4)
		10 marks

Question Number	Scheme	Marks
3. (a)	$\frac{132}{42} = 3.14...$ so lower bound is 4	M1 A1 (2)
(b)	Group 1: 8 17 9 7 Group 2: 14 18 10 Group 3: 12 22 Group 4: 15	M1 A1 (2)
(c)	e.g. middle right 8 17 9 14 18 <u>12</u> 22 10 15 7 17 14 <u>18</u> 22 15 12 8 9 <u>10</u> 7 22 18 17 <u>14</u> 15 12 10 8 <u>9</u> 7 22 18 17 <u>15</u> 14 12 10 9 8 <u>7</u> 22 18 17 15 14 12 10 9 8 7 e.g. middle left 8 17 9 14 <u>18</u> 12 22 10 15 7 22 18 8 17 9 <u>14</u> 12 10 15 7 22 18 <u>17</u> 15 14 8 9 <u>12</u> 10 7 22 18 17 15 14 12 8 <u>9</u> 10 7 22 18 17 15 14 12 10 <u>9</u> 8 7 22 18 17 15 14 12 10 9 8 7	M1 A1 A1ft A1 (4)
(d)	Group 1: 22 18 Group 2: 17 15 10 Group 3: 14 12 9 7 Group 4: 8	M1 A1 (2)
(e)	B(E)C + G(I)H = (11.2 + 14.5) + (8.3 + 17.2) = 51.2* B(F)G + C(EJ)H = (10.3 + 15.2) + (14.5 + 7.5 + 16.2) = 63.7 B(EJ)H + C(EF)G = (11.2 + 7.5 + 16.2) + (14.5 + 4.3 + 15.2) = 68.9 Repeat arcs: BE, CE, GI, HI	M1 A1 A1 A1 (4)
(f)	Route e.g. ABEBFECEJIFGIGHIJDCA Length = 227.2 + 51.2 = 278.4 (m)	B1 B1ft (2)
(g)	Finishing vertex: C Reduction in lengths: 51.2 – (10.3 + 15.2) = 25.7 (m)	B1 B1 (2)
		18 marks

Notes for Question 3

PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE FOR ASCENDING IN PART (c)

a1M1: Attempt to find the lower bound $(132 \pm 22) / 42$ (a value of 3.14 (or better) seen with no working can imply this mark)

a1A1: CSO - correct calculation seen **or** 3.14 followed by 4 – accept 3.1 if correct calculation seen. An answer of 4 with no working scores M0A0

b1M1: First six items placed correctly and at least eight items placed in bins – condone cumulative totals for M1 only (the values in bold)

b1A1: CSO (so no additional/repeated values)

c1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then M1 only**

c1A1: First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct) – so they must be choosing (if middle right) a pivot values of 18 and 10 for the second pass or (if middle left) a pivot value of 14

Question Number	Scheme	Marks
	c2A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark c3A1: CSO (correct solution only – all previous marks in this part must have been awarded) including if middle right a fourth pass with the 15 and 7 used as pivots or if middle left a fifth pass with the 8 used as a pivot Sorting list into ascending order in (c) - If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in (c) - If the list is not reversed in (c) but stated in ascending or descending order in (d) then remove the last two A marks earned in (c). If the candidate says that the list needs reversing in (c) but does not actually show the reversed list in (c) then remove the last A mark earned Note that if sorting into ascending order then a ‘sort complete’ statement is required – this could be shown by the final list being re-written or ‘sorted’ statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice) BEFORE list is reversed Middle right ascending (requires sort complete statement– see above) 8 17 9 14 18 <u>12</u> 22 10 15 7 8 9 <u>10</u> 7 12 17 14 <u>18</u> 22 15 8 <u>9</u> 7 10 12 17 <u>14</u> 15 18 22 8 <u>7</u> 9 10 12 14 17 <u>15</u> 18 22 7 8 9 10 12 14 15 17 18 22 Middle left ascending (requires sort complete statement – see above) 8 17 9 14 <u>18</u> 12 22 10 15 7 8 17 9 <u>14</u> 12 10 15 7 18 22 8 9 <u>12</u> 10 7 14 <u>17</u> 15 18 22 8 <u>9</u> 10 7 12 14 15 17 18 22 <u>8</u> 7 9 10 12 14 15 17 18 22 7 8 9 10 12 14 15 17 18 22 d1M1: First six items placed correctly and at least eight items placed in bins – condone cumulative totals for M1 only (the values in bold) d1A1: CSO (so no additional/repeated values) e1M1: Correct three pairings of the correct four odd nodes (B, C, G and H) e1A1: Any one row correct including pairings and totals e2A1: All three rows correct including pairings and totals e3A1: CAO correct arcs clearly stated: BE, CE, GI and HI – must be these arcs and not e.g. BEC, GIH, or BC via E, etc. f1B1: Any correct route (checks: 21 vertices, starting and ending at A, BE, CE, GI and HI appearing twice, A(2), B(2), C(2), D(1), E(3), F(2), G(2), H(2), I(3), J(2)) f2B1ft: For 227.2 + their smallest repeat out of a choice of at least two totals seen in (e) – this mark is dependent on M1 in (e) g1B1: CAO (C) g2B1: CAO (25.7) – note that the correct answer can come from incorrect working e.g. $11.2 + 14.5 = 25.7$ is B0 (just adding BE and EC together) so this answer need to be checked carefully – correct method is $51.2 - (10.3 + 15.2)$ (subtracting BF and FG from 51.2) but give bod on a correct answer of 25.7 with no working	

