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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Collisions (M2.4)

International A Level Mathematics · M2: Mechanics 2 (WME02)

34 questions · 325 marks · Mark schemes included

Past papers January 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

34 questions · 325 marks · 1 sub-topic

SUB-TOPIC M2.4a

IMPULSE, MOMENTUM & COEFFICIENT OF RESTITUTION

Sub-topic M2.4a · 34 questions · 325 marks · lowest tariff first



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1. A particle of mass 2kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j})\text{ms}^{-1}$ when it receives an impulse $\mathbf{I}$ Ns, such that $\mathbf{I} = a\mathbf{i} + b\mathbf{j}$

Immediately after receiving the impulse, the particle is moving with velocity $\lambda(\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$, where λ is a constant.

Given that the magnitude of $\mathbf{I}$ is $\sqrt{40}$, find the two possible impulses.

(5)

Question 1 continued

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(Total 5 marks)

Q1



Question 4 continued

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(Total 6 marks)

Q4



4. A particle P of mass 0.75 kg is moving with velocity $4\mathbf{i}\text{ ms}^{-1}$ when it receives an impulse $\mathbf{J}\text{ N s}$. Immediately after P receives the impulse, the speed of P is 8 ms^{-1}

Given that $\mathbf{J} = c(-\mathbf{i} + 2\mathbf{j})$, where c is a constant, find the two possible values of c .

(6)

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Question 4 continued

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(Total 6 marks)

Q4

Grading box for Q4.



Question 2 continued

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(Total 6 marks)

Q2



3. A particle P of mass 0.5 kg is moving with velocity $\lambda(\mathbf{i} + \mathbf{j})\text{ ms}^{-1}$ when P receives an impulse of magnitude $\sqrt{\frac{5}{2}}\text{ N s}$

Immediately after P receives the impulse, the velocity of P is $4\mathbf{i}\text{ ms}^{-1}$
Given that λ is a constant, find the two possible values of λ

(6)

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Question 3 continued

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(Total 6 marks)

Q3



3. A particle P of mass 0.2 kg is moving with velocity $(4\mathbf{i} - 3\mathbf{j})\text{ m s}^{-1}$

The particle receives an impulse $\lambda(\mathbf{i} + \mathbf{j})\text{ N s}$, where λ is a constant.

Immediately after receiving the impulse, the speed of P is 7 m s^{-1}

Find the possible values of λ

(6)

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Question 3 continued

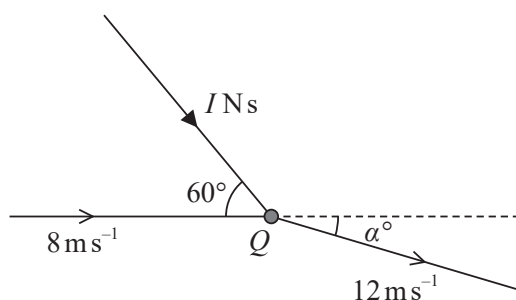
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Q3

(Total 6 marks)



3.

**Figure 2**

A particle Q of mass 0.25 kg is moving in a straight line on a smooth horizontal surface with speed 8 ms^{-1} when it receives an impulse of magnitude $I \text{ N s}$.

The impulse acts parallel to the horizontal surface and at 60° to the original direction of motion of Q .

Immediately after receiving the impulse, the speed of Q is 12 ms^{-1}

As a result of receiving the impulse, the direction of motion of Q is turned through α° , as shown in Figure 2.

Find the value of I

(6)





Question 3 continued

(Total for Question 3 is 6 marks)



Question 2 continued

(Total for Question 2 is 6 marks)



3.

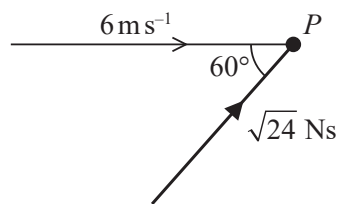


Figure 2

A particle P of mass 0.75 kg is moving along a straight line on a horizontal surface. At the instant when the speed of P is 6 ms^{-1} , it receives an impulse of magnitude $\sqrt{24}\text{ Ns}$. The impulse acts in the plane of the horizontal surface. At the instant when P receives the impulse, the line of action of the impulse makes an angle of 60° with the direction of motion of P , as shown in Figure 2.

Find

- (i) the speed of P immediately after receiving the impulse,
- (ii) the size of the angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse.

(7)

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Question 3 continued

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(Total 7 marks)

Q3



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1. A particle P of mass 1.5 kg is moving with velocity $(4\mathbf{i} + 6\mathbf{j})\text{ ms}^{-1}$ when it receives an impulse of magnitude 15 Ns . Immediately after P receives the impulse, the velocity of P is $v\mathbf{i}\text{ ms}^{-1}$.

Find the two possible values of v .

(7)

Question 1 continued

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(Total 7 marks)

Q1

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2. A particle P of mass 0.5 kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}$
The particle receives an impulse $(-2\mathbf{i} + \lambda\mathbf{j}) \text{ N s}$, where λ is a constant.
Immediately after receiving the impulse, the velocity of P is $(x\mathbf{i} + y\mathbf{j}) \text{ m s}^{-1}$
The kinetic energy gained by P as a result of receiving the impulse is 22 J .
Find the possible values of λ .

(7)



(Total for Question 2 is 7 marks)

Question 1 continued

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(Total for Question 1 = 7 marks)



Question 2 continued

(Total for Question 2 is 7 marks)



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1. A particle of mass 0.5 kg is moving with velocity $(2\mathbf{i} + 4\mathbf{j})\text{ ms}^{-1}$ when it receives an impulse of $(-4\mathbf{i} + 6\mathbf{j})\text{ N s}$.

- (a) Find the speed of the particle immediately after it receives the impulse. (5)
- (b) Find the size of the angle between the direction of motion of the particle immediately before it receives the impulse and the direction of motion of the particle immediately after it receives the impulse. (3)

Question 1 continued

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(Total 8 marks)

Q1



Question 1 continued

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(Total for Question 1 is 8 marks)

Question 5 continued

(Total for Question 5 is 8 marks)



Question 5 continued

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Question 5 continued

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Question 5 continued

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(Total for Question 5 is 8 marks)



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Question 7 continued

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Question 7 continued

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(Total 10 marks)

Q7



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6. Two particles, A and B , are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly. The mass of A is $2m$ and the mass of B is $3m$.

Immediately **after** the collision, A and B are moving in opposite directions with the same speed v .

In the collision, A receives an impulse of magnitude $5mv$.

- (a) Find the coefficient of restitution between A and B .

(6)

After the collision with A , particle B strikes a smooth fixed vertical wall and rebounds. The wall is perpendicular to the direction of motion of the particles. The coefficient of restitution between B and the wall is f .

As a result of its collision with A and with the wall, the total kinetic energy lost by B is E . As a result of its collision with B , the kinetic energy lost by A is $2E$.

- (b) Find the value of f .

(4)



Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total 10 marks)

Q6



Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total 10 marks)

Q4



Question 7 continued



Question 7 continued



Question 7 continued

(Total for Question 7 is 10 marks)



Question 5 continued

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Question 5 continued

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Question 5 continued

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(Total for Question 5 is 11 marks)



8. A particle P has mass m and a particle Q has mass km , where k is a constant.

The particles are moving in opposite directions along the same straight line on a smooth horizontal surface.

Particle P collides directly with Q .

Immediately **before** the collision, the speed of P is $6u$ and the speed of Q is u .

Immediately **after** the collision, the speed of P is $2v$ and the speed of Q is $3v$.

As a result of the collision

- the direction of motion of P is unchanged
- the direction of motion of Q is reversed

The coefficient of restitution between P and Q is $\frac{1}{3}$

(a) Find v in terms of u only.

(3)

(b) Find the value of k .

(3)

After the collision, Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q .

Particle Q rebounds and there is a second collision between P and Q .

The first collision between P and Q takes place at a distance d from the wall.

The second collision between P and Q takes place at a distance $\frac{1}{7}d$ from the wall.

The coefficient of restitution between Q and the wall is f

(c) Find the value of f

(5)



Question 8 continued



Question 8 continued

(Total for Question 8 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total 12 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

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Question 8 continued

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Question 8 continued

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Question 8 continued

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Question 8 continued

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Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END



Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total 12 marks)

Q6

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Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total for Question 4 = 12 marks)



Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total 13 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

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7. Particle A has mass m and particle B has mass $2m$.
The particles are moving in the same direction along the same straight line on a smooth horizontal surface.
Particle A collides directly with particle B .
Immediately before the collision, the speed of A is $3u$ and the speed of B is u .
The coefficient of restitution between A and B is e .

- (a) (i) Show that the speed of B immediately after the collision is $\frac{5+2e}{3}u$
(ii) Find the speed of A immediately after the collision.

(7)

After the collision, B hits a smooth fixed vertical wall that is perpendicular to the direction of motion of B .

The coefficient of restitution between B and the wall is $\frac{1}{3}$

Particle B rebounds and there is a second collision between A and B . The first collision between A and B occurs at a distance d from the wall. The time between the two collisions is T .

Given that $e = \frac{1}{2}$

- (b) find T in terms of d and u .

(6)



Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total 13 marks)

Q7
[] []



7. Particle P has mass $4m$ and particle Q has mass $2m$.

The particles are moving in opposite directions along the same straight line on a smooth horizontal surface.

Particle P collides directly with particle Q .

Immediately **before** the collision, the speed of P is $2u$ and the speed of Q is $3u$.

Immediately **after** the collision, the speed of P is x and the speed of Q is y .

The direction of motion of each particle is reversed as a result of the collision.

The total kinetic energy of P and Q after the collision is half of the total kinetic energy of P and Q before the collision.

- (a) Show that $y = \frac{8}{3}u$ (6)

The coefficient of restitution between P and Q is e .

- (b) Find the value of e . (3)

After the collision, Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q .

Particle Q rebounds.

The coefficient of restitution between Q and the wall is f .

Given that there is no second collision between P and Q ,

- (c) find the range of possible values of f . (3)

Given that $f = \frac{1}{4}$

- (d) find, in terms of m and u , the magnitude of the impulse received by Q as a result of its impact with the wall. (2)

Question 7 continued

Lined area for writing the answer to Question 7 continued.





Question 7 continued





Question 7 continued

Question 7 continued

(Total for Question 7 is 14 marks)





Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS



5. A particle P of mass $2m$ and a particle Q of mass m are at rest on a smooth horizontal plane. Particle P is projected with speed u along the plane towards Q and the particles collide. The coefficient of restitution between the particles is e , where $e > 0$

- (a) Show that the speed of Q after the collision is $\frac{2(1+e)u}{3}$
- (5)**

The total loss of kinetic energy due to the collision is $k(1 - e^2)mu^2$, where k is a constant.

- (b) Find the value of k . (5)

The magnitude of the impulse of Q on P in the collision is $\frac{8mu}{9}$

- (c) Find the value of e . (2)

After the collision, Q goes on to hit a vertical wall which is fixed at right angles to the direction of motion of Q . The coefficient of restitution between Q and the wall is f .

After Q has hit the wall, Q has the same speed as P .

- (d) Find the value of f . (3)

Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

(Total for Question 5 is 15 marks)



7.



Figure 3

Three particles, P , Q and R , lie at rest on a smooth horizontal surface.

The particles are in a straight line with Q between P and R , as shown in Figure 3.

The mass of P is $2m$, the mass of Q is $3m$ and the mass of R is $5m$.

The coefficient of restitution between each pair of particles is e .

Particle P is projected with speed $2u$ towards Q and they collide.

(a) Show that the speed of Q immediately after the collision is

$$\frac{4u}{5}(e+1) \tag{5}$$

In the collision, the direction of motion of P is reversed.

(b) Find the full range of possible values of e . (3)

In the collision, the magnitude of the impulse exerted on Q by P is $\frac{108}{25} mu$

(c) Show that $e = \frac{4}{5}$ (3)

(d) Determine, with clear reasoning, whether there will be a second collision between P and Q .

(6)



Question 7 continued

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Question 7 continued

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Question 7 continued

Lined area for writing the answer to Question 7 continued.



