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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Graphs, Networks & Shortest Paths (D1.2)

International A Level Mathematics · D1: Decision Mathematics 1 (WDM11)

12 questions · 122 marks · Mark schemes included

Past papers June 2019 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Where a question is answered in a separate answer book, those answer book pages follow the question.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
D1.2a	MINIMUM SPANNING TREES (KRUSKAL & PRIM)	8 Qs · 83 marks	2
1. Q1	IAL D1 (WDM11) October 2020	7	3
2. Q2	IAL D1 (WDM11) January 2022	7	6
3. Q2	IAL D1 (WDM11) January 2020	8	9
4. Q3	IAL D1 (WDM11) June 2024	8	12
5. Q5	IAL D1 (WDM11) January 2023	9	15
6. Q3	IAL D1 (WDM11) June 2022	14	18
7. Q7	IAL D1 (WDM11) June 2021	15	24
8. Q3	IAL D1 (WDM11) October 2021	15	25
D1.2b	DIJKSTRA'S ALGORITHM & SHORTEST PATHS	4 Qs · 39 marks	30
9. Q4	IAL D1 (WDM11) June 2025	8	31
10. Q5	IAL D1 (WDM11) June 2021	10	34
11. Q1	IAL D1 (WDM11) October 2021	10	35
12. Q2	IAL D1 (WDM11) June 2019	11	38



QUESTIONS

12 questions · 122 marks · 2 sub-topics

SUB-TOPIC D1.2a

MINIMUM SPANNING TREES (KRUSKAL & PRIM)

Sub-topic D1.2a · 8 questions · 83 marks · lowest tariff first



Write your answers in the D1 answer book for this paper.

1. The table below shows the distances, in metres, between six vertices, A, B, C, D, E and F, in a network.

	A	B	C	D	E	F
A	–	18	23	17	28	19
B	18	–	20	11	–	24
C	23	20	–	–	25	13
D	17	11	–	–	–	22
E	28	–	25	–	–	–
F	19	24	13	22	–	–

- (a) Draw the weighted network using the vertices given in Diagram 1 in the answer book. (2)
- (b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the edges in the order that you consider them and state whether you are adding them to your minimum spanning tree. (3)
- (c) Draw the minimum spanning tree on Diagram 2 in the answer book and state its total weight. (2)

(Total 7 marks)



1.

	A	B	C	D	E	F
A	–	18	23	17	28	19
B	18	–	20	11	–	24
C	23	20	–	–	25	13
D	17	11	–	–	–	22
E	28	–	25	–	–	–
F	19	24	13	22	–	–

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Diagram 1

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Question 1 continued

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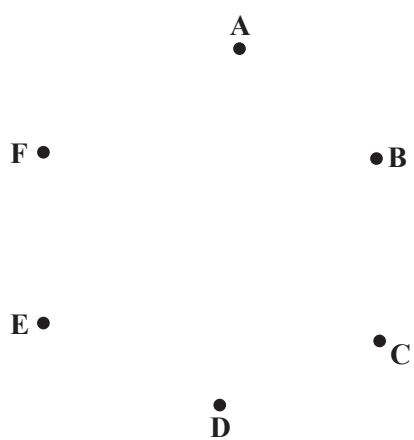


Diagram 2

Weight of minimum spanning tree _____

Q1

(Total 7 marks)



2.

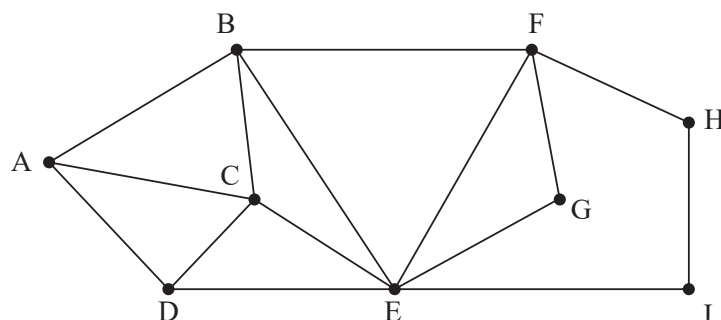


Figure 1

Figure 1 shows a graph, T.

(a) Write down an example of a path from A to J on T.

(1)

(b) State, with a reason, whether $A - B - C - D - E - G - F - H - J$ is an example of a tour on T.

(1)

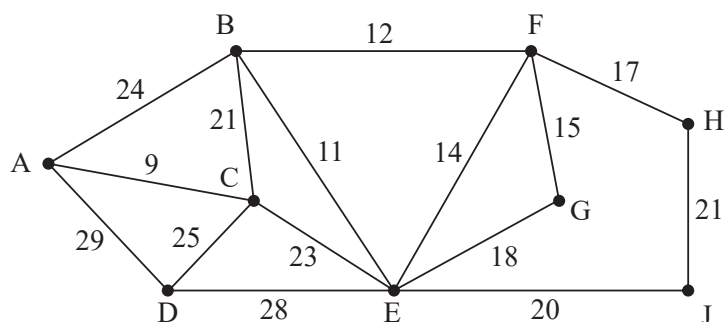


Figure 2

The numbers on the 15 arcs in Figure 2 represent the distances, in km, between nine vertices, A, B, C, D, E, F, G, H and J, in a network.

(c) Use Kruskal's algorithm to find the minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether or not you are adding the arc to the minimum spanning tree.

(3)

(d) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.

(1)

(e) State the weight of the minimum spanning tree.

(1)

(Total 7 marks)



2.

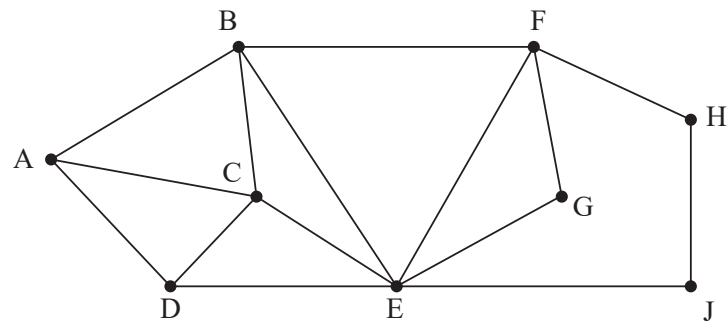


Figure 1

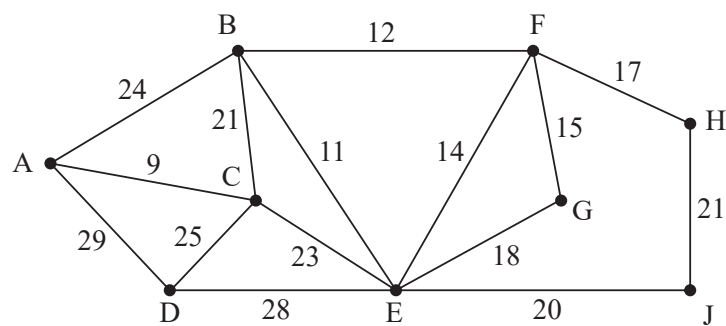


Figure 2

Question 2 continued

Diagram 1

Weight of the minimum spanning tree: _____

(Total 7 marks)



2.

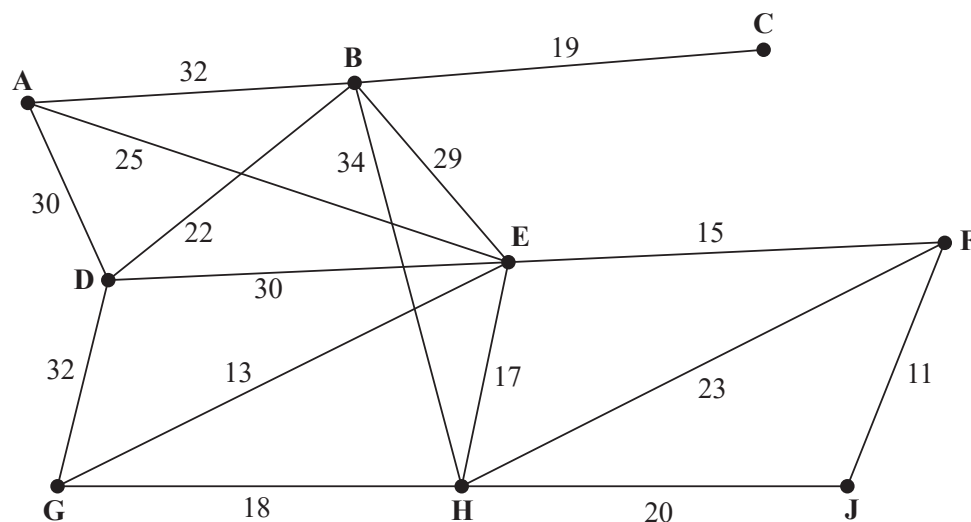


Figure 1

- (a) Define the terms
- tree,
 - minimum spanning tree.
- (3)
- (b) Use Kruskal's algorithm to find the minimum spanning tree for the network shown in Figure 1. You must clearly show the order in which you consider the edges. For each edge, state whether or not you are including it in the minimum spanning tree.
- (3)
- (c) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book and state the weight of the minimum spanning tree.
- (2)

(Total 8 marks)

2.

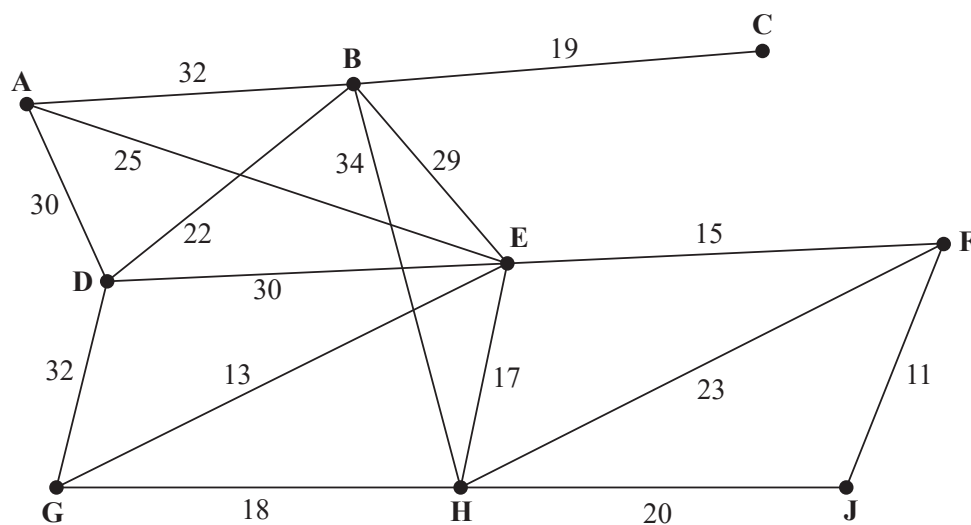


Figure 1

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Question 2 continued

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Diagram 1

Weight of the minimum spanning tree _____

(Total 8 marks)

Q2



3.

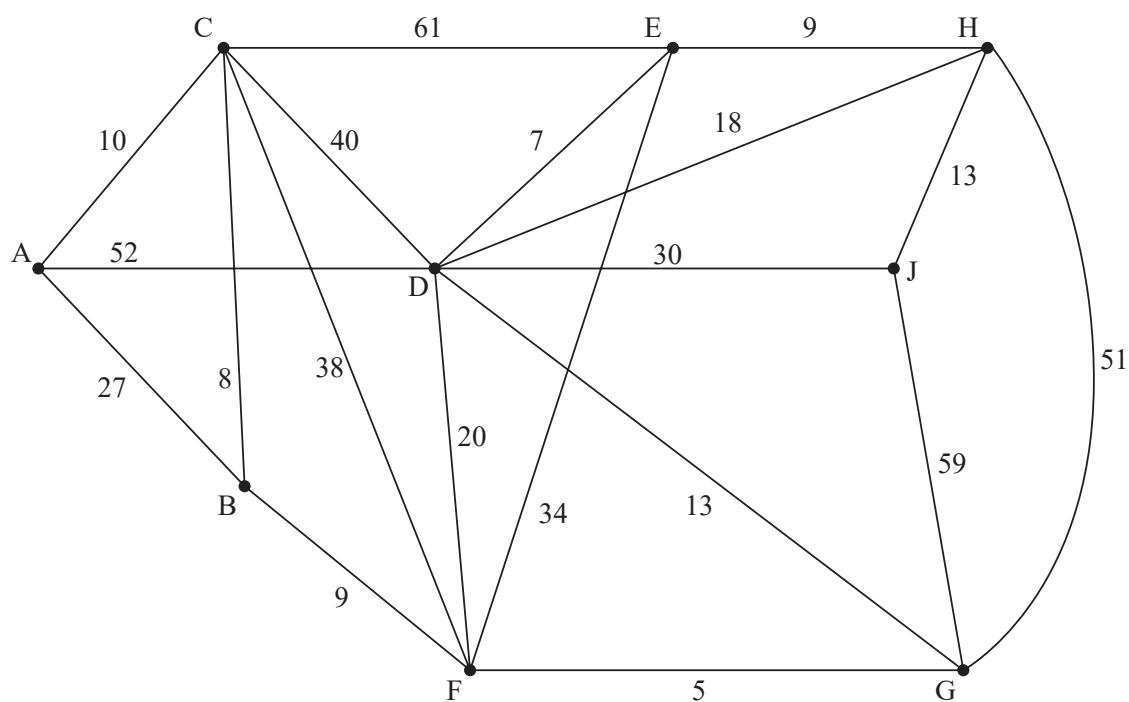


Figure 2

Figure 2 models a network of tracks between nine ranger stations, A, B, C, D, E, F, G, H and J, in a forest. The number on each edge gives the time, in minutes, to travel along the corresponding track. The forest ranger wishes to travel from A to J as quickly as possible.

