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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Linear Programming (D1.5)

International A Level Mathematics · D1: Decision Mathematics 1 (WDM11)

22 questions · 240 marks · Mark schemes included

Past papers June 2019 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Where a question is answered in a separate answer book, those answer book pages follow the question.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

22 questions · 240 marks · 1 sub-topic

SUB-TOPIC D1.5a

FORMULATING & SOLVING LINEAR PROGRAMMES

Sub-topic D1.5a · 22 questions · 240 marks · lowest tariff first



2. A restaurant sells two sizes of pizza, small and large. The restaurant owner knows that, each evening, she needs to make
- at least 85 pizzas in total
 - at least twice as many large pizzas as small pizzas

In addition, at most 80% of the pizzas must be large.

Each small pizza costs £2 to make and each large pizza costs £3 to make.

The restaurant owner wants to minimise her costs.

Let x represent the number of small pizzas made each evening and let y represent the number of large pizzas made each evening.

Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients. You should **not** attempt to solve the problem.

(Total 5 marks)



2.

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Q2

(Total 5 marks)



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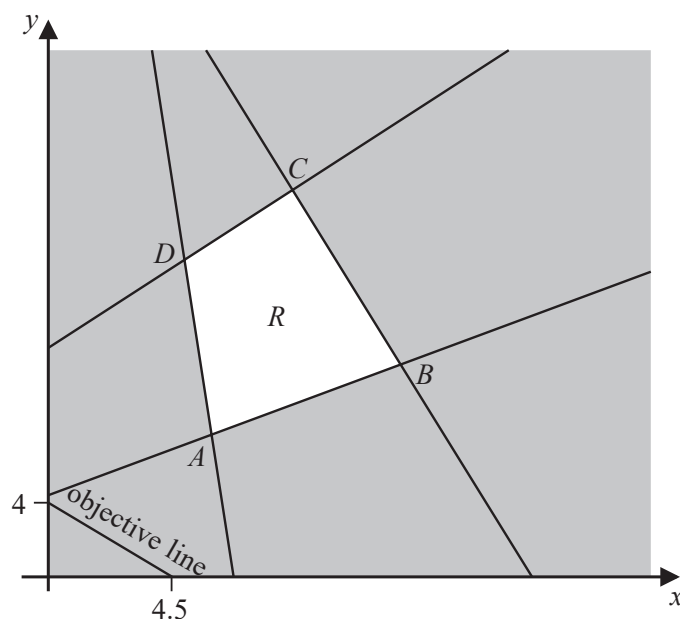
**Figure 3**

Figure 3 shows the constraints of a linear programming problem in x and y . The unshaded area, including its boundaries, forms the feasible region, R .

The four vertices of R are $A(6, 8)$, $B(13, 12)$, $C(9, 22)$ and $D(5, 18)$.

An objective line has been drawn and labelled on the graph.

When the objective function, P , is maximised, the value of P is 540

When the objective function, P , is minimised, the value of P is k

Determine the value of k . You must make your method and working clear.

(You may assume that the objective function, P , takes the form $ax + by$ where a and b are constants.)

(Total for Question 5 is 5 marks)

5.

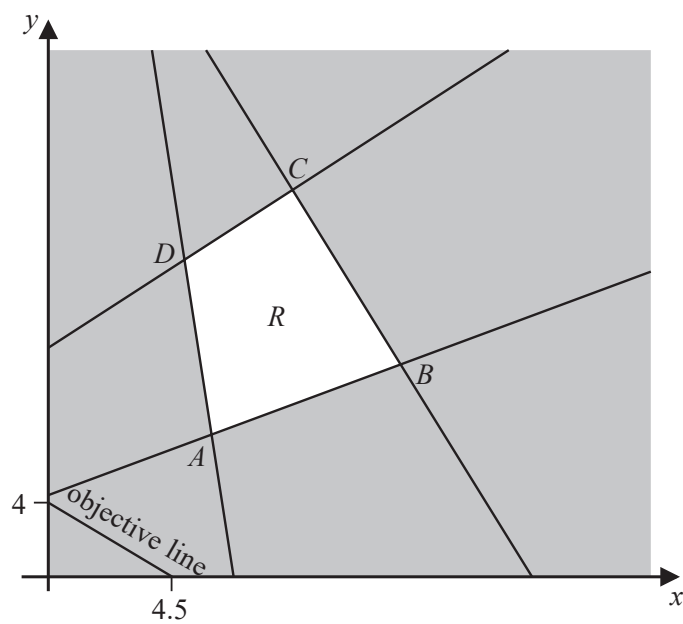


Figure 3

Question 5 continued

(Total for Question 5 is 5 marks)



2. Chris has been asked to design a badge in the shape of a triangle XYZ subject to the following constraints.

- Angle Y should be at least three times the size of angle X
- Angle Z should be at least 50° larger than angle X
- Angle Y must be at most 120°

Chris has been asked to maximise the sum of the angles X and Y .

Let x be the size of angle X in degrees.

Let y be the size of angle Y in degrees.

Let z be the size of angle Z in degrees.

Formulate this information as a linear programming problem in x and y **only**. State the objective and list the constraints as simplified inequalities with integer coefficients.

You are **not** required to solve this problem.

(6)

(Total 6 marks)



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Question 2 continued

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(Total 6 marks)

Q2



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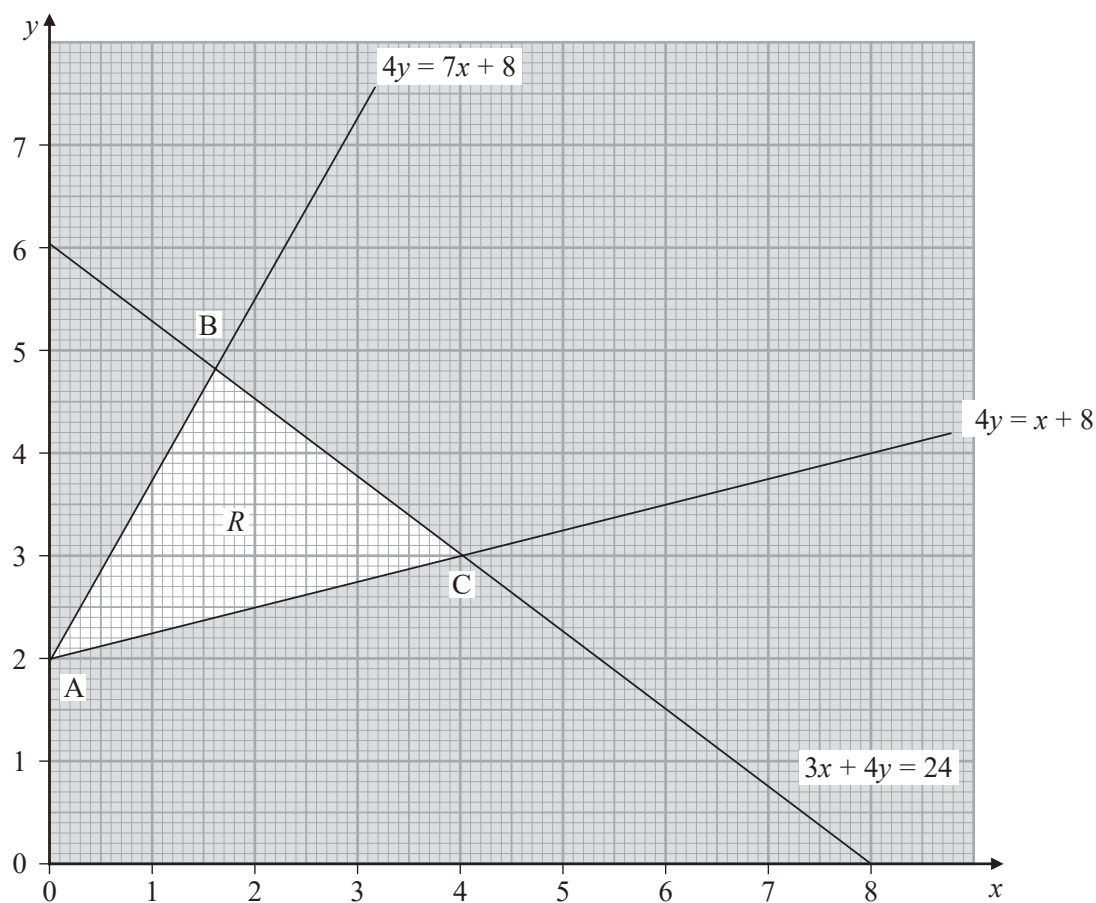


Figure 2

The graph in Figure 2 is being used to solve a linear programming problem in x and y . The three constraints have been drawn on the graph and the rejected regions have been shaded out. The three vertices of the feasible region R are labelled A, B and C.

(a) Determine the inequalities that define R .

(2)

The objective function, P , is given by

$$P = ax + by$$

where a and b are positive constants.

The minimum value of P is 8 and the maximum value of P occurs at C.

(b) Find the range of possible values of a . You must make your method clear.

(5)

(Total 7 marks)



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Question 6 continued

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(Total 7 marks)

Q6



4. A linear programming problem in x , y and z is described as follows.

Maximise $P = -x + y$

subject to

$$x + 2y + z \leq 15$$

$$3x - 4y + 2z \geq 1$$

$$2x + y + z = 14$$

$$x \geq 0, y \geq 0, z \geq 0$$

- (a) (i) Eliminate z from the first two inequality constraints, simplifying your answers.

- (ii) Hence state the maximum possible value of P

(4)

Given that P takes the maximum possible value found in (a)(ii),

- (b) (i) determine the maximum possible value of x

- (ii) Hence find a solution to the linear programming problem.

(3)

(Total for Question 4 is 7 marks)



4.

Lined area for writing the answer to question 4.



Question 4 continued

(Total for Question 4 is 7 marks)



5. A manufacturer makes three flavours of ice cream, vanilla, strawberry and chocolate. All flavours are supplied in the same size tubs.

The profit from selling each tub of vanilla ice cream is £1.50, each tub of strawberry ice cream is £2.00 and each tub of chocolate ice cream is £1.80

The manufacturer wishes to maximise their profit.

This needs to be formulated as a linear programming problem.

Let x be the number of tubs of vanilla ice cream made.

Let y be the number of tubs of strawberry ice cream made.

Let z be the number of tubs of chocolate ice cream made.

- (a) State the objective, giving the objective function in terms of x , y and z

(2)

There are sufficient ingredients to make 400 tubs of vanilla ice cream **or** 240 tubs of strawberry ice cream **or** 300 tubs of chocolate ice cream.

- (b) Show that this constraint can be written as

$$3x + 5y + 4z \leq 1200$$

(2)

The manufacturer must also meet the following three conditions

- at least 40% of the total ice cream made must be vanilla
- at most 25% of the total ice cream made must be strawberry
- for every 2 tubs of strawberry ice cream made, the manufacturer must make at least 3 tubs of chocolate ice cream

- (c) Use this information to form three further constraints, giving these as simplified inequalities with integer coefficients.

(3)

(Total for Question 5 is 7 marks)



5.

Lined area for writing the answer to question 5.



8.

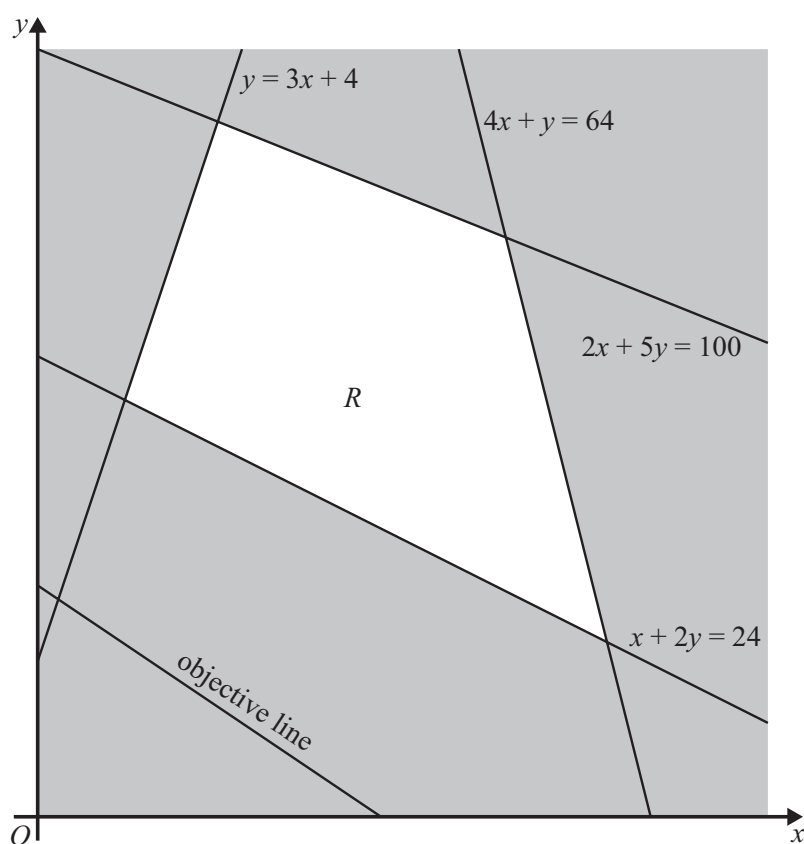


Figure 4

Figure 4 shows the constraints of a linear programming problem in x and y

The unshaded area, including its boundaries, forms the feasible region R

An objective line has been drawn and labelled on the graph.

(a) State the four inequalities that define the feasible region.

(2)

The maximum value of the objective function is $\frac{628}{9}$

The minimum value of the objective function is $\frac{260}{7}$

(b) Determine the objective function, showing your working clearly.

(5)

The practical solution to the linear programming problem requires integer values.

(c) State the minimum integer solution for this problem.

(1)

(Total for Question 8 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



8.

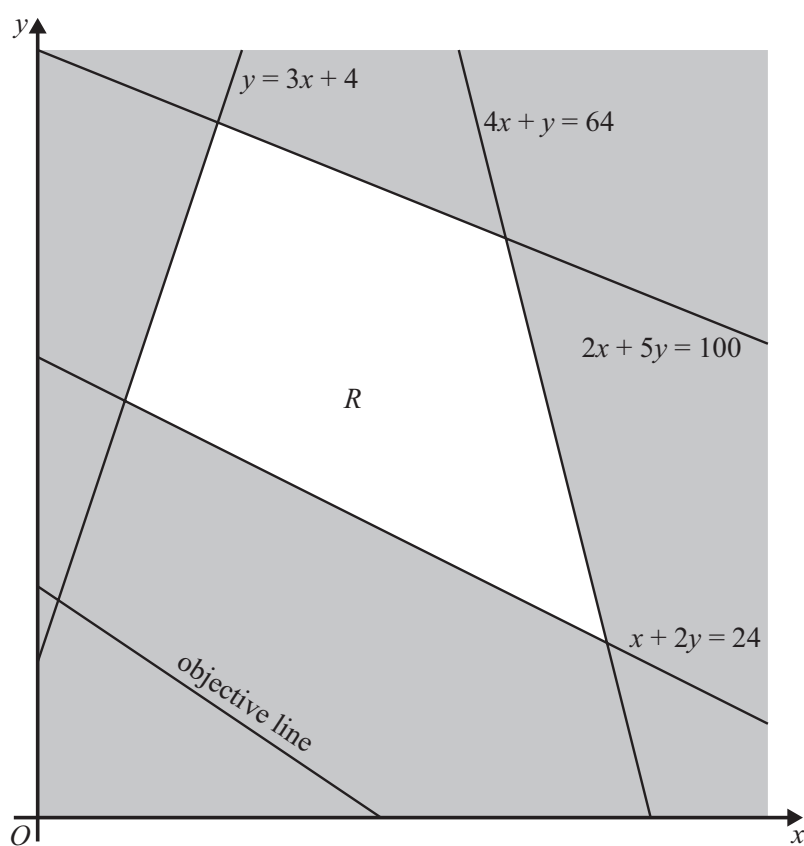


Figure 4

Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

(Total for Question 8 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



7.

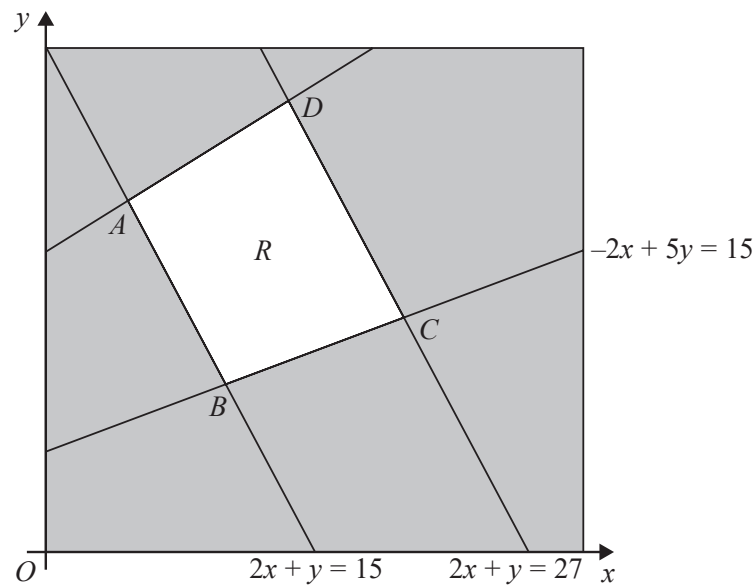


Figure 4

Figure 4 shows the constraints of a linear programming problem in x and y , where R is the feasible region.

The equations of three of the lines, and the four intersection points, A , B , C and D , are shown.

The four vertices of R are $A\left(\frac{9}{4}, \frac{21}{2}\right)$, $B(5, 5)$, $C(10, 7)$ and $D\left(\frac{27}{4}, \frac{27}{2}\right)$.

(a) Determine the inequalities that define R .

(4)

The objective function, P , is given by

$$P = 2x + ky$$

where k is a positive constant.

Given that the minimum value of P is at least half of the maximum value of P ,

(b) determine the range of possible values of k .

(5)

(Total for Question 7 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS



7.

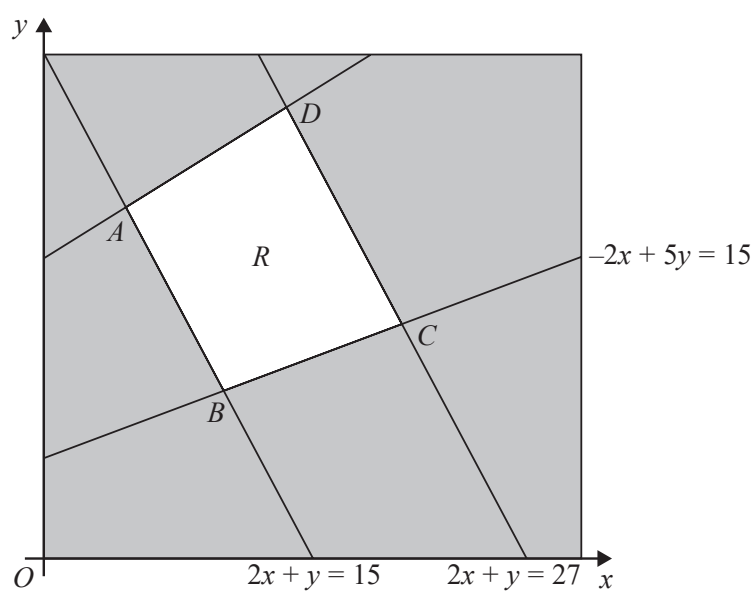


Figure 4



TOTAL FOR PAPER IS 75 MARKS



7.

