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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Elastic Strings & Springs (M3.2)

International A Level Mathematics · M3: Mechanics 3 (WME03)

31 questions · 356 marks · Mark schemes included

Past papers January 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

31 questions · 356 marks · 1 sub-topic

SUB-TOPIC M3.2a

HOOKE'S LAW & ELASTIC ENERGY

Sub-topic M3.2a · 31 questions · 356 marks · lowest tariff first



Question 1 continued

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(Total for Question 1 is 6 marks)



(Total for Question 2 is 6 marks)



2.

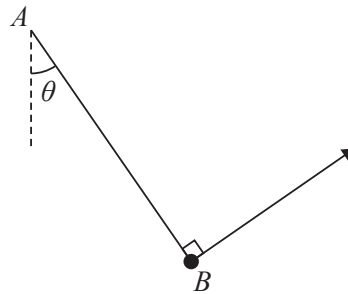


Figure 2

A light elastic string AB has modulus of elasticity $2mg$ and natural length ka , where k is a constant.

The end A of the elastic string is attached to a fixed point. The other end B is attached to a particle of mass m . The particle is held in equilibrium, with the elastic string taut, by a force that acts in a direction that is perpendicular to the string. The line of action of the force and the elastic string lie in the same vertical plane. The string makes an angle θ with the downward vertical at A , as shown in Figure 2.

Given that the length $AB = \frac{21}{10}a$ and $\tan \theta = \frac{3}{4}$, find the value of k .

(6)



Question 2 continued

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Question 2 continued

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Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 6 marks)



Question 3 continued

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(Total 7 marks)

Q3



3.

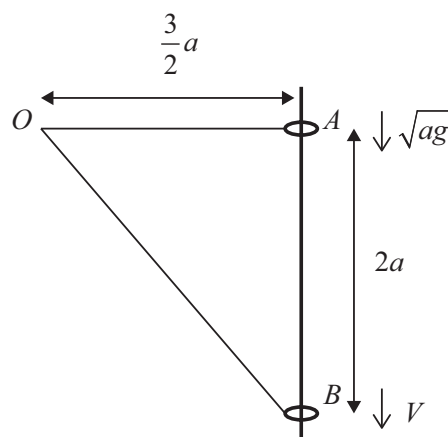


Figure 2

A light elastic string has natural length a and modulus of elasticity mg .

One end of the elastic string is attached to a fixed point O .

The other end is attached to a small smooth ring of mass m .

The ring is threaded on a fixed smooth vertical pole which is a distance $\frac{3}{2}a$ from O .

Initially, the ring is held at the point A on the pole with the elastic string horizontal.

The ring is then projected vertically downwards with speed $\sqrt{ag}$ and reaches the point B , where $AB = 2a$, with speed V , as shown in Figure 2.

Air resistance is assumed to be negligible.

Find V in terms of a and g .

(7)





(Total for Question 3 is / marks)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 7 marks)



Question 2 continued

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Q2

(Total 8 marks)



Question 1 continued

(Total for Question 1 is 8 marks)



4.

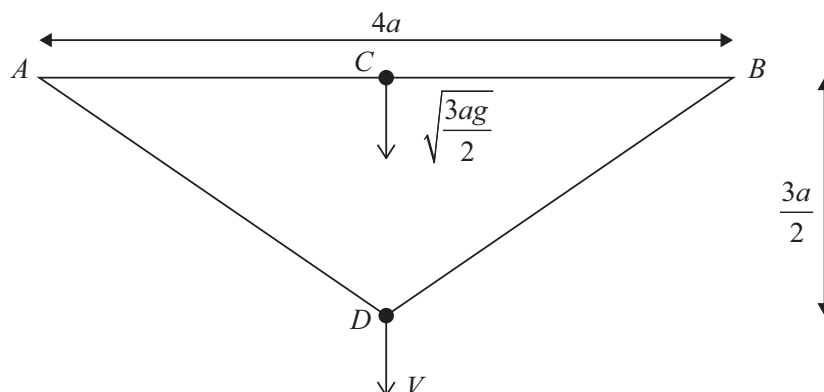


Figure 3

A light elastic string has natural length $3a$ and modulus of elasticity mg . One end of the string is attached to the point A and the other end is attached to the point B , where AB is horizontal and $AB = 4a$.

A particle P of mass m is attached to the midpoint of the string. The particle P is held at rest at C , the midpoint of AB .

The particle P is then projected **vertically downwards from C** with speed $\sqrt{\frac{3ag}{2}}$

At the instant when P reaches the point D , where $CD = \frac{3a}{2}$, the speed of P is V , as shown in Figure 3.

- (a) Show that the elastic energy stored in the string increases by $\frac{1}{2}mga$, as P moves downwards from C to D .

(3)

Air resistance is modelled as a constant force of magnitude $\frac{1}{5}mg$

Using the model and the work-energy principle,

- (b) find V in terms of a and g .

(5)





Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

(Total for Question 4 is 8 marks)







(Total for Question 2 is 5 marks)



2. A light elastic string AB has natural length $3a$ and modulus of elasticity $\frac{20mg}{7}$

When the string is unstretched, two particles, each of mass m , are attached to the string, one at P , where $AP = a$ and the other at Q , where $AQ = 2a$.

The end A of the string is then attached to a point X on a horizontal ceiling and the end B is attached to another point Y on the ceiling, where $XY > 3a$.

The particles hang at rest in equilibrium and the two portions of the string, XP and YQ , both make an angle θ with the horizontal, as shown in Figure 1.

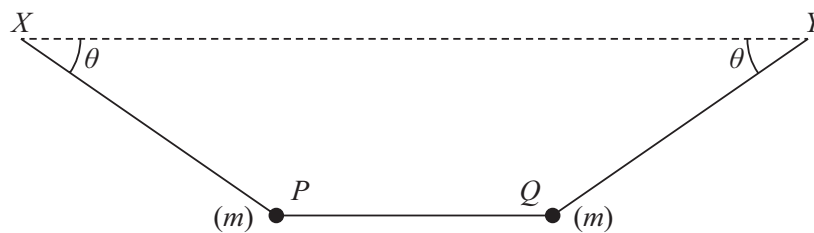


Figure 1

Given that $\tan \theta = \frac{3}{4}$, find XY in terms of a .

(10)

Question 2 continued

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Question 2 continued

Lined area for writing the answer to Question 2 continued.



Question 2 continued

(Total for Question 2 is 10 marks)



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Q4

Question 7 continued

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Question 7 continued

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Q7

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TOTAL FOR PAPER: 75 MARKS

END



Question 4 continued

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Question 4 continued

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(Total 11 marks)

Q4



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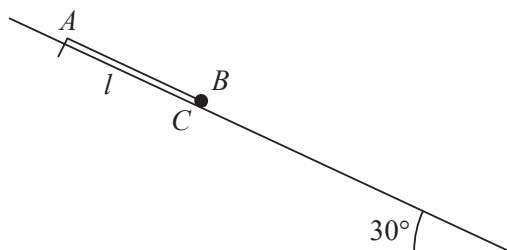


Figure 2

One end of a light elastic string, of natural length l and modulus of elasticity λ , is fixed to a point A on a smooth plane. The plane is inclined at 30° to the horizontal.

A small ball B of mass m is attached to the other end of the elastic string. Initially, B is held at rest at the point C on the plane with the elastic string lying along a line of greatest slope of the plane.

The point C is below A and $AC = l$, as shown in Figure 2.

The ball is released and comes to instantaneous rest at a point D on the plane.

The points A , C and D all lie along a line of greatest slope of the plane and $AD = \frac{5l}{4}$

The ball is modelled as a particle and air resistance is modelled as being negligible.

Using the model,

(a) show that $\lambda = 4mg$

(b) find, in terms of g and l , the greatest speed of B as it moves from C to D (7)

Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 11 marks)



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total 12 marks)

Q3
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Question 6 continuedLeave
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Question 6 continued

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(Total 13 marks)

Q6	



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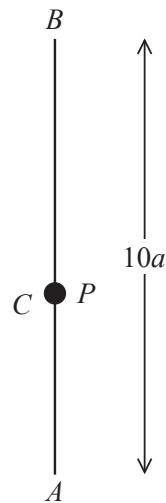


Figure 6

The fixed points, A and B , are a distance $10a$ apart, with B vertically above A .

One end of a light elastic string, of natural length $2a$ and modulus of elasticity $2mg$, is attached to a particle P of mass m and the other end is attached to A .

One end of another light elastic string, of natural length $4a$ and modulus of elasticity $6mg$, is attached to P and the other end is attached to B .

The particle P rests in equilibrium at the point C , as shown in Figure 6.

- (a) Show that each string has an extension of $2a$. (5)

The particle P is now pulled down vertically, so that it is a distance a below C and then released from rest.

- (b) Show that in the subsequent motion, P performs simple harmonic motion. (4)

- (c) Find, in terms of a and g , the speed of P when it is a distance $\frac{7}{2}a$ above A . (4)

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Question 5 continued

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Question 5 continued

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(Total 13 marks)

Q5



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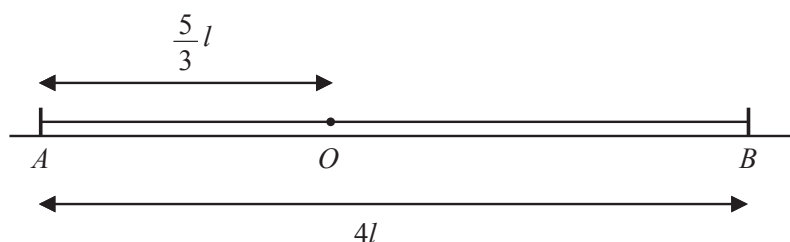


Figure 7

Two points A and B lie on a smooth horizontal table where $AB = 4l$.

A particle P of mass m is attached to one end of a light elastic spring of natural length l and modulus of elasticity $2mg$. The other end of the spring is attached to A . The particle P is also attached to one end of another light elastic spring of natural length l and modulus of elasticity mg . The other end of the spring is attached to B .

The particle P rests in equilibrium on the table at the point O , where $AO = \frac{5}{3}l$, as shown in Figure 7.

The particle P is moved a distance $\frac{1}{2}l$ along the table, from O towards A , and released from rest.

- (a) Show that P moves with simple harmonic motion of period T , where

$$T = 2\pi\sqrt{\frac{l}{3g}} \quad (6)$$

- (b) Find, in terms of l and g , the speed of P as it passes through O .

(1)

- (c) Find, in terms of g , the maximum acceleration of P .

(1)

- (d) Find the exact time, in terms of l and g , from the instant when P is released from rest to the instant when P is first moving with speed $\frac{3}{4}\sqrt{gl}$

(5)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

(Total for Question 7 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS



6. A particle P of mass 0.4 kg is attached to one end of a light elastic string, of natural length 0.8 m and modulus of elasticity 0.6 N . The other end of the string is fixed to a point A on a rough horizontal table. The coefficient of friction between P and the table is $\frac{1}{7}$

Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



(Total for Question 6 is 14 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 14 marks)



Question 6 continued

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Question 6 continued

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(Total 15 marks)

Q6



Question 6 continued

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Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 15 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 15 marks)



Question 7 continued

Lined area for writing the answer to Question 7.







Question 7 continued

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(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Handwriting practice area with horizontal lines.

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS





Question 7 continued

Question 7 continued

Lined area for writing the answer to Question 7.



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks	
2.	$F \cos \alpha = mg$	M1	A1
	$F \sin \alpha = T$		A1
	$T = \frac{2mgx}{l}$ or $T = \frac{2mg(AB-l)}{l}$	M1	
	$\frac{3}{4}mg = \frac{2mgx}{l}$	dM1	
	$AB = \frac{11l}{8}$	A1	
		(6)	
Notes			
M1	Resolve vertically or horizontally, correct no. of terms, condone sign errors and sin/cos confusion (or use trig on a right-angled triangle of forces)		
A1	Correct vertical equation		
A1	Correct horizontal equation (A2 for $T = mg \tan \alpha$ from triangle of forces)		
M1	Hooke's Law. Must clearly be an extension and not AB. Since x is not defined in the question, other extensions may be used including $(AB - l)$ or xl where x is found to be the constant $\frac{3}{8}$.		
dM1	Substitute trig (not necessarily correctly) to produce an equation in 'x' (and l) only, dependent on previous M's and on having two equations.		
A1	Cao Accept 1.375l , 1.4l, 1.38l		



Question Number	Scheme	Marks
1.	$\int_0^3 \sqrt{x+1} dx$	M1
	$= \frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_0^3$	A1
	$\frac{\int_0^3 \frac{1}{2} (\sqrt{x+1})^2 dx}{\int_0^3 \sqrt{x+1} dx}$ or $\frac{\int_0^3 \frac{1}{2} (x+1) dx}{\int_0^3 \sqrt{x+1} dx}$	M1
	$= \frac{\frac{1}{2} \left[\frac{1}{2} x^2 + x \right]_0^3}{\frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_0^3}$ or $\frac{\frac{1}{2} \left[\frac{1}{2} (x+1)^2 \right]_0^3}{\frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_0^3}$	A1
	$= \frac{45}{56}$ (0.80 or better)	A1 (5)
		(5)
	Notes	
M1	Use of $\int_0^3 \sqrt{x+1} dx$. Limits not needed. Accept $k \times \int_0^3 \sqrt{x+1} dx$ where k is a constant. To give the mark for 'use' we must see an attempt at integration. An attempt at integration is seen when the powers increase by 1.	
A1	Correct integrated expression with correct limits	
M1	Use of $\frac{\int_0^3 \frac{1}{2} (\sqrt{x+1})^2 dx}{\int_0^3 \sqrt{x+1} dx}$. Limits not needed. The formula must be correct but allow a constant multiple if it appears on both numerator and denominator. We must see the correct formula and an attempt at integrating the numerator	
A1	Correct integrated expression for the numerator in the correct formula with correct limits.	
A1	Correct answer. This question comes with a calculator warning: the correct answer must come from integrated expressions ie both previous A's must have been awarded. Numerical substitution does not need to be seen.	

Question Number	Scheme	Marks
2(a)	$\cos \theta = \frac{4}{5}$	B1
	$2T \cos \theta = 12$	M1A1
	$2T \left(\frac{4}{5} \right) = 12 \Rightarrow T = \frac{60}{8} = 7.5 \text{ (N)}$	A1
		(4)
(b)	$\text{Ext} = 2(2.5) - 3 = 2 \text{ m}$	
	$7.5 = \frac{\lambda \times 2}{3} \Rightarrow \lambda = 11.25 \text{ (N) }^*$	M1A1*
		(2)
(c)	$\text{EPE} = \frac{11.25 \times 2^2}{2 \times 3} = 7.5 \text{ (J)}$	M1A1
		(2)
		[8]

(a)**B1** $\cos \theta = \frac{4}{5}$ seen or implied.**M1** Resolving vertically, with 2 equal tensions (implied) and weight.**A1** Correct equation.**A1** $T = \frac{15}{2} = 7.5 \text{ N}$ accept any equivalent fraction, since g not used.**(b)****M1** Use of Hooke's Law with their tension to form equation in λ . Must be using natural length of 3 (or 1.5 for half string), but condone incorrect extension for M mark.**A1*** 11.25 or any equivalent fraction.**(c)****M1** Attempt at EPE in equilibrium position. Must have same extension as (b). Condone missing half in EPE formula. If using half strings, then they must include the EPE of both strings. Must be using natural length of 3 (or 1.5 for half string). Allow if an embedded term in an energy equation.**A1** 7.5 J. Accept any equivalent. Must be a clear answer (not embedded).

Question Number	Scheme	Marks
3	$\Delta\text{GPE} = 2g \times 0.7 \sin 30 (= 6.86)$	B1
	For a correct EPE term	B1
	$WD = 2g \cos 30 \times 0.3 \times 0.7 (= 3.56)$	M1A1
	$\frac{12(0.5)^2}{2 \times 0.8} + 6.86 = \frac{12(0.2)^2}{2 \times 0.8} + 0.7 \times 5.09 + \frac{1}{2} \times 2v^2$	M1A1
	$v^2 = 4.87 \Rightarrow v = 2.21 (\text{ms}^{-1}) \quad (2.2)$	A1
		[7]

B1 Correct unsimplified change in GPE.

B1 For a correct EPE term $\frac{12(0.5)^2}{2 \times 0.8}$ **or** $\frac{12(0.2)^2}{2 \times 0.8}$

M1 Complete method to find Work Done.

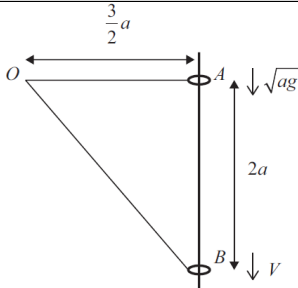
A1 Correct unsimplified expression for Work Done.

M1 Forming an energy equation. Must have 2 EPE terms, change in GPE, WD and KE. Must be dimensionally correct, but follow through their GPE and WD and condone missing half in EPE terms.

A1 Fully correct (unsimplified) equation.

A1 2.21 or 2.2 **cao**

Note: answers using constant acceleration score no marks.

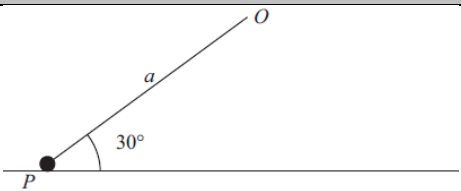
Question Number	Scheme	Marks
3		
	Attempt to find final extension: $\sqrt{\left(\left(\frac{3a}{2}\right)^2 + (2a)^2\right)} - a$	M1
	Method to find at least one expression for EPE	M1
	Two correct expressions for EPE (final and initial) $\frac{mg}{2a} \left(\frac{3a}{2}\right)^2, \quad \frac{mg}{2a} \left(\frac{a}{2}\right)^2$	A1
	GPE Loss = $mg \times 2a$	B1
	Use of conservation of mechanical energy	M1
	$mg \times 2a + \frac{1}{2} mag + \frac{mg}{2a} \left(\frac{a}{2}\right)^2 = \frac{1}{2} mV^2 + \frac{mg}{2a} \left(\frac{3a}{2}\right)^2$ $\left(2mga + \frac{1}{2} mag + \frac{mga}{8} = \frac{1}{2} mV^2 + \frac{9mga}{8}\right)$	A1
	$(V =) \sqrt{3ag}$	A1
		(7)
	Notes for question 3	
M1	Complete method to find the final extension (their $OB - a$). May see use of the 3,4,5 triangle or Pythagoras to find OB . May be implied by a correct final extension.	
M1	Method using EPE formula at least once. EPE must have the form $\frac{\lambda x^2}{ka}$ where λ is modulus of elasticity, x is their extension and k is a constant (condone $k = 1$).	
A1	Two correct expressions for EPE	
B1	GPE term seen or implied	
M1	Use of the principle of conservation of mechanical energy. All required terms present and of the correct structure with no extras (2 EPE, 2KE, GPE). Condone sign errors. Note there are different rearrangements. For example, Initial = Final $mg \times 2a + \frac{1}{2} mag + \frac{mg}{2a} \left(\frac{a}{2}\right)^2 = \frac{1}{2} mV^2 + \frac{mg}{2a} \left(\frac{3a}{2}\right)^2$	

Question Number	Scheme	Marks
	Gain = Loss $\frac{1}{2}mV^2 - \frac{1}{2}mag + \frac{mg}{2a}\left(\frac{3a}{2}\right)^2 - \frac{mg}{2a}\left(\frac{a}{2}\right)^2 = mg \times 2a$	
A1	Correct unsimplified equation	
A1	Correct answer in terms of a and g , accept $1.3\sqrt{ag}$ or better	

Question Number	Scheme	Marks
4(a)	Use of $\frac{1}{2} \int y^2 dx$ with $y = \frac{1}{a}(ax - x^2)$ to integrate the expression $\left(\frac{1}{2}\right)\left(\frac{1}{a}(ax - x^2)\right)^2$ oe	M1
	Correct integration $\left(\frac{1}{2}\right)\frac{1}{a^2}\left[\frac{a^2x^3}{3} - \frac{2ax^4}{4} + \frac{x^5}{5}\right]$	A1
	Use of $\int y dx$ with $y = \frac{1}{a}(ax - x^2)$ to integrate the expression $\frac{1}{a}(ax - x^2)$ oe	M1
	Correct integration $\frac{1}{a}\left[\frac{ax^2}{2} - \frac{x^3}{3}\right]$	A1
	Correct use of centre of mass formula for $\bar{y}$ with $y = \frac{1}{a}(ax - x^2)$ $\bar{y} = \frac{\frac{1}{2} \int_0^a \frac{1}{a^2}(ax - x^2)^2 dx}{\int_0^a \frac{1}{a}(ax - x^2) dx} \quad \left(= \frac{\frac{a^3}{60}}{\frac{a^2}{6}} \right)$	M1
	$(\bar{y}) = \frac{1}{10}a$	A1



Question Number	Scheme	Marks
1(a)	M(A),	M1
	$Mg \times \frac{1}{2} AC = T \times AC$	A1
	$(T =) \frac{1}{2} mg$	A1
		(3)
1(b)	Use of HL with T	M1
	$\frac{1}{2} mg = \frac{2mg}{L} (1.2a - L)$	A1ft
	Solve for L	M1
	$(L =) 0.96a \text{ or } \frac{24}{25} a$	A1
		(4)
Notes for question 1		
(a)		
M1	Take moments about A or another complete method to obtain a dimensionally correct equation in T and mg . All required terms present with no extras. Accept with consistent $\sin \theta$ or $\cos \theta$ present in both terms or neither. For example, $Mg \times \frac{1}{2} AC = T \times AC$, $Mga \cos \alpha = T2a \cos \alpha$, $0.8aMg = 1.6aT$, $Mga \sin \beta = T2a \sin \beta$	
A1	A correct unsimplified equation	
A1	Correct answer	
(b)		
M1	Use of Hooke's Law and their T to obtain a dimensionally correct equation in L and a only. Accept consistent omission of m and/or g . T must be of the form kmg where k is a constant. HL must be of the form $\frac{\lambda(1.2a - L)}{L}$ or $\frac{\lambda(L - 1.2a)}{L}$ If HL is seen in (a) it must be used in (b) to earn the marks.	
A1ft	A correct equation, ft on their T	
M1	Complete method using their T and an attempt at HL to solve a dimensionally correct equation and find L in terms of a only. T must have the form kmg where k is a constant. HL must have the form $\frac{\lambda(1.2a - L)}{L}$ or $\frac{\lambda(L - 1.2a)}{L}$ or $\frac{\lambda x}{L}$ where x is a multiple of a .	
A1	Correct solution including working from part (a).	

Question Number	Scheme	Marks
		
2.	Vertical equilibrium	M1
	$R + T \sin 30^\circ = mg$	A1
	N2L horizontally	M1
	$T \cos 30^\circ = mr\omega^2$	A1
	$T \cos 30^\circ = ma \cos 30^\circ \left(\frac{g}{2a} \right)$	A1
	Solve for R	dM1
	$(R =) \frac{3mg}{4}$	A1
(7)		
Notes for question 2		
M1	Resolve vertically to form a dimensionally correct equation with all required terms present and no extras. Condone $\pm$ sign errors and sin/cos confusion on the T .	
A1	Correct unsimplified equation	
M1	Use N2L to form a dimensionally correct horizontal equation of motion. All required terms present with no extras. Accept any form of circular acceleration $r\omega^2$, $\frac{v^2}{r}$. Accept correct consistent omission of $\cos 30^\circ$. Accept 'ma' if acceleration is replaced correctly with a circular form in subsequent working. Condone sin/cos confusion. An incorrect expression for the radius is marked as an accuracy error and not a method error.	
A1	Correct equation in T , m , g (and a) with at most one error.	
A1	Correct equation in T , m , g (and a)	
dM1	Dependent on both previous M marks. Solve a dimensionally correct equation to find R in terms of m and g only.	
A1	Correct answer, accept $0.75mg$. Must be in terms of m and g ,	

General Principles for Mechanics Marking

(NB specific mark schemes may sometimes override these general principles)

- Rules for M marks:
 - correct no. of terms
 - dimensionally correct
 - all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark, i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.
 - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c)...then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

Mechanics Abbreviations

M(A)	Taking moments about A
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS	Right hand side
LHS	Left hand side

Question Number	Scheme	Marks
1(a)	$T = \frac{\lambda a}{4a}$	B1



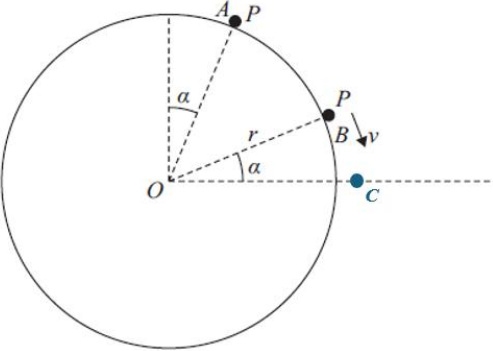
Question Number	Scheme	Marks
	$T \cos \alpha = mg$	M1A1
	$\frac{\lambda a}{4a} \times \frac{3}{5} = mg \Rightarrow \lambda = \frac{20mg}{3} *$	A1* (4)
1(b)	$T \sin \alpha = kmg$	M1A1
	$\frac{20mg}{3} \times \frac{1}{4} \times \frac{4}{5} = kmg$	M1
	$k = \frac{4}{3}$	A1 (4)
		(8)
	Notes for question 1	
	Mark parts (a) and (b) together	
1(a)		
B1	Use of Hooke's Law	
M1	For a relevant equation in T . Must be dimensionally correct with the correct number of terms, condone sign errors and sin/cos confusion. Eg - Resolve vertically: $T \cos \alpha = mg$ - Parallel to string: $T = kmg \sin \alpha + mg \cos \alpha$ - Triangle of forces: $T = \sqrt{(mg)^2 + (kmg)^2}$	
A1	Correct unsimplified equation	
A1*	Given answer obtained from complete and correct working. Must include a line of working before reaching the given answer.	
1(b)		
M1	For a relevant equation in k (a second equation). Must be dimensionally correct with the correct number of terms, condone sign errors and sin/cos confusion. Eg - Resolve horiz: $T \sin \alpha = kmg$ - Perp to string: $mg \sin \alpha = kmg \cos \alpha$ - Triangle of forces: $\tan \alpha = \frac{kmg}{mg} = \frac{4}{3}$	
A1	Correct unsimplified equation	
M1	Complete method to produce an equation in k only (replace T and trig)	
A1	Any equivalent fraction. Accept 1.3 or better	
	Lami: $\frac{T}{\sin 90} = \frac{kmg}{\sin(180-\alpha)} = \frac{mg}{\sin(90+\alpha)}$ M0 for an EPE approach	

Question Number	Scheme		Marks
	$\frac{\sqrt{a^2 - d^2}}{a} = \frac{\sin \theta}{a} = \frac{\sqrt{a^2 - d^2}}{a^2} = \sqrt{1 - \frac{d^2}{a^2}}$	$\frac{\sqrt{a^2 - d^2}}{a} = \frac{\cos \theta}{a} = \frac{\sqrt{a^2 - d^2}}{a^2} = \sqrt{1 - \frac{d^2}{a^2}}$	
2(a)	$R \cos \theta = mg$	$R \sin \theta = mg$	M1A1
	$R = \frac{mga}{d}$		A1 (3)
2(b)	$R \sin \theta = \frac{mv^2}{r}$	$R \cos \theta = \frac{mv^2}{r}$	M1A1A1
	$\frac{mga}{d} \times \frac{\sqrt{a^2 - d^2}}{a} = \frac{mv^2}{\sqrt{a^2 - d^2}}$		DM1
	$v = \sqrt{\frac{g(a^2 - d^2)}{d}}$		A1 (5)
			(8)
	Notes for question 2		
2(a)			
M1	Resolve vertically to form an equation with the correct number of terms and the correct structure. Dimensionally correct, condone sign errors and sin/cos confusion		
A1	Correct equation		
A1	Correct answer.		
2(b)			
M1	Form a horizontal equation of motion with the correct number of terms, condone sign errors and sin/cos confusion. Dimensionally correct. Accept $\frac{v^2}{r}$ or $r\omega^2$ for acceleration. Condone use of a for radius at this point but M0 if a is used for acceleration.		
A1	Equation with at most one error. An error in the acceleration term is one error (incorrect form of acceleration or radius).		
A1	Correct equation (must use the correct form of acceleration and correct radius).		
DM1	Dependent on previous M. Eliminate R and trig to form an equation in v , g , a and d		
A1	Correct answer ISW		

Question Number	Scheme	Marks
4(a)	$AD = 2.5a$ (seen or implied) Method for the difference in 2 EPE terms (at C and D) Eg Considering the whole string $\frac{mg(5a-3a)^2}{2(3a)} - \frac{mg(4a-3a)^2}{2(3a)}$ Considering two half strings $2 \times \left[\frac{mg(\frac{5}{2}a - \frac{3}{2}a)^2}{2(\frac{3}{2}a)} - \frac{mg(2a - \frac{3}{2}a)^2}{2(\frac{3}{2}a)} \right]$ $\frac{1}{2}mga^*$	B1 M1 A1* (3)
4(b)	GPE term, $mg\left(\frac{3a}{2}\right)$ Work-energy equation with all required terms $mg\left(\frac{3a}{2}\right) = \frac{1}{2}mV^2 - \frac{1}{2}m\left(\sqrt{\frac{3ag}{2}}\right)^2 + \frac{1}{2}mga + \frac{1}{5}mg\left(\frac{3a}{2}\right)$ $V = \sqrt{\frac{29ag}{10}}$	B1 M1 A1 A1 A1 (5) (8)
Notes for question 4		
(a)		
B1	Seen or implied eg $DB = 2.5a$ or $2.5a$ used correctly in EPE expression.	
M1	Complete method using 2 EPE terms to form an expression for the change in EPE, accept subtraction either way round. The EPE terms must have the form $\frac{\lambda x^2}{2l}$ where l is the relevant natural length, i.e. $\left(\frac{3}{2}a \text{ or } 3a\right)$ and x is the extension, in terms of a , at C or D respectively.	
A1*	Given answer obtained from fully correct working.	
(b)		
B1	Correct GPE term, $mg\left(\frac{3a}{2}\right)$ seen or implied	

M1	Complete method to form a dimensionally correct work-energy equation. All required terms present and no extra: GPE change, 2 KE and change in EPE. May use the EPE change from (a) or start again with 2 EPE terms of the correct structure (kmg). The work-done against resistance must have the form $\frac{1}{5}mg \times \text{distance}$ where distance is in terms of a . Condone $\pm$ sign errors on terms. Allow different rearrangements of the work-energy principle. For example, Loss = Gain + WD, Initial – Final = WD	
A1	Correct unsimplified equation with at most one error	
A1	Correct unsimplified equation	
A1	Correct answer in terms of ag . Accept $1.7\sqrt{ag}$ or better. If the answer is in decimal form it must be rounded correctly. Calculator display for $\sqrt{\frac{29}{10}}$ is 1.702938637	



Question Number	Scheme	Marks
		
5(a)	Form an energy equation from A to B	M1
	$\frac{1}{2}mv^2 = mgr \cos \alpha - mgr \sin \alpha$	A1
	$v^2 = 2gr(\cos \alpha - \sin \alpha)^*$	A1*
		(3)
5(b)	N2L towards O with $R = 0$	M1
	$mg \sin \alpha = \frac{mv^2}{r}$	A1
	Eliminate v^2	
	$\frac{1}{2}mgr \sin \alpha = mgr(\cos \alpha - \sin \alpha)$	M1
	$(3 \sin \alpha = 2 \cos \alpha \Rightarrow) \tan \alpha = \frac{2}{3}^*$	A1*
		(4)
5(c)	Expression for the horizontal component of the speed of P at C. Condone cos/sin confusion.	M1
	For example, $W \cos \theta \text{ or } W_{\text{horiz}} = v \sin \alpha \left(= \sqrt{\frac{8gr}{13\sqrt{13}}} \right)$	A1
	Method to form relevant equation for the motion from A to C or B to C. Condone cos/sin confusion, consistent with their horizontal component.	M1
	[1] Energy from A to C $\frac{1}{2}mW^2 = mgr \cos \alpha$	A1
	[2] Energy from B to C $\frac{1}{2}mW^2 - \frac{1}{2}mv^2 = mgr \sin \alpha$	
	[3] Vertical motion under gravity from B to C $(w \sin \theta)^2 = (v \cos \alpha)^2 + 2gr \sin \alpha$	
	Dependent on the previous M only. Follow their notation. Use method [1] or [2] to find an expression for the speed of P at C in terms of g and r only.	dM1

2a	Change in GPE $= mg \times 1.3l \sin \theta (= 0.5lmg)$	M1	Condone sin / cos confusion
	$EPE = \frac{\lambda l^2}{2l}$ or $EPE = \frac{\lambda (0.3l)^2}{2l}$	B1	One correct term for EPE
	Energy equation B to A	M1	Dimensionally correct with all the required terms. Condone sign errors and sin / cos confusion
	$\frac{\lambda l^2}{2l} - \frac{\lambda (0.3l)^2}{2l} = 0.5lmg$	A1	Correct unsimplified equation
	$\Rightarrow \lambda = mg \frac{1}{1-0.09} = \frac{100}{91} mg$	A1*	Obtain given answer from correct working. Must see evidence of simplification.
		[5]	
2b	Equation of motion	M1	Dimensionally correct with all the required terms. Condone sign errors and sin/cos confusion.
	$T - mg \sin \theta = ma$	A1	Correct unsimplified equation
	$\frac{\lambda \times l}{l} - mg \sin \theta = ma$ $\left(\frac{100}{91} mg - \frac{5}{13} mg = ma \right)$	A1	Correct unsimplified equation with HL used to replace T
	$a = \frac{5}{7} g$	A1	Accept 0.71g or better. If $g = 9.8$ is used, accept 7.
	Note: If $g = 9.81$ is used then penalise once per complete question. SHM equations can only be used if the motion is proven to be SHM first.		
		[4]	
		(9)	

Question Number	Scheme	Marks
2		
	First relevant force equation	M1
	Correct unsimplified equation	A1
	Relevant force equations: • Horiz $T_1 \cos \theta = T_2$ Vert $T_1 \sin \theta = mg$ or $2T_1 \sin \theta = 2mg$ • // $T_1 = T_2 \cos \theta + mg \sin \theta$ Perp $T_2 \sin \theta = mg \cos \theta$ (accept $T_2 \tan \theta = mg$) • Lami $\frac{T_2}{\sin(90+\theta)} = \frac{mg}{\sin(180-\theta)} = \frac{T_1}{\sin 90}$ $\frac{T_2}{\cos \theta} = \frac{mg}{\sin \theta} = \frac{T_1}{\sin 90}$	
	It may be useful to note the simplified expressions for tensions are $T_1 = \frac{5mg}{3}$, $T_2 = \frac{4mg}{3}$ but need not be seen explicitly.	
	Second relevant force equation	M1
	Correct unsimplified equation	A1
	Hooke's Law for either tension $T_1 = \frac{\frac{20mg}{7}x}{a} \quad \text{or} \quad T_2 = \frac{\frac{20mg}{7}y}{a}$	B1
	Solve their relevant force equation(s) and HL to find x or y	M1
	$x = \frac{7a}{12}$, $y = \frac{7a}{15}$	A1 A1
	$(XY =) 2(a+x) \cos \theta + (a+y)$	DM1
	$= 4a$	A1
		(10)
	Notes for question 2	
M1	First relevant force equation. All required forces present with no extras, condone sign errors and sin/cos confusion. M0 if $T_{XP} = T_{PQ}$ or $T_{PQ} = T_{QY}$. Condone W instead of mg .	
A1	Correct unsimplified equation. Condone W instead of mg .	

Question Number	Scheme	Marks
M1	Second relevant force equation. To be relevant, it must be possible to combine with the first force equation to find both T_1 and T_2 . All required forces present with no extras, condone sign errors and sin/cos confusion. Condone W instead of mg . M0 if $T_{XP} = T_{PQ}$ or $T_{PQ} = T_{QY}$.	
A1	Correct unsimplified equation. Condone W instead of mg .	
B1	Correct use of Hooke's Law. Accept with extension in terms of the lengths PX and QY . Eg $T_1 = \frac{20mg}{7} \frac{(PX - a)}{a}$ $T_2 = \frac{20mg}{7} \frac{(QY - a)}{a}$	
M1	Solve their relevant force equation and HL to find the unknown extension (x or y) or length PX or QY in terms of a . HL must be of the form $\frac{\lambda x}{ka}$ where k is a constant. If using W , it must be replaced with mg .	
A1	One correct (x or y) or (PX or QY) $PX = \frac{19a}{12}$ $QY = \frac{22a}{15}$	
A1	Both correct (x and y) or (PX and QY)	
DM1	Dependent on the first two method marks. Complete expression for XY with calculated x and y , condone sin/cos confusion.	
A1	cao	



Question Number	Scheme	Marks
4(a)	$mg \sin \alpha \times \left(\frac{3l}{2} + e \right) = \mu mg \cos \alpha \times \left(\frac{3l}{2} + e \right) + \frac{1}{2} \times \frac{2mg}{l} e^2$ $\frac{3}{5} \left(\frac{3l}{2} + e \right) = \frac{4\mu}{5} \left(\frac{3l}{2} + e \right) + \frac{e^2}{l}$ $\mu = \frac{9l^2 + 6le - 10e^2}{4l(3l + 2e)} \quad *$	M1B1B1A1 dM1A1cso (6)
(b)	$e = l \Rightarrow \mu = \frac{1}{4} \text{ or } 0.25$ $F = \frac{1}{5} mg$ Change in acceleration is due to change of direction of F $F_1 = 2mg - mg \sin \alpha + F_r \left(= \frac{8}{5} mg \right) \text{ and } F_2 = 2mg - mg \sin \alpha - F_r \left(= \frac{6}{5} mg \right)$ $\text{Mag of change in accel} = \frac{F_1 - F_2}{m} = \frac{2g}{5} = 3.92 \text{ or } 3.9 \text{ (ms}^{-2}\text{)}$	B1 B1ft M1 M1A1 (5) [11]
(a) M1 B1 B1 A1ft dM1 A1cso* (b) B1 B1ft M1 M1 A1	Attempt a work-energy equation with a GPE term, a single EPE term and the work done against friction. . (Allow $EPE = k \frac{\lambda x^2}{l}$) Correct EPE at C. (Ignore any extra EPE terms for this mark) Correct GPE Correct equation. Follow through their EPE and GPE terms providing they are of the correct form At least one line of correct working to rearrange towards $\mu =$. They do not need to reach $\mu =$ for this mark. Given result obtained with no errors seen and at least one line of correct rearrangement. Must be exactly as printed on paper. Correct numerical value for μ seen anywhere in (b). This might be implied by later working. Correct value for F, seen anywhere in (b). Follow through their μ but must be dimensionally correct. μ Attempt 2 equations of motion to find resultant force. (Use of $Change = 2F$) would imply this mark. Subtract and divide by m to obtain the mag of the change in the acceleration. Must be $\frac{2g}{5}$, or 3.9 or 3.92 (m s⁻²)	

Question Number	Scheme	Marks
7(a)	$0.5u = 4 \rightarrow u = 8$	B1
	$F_{max} = \frac{\sqrt{5}}{5} \times 0.5g \times \frac{\sqrt{45}}{7} (= \frac{3g}{14} = 2.1)$	B1
	$\frac{1}{2} \times 0.5 \times 8^2 = 0.5g(x+2) \sin \theta + F_r(x+2) + \frac{3x^2}{2 \times 2}$	M1A1A1
	$64 = 14(x+2) + 3x^2$	
	$3x^2 + 14x - 36 = 0$	M1
	$x = 1.8(m) \quad (1.84m)$	M1A1
		(8)
(b)	$T = \frac{3 \times 1.84}{2} (= 2.76)$	B1ft
	$0.5a = 2.76 + 1.4 - 2.1 \quad (= 2.06)$ Acceleration down slope, so particle does not remain at A.	M1A1
		(3)
		[11]

ALT (a)	$0.5u = 4 \rightarrow u = 8$	B1
	$F_{max} = \frac{\sqrt{5}}{5} \times 0.5g \times \frac{\sqrt{45}}{7} (= 2.1)$	B1
	$\frac{1}{2} \times 0.5 \times 8^2 = 0.5gd \sin \theta + F_r d + \frac{3(d-2)^2}{2 \times 2}$	M1A1A1
	$64 = 14d + 3(d-2)^2$	
	$3d^2 + 2d - 52 = 0$	M1
	$d = 3.8 \rightarrow x = 1.8(m) \quad (1.84m)$	M1A1
		(8)
ALT (b)	$T = \frac{3 \times 1.84}{2} (= 2.76)$	B1ft
	Upslope: $F_{max}(= 2.1)$, Downslope: $0.5g \sin \theta + T (= 4.16)$	M1
	$4.16 > 2.1$ so there is a resultant force down slope, so the particle does not remain at A	A1 (3)

(a)

B1 Initial speed seen.

B1 Maximum friction seen/used. Award if only seen in (b).

M1 Energy equation with KE, GPE, EPE and WD. If they split the motion up to find the speed when the string begins to extend ($= 6 \text{ ms}^{-1}$) only award this mark once they have the equation containing EPE. Allow with $EPE = k \frac{\lambda x^2}{l}$.

A1 Equation with at most one error.

A1 Fully correct equation.

M1 Produce a 3 term in x or d equalling zero (see ALT). This is independent.

M1 Solve a 3 term quadratic to find the extension or distance travelled.

A1 Correct extension. Must be 2 or 3 s.f.

(b)

B1ft Correct expression for the tension at A ft their extension.

M1 Consider the three forces parallel to the plane.

A1 Correct conclusion from comparison of the three forces. Correct working with numerical values seen. Could be an acceleration or correct statement about the forces up/down the slope.



Question Number	Scheme	Marks
3(a)	$mg = \frac{kmg}{l} \frac{2l}{5}$	M1
	$k = \frac{5}{2}^*$	A1* (2)
3(b)	$mg - T = m\ddot{x}$	M1
	$mg - \frac{5mg}{2l} \left(x + \frac{2l}{5} \right) = m\ddot{x}$	DM1A1
	$-\frac{5g}{2l}x = \ddot{x}$, hence SHM.*	A1* (4)
3(c)	$\omega = \sqrt{\frac{5g}{2l}}$; $a = \frac{1}{4}l$	B1 ft; B1
	$v = a\omega = \frac{1}{4}l \times \sqrt{\frac{5g}{2l}}$	M1
	$\frac{1}{4}\sqrt{\frac{5gl}{2}}$ oe	A1 (4)
3(d)	$\frac{1}{4} \times \frac{2\pi}{\omega}$	M1
	$\frac{\pi}{2}\sqrt{\frac{2l}{5g}}$ oe	A1 ft (2)
		(12)
	Notes for question 3	
3(a)	M1 for $mg = T$ and use of Hooke's Law	
	A1* Given answer correctly obtained	
3(b)	M1 for equation of motion, dim correct with all necessary terms, allow a for acceleration and condone sign errors. Accept T or attempt at T , which may not have a variable extension.	
	DM1 for equation of motion, dim correct with correct terms, and use of Hooke's Law with a variable extension measured from E and now need $\ddot{x}$, condone sign errors. Depends on the first M mark; both M marks can be awarded together.	
	A1 for a correct unsimplified equation	
	A1* for a correct equation and conclusion	
3(c)	B1 ft for a dimensionally correct ω or ω^2 , seen explicitly or used. B1 for $a = \frac{1}{4}l$	
	M1 for use of $v = a\omega$ or $v^2 = \omega^2(a^2 - x^2)$ with $x = 0$ later	
	A1 cao	
	Use of energy: B1 gain of GPE B1 either EPE M1 energy equation with change in GPE, change in EPE and KE. A1 cao	
3(d)	M1 for use of $\frac{1}{4} \times \frac{2\pi}{\omega}$	
	A1 cao	

Question Number	Scheme	Marks
4(a)	WD against air resistance = kmg ; PE Gain = $\frac{1}{2}mga$; KE Gain = $\frac{1}{2} \times \frac{1}{2}m \times 3ag$	B2,1,0
	Initial EPE = $\frac{2mg}{4a}(2a)^2$; Final EPE = $\frac{2mg}{4a}a^2$	B1; B1
	$kmg = \frac{2mg}{4a}((2a)^2 - a^2) - \frac{1}{2}mga - \frac{1}{2} \times \frac{1}{2}m \times 3ag$	M1A1
	$k = \frac{1}{4} *$	A1* (7)
4(b)	$\frac{1}{2}mg - \frac{1}{4}mg - T = 0$	M1
	$\frac{1}{2}mg - \frac{1}{4}mg - \frac{2mg}{2a}x = 0$	A1
	$x = \frac{1}{4}a$	A1
	$OB = \frac{9a}{4}$	A1 ft (4)
		(11)
	Notes for question 4	
4(a)	B2 for all 3 unsimplified terms. B1 B0 for 2 out of 3 correct	
	B1 for the initial EPE	
	B1 for the final EPE	
	M1 for the work-energy equation with all necessary terms, condone sign errors.	
	A1 for a correct equation.	
	A1* for the given answer correctly obtained. At least one step of working to be seen.	
4(b)	M1 for a vertical resolution with the correct terms (T does not need to be substituted) Must have acceleration = 0 for this mark	
	A1 for a correct equation with T replaced.	
	A1 cao	
	A1 ft $2a +$ their x	
	Use of uniform acceleration equations scores 0/4	
	Alternative, using work-energy	
	M1 for an equation with GPE, EPE, KE and WD terms – all but KE using a variable distance (OB or the extension).	
	A1 correct equation	
	DM1 (A1 on e-pen) Obtain an expression for v^2 in terms of their unknown distance and find their distance when this is maximum by calculus or completing the square	
	A1 cao	

Question Number	Scheme	Marks
4(a)	Energy from C to D $mg \frac{l}{4} \sin 30^\circ = \frac{\lambda}{2l} \left(\frac{l}{4} \right)^2$ $\lambda = 4mg \text{ *}$	M1A1A1 A1* (4)
(b)	The greatest speed is when the acceleration of B is zero $(\text{N}) \quad T = mg \sin 30^\circ = \frac{4mge}{l}$ $e = \frac{l}{8}$ Energy: $\frac{1}{2}mv^2 + \frac{4mg}{2l} \left(\frac{l}{8} \right)^2 = mg \frac{l}{8} \sin 30^\circ$ $v = \sqrt{\left(\frac{gl}{16} \right)} = \frac{\sqrt{gl}}{4}$	M1 A1 M1A1A1 DM1A1 (7) [11]
Notes		
(a) M1 A1 A1 A1* (b) M1 A1 M1 A1 A1 DM1 A1	Attempt the energy equation from C to D. Must use a vertical height for PE. EPE must have the form kx^2. Must have 1 PE term and 1 EPE term. Correct loss of PE Correct final EPE Correct answer correctly obtained Resolve along the plane using HL to find T Correct value for the extension Form the energy equation with an extension they have found. M0 if $l/4$ is used for the extension. Must use a vertical height for PE. EPE must have the form kx^2. Must have 1 PE term, 1 KE term and 1 EPE term. Two correct terms Completely correct equation Solve for v. Dependent on previous M. Correct expression for v	
4(b) ALT 1 M1 A1 M1 A1 A1 DM1 A1	Using integration As above, for finding correct value for e. This may be embedded in a complete method. Uses $F=ma$ to and attempts to integrate. Must have the correct number of terms and weight resolved, $\int g \sin 30 - \frac{4gx}{l} dx = \int v dv \quad \text{leading to} \quad \frac{gx}{2} - \frac{2gx^2}{l} = \frac{v^2}{2} + c$ Correct integration with at most one slip/error Completely correct integration but c may be missing Find value for c (when $x = \frac{l}{4}$, $v=0$ gives $c=0$) and sub in e to find an expression for v Correct expression for v	

4(b) ALT 2 M1 A1	Using SHM As above, for finding correct value for e. This may be embedded in a complete method.
M1 A1 A1	Correctly uses $F=ma$ to show that the motion is SHM Correct proof of SHM
M1 A1	Uses $v = aw$ to find an expression for v Correct expression for v



Question Number	Scheme		Marks
7(a)	General equation of motion for particle, measuring up from E. $T - \frac{3mg}{2} = \frac{3m\ddot{x}}{2}$	General equation of motion for particle, measuring down from E. $\frac{3mg}{2} - T = \frac{3m\ddot{x}}{2}$	M1
	Use HL in equation of motion with extension $\left(\frac{3L}{8} \pm x\right)$	Use HL in equation of motion with extension $\left(\frac{3L}{8} \pm x\right)$	DM1
	Correct equation $\frac{4mg}{L}\left(\frac{3L}{8} - x\right) - \frac{3mg}{2} = \frac{3m\ddot{x}}{2}$	Correct equation $\frac{3mg}{2} - \frac{4mg}{L}\left(\frac{3L}{8} + x\right) = \frac{3m\ddot{x}}{2}$	A1
	Complete conclusion $-\frac{8g}{3L}x = \ddot{x} \quad \therefore \text{SHM}^*$		A1*
	Identify ω from SHM method and use to find the Period		DM1
	$\omega^2 = \frac{8g}{3L} \quad \text{or} \quad \omega = \sqrt{\frac{8g}{3L}}$ $\text{Period} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3L}{8g}} = \pi\sqrt{\frac{3L}{2g}}^*$		A1*
			(6)
7(b)	Correct amplitude, $a = \frac{1}{2}L$		B1
	Use of SHM displacement equation with amplitude substituted. No need to substitute for ω . [1] $x = -a \cos \omega t$ [2] $x = a \cos \omega t$ [3] $x = a \sin \omega t$ [4] $x = -a \sin \omega t$		M1
	Complete method to find an equation for a relevant time. No need to substitute for ω . Either Use $\ddot{x} = -\omega^2 x$ with $-\frac{2g}{3}$ to find displacement. May start again with N2L, HL and $-\frac{2g}{3}$ to find displacement. Must then combine displacement of $\pm \frac{L}{4}$ with one of [1], [2], [3] or [4] above to form an equation in t (and ω).		M1

Question Number	Scheme	Marks
	Or Substitute $-\frac{2g}{3}$ and their a in the corresponding acceleration equation to form an equation in t (and ω) [1] $\ddot{x} = a\omega^2 \cos(\omega t)$ [2] $\ddot{x} = -a\omega^2 \cos(\omega t)$ [3] $\ddot{x} = -a\omega^2 \sin(\omega t)$ [4] $\ddot{x} = a\omega^2 \sin(\omega t)$	
	Dependent on both previous M marks. Complete method to find the required time. Where appropriate, must use the correct proportion of the given period to find an expression for the required time. Must use the correct ω. [1] $t = \frac{1}{\omega} \times \cos^{-1}\left(-\frac{1}{2}\right)$ [2] $t = \frac{1}{\omega} \cos^{-1}\left(\frac{1}{2}\right) - \frac{\text{period}}{2} = \frac{5\pi}{3\omega} - \frac{\text{period}}{2}$ [3] $t = \frac{1}{\omega} \sin^{-1}\left(\frac{1}{2}\right) + \frac{\text{period}}{4} = \frac{\text{period}}{4} + \frac{\pi}{6\omega}$ [4] $t = \frac{1}{\omega} \sin^{-1}\left(-\frac{1}{2}\right) - \frac{\text{period}}{4} = \frac{7\pi}{6\omega} - \frac{\text{period}}{4}$	DM1
	$t = \frac{2\pi}{3} \sqrt{\frac{3L}{8g}} \quad \text{o.e.}$	A1
		(5)
		(11)
	Notes for question 7	
7(a)		
M1	Equation of motion in a <i>general</i> position ie T does not take a particular value. Allow a for acceleration here. Required terms present with no extras. Condone sign errors but must have a difference between T and weight. Condone missing $\frac{3}{2}$ in ' ma ' term only for this mark.	
DM1	Dependent on previous M. Use of Hooke's Law in a <i>general</i> equation of motion with general position measured from E . Extension in HL must be of the appropriate form: $\left(\frac{3L}{8} - x\right)$ or $\left(\frac{3L}{8} + x\right)$. Allow a for acceleration, condone sign errors. Must use the correct mass, $\frac{3m}{2}$ in ' ma ' term.	
A1	For a fully correct unsimplified equation. Must use $\ddot{x}$ for acceleration.	

Question Number	Scheme	Marks
A1*	Correct SHM equation and conclusion. Equation must have required form, $\ddot{x} = -\omega^2 x$ with $\ddot{x}$ for acceleration. Conclusion must include 'SHM'	
DM1	Dependent on both previous M's. Use of $\frac{2\pi}{\omega}$ where ω has come from an attempt at using N2L at a general point.	
A1*	Obtain the given answer for the period. Must follow from complete and correct working. At least one line of working must be seen between $\ddot{x} = -\frac{8g}{3L}x$ and reaching the given Period. Eg - period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{8g}{3L}}} = \pi\sqrt{\frac{3L}{2g}}$ $\omega = \sqrt{\frac{8g}{3L}}$, period = $\frac{2\pi}{\omega} = \pi\sqrt{\frac{3L}{2g}}$ Note The score of M1 DM1 A1 A0* DM1 A1* is possible if there is no conclusion of 'SHM' The score of M1 DM1 A0 A0* DM1 A1* is possible if $\ddot{x}$ is not used for acceleration.	
7(b)		
B1	Correct amplitude, $\frac{L}{2}$.	
M1	Use of suitable SHM equation with their amplitude substituted. May be implied if starting with a corresponding velocity or acceleration equation. No need to replace ω .	
M1	All necessary steps completed to find an equation for a relevant time. No need to replace ω .	
DM1	Complete method to find an expression for the required time. Must use the correct ω .	
A1	Correct value for required time in exact form, in terms of L and g e.g. $\frac{2\pi}{3}\sqrt{\frac{3L}{8g}}$, $\frac{\pi}{3}\sqrt{\frac{3L}{2g}}$, $\pi\sqrt{\frac{L}{6g}}$, $\frac{\pi}{6}\sqrt{\frac{6L}{g}}$	

Question Number	Scheme	Marks
6(a)	$\frac{20(0.2-x)}{0.4} - \frac{20(0.2+x)}{0.4} = 0.4\ddot{x}$ $-100x = 0.4\ddot{x}$ $\ddot{x} = -250x \quad \therefore \text{SHM}$	M1A1 dM1A1cso (4)
(b)	Period = $\frac{2\pi}{\sqrt{250}}$ oe	B1ft (1)
(c)	$v_{\max} = \frac{2}{0.4} = 5 \text{ m s}^{-1}$ $a\omega = 5 \quad a = \frac{5}{\sqrt{250}} = \frac{1}{\sqrt{10}} (= 0.3162...) \text{ m}$	B1 M1A1ft (3)
(d)	$x = a \cos \omega t \quad 0.1 = \frac{1}{\sqrt{10}} \cos \sqrt{250}t \quad \text{or} \quad x = a \sin \omega t \quad 0.1 = \frac{1}{\sqrt{10}} \sin \sqrt{250}t$ $t = \frac{1}{\sqrt{250}} \cos^{-1}(0.1 \times \sqrt{10}) \quad \text{or} \quad t = \frac{1}{\sqrt{250}} \sin^{-1}(0.1 \times \sqrt{10})$ Time for which $AP > 0.5$ $= \frac{2\pi}{\sqrt{250}} - 2 \frac{1}{\sqrt{250}} \cos^{-1}(0.1 \times \sqrt{10}) \quad \text{or} \quad = \frac{\pi}{\sqrt{250}} + 2 \frac{1}{\sqrt{250}} \sin^{-1}(0.1 \times \sqrt{10})$ $= 0.2393... \text{ s}$	M1A1ft A1 dM1 A1cso (5) [13]
(a) M1 A1 dM1 A1ft (b) B1ft (c)B1 M1 A1ft (d) M1 A1ft A1 dM1 A1cso	Attempt an equation of motion using a difference of 2 tensions obtained from Hooke's law and having different variable extensions. $\ddot{x}$ or a allowed. Can be in algebraic form. Correct equation. $\ddot{x}$ or a allowed but if a used the signs must indicate it is in the same direction as $\ddot{x}$. Can be in algebraic form. Rearrange their equation to the required form $\ddot{x} = -\omega^2 x$. Must be $\ddot{x}$. They cannot just lose terms to get to the required form. Correct equation, can be numerical as shown or algebraic $\left(\text{e.g. } \ddot{x} = -\frac{4\lambda}{ml} x \right)$, and state conclusion. If algebraic this must include stating that their "ω^2" is positive. Correct period (numerical) as shown or equivalent. Follow through their ω from $\ddot{x}$ or $a = \pm \omega^2 x$ (0.40 or better) Correct max speed, seen explicitly or used Using $v_{\max} = a\omega$ to obtain a value for a Correct value, exact or decimal (0.32 or better) Use $x = a \cos \omega t$ or $x = a \sin \omega t$ with $x = \pm 0.1$, their ω, a. Correct equation, follow through their ω, a Correct expression for time from their choice of equation (if only decimal seen, award for 2sf or better 0.078997.. for cos, 0.020349... for sin) Must have come from fully correct work in (a) Complete correct method to obtain the required time. Dependent on previous M mark. Correct final answer. 2s.f. or better. Must have come from fully correct work in (a) and (d), although they might not have used $\ddot{x}$	



Question Number	Scheme	Marks
5(a)	$\frac{2mge_1}{2a}$ or $\frac{6mg(4a - e_1)}{4a}$	B1
	$mg + \frac{2mge_1}{2a} = \frac{6mg(4a - e_1)}{4a}$	M1A1
	Solve to find either extension	dM1
	$e_1 = 2a$ and $e_2 = 4a - e_1 = 2a^*$	A1*
		(5)
ALT (a)	$mg + \frac{2mge_1}{2a} = \frac{6mge_2}{4a}$, $e_1 + e_2 = 4a$	M1A1
	Solve simultaneously to find either extension	dM1
	$e_1 = 2a$ and $e_2 = 4a - e_1 = 2a^*$	A1*
(b)		
	$mg + \frac{2mg(2a - x)}{2a} - \frac{6mg(2a + x)}{4a} = m\ddot{x}$	M1A1A1
	$\ddot{x} = -\frac{5g}{2a}x \therefore \text{SHM}$	A1
		(4)
(c)	$\omega^2 = \frac{5g}{2a}$	B1ft
	$v^2 = \frac{5g}{2a} \left(a^2 - \left(\frac{a}{2} \right)^2 \right)$	M1A1
	$v = \sqrt{\frac{15ga}{8}} = \frac{\sqrt{30ga}}{4}$	A1 cso
		(4)
ALT (c)	$\frac{2mga^2}{4a}$ or $\frac{6mg(3a)^2}{8a}$ or $\frac{2mg(\frac{3a}{2})^2}{4a}$ or $\frac{6mg(\frac{5a}{2})^2}{8a}$	B1
	$\frac{2mga^2}{4a} + \frac{6mg(3a)^2}{8a} = \frac{2mg(\frac{3a}{2})^2}{4a} + \frac{6mg(\frac{5a}{2})^2}{8a} + \frac{mga}{2} + \frac{mv^2}{2}$	M1A1
	$v = \sqrt{\frac{15ga}{8}}$	A1
		(4)
		[13]

(a)

B1 Correct use of Hooke's law for either string. Must include an unknown extension.

M1 Resolve vertically, with two variable tensions and weight (M0 for setting both extensions as e)

A1 Correct equation.

dM1 Solve to find either extension.

A1* Correct extensions found for both strings, from fully correct working.

(b)

M1 Vertical equation of motion with two different variable tensions, weight and $m\ddot{x}$ (allow ma)

A1 Equation with at most one error (allow ma for this mark, which does not count as an error).

A1 Fully correct equation. Must now be $m\ddot{x}$

A1 $\ddot{x} = -\frac{5g}{2a}x$ $\therefore$ SHM. **Must have concluding statement.**

(c)

B1ft Use of their ω^2

M1 Complete method to find speed at $\frac{7}{2}a$ above A. Follow through their ω . Needs amplitude a and $x = \frac{1}{2}a$

A1 Correct equation. No follow through now.

A1 cso

ALT (a) using simultaneous equations

B1 Correct use of Hooke's law for either string. Must include an unknown extension.

M1 Resolve vertically with two tensions in e_1 and e_2 and weight AND give a second equation for $e_1 + e_2$

A1 Both equations correct.

dM1 Solves both equations simultaneously to find either extension.

A1* Correct extensions found for both strings, from fully correct working.

ALT (c)

B1 Use of correct EPE

M1 Complete method to find speed at $\frac{7}{2}a$ above A. Allow with $EPE = k \frac{\lambda x^2}{l}$. Must have all terms.

A1 Correct equation.

A1 Correct final answer



Question Number	Scheme	Marks
7(a)	$T_A - T_B = m\ddot{x}$	M1
	$\frac{2mg}{l} \left(\frac{2l}{3} - x \right) - \frac{mg}{l} \left(\frac{4l}{3} + x \right) = m\ddot{x}$ or $\frac{mg}{l} \left(\frac{4l}{3} - x \right) - \frac{2mg}{l} \left(\frac{2l}{3} + x \right) = m\ddot{x}$.	dM1A1
	$-\frac{3g}{l}x = \ddot{x}$, so SHM	A1
	$T = \frac{2\pi}{\sqrt{\frac{3g}{l}}} = 2\pi\sqrt{\frac{l}{3g}}$ *	M1 A1*
		(6)
7(b)	$\frac{1}{2}l \times \sqrt{\frac{3g}{l}}$ or $\frac{1}{2}\sqrt{3gl}$ or $\sqrt{\frac{3gl}{4}}$ oe	B1
		(1)
7(c)	$\frac{3g}{2}$ or $1.5g$	B1
		(1)
7(d)	$x = a \cos \omega t \Rightarrow v = -a\omega \sin \omega t$	M1
	$-\frac{3}{4}\sqrt{gl} = -a\omega \sin \omega t$ to find t	M1A1
	Solve for t	M1
	$t = \frac{\pi}{3}\sqrt{\frac{l}{3g}}$ oe	A1
		(5)
		(13)
Notes		
(a)		
M1	Equation of motion in a <i>general</i> position, allow a for acceleration, correct no. of terms, condone sign errors.	
dM1	Use Hooke's Law to sub for the two tensions, allow a for acceleration. Extensions must be different and of the form $(d \pm x)$ where d is a multiple of l .	
A1	Correct unsimplified equation, allow a for acceleration.	
A1	Correct equation using $\ddot{x}$ for acceleration.	
M1	Use of $\frac{2\pi}{\omega}$ Their ω from their equation of motion, which must be in terms of x .	
A1*cso	Given answer correctly obtained – this includes proof of SHM with conclusion and correct expression for the period.	
(b)		
B1	Cao Speed at O so must be positive. Unsimplified, ignore errors from subsequent 'simplifying' of surds.	
(c)		
B1	Cao Max acceleration so must be positive.	



(d)	
Main	
M1	Use of $x = a \cos \omega t$ to obtain $v = -a\omega \sin \omega t$ Substitution for a and ω is not required.
M1	Use $v = -a\omega \sin \omega t$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in t only, $-\frac{3}{4}\sqrt{gl} = -a\omega \sin \omega t$
A1	Correct equation in t only
M1	Solve to find the required time, t
A1	Cao for required time.
ALT 1	
M1	Use of $x = a \sin \omega t$ to obtain $v = a\omega \cos \omega t$ Substitution for a and ω is not required.
M1	Use $v = a\omega \cos \omega t$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in t only, $\frac{3}{4}\sqrt{gl} = a\omega \cos \omega t$
A1	Correct equation in t only
M1	Solve to find t and then subtract from $\frac{1}{4}$ period to find the required time. $t = \frac{\pi}{6} \sqrt{\frac{l}{3g}} \Rightarrow \text{required time} = \frac{1}{4} \left(2\pi \sqrt{\frac{l}{3g}} \right) - \frac{\pi}{6} \sqrt{\frac{l}{3g}} = \frac{\pi}{3} \sqrt{\frac{l}{3g}}$ Eg
A1	$t = \frac{\pi}{3} \sqrt{\frac{l}{3g}}$ oe Cao for required time,
ALT2	
M1	Use of $x = a \cos \omega t$ or use of $x = a \sin \omega t$. Substitution for a and ω is not required.
M1	Using $v^2 = \omega^2(a^2 - x^2)$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in x only. $\left(-\frac{3}{4}\sqrt{gl}\right)^2 = \omega^2(a^2 - x^2)$
A1	Correct equation in x only. (Solution leads onto the first M mark in (d))
M1	Solves for t and then completes the method to find the required time. $\frac{l}{4} = \frac{l}{2} \cos \left(\sqrt{\frac{3g}{l}} t \right)$ e.g. or quarter period with sin method.
A1	$t = \frac{\pi}{3} \sqrt{\frac{l}{3g}}$ oe Cao for required time,
SPECIAL CASE where $a = \frac{1}{2}$ is clearly stated as amplitude and consistently used in (b) (c) & (d)	
(b)	B1 $\frac{1}{2} \sqrt{\frac{3g}{l}}$
(c)	B1 $\frac{3g}{2l}$
(d)	Maximum M1 M1 A0 M0 A0



Question Number	Scheme	Marks
6a)	$F_r = \frac{1}{7} \times 0.4 \times 9.8 \quad (= 0.56)(\text{N})$	B1
	$\frac{1}{2} \times 0.4v^2 = \frac{1}{2} \times 0.4(1.8)^2 - 0.8 \times "0.56"$	M1A1A1 (ft their F_r)
	$v^2 = 1.00 \Rightarrow v = 1.0 \text{ or } 1.00 (\text{m s}^{-1})$	A1
		(5)
6b)	$\frac{1}{2} \times 0.4(1.0)^2 = 0.56x + \frac{0.6x^2}{2(0.8)}$	M1A1B1
	$0.375x^2 + 0.56x - 0.2 = 0$	DM1
	$x = 0.2977\dots$	A1
	Total distance = 1.1 (m) (or 1.10)	A1
		(6)
ALT 1	<i>Work from A to C with total distance as the unknown</i>	
	$\frac{1}{2} \times 0.4(1.8)^2 = 0.56y + \frac{0.6(y-0.8)^2}{2(0.8)}$	M1A1B1
	$0.375y^2 - 0.046y - 0.408 = 0$	DM1
	$y = 1.0977\dots$	A1
	$y = 1.1 \text{ or } 1.10$	A1
ALT 2	<i>Work from A to C with distance BC as the unknown</i>	
	$\frac{1}{2} \times 0.4(1.8)^2 = 0.56(y+0.8) + \frac{0.6y^2}{2(0.8)}$	M1A1B1
	Rest as main scheme	DM1A1A1
6c)	$T = \frac{0.6 \times "0.2977"}{0.8} \quad (= 0.223)$	M1A1ft
	$0.223 < 0.56$	
	Tension less than $F_{\max}$. Therefore particle will not move. cso.*	A1cso*
		(3)
		[14]

(a)		
B1	Correct friction seen. (Might be contained in WD) g or 9.8 acceptable.	
M1	Work-Energy equation with 2 KE terms and their WD by friction. All terms must be dimensionally correct.	
A1A1	One each for the KE terms.	
A1	$v = 1.0$ or 1.00 (2 or 3 sf as g has been used to obtain the speed at B)	
(b)		
M1	Work-Energy equation with KE, WD and EPE. EPE term to be of the form $\frac{\lambda x^2}{k \times \text{natural length}}$ with $k = 1$ or 2	
B1	Correct EPE term. M mark for the equation not needed for this mark.	
A1ft	Fully correct equation. Follow through their EPE	
DM1	Reducing to a 3 term quadratic in x , terms in any order. Depends on the first M of (b)	
A1	$x = 0.2977\dots$	
A1	1.1 (m) or 1.10 (m) (2 or 3 sf as g has been used to obtain the speed at B)	
ALT 1		
	First 4 marks as main scheme notes.	
A1A1	Award A1A1 if final answer is correct, from a correct equation. Award A1A0 if answer correct but not rounded.	
ALT 2	Notes as for main scheme.	
(c)		
M1	Use of Hooke's Law for their extension at C .	
A1ft	Correct tension. Follow through their extension. No need to simplify.	
A1cso*	Correct conclusion, from fully correct working including evaluation of tension.	

6a	In equilibrium	M1	Need all three forces. Dimensionally correct
	$mg + 4mg \frac{l-e}{l} = 4mg \frac{e}{l}$	A1 A1	Unsimplified equation with at most one error Correct unsimplified equation
	$5l = 8e \Rightarrow e = \frac{5l}{8},$ $AE = l + \frac{5l}{8} = \frac{13l}{8}$ *	A1*	Obtain given answer from correct working. Must see $AE =$
ALT1	$mg + 4mg \frac{(2l-AE)}{l} = 4mg \frac{(AE-l)}{l}$	M1 A1 A1	Need all three forces. Dimensionally correct Unsimplified equation with at most one error Correct unsimplified equation
	$AE = \frac{13l}{8}$ *	A1*	Obtain given answer from correct working. Must see $AE =$
ALT2	$mg + 4mg \left(\frac{\frac{l}{2} - e}{l} \right) = 4mg \left(\frac{\frac{l}{2} + e}{l} \right)$	M1 A1 A1	Need all three forces. Dimensionally correct Unsimplified equation with at most one error Correct unsimplified equation
	$e = \frac{l}{8}, AE = l + \frac{l}{2} + \frac{l}{8} = \frac{13l}{8}$ *	A1*	Obtain given answer from correct working
		[4]	
6b	Equation of motion	M1	Need all terms. Dimensionally correct. Condone use of a for acceleration.
	$4mg \frac{\frac{5l}{8} + x}{l} - 4mg \frac{\frac{3l}{8} - x}{l} - mg = -m\ddot{x}$	A1 A1	Unsimplified equation with at most one error. Correct unsimplified equation Note: the question states x is measured vertically down.
	$\Rightarrow -m\ddot{x} = \frac{8mg}{l}x, \ddot{x} = -\frac{8g}{l}x$ *	A1*	Obtain given answer from correct working. Must use $\ddot{x}$
		[4]	
6c	Use of $v^2 = \omega^2(a^2 - x^2)$ with $a = \frac{3l}{8}$	M1	Or use of equivalent correct formula
	$= \frac{8g}{l} \left(\frac{9}{64}l^2 - \frac{1}{64}l^2 \right)$	A1	Correct unsimplified expression for v or v^2
	$v = \sqrt{gl}$	A1	Correct only
		[3]	

6d	$x = \frac{3l}{8} \cos \omega t$	B1	Use of relevant formula with correct amplitude $x = \frac{3l}{8} \cos \omega t$ or $x = \frac{3l}{8} \sin \omega t$
	Use of $-\frac{l}{8} = \frac{3l}{8} \cos \omega t$ or $\frac{l}{8} = \frac{3l}{8} \sin \omega t$ and correct use of $\frac{1}{2} \times \frac{2\pi}{\omega}$ or $\frac{l}{8} = \frac{3l}{8} \cos \omega t$ and correct use of $\pi - \cos^{-1}\left(\frac{1}{3}\right)$	M1	Complete method to find t or required time $t = \frac{1}{\omega} \cos^{-1}\left(-\frac{1}{3}\right)$ or $t = \frac{1}{\omega} \sin^{-1}\left(\frac{1}{3}\right)$ with $\frac{1}{2}$ period or $t = \frac{1}{\omega} \cos^{-1}\left(\frac{1}{3}\right)$ with π
	Required time $\frac{2}{\omega} \cos^{-1}\left(\frac{-1}{3}\right) = \sqrt{\frac{l}{2g}} \cos^{-1}\left(\frac{-1}{3}\right)$ or $\frac{\pi}{\omega} + \frac{2}{\omega} \sin^{-1}\left(\frac{1}{3}\right) = \sqrt{\frac{l}{8g}} \left(\pi + 2 \sin^{-1}\left(\frac{1}{3}\right) \right)$ or $\frac{2}{\omega} \left[\pi - \cos^{-1}\left(\frac{1}{3}\right) \right] = \sqrt{\frac{l}{2g}} \left[\pi - \cos^{-1}\left(\frac{1}{3}\right) \right]$	A1	Or equivalent, accept $1.91\sqrt{\frac{l}{2g}}, 1.35\sqrt{\frac{l}{g}}, 0.43\sqrt{l}$ $3.82\sqrt{\frac{l}{8g}}$ $\cos^{-1}\left(\frac{-1}{3}\right) = 1.91\dots$
		[3]	
		(14)	



Question Number	Scheme	Marks
6(a)	$\frac{\frac{3}{4}mge}{a} = mg$	M1
	$e = \frac{4}{3}a \Rightarrow AP = \frac{7}{3}a$	A1
		(2)
(b)	$m\ddot{x} = mg - \frac{\frac{3}{4}mg\left(x + \frac{4}{3}a\right)}{a}$	M1A1
	$m\ddot{x} = mg - \frac{3mgx}{4a} - mg = -\frac{3mgx}{4a}$	
	$\ddot{x} = -\frac{3g}{4a}x = -\omega^2x \therefore \text{SHM}$	DM1A1
	amp = $\frac{2}{3}a$ KE = $-\frac{mg}{2}$	B1
	$\frac{2}{3}a < \frac{4}{3}a$ oe So string remains taut or String never goes slack, therefore always SHM	B1
		(6)
(c)	$v^2 = \frac{3g}{4a}\left(\left(\frac{2a}{3}\right)^2 - \left(\frac{a}{3}\right)^2\right)$	M1A1ft
	$v^2 = \frac{ag}{4}$ $v = \frac{\sqrt{ag}}{2}$	A1
		(3)
Alt	Energy $\frac{\frac{3mg}{4}(2a)^2}{2a} = \frac{\frac{3mg}{4}(a)^2}{2a} + mga + \frac{1}{2}mv^2$	M1A1
	$v = \frac{\sqrt{ag}}{2}$	A1
(d)	$-\frac{a}{3} = \frac{2a}{3}\cos(\omega t)$ $\left[\frac{a}{3} = \frac{2a}{3}\sin(\omega t) \quad \frac{T}{4} = \pi\sqrt{\frac{a}{3g}}\right]$	M1A1ft
	$\cos(\omega t) = -\frac{1}{2}$	

Question Number	Scheme	Marks
	$t\sqrt{\frac{3g}{4a}} = \frac{2\pi}{3}$	
	$t = \frac{2\pi}{3} \sqrt{\frac{4a}{3g}}$	DM1A1
		(4)
		[14]

- (a) **M1** Resolve vertically. Can be in either e or AP .

A1 $AP = \frac{7}{3}a$

- (b) **M1** Form an equation of motion including weight and tension. Variable must be measured from equilibrium position. If e is used, this must have been defined already in working (part (a)). Acceleration can be $\ddot{x}$ or a .

A1 Correct unsimplified equation. Can use $\ddot{x}$ or a , but if a used, it must be in same direction as x .

DM1 Solve to obtain $\ddot{x} =$ Must be $\ddot{x}$ now.

A1 $\ddot{x} = -\frac{3g}{4a}x = -\omega^2x \therefore \text{SHM}$ Correct equation **and** statement.

B1M1 Correct/sufficient values found to establish string remains taut.

B1A1 Appropriate argument

- (c) **M1** Use of $v^2 = \omega^2(a^2 - x^2)$ with their ω (ignore the dimensions of their ω)

A1ft Correct equation in v and a . ft their ω and amplitude (really their AP . Condone failing to add a to their extension in (a))

A1 Correct v . c.s.o., but award if correct ω found with only sign error in (b)

Alt for first 2 Marks

M1 Energy equation. Must contain 2 EPE terms, GPE and KE.

A1 Correct equation.

- (d) **M1** Use of $x = "a" \cos(\omega t)$ or $x = "a" \sin(\omega t)$. Amplitude must be consistent with their AP .

A1 Correct equation in ω

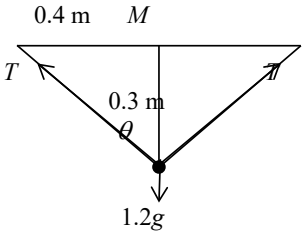
DM1 Solve equation to find expression for t . Must now have a full method to find complete time.

A1 Any correct equivalent. Must come from correct working throughout, but condone sign error on acceleration in (b).



Question Number	Scheme	Marks
6(a)	$mg = \frac{\lambda \times 4l}{2l} \Rightarrow \lambda = \frac{1}{2}mg$	M1 A1
	$mg - \frac{\frac{1}{2}mg(x+4l)}{2l} = m\ddot{x}$ (or x replaced by $-x$ on both sides)	M1 A2,1,0
	$-\frac{g}{4l}x = \ddot{x}$, hence SHM (with $\omega = \sqrt{\frac{g}{4l}}$)	A1
	$T = \frac{2\pi}{\omega}$	M1
	$2\pi\sqrt{\frac{4l}{g}} = 4\pi\sqrt{\frac{l}{g}}$ *	A1*
		(8)
6(b)	Their $\omega \times 2l$	M1
	$\sqrt{gl}$	A1
	$\frac{1}{2}mgl$	A1
		(3)
6(c)	$-l = 2l \cos \omega t$	M1A1
	$t = \frac{2\pi}{3\omega}$	M1
	$\frac{1}{3}T$	A1
		(4)
		(15)
	Notes for question 6	
6(a)	M1 Resolving vertically and using Hooke's Law	
	A1 cao	
	M1 equation of motion in a general position with correct no. of terms, allow use of accln instead of derivative	
	A2 for a correct equation, A1 for an equation with at most one error	
	A1 for correct equation in correct form	
	M1 Use of correct formula	
	A1* Given answer correctly obtained	
6(b)	M1 for use of correct formula	
	A1 for correct speed	
	A1 cao	
6(c)	M1 for complete method to find t	
	A1 for correct equation(s) (ω does not need to be substituted for this mark)	
	M1 for solving for t	
	A1 cso	



Question Number	Scheme	Marks
ALT (e)	$1.5 = 4 \sin\left(\frac{1}{5}\pi t\right) \Rightarrow t_A = \frac{5}{\pi} \sin^{-1}\left(\frac{1.5}{4}\right)$ $2.5 = 4 \sin\left(\frac{1}{5}\pi t\right) \Rightarrow t_B = \frac{5}{\pi} \sin^{-1}\left(\frac{2.5}{4}\right)$ ${}_A t_B = t_B + t_A = \frac{5}{\pi} \sin^{-1}\left(\frac{2.5}{4}\right) + \frac{5}{\pi} \sin^{-1}\left(\frac{1.5}{4}\right) = 1.6862.. \text{ 1.7 or better}$	M1A1 A1 A1
6(a)	$T = \frac{\lambda x}{l} \Rightarrow 30 = \frac{\lambda \times 0.3}{0.5}$ $\lambda = 50 *$	M1A1 A1* (3)
(b)	 Extension = 0.5 m (used in (b) or (c)) $T = \frac{50 \times 0.5}{0.5} = (50)$ $R(\uparrow) \quad 2T \cos \theta - 1.2g = 1.2a$ $100 \times \frac{3}{5} - 1.2 \times 9.8 = 1.2a$ $a = 40.2 \quad a = 40 \text{ or } 40.2 \text{ m s}^{-2} \text{ (positive)}$	B1 M1A1ft M1 A1ft A1 (6)
(c)	$\text{E.P.E.} = \frac{1}{2} \times 50 \times \frac{0.5^2}{0.5}$ $1.2g \times 0.3 + \frac{1}{2} \times 1.2v^2 = \frac{1}{2} \times 50 \times \frac{0.5^2}{0.5} - \frac{1}{2} \times 50 \times \frac{0.3^2}{0.5}$ $v^2 = \frac{1}{0.6} \left(25 \times \frac{0.5^2}{0.5} - 25 \times \frac{0.3^2}{0.5} - 1.2g \times 0.3 \right) (= 7.452)$ $v = 2.730... = 2.7 \text{ or } 2.73 \text{ ms}^{-1}$	B1ft (any correct EPE) M1A1A1 DM1 A1 (6)



Question Number	Scheme	Marks
		[15]
(a) M1 A1 A1*	Use HL with $T = 30$ Correct equation Obtain given value for λ from fully correct working	
(b) B1 M1 A1ft M1 A1ft A1	Correct extension, seen explicitly or used in (b) or (c) Extension 0.25 if half string used. Use HL to form an equation using $\lambda = 50$ and their extension. Correct equation, ft their extension Attempt a vertical equation of motion. Must have 3 terms with T resolved a could be negative. Correct equation with their value of T Correct value of the acceleration (positive). Must be 2 or 3 sf.	
(c) B1ft M1	Either EPE correct, follow through their extension. Energy equation, from C to the ceiling. PE, KE and 2 EPE terms required. EPE of the form $k \frac{x^2}{l}$	
A1 A1 DM1 A1	Both EPE terms correct. Completely correct equation Solve for v^2 or v Correct value for v . Must be 2 or 3 sf (unless penalised in (b)).	



Question Number	Scheme	Marks
7(a)	$\text{EPE Gain} = \frac{2mgx^2}{2a}$	B1
	$\text{PE loss} = mgx \sin \alpha$	B1
	$\text{WD against friction} = \mu mg \cos \alpha \times x$	B1
	$\mu mg \cos \alpha \times x = mgx \sin \alpha - \frac{2mgx^2}{2a}$	M1
	$x = AB = a(\sin \alpha - \mu \cos \alpha)^*$	A1*
		(5)
7(b)	$\mu mg \cos \alpha \times y = mgy \sin \alpha - \frac{1}{2}mv^2 - \frac{2mgy^2}{2a}$	M1A2,1,0
	At max speed, $\mu mg \cos \alpha = mg \sin \alpha - \frac{2mgy}{a}$	M1
	$y = \frac{1}{10}a$	A1
	Use their y value to find the max speed	M1
	$v = \sqrt{\frac{ag}{50}}$ oe	A1
		(7)
7(b)	$\mu mg \cos \alpha \times y = mgy \sin \alpha - \frac{1}{2}mv^2 - \frac{2mgy^2}{2a}$	M1A2,1,0
7(c)	At B , nett force down plane $= \frac{3}{5}mg - \frac{2mgx}{a} = \frac{1}{5}mg$	M1
	Max friction available $= \frac{1}{2} \times mg \times \frac{4}{5} = \frac{2}{5}mg$	B1
	Hence, friction $= \frac{1}{5}mg$ up and P remains at B .	A1
		(3)
		(15)
	Notes for question 7	
7(a)	B1 correct expression	
	B1 correct expression	
	B1 correct expression	
	M1 for energy equation dim correct with correct terms, condone sign errors	
	A1* for given answer correctly obtained	
7(b)	M1 for energy equation, dim correct with correct terms, condone sign errors	
	A2 for a correct unsimplified equation, A1 for for a correct unsimplified equation with at most one error	
	M1 for finding the resultant force parallel to the plane and equating to 0 or differentiating energy equation wrt y and equating $\frac{dv}{dy}$ to 0	



Question Number	Scheme	Marks
	A1 for correct value of y	
	M1 for using their y value to find the max speed	
	A1 cao	
7(b)	M1 for energy equation, dim correct with correct terms, condone sign errors	
7(c)	M1 for finding nett force up or down plane, correct terms, condone sign errors	
	B1 for max friction	
	A1 correct conclusion and justification	



Question Number	Scheme	Marks
7(a)	$\frac{1}{2}mU^2 - \frac{1}{2}mv^2 = \frac{2mgx^2}{2l}$	M1 A1A1
	$v^2 = U^2 - \frac{2gx^2}{l} *$	A1* (4)
7(b)	$2v \frac{dv}{dx} = -\frac{4gx}{l}$	M1A1
	$\ddot{x} = -\frac{2g}{l}x$, SHM ($\omega = \sqrt{\frac{2g}{l}}$)	A1
	Period = $\frac{2\pi}{\omega} = \pi \sqrt{\frac{2l}{g}} *$	DM1A1* (5)
7(c)	$\sqrt{\frac{gl}{2}} = a \sqrt{\frac{2g}{l}}$ OR $0 = \frac{gl}{2} - \frac{2ga^2}{l}$	M1
	$a = \frac{1}{2}l$	A1
	time from $x = a$ to $x = \frac{1}{4}l$, t given by: $\frac{1}{4}l = \frac{1}{2}l \cos \sqrt{\frac{2g}{l}}t$	M1
	$t = \frac{\pi}{3} \sqrt{\frac{l}{2g}}$	A1
	Time = $\frac{1}{4}$ period + time from $x = a$ to $x = \frac{1}{4}l$	M1
	$= \frac{5\pi}{6} \sqrt{\frac{l}{2g}}$	A1 (6)
		(15)
	Notes for question 7	
7(a)		
M1	Use conservation of energy equation with 2KE terms and 1EPE term. Note there are rearrangements. Dimensionally correct, terms of the correct structure, condone sign errors. EPE of the form $\frac{1}{2}kx^2$	
A1	For an unsimplified equation with at most one error	
A1	For a correct unsimplified equation	
A1*	Given answer obtained from complete and correct working. Must include a line of working before reaching the given answer.	
7(b)	Note: In (b) it is possible to score M1A1A0 DM1A1*	
M1	For differentiating wrt x . Powers of v and x to reduce by 1 and $\frac{dv}{dx}$ seen. M0 for an approach that does not involve differentiating with respect to x eg N2L	
A1	A correct differentiated equation	
A1	Correct SHM equation. Must use $\ddot{x}$ for acceleration and conclude SHM.	
DM1	Dependent on previous M. Correct use of period = $\frac{2\pi}{\omega}$	

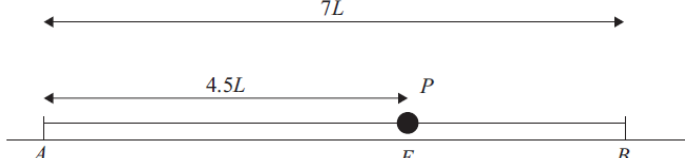
Question Number	Scheme	Marks
A1*	Given answer correctly obtained. Must include a line of working between $\ddot{x} = -\omega^2 x$ and the given answer. Eg $\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2g}{l}}} = \pi\sqrt{\frac{2l}{g}}$ Or $\omega = \sqrt{\frac{2g}{l}}, \text{ period} = 2\pi\sqrt{\frac{l}{2g}} = \pi\sqrt{\frac{2l}{g}}$	
7(c)		
M1	For use of $U = a\omega$ OR energy equation with $v = 0$ and $x = a$ to find the amplitude.	
A1	For correct amplitude, $\frac{l}{2}$	
M1	For a complete method to find the partial time with their calculated a and their ω - Use of $x = a \cos(\omega t)$ where $\frac{1}{4}l = \frac{1}{2}l \cos\sqrt{\frac{2g}{l}}t$ to give a partial time. - Use of $x = a \sin(\omega t)$ where $\frac{1}{4}l = \frac{1}{2}l \sin\sqrt{\frac{2g}{l}}t$ to give a partial time.	
A1	For a correct partial time - Use of $x = a \cos(\omega t) \Rightarrow t = \frac{1}{\omega} \frac{\pi}{3}$ - Use of $x = a \sin(\omega t) \Rightarrow t = \frac{1}{\omega} \frac{\pi}{6}$ or $\frac{1}{\omega} \frac{5\pi}{6}$	
M1	For a complete method to find the total time - Using $x = a \cos(\omega t)$ Total time = $\frac{1}{4}$ period + time from $x = a$ to $x = \frac{1}{4}l$ $= \frac{1}{4}\pi\sqrt{\frac{2l}{g}} + \frac{\pi}{3}\sqrt{\frac{l}{2g}}$ - Using $x = a \sin(\omega t)$ with $\frac{1}{\omega} \frac{5\pi}{6}$ Total time = $\frac{1}{\omega} \frac{5\pi}{6}$ - Using $x = a \sin(\omega t)$ with $\frac{1}{\omega} \frac{\pi}{6}$ Total time = $\frac{1}{2}$ period – time from $x = 0$ to $x = \frac{1}{4}l$ $\text{Total time} = \frac{1}{2}\pi\sqrt{\frac{2l}{g}} - \frac{\pi}{6}\sqrt{\frac{l}{2g}}$	
A1	Correct answer of $\frac{5\pi}{6}\sqrt{\frac{l}{2g}} = \frac{5\pi}{3}\sqrt{\frac{l}{8g}} = \pi\sqrt{\frac{25l}{72g}}$ o.e	

Question Number	Scheme	Marks
7(a)	$(T =) \frac{20(1)}{2} = \frac{\lambda \times 0.8}{1.2}$ $\lambda = 15 *$	M1A1 A1* (3)
(b)	Either $1.25\ddot{x} = \frac{15(0.8-x)}{1.2} - \frac{20(1+x)}{2}$ Or $1.25\ddot{x} = \frac{20(1-x)}{2} - \frac{15(0.8+x)}{1.2}$ $\ddot{x} = -18x$	M1A1A1 A1* (4)
(c)	$10 = a\sqrt{18} \Rightarrow a = \frac{10}{\sqrt{18}} \text{ oe}$ When string <i>PB</i> becomes slack $v^2 = 18 \left(\left(\frac{10}{\sqrt{18}} \right)^2 - 0.8^2 \right)$ $v = 9.4063... \text{ } v = 9.4 \text{ or } 9.41 \text{ ms}^{-1}$	B1 M1 A1 (3)
(d)	$0.8 = \frac{10}{\sqrt{18}} \sin \sqrt{18}t_1$ $t_1 = \frac{1}{\sqrt{18}} \sin^{-1} \left(0.8 \frac{\sqrt{18}}{10} \right) (= 0.0816...)$ PA becomes slack when $x = -1$ $(\pm 1) = \frac{10}{\sqrt{18}} \sin \sqrt{18}t_2$ $t_2 = \frac{1}{\sqrt{18}} \sin^{-1} \left(\frac{\sqrt{18}}{10} \right) (= 0.1032...)$ $T = 2(t_1 + t_2) = 2 \left(\frac{1}{\sqrt{18}} \sin^{-1} \left(0.8 \frac{\sqrt{18}}{10} \right) + \frac{1}{\sqrt{18}} \sin^{-1} \left(\frac{\sqrt{18}}{10} \right) \right)$ $= 0.3697... = 0.37 \text{ or } 0.370$	M1A1 A1 M1 A1 A1 (6)
Notes		[16]
(a) M1 A1 A1*	Form an equation by equating the 2 tensions (found using HL) Equation correct Correct answer correctly obtained	
(b) M1 A1 A1 A1*	Equation of motion for <i>P</i> . Acceleration can be <i>a</i> Correct equation of motion with at most one error, acceleration may be <i>a</i> Fully correct equation of motion, acceleration may be <i>a</i> Correct given equation, correctly obtained	

(c) B1 M1 A1	Correct amplitude, $a = \frac{10}{\sqrt{18}}, \frac{5\sqrt{2}}{3}, \frac{\sqrt{50}}{3}, 2.4$ oe Use $v^2 = \omega^2(a^2 - x^2)$ with $x = 0.8$ and their a and ω Correct speed when $x = 0.8$
(d) M1 A1 A1 M1 A1 A1	Use $x = 0.8$ to find the time until PB becomes slack using their a and ω Correct equation Correct time (seen or implied) Allow consistent use of degrees. NB There are alternative method for finding this time but a complete method for the time until PB becomes slack must be used for the M mark to be awarded. Use $x = \pm 1$ to find the time until PA becomes slack (as before, alternative methods must be complete) using their a and ω Correct time obtained. Ignore consistent use of degrees. Complete to obtain the correct value of T
ALT (c) M1 B1 (treat as A1) A1	Conservation of Energy, O to slack Dimensionally correct energy equation with 3 EPE terms and 2 KE terms $\frac{20 \times 1^2}{2 \times 2} + \frac{1.25 \times 10^2}{2} + \frac{15 \times 0.8^2}{2 \times 1.2} = \frac{20 \times 1.8^2}{2 \times 2} + \frac{1.25 \times v^2}{2}$ Correct answer. $v = 9.4063...$ $v = 9.4$ or 9.41 ms^{-1}
ALT 7 (d) M1 A1 A1 M1 A1 A1	Using cos $0.8 = \frac{10}{\sqrt{18}} \cos \sqrt{18} t_1$ $t_1 = \frac{1}{\sqrt{18}} \cos^{-1} \left(0.8 \frac{\sqrt{18}}{10} \right) (= 0.2886...)$ $-1 = \frac{10}{\sqrt{18}} \cos \sqrt{18} t_2$ $t_2 = \frac{1}{\sqrt{18}} \cos^{-1} \left(-\frac{\sqrt{18}}{10} \right) (= 0.4735...)$ $T = 2(t_2 - t_1) = 2 \left(\frac{1}{\sqrt{18}} \cos^{-1} \left(-\frac{\sqrt{18}}{10} \right) - \frac{1}{\sqrt{18}} \cos^{-1} \left(0.8 \frac{\sqrt{18}}{10} \right) \right)$ $= 0.3697... = 0.37 \text{ or } 0.370$

Question number	Scheme	Marks
7(a)	$\frac{\lambda(D-l)}{l} = mg$	M1A1
	$\frac{\lambda(2l)^2}{2l} = mg \times 3l$	M1A1A1
	$D = \frac{5l}{3} *$	A1*
		(6)
7(b)	$mg - T = m\ddot{x}$ or $T - mg = m\ddot{x}$	M1
	$mg - \frac{3mg}{2l}(\frac{2l}{3} + x) = m\ddot{x}$ or $\frac{3mg}{2l}(\frac{2l}{3} - x) - mg = m\ddot{x}$	dM1A1
	$-\frac{3g}{2l}x = \ddot{x}$ hence SHM	A1
	period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3g}{2l}}}$ { $\omega = \sqrt{\frac{3g}{2l}}$ }	M1
	$= 2\pi\sqrt{\frac{2l}{3g}} *$	A1*
		(6)
7(c)	$-\frac{2l}{3} = \frac{4l}{3} \cos\sqrt{\frac{3g}{2l}}t$	M1A1A1
	$t = \frac{2\pi}{3} \sqrt{\frac{2l}{3g}}$	A1
	OR	
	Complete method $t = \frac{1}{4}2\pi\sqrt{\frac{2l}{3g}} + t_1$ where $\frac{2l}{3} = \frac{4l}{3} \sin\sqrt{\frac{3g}{2l}}t_1$	M1A1A1
	$t = \frac{2\pi}{3} \sqrt{\frac{2l}{3g}}$ oe	A1
	OR	
	Complete method $t = \frac{1}{2}2\pi\sqrt{\frac{2l}{3g}} - t_1$ where $\frac{2l}{3} = \frac{4l}{3} \cos\sqrt{\frac{3g}{2l}}t_1$	M1A1A1
	$t = \frac{2\pi}{3} \sqrt{\frac{2l}{3g}}$ or equivalent exact form.	A1
		(4)
		(16)
	Notes	
7(a)		
M1	Use Hooke's law in D and equate to mg	
A1	Correct equation	

M1	Energy equation with correct no. of terms. EPE of the form $\frac{\lambda x^2}{kl}$, $k \neq 1$
A1	Equation with at most one error
A1	Correct equation
A1*	Given answer correctly obtained
7(b)	
M1	Equation of motion in a <i>general</i> position, allow a for acceleration, correct no. of terms, condone sign errors
dM1	Use Hooke's Law to sub for the tension with extension measured from the equilibrium position and allow a for acceleration
A1	Correct unsimplified equation, allow a for acceleration
A1	Correct SHM equation and conclusion. Must use $\ddot{x}$ for acceleration and conclude SHM.
M1	Use of $\frac{2\pi}{\omega}$ where ω has come from an attempt at using N2L at a general point.
A1*	Obtain the given answer for the period. Must follow from fully correct working, including N2L. At least one line of working must be seen between $\ddot{x} = -\omega^2 x$ and reaching the given answer. Eg - period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3g}{2l}}} = 2\pi \sqrt{\frac{2l}{3g}}$ $\omega = \sqrt{\frac{3g}{2l}}$, period = $\frac{2\pi}{\omega} = 2\pi \sqrt{\frac{2l}{3g}}$
(c)	
M1	Complete method to find the required time. Do not ISW. For example, If the sine approach is used, it must include $\frac{1}{4}T$ + their t value for M1. If the cos approach is used with $+\frac{2l}{3}$, it must include $\frac{1}{2}T$ – their t value for M1. The correct ω must be used. For the method, condone any multiple of l for the amplitude.
A1	Equation with at most one error
A1	Correct equation
A1	Cao

Question Number	Scheme	Marks
7(a)		
	Equation of motion for P at a general position $T_{PB} - T_{PA} = m\ddot{x}$	M1
	Use HL in the equation of motion	M1
	$\frac{2mg}{2L}(0.5L - x) - \frac{mg}{3L}(1.5L + x) = m\ddot{x}$	A1
		A1
	Obtain SHM equation of the correct form and conclude $-\frac{4g}{3L}x = \ddot{x}$ and SHM	A1*
	Use of $\omega^2 = \frac{4g}{3L}$ with the formula period = $\frac{2\pi}{\omega}$	dM1
	Must include working that leads to the given answer for the period. For example - Clearly identify ω within the period formula to reach the answer period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{4g}{3L}}} = \pi\sqrt{\frac{3L}{g}}$ - Clearly identify ω and state the period formula to reach the answer $\omega = \sqrt{\frac{4g}{3L}}$, period = $\frac{2\pi}{\omega} = \pi\sqrt{\frac{3L}{g}}$ $\Rightarrow$ (period =) $\pi\sqrt{\frac{3L}{g}}$	A1*
		(7)
7(b)	amplitude = $0.5L$	B1
	$v_{MAX} = a \times \sqrt{\frac{4g}{3L}}$	M1
	$= \sqrt{\frac{gL}{3}}$	A1
		(3)
7(c)	Method for use of SHM equation for displacement with their amplitude. [1] $x = a \cos \omega t$ [2] $x = a \sin \omega t$ [3] $x = -a \cos \omega t$ [4] $x = -a \sin \omega t$ [5] Any of the above with their x found after use of $v^2 = \omega^2(a^2 - x^2)$	M1

	Form SHM equation with $v = \sqrt{\frac{gL}{12}}$ [1] $v = -a\omega \sin \omega t$ [2] $v = a\omega \cos \omega t$ [3] $v = a\omega \sin \omega t$ [4] $v = -a\omega \cos \omega t$ [5] $v^2 = \omega^2 (a^2 - x^2)$	M1
	Correct equation in g, L, t (and ω). [1] $\sqrt{\frac{gL}{12}} = -\sqrt{\frac{gL}{3}} \sin \omega t$ [2] $\sqrt{\frac{gL}{12}} = \sqrt{\frac{gL}{3}} \cos \omega t$ [3] $\sqrt{\frac{gL}{12}} = \sqrt{\frac{gL}{3}} \sin \omega t$ [4] $\sqrt{\frac{gL}{12}} = -\sqrt{\frac{gL}{3}} \cos \omega t$ [5] Correct expression for x: $\frac{gL}{12} = \omega^2 \left(\frac{L^2}{4} - x^2 \right) \Rightarrow x = L \frac{\sqrt{3}}{4}$ Accept answers from correct working which may include changing methods eg using [1] initially then switching to [3] in order to use an acute angle.	A1
	Correct expression for a relevant time. No need to replace ω [1] $t = \frac{1}{\omega} \sin^{-1} \left(-\frac{1}{2} \right) \left(= -\frac{\pi}{6\omega} \right)$ [2] $t = \frac{1}{\omega} \cos^{-1} \left(\frac{1}{2} \right) \left(= \frac{\pi}{3\omega} \right)$ [3] $t = \frac{1}{\omega} \sin^{-1} \left(\frac{1}{2} \right) \left(= \frac{\pi}{6\omega} \right)$ [4] $t = \frac{1}{\omega} \cos^{-1} \left(-\frac{1}{2} \right) \left(= \frac{2\pi}{3\omega} \right)$ [5] Correct for their approach above Accept answers from correct working which may include changing methods	A1
	Complete method to find the total time in terms of L and g [1] $-4t = -4 \left(-\frac{\pi}{6} \right) \left(\frac{1}{2} \sqrt{\frac{3L}{g}} \right)$ [2] period $-4t = \pi \sqrt{\frac{3L}{g}} - 4 \left(\frac{\pi}{3} \right) \left(\frac{1}{2} \sqrt{\frac{3L}{g}} \right)$ [3] $4t = 4 \left(\frac{\pi}{6} \right) \left(\frac{1}{2} \sqrt{\frac{3L}{g}} \right)$	dM1

	[4] $4t - \text{period} = 4\left(\frac{2\pi}{3}\right)\left(\frac{1}{2}\sqrt{\frac{3L}{g}}\right) - \pi\sqrt{\frac{3L}{g}}$ [5] Correct for their approach above Accept answers from correct working which may include changing methods.	
	$\pi\sqrt{\frac{L}{3g}}$ o.e.	A1
		(6)
		(16)
Notes for question 7		
(a)		
M1	Equation of motion in a <i>general</i> position ie the tension expressions never take a fixed value. Allow <i>a</i> for acceleration here. Required terms present with no extras. Condone sign errors but must have a difference between T_{AP} and T_{BP} .	
M1	Use of Hooke's Law in an equation of motion in a general position with natural length and modulus of elasticity paired correctly. Accept extensions in terms of L and x if they sum to $2L$. Allow <i>a</i> for acceleration for the method mark only. Condone sign errors.	
A1	Unsimplified equation with at most one error. Acceleration must have the form $\ddot{x}$ seen at some stage. Incorrect acceleration notation is an accuracy error.	
A1	Fully correct unsimplified equation. Must use $\ddot{x}$ for acceleration at some stage.	
A1*	Correct SHM equation and conclusion. Equation must be in the required form, $\ddot{x} = -\omega^2 x$ with $\ddot{x}$ for acceleration and where ω is correct in terms of g and L . Conclusion must include 'SHM'	
dM1	Dependent on both previous M's. Use of $\frac{2\pi}{\omega}$ where ω has come from an attempt at using N2L at a general point.	
A1*	Obtain the given answer for the period. Must follow from a complete and correct solution. At least one line of working must be seen between $\ddot{x} = -\frac{4g}{3L}x$ and reaching the given period. N.B. The score of M1 DM1 A1 A1 A0* DM1 A1* is possible if there is no conclusion of 'SHM' The score of M1 DM1 A1 A0 A0* DM1 A1* is possible if $\ddot{x}$ is not used for acceleration.	
(b)		
B1	Amplitude = $0.5L$, seen or implied	
M1	Complete correct method, amplitude $\times \sqrt{\frac{4g}{3L}}$. No need for amplitude to be substituted.	
A1	Correct answer in terms of L and g expressed as a single term then ISW.	

	i.e. For the final A, accept $\frac{1}{2}\sqrt{\frac{4gL}{3}}$ but not $\frac{1}{2}\times\sqrt{\frac{4gL}{3}}$ until expressed as a single term.	
(c)	N.B. A score of M0M1 is only possible for the first 2 marks if [5] is used to find x but this is never substituted correctly into an SHM displacement equation.	
M1	Method using SHM equation for displacement. Award if seen in part (c) with <i>their</i> amplitude substituted. This mark may be implied by using the SHM equation for velocity in terms of t .	
M1	Method using a relevant SHM equation with velocity, $v = \sqrt{\frac{gL}{12}}$	
A1	Correct equation in g, L, t (and ω) using $v = \sqrt{\frac{gL}{12}}$	
A1	Correct expression for a relevant time. No need to replace ω .	
dM1	Dependent on both previous M marks. Complete method to find the total time in terms of L and g only.	
A1	Correct answer in terms of L and g e.g. $\pi\sqrt{\frac{L}{3g}}, \frac{\pi}{3}\sqrt{\frac{3L}{g}}$ Accept any equivalent expressed as a single term then ignore any subsequent surd manipulation. N.B. $4\times\left(\frac{\pi}{12}\sqrt{\frac{3L}{g}}\right)$ is not a single term until written without the multiplication sign i.e. $\frac{4\pi}{12}\sqrt{\frac{3L}{g}}$ Do not ISW if the method continues and is not complete at this stage.	

Question Number	Scheme	Marks
7a)	$\frac{4(x-2)}{2} = \frac{2(7-x-3)}{3}$	M1A1A1
	$6(x-2) = 2(4-x)$	
	$x = 2.5(\text{m})$ *	A1
		(4)
ALT	$(T_A =) \frac{4x}{2} = \frac{2y}{3} (= T_B) \quad x + y = 2$	M1A1
	$x = \frac{1}{2}$ or $y = 1.5$	A1
	Distance $AO = 2.5(\text{m})$	A1
7bi)	$2\ddot{y} = \frac{2(1.5-y)}{3} - \frac{4(y+0.5)}{2}$ OR $2\ddot{y} = \frac{4(-y+0.5)}{2} - \frac{2(1.5+y)}{3}$ (y measured towards B) (y measured towards A)	M1A1
	$\ddot{y} = -\frac{4}{3}y = -\omega^2 y \quad \therefore \text{SHM}$	M1A1
		(4)
7bii)	$\omega^2 = \frac{4}{3}$	B1
	$(2v_{\max} = 6 \Rightarrow) v_{\max} = 3(\text{ms}^{-1})$	B1
	$v_{\max} = a\omega = \frac{2a}{\sqrt{3}}$ Accept $1.2a$ or better	M1
	$3 = a \frac{2}{\sqrt{3}} \quad a = \frac{3\sqrt{3}}{2}(\text{m})$ Accept 2.6 or better	A1
		(4)
7c)	$v^2 = \frac{4}{3} \left(\frac{27}{4} - \left(\frac{3}{2} \right)^2 \right)$	M1
	$v = \sqrt{6}(\text{ms}^{-1})$ Accept 2.4 or better	A1 (2)

Question Number	Scheme	Marks
7d)	$\frac{3}{2} = \frac{3\sqrt{3}}{2} \sin(\omega t)$	M1A1
	$t = 0.53(\text{s})$ or better	A1
		(3)
ALT	$x = a \cos \omega t \Rightarrow 1.5 = \frac{3\sqrt{3}}{2} \cos\left(\frac{2}{\sqrt{3}}\right)t$	
	$\text{time} = \frac{\pi}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	M1A1
	$t = 0.53$ or better	A1
		[17]

(a)		
M1	Equate tension in the two strings. Must be using $k\lambda \frac{x}{l}$ and the sum of the extensions must be 2	
A1	Correct LHS	
A1	Correct RHS	
A1*	Given result found from fully correct working.	
ALT		
M1	Obtain 2 equations using the tensions in the 2 strings and sum of extensions = 2 Must be using $k\lambda \frac{x}{l}$	
A1	2 correct equations	
A1	Correct extension for either string	
A1*	Obtain given result from fully correct working.	

b i)		
M1	Equation of motion with two variable tensions. Allow a or $\ddot{y}$	
A1	Correct equation. Allow a or $\ddot{y}$	
M1	Rearrange to required form. Must now be $\ddot{y}$	
A1	Correct result, from fully correct working and concluding statement.	
(ii)		
B1	Correct ω or ω^2	
B1	$v_{\max} = 3$ or $2v_{\max} = 6$ seen explicitly or used	
M1	Use $v_{\max} = a\omega$ with their ω or use $v^2 = \omega^2(a^2 - x^2)$ with their ω and $x = 0$	
A1	$a = \frac{3\sqrt{3}}{2}$ (allow 2.6 or better)	
(c)		
M1	Use of $v^2 = \omega^2(a^2 - x^2)$ with $x = 1.5$ and their ω and a OR attempt an energy equation with the correct number of terms	
A1	$v = \sqrt{6}$ (2.4 or better)	
7(d)		
M1	Use of $x = a \sin(\omega t)$, with $x = 1.5$ and their ω and a	
A1ft	Correct equation. Ft their ω and a	
A1	$t = 0.53$ or better ($t = 0.533021.....$)	
ALT		
M1	Complete method using cosine.	
A1ft	Correct equation (or equations) follow through their ω and a	
A1	$t = 0.53$ or better ($t = 0.533021.....$)	