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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Route Inspection & the Travelling Salesperson (D1.3)

International A Level Mathematics · D1: Decision Mathematics 1 (WDM11)

23 questions · 265 marks · Mark schemes included

Past papers June 2019 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Where a question is answered in a separate answer book, those answer book pages follow the question.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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23. Q6	IAL D1 (WDM11) January 2022	14	96



QUESTIONS

23 questions · 265 marks · 2 sub-topics

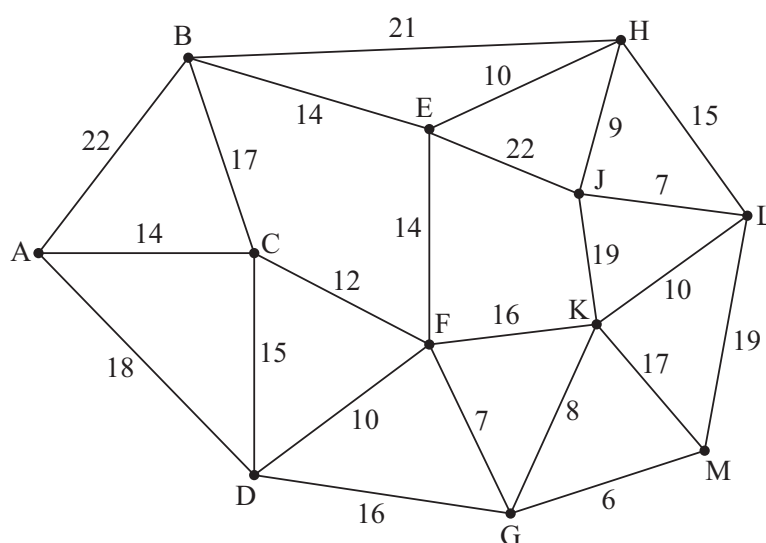
SUB-TOPIC D1.3a

ROUTE INSPECTION

Sub-topic D1.3a · 13 questions · 168 marks · lowest tariff first



6.



[The total length of the network is 338]

Figure 2

Figure 2 represents a network of roads connecting a group of villages. The number on each arc is the length, in km, of the corresponding road.

Bolin needs to inspect the network in Figure 2. He must travel along each road at least once, minimising the length of his route.

Bolin's route must **start** at A and **finish** at J.

- (a) Determine the length of Bolin's route. You must make your method clear and state the roads which need to be repeated.

(5)

A new road is constructed from F to J which has length 18 km. Bolin must inspect the changed network, starting at A and finishing at J. He must travel along each road at least once, minimising the length of his route.

- (b) Determine the change to the length of Bolin's route.

(2)

(Total for Question 6 is 7 marks)



6.

Lined area for writing the answer to question 6.



(Total for Question 6 is / marks)



5.

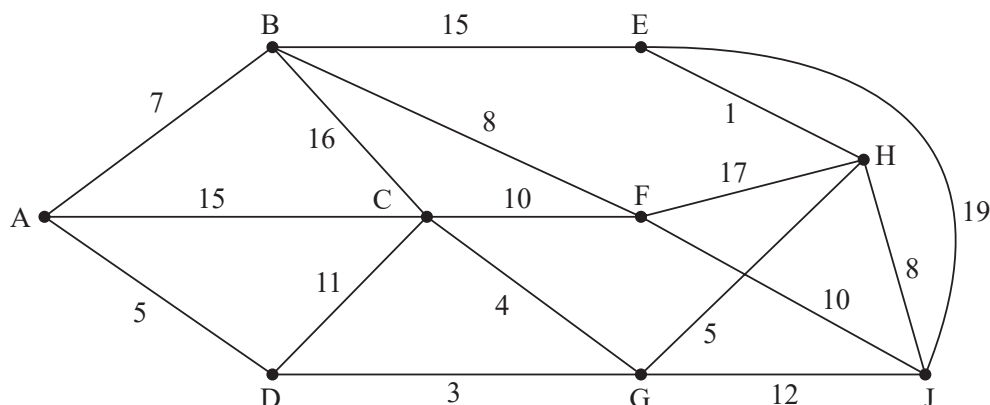


Figure 3

[The total weight of the network is 166]

Figure 3 models a network of cycle lanes that must be inspected. The number on each arc represents the length, in km, of the corresponding cycle lane. Lance needs to cycle along each lane at least once and wishes to minimise the length of his inspection route.

He must start and finish at A.

- (a) Use an appropriate algorithm to find the length of the route. State the cycle lanes that Lance will need to traverse twice. You should make your method and working clear. (6)
- (b) State the number of times that vertex C appears in Lance's route. (1)

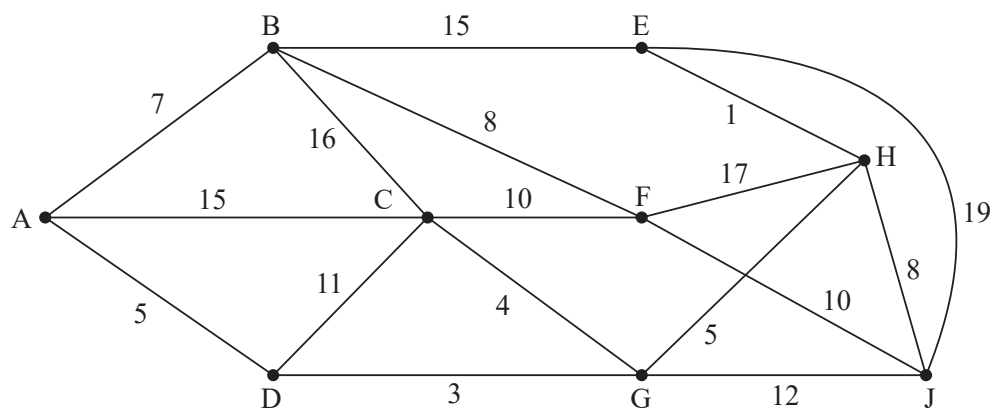
It is now decided that the inspection route may finish at any vertex. Lance will still start at A and must cycle along each lane at least once.

- (c) Determine the finishing point so that the length of the route is minimised. You must give reasons for your answer and state the length of this new minimum route. (3)

(Total 10 marks)



5.

**Figure 3**

[The total weight of the network is 166]

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Question 5 continued

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Handwriting practice lines for Question 5.

(Total 10 marks)

Q5



5.

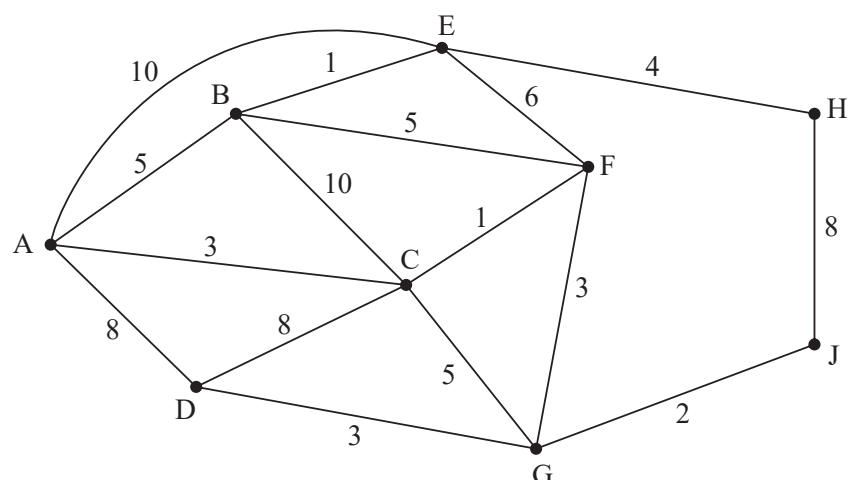


Figure 4

[The total weight of the network is 82]

Figure 4 represents a network of 16 roads in a city. The number on each arc represents the time taken, in minutes, to travel along the corresponding road.

Chan needs to check that the roads are in good repair. He must travel along each road at least once. Chan will start and finish at his office at G and must minimise the total time taken for his inspection route.

For this inspection route,

- (a) find the time taken and state a possible route. You must make your method and reasoning clear.

(3)

Chan wonders if he can reduce his travel time by starting from his home at B, travelling along each road at least once and finishing at his office at G.

- (b) By considering the pairings of all relevant nodes, find any arcs that would need to be traversed twice in the minimum inspection route from B to G. You must make your method clear, showing your working.

(5)

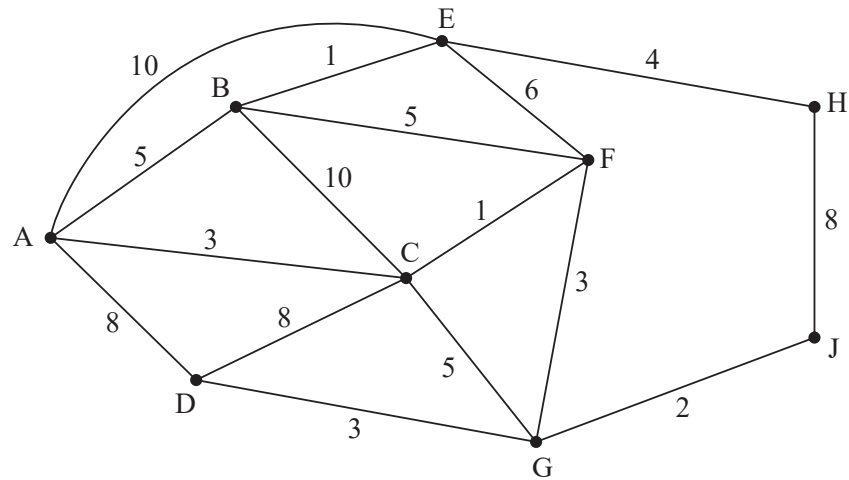
- (c) Determine which of the two routes ending at G is quicker, the one starting at G or the one starting at B. You must justify your answer.

(2)

(Total 10 marks)



5.

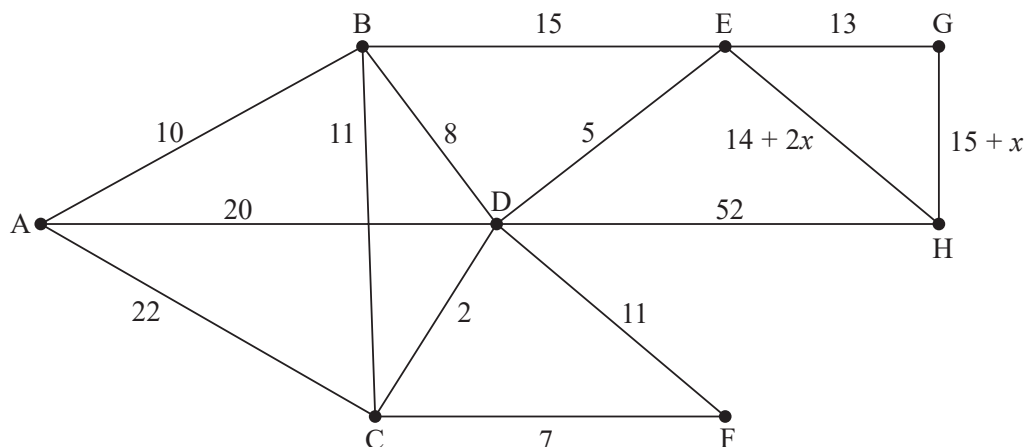
**Figure 4**

[The total weight of the network is 82]





7.

**Figure 3**

[The total weight of the network is $205 + 3x$]

Figure 3 represents a network of roads. The number on each arc represents the time taken, in minutes, to drive along the corresponding road.

Malcolm wishes to minimise the time spent driving from his home at A to his office at H. The delays from roadworks on two of the roads leading in to H vary daily, and so the time taken to drive along these roads is expressed in terms of x , where x is fixed for any given day and $x > 0$

- (a) Use Dijkstra's algorithm to find the possible routes that minimise the driving time from A to H. State the length of each route, leaving your answer in terms of x where necessary. (7)

On Monday, Malcolm needs to check each road. He must travel along each road at least once. He must start and finish at H and minimise the total time taken for his inspection route.

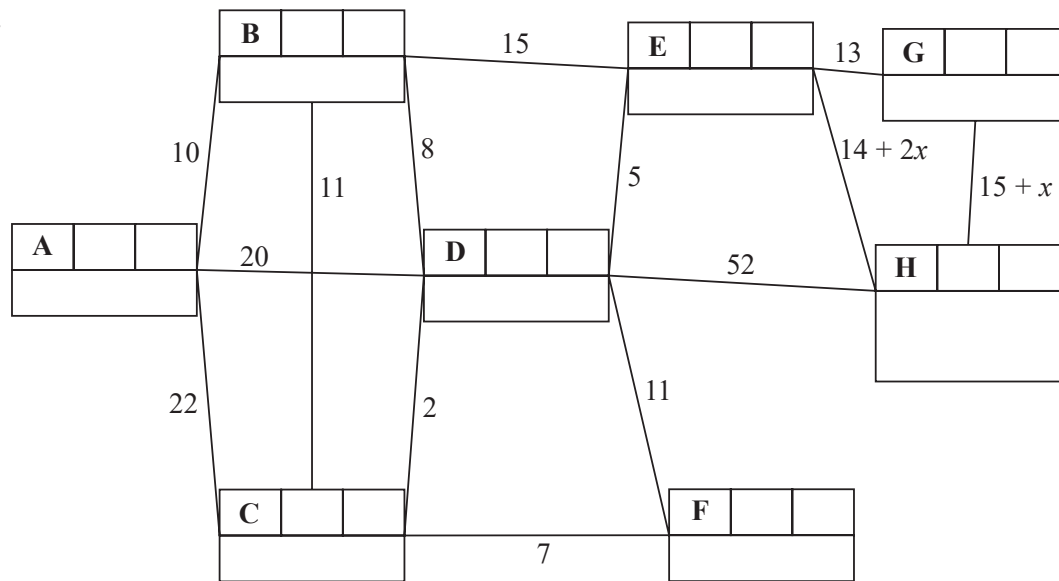
Malcolm finds that his minimum duration inspection route requires him to traverse exactly four roads twice and the total time it takes to complete his inspection route is 307 minutes.

- (b) Calculate the minimum time taken for Malcolm to travel from A to H on Monday. You must make your method and working clear. (4)

(Total 11 marks)



7.



Key:

Vertex	Order of labelling	Final value
Working values		

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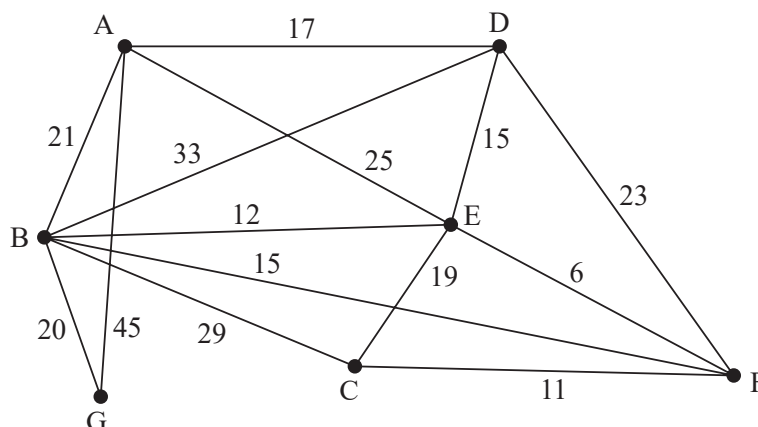


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Q7

4.

**Figure 3**

[The total weight of the network is 291]

Figure 3 models a network of roads. The number on each edge gives the length, in km, of the corresponding road. The vertices, A, B, C, D, E, F and G, represent seven towns. Derek needs to visit each town. He will start and finish at A and wishes to minimise the total distance travelled.

- By inspection, complete the two copies of the table of least distances in the answer book. (2)
- Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of Derek's route. Write down the route that gives this upper bound. (2)
- Interpret the route found in (b) in terms of the towns actually visited. (1)
- Starting by deleting A and all of its arcs, find a lower bound for the route length. (3)

Clive needs to travel along the roads to check that they are in good repair. He wishes to minimise the total distance travelled and must start at A and finish at G.

- By considering the pairings of all relevant nodes, find the length of Clive's route. State the edges that need to be traversed twice. You must make your method and working clear. (5)

(Total 13 marks)



Question 4 continued

	A	B	C	D	E	F	G
A	–	21		17	25	31	41
B	21	–	26	27	12	15	20
C		26	–		17	11	46
D	17	27		–	15		47
E	25	12	17	15	–	6	32
F	31	15	11		6	–	35
G	41	20	46	47	32	35	–

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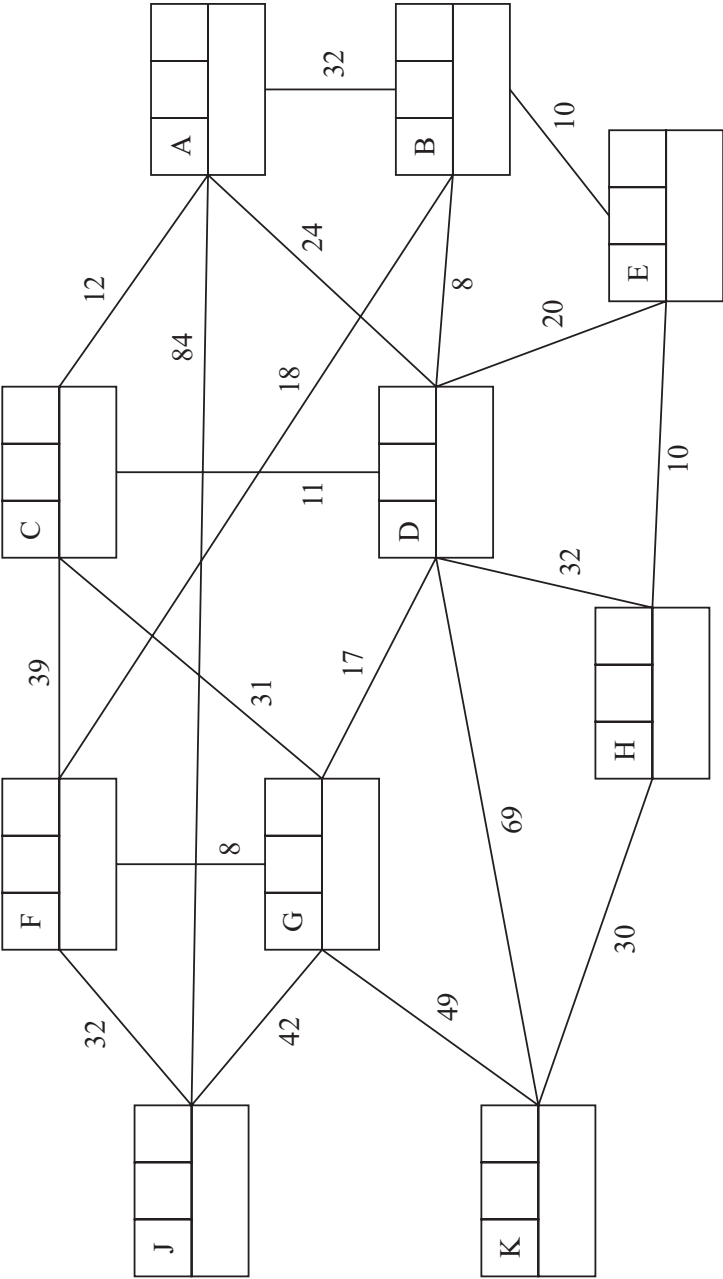
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(Total 13 marks)

Q4

5.

(a)



Key:

Vertex	Order of labelling	Final value
Working values		

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Question 5 continued

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Handwriting practice area with 30 horizontal lines.

(Total 10 marks)

Q5
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6.

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Question 6 continued

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Handwriting practice lines for Question 6.