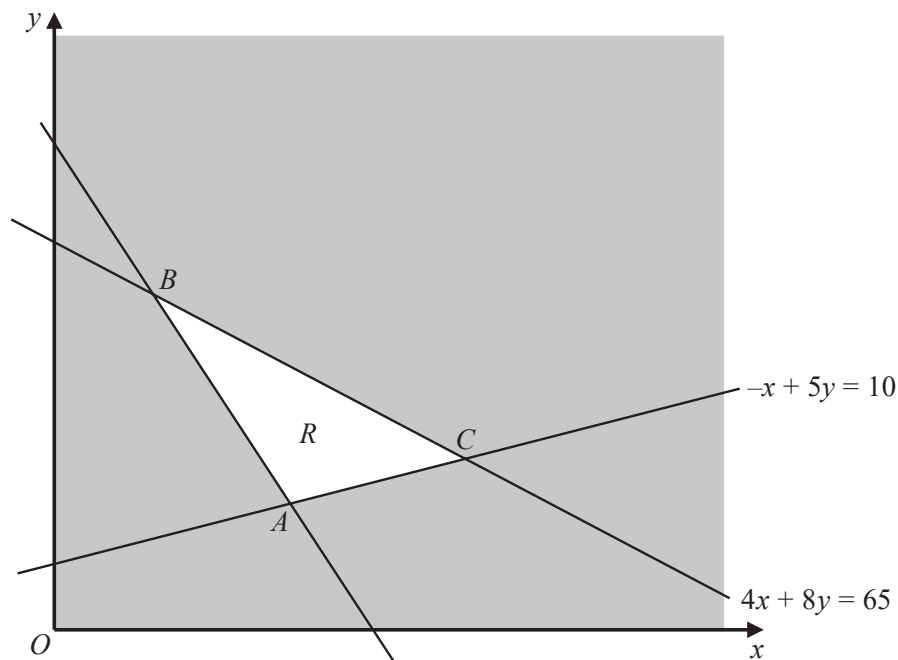


Figure 5

Figure 5 shows the constraints of a linear programming problem in x and y , where R is the feasible region. The equations of two of the lines and the three intersection points, A , B and C , are shown. The coordinates of C are $\left(\frac{35}{4}, \frac{15}{4}\right)$

The objective function is $P = x + 3y$

When the objective is to maximise $x + 3y$, the value of P is 24

When the objective is to minimise $x + 3y$, the value of P is 10

(a) (i) Find the coordinates of A and B .

(ii) Determine the inequalities that define R .

(8)

An additional constraint, $y \geq kx$, where k is a positive constant, is added to the linear programming problem.

(b) Determine the greatest value of k for which this additional constraint does not affect the feasible region.

(2)

(Total for Question 7 is 10 marks)

TOTAL FOR PAPER: 75 MARKS

END



7.

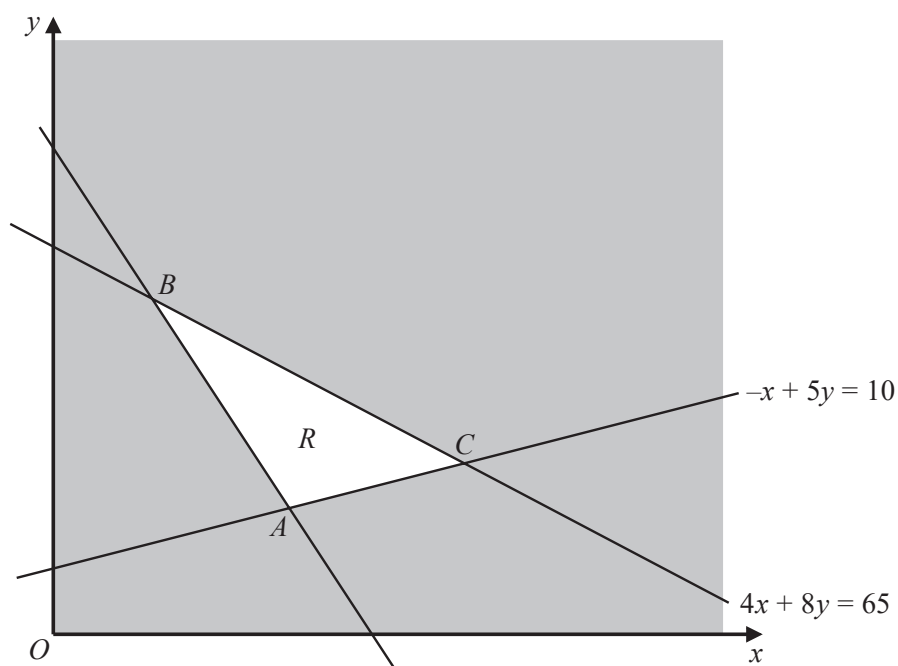


Figure 5



TOTAL FOR PAPER: 75 MARKS

END



8. A headteacher is deciding how to allocate prizes to the students who are leaving at the end of the school year.

There are three categories of prize: academic, sport, and leadership.

- Each academic prize costs £14, each sport prize costs £8, and each leadership prize costs £12. The total amount available to spend on all prizes is £976
- For every 5 academic prizes there must be at least 2 leadership prizes
- At least half the prizes must be academic
- 20% of the prizes must be for sport

The headteacher wishes to maximise the total number of prizes.

Let x , y and z represent the number of academic, sport and leadership prizes respectively.

- (a) Formulate this as a linear programming problem in x and y only, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(8)

Given that the headteacher awards 16 sport prizes,

- (b) calculate the corresponding number of leadership prizes that the headteacher awards. You must show your working.

(2)

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER: 75 MARKS

END



8.

Lined area for writing the answer to question 8.





Question 8 continued

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER: 75 MARKS

END



5. The head of a Mathematics department needs to order three types of paper. The three types of paper are plain, lined and graph.

All three types of paper are sold in reams. (A ream is 500 sheets of paper.)

Based on the last academic year the head of department formed the following constraints.

- At least half the paper must be lined
- No more than 15% of the paper must be graph paper
- The ratio of plain paper to graph paper must be 5 : 2

The cost of each ream of plain, lined and graph paper is £5, £12 and £15 respectively. The head of department has at most £834 to spend on paper.

The head of department wants to maximise the total number of reams of paper ordered.

Let x , y and z represent the number of reams of plain paper, lined paper and graph paper ordered respectively.

- (a) Formulate this information as a linear programming problem in x and y only, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(7)

The head of department decides to order exactly 42 reams of lined paper and still wishes to maximise the total number of reams of paper ordered.

- (b) Determine

- (i) the total number of reams of paper to be ordered,
- (ii) the number of reams of graph paper to be ordered.

(3)

(Total for Question 5 is 10 marks)



5.

Lined area for writing the answer to question 5.



(Total for Question 5 is 10 marks)



8.

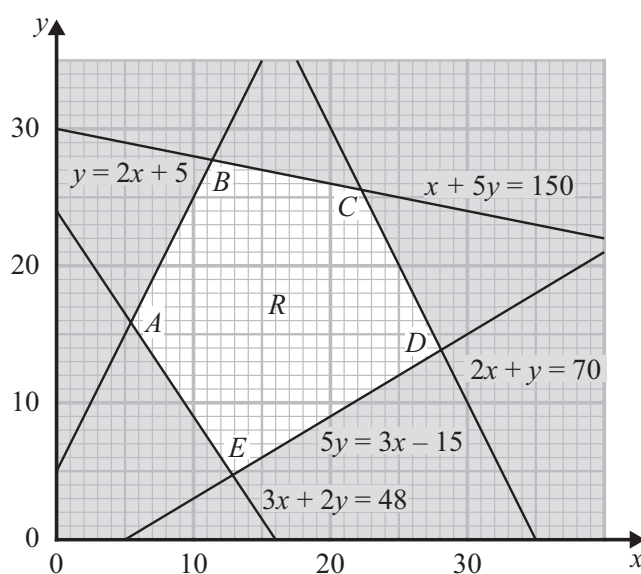


Figure 4

The feasible region, R , of a linear programming problem is shown in Figure 4.

The boundaries form part of the feasible region.

The regions excluded from the feasible region have been shaded.

Given that one of the constraints is

$$3x + 2y \geq 48$$

(a) state the remaining four constraints.

(2)

Question 8 continues on page 11



The five vertices of the feasible region are labelled A , B , C , D and E

An objective function is of the form

$$P = ax + by$$

where a and b are integers.

This objective function has

- a minimum value of $202\frac{5}{7}$ at vertex E
- a maximum value of $576\frac{2}{3}$ at vertex C

(b) Determine the value of a and the value of b , making your working clear.

(4)

A **different** objective function is of the form

$$Q = x + ky$$

where k is a positive constant.

This objective function has a minimum value at vertex A and a maximum value at vertex C

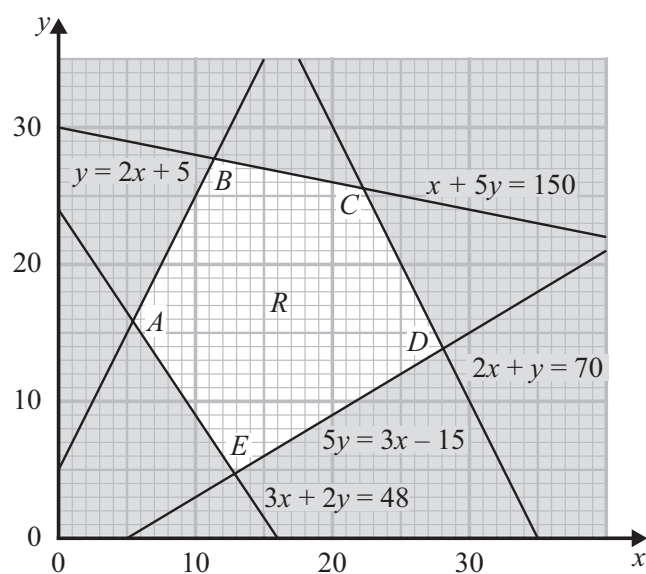
(c) Determine the range of values of k , making your working clear.

(4)

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

8.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS



8. A bakery makes three types of doughnut. These are ring, jam and custard. The bakery has the following constraints on the number of doughnuts it must make each day.

- The total number of doughnuts made must be at least 200
- They must make at least three times as many ring doughnuts as jam doughnuts
- At most 70% of the doughnuts the bakery makes must be ring doughnuts
- At least a fifth of the doughnuts the bakery makes must be jam doughnuts

It costs 8 pence to make each ring doughnut, 10 pence to make each jam doughnut and 14 pence to make each custard doughnut. The bakery wants to minimise the total daily costs of making the required doughnuts.

Let x represent the number of ring doughnuts, let y represent the number of jam doughnuts and let z represent the number of custard doughnuts the bakery makes each day.

- (a) Formulate this as a linear programming problem stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(6)

On a given day, instead of making **at least** 200 doughnuts, the bakery requires that **exactly** 200 doughnuts are made. Furthermore, the bakery decides to make the minimum number of jam doughnuts which satisfy all the remaining constraints.

Given that the bakery still wants to minimise the total cost of making the required doughnuts, use algebra to

- (b) (i) calculate the number of each type of doughnut the bakery will make on that day,
 (ii) calculate the corresponding total cost of making all the doughnuts.

(5)

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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Question 8 continued

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Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END



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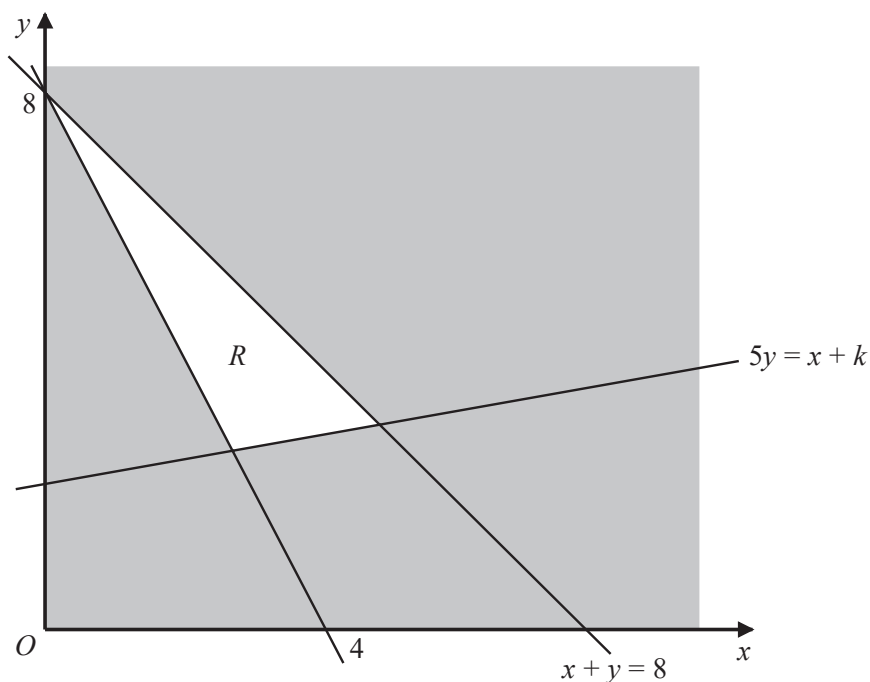


Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y , where R is the feasible region. The equations of two of the lines have been shown in Figure 3.

Given that k is a positive constant,

- (a) determine, in terms of k where necessary, the inequalities that define R . (4)

The objective is to maximise $P = 5x + ky$

Given that the value of P is 38 at the optimal vertex of R ,

- (b) determine the possible value(s) of k . You must show algebraic working and make your method clear. (7)

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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Question 7 continued

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Q7

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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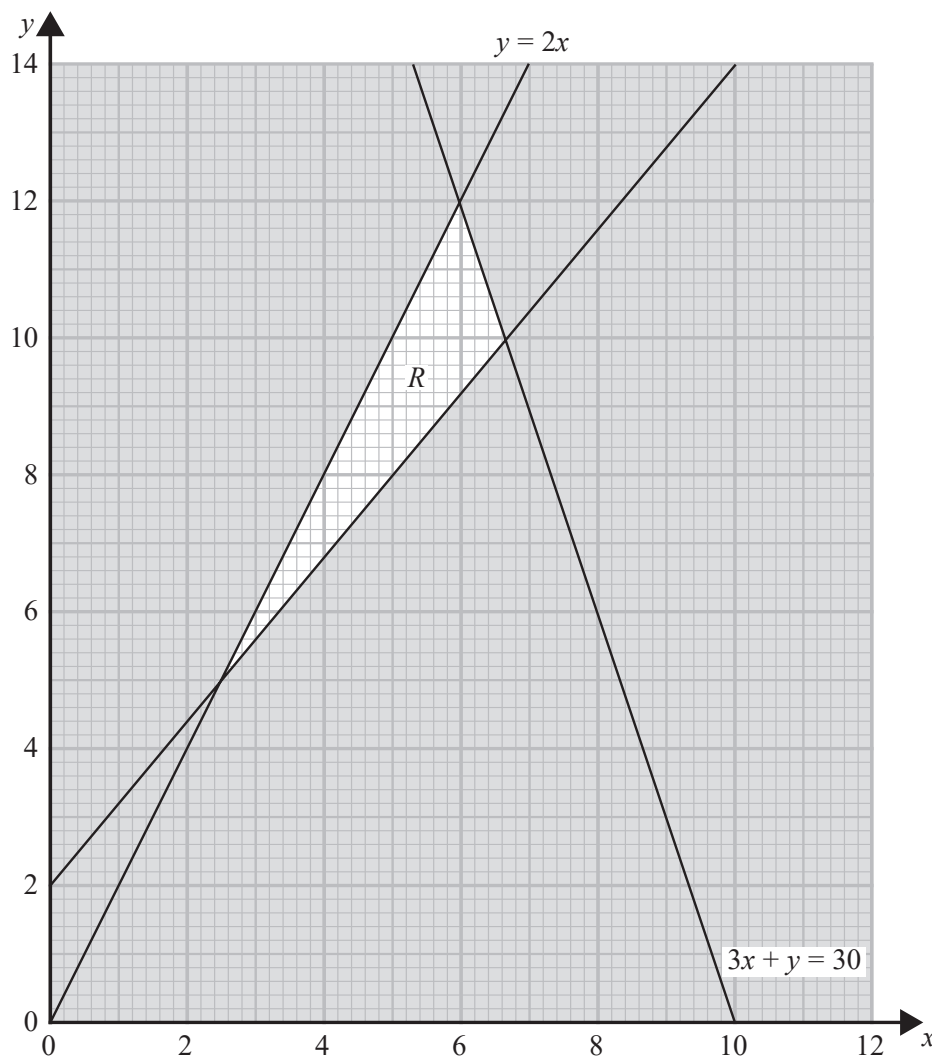


Figure 2

Figure 2 shows the constraints of a linear programming problem in x and y , where R is the feasible region. The equations of two of the lines are shown on the graph.

(a) Determine the inequalities that define the feasible region.

(3)

(b) Find the exact coordinates of the vertices of the feasible region.

(3)

The objective is to maximise P , where $P = 2x + ky$

(c) For the case $k = 3$, use the point testing method to find the optimal vertex of the feasible region and state the corresponding value of P .

(3)

(d) Determine the range of values for k for which the optimal vertex found in (c) is still optimal.

(2)

(Total for Question 4 is 11 marks)



4.

Lined area for writing the answer to question 4.



(Total for Question 4 is 11 marks)



5. A gardener wants to plant some rose bushes. She will use three different colours, red, yellow and white.
The gardener decides to use linear programming to determine how many of each colour she will plant.

Let x represent the number of red rose bushes, y represent the number of yellow rose bushes and z represent the number of white rose bushes.

The gardener wants to maximise the total number of rose bushes she will plant.

- (a) Write down the objective for this problem.

(1)

The gardener has a total of £1400 available to spend. Given that

- red rose bushes cost £12.50 each
- yellow rose bushes cost £10 each
- white rose bushes cost £15 each

- (b) show that one constraint is given by

$$5x + 4y + 6z \leq 560$$

(2)

The gardener decides that

- the number of red rose bushes must be at least 40% of the total number of rose bushes
- there must be at least three white rose bushes for every two yellow rose bushes

- (c) Use this information to formulate two further constraints, which should be fully simplified with integer coefficients.

(3)

The gardener decides to plant exactly 30 white rose bushes. This reduces two of the constraints to

$$5x + 4y \leq 380$$

$$y \leq 20$$

- (d) Show that the third constraint can be reduced to

$$3x - 2y \geq 60$$

(2)

- (e) Draw these three constraints using the axes in the answer book and hence determine the number of red rose bushes and the number of yellow rose bushes that the gardener will plant.