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Solution	Marks	Notes
1.	$\mathbf{I} = 2[\lambda \mathbf{i} + \lambda \mathbf{j} - 5\mathbf{i} - 3\mathbf{j}]$	M1	Use of $\mathbf{I} = m(\mathbf{v} - \mathbf{u})$
	$= 2(\lambda - 5)\mathbf{i} + 2(\lambda - 3)\mathbf{j}$	A1	Any equivalent form
	$ \mathbf{I} = \sqrt{40} \Rightarrow (\lambda - 5)^2 + (\lambda - 3)^2 = 10$	M1	Correct use of Pythagoras and their impulse to form an equation in λ
	$\lambda^2 - 8\lambda + 12 = 0 \Rightarrow \lambda = 2 \text{ or } \lambda = 6$	DM1	Solve to find both values for λ . Dependent on the 2 preceding M marks
	$\mathbf{I} = -6\mathbf{i} - 2\mathbf{j}$ or $\mathbf{I} = 2\mathbf{i} + 6\mathbf{j}$ ($a = -6, b = -2$ or $a = 2, b = 6$)	A1	And no others
		(5)	
	Alternative working:		
	$\mathbf{I}(= a\mathbf{i} + b\mathbf{j}) = 2(\mathbf{v} - (5\mathbf{i} + 3\mathbf{j}))$	M1A1	
	$\mathbf{v} = \frac{a+10}{2}\mathbf{i} + \frac{b+6}{2}\mathbf{j} \Rightarrow (\Rightarrow a+10 = b+6)$		
	$a^2 + b^2 = 40 \Rightarrow b^2 - 4b - 12 = 0$ or $a^2 + 4a - 12 = 0$	M1	Correct use of Pythagoras and impulse to form an equation in a or b Any equivalent form
	$b^2 - 4b - 12 = 0 \Rightarrow b = 6 \text{ or } b = -2$	DM1	
	$\mathbf{I} = -6\mathbf{i} - 2\mathbf{j}$ or $\mathbf{I} = 2\mathbf{i} + 6\mathbf{j}$	A1	Or simplified equivalent
		[5]	

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

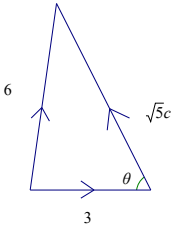
1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\surd$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - $\square$ or d... The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

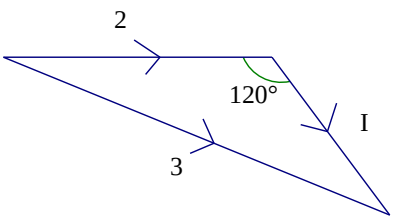


4	Use of $\mathbf{I} = m(\mathbf{v} - \mathbf{u})$	M1	As a single vector equation or two separate equations.
	$c \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \frac{3}{4} \mathbf{v} - \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $\left(\mathbf{v} = \frac{4}{3} \begin{pmatrix} 3-c \\ 2c \end{pmatrix} \right)$	A1	Any equivalent substituted form
	Use of Pythagoras	M1	
	$64 = \frac{16}{9} \left((3-c)^2 + 4c^2 \right)$	A1	Correct unsimplified equation in c or a component of $\mathbf{v}$. ($5a^2 - 32a = 0$ or $5b^2 - 16b - 192 = 0$)
	Simplify to 3 term quadratic and solve for c	M1	$5c^2 - 6c - 27 = 0$
	$c = 3$ or $c = -\frac{9}{5}(-1.8)$	A1	Correct only
		(6)	
4 Alt		M1	Form vector triangle. Dimensionally correct
		A1	Three correct lengths and $ \cos \theta = \frac{1}{\sqrt{5}}$ seen or implied
	Use of cosine rule	M1	
	$36 = 9 + 5c^2 - 2 \times 3\sqrt{5}c \cos \theta$	A1	Correct unsimplified equation in c with $\cos \theta$ or their $\cos \theta$
	Rearrange as 3 term quadratic and solve for c .	M1	$5c^2 - 6c - 27 = 0$
	$c = 3$ or $c = -\frac{9}{5}(-1.8)$	A1	Correct only
		(6)	
		[6]	

2a	Speed after first collision $= \frac{2}{3}u$	B1	Seen or implied (possibly on diagram)
	Speed after second collision $= \frac{4}{9}u$	B1	Seen or implied (possibly on diagram)
	Correct method for total time	M1	Correct formula, dimensionally correct and including all 3 elements.
	$T_1 = \frac{d}{u} + \frac{3d}{\frac{2}{3}u} + \frac{2d}{\frac{4}{9}u} \left(= \frac{d}{u} + \frac{9d}{2u} + \frac{18d}{4u} \right)$	A1	Correct unsimplified expression for T_1
	$T_1 = \frac{10d}{u}$	A1	Correct single term. Allow unsimplified fraction e.g. $T_1 = \frac{40d}{4u}$
		[5]	
2b	$T_2 = \frac{10d}{\frac{4}{9}u} = \frac{45d}{2u} \quad \left(T_2 = \frac{9}{4}T_1 \right)$	B1ft	Follow through is on their T_1 and / or their $\frac{4}{9}u$ Any equivalent form e.g. $\frac{90d}{4u}$.
		[1]	
		(6)	

3			Allow column vectors
	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	Must be subtracting
	$(\mathbf{I} =) \pm 0.5((4 - \lambda)\mathbf{i} + (-\lambda)\mathbf{j})$	A1	Accept $\pm$ correct unsimplified expression on right hand side. (Ignore the left hand side) Allow $2\mathbf{i} - \frac{\lambda}{2}(\mathbf{i} + \mathbf{j})$ or equivalent
	Use of magnitude to form an equation in one variable	M1	Correct use of Pythagoras
	$\frac{5}{2} = \frac{1}{4}((4 - \lambda)^2 + (-\lambda)^2)$	A1ft	Follow their I
	$0 = 2\lambda^2 - 8\lambda + 6 \quad (= (2\lambda - 6)(\lambda - 1))$	DM1	Form a 3 term quadratic (seen or implied). Not necessarily stated “= 0” From $\mathbf{I} = a\mathbf{i} + b\mathbf{j}$ can obtain $4a^2 - 8a + 3 = 0$ or $4b^2 + 8b + 3 = 0$ Dependent on the preceding M1 Solving not required for the M1.
	$\lambda = 3$ and $\lambda = 1$	A1cso	From correct solution only. Do not need to see method of solution.
		[6]	
3alt	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ to form a vector triangle	M1	
	Triangle with sides of length $\sqrt{\frac{5}{2}}, 2\mathbf{i} $ and $\left \frac{\lambda}{2}(\mathbf{i} + \mathbf{j})\right $	A1	
	Use of cosine rule with $45^\circ \left(\frac{\pi}{4}\right)$	M1	
	$\frac{5}{2} = 2^2 + \left(\frac{\lambda}{2}\right)^2 \times 2 - 2 \times 2 \times \frac{\lambda}{2} \sqrt{2} \cos 45^\circ$	A1ft	Correct unsimplified equation Follow their magnitudes
	$0 = \lambda^2 - 4\lambda + 3 \quad (= (\lambda - 3)(\lambda - 1))$	DM1	Form a 3 term quadratic (seen or implied) Dependent on the preceding M1
	$\lambda = 3$ and $\lambda = 1$	A1	Correct solution only
		[6]	
		(6)	

Q	Mark Scheme	Marks	Marking guidance
3.	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	Accept equivalent e.g. $\mathbf{I} + m\mathbf{u} = m\mathbf{v}$. Dimensionally correct and must be using subtraction (but could be the wrong way round). The use of 7 in place of the velocity in the impulse momentum equation is M0 unless they recover. See below
	$0.2(\mathbf{v} - 4\mathbf{i} + 3\mathbf{j}) = \lambda(\mathbf{i} + \mathbf{j})$ $((x-4)\mathbf{i} + (y+3)\mathbf{j}) = 5\lambda\mathbf{i} + 5\lambda\mathbf{j}$	A1	Correct unsimplified vector equation or pair of separate equations for the $\mathbf{i}$ and $\mathbf{j}$ components. Condone column vectors with $\mathbf{i}$ and $\mathbf{j}$ included in the components.
	Use of Pythagoras for the speed	M1	Correct use of Pythagoras and 49 for their speed
	$x^2 + y^2 = 49$	A1	Correct unsimplified equation for their x, y
	Form quadratic in x, y or λ and solve for λ	DM1	Dependent on both previous M marks. $x^2 + -(x-7)^2 = 49$ or $(y+7)^2 + -y^2 = 49$ or $(5\lambda+4)^2 + (5\lambda-3)^2 = 49$
	$\lambda = \frac{3}{5}$ or $\lambda = -\frac{4}{5}$	A1	Or equivalent
*** *** *** *** *** *** ***	Special case: Candidates who use 7 as a vector can score a maximum of M1A0M1A0 for $1.4^2 = +(\lambda \cdot 0.8)^2 + -(\lambda \cdot 0.6)^2$ or equivalent DM1A0 for forming and solving a quadratic in λ .		This maximum of 3 marks is only available for those candidates who “recover”. So, if all you see is $\lambda \lambda \mathbf{i} + -\mathbf{j}$ $1.4 \quad 0.2 \quad 4(\mathbf{i}-3\mathbf{j})$ they score M0M0M0 If they recover to go on to form a “sensible” equation using Pythagoras then they can score the first 2 M marks, and potentially the third M1 as well.
		(6)	
3 alt			
	Form vector triangle	M1	Dimensionally correct. Allow incorrect configuration
	Correct triangle and correct lengths	A1	In speeds or momentum but not a mixture
	Use scalar product to find cosine of angle	M1	Or equivalent method
	$\cos \theta = -\frac{1}{5\sqrt{2}}$	A1	Allow $\pm$
	Form equation in λ $(2\lambda^2 + .4\lambda - 0.96 = 0)$	DM1	e.g. by use of cosine rule Dependent on the first 2 M marks
	$\lambda = \frac{3}{5}$ or $\lambda = -\frac{4}{5}$	A1	Or equivalent
		(6)	

Question	Scheme	Mark	Notes
3	Form impulse-momentum equation	M1	Dimensionally correct. Accept answers in “vector” form, or as separate components. Condone sine / cosine confusion.
	One correct equation	A1	e.g. one correct component of $\begin{pmatrix} I \cos 60^\circ \\ I \sin 60^\circ \end{pmatrix} = \frac{1}{4} \left[\begin{pmatrix} 12 \cos \alpha \\ 12 \sin \alpha \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix} \right]$ or $\begin{pmatrix} 3 \cos \alpha - 2 \\ 3 \sin \alpha \end{pmatrix}$ or $\begin{pmatrix} I \cos 60^\circ \\ I \sin 60^\circ \end{pmatrix} = \frac{1}{4} \left[\begin{pmatrix} v_x \\ v_y \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix} \right]$ or $\begin{pmatrix} 3 \cos \alpha - 2 \\ 3 \sin \alpha \end{pmatrix}$ if working parallel and perpendicular to the initial direction or one of $8 \sin 60^\circ = 12 \cos(30^\circ + \alpha)$ or $I = 0.25(12 \sin(30^\circ + \alpha) - 8 \cos 60^\circ)$ if working parallel and perpendicular to the impulse
	Form a second impulse-momentum equation	M1	
	correct second equation	A1	
	Complete method to solve for I	DM1	Dependent on the two preceding M marks. e.g. from $36 = (I + 4)^2 + 3I^2 \quad (4I^2 + 8I - 20 = 0)$
	$I = \sqrt{6} - 1$ (or 1.45 or 1.4)	A1	
		[6]	
3 alt		M1	Use of $I = mv - mu$ to draw a vector triangle. Dimensionally consistent.
		A1	Correct diagram
	Form an equation in I	M1	e.g. by using cosine rule
	$4 + I^2 - 4I \cos 120^\circ = 9$	A1	Correct unsimplified equation A correct cosine rule equation can imply the first M1A1 if no diagram seen
	Solve for I	DM1	Dependent on the 2 preceding M marks $I^2 + 2I - 5 = 0$
	$I = \sqrt{6} - 1$ (or 1.45 or 1.4)	A1	
		(6)	

Q	Scheme	Mark	Notes
2a	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	NB: Column vectors are acceptable. Condone wrong order but must be subtracting. Condone 5 in place of 0.5.
	$2\mathbf{i} + 5\mathbf{j} = 0.5(\mathbf{v} - (3\mathbf{i} + \mathbf{j}))$ $(\mathbf{v} = 7\mathbf{i} + 11\mathbf{j})$	A1	Correct unsimplified equation Accept as a vector equation or as a pair of equations, one for each component. Accept alternative notations provided the meaning is clear.
	Use of Pythagoras	M1	For their $\mathbf{v}$ Independent M1 but they must have a $\mathbf{v}$
	$ \mathbf{v} = \sqrt{121 + 49} = \sqrt{170} \text{ (ms}^{-1}\text{)}$	A1	$13 \text{ (ms}^{-1}\text{)}$ or better. (13.038.....)
		[4]	
2b	Correct use of trigonometry e.g. $\theta = \tan^{-1} \frac{11}{7} - \tan^{-1} \frac{1}{3}$ $(= 57.5 - 18.4)$	M1	Condone subtraction in either order. Allow if both fractions are the other way up. Alternatives: scalar product $\theta = \cos^{-1} \left(\frac{21 + 11}{\sqrt{10}\sqrt{170}} \right)$ cosine rule $4 \times 29 = 10 + 170 - 2\sqrt{10}\sqrt{170} \cos \theta$
	$\theta = 39.1$	A1	Accept ± 39 or better (39.0938...) 0.68(2) radians is M1A0 Accept $\pm(360 - 39) = \pm 321$ or better
		[2]	
		(6)	

