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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Circular Motion (M3.3)

International A Level Mathematics · M3: Mechanics 3 (WME03)

29 questions · 327 marks · Mark schemes included

Past papers January 2020 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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2. Q1	IAL M3 (WME03) October 2020	6	5
3. Q2	IAL M3 (WME03) (unused paper) January 2022	7	7
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6. Q2	IAL M3 (WME03) June 2024	8	17
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QUESTIONS

29 questions · 327 marks · 2 sub-topics

SUB-TOPIC M3.3a

HORIZONTAL CIRCULAR MOTION

Sub-topic M3.3a · 15 questions · 144 marks · lowest tariff first



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1. A rough disc is rotating at a constant angular speed of 5 revolutions per minute about a vertical axis. The axis is perpendicular to the plane of the disc and passes through the centre of the disc. A particle, P , of mass m kg is placed on the disc at distance 0.2 m from the axis. The particle does not move relative to the disc. The coefficient of friction between P and the disc is μ .

Find the smallest possible value of μ .

(6)



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Q1

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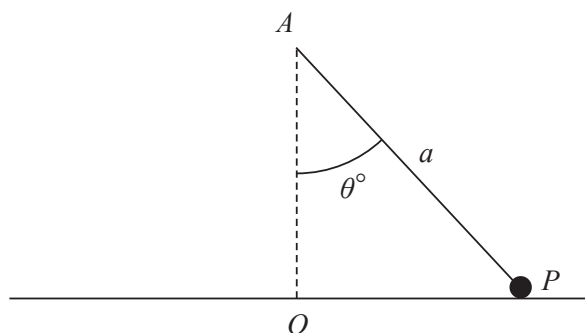


Figure 2

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a point A which lies above a smooth horizontal table. The particle P moves in a horizontal circle on the table with the string taut. The centre of the circle is the point O on the table, where AO is vertical and the string makes a constant angle θ° with AO , as shown in Figure 2.

Given that P moves with constant angular speed $\sqrt{\frac{2g}{a}}$, find the range of possible values of θ

(7)





Question 2 continued

Lined area for writing the answer to Question 2 continued.



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



(Total for Question 2 is / marks)







(Total for Question 2 is 8 marks)



Question 2 continued

(Total for Question 2 is 8 marks)





Question 2 continued

(Total for Question 2 is 9 marks)



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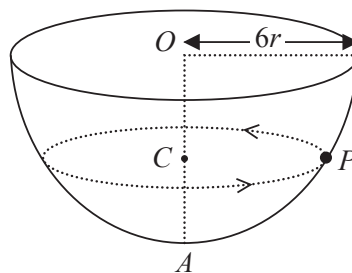


Figure 1

A hemispherical bowl of internal radius $6r$ is fixed with its circular rim horizontal. The centre of the circular rim is O and the point A on the surface of the bowl is vertically below O . A particle P moves in a horizontal circle, with centre C , on the smooth inner surface of the bowl. The particle moves with constant angular speed $\sqrt{\frac{g}{4r}}$. The point C lies on OA , as shown in Figure 1.

Find, in terms of r , the distance OC

(9)





Question 2 continued

Lined area for writing the answer to Question 2 continued.



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 9 marks)







(Total for Question 4 is 3 marks)



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Question 3 continued

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(Total 11 marks)

Q3

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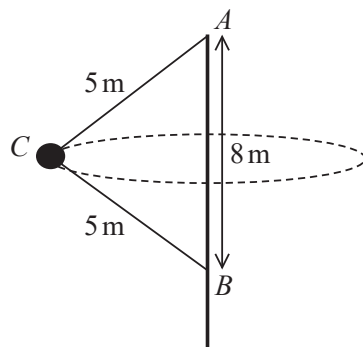


Figure 2

A fairground ride consists of a cabin C that travels in a horizontal circle with a constant angular speed about a fixed vertical central axis. The cabin is attached to one end of each of two rigid arms, each of length 5 m. The other end of the top arm is attached to the fixed point A at the top of the central axis of the ride. The other end of the lower arm is attached to the fixed point B on the central axis, where AB is 8 m, as shown in Figure 2.

Both arms are free to rotate about the central axis.

The arms are modelled as light inextensible rods.

The cabin, together with the people inside, is modelled as a particle.

The cabin completes one revolution every 2 seconds.

Given that the combined mass of the cabin and the people is 600 kg,

- (a) find
- the tension in the upper arm of the ride,
 - the tension in the lower arm of the ride.

(9)

In a refined model, it is assumed that both arms stretch to a length of 5.1 m.

- (b) State how this would affect the sum of the tensions in the two arms, justifying your answer.

(2)

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Question 3 continued

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Q3

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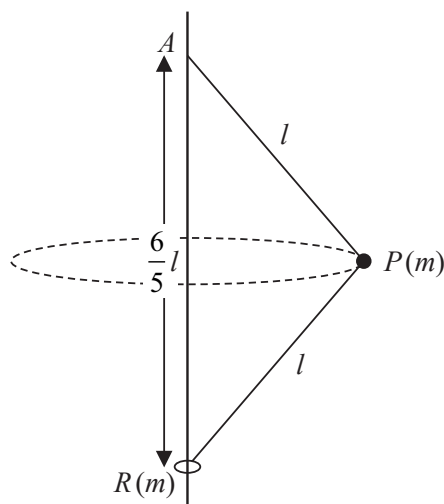
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Figure 2

A small smooth ring R of mass m is threaded on to a thin smooth fixed vertical pole. One end of a light inextensible string of length $2l$ is attached to a point A on the pole. The other end of the string is attached to R . A particle P of mass m is attached to the midpoint of the string. The particle P moves with constant angular speed in a horizontal circle, with both

halves of the string taut, and $AR = \frac{6l}{5}$, as shown in Figure 2.

It may be assumed that in this motion the string does not wrap itself around the pole and that at any instant, the triangle APR lies in a vertical plane.

- (a) Show that the tension in the lower half of the string is $\frac{5mg}{3}$ (3)
- (b) Find, in terms of l and g , the time for P to complete one revolution. (8)

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Question 5 continued

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(Total 11 marks)

Q5

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Question 4 continued

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Question 4 continued

Lined area for writing the answer to Question 4 continued.



(Total for Question 4 is 12 marks)



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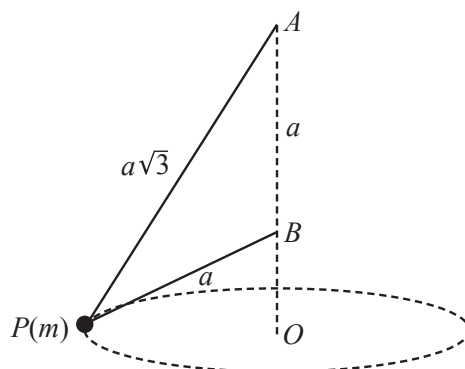


Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length $a\sqrt{3}$. The other end of the string is attached to a fixed point A . The particle P is also attached to one end of a second light inextensible string of length a . The other end of this string is attached to a fixed point B , where B is vertically below A , with $AB = a$.

The particle P moves in a horizontal circle with centre O , where O is vertically below B .