Question Number	Scheme								Marks
3.(a)	a	b	c	d	e	f	g	h	M1 A1 A1ft
	1 980	462	914 760	30/7	4	1848	132		
	462	132	(914 760)	7/2	3	396	66		
	132	66	(914 760)	2	2	132	0	13 860	
	Output 13 860								A1 (4)
(b)	Calculates the lowest common multiple of two numbers o.e. description								B1 (1)
									5 marks
Notes for Question 3									
	Note values may be written on individual rows. Please award the marks for the complete rows shown above. The values in cell c do not need to be repeated in each row, but we will condone this.								
a1M1	First row cells a to g completed – something in each cell								
a1A1	First and second row correct. Accept decimal equivalent awrt 4.29 and 3.5 (h must have no numerical value in these rows)								
a2A1ft	Third row correct – ft their first and second rows								
a3A1	13 860 in Output row CSO								
b1B1	LCM CAO (accept e.g. smallest common multiple, minimum common multiple, minimum common product, smallest number which can be divided by both a and b o.e.) (do not accept times instead of multiple or product)								

Question Number	Scheme	Marks
4.(a)	e.g.	M1 A1 A1 A1 A1 (5)
(b)	Activities A, B, C, F and H are critical	B1 (1)
Notes for Question 4		
	Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. 'H dealt with correctly' requires the correct precedences for this activity, i.e. D and E labelled correctly and leading into the same node, a dummy from the end of F into the node and H starting from that node but do not consider the end event. Activity on node is M0	
	If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be). Ignore incorrect or lack of arrows on the activities for the first four marks only	
a1M1	Nine activities (labelled on arc), one start and at least two dummies placed	
a1A1	Activities A, B, C, 1 st two dummies (including correct arrow), D, E and F dealt with correctly	
a2A1	Activities G, H and I dealt with correctly and dummy from the end of F to end of D/E (including correct arrow)	
a3A1	Activities J and K dealt with correctly and dummy from the end of F to end of G (including correct arrow)	
a4A1	CSO – all arrows correctly placed for each activity with one finish and exactly five dummies (so uniqueness dummy for J/K required)	

Question Number	Scheme	Marks																					
3.(a)	<table><tr><th>a</th><th>b</th><th>Qn</th></tr><tr><td>1</td><td>0.967168(21)</td><td>N</td></tr><tr><td>0.967168(21)</td><td>0.972789(34)</td><td>N</td></tr><tr><td>0.972789(34)</td><td>0.971833(85)</td><td>N</td></tr><tr><td>0.971833(85)</td><td>0.971996(47)</td><td>N</td></tr><tr><td>0.971996(47)</td><td>0.971968(80)</td><td>N</td></tr><tr><td>0.971968(80)</td><td>0.971973(50)</td><td>Y</td></tr></table>	a	b	Qn	1	0.967168(21)	N	0.967168(21)	0.972789(34)	N	0.972789(34)	0.971833(85)	N	0.971833(85)	0.971996(47)	N	0.971996(47)	0.971968(80)	N	0.971968(80)	0.971973(50)	Y	M1
	a	b	Qn																				
	1	0.967168(21)	N																				
	0.967168(21)	0.972789(34)	N																				
	0.972789(34)	0.971833(85)	N																				
	0.971833(85)	0.971996(47)	N																				
	0.971996(47)	0.971968(80)	N																				
	0.971968(80)	0.971973(50)	Y																				
	A1																						
	A1																						
	A1																						
Final output is 0.97197	(4)																						
(b)	Consideration of $12 - 5a$ being negative (oe)	M1																					
	$a > 2.4$	A1 (2)																					
		6 marks																					