(Total 9 marks)

Q6



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Question 7 continued

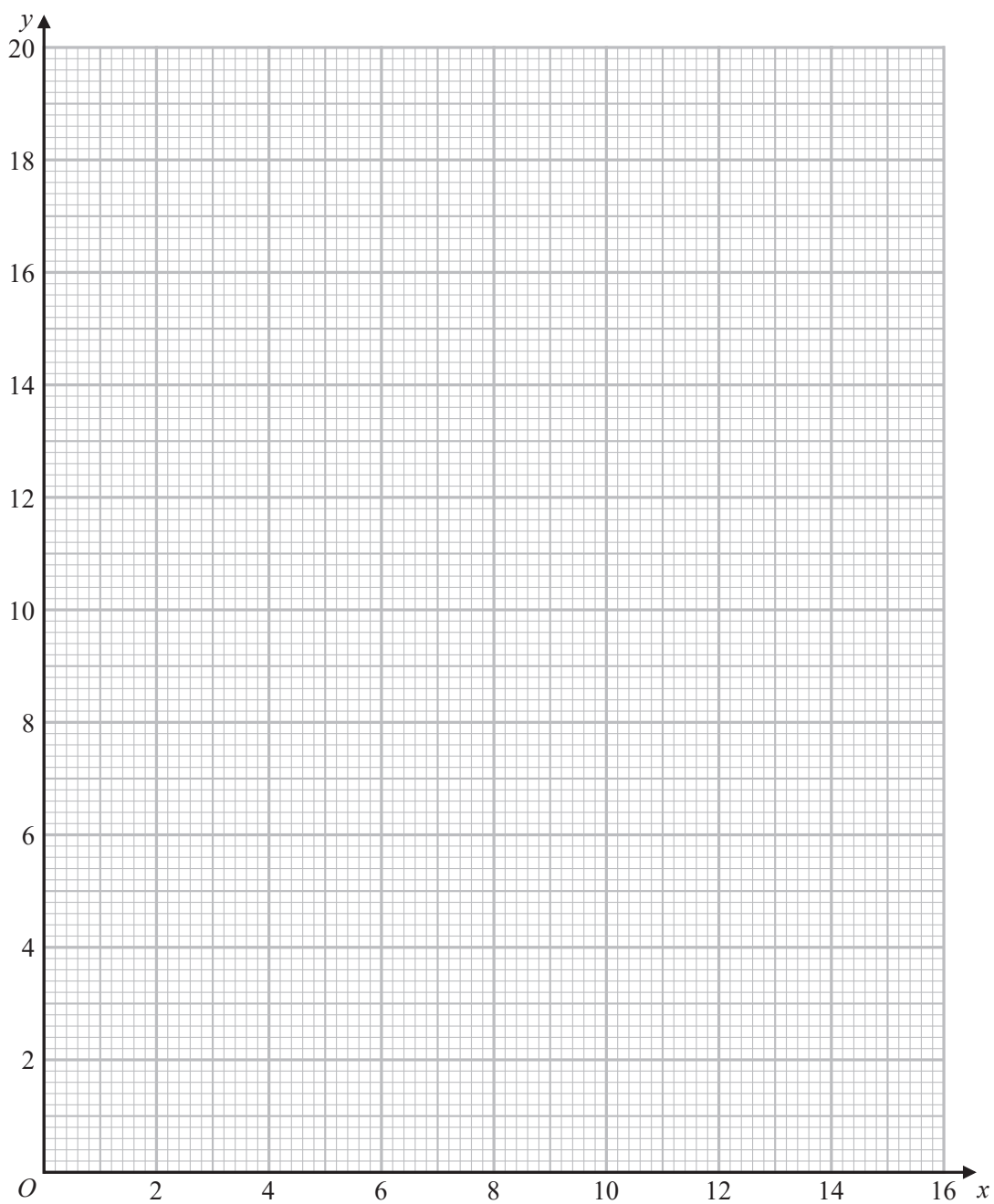
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Diagram 1



Question 7 continued

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(Total 15 marks)

Q7

TOTAL FOR PAPER: 75 MARKS

END



6.

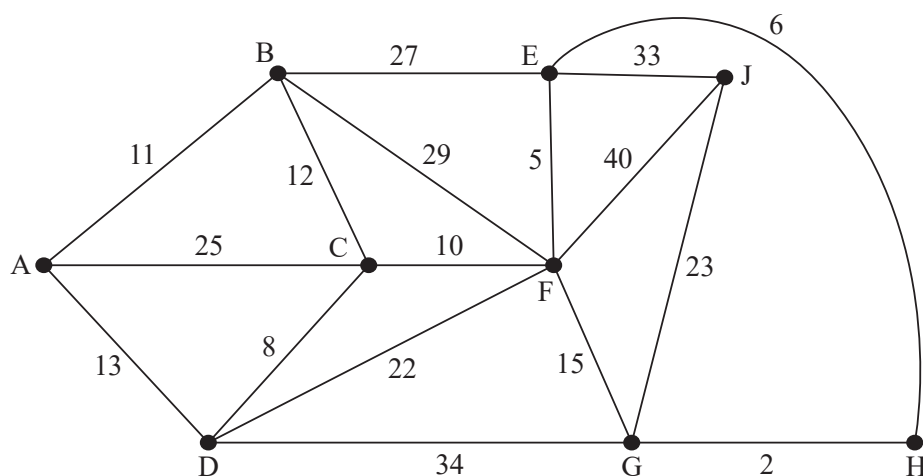


Figure 3

[The total weight of the network is 315]

Figure 3 represents a network of roads between nine parks, A, B, C, D, E, F, G, H and J. The number on each edge represents the length, in miles, of the corresponding road.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.

(ii) State the length of the shortest path from A to J.

(6)

The roads between the parks need to be inspected. Robin must travel along each road at least once. Robin wishes to minimise the length of the inspection route. Robin will start the inspection route at C and finish at E.

(b) By considering the pairings of all relevant nodes, find the length of Robin's route.

(4)

(c) State the number of times Robin will pass through G.

(1)

It is now decided to start and finish the inspection route at A. Robin must still minimise the length of the route and travel along each road at least once.

(d) Calculate the difference between the lengths of the two inspection routes.

(1)

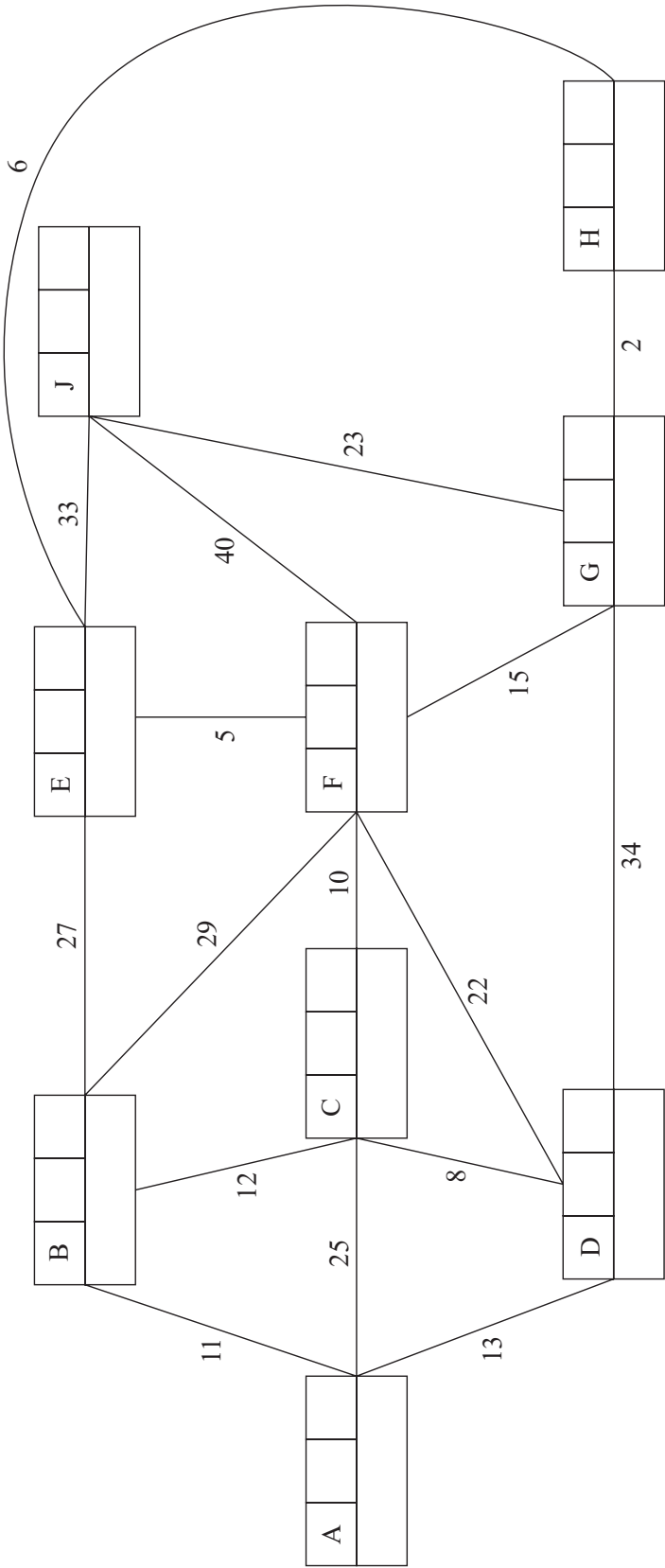
(e) State the edges that need to be traversed twice in the route that starts and finishes at A, but do not need to be traversed twice in the route that starts at C and finishes at E.

(1)

(Total for Question 6 is 13 marks)



6.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path: _____

Length of shortest path: _____

Question 6 continued

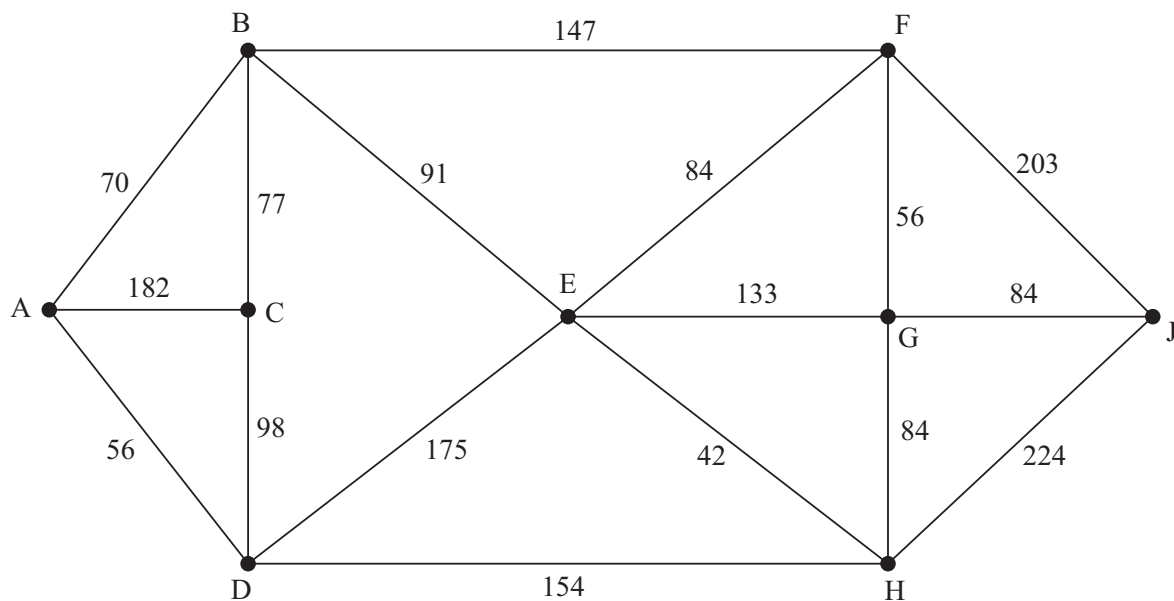
Lined area for writing the answer to Question 6.



(Total for Question 6 is 15 marks)



2.

**Figure 1**

[The total weight of the network is 1960 m]

The network shown in Figure 1 represents the paths between nine attractions in a theme park.

The number on each edge is the length, in metres, of the corresponding path.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.

(ii) State the length of the shortest path from A to J in metres.

(6)

Sarita needs to inspect the paths between the attractions. She must travel along each path at least once.

Sarita decides to start and finish her inspection route at A. She wishes to minimise the length of her route.

(b) By considering the pairings of all relevant nodes, find the length of Sarita's route.

(4)

Sarita now decides to start her inspection route at A, but finish at a different attraction. She must still minimise the length of her route and travel along each path at least once.

(c) (i) Determine where Sarita should finish her route. You must justify your answer.

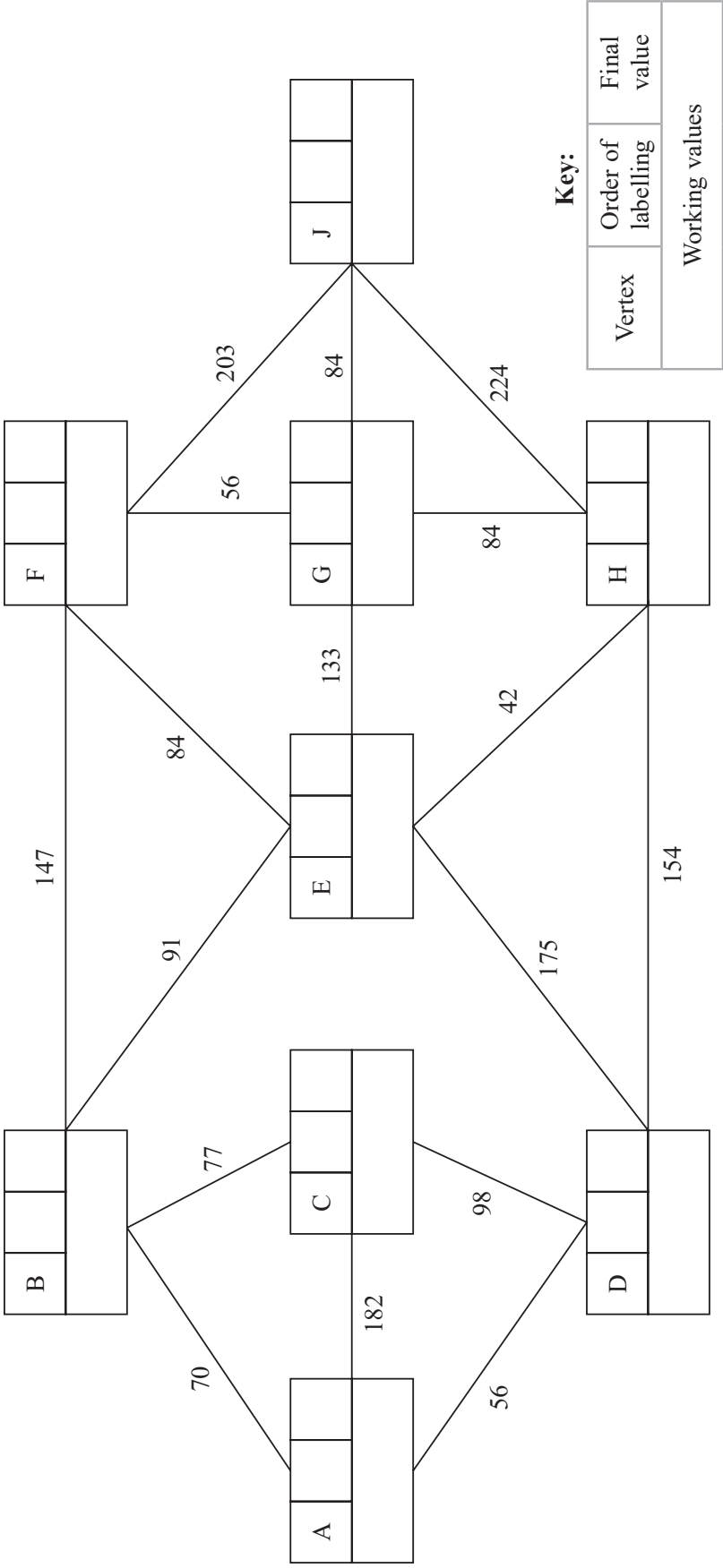
(ii) Calculate the difference between the lengths of the two inspection routes.

(3)

(Total for Question 2 is 13 marks)



2. (a)



(i) Shortest path from A to J: _____

(ii) Length of shortest path from A to J: _____



(Total for Question 2 is 15 marks)



3.

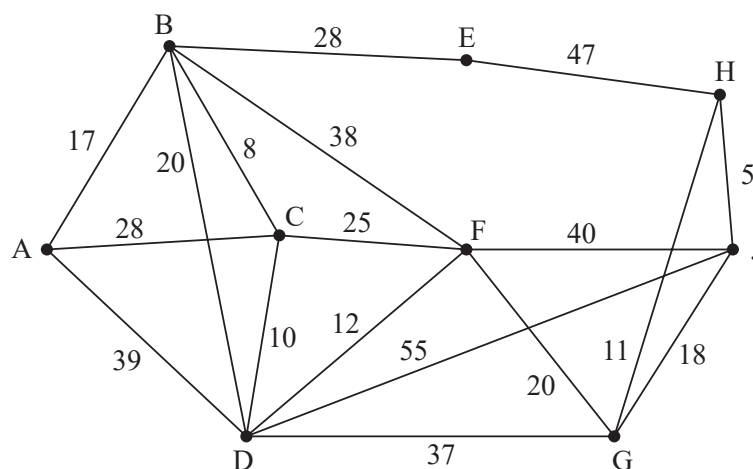


Figure 2

[The total weight of the network is 458]

Figure 2 represents a network of roads between nine towns, A, B, C, D, E, F, G, H and J. The number on each edge represents the length, in kilometres, of the corresponding road.

- (a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.
 (ii) State the length of the shortest path from A to J.

(6)

The roads between the towns must be inspected. Claude must travel along each road at least once. Claude will start the inspection route at A and finish at J. Claude wishes to minimise the length of the inspection route.

- (b) By considering the pairings of all relevant nodes, find the length of Claude's route. State the arcs that will need to be traversed twice.

(5)

If Claude does **not** start the inspection route at A and finish at J, a shorter inspection route is possible.

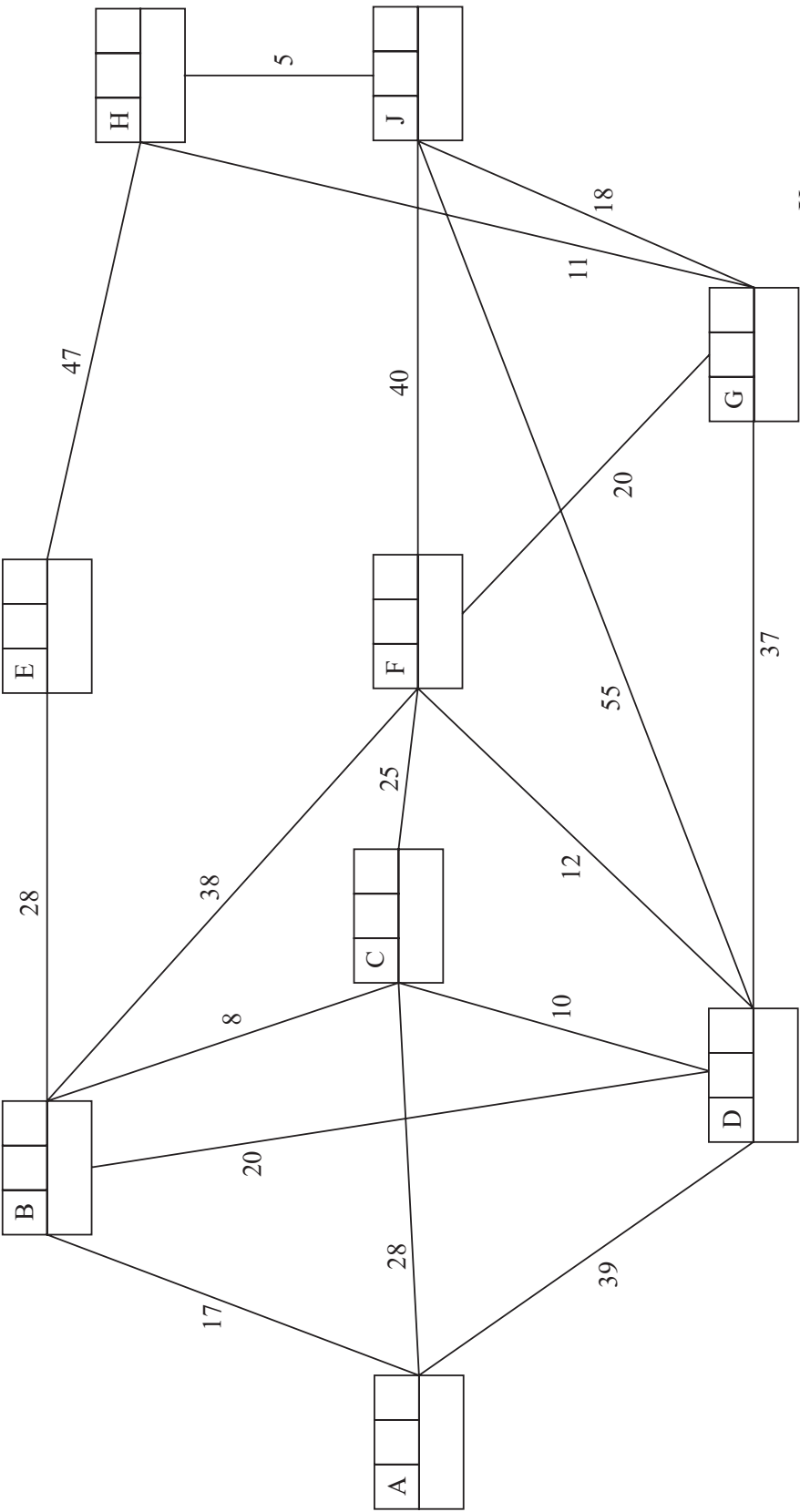
- (c) Determine the two towns at which Claude should start and finish so that the route has minimum length. Give a reason for your answer and state the length of this route.