(4)

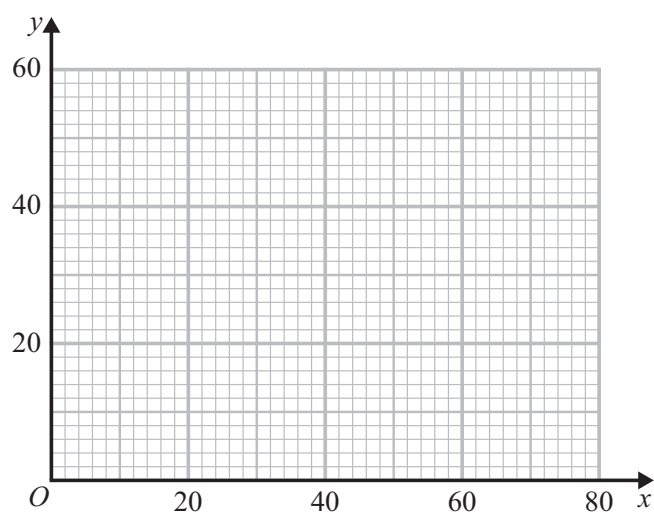
(Total for Question 5 is 12 marks)



5.

Lined area for writing the answer to question 5.



Question 5 continued

(Total for Question 5 is 12 marks)



6. A linear programming problem in x and y is described as follows.

Maximise $P = kx + y$, where k is a constant

subject to: $3y \geq x$

$$x + 2y \leq 130$$

$$4x + y \geq 100$$

$$4x + 3y \leq 300$$

- (a) Add lines and shading to Diagram 1 in the answer book to represent these constraints. Hence determine the feasible region and label it R . (4)

- (b) For the case when $k = 0.8$

- (i) use the objective line method to find the optimal vertex, V , of the feasible region. You must draw and label your objective line and label vertex V clearly. (2)

- (ii) calculate the coordinates of V and hence calculate the corresponding value of P at V . (3)

Given that for a different value of k , V is not the optimal vertex of R ,

- (c) determine the range of possible values for k . You must make your method and working clear. (4)

(Total 13 marks)

6.

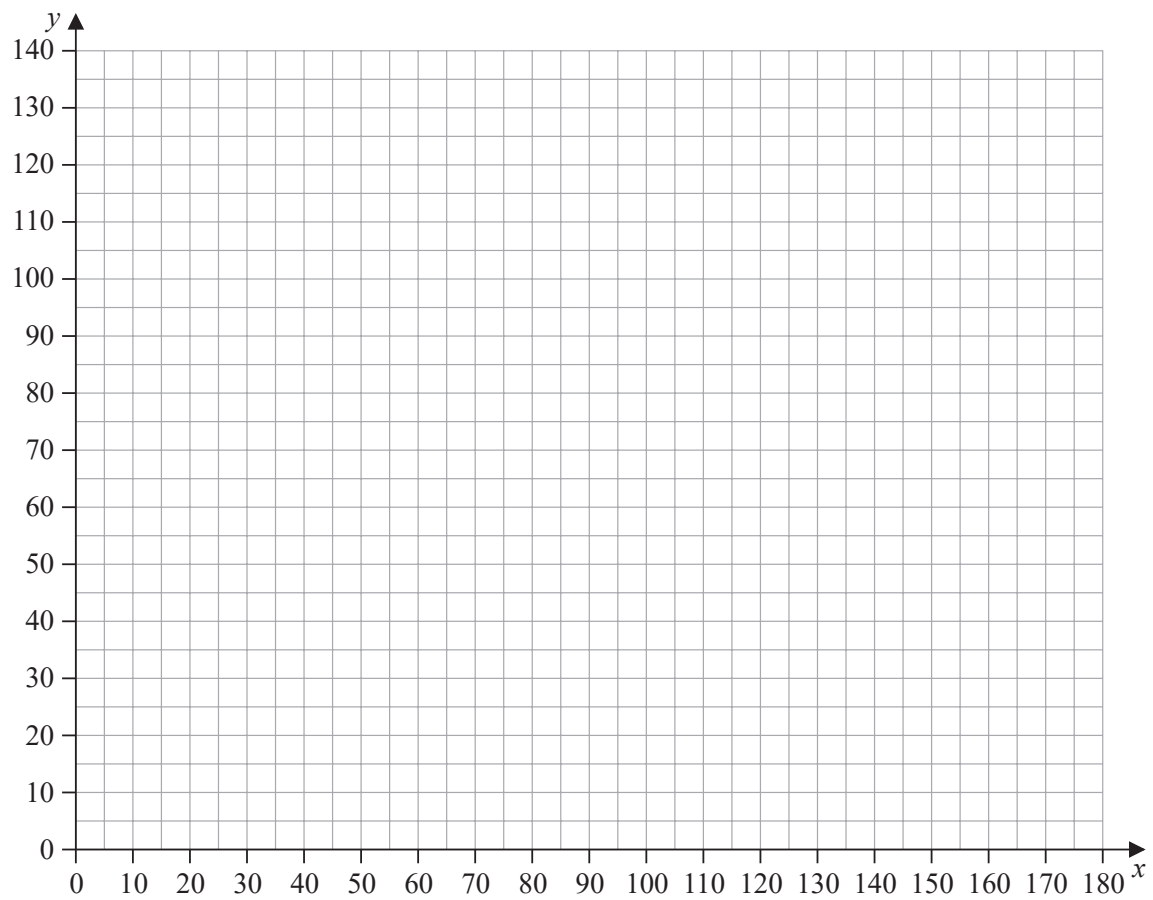


Diagram 1

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Question 6 continued

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Question 6 continued

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(Total 13 marks)

Q6



6. Martin is making three types of cake for a picnic. The three types of cake are carrot cake, apple cake and chocolate cake. Along with other ingredients,

- each carrot cake contains 275 grams of flour, 300 grams of sugar and 5 eggs
- each apple cake contains 200 grams of flour, 400 grams of sugar and 2 eggs
- each chocolate cake contains 100 grams of flour, 400 grams of sugar and 3 eggs

If Martin makes only one type of cake then he has enough time to prepare 15 carrot cakes or 20 apple cakes or 30 chocolate cakes.

Martin has 5.5 kilograms of flour and 70 eggs available and he has promised the picnic organisers that he will make at least 18 cakes in total.

Martin plans to make a selection of these cakes and wants to minimise the total amount of sugar that he uses.

Let x be the number of carrot cakes made, y the number of apple cakes made and z the number of chocolate cakes made.

- (a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(6)

A further constraint is that $y = 2z$

- (b) Explain what this constraint means in the context of the question.

(1)

The constraint $y = 2z$ reduces the problem to the following

Minimise $P = 300x + 600y$

subject to $11x + 10y \leq 220$

$$10x + 7y \leq 140$$

$$x + y \leq 15$$

$$2x + 3y \geq 36$$

$$x \geq 0, y \geq 0$$

- (c) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R .

(4)

- (d) Use the objective line method to find the optimal number of each type of cake that Martin should make, and the amount of sugar used.

(4)

- (e) Determine how much flour and how many eggs Martin will have left over after making the optimal number of cakes.

(2)

(Total for Question 6 is 17 marks)

TOTAL FOR PAPER: 75 MARKS

END



6.

Lined area for writing the answer to question 6.



Question 6 continued

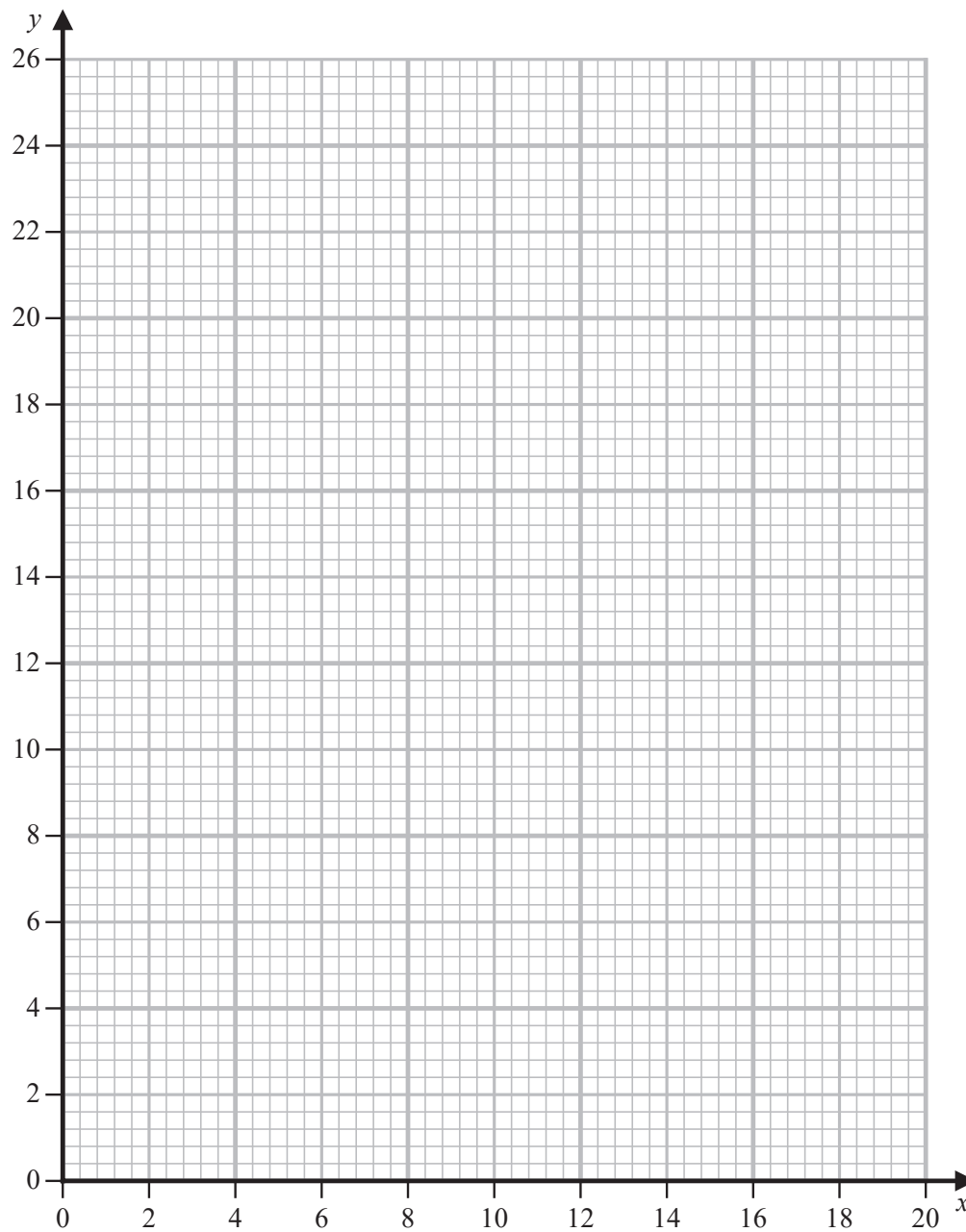


Diagram 1



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 17 marks)

TOTAL FOR PAPER: 75 MARKS

END



7. A farmer has 100 acres of land available that can be used for planting three crops: A, B and C.

It takes 2 hours to plant each acre of crop A, 1.5 hours to plant each acre of crop B and 45 minutes to plant each acre of crop C. The farmer has 138 hours available for planting.

At least one quarter of the total crops planted must be crop A.

For every three acres of crop B planted, at most five acres of crop C will be planted.

The farmer expects a profit of £160 for each acre of crop A planted, £75 for each acre of crop B planted and £125 for each acre of crop C planted.

The farmer wishes to maximise the profit from planting these three crops.

Let x , y and z represent the number of acres of land used for planting crop A, crop B, and crop C respectively.

- (a) Formulate this information as a linear programming problem. State the objective, and list the constraints as simplified inequalities with integer coefficients.

(6)

The farmer decides that all 100 acres of available land will be used for planting the three crops.

- (b) Explain why the maximum total profit is achieved when $-7x + 10y$ is minimised.

(2)

The farmer's decision to use all 100 acres reduces the constraints of the problem to the following:

$$x \geq 25$$

$$3x + 8y \geq 300$$

$$x + y \leq 100$$

$$5x + 3y \leq 252$$

$$y \geq 0$$

- (c) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R .

(4)

- (d) (i) Determine the exact coordinates of each of the vertices of R .

- (ii) Apply the vertex method to determine how the 100 acres should be used for planting the three crops.

- (iii) Hence find the corresponding maximum expected profit.

(5)

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS



7.

Lined area for writing the answer to question 7.



Question 7 continued

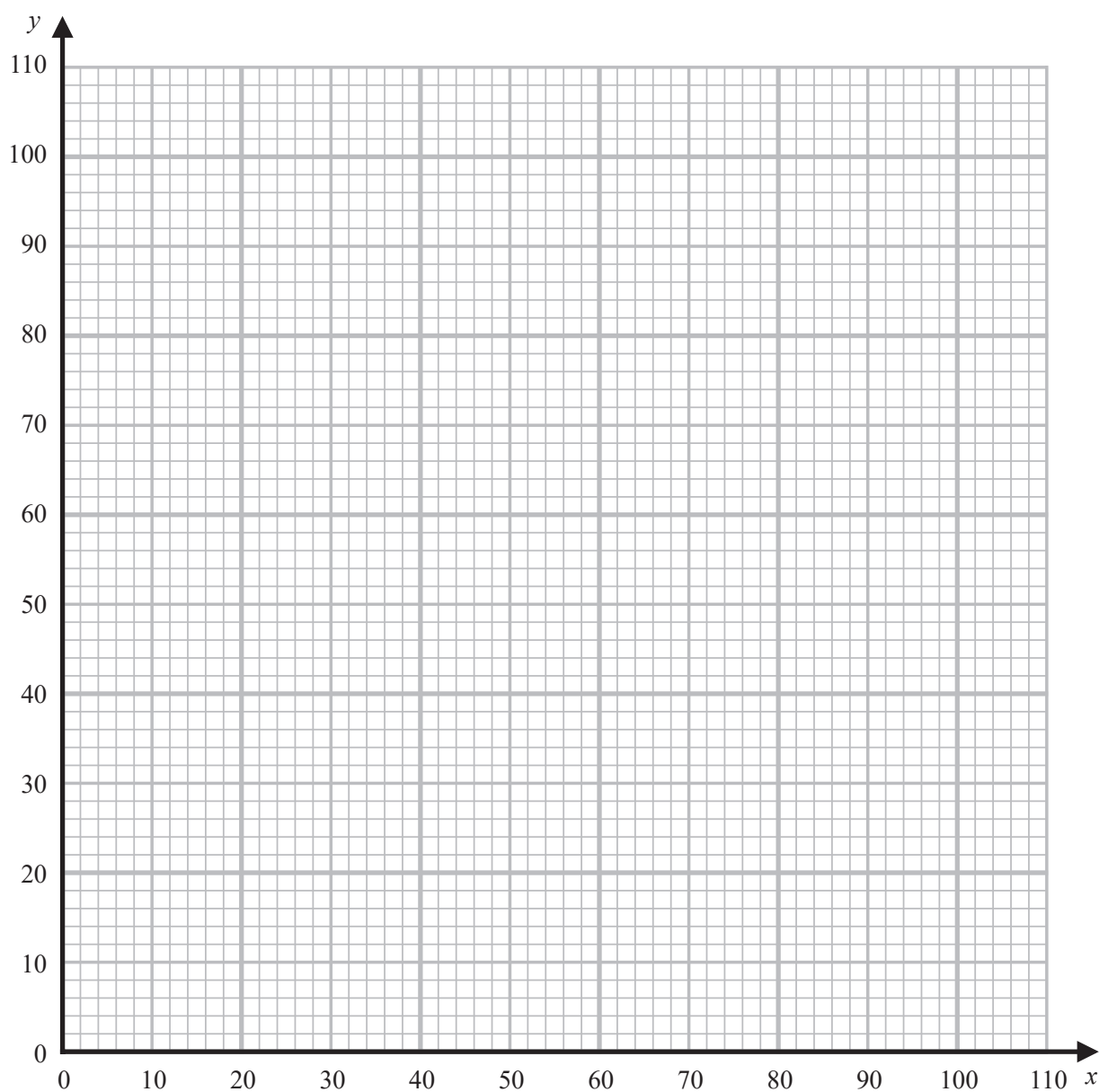


Diagram 1



Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER: 75 MARKS



5. A clothing shop sells a particular brand of shirt, which comes in three different sizes, small, medium and large.

Each month the manager of the shop orders x small shirts, y medium shirts and z large shirts.

The manager forms constraints on the number of each size of shirts he will have to order.

One constraint is that for every 3 medium shirts he will order at least 5 large shirts.

- (a) Write down an inequality, with integer coefficients, to model this constraint. (2)

Two further constraints are

$$x + y + z \geq 250 \text{ and } x \leq 0.2(x + y + z)$$

- (b) Use these two constraints to write down statements, in context, that describe the number of different sizes of shirt the manager will order. (3)

The cost of each small shirt is £6, the cost of each medium shirt is £10 and the cost of each large shirt is £15

The manager must minimise the total cost of all the shirts he will order.

- (c) Write down the objective function. (1)

Initially, the manager decides to order exactly 150 large shirts.

- (d) (i) Rewrite the constraints, as simplified inequalities with integer coefficients, in terms of x and y only.
(ii) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region R . (6)

- (e) Use the objective line method to find the optimal vertex, V , of the feasible region. You must make your objective line clear and label V . (2)

- (f) Write down the number of each size of shirt the manager should order. Calculate the total cost of this order. (2)

Later, the manager decides to order exactly 50 small shirts and exactly 75 medium shirts instead of 150 large shirts.

- (g) Find the minimum number of large shirts the manager should order and show that this leads to a lower cost than the cost found in (f). (2)

(Total 18 marks)

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Question 5 continued

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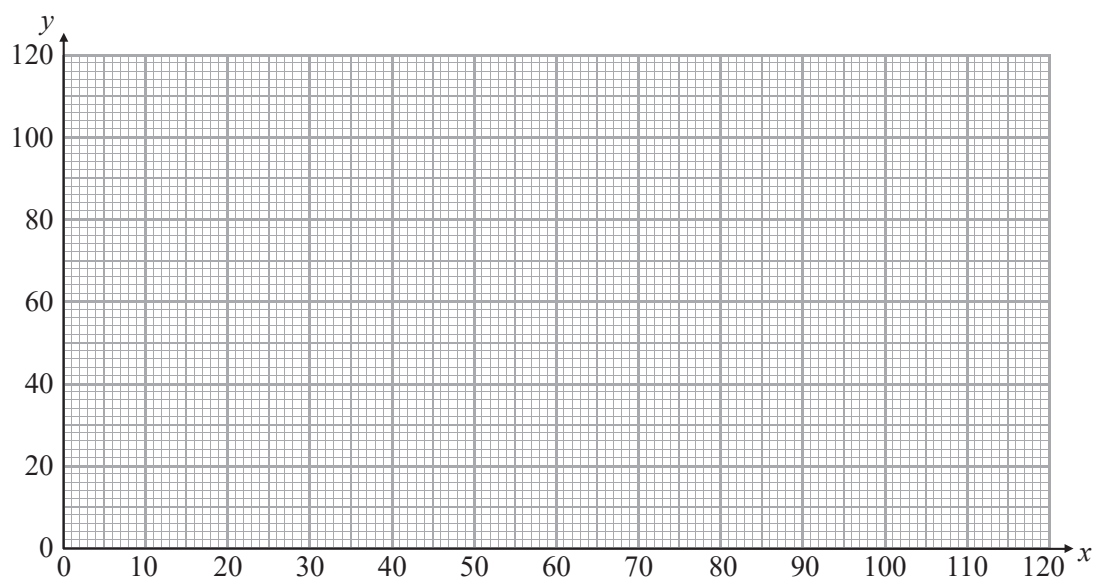


Diagram 1

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Question 5 continued

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Q5

(Total 18 marks)



7. A school is planning to run several training days next year for its new teachers, middle leaders and senior leaders.

Next year, the school will need to run

- at least 20 training days in total,
- at most twice as many training days for new teachers when compared to the total number of training days required for both middle and senior leaders,
- at most 25% of the training days for senior leaders.