Question Number	Scheme	Marks
3	Use of $mv = I + mu$ Component of momentum parallel to original direction = $6 \times 0.75 + \sqrt{24} \cos 60$ ($= 4.5 + \sqrt{6}$) Use of Pythagoras: $(\frac{3}{4}v =) \sqrt{(4.5 + \sqrt{6})^2 + 18}$ $v = 10.9 \text{ (m s}^{-1}\text{)}, 11 \text{ (m s}^{-1}\text{)}$ <i>Alternative for the 1st 5 marks:</i> Vector triangle for impulses or velocities Use of cosine rule $\left(\frac{3}{4}v\right)^2 = 4.5^2 + 24 - 2 \times 4.5 \times \sqrt{24} \times \cos 120^\circ$ $v = 10.9 \text{ (m s}^{-1}\text{)}, 11 \text{ (m s}^{-1}\text{)}$ Change in direction = $\tan^{-1} \frac{3\sqrt{2}}{4.5 + \sqrt{6}}$ $= 31.4^\circ \text{ (} 31^\circ \text{)}$ Notes	M1 A1 A1 M1 A1 M1 M1 A1 A1 A1 M1 A1 (7) [7]
M1 A1 M1 A1 A1 M1 A1 A1 M1 A1	Need to consider both components. Or equivalent Or equivalent. Correct LHS Or better Must be using correct triangle - need 120° seen or implied Correct unsimplified Or better Or equivalent use of trig. With their components to find the required angle Eg angle = $\cos^{-1} \left(\frac{4.5^2 + (mv)^2 - 24}{2 \times 4.5 \times (mv)} \right)$ Or from scalar product, $\cos^{-1} \left(\frac{6 \times 9.27...}{6 \times 10.9...} \right)$ 0.548 radians (0.55 radians) or better. Do not ISW	



Question Number	Scheme	Marks	
1	$(\mathbf{I} =) 1.5\{\mathbf{vi} - (4\mathbf{i} + 6\mathbf{j})\}$	M1	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$. Must be using $\mathbf{vi}$. Condone $\mathbf{u}, \mathbf{v}$ confusion. Ignore the left hand side
	$= 1.5\{(v-4)\mathbf{i} - 6\mathbf{j}\}$	A1	Or equivalent seen or implied Condone subtraction the wrong way round. Ignore the left hand side
	$\Rightarrow 15^2 = 1.5^2\{(v-4)^2 + 6^2\}$	M1	Use of modulus. Allow for $p^2 + q^2 = 100$
	$(100 = (v-4)^2 + 36)$	A1	Correct unsimplified equation in v
	$(v^2 - 8v - 48 = 0)$	A1	Correct simplified equation in v seen or implied.
	$\Rightarrow v = 12$	A1	One correct value
	or $v = -4$	A1	Both correct values
		[7]	
1 alt1			
	Initial momentum $= (6\mathbf{i} + 9\mathbf{j})$ Ns	M1	Impulse momentum triangle. Accept $\sqrt{117}$ Ns
	$\cos \alpha = \frac{6}{\sqrt{117}} \left(= \frac{2}{\sqrt{13}} \right)$	A1	Or equivalent
	$m^2 + 117 - 2m\sqrt{117} \cos \alpha = 225$	M1	Use of cosine formula (final momentum m)
	$m^2 - 12m - 108 = 0$	A1	Or equivalent
	$\Rightarrow m = -6$ or $m = 18$	A1	
	$\Rightarrow v = 12$	A1	One correct value
	or $v = -4$	A1	Both correct values
		[7]	
1alt2	Initial momentum $= (6\mathbf{i} + 9\mathbf{j})$ Ns	M1	Impulse momentum triangle. Accept $\sqrt{117}$ Ns
	$\sin \alpha = \frac{3}{\sqrt{13}}$	A1	Or equivalent
	$\frac{15}{\sin \alpha} = \frac{\sqrt{117}}{\sin \theta}$	M1	Use of sine formula
	$\Rightarrow \sin \theta = \frac{3}{5}$, $\theta = 36.9^\circ$ or $\theta = 143.1^\circ$	A1	
	$\frac{m}{\sin 86.8} = \frac{15}{\sin \alpha}$ or $\frac{m}{\sin 19.4} = \frac{15}{\sin (180 - \alpha)}$	A1	Correct equation in m
	$\Rightarrow v = 12$	A1	One correct value
	or $v = -4$	A1	Both correct values
		[7]	

1a	Equation of motion	M1	Dimensionally correct. Condone sign error.
	$F - R = 1500a$	A1	Correct unsimplified equation in F or P
	Use of $P = Fv : \left(\frac{30000}{20} - R = 1500 \times 0.6 \right)$	M1	Must be trying to use 30 kW but condone error in zeros
	$R = 600$	A1	Correct answer only
		4	
1b	Equation of motion	M1	Dimensionally correct. Need all relevant terms. Condone sign errors and sin/cos confusion. Allow with F .
	$\frac{30000}{V} - 1500g \times \frac{1}{8} - 500 = -1500 \times 0.2$	A1 A1	Unsimplified equation with F substituted and at most one error Correct unsimplified equation with F substituted. If F is never substituted, A0A0
	$V = 14.7 \quad (15)$	A1	3 sf or 2 sf
		4	
		(8)	
2	1 st equation e.g. Equation for change in KE	M1	Dimensionally correct. Must be subtracting but condone sign error.
	$\frac{1}{2} \times 0.5(x^2 + y^2 - (5^2 + 3^2)) = 22$ $(x^2 + y^2 = 122) \quad (1^2 + (2\lambda + 3)^2 = 122)$	A1	Correct unsimplified equation seen or implied (They might have used impulse-momentum first and done some work before substituting x and y .)
	2 nd equation e.g. Impulse-momentum equation	M1	Dimensionally correct. Must be subtracting but condone sign error.
	$0.5(\mathbf{x}\mathbf{i} + \mathbf{y}\mathbf{j}) - 0.5(5\mathbf{i} + 3\mathbf{j}) = (-2\mathbf{i} + \lambda\mathbf{j})$ $((x-5)\mathbf{i} + (y-3)\mathbf{j} = -4\mathbf{i} + 2\lambda\mathbf{j})$	A1	Correct unsimplified equation
NB: open has M1A1A1 for the final 3 marks but this should be marked DM1DM1A1			
	Form a quadratic equation in λ	DM1	e.g. $1^2 + (3 + 2\lambda)^2 = 122$ Dependent on the 2 preceding M marks
	Solve for 2 values of λ	DM1	e.g. solve $4\lambda^2 + 12\lambda - 112 = 0$ or $(3 + 2\lambda)^2 = 121$ Dependent on the preceding M1
	$\Rightarrow \lambda = 4 \quad \text{or} \quad \lambda = -7$	A1	Correct only and no errors seen (watch out for $x = -1$ used)
alt	Form a quadratic in y	DM1	e.g. $1 + y^2 = 122 \quad (y^2 = 121)$ Dependent on the 2 preceding M marks
	Solve for 2 values of y and use these to obtain 2 values of λ	DM1	Dependent on the preceding M1
	$\Rightarrow \lambda = 4 \quad \text{or} \quad \lambda = -7$	A1	
		7	

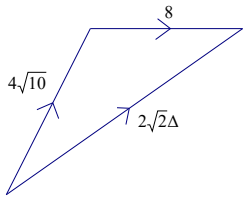
Question	Scheme	Marks	Notes
1a	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	Must be subtracting but condone subtraction in wrong order
	$= 0.3((7\mathbf{i} + 7\mathbf{j}) - 5\mathbf{i}) \quad (= 0.6\mathbf{i} + 2.1\mathbf{j})$	A1	correct unsimplified equation Allow $\pm$
	$ \mathbf{I} = \sqrt{0.6^2 + 2.1^2}$	M1	Use of Pythagoras
	$= \frac{3\sqrt{53}}{10}$	A1	2.2 or better (2.18403...)
		(4)	
1b	Correct method for a relevant angle	M1	e.g. use of trigonometry or scalar product for their $\mathbf{I}$ θ or $90 - \theta$
	Correct trig ratio for the required angle and no other angle involved.	A1	From correct $\mathbf{I}$ e.g. $\tan \theta = \frac{7}{2}$ or $\cos \theta = \frac{10}{\sqrt{53} \times 5}$
	$\theta = 74.1^\circ$	A1	74° or better ($74.0546..^\circ$) or $360 - 74$ (286) (1.29... radians)
		(3)	

Question	Scheme	Marks	Notes
Accept column vectors throughout			
2a	Use of $\mathbf{r} = \int \mathbf{v} dt$	M1	Powers going up by 1. Allow one slip in the powers
	$\mathbf{r} = \left(\frac{4}{3}t^3 - \frac{5}{2}t^2 + A \right) \mathbf{i} + (-5t^2 - 12t + B) \mathbf{j}$	A1	Allow without constant of integration
	Use $t = 2$ and $\mathbf{r} = 2\mathbf{i} + 6\mathbf{j}$ when $t = 0$: $\mathbf{r} = \left(\frac{4}{3} \times 8 - \frac{5}{2} \times 4 + 2 \right) \mathbf{i} + (-5 \times 4 - 12 \times 2 + 6) \mathbf{j}$	M1	Correct use of given value to obtain $\mathbf{r}$
	$= \frac{8}{3} \mathbf{i} - 38 \mathbf{j}$	A1	Correct answer only Allow 2.7 or better ISW if they go on to find the magnitude.
		(4)	
2b	$\mathbf{v}$ in direction of $\mathbf{i} - 2\mathbf{j}$	M1	Use velocity and direction to form an equation in T Condone if they have (-)2 on the wrong side of their equation
	$\Rightarrow -2(4T^2 - 5T) = (-10T - 12)$ $(8T^2 - 20T - 12 = 0)$	A1	Correct unsimplified equation in T (or t) only
	$\Rightarrow T = 3$	A1	Only. Allow $t = 3$.
		(3)	
2c	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ ($\mathbf{a} = (8t - 5)\mathbf{i} - 10\mathbf{j}$)	M1	Powers going down by 1 Allow one slip in the powers
	Use of Pythagoras and $t = 2.5$	M1	Correct use of their derivative to obtain acceleration
	$ a = \sqrt{(20 - 5)^2 + 10^2} = \sqrt{325} (= 5\sqrt{13}) \text{ ms}^{-2}$	A1	Any equivalent simplified exact form. Ignore decimals after exact answer seen.
		(3)	

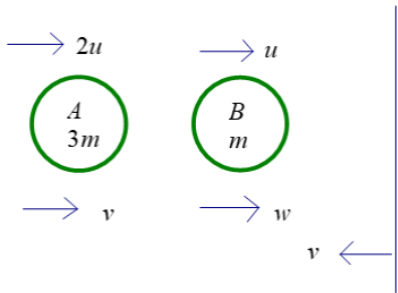
Question	Scheme	Marks
2	Use of Impulse momentum equation	M1
	$\mathbf{I} = 3(\mathbf{x}\mathbf{i} + \mathbf{y}\mathbf{j}) - 3 \times 5\mathbf{i}$ $= 3(x-5)\mathbf{i} + 3y\mathbf{j}$	A1
	$ I ^2 = 9((x-5)^2 + y^2) = 9 \times 82 (=738)$	M1
	Equation for change in KE	M1
	$\frac{1}{2} \times 3(x^2 + y^2 - 25) = 138$ $(x^2 + y^2 - 25 = 92)$	A1
	$(x-5)^2 + y^2 = 82$ $x^2 + y^2 = 117 \Rightarrow 10x = 60$	DM1
	$\mathbf{v} = 6\mathbf{i} + 9\mathbf{j} (\text{ms}^{-1})$	A1
		(7)
ALT	Use of $I = \begin{pmatrix} 3\sqrt{82} \cos \theta \\ 3\sqrt{82} \sin \theta \end{pmatrix}$ $I = \begin{pmatrix} 3\sqrt{82} \cos \theta \\ 3\sqrt{82} \sin \theta \end{pmatrix} = 3 \begin{pmatrix} x \\ y \end{pmatrix} - 3 \begin{pmatrix} 5 \\ 0 \end{pmatrix}$	M1 A1
	$ I ^2 = 9((x-5)^2 + y^2) = 9 \times 82$	M1
	Equation for change in KE $\frac{1}{2} \times 3(x^2 + y^2 - 25) = 138$	M1 A1
	Leads to $\theta = 83.66\dots$ $x = 5 + 3\sqrt{82} \cos \theta = 6$ $y = 3\sqrt{82} \sin \theta = 9$ $\mathbf{v} = 6\mathbf{i} + 9\mathbf{j} (\text{ms}^{-1})$	DM1 A1
	Notes	
M1	Find the difference in momenta. Dimensionally correct. Condone subtraction in wrong order. Must use both components. (Ignore $3\sqrt{82}$ if it appears on LHS)	
A1	Correct unsimplified expression for difference in momentum. Condone subtraction the wrong way round.	
M1	Correct use of Pythagoras with $3\sqrt{82}$ and both components of impulse. Ignore poor $\mathbf{i}\cdot\mathbf{j}$ notation, eg $\mathbf{i}^2$ for this mark, if recovered by correct subsequent working.	
M1	Use change in KE to produce an equation in terms of x and y . Dimensionally correct, requires 2 KE terms of correct structure. Condone subtraction in wrong order. Ignore poor $\mathbf{i}\cdot\mathbf{j}$ notation eg $(\mathbf{x}\mathbf{i})^2 + (\mathbf{y}\mathbf{j})^2$ for this mark, if recovered by correct subsequent working.	