(3)

(Total for Question 3 is 14 marks)



3.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to J: _____

Length of shortest path from A to J: _____

Question 3 continued

Lined area for writing the answer to Question 3.



Question 3 continued

Lined area for writing the answer to Question 3.





4.

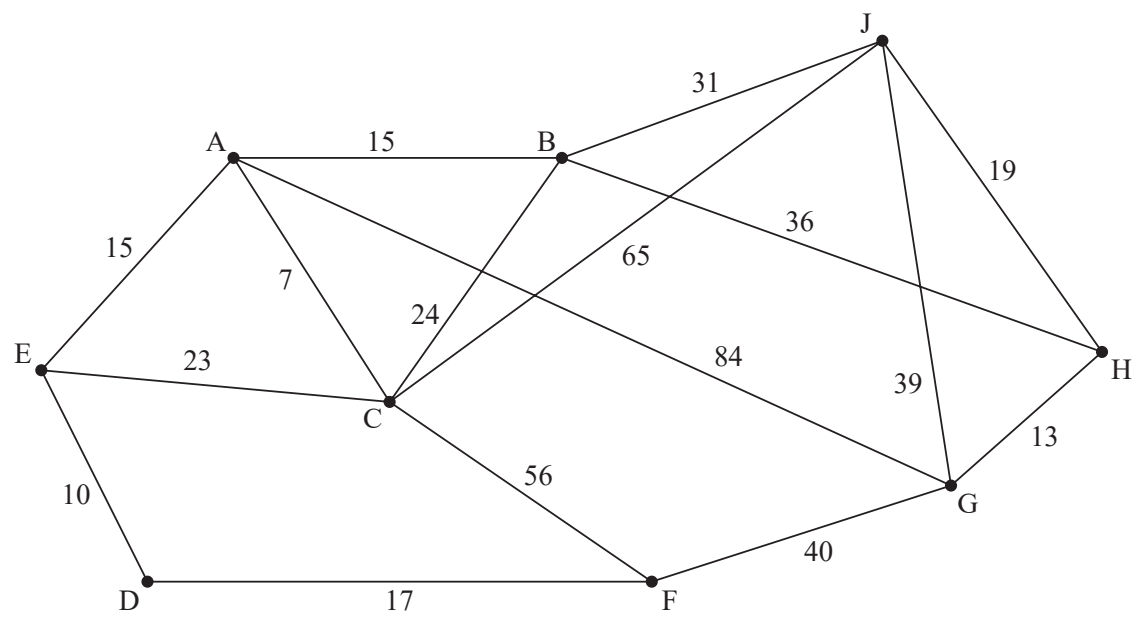


Figure 3

[The total weight of the network is 494]

Direct roads between nine factories, A, B, C, D, E, F, G, H and J, are represented in Figure 3.

The number on each arc represents the lengths, in kilometres, of the corresponding road.

The table below shows the shortest distances, in kilometres, between the nine factories.

	A	B	C	D	E	F	G	H	J
A	-	15	7	25	15	42	64	51	46
B	15	-	22	40	30	57	49	36	31
C	7	22	-	32	22	49	71	58	53
D	25	40	32	-	10	17	57	70	71
E	15	30	22	10	-	27	67	66	61
F	42	57	49	17	27	-	40	53	72
G	64	49	71	57	67	40	-	13	32
H	51	36	58	70	66	53	13	-	19
J	46	31	53	71	61	72	32	19	-

Table of shortest distances

- (a) Starting at A, use Prim’s algorithm to find a minimum spanning tree for the table of shortest distances. You must state the order in which you select the arcs of your tree. (3)
- (b) State the weight of the minimum spanning tree. (1)



A route is needed that minimises the total distance to traverse each road at least once. The route must start at E and finish at F.

- (c) Determine the length of this route. You must give a reason for your answer. (2)

It is now decided to start the route at C and finish the route at A. The route must include every road at least once and must still minimise the total distance travelled.

- (d) By considering the pairings of all relevant nodes, find the roads that need to be traversed twice. (4)

Naoko needs to visit all nine factories, starting and finishing at the same factory, and wishes to minimise the total distance travelled.

- (e) Starting at B, use the nearest neighbour algorithm on the table of shortest distances to find an upper bound for the length of Naoko's route. Write down the cycle, obtained from the table of shortest distances, which gives this upper bound. (2)
- (f) By deleting C and all of its arcs, use the values in the table of shortest distances to find a lower bound for the length of Naoko's route. (2)

(Total for Question 4 is 14 marks)



Question 4 continued

	A	B	C	D	E	F	G	H	J
A	-	15	7	25	15	42	64	51	46
B	15	-	22	40	30	57	49	36	31
C	7	22	-	32	22	49	71	58	53
D	25	40	32	-	10	17	57	70	71
E	15	30	22	10	-	27	67	66	61
F	42	57	49	17	27	-	40	53	72
G	64	49	71	57	67	40	-	13	32
H	51	36	58	70	66	53	13	-	19
J	46	31	53	71	61	72	32	19	-



Question 4 continued

	A	B	C	D	E	F	G	H	J
A	-	15	7	25	15	42	64	51	46
B	15	-	22	40	30	57	49	36	31
C	7	22	-	32	22	49	71	58	53
D	25	40	32	-	10	17	57	70	71
E	15	30	22	10	-	27	67	66	61
F	42	57	49	17	27	-	40	53	72
G	64	49	71	57	67	40	-	13	32
H	51	36	58	70	66	53	13	-	19
J	46	31	53	71	61	72	32	19	-



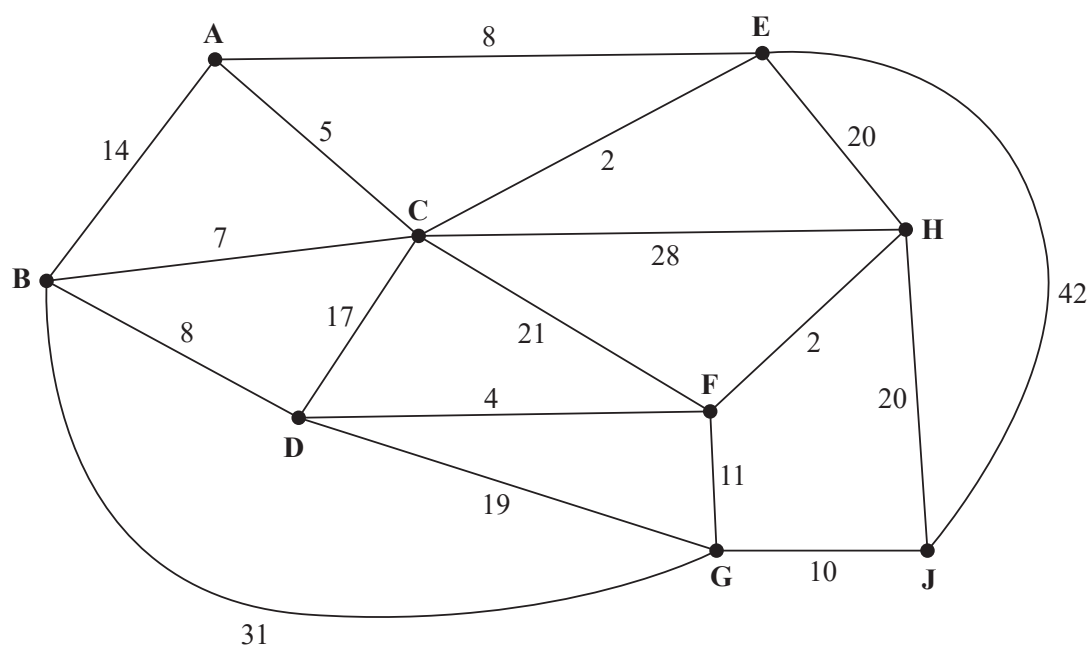
Question 4 continued

	A	B	C	D	E	F	G	H	J
A	-	15	7	25	15	42	64	51	46
B	15	-	22	40	30	57	49	36	31
C	7	22	-	32	22	49	71	58	53
D	25	40	32	-	10	17	57	70	71
E	15	30	22	10	-	27	67	66	61
F	42	57	49	17	27	-	40	53	72
G	64	49	71	57	67	40	-	13	32
H	51	36	58	70	66	53	13	-	19
J	46	31	53	71	61	72	32	19	-

(Total for Question 4 is 14 marks)



6.



[The total weight of the network is 269]

Figure 3

Figure 3 models a network of roads. The number on each edge gives the time taken, in minutes, to travel along the corresponding road.

- (a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to J. State the quickest route. (6)

Alan needs to travel along all the roads to check that they are in good repair. He wishes to complete his route as quickly as possible and will start at his home, H, and finish at his workplace, D.

- (b) By considering the pairings of all relevant nodes, find the arcs that will need to be traversed twice in Alan's inspection route from H to D. You must make your method and working clear. (5)

For Alan's inspection route from H to D

- (c) (i) state the number of times vertex C will appear,
(ii) state the number of times vertex D will appear. (2)
- (d) Determine whether it would be quicker for Alan to start and finish his inspection route at H, instead of starting at H and finishing at D. You must explain your reasoning and show all your working. (2)

(Total 15 marks)

6.

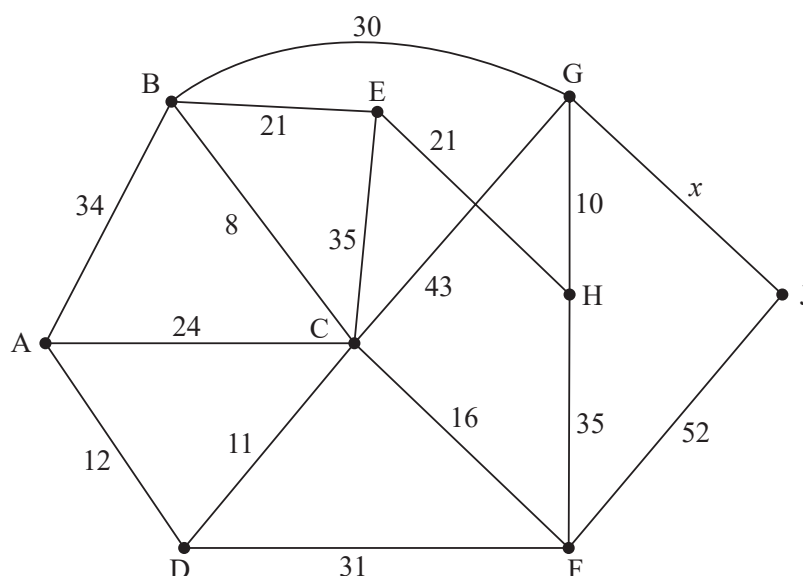


Figure 4

[The total weight of the network is $383 + x$]

Figure 4 models a network of roads. The number on each edge gives the time, in minutes, to travel along the corresponding road. The vertices, A, B, C, D, E, F, G, H and J represent nine towns. Ezra wishes to travel from A to H as fast as possible.

The time taken to travel between towns G and J is unknown and is denoted by x minutes.

Dijkstra's algorithm is to be used to find the fastest time to travel from A to H. On Diagram 1 in the answer book the "Order of labelling" and "Final value" at A and J, and the "Working values" at J, have already been completed.

- (a) Use Dijkstra's algorithm to find the fastest time to travel from A to H. State the quickest route. (6)

Ezra needs to travel along each road to check it is in good repair. He wishes to minimise the total time required to traverse the network. Ezra plans to start and finish his inspection route at A. It is given that his route will take at least 440 minutes.

- (b) Use the route inspection algorithm and the completed Diagram 1 to find the range of possible values of x . (6)

- (c) Write down a possible route for Ezra. (1)

A new direct road from D to H is under construction and will take 25 minutes to travel along. Ezra will include this new road in a minimum length inspection route starting and finishing at A. It is given that this inspection route takes exactly 488 minutes.

- (d) Determine the value of x . You must give reasons for your answer. (2)

(Total for Question 6 is 15 marks)



6.

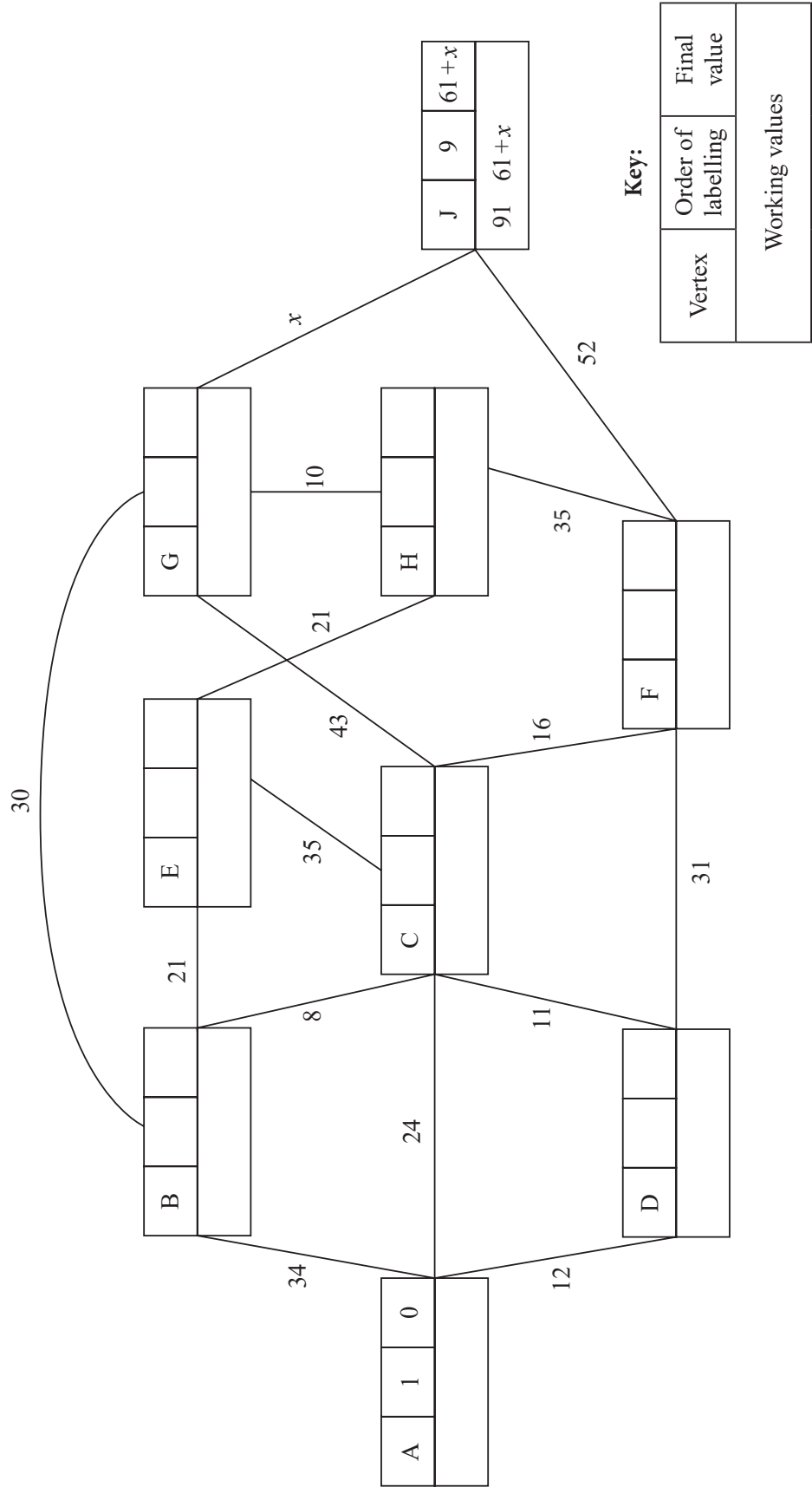


Diagram 1

Fastest time: _____

Quickest route: _____



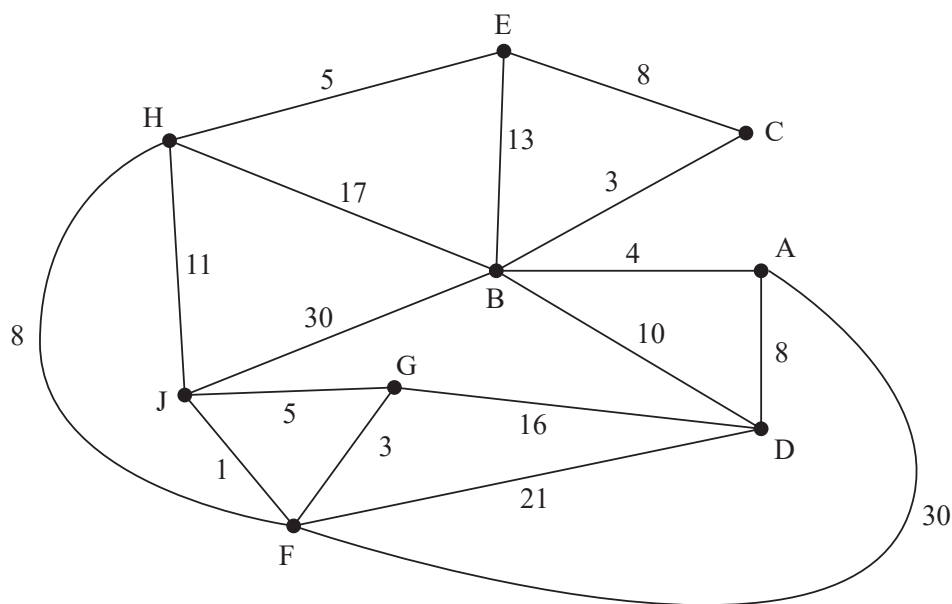
Question 6 continued

Lined area for writing the answer to Question 6.





2.

**Figure 1**

[The total weight of the network is 193]

Figure 1 represents a network of roads. The number on each edge represents the length, in miles, of the corresponding road. Jan wishes to travel from A to J. She wishes to minimise the distance she travels.

- (a) Use Dijkstra's algorithm to find the shortest path from A to J. Obtain the shortest path and state its length.

(6)

On Monday, Jan needs to travel from her gym at J to her home at H via her office at A.

- (b) State the shortest path from J to H via A and its length.

(2)

On Tuesday, Jan needs to check each road. She must travel along each road at least once. Jan must start and finish at A.

- (c) Use the route inspection algorithm to find the length of the shortest inspection route. State the roads that should be repeated. You should make your method and working clear.

(5)

On Wednesday, Jan decides to start her inspection route at G but can finish her route at a different node. The inspection route must still traverse each road at least once.