- (a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to J.

State the quickest route.

(6)

- (b) Hence determine the weight of the minimum spanning tree for the network given in Figure 2. Give a reason for your answer.

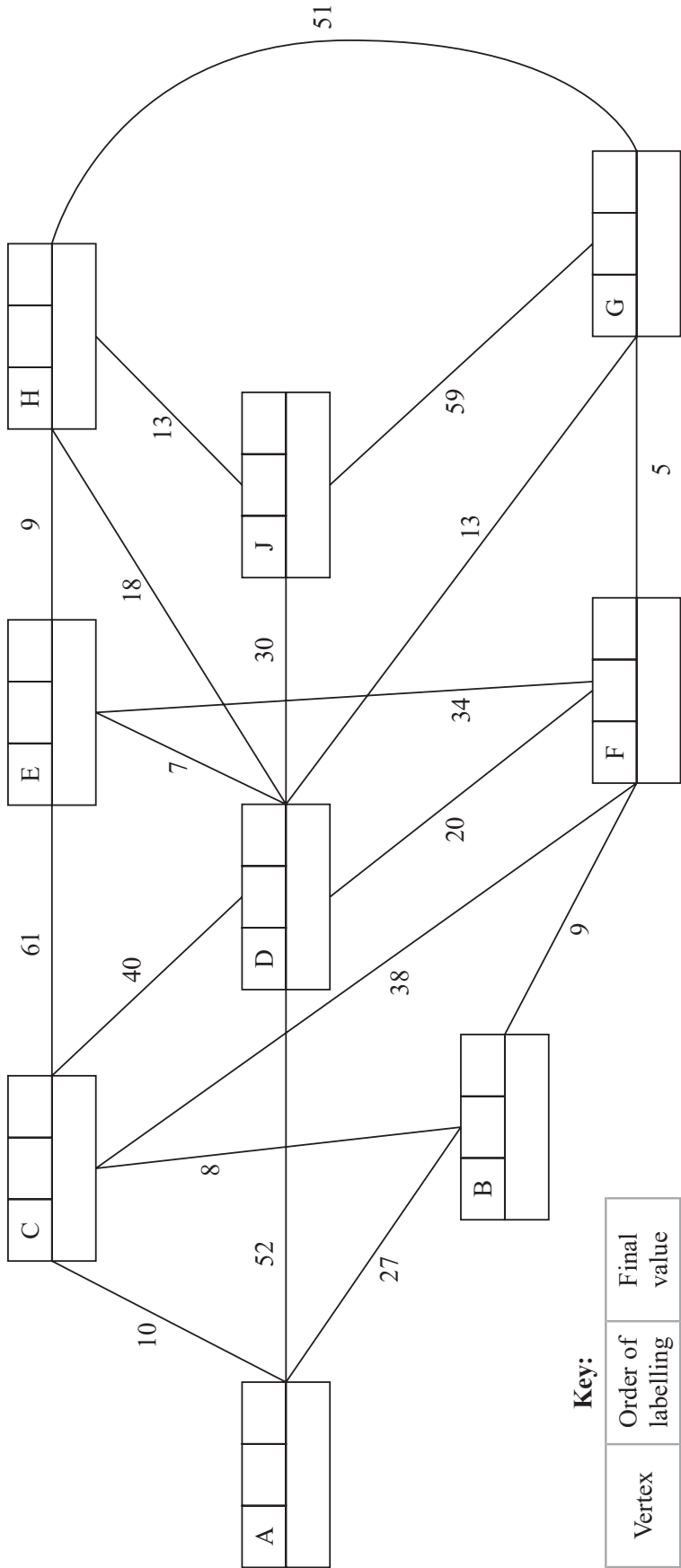
You do not need to find the minimum spanning tree.

(2)

(Total for Question 3 is 8 marks)



3.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest time to travel from A to J:

Quickest route from A to J:



Question 3 continued

(Total for Question 3 is 8 marks)



5.

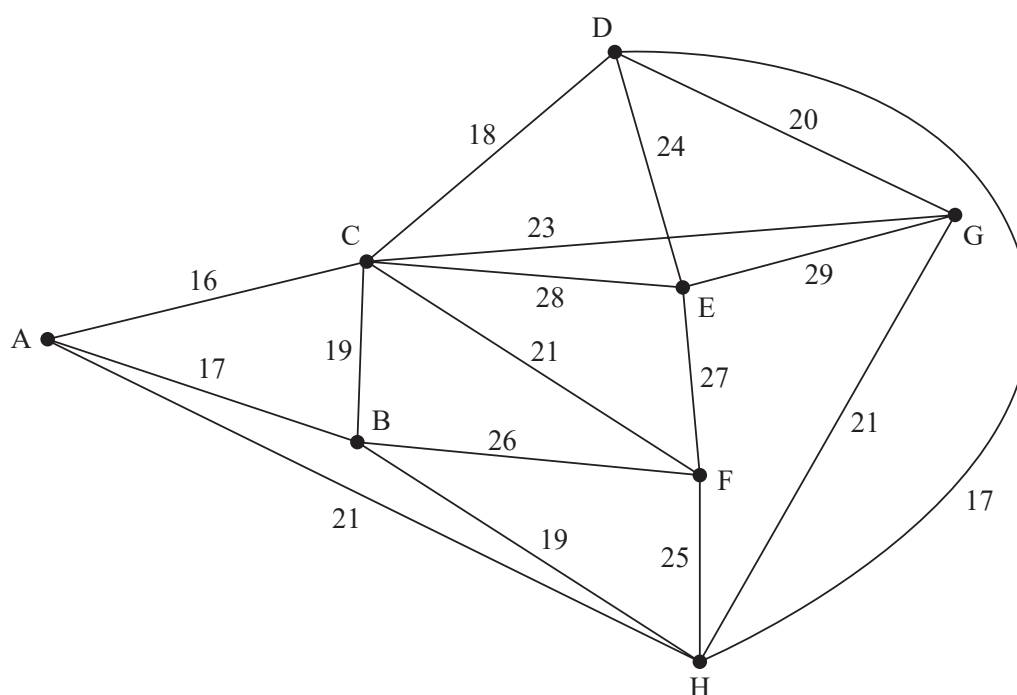


Figure 3

- (a) Explain why it is impossible to draw a graph with eight vertices in which the vertex orders are 1, 2, 2, 3, 3, 4, 4 and 6

(1)

Figure 3 shows the network T. The numbers on the arcs represent the distances, in km, between the eight vertices, A, B, C, D, E, F, G and H.

- (b) Determine whether or not $A - C - D - E - C - B - F$ is an example of a path on T. You must justify your answer.
- (c) Use Prim's algorithm, starting at A, to find the minimum spanning tree for T. You must clearly state the order in which you select the arcs of the tree.
- (d) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.

(1)

The weight of arc CF is now increased to a value of x . The minimum spanning tree for T is unique and includes the same arcs as those found in (c).

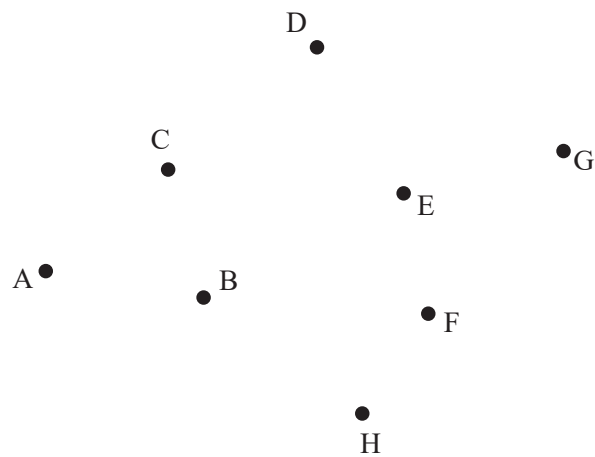
- (e) Write down the smallest interval that must contain x .

(2)

(Total for Question 5 is 9 marks)





Question 5 continued**Diagram 1****(Total for Question 5 is 9 marks)**

3.

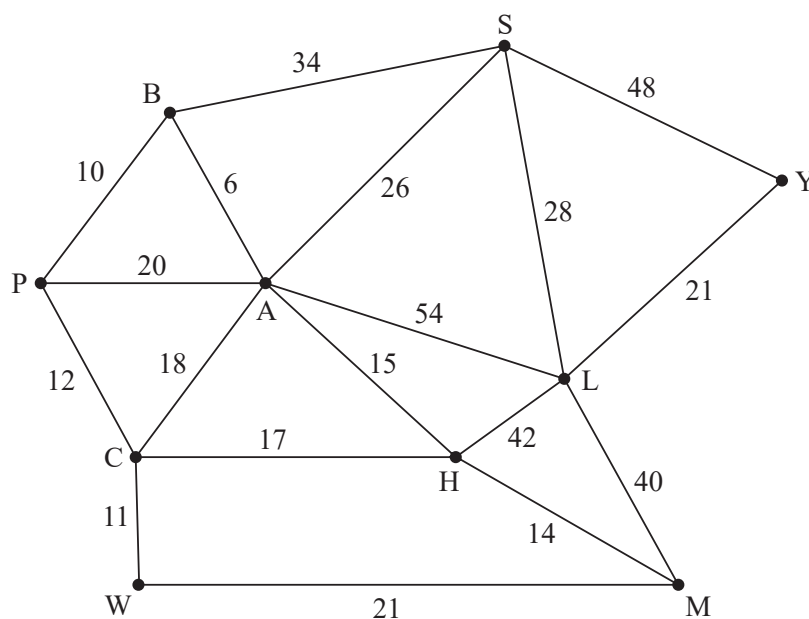


Figure 2

The network in Figure 2 shows the distances, in miles, between ten towns, A, B, C, H, L, M, P, S, W and Y.

- (a) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–

The table shows the shortest distances, in miles, between the ten towns.



- (b) Use Prim's algorithm on the table, starting at A, to find the minimum spanning tree for this network. You must clearly state the order in which you select the arcs of your tree.

(3)

- (c) State the weight of the minimum spanning tree found in (b).

(1)

Sharon needs to visit all of the towns, starting and finishing in the same town, and wishes to minimise the total distance she travels.

- (d) Use your answer to (c) to calculate an initial upper bound for the length of Sharon's route.

(1)

- (e) Use the nearest neighbour algorithm on the table, starting at W, to find an upper bound for the length of Sharon's route. Write down the route which gives this upper bound.

(2)

Using the nearest neighbour algorithm, starting at Y, an upper bound of length 212 miles was found.

- (f) State the best upper bound that can be obtained by using this information and your answers from (d) and (e). Give the reason for your answer.

(1)

- (g) By deleting W and all of its arcs, find a lower bound for the length of Sharon's route.

(2)

Sharon decides to take the route found in (e).

- (h) Interpret this route in terms of the actual towns visited.

(1)

(Total for Question 3 is 14 marks)



3.

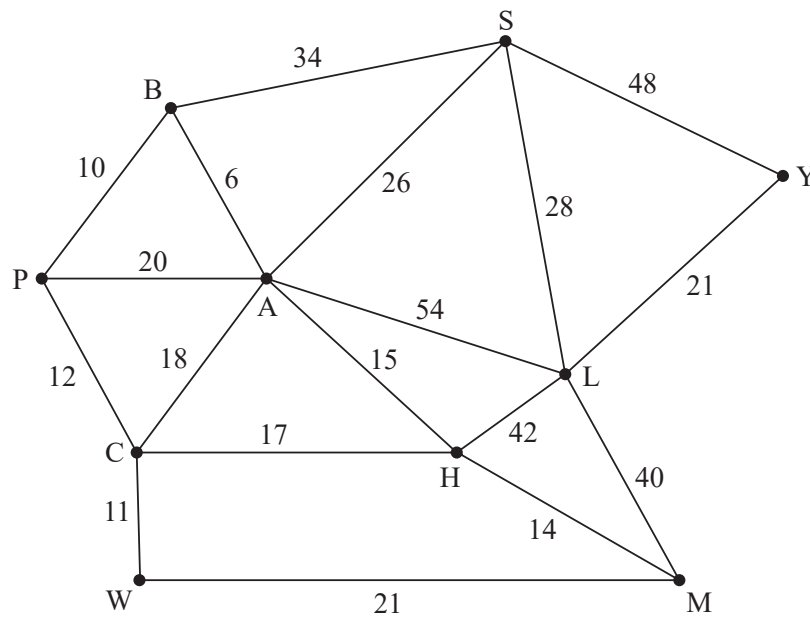


Figure 2

Question 3 continued

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–

Question 3 continues on the next page.