The costs of running a training day for new teachers, middle leaders and senior leaders are £400, £550 and £750 respectively.

The school wants to minimise the total cost of running the training days.

Let x be the number of training days required for new teachers.

Let y be the number of training days required for middle leaders.

Let z be the number of training days required for senior leaders.

- (a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients. (6)

The school decides that the number of training days for middle leaders and the number of training days for senior leaders should be in the ratio 5 : 3

This reduces two of the constraints to $5x \leq 16y$ and $4y \leq 5x$.

- (b) (i) Express the constraint representing the requirement for a total of at least 20 training days as a simplified inequality with integer coefficients in terms of x and y only.

- (ii) Express the objective in the form $ax + by$ where a and b are integers. (3)

- (c) Represent the constraints in x and y only on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R . (4)

- (d) Use the objective line method to locate the optimal vertex, V , of the feasible region. You must make your objective line clear and label the optimal vertex, V . (3)

- (e) Hence, determine

- (i) the total cost of running the training days,
- (ii) the number of training days required for senior leaders. (2)

(Total 18 marks)

TOTAL FOR PAPER: 75 MARKS

END



7. A company makes three types of storage container, small, medium and large.

The company owner knows that each week she should make

- at least 40 containers in total
- at least twice as many large containers as medium containers
- at most 60% small containers

Each small container requires 1 hour to make, each medium container requires 1.5 hours to make, and each large container requires 2.5 hours to make. The company has a total of 75 hours per week available to make all the containers.

Each small container costs £9 to make, each medium container costs £12 to make and each large container costs £16 to make.

The company owner wants to minimise her total cost.

- Let x represent the number of small containers made
- Let y represent the number of medium containers made
- Let z represent the number of large containers made

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(7)

The company owner now decides to make exactly 45 containers.

(b) Explain why the minimum total cost is achieved when $7x + 4y$ is maximised.

(3)

The requirement to make exactly 45 containers reduces the constraints of the problem to the following:

$$x + 3y \leq 45$$

$$0 \leq x \leq 27$$

$$3x + 2y \geq 75$$

$$y \geq 0$$

(c) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R .

(4)

(d) Use the objective line method to find the optimal vertex, V , of the feasible region. You must make your objective line clear and label V .

(2)

(e) Write down the number of each type of container the company should make. Calculate the corresponding total cost.

(2)

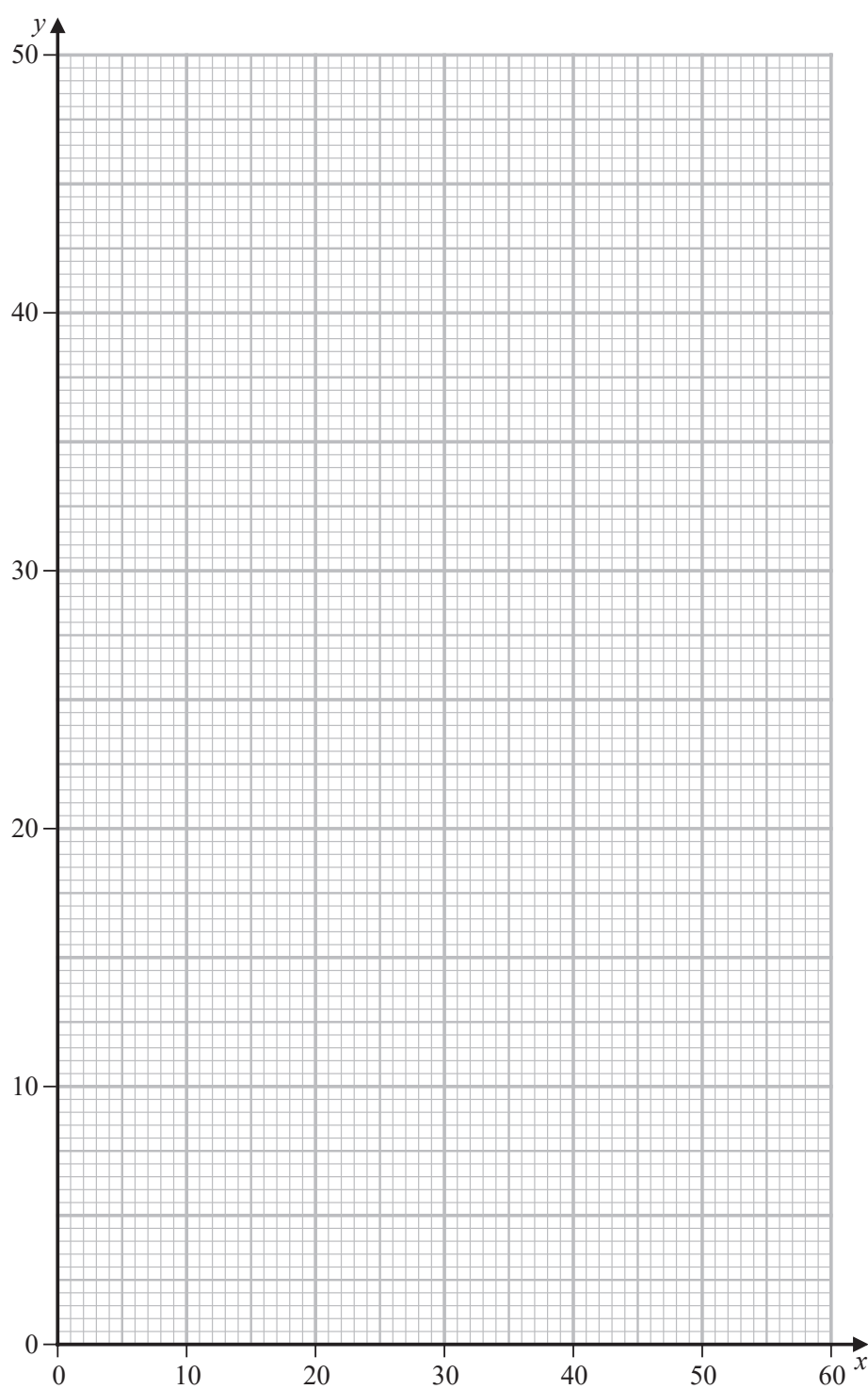
(Total 18 marks)

TOTAL FOR PAPER = 75 MARKS

END





Question 7 continued**Diagram 1**

MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
2.	Minimise $(C =) 2x + 3y$ $x + y \geq 85$ $y \geq 2x$ $y \leq \frac{4}{5}(x + y)$ $y \geq 2x$ and $y \leq 4x$	B1 B1 M1 M1 A1 (5) 5 marks
Notes for Question 2		
1B1: Expression correct $(2x + 3y)$ together with ‘minimise’ or ‘min’ (but not ‘minimum’) – if ‘simplified’ e.g. $x + 1.5y$ then must see $2x + 3y$ at some point 2B1: CAO – any equivalent form provided integer coefficients and only one term in x and one term in y e.g. $x \geq 85 - y$ 1M1: $y \square 2x$ where $\square$ is any inequality or equals. Accept $2y \geq x$ for this mark 2M1: $y \square \frac{4}{5}(x + y)$ where $\square$ is any inequality or equals – if no bracket then correct rhs must be implied by later working. $y \square 4x$ where $\square$ is any inequality or equals implies this mark. Use of % symbol only is M0 unless correctly replaced by a fraction or decimal later 1A1: Both $y \geq 2x$ and $y \leq 4x$ CAO – must be a single terms in x and y but allow any equivalent form provided integer coefficients e.g. $2x - y \leq 0$, $2y - 8x \leq 0$ etc.		

5.	Objective function is maximised at C(9, 22) and minimised at A(6, 8)	B1
	Let $P = \lambda(4x + 4.5y)$	M1
	$540 = \lambda(4(9) + 4.5(22)) \Rightarrow \lambda = \dots$	M1
	$P_{\min} = \lambda(4(6) + 4.5(8))$	dM1
	$P_{\min} = 240$	A1 (5)
		5 marks
	Notes for Question 5	
1B1	Recognises that the objective function is maximised at C and minimised at A (possibly implied by later working). Award if correct coordinates (9, 22) and (6, 8) used in their calculations. Sight of $9a + 22b = 540$ and $6a + 8b = k$ earns this mark.	
1M1	Setting up an objective function of the form $\lambda(4x + 4.5y)$ or $\lambda(4.5x + 4y)$ Allow consideration of $4x + 4.5y$ or any multiple Considers the gradient of the objective function $-\frac{8}{9}$ Considers ratio between a and b (e.g. $4b = 4.5a$)	
2M1	A correct approach to find the objective function. ($P =$) $16x + 18y$ earns this mark (if no incorrect working seen). This may be implied by consideration of e.g. 540/135 Candidates may adopt alternative algebraic approaches using $9a + 22b$ and forming simultaneous equations. Any algebraic approach leading to the correct answer with no incorrect working is acceptable. e.g. Setting up an equation in their λ using correct point C and their expression for P and solving for their λ (if correct $\lambda = 4$)	
3M1	Dependent on previous M mark – using point A and their objective function (of the form $\lambda(4x + 4.5y)$ where $\lambda \neq 1$) to get a value for $P_{\min}$	
1A1	CAO ($k = 240$)	

Question Number	Scheme	Marks
2.	$y \geq 3x$	B1
	$z - x \geq 50$	B1
	$y \leq 120$	B1
	Sub. $x + y + z = 180$	M1
	$2x + y \leq 130$	A1
	Maximise ($P =$) $x + y$	B1 (6)
		6 marks
Notes for Question 2		
1B1: cao ($y \geq 3x$) oe (two terms only with integer coefficients) 2B1: cao ($z - x \geq 50$) – may be implied by later working oe (three terms only with integer coefficients) 3B1: $y \leq 120$ oe 1M1: Eliminating z by substituting $x + y + z = 180$ into an inequality that involves z and x only 1A1: $2x + y \leq 130$ oe (three terms only with integer coefficients) 4B1: correct objective with ‘maximise’ or ‘max’ but not ‘maximum’ – either the expression $x + y$ or any other letter for P except x, y or z		

Question Number	Scheme	Marks
6.(a)	$4y \leq 7x + 8$ $4y \geq x + 8$ $3x + 4y \leq 24$	B1 B1 (2)
(b)	Min value of P is $8 \Rightarrow 2b = 8 \therefore b = 4$	B1
	Solve $4y = 7x + 8$ and $3x + 4y = 24$ simultaneously (to give $B\left(\frac{8}{5}, \frac{24}{5}\right)$)	M1
	$C(4,3) \Rightarrow P = 4a + 12, B\left(\frac{8}{5}, \frac{24}{5}\right) \Rightarrow P = \frac{8}{5}a + \frac{96}{5}$	M1
	$4a + 12 > \frac{8}{5}a + \frac{96}{5} \Rightarrow a > \dots$	M1
	$a > 3$	A1 (5)
		7 marks

Notes for Question 6**a1B1:** One correct inequality (allow strict inequality)**a2B1:** All three inequalities correct (allow any equivalent forms)**b1B1:** $b = 4$ (only)**b1M1:** Solve correct pair of simultaneous equations to find B – this mark can be implied by correct coordinates of B stated**b2M1:** Either linear expression in terms of a only (using their value of b) for either the correct C or their B (their B must be correct or a method for solving the correct simultaneous equations to find B must be seen)**b3M1:** Their linear expression in a only for C compared to their linear expression in a only for B (allow any inequality or equals) and attempting to solve for a – this mark is dependent on one correct expression in a **b1A1:** CAO ($a > 3$ only)**Alternative approach for (b)****b1B1:** $b = 4$ (only)**b1M1:** Finding the gradient of $3x + 4y = 24$ e.g. $y = -\frac{3}{4}x + 6 \Rightarrow m = -\frac{3}{4}$ the gradient must either be stated explicitly or used later**b2M1:** Gradient of the objective function stated as $-\frac{a}{b}$ (or used later) in terms of a only (so must have substituted their value of b)**b3M1:** Their gradient of $3x + 4y = 24$ compared to their gradient of the objective function (in terms of a only) – allow any inequality or equals and attempting to solve for a – this mark is dependent on one correct gradient (if correct then should be $-\frac{a}{4} < -\frac{3}{4}$)**b1A1:** CAO ($a > 3$ only)

Correct answer with no working please send to review



Question Number	Scheme	Marks
7.(a)		M1 A1 A1 A1ft
	Shortest path from A to H via DH: ABDH length: 70	A1
	Shortest path from A to H via EH: ABDEH length: $37 + 2x$	A1
	Shortest path from A to H via GH: ABDEGH length: $51 + x$	A1 (7)
(b)	A to H are the only two odd nodes in the network so repeat arcs in path ABDEH	M1
	$3x + 205 + 37 + 2x = 307$	M1
	$x = 13$	A1
	Time taken is 63 (minutes)	A1 (4)
		11 marks

Notes for Question 7

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at C the working values must be 22 21 20 in that order (22 20 21 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at any node except A, B, G or H (or once with at least two working values seen at H)

a1A1: All values in A, B, D and C correct and the working values in the correct order at D and C (including order of labelling)

a2A1: All values E and F correct and the working values in the correct order. Penalise order of labelling only once per question

a3A1ft: All values in G and H correct on the follow through and the working values in the correct order (the order at H must be correct but give bod). Penalise order of labelling only once per question. Ignore permanent label and final value at H only. Allow unsimplified expressions in x for the working values at H

a4A1: ABDH and 70

a5A1: ABDEH and $37 + 2x$

a6A1: ABDEGH and $51 + x$

If A0A0A0 for the final three marks in (a) then award A1A0A0 for all 3 routes stated correctly or all 3 correct values stated explicitly (so not just left in the working values at H)

4.(a)(i)	$z = 14 - 2x - y$ substituted into both $x + 2y + z \leq 15$ and $3x - 4y + 2z \leq 1$	M1
	$-x + y \leq 1$	A1
	$x + 6y \leq 27$	A1
(a)(ii)	The maximum possible value of P is 1	A1 (4)
(b)(i)	$-x + y = 1$ & $x + 6(1 + x) \leq 27$	M1
	$7x \leq 21$ & maximum possible value of x is 3	A1
(b)(ii)	$x = 3, y = 4$ and $z = 4$	A1 (3)
		7 marks

Notes for Question 4

Mark parts (a) and (b) together (so ignore labelling of parts in this question)

ai1M1: substituting $z = 14 - 2x - y$ into **both** correct inequality constraints – allow sign errors in re-arranging to make z the subject. This mark can also be awarded for **one** correct simplified inequality

ai1A1: $-x + y \leq 1$ – or equivalent (e.g. $x - y + 1 \leq 0$) but must be three terms only – ISW if candidates incorrectly re-arrange after a correct three term inequality seen

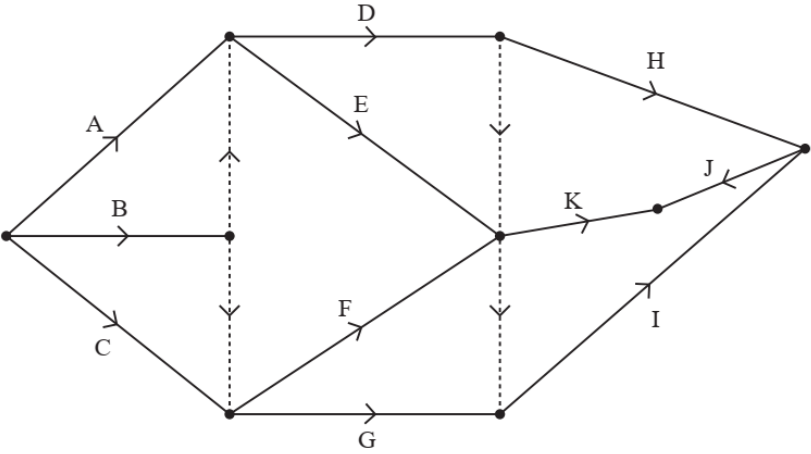
ai2A1: $x + 6y \leq 27$ – or equivalent but must be three terms only – ISW if candidates incorrectly re-arrange after a correct three term inequality seen

aii3A1: $P = 1$ – this mark can be awarded after correctly finding x and y – note that this mark is not dependent on the previous A mark. Just stating $P = 1$ is fine (so do not need to mention ‘maximum’)

bi1M1: substitute their $-x + y = 1$ into their $x + 6y \leq 27$ or their $x + 6y = 27$

bi1A1: correct value of x (if using equations then they do not need to justify that this is the maximum value)

bii2A1: $x = 3, y = 4$ and $z = 4$ – accept if seen as a coordinate

5. (a)	e.g. 	M1 A1 A1 A1 A1 (5)
(b)(i)	24 (hours)	B1
(b)(ii)	C, F, I and J	B1
(b)(iii)	Total float for G is 3 (hours) Total float for K is 1 (hour)	B1 B1 (4)
9 marks		

Notes for Question 5

Condone lack of, or incorrect, numbered events throughout. ‘Dealt with correctly’ means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. ‘D dealt with correctly’ requires the correct precedences for this activity, i.e. A and B labelled correctly and leading into the same node and D starting from that node but do not consider the end event for D. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity C is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first four marks only

a1M1: At least eight activities (labelled on arc), one start and at least two dummies placed

a1A1: Activities A, B, C **and** two of activities D, E, F or G dealt with correctly (so at least one dummy (+ correct arrow) required)

a2A1: Activities D, E, F and G dealt with correctly – so first two dummies (+ correct arrows) are required for this mark

a3A1: Activities H and I dealt with correctly (so must have the final two dummies + correct arrows)

a4A1: cso – activities J and K dealt with correctly. All arrows correctly placed for each activity with one finish and at most four dummies. Note that some candidates are drawing the graph non-planar which is fine

Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only.