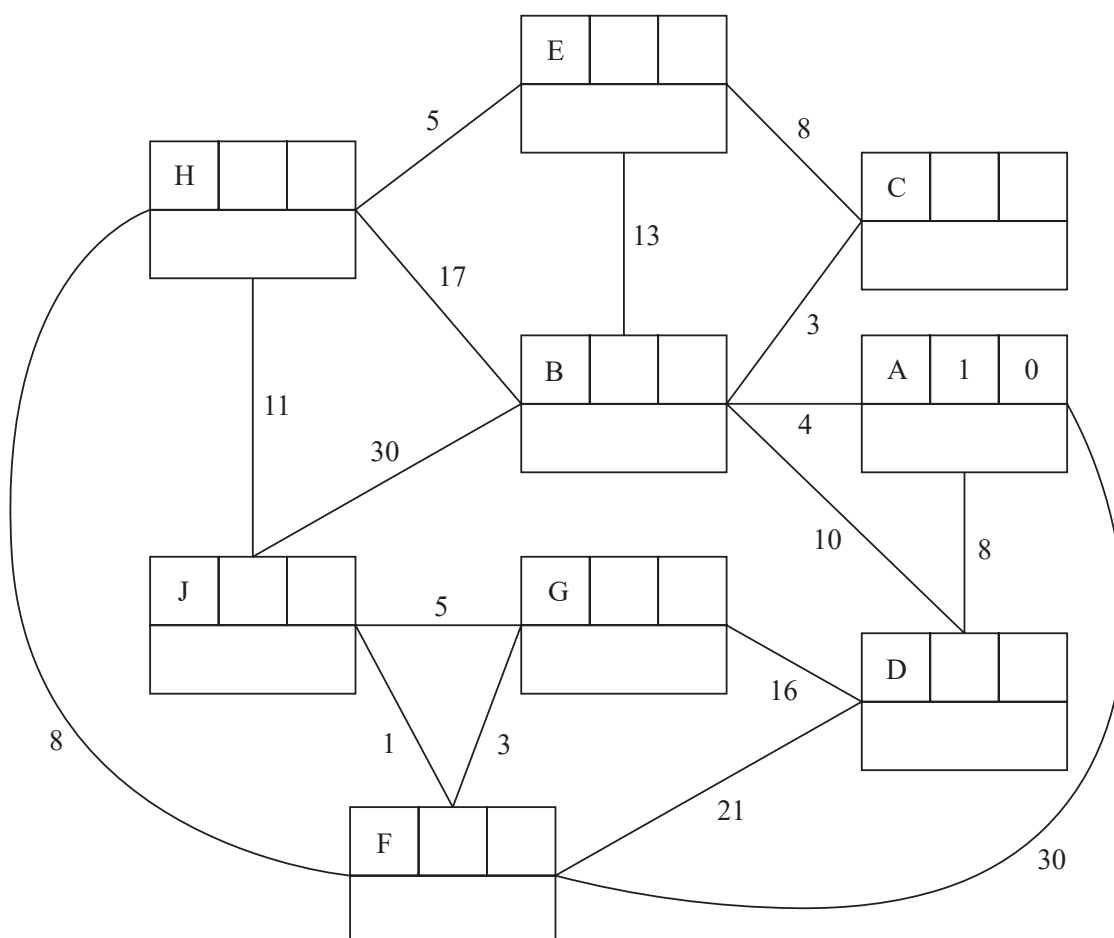
- (d) Determine where the route should finish so that the length of the inspection route is minimised. You must give reasons for your answer and state the length of the route.

(3)

(Total for Question 2 is 16 marks)



2.

**Key:**

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to J: _____

Length of shortest path from A to J: _____



Question 2 continued

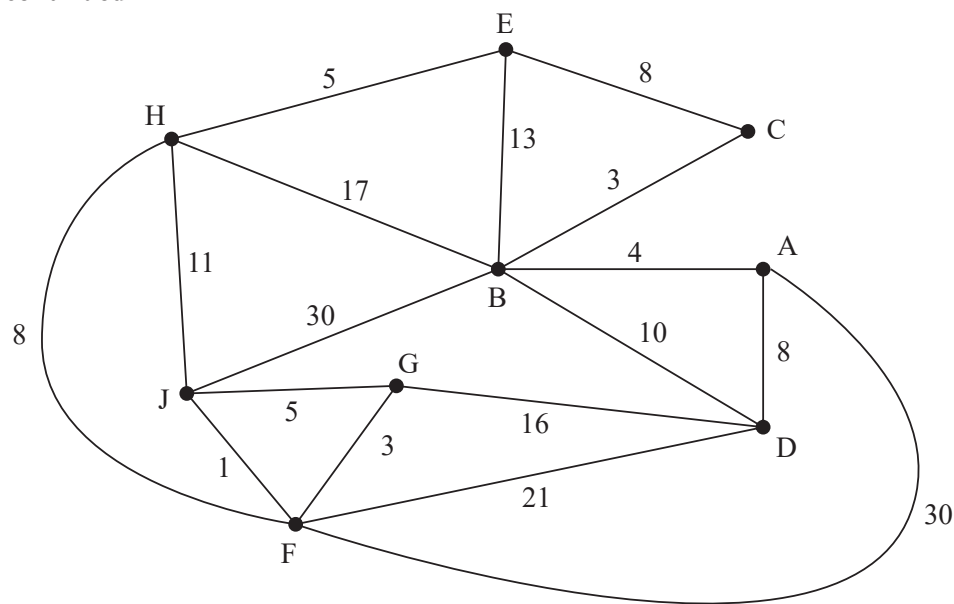


Figure 1

[The total weight of the network is 193]

Question 2 continued

Lined area for writing the answer to Question 2.





5.

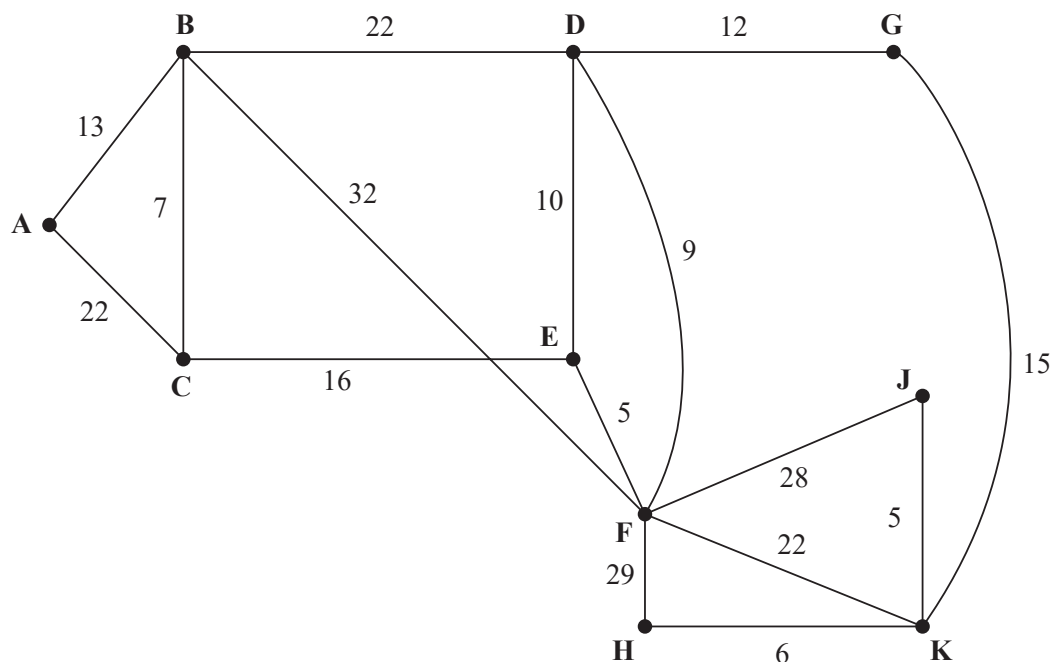


Figure 1

[The total weight of the network is 253]

Figure 1 represents a network of roads between 10 cities, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in miles, of the corresponding road.

One day, Mabintou wishes to travel from A to H. She wishes to minimise the distance she travels.

- (a) Use Dijkstra's algorithm to find the shortest path from A to H. State your path and its length. (6)

On another day, Mabintou wishes to travel from F to K via A.

- (b) Find a route of minimum length from F to K via A and state its length. (2)

The roads between the cities need to be inspected. James must travel along each road at least once. He wishes to minimise the length of his inspection route. James will start his inspection route at A and finish at J.

- (c) By considering the pairings of all relevant nodes, find the length of James' route. State the arcs that will need to be traversed twice. You must make your method and working clear. (6)
- (d) State the number of times that James will pass through F. (1)



It is now decided to start the inspection route at D. James must minimise the length of his route. He must travel along each road at least once but may finish at any vertex.

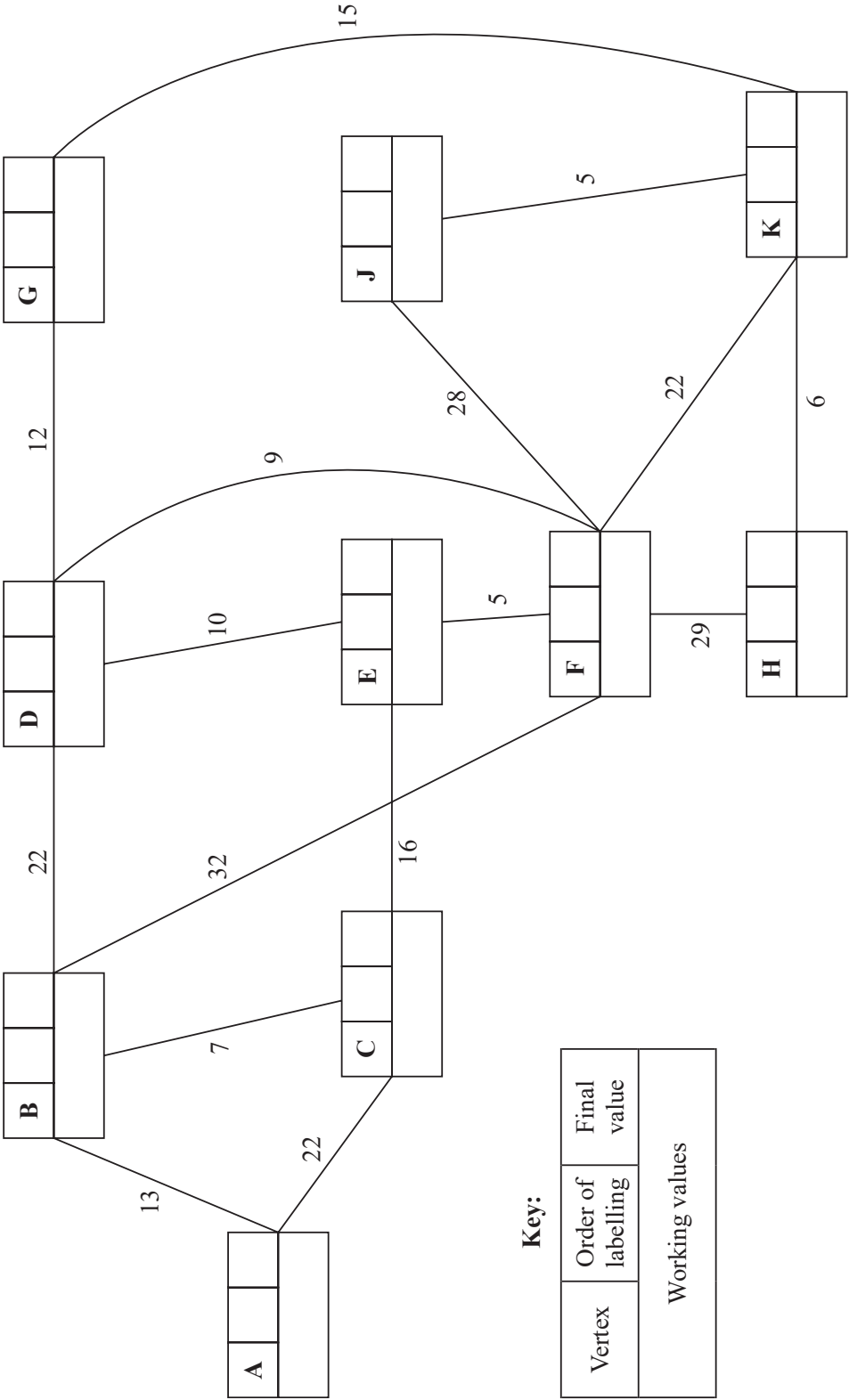
(e) State the vertex where the new inspection route will finish. (1)

(f) Calculate the difference between the lengths of the two inspection routes. (1)

(Total 17 marks)



5.



Shortest path: _____
Length of shortest path: _____

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(Total 17 marks)

Q5

SUB-TOPIC D1.3b

THE TRAVELLING SALESPERSON PROBLEM

Sub-topic D1.3b · 10 questions · 97 marks · lowest tariff first



Write your answers in the D1 answer book for this paper.

1. The table below shows the distances, in km, between six data collection points, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	35	42	55	48	50
B	35	–	40	49	52	31
C	42	40	–	47	53	49
D	55	49	47	–	39	44
E	48	52	53	39	–	52
F	50	31	49	44	52	–

Ferhana must visit each data collection point. She will start and finish at A and wishes to minimise the total distance she travels.

- (a) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the distance Ferhana must travel. Make your method clear. (2)
- (b) Starting by deleting B, and all of its arcs, find a lower bound for the distance Ferhana must travel. Make your calculation clear. (3)

(Total 5 marks)



1.

	A	B	C	D	E	F
A	–	35	42	55	48	50
B	35	–	40	49	52	31
C	42	40	–	47	53	49
D	55	49	47	–	39	44
E	48	52	53	39	–	52
F	50	31	49	44	52	–

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Question 1 continued

	A	B	C	D	E	F
A	–	35	42	55	48	50
B	35	–	40	49	52	31
C	42	40	–	47	53	49
D	55	49	47	–	39	44
E	48	52	53	39	–	52
F	50	31	49	44	52	–

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Q1

(Total 5 marks)



3. The table below shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	57	76	59	72	65
B	57	–	67	80	66	76
C	76	67	–	71	83	80
D	59	80	71	–	77	78
E	72	66	83	77	–	69
F	65	76	80	78	69	–

Mei must visit each town at least once. She will start and finish at A and wishes her route to minimise the total distance she will travel.

- (a) Starting with the minimum spanning tree in the answer book, use the shortcut method to find an upper bound below 520 km for Mei's route. You must state the shortcut(s) you use and the length of your upper bound. (2)
- (b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Mei's route. (2)
- (c) Starting by deleting E, and all of its arcs, find a lower bound for the length of Mei's route. Make your method clear. (3)

(Total 7 marks)



Question 3 continued

	A	B	C	D	E	F
A	–	57	76	59	72	65
B	57	–	67	80	66	76
C	76	67	–	71	83	80
D	59	80	71	–	77	78
E	72	66	83	77	–	69
F	65	76	80	78	69	–

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Q3

(Total 7 marks)



Write your answers in the D1 answer book for this paper.

1.

	A	B	C	D	E	F	G
A	–	43	52	47	59	53	55
B	43	–	59	45	46	52	47
C	52	59	–	51	50	55	51
D	47	45	51	–	52	49	55
E	59	46	50	52	–	57	48
F	53	52	55	49	57	–	55
G	55	47	51	55	48	55	–

The table above shows the least distances, in metres, between seven classrooms, A, B, C, D, E, F and G. A teacher needs to visit each classroom, starting and finishing at A, and wishes to minimise the total distance travelled.

- (a) Show that there are two nearest neighbour routes that start from A. State these routes and their corresponding lengths.

(3)

- (b) Starting by deleting A, and all of its arcs, find a lower bound for the length of the teacher's route.

(3)

- (c) Use your results to write down the smallest interval which you can be confident contains the optimal length of the teacher's route.

(1)

(Total for Question 1 is 7 marks)



	A	B	C	D	E	F	G
A	–	43	52	47	59	53	55
B	43	–	59	45	46	52	47
C	52	59	–	51	50	55	51
D	47	45	51	–	52	49	55
E	59	46	50	52	–	57	48
F	53	52	55	49	57	–	55
G	55	47	51	55	48	55	–

Question 1 continued

	A	B	C	D	E	F	G
A	–	43	52	47	59	53	55
B	43	–	59	45	46	52	47
C	52	59	–	51	50	55	51
D	47	45	51	–	52	49	55
E	59	46	50	52	–	57	48
F	53	52	55	49	57	–	55
G	55	47	51	55	48	55	–

(Total for Question 1 is 7 marks)



Write your answers in the D1 answer book for this paper.

1.

	A	B	C	D	E	F
A	–	73	56	27	38	48
B	73	–	58	59	43	34
C	56	58	–	46	38	42
D	27	59	46	–	25	32
E	38	43	38	25	–	21
F	48	34	42	32	21	–

The table above shows the least distances, in km, between six cities, A, B, C, D, E and F. Mohsen needs to visit each city, starting and finishing at A, and wishes to minimise the total distance he will travel.

- (a) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the length of Mohsen's route. You must state your route and its length. (3)
- (b) Starting by deleting A and all of its arcs, find a lower bound for the length of Mohsen's route. (3)
- (c) Use your answers from (a) and (b) to write down the smallest interval that you can be confident contains the optimal length of the route. (2)

(Total 8 marks)

1.

	A	B	C	D	E	F
A	–	73	56	27	38	48
B	73	–	58	59	43	34
C	56	58	–	46	38	42
D	27	59	46	–	25	32
E	38	43	38	25	–	21
F	48	34	42	32	21	–

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Question 1 continued

	A	B	C	D	E	F
A	–	73	56	27	38	48
B	73	–	58	59	43	34
C	56	58	–	46	38	42
D	27	59	46	–	25	32
E	38	43	38	25	–	21
F	48	34	42	32	21	–

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Q1

(Total 8 marks)



2.

	P	Q	R	S	T	U	V
P	–	105	90	195	270	95	155
Q	105	–	195	295	370	190	255
R	90	195	–	110	185	180	245
S	195	295	110	–	75	115	170
T	270	370	185	75	–	180	245
U	95	190	180	115	180	–	65
V	155	255	245	170	245	65	–

The table shows the least distances, in metres, between seven signposts, P, Q, R, S, T, U and V.

Aisha must visit each signpost to check that it has not been damaged. She needs to find a route which minimises the distance travelled, starting and finishing at P.

- (a) Use Prim's algorithm, starting at P, to obtain a minimum spanning tree for the network. You must clearly show the order in which you select arcs. (2)
- (b) Use the answer to part (a) to obtain an initial upper bound for the length of Aisha's route. (2)
- (c) Use the nearest neighbour algorithm, starting at P, to find a second upper bound for the length of Aisha's route. You should state both the route and its length. (3)
- (d) By deleting T and all of its arcs, and using the answer to part (a), obtain a lower bound for the length of Aisha's route. (2)

(Total for Question 2 is 9 marks)



Question 2 continued

	P	Q	R	S	T	U	V
P	–	105	90	195	270	95	155
Q	105	–	195	295	370	190	255
R	90	195	–	110	185	180	245
S	195	295	110	–	75	115	170
T	270	370	185	75	–	180	245
U	95	190	180	115	180	–	65
V	155	255	245	170	245	65	–

(Total for Question 2 is 9 marks)



2.

	A	B	C	D	E	F	G	H
A	–	34	29	35	28	30	37	38
B	34	–	32	28	39	40	32	39
C	29	32	–	27	33	39	34	31
D	35	28	27	–	35	38	41	36
E	28	39	33	35	–	36	33	40
F	30	40	39	38	36	–	34	39
G	37	32	34	41	33	34	–	35
H	38	39	31	36	40	39	35	–

Table 1

Table 1 represents a network that shows the travel times, in minutes, between eight towns, A, B, C, D, E, F, G and H.

- (a) Use Prim's algorithm, starting at A, to find the minimum spanning tree for this network. You must clearly state the order in which you select the edges of your tree.

(3)

- (b) State the weight of the minimum spanning tree.

(1)

	A	B	C	D	E	F	G	H
J	33	37	41	35	x	40	28	42

Table 2

Table 2 shows the travel times, in minutes, between town J and towns A, B, C, D, E, F, G and H.

The journey time between towns E and J is x minutes where $x > 28$

A salesperson needs to visit all of the **nine** towns, starting and finishing at J.

The salesperson wishes to minimise the total time spent travelling.

- (c) Starting at J, use the nearest neighbour algorithm to find an upper bound for the duration of the salesperson's route. Write down the route that gives this upper bound.

(2)

Using the nearest neighbour algorithm, starting at E, an upper bound of 291 minutes for the salesperson's route was found.

- (d) State the best upper bound that can be obtained by using this information and your answer to (c). Give the reason for your answer.

(1)

Starting by deleting J and all of its arcs, a lower bound of 264 minutes for the duration of the salesperson's route was found.

- (e) Determine the value of x . You must make your method and working clear.

(3)

(Total for Question 2 is 10 marks)



	A	B	C	D	E	F	G	H
A	–	34	29	35	28	30	37	38
B	34	–	32	28	39	40	32	39
C	29	32	–	27	33	39	34	31
D	35	28	27	–	35	38	41	36
E	28	39	33	35	–	36	33	40
F	30	40	39	38	36	–	34	39
G	37	32	34	41	33	34	–	35
H	38	39	31	36	40	39	35	–

	A	B	C	D	E	F	G	H
A	–	34	29	35	28	30	37	38
B	34	–	32	28	39	40	32	39
C	29	32	–	27	33	39	34	31
D	35	28	27	–	35	38	41	36
E	28	39	33	35	–	36	33	40
F	30	40	39	38	36	–	34	39
G	37	32	34	41	33	34	–	35
H	38	39	31	36	40	39	35	–

(Total for Question 2 is 10 marks)



4. (a) Explain the difference between the classical and the practical travelling salesperson problems. (2)

The table below shows the distances, in km, between seven museums, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	25	31	28	35	30	32
B	25	–	34	24	27	32	39
C	31	34	–	40	35	27	29
D	28	24	40	–	37	35	36
E	35	27	35	37	–	28	31
F	30	32	27	35	28	–	33
G	32	39	29	36	31	33	–

Fran must visit each museum. She will start and finish at A and wishes to minimise the total distance travelled.