Question 3 continued

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–



Question 3 continued

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–

(Total for Question 3 is 14 marks)



7.

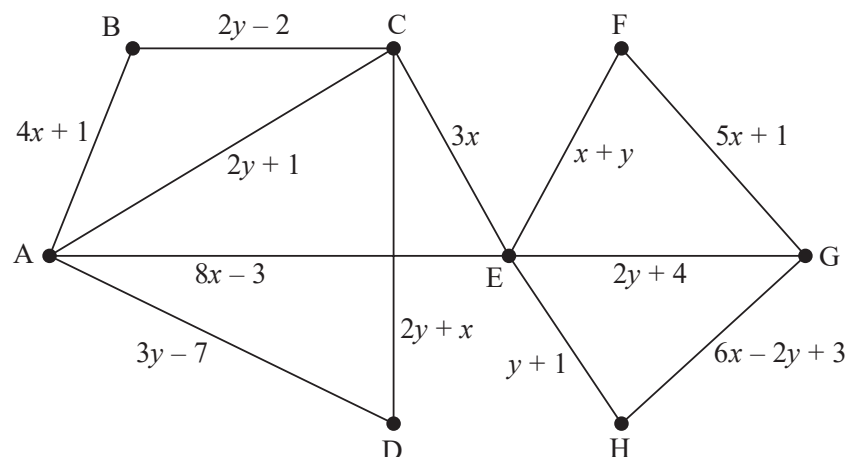


Figure 5

Figure 5 shows a weighted graph that contains 12 arcs and 8 vertices.

It is given that

- no two arcs have the same weight
- x and y are positive integers
- arc CD is not in the minimum spanning tree for the graph

(a) Explain why $y < x + 7$

(2)

It is also given that when Prim's algorithm, starting at A, is applied to the weighted graph, AB is the first arc selected.

(b) Show that $y > 2x$ and write down and simplify two further constraints on the values of x and y .

(3)

(c) Represent these four constraints on Diagram 1 in the answer book.

(4)

(d) Using Diagram 1 **only**, write down the possible pairs of values that x and y can take in the form (x, y) .

(2)

The minimum spanning tree for the weighted graph in Figure 5 has total weight 73
Six of the seven arcs in the minimum spanning tree are AB, AD, BC, CE, EF and GH.

(e) Determine the value of x and the value of y . You must make your method and working clear.

(4)

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END



3. The table below represents a complete network that shows the least costs of travelling between eight cities, A, B, C, D, E, F, G and H.

	A	B	C	D	E	F	G	H
A	–	36	38	40	23	39	38	35
B	36	–	35	36	35	34	41	38
C	38	35	–	39	25	32	40	40
D	40	36	39	–	37	37	26	33
E	23	35	25	37	–	42	24	43
F	39	34	32	37	42	–	45	38
G	38	41	40	26	24	45	–	40
H	35	38	40	33	43	38	40	–

Srinjoy must visit each city at least once. He will start and finish at A and wishes to minimise his total cost.

- Use Prim's algorithm, starting at A, to find a minimum spanning tree for this network. You must list the arcs that form the tree in the order in which you select them. (3)
- State the weight of the minimum spanning tree. (1)
- Use your answer to (b) to help you calculate an initial upper bound for the total cost of Srinjoy's route. (1)
- Show that there are two nearest neighbour routes that start from A. You must make the routes and their corresponding costs clear. (4)
- State the best upper bound that can be obtained by using your answers to (c) and (d). (1)
- Starting by deleting A and all of its arcs, find a lower bound for the total cost of Srinjoy's route. You must make your method and working clear. (3)
- Use your results to write down the smallest interval that must contain the optimal cost of Srinjoy's route. (2)

(Total 15 marks)



3.

	A	B	C	D	E	F	G	H
A	–	36	38	40	23	39	38	35
B	36	–	35	36	35	34	41	38
C	38	35	–	39	25	32	40	40
D	40	36	39	–	37	37	26	33
E	23	35	25	37	–	42	24	43
F	39	34	32	37	42	–	45	38
G	38	41	40	26	24	45	–	40
H	35	38	40	33	43	38	40	–

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	A	B	C	D	E	F	G	H
A	–	36	38	40	23	39	38	35
B	36	–	35	36	35	34	41	38
C	38	35	–	39	25	32	40	40
D	40	36	39	–	37	37	26	33
E	23	35	25	37	–	42	24	43
F	39	34	32	37	42	–	45	38
G	38	41	40	26	24	45	–	40
H	35	38	40	33	43	38	40	–

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Question 3 continued

	A	B	C	D	E	F	G	H
A	–	36	38	40	23	39	38	35
B	36	–	35	36	35	34	41	38
C	38	35	–	39	25	32	40	40
D	40	36	39	–	37	37	26	33
E	23	35	25	37	–	42	24	43
F	39	34	32	37	42	–	45	38
G	38	41	40	26	24	45	–	40
H	35	38	40	33	43	38	40	–

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Question 3 continued

	A	B	C	D	E	F	G	H
A	–	36	38	40	23	39	38	35
B	36	–	35	36	35	34	41	38
C	38	35	–	39	25	32	40	40
D	40	36	39	–	37	37	26	33
E	23	35	25	37	–	42	24	43
F	39	34	32	37	42	–	45	38
G	38	41	40	26	24	45	–	40
H	35	38	40	33	43	38	40	–

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(Total 15 marks)

Q3



SUB-TOPIC D1.2b

DIJKSTRA'S ALGORITHM & SHORTEST PATHS

Sub-topic D1.2b · 4 questions · 39 marks · lowest tariff first



4.

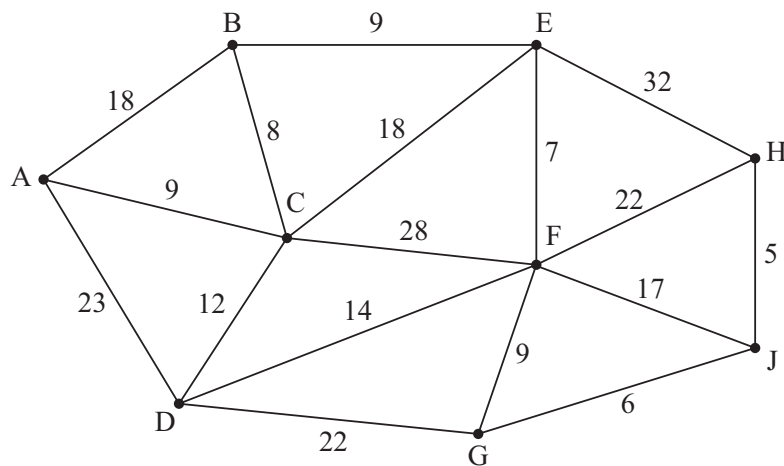
**Figure 1**

Figure 1 represents a network of roads between nine villages, A, B, C, D, E, F, G, H and J. The number on each arc is the time taken, in minutes, to drive along the corresponding road.

Liliya must drive from village A to village H.

- (a) (i) Use Dijkstra's algorithm to determine the fastest route from A to H.
 (ii) State the fastest time.

(6)

The roads EF and GJ are now closed for maintenance.

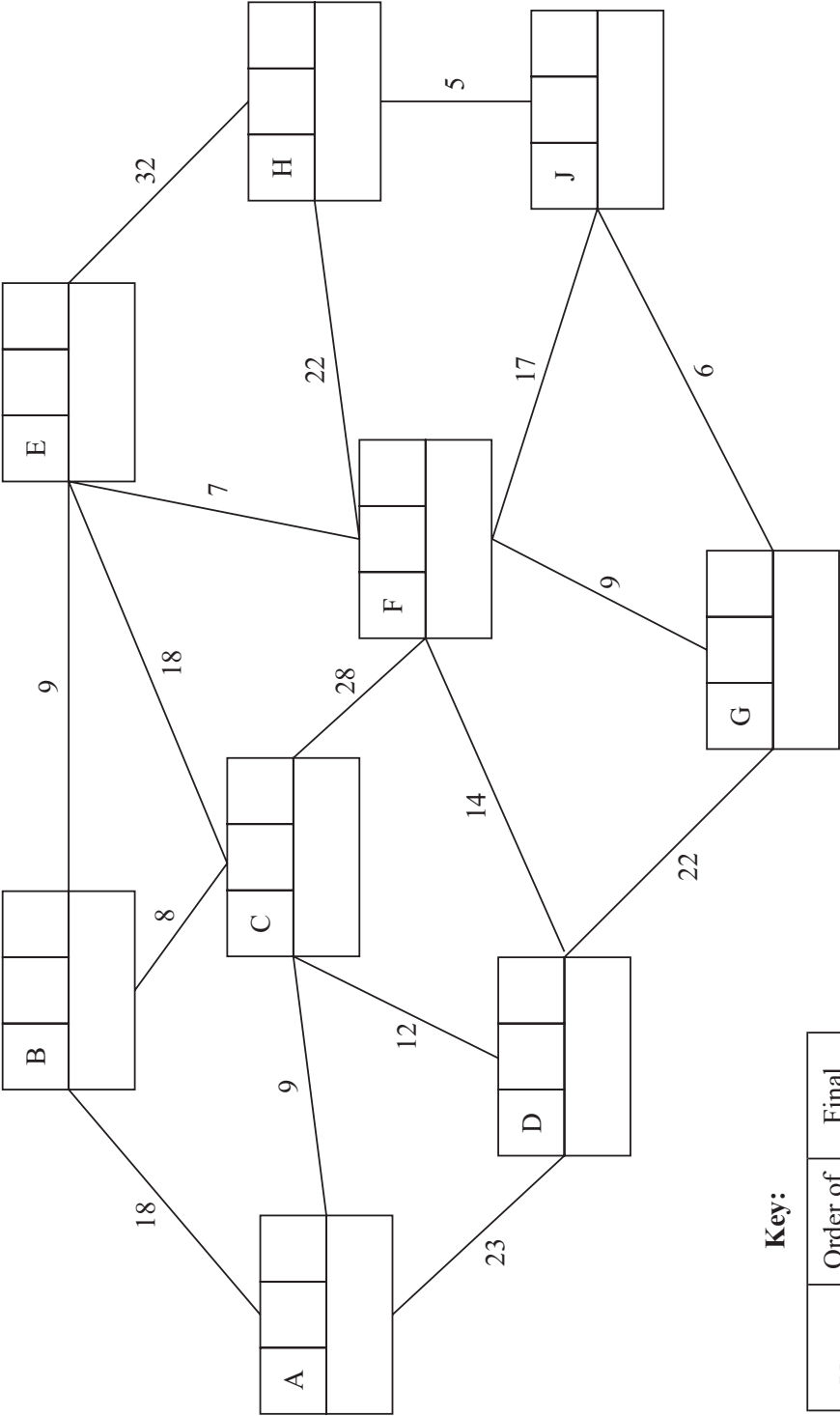
- (b) Calculate how much more time Liliya's drive will now take.

(2)

(Total for Question 4 is 8 marks)



4.



Key:

Vertex	Order of labelling	Final value
Working values		

- (i) Fastest route from A to H _____
- (ii) Fastest time _____

(Total for Question 4 is 6 marks)



5.

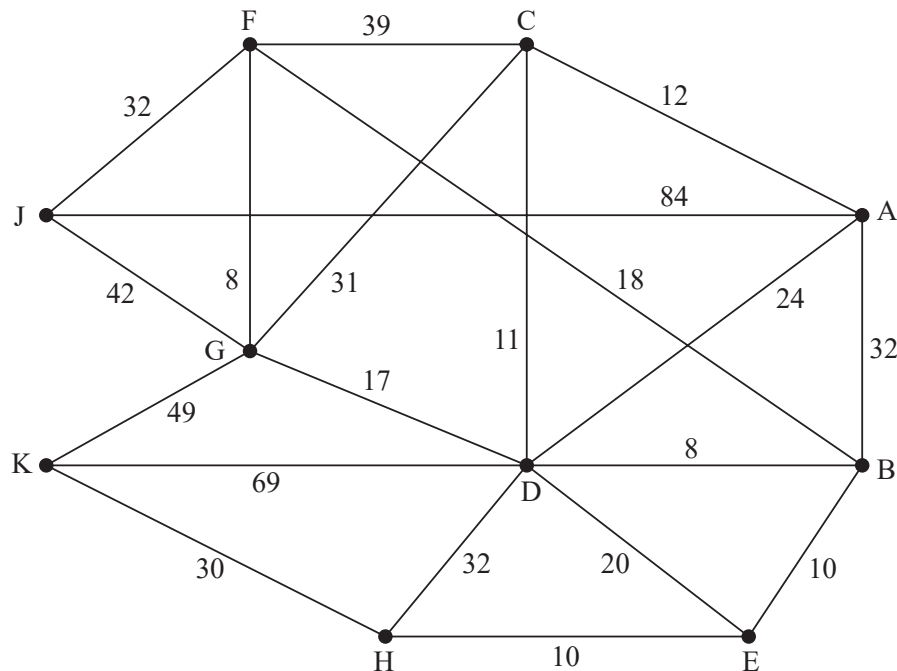


Figure 4

Figure 4 represents a network of roads. The number on each arc represents the length, in miles, of the corresponding road. Tamasi, who lives at A, needs to collect a caravan. Tamasi can collect a caravan from either J or K.