The particle P moves with constant angular speed ω , with both strings taut, as shown in Figure 5.

- (a) Show that the upper string makes an angle of 30° with the downward vertical and the lower string makes an angle of 60° with the downward vertical.

(2)

- (b) Show that the tension in the upper string is $\frac{1}{2}m\sqrt{3}(2g - a\omega^2)$.

(8)

- (c) Show that $\frac{2g}{3a} < \omega^2 < \frac{2g}{a}$

(4)



This image shows a full page of blank, lined paper. It features approximately 28 horizontal blue or grey lines spaced evenly apart, typical of notebook paper. The lines extend across the entire width of the page, leaving small margins at the top and bottom. There are no vertical lines, text, or other markings on the page.



This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(Total for Question 3 is 14 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 16 marks)



SUB-TOPIC M3.3b

VERTICAL CIRCULAR MOTION

Sub-topic M3.3b · 14 questions · 183 marks · lowest tariff first



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued



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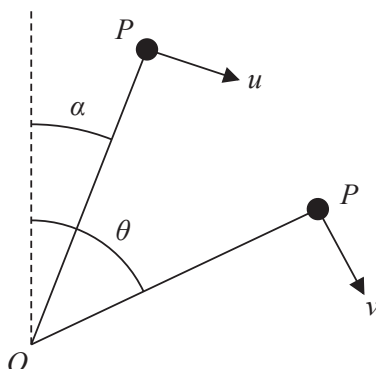


Figure 3

A light rod of length a is free to rotate in a vertical plane about a horizontal axis through one end O . A particle P of mass m is attached to the other end of the rod. The particle P is held at rest with the rod making an angle α with the upward vertical through O ,

where $\tan \alpha = \frac{3}{4}$

The particle P is then projected with speed u in a direction which is perpendicular to the rod. At the instant when the rod makes an angle θ with the upward vertical through O , the speed of P is v , as shown in Figure 3.

Air resistance is assumed to be negligible.

(a) Show that $v^2 = u^2 + \frac{2ag}{5}(4 - 5\cos\theta)$ (4)

It is given that $u^2 = \frac{6ag}{5}$ and P moves in complete vertical circles.

When $\theta = \beta$, the force exerted on P by the rod is zero.

(b) Find the value of $\cos\beta$ (6)

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Question 6 continued

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Question 6 continued

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(Total 10 marks)

Q6

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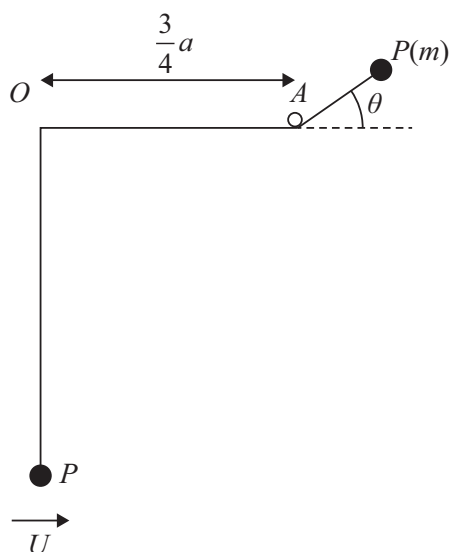


Figure 5

One end of a light inextensible string of length a is attached to a fixed point O . The other end is attached to a particle P of mass m . The particle is held at rest with the string taut and vertical.

The particle is then projected horizontally with speed U , where $U > \sqrt{2ag}$.

When the string is horizontal, it comes into contact with a small smooth peg.

The peg is fixed at the point A on the same horizontal level as O , with $OA = \frac{3}{4}a$.

After the string makes contact with A , the particle P initially moves in a vertical circle centre A .

When the string is taut and AP makes an angle θ with the horizontal, as shown in Figure 5, the speed of P is V and the tension in the string is T .

(a) Show that $V^2 = U^2 - \frac{ag}{2}(4 + \sin \theta)$ (4)

(b) Find T in terms of m , g , U , a and θ (4)

Given that $U = \sqrt{\frac{19ag}{8}}$

(c) find, in terms of a , the height of P above the level of A when the string goes slack. (2)





Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

(Total for Question 5 is 10 marks)

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Question 5 continued

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Lined area for writing the answer to Question 5.

(Total 11 marks)

Q5

Grading boxes for Question 5.



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Question 6 continued

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(Total 13 marks)

Q6

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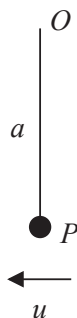


Figure 4

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P is held at rest vertically below O , with the string taut, as shown in Figure 4.

The particle is then projected horizontally with speed u , where $u > \sqrt{2ag}$

Air resistance is modelled as being negligible.

At the instant when the string makes an angle θ with the upward vertical through O , the speed of P is v and the string goes slack.

(a) Show that $3v^2 = u^2 - 2ag$ (7)

From the instant when the string goes slack to the instant when OP is next horizontal, P moves as a projectile.

The time from the instant when the string goes slack to the instant when OP is next horizontal is T

Given that $\theta = 30^\circ$

(b) show that $T = \frac{2v}{g}$ (4)

(c) Hence, show that the string goes taut again when it is next horizontal. (2)

Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



(Total for Question 5 is 13 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 13 marks)



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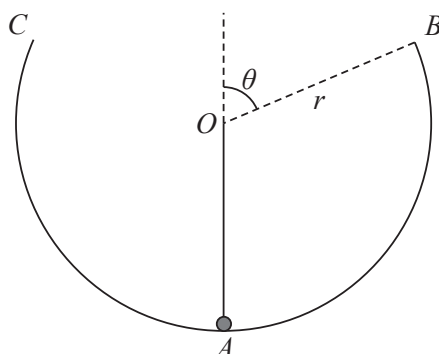


Figure 5

A thin smooth hollow spherical shell has centre O and radius r . Part of the shell is removed to form a bowl with a plane circular rim. The bowl is fixed with the circular rim uppermost and horizontal. The point A is the lowest point of the bowl, as shown in Figure 5.

The point B is on the rim of the bowl, with OB at an angle θ to the upward vertical,

where $\tan \theta = \frac{12}{5}$

A small ball is placed in the bowl at A . The ball is projected from A with horizontal speed u and moves in the vertical plane AOB . The ball stays in contact with the bowl until it reaches B .

At the instant when the ball reaches B , the speed of the ball is v .

By modelling the ball as a particle and ignoring air resistance,

(a) use the principle of conservation of mechanical energy to show that

$$v^2 = u^2 - \frac{36}{13}gr \quad (3)$$

(b) show that $u^2 \geq \frac{41}{13}gr$ (4)

The point C is such that BC is a diameter of the rim of the bowl.