- (b) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the length of Fran's route. Make your method clear. (2)

Starting at D, a second upper bound of 203 km was found.

- (c) State whether this is a better upper bound than the answer to (b), giving a reason for your answer. (1)

A reduced network is formed by deleting G and all the arcs that are directly joined to G.

- (d) (i) Use Prim's algorithm, starting at A, to construct a minimum spanning tree for the reduced network. You must clearly state the order in which you select the arcs of your tree.

- (ii) Hence calculate a lower bound for the length of Fran's route. (4)

By deleting A, a second lower bound was found to be 188 km.

- (e) State whether this is a better lower bound than the answer to (d)(ii), giving a reason for your answer. (1)

- (f) Using only the results from (c) and (e), write down the smallest interval that you can be confident contains the length of Fran's optimal route. (2)

(Total 12 marks)



4. (a)

	A	B	C	D	E	F	G
A	–	25	31	28	35	30	32
B	25	–	34	24	27	32	39
C	31	34	–	40	35	27	29
D	28	24	40	–	37	35	36
E	35	27	35	37	–	28	31
F	30	32	27	35	28	–	33
G	32	39	29	36	31	33	–

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	A	B	C	D	E	F	G
A	–	25	31	28	35	30	32
B	25	–	34	24	27	32	39
C	31	34	–	40	35	27	29
D	28	24	40	–	37	35	36
E	35	27	35	37	–	28	31
F	30	32	27	35	28	–	33
G	32	39	29	36	31	33	–

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Question 4 continued

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(Total 12 marks)

Q4



7.

	A	B	C	D	E	F	G	H
A	–	38	37	x	37	42	41	27
B	38	–	26	32	33	38	37	34
C	37	26	–	39	38	39	30	39
D	x	32	39	–	37	36	29	36
E	37	33	38	37	–	32	33	30
F	42	38	39	36	32	–	31	28
G	41	37	30	29	33	31	–	33
H	27	34	39	36	30	28	33	–

The network represented by the table shows the least distances, in km, between eight museums, A, B, C, D, E, F, G and H.

A tourist wants to visit each museum at least once, starting and finishing at A. The tourist wishes to minimise the total distance travelled. The shortest distance between A and D is x km where $32 \leq x \leq 35$

- (a) Using Prim's algorithm, starting at A, obtain a minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree. (3)
- (b) Use your answer to (a) to determine an initial upper bound for the length of the tourist's route. (1)
- (c) Starting at A, use the nearest neighbour algorithm to find another upper bound for the length of the tourist's route. Write down the route that gives this upper bound. (2)

The nearest neighbour algorithm starting at E gives a route of

$$E - H - A - D - G - C - B - F - E$$

- (d) State which of these two nearest neighbour routes gives the better upper bound. Give reasons for your answer. (2)

Starting by deleting A, and all of its arcs, a lower bound of 235 km for the length of the route is found.

- (e) Determine the smallest interval that must contain the optimal length of the tourist's route. You must make your method and working clear. (4)

(Total for Question 7 is 12 marks)



Question 7 continued

	A	B	C	D	E	F	G	H
A	–	38	37	x	37	42	41	27
B	38	–	26	32	33	38	37	34
C	37	26	–	39	38	39	30	39
D	x	32	39	–	37	36	29	36
E	37	33	38	37	–	32	33	30
F	42	38	39	36	32	–	31	28
G	41	37	30	29	33	31	–	33
H	27	34	39	36	30	28	33	–

Question 7 continued

	A	B	C	D	E	F	G	H
A	–	38	37	x	37	42	41	27
B	38	–	26	32	33	38	37	34
C	37	26	–	39	38	39	30	39
D	x	32	39	–	37	36	29	36
E	37	33	38	37	–	32	33	30
F	42	38	39	36	32	–	31	28
G	41	37	30	29	33	31	–	33
H	27	34	39	36	30	28	33	–

(Total for Question 7 is 12 marks)

6.

	A	B	C	D	E	F	G	H	J	K
A	-	16	26	19	22	34	30	41	45	36
B	16	-	10	15	38	33	40	25	29	20
C	26	10	-	12	39	30	31	15	19	10
D	19	15	12	-	33	18	25	28	31	22
E	22	38	39	33	-	15	8	33	20	29
F	34	33	30	18	15	-	7	24	19	28
G	30	40	31	25	8	7	-	25	12	21
H	41	25	15	28	33	24	25	-	13	5
J	45	29	19	31	20	19	12	13	-	9
K	36	20	10	22	29	28	21	5	9	-

- (a) Explain the difference between the **classical** Travelling Salesman Problem and the **practical** Travelling Salesman Problem.

(2)

The table shows the shortest distances, in miles, between ten towns, A, B, C, D, E, F, G, H, J and K.

Kenzo must visit each town at least once, starting and finishing at A. Kenzo wishes to minimise the total distance travelled.

- (b) Use Prim's algorithm, starting at A, to obtain a minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree.

(3)

- (c) Use your answer to part (b) to determine an initial upper bound for the length of Kenzo's route.

(1)

- (d) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Kenzo's route. Write down the route that gives this upper bound.

(3)

Using the answer to part (d), and given that the length of the nearest neighbour route starting at G is 145 miles,

- (e) state which of these two nearest neighbour routes gives the better upper bound. Give a reason for your answer.

(1)

- (f) By deleting A and all of its arcs, obtain a lower bound for the length of Kenzo's route.

(2)

- (g) State the smallest interval that must contain the optimal length of Kenzo's route.

(1)

(Total for Question 6 is 13 marks)



	A	B	C	D	E	F	G	H	J	K
A	-	16	26	19	22	34	30	41	45	36
B	16	-	10	15	38	33	40	25	29	20
C	26	10	-	12	39	30	31	15	19	10
D	19	15	12	-	33	18	25	28	31	22
E	22	38	39	33	-	15	8	33	20	29
F	34	33	30	18	15	-	7	24	19	28
G	30	40	31	25	8	7	-	25	12	21
H	41	25	15	28	33	24	25	-	13	5
J	45	29	19	31	20	19	12	13	-	9
K	36	20	10	22	29	28	21	5	9	-



Question 6 continued

	A	B	C	D	E	F	G	H	J	K
A	-	16	26	19	22	34	30	41	45	36
B	16	-	10	15	38	33	40	25	29	20
C	26	10	-	12	39	30	31	15	19	10
D	19	15	12	-	33	18	25	28	31	22
E	22	38	39	33	-	15	8	33	20	29
F	34	33	30	18	15	-	7	24	19	28
G	30	40	31	25	8	7	-	25	12	21
H	41	25	15	28	33	24	25	-	13	5
J	45	29	19	31	20	19	12	13	-	9
K	36	20	10	22	29	28	21	5	9	-



Question 6 continued

	A	B	C	D	E	F	G	H	J	K
A	-	16	26	19	22	34	30	41	45	36
B	16	-	10	15	38	33	40	25	29	20
C	26	10	-	12	39	30	31	15	19	10
D	19	15	12	-	33	18	25	28	31	22
E	22	38	39	33	-	15	8	33	20	29
F	34	33	30	18	15	-	7	24	19	28
G	30	40	31	25	8	7	-	25	12	21
H	41	25	15	28	33	24	25	-	13	5
J	45	29	19	31	20	19	12	13	-	9
K	36	20	10	22	29	28	21	5	9	-



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 13 marks)



6.

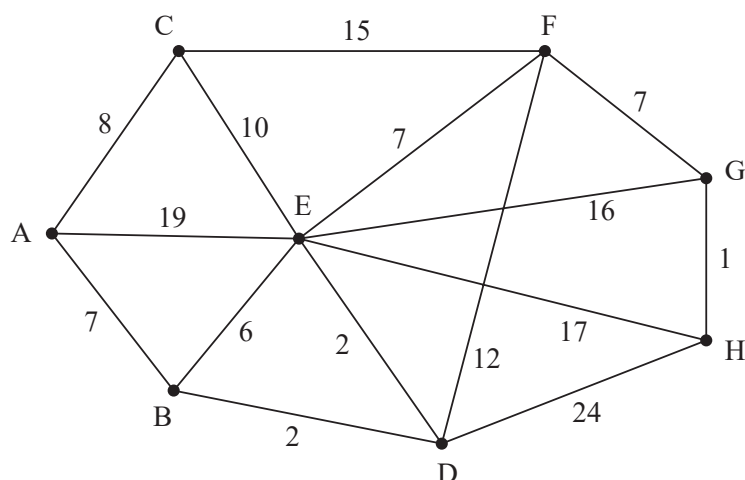


Figure 5

Figure 5 models a network of roads. The number on each edge gives the length, in km, of the corresponding road. The vertices, A, B, C, D, E, F, G and H, represent eight towns. Bronwen needs to visit each town. She will start and finish at A and wishes to minimise the total distance travelled.

- (a) By applying Dijkstra's algorithm, starting at A, complete the table of least distances in the answer book.

(6)

- (b) Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of Bronwen's route. Write down the route that gives this upper bound.

(2)

A reduced network is formed by deleting A and all arcs that are directly joined to A.

- (c) (i) Use Prim's algorithm, starting at C, to construct a minimum spanning tree for the reduced network. You must clearly state the order in which you select the arcs of your tree.

- (ii) Hence, calculate a lower bound for the length of Bronwen's route.

(4)

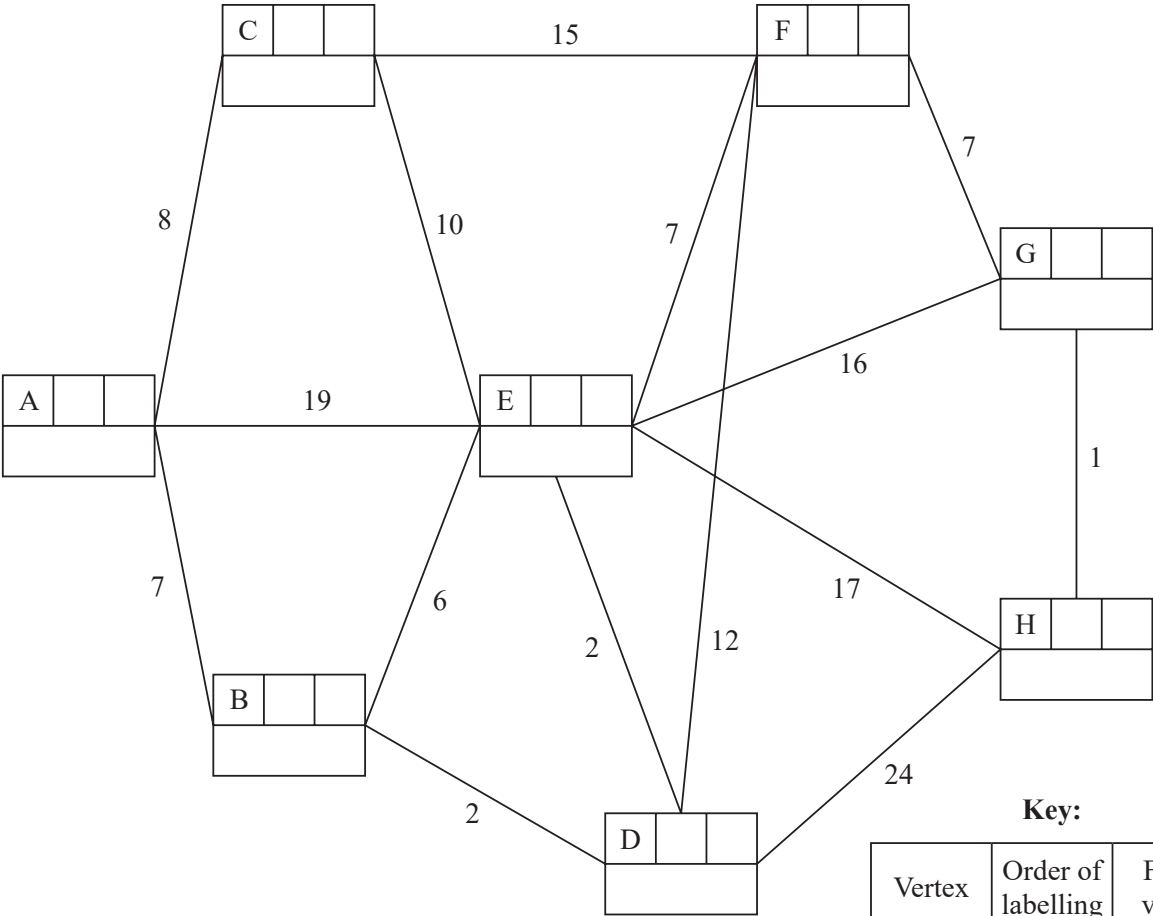
- (d) Using only the results from (b) and (c), write down the smallest interval that you can be confident contains the length of Bronwen's optimal route.

(2)

(Total 14 marks)



6.



	A	B	C	D	E	F	G	H
A	–							
B		–	14	2	4	11	18	19
C		14	–	12	10	15	22	23
D		2	12	–	2	9	16	17
E		4	10	2	–	7	14	15
F		11	15	9	7	–	7	8
G		18	22	16	14	7	–	1
H		19	23	17	15	8	1	–

	B	C	D	E	F	G	H
B	–	14	2	4	11	18	19
C	14	–	12	10	15	22	23
D	2	12	–	2	9	16	17
E	4	10	2	–	7	14	15
F	11	15	9	7	–	7	8
G	18	22	16	14	7	–	1
H	19	23	17	15	8	1	–

Question 6 continued

(Total 14 marks)

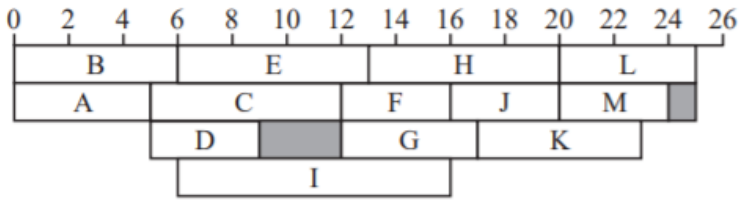


MARK SCHEMES

official mark schemes, in the same order as the questions



Question	Scheme	Marks
6 (a)	Considers pairings of nodes F, J, K and M	M1
	$FJ + KM = 32 + 14 = 46$	A1
	$FK + JM = 15 + 26 = 41$	A1
	$FM + JK = 13 + 17 = 30$	A1
	Repeat FG GM JL LK Total length = $338 + 30 = 368$ (km)	A1ft
	Special Case Max M1 A1 A0 A0 A0 Considers pairings of A, F, K and M $AF + KM = 26 + 14 = 40$ $AK + FM = 41 + 13 = 54$ $AM + FK = 39 + 15 = 54$	
		(5)
(b)	Now only repeat needed is KM, length 14 e.g. New length $338 + 18 + 14 = 370$ (km) (Increases length by) 2 (km)	M1
		A1
		(2)
(7 marks)		
Notes		
a1M1	The correct three pairings of the correct four nodes F, J, K and M with one row correct (including total)	
a1A1	Two rows correct including pairings and totals	
a2A1	All three rows correct including pairings and totals	
a3A1	CAO for the repeated arcs	
a4A1ft	$338 +$ their smallest from a choice of three totals	
M1 A1 A0 A0 A0	Special Case – considers A, F, K, M Three pairings of the four nodes with one row correct (including total) All three rows correct including pairings and totals No further marks available	
b1M1	Considers KM (may be implied by a correct calculation) as only pair of odd nodes and calculates new length $338 + 18 + 14$ or calculates $18 + 14$ as new additional length	
b1A1	CAO – 2 (km) (must not come from any incorrect working)	

Question Number	Scheme	Marks																																
4. (a)	<table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>-</td></tr><tr><td>C</td><td>A</td></tr><tr><td>D</td><td>A</td></tr><tr><td>E</td><td>A, B</td></tr></tbody></table> <table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>F</td><td>C</td></tr><tr><td>G</td><td>C</td></tr><tr><td>H</td><td>C, D, E</td></tr><tr><td>I</td><td>A, B</td></tr><tr><td>J</td><td>A, B</td></tr></tbody></table> <table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>K</td><td>F</td></tr><tr><td>L</td><td>F, G, H, I</td></tr><tr><td>M</td><td>F, G, H, I, J</td></tr></tbody></table>	Activity	IPA	A	-	B	-	C	A	D	A	E	A, B	Activity	IPA	F	C	G	C	H	C, D, E	I	A, B	J	A, B	Activity	IPA	K	F	L	F, G, H, I	M	F, G, H, I, J	B2, 1, 0 (2)
Activity	IPA																																	
A	-																																	
B	-																																	
C	A																																	
D	A																																	
E	A, B																																	
Activity	IPA																																	
F	C																																	
G	C																																	
H	C, D, E																																	
I	A, B																																	
J	A, B																																	
Activity	IPA																																	
K	F																																	
L	F, G, H, I																																	
M	F, G, H, I, J																																	
(b)	$v = 7, w = 4, x = 6, y = 16, z = 19$	B3, 2, 1, 0 (3)																																
(c)	$\frac{74}{25} = 2.96$ so 3 workers	B1 (1)																																
(d)	e.g. 	M1 A1 A1 (3)																																
(e)	New minimum project completion time: 27 (days)	B1																																
	Critical path: ACFK	B1 (2)																																
		11 marks																																
Notes for Question 4																																		
a1B1: Any 6 rows correct (not including A and B) a2B1: cao (condone blank rows for A and B) b1B1: Any 2 correct values b2B1: Any 3 correct values b3B1: All 5 values correct c1B1: cao (3 from correct working) – as a minimum for correct working accept either 2.96 or $\frac{5 + 6 + 7 + 4 + 7 + 4 + 5 + 7 + 10 + 4 + 6 + 5 + 4}{25}$ or $\frac{74}{25}$ d1M1: Not a cascade chart. 4 workers used at most, at least 9 activities placed d1A1: 4 workers. All 13 activities present (just once). Condone two errors either precedence or time interval or activity length. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA d2A1: 4 workers. All 13 activities present (just once). No errors e1B1: cao (27 only) e2B1: cao (ACFK or KFCA only)																																		

Question Number	Scheme			Marks
Activity	Duration	Time	IPA	
A	5	0 – 6	-	
B	6	0 – 6	-	
C	7	5 – 13	A	
D	4	5 – 13	A	
E	7	6 – 13	A, B	
F	4	12 – 19	C	
G	5	12 – 20	C	
H	7	13 – 20	C, D, E	
I	10	6 – 20	A, B	
J	4	6 – 21	A, B	
K	6	16 – 25	F	
L	5	20 – 25	F, G, H, I	
M	4	20 – 25	F, G, H, I, J	



Question Number	Scheme	Marks
5.(a)	$A(DG)C + D(GH)E = 12 + 9 = 21$	M1 A1
	$AD + C(GH)E = 5 + 10 = 15^*$	A1
	$A(DGH)E + C(G)D = 14 + 7 = 21$	A1
	Repeated arcs: AD, CG, GH, EH	A1
	Length of route: $166 + 15 = 181$ (km)	A1ft (6)
(b)	Vertex C: 3 times	B1 (1)
(c)	CD (7) is the shortest path between two odd nodes excluding A	M1
	Repeat CGD (7) since this is the shortest path excluding A The route finishes at E	A1
	Length of route = $166 + 7 = 173$ (km)	A1 (3)
		10 marks
	Notes for Question 5	
	a1M1: Three distinct pairings of the correct four odd nodes (A, C, D, E) a1A1: One row correct including pairings and totals a2A1: Two rows correct including pairings and totals a3A1: All three rows correct including pairings and totals a4A1: The smallest repeat arcs (accept AD, CG, GH, EH only) a5A1ft: Correct answer of 181 or $166 +$ their least b1B1: cao (3) c1M1: Identifies the need to repeat one path of the three (DE, CE, CD) which does not include A (this maybe implicit) or listing of only these three possible repeats. This mark is dependent on either scoring the M mark in (a) or stating all three possible paths in this part. As a minimum accept the stating of one of these three paths c1A1: Identifies C(G)D as the least and E as the finishing point. They have to <u>explicitly state</u> that C(G)D is the <u>least</u> path of those that <u>do not include A</u> (this can be done by stating that CD is the least of CD, CE, DE only (so with no others) or stating that CD is the least of those that don't include A but not for just 'CD is the least') c2A1: cao (173)	