1a	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	M1	Condone subtraction in the wrong order.
	$\begin{pmatrix} -4 \\ 6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x-2 \\ y-4 \end{pmatrix}$	A1	Correct unsimplified equation Any equivalent form. Allow with $\mathbf{v}$
	$\mathbf{v} = -6\mathbf{i} + 16\mathbf{j} \text{ (ms}^{-1}\text{)}$	A1	Correct only. Seen or implied SR: Allow 3/3 if stop at $\mathbf{v} = 6\mathbf{i} - 16\mathbf{j} \text{ (ms}^{-1}\text{)}$
	$ \mathbf{v} = \sqrt{(-6)^2 + 16^2}$	M1	Correct use of Pythagoras with their $\mathbf{v}$
	$= \sqrt{292} \text{ (= } 2\sqrt{73}\text{)} \text{ (ms}^{-1}\text{)}$	A1	Correct simplified value. 17 or better (17.088.....)
			Allow 5/5 if working from the negative of the velocity.
		[5]	
1b	Correct use of trigonometry to find 2 relevant angles - as values or in inverse tangent form	M1	For their $\mathbf{v}$ e.g. $\pm 69.44..^\circ, 63.43..^\circ$ or $\pm 1.212..., 0.4636..$
	$\theta = \left(180^\circ - \tan^{-1} \frac{16}{6} \right) - \tan^{-1} \frac{4}{2}$	A1ft	Correct unsimplified expression for θ Any equivalent form
	$= 47^\circ$	A1	47° or better (47.121..) 312.9° Accept radians (0.8224..)
		[3]	
1b alt	Use of scalar product with two relevant vectors	M1	For their $\mathbf{v}$
	$\theta = \cos^{-1} \left(\frac{-12 + 64}{\sqrt{20}\sqrt{292}} \right)$	A1ft	Correct unsimplified expression for $\cos \theta$ or equivalent
	$= 47^\circ$	A1	47° or better (47.121..) 312.9° Accept radians (0.8224..)
		[3]	
		(8)	

Q	Scheme	Marks	Notes
1(a)	Expression for total KE before collision	M1	Dimensionally correct. Condone confusion between before and after for A. Allow if vectors seen in the working but the modulus is used correctly. The two parts must be added together Allow confusion of 2 kg and 3 kg
	$\frac{1}{2} \times 2 \times 5^2 + \frac{1}{2} \times 3 \times (3^2 + (-1)^2)$	A1	correct unsimplified expression
	= 40 (J)	A1	cao
		(3)	
1(b)	$2((3\mathbf{i} + 2\mathbf{j}) - 5\mathbf{j})$	M1	change in momentum of A, must be a difference but allow subtraction in either order Must be using the correct mass, 2 kg
	= $(6\mathbf{i} - 6\mathbf{j})$ (N s)	A1	Cao The final answer should be in terms of $\mathbf{i}$ and $\mathbf{j}$ because this is asked for in the question. Accept $2(3\mathbf{i} - 3\mathbf{j})$ ISW
		(2)	
1(c)	impulse-momentum equation for B	M1	must use negative of their answer to (b) and the initial velocity of B Must be using the correct mass, 3 kg or CLM with correct terms (allow slip) and plus signs
	$3(\mathbf{v}_B - (3\mathbf{i} - \mathbf{j})) = (-6\mathbf{i} + 6\mathbf{j})$ or $2 \times 5\mathbf{j} + 3(3\mathbf{i} - \mathbf{j}) = 2(3\mathbf{i} + 2\mathbf{j}) + 3\mathbf{v}_B$	A1ft	correct unsimplified equation ft on their impulse from (b)
	$\mathbf{v}_B = (\mathbf{i} + \mathbf{j})$ (m s ⁻¹)	A1	Cao Accept column vector ISW
		(3)	
		(8)	

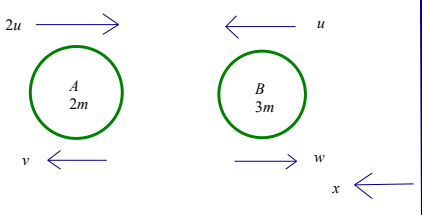
5a	Impulse-momentum equation.	M1	Dimensionally correct. Subtraction seen or implied. Condone subtraction in wrong order.
	$(\pm \mathbf{I} =)$ $2(\lambda \mathbf{i} + \lambda \mathbf{j}) - 2(4\mathbf{i}) = (2\lambda - 8)\mathbf{i} + 2\lambda \mathbf{j}$	A1	Or equivalent Ignore $4\sqrt{10}$ if seen here
	$(\mathbf{I} ^2 =) 160 = (2\lambda - 8)^2 + (2\lambda)^2$	DM1	Use of Pythagoras to obtain an equation in λ Dependent on the previous M1
	$(\Rightarrow 0 = \lambda^2 - 4\lambda - 12)$	A1	Or any correct unsimplified equation in λ
	$\Rightarrow (\lambda =) 6$	A1	Correct only.
SC Allow 5/5 in (a) if working with $-\mathbf{I}$. They will lose marks later.			
		[5]	
5a alt	Form vector triangle for impulse or for momentum.	M1	Dimensionally correct. Must be subtracting. Condone subtraction in wrong order.
	Correct triangle	A1	 e.g.
	$160 = 64 + 8\lambda^2 - 32\sqrt{2}\lambda \times \frac{1}{\sqrt{2}}$	DM1	Use of Cosine Rule to obtain an equation in λ Dependent on the previous M1
	$\Rightarrow 0 = 8\lambda^2 - 32\lambda - 96$	A1	Or equivalent equation in λ
	$\Rightarrow (\lambda =) 6$	A1	Correct only
		[5]	
5b	$\mathbf{I} = 4\mathbf{i} + 12\mathbf{j}$	B1ft	Follow their λ $(\mathbf{I} = (2\lambda - 8)\mathbf{i} + 2\lambda \mathbf{j})$ B0 for a column vector. B0 if still in terms of lambda. Ignore second solution for negative lambda if seen
		[1]	
5c	$\tan \theta^\circ = \frac{12}{4}$ or $\cos \theta^\circ = \frac{16}{4 \times 4\sqrt{10}}$	M1	Correct use of trig or scalar product for the required angle with <i>their</i> $\mathbf{I}$ provided both components are non-zero Do not allow for the reciprocal
	$\theta = 72$	A1	72 or better (71.56505...) from correct work only Ignore second solution for negative lambda if seen
		[2]	

QUESTION NUMBER	SCHEME	MARKS
5	Accept column vectors throughout the question.	
	Velocity in component form eg $\mathbf{v} = (v \cos 45^\circ)\mathbf{i} \pm (v \sin 45^\circ)\mathbf{j}$ OR $\mathbf{v} = \lambda\mathbf{i} \pm \lambda\mathbf{j}$	M1 A1
	Change in momentum in vector form $0.5\mathbf{v} - 0.5(6\mathbf{i})$	M1 A1
	Use of magnitude of impulse to form an equation in one unknown . eg $\frac{3\sqrt{2}}{2} = \sqrt{(0.5v \cos 45^\circ - 3)^2 + (0.5v \sin 45^\circ)^2}$ OR $\frac{3\sqrt{2}}{2} = \sqrt{(0.5\lambda - 3)^2 + (0.5\lambda)^2}$	M1 A1
	$-\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j}$ $-\frac{3}{2}\mathbf{i} - \frac{3}{2}\mathbf{j}$	A1 A1
		(8)
	Notes for question	
	Accept column vectors throughout the question.	
M1	Velocity after impact as components in variable form.	
A1	Correct unsimplified expressions Eg $\mathbf{v} = (v \cos 45^\circ)\mathbf{i} \pm (v \sin 45^\circ)\mathbf{j}$ or $\mathbf{v} = \lambda\mathbf{i} \pm \lambda\mathbf{j}$	
M1	$m(\mathbf{v} - \mathbf{u})$ used in vector form.	
A1	Correct unsimplified expression, $0.5\mathbf{v} - 0.5(6\mathbf{i})$	
M1	Use of magnitude of impulse to form an equation in one unknown. Eg $\frac{3\sqrt{2}}{2} = \sqrt{(0.5v \cos 45^\circ - 3)^2 + (0.5v \sin 45^\circ)^2} \quad (\Rightarrow v = 3\sqrt{2})$ Or $\frac{3\sqrt{2}}{2} = \sqrt{(0.5\lambda - 3)^2 + (0.5\lambda)^2} \quad (\Rightarrow \lambda = 3)$	
A1	Correct unsimplified equation.	
A1	At least one correct expression for impulse must be in $p\mathbf{i} + q\mathbf{j}$ form for this mark	
A1	Two correct expressions for impulse. Do not penalise lack of $p\mathbf{i} + q\mathbf{j}$ form twice.	
	N.B. Case1: If solution appears as per scheme, then full marks possible Case 2: If no $\pm$ in line one but two solutions emerge at end, then full marks available Case 3: If only one sign used in line one, and only one solution appears at end, then max M1A0M1A1 M1A1A1A0	

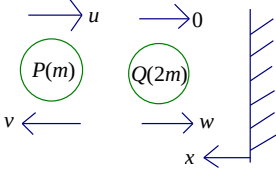
Question Number	Solution	Marks	Notes
7			
7a	KE gain = final KE – initial KE	M1	KE equation for B. Allow for change in KE
	$\frac{48}{25}mu^2 = \frac{1}{2}mw^2 - \frac{1}{2}mu^2$	A1	Correct unsimplified equation to find w
	$\left(w^2 = \frac{121}{25}u^2, \quad w = \frac{11}{5}u\right)$		
	CLM: $3m \times 2u + mu = 3mv + mw$	M1	All terms required. Condone sign errors
	$\left(7mu = 3mv + \frac{11}{5}mu\right) \left(v = \frac{8}{5}u\right)$	A1	Correct unsimplified equation in v and w or their w
	Impact law:	M1	Used correctly
	$w - v = e(2u - u)$	A1	Correct unsimplified equation in v and w or their v and w
	Solve for e	DM1	Dependent on the preceding M marks
	$\frac{3}{5}u = eu, \quad e = \frac{3}{5}$	A1	
		(8)	
7b	Impact law: $fw = v$	M1	Condone sign error
	$f = \frac{8}{11}$	A1	0.73 or better Final answer must be positive
		(2)	
		[10]	

6a			
	$5mv = 2m(v - (-x))$	M1	Use of $I = mv - mu$
	$x = \frac{3v}{2}$	A1	Seen or implied
	$5mv = 3m(v - (-y))$ or $2mx - 3my = 3mv - 2mv$	M1	Use of $I = mv - mu$ or use of CLM
	$y = \frac{2v}{3}$	A1	Seen or implied
	$2v = e\left(\frac{3v}{2} + \frac{2v}{3}\right)$	M1	Correct use of impact law (not necessarily with values in terms of v) Allow $v - v$ on LHS
	$e = \frac{12}{13}$	A1	0.92 or better
		(6)	
6b	Speed of B after collision with wall = vf	B1	Seen or implied
	$2 \times \frac{1}{2} \times 3m(y^2 - (vf)^2) = \frac{1}{2} \times 2m(x^2 - v^2)$	M1	Use KE to form an equation in f . Condone use of change in KE rather than loss Condone 2 on wrong side
	$3\left(\frac{4}{9} - f^2\right) = \left(\frac{9}{4} - 1\right)$	A1	Correct unsimplified equation for f
	$\left(f^2 = \frac{1}{36}\right) \quad f = \frac{1}{6}$	A1	cao NB: $\frac{\sqrt{31}}{6}$ comes from inconsistent subtraction.
		(4)	
		[10]	

3a	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ $(\mathbf{a} = 18 \cos 3t \mathbf{i} - 2 \sin t \mathbf{j})$	M1	Differentiate to obtain $\mathbf{a} = \lambda \cos 3t \mathbf{i} + \mu \sin t \mathbf{j}$
	Use of $\mathbf{F} = m\mathbf{a} : \mathbf{F} = \frac{1}{4}\mathbf{a}$	M1	Must be working in vectors
	$\mathbf{F} = \frac{9}{2} \cos 3t \mathbf{i} - \frac{1}{2} \sin t \mathbf{j}$	A1	Or equivalent. e.g. as a column vector
		[3]	
3b	$2 \cos t + 1 = 0$	M1	Set $\mathbf{j}$ component of $\mathbf{v} = 0$ and solve for t
	$\Rightarrow t = \frac{2\pi}{3}$	A1	ISW if correct answer seen. Only answer 120° scores A0 here and the final A0
	Use of $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $(\mathbf{r} = -2 \cos 3t \mathbf{i} + (t + 2 \sin t) \mathbf{j} (+\mathbf{C}))$	M1	Integrate $\mathbf{v}$ with respect to t to obtain $\mathbf{r} = p \cos 3t \mathbf{i} + (t + q \sin t) \mathbf{j} (+\mathbf{C})$ Condone if there is no constant of integration.
	$t = 0, \quad \mathbf{r} = (4\mathbf{i} - \sqrt{3}\mathbf{j})\text{m}$ $\mathbf{r} = (-2 \cos 3t + 6) \mathbf{i} + (t + 2 \sin t - \sqrt{3}) \mathbf{j}$	M1	Correct use of boundary condition to find their $\mathbf{C}$. Could be part of a definite integral e.g. $4\mathbf{i} - \sqrt{3}\mathbf{j} + \int_0^t 6 \sin 3t \mathbf{i} + (1 + 2 \cos t) \mathbf{j} dt$ for their upper limit
	$= 4\mathbf{i} + \frac{2\pi}{3} \mathbf{j} \quad (\text{m})$	A1 A1	Accept $4\mathbf{i} + 2.1\mathbf{j}$ or better one component correct both components correct ISW if they also offer $4\mathbf{i} + 120\mathbf{j}$ “correct” components after an M0 are fortuitous – A0
		[6]	
		(9)	