Given that $u^2 = 4gr$

(c) use the model to show that, after leaving the inner surface of the bowl at B , the ball falls back into the bowl before reaching C . (6)



Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued



Question 7 continued

(Total for Question 7 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS



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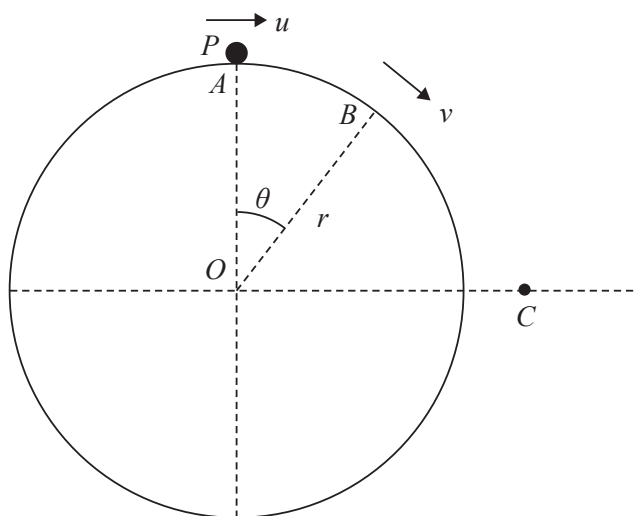


Figure 4

A fixed solid sphere has centre O and radius r .

A particle P of mass m is held at rest on the smooth surface of the sphere at A , the highest point of the sphere.

The particle P is then projected horizontally from A with speed u and moves on the surface of the sphere.

At the instant when P reaches the point B on the sphere, where angle $AOB = \theta$, P is moving with speed v , as shown in Figure 4.

At this instant, P loses contact with the surface of the sphere.

(a) Show that

$$\cos \theta = \frac{2gr + u^2}{3gr} \quad (7)$$

In the subsequent motion, the particle P crosses the horizontal through O at the point C , also shown in Figure 4.

At the instant P passes through C , P is moving at an angle α to the horizontal.

Given that $u^2 = \frac{2gr}{5}$

(b) find the exact value of $\tan \alpha$.

(6)

Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 13 marks)



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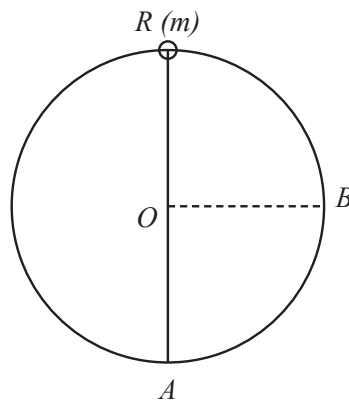


Figure 6

A small smooth ring R of mass m is threaded on to a smooth wire in the shape of a circle with centre O and radius l . The wire is fixed in a vertical plane. The ring R is attached to one end of a light elastic string of natural length l and modulus of elasticity mg . The other end of the elastic string is attached to A , the lowest point of the wire. The point B is on the wire and OB is horizontal.

The ring R is at rest at the highest point of the wire, as shown in Figure 6.

The ring R is slightly disturbed from rest and slides along the wire.

At the instant when R reaches the point B , the speed of R is v and the magnitude of the force exerted on R by the wire is N .

(a) Show that

$$v^2 = 2gl\sqrt{2} \quad (7)$$

(b) Show that

$$N = \frac{1}{2}mg(5\sqrt{2} - 2) \quad (7)$$

[illegible]

[illegible]

Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 14 marks)



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

(Total for Question 5 is 14 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

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Question 6 continued

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(Total for Question 6 is 16 marks)

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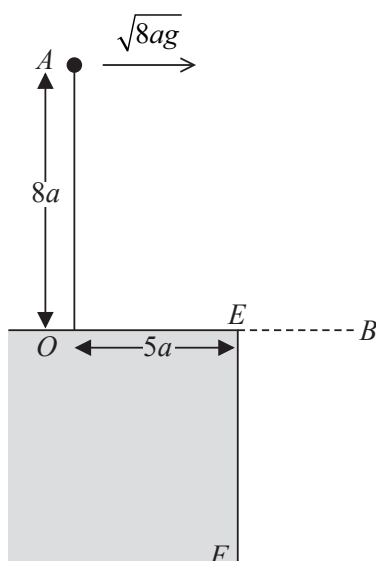
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Figure 5

A particle of mass m is attached to one end of a light inextensible string of length $8a$. The other end of the string is fixed to the point O on the smooth horizontal surface of a desk. The point E is on the edge of the desk, where $OE = 5a$ and OE is perpendicular to the edge of the desk. The particle is held at the point A , vertically above O , with the string taut.

The particle is projected horizontally from A with speed $\sqrt{8ag}$ in the direction OE , as shown in Figure 5.

When the particle is above the level of OE the particle is moving in a vertical circle with radius $8a$.

Given that, when the string makes an angle θ with the upward vertical through O , the tension in the string is T ,

(a) show that $T = 3mg(1 - \cos \theta)$ (7)

At the instant when the string is horizontal, the particle passes through the point B .

(b) Find the instantaneous change in the tension in the string as the particle passes through B . (3)

The particle hits the vertical side EF of the desk and rebounds. As a result of the impact, the particle loses one third of the kinetic energy it had immediately before the impact.

In the subsequent motion the string becomes slack when it makes an angle α with the upward vertical through O .

(c) Show that $\cos \alpha = \frac{7}{12}$ (7)

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Question 7 continued

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(Total 17 marks)

Q7

TOTAL FOR PAPER = 75 MARKS

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Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
1.	$\omega = \frac{10\pi}{60} \text{ (rad s}^{-1}\text{)}$ $F = mg\mu \text{ (N)}$ $F = m \times 0.2 \left(\frac{\pi}{6} \right)^2 = \frac{m\pi^2}{180}$ $mg\mu \geq \frac{m\pi^2}{180}$ $\mu_{\min} = \frac{\pi^2}{180g}, \text{ (0.0056, 0.00560)}$	B1 B1 M1A1ft dM1 A1 [6]
B1 B1 M1 A1ft dM1 A1	Correct angular speed in radians per second, seen anywhere Correct inequality or equation for Friction, seen or used anywhere Attempt the equation of motion along the radius. Must only contain friction and resultant force (give BOD unless clearly not friction). Allow with their ω or just ω . Correct equation. Follow through their ω Eliminate F and solve to find μ . Allow with an inequality or equation. Dependent on previous M1. Correct answer, as shown or 2/3 sf decimal (0.00560). Must not be an inequality now.	

Special Case: If $F \geq mg\mu$ or $F < mg\mu$ used, leading to $\mu = \frac{\pi^2}{180g}$ award max B1B0 M1A1

M1A0

Question Number	Scheme	Marks
1	$\omega = \frac{2\pi}{0.5} (= 4\pi)$	B1
	$R = m\omega^2 \times 0.35 \quad \left(= \frac{28m\pi^2}{5} \right)$	M1
	$F_r = mg$	B1
	$F_r \leq \mu R$	M1
	$mg \leq \mu \frac{28m\pi^2}{5}$	DM1
	$\mu \geq 0.18 \quad (\mu \geq 0.177)$	A1
		[6]

B1 Use of period to find ω .

M1 Equation of motion horizontally. Must be considering R . Acceleration can be in either circular motion form.

B1 Resolve vertically. If only seen within friction inequality, then must be correct way round.

M1 Use of $F_r \leq \mu R$. Condone use of strict inequality for this mark.

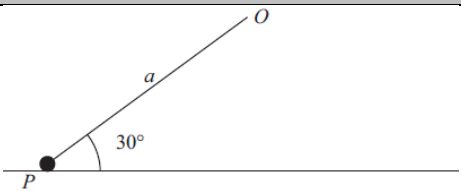
DM1 Substitute in F_r and R . Could be an equation. Dependent on first M mark.