Tamasi decides to use Dijkstra's algorithm once to find the shortest routes between A and J and between A and K.

(a) State, with a reason, which vertex should be chosen as the starting vertex for the algorithm. (2)

(b) Use Dijkstra's algorithm to find the shortest routes from A to J and from A to K. You should state the routes and their corresponding lengths. (7)

Tamasi's brother lives at F. He needs to visit Tamasi at A and then visit their mother who lives at H.

(c) Find a route of minimal length that goes from F to H via A. (1)

(Total 10 marks)



Write your answers in the D1 answer book for this paper.

1. (a) Explain what is meant by the term 'path'.

(2)

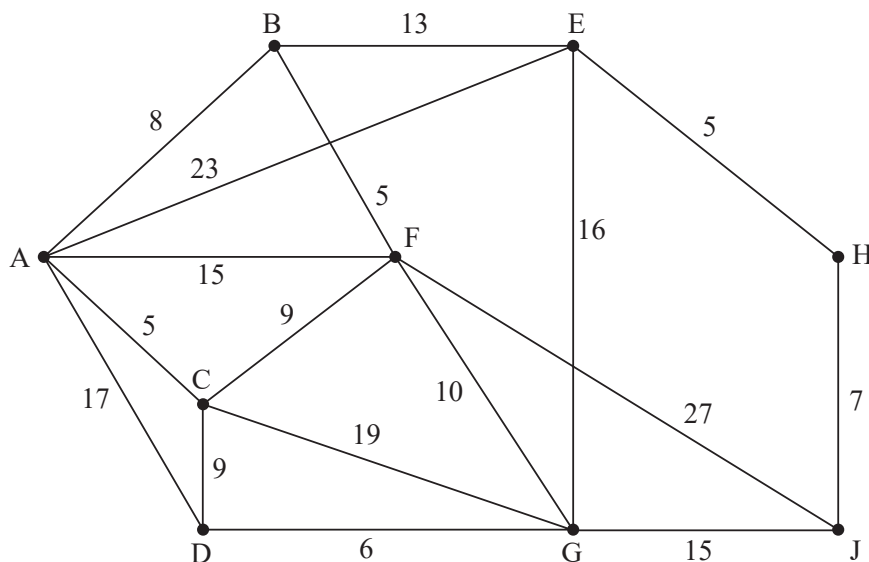


Figure 1

Figure 1 represents a network of roads. The number on each arc represents the length, in km, of the corresponding road. Piatrice wishes to travel from A to J.

- (b) Use Dijkstra's algorithm to find the shortest path Piatrice could take from A to J. State your path and its length.

(6)

Piatrice needs to return from J to A via G.

- (c) Find the shortest path Piatrice could take from J to A via G and state its length.

(2)

(Total 10 marks)



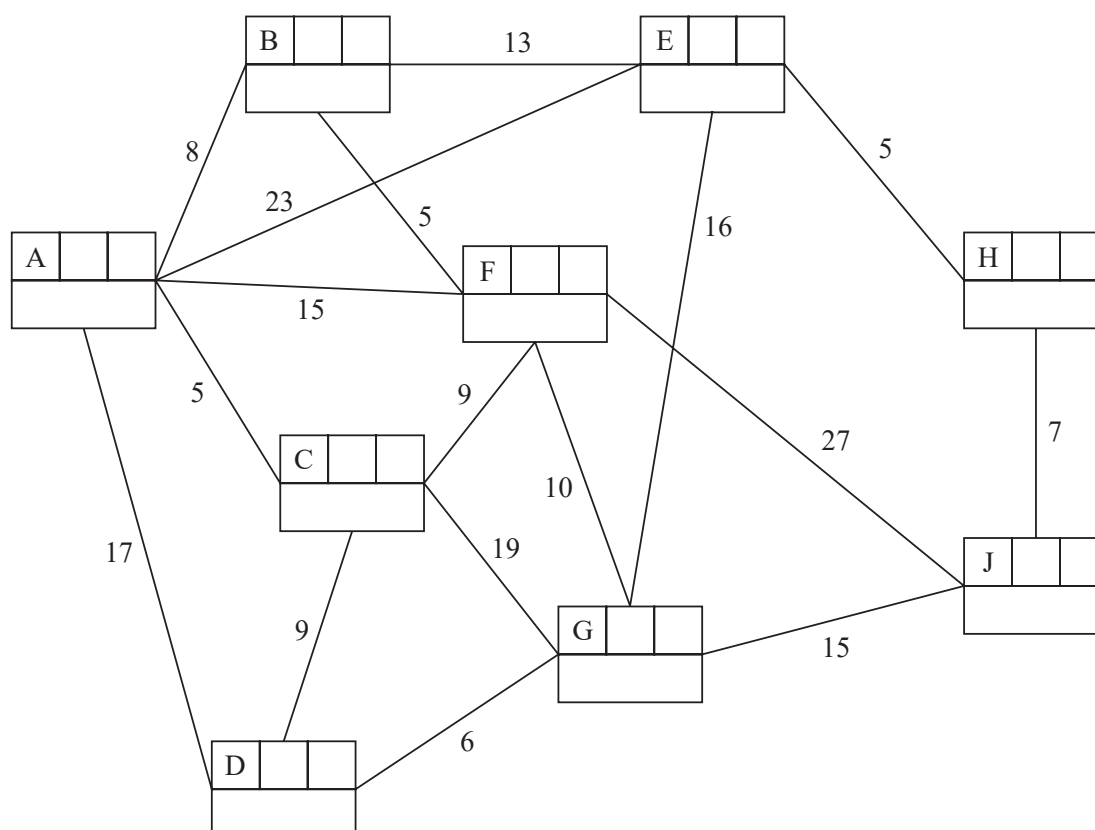
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1.

(a)

Key:

Vertex	Order of labelling	Final value
Working values		



Shortest path from A to J: _____

Length of shortest path from A to J: _____



Question 1 continued

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(Total 10 marks)

Q1



2.

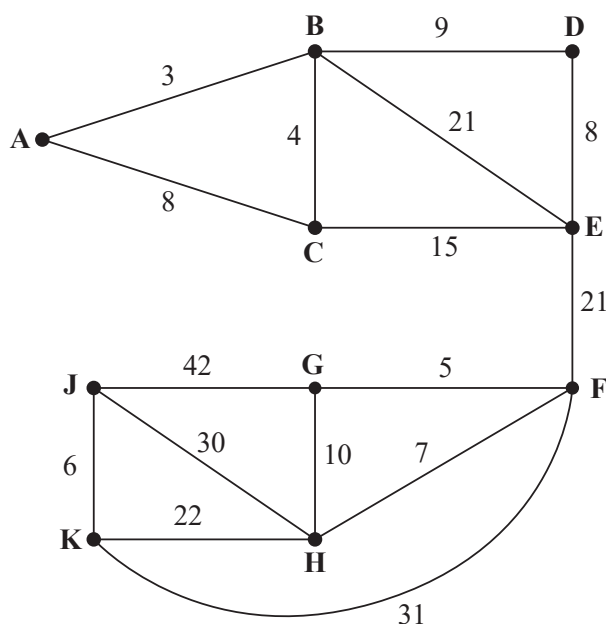


Figure 1

Figure 1 represents a network of roads between ten villages, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in kilometres, of the corresponding road. The local council needs to find the shortest route from A to J.

- (a) Use Dijkstra's algorithm to find the shortest route from A to J. State the route and its length. (6)

During the winter, the council needs to ensure that all ten villages are accessible by road even if there is heavy snow. The council wishes to minimise the total length of road it needs to keep clear.

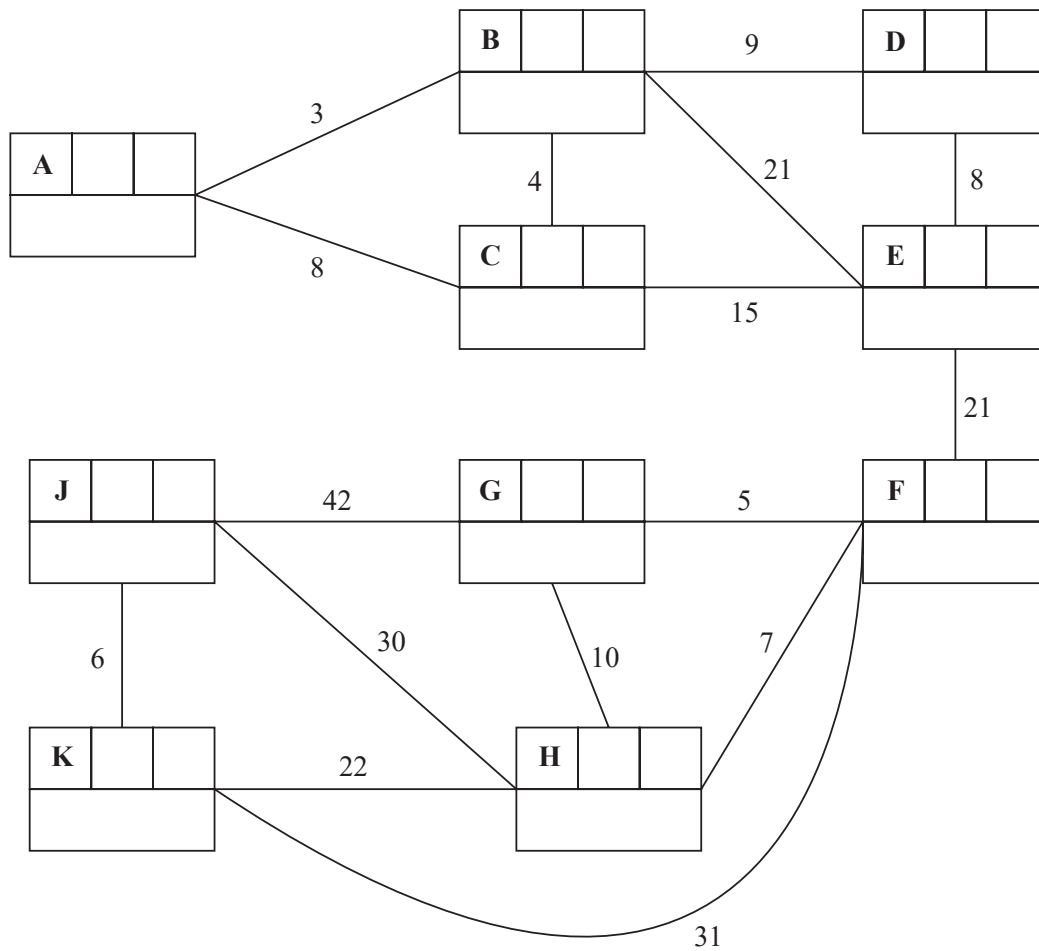
- (b) Use Prim's algorithm, starting at A, to find a minimum connector for the five villages A, B, C, D and E. You must clearly state the order in which you select the edges of your minimum connector. (2)
- (c) Use Kruskal's algorithm to find a minimum connector for the five villages F, G, H, J and K. You must clearly show the order in which you consider the edges. For each edge, state whether or not you are including it in your minimum connector. (2)
- (d) Calculate the total length of road that the council must keep clear of snow to ensure that all ten villages are accessible. (1)

(Total 11 marks)



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2.

**Key:**

Vertex	Order of labelling	Final value
Working values		

Shortest route: _____

Length of shortest route: _____



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This image shows a full page of blank, lined paper. It features approximately 28 horizontal blue or grey lines spaced evenly apart, typical of notebook paper. The lines extend across the entire width of the page, leaving small margins at the top and bottom. There are no vertical lines, text, or other markings on the page.

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This image shows a full page of blank, lined paper. It features approximately 20 evenly spaced horizontal blue or grey lines across its entire width, typical of notebook paper. The lines are uniform in thickness and spacing, providing a guide for writing. There are no margins, text, or other markings on the page.