Question Number	Scheme	Marks
6. (a)		B1 B1 B1 B1 (4)
(b)(i)	A correct objective line	B1
	Correct optimal vertex labelled as V	B1 (2)
(b)(ii)	Solve correct set of simultaneous equations consistent with either their V or objective line	M1
	$(42, 44)$	A1
	77.6	A1 (3)
(c)	$(10, 60) > V \Rightarrow 10k + 60 > 42k + 44$ or $(60, 20) > V \Rightarrow 60k + 20 > 42k + 44$ $-\frac{1}{2} < -k$ or $-\frac{4}{3} > -k$	M1
	$k < \frac{1}{2}$ or $k > \frac{4}{3}$	A1
	$(10, 60) > V \Rightarrow 10k + 60 > 42k + 44$ and $(60, 20) > V \Rightarrow 60k + 20 > 42k + 44$ $-\frac{1}{2} < -k$ and $-\frac{4}{3} > -k$	M1dep
	$k < \frac{1}{2}$ and $k > \frac{4}{3}$ only	A1 (4)
		13 marks

Question Number	Scheme	Marks
5. (a)	Pair the odd nodes: C, D or repeated arcs are CF, FG, DG	B1
	Time = $82 + 7 = 89$	B1
	e.g. route GDGJHEADCABEFBCFCGFG	B1 (3)
(b)	$BC + DG = B(F)C + DG = 6 + 3 = 9^*$	M1 A1
	$BD + CG = B(FG)D + C(F)G = 11 + 4 = 15$	A1
	$BG + CD = B(F)G + C(FG)D = 8 + 7 = 15$	A1
	Repeat arcs: BF, CF, DG	A1 (5)
(c)	Route starting from G is quicker	B1
	e.g. difference = $(82 + 9) - 89 = 2$ or $9 - 7 = 2$	B1 (2)
		10 marks

Notes for Question 5

a1B1: cao (correctly stating the two odd nodes **or** correct repeated arcs stated) – so must either state that C and D are **odd** **or** state the arcs CF, FG, DG only, but B0 if only stating C and D or CD (without mention of ‘odd’)

a2B1: cao (89)

a3B1: Correct route: checks – starts and finishes at G, 20 nodes, CF, FG and DG repeated, A(2), B(2), C(3), D(2), E(2), F(3), G(4), H(1), J(1)

b1M1: Correct three distinct pairings of the correct four odd nodes B, C, D and G

b1A1: Any one row correct including pairing **and** total

b2A1: Any two rows correct including pairings **and** totals

b3A1: All three rows correct including pairings **and** totals

b4A1: cao – correct arcs clearly stated and not just in their working as BF, CF and DG – must be these arcs. Do not accept BC, BFC or BC via F

c1B1: cao (oe e.g. B to G is slower) – dependent on the correct repeats arcs (possibly implied) in (a) and (b) **or** clearly implied in (c) (e.g. correct values compared in this part) – must be clear that it is the route starting at G which is quicker

c2B1: cao (difference of 2 **or** comparing 89 and 91 **or** comparing 7 with 9)

6.(a)		M1 A1 (ABCD) A1 (EF) A1ft (GH) dM1 A1 (6)
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Question Number	Scheme	Marks
7.(a)		M1 A1 A1 A1ft
	Shortest path from A to H via DH: ABDH length: 70	A1
	Shortest path from A to H via EH: ABDEH length: $37 + 2x$	A1
	Shortest path from A to H via GH: ABDEGH length: $51 + x$	A1 (7)
(b)	A to H are the only two odd nodes in the network so repeat arcs in path ABDEH	M1
	$3x + 205 + 37 + 2x = 307$	M1
	$x = 13$	A1
	Time taken is 63 (minutes)	A1 (4)
		11 marks

Notes for Question 7

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at C the working values must be 22 21 20 in that order (22 20 21 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at any node except A, B, G or H (or once with at least two working values seen at H)

a1A1: All values in A, B, D and C correct and the working values in the correct order at D and C (including order of labelling)

a2A1: All values E and F correct and the working values in the correct order. Penalise order of labelling only once per question

a3A1ft: All values in G and H correct on the follow through and the working values in the correct order (the order at H must be correct but give bod). Penalise order of labelling only once per question. Ignore permanent label and final value at H only. Allow unsimplified expressions in x for the working values at H

a4A1: ABDH and 70

a5A1: ABDEH and $37 + 2x$

a6A1: ABDEGH and $51 + x$

If A0A0A0 for the final three marks in (a) then award A1A0A0 for all 3 routes stated correctly or all 3 correct values stated explicitly (so not just left in the working values at H)

b1M1: Indication of repeating arcs in a path from A to H. As a minimum: stating A and H as the odd nodes for the network (not just stating A and H) **or** stating a route from A to H with 5 nodes only **or** stating the need to repeat a path/route from A to H - this mark is for making their method clear

b2M1: $3x + 205 + (\text{one of their paths involving } x) = 307$ – this mark is for making their working clear

b1A1: CAO ($x = 13$) – this mark is dependent on the second M mark only

b2A1: CAO (63) – this mark is dependent on the second M mark only

SC If M0M0 then **both** correct answers of $x = 13$ and 63 score M0M0A0A1 only (so treating the final mark as a B mark)



Question Number	Scheme								Marks
4. (a)		A	B	C	D	E	F	G	B2, 1, 0 (2)
	A	-	21	42	17	25	31	41	
	B	21	-	26	27	12	15	20	
	C	42	26	-	32	17	11	46	
	D	17	27	32	-	15	21	47	
	E	25	12	17	15	-	6	32	
	F	31	15	11	21	6	-	35	
	G	41	20	46	47	32	35	-	
(b)	NNA starting at A: A – D – E – F – C – B – G – A 17 + 15 + 6 + 11 + 26 + 20 + 41 = 136 (km)								M1 A1 (2)
(c)	A – D – E – F – C – F – B – G – B – A								B1 (1)
(d)	RMST weight = 12 + 6 + 11 + 15 + 20 = 64								B1
	64 + 17 (AD) + 21 (AB) = 102 (km)								M1 A1 (3)
(e)	AC + EG = 42 + 32 = 74								M1
	AE + CG = 25 + 46 = 71								A1
	AG + CE = 41 + 17 = 58*								A1
	Repeat arcs: AB, BG, CF, EF								A1
	Length: 291 + 58 = 349 (km)								A1ft (5)
									13 marks

Notes for Question 4

Condone lack of, or incorrect units throughout this question

a1B1: At least two of the six values correct (in either table) – these are the bold values. The two values can be the same (for example, 42 in both cells AC and CA would score this mark)

a2B1: Fully correct (the six bold values in both tables)

b1M1: Nearest neighbour route starting at A – must have at least A – D – E – F – C – B – ...

Allow if stated in terms of arcs (AD, DE, EF, FC, CB,...) rather than nodes (note that arcs AD and DA are equivalent and so therefore AD, ED, EF, FC, CB,... is acceptable)

b1A1: CAO on length (136) and route (must return to A and can be stated in terms of arcs rather than nodes)

c1B1: CAO (ADEFCEFBGBA or in terms of arcs, e.g., AD, DE, EF, FC, CF, FB, BG, GB, BA)

d1B1: CAO on the weight of the RMST (using arcs BE, EF, CF, DE, BG) – accept either 64 or $12 + 6 + 11 + 15 + 20$ – could be seen or implied by later working as part of the lower bound calculation

d1M1: Adding correct two least weighted arcs from A (17 (AD) + 21 (AB)) to their RMST length where their RMST length x is $58 \leq x \leq 70$

d1A1: CAO (102). If this answer is stated with no working then award B0M1A1

e1M1: Correct three distinct pairings of the correct four odd nodes (A, C, E and G)

e1A1: Any two rows correct including pairings and totals

e2A1: All three rows correct including pairings and totals

e3A1: CAO correct edges clearly stated and not just in their working as AB, BG, CF and EF – must be these arcs. Do not accept AG, ABG or AG via B (and similarly for CE)

e4A1ft: Follow through their value of their smallest pairing total + 291

Question Number	Scheme	Marks
5.(a)	Start at A e.g. we would be able to find the shortest distance from A to every other vertex e.g. A appears in both required routes	B1 B1dep (2)
(b)	Shortest path to J: A C D G F J Length: 80 (miles) Shortest path to K: A C D B E H K Length: 81 (miles)	M1 A1 (ACDBG) A1 (EFH) A1ft (JK) A1 A1 A1ft (7)
(c)	Route from F to H via A: F G D C A C D B E H	B1 (1)
		10 marks
Notes for Question 5		

Question Number	Scheme	Marks
6.(a)(i)	Shortest path from A to J: ADCFEHGJ	M1 A1 (ABDC) A1 (FE) A1ft (HGJ) A1
(a)(ii)	Length of shortest path from A to J: 67 (miles)	A1ft (6)
(b)	$AC + EJ = A(D)C + E(HG)J = 21 + 31 = 52$	M1 A1
	$AE + CJ = A(DCF)E + C(FE)HG)J = 36 + 46 = 82$	
	$AJ + CE = A(DCFE)HG)J + C(F)E = 67 + 15 = 82$	A1
	Route length is $315 + 52 = 367$ (miles)	A1ft (4)
(c)	Pass through G a total of 3 times	B1 (1)
(d)	Difference in inspection routes is $67 - 52 = 15$ (miles)	B1ft (1)
(e)	Arcs CF and EF do not need to be repeated	B1 (1)
		13 marks
	Notes for Question 6	
	In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 40 35 31 in that order (so 40 31 35 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling	

a1M1: A larger value replaced by a smaller value at least twice in the working values at either C, E, F, G or J

a1A1: All values at A, B, D and C correct and the working values in the correct order

a2A1: All values at F and E correct and the working values in the correct order

a3A1ft: All values in H, G and J correct on the follow through and the working values in the correct order.

To follow through G say check that the working value(s) at G follow from the candidate's final values for the nodes that are directly attached to G (which are D, F, H and J). For example, **if** correct then the order of labelling of nodes D, F and H are 3, 5 and 7 respectively so the working values at G should come from D, F and H in that order. The first working value at G should be their 23 (the Final value at D) + 34 (the weight of the arc DG), the second working value at G should be their 31 (the Final value at F) + 15 (the weight of the arc FG) and the third working value at G should be their 42 (the Final value at H) + 2 (the weight of arc GH). Repeat this exact process for H and J for the follow through for this mark

a4A1: Correct shortest path from A to J (ADCFEHGJ) **only** – not from J to A

a5A1: Follow through their final value at J **only** (condone lack of units) – if their answer is 67 but this is not their Final Value at J then **A0**

Condone for the final two marks in (a) the 'shortest path' and 'length of shortest path' written on the wrong lines

b1M1: Three distinct pairings of nodes A, C, E and J

b1A1: Any one row correct including pairing **and** total

b2A1: All three rows correct including pairings **and** totals

b3A1ft: Correct route length (367) from the correct pairing or follow through 315 + their least total from a choice of three

c1B1: CAO (3)

d1B1ft: Correct answer of 15 either from correct or no incorrect working **or** follow through (their Final value at J from (a) – their least repeat from (b)) – **this mark is dependent on having scored both M marks in (a) and (b)**

e1B1: CAO (CF, EF only)

Question Number	Scheme	Marks
2.(a) (i)		M1 A1 (ADBCE) A1 (HF) A1ft (GJ)
	Shortest path from A to J is ABFGJ	A1
(a) (ii)	Shortest length is 357 (metres)	A1ft (6)
(b)	Consider all pairings of A, C, E and J	M1
	$AC + EJ \quad 147 + 210 = 357$ $AE + CJ \quad 161 + 364 = 525$ $AJ + CE \quad 357 + 168 = 525$	A1ft A1ft
	Length of route = $1\,960 + 357 = 2\,317$ (metres)	A1 ft (4)
(c)(i)	Consider arcs between odd nodes not using A – shortest CE so repeat. Finish at J	M1 A1
(c)(ii)	Difference in length their “357” – 168 = 189 metres or their “2317” – 2128 = 189 (metres)	A1ft (3)
		13 Marks



Notes for Question 2	
	In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at C the working values must be 182 154 147 in that order (so 182 147 154 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling
ai1M1	A larger value replaced by a smaller value at least twice in the working values at either C, E, G, H, J
ai1A1	All values at A, D, B, C and E correct and the working values in the correct order
ai2A1	All values at H and F correct and the working values in the correct order
ai3A1ft	All values in G and J correct on the follow through and the working values in the correct order. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are E, F and H (and J)). For example, if correct then the order of labelling of nodes E, F and H are 5, 7 and 6 respectively so the working values at H should come from E, H and F in that order. The first working value at G should be their 161 (the Final value at E) + 133 (the weight of the arc EG), the second working value at G should be their 203 (the Final value at H) + 84 (the weight of the arc HG) and the third working value should be their 217 (the Final value at F) + 56 (the weight of the arc FG) . Repeat the process for J (which will have working values from F, G and H with the order of these nodes determined by the candidate's order of labelling at F, G and H)
ai4A1	CAO (accept route written in arcs)
aii1A1ft	Follow through their final value at J only – if answer is 357 but the final value at J is not 357 then A0
b1M1	The correct three pairings of the correct four nodes (A, C, E and J)
b1A1ft	Two rows correct including pairings and totals. Ft for AC, AE and AJ only using their values from (a) but allow recovery if correct totals seen here
b2A1ft	All three rows correct including pairings and totals. Ft for AC, AE and AJ only using their values from (a) but allow recovery if correct totals seen here
b3A1ft	1960 + their smallest from a choice of three totals
ci1M1	Considers arcs between odd nodes not including A (e.g. listing possible arcs to be repeated CE CJ and EJ) (A statement that CE is the shortest implies this mark)

ci1A1	Clear indication that CE is shortest (e.g. a statement or tick or starring of CE, not just using 168) (we do not need to see all values explicitly listed here) and therefore finish at J
cii1A1ft	Dependent on M mark in (b) 189 or their “357” – 168



Question Number	Scheme								Marks
3.(a)	a	b	c	d	e	f	g	h	M1 A1 A1ft
	1 980	462	914 760	30/7	4	1848	132		
	462	132	(914 760)	7/2	3	396	66		
	132	66	(914 760)	2	2	132	0	13 860	
	Output 13 860								A1 (4)
(b)	Calculates the lowest common multiple of two numbers o.e. description								B1 (1)
									5 marks
Notes for Question 3									
	Note values may be written on individual rows. Please award the marks for the complete rows shown above. The values in cell c do not need to be repeated in each row, but we will condone this.								
a1M1	First row cells a to g completed – something in each cell								
a1A1	First and second row correct. Accept decimal equivalent awrt 4.29 and 3.5 (h must have no numerical value in these rows)								
a2A1ft	Third row correct – ft their first and second rows								
a3A1	13 860 in Output row CSO								
b1B1	LCM CAO (accept e.g. smallest common multiple, minimum common multiple, minimum common product, smallest number which can be divided by both a and b o.e.) (do not accept times instead of multiple or product)								

3.(a)(i)		M1 A1 (ABCDE) A1 (FG) A1ft (HJ)
	Shortest path from A to J is ABCDFGHJ	A1
(a)(ii)	Length of shortest path from A to J is 83 (km)	A1ft (6)
(b)	$B(CD)F + HJ = 30 + 5 = 35^*$ $B(CDFG)H + F(GH)J = 61 + 36 = 97$ $B(CDFGH)J + F(G)H = 66 + 31 = 97$	M1 A1 A1
	Repeat arcs: BC, CD, DF and HJ	A1
	Route length is $458 + 35 = 493$ (km)	A1ft (5)
(c)	The shortest path between any of the pairs of the four odd nodes (A, B, F and H) is AB (17)	M1
	So start at F and finish at H (or vice-versa)	A1
	Length of route is $458 + 17 = 475$ (km)	B1 (3)
		14 marks
	Special Case for Part B – using A B F H – Mark as Misread	
(b)	$AB + F(G)H = 17 + 31 = 48^*$ $A(BCD)F + B(CDFG)H = 47 + 61 = 108$ $A(BCDFG)H + B(CD)F = 78 + 30 = 108$	M1 A1 A1
	Repeat arcs: AB, FG and GH	A1
	Route length is $458 + 48 = 506$ (km)	A1ft (5)
	Remove final two A marks earned in this section so max 3/5	

	Notes for Question 3	
	In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at D the working values must be 39 37 35 in that order (so 39 35 37 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling	
a1M1	A larger value replaced by a smaller value in the working values of at least two of the nodes C, D, F, G, H, J	
a1A1	All values at A, B, C, D and E correct and the working values in the correct order	
a2A1	All values at F and G correct and the working values in the correct order	
a3A1ft	All values in H and J correct on the follow through and the working values in the correct order. To follow through H check that the working values at H follow from the candidate's final values for the nodes that are directly attached to H (which are E and G). For example, if correct then the order of labelling of nodes E and G are 5 and 7 respectively so the working values at H should come from E and G in that order. The first working value at H should be their 45 (the Final value at E) + 47 (the weight of the arc EH), the second working value at H should be their 67 (the Final value at G) + 11 (the weight of the arc GH). Repeat the process for J (which will have working values from D, F, G and H with the order of these nodes determined by the candidate's order of labelling at D, F, G and H)	
a4A1	CAO (ABCDFGHJ)	
a5A1ft	Follow through their final value at J only – if answer is 83 but this is not the Final Value at J then A0	
b1M1	Three distinct pairings of the nodes B, F, H and J with one row correct (including total)	
b1A1	Any two rows correct including pairings and totals	
b2A1	All three rows correct including pairings and totals	
b3A1	CAO - correct arcs clearly stated and not just in their working as BC, CD, DF and HJ – must be these arcs. Do not accept BF or B(CD)F or BF via C and D	
b4A1ft	Correct route length (493) or follow through their least repeat + 458	
c1M1	Indicates the need to find the shortest path between any pair of the correct four odd nodes (A, B, F, H)	
c1A1	CAO (F, H)	
c1B1	CAO (475)	

	Special Case for Part B – using A B F H – Mark as Misread	
b1M1	Three distinct pairings of the nodes A, B, F and H with one row correct (including total)	
b1A1	Any two rows correct including pairings and totals	
b2A1	All three rows correct including pairings and totals	
b3A1	CAO - correct arcs clearly stated and not just in their working as AB, FG and GH – must be these arcs. Do not accept FH or F(G)H or FH via G	
b4A1ft	Correct route length (506) or follow through their least repeat + 458	
	Remove final two A marks earned in this section so max 3/5	

Question Number	Scheme	Marks
4.(a)	Prim: AC, AB, AE; DE, DF, BJ; HJ, GH or AC, AE, DE; AB, DF, BJ; HJ, GH	M1 A1 A1 (3)
(b)	127 (km)	B1 (1)
(c)	If starting at E and finishing at F then the shortest path between C and H needs to be traversed twice. So the length of the route is $58 + 494 = 552$ (km)	M1 A1 (2)
(d)	Route must start at C and finish at A therefore need to consider pairings of the nodes A, E, F and H	M1
	AE + FH = $15 + 53 = 68$ AF + EH = $42 + 66 = 108$ AH + EF = $51 + 27 = 78$	A1 A1
	Repeat roads: AE, FG, GH	A1 (4)
ALT	Special Case – considers C, E, F and H – mark as misread removing final 2 A marks earned in this section	M1
	CE + FH = $22 + 53 = 75$ CF + EH = $49 + 66 = 115$ CH + EF = $58 + 27 = 85$	A1 A1
	Repeat roads: CA, AE, FG, GH	A1 (4)
(e)	B – A – C – E – D – F – G – H – J – B $15 + 7 + 22 + 10 + 17 + 40 + 13 + 19 + 31 = 174$ (km)	M1 A1 (2)
(f)	$(127 - 7) + 7 + 22 = 149$ (km)	M1 A1 (2)
		14 marks