A1 $\mu \geq 0.18 \quad (\mu \geq 0.177)$ **cao**

Note: If equation used throughout and correct inequality added for final answer, full marks are available if no incorrect working seen.

Question Number	Scheme	Marks
2.	$T \cos \theta (+R) = mg$	M1A1
	$T \sin \theta = ma \sin \theta \frac{2g}{a} \quad (T = 2mg)$	M1A1
	$\cos \theta < \frac{1}{2} \text{ or } \cos \theta \leq \frac{1}{2} \text{ or } \cos \theta = \frac{1}{2}$	M1
	$\theta > 60 \text{ or } \theta \geq 60$	A1
	$90 > \theta > 60 \text{ or } 90 > \theta \geq 60$	A1
		(7)
	Notes for question 2	
	M1 for resolving vertically, correct no. of terms, T resolved	
	A1 for a correct equation	
	M1 for equation of motion horizontally	
	A1 for a correct unsimplified equation	
	M1 for producing an appropriate inequality in $\cos \theta$ Allow an equation	
	A1 Must come from an inequality.	
	A1 cao	
3(a)	$a = v \frac{dv}{dx} = -\frac{2}{(2x+1)^3} \quad \text{separate and integrate}$	M1
	$\frac{1}{2} v^2 = \frac{1}{2(2x+1)^2} + (C)$	A1
	$x = 0, v = 1 \Rightarrow C = 0$	M1
	$v = \frac{1}{(2x+1)}$	A1
		(4)
3(b)	$\frac{dx}{dt} = \frac{1}{(2x+1)} \quad \text{separate and integrate}$	M1
	$x^2 + x + (D) = t$	A1
	Complete the square: $(x + \frac{1}{2})^2 - \frac{1}{4} = t$ or use quadratic formula: $x = \frac{-1 \pm \sqrt{1+4t}}{2}$	M1
	$x = \frac{1}{2}(\sqrt{4t+1} - 1) *$	A1* (4)
		(8)
	Notes for question 3	
3(a)	M1 allow omission of - sign, powers increasing by 1	
	A1 correct equation, but allow omission of C	
	M1 use of initial conditions to find C	
	A1 cso	
3(b)	M1 powers increasing by 1	
	A1 correct equation, but allow omission of D	



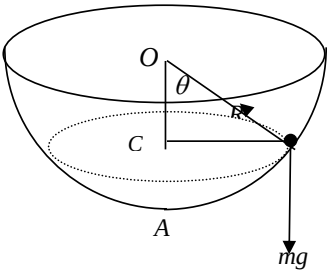
Question Number	Scheme	Marks
		
2.	Vertical equilibrium	M1
	$R + T \sin 30^\circ = mg$	A1
	N2L horizontally	M1
	$T \cos 30^\circ = mr\omega^2$	A1
	$T \cos 30^\circ = ma \cos 30^\circ \left(\frac{g}{2a} \right)$	A1
	Solve for R	dM1
	$(R =) \frac{3mg}{4}$	A1
(7)		
Notes for question 2		
M1	Resolve vertically to form a dimensionally correct equation with all required terms present and no extras. Condone $\pm$ sign errors and sin/cos confusion on the T .	
A1	Correct unsimplified equation	
M1	Use N2L to form a dimensionally correct horizontal equation of motion. All required terms present with no extras. Accept any form of circular acceleration $r\omega^2$, $\frac{v^2}{r}$. Accept correct consistent omission of $\cos 30^\circ$. Accept ' ma ' if acceleration is replaced correctly with a circular form in subsequent working. Condone sin/cos confusion. An incorrect expression for the radius is marked as an accuracy error and not a method error.	
A1	Correct equation in T, m, g (and a) with at most one error.	
A1	Correct equation in T, m, g (and a)	
dM1	Dependent on both previous M marks. Solve a dimensionally correct equation to find R in terms of m and g only.	
A1	Correct answer, accept $0.75mg$. Must be in terms of m and g ,	

Question Number	Scheme	Marks
1.	$\text{Area} = \int_0^a (x^2 + ax) dx = \left[\frac{1}{3}x^3 + \frac{1}{2}ax^2 \right]_0^a = \frac{5a^3}{6}$ $\int \frac{1}{2}y^2 dx = \int_0^a \frac{1}{2}(x^4 + 2ax^3 + a^2x^2) dx$ $= \frac{1}{2} \left[\frac{1}{5}x^5 + \frac{a}{2}x^4 + \frac{a^2}{3}x^3 \right]_0^a \left(= \frac{31a^5}{60} \right)$ $\bar{y} = \frac{\int \frac{1}{2}y^2 dx}{\int y dx} = \frac{31a^5}{60} \div \frac{5a^3}{6} = \frac{31a^2}{50}$	M1A1 M1 DM1A1 M1A1 (7) [7]
M1 A1 M1 DM1 A1 M1 A1	Attempt the area by integration. Powers of both terms to increase by 1. Correct area. Use $\int \frac{1}{2}y^2 dx$ to give $\int_0^a \frac{1}{2}(x^4 + 2ax^3 + a^2x^2) dx$. Limits not needed. Squaring to be correct. For method mark, condone missing $\frac{1}{2}$ or any multiple. Attempt the integration (powers of at least 2 terms to increase by 1). Depends on second M mark. Correct integration and correct limits shown. Limits needed but substitution does not need to be seen. Use $\bar{y} = \frac{\int \frac{1}{2}y^2 dx}{\int y dx}$ Note: This independent method mark is for use of the correct formula. Correct answer.	

Question Number	Scheme	Marks
2	Any correct sin or cos ratio. $T \cos 60^\circ + N = mg$ $T \sin 60^\circ = mr\omega^2 = m\omega^2 \times 2l \sin 60^\circ$ $\frac{1}{2}T + N = mg \quad \frac{1}{2}T = ml\omega^2$ $\Rightarrow ml\omega^2 + N = mg$ $N \geq 0 \Rightarrow l\omega^2 \leq g$ $\omega \leq \sqrt{\frac{g}{l}} \quad *$	B1 M1A1 M1A1 DM1 DM1 A1 * (8) [8]
B1 M1 A1 M1 A1 DM1 DM1 A1*	May be seen explicitly or used in an equation. Attempt at vertical resolution, 3 terms needed. Correct equation. Attempt an equation for NL2 along the radius, acceleration in either form but not 'a'. May have r and v in the equation. Fully correct equation with the acceleration in $r\omega^2$ form and radius in terms of l. Eliminate T Depends on both M marks above. Must see an equation still involving N. Use $N \geq 0$ Depends on all 3 M marks above. Must see correct inequality stated, not $N = 0$ or $N > 0$. Reach the given result from fully correct working.	
ALT B1 M1 A1 DM1 M1 A1 DM1 A1*	For solutions that do not use vertical equilibrium but go straight to a vertical inequality. As above. Forming a correct inequality $T \cos 60 \leq mg$ Attempt NL2 as the main mark scheme. Eliminate T Depends on M marks above. Reach the given result from fully correct working.	

Question Number	Scheme		Marks
	$\frac{\sqrt{a^2 - d^2}}{a} = \frac{\sin \theta = \sqrt{a^2 - d^2}}{a^2} = \sqrt{1 - \frac{d^2}{a^2}}$	$\frac{\sqrt{a^2 - d^2}}{a} = \frac{\cos \theta = \sqrt{a^2 - d^2}}{a^2} = \sqrt{1 - \frac{d^2}{a^2}}$	
2(a)	$R \cos \theta = mg$	$R \sin \theta = mg$	M1A1
	$R = \frac{mga}{d}$		A1 (3)
2(b)	$R \sin \theta = \frac{mv^2}{r}$	$R \cos \theta = \frac{mv^2}{r}$	M1A1A1
	$\frac{mga}{d} \times \frac{\sqrt{a^2 - d^2}}{a} = \frac{mv^2}{\sqrt{a^2 - d^2}}$		DM1
	$v = \sqrt{\frac{g(a^2 - d^2)}{d}}$		A1 (5)
			(8)
	Notes for question 2		
2(a)			
M1	Resolve vertically to form an equation with the correct number of terms and the correct structure. Dimensionally correct, condone sign errors and sin/cos confusion		
A1	Correct equation		
A1	Correct answer.		
2(b)			
M1	Form a horizontal equation of motion with the correct number of terms, condone sign errors and sin/cos confusion. Dimensionally correct. Accept $\frac{v^2}{r}$ or $r\omega^2$ for of acceleration. Condone use of a for radius at this point but M0 if a is used for acceleration.		
A1	Equation with at most one error. An error in the acceleration term is one error (incorrect form of acceleration or radius).		
A1	Correct equation (must use the correct form of acceleration and correct radius).		
DM1	Dependent on previous M. Eliminate R and trig to form an equation in v , g , a and d		
A1	Correct answer ISW		

Question Number	Scheme	Marks
2a)	$\sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}$	
	$T \cos \theta = mg$	M1A1
	$\frac{4}{5}T = mg \rightarrow T = \frac{5mg}{4}$	A1
		(3)
2b)	$r = a + \frac{5a}{4} \times \frac{3}{5} = \frac{7a}{4}$	B1
	$T \sin \theta = m\omega^2 \left(\frac{7a}{4} \right)$	M1A1A1
	$\frac{3}{5} \left(\frac{5mg}{4} \right) = \frac{7}{4} m a \omega^2$	DM1
	$\omega = \sqrt{\frac{3g}{7a}}$	A1
		(6)
		[9]
a)		
M1	Resolving vertically. $T \cos \theta$ or $T \sin \theta$ accepted	
A1	Correct equation	
A1	Correct tension	
b)		
B1	Correct radius of motion seen explicitly or used in N2L	
M1	Attempt at horizontal equation of motion. Allow either form of acceleration. $T \cos \theta$ or $T \sin \theta$ accepted. Allow this mark if $r = \frac{3a}{4}$ used.	
A1	Correct LHS	
A1	Correct RHS. Acceleration must now be in $r\omega^2$ form.	
DM1	Substitute in trig and eliminate T to find value for ω or ω^2 . Depends on first M mark in (b)	
A1	Correct value for ω . (Square root sign must cover all terms)	

Question Number	Scheme	Marks
2.	 $R \sin \theta = m \times 6r \sin \theta \times \frac{g}{4r}$ $R = \frac{3}{2} mg$ $R \cos \theta = mg$ $\frac{3}{2} mg \cos \theta = mg$ $\cos \theta = \frac{2}{3}$ $OC = 6r \cos \theta = 6r \times \frac{2}{3} = 4r$	M1A1A1 M1A1 DM1 A1 M1A1 [9]
Notes		
M1 A1 A1	Attempt NL2 along CP with correct number of terms and forces resolved. Either side correct Fully correct equation Note: If R is not resolved then M0 but do allow if $\sin \theta$ is cancelled from both sides: $R = m \times 6r \times \frac{g}{4r}$ would score M1A1A1 If r is used instead of the radius: $R \sin \theta = m \times r \times \frac{g}{4r}$ would score M1A1A0 (force resolved correctly on LHS but error in radius on RHS)	
M1 A1	Resolve vertically Correct equation	
DM1 A1 M1 A1	Eliminate R between the two equations. Depends on both M marks above Correct value for $\cos \theta$ seen or implied Attempt to obtain OC (allow sin/cos confusion) $OC = 4r$ Note: If θ is the angle with the horizontal then all equations above will appear with $\sin \theta$ and $\cos \theta$ reversed.	

4			If angle is between incline and vertical then $\sin \theta = \frac{4}{5}, \cos \theta = \frac{3}{5}$
	Resolve vertically	M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion.
	$R \sin \theta = mg + F \cos \theta$	A1 A1	Unsimplified equation with at most one error. Correct unsimplified equation
	Equation for horizontal motion	M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion. Accept any form of acceleration for the method mark only.
	$R \cos \theta + F \sin \theta = mr\omega^2$	A1 A1	Unsimplified equation with at most one error. Direction of F consistent with vertical resolution. Incorrect form of acceleration is one error. Correct unsimplified equation
	Use of $F = \mu R$	M1	Used, not just quoted. $F = \frac{1}{4}R$
	Substitute for trig and solve for max ω	DM1	Dependent on all preceding M marks. If more than two equations are produced, the correct two must be used. $\left(R = \frac{20mg}{13}, F = \frac{5mg}{13} \right)$
	$\Rightarrow \omega = \sqrt{\frac{16g}{13r}}$	A1*	Obtain given answer from correct working
		[9]	
		(9)	
Alt1	Using N2L parallel and perpendicular to the incline. Perpendicular $R - mg \sin \theta = mr\omega^2 \cos \theta$ Parallel $F + mg \cos \theta = mr\omega^2 \sin \theta$	M1 A1A1 M1 A1A1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion. Note that the acceleration must have a sin/cos component. Accept any form of acceleration for the method mark only. Mark A's as above. A1A0 Unsimplified equation with at most one error A1A1 Correct unsimplified equation

Question Number	Scheme	Marks
2(a)	$v = \frac{dx}{dt} = \frac{1}{(4x+3)}$ $\frac{dt}{dx} = 4x+3$ $t = \int (4x+3)dx = \frac{1}{2} \times 4x^2 + 3x + c$ OR $\int_0^2 dt = \int_0^x (4x+3)dx = \left[\frac{1}{2} \times 4x^2 + 3x \right]_0^x$ $c = 0$ $t = 2 = \frac{1}{2} \times 4x^2 + 3x \quad 2x^2 + 3x - 2 = 0$ $x = \frac{1}{2} \quad (x = -2)$	M1,dM1A1
(b)	$a = v \frac{dv}{dx} \quad alt: \quad a = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$ $= \frac{1}{4x+3} \times \frac{-4}{(4x+3)^2}$ $ F = \frac{1}{2} \times \frac{4}{(2+3)^3} = \frac{2}{125} = 0.016 \text{ N}$	dM1 A1cso (5) M1 dM1A1 M1 A1cso (5) [10]
(a)	Rewrite as $\frac{dx}{dt}$ and separate variables to reach a form ready for integration	
M1		
dM1	Attempt the integration (at least one power going up).	
A1	Correct integration. Constant/limits not needed.	
dM1	Use $t = 2$ in their expression or substitute correct limits, and solve their 3 term quadratic to find x . If solving an incorrect quadratic, evidence of a correct method must be seen. Depends on the previous M mark.	
A1cso	Obtain $x = \frac{1}{2}$ (and reject -2 if seen) from completely correct work. Constant of integration must have been seen, although we do not need to see evidence of evaluation.	
(b)		
M1	Use $a = v \frac{dv}{dx}$	
dM1	Differentiate the given expression for v and obtain an expression for a . We need to see a power of 2 (or a power of 3 if using $\frac{d}{dx} \left(\frac{1}{2} v^2 \right)$). Depends on the previous M mark.	
A1	Correct expression, any form.	
M1	Use their acceleration in an equation of motion to obtain a value for F . Mass must be included and they must use their value of x . Independent, but must have found an expression for acceleration.	
A1cso	Correct magnitude of F . Correct solution only. Can be fraction or decimal. Must be positive.	
3		
(a)	$\angle PBA = 30^\circ$	B1

Question Number	Scheme	Marks
	$R(\uparrow) \quad T \cos 30^\circ + R \cos 60^\circ = mg$ NL2 horizontally: $T \cos 60^\circ + R \cos 30^\circ = m\omega^2 r, = ma\omega^2 \cos 30^\circ$ $T = \frac{m\sqrt{3}}{2}(2g - a\omega^2)_{\text{o.e.}}$	M1 M1A1,A1 dM1A1 (7)
(b)	$R = 2mg - \frac{3m}{2}(2g - a\omega^2) = \frac{3ma\omega^2}{2} - mg$ Use $R \geq 0$ $\omega \geq \sqrt{\frac{2g}{3a}} \quad *$	M1A1 M1 A1cso (4)
		[11]
(a) B1 M1 M1 A1 A1 dM1 A1	Correct angle, seen explicitly, implied by a correct trig ratio, or used. Attempt a vertical equation with 3 forces, T and R resolved. Angles can be algebraic. Condone sin/cos confusion and use of the same angle for both forces. Equation of motion horizontally, two forces resolved and acceleration in either form. Attempt at radius not needed. Angles can be algebraic. Condone sin/cos confusion and use of the same angle for both forces. Correct LHS Correct acceleration with correct radius (which might be seen later in part (a)). Eliminate R and solve to find expression for T . Depends on both previous M marks. Allow this mark even if they have not found an angle. Correct expression for T (any correct equivalent).	
(b) M1 A1 M1 A1cso*	Attempt to obtain an expression in R . Independent of the M marks in (a), but must have come from 2 equations in T and R . Correct unsimplified expression in R Use of the correct inequality for R Obtain given result from fully correct working.	



Question Number	Scheme	Marks
3(a)	$\cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}$	B1
	$\omega = \pi$	B1
	$T_A \cos \theta - T_B \cos \theta = 600g$	M1A1
	$(T_A - T_B = 750g)$	
	$T_A \sin \theta + T_B \sin \theta = 600 \times \omega^2 \times (5 \sin \theta)$	M1A1
	$(T_A + T_B = 3000\pi^2)$	
	Solve their two equations simultaneously	dM1
	$T_A = 1500\pi^2 + 375g = 18000(N), \quad 18500(N), \quad 18\text{kN}, \quad 18.5\text{kN}$	A1
	$T_B = 1500\pi^2 - 375g = 11000(N), \quad 11100(N), \quad 11\text{kN}, \quad 11.1\text{kN}$	A1
		(9)
(b)	If the length of the arms increased, then the radius of the circle would increase.	B1
	Therefore the total tension would increase.	dB1
		(2)
		[11]



(a)

B1 Correct trig **used** anywhere.

B1 Angular speed **seen**.

M1 Attempt at vertical resolution. Allow mass as m

A1 Correct equation in θ . $m = 600$ needs to be used now or later.

M1 Attempt at horizontal resolution. Acceleration in either form. Attempt at R not needed. Condone sin/cos confusion and use of the same angle for both forces. Allow mass as m

A1 Correct equation in w and θ . m and R must be substituted ($= 3$ or $5\sin \theta$ may be seen later on)

dM1 Solve their equations to find at least one tension. Dependent on both previous M marks.

A1 Correct T_A (must be 2/3 s.f.)

A1 Correct T_B (must be 2/3 s.f.) (only penalise over accuracy on T_A if in both)

(b)

B1 Correct statement about the effect on the radius of the motion.

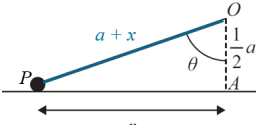
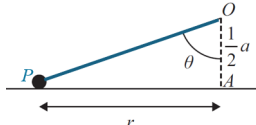
dB1 Conclusion that **total tension** would be greater (must reference the total tension) following a correct statement about the radius.



Question Number	Scheme	Marks
2.	$T = mg \cos \theta$	M1A1
	$T = \frac{2mg \left(\frac{21}{10}a - ka \right)}{ka}$	M1A1
	$\frac{4}{5}mg = \frac{2mg \left(\frac{21}{10}a - ka \right)}{ka}$	dM1
	$k = \frac{3}{2}$ or 1.5	A1
		(6)
	Notes	
M1	Resolve parallel to the string, correct no. of terms, condone sign errors and sin/cos confusion (or resolve in two directions and eliminate the unknown force or use trig on a right-angled triangle of forces) to give an equation in T , mg and θ only	
A1	Correct equation. Trig does not need to be substituted.	
M1	Use Hooke's Law with correct structure.	
A1	Correct equation	
dM1	Substitute trig and eliminate T to produce equation in k only, dependent on previous M's. If x is used for extension, should see $x = \frac{2k}{5}a \rightarrow ka + \frac{2k}{5}a = \frac{21}{10}a$	
A1	cao	
ALT 1	First M1A1	
M1A1	Complete method to form an equation in T and θ Vert: $mg = T \cos \theta + F \sin \theta$ Horiz: $F \cos \theta = T \sin \theta$ Eliminate F eg $\frac{\sin \theta}{\cos \theta} = \frac{mg - T \cos \theta}{T \sin \theta}$	

Question Number	Scheme	Marks
5(a)	Use of cosine rule on triangle APB OR trig. on 'half' of the triangle APB to find one relevant angle.	M1
	Given answers correctly obtained.*	A1*
		(2)
5(b)	$T_A \cos 30^\circ + T_B \cos 60^\circ = mg$	M1 A1
	$T_A \sin 30^\circ + T_B \sin 60^\circ = mr\omega^2$	M1A1A1
	$r = a \sin 60^\circ$ (or $r = a\sqrt{3} \cos 30^\circ$ or $r = a \frac{\sqrt{3}}{2}$)	B1
	Solve for T_A	dM1
	$T_A = \frac{1}{2} m\sqrt{3}(2g - a\omega^2)$ *	A1*
		(8)
5(c)	Attempt to obtain one inequality on ω^2	M1
	Correct inequality	A1
	Attempt to obtain another inequality on ω^2 and use both to obtain answer	M1
	$\frac{2g}{3a} < \omega^2 < \frac{2g}{a}$ *	A1 *
		(4)
		(14)
Notes		
(a)		
M1	Either complete method to obtain one relevant angle.	
A1*	Correct GIVEN angles correctly obtained. Sufficient annotation/justification leading to both given answers eg Stating $\angle OBP = 2 \times \angle OAP$ alone is not sufficient – additional annotation or justification is required. Use of triangles to verify is acceptable.	
(b)		
M1	Resolve vertically, dimensionally correct equation with correct no. of terms, condone sign errors and sin/cos confusion.	
A1	Correct equation	
M1	Equation of motion horizontally: dimensionally correct equation with correct no. of terms, condone sign errors and sin/cos confusion.	
A1	Correct equation, with at most one error. If $r\omega^2$ is never seen, this is an A error.	
A1	Correct equation	
B1	Cao If this is seen in (a) it must be used in (b) for this mark.	
dM1	Solve for T_A in terms of m, a, g and ω	
A1*	Given answer correctly obtained. Must see exactly.	
(c)		
M1	Correct use of either $T_A > 0$ or their $T_B > 0$ oe to obtain one inequality on ω^2 . Could be their expression for either Tension > 0 .	
A1	Correct inequality	
M1	Use both $T_A > 0$ and their $T_B > 0$ to form inequalities in attempt to obtain answer. Could be their expression for either Tension > 0 . Note: $T_B = \frac{3}{2} m a \omega^2 - mg$	
A1*	Given answer correctly obtained	



Question Number	Scheme	Marks
6(a)	Horizontal equation of motion	M1
	$T \sin \theta = \frac{mgr}{a}$	A1
	Use triangle AOP to form an equation in θ , r , a and x 	M1
	Use triangle AOP to form an equation in θ , r and OP 	
	Eg $\sin \theta = \frac{r}{a+x}$	A1
	Eg $\sin \theta = \frac{r}{OP}$ $r = OP \sin \theta$	
	Use of HL with extension $T = \frac{3mgx}{a}$	M1
	Use of HL with OP $T = \frac{3mg(OP-a)}{a}$	
6(b)	Eliminate θ and T	DM1
	Eg $\frac{3mgx}{a} \left(\frac{r}{a+x} \right) = \frac{mgr}{a}$	A1
	Eg $\frac{3mg(OP-a)}{a} \left(\frac{r}{OP} \right) = \frac{mgr}{a}$	
	$(OP =) \frac{3}{2}a$ *	A1*
		(8)
6(b)	Vertical equilibrium	M1
	"R" + $T \cos \theta = mg$	A1
	Use $T = \frac{3mg}{2}$ and $\cos \theta = \frac{1}{3}$ to find an expression for "R" in terms of m and g only	DM1
	$(R =) \frac{1}{2}mg$	A1
		(4)
6(c)	$EPE = \frac{3mg}{2a} \left(\frac{1}{2}a \right)^2 \quad \left(= \frac{3mga}{8} \right)$	B1
	Expression for sum of EPE and KE KE of the form $\frac{1}{2}m(r\omega)^2$ with $\omega = \sqrt{\frac{g}{a}}$ and $r = \sqrt{\left(\frac{3a}{2} \right)^2 - \left(\frac{a}{2} \right)^2}$ $(r\omega = \sqrt{2ag})$	M1



Question Number	Scheme	Marks
	$= \frac{1}{2}m \left(a\sqrt{2}\sqrt{\frac{g}{a}} \right)^2 + \frac{3mg}{2a} \left(\frac{1}{2}a \right)^2$	A1
	$= \frac{11mga}{8}$	A1
		(4)
		(16)
	Notes for question 6	
6(a)		
M1	Horizontal equation of motion. All required terms and no extras. Condone sin/cos confusion. Acceleration does not need to be replaced. Accept any form of acceleration, $r\omega^2$ or $\frac{v^2}{r}$. Condone a for acceleration for this M mark only.	
A1	Correct equation with acceleration correctly replaced and correct radius. A0 if OP is the radius.	
M1	Use triangle OPA to form an equation in θ , r and either OP or a and x . Condone sin/cos confusion. (Pythagoras alone is M0)	
A1	Correct equation	
M1	Use of Hooke's Law in terms of extension or OP	
DM1	Dependent on all 3 previous M marks. Eliminate T and θ to produce an equation in m , g , a , r and either OP or x (r 's may have been cancelled)	
A1	Correct equation	
A1*	Given answer obtained from complete and correct working. Condone missing OP . Accept $\frac{3}{2}a$, $\frac{3a}{2}$.	
6(b)		
M1	Method using vertical equilibrium. All required terms present and no extras. Condone sign errors and sin/cos confusion. If seen in part (a), it must be used in (b) to earn the marks. Note that R is not defined in the question, may use another letter or even the wording from the question.	
A1	Correct unsimplified equation.	
DM1	Dependent on previous M. Substitute for T and $\cos\theta$ to obtain an expression for " R " in terms of m and g only.	
A1	Correct answer in terms of m and g	
6(c)		
B1	Correct expression for EPE	
M1	Method to find total, KE + EPE. Must have terms added together and not just listed. Required terms only. Dimensionally correct expression with energy terms of the correct structure. KE term, must use $v = r\omega$ with $\omega = \sqrt{\frac{g}{a}}$ and $r = \sqrt{\left(\frac{3a}{2}\right)^2 - \left(\frac{a}{2}\right)^2}$	
A1	Correct unsimplified expression.	
A1	Correct answer. Must be simplified to one term in m , g and a . Accept $1.375mga$, $1.38mga$, $1.4mga$.	



Question Number	Scheme	Marks
4a)	Initially $T_{IN} = mg \cos 60$	M1
	$T_{IN} = \frac{mg}{2}$	A1
		(2)
4b)	Energy to the lowest point $\frac{1}{2}mv^2 - mg(l) = -mg(l \sin 30)$	M1A1A1
	$T_{FI} - mg = m\frac{v^2}{r}$	M1A1
	$T_{FI} = m\left(\frac{gl}{l}\right) + mg$	DM1
	$T_{FI} = 2mg = 4T_{IN}^*$	A1* (7)
		[9]
(a)		
M1	Equation of motion towards centre at <i>A</i> . Must have $v = 0$. Weight must be resolved and tension not resolved. Allow with cos or sin of 60° or 30° .	
A1	Correct tension	
(b)		
M1	Energy equation from <i>A</i> to the lowest point. One KE and a difference in GPE required.	
A2	Correct equation. -1 for each error.	
M1	Equation of motion at the lowest point, with acceleration in either form. Tension and weight needed.	
A1	Correct equation. Acceleration must now be in the correct form.	
DM1	Solve to find tension at the lowest point. Must reach $T = \dots$ but need not be simplified. Depends on both M marks in (b)	
A1*	Achieve the given result, from fully correct working. c.s.o.	
NB	If the equations in (b) are found at a general point, mark the equations as above. The final M mark will require some evidence of maximising the tension.	