(Total 11 marks)

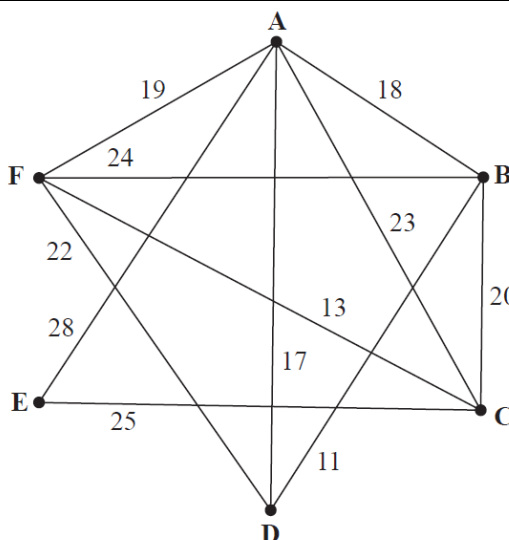
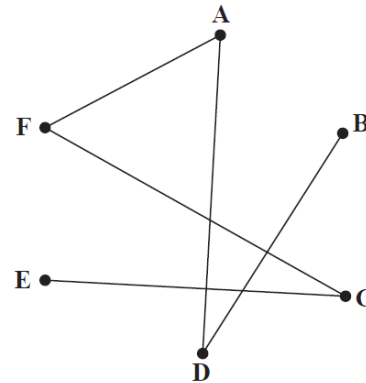
Q2



MARK SCHEMES

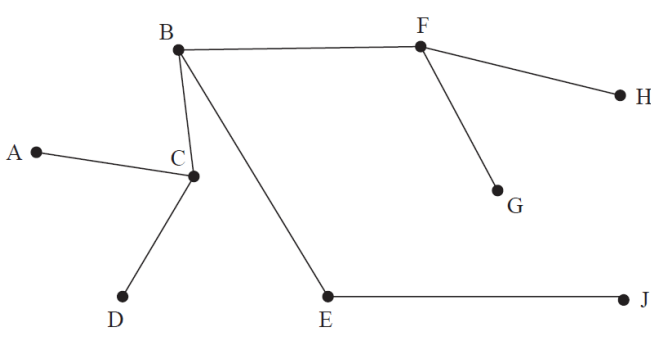
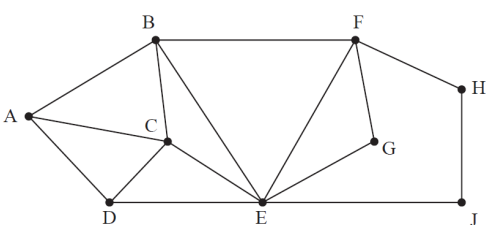
official mark schemes, in the same order as the questions

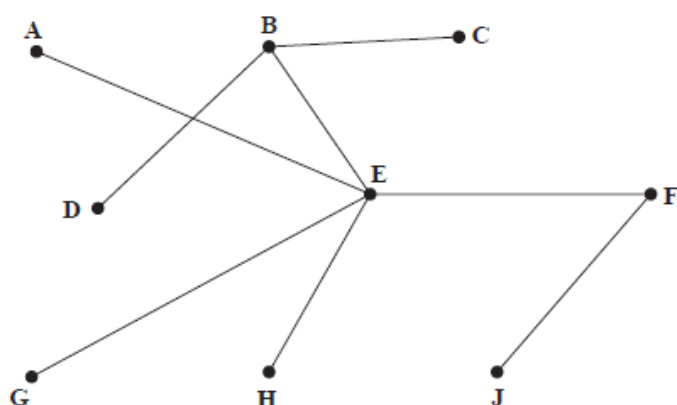


Question Number	Scheme	Marks
1. (a)		M1 A1 (2)
(b)	Kruskal: BD(11), CF(13), AD(17), reject AB(18), AF(19), reject BC(20), reject DF(22), reject AC(23), reject BF(24), CE(25) (not AE)	M1 A1 A1 (3)
(c)		B1
	Weight of MST = 85 (metres)	B1 (2)
		7 marks
Notes for Question 1		
a1M1: At least 8 correct arcs with corresponding correct values or all 11 correct arcs a1A1: CSO (11 arcs only + correct values) – give bod b1M1: Kruskal's: first three arcs (BD, CF, AD) correctly chosen and at least one rejection seen at some point b1A1: All arcs in tree selected correctly and in the correct order (BD, CF, AD, AF, CE) – no other arcs in MST b2A1: CSO including all rejections correct and at the correct time – AE need not be considered but if AE is considered then it must be rejected after CE has been added to the MST c1B1: CAO (tree) c2B1: CAO (85)		

Question Number	Scheme	Marks
1.(a)	$\frac{131}{40} = 3.275$ so lower bound is 4	M1 A1 (2)
(b)	Container 1: <u>17</u> <u>9</u> <u>8</u> 4 Container 2: <u>15</u> <u>20</u> 5 Container 3: <u>13</u> 12 Container 4: <u>28</u>	<u>M1</u> <u>A1</u> A1 (3)
(c)	middle right 17 9 15 8 20 13 28 4 12 5 9 8 4 12 5 <u>13</u> 17 15 20 28 <u>4</u> 9 8 12 5 <u>13</u> 17 15 <u>20</u> 28 <u>4</u> 9 8 5 <u>12</u> <u>13</u> <u>15</u> 17 <u>20</u> 28 <u>4</u> 5 <u>8</u> 9 <u>12</u> <u>13</u> <u>15</u> 17 <u>20</u> 28 Pivots 13 4, 20 12, 15, (28) 8, (17) Sort complete	M1 A1 A1 (3)
	middle left 17 9 15 8 20 13 28 4 12 5 17 9 15 8 13 4 12 5 <u>20</u> 28 4 5 <u>8</u> 17 9 15 13 12 <u>20</u> 28 <u>4</u> 5 <u>8</u> 9 13 12 <u>15</u> 17 <u>20</u> 28 <u>4</u> 5 <u>8</u> 9 12 <u>13</u> <u>15</u> 17 <u>20</u> 28 <u>4</u> 5 <u>8</u> <u>9</u> 12 <u>13</u> <u>15</u> 17 <u>20</u> 28 Pivots 20 8, (28) 4, 15 (5), 13, (17) 9 (Sort Complete)	
(d)	$\left\lceil \frac{1+10}{2} \right\rceil = 6$ 13 – reject 13 – 28 $\left\lceil \frac{1+5}{2} \right\rceil = 3$ 8 – reject 4 – 8 $\left\lceil \frac{4+5}{2} \right\rceil = 5$ 12 – reject 12 $\lceil 4 \rceil = 4$ 9 – found	M1 A1 A1 (3)
		11 marks
Notes for Question 1		
a1M1: Attempt to find the lower bound $(131 \pm 28) / 40$. A value of 3.275 seen with no corresponding calculation can imply this mark. Allow this mark for a clear intention of adding all ten values and dividing by 40, for example, $\frac{17 + 9 + 15 + 8 + 20 + 13 + 28 + 4 + 12 + 5}{40} \text{ scores M1}$ a1A1: cao - correct calculation seen followed by 4 or the correct value of 3.275 followed by 4. However, an answer of 4 with no working scores no marks in (a) b1M1: First four values placed correctly and at least eight values placed in containers. The first four values are those that are boxed. Condone cumulative totals for M1 only (e.g. for Container 1: 17 26 34 38)		

Question Number	Scheme	Marks
	b1A1: First eight values placed correctly (the boxed and underlined values). This mark cannot be awarded if you see any repeated values or more than ten values placed in containers (even if the first eight items have been placed correctly) b2A1: cso – no additional or repeated values (dependent on both previous marks)	
	c1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values less than the pivot), pivot, (values greater than the pivot). If sorting into descending order then M1 only (even if the list is reversed at the end of the sort). This mark can be scored if one number only is either missing or incorrect or an additional number is added to the list c1A1: First two passes correct and next pivots chosen correctly for third pass (but third pass does not need to be attempted or correct) c2A1: cso (correct solution only – all previous marks in this part must have been awarded) – if middle right then either a fifth pass or a 'sort complete' statement (e.g. 'sorted', 'complete', etc. but not just underlining the fourth pass) is required after the fourth pass. If middle left then a fifth pass (with the value of 9) is required (but no 'sort complete' statement is required)	
	d1M1: Choosing middle right pivot (choosing middle left '12' is M0) and an attempt at discarding/retaining half the list (condone if retaining the wrong half of the list or if retaining 1 – 6) d1A1: First and second passes correct i.e. selecting the 6th item in the first pass and using 1st to 5th items in the second pass (must not be using the 6th item for the second pass) and then correctly selecting the 3rd item (the 8) in the second pass and rejecting the 1st to 3rd items d2A1: cao - search completed correctly (so rejecting the 12 in the third pass) together with 'found'. Condone candidates who say that after the 12 in the third pass has been rejected the only value left is the 9 so it has been found. It must be clear that the 9 has been 'found' and not just stated as the final value	
	In (d) candidates must be using a correct ordered list (4 5 8 9 12 13 15 17 20 28) – if it is clear that the candidates are not using this list then M0. With regards to using the original (unsorted) list the 6th value is 13 too so what the candidates do next will most likely indicate if the correct list is being used (e.g. if the next pivot is 15 then M0) Candidates who have sorted the list into descending order can earn full marks in (d) – scheme above applies in the same way for descending – must be choosing middle right (the 12) not middle left (the 13) for M1 28 20 17 15 13 12 9 8 5 4 $\left\lceil \frac{1+10}{2} \right\rceil = 6 \quad 12 - \text{reject } 28 - 12$ $\left\lceil \frac{7+10}{2} \right\rceil = 9 \quad 5 - \text{reject } 5 - 4$ $\left\lceil \frac{7+8}{2} \right\rceil = 8 \quad 8 - \text{reject } 8$ $\lceil 7 \rceil = 7 \quad 9 - \text{found}$	
2.(a)	e.g. A – B – F – H – J	B1 (1)
(b)	A – B – C – D – E – G – F – H – J is not an example of a tour on T as although it contains every vertex it does not return to A	B1 (1)

Question Number	Scheme	Marks
(c)	Kruskal: AC(9), BE(11), BF(12), not EF(14), FG(15), FH(17), not EG(18), EJ(20), $\left\{ \begin{array}{l} \text{not HJ (21)} \\ \text{BC (21)} \end{array} \right\}$, not CE(23), not AB(24), CD(25) (not DE, AD)	M1 A1 A1 (3)
(d)		B1 (1)
(e)	130 (km)	B1 (1)
		7 marks
Notes for Question 2		
a1B1: Any correct example of a path (so no vertex appearing more than once) from A to J b1B1: No with a correct reason i.e. must mention that a tour must begin and start at the same vertex, e.g. the route given does not finish at A scores B1 c1M1: Kruskal's algorithm - first four arcs (AC, BE, BF, FG) correctly chosen and at least one rejection seen at some point (the rejection need not be correct or at the correct time) c1A1: All arcs in tree selected correctly in the correct order (AC, BE, BF, FG, FH, EJ, BC, CD) with no additional arcs included in MST c2A1: cso - including all rejections correct and at the correct time (do not need to see DE and/or AD rejected but if they are rejected then must be after CD has been selected). Note that BC can be included before HJ rejected d1B1: cao e1B1: cao (130) – no units required/ignore units even if incorrect		
For reference:		
		
3.	e.g.	M1 A1

Question Number	Scheme	Marks
2. (a)(i)	A tree is a connected graph with no cycles	B1
(ii)	A minimum spanning tree is a tree that contains all vertices The total length of its arcs is as small as possible	B1 B1 (3)
(b)	Kruskal: FJ(11), EG(13), EF(15), EH(17), not GH (18), BC (19), not HJ (20), BD (22), not FH(23), AE (25), BE (29) (not AD, DE, DG, AB, BH)	M1 A1 A1 (3)
(c)	 (Weight of the tree is) 151	B1 B1 (2)
		8 marks

Notes for Question 2

ai1B1: Connected + no cycle(s) (must contain these two points – do not allow ‘circle’, ‘loop’ etc. for cycle(s)) – if not using the word ‘connected’ then allow ‘a graph that connects the vertices/nodes’ (condone issues with plural or singular e.g. cycle for cycles)

aii2B1: Contains **all vertices/nodes** (must be clear that all vertices (or nodes) are in a MST)

aii3B1: Total **length of arcs** is **minimised** (must contain the three points regarding weight/length, arcs/edges and minimised/smallest (oe))

b1M1: Kruskal’s algorithm - first four arcs (FJ, EG, EF, EH) correctly chosen **and** at least one rejection seen at some point

b1A1: All arcs in tree selected correctly in the correct order (FJ, EG, EF, EH, BC, BD, AE, BE) with no additional arcs included in MST

b2A1: CSO including all rejections correct and at the correct time (do not need to see AD, DE, DG, AB, BH rejected but if they are they must be rejected correctly (i.e. in this order) but note that AD, DE have the same weight as do DG and AB so they could appear in either order)

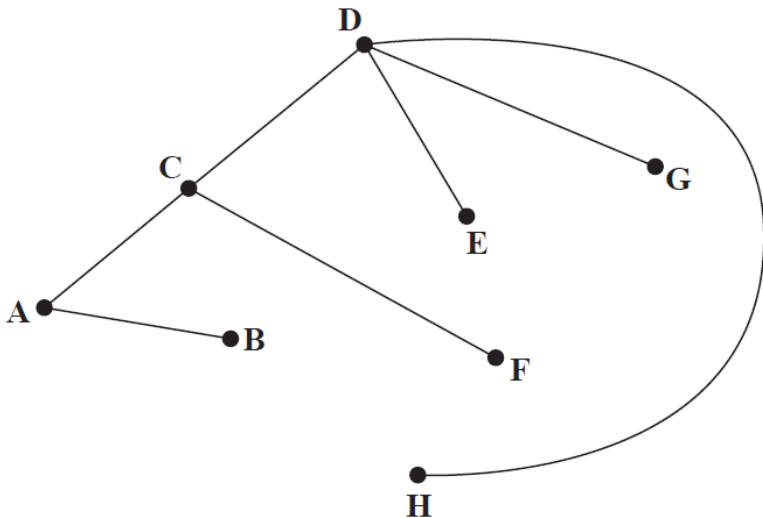
Note that stating all the arcs in order (e.g. FJ, EG, EF, EH, GH, BC, HJ, BD, FH, AE, BE, AD, DE, DG, AB, BH) and then stating only those in the tree in the correct order is fine for all three marks in this part

c1B1: CAO (tree)

c2B1: CAO (151)

	Notes for Question 2																																																												
ai1M1	All top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue																																																												
ai1A1	CAO (top boxes)																																																												
ai2M1	All bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue. Condone missing 0 and/or their 20 (at the end event) for the M mark only																																																												
ai2A1	CAO (bottom boxes)																																																												
aii3A1ft	Follow through the top value at the end node – dependent on both M marks – if labelled must be answered in the correct part of the question																																																												
b1B1	CSO – requires both a correct calculation or 2.95 seen and 3. An answer of 3 with no working scores no marks																																																												
c1M1	Not a cascade (Gantt) chart. 5 'workers' used at most and at least 8 activities placed. End time must be the same as their end time from the diagram in (a)(i)																																																												
c1A1	3 or 4 workers. All 14 activities present (just once). Condone at most three errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA																																																												
c2A1	3 or 4 workers. All 14 activities present (just once). Condone one error either precedence or time interval or activity length; An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA																																																												
c3A1	3 workers. All 14 activities present (just once). No errors.																																																												
For (c) the following may be useful in checking their schedule Note J and K are interchangeable																																																													
<table><tr><th>Activity</th><th>Duration</th><th>Time interval</th><th>IPA</th></tr><tr><td>A</td><td>5</td><td>0 – 5</td><td>-</td></tr><tr><td>B</td><td>4</td><td>0 – 12</td><td>-</td></tr><tr><td>C</td><td>7</td><td>0 – 11</td><td>-</td></tr><tr><td>D</td><td>4</td><td>5 – 9</td><td>A</td></tr><tr><td>E</td><td>3</td><td>5 – 12</td><td>A</td></tr><tr><td>F</td><td>3</td><td>9 – 12</td><td>D</td></tr><tr><td>G</td><td>2</td><td>9 – 17</td><td>D</td></tr><tr><td>H</td><td>4</td><td>9 – 18</td><td>D</td></tr><tr><td>I</td><td>6</td><td>12 – 18</td><td>B, C, E, F</td></tr><tr><td>J</td><td>7</td><td>7 – 18</td><td>C</td></tr><tr><td>K</td><td>7</td><td>7 – 18</td><td>C</td></tr><tr><td>L</td><td>3</td><td>11 – 20</td><td>G</td></tr><tr><td>M</td><td>2</td><td>18 – 20</td><td>G, H, I, J, K</td></tr><tr><td>N</td><td>2</td><td>14 – 20</td><td>K</td></tr></table>		Activity	Duration	Time interval	IPA	A	5	0 – 5	-	B	4	0 – 12	-	C	7	0 – 11	-	D	4	5 – 9	A	E	3	5 – 12	A	F	3	9 – 12	D	G	2	9 – 17	D	H	4	9 – 18	D	I	6	12 – 18	B, C, E, F	J	7	7 – 18	C	K	7	7 – 18	C	L	3	11 – 20	G	M	2	18 – 20	G, H, I, J, K	N	2	14 – 20	K
Activity	Duration	Time interval	IPA																																																										
A	5	0 – 5	-																																																										
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C	7	0 – 11	-																																																										
D	4	5 – 9	A																																																										
E	3	5 – 12	A																																																										
F	3	9 – 12	D																																																										
G	2	9 – 17	D																																																										
H	4	9 – 18	D																																																										
I	6	12 – 18	B, C, E, F																																																										
J	7	7 – 18	C																																																										
K	7	7 – 18	C																																																										
L	3	11 – 20	G																																																										
M	2	18 – 20	G, H, I, J, K																																																										
N	2	14 – 20	K																																																										

Question Number	Scheme	Marks
3.(a)		M1 A1 (ACBFG) A1 (DE) A1ft (HJ)
	Shortest time to travel from A to J is 74 (mins)	A1ft
	Quickest route is ACBFGDEHJ	A1 (6)
(b)	As the quickest route found in (a) is a path through all vertices in the network the weight of this path is equal to the weight of the MST for the network	M1
	Weight of MST is 74 (mins)	A1ft (2)
		8 marks
Notes for Question 3		
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at E the working values must be 71 61 52 in that order (so 71 52 61 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling		
a1M1	A larger value replaced by a smaller value at least twice in the working values at either B, D, E, F, H, J	
a1A1	All values at A, C, B, F and G correct and the working values in the correct order	
a2A1	All values at D and E correct and the working values in the correct order	
a3A1ft	All values in H and J correct on the follow through and the working values in the correct order. To follow through H check that the working values at H follow from the candidate's final values for the nodes that are directly attached to H (which are D, E and G (and J)). For example, if correct then the order of labelling of nodes D, E and G are 6, 7 and 5 respectively so the working values at H should come from G, D and E in that order. The first working value at H should be their 32 (the Final value at G) + 51 (the weight of the arc GH), the second working value at H should be their 45 (the Final value at D) + 18 (the weight of the arc DH) and the third working value should be their 52 (the Final value at E) + 9 (the weight of the arc EH) . Repeat the process for J (which will have working values from D, G and H with the order of these nodes determined by the candidate's order of labelling at D, G and H)	
a4A1ft	Follow through their final value at J only – if answer is 74 but this is not the Final Value at J then A0	
a5A1	CAO (ACBFGDEHJ)	
b1M1	A statement that the route found in (a) is the MST, either stated directly, or an indication that the route has visited all 9 nodes.	
b1A1ft	Follow through their answer to (a) provided that the route from (a) passed through all nine nodes in the network (a weight of 74 with no reason scores M0)	

Question number	Scheme	Marks
5.(a)	E.g. You cannot have a graph with an odd number of odd vertices E.g. $\frac{1+2+2+3+3+4+4+6}{2} = 12.5$ which is not an integer and so therefore not possible to have a graph with the given vertex orders	B1 (1)
(b)	As vertex C appears more than once A – C – D – E – C – B – F is not an example of a path on T	B2, 1, 0 (2)
(c)	AC, AB, CD; DH, DG; CF, DE	M1; A1; A1 (3)
(d)		B1 (1)
(e)	$21 < x < 25$	B2, 1, 0 (2)
		(9 marks)

Notes for Question 5**a1B1:** CAO – common examples that score B1:

- Cannot have (a graph with an) odd number of odd vertices
- Cannot have a graph with three odd vertices
- The sum of the degrees/order (of the vertices) is 25 which is not even therefore not possible (but not just for obtaining 25 and saying ‘impossible’). The 25 must be linked either in words to the ‘sum of the degrees/order’ **or** explicitly showing $1+2+2+3+3+4+4+6=25$ so just ‘25 is not even’ scores B0
- The sum of the degrees/order (of the vertices) is 25 which is odd therefore not possible (with equivalent justification of the 25 as in the previous bullet-point)
- $\frac{1+2+2+3+3+4+4+6}{2} = 12.5$ which is not an integer so therefore impossible. They do not have to explain that they are using the result that $\sum \text{vertex degrees} = 2(\text{no of arcs})$ but they must explain why a value of 12.5 leads to the required graph not being possible. A value of 12.5 with no working (or explanation) scores B0

In (a) do not condone clearly incorrect technical language e.g. using ‘arc’ when it should be ‘vertex’

b1B1: No + attempt at a reason which includes **either** the mention of a cycle/circle/loop etc. **or** the repeating of a vertex/node/point etc. is sufficient for this mark (condone incorrect technical language) – give bod (but ‘no because there is a repeated arc’ **only** scores B0 unless we also see mention of a repeated vertex (oe) as well)

b2DB1: No + correct reason (dependent on first B mark in (b)) – no bod – must refer to C appearing twice (**not** just that a vertex is repeated) or that it contains the cycle C – D – E – C (**not** just that it contains a cycle). **All technical language must be correct if used for this mark and do not isw any incorrect reasoning**

The minimum acceptable answer for both marks in this part is, ‘it is not a path as C appears twice’

c1M1: Prim’s – first three arcs correctly chosen in order (AC, AB, CD) **or** first four nodes (A, C, B, D) correctly chosen in order. **If any explicit rejections seen at some point then M1 (max) only.** Order of nodes may be seen at the top of a matrix/table {1, 3, 2, 4, -, -, -, -}. However, do not accept a list of weights only (as the weights in the network are not unique)

c1A1: First five arcs correctly chosen in order (AC, AB, CD, DH, DG) **or** all eight nodes {A, C, B, D, H, G, F, E} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 3, 2, 4, 8, 7, 6, 5} (**no** missing numbers). However, do not accept a list of weights only (as the weights in the network are not unique)

c2A1: CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional incorrect arcs). They must be considering arcs for this final mark (do not accept a list of the weights of each arc, nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

Misread in (c): Starting at a node other than A scores **M1 only** – **must** have the first three arcs (or four nodes) correct (and in the correct order) – condone any rejections seen for this mark

d1B1: CAO (ignore weights on arcs even if incorrect)

e1B1: $x < 25$ or $x \leq 25$ or $x < 24$ or $x \leq 24$ or equivalent notation e.g. (... , 25)

e2B1: CAO ($21 < x < 25$) or equivalent notation (e.g. (21, 25))

3.(a)	Kruskal: AB(6), BP(10), CW(11), CP(12), HM(14), AH(15), reject CH(17), reject AC(18), reject AP(20), reject MW(21), LY(21), AS(26), LS(28) (not BS, LM, HL, SY, AL)	M1 A1 A1 (3)
(b)	Prim: AB, BP, CP, CW, AH, HM, AS, LS, LY	M1 A1 A1 (3)
(c)	143 (miles)	B1 (1)
(d)	286 (miles)	B1ft (1)
(e)	NNA starting at W: W – C – P – B – A – H – M – L – Y – S – W $11 + 12 + 10 + 6 + 15 + 14 + 40 + 21 + 48 + 55 = 232$	M1 A1 (2)
(f)	The best upper bound is the one starting at Y as 212 is less than both 232 and 286	B1 (1)
(g)	$(143 - 11) + 11 + 21 = 164$ (miles)	M1 A1 (2)
(h)	WCPBAHMLYSACW	B1 (1)
		14 marks

Notes for Question 3

a1M1: First four arcs (AB, BP, CW, CP) correctly chosen and at least one rejection seen at some point

a1A1: All arcs in tree selected correctly and in the correct order (AB, BP, CW, CP, HM, AH, LY, AS, LS) – no other arcs in MST

a2A1: cso including all rejections correct and at the correct time – **note that LY can be accepted before MW is rejected.** We do not need to see the explicit rejection of arcs BS to AL but if these are explicitly rejected then they must be in the correct order. Note that a list of all the arcs in the correct order followed by a list of the arcs in the MST can score full marks

b1M1: First three arcs correctly chosen in order (AB, BP, CP,...) **or** first four nodes {A, B, P, C, ...} correctly chosen in order. If any **explicit** rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, 2, 4, -, -, -, 3, -, -, -} so do check carefully for this. Starting at any other node can score M1 only for first three arcs chosen correctly

b1A1: First six arcs correctly chosen in order (AB, BP, CP, CW, AH, HM,...) **or** all ten nodes {A, B, P, C, W, H, M, S, L, Y} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 2, 4, 6, 9, 7, 3, 8, 5, 10} (**no** missing numbers)

b2A1: cso – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

c1B1: cao (143) – this mark can be awarded if seen in (b) (although if answered in (c) too then mark according to the answer given in (c))

d1B1ft: Follow through double their answer from (c)

e1M1: Nearest neighbour route starting at W – must have at least W – C – P – B – A – H – ... allow if stated in terms of arcs

e1A1: CAO on length (232) **and** route (must return to W and can be stated in terms of arcs)

f1B1: An indication that 212 is the minimum (of 212 and the answers to (d) and (e)) – this mark is dependent on the correct values in (d) and (e) so accept an answer of the form ‘the one starting at Y (or the route with weight 212) as it is the least’ – we do not need to see explicit mention of the values in (d) and (e) provided they are correct in (d) and (e)

g1M1: (weight of their MST from (c) **or** (b) **or** 132 **only**) – $11 + 11(WC) + 21(MW)$ (oe so may not see the $-11 + 11$). A correct answer of 164 can imply this (and the next) mark

g1A1: 164

h1B1: cao – either the route must be written out in full (in terms of nodes or arcs) **or** they must make it absolutely clear that the route begins exactly as in (e) (which must therefore be correct) but after S, towns A and C are visited before (returning to) W. Just stating that A, C, W are visited twice (or similar) is B0

c1B1: Correct answer only (12)

c2B1: Correct answer only (C G H I L M)

Question Number	Scheme	Marks
7.(a)	If CD is not in the tree then AD must be as these are the only two arcs incident to D or the weight of CD is greater than the weight of AD	M1
	$2y + x > 3y - 7 \Rightarrow y < x + 7^*$	A1 (2)
(b)	$4x + 1 < 2y + 1 \Rightarrow y > 2x^*$	B1
	$4x + 1 < 8x - 3 \Rightarrow x > 1$	B1
	$4x + 1 < 3y - 7 \Rightarrow 3y > 4x + 8$	B1 (3)
(c)		B1
		B1
		B1
		B1 (4)

(d)	$(2,6), (2,7), (2,8)$ $(3,7), (3,8), (3,9)$ $(4,9), (4,10)$ $(5,11)$	B1ft B1 (2)
(e)	Arcs in the tree are given as AB, AD, BC, CE, EF and GH – the one remaining arc in the MST is EH (because the weight of arcs EG and FG > weight of arc EH)	M1
	Total weight of the MST is therefore $14x + 5y - 4$	A1
	$14x + 5y - 4 = 73$ and testing integer value points inside FR	M1dep

	$x = 3$ and $y = 7$	A1 (4)
		15 marks
Notes for Question 7		
a1M1: Explaining that if CD is not in the tree then AD must be e.g. ‘the MST must contain D so if CD is not in the tree then AD is’. Must explicitly mention arc AD for this mark, so as a minimum accept, ‘AD must be in the MST’ a1A1: Correct reasoning and derivation of the given result ($2y + x > 3y - 7 \Rightarrow y < x + 7$) – as the answer is given we must see at least $2y + x > 3y - 7$ or $3y - 7 < 2y + x$ before the required answer SC (Special Case) in (a): $2y + x > 3y - 7 \Rightarrow y < x + 7$ without any explanation given (or if explanation is incorrect) can score M1A0 b1B1: CAO – must see at least $4x + 1 < 2y + 1$ before the given answer of $y > 2x$ and not just arc $AB < \text{arc } AC$ or $4x < 2y$ b2B1: CAO ($x > 1$) – but allow equivalents, e.g., $x - 1 > 0$, $1 < x$, $4x > 4$, etc. but must be two terms only b3B1: CAO ($3y > 4x + 8$) – but allow exact equivalents e.g. $y > \frac{4}{3}x + \frac{8}{3}$, $4x - 3y < -8$, $4x + 8 - 3y < 0$, or equivalent but must be three terms only In (c), the lines can be drawn as either dashed or non-dashed lines (or a combination of the two). The lines must be long enough to define the correct feasible region and pass through one small square of the points stated below: $y = 2x$ must pass within one small square of (0, 0) and (7, 14) $y = x + 7$ must pass within one small square of (0, 7) and (7, 14) $x = 1$ must pass within one small square of (1, 0) and (1, 10) $3y = 4x + 8$ must pass within one small square of (1, 4) and (7, 12) c1B1: Any one line correctly drawn (ignore any shading) c2B1: Any two lines correctly drawn (ignore any shading) c3B1: Any three lines correctly drawn (ignore any shading) c4B1: All four lines correctly drawn and shading which implies the correct region (but region need not be labelled) d1B1ft: At least 4 pairs of integer coordinates correctly stated for points inside their region. This mark is dependent on scoring at least the first two marks in (c) (so must have drawn at least two lines correctly) and the candidate must have drawn exactly four lines. The region must not be infinite but need not necessarily be bounded by all four lines. If the candidate’s region does not contain 4 integer coordinates then B0. Note that integer points on the lines that define the boundary of the region are not counted as being inside the region (regardless of if the candidate has strict inequalities or not) d2B1: All 9 coordinates correct (and no others) – dependent on all four lines correctly drawn in (c) e1M1: States that the remaining arc (in the MST) is one of either EH, EG or FG (and no others). Only one of these three arcs need to be stated for this mark. Allow this mark for either the expression $(4x + 1) + (3y - 7) + (2y - 2) + (3x) + (x + y) + (6x - 2y + 3) + X$ or the equation		

$(4x+1)+(3y-7)+(2y-2)+(3x)+(x+y)+(6x-2y+3)+X=73$ where $X = y+1$ or $2y+4$ or $5x+1$ (or equivalent equations/expressions). Their expression/equation need not be simplified but for reference (if correct) they are $14x+4y-5+X$ and $14x+4y-5+X=73$ (with the expression for X as before). Note that stating $14x+4y-5$ (or equivalent) and then separately stating one of the expressions for X would imply this mark

e2A1: A correct expression for the weight of the MST either simplified ($14x+5y-4$) or not $((4x+1)+(3y-7)+(2y-2)+(3x)+(x+y)+(6x-2y+3)+(y+1))$. This mark can be implied if a correct equation is seen, e.g. $14x+5y=77$ (or equivalent and again need not be simplified). **If more than one equation or expression seen then they must clearly select the correct expression/equation for this mark** (so stating more than one expression for the weight of the MST is A0)

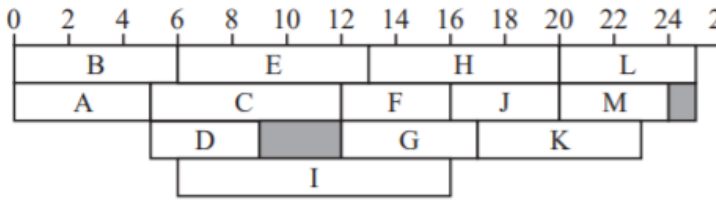
e1M1dep: This mark is dependent on the first M mark in this part and the first B mark in (d). Setting their linear expression (in x and y) for the weight of the MST equal to 73 and then substituting into this equation at least one integer pair of values of x and y from (d). This mark can also be awarded for substituting at least one integer pair of values of x and y into their linear expression. The correct answers can imply this mark

e1A1: Correct answers only ($x=3$ and $y=7$) from correct working – do not accept any other answers stated as well but accept as a coordinate (3, 7) – **must have drawn the correct four lines in (c) but need not have stated all nine correct coordinates in (d)**. As a minimum for full marks in (e) the candidate must have stated a correct expression (e.g. $14x+5y-4$ or equivalent) or equation (e.g. $14x+5y=77$ or equivalent) before then stating the correct answer

The correct answer with no method or working scores no marks in (e)



Question Number	Scheme	Marks
3. (a)	Prim: AE, EG, CE; DG, CF; DH, BF	M1 A1 A1 (3)
(b)	Weight of MST = 197	B1 (1)
(c)	Initial upper bound = $2(197) = 394$	B1ft (1)
(d)	A – E – G – D – H – B – F – C – A $23 + 24 + 26 + 33 + 38 + 34 + 32 + 38 = 248$	M1 A1
	A – E – G – D – H – F – C – B – A	A1
	$23 + 24 + 26 + 33 + 38 + 32 + 35 + 36 = 247$	A1 (4)
(e)	247	B1ft (1)
(f)	Weight of RMST is 174	B1ft
	Lower bound = $174 + 23 + 35 = 232$	M1 A1 (3)
(g)	$232 \square$ optimal value $\square$ 247	M1 A1 (2)
		15 marks
	Notes for Question 3	
a1M1: Prim's – first three arcs correctly chosen in order (AE, EG, CE, ...) or first four nodes {A, E, G, C, ...} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, 4, -, 2, -, 3, -}. Starting at any other node can score M1 only for first three arcs chosen correctly a1A1: First five arcs correctly chosen in order (AE, EG, CE, DG, CF, ...) or all eight nodes {A, E, G, C, D, F, H, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 4, 5, 2, 6, 3, 7} (no missing numbers) a2A1: cso – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen) b1B1: cao ($197 - \text{ignore units}$) should come from $23 + 24 + 25 + 26 + 32 + 33 + 34$ c1B1ft: Follow through double their answer to (b) Mark (d) and (e) together d1M1: Nearest neighbour starting at A with first five nodes correct (A – E – G – D – H –) d1A1: One correct route (must return to A) d2A1: One correct value or both correct routes d3A1: Both correct values (do not isw if values doubled) and both correct routes (must both return to A) SC in (d) correct Hamiltonian paths and corresponding weights (AEGDHBFC (210) and AEGDHFCB (211)) scores M1A1A0A0 e1B1ft: Follow through their least weight route from (d) – must have or imply two Hamiltonian cycles in (d) or (e) f1B1ft: Either 174 or $24 + 25 + 26 + 32 + 33 + 34$ or $197 - 23$ or the weight of their MST from (b) – 23 f1M1: Weight of RMST + 23 + 35 (two smallest arcs incident to A) with 151£ RMST £ 197 (if clearly not six arcs in RMST then M0) f1A1: cao (232) – if correct answer with no working then awarded B0M1A1 – as a minimum for full marks accept $174 + 23 + 35 = 232$ but $174 + 58 = 232$ scores B1M1A0 g1M1: Any indication of an interval from their answer to (f) to their answer to (e) with one value correct g1A1: cao (either 232£ optimal value £ 247 or $232 < \text{optimal value } £ 247$)		

Question Number	Scheme	Marks																																
4. (a)	<table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>-</td></tr><tr><td>C</td><td>A</td></tr><tr><td>D</td><td>A</td></tr><tr><td>E</td><td>A, B</td></tr></tbody></table> <table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>F</td><td>C</td></tr><tr><td>G</td><td>C</td></tr><tr><td>H</td><td>C, D, E</td></tr><tr><td>I</td><td>A, B</td></tr><tr><td>J</td><td>A, B</td></tr></tbody></table> <table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>K</td><td>F</td></tr><tr><td>L</td><td>F, G, H, I</td></tr><tr><td>M</td><td>F, G, H, I, J</td></tr></tbody></table>	Activity	IPA	A	-	B	-	C	A	D	A	E	A, B	Activity	IPA	F	C	G	C	H	C, D, E	I	A, B	J	A, B	Activity	IPA	K	F	L	F, G, H, I	M	F, G, H, I, J	B2, 1, 0 (2)
Activity	IPA																																	
A	-																																	
B	-																																	
C	A																																	
D	A																																	
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K	F																																	
L	F, G, H, I																																	
M	F, G, H, I, J																																	
(b)	$v = 7, w = 4, x = 6, y = 16, z = 19$	B3, 2, 1, 0 (3)																																
(c)	$\frac{74}{25} = 2.96$ so 3 workers	B1 (1)																																
(d)	e.g. 	M1 A1 A1 (3)																																
(e)	New minimum project completion time: 27 (days)	B1																																
	Critical path: ACFK	B1 (2)																																
		11 marks																																
	Notes for Question 4																																	
a1B1: Any 6 rows correct (not including A and B) a2B1: cao (condone blank rows for A and B) b1B1: Any 2 correct values b2B1: Any 3 correct values b3B1: All 5 values correct c1B1: cao (3 from correct working) – as a minimum for correct working accept either 2.96 or $\frac{5 + 6 + 7 + 4 + 7 + 4 + 5 + 7 + 10 + 4 + 6 + 5 + 4}{25}$ or $\frac{74}{25}$ d1M1: Not a cascade chart. 4 workers used at most, at least 9 activities placed d1A1: 4 workers. All 13 activities present (just once). Condone two errors either precedence or time interval or activity length. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA d2A1: 4 workers. All 13 activities present (just once). No errors e1B1: cao (27 only) e2B1: cao (ACFK or KFCA only)																																		

Question	Scheme	Marks
4. (a)(i)	Fastest route: ACBEFGJH	M1 A1 (ACB D) A1 (EFG) A1ft (JH)
(ii)	Fastest time: 53 (minutes)	A1ft
		(6)
(b)	Either calculates new fastest route $AF(35) + FH(22) = 57$ Or calculates the change to sections of the route e.g. ACDF compared with ACBEF and FGJH compared with FJH Difference $57 - "53" = 4$ (minutes) or $2 + 2 = 4$ (minutes)	M1 A1ft
		(2)
(8 marks)		



Notes	
	In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at E the working values must be 27 26 in that order (so 26 27 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling
ai1M1	A larger value replaced by a smaller value at least twice in the working values at either B, D, E, F, G, H, J
ai1A1	All values at A, C, B and D correct and the working values in the correct order
ai2A1	All values at E, F and G correct and the working values in the correct order
ai3A1ft	All values in J and H correct on the follow through and the working values in the correct order. To follow through J check that the working values at J follow from the candidate's final values for the nodes that are directly attached to J (which are F and G). For example, if correct then the order of labelling of nodes F and G are 6 and 7 respectively so the working values at J should come from F and G in that order. The first working value at J should be their 50 (the Final value at F) + 17 (the weight of the arc FJ), the second working value at J should be their 48 (the Final value at G) + 6 (the weight of the arc GJ). Repeat the process for H (which will have working values from E, F and J with the order of these nodes determined by the candidate's order of labelling at E, F and J)
ai4A1	CAO
aiiA1ft	Follow through their final value at H only – if answer is 53 but the final value at H is not 53 then A0 (condone lack of minutes)
bM1	Either calculates the revised time of 57 minutes Or compares the route in stages e.g. ACBEF against ACDF difference 2 and FGJH against FJH difference 2. If their final value at H is not 53 follow through their final value at F and shortest from F to H for these calculations
bA1ft	4 (minutes) (may be seen as $2 + 2 = 4$) or follow through 57 - their fastest to H (which may be in sections AF + FH)

e4A1ft: Follow through their value of their smallest pairing total + 291

Question Number	Scheme	Marks
5.(a)	Start at A e.g. we would be able to find the shortest distance from A to every other vertex e.g. A appears in both required routes	B1 B1dep (2)
(b)	Shortest path to J: A C D G F J Length: 80 (miles) Shortest path to K: A C D B E H K Length: 81 (miles)	M1 A1 (ACDBG) A1 (EFH) A1ft (JK) A1 A1 A1ft (7)
(c)	Route from F to H via A: F G D C A C D B E H	B1 (1)
		10 marks
Notes for Question 5		



a1B1: CAO (A) – if more than (vertex) A stated then B0

a2B1dep: Correct reason for starting at A (dependent on first B mark) – either need to explicitly mention that A appears on both routes **or** if starting at A then the shortest route to all other vertices (or just to vertices J and K) can be found

In (b) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 51 49 48 in that order (so 51 48 49 is incorrect)

It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

b1M1: Any larger working value replaced by any smaller working value at any two nodes except A and C (for example, **if** correct at B, 32 is replaced by 31 which is a larger value being replaced by a smaller value at one node – as this is a method mark the values do not need to be correct)

b1A1: All values at A, C, D, B and G correct and the working values in the correct order (including order of labelling so nodes must be labelled in the order A, then C, then D, then B, then G). Condone lack of a zero as a working value at A

b2A1: All values at E, F and H correct and the working values in the correct order. Penalise order of labelling only once per question (so E, F and H must be labelled in that order and E must be labelled after A, C, D, B and G)

b3A1ft: All values in J and K correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through J check that the working values at J follow from the candidate's final values for the nodes that are directly attached to J (which are A, F and G). For example, **if** correct then the order of labelling of nodes A, F and G is 1, 7 and 5 respectively so the working values at J should come from A, G and F in that order. The first working value at J should be 0 (the Final value at A) + 84 (the weight of the arc AJ), the second working value at J should be 40 (the Final value at G) + 42 (the weight of the arc GJ) and the final working value at J should be 48 (the Final value at F) + 32 (the weight of the arc FJ). Repeat the process for K (which will have working values from D, G and H with the order of these nodes determined by the candidate's order of labelling at D, G and H)

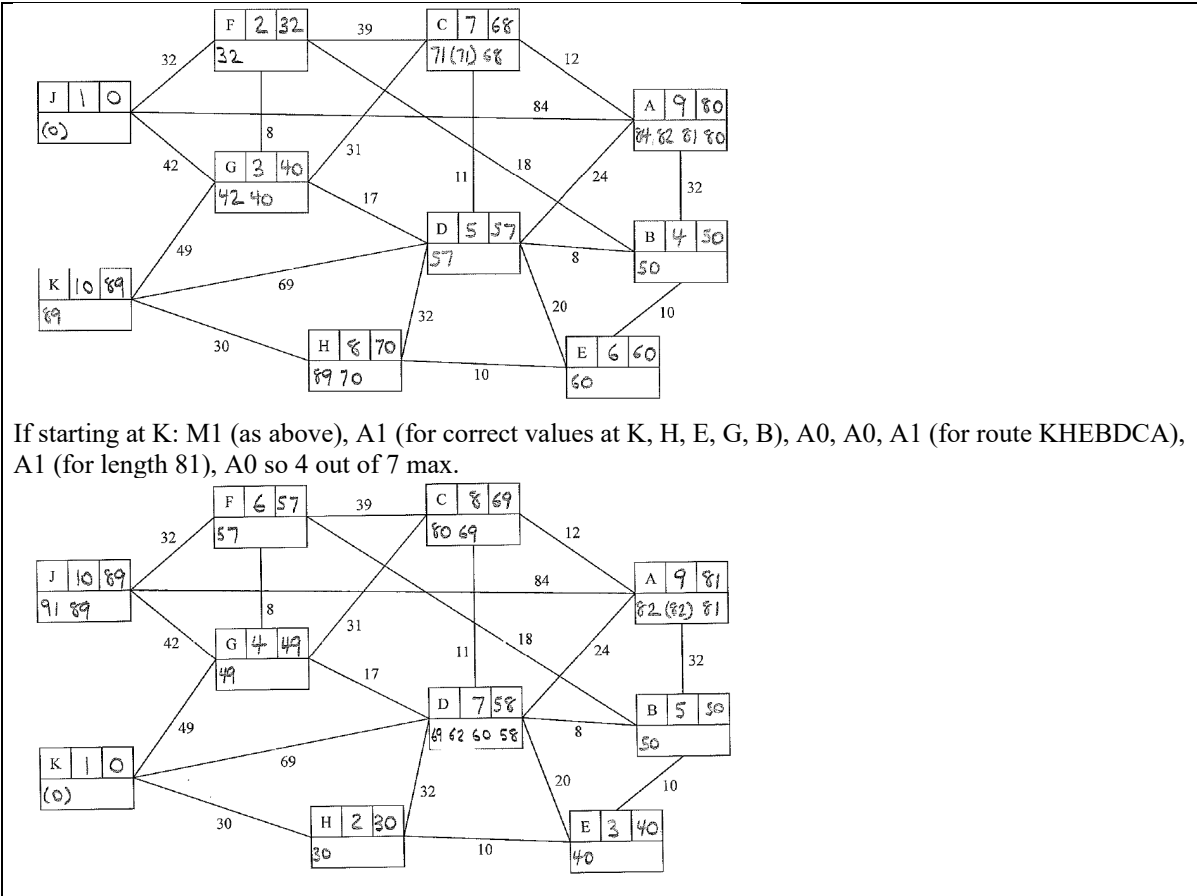
b4A1: One correct route (either ACDGFJ or ACDBEHK) – allow if reversed (e.g. JFGDCA) and allow if stated in terms of arcs (e.g. AC, CD, DG, GF, FJ)

b5A1: Both routes correct (as for **b4A1** – routes can be reversed and accept in terms of arcs)

b6A1ft: Both lengths correct following through **their** Final values at J and K. Condone correct answers or correct answers following through their diagram even if not explicitly clear which value refers to which path

c1B1: Correct answer only (FGDCACDBEH or FG, GD, DC, CA, AC, CD, DB, BE, EH) – if stated in terms of arcs then arcs AC and CD must appear twice in their route

If Dijkstra is completed twice (from J **and** K) then full marks can be awarded. If the candidate uses **just** J or K as the starting vertex in **(b)** then this is not a misread. The candidate can score if starting at J: M1 (as above), A1 (for correct values at J, F, G, B, D), A0, A0, A1 (for route JFGDCA), A1 (for length 80), A0 so 4 out of 7 max.



If starting at K: M1 (as above), A1 (for correct values at K, H, E, G, B), A0, A0, A1 (for route KHEBDCA), A1 (for length 81), A0 so 4 out of 7 max.

Question Number	Scheme	Marks
6.(a)	c.g. The diagram shows a network of nodes and edges. The path A-B-F-I-K-M is highlighted with solid arrows. Other nodes and edges are shown but not part of the path. Dashed lines indicate other possible paths or connections.	M1 A1 A1 A1 A1 (5)

Question Number	Scheme	Marks
1.(a)	A path is a (i) finite sequence of edges, such that (ii) the end vertex of one edge in the sequence is the start vertex of the next, and in which (iii) no vertex appears more than once	B2, 1, 0 (2)
(b)	Shortest path: A B E H J Length: 33 (km)	M1 A1 (ACBFD) A1 (GE) A1ft (HJ) A1 A1ft (6)
(c)	Shortest path from J to A via G: J G D C A Length: $20 + 15 = 35$ (km)	B1 B1ft (2)
		10 marks

Question Number	Scheme	Marks
	Notes for Question 1	
a1B1: One of the three points made clearly ('finite, edges', 'end vertex of one edge is the start vertex of the next', 'no vertex appears more than once') a2B1: All three points made clearly. Candidates who state that a path is a walk in which no vertex appears more than once can score B1B0 only In (b) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 15 14 13 in that order (so 15 13 14 is incorrect). It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling b1M1: Working values - a larger value replaced by a smaller value for at least two of the five activities D, E, F, G, J b1A1: All values at A, C, B, F and D correct and the working values in the correct order b2A1: All values at G and E correct and the working values in the correct order b3A1ft: All values in H and J correct on the follow through and the working values in the correct order b4A1: cao (A B E H J only) b5A1ft: Follow through on their final value at J only (condone lack of units) c1B1: cao (J G D C A only) c2B1ft: 35 or follow through their final value at G + 15		

Question Number	Scheme	Marks
2. (a)	Route: ABDEFHKJ Length: 76 (km)	M1 A1 (ABCDE) A1 (FGH) A1ft (KJ) A1 A1ft (6)
(b)	Prim: AB, BC; BD, DE	M1; A1 (2)
(c)	Kruskal: FG, JK, FH, not GH, HK, (not HJ), (not FK), (not GJ)	M1; A1 (2)
(d)	Total length: 85 (km)	B1 (1)
		11 marks

Notes for Question 2

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at E the working values must be 24 22 20 in that order (so 24 20 22 is incorrect)

It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at either C or E or J or K

a1A1: All values in A, B, C, D and E correct and the working values in the correct order at C and E (including order of labelling). Condone lack of 0 in A's working value

a2A1: All values in F, G and H correct and the working values in the correct order. Penalise order of labelling only once per question (F, G and H must be labelled in that order and F must be labelled after A, B, C, D and E). Note that an additional working value of 56 at H after the 48 is not an error so 48 56 is fine, however, any other number or 56 48 in this order is incorrect and scores A0 in this part

a3A1ft: All values in K and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through K check that the working value at K follows from the candidate's final values from their feeds into K (which will come from nodes F, H and possibly even J (in the order in which the candidate has labelled them)) and that the final value, and order of

Question Number	Scheme	Marks
	labelling, follows through correctly. Repeat this process for J (which will possibly have working values from G, H and K with the order of these values determined by the candidate's order of labelling at G, H and K) a4A1: CAO - correct route (ABDEFHKJ) not from J to A a5A1ft: Follow through on their final value at J only (so if 76 given as the answer and the final value at J is not 76 then A0) b1M1: First two arcs (AB, BC) chosen correctly in order, or first three nodes (ABC) chosen correctly in order. If any rejections seen at any point, or just a list of all the arcs in order, or only a list of weights then M0 (condone for M1 only those who find the MST for the entire network) b1A1: CSO (must be considering arcs so must be AB, BC, BD, DE or BA, BC, etc.) – do not isw if candidates continue and find the MST for the entire network c1M1: First two arcs (FG, JK) chosen correctly in order and at least one rejection seen at some point – no marks in this part if candidates apply Kruskal to the entire network or if only a list of weights given c1A1: CSO – all selections and rejections correct in the correct order and at the correct time. Note that stating all the arcs in order (e.g. GF, JK, FH, GH, KH, JH, FK, GJ) and then stating only those in the tree in the correct order is fine for both marks in this part d1B1: CAO (85)	