4a			
	Use of CLM	M1	Need all terms, dimensionally correct. Condone sign errors.
	$4mu - 3mu = 3mw - 2mv$ $(u = 3w - 2v)$	A1	Correct unsimplified equation
	Use of impact law	M1	Used correctly. Condone sign errors
	$v + w = 3eu$	A1	Correct unsimplified equation. Signs consistent with their CLM equation
	$\begin{cases} u = 3w - 2v \\ 6eu = 2w + 2v \end{cases}$	DM1	Dependent on both preceding M marks. Solve to find speed of B.
	$\Rightarrow 5w = u + 6eu, \quad w = \frac{1}{5}u(1 + 6e) \quad *$	A1*	Obtain given answer from correct working
		[6]	
4b	$v = 3eu - w = \frac{u}{5}(9e - 1)$	B1	Check their diagram / directions and allow $v = \frac{u}{5}(1 - 9e)$ if correct for their working. Any equivalent form. Must be seen or used in (b)
	$x = \frac{u}{7}(1 + 6e)$	B1	Seen or implied. Accept $\pm$
	Second collision if $\frac{u}{7}(1 + 6e) > \frac{u}{5}(9e - 1)$	M1	Correct inequality to find the upper limit for e , using their v and x
	$(0 <) e < \frac{4}{11}$	A1	Final answer. Or equivalent. Do not need to mention the lower limit, but if they do it must be stated correctly (strict inequality).
		[4]	

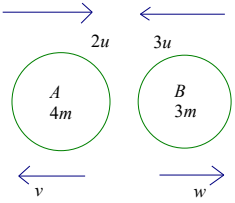
7a			
	Use CLM	M1	Need all terms and dimensionally correct. Condone sign errors. Might see them using equal (and opposite) impulses.
	$6mu - 3kmu = 3mu + kmv \quad ((3-3k)u = kv)$	A1	Correct unsimplified equation
	$\Rightarrow v = \frac{(3-3k)}{k}u \quad *$	A1*	Obtain given answer from full and correct working
		3	
7b	Use of Impulse = change in momentum	M1	Must be subtracting. Can be for either particle.
	$ I_Q = I_P = 3mu - 3m \cdot 2u = 3mu$ or $ kmv - (-3mku) = \left km \cdot \frac{3-3k}{k}u + 3mku \right = 3mu$	A1	Correct only (Do not need to state that $ I_Q = I_P $ if find $ I_P $)
		2	
7c	Use impact law:	M1	Seen or implied. If stated in (a) must be used here. Must be used correctly but condone sign errors
	$\frac{v-u}{5u} = e$ or $\frac{3-3k}{k}u - u = 5ue$	A1	Correct unsimplified equation
	NB: the second and third M mark are not dependent on the first M mark		
	Use $v > u$ or $e > 0$ to form an inequality in k	M1	Could use $e \dots 0$ followed by $v \neq u$
	Use $e, 1$ to form an inequality in k	M1	
	$\frac{3-3k}{k} > 1$ and $3-3k \geq 6k \Rightarrow \frac{1}{3}, k < \frac{3}{4}$	A1	Correct answer only.
		5	
		(10)	

Q	Scheme	Marks	Notes
5(a)			
	Use of CLM (or equal and opposite impulses):	M1	correct no. of terms, dim correct, condone sign errors
	$mu = -mv + 2mw$	A1	Or equivalent
	Use of NEL:	M1	correct way round, condone sign errors
	$eu = v + w$	A1	Or equivalent
	Solve for v	DM1	Dependent on both preceding M marks
	$v = \frac{u(2e-1)}{3}$	A1	Or equivalent
	v consistently in the wrong direction gives $v = \frac{u(1-2e)}{3}$		Mark as a misread and allow M1A0M1A0M1A1, but full marks if they later take account of the change in direction to give the correct final answer
	If the direction of v is correct in one equation and incorrect in the other then mark as seen		
		(6)	
5(b)	NEL at the wall: $x = \frac{1}{3}w$	B1	Allow + / -: they might be working with velocities
	$w = \frac{u(e+1)}{3}$	B1	Or equivalent expression for w
	$\frac{1}{3} \times \frac{u(e+1)}{3} > \frac{u(2e-1)}{3}$	M1	use of <i>their</i> $x > \text{their } v$
	$e < \frac{4}{5}$	A1	cao
	$\frac{1}{2} < e < \frac{4}{5}$	A1	cao
		(5)	
		(11)	

8a			
	Use of impact law	M1	Used the right way round. Condone sign errors
	$\frac{3v - 2v}{6u + u} = \frac{1}{3}$	A1	Correct unsimplified equation e.g. If see just $\frac{v}{5u} = \frac{1}{3}$ assume a sign slip and allow M1A0A0
	$v = \frac{7}{3}u$	A1	Correct only. CSO
		[3]	
8b	Use of CLM (or equal and opposite impulses)	M1	Dimensionally consistent. Need all terms. Condone sign errors. Condone one slip in matching speeds and masses. Condone consistent omission of m .
	$6mu - kmv = 3kmv + 2mv$ or $6u - kv = 3kv + 2v$	A1	Correct unsimplified equation Allow the marks if CLM stated correctly in (a) and used here.
	$6 - k = 3k \times \frac{7}{3} + 2 \times \frac{7}{3}, \quad 8k = \frac{4}{3}, \quad k = \frac{1}{6}$	A1	Correct only from correct work only
		[3]	
8c	This method looks at the total time between the two collisions between P and Q		
	Speed of Q after rebound $= f \times 3v (= f \times 7u)$	B1ft	Seen or implied ft is for correct use of their v
	t_p between collisions $= \frac{6d}{7 \times 2v} \left(= \frac{3d}{7v} = \frac{9d}{49u} \right)$	B1ft	Seen or implied For P distance $6d/7$ at $2v$ ft is for correct use of their v
	t_Q between collisions $= \frac{d}{3v} + \frac{d}{7 \times 3fv}$ $\left(= \frac{d}{7u} + \frac{d}{49fu} \right)$	M1	For Q distance d at $3v$ and distance $d/7$ at $3vf$
	$t_Q = t_P \Rightarrow \frac{3d}{7v} = \frac{d}{3v} + \frac{d}{21fv}$	DM1	Equate times and solve for f Dependent on preceding M1
	$\frac{3}{7} = \frac{1}{3} + \frac{1}{21f}, \quad \frac{2}{21} = \frac{1}{21f}, \quad f = \frac{1}{2}$	A1	Correct only from correct working
		[5]	
	See over for alternatives		

8c alt	This method looks at the time between the collision between Q and the wall and the second collision between P and Q		
	Speed of Q after rebound $= f \times 3v (= f \times 7u)$	B1ft	Seen or implied ft is for correct use of their v
	Distance apart when Q hits wall $= d - \frac{14u}{3} \times \frac{d}{7u} \left(= \frac{d}{3} \right)$	B1ft	Seen or implied ft is for correct use of their v Distance moved by Q – distance moved by P
	t_p for extra distance $= \frac{4d}{21} \div \frac{14u}{3} \left(= \frac{4d}{21} \div 2v \right)$	M1	Additional time to second collision = extra distance divided by speed of P
	$t_Q = t_p \Rightarrow \frac{4d}{21} \div \frac{14u}{3} = \frac{d}{7} \div 7uf$	DM1	Equate times to second collision and solve for f Dependent on preceding M1
	$\frac{12}{3 \times 2 \times 49} = \frac{1}{49f}, \quad f = \frac{1}{2}$	A1	Correct only from correct working
		[5]	
8c alt	This method looks at how far Q travels after the rebound		
	Speed of Q after rebound $= f \times 3v (= f \times 7u)$	B1ft	Seen or implied ft is for correct use of their v
	t_p between collisions $= \frac{6d}{7 \times 2v} \left(= \frac{3d}{7v} \right)$	B1ft	
	Distance travelled by Q if $f = 1$ $= \frac{3d}{7v} \times 3v = \frac{9}{7}d$	M1	
	$f = \frac{\text{actual distance after rebound}}{\frac{9}{7}d - d}$	M1	This is equivalent to $\frac{3d}{7v} \times 3v = \frac{d}{3v} \times 3v + \frac{d}{21fv} \times 3v$ or $\frac{9d}{7} = d + \frac{d}{7f}$
	$= \frac{1}{2}$	A1	Correct only from correct working
		[5]	
8c alt	This method looks at distances		
	Speed of Q after rebound $= f \times 3v (= f \times 7u)$	B1ft	Seen or implied ft is for correct use of their v
	If t_1 is the time for Q to the wall and t_2 is the time between wall and second collision distance travelled by P is $(t_1 + t_2) \frac{14}{3}u$	B1ft	

	$(t_1 + t_2) \frac{14}{3} u = \frac{6}{7} \times 7 u t_1$	M1	Equate distances for P and Q
	Use $\frac{d}{7} = 7 u f \times t_2$ and solve	M1	
	Obtain $f = \frac{1}{2}$	A1	Correct only from correct working
		[5]	

Question Number	Scheme	Marks
8	 Change in KE $\frac{4m}{2}(4u^2 - v^2) + \frac{3m}{2}(9u^2 - w^2) = \frac{473}{24}mu^2$ $(48v^2 + 36w^2 = 43u^2)$ Equation for CLM Need all terms. Dimensionally correct. Condone sign errors. $8mu - 9mu = -4mv + 3mw$ $(u = 4v - 3w)$ Impact law $w + v = 5eu$ $48v^2 + 36\left(\frac{4v - u}{3}\right)^2 = 43u^2 \text{ Or } 48\left(\frac{u + 3w}{4}\right)^2 + 36w^2 = 43u^2$ $\text{Or } \frac{48}{49}(1 + 15e)^2 + \frac{36}{49}(20e - 1)^2 = 43$ $112v^2 - 32uv - 39u^2 = 0 \quad \text{Or} \quad 63w^2 + 18uw - 40u^2 = 0$ $= (4v - 3u)(28v + 13u) \quad \text{Or} \quad = (21w + 20u)(3w - 2u)$ $\text{Or } 25200e^2 = 2023$ Notes	M1 A1 M1 A1 M1 A1 DM1 DM1
M1 A1 M1 A1 M1 A1 DM1 DM1	The first 8 marks are available if they have ignored the information about the final directions. Work with their directions. Ignore the diagram if that is to the candidate's advantage. Need all terms. Dimensionally correct. Accept $\pm$ Correct unsimplified equation in v, w or their v, w Need all terms. Dimensionally correct. Condone sign errors. Correct unsimplified equation with their correct signs Must be used the right way round Or equivalent in their w, v. Signs for v, w consistent with CLM eqn Form equation for v or w or e Dependent on M marks scored for the equations used. Solve for v or w or e. Dependent on the preceding M	

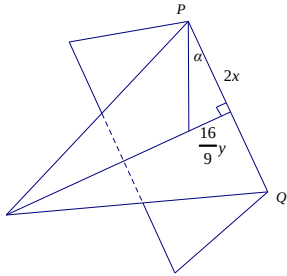
Question Number	Scheme	Marks
8	$v = \frac{3u}{4} \quad w = \frac{2u}{3}$ $\frac{3u}{4} + \frac{2u}{3} = 5eu, \quad e = \frac{17}{60}$ Use of $I = m(v - u)$ $4m\left(2u + \frac{3u}{4}\right) = 11mu$	A1 A1 DM1 A1 (12)
	Notes	[12]
A1	v or w correct	
A1	$\frac{3u}{4} + \frac{2u}{3} = 5eu$	
DM1	Must be attempting to subtract corresponding values for u and v Dependent on the first 4 M marks	
A1	Or $3m\left(3u + \frac{2u}{3}\right)$ from correct solution only	

Question Number	Scheme	Marks	Notes
8a			
	Impulse on A: $8mu = 3mv - 3m \times \frac{u}{3}$	M1	Terms dimensionally correct. Must be subtracting. Condone sign errors. Must be combining correct mass and speed
	$v = 3u$	A1	
	Impulse on B: $8mu = 4mw + 4mu$	M1	Terms dimensionally correct. Condone sign errors Or use CLM: $9mu - 4mw = 3m \frac{u}{3} + 4mu$ Must be combining correct mass and speed
	$w = u$	A1	
	Impact law: $u - \frac{u}{3} = e(3u + u)$	M1	Used the right way round. Condone sign errors
	$e = \frac{1}{6}$	A1	
	Award first 4 marks in order on the scheme. Marks for CLM equation, if used, should be given in place of whichever impulse equation is not used.		
	Watch out for sign errors in the equations If they have $3mv + 4mw$ in the equation for CLM they might combine this with $w = -u$ to obtain a "correct" answer. The sign error in the CLM is due to a misread so the maximum score for this double sign error is 4/6		
		[6]	
8b	Gap when B hits wall = $\frac{2d}{3}$	B1	Or find distances from the first impact: $s_A = \frac{d}{3} + \frac{u}{3}t$ and $s_B = d - \frac{u}{4}t$
	Speed of rebound from wall = $\frac{u}{4}$	B1	Allow + / -
	Time to close gap = $\frac{\frac{2d}{3}}{\frac{u}{3} + \frac{u}{4}}$	M1	
	$= \frac{8d}{7u}$	A1	
	Distance from wall = $\frac{8d}{7u} \times \frac{u}{4}$	DM1	Dependent on the preceding M1
	$= \frac{2d}{7}$	A1	
		[6]	
8balt	Time for A $\frac{d-x}{\frac{u}{3}} \left(= \frac{3d-3x}{u} \right)$	B1	
	Speed of rebound from wall = $\frac{u}{4}$	B1	