Note that additional (but unnecessary) ‘correct’ dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

bi1B1: cao (24)

bii1B1: cao (C, F, I and J with no others)

biii1B1: cao (total float for G as 3)

Question Number	Scheme	Marks
5. (a)	Maximise (P) = $1.50x + 2.00y + 1.80z$	B1 B1
		(2)
(b)	$\frac{x}{400} + \frac{y}{240} + \frac{z}{300} \leq 1 \Rightarrow 3x + 5y + 4z \leq 1200$ * (result given in question)	M1 A1
		(2)
(c)	$x \geq \frac{40}{100} (x + y + z) \Rightarrow 3x \geq 2y + 2z$ $y \leq \frac{25}{100} (x + y + z) \Rightarrow 3y \leq x + z$ $3y \leq 2z$ $(x, y, z \geq 0)$	M1 A1 A1
		(3)
		7 marks
Notes for Question 5		
a1B1	Maximise stated (accept max but not maximum)	
a2B1	Objective function correct (we do not need to see $P =$) accept $1.5x + 2y + 1.8z$ or if they work in pence so $150x + 200y + 180z$ accept the use of correct fractions Do not ISW if they multiply by 10 and $P = 15x + 20y + 18z$ is seen – this is B0	
bM1	Attempts inequality for total quantity of ice cream: Sum of the correct 3 fractions ≤ 1	
bA1	CSO * (result given in question)	
cM1	Attempts at least two non-trivial constraints (any inequality symbol or equals) (must be using fractions or decimals but not just e.g. 40%) Each inequality must have the correct ingredients (e.g. x , $40/100$, and $x + y + z$) correct simplified inequalities can imply this. Accept $2y \leq 3z$ for this mark	
c1A1	Two non-trivial constraints correct – simplified with integer coefficients (so accept multiples)	
c2A1	All non-trivial constraints correct – simplified with integer coefficients (so accept multiples) (note we do not need to see $x, y, z \geq 0$)	



Question Number	Scheme	Marks
8. (a)	$y \leq 3x + 4$ $x + 2y \geq 24$ $2x + 5y \leq 100$ $4x + y \leq 64$	B1 B1 (2)
(b)	$4x + y = 64$ $x + 2y = 24$ $2x + 5y = 100 \Rightarrow \left(\frac{110}{9}, \frac{136}{9}\right)$ $y = 3x + 4 \Rightarrow \left(\frac{16}{7}, \frac{76}{7}\right)$	M1 A1 A1
	$P = ax + by$ $\frac{110}{9}a + \frac{136}{9}b = \frac{628}{9} \Rightarrow 55a + 68b = 314$ $\frac{16}{7}a + \frac{76}{7}b = \frac{260}{7} \Rightarrow 4a + 19b = 65$ $a = 2 \quad b = 3$ $(P =) 2x + 3y$	dM1 A1
		(5)
(c)	(3, 11)	B1
		(1)
		8 marks

Notes for Question 8		
a1B1	Any two inequalities correct – condone strict inequalities here (accept equivalent forms)	
a2B1	All four inequalities correct (accept equivalent forms)	
b1M1	Attempts to solve either pair of simultaneous equations to find min and max points (they may attempt to find all 4 vertices) Note they may use calculators to solve these equations, with no working shown. The correct two pairs of coordinates implies M1 A1 A1 (These values may be seen on the diagram) If the correct coordinates are not seen and no working is shown, this is M0	
b1A1	One pair correctly solved (accept decimal equivalents awrt (12.22, 15.11) and (2.29, 10.86))	
b2A1	Both pairs correctly solved (accept decimal equivalents)	
b2dM1 (cM1 on ePen)	Forms a pair of simultaneous equations using $P = ax + by$ and their minimum and maximum values and solves to obtain a and b (dependent on the previous M mark) Condone use of alternative letters instead of a and b (but not x and y)	
b3A1 (cA1 on ePen)	CAO – $(P =) 2x + 3y$ (must follow from correct working) Note we do not need $P =$, but if they write e.g. $2x + 3y = 0$ this is A0	
c1B1 (dB1 on ePen)	CAO (note the correct coordinates may be embedded in a calculation of the minimum profit e.g. $2(3) + 3(11) = 39$ (we will accept a value of 39 with no obvious wrong working as evidence of the correct point) Accept 3, 11 here	

Question Number	Scheme	Marks
7. (a)	e.g. Equation of line through A and D is $\frac{y - \frac{21}{2}}{\frac{27}{2} - \frac{21}{2}} = \frac{x - \frac{9}{4}}{\frac{27}{4} - \frac{9}{4}}$	M1
	$-2x + 3y = 27$	A1
	$2x + y \geq 15, -2x + 5y \geq 15, -2x + 3y \leq 27, 2x + y \leq 27$	B1 A1 (4)
(b)	A: $P = 2\left(\frac{9}{4}\right) + \frac{21}{2}k$, B: $P = 2(5) + 5k$, C: $P = 2(10) + 7k$, D: $P = 2\left(\frac{27}{4}\right) + \frac{27}{2}k$	B1
	First possibility is that min is A and max is C $2\left(\frac{9}{2} + \frac{21}{2}k\right) \geq 20 + 7k$	M1
	$k \geq \frac{11}{14}$	A1
	Second possibility is that min is B and max is D $2(10 + 5k) \geq \frac{27}{2} + \frac{27}{2}k$	M1
	$k \leq \frac{13}{7}$	A1 (5)
	Special Case Candidates may consider $k < 1$ and $k > 1$ separately and obtain two ranges for k $\frac{11}{14} \leq k < 1$ and $1 < k \leq \frac{13}{7}$ scores M1 A1 M1 A0 $\frac{11}{14} \leq k \leq 1$ and $1 \leq k \leq \frac{13}{7}$ scores M1 A1 M1 A1	
		9 marks
	Notes for Question 7	
a1M1	Correct method for finding the equation of the line through A and D (condone a slip in values for this mark)	
a1A1	Correct equation of the line through A and D (a correct 3 term equation, in any equivalent form, implies M1 A1) (may also be implied by any inequality with the correct 3 terms)	
a1B1	Any two correct inequalities (condone strict inequalities)	
a2A1	CAO (all four correct, accept any equivalent 3 term form or e.g. $2x + y - 15 \geq 0$)	
b1B1	Correct objective applied at any of A, B, C or D	
b1M1	Setting up a linear inequality using A and C – allow any inequality or equals and the 2 on the wrong side	
b1A1	CAO ($k \geq \frac{11}{14}$)	
b2M1	Setting up a linear inequality using B and D – allow any inequality or equals and the 2 on the wrong side	
b2A1	CAO ($k \leq \frac{13}{7}$) Note we do not need to see the inequalities combined to form a single range of values. If they do combine the two correct inequalities, they must give a correct range $\frac{11}{14} \leq k \leq \frac{13}{7}$ do not ISW, remove the final A mark	

7.(a)(i)	Let the point of intersection of $-x + 5y = 10$ and the unknown line be $A(a_1, a_2)$ Let the point of intersection of $4x + 8y = 65$ and the unknown line be $B(b_1, b_2)$	
	$a_1 + 3a_2 = 10$ or $b_1 + 3b_2 = 24$ $- a_1 + 5a_2 = 10$ $4b_1 + 8b_2 = 65$	M1 A1
	$A(\frac{5}{2}, \frac{5}{2})$ or $B(\frac{3}{4}, \frac{31}{4})$	A1
	$a_1 + 3a_2 = 10$ and $b_1 + 3b_2 = 24$ $- a_1 + 5a_2 = 10$ $4b_1 + 8b_2 = 65$	depM1
	$A(\frac{5}{2}, \frac{5}{2})$ and $B(\frac{3}{4}, \frac{31}{4})$	A1
(ii)	$y - \frac{5}{2} = \frac{\frac{31}{4} - \frac{5}{2}}{\frac{3}{5} - \frac{1}{2}}(x - \frac{5}{2})$ ($y = -3x + 10$)	ddM1
	$3x + y \dots 10$	A1
	$-x + 5y \dots 10, 4x + 8y \dots 65$	B1 (8)
(b)	$k = \frac{15/4}{35/4} = \frac{3}{7}$	M1 A1 (2)
		10 marks

Notes for Question 7

ai1M1: Form simultaneous equations to find one of the points of intersection of the unknown line with one of the given lines – allow sign slips only

ai1A1: One correct pair of simultaneous equations (allow any choice of letters for their coordinates)

ai2A1: One correct point (need not be stated as coordinates so $x = \dots, y = \dots$ is fine)

ai2dM1: Forming both pairs of simultaneous equations (dependent on previous M mark) – allow sign slips only

ai3A1: Both correct points (need not be stated explicitly as coordinates)

aii3ddM1: Find the correct equation of the third line for **their** A and B (dependent on both previous M marks). Allow unsimplified but must be the correct equation for the line passing through their two points. Condone any inequality sign instead of equals

aii4A1: cao (for the third line) – must be three terms but accept any equivalent form e.g. $6x + 2y - 20 \dots 0$

aii1B1: cao (for the other two given lines) – must be three terms only but accept any equivalent forms

b1M1: Attempt to find the gradient of the line through O and $(\frac{35}{4}, \frac{15}{4})$ (condone reciprocal) – ignore use of

inequalities or k for this mark e.g. seeing $\frac{15/4}{35/4}$ **or** $\frac{15}{35}$ **or** $\frac{35}{15}$ etc. scores M1

b1A1: cao – need not be simplified e.g. $k = \frac{15}{35}$ scores both marks. Allow $y \dots \frac{3}{7}x$, or just $\frac{3}{7}$ but not $k \dots \frac{3}{7}$ only. If more than one value of k implied then A0



Question Number	Scheme	Marks
8.(a)	(Maximise) $x + y + z$	B1
	Subject to:	
	$14x + 8y + 12z \leq 976$	B1
	$5z \geq 2x$	B1
	$\frac{1}{2}(x + y + z) \leq x (\Rightarrow -x + y + z \leq 0)$	M1
	$\frac{1}{5}(x + y + z) = y (\Rightarrow x - 4y + z = 0)$	B1
	Substituting $z = 4y - x$ into objective and constraints	M1
	Maximise $(P =) 5y$ Subject to: $x + 28y \leq 488$ $7x - 20y \leq 0$ $2x - 5y \geq 0$ $x - 4y \leq 0$ $x \geq 0 \quad y \geq 0$	A1 A1 (8)
(b)	Substituting $y = 16$ into constraints gives $x \leq 40, x \leq \frac{320}{7}, x \geq 40, x \leq 64$	M1
	$\Rightarrow x = 40$ and therefore the maximum number of leadership prizes is 24	A1 (2)
		10 marks

Notes for Question 8

a1B1: CAO $(x + y + z)$ – can be implied by seeing $5y$ as the objective – do not penalise lack of ‘maximise’ here. Allow equal to any letter but not equal to a numerical value

a2B1: CAO $(14x + 8y + 12z \leq 976$ or an unsimplified inequality equivalent to $x + 28y \leq 488$)

a3B1: CAO $(5z \geq 2x$ (or equivalent) or an unsimplified inequality equivalent to $7x - 20y \leq 0$)

a1M1: Correct method - must see $\frac{1}{2}(x + y + z) \bullet x$ where $\bullet$ is any inequality or $=$. The bracket must be present or implied by later working – this mark is implied by the stating of a correct/incorrect inequality that would come from $\frac{1}{2}(x + y + z) \bullet x$ (possibly with $z = 4y - x$ substituted)

a4B1: CAO $\frac{1}{5}(x + y + z) = y$ (allow any equivalent unsimplified/simplified form for this mark) – simply stating that e.g. $x + z = 4y$ scores this mark

a2M1: Eliminating z using the correct equation $z = 4y - x$ from all three non-trivial constraints and objective

a1A1: At least two simplified constraints correct + correct objective (in x/y only) – condone lack of ‘maximise’ for this mark - accept any equivalent forms for the constraints provided that coefficients are integers and only a single term in x and/or y

a2A1: CAO – must include ‘maximise’ or ‘max’ (but not ‘maximum’) and all 6 constraints – accept any equivalent forms for the constraints provided that coefficients are integers and only a single term in x and/or y

Condone using a for x , s for y and l for z – if any other letter used then send to review

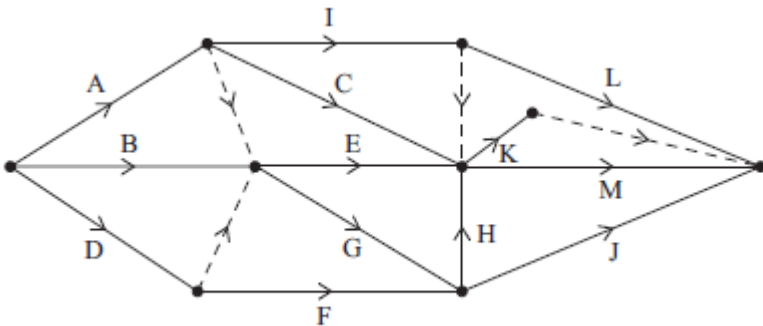


Must be using inequalities in x and y only in (b) – if inequalities are still in terms of z then z must be eliminated using the correct equation (e.g. $z = 64 - x$). Attempting part (b) with equations (rather than inequalities) scores no marks (even if the correct answer of 24 is seen).

b1M1: Substituting $y = 16$ into at least two correct constraints from the following four: $x + 28y \leq 488$, $7x - 20y \leq 0$, $2x - 5y \geq 0$ and $x - 4y \leq 0$

b1A1: Correctly obtaining $x \leq 40$ from $x + 28y \leq 488$ **and** $x \leq \frac{320}{7}$ from $7x - 20y \leq 0$ **and** $x \geq 40$ from $2x - 5y \geq 0$ (condone lack of $x \leq 64$ from $x - 4y \leq 0$) then stating/implies that $x = 40$ and therefore $z = 24$ (or equivalent in context e.g. 24 leadership prizes)

Question Number	Scheme	Marks
5.(a)	$\frac{1}{2}(x + y + z) \leq y \quad (\Rightarrow x - y + z \leq 0)$	M1
	$\frac{3}{20}(x + y + z) \geq z \quad (\Rightarrow 3x + 3y - 17z \geq 0)$	M1
	$2x = 5z$	B1
	$5x + 12y + 15z \leq 834$	B1
	Eliminating z from the objective $x + y + z$ and at least one correct constraint or states the objective and eliminates z from at least two correct constraints	M1
	Maximise ($P =$) $1.4x + y$ subject to $11x + 12y \leq 834$ $7x - 5y \leq 0$ $19x - 15y \leq 0$ $(x \geq 0, y \geq 0)$	A1 A1 (7)
(b)(i)	Substitute $y = 42$ into LP gives $x \leq 30, x \leq 30, x \leq \frac{630}{19}$ which implies that $x = 30$	M1
	Total number of reams ordered is $1.4(30) + 42 = 84$	A1
(b)(ii)	12 reams of graph paper ordered	A1 (3)
		10 marks
Notes for Question 5		
a1M1	$\frac{1}{2}(x + y + z) \bullet y$ where $\bullet$ is any inequality or equals sign (accept any equivalent form)	
a2M1	$\frac{3}{20}(x + y + z) \bullet z$ where $\bullet$ is any inequality or equals sign (accept any equivalent form)	
a1B1	Correct relationship between x and z (accept any equivalent form)	
a2B1	CAO ($5x + 12y + 15z \leq 834$)	
a3M1	Either eliminating z (using either $2x = 5z$ or $2z = 5x$ oe) from the correct objective and at least one correct constraint or states objective in the form ($P =$) $x + y + z$ and eliminates z from at least two correct constraints	
a1A1	Either two correct inequalities in x and y only or one correct inequality and the correct objective (but do not need 'maximise' or 'max') in x and y only – note the correct inequality must have integer coefficients (accept equivalent forms for the objective e.g. $5P = 7x + 5y$ (must include $5P =$) if $P = 7x + 5y$ stated do not ISW)	
a2A1	Correct LP formulation in x and y including 'maximise' or 'max' with objective (condone if previously stated) – note that all constraints must have integer coefficients (condone lack of $x \geq 0, y \geq 0$). Do not ISW if any incorrect simplifications seen.	
bi1M1	Substituting $y = 42$ into at least two of their constraints (must only be in terms of x and y) leading to a value of x (if they over simplify and assume that 42 is exactly $\frac{1}{2}$ of the total number of reams, they must still substitute into at least 2 constraints)	
bi1A1	CAO (84) – must come from correct working	
bii1A1	CAO (12) – must come from correct working	

Question Number	Scheme	Marks
6.(a)	e.g. 	M1 A1 (A B D I C F) A1 (E G H J) A1 (K L M) A1 (5)
(b)	Activities B, G, H and M are critical	B1 (1)
(c)	At time 9.5, activities G, C, E and I must all be happening therefore a minimum of 4 workers is required	M1 A1 (2)
(d)	$2 \leq x \leq 6$	B2, 1, 0 (2)
	Notes for Question 6	10 marks
	Condone lack of, or incorrect, numbered events throughout. ‘Dealt with correctly’ means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. ‘J dealt with correctly’ requires the correct precedences for this activity, i.e. F and G labelled correctly and leading into the same node and J starting from that node but do not consider the end event for J. Activity on node is M0 If an arc is not labelled, for example, if the arc for activity F is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be). Ignore missing arrows on the activities for the first four marks only	
a1M1	Nine activities (labelled on arc), one start and at least two dummies placed	
a1A1	Activities A, B, D, 1 st two dummies (including correct arrows), I, C and F dealt with correctly	
a2A1	Activities E, G, H and J dealt with correctly	
a3A1	Activities K, L and M dealt with correctly (so dummy at the end of I required + arrow)	
a4A1	CSO – all arrows correctly placed for each activity with one finish and exactly four dummies (so uniqueness dummy for K/M required)	
	Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only. Note that additional (but unnecessary) ‘correct’ dummies that still maintain precedence for the network should only be penalised with the final A mark if earned. Note that this answer is not unique	
b1B1	CAO (B, G, H and M and no others)	
c1M1	Either a statement with the correct number of workers (4) and stating the correct activities (G, C, E and I) with any numerical time stated or the correct number of workers (4) and a time in the interval $9 \leq t \leq 10$ – mark the numerical value only not their use of the words ‘day/time’ (or equivalent)	
c1A1	A completely correct statement with details of both time and activities. Candidates must give a time within the correct interval of $9 < t < 10$, e.g. 9.5 (or ‘on/during hour 10’) and state the correct activities (G, C, E and I). Please note the strict inequalities for the time interval (e.g. implying a time of 10 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 9 – 10 or ‘between 9 and 10’ is A0). A completely correct statement with an additional incorrect statement scores A0 (so do not ignore subsequent working)	
d1B1	For either $x \geq 2$ or $x \leq 6$ (must be x not L)	
d2B1	CAO ($2 \leq x \leq 6$) – SC in (d) B1B0 for $2 < x < 6$	

Question	Scheme	Marks
8. (a)	$y \leq 2x + 5$ $x + 5y \leq 150$ $2x + y \leq 70$ $5y \geq 3x - 15$	B1 B1
		(2)
(b)	$3x + 2y = 48$ $x + 5y = 150$ $5y = 3x - 15 \Rightarrow \left(\frac{90}{7}, \frac{33}{7}\right)$ $2x + y = 70 \Rightarrow \left(\frac{200}{9}, \frac{230}{9}\right)$ $\frac{90}{7}a + \frac{33}{7}b = \frac{1419}{7} \Rightarrow 30a + 11b = 473$ $\frac{200}{9}a + \frac{230}{9}b = \frac{1730}{3} \Rightarrow 20a + 23b = 519$ $a = 11 \quad b = 13 \quad \text{or} \quad P = 11x + 13y$	M1 A1 M1 A1
		(4)
(c)	Considers the gradient of $Q = 3x + 2y$ and attempts to find a limiting value for k Considers the gradient of $Q = 2x + y$ and attempts to find a limiting value for k e.g. $-\frac{1}{k} < -\frac{3}{2} \quad \text{and} \quad -\frac{1}{k} > -2$ $\Rightarrow k < \frac{2}{3} \quad \text{or} \quad k > \frac{1}{2}$ $\frac{1}{2} < k < \frac{2}{3}$	M1 M1 A1 A1

