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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

## Further Kinematics (M3.1)

**International A Level Mathematics · M3: Mechanics 3 (WME03)**

14 questions · 135 marks · Mark schemes included

Past papers January 2020 - June 2025 · 1 sub-topic

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**HOW TO USE THIS BOOKLET**

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

| QUESTION                                                            | SOURCE                                     | MARKS             | PAGE      |
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| <b>M3.1a ACCELERATION AS A FUNCTION OF DISPLACEMENT OR VELOCITY</b> |                                            | 14 Qs · 135 marks | <b>2</b>  |
| 1. <b>Q1</b>                                                        | IAL M3 (WME03) January 2024                | 6                 | <b>3</b>  |
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| 9. <b>Q3</b>                                                        | IAL M3 (WME03) June 2022                   | 10                | <b>29</b> |
| 10. <b>Q4</b>                                                       | IAL M3 (WME03) January 2023                | 10                | <b>33</b> |
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| 12. <b>Q2</b>                                                       | IAL M3 (WME03) October 2021                | 11                | <b>39</b> |
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# QUESTIONS

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14 questions · 135 marks · 1 sub-topic

SUB-TOPIC M3.1a

## ACCELERATION AS A FUNCTION OF DISPLACEMENT OR VELOCITY

Sub-topic M3.1a · 14 questions · 135 marks · lowest tariff first



1. A spacecraft  $S$  of mass  $m$  moves in a straight line towards the centre,  $O$ , of a planet.

The planet is modelled as a fixed sphere of radius  $R$ .

The spacecraft  $S$  is modelled as a particle.

The gravitational force of the planet is the only force acting on  $S$ .

When  $S$  is a distance  $x$  ( $x \geq R$ ) from  $O$

- the gravitational force is directed towards  $O$  and has magnitude  $\frac{mgR^2}{2x^2}$
- the speed of  $S$  is  $v$

- (a) Show that

$$v^2 = \frac{gR^2}{x} + C$$

where  $C$  is a constant.

(3)

When  $x = 3R$ ,  $v = \sqrt{3gR}$

- (b) Find, in terms of  $g$  and  $R$ , the speed of  $S$  as it hits the surface of the planet.

(3)

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(Total for Question 1 is 6 marks)



- (4)



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 8 marks)



3. A particle  $P$  of mass  $m$  kg is initially held at rest at the point  $O$  on a smooth inclined plane. The plane is inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{2}{5}$

The particle is released from rest and slides down the plane against a force which acts towards  $O$ . The force has magnitude  $\frac{1}{3}mx^2$  N, where  $x$  metres is the distance of  $P$  from  $O$ .

- (a) Find the speed of  $P$  when  $x = 2$  (6)

The particle first comes to instantaneous rest at the point  $A$ .

- (b) Find the distance  $OA$ . (2)



**Question 3 continued**



**Question 3 continued**



**Question 3 continued**

**(Total for Question 3 is 8 marks)**



3. A particle  $P$  is moving along the  $x$ -axis.

At time  $t$  seconds, where  $t \geq 0$ , the displacement of  $P$  from the origin  $O$  is  $x$  metres and  $P$  is moving with velocity  $v \text{ m s}^{-1}$  in the positive  $x$  direction.

The acceleration of  $P$  is  $\frac{3\sqrt{x+1}}{4} \text{ m s}^{-2}$  in the positive  $x$  direction.

When  $t = 0$ ,  $x = 15$  and  $v = 8$

(a) Show that  $v = (x+1)^{\frac{3}{4}}$  (4)

(b) Find  $t$  in terms of  $v$ . (5)

**Question 3 continued**





**Question 3 continued**

**(Total for Question 3 is 9 marks)**





**Question 1 continued**

**(Total for Question 1 is 9 marks)**





### Question 3 continued

**(Total for Question 3 is 9 marks)**



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2. A particle,  $P$ , of mass  $0.5\text{ kg}$  is moving along the positive  $x$ -axis. At time  $t$  seconds,  $t \geq 0$ ,  $P$  is  $x$  metres from the origin  $O$  and is moving away from  $O$  with velocity  $v\text{ m s}^{-1}$ , where  $v = \frac{1}{(4x + 3)}$

When  $t = 0$ ,  $P$  is at  $O$ .

- (a) Find the distance of  $P$  from  $O$  when  $t = 2$  (5)
- (b) Find the magnitude of the resultant force acting on  $P$  when  $t = 2$  (5)



**Question 2 continued**

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**Question 2 continued**

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**(Total 10 marks)**

**Q2**





**Question 2 continued**

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**Question 2 continued**

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**Question 2 continued**

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**(Total 10 marks)**

**Q2**



3.

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A particle  $P$  is moving along a straight line.

At time  $t$  seconds,  $P$  is a distance  $x$  metres from a fixed point  $O$  on the line and is moving away from  $O$  with speed  $\frac{50}{2x+3} \text{ m s}^{-1}$

- (a) Find the deceleration of  $P$  when  $x = 12$

(5)

Given that  $x = 4$  when  $t = 1$

- (b) find the value of  $t$  when  $x = 12$

(5)

Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



**Question 3 continued**

**(Total for Question 3 is 10 marks)**





**Question 4 continued**

**(Total for Question 4 is 10 marks)**





Question 5 continued

Lined area for writing the answer to Question 5 continued.



**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 11 marks)**



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**2. In this question solutions relying on calculator technology are not acceptable.**

A particle  $P$  of mass  $2\text{ kg}$  is moving along the positive  $x$ -axis.

At time  $t$  seconds, where  $t \geq 0$ ,  $P$  is  $x$  metres from the origin  $O$  and is moving away from

$O$  with speed  $v \text{ ms}^{-1}$  where  $v = \frac{1}{\sqrt{(2x+1)}}$

(a) Find the magnitude of the resultant force acting on  $P$  when its speed is  $\frac{1}{3} \text{ m s}^{-1}$

(6)

When  $t = 0$ ,  $P$  is at  $O$

(b) Find the value of  $t$  when  $P$  is 7.5 m from  $O$

### Question 2 continued

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Question 2 continued

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### Question 2 continued

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**(Total 11 marks)**

**Q2**



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5. A particle  $P$  of mass  $0.5\text{ kg}$  moves along the positive  $x$ -axis in the positive  $x$  direction.

At time  $t$  seconds,  $t \geq 1$ ,  $P$  is  $x$  metres from the origin  $O$  and is moving with speed  $v \text{ ms}^{-1}$ .

The resultant force acting on  $P$  has magnitude  $\frac{2}{x^3}$  N and is directed towards  $O$ .

When  $t = 1$ ,  $x = 1$  and  $v = 3$

Show that

$$(a) \quad v^2 = \frac{4}{x^2} + 5 \tag{5}$$

$$(b) \quad t = \frac{a + \sqrt{bx^2 + c}}{d}, \text{ where } a, b, c \text{ and } d \text{ are integers to be found.} \quad (7)$$

Question 5 continued

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Question 5 continued

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**Question 5 continued**

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**(Total 12 marks)**

**Q5**





**Question 5 continued**

Question 5 continued

Lined area for writing the answer to Question 5 continued.



**Question 5 continued**

Lined area for writing the answer to Question 5.

(Total for Question 5 is 12 marks)



# MARK SCHEMES

official mark schemes, in the same order as the questions



|          |                                                                            |     |                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------|----------------------------------------------------------------------------|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a       | $-\frac{mgR^2}{2x^2} = mv \frac{dv}{dx}$                                   | M1  | Form differential equation in $v$ and $x$ only. Need to see $\frac{dv}{dx}$ or $v \frac{dv}{dx}$<br>Cannot get this mark using $t$ .<br>Allow with both $m$ 's cancelled.<br>Condone sign error.                                                                                                                                                                                                       |
|          | $-\frac{gR^2}{2} \int \frac{1}{x^2} dx = \int v dv$                        | M1  | Separate variables correctly and integrate at least one side.<br>Cannot get this mark using $t$ .<br>Condone sign error.                                                                                                                                                                                                                                                                               |
|          | $v^2 = \frac{gR^2}{x} + C \quad *$                                         | A1* | Obtain given answer from correct work. Must include at least one line of working between integral and final answer. Correct signs seen throughout working.<br>Condone $\frac{v^2}{2} = \frac{gR^2}{2x} + C$ followed by<br>$v^2 = \frac{gR^2}{x} + C$<br>Note: If the first line of working is<br>$\frac{1}{2}v^2 = -\int \frac{gR^2}{2x^2} dx$ followed by<br>integration of RHS, this scores M0M1A0* |
| ALT1 (a) | $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \int \frac{gmR^2}{x^2} dx$            | M1  | Form an energy equation with 2 KE terms and the integral of the variable force. Condone sign errors.                                                                                                                                                                                                                                                                                                   |
|          | $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = -\frac{gmR^2}{x} + A$                 | M1  | Integrate the force wrt $x$ . Condone sign errors.                                                                                                                                                                                                                                                                                                                                                     |
|          | $v^2 = \frac{gR^2}{x} + C \quad *$                                         | A1* | Obtain given answer from correct work. Must include at least one line of working and correct signs seen throughout working.                                                                                                                                                                                                                                                                            |
|          |                                                                            | [3] |                                                                                                                                                                                                                                                                                                                                                                                                        |
| b        | $x = 3R, v^2 = 3gR$                                                        | M1  | Use initial conditions to evaluate $C$ in the given answer.                                                                                                                                                                                                                                                                                                                                            |
|          | $\Rightarrow C = 3gR - \frac{gR^2}{3R} \left( = \frac{8gR}{3} \right)$     | A1  | Or equivalent                                                                                                                                                                                                                                                                                                                                                                                          |
|          | $x = R \Rightarrow v = \sqrt{\frac{11gR}{3}}$                              | A1  | Accept $\frac{\sqrt{33gR}}{3}$<br>Answer must be in terms of $g$ and $R$                                                                                                                                                                                                                                                                                                                               |
|          |                                                                            | [3] |                                                                                                                                                                                                                                                                                                                                                                                                        |
| ALT1 (b) | Use of definite integral instead of finding $+ C$                          |     |                                                                                                                                                                                                                                                                                                                                                                                                        |
|          | $\left[ v^2 \right]_{\sqrt{3gR}}^v = \left[ \frac{gR^2}{x} \right]_{3R}^R$ | M1  | Use initial conditions in a definite integral.                                                                                                                                                                                                                                                                                                                                                         |

|  |                                                |     |                                                                          |
|--|------------------------------------------------|-----|--------------------------------------------------------------------------|
|  | $v^2 - 3gR = \frac{gR^2}{R} - \frac{gR^2}{3R}$ | A1  | Or equivalent                                                            |
|  | $v = \sqrt{\frac{11gR}{3}}$                    | A1  | Accept $\frac{\sqrt{33gR}}{3}$<br>Answer must be in terms of $g$ and $R$ |
|  |                                                | (6) |                                                                          |