	Time for $B = \frac{d}{u} + \frac{x}{u/4}$	M1	
	$\left(= \frac{d + 4x}{u} \right)$	A1	
	$3d - 3x = d + 4x$	DM1	Solve for x Dependent on the preceding M1
	$x = \frac{2d}{7}$	A1 [6]	
		[12]	

6a			
	They need to form three equations, one of which must be the impact law. Mark them as you see them, so the first M1A1 on open is available for the first equation seen, the second M1A1 is for the second equation seen etc. If there are more than 3 equations, mark this as multiple attempts and all the marks for the equations actually used in the solution. Treat the second and third A marks as follow through marks if they are substituting values they have already found.		
	Use of $I = mv - mu$ for P or Q	M1	Dimensionally correct. Need all terms. M0 if m is missing on RHS
	$5mv = m(2v - (-y))$ or $-5mv = km(v - x)$	A1	Correct unsimplified equation
	Use of CLM or second use of $I = mv - mu$	M1	Dimensionally correct. Need all terms. In CLM allow cancelled m and extra common factor (eg g) throughout
	$kmx - my = kmv + 2mv$ $(kx - y = kv + 2v)$ or $-5mv = km(v - x)$	A1	Correct unsimplified equation
	Use of impact law	M1	Must be used with e on the correct side. Condone sign errors
	$2v - v = \frac{1}{5}(x + y)$	A1	Correct unsimplified equation
	$y = 3v$	A1	cao
	$x = 2v$	A1	cao
	$k = 5$	A1	cao
		[9]	
6b	KE lost	M1	Dimensionally correct. Accept change in KE. Not scored until they form the complete substituted equation.
	$= \frac{1}{2} \times km(x^2 - v^2) + \frac{1}{2} \times m(y^2 - 4v^2)$ $\left(= \frac{15}{2}mv^2 + \frac{5}{2}mv^2 \right)$	A1ft	Correct unsimplified expression. Follow their x, y, k Condone sign change without explanation. $\left(\text{KE before} = 14.5mv^2 \right)$ $\left(\text{KE after} = 4.5mv^2 \right)$
	$= 10mv^2$	A1	Only
		[3]	
		(12)	

Question	Scheme					Marks	Notes
They must have a dissection for which they should know or find the position of the centre of mass (e.g. triangles and rectangles). A false assumption about the position of the centre of mass of a trapezium results in 0/5.							
3a		Large tri	Small tri	Small tri	Whole	B1	Correct distances from PQ or a parallel axis for their complete dissection
	Dist PQ	0	-2y	2y	d	B1	Correct mass ratios for a complete dissection
	Mass ratio	27xy	12xy	12xy	27xy		Correct distances from PQ or a parallel axis for their complete dissection
	Moments about PQ:					M1	Or a parallel axis. Dimensionally correct. Need all non-zero terms and no extras. Condone sign error(s). Allow for ±d. Check the logic carefully.
	$(27xy \times 0) - 12xy \times (-2y) + 12xy \times 2y = 27xyd$					A1	Correct unsimplified equation. Allow for ±d. Allow for correct distance from a parallel axis
	$d = \frac{48}{27}y = \frac{16}{9}y$ *					A1*	Obtain given result from fully correct working.
There are many different approaches to this. NB If they are using a trapezium they must show method for the distance. For PQBC the correct value for distance centre of mass from PQ is $\frac{8y}{5}$ Possible alternative moments equations include: $15xy \times \frac{8y}{5} + 9xy \times \frac{4y}{3} + 3xy \times 4y = 27xyd$ using PQBC, PQDE and DEA $12xy \times 2y + 15xy \times \frac{8y}{5} = 27xyd$ using PQA and PQBC $2 \times 3xy \times y - 3xy \times y + 2 \times 6xy \times 1.5y + 2 \times 3xy \times 2y = 27xyd$ working from BC for the folded figure. $2 \times 3xy \times 2y + 4 \times \frac{1}{2} 3xy \times y + 2 \times 6xy \times 1.5y + 3xy \times 4y = 27xyd$ working down from PQ							
						(5)	

Question	Scheme	Marks	Notes
3b			
	Use of trigonometry	M1	Trig ratio for a relevant angle In their working they need a valid attempt to find α or $90^\circ - \alpha$.
	$\tan \alpha = \frac{\frac{16}{9}y}{2x} = \frac{64}{81}$	A1	Correct unsimplified equation in x and y
	$\Rightarrow x = \frac{9}{8}y$	A1	Correct only. ($x = 1.125y$) (Accept $x = 1.1y$ or better)
		(3)	

4a	Impulse-momentum equation for P:	M1	Correct use of $I = mv - mu$: Evidence of subtraction (can go straight to + you do not need to see $-(-)$) and dimensionally correct. Use of $3m$
	$15mv = 3m(2v - (-u))$	A1	Correct unsimplified equation
	$9mv = 3mu \Rightarrow u = 3v$ *	A1*	Obtain given answer from correct working
4a alt	Impulse-momentum equation for Q and CLM:	M1	CLM dimensionally consistent, all 4 terms, condone sign error(s). Correct use of $I = mv - mu$: Evidence of subtraction and dimensionally correct. Use of $5m$
	$15mv = 5m(v + ku)$, $k = 2\frac{v}{u}$ and substitute into CLM: $3mu - 5m\frac{2v}{u}u = 5mv - 6mv$	A1	Correct unsimplified equation in u and v
	$\Rightarrow u = 3v$ *	A1*	Obtain given answer from correct working
		(3)	
4b	Impulse-momentum equation for Q or use of CLM:	M1	Dimensionally consistent. All relevant terms.
	$15mv = 5m(v - (-ku))$ or $3mu - 5mku = 5mv - 6mv$	A1	Correct unsimplified equation
	$10v = 5ku = 15kv \Rightarrow k = \frac{2}{3}$	A1	Correct only. Accept 0.67 or better
		(3)	
4c	Use of impact law:	M1	Must be used the right way round. Condone sign error(s)
	$2v + v = e(u + ku) \quad \left(= e \times 3v \times \frac{5}{3} \right)$	A1ft	Correct unsimplified equation. Follow their k .
	$\Rightarrow e = \frac{3}{5}$	A1	Correct only
		(3)	

3a		square	triangle	circle	T			
	mass	36	8	π	$28 - \pi$			
	c from AD	$3a$	$\frac{7}{3}a$	$4a$	d			
	Mass ratio					B1		
	Distances from AD or a parallel axis					B1		
	M(AD or parallel axis):					M1	Moments equation. Need all terms and dimensionally correct. Condone sign errors.	
	$36 \times 3a - 8 \times \frac{7}{3}a - \pi \times 4a = (28 - \pi)d$					A1	Correct unsimplified equation for their parallel axis	
	$\left(108a - \frac{56}{3}a - 4\pi a = (28 - \pi)d\right)$							
	$d = \frac{324 - 56 - 12\pi}{3(28 - \pi)}a = \frac{4(67 - 3\pi)}{3(28 - \pi)}a$ *					A1*	Obtain given answer from correct working	
							Distance from BC is $\frac{(236 - 6\pi)a}{3(28 - \pi)}$ Allow 4/5 if seen.	
						(5)		
3b	$M(A): W \times \frac{4(67 - 3\pi)}{3(28 - \pi)}a = kW \times 6a$ Or resolve vertically and use $M(G)$, where G is the centre of mass, $T_A + kW = W$ $T_A \left(\frac{4(67 - 3\pi)}{3(28 - \pi)}a \right) = kW \left(6a - \frac{4(67 - 3\pi)}{3(28 - \pi)}a \right)$					M1	Complete method to form an equation in k and W only Dimensionally correct but condone use of incorrect distance(s)	
						A1	Correct unsimplified equation $\left(\text{NB } \frac{4(67 - 3\pi)}{3(28 - \pi)}a = 3.088a \right)$	
	$k = 0.51$					A1	Q asks for 2dp	
						(3)		
						[8]		

[illegible]

4 Alt 1			Resolving parallel and perpendicular to the direction of the impulse.
	Use of $J = m(v - u)$	M1	Use of $J = m(v - u)$ in any direction
	$J = 0.3(8 \cos \alpha - 5 \cos 60^\circ)$ Or $5 \sin 60^\circ = 8 \sin \alpha$	A1	Correct unsimplified equation $\left(\begin{array}{l} 2.4 \cos \alpha = J + 1.5 \cos 60^\circ \\ 2.4 \sin \alpha = 1.5 \sin 60^\circ \end{array} \right)$
	Use of $J = m(v - u)$	M1	Use of $J = m(v - u)$ in perpendicular direction
		A1	Correct unsimplified equation
	$2.4^2 = \left(J + \frac{3}{4}\right)^2 + \left(\frac{3}{2}\right)^2 \times \frac{3}{4}$ $(J^2 + 1.5J - 3.51 = 0)$	DM 1	Form an equation in J only Dependent on previous two M1 marks
	$J = 1.3$	A1	1.3 or better (1.268....)
	Could have a mixture of the first 2 alternatives. M1A1M1A1 for 2 independent equations. DM1A1 for solving		
		(6)	
4 Alt 2			Using vector triangle.
	Impulse momentum triangle	M1	Form dimensionally correct vector triangle (for impulse or momentum)
	Use of cosine rule	M1	Use of cosine rule in momentum or velocity triangle
	$2.4^2 = J^2 + 1.5^2 - 3J \cos 120^\circ$	A1	unsimplified equation in v or mv with at most one error
		A1	Correct unsimplified equation
	$J^2 + 1.5J - 3.51 = 0$	DM 1	Form a simplified equation in J Dependent on previous two M1 marks
	$J = 1.3$	A1	1.3 or better (1.268....)
		(6)	
		[6]	

5a			
	Moments about A:	M1	Need all terms and dimensionally correct. Condone sign errors, incorrect angles and sin/cos confusion Or complete method to form equation in T (and M).
	$5a \times T \sin 55^\circ = 4a \cos 20^\circ \times Mg$	A1	Correct unsimplified equation in T (and M).
	$T = \frac{4 \cos 20^\circ}{5 \sin 55^\circ} Mg (= 0.918Mg)$	A1	Or equivalent (Exact or $0.92Mg$ or better)
		(3)	
5b	Resolve vertically	M1	Need all terms. Condone sign errors, incorrect angle and sin/cos confusion
	$\uparrow: Mg = V + T \cos 55^\circ$ ($V = 0.47...Mg$)	A1	Correct unsimplified equation in T or their T
	Resolve horizontally	M1	Condone consistent sin/cos confusion
	$H = T \sin 55^\circ$ ($H = 0.75...Mg$)	A1	Correct unsimplified equation in T or their T
	Resultant $\lambda = \sqrt{(0.4736..)^2 + (0.7517..)^2}$	M1	Substitute for T and use Pythagoras
	$= 0.89$	A1	The Q asks for 2 sf
		(6)	
			See over for further alternative

5b alt	Moments about B	M1	Dimensionally correct. Need all terms. Condone sign errors and sin/cos confusion
	$Mga \cos 20^\circ + 5aH \cos 70^\circ = 5aV \cos 20^\circ$	A1	Correct unsimplified equation
	Moments about C	M1	Dimensionally correct. Condone sign errors and sin/cos confusion
	$5aH = 4aMg \cos 20^\circ$	A1	Correct unsimplified equation
	Resultant $\lambda = \sqrt{(0.4736..)^2 + (0.7517..)^2}$	M1	Use Pythagoras
	$= 0.89$	A1	The Q asks for 2 sf
	M1A1M1A1 for 2 independent equations M1A1 to solve for λ		

6a	GPE lost	M1	Need all terms. Condone sign errors and sin/cos confusion
	$= 3g \times 2 - 2g \times 2 \sin \theta$ $(= 6g - 4g \times \frac{5}{13})$	A1	Correct unsimplified. Accept $\pm$
	$= \frac{58}{13}g = 43.7(44)(J)$	A1	Must be positive. Exact multiple of g or 3 sf or 2 sf
		(3)	
6b	Normal reaction $= 2g \cos \theta \left(= \frac{24}{13}g \right)$	B1	Condone $\frac{1176}{65}$
	$F_{\max} = \frac{3}{8} \times R \left(= \frac{9g}{13} \right)$	M1	Use $F = \mu R$ with their R $\left(\frac{441}{65} \right)$
	Work done $= 2 \times F_{\max}$	M1	Their $F_{\max}$
	$\left(= \frac{18g}{13} \right) = 13.6(J) \ 14(J)$	A1	Exact multiple of g or 3 sf or 2 sf. Not $\frac{882}{65}$
		(4)	
6c	Total KE gained = GPE lost - total WD against friction	M1	Must be using work-energy. Dimensionally correct. Required terms and no extras. Condone sign errors.
	$\frac{1}{2}(2+3)v^2 = (their(a)) - (their(b))$ $\left(\frac{5}{2}v^2 = \frac{58}{13}g - \frac{18}{13}g = \frac{40}{13}g \right)$	A2ft	Follow their (a) and (b) -1 each error
	$v = \sqrt{\frac{16}{13}}g = 3.47(\text{ms}^{-1}) \text{ or } 3.5(\text{ms}^{-1})$	A1	3 sf or 2 sf (need to substitute for g)
		(4)	
6d	KE lost = GPE gained + WD against friction	M1	Must be using work-energy. Dimensionally correct. Required terms and no extras. Condone sign errors.
	$\frac{1}{2} \times 2 \times \frac{16}{13}g = 2g \times d \sin \theta + \frac{3}{8} \times 2g \times \frac{12}{13}d$ $\frac{1}{2} \times 2 \times v^2 = 2g \times d \sin \theta + d \times F_{\max}$ $\frac{16}{13}g = \left(\frac{10}{13}g + \frac{9}{13}g \right)d$	A2ft	Follow their (c) and their $F_{\max}$ -1 each error
	$d = \frac{16}{19}$	A1	g cancels. 0.84 or better (0.8421....)
		[15]	