	Notes for Question 4
a1M1:	Prim's – first three arcs correctly chosen in order (AC, AB, AE, ... or AC, AE, DE) or first four nodes {A, C, B, E, ... or A, C, E, D,...} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, 3, 2, -, 4, -, -, -, - or 1, -, 2, 4, 3, -, -, -, -}. Starting at any other node can score M1 only for first three arcs chosen correctly
a1A1:	First six arcs correctly chosen in order {AC, AB, AE, DE, DF, BJ, ...} or all nine nodes {A, C, B, E, D, F, J, H, G} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 3, 2, 5, 4, 6, 9, 8, 7} (no missing numbers). Or the alternative six arcs {AC, AE, DE, AB, DF, BJ,...}, nine nodes {A, C, E, D, B, F, J, H, G} or numbers {1, 5, 2, 4, 3, 6, 9, 8, 7}
a2A1:	CSO – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)
b1B1:	CAO (127)
c1M1:	Indication that the shortest path between C and H needs to be traversed twice (correct answer can imply this mark)
c1A1:	CAO (552)
d1M1:	The correct three pairings of the correct four nodes (A, E, F and H)
d1A1:	Two rows correct including pairings and totals
d2A1:	All three rows correct including pairings and totals
d3A1:	CAO (AE, FG, GH only but in any order) – must be stated as edges so A0 for AE, FH or AE F(G)H
	Special Case – considers C, E, F and H – mark as misread removing last two A marks earned in this section, so max 2/4
d1M1:	The correct three pairings of the four nodes (C, E, F and H)
d1A1:	Two rows correct including pairings and totals
d2A1:	All three rows correct including pairings and totals
d3A1:	CAO (CA, AE, FG, GH only but in any order) – must be stated as edges so A0 for CE, FH or C(A)E F(G)H
e1M1:	Nearest neighbour starting at B with first five nodes correct (B – A – C – E – D –) Accept arcs so BA, AC, CE, ED,
e1A1:	Correct nearest neighbour route (must return to B) and correct length (174) Accept arcs so BA, AC, CE, ED, DF, FG, GH, HJ, JB
f1M1:	(weight of their MST from (b) or 127 only) – $7 + 7(AC) + 22(CE \text{ or } CB)$ (oe so may not see the $-7 + 7$). A correct answer of 149 with no working can imply this (and the next) mark. If (b) is incorrect a value of their “127” from (b) plus 22 scores M1 (Note new calculation to find RMST $120 + 7 + 22 = 149$ may be seen)
f1A1:	CAO (149)

Question Number	Scheme	Marks
6. (a)	Shortest time: 45 (minutes) Quickest route: A C B D F G J	M1 A1 (CEBD) A1 (FH) A1ft (GJ) A1ft A1 (6)
(b)	$A(CB)D + HJ = 20 + 20 = 40$ $A(CBDF)H + D(FG)J = 26 + 25 = 51$ $A(CBDFG)J + D(F)H = 45 + 6 = 51$ Repeated arcs: AC, BC, BD, HJ	M1 A1 A1 A1 A1 (5)
(c)(i)	Vertex C: 4 times	B1
(ii)	Vertex D: 3 times	B1 (2)
(d)	H to H requires the consideration of the shortest path from A to J only (as these are the only two odd nodes) $314 > 309$ or $45 > 40$ so quicker to start at H and finish at D	B1 B1 (2)
		15 marks
Notes for Question 6		
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at J the working values must be 49 46 45 in that order (so 49 45 46 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at any node except A or C a1A1: All values at C, E, B and D correct and the working values in the correct order (including order of labelling) a2A1: All values at F and H correct and the working values in the correct order. Penalise order of labelling only once per question (F and H must be labelled in that order and F must be labelled after C, E, B and D) a3A1ft: All values in G and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working value at G follows from the candidate's final values from their feeds into G (which will mostly likely come from nodes B, D and F (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will possibly have working values from E, H and G with the order of these values determined by the candidate's order of labelling at E, H and G)		

Question Number	Scheme	Marks
	a4A1ft: Follow through on their final value at J only (condone lack of units) - so if 45 given as the answer and the final value at J is not 45 then A0 a5A1: CAO - correct route (from either A to J or J to A) – ACBDFGJ or JGFDBCA b1M1: Correct three distinct pairings of the correct four odd nodes A, D, H and J b1A1: Any row correct including pairing and total b2A1: Any two rows correct including pairings and totals b3A1: All three rows correct including pairings and totals b4A1: CAO correct edges clearly stated and not just in their working as AC, BC, BD, HJ. Must be these arcs and not AD, ACBD or AD via B and C ciB1: CAO (Vertex C: 4) ciiB1: CAO (Vertex D: 3) d1B1: Correct reasoning (that to travel from H to H) only the shortest path between A and J needs traversing twice – as a minimum must mention either ‘A to J <u>only</u>’ or refer to A and J being the only odd nodes (e.g. odd nodes: A and J is fine but not ‘A and J are odd nodes’) d2B1: Either ‘it will be slower’ or ‘it will be quicker from H to D’ (if saying ‘quicker’ then it must be clear that they are talking about H to D) + correct numerical <u>argument</u> (not just stating the values 314 and 309 (or 45 and 40) and saying ‘slower’ - there must be some comparison of these two values)	

6.(a)	Fastest time: 71 (minutes) Quickest route: ADCBGH	M1 A1 (DCB) A1 (FE) A1ft (GH)	
		A1ft A1	
		(6)	
	(b)	$AD + EH = 12 + 21 = 33^*$	M1 A1
		$A(DCB)E + D(CBG)H = 52 + 59 = 111$	A1
	$A(DCBG)H + D(CB)E = 71 + 40 = 111$	A1	
	$(383 + x) + 33 \dots 440 \text{ P } x \dots$	depM1	
	$24,, x < 30$	A1 (6)	
(c)	e.g. ABCEBGCDADFJGHEHFCA	B1 (1)	
(d)	If a direct road DH opens then only A and E are odd therefore the shortest inspection route is $(383 + x) + 25$ (DH) + 52 (AE)	M1	
	$460 + x = 488$ therefore $x = 28$	A1 (2)	
		15 marks	
	Notes for Question 6		

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 74 73 71 in that order (so 74 71 73 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling
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a1M1: A larger value replaced by a smaller value at least twice in the working values at either B, C, E, F, G, H

a1A1: All values at D, C and B correct and the working values in the correct order

a2A1: All values at F and E correct and the working values in the correct order

a3A1ft: All values in G and H correct on the follow through and the working values in the correct order. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are C and B). For example, **if** correct then the order of labelling of nodes C and B are 3 and 4 respectively so the working values at G should come from C and B in that order. The first working value at G should be their 23 (the Final value at C) + 43 (the weight of the arc CG), the second working value at G should be their 31 (the Final value at B) + 30 (the weight of the arc BG). Repeat the process for H (which will have working values from F, E and G with the order of these nodes determined by the candidate's order of labelling at F, E and G)

a4A1ft: Follow through their final value at H **only** – if answer is 71 but this is not the Final Value at H then A0

a5A1: CAO (ADCBGH)

b1M1: Three distinct pairings of the nodes A, D, E and H

b1A1: Any one row correct including pairing **and** total

b2A1: Any two rows correct including pairings **and** totals

b3A1: All three rows correct including pairings **and** totals

b2dM1: $(383 + x) + (\text{their least pairing total})$ with any inequality sign or equal to 440 – dependent on first M mark in (b). Give bod if not all totals are shown (so if they only give two totals then they should be using the least of these two) but they must have shown all three distinct pairings of the four odd nodes

b4A1: cao (24,, $x < 30$) – condone 24,, x ,, 29

c1B1: cao (check: starting and finishing at A, 19 nodes, AD and EH repeated in route, with A(3), B(2), C(3), D(2), E(2), F(2), G(2), H(2), J(1)) – can be given in terms of arcs

d1M1: $(383 + x) + 25 + \text{their } 52$ (where 'their 52' must be the length of their shortest path from A to E in either (a) **or** (b) **or** they state/imply the shortest path from A to E is 52) - a correct value of 28 with no working can imply this mark only

d1A1: cao (28) from correct working **and** correct reasoning that A and E are the only odd nodes **or** that we only need to pair A and E (as a minimum accept mention of A and E only but ignore any mention of the new direct road from D to H)

Question number	Scheme	Marks
2. (a)	The graph shows 10 nodes: A, B, C, D, E, F, G, H, J. Each node is represented by a box with four compartments: the top-left for the node label, and the top-right, middle-left, and middle-right for working values. The bottom compartment contains the final value. The edges and their weights are: A-B (3), A-D (8), B-C (3), B-E (5), B-F (17), C-E (8), D-F (16), D-G (10), E-H (5), F-G (5), F-J (1), G-H (13), H-J (11), J-I (30), I-D (21), I-F (30).	M1 A1 (BCDE) A1 (HG) A1ft (FJ)
	Shortest path: A – D – G – F – J	A1
	Length: 28 (miles)	A1ft (6)
(b)	Shortest path: J – F – G – D – A – B – C – E – H	B1
	Length: $28 + 20 = 48$ (miles)	B1ft (2)
(c)	$A(BC)E + FG = 15 + 3 = 18^*$ $A(DG)F + E(HF)G = 27 + 16 = 43$ $A(D)G + E(H)F = 24 + 13 = 37$	M1 A1 A1
	Repeat arcs: AB, BC, CE, FG	A1
	Length: $193 + 18 = 211$ (miles)	A1ft (5)
(d)	EF (13) is the shortest link between two odd nodes excluding G	M1
	Repeat EF (13) since this is the shortest path excluding G	
	We finish at A	A1
	Length of route = $193 + 13 = 206$ (miles)	A1 (3)
		(16 marks)
Notes for Question 2		
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 21 20 in that order (so 20 21 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling		

a1M1: A larger value replaced by a smaller value at least twice in the working values at either E, F, H or J

a1A1: All values at B, C, D and E correct and the working values in the correct order

a2A1: All values at H and G correct and the working values in the correct order

a3A1ft: All values in F and J correct on the follow through and the working values in the correct order. To follow through F check that the working values at F follow from the candidate's final values for the nodes that are directly attached to F (which are A, D, H, G (and J)). For example, **if** correct then the order of labelling of nodes A, D, H and G are 1, 4, 6 and 7 respectively so the working values at F should come from A, D, H and G in that order. The first working value at F should be 30 (from A), the second working value at F should be their 29 (the Final value at D) + 21 (the weight of the arc DF), the third working value at F should be their 20 (the Final value at H) + 8 (the weight of the arc HF) and the fourth working value at F should be their 24 (the Final value at G) + 3 (the weight of the arc FG). Repeat the process for J (which will have working values from B, H, G and F with the order of these nodes determined by the candidate's order of labelling at B, H, G and F)

a4A1: CAO (ADGFJ or AD, DG, GF, FJ but **not** JFGDA or equivalent from J to A)

a5A1ft: Follow through their final value at J **only** – if their answer is 28 but this is not the Final Value at J then A0

b1B1: CAO for the route (JFGDABCEH or JF, FG, GD, DA, AB, BC, CE, EH)

b2B1ft: 48 or follow through their final value at J + their final value at H

c1M1: Three distinct pairings of the nodes A, E, F and G

c1A1: Any two rows correct including pairings **and** totals

c2A1: All three rows correct including pairings **and** totals

c3A1: CAO - correct arcs clearly stated and not just in their working as AB, BC, CE and FG (allow BA, CB, etc.) – must be these arcs. Do not accept ABCE or AE via B and C

c4A1ft: Correct answer of 211 **or** follow through 193 + their least total from a choice of three

d1M1: Identifies the need to repeat one path of the three (AE, AF, EF) which does not include G (maybe implicit) or listing of only these possible repeats – **this mark is dependent on either scoring the M mark in (c) or stating all three possible paths**. If stating more than these three paths (AE, AF, EF) then it must be clear from later working that they are only considering these three. **As a minimum stating just one of these three paths (or any combination of these three paths with no others) can score this mark (so, for example, just stating AE and AF scores this mark) provided that they do not further imply that a path including G should be repeated (as this would indicate that mentioning one (or more) of these paths is for the purpose of not repeating it)**

d1A1: Identifies EF as the least **and** A as the finishing point. They have to explicitly state that **EF** is the least path that **does not include G**

d2A1: CAO (206)

Question number	Scheme	Marks
3.(a)	Bin 1: 1.8 1.4 1.6 Bin 2: 2.6 <u>0.9</u> <u>0.8</u> 0.6 Bin 3: <u>2.8</u> 1.2 Bin 4: <u>3.1</u> Bin 5: 2.4	M1 A1 A1 (3)
(b)(i)	1.8 2.6 1.6 2.8 1.4 3.1 0.9 1.2 2.4 0.8 0.6	B1
(ii)	Comparisons: 10 Swaps: 6	B1 B1 (3)
(c)	e.g. using middle right 2.6 1.8 2.8 1.6 3.1 <u>1.4</u> 1.2 2.4 0.9 0.8 0.6 pivot 1.4 2.6 1.8 2.8 <u>1.6</u> 3.1 2.4 <u>1.4</u> 1.2 0.9 <u>0.8</u> 0.6 pivots 1.6 0.8 2.6 1.8 <u>2.8</u> 3.1 2.4 <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivots 2.8 0.9 (0.6) 3.1 <u>2.8</u> 2.6 <u>1.8</u> 2.4 <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivot(s) (3.1) 1.8 (1.2) 3.1 <u>2.8</u> 2.6 <u>2.4</u> <u>1.8</u> <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivot 2.4 3.1 <u>2.8</u> 2.6 <u>2.4</u> <u>1.8</u> <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 (sort complete) e.g. using middle left 2.6 1.8 2.8 1.6 3.1 <u>1.4</u> 1.2 2.4 0.9 0.8 0.6 pivot 1.4 2.6 1.8 <u>2.8</u> 1.6 3.1 2.4 <u>1.4</u> 1.2 <u>0.9</u> 0.8 0.6 pivots 2.8 0.9 3.1 <u>2.8</u> 2.6 <u>1.8</u> 1.6 2.4 <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivots (3.1) 1.8 (1.2) 0.8 3.1 <u>2.8</u> <u>2.6</u> 2.4 <u>1.8</u> 1.6 <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 pivot(s) 2.6 (1.6) (0.6) 3.1 <u>2.8</u> <u>2.6</u> 2.4 <u>1.8</u> <u>1.6</u> <u>1.4</u> 1.2 <u>0.9</u> <u>0.8</u> 0.6 (sort complete)	M1 A1 A1ft A1
(d)	Bin 1: 3.1 1.8 Bin 2: 2.8 <u>1.6</u> 0.6 Bin 3: 2.6 2.4 Bin 4: <u>1.4</u> <u>1.2</u> 0.9 0.8	M1 A1 A1 (3)
		(13 marks)

Notes for Question 3

PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE IN PART (c) AND THE GUIDANCE FOR THE M MARK IN (d)

a1M1: The **correct** first four items placed correctly (the bold values) and at least eight values placed in bins (allow repeated values). Condone cumulative totals for M1 only

a1A1: First eight values placed correctly (the bold **and** underlined values) with all eleven correct values only placed in bins. This mark cannot be awarded if **any** repeated values or incorrect values are seen (even if the first eight values have been placed correctly)

a2A1: CSO – no additional or repeated values (dependent on both previous marks)

bi1B1: CAO – isw after one complete pass. Please check these carefully as some candidates show all the swaps and comparisons in the first pass and some show more than one complete pass. As a guide consider the placement of the 0.8 (when this is the second value from the right-hand side this will indicate the completion of the first pass)



Question Number	Scheme	Marks
5. (a)	Shortest path: A B D G K H Length: 68 (miles)	M1 A1 (ABCDE) A1 (FGK) A1ft (JH) A1 A1ft (6)
(b)	Route from F to K via A: F E C B A B D G K Length: $41 + 62 = 103$ (miles)	B1 B1ft (2)
(c)	$AJ + CE = 67 + 16 = 83$ $AC + EJ = 20 + 32 = 52$ $AE + CJ = 36 + 48 = 84$ Repeated arcs: AB, BC, EF, FK, JK Length: $253 + 52 = 305$ (miles)	M1 A1 A1 A1 A1 A1ft (6)
(d)	Vertex F: 4 times	B1 (1)
(e)	(Start at D and) finish at C	B1 (1)
(f)	Difference = $305 - (253 + 10) = 42$ (miles)	B1 (1)
		17 marks

Notes for Question 5

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 45 44 41 in that order (so 45 41 44 is incorrect)

It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at either C, F, K, J, or H

a1A1: All values at A, B, C, D and E correct and the working values in the correct order (including order of labelling) – if a working value of 45 appears at E then it must appear after the 36 so therefore 45 36 at E (in this order) is A0

a2A1: All values at F, G and K correct and the working values in the correct order (F, G and K must be labelled in that order and F must be labelled after A, B, C, D and E)

a3A1ft: All values in J and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through J check that the working value at J follows from the candidate's final values from their feeds into J (which will mostly likely come from nodes F and K (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for H (which will possibly have working values from F and K with the order of these values determined by the candidate's order of labelling at F and K)

a4A1: CAO (ABDGKH or AB, BD, DG, GK, KH)

a5A1ft: Follow through on their final value at H **only** (condone lack of units) so if answer given as 68 but final value at H is not 68 then A0

b1B1: CAO (FECBABDGK or FE, EC, CB, BA, AB, BD, DG, GK)

b2B1ft: Follow through their final value at F + their final value at K **or** 103

c1M1: Correct three distinct pairings of the correct four odd nodes of A, C, E and J

c1A1: **Any** row correct including pairing **and** total

c2A1: **Any** two rows correct including pairings **and** totals

c3A1: **All** three rows correct including pairings **and** totals

c4A1: CAO correct edges clearly stated and not just in their working as AB, BC, EF, FK and JK – must be these arcs

c5A1ft: Follow through their value of their smallest pairing total + 253

d1B1: CAO (4 only)

e1B1: CAO (C)

f1B1: CAO (42)

Question Number	Scheme	Marks
6. (a) and (b)		B1 B1 B1 (3) M1 A1 M1 A1 (4)
(c)	Critical activities: A, E, K, M, P	B1 (1)
(d)	Lower bound = $\frac{97}{28} = 3.46\dots$ so 4 workers	M1 A1 (2)
(e)	e.g. 	M1 A1 A1 (3)
		13 marks

Question Number	Scheme	Marks
1. (a)	Nearest neighbour: A – B – F – D – E – C – A 35 31 44 39 53 42 = 244 (km)	M1 A1 (2)
(b)	MST with B removed: AC, CD, DF, DE gives a RMST weight of 172 (km)	B1
	172 + 31 + 35 = 238 (km)	M1 A1 (3) 5 marks
	Notes for Question 1	

a1M1: Nearest neighbour A – B – F – D – E – C – (condone lack of return to start) **or** correct route length of 244. Accept AB, BF, FD, DE, EC but do not accept weights only. Accept 1 2 6 4 5 3 across the top of the table

a1A1: CAO both route (either in terms of vertices (ABFDECA) or arcs (AB, BF, FD, DE, EC, CA) but not weights) **and** length correct (244) do not ISW if this value is then doubled to 488

b1B1: CAO for RMST weight (either 172 or $42 + 47 + 44 + 39$) – maybe implied by later working

b1M1: Adding $31 + 35$ (the two least weighted arcs) to their RMST length – this mark maybe implied by the correct value for the lower bound – note that their RMST must contain only four arcs

b1A1: CAO - if 238 seen without working then award B0M1A1

Question Number	Scheme	Marks
3. (a)	e.g. add CD and remove AD, BA and BC gives 516 (km) e.g. add EF and remove EB, BA and AF gives 509 (km)	M1 A1 (2)
(b)	NNA: A – B – E – F – D – C – A	B1
	57 66 69 78 71 76 = 417 (km)	B1 (2)
(c)	Length of RMST = 248	B1
	248 + 66 + 69 = 383 (km)	M1 A1 (3)
		7 marks
Notes for Question 3		
a1M1: Must clearly start with 2(length of given MST) and add and subtract at least one arc (to give a network of weight < 628) – graph must be connected and Eulerian a1A1: CAO – shortcut(s) and length must be consistent (with length stated < 520). The shortcuts must be clearly stated (that is the arcs added and subtracted) and network must be connected and Eulerian b1B1: CAO (must return to A) – must be stated in terms of either the nodes or arcs (e.g. AB, BE, EF,...) but not just the weights of the arcs b2B1: CAO (417) c1B1: Correct length of RMST (248) – maybe implied by later working c1M1: Adding the two correct least weighted arcs (66 and 69) to their RMST length ($231 \leq \text{length} \leq 265$) – give bod but their RMST must only contain 4 arcs – this mark can be implied by the correct value for the lower bound c1A1: CAO (383) – if correct answer with no working then award B0M1A1		



Question Number	Scheme	Marks
4.(a)		M1 A1 A1 A1 A1 (5)
(b)	D and F are guaranteed to be critical	B1 (1)
(c)	Critical path: A – C – G – I – J	B1 (1)
		7 marks

Notes for Question 4

In (a) condone lack of, or incorrect, numbered events throughout – also ‘dealt with correctly’ means that the activity starts from the correct event but may not finish at the correct event (use the table below to check this). **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity D is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first four marks only

a1M1: Eight activities (labelled on arc), one start and at least two dummies placed

a1A1: Activities A, B, C, 1st dummy (+ correct arrow on this dummy – this is the dummy at the end of A) and D dealt with correctly

a2A1: 2nd and 3rd dummies (+ correct arrows on these dummies – these dummies are the ones at the end of C and D) and E, F and G dealt with correctly

a3A1: 4th dummy (+ correct arrow on this dummy – this is the dummy at the end of F) and activities H, I, J and K dealt with correctly

a4A1: CSO – all arrows present and correctly placed with one finish

Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only.