| Question Number | Scheme                                                                                                                       | Marks          |
|-----------------|------------------------------------------------------------------------------------------------------------------------------|----------------|
| <b>2.</b>       | $T \cos \theta (+R) = mg$                                                                                                    | M1A1           |
|                 | $T \sin \theta = ma \sin \theta \frac{2g}{a} \quad (T = 2mg)$                                                                | M1A1           |
|                 | $\cos \theta < \frac{1}{2} \text{ or } \cos \theta \leq \frac{1}{2} \text{ or } \cos \theta = \frac{1}{2}$                   | M1             |
|                 | $\theta > 60 \text{ or } \theta \geq 60$                                                                                     | A1             |
|                 | $90 > \theta > 60 \text{ or } 90 > \theta \geq 60$                                                                           | A1             |
|                 |                                                                                                                              | <b>(7)</b>     |
|                 | <b>Notes for question 2</b>                                                                                                  |                |
|                 | M1 for resolving vertically, correct no. of terms, $T$ resolved                                                              |                |
|                 | A1 for a correct equation                                                                                                    |                |
|                 | M1 for equation of motion horizontally                                                                                       |                |
|                 | A1 for a correct unsimplified equation                                                                                       |                |
|                 | M1 for producing an appropriate inequality in $\cos \theta$<br>Allow an equation                                             |                |
|                 | A1 Must come from an inequality.                                                                                             |                |
|                 | A1 cao                                                                                                                       |                |
|                 |                                                                                                                              |                |
|                 |                                                                                                                              |                |
| <b>3(a)</b>     | $a = v \frac{dv}{dx} = -\frac{2}{(2x+1)^3} \quad \text{separate and integrate}$                                              | M1             |
|                 | $\frac{1}{2} v^2 = \frac{1}{2(2x+1)^2} + (C)$                                                                                | A1             |
|                 | $x = 0, v = 1 \Rightarrow C = 0$                                                                                             | M1             |
|                 | $v = \frac{1}{(2x+1)}$                                                                                                       | A1             |
|                 |                                                                                                                              | <b>(4)</b>     |
| <b>3(b)</b>     | $\frac{dx}{dt} = \frac{1}{(2x+1)} \quad \text{separate and integrate}$                                                       | M1             |
|                 | $x^2 + x + (D) = t$                                                                                                          | A1             |
|                 | Complete the square: $(x + \frac{1}{2})^2 - \frac{1}{4} = t$<br>or use quadratic formula: $x = \frac{-1 \pm \sqrt{1+4t}}{2}$ | M1             |
|                 | $x = \frac{1}{2}(\sqrt{4t+1} - 1) *$                                                                                         | A1* <b>(4)</b> |
|                 |                                                                                                                              | <b>(8)</b>     |
|                 | <b>Notes for question 3</b>                                                                                                  |                |
| <b>3(a)</b>     | M1 allow omission of - sign, powers increasing by 1                                                                          |                |
|                 | A1 correct equation, but allow omission of $C$                                                                               |                |
|                 | M1 use of initial conditions to find $C$                                                                                     |                |
|                 | A1 cso                                                                                                                       |                |
| <b>3(b)</b>     | M1 powers increasing by 1                                                                                                    |                |
|                 | A1 correct equation, but allow omission of $D$                                                                               |                |



[illegible]

| Question Number | Scheme                                                                                                                                                                                                                                                     | Marks                                     |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------|
| <b>3</b>        |                                                                                                                                                                                                                                                            |                                           |
| (a)             | $mv \frac{dv}{dx} = mg \sin \alpha - \frac{1}{3}mx^2$ $\frac{1}{2}v^2 = xg \sin \alpha - \frac{1}{9}x^3 \quad (+c)$ $x = 2 \quad \frac{1}{2}v^2 = 2g \sin \alpha - \frac{8}{9}$ $(v = 3.728\dots)$ $v = 3.7 \text{ or } 3.73 \text{ (ms}^{-1}\text{)}$     | M1A1<br>DM1A1<br><br>DM1<br><br>A1cso (6) |
| <b>ALT</b>      | By energy:<br>$mg \sin \alpha x = \int \frac{1}{3}mx^2 dx + \frac{1}{2}mv^2$ $xg \sin \alpha = \frac{1}{9}x^3 + \frac{1}{2}v^2 \quad (+c)$ $x = 2 \quad \frac{1}{2}v^2 = 2g \sin \alpha - \frac{8}{9}$ $v = 3.7 \text{ or } 3.73 \text{ (ms}^{-1}\text{)}$ | M1A1<br>DM1A1<br><br>DM1<br>A1            |
| (b)             | $v = 0 \Rightarrow x^2 = 9g \sin \alpha = 9g \times \frac{2}{5} \quad (x \neq 0)$ $x = 5.939\dots \Rightarrow OA = 5.9 \text{ or } 5.94 \text{ (m)}$                                                                                                       | M1A1 (2)<br><b>[8]</b>                    |

| Question Number | Scheme                                                                                                             | Marks |
|-----------------|--------------------------------------------------------------------------------------------------------------------|-------|
| <b>(a)</b>      |                                                                                                                    |       |
| <b>M1</b>       | Attempt an equation of motion parallel to the plane with acceleration in any form (including $a$ )                 |       |
| <b>A1</b>       | Correct equation with the acceleration in $v \frac{dv}{dx}$ form                                                   |       |
|                 | Attempt the integration, powers increase by 1 in 2 terms – the constant may be missing.                            |       |
| <b>DM1</b>      | Acceleration must be in $v \frac{dv}{dx}$ form.                                                                    |       |
| <b>A1</b>       | Correct integration.                                                                                               |       |
| <b>DM1</b>      | Sub $x = 2$ in their expression for $v^2$ Depends on all previous M marks.                                         |       |
| <b>A1</b>       | Correct value for $v$ and $+c$ should be dealt with. Must be 2 or 3 sf                                             |       |
| <b>ALT</b>      |                                                                                                                    |       |
| <b>M1</b>       | Attempt a 3 term energy equation – KE, GPE, work done. Integral form is not required here.                         |       |
| <b>A1</b>       | Fully correct equation with integral form for work done.<br>Rest as main scheme.                                   |       |
| <b>(b)</b>      |                                                                                                                    |       |
| <b>M1</b>       | Use $v = 0$ in their expression for $v$ and obtain a value of $x$                                                  |       |
| <b>A1</b>       | Correct value of length $OA$ . (Allow if $x$ instead of $OA$ ) Must be 2 or 3 sf (unless already penalised in (a)) |       |
| <b>ALT (b)</b>  |                                                                                                                    |       |
| <b>M1</b>       | Start again with energy and integrate to obtain a value of $x$                                                     |       |
| <b>A1</b>       | See mark scheme.                                                                                                   |       |

| Question Number | Scheme                                                                                                                                                                                                                                         | Marks      |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| <b>3(a)</b>     | $v \frac{dv}{dx} = \frac{3\sqrt{x+1}}{4}$                                                                                                                                                                                                      | M1         |
|                 | $\frac{1}{2}v^2 = \frac{1}{2}(x+1)^{\frac{3}{2}} (+C)$                                                                                                                                                                                         | M1A1       |
|                 | $x = 15, v = 8 \Rightarrow C = 0$ so $v = (x+1)^{\frac{3}{4}} *$                                                                                                                                                                               | A1* (4)    |
| <b>3(b)</b>     | $\frac{dx}{dt} = (x+1)^{\frac{3}{4}}$                                                                                                                                                                                                          | M1         |
|                 | $4(x+1)^{\frac{1}{4}} = t (+C)$                                                                                                                                                                                                                | M1A1       |
|                 | $x = 15, t = 0 \Rightarrow C = 8$ so $4v^{\frac{1}{3}} = t + 8$                                                                                                                                                                                | M1         |
|                 | $t = 4v^{\frac{1}{3}} - 8$                                                                                                                                                                                                                     | A1 (5)     |
|                 | <b>OR</b>                                                                                                                                                                                                                                      |            |
|                 | $\frac{dv}{dt} = \frac{3}{4}v^{\frac{2}{3}}$                                                                                                                                                                                                   | M1         |
|                 | $3v^{\frac{1}{3}} = \frac{3}{4}t (+C)$                                                                                                                                                                                                         | M1A1       |
|                 | $t = 0, v = 8 \Rightarrow C = 6$ so $3v^{\frac{1}{3}} = \frac{3}{4}t + 6$                                                                                                                                                                      | M1         |
|                 | $t = 4v^{\frac{1}{3}} - 8$                                                                                                                                                                                                                     | A1 (5)     |
|                 |                                                                                                                                                                                                                                                | <b>(9)</b> |
|                 | <b>Notes for question 3</b>                                                                                                                                                                                                                    |            |
| <b>3(a)</b>     |                                                                                                                                                                                                                                                |            |
| <b>M1</b>       | Set up a differential equation in $v$ and $x$ only<br>M0 if acceleration is $\frac{dv}{dx}$ or $\frac{dv}{dt}$<br>M0 if there is no differential equation eg starting with $\frac{1}{2}v^2 = \int \frac{3\sqrt{x+1}}{4} dx$                    |            |
| <b>M1</b>       | Clear attempt to separate variables and integrate acceleration in terms of $v$ and $x$ .<br>At least one of the powers must increase by 1.                                                                                                     |            |
| <b>A1</b>       | Correct integration, condone missing $+ C$                                                                                                                                                                                                     |            |
| <b>A1*</b>      | Given answer obtained from complete and correct working. Must include use of the boundary conditions and the initial differential equation. A0 if $+C$ is not dealt with correctly eg If $+C$ is only considered <i>after</i> the square root. |            |
| <b>3(b)</b>     |                                                                                                                                                                                                                                                |            |
| <b>M1</b>       | Set up a differential equation in $x$ and $t$ only. Using the given answer in (a).                                                                                                                                                             |            |
| <b>M1</b>       | Clear attempt to separate the variables and integrate in terms of $x$ and $t$ . At least one of the powers must increase by 1                                                                                                                  |            |
| <b>A1</b>       | Correct integration, condone missing $+ C$                                                                                                                                                                                                     |            |
| <b>M1</b>       | Use of boundary conditions in an integrated equation and use of (a) to form an equation in $v$ and $t$ . M0 if boundary conditions are not used.                                                                                               |            |
| <b>A1</b>       | Correct answer                                                                                                                                                                                                                                 |            |
|                 | <b>OR</b>                                                                                                                                                                                                                                      |            |
| <b>M1</b>       | Set up a differential equation in $v$ and $t$ only                                                                                                                                                                                             |            |

| Question Number                                                                                           | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Marks |
|-----------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| 1(a)                                                                                                      | Differentiate the second expression, $v(t \dots 4)$ wrt $t$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | M1    |
|                                                                                                           | Correct derivative<br>$(a =) \frac{1}{4}t + \frac{8}{t^2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | A1    |
|                                                                                                           | $t = 8,$<br>$(a =) \frac{17}{8} = 2\frac{1}{8}$ oe                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | A1    |
|                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (3)   |
| 1(b)                                                                                                      | Equate velocity expressions, substitute $t = 4$ and solve for $k$<br>$\frac{1}{8}(4)^2 = \frac{1}{8}(4)^2 - \frac{8}{(4)} + k \Rightarrow k = \dots$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | M1    |
|                                                                                                           | $k = 2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | A1    |
|                                                                                                           | Integrate $v(0, t < 4)$ wrt $t$<br>$\int \frac{1}{8}t^2 dt = \frac{1}{24}t^3 (+ C)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | M1    |
|                                                                                                           | Integrate $v(t \dots 4)$ wrt $t$<br>$\int \frac{1}{8}t^2 - \frac{8}{t} + "2" dt = \frac{1}{24}t^3 - 8 \ln t + "2"t (+ C)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | M1    |
|                                                                                                           | <div> <b>Either</b> <ul style="list-style-type: none"> <li>a correct <b>definite</b> integral</li> <li><b>and</b> the correct limits, 4 and 8.<br/> <math display="block">\left[ \frac{1}{24}t^3 - 8 \ln t + "2"t \right]_4^8</math> </li> </ul> </div> <div> <b>Or</b> <ul style="list-style-type: none"> <li>a correct <b>indefinite</b> integral<br/> <math display="block">\frac{1}{24}t^3 - 8 \ln t + "2"t + C</math> </li> <li><b>and</b> use <math>t = 4</math> to equate distances to find an expression for the constant of integration<br/> <math display="block">\frac{4^3}{24} = \frac{4^3}{24} - 8 \ln 4 + 4("2") + C</math> </li> </ul> </div> | A1 ft |
|                                                                                                           | $(x =) \frac{88}{3} - 8 \ln 2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | A1    |
|                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (6)   |
|                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (9)   |
| <b>Notes for question 1</b>                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |
| <b>Note: There is a calculator warning so candidates must show their differentiation and integration.</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |
| (a)                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |
| M1                                                                                                        | Differentiate the second expression wrt $t$ , with both powers decreasing by 1. M0 if numerical answer appears without sight of the derivative.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |
| A1                                                                                                        | Correct derivative, accept $\frac{2t}{8} + 8t^{-2}$ , $\frac{1}{4}t + \frac{8}{t^2}$ o.e.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |       |
| A1                                                                                                        | Correct answer, accept 2.125, 2.13, 2.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |       |
| (b)                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |



| Question Number | Scheme                                                                                                                                                                                                                                                                             | Marks |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| <b>M1</b>       | Equate the given expressions for $v$ , substitute $t = 4$ and solve for $k$ . If $k$ is found in part (a) it must be used in (b) to earn the marks.                                                                                                                                |       |
| <b>A1</b>       | Correct answer for $k$                                                                                                                                                                                                                                                             |       |
| <b>M1</b>       | Attempt to integrate $v(0..t < 4)$ wrt $t$ , with power of $t$ increasing by 1.<br>M0 if numerical answer appears without sight of the integral.<br>$\left( \text{no need to evaluate to } \frac{8}{3} \text{ at this stage} \right)$                                              |       |
| <b>M1</b>       | Attempt to integrate $v(t..4)$ wrt $t$ , with a power of $t$ increasing by 1 and a $\ln t$ . Condone working with $k$ or their numerical $k$ . M0 if numerical answer appears without sight of the integral.                                                                       |       |
| <b>A1ft</b>     | Integrate $v(t..4)$ wrt $t$ . Condone working in terms of $k$ or ft on their numerical $k$ .<br>$\left( \frac{80}{3} - 8 \ln 2 \text{ oe may be seen but is not necessary for this mark} \right)$<br>$(C = 8 \ln 4 - 8 \text{ oe may be seen but is not necessary for this mark})$ |       |
| <b>A1</b>       | Correct answer, $\frac{88}{3} - 8 \ln 2$ , $\frac{88}{3} + 8 \ln \frac{1}{2}$ , $\frac{88}{3} + 8 \ln 0.5$<br>Accept equivalent exact form but must have fractions combined and $\ln$ terms combined                                                                               |       |



| Question Number             | Scheme                                                                                                                                                                                                                                                                                                                                                                       | Marks |
|-----------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| <b>3(a)</b>                 | Correct method to form a differential equation in $v$ and $x$ , separate the variables and integrate to the form<br>$\frac{1}{2}v^2 = Ax^2 + Bx \quad (+C) \quad \text{o.e.}$                                                                                                                                                                                                | M1    |
|                             | $\left( v \frac{dv}{dx} = 4x - 2 \Rightarrow \right) \quad \frac{1}{2}v^2 = 2x^2 - 2x \quad (+C)$                                                                                                                                                                                                                                                                            | A1    |
|                             | $\frac{1}{2}v^2 = 2x^2 - 2x + \frac{1}{2} \quad \text{o.e.}$                                                                                                                                                                                                                                                                                                                 | A1    |
|                             | $v = 1 - 2x$                                                                                                                                                                                                                                                                                                                                                                 | A1    |
|                             |                                                                                                                                                                                                                                                                                                                                                                              | (4)   |
| <b>3(b)</b><br><b>ALT1</b>  | Method to form a differential equation in $x$ and $t$ , separate the variables and integrate to the form<br>$D \ln(1 - 2x) = t \quad (+C) \quad \text{o.e.}$                                                                                                                                                                                                                 | M1    |
|                             | $\left( \int \frac{1}{1-2x} dx = \int dt \Rightarrow \right) \quad -\frac{1}{2} \ln(1 - 2x) = t \quad (+C)$                                                                                                                                                                                                                                                                  | A1    |
|                             | $(t = 1, x = 0 \Rightarrow C = -1)$<br>$-\frac{1}{2} \ln(1 - 2x) = t - 1 \quad \text{o.e.}$                                                                                                                                                                                                                                                                                  | A1    |
|                             | Complete method to obtain $v$ in terms of $t$<br>For example<br><ul style="list-style-type: none"> <li><math>-\frac{1}{2} \ln(v) = t - 1 \quad (+C) \Rightarrow v = \dots</math> in terms of <math>t</math></li> <li><math>x = \frac{1}{2}(1 - e^{2-2t})</math> then differentiate wrt <math>t</math> to obtain <math>v = \dots</math> in terms of <math>t</math></li> </ul> | dM1   |
|                             | $(v =) e^{2-2t}$                                                                                                                                                                                                                                                                                                                                                             | A1    |
|                             |                                                                                                                                                                                                                                                                                                                                                                              | (5)   |
| <b>3(b)</b><br><b>ALT2</b>  | Method to form a differential equation in $v$ and $t$ , separate the variables and integrate to the form<br>$\ln v = Et \quad (+C) \quad \text{o.e.}$                                                                                                                                                                                                                        | M1    |
|                             | $\left( \int \frac{1}{v} dv = -2 \int dt \Rightarrow \right) \quad \ln v = -2t \quad (+C)$                                                                                                                                                                                                                                                                                   | A1    |
|                             | $t \quad (t = 1, v = 1 \Rightarrow C = 2)$<br>$\ln(v) = 2 - 2t \quad \text{o.e.}$                                                                                                                                                                                                                                                                                            | A1    |
|                             | Obtain $v$ in terms of $t$                                                                                                                                                                                                                                                                                                                                                   | dM1   |
|                             | $v = e^{2-2t}$                                                                                                                                                                                                                                                                                                                                                               | A1    |
|                             |                                                                                                                                                                                                                                                                                                                                                                              | (5)   |
|                             |                                                                                                                                                                                                                                                                                                                                                                              | (9)   |
| <b>Notes for question 3</b> |                                                                                                                                                                                                                                                                                                                                                                              |       |
| <b>(a)</b>                  |                                                                                                                                                                                                                                                                                                                                                                              |       |



|                  |                                                                                                                                                                                                                                                                                                                                                                                                                |  |
|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| <b>M1</b>        | Method using $v \frac{dv}{dx}$ to form a differential equation in $v$ and $x$ , separate the variables and integrate to the form $\frac{1}{2}v^2 = Ax^2 + Bx (+C)$ where $A$ and $B$ are non-zero constants. Condone a missing constant of integration. $\int v dv$ may be implied by sight of $\frac{1}{2}v^2$ . M0 for use of <i>suvat</i>                                                                   |  |
| <b>A1</b>        | A correct equation with or without $+C$                                                                                                                                                                                                                                                                                                                                                                        |  |
| <b>A1</b>        | A fully correct equation including the evaluated constant of integration.                                                                                                                                                                                                                                                                                                                                      |  |
| <b>A1</b>        | Correct only                                                                                                                                                                                                                                                                                                                                                                                                   |  |
|                  | <b>N.B.</b> If the answer to (a) is $v = 2x - 1$ , the maximum score for (b) is M1A1A0M1A0                                                                                                                                                                                                                                                                                                                     |  |
| <b>(b) ALT1</b>  |                                                                                                                                                                                                                                                                                                                                                                                                                |  |
| <b>M1</b>        | Method using $v = \frac{dx}{dt}$ to form a differential equation in $x$ and $t$ , separate the variables and solve to reach the form $D \ln(1 - 2x) = t (+C)$ where $D$ is a non-zero constant. Condone a missing constant of integration. M0 for use of <i>suvat</i>                                                                                                                                          |  |
| <b>A1</b>        | Correct equation in $x$ and $t$ , with or without $C$<br><b>N.B.</b> if they start with $v = 2x - 1$ , allow M1A1 for $\frac{1}{2} \ln(2x - 1) = t (+C)$                                                                                                                                                                                                                                                       |  |
| <b>A1</b>        | Correct equation in $x$ and $t$ with the evaluated constant of integration.                                                                                                                                                                                                                                                                                                                                    |  |
| <b>dM1</b>       | Dependent on previous M. A complete method to obtain $v$ as an exponential function in terms of $t$ only.<br><ul style="list-style-type: none"> <li><math>-\frac{1}{2} \ln(v) = t - 1 \Rightarrow v = \dots</math> in terms of <math>t</math></li> <li><math>x = \frac{1}{2}(1 - e^{2-2t})</math> then differentiate wrt <math>t</math> to obtain <math>v = \dots</math> in terms of <math>t</math></li> </ul> |  |
| <b>A1</b>        | Correct answer only                                                                                                                                                                                                                                                                                                                                                                                            |  |
| <b>(b) ALT 2</b> |                                                                                                                                                                                                                                                                                                                                                                                                                |  |
| <b>M1</b>        | Method to form a differential equation in $v$ and $t$ , separate the variables and integrate to the form $\ln v = Et (+C)$ where $E$ is a non-zero constant. Condone a missing constant of integration.<br>$\left\{ \frac{dv}{dt} = -2 \frac{dx}{dt} \Rightarrow \frac{dv}{dt} = -2v \Rightarrow \ln v = -2t (+C) \right\}$<br>M0 for use of <i>suvat</i>                                                      |  |
| <b>A1</b>        | Correct equation in $v$ and $t$ , with or without $C$<br><b>N.B.</b> if they start with $v = 2x - 1$ , allow M1A1 for $\ln v = Et (+C)$                                                                                                                                                                                                                                                                        |  |
| <b>A1</b>        | A correct equation in $v$ and $t$                                                                                                                                                                                                                                                                                                                                                                              |  |
| <b>dM1</b>       | Dependent on previous M. A complete method to obtain $v$ as an exponential function in terms of $t$ only.                                                                                                                                                                                                                                                                                                      |  |
| <b>A1</b>        | Correct answer only                                                                                                                                                                                                                                                                                                                                                                                            |  |

| Question Number             | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Marks                                                                   |
|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------|
|                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                         |
| <b>4(a)</b>                 | <p><math>AD = 2.5a</math> (seen or implied)</p> <p>Method for the difference in 2 EPE terms (at <math>C</math> and <math>D</math>)</p> <p>Eg</p> <p>Considering the whole string <math>\frac{mg(5a-3a)^2}{2(3a)} - \frac{mg(4a-3a)^2}{2(3a)}</math></p> <p>Considering two half strings <math>2 \times \left[ \frac{mg(\frac{5}{2}a - \frac{3}{2}a)^2}{2(\frac{3}{2}a)} - \frac{mg(2a - \frac{3}{2}a)^2}{2(\frac{3}{2}a)} \right]</math></p> <p><math>\frac{1}{2}mga^*</math></p> | <p>B1</p> <p>M1</p> <p>A1*</p> <p>(3)</p>                               |
| <b>4(b)</b>                 | <p>GPE term, <math>mg\left(\frac{3a}{2}\right)</math></p> <p>Work-energy equation with all required terms</p> $mg\left(\frac{3a}{2}\right) = \frac{1}{2}mV^2 - \frac{1}{2}m\left(\sqrt{\frac{3ag}{2}}\right)^2 + \frac{1}{2}mga + \frac{1}{5}mg\left(\frac{3a}{2}\right)$ $V = \sqrt{\frac{29ag}{10}}$                                                                                                                                                                            | <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>(5)</p> <p>(8)</p> |
| <b>Notes for question 4</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                         |
| <b>(a)</b>                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                         |
| <b>B1</b>                   | Seen or implied eg $DB = 2.5a$ or $2.5a$ used correctly in EPE expression.                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                         |
| <b>M1</b>                   | Complete method using 2 EPE terms to form an expression for the change in EPE, accept subtraction either way round. The EPE terms must have the form $\frac{\lambda x^2}{2l}$ where $l$ is the relevant natural length, i.e. $\left(\frac{3}{2}a \text{ or } 3a\right)$ and $x$ is the extension, in terms of $a$ , at $C$ or $D$ respectively.                                                                                                                                   |                                                                         |
| <b>A1*</b>                  | Given answer obtained from fully correct working.                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                         |
| <b>(b)</b>                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                         |
| <b>B1</b>                   | Correct GPE term, $mg\left(\frac{3a}{2}\right)$ seen or implied                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                         |



| Question Number | Scheme                                                                                                                                                                                                                                                                                                                        | Marks                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                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| 2(a)            | $v = \frac{dx}{dt} = \frac{1}{(4x+3)}$ $\frac{dt}{dx} = 4x+3$ $t = \int (4x+3)dx, = \frac{1}{2} \times 4x^2 + 3x + c$ $\text{OR } \int_0^2 dt = \int_0^x (4x+3)dx, = \left[ \frac{1}{2} \times 4x^2 + 3x \right]_0^x$ $c = 0$ $t = 2 = \frac{1}{2} \times 4x^2 + 3x \quad 2x^2 + 3x - 2 = 0$ $x = \frac{1}{2} \quad (x = -2)$ | M1,dM1A1<br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br>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| Question Number | Scheme                                                                                                                                   | Marks       |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------|-------------|
| <b>2(a)</b>     | $F = \frac{k}{(x+R)^2}$                                                                                                                  | M1          |
|                 | $x = 0, F = mg \rightarrow mg = \frac{k}{R^2}$                                                                                           | M1          |
|                 | $k = mgR^2 \rightarrow F = \frac{mgR^2}{(x+R)^2}$ *                                                                                      | A1*         |
|                 |                                                                                                                                          | <b>(3)</b>  |
| <b>(b)</b>      |                                                                                                                                          |             |
|                 | $mv \frac{dv}{dx} = -\frac{mgR^2}{(x+R)^2} \quad \text{or} \quad m \frac{d}{dx} \left( \frac{1}{2} v^2 \right) = -\frac{mgR^2}{(x+R)^2}$ | M1          |
|                 | $\frac{1}{2} v^2 = -\int \frac{gR^2}{(x+R)^2} dx$                                                                                        | dM1         |
|                 | $\frac{1}{2} v^2 = \frac{gR^2}{x+R} (+c)$                                                                                                | A1          |
|                 | $x = R, v = U$                                                                                                                           | M1          |
|                 | $\frac{U^2}{2} = \frac{gR^2}{2R} + c \rightarrow c = \frac{U^2 - gR}{2}$                                                                 | A1          |
|                 | $x = 0 \rightarrow \frac{1}{2} v^2 = gR + \frac{U^2 - gR}{2}$                                                                            |             |
|                 | $v^2 = U^2 + gR \rightarrow v = \sqrt{U^2 + gR}$                                                                                         | M1, A1      |
|                 |                                                                                                                                          | <b>(7)</b>  |
|                 |                                                                                                                                          | <b>[10]</b> |
| <b>ALT1 (b)</b> | $\frac{mv^2}{2} - \frac{mU^2}{2} = -m \int_R^0 \frac{gR^2}{(x+R)^2} dx$                                                                  | M1          |
|                 | $\frac{v^2}{2} - \frac{U^2}{2} = \left[ \frac{gR^2}{x+R} \right]_R^0$                                                                    | dM1 A1      |
|                 | $\frac{v^2}{2} - \frac{U^2}{2} = \frac{gR^2}{R} - \frac{gR^2}{2R}$                                                                       | M1 A1       |
|                 | $v^2 = U^2 + gR \rightarrow v = \sqrt{U^2 + gR}$                                                                                         | M1, A1      |
|                 |                                                                                                                                          |             |
|                 |                                                                                                                                          |             |
|                 |                                                                                                                                          |             |



| Question Number     | Scheme                                                                   | Marks      |
|---------------------|--------------------------------------------------------------------------|------------|
| <b>ALT2<br/>(b)</b> | $mv \frac{dv}{dx} = -\frac{mgR^2}{(x+R)^2}$                              | M1         |
|                     | $\int_U^V v dv = - \int_R^0 \frac{gR^2}{(x+R)^2} dx$                     | dM1        |
|                     | $\left[ \frac{v^2}{2} \right]_U^V = \left[ \frac{gR^2}{x+R} \right]_R^0$ | A1         |
|                     | $\frac{v^2}{2} - \frac{U^2}{2} = \frac{gR^2}{R} - \frac{gR^2}{2R}$       | M1 A1      |
|                     | $v^2 = U^2 + gR \rightarrow v = \sqrt{U^2 + gR}$                         | M1, A1     |
|                     |                                                                          | <b>(7)</b> |
|                     |                                                                          |            |



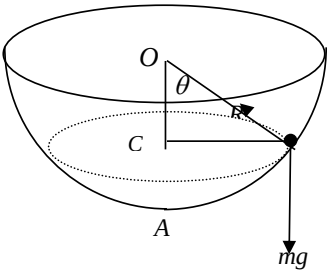
**(a)****M1** Setting up an inverse square relationship between  $F$  and  $(x + R)$ . Can be negativeAllow with  $d = x + R$  or  $k = GMm$ **dM1** Clear use of  $x = 0$  and  $F = mg$  to find value of constant ( $k$  or  $GM$ )**A1\*** Given result reached with both M marks clearly earned. Must be positive**(b)****M1** Use of  $mv \frac{dv}{dx}$  or  $m \frac{d}{dx} (\frac{1}{2} v^2)$  to form equation. Condone sign error.**dM1** Separate variables to produce form ready for integration. Condone sign error.**A1** Correct integration. Sign must be correct now. Constant of integration not needed.**M1** Use of initial conditions in the result of an integration to find constant.**A1** Correct value for their  $c$  (for the side they place  $c$  on).**M1** Finding a value for  $v$  (or  $v^2$ ) using  $x = 0$ .  $v^2$  must come from a dimensionally correct expression.**A1** Correct expression for  $v$ .**ALT1 (b) uses the change in KE = work done****M1** Equates change in KE to the integral of  $F$ . Condone sign error**dM1** Integrates (power of  $(x + R)$  must increase). Condone sign errors. Limits not needed.**A1** Correct integration. Sign must be correct now for their LHS. Limits not needed.**M1** Substitution of both limits  $R$  and  $0$  into definite integration.**A1** Correct limits, the correct way round for their equation.

Final two marks are the same as the main scheme.

**ALT2 (b) uses definite integration****M1** Use of  $mv \frac{dv}{dx}$  to form equation. Condone sign error.**dM1** Separate variables to produce form ready for integration. Condone sign error. Limits not needed.**A1** Correct integration. Sign must be correct now. Limits not needed.**M1** Substitution of both limits in definite integration. Must be  $0$ ,  $R$ ,  $v$  and  $U$ **A1** Correct limits, the correct way round.

Final two marks are the same as the main scheme.

**S.C.** If they redefine  $x$  as the distance from the centre of the earth for (b) and use limits  $R$  and  $2R$  correctly, full marks can still be gained in (b)

| Question Number                                   | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Marks                                                                 |
|---------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------|
| 2.                                                |  $R \sin \theta = m \times 6r \sin \theta \times \frac{g}{4r}$ $R = \frac{3}{2} mg$ $R \cos \theta = mg$ $\frac{3}{2} mg \cos \theta = mg$ $\cos \theta = \frac{2}{3}$ $OC = 6r \cos \theta = 6r \times \frac{2}{3} = 4r$                                                                                                                                                                                                                                                | <p>M1A1A1</p> <p>M1A1</p> <p>DM1</p> <p>A1</p> <p>M1A1</p> <p>[9]</p> |
| <b>Notes</b>                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                       |
| <b>M1</b><br><b>A1</b><br><b>A1</b>               | <p>Attempt NL2 along CP with correct number of terms and forces resolved.<br/>         Either side correct<br/>         Fully correct equation<br/>         Note:</p> <p>If R is not resolved then M0 <b>but</b> do allow if <math>\sin \theta</math> is cancelled from <b>both</b> sides: <math>R = m \times 6r \times \frac{g}{4r}</math> would score M1A1A1</p> <p>If r is used instead of the radius: <math>R \sin \theta = m \times r \times \frac{g}{4r}</math> would score M1A1A0 (force resolved correctly on LHS but error in radius on RHS)</p> |                                                                       |
| <b>M1</b><br><b>A1</b>                            | <p>Resolve vertically<br/>         Correct equation</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                       |
| <b>DM1</b><br><b>A1</b><br><b>M1</b><br><b>A1</b> | <p>Eliminate R between the two equations. Depends on both M marks above<br/>         Correct value for <math>\cos \theta</math> seen or implied<br/>         Attempt to obtain OC (allow sin/cos confusion)<br/> <math>OC = 4r</math></p> <p>Note: If <math>\theta</math> is the angle with the horizontal then all equations above will appear with <math>\sin \theta</math> and <math>\cos \theta</math> reversed.</p>                                                                                                                                  |                                                                       |

| Question Number | Scheme                                                                                                                                                                                                                                                                                                                                                                                                           | Marks |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| <b>ALT 1</b>    | <p><b>Case: using trig ratios where radius, L, and <math>\omega^2</math> are never replaced</b></p> <p>M1 A1 A1: <math>R \sin \theta = m L \omega^2</math><br/> M1 A1: <math>R \cos \theta = mg</math><br/> <math display="block">\frac{L \omega^2}{g} = \frac{L}{4r}</math> DM1 A1: <math>\tan \theta = \frac{L \omega^2}{g} \Rightarrow OC = 4r</math><br/> M1 A1: <math>\tan \theta = \frac{L}{OC}</math></p> |       |
| <b>ALT 2</b>    | <p><b>Case: resolving tangentially where R is never seen</b></p> <p><math>mg \sin \theta = m \times (6r \sin \theta) \times \frac{g}{4r} \cos \theta</math> scores M1A1A1 M1A1 DM1<br/> <math display="block">\cos \theta = \frac{2}{3}</math> leads straight to A1</p>                                                                                                                                          |       |

| Question Number         | Scheme                                                                                                                                                                                                                                                                                                                                                                                    | Marks                                          |
|-------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------|
| 3(a)                    | $v = \frac{50}{2x+3}$ $\frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}$ $= \frac{-100}{(2x+3)^2} \times \frac{50}{2x+3} \left( = \frac{-5000}{(2x+3)^3} \right)$ $x = 12 \quad \frac{dv}{dt} = -\frac{5000}{27^3} = -0.2540... = -0.25 \text{ or } -0.254 \text{ ms}^{-2}$ <p>deceleration = 0.25 (m s<sup>-2</sup>) or better</p>                                                     | <p>M1</p> <p>DM1A1</p> <p>M1</p> <p>A1 (5)</p> |
| (b)                     | $v = \frac{dx}{dt} = \frac{50}{2x+3}$ $\int (2x+3) dx = \int 50 dt$ $x^2 + 3x = 50t \text{ (+c)}$ $t = 1, x = 4 \Rightarrow 28 = 50 + c, c = -22$ $x = 12 \Rightarrow 50t = 12^2 + 36 + 22 \quad t = \frac{202}{50} = 4.04 \text{ (accept 4.0)}$                                                                                                                                          | <p>M1</p> <p>M1A1</p> <p>A1</p> <p>A1 (5)</p>  |
| <b>[10]</b>             |                                                                                                                                                                                                                                                                                                                                                                                           |                                                |
| <b>Notes</b>            |                                                                                                                                                                                                                                                                                                                                                                                           |                                                |
| (a)                     | <p><b>M1</b> Uses chain rule of the form <math>\frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}</math> or <math>\frac{d(\frac{1}{2}v^2)}{dx}</math></p> <p>Note, <math>\frac{1}{2}v^2 = \frac{1250}{(2x+3)^2} \Rightarrow \text{acc} = \frac{d(\frac{1}{2}v^2)}{dx} = -\frac{2500}{(2x+3)^3} \times 2</math></p> <p>However, <b>M0</b> for <math>\text{acc} = \frac{1}{2}v^2</math></p> |                                                |
| <b>DM1</b><br><b>A1</b> | <p>Differentiate v wrt x</p> <p>Correct differentiation.</p>                                                                                                                                                                                                                                                                                                                              |                                                |
| <b>M1</b><br><b>A1</b>  | <p>Sub <math>x = 12</math> into their expression for acceleration to obtain the deceleration. Must have attempted to differentiate.</p> <p>Correct deceleration – must be positive</p>                                                                                                                                                                                                    |                                                |
| (b)                     | <p><b>M1</b> Use <math>v = \frac{dx}{dt}</math></p> <p><b>M1</b> Attempt at integration</p> <p><b>A1</b> Correct integration but c may be missing</p> <p><b>A1</b> Use <math>t = 1, x = 4</math> to obtain the correct value of c for their correct integration</p> <p><b>A1</b> Sub <math>x = 12</math> to obtain the correct value of t</p>                                             |                                                |

|             |                                                                       |
|-------------|-----------------------------------------------------------------------|
| <b>ALT</b>  | Using definite integration: $\int_4^{12} (2x + 3)dx = \int_1^T 50 dt$ |
| <b>3(b)</b> |                                                                       |
| <b>M1</b>   | Integrate $\left[ x^2 + 3x \right]_4^{12} = \left[ 50t \right]_1^T$   |
| <b>A1</b>   | Correct integration                                                   |
| <b>A1</b>   | Sub in limits $12^2 + 3(12) - 4^2 - 3(4) = 50T - 50$                  |
| <b>A1</b>   | Obtain correct value                                                  |



| Question Number | Scheme                                                                                                                                                                                                      | Marks |    |
|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|----|
| 2.              | $F \cos \alpha = mg$                                                                                                                                                                                        | M1    | A1 |
|                 | $F \sin \alpha = T$                                                                                                                                                                                         |       | A1 |
|                 | $T = \frac{2mgx}{l}$ or $T = \frac{2mg(AB-l)}{l}$                                                                                                                                                           | M1    |    |
|                 | $\frac{3}{4}mg = \frac{2mgx}{l}$                                                                                                                                                                            | dM1   |    |
|                 | $AB = \frac{11l}{8}$                                                                                                                                                                                        | A1    |    |
|                 |                                                                                                                                                                                                             | (6)   |    |
| Notes           |                                                                                                                                                                                                             |       |    |
| M1              | Resolve vertically or horizontally, correct no. of terms, condone sign errors and sin/cos confusion (or use trig on a right-angled triangle of forces)                                                      |       |    |
| A1              | Correct vertical equation                                                                                                                                                                                   |       |    |
| A1              | Correct horizontal equation (A2 for $T = mg \tan \alpha$ from triangle of forces)                                                                                                                           |       |    |
| M1              | Hooke's Law. Must clearly be an extension and not AB. Since x is not defined in the question, other extensions may be used including $(AB - l)$ or $xl$ where x is found to be the constant $\frac{3}{8}$ . |       |    |
| dM1             | Substitute trig (not necessarily correctly) to produce an equation in 'x' (and l) only, dependent on previous M's and on having two equations.                                                              |       |    |
| A1              | Cao Accept 1.375l , 1.4l, 1.38l                                                                                                                                                                             |       |    |

| Question Number | Scheme                                                                                                                                                                                                                                                   | Marks                                            |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------|
| <b>3(a)</b>     | Slant height, $l = \sqrt{\left(\frac{7a}{4}\right)^2 + (6a)^2} (= \frac{25a}{4})$                                                                                                                                                                        | M1                                               |
|                 | <b>Masses</b><br>Square $16a^2$<br>Circle $\pi\left(\frac{7a}{4}\right)^2$<br>Conical shell $\pi \times \frac{7a}{4} \times \frac{25a}{4}$<br>Total $\left[16a^2 - \pi\left(\frac{7a}{4}\right)^2 + \pi \times \frac{7a}{4} \times \frac{25a}{4}\right]$ | B1 square<br>B1 circle<br>B1ft (shell and total) |
|                 | <b>Distances</b> Square      Circle      Conical shell      Total<br>0              0              2a              : $\bar{x}$                                                                                                                           | B1                                               |
|                 | $\pi \times \frac{7a}{4} \times \frac{25a}{4} \times 2a = \left[16a^2 - \pi\left(\frac{7a}{4}\right)^2 + \pi \times \frac{7a}{4} \times \frac{25a}{4}\right] \bar{x}$                                                                                    | M1 A1                                            |
|                 | $\bar{x} = \frac{175\pi a}{(63\pi + 128)} *$                                                                                                                                                                                                             | A1*                                              |
|                 |                                                                                                                                                                                                                                                          | (8)                                              |
| <b>3(b)</b>     | $\tan \alpha = \frac{2a}{\left(\frac{175\pi a}{(63\pi + 128)}\right)}$                                                                                                                                                                                   | M1                                               |
|                 | $\tan \alpha = \frac{126\pi + 256}{175\pi} \quad (\text{or } \frac{2(63\pi + 128)}{175\pi})$                                                                                                                                                             | A1                                               |
|                 |                                                                                                                                                                                                                                                          | (2)                                              |
|                 |                                                                                                                                                                                                                                                          | (10)                                             |
| <b>Notes</b>    |                                                                                                                                                                                                                                                          |                                                  |
| <b>(a)</b>      |                                                                                                                                                                                                                                                          |                                                  |
| <b>M1</b>       | Use of Pythagoras (unsimplified). May be seen on the diagram.                                                                                                                                                                                            |                                                  |
| <b>B1</b>       | Mass/area of square                                                                                                                                                                                                                                      |                                                  |
| <b>B1</b>       | Mass/area of circle                                                                                                                                                                                                                                      |                                                  |
| <b>B1 ft</b>    | Mass/area of conical shell and total. A common error is to use 6a as slant height, only ft on their calculated slant height. May derive conical shell formula from area of a sector.                                                                     |                                                  |
| <b>B1</b>       | All distances correct                                                                                                                                                                                                                                    |                                                  |
| <b>M1</b>       | Dimensionally correct moments equation. Must have correct number of terms including an attempt to subtract the circle. Condone a slip with an 'a' in one term.                                                                                           |                                                  |
| <b>A1</b>       | Correct equation (no ft)                                                                                                                                                                                                                                 |                                                  |
| <b>A1*</b>      | Given answer correctly obtained. Condone missing brackets from denominator and terms reversed.                                                                                                                                                           |                                                  |
| <b>(b)</b>      |                                                                                                                                                                                                                                                          |                                                  |
| <b>M1</b>       | Allow reciprocal. Must use 2a and given $\bar{x}$ .                                                                                                                                                                                                      |                                                  |
| <b>A1</b>       | Exact fraction required.                                                                                                                                                                                                                                 |                                                  |

| Question Number | Scheme                                                                                                                                  | Marks |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------|-------|
| <b>4(a)</b>     | $a = v \frac{dv}{dx}$                                                                                                                   | M1    |
|                 | $= \frac{3}{2} (2x+1)^{\frac{1}{2}} \times 2 \times (2x+1)^{\frac{3}{2}} = 3(2x+1)^2$                                                   | A1    |
|                 | $3(2x+1)^2 = 243$                                                                                                                       | M1    |
|                 | $x = 4$                                                                                                                                 | A1    |
|                 |                                                                                                                                         | (4)   |
| <b>4(b)</b>     | $(2x+1)^{\frac{3}{2}} = \frac{dx}{dt}$ <b>OR</b> $a = 3v^{\frac{4}{3}} = \frac{dv}{dt}$                                                 | M1 A1 |
|                 | $\int dt = \int (2x+1)^{\frac{3}{2}} dx$ $\int 3dt = \int v^{-\frac{4}{3}} dv$                                                          | M1    |
|                 | $t = -(2x+1)^{\frac{1}{2}} (+C)$ $3t + (C) = -3v^{-\frac{1}{3}}$                                                                        | A1    |
|                 | $t = 0, x = 0 \Rightarrow C = 1$ $t = 0, x = 0 \Rightarrow v = 1 \Rightarrow C = -3$<br>and obtain an equation in $v$ and $t$ only.     | M1    |
|                 | $v = \frac{1}{(1-t)^3}$                                                                                                                 | A1    |
|                 |                                                                                                                                         | (6)   |
|                 |                                                                                                                                         | (10)  |
| <b>Notes</b>    |                                                                                                                                         |       |
| <b>(a)</b>      |                                                                                                                                         |       |
| <b>M1</b>       | Use of $a = v \frac{dv}{dx}$ or $a = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$ . Evidence of differentiation, power decreasing by 1. |       |
| <b>A1</b>       | Should see a product of terms to imply 'use of'.                                                                                        |       |
| <b>A1</b>       | Correct differentiation                                                                                                                 |       |
| <b>M1</b>       | Independent. Use their result from differentiation and put $a = 243$ then solve for $x$                                                 |       |
| <b>A1</b>       | Cao If -5 is seen then it must be rejected or 4 must be clearly identified.                                                             |       |
| <b>(b)</b>      |                                                                                                                                         |       |
| <b>M1</b>       | Use of $v = \frac{dx}{dt}$ to obtain DE in $x$ and $t$ <b>OR</b> Use of $a = \frac{dv}{dt}$ to obtain DE in $v$ and $t$                 |       |
| <b>A1</b>       | Correct equation                                                                                                                        |       |
| <b>M1</b>       | Separate and integrate (evidence of integration, power increasing by 1)                                                                 |       |
| <b>A1</b>       | Correct integration, condone missing $C$                                                                                                |       |
| <b>M1</b>       | Use $t = 0, x = 0$ to obtain a value of $C$ and obtain an equation in $v$ and $t$ only.                                                 |       |
| <b>A1</b>       | Cao Accept $v = (1-t)^{-3}$ or $v = \frac{-1}{(t-1)^3}$ or $v = -(t-1)^{-3}$                                                            |       |
|                 | Note: No marks in (b) for use of $a = 243$                                                                                              |       |

| Question Number | Scheme                                                                                                                                        | Marks |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------------|-------|
| <b>5a)</b>      | $0.5v \frac{dv}{dx} = -\sin 2x$                                                                                                               | M1    |
|                 | $\int 0.5v \, dv = \int -\sin 2x \, dx$                                                                                                       | DM1   |
|                 | $0.25v^2 = \frac{1}{2} \cos 2x (+c)$                                                                                                          | A1    |
|                 | $v^2 = 2 \cos 2x + c$                                                                                                                         |       |
|                 | $x = 0, v = 2 \Rightarrow 4 = 2 + c$                                                                                                          | DM1   |
|                 | $v^2 = 2 \cos 2x + 2 (= 4 \cos^2 x)$                                                                                                          | A1    |
|                 | $v = 2 \cos x$ *                                                                                                                              | A1 *  |
|                 |                                                                                                                                               | (6)   |
| <b>ALT</b>      | <i>Using definite integration</i>                                                                                                             |       |
|                 | $0.5v \frac{dv}{dx} = -\sin 2x$                                                                                                               | M1    |
|                 | $\int_2^v 0.5v \, dv = \int_0^x -\sin 2x \, dx \quad \left( \text{or } \int_0^x \sin 2x \, dx \right)$                                        | DM1   |
|                 | $\left[ 0.25v^2 \right]_2^v = \left[ \frac{1}{2} \cos 2x \right]_0^x \quad \left( \text{or } \left[ -\frac{1}{2} \cos 2x \right]_x^0 \right)$ | A1    |
|                 | $0.25(v^2 - 4) = \frac{1}{2} \cos 2x - \frac{1}{2}$                                                                                           | DM1A1 |
|                 | $v = 2 \cos x$ *                                                                                                                              | A *   |
|                 |                                                                                                                                               | (6)   |
|                 |                                                                                                                                               |       |
|                 |                                                                                                                                               |       |
|                 |                                                                                                                                               |       |
|                 |                                                                                                                                               |       |
|                 |                                                                                                                                               |       |
|                 |                                                                                                                                               |       |

| Question Number | Scheme                                                                                                           | Marks |
|-----------------|------------------------------------------------------------------------------------------------------------------|-------|
| <b>5b)</b>      | $\frac{dx}{dt} = 2 \cos x$                                                                                       | M1    |
|                 | $\int \sec x dx = \int 2 dt$                                                                                     |       |
|                 | $\ln  \sec x + \tan x  = 2t + k$                                                                                 | DM1   |
|                 | $t = 0, x = 0 \ln 1 = 2(0) + k \Rightarrow k = 0$                                                                | A1    |
|                 | $t = \frac{1}{2} \ln  \sec x + \tan x  = \frac{1}{2} \ln \left( \sec \frac{\pi}{4} + \tan \frac{\pi}{4} \right)$ | DM1   |
|                 | $t = \frac{1}{2} \ln (\sqrt{2} + 1) *$                                                                           | A1 *  |
|                 |                                                                                                                  | (5)   |
| <b>ALT</b>      | <i>Using definite integration</i>                                                                                |       |
|                 | $\frac{dx}{dt} = 2 \cos x$                                                                                       | M1    |
|                 | $\int_0^{\frac{\pi}{4}} \sec x dx = \int_0^t 2 dt$                                                               |       |
|                 | $\left[ \ln  \sec x + \tan x  \right]_0^{\frac{\pi}{4}} = [2t]_0^t$                                              | DM1A1 |
|                 | $2t = \ln \left( \sec \frac{\pi}{4} + \tan \frac{\pi}{4} \right)$                                                | DM1   |
|                 | $\therefore t = \frac{1}{2} \ln (\sqrt{2} + 1) .. *$                                                             | A1 *  |
|                 |                                                                                                                  | (5)   |
|                 |                                                                                                                  |       |
|                 |                                                                                                                  | [11]  |

|            |                                                                                                                                                                               |  |
|------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| <b>(a)</b> | <i>Indefinite integration</i>                                                                                                                                                 |  |
| <b>M1</b>  | Equation of motion, with acceleration in the form $v \frac{dv}{dx}$ . Condone sign error.                                                                                     |  |
| <b>DM1</b> | Separate variables to prepare for integration. Depends on the M mark above.                                                                                                   |  |
| <b>A1</b>  | Correct integration. Constant not needed.                                                                                                                                     |  |
| <b>DM1</b> | Substitute $x = 0$ , $v = 2$ to find the constant. Depends on both M marks above.                                                                                             |  |
| <b>A1</b>  | A correct result for $v^2$                                                                                                                                                    |  |
| <b>A1*</b> | <b>Given</b> result reached through use of double angle formula. (Formula need not be shown.).                                                                                |  |
| <b>ALT</b> | <i>Definite integration</i>                                                                                                                                                   |  |
| <b>M1</b>  | Equation of motion, with acceleration in the form $v \frac{dv}{dx}$ . Condone sign error.                                                                                     |  |
| <b>DM1</b> | Separate variables, to prepare for integration. Limits not needed for this mark.<br>Depends on the M mark above.                                                              |  |
| <b>A1</b>  | Correct integration – limits not needed                                                                                                                                       |  |
| <b>DM1</b> | Correct substitution of correct limits in their integrated function. Limits must be “paired” correctly. Depends on both previous M marks in (a) (Formula need not be shown.). |  |
| <b>A1</b>  | Correct expression which can yield $v^2$                                                                                                                                      |  |
| <b>A1*</b> | <b>Given</b> result reached through use of double angle formula. (Formula need not be shown.).                                                                                |  |
| <b>(b)</b> |                                                                                                                                                                               |  |
| <b>M1</b>  | Use of $v = \frac{dx}{dt}$                                                                                                                                                    |  |
| <b>DM1</b> | Correct separation of variables and attempt integration (integral is in the formula book). Depends on first M of (b)<br>Modulus signs may be missing.                         |  |
| <b>A1</b>  | Correct integration and use limits to find correct value for constant.                                                                                                        |  |
| <b>DM1</b> | Substitute $x = \frac{\pi}{4}$ and solve for $t$ . Depends on both previous M marks in (b)                                                                                    |  |
| <b>A1*</b> | Given result reached from fully correct working. (Modulus signs may be missing throughout.).                                                                                  |  |
|            |                                                                                                                                                                               |  |
|            |                                                                                                                                                                               |  |

|            |                                                                                                                                      |  |
|------------|--------------------------------------------------------------------------------------------------------------------------------------|--|
| <b>ALT</b> | <i>Definite integration</i>                                                                                                          |  |
| <b>M1</b>  | Use of $v = \frac{dx}{dt}$                                                                                                           |  |
| <b>DM1</b> | Correct separation of variables and attempt integration. Limits not needed. Depends on first M of (b). Modulus signs may be missing. |  |
| <b>A1</b>  | Correct integration including correct limits.                                                                                        |  |
| <b>DM1</b> | Substitute their limits and solve for $t$ . Depends on both previous M marks in (b)                                                  |  |
| <b>A1*</b> | Given result reached from fully correct working. (Modulus signs may be missing throughout.).                                         |  |

| Question Number | Scheme                                                                                                                                                                                                                                              | Marks      |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| <b>1(a)</b>     | $\frac{2\pi}{\omega} = 4$                                                                                                                                                                                                                           | M1         |
|                 | $\omega = \frac{\pi}{2}$                                                                                                                                                                                                                            | A1         |
|                 | $2 = a \frac{\pi}{2} \cos\left(\frac{\pi}{2} \times 0.5\right) \Rightarrow 2 = a \frac{\pi}{2} \times \frac{1}{\sqrt{2}}$                                                                                                                           | M1         |
|                 | $a = \frac{4\sqrt{2}}{\pi} \text{ m s}^{-2}$                                                                                                                                                                                                        | A1* (4)    |
| <b>1(b)</b>     | $v_{\text{MAX}} = \frac{4\sqrt{2}}{\pi} \times \text{their } \omega$                                                                                                                                                                                | M1         |
|                 | $2\sqrt{2} \text{ (m s}^{-1}\text{)}$                                                                                                                                                                                                               | A1 (2)     |
|                 |                                                                                                                                                                                                                                                     | <b>(6)</b> |
|                 | <b>Notes for question 1</b>                                                                                                                                                                                                                         |            |
| <b>1(a)</b>     | M1 Need to see this equation, as it's a 'show that'. Allow with 4 or $T$ or in a rearranged form.                                                                                                                                                   |            |
|                 | A1 seen                                                                                                                                                                                                                                             |            |
|                 | M1 Complete method to obtain an equation in $a$ only<br>Use of $x = a \sin \omega t$ to find $x$ followed by $v^2 = \omega^2 (a^2 - x^2)$ may be seen.<br>(Use of $v = \pm a\omega \sin \omega t$ scores M0 (this implies $t = 0$ at an end-point.) |            |
|                 | A1* Correct answer correctly obtained                                                                                                                                                                                                               |            |
| <b>1(b)</b>     | M1 Use of $a\omega$ with the given value of $a$                                                                                                                                                                                                     |            |
|                 | A1 Allow 2.8 or better. Ignore units but must be positive.                                                                                                                                                                                          |            |

| Question Number             | Scheme                                                                                                                                | Marks       |
|-----------------------------|---------------------------------------------------------------------------------------------------------------------------------------|-------------|
| <b>2(a)</b>                 | $\frac{1}{\sqrt{(2x+1)}} = \frac{1}{3} \Rightarrow x = 4$                                                                             | M1A1        |
|                             | $a = v \frac{dv}{dx} = \frac{1}{\sqrt{(2x+1)}} \times -\frac{1}{2} \times 2(2x+1)^{-\frac{3}{2}} \quad \left( = -(2x+1)^{-2} \right)$ | M1A1        |
|                             | $a = (-)\frac{1}{81}$                                                                                                                 | A1          |
|                             | $\frac{2}{81}$ (N), 0.025 or better                                                                                                   | A1 (6)      |
| <b>2(b)</b>                 | $t = \int \sqrt{(2x+1)} \, dx$                                                                                                        | M1          |
|                             | $t = \frac{2}{3} \times \frac{1}{2} (2x+1)^{\frac{3}{2}} + (C)$                                                                       | A1          |
|                             | Use of $t = 0, x = 0$ to obtain a value for $C$ ( $= -\frac{1}{3}$ )                                                                  | M1          |
|                             | Substitute $x = 7.5$                                                                                                                  | dM1         |
|                             | $t = 21$                                                                                                                              | A1 (5)      |
|                             |                                                                                                                                       | <b>(11)</b> |
| <b>Notes for question 2</b> |                                                                                                                                       |             |
| <b>2(a)</b>                 | M1 for putting $v = \frac{1}{3}$ and solving for $x$ or e.g. $(2x+1) = 9$                                                             |             |
|                             | A1 for $x = 4$ or                                                                                                                     |             |
|                             | M1 for use of $v \frac{dv}{dx}$ with clear attempt at differentiation. (Power $-\frac{3}{2}$ needed)                                  |             |
|                             | A1 for a correct unsimplified expression for $a$ in terms of $x$ only                                                                 |             |
|                             | A1 for a correct value for $a$ (ignore sign)                                                                                          |             |
|                             | A1 cao (must be positive)                                                                                                             |             |
| <b>2(b)</b>                 | M1 for use of $\frac{dx}{dt} = \frac{1}{\sqrt{(2x+1)}}$ , separate variables and attempt to integrate.                                |             |
|                             | $k(2x+1)^{\frac{3}{2}}$ should be seen                                                                                                |             |
|                             | A1 for correct unsimplified integration, $C$ not needed.                                                                              |             |
|                             | If definite integration used, ignore limits for these 2 marks                                                                         |             |
|                             | M1 for use of initial conditions to find a value for $C$                                                                              |             |
|                             | dM1 dependent on first M1, for substituting in $x = 7.5$ and evaluating                                                               |             |
|                             | For definite integration award M1 for substitution of the lower limits and DM1 for substitution of the upper limits.                  |             |
|                             | A1 cao                                                                                                                                |             |
|                             |                                                                                                                                       |             |

| Question Number | Scheme                                                 | Marks      |
|-----------------|--------------------------------------------------------|------------|
| <b>5(a)</b>     | $0.5v \frac{dv}{dx} = -\frac{2}{x^3}$                  | M1         |
|                 | $\int v \, dv = \int -\frac{4}{x^3} \, dx$             |            |
|                 | $\frac{v^2}{2} = \frac{2}{x^2} \quad (+k)$             | DM1A1      |
|                 | $v^2 = \frac{4}{x^2} + 2k$                             |            |
|                 | $v = 3, x = 1 \Rightarrow 2k = 5$                      | DM1        |
|                 | $v^2 = \frac{4}{x^2} + 5 \quad *$                      | A1 *       |
|                 |                                                        | <b>(5)</b> |
|                 | Alt for first 2 marks using Energy                     |            |
|                 | Use of $W = \int F dx$                                 | M1         |
|                 | Attempt integration <b>and</b> equate to change in KE. | M1         |
| <b>(b)</b>      | $\frac{dx}{dt} = \sqrt{\frac{4}{x^2} + 5}$             | M1         |
|                 | $\int \frac{x}{\sqrt{5x^2 + 4}} \, dx = \int dt$       | M1A1       |
|                 | $\frac{1}{5} \sqrt{5x^2 + 4} = t \quad (+k_1)$         | DM1A1      |
|                 | $x = 1, t = 1 \Rightarrow k_1 = -\frac{2}{5}$          | DM1        |
|                 | $t = \frac{2 + \sqrt{5x^2 + 4}}{5}$                    | A1         |
|                 |                                                        | <b>(7)</b> |

**(a)**

**M1** Dimensionally correct equation of motion. Acceleration must be in form  $v \frac{dv}{dx}$ . Condone missing minus sign.

**DM1** Separate variables **and** attempt integration. Condone missing minus sign.

**A1** Correct integrals. Condone missing constant.

**DM1** Use  $v = 3, x = 1$  to find constant of integration. Dependent on both previous M marks.

**A1\*** Reach given result with no errors.

Alt – Definite Integration/Energy Work.

**M1** – Equate change in KE to Integral for WD. Integration not needed for this mark and condone inconsistent signs.

$$\frac{1}{2} \times 2v^2 - \frac{1}{2} \times 2 \times 3^2 = \int (\pm) \frac{2}{x^3} dx$$

**DM1** Attempt integration. Condone inconsistent signs.

**A1** Correct integration, with consistent signs for KE and WD.

**DM1** Substitute in limits.

**A1** Reach given result with no errors.

**(b)**

**M1** Use  $v = \frac{dx}{dt}$  to form differential equation in  $x$  and  $t$ .

**M1** Separate variables and to produce functions ready to be integrated.

**A1** Correct integrands, written in a form that can be integrated.

**DM1** Valid attempt to integrate their expression. If they have an incorrect expression, the integration must not be significantly simplified. Dependent on first 2 M marks.

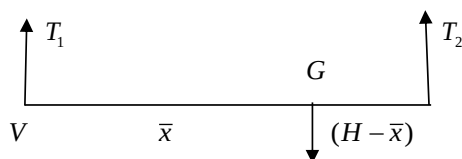
**A1** Correct integration. Condone missing constant.

**DM1** Use  $x = 1, t = 1$  to find constant of integration. Dependent on all 3 M marks.

**A1** Correct result. (cso)



| Question Number | Scheme                                                                                                                                                                                                                                                                     | Marks |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| 2.              | $T = mg \cos \theta$                                                                                                                                                                                                                                                       | M1A1  |
|                 | $T = \frac{2mg \left( \frac{21}{10}a - ka \right)}{ka}$                                                                                                                                                                                                                    | M1A1  |
|                 | $\frac{4}{5}mg = \frac{2mg \left( \frac{21}{10}a - ka \right)}{ka}$                                                                                                                                                                                                        | dM1   |
|                 | $k = \frac{3}{2}$ or 1.5                                                                                                                                                                                                                                                   | A1    |
|                 |                                                                                                                                                                                                                                                                            | (6)   |
|                 | <b>Notes</b>                                                                                                                                                                                                                                                               |       |
| <b>M1</b>       | Resolve parallel to the string, correct no. of terms, condone sign errors and sin/cos confusion (or resolve in two directions and eliminate the unknown force or use trig on a right-angled triangle of forces) to give an equation in $T$ , $mg$ and $\theta$ <b>only</b> |       |
| <b>A1</b>       | Correct equation. Trig does not need to be substituted.                                                                                                                                                                                                                    |       |
| <b>M1</b>       | Use Hooke's Law with correct structure.                                                                                                                                                                                                                                    |       |
| <b>A1</b>       | Correct equation                                                                                                                                                                                                                                                           |       |
| <b>dM1</b>      | Substitute trig and eliminate $T$ to produce equation in $k$ only, dependent on previous M's.<br>If $x$ is used for extension, should see $x = \frac{2k}{5}a \rightarrow ka + \frac{2k}{5}a = \frac{21}{10}a$                                                              |       |
| <b>A1</b>       | cao                                                                                                                                                                                                                                                                        |       |
| <b>ALT 1</b>    | <b>First M1A1</b>                                                                                                                                                                                                                                                          |       |
| <b>M1A1</b>     | Complete method to form an equation in $T$ and $\theta$<br><br>Vert: $mg = T \cos \theta + F \sin \theta$<br>Horiz: $F \cos \theta = T \sin \theta$<br>Eliminate $F$ eg $\frac{\sin \theta}{\cos \theta} = \frac{mg - T \cos \theta}{T \sin \theta}$                       |       |

| Question Number             | Scheme                                                                                                                                                                                                                                           | Marks     |
|-----------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|
| <b>3(a)</b>                 | Mass: $\frac{1}{3}\pi r^2 H$ $\frac{1}{3}\pi r^2 h$ $\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h$                                                                                                                                                | B1        |
|                             | Distance from V: $\frac{3H}{4}$ $H - \frac{1}{4}h$ $\bar{x}$                                                                                                                                                                                     | B1        |
|                             | $\frac{1}{3}\pi r^2 H \times \frac{3H}{4} - \frac{1}{3}\pi r^2 h \times \left(H - \frac{1}{4}h\right) = \left(\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h\right) \bar{x}$                                                                        | M1A1      |
|                             | $\bar{x} = \frac{(3H-h)(H-h)}{4(H-h)} = \frac{1}{4}(3H-h) *$                                                                                                                                                                                     | A1*       |
|                             |                                                                                                                                                                                                                                                  | (5)       |
| <b>3(b)</b>                 |                                                                                                                                                                 |           |
|                             | $M(G), T_1 \bar{x} = T_2 (H - \bar{x})$                                                                                                                                                                                                          | M1        |
|                             | $\frac{T_1}{T_2} = \frac{H - \frac{1}{4}(3H-h)}{\frac{1}{4}(3H-h)}$                                                                                                                                                                              | A1        |
|                             | $\frac{T_1}{T_2} = \frac{H+h}{3H-h}$                                                                                                                                                                                                             | A1<br>(3) |
|                             |                                                                                                                                                                                                                                                  | (8)       |
|                             | <b>Notes</b>                                                                                                                                                                                                                                     |           |
| <b>3(a)</b>                 |                                                                                                                                                                                                                                                  |           |
| <b>B1</b>                   | Three correct mass ratios: $H$ $h$ $H-h$                                                                                                                                                                                                         |           |
| <b>B1</b>                   | Three correct distances (Allow if measured from some other axis)                                                                                                                                                                                 |           |
| <b>M1</b>                   | Moments equation with correct no. of terms, dim correct. Condone addition, treat as a sign error. Must be working with solids eg not a conical shell.                                                                                            |           |
| <b>A1</b>                   | Correct unsimplified equation (For their axis)                                                                                                                                                                                                   |           |
| <b>A1*</b>                  | Given answer correctly obtained, including cancelling $(H-h)$ . A factorised expression should be seen to reach the GIVEN answer.                                                                                                                |           |
| <b>3(b)</b>                 |                                                                                                                                                                                                                                                  |           |
| <b>M1</b>                   | Complete method to obtain an equation in $H, h, T_1$ and $T_2$ only. Must be dimensionally correct. May take moments about G and substitute for $\bar{x}$ . Alternatively may use two equations, eliminate weight and substitute for $\bar{x}$ . |           |
| <b>A1</b>                   | Correct equation in $T_1, T_2, H$ and $h$ only                                                                                                                                                                                                   |           |
| <b>A1</b>                   | Correct answer. The question asks for simplest form.                                                                                                                                                                                             |           |
| <b>3(a)</b><br><b>ALT 1</b> | <b>Distances measured from circular face</b>                                                                                                                                                                                                     |           |
| <b>B1</b>                   | Mass: $\frac{1}{3}\pi r^2 H$ $\frac{1}{3}\pi r^2 h$ $\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h$                                                                                                                                                |           |
| <b>B1</b>                   | Dist from Circular face: $\frac{H}{4}$ $\frac{1}{4}h$ $d$                                                                                                                                                                                        |           |
| <b>M1A1</b>                 | $\frac{1}{3}\pi r^2 H \times \frac{H}{4} - \frac{1}{3}\pi r^2 h \times \frac{1}{4}h = \left(\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h\right) d$                                                                                                |           |

| Question Number       | Scheme                                                                                                                                                                                                                                          | Marks |
|-----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| <b>A1*</b>            | Leads to $d = \frac{H+h}{4}$ which must be subtracted from $H$ to reach the required distance:<br>$\bar{x} = H - \frac{H+h}{4} = \frac{1}{4}(3H-h)$ *                                                                                           |       |
| <b>3(b)<br/>ALT 1</b> |                                                                                                                                                                                                                                                 |       |
| <b>M1</b>             | May use two equations and eliminate weight/mass ratio to find an equation in terms of $T_1$ , $T_2$ , $H$ and $h$ only.<br><br>Vert $T_1 + T_2 = W$<br>M(V), $W \bar{x} = T_2 H$<br>M(circular face), $T_1 H = W(H - \bar{x})$<br>Eliminate $W$ |       |

| Question Number | Scheme                                                                                                                                                                                    | Marks |
|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| <b>4(a)</b>     | $R \cos \alpha = mg$                                                                                                                                                                      | M1 A1 |
|                 | $R \sin \alpha = \frac{m \left( \frac{1}{4} gr \right)}{r}$ <p><b>OR:</b> <math>mg \sin \alpha = \frac{m \left( \frac{1}{4} gr \right)}{r} \cos \alpha</math> M2 A2</p>                   | M1A1  |
|                 | $\tan \alpha = \frac{1}{4}^*$                                                                                                                                                             | A1*   |
|                 |                                                                                                                                                                                           | (5)   |
| <b>4(b)</b>     | Vert equil: $S \cos \alpha - F \sin \alpha = mg$<br>Perp N2L: $S - mg \cos \alpha = \frac{mV^2}{r} \sin \alpha$                                                                           | M1A1  |
|                 | N2L towards O: $S \sin \alpha + F \cos \alpha = \frac{mV^2}{r}$<br>Parallel N2L: $F + mg \sin \alpha = \frac{mV^2}{r} \cos \alpha$                                                        | M1A1  |
|                 | $F = \mu S$                                                                                                                                                                               | B1    |
|                 | Eliminate $F$ , sub for trig and solve for $V$ in terms of $\mu$ , $r$ and $g$ .                                                                                                          | dM1   |
|                 | $V = \sqrt{rg \frac{(1+4\mu)}{(4-\mu)}} \text{ oe}$                                                                                                                                       | A1    |
|                 |                                                                                                                                                                                           | (7)   |
|                 |                                                                                                                                                                                           | (12)  |
|                 | <b>Notes</b>                                                                                                                                                                              |       |
| <b>4(a)</b>     |                                                                                                                                                                                           |       |
|                 | Note: For use of $\theta$ instead of $\alpha$ in (a) penalise only the last mark in (a). The maximum score is M1A1M1A1A0*                                                                 |       |
| <b>M1</b>       | Resolve vertically correct no. of terms, condone sign errors and sin/cos confusion.                                                                                                       |       |
| <b>A1</b>       | Correct equation                                                                                                                                                                          |       |
| <b>M1</b>       | Equation of motion horizontally correct no. of terms, condone sign errors and sin/cos confusion. $V$ does not need to be substituted. Allow $r\omega^2$ for acceleration but not ' $a$ '. |       |
| <b>A1</b>       | Correct equation. May still contain $V$ and either form of acceleration (circular).                                                                                                       |       |
| <b>OR</b>       | M2 Equation of motion down the plane correct no. of terms, condone sign errors and sin/cos confusion.<br>A1 Correct equation with at most one error<br>A1 Correct equation                |       |
| <b>A1*</b>      | Correctly obtain given answer, written exactly.                                                                                                                                           |       |
| <b>4(b)</b>     |                                                                                                                                                                                           |       |
| <b>M1</b>       | Resolve vertically or equation of motions perpendicular. Correct no. of terms, condone sign errors and sin/cos confusion. M0 if $R$ from (a) is used.                                     |       |

| Question Number | Scheme                                                                                                                                             | Marks |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| <b>A1</b>       | Correct equation                                                                                                                                   |       |
| <b>M1</b>       | Equation of motion horizontally or parallel to slope. Correct no. of terms, condone sign errors and sin/cos confusion. M0 if $R$ from (a) is used. |       |
| <b>A1</b>       | Correct equation                                                                                                                                   |       |
| <b>B1</b>       | $F = \mu S$ seen where $S$ is the normal reaction in (b).                                                                                          |       |
| <b>dM1</b>      | Eliminate $F$ , sub for trig and solve for $V$ in terms of $\mu$ , $r$ and $g$ . Dependent on both previous M marks.                               |       |
| <b>A1</b>       | Correct answer.                                                                                                                                    |       |



| Question number | Scheme                                                                                                                                                                                                                                                 | Marks |
|-----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| <b>5(a)</b>     | $v \frac{dv}{dx} = -\frac{gR^2}{x^2}$                                                                                                                                                                                                                  | M1 A1 |
|                 | $\int v \, dv = -\int \frac{gR^2}{x^2} \, dx$ or $\frac{1}{2} V^2 = -\int \frac{gR^2}{x^2} \, dx$<br>Or the Energy alternative below                                                                                                                   | M1    |
|                 | $\frac{1}{2} v^2 = \frac{gR^2}{x} + C$                                                                                                                                                                                                                 | A1    |
|                 | Use of $x = R, v = U$ to find $C$ ( $C = \frac{1}{2} u^2 - gR$ )                                                                                                                                                                                       | M1    |
|                 | $v^2 = \frac{2gR^2}{x} + U^2 - 2gR$ *                                                                                                                                                                                                                  | A1*   |
|                 |                                                                                                                                                                                                                                                        | (6)   |
| <b>5(b)</b>     | $\frac{1}{4} gR = \frac{2gR^2}{x} + gR - 2gR$                                                                                                                                                                                                          | M1    |
|                 | $x = \frac{8R}{5}$                                                                                                                                                                                                                                     | A1    |
|                 | $AB = \frac{3R}{5}$ oe                                                                                                                                                                                                                                 | A1    |
|                 |                                                                                                                                                                                                                                                        | (3)   |
| <b>5(c)</b>     | Correct statement regarding $\frac{2gR^2}{x}$ for example<br><ul style="list-style-type: none"> <li><math>\frac{2gR^2}{x} &gt; 0</math> for <math>x \geq R</math></li> <li><math>x \rightarrow \infty, \frac{2gR^2}{x} \rightarrow 0</math></li> </ul> | M1    |
|                 | Correct reasoning.<br><ul style="list-style-type: none"> <li><math>U^2 - 2gR = 0</math></li> <li><math>U^2 \rightarrow 2gR</math></li> <li><math>U^2 \geq 2gR</math></li> </ul>                                                                        | dM1   |
|                 | $U_{\text{MIN}} = \sqrt{2gR}$                                                                                                                                                                                                                          | A1    |
|                 |                                                                                                                                                                                                                                                        | (3)   |
|                 |                                                                                                                                                                                                                                                        | (12)  |
| <b>Notes</b>    |                                                                                                                                                                                                                                                        |       |
| <b>5(a)</b>     |                                                                                                                                                                                                                                                        |       |
| <b>M1</b>       | Equation with or without -ve sign and any derivative form for the acceleration                                                                                                                                                                         |       |
| <b>A1</b>       | Correct equation with -ve sign                                                                                                                                                                                                                         |       |
| <b>M1</b>       | Separate variables and clear attempt to integrate acceleration in terms of $v$ and $x$ .                                                                                                                                                               |       |
| <b>A1</b>       | Correct equation; allow without $C$                                                                                                                                                                                                                    |       |
| <b>M1</b>       | Use of initial conditions or limits                                                                                                                                                                                                                    |       |
| <b>A1*</b>      | Given answer correctly obtained                                                                                                                                                                                                                        |       |

|               |                                                                                                                                                                       |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>5(b)</b>   |                                                                                                                                                                       |
| <b>M1</b>     | Substitution of $v^2$ and $U^2$ into (a) to produce a correct equation                                                                                                |
| <b>A1</b>     | Correct value of $x$                                                                                                                                                  |
| <b>A1</b>     | cao                                                                                                                                                                   |
| <b>5(c)</b>   |                                                                                                                                                                       |
| <b>M1</b>     | Correct reasoning for the term $\frac{2gR^2}{x}$<br>Accept $x = \infty$ , $x \rightarrow \infty$ , $\frac{2gR^2}{x} = 0$                                              |
| <b>dM1</b>    | Dependent on previous M. Correct reasoning leading to correct equation or inequality.                                                                                 |
| <b>A1</b>     | cs0                                                                                                                                                                   |
| <b>ALT 5a</b> | <b>Energy approach must use integration</b>                                                                                                                           |
| <b>M1 A1</b>  | Energy equation with variable force.<br>The sign may be missing for the M mark.<br>$\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = \int F \, dx = \int -\frac{mgR^2}{x^2} \, dx$ |
| <b>M1 A1</b>  | Clear attempt to integrate. Limits may be missing or incorrect.<br>$\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = \left[ \frac{mgR^2}{x} \right]_R^x$                           |
| <b>M1</b>     | Correct limits substituted the right way round.<br>$\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = \frac{mgR^2}{x} - \frac{mgR^2}{R}$                                            |
|               | $v^2 - U^2 = \frac{2gR^2}{x} - 2gR$                                                                                                                                   |
| <b>A1*</b>    | $v^2 = \frac{2gR^2}{x} + U^2 - 2gR$                                                                                                                                   |