	Alternative 1 Compares value of objective function at A and E Compares value of objective function at C and D $\frac{38}{7} + \frac{111}{7}k < \frac{90}{7} + \frac{33}{7}k \Rightarrow k < \frac{2}{3}$ $\frac{200}{9} + \frac{230}{9}k > \frac{365}{13} + \frac{180}{13}k \Rightarrow k > \frac{1}{2} \text{ giving } \frac{1}{2} < k < \frac{2}{3}$	M1 M1 A1 A1
	Alternative 2 Rearranges $3x + 2y = 48$ to compare with $x + ky$ Rearranges $2x + y = 70$ to compare with $x + ky$ $x + \frac{2}{3}y = 16 \Rightarrow k < \frac{2}{3}$ $x + \frac{1}{2}y = 35 \Rightarrow k > \frac{1}{2} \quad \frac{1}{2} < k < \frac{2}{3}$	M1 M1 A1 A1
		(4)
	For reference the coordinates of all points of intersection A $\left(\frac{38}{7}, \frac{111}{7}\right)$ $3x + 2y = 48$ and $y = 2x + 5$ B $\left(\frac{125}{11}, \frac{305}{11}\right)$ $y = 2x + 5$ and $x + 5y = 150$ C $\left(\frac{200}{9}, \frac{230}{9}\right)$ $x + 5y = 150$ and $2x + y = 70$ D $\left(\frac{365}{13}, \frac{180}{13}\right)$ $2x + y = 70$ and $5y = 3x - 15$ E $\left(\frac{90}{7}, \frac{33}{7}\right)$ $5y = 3x - 15$ and $3x + 2y = 48$	
(10 marks)		

Notes	
a1B1 a2B1	Any two inequalities correct (accept equivalent rearrangements) CAO – all four correct (ignore any mention of $3x + 2y \geq 48$ (SC B1 B0 for all four strict inequalities)
b1M1	Attempts to solve both correct pairs of simultaneous equations (candidates may use their calculator to solve these, so the correct values for both points implies both marks) Ignore any additional work to find other points of intersection
b1A1	CAO – both pairs of coordinates (accept awrt (12.9, 4.7) and (22.2, 25.6)) Ignore additional points of intersection
b2M1	Forms a pair of simultaneous equations using their minimum and maximum points and solves to obtain a and b (may be unsimplified). If they have calculated additional points of intersection, then they must use their answer for the correct two points here
b2A1	CAO – must be exact values (either stating a and b or as function $P =$)
c1M1	Considers $3x + 2y$ and attempts to find a value for k by comparing gradients or compares the value of the objective function at A and E (accept the use of m for $-1/k$ to compare gradients even if not clearly defined)
c2M1	Considers $2x + y$ and attempts to find a value for k by comparing gradients or compares the value of the objective function at C and D
c1A1	Either $k < \frac{2}{3}$ or $k > \frac{1}{2}$ (allow use of $\leq$ and $\geq$) (If gradients are compared please check inequalities carefully to ensure that any working shown is fully correct) (this mark can be awarded following either of the M marks so M0 M1 A1 A0 or M1 M0 A1 A0 are possible)
c2A1	Both values correct and correct interval stated (allow use of $\leq$) (both M marks must be awarded and must not come from incorrect working)

Question Number	Scheme	Marks
8.(a)	Minimise $P = 8x + 10y + 14z$	B1
	Subject to $x + y + z \geq 200$	B1
	$3y \leq x$	B1
	$\frac{7}{10}(x + y + z) \geq x$ or $\frac{1}{5}(x + y + z) \leq y$	M1
	$x + z \leq 4y$ $3x \leq 7y + 7z$ $(x, y, z \geq 0)$	A1 A1 (6)
(b)(i)	$z = 200 - x - y$ substituted into constraints gives	M1
	$x \leq 140, y \geq 40$	A1
	Using their least value of y and greatest value of x to find z	dM1
	140 ring doughnuts, 40 jam doughnuts and 20 custard doughnuts	A1
(b)(ii)	£18 or 1800	A1 (5)
		11 marks

Notes for Question 8

a1B1: Expression correct together with ‘minimise’ or ‘min’ but not ‘minimum’ – isw if coefficients are subsequently simplified but $8x + 10y + 14z$ must be seen at some point for this mark to be awarded. The ‘min’ must appear beside (or near to) the correct expression

a2B1: CAO ($x + y + z \geq 200$) oe

a3B1: CAO ($3y \leq x$) oe

a1M1: $\frac{7}{10}(x + y + z) \square x$ oe or $\frac{1}{5}(x + y + z) \square y$ oe where $\square$ is any inequality or equals – allow 0.7 and 0.2 but not 70% or 20% unless recovered to a fraction or decimal later

a1A1: Either $x + z \leq 4y$ or $3x \leq 7y + 7z$ or both correct but not simplified or with integer coefficients (so

$\frac{7}{10}(x + y + z) \geq x$ and $\frac{1}{5}(x + y + z) \leq y$ can score this mark)

a2A1: Both correct – must be simplified (e.g. only one term in each variable) and integer coefficients but allow positive integer multiplies (e.g. $2x + 2z - 8y \leq 0$)

bi1M1: Using $x + y + z = 200$ to obtain either an inequality (or value) for either x or y

bi1A1: Any one of $x \leq 140$ or $y \geq 40$ correct (this mark can be implied for either $x = 140$ or $y = 40$ seen provided not from incorrect working)

bi2dM1: Using their least value of y and greatest value of x to find z (so not from the inequality $3y \leq x$) - the total of x, y and z must be 200 (and dependent on first M mark)

bi2A1: All three types of doughnuts correct (in context) - so not just in terms of x, y and z

bi1A1: CAO for cost – if 1800 given then this is fine (without pence) but if 18 then must be £

SC in (b): If correct answers from either no working or from explicitly solving three correct equations e.g. $x - 4y + z = 0$

$3x - 7y - 7z = 0$ with no algebraic working then award M0A0 M1 (for correct values of x, y and z) A1 (for $x + y + z = 200$

answers in context) and A1 for 1800. If solving these three equations with algebraic working then full marks can be awarded. If no marks awarded according to the notes above and solving any other (e.g. incorrect) sets of equations then no marks in (b).

Notes for Question 6

a1B1: Any two of the five arcs (G, H, I or the two dummies) drawn correctly (from correct vertex to correct vertex) – activities labelled with the correct letter (but condone no or wrong arrows) and the dummies must be shown as dashed lines (or labelled as ‘dummy’) with no weight (but condone no or wrong arrows)

a2B1: Four of the five arcs (G, H, I and the two dummies) drawn correctly – activities must be labelled with the correct letter (but condone no or wrong arrow) and the dummies must be shown as dashed lines (or labelled ‘dummy’) with no weight (but condone no or wrong arrows)

a3B1: CSO - all three activities (G, H and I) and the two dummies drawn correctly with no extras. Activities must be labelled with the correct letter and weights. The activities **and** dummies (as dashed lines with zero weight) must have the correct arrows (**do check carefully that all arrows are present**)

In (a) condone for full marks activities which are shown as dashed lines provided they are labelled with the correct letter. Condone lack of (or incorrect) weights on the activity arcs for the first two marks only.

b1M1: All top boxes complete (condone lack of 0 for the M mark only), values generally increasing in the direction of the arrows (‘left to right’), condone one ‘rogue’ value (if values do not increase in the direction of the arrows then if one value is ignored and the remaining values do increase in the direction of the arrows then this is considered to be a single rogue value). Note that all values in the top boxes could be incorrect but it can still score the M mark if the values are **increasing** in the way stated above – **this mark is dependent on the first mark having being awarded in (a)**

b1A1: CAO – all values correct in the top boxes

b2M1: All bottom boxes complete (condone lack of 28 and/or 0 for the M mark only), values generally decreasing in the opposite direction of the arrows (‘right to left’), condone one ‘rogue’ – **this mark is dependent on the first mark having being awarded in (a)**

b2A1: CAO – all values correct in the bottom boxes

For full marks in (b) all three activities (G, H and I) and the two dummies must have been added correctly in (a) – condone lack of arrows only. If all values in the bottom and top boxes are correct but any arc or dummy is missing or incorrect then award M1A1M1A0 – if all values are not correct (and some arcs are missing) then mark to the scheme above

c1B1: CAO (A, E, K, M, P only)

d1M1: Attempt to find lower bound: (a value in the interval [87 – 107] / their finish time) **or** showing the summing of **all** 15 activities divided by their finish time **or** (as a minimum) an awrt 3.5

d1A1: CSO – either a **correct** calculation seen **or** awrt 3.5 **then** 4 (with no incorrect working seen). An answer of 4 with no working scores M0A0

e1M1: Not a cascade (Gantt) chart. 5 ‘workers’ used at most and at least 11 activities placed

e1A1: 4 workers. All 15 activities present (just once). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA

e2A1: 4 workers. All 15 activities present (just once). No errors

Activity	Duration	Time	IPA
A	4	0 – 4	-
B	7	0 – 9	-
C	6	0 – 9	-
D	10	4 – 19	A
E	5	4 – 9	A
F	7	6 – 16	C
G	6	9 – 16	B, C, E
H	6	9 – 19	B, C, E

Activity	Duration	Time	IPA
I	7	9 – 28	B, C, E
J	9	15 – 28	D, H
K	8	9 – 17	B, C, E
L	4	17 – 28	F, G, K
M	6	17 – 23	F, G, K
N	7	15 – 23	F, G
P	5	23 – 28	M, N



Question Number	Scheme	Marks
7. (a)	$x + y \leq 8$ (oe) $5y \geq x + k$ (oe) $y = -\frac{8}{4}x + 8$ (or any inequality symbol replacing 'equals') $2x + y \geq 8$ (oe)	B1 B1 M1 A1 (4)
(b)	$P = 5x + ky$ If $(0, 8)$ is the optimal vertex then $k = \frac{19}{4}$ ($= 4.75$) Other possible optimal vertex is the point of intersection of $x + y = 8$ and $5y = x + k$ and attempting to solve simultaneously or attempt to express x and k in terms of y or attempt to express y and k in terms of x $\left(\frac{40-k}{6}, \frac{8+k}{6}\right)$ or stating $y = 8 - x$ or stating $x = 8 - y$ $k = 40 - 6x$ or stating $k = 6y - 8$ $5\left(\frac{40-k}{6}\right) + k\left(\frac{8+k}{6}\right) = 38$ or $5x + (40 - 6x)(8 - x) = 38$ or $5(8 - y) + (6y - 8)y = 38$ $k^2 + 3k - 28 = 0 \Rightarrow (k - 4)(k + 7) = 0$ or $6x^2 - 83x + 282 = 0 \Rightarrow (6x - 47)(x - 6) = 0$ then $k = \dots$ or $6y^2 - 13y + 2 = 0 \Rightarrow (6y - 1)(y - 2) = 0$ then $k = \dots$ $k = 4$ If $k = 4$ then $(6, 2) \rightarrow P = 38$ and $(0, 8) \rightarrow P = 32$ If $k = \frac{19}{4}$ then $(0, 8) \rightarrow P = 38$ and $\left(\frac{47}{8}, \frac{17}{8}\right) \rightarrow P = \frac{1263}{32} (= 39.468\dots)$ and so $k = 4$ only	B1 M1 A1 dM1 ddM1 A1 A1 (7)
		11 marks

Notes for Question 7

a1B1: CAO ($x + y \leq 8$) – any equivalent inequality (but not strict inequality)

a2B1: CAO ($5y \geq x + k$) – any equivalent inequality (but not strict inequality)

a1M1: **Correct** equation (or with any inequality symbol) of the line through (0, 8) and (4, 0)

a1A1: CAO (any equivalent form with three terms only – condone coefficients that are not integers) – any equivalent inequality (but not strict inequality)

b1B1: $k = \frac{19}{4}$ (oe exact value) seen

b1M1: Correct method for solving the correct pair of simultaneous equations ($x + y = 8$ and $5y = x + k$) to find both x and y in terms of k **or** attempt to express x and k in terms of y only **or** attempt to express y and k in terms of x only (this mark can be implied by a **correct** equation in either k , x or y only)

b1A1: CAO for coordinates of possible optimal vertex in terms of k – allow correct unsimplified, for example, $x = \frac{40-k}{6}$, $y = 8 - \frac{40-k}{6}$ **or** for $y = 8 - x$ **or** for $x = 8 - y$ (this mark can be implied by a **correct** equation in either k , x or y only)

b2dM1: Setting up an equation in either k , x or y only using the intersection point of $x + y = 8$ and $5y = x + k$, together with $5x + ky$ and 38 (dependent on previous M mark)

b3ddM1: Solving a three term quadratic in k , x or y and finding at least one positive value of k (dependent on both previous M marks) – if the method for solving their quadratic is not shown then this mark can be implied if their value of k satisfies their equation. If solving using the quadratic formula then they must be using the correct formula with their values, if using factorisation then when expanding their brackets must give two terms of their 3-term quadratic. Note that $5\left(\frac{40-k}{6}\right) + k\left(\frac{8+k}{6}\right) = 38$ (oe) $\Rightarrow k = 4$ can imply this and the next A mark

b2A1: $k = 4$ (ignore mention of other values of k , e.g. $k = -7$) but this value must have come from correct working

b3A1: Clear rejection of $k = 19/4$ (by showing that for this value of k , $P > 38$) **and evidence** of the second root or factor in k (e.g. stating at some point that $k = -7$ or $(k+7)(k-4) = 0$ seen) which is also rejected (possibly done implicitly so e.g. $(k+7)(k-4) = 0 \Rightarrow k = 4$ would be sufficient), **and** stating $k = 4$ only (so not giving more than one value for k) – dependent on all previous marks in (b)

The correct value of k with no working scores no marks – correct value with minimal working then please send to review



Question Number	Scheme	Marks
4.(a)	$y \leq 2x$ and $3x + y \leq 30$	B1
	Correct method for finding the third boundary of the feasible region	M1
	$5y \geq 6x + 10$	A1 (3)
(b)	(6, 12)	B1
	Solving simultaneous equations to find the other two vertices	M1
	$\left(\frac{5}{2}, 5\right)$ and $\left(\frac{20}{3}, 10\right)$	A1 (3)
(c)	$\left(\frac{5}{2}, 5\right) \rightarrow P = 20$ $\left(\frac{20}{3}, 10\right) \rightarrow P = \frac{130}{3}$ $(6, 12) \rightarrow P = 48$	M1 A1
	Optimal vertex is (6,12) with $P = 48$	A1 (3)
(d)	$12 + 12k \square 2\left(\frac{20}{3}\right) + 10k$ where $\square$ is any inequality or equals	M1
	$k \geq \frac{2}{3}$	A1 (2)
		11 marks

Notes for Question 4

a1B1: Both correct inequalities: $y \leq 2x$ and $3x + y \leq 30$ (or equivalent if they have been rearranged)

a1M1: Correct equation for the third boundary of the feasible region, e.g. $\frac{y-20}{14-20} = \frac{x-15}{10-15}$ would score

M1 (so does not need to be simplified). Condone any inequality sign used (provided that if the inequality sign was replaced with an 'equals' then the equation ($5y = 6x + 10$) would be correct). ISW once the correct equation has been seen (so condone the correct unsimplified equation if not simplified correctly)

a1A1: CAO for third inequality – must be three terms only but need not be further simplified (e.g. $12x - 10y + 20 \leq 0$, $y \geq 1.2x + 2$ etc. scores **A1**)

b1B1: (6, 12) or $x = 6$, $y = 12$

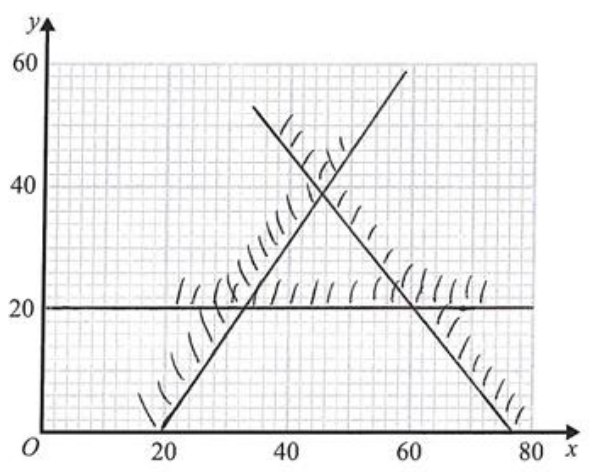
b1M1: Correct method for solving their simultaneous equations (so $y = 2x$ with their $5y = 6x + 10$ or $3x + y = 30$ with their $5y = 6x + 10$) to find both the x and y -coordinates of at least one of the other two vertices (can be implied by either vertex stated correctly – condone non-exact values (to at least 3 sf) for this mark if no working seen)

b1A1: CAO $(2.5, 5)$ and $\left(\frac{20}{3}, 10\right)$ (must be exact, so accept $(6\frac{2}{3}, 10)$, $(6.\dot{6}, 10)$, etc. but not $(6.6666\dots, 10)$)

c1M1: Testing their three **vertices** (not just points in the FR) in the correct objective function ($P = 2x + 3y$). Condone one slip only when applying the objective function to one of their three vertices (so they must apply the correct objective to at least two of their three points but condone a single slip in the third)

c1A1: At least two correct (therefore exact) points tested correctly (so at least two correct values of P explicitly stated – allow awrt 43.3 when testing $(20/3, 10)$)

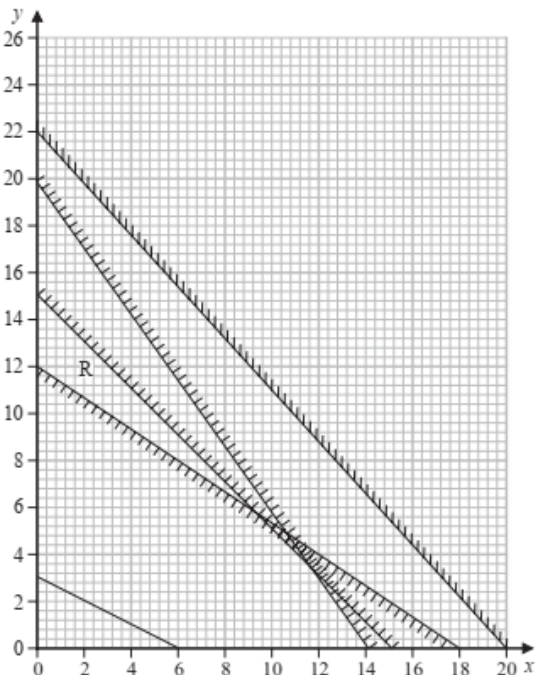
c2A1: CSO all three correct (therefore exact) points tested correctly (so all three correct values of P explicitly seen – allow awrt 43.3 when testing $(20/3, 10)$) with a clear indication of which is the optimal vertex (this can be achieved by either making it clear that (6, 12) is the optimal point or that $P = 48$ is the maximum (as one implies the other))

Question	Scheme	Marks
5 (a)	Maximise ($P = $) $x + y + z$	B1
		(1)
(b)	$12.50x + 10y + 15z \leq 1400$ $\Rightarrow 5x + 4y + 6z \leq 560$ *	M1 A1
		(2)
(c)	$x \geq \frac{40}{100}(x + y + z)$ $\Rightarrow 3x \geq 2y + 2z$ $3y \leq 2z$	M1 A1 B1
		(3)
(d)	$3x \geq 2y + 2 \times 30$ $\Rightarrow 3x - 2y \geq 60$ *	M1 A1
		(2)
(e)	 Optimal point (60, 20) 60 red (rose bushes), 20 yellow (rose bushes)	M1 A1 A1 A1
		(4)
		(12 marks)

Notes	
aB1	CAO – must have maximise (accept max but not maximum) and $x + y + z$
bM1	Forms inequality using the costs and attempts to simplify. If a correct equation is formed and then changed to an inequality this is M1 A0
bA1	CSO *
cM1	Attempts to form inequality with either 0.4 or 40/100 (o.e. fraction) combined with $(x + y + z)$. Accept any inequality or equals for this mark
cA1	CAO – fully simplified with integer coefficients
cB1	CAO
dM1	Substitutes $z = 30$ into their $3x \geq 2y + 2z$ and attempts to simplify (Allow recovery if their inequality is not fully simplified e.g. $6x \geq 4y + 4z$)
dA1	CSO *
eM1	Draws at least two of the three lines with at least one correct (Condone no consideration of $y \geq 0$)
e1A1	Any two lines correct $y = 20$ $5x + 4y = 380$ passes through (60, 20) and (76, 0) $3x - 2y = 60$ passes through (20, 0) and (40, 30) (must be within $\frac{1}{2}$ small square from these points)
e2A1	All three lines correct
e3A1	CSO – must have drawn the correct region (we do not require shading or for the feasible region to be marked with R , but if this is done, the region selected must be correct) The answer must be in context so $x = 60, y = 20$ is A0 Accept 60 red and 20 yellow with no mention of rose bushes. We do not need to see any mention of white roses here.