Notes for Question 3

Candidates may write each value for a , b and N/Y (or No/Yes) in a new row which is fine. Assume that each row begins and ends when a value is changed. For example, the values in row 2 in the table above consists of the a value of 0.967168 and the b value of 0.972789. In (a) for the first three marks the values for a and b must be given to at least 6 decimal places (but accepted either rounded or truncated values)

a1M1: At least **three** rows of cells for columns a and b completed with a correct first row (so 1 for a and 0.967168 for b)

a1A1: CAO for the first three rows (for just the a and b columns) – to at least 6 dp

a2A1: CAO for the fourth and fifth rows (for just the a and b columns) – to at least 6 dp

a3A1: CSO - Output **must** be correct and written in the 'Final Output' box at the bottom of the page. This value must be given as 0.97197 (do not accept any other value). Candidates **must** have completed the third column of the table correctly to score this mark

b1M1: Consideration that the fourth root cannot be negative – allow just stating the critical value of 2.4 for this mark or stating or implying one of $f(a) \leq 0$ or $f(a) = 0$ or $f(a) < 0$ where

$$f(a) = 12 - 5a \text{ or } f(a) = \frac{12 - 5a}{8} \text{ or } f(a) = (12 - 5a)^{\frac{1}{4}} \text{ or } f(a) = \left(\frac{12 - 5a}{8}\right)^{\frac{1}{4}}$$

Allow any letter for a (e.g. x)

b1A1: CAO (must be a strict inequality and must be using a)

Question Number	Scheme								Marks
4. (a)		A	B	C	D	E	F	G	B2, 1, 0 (2)
	A	-	21	42	17	25	31	41	
	B	21	-	26	27	12	15	20	
	C	42	26	-	32	17	11	46	
	D	17	27	32	-	15	21	47	
	E	25	12	17	15	-	6	32	
	F	31	15	11	21	6	-	35	
	G	41	20	46	47	32	35	-	
(b)	NNA starting at A: A – D – E – F – C – B – G – A 17 + 15 + 6 + 11 + 26 + 20 + 41 = 136 (km)								M1 A1 (2)
(c)	A – D – E – F – C – F – B – G – B – A								B1 (1)
(d)	RMST weight = 12 + 6 + 11 + 15 + 20 = 64								B1
	64 + 17 (AD) + 21 (AB) = 102 (km)								M1 A1 (3)
(e)	AC + EG = 42 + 32 = 74								M1
	AE + CG = 25 + 46 = 71								A1
	AG + CE = 41 + 17 = 58*								A1
	Repeat arcs: AB, BG, CF, EF								A1
	Length: 291 + 58 = 349 (km)								A1ft (5)
									13 marks
Notes for Question 4									
Condone lack of, or incorrect units throughout this question									
a1B1: At least two of the six values correct (in either table) – these are the bold values. The two values can be the same (for example, 42 in both cells AC and CA would score this mark)									
a2B1: Fully correct (the six bold values in both tables)									
b1M1: Nearest neighbour route starting at A – must have at least A – D – E – F – C – B – ... Allow if stated in terms of arcs (AD, DE, EF, FC, CB,...) rather than nodes (note that arcs AD and DA are equivalent and so therefore AD, ED, EF, FC, CB,... is acceptable)									
b1A1: CAO on length (136) and route (must return to A and can be stated in terms of arcs rather than nodes)									
c1B1: CAO (ADEFCFBGBA or in terms of arcs, e.g., AD, DE, EF, FC, CF, FB, BG, GB, BA)									
d1B1: CAO on the weight of the RMST (using arcs BE, EF, CF, DE, BG) – accept either 64 or 12 + 6 + 11 + 15 + 20 – could be seen or implied by later working as part of the lower bound calculation									
d1M1: Adding correct two least weighted arcs from A (17(AD) + 21(AB)) to their RMST length where their RMST length x is $58 \leq x \leq 70$									
d1A1: CAO (102). If this answer is stated with no working then award B0M1A1									
e1M1: Correct three distinct pairings of the correct four odd nodes (A, C, E and G)									
e1A1: Any two rows correct including pairings and totals									
e2A1: All three rows correct including pairings and totals									
e3A1: CAO correct edges clearly stated and not just in their working as AB, BG, CF and EF – must be these arcs. Do not accept AG, ABG or AG via B (and similarly for CE)									