



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Kinematics & Projectiles (M2.1)

International A Level Mathematics · M2: Mechanics 2 (WME02)

34 questions · 363 marks · Mark schemes included

Past papers January 2020 - June 2025 · 2 sub-topics

 guidemygrades.com

 [@Guidemygrades](https://www.instagram.com/Guidemygrades)

 [@Guidemygrades](https://twitter.com/Guidemygrades)

 [@Guidemygrades](https://www.youtube.com/Guidemygrades)

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
M2.1a VARIABLE ACCELERATION (CALCULUS)		18 Qs · 168 marks	3
1. Q1	IAL M2 (WME02) October 2023	7	4
2. Q2	IAL M2 (WME02) June 2025	7	6
3. Q2	IAL M2 (WME02) June 2021	8	8
4. Q1	IAL M2 (WME02) June 2022	8	10
5. Q1	IAL M2 (WME02) January 2024	8	12
6. Q1	IAL M2 (WME02) June 2025	8	14
7. Q3	IAL M2 (WME02) October 2021	9	16
8. Q3	IAL M2 (WME02) January 2022	9	18
9. Q1	IAL M2 (WME02) June 2025	9	22
10. Q5	IAL M2 (WME02) January 2020	10	24
11. Q5	IAL M2 (WME02) October 2020	10	28
12. Q4	IAL M2 (WME02) October 2022	10	32
13. Q4	IAL M2 (WME02) January 2023	10	36
14. Q2	IAL M2 (WME02) June 2023	10	38
15. Q1	IAL M2 (WME02) October 2024	10	42
16. Q5	IAL M2 (WME02) January 2021	11	44
17. Q3	IAL M2 (WME02) January 2025	11	46
18. Q2	IAL M2 (WME02) June 2024	13	48
M2.1b PROJECTILES		16 Qs · 195 marks	50
19. Q4	IAL M2 (WME02) January 2025	8	51
20. Q3	IAL M2 (WME02) June 2025	10	53
21. Q7	IAL M2 (WME02) June 2025	10	57
22. Q7	IAL M2 (WME02) June 2021	11	60
23. Q8	IAL M2 (WME02) January 2024	11	64
24. Q7	IAL M2 (WME02) June 2024	11	67
25. Q7	IAL M2 (WME02) January 2021	12	70
26. Q8	IAL M2 (WME02) January 2023	12	74
27. Q4	IAL M2 (WME02) October 2023	12	79
28. Q5	IAL M2 (WME02) October 2024	12	83
29. Q8	IAL M2 (WME02) October 2020	13	87
30. Q7	IAL M2 (WME02) January 2020	14	92
31. Q8	IAL M2 (WME02) October 2021	14	96
32. Q8	IAL M2 (WME02) October 2022	14	101
33. Q7	IAL M2 (WME02) June 2023	15	106



QUESTION	SOURCE	MARKS	PAGE
34. Q7	IAL M2 (WME02) January 2022	16	111



QUESTIONS

34 questions · 363 marks · 2 sub-topics

SUB-TOPIC M2.1a

VARIABLE ACCELERATION (CALCULUS)

Sub-topic M2.1a · 18 questions · 168 marks · lowest tariff first



- $$\mathbf{r} = (t^4 - 8t^2)\mathbf{i} + \left(6t^2 - 2t^{\frac{3}{2}}\right)\mathbf{j}$$

- (b) Find the acceleration of P when $t = 4$

- $$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$



Question 1 continued

(Total for Question 1 is 7 marks)



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



Leave
blank

2. A particle P of mass 1.5 kg moves under the action of a single force $\mathbf{F}$ newtons.

At time t seconds, $t \geq 0$, P has velocity $\mathbf{v} \text{ m s}^{-1}$, where

$$\mathbf{v} = (5t^2 - t^3)\mathbf{i} + (2t^3 - 8t)\mathbf{j}$$

- (a) Find $\mathbf{F}$ when $t = 2$

(4)

At time $t = 0$, P is at the origin O .

- (b) Find the position vector of P relative to O at the instant when P is moving in the direction of the vector $\mathbf{j}$

(4)

Question 2 continued

Leave
blank

(Total 8 marks)

Q2



Leave
blank

1. At time t seconds, $t \geq 0$, a particle P has position vector $\mathbf{r}$ metres with respect to a fixed origin O , where

$$\mathbf{r} = (t^3 - 8t)\mathbf{i} + \left(\frac{1}{3}t^3 - t^2 + 2t\right)\mathbf{j}$$

- (a) Find the acceleration of P when $t = 4$

(5)

At time T seconds, $T \geq 0$, P is moving in the direction of $(2\mathbf{i} + \mathbf{j})$

- (b) Find the value of T

(3)

Question 1 continued

Leave
blank

Lined area for writing the answer to Question 1.

(Total 8 marks)

Q1

Box for marking the question.



Question 1 continued

(Total for Question 1 is 8 marks)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 8 marks)



Question 3 continued

Leave
blank

(Total 9 marks)

Q3



Question 3 continued

Leave
blank



Question 3 continued

Leave
blank



Question 3 continued

Leave
blank

Lined area for writing the answer to Question 3.

(Total 9 marks)

Q3

Mark box for Question 3.



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 9 marks)



Question 5 continued

Leave
blank



Question 5 continued

Leave
blank

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Leave
blank

Lined area for writing the answer to Question 5.

(Total 10 marks)

Q5

Grading boxes for Question 5.



Leave
blank

5. At time t seconds ($t \geq 0$), a particle P has velocity $\mathbf{v} \text{ m s}^{-1}$, where

$$\mathbf{v} = (3t^2 - 9t + 6)\mathbf{i} + (t^2 + t - 6)\mathbf{j}$$

- (a) Find the acceleration of P when $t = 3$

(3)

When $t = 0$, P is at the fixed point O .

The particle comes to instantaneous rest at the point A .

- (b) Find the distance OA .

(7)

Question 5 continued

Leave
blank



Question 5 continued

Leave
blank



Question 5 continued

Leave
blank

(Total 10 marks)

Q5



Question 4 continued

Leave
blank



Question 4 continued

Leave
blank



Question 4 continued

Leave
blank

Lined area for writing the answer to Question 4.

(Total 10 marks)

Q4



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 10 marks)



Question 2 continued

Lined area for writing the answer to Question 2 continued.



Question 2 continued

Lined area for writing the answer to Question 2 continued.



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 = 10 marks)



Question 1 continued

(Total for Question 1 is 10 marks)



Question 5 continued

Leave
blank

(Total 11 marks)

Q5



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 11 marks)



Solutions relying on calculator technology are not acceptable.

At time t seconds, the speed, $v \text{ m s}^{-1}$, of P is given by the continuous function



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 13 marks)



SUB-TOPIC M2.1b

PROJECTILES

Sub-topic M2.1b · 16 questions · 195 marks · lowest tariff first



(Total for Question 4 is 6 marks)



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 10 marks)



7. A small stone is projected from a point O on horizontal ground.

The stone is projected with speed 20 m s^{-1} at an angle α to the ground.

When the stone is 11 m above the ground

- the vertical velocity component of the stone has magnitude $V \text{ m s}^{-1}$
- the horizontal velocity component of the stone has magnitude $2V \text{ m s}^{-1}$

In a model of the motion

- the stone is modelled as a particle moving freely under gravity
- **the acceleration due to gravity is modelled as being 10 m s^{-2}**

Using the model and the principle of conservation of mechanical energy,

(a) show that $V = 6$

(4)

Figure 3 shows a plane fixed to the ground at an angle of 60°

The stone hits this plane at the point A .

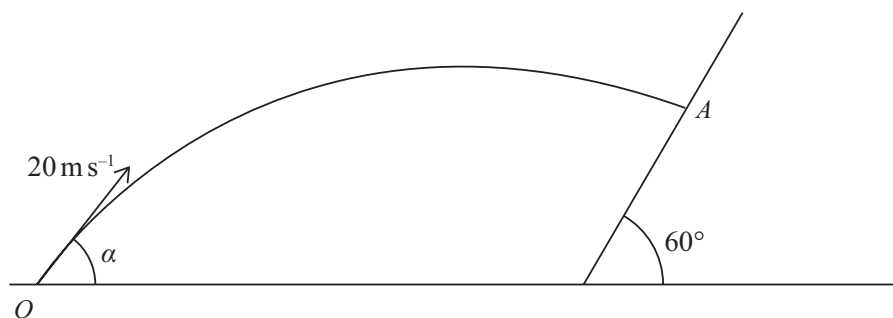


Figure 3

The direction of the motion of the stone is perpendicular to the plane at the instant the stone hits A .

Using the model,

(b) find the height of A above the ground, giving your answer to 2 significant figures.

(6)

Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 7 continued

Leave
blank



Leave
blank



Question 7 continued

Leave
blank

(Total 11 marks)

Q7



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS



Leave
blank

Leave
blank



Question 7 continued

Leave
blank

Handwriting practice area with 25 horizontal lines.

(Total 12 marks)

Q7	
----	--



8. A particle P is projected from a fixed point O . The particle is projected with speed $u \text{ ms}^{-1}$ at angle α above the horizontal. The particle moves freely under gravity. At the instant when the horizontal distance of P from O is x metres, P is y metres vertically above the level of O .

(a) Show that $y = x \tan \alpha - \frac{gx^2}{2u^2} (1 + \tan^2 \alpha)$

(6)

A small ball is projected from a fixed point A with speed $U \text{ m s}^{-1}$ at θ° above the horizontal.

The point B is on horizontal ground and is vertically below the point A , with $AB = 20$ m.

The ball hits the ground at the point C , where $BC = 30\text{m}$, as shown in Figure 4.

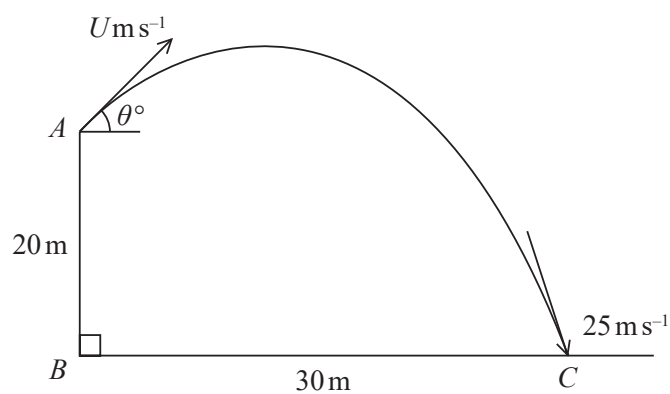


Figure 4

The speed of the ball immediately before it hits the ground is 25 m s^{-1}

The motion of the ball is modelled as that of a particle moving freely under gravity.

- (b) Use the principle of conservation of mechanical energy to find the value of U .

(3)

- (c) Find the value of θ

(3)



Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4 continued.



(Total for Question 4 is 12 marks)



Question 5 continued

Lined area for writing the answer to Question 5.



Question 5 continued

Lined area for writing the answer to Question 5.



(Total for Question 3 is 12 marks)



Leave
blank

Leave
blank



Leave
blank

Question 8 continued

Leave
blank

Lined area for writing the answer to Question 8.

(Total 13 marks)

Q8

END

TOTAL FOR PAPER: 75 MARKS



Question 7 continued

Leave
blank



Question 7 continued

Leave
blank



Question 7 continued

Leave
blank

Handwriting practice area with 25 horizontal lines.

(Total 14 marks)

Q7	
----	--



Leave
blank



Question 8 continued

Leave
blank

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Leave
blank

Lined area for writing the answer to Question 8 continued.



Leave
blank

Q8

TOTAL FOR PAPER IS 75 MARKS



Leave
blank

8. [In this question, the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane, with $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertically upwards.]

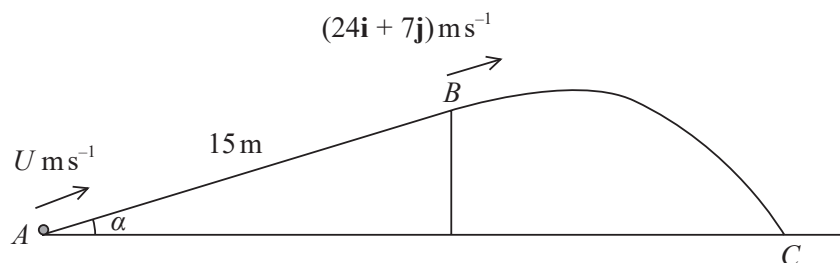


Figure 3

A rough ramp is fixed to horizontal ground.

The ramp is inclined to the ground at an angle α , where $\tan \alpha = \frac{7}{24}$

The point A is at the bottom of the ramp and the point B is at the top of the ramp. The line AB is a line of greatest slope of the ramp and $AB = 15 \text{ m}$, as shown in Figure 3.

A particle P of mass 0.3 kg is projected with speed $U \text{ m s}^{-1}$ from A directly towards B .

At the instant P reaches the point B , the velocity of P is $(24\mathbf{i} + 7\mathbf{j}) \text{ m s}^{-1}$

The particle leaves the ramp at B , and moves freely under gravity until it hits the horizontal ground at the point C .

The coefficient of friction between P and the ramp is $\frac{1}{5}$

- (a) Find the work done against friction as P moves from A to B . (3)

- (b) Use the work-energy principle to find the value of U . (4)

- (c) Find the time taken by P to move from B to C . (3)

At the instant immediately before P hits the ground at C , the particle is moving downwards at θ° to the horizontal.

- (d) Find the value of θ (4)

Question 8 continued

Leave
blank

Handwriting practice area with 30 horizontal lines.



Question 8 continued

Leave
blank

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Leave
blank

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Leave
blank

Handwriting practice area with horizontal lines.

Q8

(Total 14 marks)

TOTAL FOR PAPER IS 75 MARKS

END



7. [In this question, the perpendicular unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane with $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertically upwards.]

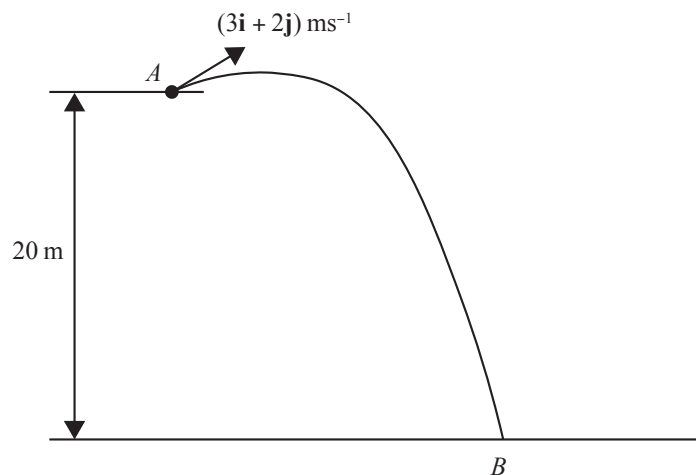


Figure 4

A small ball is projected with velocity $(3\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$ from the fixed point A .

The point A is 20 m above horizontal ground.

The ball hits the ground at the point B , as shown in Figure 4.

The ball is modelled as a particle moving freely under gravity.

- (a) By considering energy, find the speed of the ball at the instant immediately before it hits the ground.

(3)

- (b) Find the direction of motion of the ball at the instant immediately before it hits the ground.

(3)

- (c) Find the time taken for the ball to travel from A to B .

(3)

At the instant when the direction of motion of the ball is perpendicular to $(3\mathbf{i} + 2\mathbf{j})$ the ball is h metres above the ground.

- (d) Find the value of h .

(6)

Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 = 15 marks)

TOTAL FOR PAPER IS 75 MARKS

END



Leave
blank

7. A particle P is projected from a fixed point O on horizontal ground. The particle is projected with speed u at an angle α above the horizontal. At the instant when the horizontal distance of P from O is x , the vertical distance of P above the ground is y . The motion of P is modelled as that of a particle moving freely under gravity.

(a) Show that $y = x \tan \alpha - \frac{gx^2}{2u^2} (1 + \tan^2 \alpha)$ (6)

A small ball is projected from the fixed point O on horizontal ground. The ball is projected with speed 20 m s^{-1} at angle θ° above the horizontal. A vertical pole AB , of height 2 m , stands on the ground with $OA = 10 \text{ m}$, as shown in Figure 3.

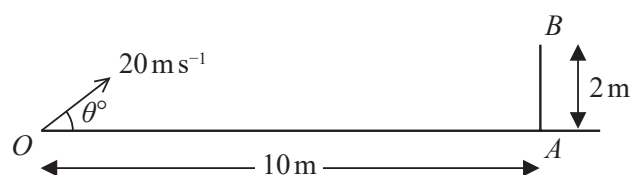


Figure 3

The ball is modelled as a particle moving freely under gravity and the pole is modelled as a rod.

The path of the ball lies in the vertical plane containing O , A and B .

Using the model,

- (b) find the range of values of θ for which the ball will pass over the pole. (3)

Given that $\theta = 40^\circ$ and that the ball first hits the ground at the point C

- (c) find the speed of the ball at the instant it passes over the pole, (5)

- (d) find the distance OC . (2)

Leave
blank

Question 7 continued

Leave
blank

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Leave
blank

Handwriting practice area with 30 horizontal lines.



Question 7 continued

Leave
blank

Lined area for writing the answer to Question 7.

(Total 16 marks)

Q7

END

TOTAL FOR PAPER IS 75 MARKS



MARK SCHEMES

official mark schemes, in the same order as the questions



Question	Scheme	Mark	Notes
1	Accept column vectors throughout this question		
1a	Differentiate $\mathbf{r}$ (both components)	M1	In each component at least one power going down by 1
	$\mathbf{v} = (4t^3 - 16t)\mathbf{i} + (12t - 3\sqrt{t})\mathbf{j}$	A1	Accept as two separate components
	Equate $\mathbf{i}$ component of $\mathbf{v}$ to zero and solve for t	DM1	Dependent on the first M1. Must start with a component of the vector for $\mathbf{v}$ Can have more than one value at this stage.
	Obtain $(24 - 3\sqrt{2})\mathbf{j} \text{ (ms}^{-1}\text{)}$	A1	Accept $20\mathbf{j} \text{ (ms}^{-1}\text{)}$ or better. (19.757359...) Correct answer only Answer must be a vector
		[4]	
1b	Differentiate $\mathbf{v}$ (both components)	M1	For differentiating their $\mathbf{v}$, even if the method for obtaining it was incorrect. Their $\mathbf{v}$ must be a vector. In each component at least one power going down by 1
	Obtain $\mathbf{a} = (12t^2 - 16)\mathbf{i} + (12 - \frac{3}{2}t^{-\frac{1}{2}})\mathbf{j}$	A1	Any equivalent form for acceleration
	Obtain $176\mathbf{i} + \frac{45}{4}\mathbf{j} \text{ (ms}^{-2}\text{)}$	A1	Accept $180\mathbf{i} + 11\mathbf{j} \text{ (ms}^{-2}\text{)}$ or better ISW
		[3]	
		(7)	

2.	$\frac{d}{dt}(4t^{\frac{1}{2}}\mathbf{i} - 3t\mathbf{j})$	M1
	$2t^{-\frac{1}{2}}\mathbf{i} - 3\mathbf{j}$	A1
	$t = 4: \mathbf{i} - 3\mathbf{j}$	M1
	$(6\mathbf{i} - 3\mathbf{j}) = 3(\mathbf{v} - (\mathbf{i} - 3\mathbf{j}))$	M1
	$\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$	A1
	Speed = $\sqrt{3^2 + (-4)^2} = 5 \text{ (ms}^{-1}\text{)}$	M1A1
		(7)
	Notes for question 2	
	N.B. Column vectors are acceptable throughout.	
2.	M1 for differentiating $\mathbf{r}$ wrt to t with both powers decreasing by 1, must be a vector. M0 if $\mathbf{i}$ or $\mathbf{j}$ is missing and never reappear(s).	
	A1 Correct vector	
	M1 for putting $t = 4$ in their $\mathbf{v}$, allow a slip, must have attempted to differentiate e.g. $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ seen.	
	M1 for use of impulse-momentum (M0 if g included or 3 missing) to form an equation in $\mathbf{v}$ only using their $\mathbf{u}$, which must have come from an attempt at finding a velocity at $t = 4$. M0 if they use $\mathbf{r}$ at $t = 4$. Condone $(6\mathbf{i} - 3\mathbf{j}) = 3((\mathbf{i} - 3\mathbf{j}) - \mathbf{v})$ and a slip.	
	A1 correct $\mathbf{v}$, seen or implied.	
	M1 Use of Pythagoras on their $\mathbf{v}$	
	A1 cao from a correct v	

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\surd$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - $\square$ or d... The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Q	Solution	Mark	Guidance
1a			Allow column vectors.
	Use of $\mathbf{v} = \frac{d\mathbf{r}}{dt}$	M1	Powers going down by 1. At least 2 powers going down .
	$\mathbf{v} = (3t^2 - 8)\mathbf{i} + (t^2 - 2t + 2)\mathbf{j}$	A1	Any equivalent form
	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$	M1	Powers going down by 1. At least 2 powers going down .
	$\mathbf{a} = 6t\mathbf{i} + (2t - 2)\mathbf{j}$	A1	Any equivalent form
	$= 24\mathbf{i} + 6\mathbf{j}(\text{ms}^{-2})$	A1	Must see acceleration stated as a correct simplified vector. ISW
		[5]	
1b	Direction $2\mathbf{i} + \mathbf{j}$	M1	Form equation in t or T only using direction. Condone use of 2 on the wrong side. Using their $\mathbf{v}$
	$\Rightarrow (3T^2 - 8) = 2(T^2 - 2T + 2)$ $(T^2 + 4T - 12 = 0)$	A1ft	Correct unsimplified equation in t or T . Solving not required for the M1 Follow their $\mathbf{v}$: $\mathbf{i}$ component = $2(\mathbf{j}$ component)
	$T = 2$	A1	Only Do not need to see method of solution.
		[3]	
		(8)	

Q	Scheme	Mark	Notes
1a	Use of $v = \frac{dx}{dt}$	M1	At least 2 powers going down by 1. Clear division by t is M0
	$v = 6t^2 - 42t + 60$	A1	Correct only
	Set $v = 0$ and correctly solves to obtain 2 values for t	M1	Complete method to obtain both values (implied by correct answers seen) $(0 = t^2 - 7t + 10 = (t - 2)(t - 5))$
	Obtain $t = 2$ and $t = 5$	A1	Correct only. Allow 2.0, 5.0
		[4]	
1b	Distance = $ x_2 - x_1 + x_3 - x_2 $ $(= 45 - 52 + 52 - 41)$	M1	Correct strategy dependent on their t being in $1 < t < 3$
	$= 11 + 7 = 18 \text{ (m)}$	A1	Correct only
		[2]	
1c	Use of $a = \frac{dv}{dt}$	M1	Differentiate their v . Clear division by t is M0. A power going down by 1 $(a = 12t - 42)$
	Obtain $6 \text{ (ms}^{-2}\text{)}$	A1	Must be positive – the Q asks for magnitude
		[2]	
		(8)	

QUESTION NUMBER	SCHEME	MARKS
1	N.B. Calculator warning : 'Not entirely'	
1(a)	Differentiate displacement to find an expression for velocity.	M1
	$v = 8t^3 - 42t^2 + 45t + 14$	A1
	Substitutes $t = 2$ to find $v = 0$ Or solves $v = 0$ to give $t = -\frac{1}{4}, 2, 3.5$ N.B. Only stating with no working $t=2, v=0$ or $v=0, t=2$ is A0	A1*
		(3)
1(b)	Complete method to find total distance = $ x_2 - x_0 + x_3 - x_2 $	M1
	$\frac{95}{2}$ (m)	A1
		(2)
1(c)	Differentiates velocity $24t^2 - 84t + 45$	M1
	Substitute $t = 1.5$ into differentiated expression	M1
	Correct magnitude of acceleration $27 \text{ (ms}^{-2}\text{)}$	A1
		(3)
		(8)
	Notes for question	
1(a)		
M1	Differentiate given expression with at least two powers of t decreasing by 1.	
A1	Correct differentiated expression.	
A1*	Obtain $t=2$ from correct and complete working. If solving equation, and if any incorrect roots appear A0.	
1(b)		
M1	Complete method to find the total distance. Do not condone sign errors. ($x_0=0, x_2=38, x_3=28.5$)	
A1	Correct distance, must be positive	
1(c)		
M1	Differentiate velocity to find an expression for acceleration with at least two powers of t decreasing by 1.	
M1	Substitute 1.5 into their differentiated velocity.	
A1	Correct answer, must be positive.	

3a	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ $(\mathbf{a} = 18 \cos 3t \mathbf{i} - 2 \sin t \mathbf{j})$	M1	Differentiate to obtain $\mathbf{a} = \lambda \cos 3t \mathbf{i} + \mu \sin t \mathbf{j}$
	Use of $\mathbf{F} = m\mathbf{a} : \mathbf{F} = \frac{1}{4}\mathbf{a}$	M1	Must be working in vectors
	$\mathbf{F} = \frac{9}{2} \cos 3t \mathbf{i} - \frac{1}{2} \sin t \mathbf{j}$	A1	Or equivalent. e.g. as a column vector
		[3]	
3b	$2 \cos t + 1 = 0$	M1	Set $\mathbf{j}$ component of $\mathbf{v} = 0$ and solve for t
	$\Rightarrow t = \frac{2\pi}{3}$	A1	ISW if correct answer seen. Only answer 120° scores A0 here and the final A0
	Use of $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $(\mathbf{r} = -2 \cos 3t \mathbf{i} + (t + 2 \sin t) \mathbf{j} (+\mathbf{C}))$	M1	Integrate $\mathbf{v}$ with respect to t to obtain $\mathbf{r} = p \cos 3t \mathbf{i} + (t + q \sin t) \mathbf{j} (+\mathbf{C})$ Condone if there is no constant of integration.
	$t = 0, \quad \mathbf{r} = (4\mathbf{i} - \sqrt{3}\mathbf{j})\text{m}$ $\mathbf{r} = (-2 \cos 3t + 6) \mathbf{i} + (t + 2 \sin t - \sqrt{3}) \mathbf{j}$	M1	Correct use of boundary condition to find their $\mathbf{C}$. Could be part of a definite integral e.g. $4\mathbf{i} - \sqrt{3}\mathbf{j} + \int_0^t 6 \sin 3t \mathbf{i} + (1 + 2 \cos t) \mathbf{j} dt$ for their upper limit
	$= 4\mathbf{i} + \frac{2\pi}{3} \mathbf{j} \quad (\text{m})$	A1 A1	Accept $4\mathbf{i} + 2.1\mathbf{j}$ or better one component correct both components correct ISW if they also offer $4\mathbf{i} + 120\mathbf{j}$ “correct” components after an M0 are fortuitous – A0
		[6]	
		(9)	

Question Number	Scheme	Marks
1(a)	$\int k(3-t^2)dt$	M1
	$= k(3t - \frac{1}{3}t^3) + (C)$	A1
	$t = 0, v = 0 \Rightarrow C = 0$ and $t = 1.5, v = 13.5$ OR $\left[k(3t - \frac{1}{3}t^3) \right]_0^{1.5} = 13.5$ $\Rightarrow k(3 \times 1.5 - \frac{1}{3} \times 1.5^3) = 13.5 \Rightarrow k = 4^*$	A1*
		(3)
1(b)	$s = \int 4(3t - \frac{1}{3}t^3)dt$	M1
	$s = 4(3 \times \frac{1}{2}t^2 - \frac{1}{3} \times \frac{1}{4}t^4) = \frac{1}{3}t^2(18-t^2) + D$ and $t = 0, s = 0 \Rightarrow D = 0$ OR $s = \left[4(3 \times \frac{1}{2}t^2 - \frac{1}{3} \times \frac{1}{4}t^4) \right]_0^t$ so $s = \frac{1}{3}t^2(18-t^2)^*$ Accept any equivalent factorised form e.g. $s = \frac{(18-t^2)t^2}{3}$	A1*
		(2)
1(c)	$v = 0 \Rightarrow (k)(3t - \frac{1}{3}t^3) = 0$	M1
	$t = 3$	A1
	When $t = 3, s = 27$	M1
	Total distance = 54 (m)	A1
		(4)
	Notes for question 1	(9)
1(a)	M1 Integrate a wrt t , with both powers increasing by 1	
	A1 Correct integral	
	A1* $k = 4$ correctly obtained. $t = 0$ and $v = 0$ must be referred to but do not need to be actually substituted in. We must see the substitution of 1.5 and 13.5	
1(b)	M1 Integrate <i>their</i> v wrt t , with both powers increasing by 1, where v has come from a valid attempt at an integral of a .	
	N.B. Allow without k being substituted.	
	A1* Given answer correctly obtained including “ $s =$ ” Must see reference to $t = 0, s = 0$ but do not need to be actually substituted in.	
1(c)	M1 Equate their v to 0 and solve for t	
	A1 cao	
	M1 Put their non-zero t , which must have come from $v = 0$, into s	

Question Number	Scheme	Marks
	A1 cao	
	N.B. $t = 3$ could come from a correct sketch of the graph of their v	

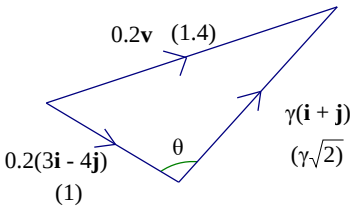


Question Number	Scheme	Marks
5a	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$: $\mathbf{a} = 6t\mathbf{i} + 2\mathbf{j}$ $t = 0 \Rightarrow \mathbf{a} = 2\mathbf{j} \text{ (ms}^{-2}\text{)}$	M1 A1 (2)
5b	$11(2t - 4) = (3t^2 - 4)$ $3t^2 - 22t + 40 = 0 \Rightarrow \left(t = \frac{10}{3}\right) t = 4$ $\mathbf{v} = 44\mathbf{i} + 4\mathbf{j}$, speed = $\sqrt{44^2 + 4^2}$ $= 4\sqrt{122} \text{ (m s}^{-1}\text{)}$	M1 M1 DM1 A1 (4)
5c	Use of $\mathbf{r} = \int \mathbf{v} dt$ $\mathbf{r} = (t^3 - 4t)\mathbf{i} + (t^2 - 4t)\mathbf{j}$ Set $\mathbf{r} = 0$ and solve for t $t^3 - 4t = 0 \Rightarrow t = 0, 2, (-2)$ $t^2 - 4t = 0 \Rightarrow t = 0, 4$ the only common value is $t = 0$, so does not return to O .*	M1 A1 M1 A1* (4)
Notes		[10]
5a M1 A1	Powers going down Must see vector answer but ISW if go on to state the magnitude.	
5b M1	Use of velocity parallel to $11\mathbf{i} + \mathbf{j}$ 11 must be on the correct side.	
DM1	Select the larger root (dependent on the previous 2 M1 marks and on 2 positive roots) and use Pythagoras. Condone if they find both speeds	
A1	Any equivalent simplified surd form $(\sqrt{1952})$ ISW 44.18... implies M1 if correct surd form not seen. Both values for speed given is A0	
M1	Powers going up	
A1	If a constant of integration is introduced, they must conclude it is equal to the zero vector	
M1	Consider both components	
A1*	Or equivalent clear explanation of given result . Condone if they ignore $t = 0$. Do not need to see the roots. But do need to see the factorised form for each component if using this method.	

Question	Solution	Marks	Notes
3			
	Use of $F = \mu R$	B1	At least once
	Resolve horizontally	M1	Allow with their horizontal friction
	$S = \frac{4}{5} R \quad (S = F_A)$	A1	Correct unsimplified equation
	Resolve vertically	M1	Allow with their vertical friction
	$\frac{3}{5} S + R = 25g \quad F_B + R = 25g$ $\left(\frac{3}{5} S + \frac{5}{4} S = 25g, \quad S = \frac{500}{37} g \right)$	A1	Correct unsimplified equation
	Moments equation	M1	Any moments equation. Need all terms & dimensionally correct
	$M(A): 25g \times 1.5 \cos \theta = S \times 3 \sin \theta + \frac{3}{5} S \times 3 \cos \theta$ $\left(25g \cos \theta - \frac{6}{5} S \cos \theta = 2S \sin \theta \right)$ $M(B): R \times 3 \cos \theta = 25g \times 1.5 \cos \theta + \frac{4}{5} R \times 3 \sin \theta$	A1	Correct unsimplified equation
	M1A1 for first equation, M1A1 for second equation, M1A1 for third equation (i.e. mark in the order in which they appear rather than as listed on the mark scheme). If there are more than 3 equations, mark the 3 used or the best 3 if they go no further. Can also be solved using one resolution and two moments equations. Friction acting in the wrong direction scores A0.		
	$\tan \theta = \left(\frac{25g - \frac{6}{5} S}{2S} \right) = \frac{25 - \frac{600}{37}}{\frac{1000}{37}}$	DM1	Substitute to form equation in $\tan \theta$ only Condone in decimals Dependent on M marks for the equations
	$= \frac{325}{1000} \left(= \frac{13}{40} \right)$	A1	Or exact equivalent (0.325)
		(9)	
SC	It is possible to solve by resolving horizontally or vertically and taking moments about the centre: $1.5 \cos \theta \times R = 1.5 \cos \theta \times \frac{3}{5} S$ $+ 1.5 \sin \theta \times S + 1.5 \sin \theta \times \frac{4}{5} R$		M1A1 for a correct resolution M2A2 for a complete sets of equations to solve
		[9]	

Question Number	Solution	Marks	Notes															
4a	<table><tr><td></td><td><i>ABCD</i></td><td><i>PQRV</i></td><td><i>RSTU</i></td><td><i>L</i></td></tr><tr><td>Mass ratio</td><td>64</td><td>4</td><td>16</td><td>44</td></tr><tr><td>c of m from <i>AD</i></td><td>$4a$</td><td>$2a$</td><td>$5a$</td><td>(d)</td></tr></table>		<i>ABCD</i>	<i>PQRV</i>	<i>RSTU</i>	<i>L</i>	Mass ratio	64	4	16	44	c of m from <i>AD</i>	$4a$	$2a$	$5a$	(d)	B1	Correct mass ratios for their split
		<i>ABCD</i>	<i>PQRV</i>	<i>RSTU</i>	<i>L</i>													
	Mass ratio	64	4	16	44													
c of m from <i>AD</i>	$4a$	$2a$	$5a$	(d)														
B1	Correct distances from vertical axis for their split Must be multiples of a																	
	M(<i>AD</i>)	M1	Moments about <i>AD</i> or a parallel axis. Need all terms and dimensionally consistent.															
	$64 \times 4a - 4 \times 2a - 16 \times 5a = 44d$	A1	Correct unsimplified equation Accept as part of a vector equation															
	$\Rightarrow d = \frac{168}{44}a = \frac{42}{11}a$ *	A1*	Obtain given answer from correct working															
		(5)																
4b	C of M of <i>L</i> lies at midpt of <i>AC</i>	B1	Seen or implied															
	M(Mid pt <i>AB</i>)	M1	Use of moments to form equation in k .															
	$\left(4 - \frac{42}{11}\right)aM = 4akM$	A1	Correct unsimplified equation. Allow with a not seen															
	$k = \frac{1}{22}$	A1	0.05 or better (0.0454545...) Allow with a not seen															
		(4)																
4b alt	C of M of <i>L</i> lies at midpt of <i>AC</i>	B1	Seen or implied by use of $\bar{x} = \bar{y}$ or $\tan 45^\circ = 1$															
	Find $\bar{x}$ and $\bar{y}$ for system	M1																
	From <i>AB</i> : $\frac{42}{11}Ma + 8akM = (1+k)M\bar{y}$ From <i>BC</i> : $\frac{46}{11}aM = (1+k)M\bar{x}$	A1	Correct unsimplified equations in $\bar{x}$ and $\bar{y}$ Allow with a not seen															
	$\bar{x} = \bar{y} \Rightarrow \frac{42}{11} + 8k = \frac{46}{11} \Rightarrow k = \frac{1}{22}$	A1	Allow with a not seen															
4b alt	C of M of <i>L</i> lies at midpt of <i>AC</i>	B1	Seen or implied in moments equation															
	If <i>G</i> is c of m of <i>L</i> then $\tan ABG = \frac{42}{46}$ and take moments about <i>B</i>	M1	Complete method for moments about <i>B</i>															
	$8a \sin 45^\circ \times kM$ $= \frac{Ma\sqrt{46^2 + 42^2}}{11} \sin(45^\circ - ABG)$	A1	Correct unsimplified equation in k Allow with a not seen															
	$\Rightarrow k = \frac{1}{22}$	A1	Allow with a not seen															
4b alt	C of M of <i>L</i> lies at midpt of <i>AC</i>	B1	Seen or implied in moments equation															

	Take moments about the centre of $ABCD$	M1	
	$M \times \frac{2\sqrt{2}}{11}a = kM \times 4\sqrt{2}a$	A1	Correct unsimplified equation in k Allow with a not seen
	$\Rightarrow k = \frac{1}{22}$	A1	Allow with a not seen
		[9]	
Question Number	Solution	Marks	Notes
5a	$\mathbf{a} = \frac{d\mathbf{v}}{dt}$	M1	Differentiate to obtain $\mathbf{a}$ – powers going down
	$= (6t - 9)\mathbf{i} + (2t + 1)\mathbf{j}$	A1	differentiation correct
	$= 9\mathbf{i} + 7\mathbf{j} \text{ (m s}^{-2}\text{)}$	A1	ISW if go on to find $ \mathbf{a} $
		(3)	
5b	Instantaneous rest $\Rightarrow \mathbf{v} = 0\mathbf{i} + 0\mathbf{j}$ $\Rightarrow 3(t - 1)(t - 2) = 0$ and $(t - 2)(t + 3) = 0$	M1	Set $\mathbf{v} = 0$ and solve for t (Need both components equal to zero)
	$\Rightarrow t = 2$	A1	
	$\mathbf{r} = \int \mathbf{v} dt$	M1	Integrate to obtain $\mathbf{r}$ – powers going up. Condone if no constant of integration seen.
	$= \left(t^3 - \frac{9}{2}t^2 + 6t\right)\mathbf{i} + \left(\frac{1}{3}t^3 + \frac{1}{2}t^2 - 6t\right)\mathbf{j}$	A1 A1	At most one error Correct integration Allow column vector. Allow A1A0 for correct integration and non-zero constants(s) of integration
	$= 2\mathbf{i} - \frac{22}{3}\mathbf{j}$, distance $= \sqrt{2^2 + \left(\frac{22}{3}\right)^2}$	DM1	Correct strategy to find the distance, i.e. substitute their value for t and use Pythagoras Dependent on the two preceding M marks
	$= \frac{2\sqrt{130}}{3} = 7.60 \text{ (m)}$	A1	7.6 or better from correct work
		(7)	
		[10]	

Q	Mark Scheme	Marks	Marking guidance
3.	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	Accept equivalent e.g. $\mathbf{I} + m\mathbf{u} = m\mathbf{v}$. Dimensionally correct and must be using subtraction (but could be the wrong way round). The use of 7 in place of the velocity in the impulse momentum equation is M0 unless they recover. See below
	$0.2(\mathbf{v} - 4\mathbf{i} + 3\mathbf{j}) = \lambda(\mathbf{i} + \mathbf{j})$ $((x-4)\mathbf{i} + (y+3)\mathbf{j}) = 5\lambda\mathbf{i} + 5\lambda\mathbf{j}$	A1	Correct unsimplified vector equation or pair of separate equations for the $\mathbf{i}$ and $\mathbf{j}$ components. Condone column vectors with $\mathbf{i}$ and $\mathbf{j}$ included in the components.
	Use of Pythagoras for the speed	M1	Correct use of Pythagoras and 49 for their speed
	$x^2 + y^2 = 49$	A1	Correct unsimplified equation for their x, y
	Form quadratic in x, y or λ and solve for λ	DM1	Dependent on both previous M marks. $x^2 + -(x-7)^2 = 49$ or $(y+7)^2 + -y^2 = 49$ or $(5\lambda+4)^2 + (5\lambda-3)^2 = 49$
	$\lambda = \frac{3}{5}$ or $\lambda = -\frac{4}{5}$	A1	Or equivalent
** ** ** ** ** ** **	Special case: Candidates who use 7 as a vector can score a maximum of M1A0M1A0 for $1.4^2 = +(\lambda \cdot 0.8)^2 + -(\lambda \cdot 0.6)^2$ or equivalent DM1A0 for forming and solving a quadratic in λ .		This maximum of 3 marks is only available for those candidates who “recover”. So, if all you see is $\lambda \cdot \mathbf{i} + -\mathbf{j}$ $1.4 \quad 0.2 \quad 4(\mathbf{i}-3\mathbf{j})$ they score M0M0M0 If they recover to go on to form a “sensible” equation using Pythagoras then they can score the first 2 M marks, and potentially the third M1 as well.
		(6)	
3 alt			
	Form vector triangle	M1	Dimensionally correct. Allow incorrect configuration
	Correct triangle and correct lengths	A1	In speeds or momentum but not a mixture
	Use scalar product to find cosine of angle	M1	Or equivalent method
	$\cos \theta = -\frac{1}{5\sqrt{2}}$	A1	Allow $\pm$
	Form equation in λ $(2\lambda^2 + .4\lambda - 0.96 = 0)$	DM1	e.g. by use of cosine rule Dependent on the first 2 M marks
	$\lambda = \frac{3}{5}$ or $\lambda = -\frac{4}{5}$	A1	Or equivalent
		(6)	

4a	Use $t = 2$ and $3t^2 + 2t = t^3 + kt$ $(12 + 4 = 8 + 2k)$	M1	Allow verification.
	$k = 4$ *	A1*	Obtain given answer from correct working. Verification requires a clear conclusion.
		2	
4b	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$	M1	Differentiate the vector $\mathbf{v}$ Majority of powers going down
	$\mathbf{a} = (6t + 2)\mathbf{i} + (3t^2 + 4)\mathbf{j}$	A1	Correct only
	Use $ \mathbf{F} = m \mathbf{a} $	DM1	Correct use of Pythagoras and N2L Dependent on the preceding M1
	$ \mathbf{F} = 1.5 \times \sqrt{14^2 + 16^2} = 3\sqrt{113}$	A1	Or $\frac{3}{2}\sqrt{452}$ or 32 or better (31.89....)
		4	
4c	Use of $\mathbf{r} = \int \mathbf{v} dt$	M1	Majority of powers going up
	$\mathbf{r} = (t^3 + t^2(+A))\mathbf{i} + \left(\frac{1}{4}t^4 + \frac{4}{2}t^2(+B)\right)\mathbf{j}$	A1	Allow without constant of integration
	Correct use of $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$ when $t = 0$ to find $\mathbf{r}$ when $t = 2$	DM1	$\left(\mathbf{r} = (t^3 + t^2 + 3)\mathbf{i} + \left(\frac{1}{4}t^4 + \frac{4}{2}t^2 + 4\right)\mathbf{j}\right)$ Dependent on the preceding M1 Use of $\mathbf{r} = -3\mathbf{i} - 4\mathbf{j}$ is M0
	$\mathbf{r} = 15\mathbf{i} + 16\mathbf{j}$	A1	Correct answer only. Accept column vector
		4	
		(10)	

Question	Scheme	Marks	Notes
Accept column vectors throughout			
2a	Use of $\mathbf{r} = \int \mathbf{v} dt$	M1	Powers going up by 1. Allow one slip in the powers
	$\mathbf{r} = \left(\frac{4}{3}t^3 - \frac{5}{2}t^2 + A \right) \mathbf{i} + (-5t^2 - 12t + B) \mathbf{j}$	A1	Allow without constant of integration
	Use $t = 2$ and $\mathbf{r} = 2\mathbf{i} + 6\mathbf{j}$ when $t = 0$: $\mathbf{r} = \left(\frac{4}{3} \times 8 - \frac{5}{2} \times 4 + 2 \right) \mathbf{i} + (-5 \times 4 - 12 \times 2 + 6) \mathbf{j}$	M1	Correct use of given value to obtain $\mathbf{r}$
	$= \frac{8}{3} \mathbf{i} - 38 \mathbf{j}$	A1	Correct answer only Allow 2.7 or better ISW if they go on to find the magnitude.
		(4)	
2b	$\mathbf{v}$ in direction of $\mathbf{i} - 2\mathbf{j}$	M1	Use velocity and direction to form an equation in T Condone if they have (-)2 on the wrong side of their equation
	$\Rightarrow -2(4T^2 - 5T) = (-10T - 12)$ $(8T^2 - 20T - 12 = 0)$	A1	Correct unsimplified equation in T (or t) only
	$\Rightarrow T = 3$	A1	Only. Allow $t = 3$.
		(3)	
2c	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ ($\mathbf{a} = (8t - 5)\mathbf{i} - 10\mathbf{j}$)	M1	Powers going down by 1 Allow one slip in the powers
	Use of Pythagoras and $t = 2.5$	M1	Correct use of their derivative to obtain acceleration
	$ a = \sqrt{(20 - 5)^2 + 10^2} = \sqrt{325} (= 5\sqrt{13}) \text{ ms}^{-2}$	A1	Any equivalent simplified exact form. Ignore decimals after exact answer seen.
		(3)	

Question	Scheme					Marks	Notes
They must have a dissection for which they should know or find the position of the centre of mass (e.g. triangles and rectangles). A false assumption about the position of the centre of mass of a trapezium results in 0/5.							
3a		Large tri	Small tri	Small tri	Whole	B1	Correct distances from PQ or a parallel axis for their complete dissection
	Dist PQ	0	-2y	2y	d	B1	Correct mass ratios for a complete dissection
	Mass ratio	27xy	12xy	12xy	27xy		Correct distances from PQ or a parallel axis for their complete dissection
	Moments about PQ:					M1	Or a parallel axis. Dimensionally correct. Need all non-zero terms and no extras. Condone sign error(s). Allow for ±d. Check the logic carefully.
	$(27xy \times 0) - 12xy \times (-2y) + 12xy \times 2y = 27xyd$					A1	Correct unsimplified equation. Allow for ±d. Allow for correct distance from a parallel axis
	$d = \frac{48}{27}y = \frac{16}{9}y$ *					A1*	Obtain given result from fully correct working.
There are many different approaches to this. NB If they are using a trapezium they must show method for the distance. For PQBC the correct value for distance centre of mass from PQ is $\frac{8y}{5}$ Possible alternative moments equations include: $15xy \times \frac{8y}{5} + 9xy \times \frac{4y}{3} + 3xy \times 4y = 27xyd$ using PQBC, PQDE and DEA $12xy \times 2y + 15xy \times \frac{8y}{5} = 27xyd$ using PQA and PQBC $2 \times 3xy \times y - 3xy \times y + 2 \times 6xy \times 1.5y + 2 \times 3xy \times 2y = 27xyd$ working from BC for the folded figure. $2 \times 3xy \times 2y + 4 \times \frac{1}{2} 3xy \times y + 2 \times 6xy \times 1.5y + 3xy \times 4y = 27xyd$ working down from PQ							
						(5)	

Question	Scheme	Marks
1(a)	Use $\mathbf{r} = \int \mathbf{v} dt$	M1
	Correct integration Eg $(t^3 + 6t^2 + 12t)\mathbf{i} + \left(\frac{5}{3}t^3 + 5t^2\right)\mathbf{j} + (\mathbf{C})$ Or $(t+2)^3\mathbf{i} + \left(\frac{5}{3}t^3 + 5t^2\right)\mathbf{j} + (\mathbf{K})$	A1
	Complete method using $t = 0, \mathbf{r} = -30\mathbf{i} - 45\mathbf{j}(\text{m})$ and substitute $t = 3$ Indefinite integration Use of $t = 0, \mathbf{r} = -30\mathbf{i} - 45\mathbf{j}(\text{m})$ to find constant of integration and substitute $t = 3$. Definite integration Use of $\mathbf{r} = (-30\mathbf{i} - 45\mathbf{j}) + \int_0^3 \mathbf{v} dt$	DM1
	$\mathbf{r} = 87\mathbf{i} + 45\mathbf{j}(\text{m})$	A1
		(4)
1(b)	Use $\mathbf{a} = \frac{d\mathbf{v}}{dt}$	M1
	Correct differentiation $\mathbf{a} = (6t + 12)\mathbf{i} + (10t + 10)\mathbf{j}$	A1
	Substitute $t = 3$ and find magnitude	DM1
	$ \mathbf{a} = 50(\text{ms}^{-2})$	A1
		(4)
1(c)	$3(T+2)^2 = 10T(T+2)$ $(3T+6=10T) \quad (7T^2+8T-12=0)$	M1
	$T = \frac{6}{7}$	A1
		(2)
		(10)
	Notes	
1(a)		
M1	Integrate to obtain $\mathbf{r}$. Powers increase by 1 in both components. Condone working with separated components. Condone missing brackets with $\mathbf{i}$ - $\mathbf{j}$ notation. M0 for <i>suvat</i> .	
A1	Correct integration. Condone missing constant of integration.	
DM1	- Dependent on the preceding M1. - Must have a constant of integration before using $t = 3$ (unless using definite integration) - Must use $(-30\mathbf{i} - 45\mathbf{j})$ - Must substitute $t = 3$ Condone working with separated components. DM0 if substitution of $t = 3$ occurs before finding $+ C$.	

	$\left(t^3 + 6t^2 + 12t - 30\right)\mathbf{i} + \left(\frac{5}{3}t^3 + 5t^2 - 45\right)\mathbf{j} \quad C = (-30\mathbf{i} - 45\mathbf{j})$ $\left((t+2)^3 - 38\right)\mathbf{i} + \left(\frac{5}{3}t^3 + 5t^2 - 45\right)\mathbf{j} \quad K = (-38\mathbf{i} - 45\mathbf{j})$	
A1	Correct answer, accept column vector $\begin{pmatrix} 87 \\ 45 \end{pmatrix}$ A0 for poor notation in final answer eg $\begin{pmatrix} 87\mathbf{i} \\ 45\mathbf{j} \end{pmatrix}$ ISW if they continue and find $\mathbf{r}$	
1(b)		
M1	Differentiate to obtain a . Powers decrease by 1 in both components. Condone working with separated components and missing brackets with i-j notation. M0 for <i>suvat</i>	
A1	Correct differentiation	
M1	Dependent on the preceding M1	
	Use of Pythagoras seen or implied $\sqrt{30^2 + 40^2}$	
A1	Correct only	
1(c)		
M1	Correct method using the ratio of i and j components of v . Form an equation in <i>T</i> only. Accept working in <i>t</i> or <i>T</i> .	
A1	0.86 or better. Condone <i>t</i> instead of <i>T</i> .	

[illegible]

Question Number	Scheme	Marks	Notes
5(a)	$5T^2 - 12T + 15 = T^2 + 8T - 10$	M1	Parallel to $\mathbf{i} + \mathbf{j}$
	$\Rightarrow 4T^2 - 20T + 25 = 0$	A1	Correct quadratic in T
	$\Rightarrow T = \frac{5}{2}$	A1	
		[3]	
5(b)	$\mathbf{a} = (10t - 12)\mathbf{i} + (2t + 8)\mathbf{j}$	M1	Correct differentiation (at least 2 powers going down by one)
	$= 18\mathbf{i} + 14\mathbf{j}$	A1	
	$ \mathbf{a} = \sqrt{18^2 + 14^2}$	DM1	Use of Pythagoras to find magnitude. Dependent on preceding M1
	$= \sqrt{520} = 22.8 \text{ (m s}^{-2}\text{)}$	A1	23 or better e.g. $2\sqrt{130}$
		[4]	
5(c)	$\mathbf{s} = \left(\frac{5}{3}t^3 - 6t^2 + 15t\right)\mathbf{i} + \left(\frac{1}{3}t^3 + 4t^2 - 10t\right)\mathbf{j}$	M1	Integrate (at least 2 powers going up by one)
		A1	At most one error
		A1	All correct
	$= (45 - 54 + 45)\mathbf{i} + (9 + 36 - 30)\mathbf{j}$		
	$= 36\mathbf{i} + 15\mathbf{j} \text{ (m)}$	A1	
		[4]	
		[11]	

3a	Use $\mathbf{v} = \frac{d\mathbf{r}}{dt}$	M1	Differentiate the vector. At least 3 powers going down
	$\mathbf{v} = \left(3 - (t+1)^{-\frac{1}{2}}\right)\mathbf{i} + (2t-6)\mathbf{j}$	A1 A1	one component correct both components correct.
	$(2t-6) = 0 \Rightarrow t = 3$	M1	Equate their $\mathbf{j}$ component of velocity to zero and solve for t Must have seen a clear attempt to differentiate
	Speed = $2.5(\text{ms}^{-1})$ or equivalent	A1	Must be a scalar. A0 for $2.5\mathbf{i}$
		[5]	
3b	Use $\mathbf{a} = \frac{d\mathbf{v}}{dt}$	M1	Differentiate the vector. Powers going down and at least one constant goes to zero.
	$\mathbf{a} = \frac{1}{2}(t+1)^{-\frac{3}{2}}\mathbf{i} + 2\mathbf{j}$	A1	Or equivalent correct expression Allow if correct derivative implied by correct substitution
	$\left(\mathbf{a} = \frac{4}{\sqrt{125}}\mathbf{i} + 2\mathbf{j}\right) \mathbf{a} = \sqrt{\frac{16}{125} + 2^2}$	DM1	Correct use of Pythagoras Dependent on the preceding M1
	$= 2.03(\text{ms}^{-2})$ or better	A1	$\frac{2\sqrt{645}}{25}$ or 2.0317...
		[4]	
3c	For $\mathbf{r} = \mathbf{0}$, $(3t+2-2\sqrt{t+1}) = 0$ and $(t^2-6t) = 0$	M1	No need to consider $t = 0$ as this is excluded in the Q
	$t \neq 0 \Rightarrow t = 6$ but $(3 \times 6 + 2 - 2\sqrt{6+1}) \neq 0$ Hence no solution and does not return	A1	Clear explanation of the given result with no errors seen. e.g. $(3t+2-2\sqrt{t+1}) = 0$ $\Rightarrow 9t^2 + 8t = 0$ has no solution for $t > 0$ (need something to indicate impossible)
	There will be other alternatives e.g. Show that the horizontal component of the velocity is always > 2 , so no return M1 for correct strategy A1 for correct conclusion with no errors seen and sufficient justification. Conclusion needs to be clear but does not need to be the exact wording from the question. An argument dependent solely on acceleration is unlikely to work – do send to review if you find one worthy of merit.		
		[2]	
		(11)	

Q	Scheme	Marks	Notes
1(a)	Expression for total KE before collision	M1	Dimensionally correct. Condone confusion between before and after for A. Allow if vectors seen in the working but the modulus is used correctly. The two parts must be added together Allow confusion of 2 kg and 3 kg
	$\frac{1}{2} \times 2 \times 5^2 + \frac{1}{2} \times 3 \times (3^2 + (-1)^2)$	A1	correct unsimplified expression
	$= 40 \text{ (J)}$	A1	cao
		(3)	
1(b)	$2((3\mathbf{i} + 2\mathbf{j}) - 5\mathbf{j})$	M1	change in momentum of A, must be a difference but allow subtraction in either order Must be using the correct mass, 2 kg
	$= (6\mathbf{i} - 6\mathbf{j}) \text{ (N s)}$	A1	Cao The final answer should be in terms of $\mathbf{i}$ and $\mathbf{j}$ because this is asked for in the question. Accept $2(3\mathbf{i} - 3\mathbf{j})$ ISW
		(2)	
1(c)	impulse-momentum equation for B	M1	must use negative of their answer to (b) and the initial velocity of B Must be using the correct mass, 3 kg or CLM with correct terms (allow slip) and plus signs
	$3(\mathbf{v}_B - (3\mathbf{i} - \mathbf{j})) = (-6\mathbf{i} + 6\mathbf{j})$ or $2 \times 5\mathbf{j} + 3(3\mathbf{i} - \mathbf{j}) = 2(3\mathbf{i} + 2\mathbf{j}) + 3\mathbf{v}_B$	A1ft	correct unsimplified equation ft on their impulse from (b)
	$\mathbf{v}_B = (\mathbf{i} + \mathbf{j}) \text{ (m s}^{-1}\text{)}$	A1	Cao Accept column vector ISW
		(3)	
		(8)	

4	NB: For the whole of this question, confusion between horizontal and vertical is not a misread		
4a	$x = 3$	B1	Seen or implied anywhere Do not accept $x = 3\mathbf{i}$
	Use of $v^2 = u^2 + 2as$	M1	Complete method using <i>suvat</i> or energy to form an equation in y . Condone sign errors
	$15^2 = y^2 + 2 \times g \times 10$	A1	Correct unsimplified equation
	$y^2 = 29$, $y = 5.4$ or 5.39	A1	2 sf or 3 sf . If final answer is $y = 5.4\mathbf{j}$ do not penalise inclusion of a vector a second time.
			SC allow 4/4 for $x\mathbf{i} + y\mathbf{j} = 3\mathbf{i} + 5.4\mathbf{j}$
		[4]	
4a alt	$x = 3$	B1	Seen or implied anywhere Do not accept $x = 3\mathbf{i}$
	Equation for conservation of energy	M1	Require all 3 terms and no extras. Dimensionally correct. Condone sign errors. Must include m
	$\frac{1}{2}m \times (3^2 + 15^2) = mg \times 10 + \frac{1}{2}m(x^2 + y^2)$	A1	Correct unsimplified equation – any equivalent form
	$y^2 = 29$, $y = 5.4$ or 5.39	A1	2 sf or 3 sf . If final answer is $y = 5.4\mathbf{j}$ do not penalise inclusion of a vector a second time.
		[4]	
4b	Time from B to C :	M1	Complete method using <i>suvat</i> and their vertical speed. Condone sign errors
	$-15 = 5.39 - gt$ ($t = 2.08$)	A1ft	Correct equation in t only e.g. $10 = 15t - \frac{1}{2}gt^2$ ft on their 5.39 if used
	Horizontal distance $= 3t$ ($= \text{their } x \times \text{their } t$)	DM1	Complete method using <i>suvat</i> and their x value. Dependent on preceding M1
	$(AC =) 6.2(\text{m})$ or $6.24(\text{m})$	A1	2 sf or 3 sf NB Penalise over-accuracy only once per question
		[4]	
		(8)	



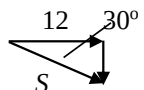
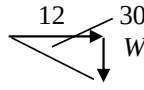
QUESTION NUMBER	SCHEME	MARKS
3	Accept column vectors throughout this question	
3(a)	Complete method to find greatest height (h) e.g. $0 = 14^2 + 2(-g)h$ $h = 10$	M1 A1 (2)
3(b)	Vertical component $v = 14 - g(2.4)$	M1
	Use Pythagoras to find $\text{Speed} = \sqrt{8^2 + (14 - 2.4g)^2}$	M1
	12.4 or 12 (ms^{-1})	A1
		(3)
3(c)	Relevant equation in t formed using vertical motion. e.g. $3 = 14t + \frac{1}{2}(-g)t^2$	M1 A1
	Use Horizontal motion to find the required distance $8t$	M1
	8x 2.6..	A1
	21 or 21.0 (m)	A1
		(5)
	ALT method forming trajectory equation	
	Relevant equation in t and y formed using vertical motion. $y = 14t - \frac{1}{2}gt^2$	M1
	Form relevant horizontal equation in x and t $x = 8t$	M1
	Eliminate t to form correct equation in x and y $y = 14 \times \frac{x}{8} - \frac{1}{2}g\left(\frac{x}{8}\right)^2$	A1
	Substitute $y=3$ into correct equation and solve for x	A1
	21 or 21.0 (m)	A1
		(5)
	ALT method using Energy	
	Vertically: $\frac{1}{2}(m)(14^2 - v^2) = 3(m)g$	M1
	Form vertical suvat equation in their v and t $v = 14 - gt$	M1
	$\sqrt{\frac{686}{5}} = 14 - gt$	A1

	8x2.6...	A1
	21 or 21.0 (m)	A1
		(5)
		(10)
	Notes for question	
3(a)		
M1	Complete method to find greatest height. Condone sign errors.	
A1	cao	
3(b)		
M1	Complete method to find the vertical component at $t=2.4$ Condone sign errors.	
M1	Use of Pythagoras with both components to find speed	
A1	Correct answer, 2/3sf	
3(c)		
M1	Relevant equation formed using vertical motion. Condone sign errors.	
A1	Correct unsimplified equation(s). (Note $t = 2.62$ (3sf) but does not need to be seen for this mark)	
M1	Use horizontal motion to find the required distance	
A1	Uses 'larger' $t=2.6...$ to calculate distance AB	
A1	Correct answer with 2 or 3sf. Accept 21, 21.0.	
	Alt method Trajectory Enter marks in correct M and A spaces	
M1	Relevant equation formed in t and y using vertical motion.	
M1	Form relevant horizontal equation in x and t	
A1	Eliminate t to form correct equation in x and y	
A1	Substitute $t=3$ into correct equation and solve for x	
A1	Correct answer with 2 or 3sf. Accept 21, 21.0.	
	Alt method Energy	
M1	Form (vertical) energy equation, m 's may have been cancelled and 8^2 may have been added to both velocity parts	
M1	Form vertical suvat equation in their v and t	
A1	Substitute correct value for v ($\sqrt{\frac{686}{5}}=11.71..$) and solve for t	
A1	Uses $t=2.62...$ to calculate distance AB	
A1	Correct answer with 2 or 3sf. Accept 21, 21.0.	



6(a)	M(A), $S \times 1.5a = mga \cos \theta$	M1A1
	$S = \frac{2mg \cos \theta}{3} *$ N.B. Allow RHS in any equivalent form with the same terms in any order.	A1*
		(3)
6(b)	$V = mg - S \cos \theta$	M1A1
	$*V = \frac{mg}{3} (3 - 2 \cos^2 \theta)$	A1*
		(3)
6(c)	$\frac{4}{7}V$ seen or implied	B1
	Horizontal: $F = S \sin \theta$ Other possible equations: Perp to rod: $F \cos \theta + V \sin \theta = mg \sin \theta$ Parallel to rod: $F \sin \theta + mg \cos \theta = V \cos \theta + S$ M(B): $F \times 2a \sin \theta + mga \cos \theta = V \times 2a \cos \theta + S \times 0.5a$ M(C): $F \times 1.5a \sin \theta + mg \times 0.5a \cos \theta = V \times 1.5a \cos \theta$ M(G): $Fa \sin \theta + S \times 0.5a = Va \cos \theta$	M1A1
	e.g. $\frac{4}{7} \times \frac{mg}{3} (3 - 2 \cos^2 \theta) = \frac{2mg \cos \theta}{3} \times \sin \theta$	DM1
	Divide by $\cos^2 \theta$ to produce an equation in $\tan \theta$	DM1
	$6 \tan^2 \theta - 7 \tan \theta + 2 = 0$	A1
		(6)
		(12)
	Notes for question 6	
6(a)	M1 for moments about A equation, dimensionally correct, correct no. of terms, condone sin/cos confusion and sign errors N.B. M0 if a's missing	
	A1 for a correct equation (allow a different letter for S e.g. R_C provided it's clear that $R_C = S$)	
	A1* for given answer correctly obtained including "S = "but not necessarily in the final line of working..	
6(b)	M1 for resolving vertically, correct no. of terms, condone sin/cos confusion and sign errors	
	A1 for a correct equation (allow a different letter for V e.g. R_A provided it's clear that $R_A = V$)	
	A1* for obtaining the given answer from fully correct working including "V = .." but not necessarily in the final line of working. N.B. If S is never seen, can score max M1A1A0*	
6(c)	N.B. The first 3 marks below can be earned if the 'equations' appear in (a) or (b).	

	B1 $\frac{4}{7}V$ seen or implied	
	M1 for another equation: either resolving or taking moments, correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors	
	A1 for a correct equation	
	DM1, dependent on previous M, for substituting for V and S and using $F = \frac{4}{7}V$ to give an equation in θ and m or g or both only	
	DM1, dependent on previous M, for dividing by $\cos^2 \theta$ and using $\sec^2 \theta = 1 + \tan^2 \theta$ to produce an equation in $\tan \theta$	
	A1 for answer, or an integer multiple of answer, correctly obtained N.B. Allow the terms in a different order.	

7(a)	Use of conservation of energy	M1
	$\frac{1}{2}m \times 20^2 - \frac{1}{2}mv^2 = mg \times 11$	A1
	$\frac{1}{2}m \times 20^2 - \frac{1}{2}m(V^2 + (2V)^2) = mg \times 11$	A1
	$V = 6 *$	A1*
		(4)
7(b)	At A,  $S = \frac{12}{\cos 30^\circ} \quad (8\sqrt{3})$	M1A1
	$\frac{1}{2}m \times 20^2 - \frac{1}{2}mS^2 = mgH$ (from O to A)	M1A1
	OR $\frac{1}{2}mS^2 - \frac{1}{2}m \times (6^2 + 12^2) = mgh$ (from ht. 11m to A)	
	Solve for H or h (0.6)	DM1
	$H = 10 \text{ m}$ (2sf)	A1
		(6)
	OR	
	At A,  $W = 12 \tan 30^\circ \quad (4\sqrt{3}) \quad (\text{OR: } 12 = S \cos 30^\circ \text{ AND } W = S \sin 30^\circ)$	M1A1
	$W^2 = \left(20 \times \frac{4}{5}\right)^2 - 2gH$ (from O to A)	M1A1
	OR $W^2 = (20^2 - 12^2) - 2gH$ (from O to A)	
	OR $W^2 = 6^2 - 2gs$ (from ht. 11m to A)	
	N.B. All 3 equations are using positive UP	
	Solve for H (10.4) OR $s = -0.6$ so height is $11 - 0.6 = 10.4$	DM1
	Height is 10 m (2sf)	A1
		(6)
		(10)
	Notes for question 7	
7(a)	M1 for an energy equation, dimensionally correct with correct terms, condone sign errors N.B. M0 if not using energy. N.B. If clearly using V, must include both horizontal and vertical cpts.	
	A1 correct equation in v May find v first: $(v^2 = 180)$	
	A1 correct equation in V then $V^2 + (2V)^2 = 180$	
	A1* for correct given answer correctly obtained, by putting $g = 10$,	

	cancelling m 's and solving for V .	
7(b)	M1 for using perpendicularity at A to find S , the speed at A Condone sin/cos confusion.	
	A1 for correct unsimplified speed	
	M1 for complete method using energy to find an equation in S and H , or S and h , correct no. of terms, condone sign errors	
	A1 correct equation without S replaced	
	DM1, dependent on both M marks, solve for H	
	A1 cao	
	OR	
	M1 for using perpendicularity at A to find W , the vertical velocity component at A. Condone $30^\circ / 60^\circ$ confusion.	
	A1 for correct unsimplified vertical component. Allow + or -.	
	M1 for complete method using <i>suvat</i> to find an equation in W and H , condone sign errors, N.B. They may find t first and then use that to form this equation.	
	A1 correct equation without W replaced	
	DM1, dependent on both M marks, solve for H	
	A1 cao	

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.



General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

RHS, LHS Right hand side, left hand side.

Q	Solution	Mark	Notes
1	Driving force $(F) = \frac{3500}{V}$	B1	Use of $P = Fv$
	Equation of motion: $F - 20V + 480g \sin \theta = 0$	M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion
	$\frac{3500}{V} - 20V + 40g = 0$	A1	Correct unsimplified equation in V .
	$20V^2 - 392V - 3500 = 0$	M1	Form a 3 term quadratic equation ($= 0$) in V
	$V = 26.3 \quad (26)$	A1	3 sf or 2 sf Not $\frac{49 + 22\sqrt{14}}{5}$ (follows use of 9.8)
		(5)	
		[5]	

3a		square	triangle	circle	T			
	mass	36	8	π	$28 - \pi$			
	c from AD	$3a$	$\frac{7}{3}a$	$4a$	d			
	Mass ratio					B1		
	Distances from AD or a parallel axis					B1		
	M(AD or parallel axis):					M1	Moments equation. Need all terms and dimensionally correct. Condone sign errors.	
	$36 \times 3a - 8 \times \frac{7}{3}a - \pi \times 4a = (28 - \pi)d$					A1	Correct unsimplified equation for their parallel axis	
	$\left(108a - \frac{56}{3}a - 4\pi a = (28 - \pi)d\right)$							
	$d = \frac{324 - 56 - 12\pi}{3(28 - \pi)}a = \frac{4(67 - 3\pi)}{3(28 - \pi)}a$ *					A1*	Obtain given answer from correct working	
							Distance from BC is $\frac{(236 - 6\pi)a}{3(28 - \pi)}$ Allow 4/5 if seen.	
						(5)		
3b	$M(A): W \times \frac{4(67 - 3\pi)}{3(28 - \pi)}a = kW \times 6a$ Or resolve vertically and use $M(G)$, where G is the centre of mass, $T_A + kW = W$ $T_A \left(\frac{4(67 - 3\pi)}{3(28 - \pi)}a \right) = kW \left(6a - \frac{4(67 - 3\pi)}{3(28 - \pi)}a \right)$					M1	Complete method to form an equation in k and W only Dimensionally correct but condone use of incorrect distance(s)	
						A1	Correct unsimplified equation $\left(\text{NB } \frac{4(67 - 3\pi)}{3(28 - \pi)}a = 3.088a \right)$	
	$k = 0.51$					A1	Q asks for 2dp	
						(3)		
						[8]		

Q	Scheme	Mark	Notes
8a	Use symmetry to find time taken: $-7 = 7 - gt$	M1	Or equivalent complete method using <i>suvat</i> to find the time taken e.g. find the time for vertical distance = 0
	$t = \frac{14}{g} (= 1.428...)$	A1	Correct value seen or implied
	Horizontal distance = $4t$	DM1	Complete method using <i>suvat</i> to find the distance. Dependent on the preceding M1
	$= 5.71(\text{m})$ or $5.7(\text{m})$	A1	3 sf or 2 sf only $\frac{40}{7}$ scores A0 $\frac{56}{g}$ scores A0 (incorrect units)
		[4]	
8a alt	Find speed and angle of projection	M1	Correct use of Pythagoras and trig.
	Speed = $\sqrt{16 + 49} = \sqrt{65} (\text{ms}^{-1})$ Direction = $\tan^{-1} \frac{7}{4} (= 60.3^\circ)$	A1	Both values seen or implied.
	Use of $R = \frac{u^2 \sin 2\alpha}{g}$	DM1	Or equivalent. Dependent on the preceding M1
	$= 5.71(\text{m})$ or $5.7(\text{m})$	A1	3 sf or 2 sf only
		[4]	
8b	$ \mathbf{v} = 5 \Rightarrow \mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$ or $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$	B1	Correct vertical component seen or implied
	$-3 = 3 - gT$	M1	Complete method to find T e.g. $T = \frac{14}{g} - 2 \times \frac{4}{g}$
	$T = 0.612$ or $T = 0.61$	A1	3 sf or 2 sf only $\frac{30}{49}$ scores A0 $\frac{6}{g}$ scores A0 (incorrect units)
		[3]	
8c	$\begin{pmatrix} 4 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ p \end{pmatrix} = 0$	M1	Or equivalent method to find perpendicular velocity
	$\Rightarrow p = -\frac{16}{7}, \quad \mathbf{v} = 4\mathbf{i} - \frac{16}{7}\mathbf{j}$	A1	Correct vertical component Allow -2.28....
	$\left((-)\frac{16}{7}\right)^2 = 7^2 - 2gh$	DM1	Complete method using <i>suvat</i> or energy to form an equation in h only. Dependent on the preceding M1
	$h = 2.23$ or $h = 2.2$	A1	3 sf or 2 sf only cso (negative vertical component seen at some point)
		[4]	
8c alt	$\begin{pmatrix} 4 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 7 - gt \end{pmatrix} = 0$	M1	Or equivalent method to find time when velocity perpendicular
	$t = \frac{65}{7g} (= 0.947...)$	A1	Correct time
	$h = 7t - \frac{1}{2}gt^2$	DM1	Complete method using <i>suvat</i> to form an equation in h only.
	$h = 2.23$ or $h = 2.2$	A1	3 sf or 2 sf only cso
		[4]	
		(11)	



Q	Scheme	Marks	Notes
7(a)	Horizontal distance	M1	equation with correct terms, condone sign errors
	$2ut = 80$	A1	correct equation
	Vertical distance or vertical speed	M1	equation with correct terms, condone sign errors
	$0 = ut - \frac{1}{2}gt^2$	A1	correct equation in t Alternatives include $-u = u - gt$ or $0 = u - g\frac{1}{2}t$
	Solve for u $\left(\text{e.g. } u \times \frac{80}{2u} = \frac{1}{2}g \frac{80^2}{4u^2} \right)$	DM1	Dependent on the two previous M marks
	$u = 14^*$	A1*	obtain given answer correctly
	If they consistently have u horizontal and $2u$ vertically, then mark as a misread. M1A0M1A0M1A1 Fortunately, they do obtain the given answer		
		(6)	
7(b)	$v^2 = (7\sqrt{17})^2 - 28^2$	M1	form an equation in v only (v is vertical component)
	$\Rightarrow v = 7$ (or -7)	A1	second value not needed
	Use of <i>suvat</i> to find the required time Check their logic. Have they found the time speed is $< 7\sqrt{17}$ or the time the speed is $> 7\sqrt{17}$?	DM1	Dependent on the first M mark. Complete method to obtain the required time. condone sign errors
	$7 = 14 - gt \Rightarrow t = \frac{5}{7} = 0.71\dots$	A1	Obtain a relevant value of t
	Total time = $2 \times \frac{5}{7} = 1.4$ or 1.43 (s)	A1	For the required time to 2 sf or 3 sf A0 for $\frac{10}{7}$; follows the use of an approximate value for g
	The misread from (a) will give $v = \pm\sqrt{637} = \pm 7\sqrt{13}$ (± 25.2), critical value of time $t = 0.282$, required time 0.56 (s)		No further penalty for the misread if the penalty is already applied in (a)
		(5)	

7(b) alt	$\frac{1}{2}m(28^2 + 14^2) - \frac{1}{2}m(7\sqrt{17})^2 = mgh$	M1	form an equation in h only
	$\Rightarrow h = 7.5$	A1	Correct only
	Use of <i>suvat</i> to find the required time Check their logic. Have they found the time speed is $< 7\sqrt{17}$ or the time the speed is $> 7\sqrt{17}$?	DM1	Dependent on the first M mark. Complete method to obtain the required time, condone sign errors
	$7.5 = 14t - \frac{1}{2} \times 9.8t^2 \Rightarrow t = \frac{5}{7}, t = \frac{15}{7}$	A1	Obtain at least one relevant value for t
	Total time = $\frac{20}{7} - \left(\frac{15}{7} - \frac{5}{7}\right) = 1.4$ or 1.43(s)	A1	For the required time to 2 sf or 3 sf A0 for $\frac{10}{7}$; follows the use of an approximate value for g
	The misread from (a) will give the same value for h (7.5), $t = 5.43$ and $t = 0.28$, so required time 0.56 (s)		
		(5)	
7b alt	Use $7\sqrt{17}$ to form an equation in t only	M1	
	$\Rightarrow 7\sqrt{17} = \sqrt{(14 - gt)^2 + 28^2}$	A1	Or equivalent
	Solve to find the required time Check their logic. Have they found the time speed is $< 7\sqrt{17}$ or the time the speed is $> 7\sqrt{17}$?	DM1	Dependent on the first M mark. Complete method to obtain the required time, condone sign errors
	$147 = 2gt - g^2t^2 \Rightarrow t = \frac{5}{7}, t = \frac{15}{7}$	A1	Obtain at least one relevant value for t
	Total time = $\frac{20}{7} - \left(\frac{15}{7} - \frac{5}{7}\right) = 1.4$ or 1.43(s)	A1	For the required time to 2 sf or 3 sf A0 for $\frac{10}{7}$; follows the use of an approximate value for g
		(5)	
		(11)	

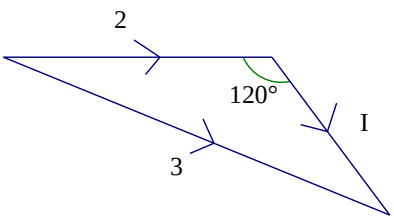
		[1]	[11]
--	--	-----	------

Question Number	Scheme	Marks	Notes
7a	Equation for conservation of energy.	M1	Need all terms. Condone sign errors
	$\frac{1}{2} \times m \times 144 + m \times g \times 20 = \frac{1}{2} m v^2$	A1	Correct unsimplified equation with at most one error
		A1	Correct equation (with or without m)
	$v = 23$ or 23.2	A1	Max 3 s.f.
		[4]	
7b	$12 \cos \theta \times 5 = 40$	M1	Horizontal motion Condone sine/cosine confusion
	(minimum=) $12 \cos \theta = 8 \text{ (m s}^{-1}\text{)}$	A1	Final answer : do not ignore subsequent working
		[2]	
7c	Speed = 10 $\Rightarrow$ Vertical component = 6 (m s ⁻¹)	B1ft	Follow their horizontal component
	$(\pm) 6 = 12 \sin \theta - gt$	M1	Vertical speed
	$= 12 \times \frac{\sqrt{5}}{3} - gt$ ($t = 0.30..$ and $t = 1.52..$)	A1	Correct equation for one value of t or for the time interval. Correct trig value seen or implied
	Time = $1.52.. - 0.30.. = 1.22... \text{ (s)}$	A1	Correct interval
	Required time = $5 - 1.22 \text{ (s)}$	M1	Find required time – follow their 1.22
	$= 3.78 \text{ (s)}$	A1	Or 3.8. Max 3 s.f.
		[6]	
	Alternatives for M1A1A1		
	Use of $v = u + at$	(M1)	
	$-6 = 6 - gt$	(A1)	Or find time to top and double it
	$t = \frac{12}{g}$	(A1)	
	Vertical speed: $6 = 12 \sin \theta - gt_1$	(M1)	
	$-6 = 12 \sin \theta - gt_2$	(A1)	
	$12 = g(t_2 - t_1), t_2 - t_1 = \frac{12}{g}$	(A1)	
	Alternatives for B1M1A1A1		
	ht above A $\frac{22}{g}$	(B1)	Using energy 2.24... seen or implied e.g. by 22.24...
	Use of $s = ut + \frac{1}{2}at^2$	(M1)	$20 + \frac{22}{g}$ used with $12 \sin \theta$ is M0
	$\frac{22}{g} = 12 \sin \theta t - \frac{1}{2}gt^2$	(A1)	
	Time = $1.52.. - 0.30.. = 1.22... \text{ (s)}$	(A1)	Correct interval
	Speed 10, angle to horizontal $\alpha \Rightarrow 10 \cos \alpha = 8$	(B1)	
	Time to top: $0 = 10 \sin \alpha - gt$	(M1)	
	$10 \times 0.6 = gt$	(A1)	

	Total time = $\frac{12}{g}$	(A1)	
		[11]	



8a	Condone use of θ or a mixture of θ and α throughout but final answer should be in one variable.		
	Equation for horizontal distance	M1	Complete method using <i>suvat</i> . Condone sine / cosine confusion
	$x = u \cos \alpha t$	A1	Correct only
	Equation for vertical distance	M1	Complete method using <i>suvat</i> . Condone sine / cosine confusion and sign error
	$y = u \sin \alpha t - \frac{1}{2}gt^2$	A1	Correct only
	$t = \frac{x}{u \cos \alpha} \Rightarrow$ $y = u \sin \alpha \cdot \frac{x}{u \cos \alpha} - \frac{g}{2} \left(\frac{x}{u \cos \alpha} \right)^2$	DM1	Substitute for t to obtain y in terms of x and α Dependent on the 2 preceding M marks
	$\Rightarrow y = x \tan \alpha - \frac{gx^2}{2u^2} (1 + \tan^2 \alpha) *$	A1*	Obtain given answer from full and correct working. Need some evidence for the final step. $\frac{1}{\cos^2 \alpha} = 1 + \tan^2 \alpha$ is not sufficient.
		6	
8b	Conservation of energy:	M1	Method specified in the question. Need all terms and dimensionally correct. Condone sign errors
	$\frac{1}{2}m \times 25^2 = \frac{1}{2}mU^2 + mg \times 20$	A1	Correct unsimplified equation
	$U = 15.3 \text{ (15)}$	A1	3 sf or 2 sf only
		3	
8c	Use part (a) or work from first principles to form an equation in $\tan \theta$	M1	$\left(-20 = 30 \tan \theta - \frac{9.8 \times 900}{2U^2} (1 + \tan^2 \theta) \right)$
	Obtain $18.9 \tan^2 \theta - 30 \tan \theta - 1.07 = 0$ $\left(\frac{4410}{233} \tan^2 \theta - 30 \tan \theta - \frac{250}{233} = 0 \right)$	A1ft	Or 3 term equivalent Follow their U Can be implied by a correct final answer
	$\Rightarrow \theta = 58.3^\circ \text{ or } 58^\circ$	A1	3 sf or 2 sf only
		3	
		(12)	

Question	Scheme	Mark	Notes
3	Form impulse-momentum equation	M1	Dimensionally correct. Accept answers in “vector” form, or as separate components. Condone sine / cosine confusion.
	One correct equation	A1	e.g. one correct component of $\begin{pmatrix} I \cos 60^\circ \\ I \sin 60^\circ \end{pmatrix} = \frac{1}{4} \left[\begin{pmatrix} 12 \cos \alpha \\ 12 \sin \alpha \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix} \right]$ or $\begin{pmatrix} 3 \cos \alpha - 2 \\ 3 \sin \alpha \end{pmatrix}$ $\begin{pmatrix} I \cos 60^\circ \\ I \sin 60^\circ \end{pmatrix} = \frac{1}{4} \left[\begin{pmatrix} v_x \\ v_y \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix} \right]$ $\begin{pmatrix} 3 \cos \alpha - 2 \\ 3 \sin \alpha \end{pmatrix}$ if working parallel and perpendicular to the initial direction or one of $8 \sin 60^\circ = 12 \cos(30^\circ + \alpha)$ or $I = 0.25(12 \sin(30^\circ + \alpha) - 8 \cos 60^\circ)$ if working parallel and perpendicular to the impulse
	Form a second impulse-momentum equation	M1	
	correct second equation	A1	
	Complete method to solve for I	DM1	Dependent on the two preceding M marks. e.g. from $36 = (I + 4)^2 + 3I^2 \quad (4I^2 + 8I - 20 = 0)$
	$I = \sqrt{6} - 1$ (or 1.45 or 1.4)	A1	
		[6]	
3 alt		M1	Use of $I = mv - mu$ to draw a vector triangle. Dimensionally consistent.
		A1	Correct diagram
	Form an equation in I	M1	e.g. by using cosine rule
	$4 + I^2 - 4I \cos 120^\circ = 9$	A1	Correct unsimplified equation A correct cosine rule equation can imply the first M1A1 if no diagram seen
	Solve for I	DM1	Dependent on the 2 preceding M marks $I^2 + 2I - 5 = 0$
	$I = \sqrt{6} - 1$ (or 1.45 or 1.4)	A1	
		(6)	

Question	Scheme	Mark	Notes
4a	$4 - gT_1 = 0$ or $T_1 = \frac{\sqrt{32} \sin 45^\circ}{g}$	M1	Complete method using <i>suvat</i>
	$T_1 = 0.408$ (0.41)	A1	3 sf or 2 sf only. Not $\frac{20}{49}$
		[2]	
4b	Height of Q above P:	M1	Complete method using <i>suvat</i> and 7 and 4 for the initial vertical components
	$h = (7T_1 - \frac{1}{2}gT_1^2) - (4T_1 - \frac{1}{2}gT_1^2) \quad (= 3T_1)$	A1	Correct unsimplified expression in T_1 or their T_1 . They do not need to have substituted for T_1 (2.0408... - 0.8163...)
	$h = 1.2$ (m)	A1ft	2 sf only ($3 \times$ their T_1)
		[3]	
4c	Correct time for P to reach B. ($\frac{40}{49}$, 0.816, or $\frac{8}{g}$ or better)	B1	Seen or implied.
	Vertical component of speed $= 7 - g \times 2T_1 \quad (= -1)$	M1	Complete method using <i>suvat</i> with $2T_1$ or their t for the time at B M0 if not using 7
	$\tan \alpha = \pm \frac{\text{their } 1}{5}$	M1	Correct use of their 1 and 5 to find an equation in a relevant angle (e.g. $90 - \alpha$)
	$\alpha = 11$	A1	11 or better (e.g. 11.3)
If they use T_1 in place of $2T_1$ can score B0M0M1A0			
		[4]	
4d	Form an equation in T_2 only	M1	Complete method using <i>suvat</i> and perpendicular gradients. e.g. $\begin{pmatrix} 5 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 7 - gT_2 \end{pmatrix} = 0$ Condone sign errors (Vertical component of speed $= \pm \frac{25}{7}$) (perpendicular direction is downwards at 35.5° to the horizontal)
	$-\frac{25}{7} = 7 - gT_2$	A1	Correct unsimplified equation
	$T_2 = 1.08$ or $T_2 = 1.1$	A1	3 sf or 2 sf only
		[3]	
		(12)	

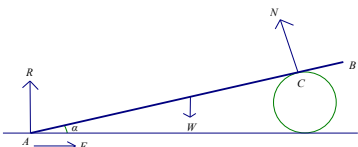


Question	Scheme	Marks
5(a)	Horizontal motion to find expression for the distance	M1
	$(XY =) u \cos \theta t$	A1
	Method using vertical motion to find relevant equation in t, u, g, θ . Eg - Using vertical distance = 0 to find expression for time. - Find expression for time to max height and $\times 2$	M1
	Correct unsimplified equation in t, u, g, θ . $u \sin \theta t - \frac{1}{2} g t^2 = 0$	A1
	Solve to obtain the distance in terms of u, g and θ	DM1
	$XY = u \cos \theta \times \frac{2u \sin \theta}{g} = \frac{u^2 \sin 2\theta}{g} *$	A1*
		(6)
5(b)	$CB = \frac{20^2 \times \sin 120^\circ}{g} - 12$	M1
	$CB = 23(\text{m}) \text{ or } CB = 23.3(\text{m})$	A1
		(2)
5(c)	Horizontal motion	M1
	$12 = 20 \cos 60^\circ t$	A1
	Vertical motion $y = 20 \sin 60^\circ \times 1.2 - \frac{1}{2} \times g \times 1.2^2 (-5)$	M1
	Height above D = 8.7(m) or 8.73(m)	A1
		(4)
		(12)
	Notes	
	Over-accuracy or under-accuracy is penalised max once per complete question. Penalise final A mark in the appropriate part. Use of $g = 9.81$ or 10 is penalised max once per complete question. Penalise final A mark in the appropriate part. If both errors occur they could lose the final two A marks.	
5(a)		
M1	Use horizontal motion to find an expression for the horizontal distance.	
A1	Correct unsimplified equation.	
M1	Method using vertical motion with relevant <i>suvat</i> to find an equation in t, u, g, θ (condone use of 9.8). M0 if $t = \frac{2u \sin \theta}{g}$ or $t = \frac{u \sin \theta}{g}$ is quoted (ie if it appears without any evidence of method)	
A1	Correct equation, (condone use of 9.8) Eg $u \sin \theta t - \frac{1}{2} g t^2 = 0$	

	$-u \sin \theta = u \sin \theta - gt$ $u \sin \theta - gt = 0$ double the time.	
DM1	Dependent on the preceding M marks. Solve to obtain the distance in terms of u , g and θ .	
A1*	Obtain given answer from correct working. Must recover g if 9.8 is used. Must have $XY =$ at this stage.	
5(b)		
M1	Complete method using result in (a), or using horizontal motion from first principles, to find the horizontal distance CB .	
A1	Correct answer, 2sf or 3sf	
5(c)		
M1	Complete method for horizontal motion to obtain an equation in t . If $t = 1.2$ o.e. is seen in earlier working, it must be used in (c) to earn the marks.	
A1	Correct unsimplified equation ($t = 1.2$)	
M1	Method using <i>suvat</i> with vertical motion to obtain a relevant vertical distance. Height of pole not required.	
A1	Correct answer, 2 sf or 3 sf	



Question Number	Solution	Marks	Notes
8a	Horizontal component: $p = 8$	B1	
	Vertical component: $-12 = q - 3g$	M1	Complete method to find q using <i>suvat</i> . Condone sign errors.
	$q = 17.4$	A1	17 or better
	Speed = $\sqrt{8^2 + 17.4^2}$	M1	Use of Pythagoras to find speed using their velocity. Independent M mark
	$= 19.2 \quad (19)(\text{ms}^{-1})$	A1	3 sf or 2 sf
		(5)	
8b	Use of Pythagoras to find vertical component	M1	
	vertical component = ± 6	A1	Seen or implied Accept without +/-
	$-6 = 6 - 9.8T$	DM1	Complete method using <i>suvat</i> to find required time Dependent on the previous M1
	$T = 1.22 \quad (1.2)$	A1	3 sf or 2 sf. Not $\frac{60}{49}$
		(4)	
8b alt	Use <i>suvat</i> and Pythagoras to form an equation in t	M1	Or an inequality
	$8^2 + (17.4 - gt)^2 = 100$	A1	Correct unsimplified equation for t Accept inequality
	Solve for T	DM1	Complete method to obtain T Dependent on the previous M1
	$T = 1.22 \quad (1.2)$	A1	3 sf or 2 sf. Not $\frac{60}{49}$
		(4)	
8c	Velocity perpendicular $\Rightarrow$ vertical component = $\frac{2}{3} \times 8$	M1	Complete method to find vertical component of velocity at B
	$= \frac{16}{3}$	A1	
	$(-12)^2 = \left(\frac{16}{3}\right)^2 - 2g(-h)$	DM1	Complete method to find the required vertical distance using their vertical component of the velocity Dependent on the previous M1
	$h = 5.90 \quad (5.9)(\text{m})$	A1	Max 3 sf
		(4)	
8c alt	$\begin{pmatrix} 8 \\ 17.4 - gt \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -12 \end{pmatrix} = 0$ and time = $3 - t$	M1	Complete method to find the time from B to A
	Time = $3 - 1.23... = 1.768...$	A1	
	$s = vt - \frac{1}{2}gt^2 = 12t - 4.9t^2$	DM1	Complete method to find the required vertical distance using their time Dependent on the previous M1
	$s = 5.9 \text{ (m)}$	A1	Max 3 sf

Question Number	Scheme	Marks
6a	 Resolve vertically $\uparrow R + N \cos \alpha = W$ Take moments about A $7aN = 4a \cos \alpha \times W$ Obtain equation in R, W and α $N = W \times \frac{4}{7} \cos \alpha \Rightarrow$ $R = W - \frac{4}{7} W \cos^2 \alpha$ $= W \left(1 - \frac{4}{7} \cos^2 \alpha \right) \quad *$ Alternative equations $R \sin \alpha + F \cos \alpha = W \sin \alpha$ $N + R \cos \alpha = W \cos \alpha + F \sin \alpha$ $W.3a \cos \alpha + F.7a \sin \alpha = R.7a \cos \alpha$ First 4 marks for alternative methods	M1 A1 M1 A1 DM1 A1* (6) M1 A1 M1 A1 DM1 A1* (6)
6a M1	A1 Correct unsimplified equation M1 A1 Correct unsimplified equation DM1 Solve for R in terms of W. Dependent on the 2 preceding M marks A1* Obtain given answer from correct working Alt: Parallel to the rod Perpendicular to the rod Moments about C M1 Equation in R. All terms needed. Condone sin/cos confusion and sign errors A1 Correct unsimplified equation M1 Sufficient additional equations to solve for R in terms of W. Dimensionally correct. All terms needed. Condone sin/cos confusion and sign errors A1 Correct unsimplified equation	

Question Number	Scheme	Marks
6b	$R = W \left(1 - \frac{4}{7} \times \frac{9}{10} \right) = \frac{17W}{35}$ Resolve horizontally $F = N \sin \alpha = \frac{4}{7} \times \frac{3}{\sqrt{10}} \times \frac{1}{\sqrt{10}} W$ $\left(= \frac{6}{35} W \right)$ Use of $F \leq \mu R$ $\Rightarrow \mu \geq \frac{6}{17}$	B1 M1 A1 M1 A1 (5)
	Notes	[11]
6b B1	Seen or implied	
M1	Obtain equation in	
A1	Correct unsimplified equation in F and W (trig. substituted) ($0.171W$)	
M1	Correct method to find the critical value. Condone with any symbol.	
A1	0.35 or better (0.3529.....) from correct working Final answer. Do not ISW	

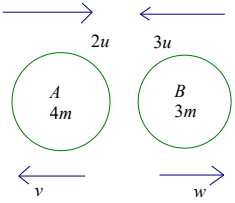


Question Number	Scheme	Marks
7a	NB: sine/cosine confusion is not condoned in projectile questions Use of conservation of energy $\frac{1}{2}m \times 25^2 = \frac{1}{2}m \times 15^2 + mgh$ $\Rightarrow h = 20$ or 20.4 (m)	M1 A1 A1 (3)
7b	Vertical distance $20.4 = 25 \sin \alpha \times 3 - 4.5 \times 9.8$ $\alpha = 59^\circ$ or 59.3°	M1 A1ft A1 (3)
7c	Horizontal component of speed is constant $\Rightarrow 25 \cos \alpha = 15 \cos \beta$ $\beta = 32^\circ$ or 31.8°	M1 A1ft A1 (3)
7c alt	Vertical distance $20.4 = -15 \sin \beta \times 3 + 4.5 \times 9.8$ $\beta = 32^\circ$ or 31.8°	M1 A1ft A1 (3)
Notes		
7a M1 A1 A1 7b M1 A1ft A1 7c M1 A1ft A1 7c alt M1 A1ft A1	Need energy equation with all 3 terms. Must be dimensionally correct. Condone sign errors. Correct unsimplified equation Max 3 sf Not $\frac{1000}{49}$ nor $\frac{200}{g}$ Use of <i>suvat</i> to find α Correct unsimplified equation in their h 0.554 rads. Max 3 sf From CWO Or horizontal distance travelled Correct unsimplified in α or their α 0.554 rads. Max 3 sf From CWO Use of <i>suvat</i> to find β e.g. using $s = vt - \frac{1}{2}gt^2$. Correct unsimplified equation in their h 0.554 rads. Max 3 sf From CWO	



Question Number	Scheme	Marks
7d	Min speed = horizontal component = $25 \cos \alpha (= 15 \cos \beta)$ = 13 or 12.8 (m s ⁻¹)	M1 A1 (2)
7e	By considering vertical component of speed at B: $15 \sin 31.8^\circ - gT = -15 \sin 31.8^\circ$ $T = 1.6$ or 1.61 (s)	M1 A1ft A1 (3)
Notes		[14]
7d M1 A1	Follow their angle. Must show working if using incorrect angle. Max 3 sf From CWO	
7e M1 A1ft A1	Complete method using <i>suvat</i> to find T Correct unsimplified equation in T - follow their angles. Max 3 sf From CWO	



Question Number	Scheme	Marks
8	 Change in KE $\frac{4m}{2}(4u^2 - v^2) + \frac{3m}{2}(9u^2 - w^2) = \frac{473}{24}mu^2$ $(48v^2 + 36w^2 = 43u^2)$ Equation for CLM Need all terms. Dimensionally correct. Condone sign errors. $8mu - 9mu = -4mv + 3mw$ $(u = 4v - 3w)$ Impact law $w + v = 5eu$ $48v^2 + 36\left(\frac{4v - u}{3}\right)^2 = 43u^2 \text{ Or } 48\left(\frac{u + 3w}{4}\right)^2 + 36w^2 = 43u^2$ $\text{Or } \frac{48}{49}(1 + 15e)^2 + \frac{36}{49}(20e - 1)^2 = 43$ $112v^2 - 32uv - 39u^2 = 0 \quad \text{Or} \quad 63w^2 + 18uw - 40u^2 = 0$ $= (4v - 3u)(28v + 13u) \quad \text{Or} \quad = (21w + 20u)(3w - 2u)$ $\text{Or } 25200e^2 = 2023$ Notes	M1 A1 M1 A1 M1 A1 DM1 DM1
M1 A1 M1 A1 M1 A1 DM1 DM1	The first 8 marks are available if they have ignored the information about the final directions. Work with their directions. Ignore the diagram if that is to the candidate's advantage. Need all terms. Dimensionally correct. Accept $\pm$ Correct unsimplified equation in v, w or their v, w Need all terms. Dimensionally correct. Condone sign errors. Correct unsimplified equation with their correct signs Must be used the right way round Or equivalent in their w, v. Signs for v, w consistent with CLM eqn Form equation for v or w or e Dependent on M marks scored for the equations used. Solve for v or w or e. Dependent on the preceding M	



8a	Conservation of energy	M1	Need all three terms and dimensionally correct. Condone sign errors.
	$\frac{1}{2}m \times 10^2 + mgh = \frac{1}{2}m \times 18^2$	A1	Correct unsimplified equation
	$h = 11.4 \quad (11)$	A1	3 sf or 2 sf only (not $\frac{80}{7}$)
		(3)	
8b	Vertical distance	M1	Complete method using <i>suvat</i> to find angle of projection
	$10 \sin \alpha \times 2.5 - \frac{1}{2}g \times 2.5^2 = -11.4$	A1ft	Follow their h
	$\alpha = 50.2^\circ$ or $10 \sin \alpha = v_v = 7.7678....$	A1	50° or better (50.1618...) Accept 50.3° from 11.4 Seen or implied Might see $\sin \alpha = \frac{43}{56}$ or $v_v = \frac{215}{28}$
	Horizontal distance = $10 \cos \alpha \times 2.5$ or $\sqrt{100 - v_v^2} \times 2.5$	M1	
	$= 16.0 \quad (16)(\text{m})$	A1	3 sf or 2 sf only
		(5)	
8c	Using energy: $\frac{1}{2}m \times 64 + mgs = \frac{1}{2}m \times 100$	M1	Complete method to find height above A
	$s = 1.8367...$	A1	1.8 or better
	Use of <i>suvat</i> to form equation in t	M1	
	$1.84 = 10 \sin 50.2^\circ \times t - 4.9t^2$	A1	Correct unsimplified equation
	Solve for t and find difference between roots	DM1	Complete method to find the required time Dependent on 2 previous M marks
	$T = 0.98$ or 0.978	A1	2 sf or 3 sf
		(6)	
8c alt	Use of Pythagoras	M1	Complete method to find vertical component of speed
	Vertical speed $\sqrt{64 - (10 \cos \alpha)^2} = 4.8...$	A1	Awrt 4.8 or better
	Use of $10 \sin \alpha - gt = \pm v$ to find t	M1	
	$\begin{cases} 10 \sin 50.2^\circ - gt_1 = 4.8 \\ 10 \sin 50.2^\circ - gt_2 = -4.8 \end{cases}$	A1	Correct unsimplified equations Could also find time to top
	$T = t_2 - t_1 = 1.27.. - 0.29..$	DM1	Complete method to find the required time Dependent on 2 previous M marks
	$= 0.98 \quad \text{or } 0.978$	A1	Final answer. 2 sf or 3 sf
		(6)	
8calt	Use of Pythagoras to form quadratic in t	M1	
	$(10 \sin \theta - gt)^2 + (10 \cos \theta)^2 = 64$	A1	

	Simplify and substitute for trig	M1	
	$36 + 9.8^2 t^2 - 150.5t = 0$	A1	
	$T = t_2 - t_1 = 1.27.. - 0.29..$	DM1	Complete method to find the required time Dependent on 2 previous M marks
	$= 0.98 \text{ or } 0.978$	A1	Final answer. 2 sf or 3 sf
		[14]	



Q	Mark Scheme	Marks	Marking guidance
8 (a)	Normal reaction between P and ramp $(R) = 0.3g \cos \alpha = \left(0.3g \times \frac{24}{25} = 2.82\ldots\right)$	M1	Seen or implied. Condone sin / cos confusion (implied by use of $\frac{7}{25}$)
	Work done against friction $= \frac{1}{5} R \times 15$	M1	Use of $WD = \mu R \times \text{distance}$ with their R
	$= 8.47(8.5)(J)$	A1	3 sf or 2 sf
		3	
8 (b)	Work-energy equation	M1	All terms required. Dimensionally correct. Condone sign errors.
	$\frac{1}{2} \times 0.3U^2 = \frac{1}{2} \times 0.3 \times 25^2 + (a) + 0.3 \times g \times (15 \sin \alpha)$	A1ft A1ft	Follow their answer to (a) Correct unsimplified equation with at most one error. Correct unsimplified equation
	$U = 27.6 \quad (28)$	A1	3 sf or 2 sf
		4	
8 (c)	Time to ground:	M1	Complete method using <i>suvat</i> to form an equation in t
	$-15 \sin \alpha = 7t - \frac{1}{2}gt^2$	A1ft	Correct unsimplified equation in t ft their 4.2
	$t = 1.88 \quad (1.9) \quad (s)$	A1	3 sf or 2 sf $\frac{5+\sqrt{67}}{7}$ is A0
		3	
8 (d)	Vertical component of speed	M1	Or use energy to find the speed
	$= \pm(7 - (\text{their } t) \times 9.8) \quad (\pm 11.459\ldots)$	A1ft	or $0.15 \times 625 + .3 \times 9.8 \times \text{their } 4.2 = 0.15v^2$ ($v = 26.59\ldots$) condone $v = \frac{7\sqrt{67}}{5}$
	Correct use of trig: $\tan \theta^\circ = \frac{\text{their vertical}}{24}$	M1	or $\cos \theta^\circ = \frac{24}{\text{their speed}}$
	$\theta = 25.5(26)$	A1	3 sf or 2 sf
		4	
	Reminder: The accuracy penalty for overspecified answers should be applied only once in any question (the first time seen). Similarly for the use of $g = 9.81$. If they make both of these errors they lose 2 A marks. The penalty applies to the final mark in any part.		
		(14)	

Question	Scheme	Marks	Notes
7a	Energy equation	M1	Q requires energy. Need all terms and dimensionally correct. Condone sign error.
	$\frac{1}{2}mv^2 = \frac{1}{2}m(9+4) + mg \times 20$	A1	Correct unsimplified equation
	$v = 20(20.1)(\text{ms}^{-1})$	A1	2 sf or 3 sf only. Not $9\sqrt{5}$
		(3)	
7b	Complete method to find the direction as an angle	M1	Complete method to find trig ratio for a relevant angle
	$\cos \alpha = \frac{3}{\text{their (a)}}$	A1ft	Correct unsimplified equation for a relevant angle. Follow their part (a)
	$\alpha = 81^\circ (81.4^\circ)$ below the horizontal	A1	Or equivalent. 2 sf or 3 sf. Needs to be clear on a diagram or in words where the angle is measured. Accept “to the horizontal”
		(3)	
7b alt	Complete method to find the direction as a vector in i and j or as a column vector	M1	
	Component = $\sqrt{(a)^2 - 9}$	A1ft	Correct unsimplified equation. Follow their part (a)
	Direction $3\mathbf{i} - 19.9\mathbf{j}$	A1	2 sf or 3 sf. ISW after correct vector seen
		(3)	
7c	Form an equation in t	M1	Complete method using <i>suvat</i> Condone sign errors.
	e.g. $-20 = 2t - \frac{1}{2}gt^2$ or $(-20.1\dots)\sin \alpha = 2 - gt$	A1	Correct unsimplified equation
	$t = 2.2(2.23)(\text{s})$	A1	2 sf or 3 sf only
		(3)	
7d	Perpendicular velocity = $3\mathbf{i} - \lambda\mathbf{j}$	B1	Horizontal component unchanged and vertical not equal to ± 2 . Seen or implied
	$(3\mathbf{i} + 2\mathbf{j}) \cdot (3\mathbf{i} - \lambda\mathbf{j}) = 0$	M1	Complete method to solve for vertical component If using angles, they should be using 56.3° for the perpendicular direction.
	$\Rightarrow \mathbf{v} = \left(3\mathbf{i} - \frac{9}{2}\mathbf{j}\right)(\text{ms}^{-1})$	A1	Correct vertical component seen or implied
	Use of <i>suvat</i> or use of energy to find relevant distance	dM1	Complete method to find the vertical component of perpendicular velocity. Dependent on the previous M1 Working with $3\mathbf{i} - 2\mathbf{j}$ is not equivalent work
	$\left(\frac{9}{2}\right)^2 = 2^2 + 2gs$ or $\frac{1}{2}m(13) + mgs = \frac{1}{2}m\left(9 + \frac{81}{4}\right)$	A1	Correct unsimplified equation for their distance
	$h = 20 - s = 19(19.2)$	A1	2 sf or 3 sf
		(6)	

7a	Horizontal distance	M1	Correct use of <i>suvat</i>
	$x = u \cos \alpha t$	A1	Correct equation
	Vertical distance	M1	Correct use of <i>suvat</i>
	$y = u \sin \alpha t - \frac{1}{2} g t^2$	A1	Correct equation. Correct signs. Condone if not using “y”
	$t = \frac{x}{u \cos \alpha} \Rightarrow$ $y = u \sin \alpha \cdot \frac{x}{u \cos \alpha} - \frac{g}{2} \left(\frac{x}{u \cos \alpha} \right)^2$ $\left(= x \tan \alpha - \frac{g x^2}{2 u^2} \sec^2 \alpha \right)$	DM1	Dependent on the first 2 M marks. Substitute for t to obtain y in terms of x and α
	$y = x \tan \alpha - \frac{g x^2}{2 u^2} (1 + \tan^2 \alpha) \quad *$	A1*	Obtain given answer from correct working (final step needs to be explained). Allow if $\sec^2 \alpha$ seen. Must be “y” here
		[6]	
7b	$u = 20, x = 10, y > 2 \Rightarrow$ $2 = 10 \tan \theta - \frac{100g}{800} (1 + \tan^2 \theta)$ $\left(\frac{g}{8} \tan^2 \theta - 10 \tan \theta + \left(2 + \frac{g}{8} \right) = 0 \right)$	M1	Use given values to form quadratic in $\tan \theta$ or equivalent equation in one trig function. Allow working with $=, < \text{ or } > 2$
	Critical values: $\theta^\circ = 18.6^\circ$ or $\theta^\circ = 82.7^\circ$	A1	One correct value to 2 sf or better
	Range: $18.6 < \theta < 82.7$	A1	Accept $<$ or $\leq$ $(19 \leq \theta \leq 82 \text{ or } 83)$ max 3 sf
		[3]	
7c	$y = 10 \tan 40^\circ - \frac{9.8 \times 100}{2 \times 400} (1 + \tan^2 40^\circ)$	M1	Use given formula to find vertical height
	$y = 6.3(03\dots) \text{ (m)}$	A1	Can be implied by correctly substituted formula
	Conservation of energy	DM1	Dependent on the first M1. Need all 3 terms. Dimensionally correct. Condone sign errors.
	$\frac{1}{2} m v^2 = \frac{1}{2} m \times 400 - mgy$	A1ft	Correct unsimplified equation in y or their y
	$v = 17 \quad (16.6) \text{ (ms}^{-1}\text{)}$	A1	2sf or 3sf only
		[5]	
7c alt	$20 \cos 40^\circ t = 10, t = \frac{1}{2 \cos 40^\circ} = 0.653\dots$ $\updownarrow v_y = 20 \sin 40^\circ - gt$	M1	Complete method using <i>suvat</i> to vertical component of speed e.g. by finding time taken then use of $v = u - gt$ or finding vertical distance and using <i>suvat</i>
	$= 6.5 \quad (6.459\dots)$	A1	6.5 or better (not final answer so allow $> 3\text{sf}$ or a correct unsimplified expression)
	$v^2 = (v_H)^2 + (v_V)^2$	DM1	Correct use of Pythagoras

			Dependent on preceding M mark
	$\leftrightarrow v_H = 20 \cos 40^\circ (= 15.3....)$	A1	Horizontal component of speed seen or implied
	$v = 17 \quad (16.6) \text{ (ms}^{-1}\text{)}$	A1	2sf or 3sf only
		[5]	
7d	$0 = x \tan 40^\circ - \frac{9.8x^2}{800} (1 + \tan^2 40^\circ)$	M1	Complete method to solve for x.
	$x = 40 \text{ (40.2) (m)}$	A1	2sf or 3sf only
		[2]	
7d Alt1	$y = 0 \Rightarrow t = \frac{40 \sin 40^\circ}{g} (= 2.623...)$ $x = 20 \cos 40^\circ \times t$	M1	Complete method to solve for x.
	$x = 40 \text{ (40.2) (m)}$	A1	2sf or 3sf only
		[2]	
7d Alt2	$\text{Range} = \frac{20^2 \sin 80^\circ}{g}$	M1	Complete method to solve for x.
	$= 40 \text{ (40.2) (m)}$	A1	2sf or 3sf only
		[2]	
		(16)	