Question Number	Scheme	Marks
6. (a)		B1 B1 B1 B1 (4)
(b)(i)	A correct objective line	B1
	Correct optimal vertex labelled as V	B1 (2)
(b)(ii)	Solve correct set of simultaneous equations consistent with either their V or objective line	M1
	$(42, 44)$	A1
	77.6	A1 (3)
(c)	$(10, 60) > V \Rightarrow 10k + 60 > 42k + 44$ or $(60, 20) > V \Rightarrow 60k + 20 > 42k + 44$ $-\frac{1}{2} < -k$ or $-\frac{4}{3} > -k$	M1
	$k < \frac{1}{2}$ or $k > \frac{4}{3}$	A1
	$(10, 60) > V \Rightarrow 10k + 60 > 42k + 44$ and $(60, 20) > V \Rightarrow 60k + 20 > 42k + 44$ $-\frac{1}{2} < -k$ and $-\frac{4}{3} > -k$	M1dep
	$k < \frac{1}{2}$ and $k > \frac{4}{3}$ only	A1 (4)
		13 marks

Question Number	Scheme		Marks																																												
	Notes for Question 6																																														
The lines in (a) must define the correct FR and pass within half a square of the points stated: $4x + 3y = 300$ with points (0, 100) and (75, 0) $4x + y = 100$ with points (0, 100) and (25, 0) $x + 2y = 130$ with points (0, 65) and (130, 0) $3y = x$ with points (0, 0) and (60, 20) a1B1: Any two lines correctly drawn a2B1: Any three lines correctly drawn a3B1: All four lines correctly drawn a4B1: Correct R labelled – dependent on all three previous B marks bi1B1: A correct objective line drawn on the graph with a gradient of -0.8 – intersections points with each axes given below																																															
<table border="1"> <tr><th>x</th><th>y</th></tr> <tr><td>10</td><td>8</td></tr> <tr><td>20</td><td>16</td></tr> <tr><td>30</td><td>24</td></tr> <tr><td>40</td><td>32</td></tr> <tr><td>50</td><td>40</td></tr> <tr><td>60</td><td>48</td></tr> <tr><td>70</td><td>56</td></tr> <tr><td>80</td><td>64</td></tr> <tr><td>90</td><td>72</td></tr> <tr><td>100</td><td>80</td></tr> </table>		x	y	10	8	20	16	30	24	40	32	50	40	60	48	70	56	80	64	90	72	100	80	<table border="1"> <tr><th>x</th><th>y</th></tr> <tr><td>12.5</td><td>10</td></tr> <tr><td>25</td><td>20</td></tr> <tr><td>37.5</td><td>30</td></tr> <tr><td>50</td><td>40</td></tr> <tr><td>62.5</td><td>50</td></tr> <tr><td>75</td><td>60</td></tr> <tr><td>87.5</td><td>70</td></tr> <tr><td>100</td><td>80</td></tr> <tr><td>112.5</td><td>90</td></tr> <tr><td>125</td><td>100</td></tr> </table>		x	y	12.5	10	25	20	37.5	30	50	40	62.5	50	75	60	87.5	70	100	80	112.5	90	125	100
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112.5	90																																														
125	100																																														
bi2B1: cao V labelled – dependent on first three B marks in (a) and the first B mark in (b) bii1M1: Solving correct pair of simultaneous equations for their V (or if not labelled then the vertex consistent with their objective line) – this mark can be implied by (42, 44) but in all cases they must have drawn four lines with at least two correct and an attempt at an objective line bii1A1: cao ((42, 44) only) – dependent on first three B marks in (a) and the first B mark in (b) bii2A1: cao (77.6 only) – dependent on first three B marks in (a) and the first B mark in (b)																																															
SC in (b) if no objective line drawn then can score in (b) B0B0M1A1A0 for both (42, 44) and 77.6 only provided that the first three B marks earned in (a) Marks in part (c) are not dependent on the marks in (a) and/or (b) c1M1: <u>Point testing method:</u> $10k + 60 \square kx_1 + y_1$ or $60k + 20 \square kx_1 + y_1$ or $42k + 44 \square kx_1 + y_1$ where $\square$ is any inequality sign or the equals sign and (x_1, y_1) is their numerical V or (42, 44). <u>Objective line method:</u> $-\frac{1}{2} \square - k$ or $-\frac{4}{3} \square - k$ or $\frac{1}{3} \square - k$ where $\square$ is any inequality or equals. Or one correct answer stated c1A1: One correct answer ($k < \frac{1}{2}, k \leq \frac{1}{2}, k > \frac{4}{3}, k \geq \frac{4}{3}$) - if no method or working (as shown above) then A0 c2M1dep: <u>Point testing:</u> $10k + 60 \square kx_1 + y_1$ and $60k + 20 \square kx_1 + y_1$ where $\square$ is any inequality sign or the equals sign and (x_1, y_1) is (42, 44) or their V (but not ((10, 60) or (60, 20)) (so V must now be the intersection of the two lines $4x + 3y = 300$ and $x + 2y = 130$). Objective line: $-\frac{1}{2} \square - k$ and $-\frac{4}{3} \square - k$ where $\square$ is any inequality or equals. Or both correct answers stated with no working c2A1: Both correct answers only ($k < \frac{1}{2}$ or $k \leq \frac{1}{2}$ and $k > \frac{4}{3}$ or $k \geq \frac{4}{3}$) with working as shown above																																															

Question number	Scheme	Marks
6. (a)	Minimise ($P =$) $300x + 400y + 400z$	B1
	Subject to:	
	$275x + 200y + 100z \leq 5500 (\Rightarrow 11x + 8y + 4z \leq 220)$	B1
	$5x + 2y + 3z \leq 70$	B1
	$\frac{x}{15} + \frac{y}{20} + \frac{z}{30} \leq 1 \Rightarrow 4x + 3y + 2z \leq 60$	M1 A1
	$x + y + z \geq 18$ ($x, y, z \geq 0$)	B1 (6)
(b)	e.g. Martin makes apple cakes and chocolate cakes in the ratio of 2:1 e.g. for every one chocolate cake that Martin makes he has to make 2 apple cakes (assuming that he makes chocolate/apple cakes) e.g. Martin should make twice as many apple cakes as chocolate cakes e.g. The number of apple cakes that Martin makes should be double the number of chocolate cakes he makes	B1 (1)
(c)		B1 B1 B1 B1 (4)
(d)	Drawing an objective line accept reciprocal gradient	M1
	Correct objective line	A1
	Martin should make 9 carrot cakes, 6 apple cakes and 3 chocolate cakes	A1
	Minimum amount of sugar is 6300 grams (or 6.3 kg)	A1 (4)
(e)	Martin has 1525 grams of flour remaining	B1
	Martin also has 4 eggs remaining	B1 (2)
		(17 marks)

Notes on Question 6

a1B1: CAO – expression correct ($300x + 400y + 400z$ or $0.3x + 0.4y + 0.4z$ **only**) and ‘minimise’ or ‘min’ but not ‘minimum’. ISW if either of these two expressions are seen and then ‘simplified’

a2B1: CAO ($275x + 200y + 100z \leq 5500$) oe but must be four terms only with integer coefficients e.g. $11x + 8y \leq 220 - 4z$

a3B1: CAO ($5x + 2y + 3z \leq 70$) oe but must be four terms only with integer coefficients

a1M1: Correct method $\frac{x}{15} + \frac{y}{20} + \frac{z}{30} \bullet 1$ where $\bullet$ is any inequality symbol or =

a1A1: CAO ($4x + 3y + 2z \leq 60$) oe must be four terms only with integer coefficients

a4B1: CAO ($x + y + z \geq 18$) oe must be four terms only with integer coefficients

b1B1: CAO but give bod if intention is correct. Some correct examples include:

- (Martin makes) apple (cakes) and chocolate (cakes) in the ratio of 2:1
- For every one chocolate (cake that Martin makes he has to) make 2 apple (cakes)
- (Martin should make) twice as many apple (cakes) as chocolate (cakes)
- The number of apple cakes (that Martin makes should be) double the number of chocolate cakes (he makes)

Please check these carefully for those candidates that imply incorrectly that Martin should make two chocolate cakes for every one apple cake. Furthermore, do not condone an answer that implies an inequality (e.g. use of words such as, ‘at least’, ‘at most’, etc.)

The lines in (c) must define the correct FR **and if extended** would pass within a small square of their point of intersection with the axes

c1B1: Any two lines correctly drawn

c2B1: Any three lines correctly drawn

c3B1: All four lines correctly drawn

c4B1: Correct R labelled – not just implied by shading – dependent on scoring the first three marks in this part **and all four lines being drawn from axis to axis (within one small square)**

d1M1: Drawing the correct objective line (gradient -0.5) or its reciprocal (gradient -2) on the graph. Line must be correct to within one small square if extended from axis to axis. If line is shorter than $(0, 1)$ to $(2, 0)$ (or for the reciprocal $(0, 2)$ to $(1, 0)$) then M0

d1A1: Correct objective line – same condition that the line must be correct to within one small square if extended from axis to axis and be no shorter than the line from $(0, 1)$ to $(2, 0)$

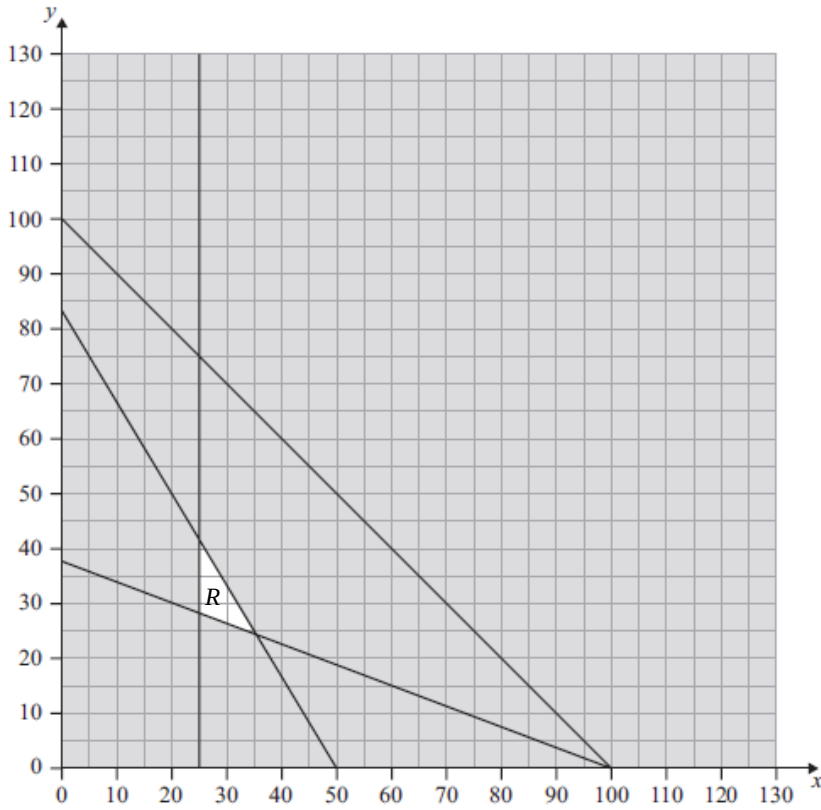
The final 4 marks are all dependent on the first three B marks in (c), the first two marks in part (d) and they must not have implied an incorrect R in (c) (but give bod if region not labelled in (c) or if the lines did not go axis to axis in (c) as this was penalised with the final mark in (c))

d2A1: CAO (in context) – as a minimum accept 9 carrot, 6 apple and 3 chocolate

d3A1: CAO (6300 or 6.3) – no units required but if stated then must be correct – so 6300 kg is A0

e1B1: CAO (1525 (grams) or 1.525 (kg) of flour) – no units required but if stated then must be correct – so 1525 kg is A0

e2B1: CAO 4 (eggs)

7.(a)	$(P=) 160x + 75y + 125z$ and maximise	B1
	Subject to:	
	$x + y + z \leq 100$	B1
	$\frac{1}{4}(x + y + z) \leq x \Rightarrow 3x - y - z \geq 0$	M1
	$3z \leq 5y$	M1
	$2x + 1.5y + 0.75z \leq 138 \Rightarrow 8x + 6y + 3z \leq 552 \quad (120x + 90y + 45z \leq 8280)$	M1
	$3x - y - z \geq 0$ $3z \leq 5y$ $8x + 6y + 3z \leq 552$ $x \geq 0 \quad y \geq 0 \quad z \geq 0$	A1 (6)
(b)	Substitute $x + y + z = 100$ into $P = 160x + 75y + 125z$ and simplify	M1
	$P = 5(7x - 10y) + 12500$ so maximising a (positive) multiple of $7x - 10y$ is equivalent to minimising the negative of this expression, that is, $-(7x - 10y) = -7x + 10y^*$	A1* (2)
(c)		B1 B1 B1 B1 (4)
(d)(i)	Vertices of R are $\left(25, \frac{225}{8}\right), \left(25, \frac{127}{3}\right), (36, 24)$	M1 A1

(ii)		$-7x + 10y$	$160x + 75y + 125z$	dM1
	$\left(25, \frac{225}{8}\right)$	$\frac{425}{4} (106.25)$	$\frac{47875}{4} (11,968.75)$	
	$\left(25, \frac{127}{3}\right)$	$\frac{745}{3} (248.3)$	$\frac{33775}{3} (11,258.3)$	
	$(36, 24)$	-12	12560	
	So 36 acres for crop A, 24 for crop B and 40 acres for crop C			A1
(iii)	Maximum expected profit is (£) 12 560			A1 (5)
				17 marks
	Notes for Question 7			
	If using A, B and C throughout (a) instead of x , y and z send to review			
a1B1	CAO – expression $(160x + 75y + 125z)$ together with ‘max’ or ‘maximise’ not ‘maximum’			
a2B1	CAO $(x + y + z \leq 100)$			
a1M1	$\frac{1}{4}(x + y + z) \square x$ where $\square$ is any inequality or equals – brackets must be present or implied by later working (accept correct equivalent unsimplified forms)			
a2M1	$3z \square 5y$ where $\square$ is any inequality or equals. Also allow $5z \leq 3y$ for this mark (accept correct equivalent unsimplified forms)			
a3M1	$2x + 1.5y + 0.75z \square 138$ (oe) where $\square$ is any inequality or equals. Time may be converted to minutes (must be all 4 values) (accept correct equivalent unsimplified forms)			
a1A1	All three constraints $(3x \geq y + z, 3z \leq 5y, 8x + 6y + 3z \leq 552)$ correct – must have integer coefficients with only one term in each variable. Condone omission of the trivial constraints $x \geq 0, y \geq 0, z \geq 0$ (Accept e.g. $24x + 18y + 9z \leq 1656$ oe if working in minutes)			
b1M1	Substitute $x + y + z = 100$ into their linear objective function and simplify to a single term in x and a single term in y only			
b1A1*	Explaining why maximising the correct objective $35x - 50y (+12500)$ is equivalent to minimising $-7x + 10y$			
c1B1	Any two lines correctly drawn ($5x + 3y = 252$ should pass within half a small square of (18, 54) and (42, 14)) ($3x + 8y = 300$ should pass within half a small square of (20, 30) and (100, 0)) ($x + y = 100$ should pass within half a small square of (0, 100), (50, 50) and (100, 0)) ($x = 25$ must be drawn through the middle of the small square from (24, 0) to (26, 0))			
c2B1	Any three lines correctly drawn			
c3B1	All four lines correctly drawn (penalise any poorly drawn lines with the loss of this mark)			
c4B1	Correct R labelled – dependent on all three previous B marks			
d1M1	Attempt to find the exact coordinates (must be finding at least two pairs of coordinates) of their R – dependent on at least two B marks in (c)			
d1A1	CAO - all three exact coordinates of the correct R (accept 28.125 and $42.\dot{3}$ but not 42.3 or 42.33)			
d2M1	Point testing (objective line method is M0) – testing at least two of their vertices of their R in either $-7x + 10y$ or $160x + 75y + 125z$ - dependent on previous M mark			
d2A1	All three values of P found (either exact or decimal equivalent for either objective function) and stating the correct allocation of the three crops. Allocation must be in context for crop A, B, C not $x =, y =, z =$			
d3A1	Maximum expected profit stated correctly (units not required)			

Question Number	Scheme	Marks
5. (a)	$5y \leq 3z$	M1 A1 (2)
(b)	The total number of shirts must be at least 250	B1
	At most 20% of all the shirts should be small	M1 A1 (3)
(c)	(Minimise) $6x + 10y + 15z$	B1 (1)
(d)(i)	$z = 150 \Rightarrow x + y \geq 100$	M1
	$y \leq 90$	
	$4x - y \leq 150$	A1
(d)(ii)		B1 B1 B1 B1 (6)
(e)	Correct objective line	B1
	V correctly labelled	B1 (2)
(f)	50 small and 50 medium shirts	B1
	Cost = £3050	B1 (2)
(g)	$x = 50, y = 75 \Rightarrow z \geq 125$ therefore minimum number of large shirts is 125	M1
	This leads to a cost of £2925 which is less than the cost in (f)	A1 (2)
		18 marks

Notes for Question 5

a1M1: Correct method: $5y \square 3z$ where $\square$ is any inequality or equals. An exact equivalent answer (with or without integer coefficients) can score M1 **or** M1 for $3y \leq 5z$ only

a1A1: CAO (or equivalent e.g. $k(5y \leq 3z)$ where k is any positive integer only)

b1B1: CAO or e.g. the minimum number of shirts is 250 is fine for this mark (note that they must imply that the total number (and not one particular brand of shirt) is **at least** 250)

b1M1: Three of 'at most', '20%', 'all' and 'small' (allow equivalents e.g. fifth or 0.2 for 20%) allow those who imply 'all' provided that it is clear that they aren't talking about one particular brand only

b1A1: CAO (o.e. e.g. the number of small shirts is less than or equal to a fifth of the total number of shirts, the number of small shirts is at most 20% of all the shirts sold) – give bod of these that clearly imply 'all' provided that they aren't talking about only one particular brand. Do not allow statements which contain use of 0.2 or $\frac{1}{5}$ for this mark, e.g. the number of small shirts is at most 0.2 of all the shirts is A0

Question Number	Scheme	Marks
	c1B1: Expression correct (or $600x + 1000y + 1500z$) di1M1: Eliminating z from all their inequalities by using the substitution $z = 150$ – accept unsimplified (e.g. $x + y + 150 \geq 250, x \leq 0.2(x + y + 150)$ and their $5y \leq 3(150)$) di1A1: CAO e.g. $x + y \geq 100, 4x \leq y + 150, 5y \leq 450$ (oe) - all constraints must be correct with integer coefficients but allow positive multiplies – ignore $x \geq 0, y \geq 0$ but any other additional constraints is A0 – allow recovery in this part if $y \leq 90$ (oe) seen in (d) even if their $5y \leq 3z$ is incorrect in (a) In (d), lines must be long enough to define the correct feasible region and would pass through one small square of the points stated: $x + y = 100$ must pass within one small square of its intersection with the axes – (0, 100) and (100, 0) $y = 90$ must pass within one small square of its intersection with the y-axis and (60, 90) $4x - y = 150$ must pass within one small square of (37.5, 0) and (60, 90) dii1B1: Any one line correctly drawn dii2B1: Any two lines correctly drawn dii3B1: All three lines correctly drawn dii4B1: Region, R, correctly labelled – not just implied by shading – dependent on scoring the three previous B marks in this part e1B1: Drawing the correct objective line on the graph with gradient of -0.6. Line must be correct to within one small square if extended from axis to axis. If line is shorter than (0, 6) to (10, 0) then B0 e2B1: V correctly labelled – note that this mark is dependent on the correct feasible region in (d) (so must have scored at least B1B1B1B0) and the previous B mark in (e) f1B1: CAO – must be in context (50 small and 50 medium and not for $x = y = 50$) note that this mark is dependent on the correct feasible region in (d) (so must have scored at least B1B1B1B0 in (d)) and the first B mark (for a correct objective line) in (e) f2B1: CAO (3050) – units not required - note that this mark is dependent on the correct feasible region in (d) (so must have scored at least B1B1B1B0 in (d)) and the first B mark (for a correct objective line) in (e) – condone if stated/seen in (g) g1M1: Substitute to obtain the correct value for z of 125 (accept $z \geq 125$ or $>$) – if no method allow 125 seen but M0 if 125 found but a different value of z stated and subsequently used g1A1: Correct cost (2925) and dependent on the final B mark in (f)	

Question Number	Scheme	Marks
6.(a)		M1 A1 A1 A1 A1 (5)
(b)	Critical path: A – E – K	B1 (1)
(c)	First critical path: C – H – I – M Second critical path: C – H – I – L	B1 B1 (2)
		8 marks

Notes for Question 6

Condone lack of, or incorrect, numbered events throughout. ‘Dealt with correctly’ means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. ‘F dealt with correctly’ requires the correct precedences for this activity, i.e. A, B and C labelled correctly and leading into the same node and F starting from that node but do not consider the end event for F. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity G is not labelled (but the arc is present) then this will lose the second A mark and the final (CSO) A mark – they can still earn the third A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first four marks only

a1M1: At least eight activities (labelled on arc), one start and at least two dummies placed

a1A1: Activities A, B, C, first two dummies (+ correct arrows on these two dummies) and D dealt with correctly – the first two dummies are those that meet at the end of activity B

a2A1: Activities E, F, G, H and I dealt with correctly

a3A1: Activities J, K, L and M and 3rd dummy (+ correct arrow on this dummy) dealt with correctly – the 3rd dummy is the one that begins at the end of activity G

a4A1: CSO – Final dummy + arrow, all arrows correctly placed for each activity with one finish and no additional dummies. Note that this is not a unique solution e.g. J and K could be interchanged, or the dummy could come immediately after E, etc. so please check these carefully. **Please check all arcs carefully for arrows – if there are no arrows on dummies then M1 only.**

Note that additional (but unnecessary) ‘correct’ dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

b1B1: CAO (A, E and K only)

c1B1: One correct path (with at most three paths stated)

c2B1: Both correct with no others

Question Number	Scheme	Marks
7. (a)	Minimise $400x + 550y + 750z$ Subject to $x + y + z \geq 20$ $x \leq 2(y + z)$ $z \leq \frac{1}{4}(x + y + z) \Rightarrow 3z \leq x + y$	B1 B1 M1 A1 M1 A1 (6)
(b)(i)(ii)	Ratio of 5:3 $\Rightarrow 3y = 5z$ $x + y + z \geq 20 \Rightarrow x + y + \frac{3}{5}y \geq 20$ and $400x + 550y + 750\left(\frac{3y}{5}\right)$ $5x + 8y \geq 100$ and $400x + 1000y$	B1 M1 A1 (3)
(c)		B1 B1 B1 B1 (4)
(d)	Drawing an objective line accept reciprocal gradient Correct objective line V correctly labelled	M1 A1 A1 (3)
(e)(i)	Cost is £9800	B1
(ii)	3 days for senior leaders (12 training days for new teachers, 5 days for middle leaders)	B1 (2)
		18 marks

Question Number	Scheme	Marks
Notes for Question 7		
a1B1: Expression correct <u>together</u> with ‘minimise’ or ‘min’ but not ‘minimum’ – isw if coefficients are subsequently simplified but $400x + 550y + 750z$ must be seen at some point for this mark to be awarded. The ‘min’ must appear beside the correct expression a2B1: CAO ($x + y + z \geq 20$) a1M1: $x \square 2(y + z)$ where $\square$ is any inequality or equals. Accept $2x \leq y + z$ for this mark a1A1: CAO $x \leq 2(y + z)$ or equivalent but only one term in each variable and integer coefficients a2M1: $z \square \frac{1}{4}(x + y + z)$ where $\square$ is any inequality or equals – allow 0.25 but do not allow $z \square 25\%(x + y + z)$ unless recovered to a fraction or decimal later a2A1: CAO $3z \leq x + y$ or equivalent but only one term in each variable and integer coefficients bi1B1: Ratio expressed correctly as an equation $3y = 5z$ (oe) (possibly implied by subsequent working) bii1M1: Their linear equation in y and z only (must be of the form $ay = bz$ oe) substituted into their constraint and their objective to eliminate z or one correct answer simplified b1A1: CAO $5x + 8y \geq 100$ and $400x + 1000y$ (allow ISW if correct objective simplified) In (c) $5x + 8y = 100$ must pass if extended within one small square of its intersection with the axes – (0, 12.5) and (20, 0) but must be long enough to define the correct feasible region $4y = 5x$ must pass within one small square of its intersection with the origin and (10, 12.5) $5x = 16y$ must pass within one small square of its intersection with the origin and (20, 6.25) c1B1: One line correct c2B1: Two lines correct c3B1: All three lines correct c4B1: Region, R, correctly labelled – not just implied by shading – dependent on scoring the first three marks in this part d1M1: Drawing their objective line (based on their answer to (b)) or its reciprocal - line must be correct to within one small square if extended from axis to axis. If their line is shorter than (0, 1) to (2.5, 0) then M0 d1A1: Drawing a correct objective line subject to the conditions above regarding length and if extended from axis to axis d2A1: V labelled clearly on their graph (so dependent on the first three marks in (c) and the previous A mark in this part) eiB1: CAO – condone lack of units – dependent on the first three marks in (c) and the first two marks in (d) eiiB1: CAO (3) – dependent on the first three marks in (c) and the first two marks in (d)		

Question Number	Scheme									Marks																																																																																	
	<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>A</td><td>-</td><td>7</td><td>8</td><td>9</td><td>11</td><td>18</td><td>25</td><td>26</td></tr><tr><td>B</td><td>7</td><td>-</td><td>14</td><td>2</td><td>4</td><td>11</td><td>18</td><td>19</td></tr><tr><td>C</td><td>8</td><td>14</td><td>-</td><td>12</td><td>10</td><td>15</td><td>22</td><td>23</td></tr><tr><td>D</td><td>9</td><td>2</td><td>12</td><td>-</td><td>2</td><td>9</td><td>16</td><td>17</td></tr><tr><td>E</td><td>11</td><td>4</td><td>10</td><td>2</td><td>-</td><td>7</td><td>14</td><td>15</td></tr><tr><td>F</td><td>18</td><td>11</td><td>15</td><td>9</td><td>7</td><td>-</td><td>7</td><td>8</td></tr><tr><td>G</td><td>25</td><td>18</td><td>22</td><td>16</td><td>14</td><td>7</td><td>-</td><td>1</td></tr><tr><td>H</td><td>26</td><td>19</td><td>23</td><td>17</td><td>15</td><td>8</td><td>1</td><td>-</td></tr></table>										A	B	C	D	E	F	G	H	A	-	7	8	9	11	18	25	26	B	7	-	14	2	4	11	18	19	C	8	14	-	12	10	15	22	23	D	9	2	12	-	2	9	16	17	E	11	4	10	2	-	7	14	15	F	18	11	15	9	7	-	7	8	G	25	18	22	16	14	7	-	1	H	26	19	23	17	15	8	1	-	
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G	25	18	22	16	14	7	-	1																																																																																			
H	26	19	23	17	15	8	1	-																																																																																			
(b)	NNA: A – B – D – E – F – G – H – C – A									B1																																																																																	
	$7 + 2 + 2 + 7 + 7 + 1 + 23 + 8 = 57$ (km)									B1 (2)																																																																																	
(c)(i)	Prim (starting at C): CE, DE, BD, EF, FG, GH									M1 A1																																																																																	
	RMST weight = $10 + 2 + 2 + 7 + 7 + 1 = 29$																																																																																										
(c)(ii)	$29 + 7(AB) + 8(AC) = 44$ (km)									M1 A1 (4)																																																																																	
(d)	$44 \leq \text{optimal distance} \leq 57$									B2, 1, 0 (2)																																																																																	
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Notes for Question 6																																																																																											
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 23 21 18 in that order (so 23 18 21 is incorrect). It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling																																																																																											

Question Number	Scheme	Marks
	a1M1: Any larger working value replaced by any smaller working value at at least two nodes except A, B, C and D a1A1: All values at A, B, C and D correct and the working values in the correct order. Condone lack of a zero as a working value at A a2A1: All values at E and F correct and the working values in the correct order. Penalise order of labelling only once per question (so E and F must be labelled in that order and E must be labelled after D) a3A1ft: All values at G and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are E, F and H). For example, if correct then the order of labelling of nodes E, F and H are 5, 6 and 8 respectively so the working values at G should come from E and F in that order. The first working value at G should be 18 (the Final value at F) + 7 (the weight of the arc FG) and the second working value at G should be 11 (the Final value at E) + 16 (the weight of the arc EG). Repeat the process for H (which will have working values from D, E and G with the order of these nodes determined by the candidate's order of labelling at D, E and G) a2M1: Correct entries in the table following through their final values – dependent on the previous M mark (need only fill in either the A row or A column) a4A1: cao b1B1: Correct nearest neighbour route starting and finishing at A (A – B – D – E – F – G – H – C – A) b2B1: cao (57) on length of route ci1M1: First three arcs (CE, DE, BD) or all 7 nodes {C, E, D, B, F, G, H} correctly chosen in order. If any explicit rejections seen then M1 only in (c)(i). Order of nodes may be seen at the top of the matrix/table {4, 1, 3, 2, 5, 6, 7}. Award M0 for a correct tree with no working. Award M1 only for the first three arcs (oe) selected correctly if starting at a different node than C. If correct values circled in the table but no indication of order of selection then M0 ci1A1: cso – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen). If AB and AC added explicitly in (c)(i) then A0 but can score both marks in (c)(ii) cii2M1: Weight of RMST + 7 + 8 (two smallest arcs incident to A) with $19 \leq \text{RMST} \leq 39$ (if clearly not six arcs in RMST then M0). Give bod if 15 is added to the total of six values circled in the table provided those six values sum to a value between 19 and 39 inclusive cii2A1: cao (44) – if correct RMST stated in (c)(i) followed by 44 (with no additional working) then award M1A1 in (c)(ii). This mark is dependent on Prim's algorithm being used to find the RMST (allow this mark if rejections seen in (c)(i) when applying Prim). So in (c) M1A0M1A1 is possible e.g. if only stating the node (instead of the arc) selection in order when applying Prim. If the correct six values are circled in the table and added to 15 to give 44 but the order of arc/node selection is not stated (so no indication that Prim has been applied) then A0 (as the qu. says, 'Hence...') d1B1: Any indication of an interval from their answer to (c)(ii) to their answer to (b) with one value correct (e.g. $44 \sim 57$ scores B1 but $57 \sim 44$ or $57 \leq \text{optimal distance} \leq 44$ scores B0). If correct route seen in (b) but the upper bound not stated in (b) allow recovery in part (d) it stated here (but still withhold the second mark in (b)). d2B1: cao ($44 \leq \text{optimal distance} \leq 57$) including correct inequalities (allow $44 < \text{optimal distance} \leq 57$) – allow interval notation e.g., [44, 57] or (44, 57]	
7.(a)	Minimise $(P=)9x+12y+16z$	B1
	$x+y+z \geq 40$	B1
	$z \geq 2y$	B1
	$\frac{3}{5}(x+y+z) \geq x$ simplifies to $2x \leq 3y+3z$	M1 A1
	$x+1.5y+2.5z \leq 75$ simplifies to $2x+3y+5z \leq 150$	M1 A1
	$(x \geq 0, y \geq 0, z \geq 0)$	(7)
(b)	$9x+12y+16(45-x-y)$	M1

Question Number	Scheme	Marks
	which leads to $-7x - 4y + 720$	A1
	$+720$ is a constant so the total value is minimised when $-7x - 4y$ is and $-7x - 4y = -(7x + 4y)$, as $-(7x + 4y)$ is negative (for all positive values of x and y) and so minimising an expression which is negative is equivalent to maximising the corresponding positive expression $7x + 4y$	A1 (3)
(c)		B1 B1 B1 B1 (4)
(d)	Correct objective line	B1
	V correctly labelled	B1 (2)
(e)	27 small containers, 6 medium containers and 12 large containers	B1
	Total cost is (£)507	B1 (2)
		18 marks
Notes for Question 7		
a1B1: cao – expression correct and ‘minimise’ or ‘min’ but not ‘minimum’ a2B1: cao $(x + y + z \geq 40$ oe but must be four terms only with integer coefficients e.g. $x + y + z - 40 \geq 0$) a3B1: cao $(z \geq 2y$ oe (e.g. $4y - 2z \leq 0$) but must be two terms only with integer coefficients) a1M1: correct method - must see $\frac{3}{5}(x + y + z) \bullet x$ where $\bullet$ is any inequality or $=$. The bracket must be present or implied by later working. Allow 0.6 but not 60% (unless implied by later working) a1A1: cao – simplified (one term only in x , y and z) – answer must have integer coefficients (e.g. $4x - 6y - 6z \leq 0$) - the correct simplified inequality with either no working or working with % sign implies M1A1 a2M1: correct method – must see $x + 1.5y + 2.5z \bullet 75$ (oe) where $\bullet$ is any inequality or $=$		

Question Number	Scheme	Marks																																				
a2A1: cao – simplified (one term only in x, y and z) – answer must have integer coefficients (e.g. $4x + 6y \leq 300 - 10z$) – the correct simplified inequality with no working implies M1A1 b1M1: substituting $z = 45 - x - y$ into $9x + 12y + 16z$ b1A1: cao of $-7x - 4y + 720$ and <u>any</u> attempt at <u>explaining</u> why the minimum total cost is achieved when $7x + 4y$ is maximised b2A1: stating that 720 is a constant (and so doesn't impact on the optimal values of x, y and z) and a correct deduction that minimising a negative expression is equivalent to maximising the corresponding positive expression (so just stating that $-7x - 4y$ is minimised when $7x + 4y$ is maximised is A0) The lines in (c) must define the correct FR and if extended pass within a small square of the points stated: $x + 3y = 45$ with points (0, 15) and (45, 0) $3x + 2y = 75$ with points (0, 37.5) and (25, 0) $x = 27$ with points (27, 0) and (27, 40) – a common wrong response is to draw either $y = 27$ or $x = 28$ – these are both B0 c1B1: Any one line drawn correctly c2B1: Any two lines drawn correctly c3B1: Any three lines drawn correctly c4B1: Correct R labelled – not just implied by shading – dependent on scoring the first three marks in this part (condone if no shading below the x-axis) d1B1: A correct objective line drawn on the graph with a gradient of -1.75 – line must be at least the length of (2, 0) to (0, 3.5) and within one small square - for reference common intersections points with each axes are given below <table><tr><th>x</th><th>y</th></tr><tr><td>4</td><td>7</td></tr><tr><td>5</td><td>8.75</td></tr><tr><td>8</td><td>14</td></tr><tr><td>10</td><td>17.5</td></tr><tr><td>12</td><td>21</td></tr><tr><td>15</td><td>26.25</td></tr><tr><td>16</td><td>28</td></tr><tr><td>20</td><td>35</td></tr></table> <table><tr><th>x</th><th>y</th></tr><tr><td>2.85...</td><td>5</td></tr><tr><td>5.71...</td><td>10</td></tr><tr><td>8.57...</td><td>15</td></tr><tr><td>11.4...</td><td>20</td></tr><tr><td>14.3...</td><td>25</td></tr><tr><td>17.1...</td><td>30</td></tr><tr><td>20</td><td>35</td></tr><tr><td>22.9...</td><td>40</td></tr></table> d2B1: V labelled clearly on their graph. This mark is dependent on - the first three marks in (c) - not labelling or implying that any other region is the FR - the first B mark in (d) By clearly labelled the vertex should either be labelled 'V' or circled or clearly distinguishable from any other vertex (but B0 if not clear e.g. another vertex circled too) (note that $V(27, 6)$) e1B1: cao (must be in context – so not in terms of x, y and z) – dependent on first three marks in (c) and the first mark in (d) (27 small, 6 medium and 12 large) e2B1: cao (507) - dependent on first three marks in (c) and the first mark in (d) – units not required. Condone incorrect units e.g. \$			x	y	4	7	5	8.75	8	14	10	17.5	12	21	15	26.25	16	28	20	35	x	y	2.85...	5	5.71...	10	8.57...	15	11.4...	20	14.3...	25	17.1...	30	20	35	22.9...	40
x	y																																					
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