Note that additional (but unnecessary) ‘correct’ dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

b1B1: CAO (D and F) and no other activities stated as critical (however, ignore K if stated too)

c1B1: CAO (A – C – G – I – J)

Activity	A	B	C	D	E	F	G	H	I	J	K
IPA	-	-	A	A, B	C, D	D	C	G	G	E, F, I	F

Question number	Scheme	Marks
1. (a)	A – B – D – F – C – E – G – A 43 + 45 + 49 + 55 + 50 + 48 + 55 = 345 (m)	M1
	A – B – D – F – G – E – C – A 43 + 45 + 49 + 55 + 48 + 50 + 52 = 342 (m)	A1
		A1 (3)
(b)	RMST weight = 237 (m) or arcs in RMST are BD, BE, BG, DF, CE 237 + 43 + 47 = 327 (m)	B1 M1 A1 (3)
(c)	$327 \leq \text{optimal distance} \leq 342$	B1ft (1)
		(7 marks)

Notes for Question 1

a1M1: First four nodes correct for a nearest neighbour route starting at A – so must have at least A – B – D – F – ... **or** in terms of arcs (AB, BD, DF,...) **or** in terms of weights (43, 45, 49,...)

a1A1: One correct route (in terms of arcs or nodes but not just weights), must return to A and corresponding correct length (units not required)

a2A1: Both routes correct (in terms of arcs or nodes but not just weights) and their corresponding correct lengths (units not required)

b1B1: cao for RMST weight (237) **or** correct arcs (BD(45), BE(46), BG(47), DF(49), CE(50) only) **or** 45 + 46 + 47 + 49 + 50 stated

b1M1: Adding the two correct least weighted arcs (AB(43) and AD(47)) to their attempt at RMST weight where their attempt at the RMST has a weight in the interval $224 \leq \text{RMST weight} \leq 250$ (give bod if not clear where their attempt at RMST weight comes from but if working shown then must be from summing the weight of exactly five arcs). Allow unsimplified answers which imply the correct two arcs added to the weight of the RMST (e.g. 45 + 46 + 47 + 49 + 50 + 43 + 47 is equivalent to the two smallest arcs (43, 47) added to five arcs which sum to a value in the given interval).

If a candidate uses one of their NN routes from (a) and removes the arcs incident to A from this route and adds on the 43 and 47 (e.g. you may see 45 + 49 + 55 + 50 + 48 + 43 + 47) then this can still score M1 as they have added the weight of five arcs that form a spanning tree (not minimal but this is their attempt - with a total in the required interval) and they have then added on the two correct least weighted arcs

b1A1: CAO (327) – the correct answer of 327 with no working can score all three marks in this part

c1B1ft: Their numbers correctly used (their answer to (b) and their least value from (a)) with correct inequalities (**allow strict inequality for lower bound**) so an answer of 327 – 342 is B0. Lower bound must be less than upper bound. The LB must be $314 \leq \text{LB} \leq 340$ and is dependent on scoring the M mark in (b). The UB is dependent on the M mark in (a) and there must be two different values in (a) stated (and they must have chosen the smaller of the two). Allow equivalent notation e.g. [314, 340] or (314, 340]

Due to the original question paper showing H in the first vertical column on the table instead of G, use the following marking guidance.

In (a) allow use of H for G or a combination of Gs and Hs. The following are all examples that would be acceptable for the correct route A – B – D – F – C – E – G – A in (a)



- $A - B - D - F - C - E - H - A$
- $A - B - D - F - C - E - G/H - A$
- $A - B - D - F - C - E - H - G - A$
- $A - B - D - F - C - E - G - H - G - A$
- $A - B - D - F - C - E - H$ or $G - G - A$
- $AB, BD, DF, FC, CE, EH/G, G/HA$
- AB, BD, DF, FC, CE, EG (or EH), HA (or GA)
- $AB, BD, DF, FC, CE, EH, HG, GA$
- $AB, BD, DF, FC, CE, EH, GA$

In general accept a cycle of the form $A - B - D - F - C - E - X - A$ where X is G or H or any combination of G s and H s (and similarly $A - B - D - F - X - E - C - A$ for the second cycle or their equivalents in terms of arcs).

Question Number	Scheme	Marks
1. (a)	NNA: A – D – E – F – B – C – A $27+25+21+34+58+56 = 221$ (km)	M1 A1 A1 (3)
(b)	RMST weight = 118 (km)	B1
	$118 + 27 + 38 = 183$ (km)	M1 A1 (3)
(c)	$183 \leq \text{length} \leq 221$	M1 A1 (2)
		8 marks
Notes for Question 1		
a1M1: Nearest neighbour A – D – E – F – B or accept 1 5 6 2 3 4 across the top of the table a1A1: Route correctly stated, must return to A, accept link back to A a2A1: Length correctly stated. Do not ISW if candidates then go on to double the route length b1B1: CAO for RMST weight (either 118 or $34 + 21 + 25 + 38$) – maybe implied by later working b1M1: Adding $27 + 38$ (the two least weighted arcs) to their RMST length – this mark maybe implied by the correct value for the lower bound – note that their RMST must contain only four arcs b1A1: CAO - if 183 seen without working then award all 3 marks in (b) c1M1: Their answers from (a) and (b) correctly used, accept any inequalities or any indication of an interval from their 183 to their 221 (so $183 - 221$ can score this mark). Please note that $UB > LB$ for this mark c1A1: CAO (no follow through on their values) including correct inequalities or equivalent set notation (but condone $183 < \text{length} \leq 221$)		

Question	Scheme	Marks
2 (a)	PR PU UV PQ RS ST	M1 A1
		(2)
(b)	“540” x 2 = 1080 (metres)	M1 A1ft
		(2)
(c)	PRSTUVQP 880 (metres)	M1 A1 A1
		(3)
(d)	$540 - 75 + (75 + 180) = 720$ (metres)	M1 A1
		(2)
(9 marks)		

Notes	
a1M1	Prim - first three arcs correctly chosen in order (PR, PU, UV, ...) or first four nodes {P, R, U, V, ...}. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, 2, -, -, 3, 4 }. Starting at any other node can score M1 only for first three arcs chosen correctly
a1A1	CSO – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)
b1M1	Calculates the weight of their MST and doubles If their MST is incorrect, follow through the total for their tree (must be exactly 6 arcs)
b1A1ft	1080 or 2 x “their 540” (if any subsequent calculations are seen do not isw this is A0) (1080 implies both marks)
c1M1	NN route attempted, first five nodes correct PRSTU (accept arcs for this mark PR, RS, ST, TU, ...) (do not accept numbering across the matrix here unless nodes or arcs are stated in the correct order)
c1A1	CAO – Route correct including return to P (accept arcs PR, RS, ST, TU, UV, VQ, QP) (condone if arcs stated alphabetically instead of in route order)
c2A2	CAO – 880 (metres) (if they double the length of the NN route do not isw, this is A0)
d1M1	Their “540” – 75 + two shortest from T (a new calculation to find the RMST may be seen e.g. new table with T deleted and a new MST calculation, with the correct 5 arcs circled PR PU UV PQ RS plus two shortest from T)
d1A1	CAO – A final answer of 720 implies both marks (if additional calculations seen do not isw)

2.(a)	Prim: AE, AC, CD; BD, AF; CH, BG	M1 A1 A1 (3)
(b)	Weight of MST is 205 (minutes)	B1 (1)
(c)	J – G – B – D – C – A – E – F – H – J $28 + 32 + 28 + 27 + 29 + 28 + 36 + 39 + 42 = 289$ (minutes)	M1 A1 (2)
(d)	The best upper bound is the one starting at J as 289 is less than 291	B1 (1)
(e)	The two smallest arcs incident to J are 28 and the $\min(x, 33)$ but $28 + 33 + 205 \neq 264$	B1
	$205 + 28 + x = 264$	M1
	$x = 31$	A1 (3)
		10 marks
	Notes for Question 2	
a1M1:	Prim's – first three arcs correctly chosen in order (AE, AC, CD, ...) or first four nodes {A, E, C, D, ...} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, 3, 4, 2, -, -, -}. Starting at any other node can score M1 only for first three arcs chosen correctly	
a1A1:	First five arcs correctly chosen in order (AE, AC, CD, BD, AF, ...) or all eight nodes {A, E, C, D, B, F, H, G} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 5, 3, 4, 2, 6, 8, 7} (no missing numbers)	
a2A1:	CSO – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)	
b1B1:	CAO for weight of MST (205) – no units required	
c1M1:	Nearest neighbour starting at J with first five nodes correct (J – G – B – D – C –) Accept arcs JG GB BD DC	
c1A1:	Correct nearest neighbour route (must return to J) (may be listed as arcs JG GB BD DC CA AE EF FH HJ) and correct length (289)	
d1B1:	Accept any wording indicating that the answer from (c) is smaller than 291 and therefore the better upper bound An indication that 289 is the minimum (of 291 and 289) – this mark is dependent on the correct value in (c) so accept an answer of the form 'the one starting at J (or the route with weight 289) as it is the least'	
e1B1:	Correct justification that the two smallest arcs incident to J are 28 and x - may calculate that the two smallest arcs must be 28 and 31 and then state that as no other arc has length 31, this must be the value of x	
e1M1:	Forming the equation: weight of MST from (b) + $28 + x = 264$	
e1A1:	CAO for x	

Question Number	Scheme	Marks
4. (a)	e.g. in the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited exactly once	B2, 1, 0 (2)
(b)	NNA starting at A: A – B – D – F – C – G – E – A $25 + 24 + 35 + 27 + 29 + 31 + 35 = 206$ (km)	B1 B1 (2)
(c)	The better upper bound is the one starting at D as it is smaller	B1dep (1)
(d)(i)	Prim (starting at A): AB, BD, BE, EF, CF RMST weight = $25 + 24 + 27 + 28 + 27 = 131$	M1 A1
(d)(ii)	$131 + 29$ (CG) + 31 (EG) = 191 (km)	M1 A1 (4)
(e)	The better lower bound is the one found by deleting G as this is the larger of the two	B1dep (1)
(f)	$191 \leq \text{optimal distance} \leq 203$	B1ft B1 dep (2)
		12 marks

Notes for Question 4

a1B1: Understands the difference is connected to the number of times each vertex may be visited – condone ‘point’ (oe) for vertex (must refer to both problems in their answer but not necessarily by name)

a2B1: Correctly identifies which is classical (each node visited ‘exactly once’ or ‘once’) and which is practical (each node visited ‘at least once’ but B0 for ‘more than once’ oe – it must be clear that for the practical case that a node may be visited more than once but not necessarily more than once). Must use correct language (e.g. vertex or node) but condone singular/plural confusion e.g. vertex for vertices, or poor spelling (in this part a mark of B0B1 is not possible)

b1B1: Correct nearest neighbour route starting at A (must return to A) – possibly stated in terms of arcs e.g. AB, BD, DF, CF, CG, EG, EA

b2B1: CAO (206) on length of route

c1B1dep: CAO dependent on the correct UB in (b) – allow ‘yes it is’ (as question asks, ‘state whether this (an upper bound of 203) is a better upper bound than the answer to (b)’)) **and** with some indication that this value is smaller than the one in (b) e.g. ‘ $203 < 206$ so yes it is’ scores B1

di1M1: Must be using Prim’s algorithm not NNA. First three arcs (or all 6 nodes / or numbers across the top of the matrix) selected correctly. First three arcs are AB, BD, BE, first six nodes are A, B, D, E, F, C and so numbers across the matrix would be 1, 2, 6, 3, 4, 5. Award M1 only for a correct tree with either no working or if starting at a different node than A

di1A1: CAO (order of arc selection clear) – in terms of arcs only for this mark - AB, BD, BE, EF, CF – condone those that state AB, BD, BE, EF, CF, CG, EG **or** AB, BD, BE, EF, CF, EG, CG (these candidates are most likely adding on the two smallest arcs incident to G for the next part of the question)

dii2M1: Adding two least weighted arcs (CG(29) + EG(31)) to the length of their answer from d(i) (where $100 \leq d(i) \leq 160$) - condone if parts (d)(i) and (d)(ii) are combined together as a single part (d)

dii2A1: CAO (191)

e1B1dep: CAO dependent on the correct LB in (d)(ii) – allow ‘no it isn’t’ (as question asks, ‘state whether this (a lower bound of 188) is a better lower bound than the answer to (d)(ii)’)) and with some indication that this value is smaller than the one in (d)(ii) e.g. ‘ $188 < 191$ so no it isn’t’ scores B1



If the candidate's answer to (b) is less than 188 then no marks can be awarded in (f)

f1B1ft: Their numbers correctly used, accept any inequalities or any indication of an interval from

their largest of the two values (188 or d(ii)) to their smallest of the two values (203 or (b))

e.g. condone for B1 only $203 - 191 = 12$

f2B1dep: This mark is dependent on the previous B mark - CAO including correct inequalities (accept either $191 \leq \text{optimal distance} \leq 203$ or $191 < \text{optimal distance} \leq 203$) or equivalent notation e.g. $[191, 203]$ or $(191, 203]$

Question Number	Scheme	Marks
7. (a)	Prim: AH, FH, EH, FG, DG, CG, BC	M1 A1 A1 (3)
(b)	Initial upper bound $2(201) = 402$ (km)	B1ft (1)
(c)	A – H – F – G – D – B – C – E – A $27 + 28 + 31 + 29 + 32 + 26 + 38 + 37 = 248$	M1 A1 (2)
(d)	Nearest neighbour starting at E has a length of $212 + x$ As $x \leq 35 \Rightarrow$ the NN route starting at E is at most 247 (km) and therefore the NN starting at E gives the better upper bound as it is less than the one starting at A (which was 248 (km))	M1 A1 (2)
(e)	Lower bound is given by $(201 - 27) + 27 + x = 235$	M1
	$x = 34$	A1
	$235 \leq \text{optimal length} \leq 246$	M1 A1 (4)
		12 marks

Notes for Question 7

a1M1: Prim's – first three arcs correctly chosen in order (AH, FH, EH, ...) **or** first four nodes {A, H, F, E, ...} correctly chosen in order. If any explicit rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, -, -, 4, 3, -, 2} so check there too. Starting at any other node apart from A can score M1 only for first three arcs chosen correctly

a1A1: First five arcs correctly chosen in order (AH, FH, EH, FG, DG, ...) **or** all eight nodes {A, H, F, E, G, D, C, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 7, 6, 4, 3, 5, 2} (**no** missing numbers)

a2A1: CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

b1B1ft: Follow through double the stated length of their MST

c1M1: NN starting at A – must have at least A – H – F – G – D – ... allow if stated in terms of arcs

c2A1: CAO on length (248) **and** route (must return to A but can be stated in terms of arcs)

d1M1: Calculating the correct length of the NN route starting at E ($212 + x$) **and** attempting to use the range of values for x to determining the better upper bound (implied by 247 seen or [244, 247]).

d1A1: Correct best upper bound stated (the one starting at E) together with a correct comparison of **248** (possibly implicit - if this value is not explicitly stated in (d) then 248 must have been seen in (c)) with 247 (or an indication of 'at most' 247). **For those who obtained an answer of 211 in (c) and say that $212 + x$ is always bigger (without using the given interval for x to find the UB) then no marks in this part**

e1M1: Correct method for calculating x (which is the weight of MST from (a)/(b) – $27 +$ two smallest arcs incident to A (the 27 and x) equal to 235). If using the doubled value from (b) then **M0**. If not using the weight of the MST from (a) then they must be using either 174 **or** $26 + 30 + 29 + 31 + 28 + 30$ **or** explicitly using the correct six arcs only (BC, CG, GD, GF, FH, HE) so not just circled in one of the tables. The correct value of x (with either no working or no incorrect working) clearly stated can imply this mark

e1A1: CAO for x (34) – **as a minimum** must have seen the calculation $201 + x = 235$ to award this mark

e2M1: Any indication of an interval from 235 to either 246, 247 or 248 (this mark is **not** dependent on the previous M mark)



e2A1: CAO (condone $235 < \text{optimal length} \leq 246$ and allow equivalent interval notation e.g. (235, 246] or [235, 246]) – this mark is dependent on all previous marks in (e) (so must have found that x equals 34).

The correct interval (with no others) with no supporting working scores **M0A0M1A0**

The minimum requirement for full marks is: $201 + x = 235 \Rightarrow x = 34 \therefore [235, 246]$

For those who simply state $x = 34$ (only) followed by the correct interval they score **M1A0M1A0**

Question Number	Scheme	Marks
6.(a)	Classical problem must visit every vertex exactly once and returns to start but practical problem must visit every vertex at least once and returns to start	B1 B1
		(2)
(b)	Either AB, BC, CK, HK, JK, CD, GJ, FG, EG or AB, BC, CK, HK, JK, GJ, FG, EG, CD	M1 A1 A1
		(3)
(c)	$89 \times 2 = 178$ (miles)	B1
		(1)
(d)	ABCKHJGFEDA 140 (miles)	M1 A1 A1
		(3)
(e)	Route from A as it is the smaller value or 140 because $140 < 145$	B1ft
		(1)
(f)	Lower Bound = $89 - 16 + (16 + 19) = 108$ (miles)	M1 A1
		(2)
(g)	$108 \leq L \leq 140$	B1ft
		(1)
		13 marks

Notes for Question 6		
a1B1	General idea that Classical visits vertices once but practical is at least once. Any mention of travelling along every arc is B0	
a2B1	Need to see all the words in bold (accept equivalent technical language e.g. node instead of vertex) – must include return to start	
b1M1	Prim's – first three arcs correctly chosen in order (AB, BC, CK, ...) or first four nodes {A, B, C, K, ...} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, 2, 3, -, -, -, -, -, -, 4}. Starting at any other node can score M1 only for first three arcs chosen correctly	
b1A1	First six arcs correctly chosen in order {AB, BC, CK, HK, JK, CD, ...} or {AB, BC, CK, HK, JK, GJ, ...} or all 10 nodes {A, B, C, K, H, J, D, G, F, E} or {A, B, C, K, H, J, G, F, E, D} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 2, 3, 7, 10, 9, 8, 5, 6, 4} or {1, 2, 3, 10, 9, 8, 7, 5, 6, 4} (no missing numbers).	
b2A1	CSO – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)	
c1B1	CAO	
d1M1	NN route attempted – first seven nodes correct ABCKHJG (accept arcs for this mark AB, BC, CK, KH, HJ, JG or complete numbering on table {1, 2, 3, 10, 9, 8, 7, 5, 6, 4} which is clearly a separate calculation from part (b))	
d1A1	CAO – route correct including return to A (accept arcs AB, BC, CK, KH, HJ, JG, GF, FE, ED, DA) (condone if arcs stated alphabetically instead of in route order)	
d2A1	CAO – distance 140	
e1B1ft	States A or 140 and reason correct (smaller value) but fit their answer to (d) dependent on M1 in (d)	
f1M1	Their “89” – 16 + two shortest from A (a new calculation to find the RMST may be seen e.g. new table with A deleted and a new MST calculation, with the correct 8 arcs circled BC CD CK EG FG GJ JK KH plus two shortest from A)	
f1A1	CAO – an answer of 108 implies both of these marks	
g1B1ft	FT their answers to (e) and (f) but dep on B1 in (e) and M1 in (f) and their (f) < their (e) (accept $108 < L \leq 140$ or interval notation $[108, 140]$ or $(108, 140]$)	

Question Number	Scheme									Marks
		A	B	C	D	E	F	G	H	
	A	-	7	8	9	11	18	25	26	
	B	7	-	14	2	4	11	18	19	
	C	8	14	-	12	10	15	22	23	
	D	9	2	12	-	2	9	16	17	
	E	11	4	10	2	-	7	14	15	
	F	18	11	15	9	7	-	7	8	
	G	25	18	22	16	14	7	-	1	
	H	26	19	23	17	15	8	1	-	
(b)	NNA: A – B – D – E – F – G – H – C – A									B1
	$7 + 2 + 2 + 7 + 7 + 1 + 23 + 8 = 57$ (km)									B1 (2)
(c)(i)	Prim (starting at C): CE, DE, BD, EF, FG, GH									M1 A1
	RMST weight = $10 + 2 + 2 + 7 + 7 + 1 = 29$									
(c)(ii)	$29 + 7(AB) + 8(AC) = 44$ (km)									M1 A1 (4)
(d)	$44 \leq \text{optimal distance} \leq 57$									B2, 1, 0 (2)
										14 marks
Notes for Question 6										
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 23 21 18 in that order (so 23 18 21 is incorrect). It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling										