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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Centres of Mass (M2.2)

International A Level Mathematics · M2: Mechanics 2 (WME02)

19 questions · 184 marks · Mark schemes included

Past papers January 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

19 questions · 184 marks · 1 sub-topic

SUB-TOPIC M2.2a

CENTRES OF MASS OF LAMINAS, FRAMEWORKS & SYSTEMS

Sub-topic M2.2a · 19 questions · 184 marks · lowest tariff first



Question 2 continued

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(Total 5 marks)

Q2



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1. Three particles of masses $2m$, $3m$ and $4m$ are placed at the points with coordinates $(-2, 5)$, $(2, -3)$ and $(3k, k)$ respectively, where k is a constant. The centre of mass of the three particles is at the point $(\bar{x}, \bar{y})$.

(a) Show that $\bar{x} = \frac{2 + 12k}{9}$ (2)

The centre of mass of the three particles lies at a point on the straight line with equation $x + 2y = 3$

(b) Find the value of k . (4)

Question 1 continued

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Q1

(Total 6 marks)



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3.

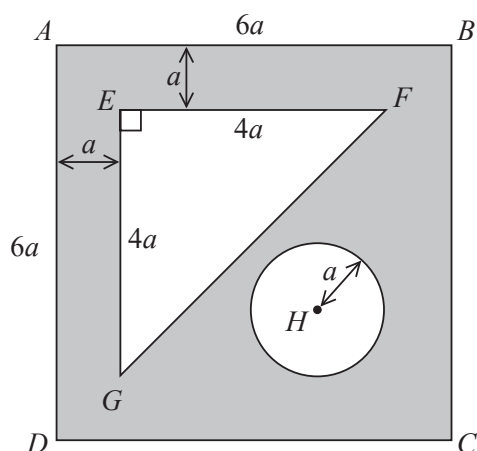


Figure 1

The uniform lamina $ABCD$ is a square of side $6a$. The template T , shown shaded in Figure 1, is formed by removing the right-angled triangle EFG and the circle, centre H and radius a , from the square lamina.

Triangle EFG has $EF = EG = 4a$, with EF parallel to AB and EG parallel to AD . The distance between AB and EF is a and the distance between AD and EG is a .

The point H lies on AC and the distance of H from BC is $2a$.

- (a) Show that the centre of mass of T is a distance $\frac{4(67 - 3\pi)}{3(28 - \pi)}a$ from AD . (5)

The template T is suspended from the ceiling by two light inextensible vertical strings. One string is attached to T at A and the other string is attached to T at B so that T hangs in equilibrium with AB horizontal.

The weight of T is W . The tension in the string attached to T at B is kW , where k is a constant.

- (b) Find the value of k , giving your answer to 2 decimal places. (3)

Question 3 continued

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Question 3 continued

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Lined area for writing the answer to Question 3 continued.



Question 3 continued

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Lined area for writing the answer to Question 3.

(Total 8 marks)

Q3





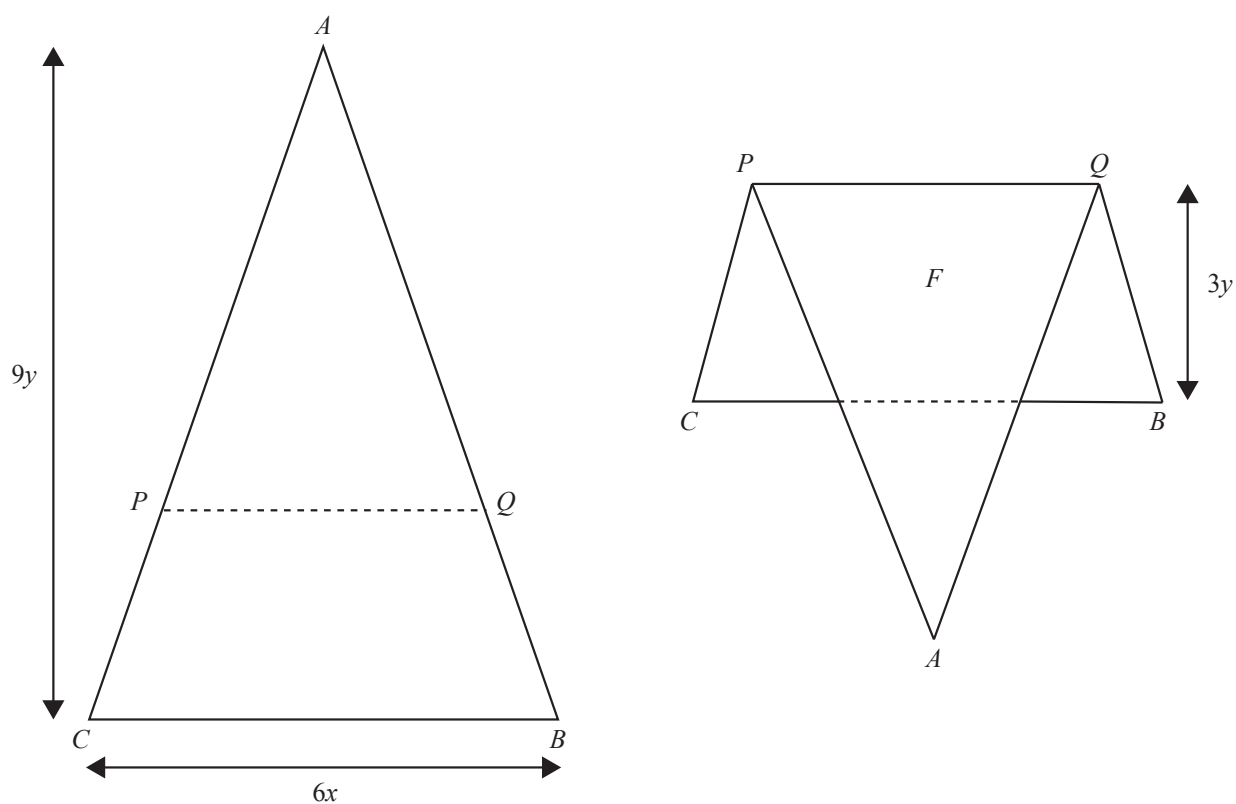


Question 3 continued

(Total for Question 3 is 8 marks)



3.

**Figure 1**

The uniform triangular lamina ABC , shown in Figure 1, has height $9y$, base $BC = 6x$, and $AB = AC$

The points P and Q are such that $AP : PC = AQ : QB = 2 : 1$

The lamina is folded along PQ to form the folded lamina F

The distance of the centre of mass of F from PQ is d

- (a) Show that $d = \frac{16}{9}y$ (5)

The folded lamina is suspended from P and hangs freely in equilibrium with PQ at an angle α to the downward vertical.

Given that $\tan \alpha = \frac{64}{81}$

- (b) find x in terms of y (3)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



[illegible]

(Total for Question 3 = 8 marks)



Question 4 continued

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Question 4 continued

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Question 4 continued

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Q4

(Total 9 marks)



Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total 9 marks)

Q4



Question 6 continued

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Question 6 continued

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Question 6 continued

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Q6

(Total 9 marks)







Question 4 continued

(Total for Question 4 is 9 marks)



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4.

[The centre of mass of a uniform semicircular lamina of radius r is $\frac{4r}{3\pi}$ from the centre.]

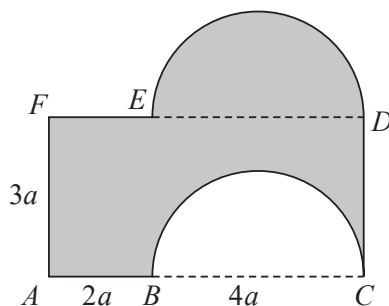


Figure 3

The uniform rectangular lamina $ABCDEF$ has sides $AC = FD = 6a$ and $AF = CD = 3a$. The point B lies on AC with $AB = 2a$ and the point E lies on FD with $FE = 2a$.

The template, T , shown shaded in Figure 3, is formed by removing the semicircular lamina with diameter BC from the rectangular lamina and then fixing this semicircular lamina to the opposite side, FD , of the rectangular lamina. The diameter of the semicircular lamina coincides with ED and the semicircular arc ED is outside the rectangle $ABCDEF$. All points of T lie in the same plane.

(a) Show that the centre of mass of T is a distance $\left(\frac{9+2\pi}{6}\right)a$ from AC . (4)

The mass of T is M . A particle of mass kM is attached to T at C . The loaded template is freely suspended from A and hangs in equilibrium with AF at angle ϕ to the downward vertical through A .

Given that $\tan \phi = \frac{3}{2}$

(b) find the value of k . (6)



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Question 4 continued

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(Total 10 marks)

Q4



2. [In this question you may use, without proof, the formula for the centre of mass of a uniform sector of a circle.]

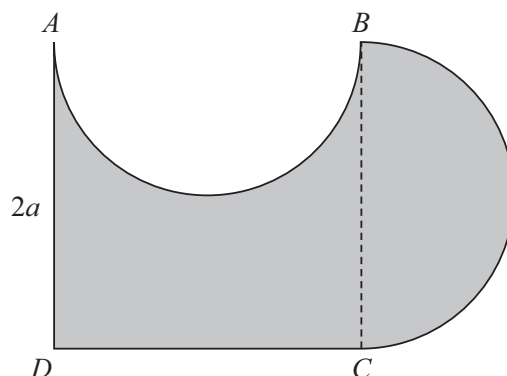


Figure 1

A uniform square lamina $ABCD$ has side $2a$.

A semicircular lamina, with diameter AB , is removed from the square lamina.

The semicircular lamina is then attached to BC to form the uniform lamina L shown shaded in Figure 1.

The centre of mass of L is a distance $\bar{x}$ from AD .

- (a) Show that

$$\bar{x} = \frac{a}{24}(28 + 3\pi) \quad (5)$$

The lamina L has weight W .

A particle of weight kW is attached to L at D to form a loaded lamina.

The loaded lamina is now suspended by two light vertical strings with the first string attached at A and the second string attached at B .

The loaded lamina hangs freely in equilibrium with DC horizontal.

Given that the tension in the string attached at A is $4T$ and the tension in the string attached at B is T

- (b) find the exact value of k .

(5)

Question 2 continued

Lined area for writing the answer to Question 2.



Question 2 continued

Lined area for writing the answer to Question 2.



Question 2 continued

(Total for Question 2 is 10 marks)



3. [The distance of the centre of mass of a semicircular arc of radius r is $\frac{2r}{\pi}$ from its centre.]

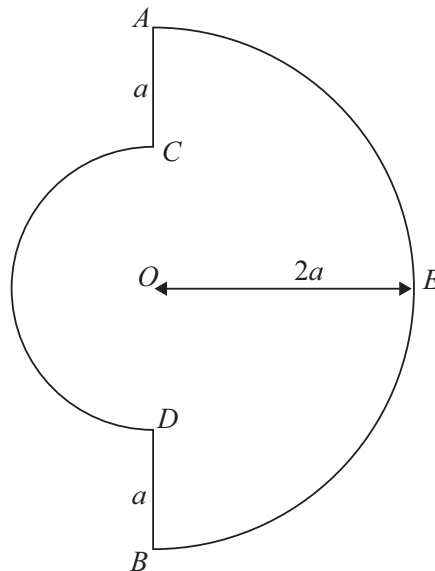


Figure 1

A uniform rigid wire **framework**, shown in Figure 1, consists of two wire semicircular arcs AB and CD , both with centre O , joined by two straight pieces of wire, AC and DB .

Arc AB has radius $2a$, arc CD has radius a , $AC = DB = a$, and $ACDB$ is a straight line.

The framework lies in a single plane.

- (a) Show that the centre of mass of the framework is a distance $\frac{6a}{(3\pi + 2)}$ from AB .

(5)

The point E is on the arc AB and the angle AOE is a right angle.

The framework is freely suspended from A .

The framework hangs at rest in a vertical plane with AE making an angle θ with the downward vertical.

- (b) Find the exact value of $\tan \theta$, simplifying your answer.

(5)

Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



(Total for Question 3 is 10 marks)



Question 7 continued

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Question 7 continued

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Question 7 continued

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Q7

(Total 11 marks)



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6. [The centre of mass of a semicircular arc of radius r is $\frac{2r}{\pi}$ from the centre.]

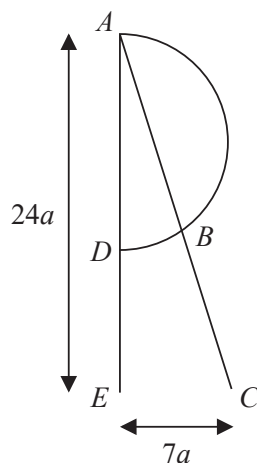


Figure 2

Uniform wire is used to form the framework shown in Figure 2.

In the framework,

- ABC is straight and has length $25a$
- ADE is straight and has length $24a$
- ABD is a semicircular arc of radius $7a$
- $EC = 7a$
- angle $AEC = 90^\circ$
- the points A, B, C, D and E all lie in the same plane

The distance of the centre of mass of the framework from AE is d .

- (a) Show that $d = \frac{53}{2(7 + \pi)}a$ (4)

The framework is freely suspended from A and hangs in equilibrium with AC at angle α° to the downward vertical.

- (b) Find the value of α . (7)

Question 6 continued

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Question 6 continued

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Question 6 continued

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Lined area for writing the answer to Question 6.

(Total 11 marks)

Q6

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6.

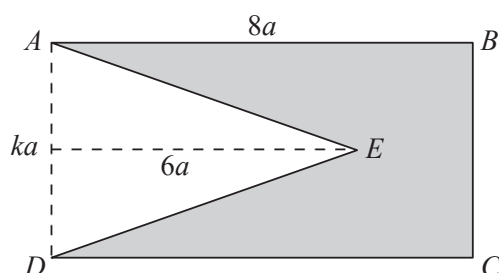


Figure 4

The uniform rectangular lamina $ABCD$ has $AB = 8a$ and $AD = ka$.

The isosceles triangle AED has $AE = DE$ and the perpendicular height of the triangle is $6a$.

The triangle AED is removed from the rectangle to form the template shown shaded in Figure 4.

The distance of the centre of mass of the template from AD is d .

(a) Show that $d = \frac{26}{5}a$.

(5)

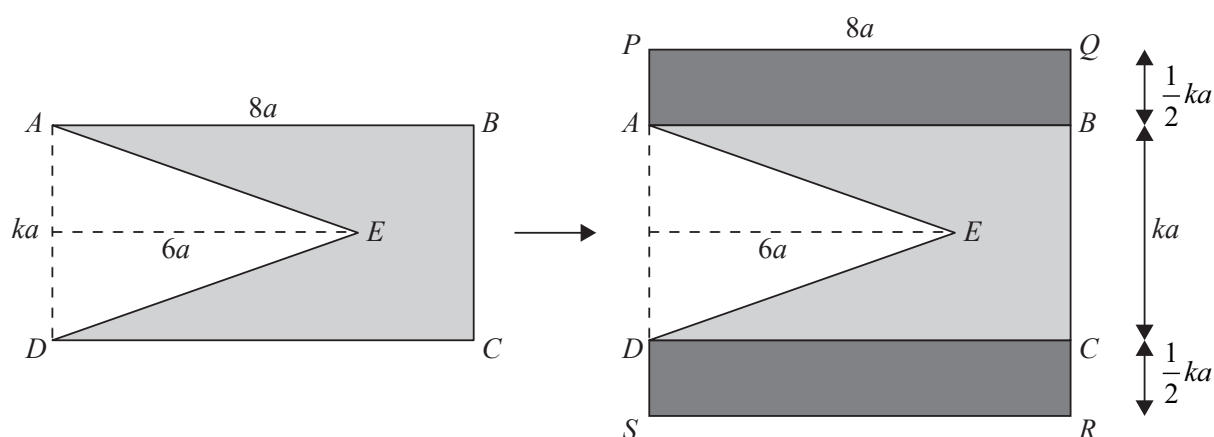


Figure 5

Two identical rectangular laminas, each with length $8a$ and width $\frac{1}{2}ka$, are attached to the template to form the model $PQRSDEA$ shown in Figure 5.

- the two rectangular laminas and the template all lie in the same plane
- the mass per unit area of rectangular lamina $PQBA$ is **three times** the mass per unit area of the template
- the mass per unit area of rectangular lamina $DCRS$ is **three times** the mass per unit area of the template

The model $PQRSDEA$ is freely suspended from P and hangs in equilibrium with PS at an angle of θ to the downward vertical.

(b) Find, in terms of k , the exact value of $\tan \theta$.

(6)



[illegible]



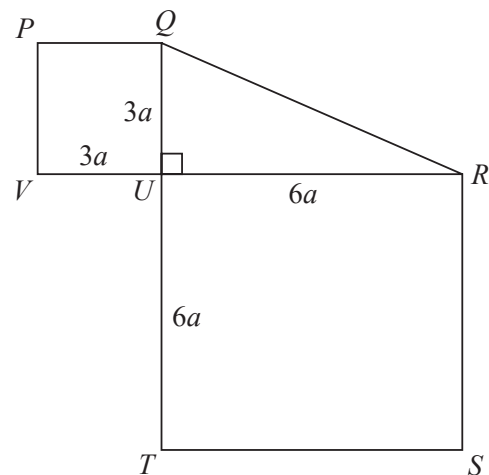
Question 6 continued

(Total for Question 6 is 11 marks)



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7.

**Figure 3**

The template shown in Figure 3 is formed by joining together three separate laminas. All three laminas lie in the same plane.

- $PQUV$ is a uniform square lamina with sides of length $3a$
- $URST$ is a uniform square lamina with sides of length $6a$
- QRU is a uniform triangular lamina with $UQ = 3a$, $UR = 6a$ and angle $QUR = 90^\circ$

The mass per unit area of $PQUV$ is k , where k is a constant.

The mass per unit area of $URST$ is k .

The mass per unit area of QRU is $2k$.

The distance of the centre of mass of the template from QT is d .

- (a) Show that $d = \frac{29}{14}a$ (5)

The template is freely suspended from the point Q and hangs in equilibrium with QR at θ° to the downward vertical.

- (b) Find the value of θ (7)

Question 7 continued

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Lined area for writing the answer to Question 7 continued.



Question 7 continued

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Question 7 continued

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Handwriting practice area with horizontal lines.

(Total 12 marks)

Q7
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3.

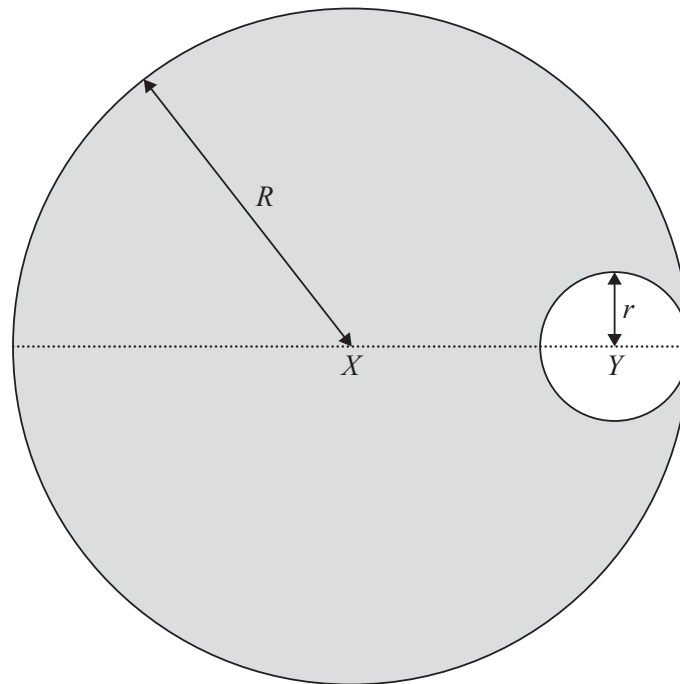


Figure 1

A uniform circular disc C has centre X and radius R .

A disc with centre Y and radius r , where $0 < r < R$ and $XY = R - r$, is removed from C to form the template shown shaded in Figure 1.

The centre of mass of the template is a distance kr from X .

(a) Show that $r = \frac{k}{1-k} R$ (4)

(b) Hence find the range of possible values of k . (2)

The point P is on the outer edge of the template and PX is perpendicular to XY .

The template is freely suspended from P and hangs in equilibrium.

Given that $k = \frac{4}{9}$

(c) find the angle that XY makes with the vertical. (3)

The mass of the template is M .

(d) Find, in terms of M , the mass of the lightest particle that could be attached to the template so that it would hang in equilibrium from P with XY horizontal. (3)

Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

(Total for Question 3 is 12 marks)



4.

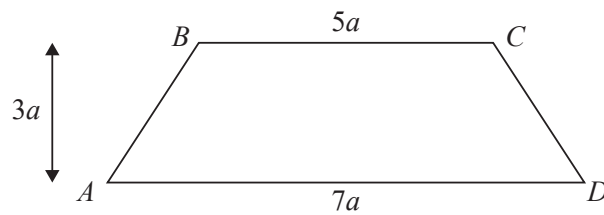


Figure 2

The uniform lamina $ABCD$ shown in Figure 2 is in the shape of an isosceles trapezium.

- BC is parallel to AD and angle BAD is equal to angle ADC
- $BC = 5a$ and $AD = 7a$
- the perpendicular distance between BC and AD is $3a$
- the distance of the centre of mass of $ABCD$ from AD is d

(a) Show that $d = \frac{17}{12}a$

(5)

The uniform lamina $PQRS$ is a rectangle with $PQ = 5a$ and $QR = 9a$.

The lamina $ABCD$ in Figure 2 is used to cut a hole in $PQRS$ to form the template shown shaded in Figure 3.

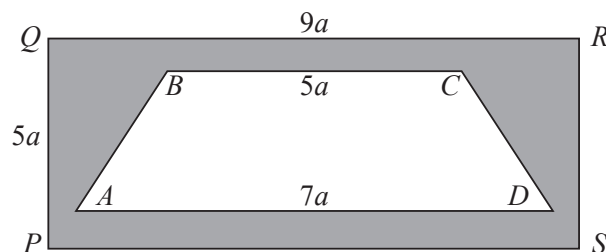


Figure 3

- PS is parallel to AD
- the perpendicular distance between PS and AD is a
- the perpendicular distance of A from PQ is a

The template is freely suspended from P and hangs in equilibrium with PS at an angle of θ° to the downward vertical.

(b) Find the value of θ

(7)



[illegible]



(Total for Question 4 is 12 marks)



2.

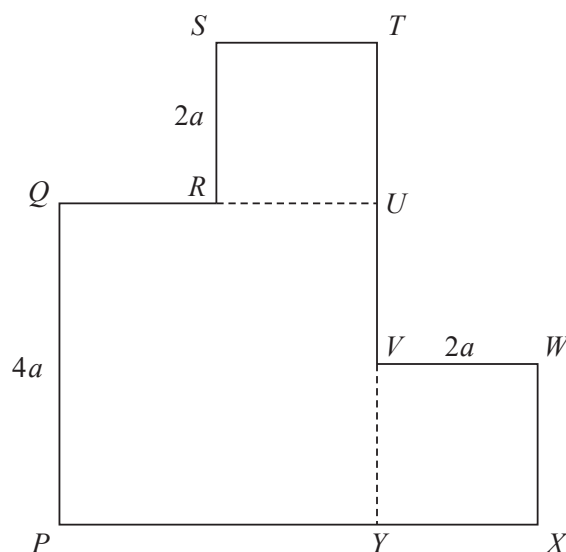


Figure 1

Figure 1 shows a template where

- $PQUY$ is a uniform square lamina with sides of length $4a$
- $RSTU$ is a uniform square lamina with sides of length $2a$
- $VWXY$ is a uniform square lamina with sides of length $2a$
- the three squares all lie in the same plane
- the mass per unit area of $VWXY$ is **double** the mass per unit area of $PQUY$
- the mass per unit area of $RSTU$ is **double** the mass per unit area of $PQUY$
- the distance of the centre of mass of the template from PX is d

(a) Show that $d = \frac{5}{2}a$ (5)

The template is freely pivoted about Q and hangs in equilibrium with PQ at an angle of θ to the downward vertical.

(b) Find the value of $\tan \theta$ (6)

The mass of the template is M

The template is still freely pivoted about Q , but it is now held in equilibrium, with PQ vertical, by a horizontal force of magnitude F which acts on the template at X . The line of action of the force lies in the same plane as the template.

(c) Find F in terms of M and g (3)





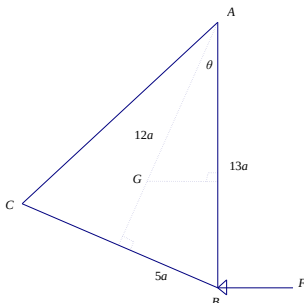
(Total for Question 2 is 14 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks	Notes
2.			
	Centre of mass of triangle is at G, where $AG = 8a$	B1	Or equivalent. Seen or implied e.g. $\frac{2}{3} \times 5a \cos \theta \left(= \frac{40a}{13} \right)$ from AB
	$\sin \theta = \frac{5}{13}$	B1	Or equivalent. Any correct trig ratio for an angle in the triangle. Seen or implied
	M(A): $13aF = W \times 8a \times \frac{5}{13}$	M1	Dimensionally correct with resolved component of their $8a$ Condone sin/cos confusion. If g appears, mark as an accuracy error
		A1	Correct substituted equation (any form)
	$F = \frac{40W}{169} \text{ (N)}$	A1	$0.24W$ or better
		[5]	

Q	Mark Scheme	Marks	Marking guidance
1 (a)	M(x axis)	M1	Need all terms. Dimensionally consistent. Condone if m missing throughout. Accept as part of a vector equation
	$2m \times (-2) + 3m \times 2 + 4m \times 3k = 9m \times \bar{x}$ $\bar{x} = \frac{2+12k}{9}$ *	A1*	Obtain given result
		2	
(b)	M(y axis)	M1	Need all terms. Dimensionally consistent. Might be seen as part of a vector equation in (a). It does not score any marks until referred to in part (b). Condone if m missing throughout.
	$2m \times 5 + 3m \times (-3) + 4m \times k = 9m \times \bar{y}$ $\left(\bar{y} = \frac{1+4k}{9} \right)$	A1	Correct unsimplified equation. Allow if m missing throughout.
	Form and solve equation in k ($2+12k+2+8k=27$)	DM1	Use their $\bar{y}$ and $\bar{x}+2\bar{y}=3$ Dependent on the two preceding M marks
	$k = \frac{23}{20}$ (1.15)	A1	Correct answer only
		4	
		(6)	

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

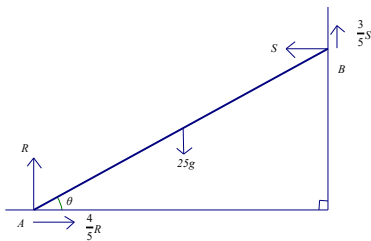
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\surd$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - $\square$ or d... The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

3a		area	distance from AE		
	rectangle	$48a^2$	$4a$	B1	Mass ratio correct
	triangle	$18a^2$	$8a - 2a (= 6a)$	B1	Distances from AE (or parallel axis) correct
	lamina	$30a^2$			
	$M(AE)$			M1	Allow use of a parallel axis. The moments equation should include a but condone if the mass ratio does not include a factor of a^2 . Dimensionally correct.
	$48a^2 \times 4a - 18a^2 \times 6a = 30a^2 \bar{x}$			A1	Correct unsimplified equation for their axis. Accept as part of a vector equation.
	$\bar{x} = \frac{84}{30}a = \frac{14}{5}a$ *			A1*	Obtain given answer from correct working (including correct use of a)
					If they take moments about BD they get $d = 5.2a$ Allow B1B1M1A1A0 if they get this far.
				5	
3b	Find trig ratio of a relevant angle			M1	Correct use of trig.
	$\tan \theta^\circ = \frac{3a}{2.8a}$			A1	Correct equation for the required angle. (DO NOT ISW: If they obtain 47 and then use $90 - 47 = 43$ they score M1A0A0)
	$\theta = 47$			A1	The Q asks for a whole number of degrees. 0.82 radians scores M1A1A0
				3	
				(8)	

Question	Scheme					Marks	Notes
They must have a dissection for which they should know or find the position of the centre of mass (e.g. triangles and rectangles). A false assumption about the position of the centre of mass of a trapezium results in 0/5.							
3a		Large tri	Small tri	Small tri	Whole	B1	Correct distances from PQ or a parallel axis for their complete dissection
	Dist PQ	0	-2y	2y	d	B1	Correct mass ratios for a complete dissection
	Mass ratio	27xy	12xy	12xy	27xy		Correct distances from PQ or a parallel axis for their complete dissection
	Moments about PQ:					M1	Or a parallel axis. Dimensionally correct. Need all non-zero terms and no extras. Condone sign error(s). Allow for ±d. Check the logic carefully.
	$(27xy \times 0) - 12xy \times (-2y) + 12xy \times 2y = 27xyd$					A1	Correct unsimplified equation. Allow for ±d. Allow for correct distance from a parallel axis
	$d = \frac{48}{27}y = \frac{16}{9}y$ *					A1*	Obtain given result from fully correct working.
There are many different approaches to this. NB If they are using a trapezium they must show method for the distance. For PQBC the correct value for distance centre of mass from PQ is $\frac{8y}{5}$ Possible alternative moments equations include: $15xy \times \frac{8y}{5} + 9xy \times \frac{4y}{3} + 3xy \times 4y = 27xyd$ using PQBC, PQDE and DEA $12xy \times 2y + 15xy \times \frac{8y}{5} = 27xyd$ using PQA and PQBC $2 \times 3xy \times y - 3xy \times y + 2 \times 6xy \times 1.5y + 2 \times 3xy \times 2y = 27xyd$ working from BC for the folded figure. $2 \times 3xy \times 2y + 4 \times \frac{1}{2} 3xy \times y + 2 \times 6xy \times 1.5y + 3xy \times 4y = 27xyd$ working down from PQ							
						(5)	

Question	Solution	Marks	Notes
3			
	Use of $F = \mu R$	B1	At least once
	Resolve horizontally	M1	Allow with their horizontal friction
	$S = \frac{4}{5} R \quad (S = F_A)$	A1	Correct unsimplified equation
	Resolve vertically	M1	Allow with their vertical friction
	$\frac{3}{5} S + R = 25g \quad F_B + R = 25g$ $\left(\frac{3}{5} S + \frac{5}{4} S = 25g, \quad S = \frac{500}{37} g \right)$	A1	Correct unsimplified equation
	Moments equation	M1	Any moments equation. Need all terms & dimensionally correct
	$M(A): 25g \times 1.5 \cos \theta = S \times 3 \sin \theta + \frac{3}{5} S \times 3 \cos \theta$ $\left(25g \cos \theta - \frac{6}{5} S \cos \theta = 2S \sin \theta \right)$ $M(B): R \times 3 \cos \theta = 25g \times 1.5 \cos \theta + \frac{4}{5} R \times 3 \sin \theta$	A1	Correct unsimplified equation
	M1A1 for first equation, M1A1 for second equation, M1A1 for third equation (i.e. mark in the order in which they appear rather than as listed on the mark scheme). If there are more than 3 equations, mark the 3 used or the best 3 if they go no further. Can also be solved using one resolution and two moments equations. Friction acting in the wrong direction scores A0.		
	$\tan \theta = \left(\frac{25g - \frac{6}{5} S}{2S} \right) = \frac{25 - \frac{600}{37}}{\frac{1000}{37}}$	DM1	Substitute to form equation in $\tan \theta$ only Condone in decimals Dependent on M marks for the equations
	$= \frac{325}{1000} \left(= \frac{13}{40} \right)$	A1	Or exact equivalent (0.325)
		(9)	
SC	It is possible to solve by resolving horizontally or vertically and taking moments about the centre: $1.5 \cos \theta \times R = 1.5 \cos \theta \times \frac{3}{5} S$ $+ 1.5 \sin \theta \times S + 1.5 \sin \theta \times \frac{4}{5} R$		M1A1 for a correct resolution M2A2 for a complete sets of equations to solve
		[9]	

Question Number	Scheme	Marks	Notes
4	$6m \times 3a + 4m \times 4a + 5m \times 2a = 15m \times y$	M1	Moments about a horizontal axis. Terms dimensionally consistent. Condone slip with a Needs all terms. Condone sign errors
	$(44ma = 15my)$	A1	Correct unsimplified
	$y = \frac{44a}{15}$ from B	A1	Or equivalent. Correct for their axis $\frac{46a}{15}$ from A $\frac{16a}{15}$ from E (CD)
	$5m \times \frac{3a}{2} + 4m \times a = 15m \times x$	M1	Moments about a vertical axis Terms dimensionally consistent. Condone slip with a Needs all terms. Condone sign errors
	$\left(\frac{23ma}{2} = 15mx\right)$	A1	Correct unsimplified
	$x = \frac{23a}{30}$ from E (AB)	A1	Or equivalent. Correct for their axis $\frac{67a}{30}$ from C
	$\tan \theta = \frac{23}{30} / \frac{44}{15}$	M1	Find a relevant angle using distances measured from B (Allow for $\tan \theta = \frac{88}{23}$)
	$= \frac{23}{88} (= 0.261...)$	A1ft	Correct for their distances from B. $\left(\frac{\text{horizontal}}{\text{vertical}}\right)$
	$\theta = 14.64.... \approx 15^\circ$	A1	From correct working. The question asks for the answer to the nearest degree.
		[9]	
	SR1: If a candidate has not used a in their working at all, marks as a misread. Maximum marks available: M1A0A0 M1A1A1 M1A1A1		
	SR2: If a candidate has a in their working, but not as part of their values for $\bar{x}$ and $\bar{y}$ the maximum marks available are M1A1A0 M1A1A0 M1A1A1		

Q	Mark Scheme	Marks	Marking guidance
6 (a)	M(PV)	M1	Allow use of a parallel axis. Terms dimensionally consistent. Could be seen as part of a vector equation. Condone error(s) in distance(s).
	$a \times 2ka^2 + \left(2a + \frac{1}{2}ka\right)2ka^2 = \bar{x} \times (2ka^2 + 2ka^2)$	A1	Correct unsimplified equation
	$2\bar{x} = a + 2a + \frac{1}{2}ka \Rightarrow \bar{x} = \frac{6+k}{4}a$ *	A1*	Obtain given answer from correct working
		3	
6 (b)	M(PR)	M1	Allow use of a parallel axis. Terms dimensionally consistent. Could be seen as part of a vector equation in (a) but needs to be used here to score mark(s) in (b). Condone error(s) in distance(s). If working from VU they might assume that c of m of QRST lies on their axis. So long as they say that this is what they have done (e.g. in a table of values) this can score M1A0A0M1A1ftA0.
	$\frac{1}{2}ka \times 2ka^2 + a \times 2ka^2 = \bar{y} \times 4ka^2$	A1	Correct unsimplified equation
	$\bar{y} = \frac{k+2}{4}a$	A1	Correct answer ($\pm$) seen or implied Accept distance from $VU = \pm \frac{3k-2}{4}a$ Or distance from TS = $\pm \frac{6-k}{4}a$
	Use angle to form equation in k	M1	Correct use of given ratio. Allow reciprocal
	$\frac{7}{15} = \frac{\bar{y}}{\bar{x}} = \frac{(k+2)a}{4} \times \frac{4}{(6+k)a}$	A1	Correct unsimplified equation using given $\bar{x}$ and their $\bar{y}$ e.g. $\frac{ka - \bar{y}_{VU}}{\bar{x}}$ or $\frac{2a - \bar{y}_{TS}}{\bar{x}}$
	$\Rightarrow k = \frac{3}{2} (=1.5)$	A1	Correct only
		6	
		(9)	

Q	Scheme	Mark	Notes																											
4a	If the division of the shape involves non-standard shapes (e.g. a trapezium) the centres of mass must be quoted correctly or a correct method used to find the position to score any marks.																													
	<table><tr><td>rectangle</td><td>-triangle</td><td>+triangle</td></tr><tr><td>$20a^2$</td><td>$-\frac{9}{2}a^2$</td><td>$\frac{9}{2}a^2$</td></tr><tr><td>$2a$</td><td>$3a$</td><td>$2a$</td></tr></table> or <table><tr><td>rectangle</td><td>rectangle</td><td>Double triangle</td></tr><tr><td>$3a^2$</td><td>$8a^2$</td><td>$9a^2$</td></tr><tr><td>$\frac{1}{2}a$</td><td>$2a$</td><td>$2a$</td></tr></table> or <table><tr><td>rectangle</td><td>trapezium</td><td>triangle</td></tr><tr><td>$5a^2$</td><td>$\frac{21}{2}a^2$</td><td>$\frac{9}{2}a^2$</td></tr><tr><td>$\frac{1}{2}a$</td><td>$\frac{48}{21}a$</td><td>$2a$</td></tr></table>	rectangle	-triangle	+triangle	$20a^2$	$-\frac{9}{2}a^2$	$\frac{9}{2}a^2$	$2a$	$3a$	$2a$	rectangle	rectangle	Double triangle	$3a^2$	$8a^2$	$9a^2$	$\frac{1}{2}a$	$2a$	$2a$	rectangle	trapezium	triangle	$5a^2$	$\frac{21}{2}a^2$	$\frac{9}{2}a^2$	$\frac{1}{2}a$	$\frac{48}{21}a$	$2a$	B1 B1	Correct mass ratios for a correct division of the folded template and correct total of $20a^2$ Correct distances from AD seen or implied. B0B1 is possible if they have incorrect masses but a full set of correct distances. e.g. if they use the second alternative but have not doubled the triangle. Or they might have a correct split with an error in one of the areas or an incorrect (or missing) total or equivalent
rectangle	-triangle	+triangle																												
$20a^2$	$-\frac{9}{2}a^2$	$\frac{9}{2}a^2$																												
$2a$	$3a$	$2a$																												
rectangle	rectangle	Double triangle																												
$3a^2$	$8a^2$	$9a^2$																												
$\frac{1}{2}a$	$2a$	$2a$																												
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$\frac{1}{2}a$	$\frac{48}{21}a$	$2a$																												
	Moments about AD or a parallel axis.	M1	Dimensionally consistent. All terms for a correct division of L and no extras. Accept as part of a vector equation																											
	$40a^3 - \frac{27}{2}a^3 + 9a^3 = 20a^2d$ or $\frac{3}{2}a^3 + 16a^3 + 18a^3 = 20a^2d$ or $\frac{2}{2}a^3 + \frac{48}{2}a^3 + 9a^3 = 20a^2d$	A1	Correct unsimplified equation for their axis. Allow for correct component in a vector equation.																											
	$d = \frac{71}{40}a$ *	A1*	Obtain given answer from correct working. Need at least one line of working to collect like terms e.g. $20d = \frac{71}{2}a$ Final answer must be as printed i.e. $d = \dots$																											
		[5]																												
4b	Moments about S	M1	A complete method to get an equation in W and F only. Need all terms and no extras. Dimensionally consistent.																											
NB If they start by finding the centre of mass for the system they do not score marks until they form the moments equation. If they are clearly using moments about A (e.g. d and $4a$ used as distances in their equation) this is M0 unless they include the reaction at S and resolve to form the required equation. If they say they are using moments about S and have just one incorrect distance allow M1A1A0A0																														
	$4W \times \frac{31}{40}a + W \times 3a = F \times 5a$ or $(4W + W)(2.22a - a) = 5aF$	A1 A1	Unsimplified equation with at most one error Correct unsimplified equation																											
	$F = \frac{61}{50}W$	A1	Accept 1.22W or 1.2W																											
		[4]																												
		(9)																												

Question Number	Scheme	Marks
3	Use of $mv = \mathbf{I} + m\mathbf{u}$ Component of momentum parallel to original direction $= 6 \times 0.75 + \sqrt{24} \cos 60$ ($= 4.5 + \sqrt{6}$) Use of Pythagoras: $(\frac{3}{4}v =) \sqrt{(4.5 + \sqrt{6})^2 + 18}$ $v = 10.9 \text{ (m s}^{-1}\text{)}, 11 \text{ (m s}^{-1}\text{)}$ <i>Alternative for the 1st 5 marks:</i> Vector triangle for impulses or velocities Use of cosine rule $\left(\frac{3}{4}v\right)^2 = 4.5^2 + 24 - 2 \times 4.5 \times \sqrt{24} \times \cos 120^\circ$ $v = 10.9 \text{ (m s}^{-1}\text{)}, 11 \text{ (m s}^{-1}\text{)}$ Change in direction $= \tan^{-1} \frac{3\sqrt{2}}{4.5 + \sqrt{6}}$ $= 31.4^\circ \text{ (} 31^\circ \text{)}$ Notes	M1 A1 A1 M1 A1 M1 M1 A1 A1 A1 M1 A1 (7) [7]
M1 A1 M1 A1 A1 M1 A1 A1 M1 A1	Need to consider both components. Or equivalent Or equivalent. Correct LHS Or better Must be using correct triangle - need 120° seen or implied Correct unsimplified Or better Or equivalent use of trig. With their components to find the required angle Eg angle $= \cos^{-1} \left(\frac{4.5^2 + (mv)^2 - 24}{2 \times 4.5 \times (mv)} \right)$ Or from scalar product, $\cos^{-1} \left(\frac{6 \times 9.27...}{6 \times 10.9...} \right)$ 0.548 radians (0.55 radians) or better. Do not ISW	

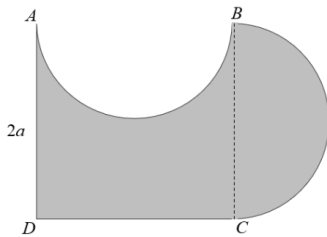


Question Number	Scheme	Marks
4(a)	Moments about AC $18 \times \frac{3a}{2} - 2\pi \times \frac{8a}{3\pi} + 2\pi \left(3a + \frac{8a}{3\pi} \right) = 18\bar{y}$ NB: valid to use $18 \times \frac{3a}{2} - 2\pi \times d + 2\pi(3a + d) = 18\bar{y}$ for $d \neq 0$ without stating value for d Use of $d = 0 \Rightarrow$ M0 $(27a + 6\pi a = 18\bar{y})$ The same incorrect distance used twice in place of $\frac{8a}{3\pi}$ is one error The same incorrect area for the semicircle used twice is one error. $27a + 6\pi a = 18\bar{y} \Rightarrow \bar{y} = \frac{9 + 2\pi}{6} a \quad *$	M1 A1 A1 (4)
4b	$M\bar{x} + kM \times 6a = (1+k)M\bar{x}_T$ $3a + 6ak = (1+k)\bar{x}_T \quad \text{o.e.}$ $M\bar{y} = (1+k)M\bar{y}_T$ $(1+k)\bar{y}_T = \frac{9 + 2\pi}{6} a$	M1 A1 M1 A1
NB	For their second equation they could use $\tan \phi$ and their $\bar{x}_T$ or $\bar{y}_T$ to form an expression for $\bar{y}_T$ or $\bar{x}_T$ $\tan \phi = \frac{3}{2} = \frac{\bar{x}_T}{\bar{y}_T} \Rightarrow \frac{3}{2} = \frac{6(3a + 6ak)}{(9 + 2\pi)a}$ $\Rightarrow k = \frac{\pi}{12} - \frac{1}{8} \quad \text{or equivalent}$	DM1 A1 (6)
Notes		
M1	All terms. Dimensionally correct. Condone sign errors	
A1	Unsimplified equation with at most one error.	
A1	Correct unsimplified equation	
A1	Obtain given answer from sufficient correct exact working. Must see a separate conclusion for $\bar{y}$.	
NB	e.g. $\bar{y}_T = \frac{2(3 + 6k)a}{3(1+k)}$	
DM1	Form equation in k and solve for k . Dependent on the previous 2 M marks.	
A1	$k = 0.137$ (0.14) or better See over for alternative solution to 4(b)	



Question Number	Scheme	Marks
4(b) alt	Distance of original c of m from vertical through A $\left(\frac{9+2\pi}{6}a - 2a\right) \times \sin \phi = \frac{\sqrt{13}(2\pi-3)a}{26}$ Distance of additional particle from vertical through A $6a \times \cos \phi = \frac{12a}{\sqrt{13}}$ $mg \times \frac{\sqrt{13}(2\pi-3)a}{26} = kmg \times \frac{12a}{\sqrt{13}}$ $k = 0.137 \quad (0.14)$ Notes	M1 A1 M1 A1 DM1 A1 (6) [10]
M1	Or equivalent Distance of additional particle from vertical through A Or equivalent Moments about A Dependent on the 2 previous M marks (0.14 or better)	
A1		
M1		
A1		
DM1		
A1		



QUESTION NUMBER	SCHEME	MARKS															
2																	
2(a)	<table border="1"> <thead> <tr> <th></th><th>Mass ratio</th><th>Distance from AD</th></tr> </thead> <tbody> <tr> <td>Square</td><td>$4a^2$</td><td>a</td></tr> <tr> <td>Semicircle removed</td><td>$\frac{\pi a^2}{2}$</td><td>a</td></tr> <tr> <td>Semicircle attached</td><td>$\frac{\pi a^2}{2}$</td><td>$\frac{4a}{3\pi} + 2a$</td></tr> <tr> <td>Lamina</td><td>$4a^2$</td><td>$\bar{x}$</td></tr> </tbody> </table>		Mass ratio	Distance from AD	Square	$4a^2$	a	Semicircle removed	$\frac{\pi a^2}{2}$	a	Semicircle attached	$\frac{\pi a^2}{2}$	$\frac{4a}{3\pi} + 2a$	Lamina	$4a^2$	$\bar{x}$	B1 (mass) B1 (distances)
	Mass ratio	Distance from AD															
Square	$4a^2$	a															
Semicircle removed	$\frac{\pi a^2}{2}$	a															
Semicircle attached	$\frac{\pi a^2}{2}$	$\frac{4a}{3\pi} + 2a$															
Lamina	$4a^2$	$\bar{x}$															
	Moments equation about AD or a parallel axis	M1															
	About AD $(4a^2 \times a) - \left(\frac{\pi a^2}{2} \times a\right) + \frac{\pi a^2}{2} \left(\frac{4a}{3\pi} + 2a\right) = 4a^2 \bar{x}$ About BC $4a^2(a) - \frac{\pi a^2}{2}(a) - \frac{\pi a^2}{2} \left(\frac{4a}{3\pi}\right) = 4a^2(2a - \bar{x})$ About line joining midpoints of AB and DC $4a^2(0) - \frac{\pi a^2}{2}(0) + \frac{\pi a^2}{2} \left(a + \frac{4a}{3\pi}\right) = 4a^2(\bar{x} - a)$	A1															
	Obtain given answer from correct working. $\bar{x} = \frac{a}{24}(28 + 3\pi)^*$	A1*															
		(5)															
2(b)	Relevant moments equation	M1															
	$T \times 2a = W \frac{a}{24}(28 + 3\pi)$	A1															
	Second relevant equation	M1															
	e.g. $5T = W + kW$	A1															
	$k = \left(\frac{92 + 15\pi}{48}\right)$	A1															
		(5)															
		(10)															
	Notes for question																
2(a)	Vector form is acceptable for part a																

B1	Correct mass ratios for all 4 sections.	
B1	Correct distances for square and two semicircle sections measured from their moments line. (or shape to LHS of BC and semicircle)	
M1	Moments taken about AD or a parallel axis. Dimensionally correct equation. All terms required. 'a ² 's may have been cancelled. LHS correct and RHS uses their $\bar{x}$.	
A1	Correct unsimplified equation in terms of a , $\bar{x}$ and π .	
A1*	Obtain given answer from correct working. At least one stage of simplification must be seen. Must be factorised with Allow $(28+3\pi)$ and $(3\pi+28)$	
2(b)		
M1	Form a moments equation about AD or a parallel axis. Dimensionally correct equation. All terms required.	
A1	Correct unsimplified equation. Allow in terms of $\bar{x}$. Accept decimal form for $\bar{x}$	
M1	Second relevant equation (moments or vertical equilibrium). Dimensionally correct equation. All terms required.	
A1	Correct unsimplified equation. Accept decimal form for $\bar{x}$ Other possible moments equations include: About B: $4T \times 2a - kW \times 2a - W(2a - \bar{x}) = 0$ About midpoint AB: $4T \times a - kW a - W(\bar{x} - a) - Ta = 0$ About G: $4T \bar{x} - kW \bar{x} = T(2a - \bar{x})$	
A1	Accept equivalent forms in the form $p + q\pi$, but must be exact	

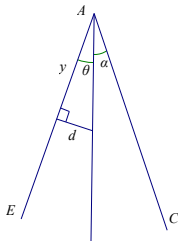
3(a)	arc AB	arc CD	$AC + DB$	L	
	Length : $\pi(2a)$	πa	$2a$	$3\pi a + 2a$	B1
	(mass)				
	or ratios: 2π	π	2	$3\pi + 2$	
	Distance : $\frac{4a}{\pi}$	$\frac{2a}{\pi}$	0	d	B1
	from AB				
	$\pi(2a) \times \frac{4a}{\pi} - \pi a \times \frac{2a}{\pi} = (3\pi a + 2a)d$				M1A1
	$*(d =) \frac{6a}{(3\pi + 2)}$				A1*
	N.B. Accept any equivalent form with whole numbers.				
					(5)
3(b)	$\tan OAG(\alpha) = \frac{\frac{6a}{(3\pi + 2)}}{2a} = \frac{3}{(3\pi + 2)}$				B1
	$\tan \theta = \tan(45^\circ - \alpha)$ or $\tan(\theta + \alpha) = \tan 45^\circ$ or $\tan \alpha = \tan(45^\circ - \theta)$				M1
	$\tan \theta = \frac{1 - \tan \alpha}{1 + \tan \alpha}$	$\frac{\tan \theta + \tan \alpha}{1 - \tan \theta \tan \alpha} = 1$	$\tan \alpha = \frac{1 - \tan \theta}{1 + \tan \theta}$		DM1
	Substitute for $\tan \alpha$ e.g. $\tan \theta = \frac{1 - \frac{3}{(3\pi + 2)}}{1 + \frac{3}{(3\pi + 2)}}$				DM1
	$\tan \theta = \frac{3\pi - 1}{3\pi + 5}$ (must have π 's and integers)				A1
	OR				
	Let F be on AE where GF is perpendicular to AE .				
	$GF = GE \sin 45^\circ$				B1
	$\tan \theta = \frac{GF}{AF} = \frac{GE \sin 45^\circ}{AE - GE \cos 45^\circ}$				M1
	$= \frac{(2a - \bar{x}) \sin 45^\circ}{2a\sqrt{2} - (2a - \bar{x}) \sin 45^\circ}$				DM1
	$= \frac{(2a - \bar{x})}{(2a + \bar{x})}$ where $\bar{x} = \frac{6a}{(3\pi + 2)}$ is substituted in.				DM1
	$= \frac{3\pi - 1}{3\pi + 5}$				A1
					(5)
					(10)
	Notes for question 3				



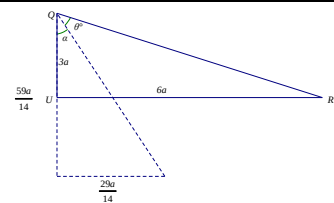
3(a)	N.B. If they treat it as a lamina, can score max B0B0M1A0A0*	
	B1 Mass (length) ratios	
	B1 Distances from AB or a parallel axis	
	M1 Moments about AB or parallel axis, dimensionally correct, condone sign errors, using their ‘masses’ and distances, including all terms.	
	A1 Correct unsimplified equation	
	N.B. Condone missing brackets on RHS here but penalise the A1*	
	A1* Given answer correctly obtained with no errors seen	
	N.B. Need to see reference to AC and DB , either in the table or in the equation.	
3(b)	B1 Correct unsimplified expression for $\tan OAG$, seen or implied	
	N.B. B0 if they have $OAG = \theta$	
	M1 Correct method seen or implied. May see $\arctan 1$ instead of 45°	
	N.B. If their first line of working is: $\tan \theta = \tan 45^\circ - \tan \alpha$ oe, treat as a correct method but wrong formula.	
	DM1 Dependent on previous M for use of a correct formula used to give an equation in $\tan \theta$ and $\tan \alpha$ only	
	DM1 Dependent on previous M for substitution for their $\tan \alpha$ to give an equation in $\tan \theta$ only	
	A1 cao. Must be simplified.	



7a	Use of $\frac{2a \times \frac{1}{2}}{3 \times \frac{\pi}{6}} \left(= \frac{2a}{\pi} \right)$	B1	Seen or implied
	Moments about EC:	M1	Dimensionally correct Condone use of a parallel axis
	$ad \times \frac{d}{2} = \frac{1}{6} \pi a^2 \times \frac{2a}{\pi} \times \sin \frac{\pi}{6}$	A1	Correct unsimplified equation
	$\Rightarrow \left(d^2 = \frac{a^2}{3} \right) \quad a = \sqrt{3}d \quad *$	A1*	Obtain given answer from correct working
		(4)	
7b	Mass ratios $\frac{a^2}{\sqrt{3}} : \frac{\pi a^2}{6} : \frac{a^2}{\sqrt{3}} + \frac{\pi a^2}{6}$	B1	Or equivalent. Seen or implied
	Moments about BC	M1	Dimensionally correct Condone use of a parallel axis
	$\frac{1}{\sqrt{3}} \times \frac{a}{2} + \frac{\pi}{6} \times \frac{2a}{\pi} \times \frac{\sqrt{3}}{2} = \left(\frac{1}{\sqrt{3}} + \frac{\pi}{6} \right) y$	A1ft	Correct unsimplified. Follow their $\frac{2a}{\pi}$
	Distance from BC = $y = \frac{6a}{6 + \sqrt{3}\pi}$	A1	Or equivalent $\left(y = \frac{6d}{2\sqrt{3} + \pi} \right)$
	Use of trig to find a relevant angle	M1	
	$\tan \beta^c = \frac{6}{6 + \sqrt{3}\pi} \times \sqrt{3} \quad \left(\frac{y}{d} \right)$	A1ft	Or equivalent correct unsimplified equation for the required angle
	$\beta = 0.737 \quad (0.74)$	A1	0.74 or better 42.2° implies correct method
		(7)	
		[11]	

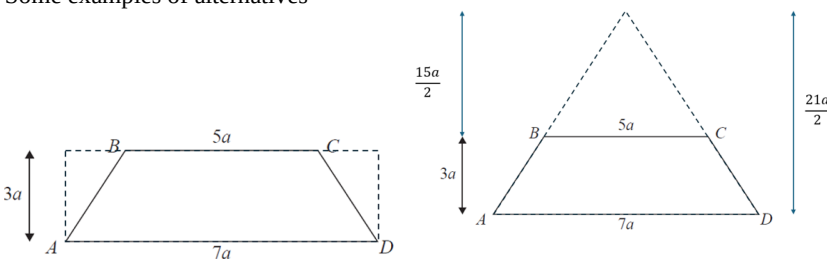
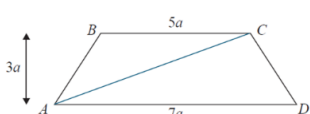
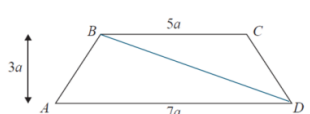
6a	Mass ratio $24a : 25a : 7\pi a : 7a(7 + \pi)$	B1	Correct ratio seen or implied
	Moments about AE	M1	Need all terms, with their masses and horizontal distances Allow use of a parallel axis.
	$25a \times \frac{7}{2}a + 7\pi a \times \frac{14a}{\pi}$ $= 7a(7 + \pi)d$	A1	Correct unsimplified equation
	$\frac{371}{2}a^2 = 7a(7 + \pi)d$ $\Rightarrow d = \frac{53}{2(7 + \pi)}a$ *	A1*	Obtain given answer from correct working Condone if they call it $\bar{x}$
		[4]	
6b	Centre of mass of semicircle lies $7a$ “vertically below” A	B1	Seen or implied e.g. $17a$ above E
	Moments about “horizontal” axis through A :	M1	Or a parallel axis. Need all terms, with their masses and distances.
	$24a \times 12a + 25a \times 12a + 7\pi a \times 7a$ $= 7a(7 + \pi)y$	A1	Correct unsimplified equation
	$y = \frac{49a(12 + \pi)}{7(7 + \pi)} \left(= \frac{7a(12 + \pi)}{7 + \pi} \right)$	A1	Any equivalent form. Accept $\frac{84 + 17\pi}{7 + \pi}a$ from E
NB: A candidate might have a vector equation in (a) which provides evidence for some or all or the first 4 marks in (b).			
			
	$\theta^\circ = \tan^{-1} \frac{d}{y} = \tan^{-1} \frac{53}{14(12 + \pi)} (= 14.037...^\circ)$	DM1	Use trig to find relevant angle (θ or $90 - \theta$) in a triangle with d and A (must now be working with vertical distance of C of M from A) Dependent on first M
	$\alpha^\circ = \tan^{-1} \frac{7}{24} - \theta^\circ$	DM1	Dependent on the previous M1. Complete method for the required angle
	$\alpha = 2.2$	A1	2.2 or better (2.22...)
		[7]	
		(11)	

6a		rectangle	triangle	lamina		
	area	$8ka^2$	$3ka^2$	$5ka^2$	B1	Correct area ratio seen or implied
	From AD	$4a$	$2a$	d	B1	Correct distances from AD or a parallel axis seen or implied. Condone if d not used
	Moments about AD				M1	Or a parallel axis. Need all terms. Dimensionally consistent. Condone sign error.
	$8ka^2 \times 4a - 3ka^2 \times 2a = 5ka^2 \times d$				A1	Correct unsimplified equation
	$26a = 5d \Rightarrow d = \frac{26}{5}a$ *				A1*	Obtain given answer from correct working. Must obtain $d = \dots$
					[5]	
6b	Moments about PS				M1	Or a parallel axis. Need all terms. Dimensionally consistent. Must be using the 3 correctly with areas, so $5ka^2, 15ka^2, 15ka^2$ is M0. Allow a slip on one value. Condone sign error.
	$5k \times \frac{26}{5}a + 2 \times 3 \times 4k \times 4a = (5k + 24k)\bar{x}$				A1	Unsimplified equation with a slip on at most one value
					A1	Correct unsimplified equation. Allow with common factors cancelled
	$\bar{x} = \frac{122}{29}a$				A1	Correct only
	$\bar{y} = ka$				B1	Distance from PQ seen or implied
	$\tan \theta = \frac{122}{29k}$				A1ft	Follow their $\bar{x} \cdot \left(\frac{\bar{x}}{ka}\right)$
					[6]	
					(11)	

7a		PQUV	URST	QRU	total	B1	Correct mass ratios (1:4:2:7)
	Mass ratio	$9a^2$	$36a^2$	$18a^2$	$63a^2$		
	Displacement From QT	$-\frac{3a}{2}$	$3a$	$2a$	d		
	Equation for moments about QT					M1	(or a parallel axis) Dimensionally correct. Condone sign errors
	$18 \times 2a + 36 \times 3a - 9 \times \frac{3a}{2} = 63d$ $\left(4a + 12a - \frac{3a}{2} = 7d\right)$					A1	Or equivalent Correct unsimplified equation Check consistent in a .
	$d = \frac{29a}{2} \div 7 \left(= \frac{261a}{2} \div 63 \right) = \frac{29a}{14} *$					A1*	Obtain given answer from correct working. Need to see at least one interim step with all the a terms collected. Check a is in final answer.
						[5]	
7b	Condone if “ a ” is missing throughout the working in part (b) because they have not been asked for the distance here.						
	Vertical distances from Q: $\frac{3a}{2}, 6a (= 3a + 3a), 2a, (v)$ From T: $7.5a, 3a, 7a$					B1	Seen or implied
	Equation for moments about PQ					M1	(Or a parallel axis) Dimensionally correct. Condone sign errors
	$9 \times \frac{3a}{2} + 18 \times 2a + 36 \times 6a = 63v$ $\left(\frac{3a}{2} + 2 \times 2a + 4 \times 6a = 7v\right)$					A1	Correct unsimplified equation
	$v = \frac{59a}{14} \left(\frac{67}{14}a \text{ above } T, \frac{17}{14}a \text{ below } U \right)$					A1	4.2a or better (4.214...)
	The working for (a) and (b) might be combined in a vector equation. The marks for (b) are scored if the work is used in (b).						
							
	$\tan \alpha = \frac{29}{59} (= 26.175...^\circ)$					M1	Use trig and their v to find a relevant angle Allow for $90^\circ - 26.17...^\circ$

	$\theta^\circ = \tan^{-1} 2 - \tan^{-1} \left(\frac{29}{59} \right)$	M1	Use their v to find the required angle ($63.43^\circ - 26.175^\circ$)
	$\theta = 37.3$	A1	37 or better
		[7]	
		(12)	
8a	Normal reaction between P and the ramp $= 3g \cos \alpha \quad \left(= \frac{18g}{\sqrt{37}} = 29.0 \right)$	B1	cao ISW
	Use of $F = \frac{3}{4} R$	M1	$\frac{3}{4} \times$ their R (Must have an R)
	Work done $= 4F$	M1	Their F (Must have an F)
	$= 87.0 (87) (J)$	A1	3 sf or 2 sf only (follows 9.8) do not allow $\frac{54}{\sqrt{37}} g$ (this is an acceleration)
		[4]	
8b	Work-energy equation	M1	M0 if not using work-energy. All terms required. Condone sign errors Condone sin/cos confusion
	$\frac{1}{2} \times 3U^2 - \text{their}(a) - 3g \times 4 \sin \alpha = \frac{1}{2} \times 3 \times 25$	A1ft A1ft	Unsimplified equation with at most one error. Follow their (a) Correct unsimplified equation Follow their (a)
	$U = 9.79$ or $U = 9.8$	A1	3 sf or 2 sf only (follows 9.8)
		[4]	
8c	Time taken:	M1	Complete method using <i>suvat</i> and $u = 5$ to form an equation in t only
	$-4 \sin \alpha = (5 \sin \alpha) t - \frac{1}{2} g t^2$ $(4.9\sqrt{37}t^2 - 5t - 4 = 0)$	A1	Correct unsimplified equation for t .
	$t = 0.45969....$	A1	Seen or implied
	Horizontal distance	M1	Complete method using <i>suvat</i> and $u = 5$
	$= (5 \cos \alpha) t \quad \left(= \frac{30}{\sqrt{37}} t \right)$	A1ft	Follow their t
	$= 2.27$ or $2.3 (m)$	A1	3 sf or 2 sf only
		[6]	
	Alternative: First M1A1 as above Second M1A1 as above Second A1 correct quadratic in horizontal distance e.g. $\frac{37 \times 4.9}{35 \times 25} d^2 - \frac{1}{6} d - \frac{4}{\sqrt{37}} = 0$ Final A1 as above		
		(14)	

Q	Scheme	Marks	Notes
1(a)	Expression for total KE before collision	M1	Dimensionally correct. Condone confusion between before and after for A. Allow if vectors seen in the working but the modulus is used correctly. The two parts must be added together Allow confusion of 2 kg and 3 kg
	$\frac{1}{2} \times 2 \times 5^2 + \frac{1}{2} \times 3 \times (3^2 + (-1)^2)$	A1	correct unsimplified expression
	= 40 (J)	A1	cao
		(3)	
1(b)	$2((3\mathbf{i} + 2\mathbf{j}) - 5\mathbf{j})$	M1	change in momentum of A, must be a difference but allow subtraction in either order Must be using the correct mass, 2 kg
	= $(6\mathbf{i} - 6\mathbf{j})$ (N s)	A1	Cao The final answer should be in terms of $\mathbf{i}$ and $\mathbf{j}$ because this is asked for in the question. Accept $2(3\mathbf{i} - 3\mathbf{j})$ ISW
		(2)	
1(c)	impulse-momentum equation for B	M1	must use negative of their answer to (b) and the initial velocity of B Must be using the correct mass, 3 kg or CLM with correct terms (allow slip) and plus signs
	$3(\mathbf{v}_B - (3\mathbf{i} - \mathbf{j})) = (-6\mathbf{i} + 6\mathbf{j})$ or $2 \times 5\mathbf{j} + 3(3\mathbf{i} - \mathbf{j}) = 2(3\mathbf{i} + 2\mathbf{j}) + 3\mathbf{v}_B$	A1ft	correct unsimplified equation ft on their impulse from (b)
	$\mathbf{v}_B = (\mathbf{i} + \mathbf{j})$ (m s ⁻¹)	A1	Cao Accept column vector ISW
		(3)	
		(8)	

Question	Scheme				Marks																																																
4(a)		Rectangle	2 x Triangle	Trapezium																																																	
	Mass	$15a^2$	$2 \times \frac{3}{2}a^2$	$18a^2$	B1																																																
	From AD	$\frac{3}{2}a$	a	d	B1																																																
Some examples of alternatives																																																					
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Expression for relevant angle					M1																																																



	$\tan \theta^\circ = \frac{\bar{y}}{\left(9a/2\right)} \quad \left(= \frac{46}{81} \right)$	
	$\theta = 29.6$	A1
		(7)
		(12)
	Notes	
4(a)		
B1	Correct mass ratio.	
B1	Correct distances from <i>AD</i> , for an appropriate division of <i>ABCD</i> Distances may be measured from a parallel axis	
M1	Dimensionally consistent equation with all required terms. Accept working from a parallel axis. Accept an equation embedded in vector form.	
A1	Correct unsimplified equation. Accept embedded in vector form.	
A1*	Obtain given answer from correct working. Working must include simplification or rearrangement (may be seen from table to equation). Answer must be extracted from vector form and have $d =$	
4(b)		
B1	Distance of c of m of lamina from <i>PQ</i> seen or implied. May not be seen until the trig ratio.	
M1	Complete method to find c of m of Lamina (remaining mass) from <i>PS</i> or a parallel axis. Dimensionally consistent equation containing all required terms. If one distance is from <i>PS</i> and the other is from <i>AD</i> , treat as an accuracy error.	
A1	Unsimplified equation for <i>PS</i> or their parallel axis, with at most one error	
A1	Correct unsimplified equation for <i>PS</i> or their parallel axis.	
A1	Correct distance of c of m from <i>PS</i> , $2.6a$ or better	
M1	Correct use of trig for a relevant angle where $\left(9a/2\right)$ is their distance of the c of m of the lamina from <i>PQ</i> and $\bar{y}$ is their calculated distance from <i>PS</i> . Allow with both a 's or neither. Allow reciprocal.	
A1	30 or better (29.59229...) cao	

Question	Scheme				Mark	Notes
2a		$PQUY$	$RSTU$	$VWXY$	B1 B1	Correct mass ratios (accept 2:1:1) Correct vertical distances
	Mass	$16a^2$	$2 \times 4a^2$	$2 \times 4a^2$		
	From PX	$2a$	$5a$	a		
	Moments about PX or a parallel axis				M1	Dimensionally correct equation. All terms required Allow for an equation within a vector equation.
	$16a^2 \times 2a + 8a^2 \times 5a + 8a^2 \times a = 32a^2 d$ $\quad (= (16+8+8)a^2 \times d)$ or equivalent for a parallel axis				A1	Correct unsimplified equation Allow for an equation within a vector equation. Could have y for d here o.e.
	$80a = 32d \Rightarrow d = \frac{5}{2}a$ *				A1*	Obtain given answer from correct working. At least one stage of simplifying the moments equation is required. e.g. $32a^3 + 40a^3 + 8a^3$ seen, or they might have simplified the mass ratios at the start. Must get to $d = \dots$ in the final line
					[5]	
2b	Moments about PQ or a parallel axis				M1	Dimensionally correct equation. All terms required
	$16a^2 \times 2a + 8a^2 \times 3a + 8a^2 \times 5a$ $\quad = (16+8+8)a^2 \times h$ or equivalent for a parallel axis				A1ft A1	Unsimplified equation with at most one error. Follow their mass ratios. Correct unsimplified equation
	$\Rightarrow h = 3a$ from PQ				A1	a from YT , $3a$ from XW
The working for the first 4 marks must be seen or used in part (b)						
	Correct use of trig. to find the tangent of a relevant angle.				M1	With <i>their</i> $3a$ e.g. $\tan \theta = \frac{3}{4 - \frac{5}{2}}$
	$\tan \theta = 2$				A1	Correct only
					[6]	
2c	Complete method to obtain an equation in M and F				M1	e.g. Moments about Q Dimensionally correct equation.
	$3a \times Mg = 4a \times F$				A1	Correct unsimplified equation Condone if a missing throughout
	$F = \frac{3}{4}Mg$				A1	Correct only
					[3]	
					(14)	