7a	$-12 = 12 - gt$	M1	Use <i>suvat</i> to find time taken
	$t = \frac{24}{g} (= 2.45)$	A1	
	$AB = 6t$	M1	Horizontal distance
	$= 14.7 (15) (\text{m})$	A1	3 sf or 2 sf Not $\frac{720}{49}$ (follows use of 9.8) Not $\frac{144}{g}$ (do not accept g in the denominator)
		(4)	
7b	Vertical component of velocity $= (\pm)8$	B1	
	$v^2 = u^2 + 2as$	M1	Complete method using <i>suvat</i> to find h
	$\Rightarrow 8^2 = 12^2 - 2gh$	A1	Correct unsimplified equation
	$h = 4.08 \quad (4.1)$	A1	3 sf or 2 sf Not $\frac{200}{49}$ (follows use of 9.8) Not $\frac{40}{g}$ (do not accept g in the denominator)
		(4)	
7b alt	$\mathbf{v} = \begin{pmatrix} 6 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ g \end{pmatrix} t \Rightarrow 12 - gt = (\pm)8$	B1	Correct expression for critical value(s) of t
	$h = 12t - \frac{1}{2}gt^2$	M1	Complete method using <i>suvat</i> to find h
	$= \frac{48}{g} - \frac{8}{g} \quad \text{or} \quad = \frac{240}{g} - \frac{200}{g}$	A1	Correct unsimplified equation
	$h = 4.08 \quad (4.1)$	A1	3 sf or 2 sf
		(4)	
7b alt	Conservation of energy	M1	Need all terms and dimensionally correct
	$mgh + \frac{1}{2}m \times 10^2 = \frac{1}{2}m(12^2 + 6^2)$	A(B)1	Unsimplified equation with at most one error
		A1	Correct unsimplified equation
	$h = 4.08 \quad (4.1)$	A1	3 sf or 2 sf
		(4)	
			See over for (c)

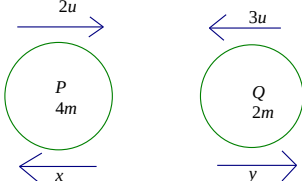
7c	$\begin{pmatrix} 6 \\ -12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ v \end{pmatrix} = 0$	M1	Complete method to find vertical component at C.
	$\Rightarrow v = 3$	A1	
	$\mathbf{v} = 6\mathbf{i} + 3\mathbf{j} \text{ (ms}^{-1}\text{)}$	A1	Must be a vector in terms of $\mathbf{i}$ and $\mathbf{j}$
	If see $\begin{pmatrix} 6 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ v \end{pmatrix} = 0$ leading to $\mathbf{v} = 6\mathbf{i} - 3\mathbf{j}$ mark as a misread: M1A0A0		
		(3)	
		[11]	
	Accept working in column vectors throughout apart from the final A1		

8a			
	Use CLM: $4mu = 2mv + mw$	M1	Need all terms. Condone sign errors. Dimensionally correct but allow with m cancelled
	$(4u = 2v + w)$	A1	Correct unsimplified. Signs correct for their v, w
	Use Impact law	M1	Used the right way round. Condone sign errors.
	$w - v = 2ue$	A1	Correct unsimplified. Signs consistent with CLM equation.
	$\Rightarrow 4u = 2(w - 2ue) + w$	DM1	Solve for v or w . Dependent on previous 2 M marks
	$3w = 4u + 4ue, \quad w = \frac{4}{3}u(1 + e) \quad *$	A1*	Obtain given result from correct working
	$v = \frac{2}{3}u(2 - e)$	A1	Or equivalent. Must be positive
		(7)	
8b	$2 > e$ so A moving towards centre	B1	Correct statement about direction of travel for A
	$mw - 3mu = mx + 3my$ $y - x = e\left(u + \frac{4u}{3} + \frac{4eu}{3}\right)$	M1	Use CLM and impact law correctly to form simultaneous equations in x and y .
	$\frac{4}{3}eu - \frac{5}{3}u = x + 3y$ $3y - 3x = e(7u + 4ue)$	A1	Both equations correct unsimplified in u, e, x and y
	$4x = \frac{4}{3}ue - \frac{5}{3}u - 7ue - 4ue^2$	DM1	Solve for x
	$x = -\frac{5}{12}u - \frac{17}{12}ue - ue^2$	A1	Allow for a correct constant multiple of x
	$e > 0, u > 0$ so B moving towards centre from opposite direction, hence they collide.*	A1*	Obtain given answer from correct working
		(6)	
	Alternative for last 3 marks;		
	C moving towards centre implies B moving towards centre, so collision. C moving away from centre, so $y > 0$, $x = w - 3u - 3y = -\frac{8u}{3} + \frac{4eu}{3} - 3y$	DM1	Consider direction of C

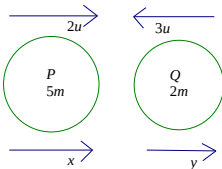
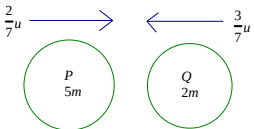
	$= -\frac{u}{3}(8-4e)-3y$	A1	
	< 0 because $e \leq 1$ and $y > 0$ hence B moving towards centre from opposite direction, and they will collide.*	A1*	Obtain given answer from correct working
		[13]	

Q	Mark Scheme	Marks	Marking guidance
6 (a)	M(PV)	M1	Allow use of a parallel axis. Terms dimensionally consistent. Could be seen as part of a vector equation. Condone error(s) in distance(s).
	$a \times 2ka^2 + \left(2a + \frac{1}{2}ka\right)2ka^2 = \bar{x} \times (2ka^2 + 2ka^2)$	A1	Correct unsimplified equation
	$2\bar{x} = a + 2a + \frac{1}{2}ka \Rightarrow \bar{x} = \frac{6+k}{4}a$ *	A1*	Obtain given answer from correct working
		3	
6 (b)	M(PR)	M1	Allow use of a parallel axis. Terms dimensionally consistent. Could be seen as part of a vector equation in (a) but needs to be used here to score mark(s) in (b). Condone error(s) in distance(s). If working from VU they might assume that c of m of QRST lies on their axis. So long as they say that this is what they have done (e.g. in a table of values) this can score M1A0A0M1A1ftA0.
	$\frac{1}{2}ka \times 2ka^2 + a \times 2ka^2 = \bar{y} \times 4ka^2$	A1	Correct unsimplified equation
	$\bar{y} = \frac{k+2}{4}a$	A1	Correct answer ($\pm$) seen or implied Accept distance from $VU = \pm \frac{3k-2}{4}a$ Or distance from TS = $\pm \frac{6-k}{4}a$
	Use angle to form equation in k	M1	Correct use of given ratio. Allow reciprocal
	$\frac{7}{15} = \frac{\bar{y}}{\bar{x}} = \frac{(k+2)a}{4} \times \frac{4}{(6+k)a}$	A1	Correct unsimplified equation using given $\bar{x}$ and their $\bar{y}$ e.g. $\frac{ka - \bar{y}_{VU}}{\bar{x}}$ or $\frac{2a - \bar{y}_{TS}}{\bar{x}}$
	$\Rightarrow k = \frac{3}{2} (=1.5)$	A1	Correct only
		6	
		(9)	

Q	Mark Scheme	Marks	Marking guidance
7 (a)			Check their diagram but remember that the directions used in their equations might not be consistent with the diagram. In this case, ignore their diagram.
	Conservation of momentum	M1	Need all terms. Dimensionally correct. Condone sign errors. Condone m missing throughout or g present throughout.
	$3mu + 2mu = mv_A + 2mv_B \quad (5u = v_A + 2v_B)$	A1	Correct unsimplified equation. Allow with v_A negative
	Use of NEL	M1	Used the right way round. Condone sign errors
	$v_B - v_A = e(3u - u) \quad (2ue = v_B - v_A)$	A1	Correct unsimplified equation. Allow with v_A negative. Signs consistent between the two equations.
	Solve for v_A or v_B	DM1	Dependent on two previous M marks
	Obtain $v_B = \frac{5+2e}{3}u$ *	A1*	Obtain given answer from correct working
	Obtain $v_A = \frac{5-4e}{3}u$	A1	Or equivalent. v_A must be positive
		7	
7 (b)	Time for B to reach the wall $t_B = \frac{d}{2u}$	B1	Seen or implied. Allow $\frac{d \times 3}{(5+2e)u}$
	Speed of B after impact with wall $= \frac{2}{3}u$	B1	Seen or implied. Allow $\frac{1}{3}\left(\frac{5+2e}{3}\right)$
	Distance travelled by A before B hits the wall $= u \times \frac{d}{2u} \left(= \frac{d}{2} \right)$	M1	Substitute $e = \frac{1}{2}$ and use their v_A and their t_b to find distance
	Time to close the gap	M1	Correct formula with their relevant speeds
	$\frac{d}{2} = u \times t + \frac{2u}{3} \times t \left(= \frac{5ut}{3} \right) \quad \left(t = \frac{3d}{10u} \right)$	A1	Correct unsimplified equation
	Total time $= \frac{d}{2u} + \frac{3d}{10u} = \frac{8d}{10u} \left(= \frac{4d}{5u} \right)$	A1	ISW Any equivalent form
		6	
7(b) alt	In time T , A travels x metres $x = uT$ B travels d metres in t sec $d = 2ut$ First B1 B travels $d - x$ metres in t' sec $d - x = 2ut'/3$ Second B1 and first M1 $t + t' = T$ $(d + 3d - 3uT)/2u = T$ Second M1 and first A1 $T = 4d/5u$ Second A1		Equivalent statement Correct value implied and Distance travelled by B after it hits the wall Correct formula for time and Correct unsimplified equation Correct answer
		(13)	

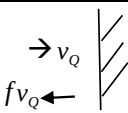
Question	Scheme	Mark	Notes
7a			
	Equation for CLM	M1	Dimensionally correct. All terms required. Condone sign errors.
	$8mu - 6mu = 2my - 4mx$ ($u = y - 2x$)	A1	Correct unsimplified equation
	Equation for kinetic energy ($\frac{1}{2}$ or 2 must be used)	M1	Dimensionally correct. Correct masses paired with correct velocities. All terms required. No sign errors. Condone 2 on the wrong side.
	$2mx^2 + my^2 = \frac{1}{2}(2m \times 4u^2 + m \times 9u^2)$ ($17u^2 = 4x^2 + 2y^2$)	A1	Correct unsimplified equation
	Solve for y: $17u^2 = 2y^2 + (y - u)^2$ $\Rightarrow 3y^2 - 2yu - 16u^2 = 0$	DM1	Some working must be shown to obtain the quadratic in y (and u). Dependent on the preceding M marks ($(3y - 8u)(y + 2u) = 0$)
	$\Rightarrow y = \frac{8}{3}u$ *	A1*	Obtain given answer from correct working
		[6]	
7b	Use of Impact Law: $x + y = e \times 5u$	M1	Condone sign errors but must be used the right way round.
	$e = \frac{\frac{1}{2}\left(\frac{8}{3}u - u\right) + \frac{8}{3}u}{5u}$	A1	Correct unsimplified equation. ($x = \frac{5u}{6}$)
	$= \frac{7}{10}$	A1	Correct only
		[3]	
7c	Velocity of Q after impact = $f \times \frac{8}{3}u$	B1	Allow $\pm$
	No collision if $f \times \frac{8}{3}u \leq \frac{5}{6}u$ i.e. speed of P $\geq$ speed of Q	M1	Correct inequality with their values Accept strict inequality. Dimensionally correct.
	$\Rightarrow 0 < f \leq \frac{5}{16}$	A1	Both ends required. ($0 < f \leq 0.3125$)
		[3]	
7d	Use of $I = \pm 2m\left(y - \left(-\frac{1}{4}y\right)\right)$	M1	Subtraction seen or implied with their $\frac{1}{4}y$ Requires correct mass Requires correct impact law
	$ I = \frac{20}{3}mu$	A1	Or equivalent. Must be positive 6.7mu or better Condone $-\frac{20}{3}mu \rightarrow \frac{20}{3}mu$ with no explanation
		[2]	
		(14)	

Q	Scheme	Mark	Notes
7a			If $6u$ and u are in opposite directions, mark as a sign error.
	Use of CLM	M1	Need all 4 terms. Dimensionally consistent. Condone sign errors Condone x in the wrong direction
	$6mu + 5mu = 5my - mx$ $(11u = 5y - x)$	A1	Correct unsimplified equation
	Use of impact law	M1	Used correctly. Dimensionally correct. Condone sign errors
	$x + y = 5eu$	A1	Correct unsimplified equation. Signs consistent with their CLM equation
	Solve for x in terms of e and u : $6x = 25eu - 11u$ or solve for e in terms of y and u : $e = \frac{6y-11u}{5u}$	DM1	Dependent on the first 2 M marks. As far as $kx = ..$ Dependent on the previous 2 M marks
	Use $x > 0$ ($\Rightarrow y > \frac{11}{5}u$): $25e > 11$	DM1	Use correct inequality for their x
	$\frac{11}{25} < e (\leq 1)$	A1	Or equivalent. Condone if 1 not mentioned. Allow with <1 . A0 if incorrect upper limit. cso
		[7]	
7b	$x = \frac{2}{3}u$ and $y = \frac{7}{3}u$	B1	Seen or implied
	Total KE lost $= \left(\frac{1}{2}m \times 36u^2 + \frac{1}{2}5m \times u^2 \right)$ $- \left(\frac{1}{2}m \times x^2 + \frac{1}{2}5m \times y^2 \right)$	M1	Complete expression. Dimensionally correct. Correct masses connected to correct speeds. Condone subtraction in the wrong order. Allow in x and y
	$= \left(\frac{1}{2}m \times 36u^2 + \frac{1}{2}5m \times u^2 \right)$ $- \left(\frac{1}{2}m \times \frac{4}{9}u^2 + \frac{1}{2}5m \times \frac{49}{9}u^2 \right)$	A1ft	Correct unsimplified expression in m and u . Follow their x, y with e substituted
	$= \frac{20}{3}mu^2$	A1	Or single term equivalent. Accept $6.7mu^2$ or better
		[4]	
7c	velocity of Q after collision with wall $= \pm fy \quad \left(= \pm f \times \frac{7}{3}u \right)$	B1ft	Follow their y (in terms of u)
	Second collision if $fy > x$ $\frac{7}{3}fu > \frac{2}{3}u$	DM1	Correct inequality for their x, y Dependent on the B1 and P moving away from the wall
	$\frac{2}{7} < f \leq 1$	A1	Correct only Need both limits
		[3]	
		(14)	

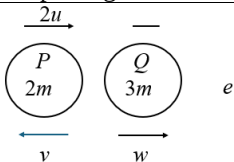
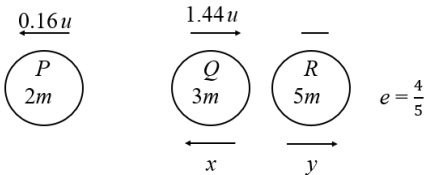
Question	Scheme	Marks	
			
7(a)	CLM	M1	
	$10mu - 6mu = 5mx + 2my$ $(4u = 5x + 2y)$	A1	
	Impact law	M1	
	$y - x = 5ue$	A1	
	$4u = 5x + 2x + 10ue (= 7x + 10ue)$	DM1	
	$(x > 0 \Rightarrow) 10ue < 4u$	DM1	
	$\Rightarrow 0, e < \frac{2}{5}$ o.e.	A1	
		(7)	
7(b)	Impulse momentum equation	M1	
	$\frac{60}{7}mu = 2m(3u - (-y))$ or $-\frac{60}{7}mu = 5m(x - 2u)$	A1	
	Solve to find a correct expression for either x or y $\left(y = \frac{9}{7}u, x = \frac{2}{7}u \text{ but may not be seen explicitly}\right)$	DM1	
	Use both impulse equations and impact law with their x and y to form an equation in e (and u). $5ue = \frac{9}{7}u - \frac{2}{7}u$	Using CLM and impact law from part (a) to form an equation in e (and u). For example $\frac{2u}{7} = \frac{4u - 10eu}{7}$	M1
	$e = \frac{1}{5u} \left(\frac{9}{7}u - \frac{2}{7}u \right) = \frac{1}{5} *$	A1*	
		(5)	
7(c)			
	Speed of $Q = \frac{1}{3} \times \frac{9}{7}u \left(= \frac{3}{7}u \right)$	B1ft	
	Magnitude of impulse = $\frac{1}{7} \times \frac{60}{7}mu$	M1	
	$= \frac{60}{49}mu$	A1	

		(3)
		(15)
	Notes	
7(a)		
M1	Form CLM equation. All terms required. Mass and velocity correctly paired. Dimensionally consistent, condone consistent additional g in each term. Condone sign errors.	
A1	Correct equation or equivalent. Condone consistent additional g in each term. Condone $\pm y$.	
M1	Use Impact Law. Dimensionally correct. Used the right way round (separation and approach must not be interchanged). Condone sign errors.	
A1	Correct equation or equivalent. Directions of P and Q after impact must be consistent with CLM.	
DM1	Dependent on the preceding M marks. Eliminate velocity of Q to form an equation in x , e and u only. $\left(x = \frac{4u - 10eu}{7} \right)$	
DM1	Dependent on all preceding M marks. May be implied by $e < 0.4$ Use direction of P to form an inequality in e (and u). Use correct inequality for their diagram: if they had P changing direction should now be using $x < 0$	
A1	Both ends required $0 \leq e < 0.4$ but condone $0 < e < 0.4$	
7(b)		
M1	Impulse-momentum equation. Dimensionally correct, using the correct mass and velocity pair for a single particle. Must be subtracting momenta but allow incorrect order. May use working in terms of e from (a) $x = \frac{4u - 10eu}{7}$ $y = \frac{25eu + 4u}{7}$	
A1	At least one correct unsimplified equation.	
DM1	Solve impulse-momentum equation to find either $y = \frac{9}{7}u$ or $x = \frac{2}{7}u$	
M1	Complete method to form an equation in e (and u) only with the usual rules for CLM and Impact Law.	
A1*	Obtain given answer from complete and correct working. A0 for 0.2	
7(c)		
B1	Follow $\frac{1}{3} \times$ their y from part (b). If y is given in terms of e then $e = \frac{1}{5}$ must be substituted at some point. Seen or implied. May appear on diagram. Accept $\pm$.	

M1	Complete method using (a) and (b) and working from first principles to find the impulse in the second collision between <i>P</i> and <i>Q</i>. Must see: - Usual rules for CLM and Impact Law to find V_p or V_q after second collision. CLM $\frac{10mu}{7} - \frac{6mu}{7} = 5mV_p + 2mV_q$ Impact Law $\frac{1}{5} \times \left(\frac{2u}{7} + \frac{3u}{7} \right) = V_q - V_p$ Then use of Impulse – momentum equation with usual rules to find magnitude of impulse. Impulse – momentum Either $I = 2m \left(\frac{9u}{49} - \frac{3u}{7} \right)$ or $I = 5m \left(\frac{2u}{49} - \frac{2u}{7} \right)$	
A1	1.2mu or better (1.2244897...)	

5(a)	$\begin{array}{cc} \rightarrow u & \rightarrow 0 \\ P(2m) & Q(m) \\ \rightarrow v_P & \rightarrow v_Q \end{array}$ 	
	CLM: $2mu = 2mv_P + mv_Q$	M1A1
	NEL: $eu = -v_P + v_Q$	M1A1
	$(v_Q =) \frac{2(1+e)u}{3} *$	A1*
		(5)
5(b)	e.g. $v_P = \frac{2(1+e)u}{3} - eu \left(= \frac{u}{3}(2-e) \right)$	B1
	KE After = $\frac{1}{2} \times 2m \left(\frac{u}{3}(2-e) \right)^2 + \frac{1}{2} m \left(\frac{2(1+e)u}{3} \right)^2$	M1A1
	KE Loss = $\frac{1}{2} \times 2mu^2 - \text{their KE after}$	DM1
	$= \frac{1}{3}(1-e^2)mu^2$ so $k = \frac{1}{3}$	A1
		(5)
5(c)	$\frac{8mu}{9} = m \times \frac{2(1+e)u}{3}$ OR $-\frac{8mu}{9} = 2m \left(\frac{u}{3}(2-e) - u \right)$	M1
	$e = \frac{1}{3}$	A1
		(2)
5(d)	$\frac{2(1+e)uf}{3}$ seen or implied, e does not need to be substituted.	B1
	$\frac{2 \left(1 + \frac{1}{3} \right) uf}{3} = \text{their speed of } P, \text{ with their value of } e \text{ used}$ to give an equation in f only.	M1
	$f = \frac{5}{8}$ or 0.625	A1
		(3)
		(15)
	Notes for question 5	
5(a)	N.B. Mark CLM equation first.	
	M1 for a CLM equation, dimensionally correct with correct no. of terms, condone sign errors, consistent extra g 's or cancelled m 's	
	A1 for a correct equation	
	M1 for a NEL equation, with e on the correct side, condone sign errors	
	A1 for a correct equation, consistent with the CLM equation.	
	A1* for given answer correctly obtained, with at least one line of intermediate working.	
	N.B. Allow any fully factorised equivalent form.	
5(b)	B1 for a correct unsimplified expression in e and u for v_P seen or	

	implied.	
	M1 for correct unsimplified expression, using their v_p and the given v_Q for KE after (must be adding the KE's)	
	A1 for a correct unsimplified expression for the KE After	
	DM1 for $\left(\frac{1}{2} \times 2mu^2 - \text{their KE after}\right)$ using their v_p and the given v_Q , condone Final KE – Initial KE.	
	A1 Accept 0.33 or better. N.B. Must be from correct working.	
5(c)	M1 for a correct impulse-momentum equation for either Q or P i.e. do NOT condone sign errors.	
	A1 Accept 0.33 or better.	
5(d)	B1 cao	
	M1 for equating their speeds of P and Q , with their e substituted, to give an equation in f only , allow slip when substituting in their e value, provided the method is clear. N.B. Must be using a value of e where $0 < e \leq 1$	
	A1 Accept 0.63	

QUESTION NUMBER	SCHEME	MARKS
7	N.B. throughout this question refer to diagram for the direction of speeds and then apply to equations. N.B. Correct mass-speed pairings are needed for all M marks	
		
7(a)	CLM	M1
	$2m(2u) = 2m(-v) + 3m(w)$	A1
	Use of impact law	M1
	$e(2u) = w + v$	A1
	$w = \frac{4u}{5}(e+1)^*$	A1*
		(5)
7(b)	Find an expression involving v , e and u	M1
	Correct expression Eg $v = \frac{2u}{5}(3e - 2)$	A1
	Use $v > 0$ to form an inequality in e , $\Rightarrow \frac{2}{3} < e \leq 1$	A1
		(3)
7(c)	Use of Impulse-momentum	M1
	$\frac{108mu}{25} = 3m \times \frac{4u}{5}(e+1)$	A1
	$e = \frac{4}{5}^*$	A1*
		(3)
7(d)		
	$v = 0.16u$, $4/25u$ Allow +/-	B1
	Use of CLM with their v $3m(1.44u) = 3m(-x) + 5m(y)$ $3m\left(\frac{4u}{5}(e+1)\right) = 3m(-x) + 5m(y)$	M1

	Use of impact law with their v $0.8(1.44u) = x + y$ $e\left(\frac{4u}{5}(e+1)\right) = x + y$	M1
	Correct expression $x = 0.18u$ o.e.	A1
	Compare the speed of P after the first collision with the speed of Q after the second collision. ($v = 0.16u$ and $x = 0.18u$)	dM1
	Since $0.18u > 0.16u$ a third collision will occur (between P and Q)	A1
		(6)
		(17)
	Notes for question	
7(a)		
M1	Equation for CLM. Dimensionally correct, all terms required. Condone sign errors.	
A1	Correct unsimplified equation.	
M1	Use of impact law. Condone sign errors but must be used the right way round.	
A1	Correct unsimplified equation, signs consistent with CLM.	
A1*	Obtain given answer from correct working. Must be factorised with $(e+1)$ for this mark. Allow $(1+e)$	
7(b)		
M1	Find an expression involving their v_p after the first collision with u and e only for their direction	
A1	Correct unsimplified expression for v_p after the first collision with u and e for their direction.	
A1	Use the direction of P and their expression for v to form a correct inequality in e , both ends required.	
7(c)		
M1	Use of impulse-momentum equation. Dimensionally correct, condone sign errors.	
A1	Correct unsimplified equation.	
A1*	Obtain given answer from complete and correct working. At least one line of working from equation to answer as a 'Show that'	
7(d)		
B1	Correct expression seen for speed of Q after the first collision. Allow $-0.16u$, $-4/25u$	
M1	Form relevant CLM equation using given answer from (a). Dimensionally correct, all terms required. Condone sign errors. No need for e to be replaced.	
M1	Use of impact law with given answer from (a). Condone sign errors but must be used the right way round. No need for e to be replaced.	

A1	Correct expression from correct working for the speed and direction of Q after the second collision, e must be replaced. Allow +/- answer o.e	
dM1	Compare the speed of P after the first collision with the speed of Q after the second collision ($v = 0.16u$ and $x = 0.18u$) Dependent on two previous M marks. Velocities must have been found in terms of u prior awarding this mark	
A1	Cso. Since $0.18u > 0.16u$ a second collision could occur between P and Q	
	N.B. if all that is seen is ' $v = 0.16u$ and $x = 0.18u$ ' and then so second collision'. This is M0A0 as no comparison has occurred.	