Question Number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga(\cos \alpha - \cos \theta)$	M1A2,1,0
	$v^2 = u^2 + \frac{2ag}{5}(4 - 5 \cos \theta) *$	A1* (4)
6(b)	$T + mg \cos \theta = \frac{mv^2}{a}$ (T may be omitted here)	M1A2,1,0
	Use of $T = 0$ and substitute for v^2 and u^2	DM1
	$mg \cos \beta = \frac{m}{a} \left(\frac{6ag}{5} + \frac{2ag}{5}(4 - 5 \cos \beta) \right)$	A1
	$\cos \beta = \frac{14}{15}$ (0.93 or better)	A1 (6)
		(10)
	Notes for question 6	
6(a)	M1 for an energy equation with the 2 KE terms and 2 PE terms. $\cos \alpha$ must be seen.	
	A2 for a correct equation, A1A0 for an equation with at most one error	
	A1* for the given answer correctly obtained.	
6(b)	M1 for an equation of motion towards O with all necessary terms, condone sign errors and sin/cos confusion mg must be resolved	
	A2 for a correct equation (allow $-T$), A1A0 for an equation with at most one error	
	DM1 for use of $T = 0$ and substitute for v^2 and u^2 to obtain an equation in $\cos \beta$ Depends on the first M mark of (b)	
	A1 Correct unsimplified equation following substitution	
	A1 cao	



Question Number	Scheme	Marks
5(a)	$\frac{1}{2}mU^2 - \frac{1}{2}mV^2 = mg\left(a + \frac{a}{4}\sin\theta\right)$	M1A1A1
	$V^2 = U^2 - \frac{ag}{2}(4 + \sin\theta)^*$	A1*
		(4)
5(b)	Equation of motion towards the peg	M1
	$T + mg\sin\theta = m\frac{V^2}{\left(\frac{a}{4}\right)}$	A1
	Eliminate V^2	DM1
	$T + mg\sin\theta = \frac{4m}{a}\left(U^2 - \frac{ag}{2}(4 + \sin\theta)\right)$	
	$T = \frac{4mU^2}{a} - 8mg - 3mg\sin\theta$	A1
		(4)
5(c)	Substitute $U = \sqrt{\frac{19ag}{8}}$ and $T = 0$ in their equation of motion and solve for $\sin\theta$	M1
	$\frac{4m}{a}\left(\sqrt{\frac{19ag}{8}}\right)^2 - mg(8 + 3\sin\theta) = 0 \Rightarrow \sin\theta = \dots$	
	$\sin\theta = \frac{1}{2} \Rightarrow \text{height} = \frac{1}{8}a$	A1
		(2)
		(10)
	Notes for question 5	
5(a)		
M1	Energy equation with the correct number of terms (2KE, GPE). All terms dimensionally correct and with the correct structure. Condone sign errors and sin/cos confusion. Must include m in each term.	
A1	Correct unsimplified equation with at most one error	
A1	Correct unsimplified equation	
A1*	Answer obtained from complete and correct working. There must be at least one line of working or simplification between the initial equation and the given answer. Accept $V^2 = U^2 - \frac{ag}{2}(\sin\theta + 4)$, $V^2 = U^2 - \frac{1}{2}ag(4 + \sin\theta)$, $V^2 = U^2 - \frac{1}{2}ag(\sin\theta + 4)$	
5(b)		
M1	Equation of motion towards the peg. Must have all required terms and no extras. Condone sin/cos confusion and sign errors. Accept any form of acceleration, condone 'a' for acceleration for the method mark only.	



Question Number	Scheme	Marks
A1	Correct equation using the form $\frac{v^2}{r}$ for acceleration where $r = \frac{a}{4}$	
DM1	Dependent on the first M. Use the given answer from (a) to eliminate V^2 from their equation of motion and form an equation in T, m, g, u, a and θ . Condone a slip when transferring the expression for V^2 as long as the intention to use the expression given in (a) is clear.	
A1	A correct expression for T with $\sin \theta$ terms collected. ISW once the correct expression is seen with collected terms.	
5(c)		
M1	Substitute $U^2 = \frac{19ag}{8}$ and $T = 0$ in their equation of motion to find a value for $\sin \theta$. M0 for using $V = 0$.	
A1	Correct answer, accept $0.13a$ and $0.125a$	

Question Number	Scheme	Marks
4(a)	$mg \sin \alpha \times \left(\frac{3l}{2} + e \right) = \mu mg \cos \alpha \times \left(\frac{3l}{2} + e \right) + \frac{1}{2} \times \frac{2mg}{l} e^2$ $\frac{3}{5} \left(\frac{3l}{2} + e \right) = \frac{4\mu}{5} \left(\frac{3l}{2} + e \right) + \frac{e^2}{l}$ $\mu = \frac{9l^2 + 6le - 10e^2}{4l(3l + 2e)} \quad *$	M1B1B1A1 dM1A1cso (6)
(b)	$e = l \Rightarrow \mu = \frac{1}{4} \text{ or } 0.25$ $F = \frac{1}{5} mg$ Change in acceleration is due to change of direction of F $F_1 = 2mg - mg \sin \alpha + F_r \left(= \frac{8}{5} mg \right) \text{ and } F_2 = 2mg - mg \sin \alpha - F_r \left(= \frac{6}{5} mg \right)$ $\text{Mag of change in accel} = \frac{F_1 - F_2}{m} = \frac{2g}{5} = 3.92 \text{ or } 3.9 \text{ (ms}^{-2}\text{)}$	B1 B1ft M1 M1A1 (5) [11]
(a) M1 B1 B1 A1ft dM1 A1cso* (b) B1 B1ft M1 M1 A1	Attempt a work-energy equation with a GPE term, a single EPE term and the work done against friction. . (Allow $EPE = k \frac{\lambda x^2}{l}$) Correct EPE at C. (Ignore any extra EPE terms for this mark) Correct GPE Correct equation. Follow through their EPE and GPE terms providing they are of the correct form At least one line of correct working to rearrange towards $\mu =$. They do not need to reach $\mu =$ for this mark. Given result obtained with no errors seen and at least one line of correct rearrangement. Must be exactly as printed on paper. Correct numerical value for μ seen anywhere in (b). This might be implied by later working. Correct value for F, seen anywhere in (b). Follow through their μ but must be dimensionally correct. μ Attempt 2 equations of motion to find resultant force. (Use of $Change = 2F$) would imply this mark. Subtract and divide by m to obtain the mag of the change in the acceleration. Must be $\frac{2g}{5}$, or 3.9 or 3.92 (m s⁻²)	

Question Number	Scheme	Marks
5(a)	$3amg = \frac{1}{2}m \times 7ag - \frac{1}{2}mv^2$ $v^2 = ag \quad v = \sqrt{ag}$	M1A2 A1 (4)
(b)	$amg = \frac{1}{2}mw^2 - \frac{1}{2}m \times 7ag$ $w^2 = 9ag$ $T_1 - mg = \frac{mw^2}{4a}$ $T_1 = \frac{13mg}{4}$ $\text{Speed immediately after impact} = \frac{1}{2}\sqrt{ag}$ $4amg = \frac{1}{2}mV^2 - \frac{1}{2}m \times \frac{1}{4}ag$ $V^2 = \frac{33}{4}ag$ $T_2 - mg = \frac{mV^2}{4a}$ $T_2 = \frac{49}{16}mg$ $T_1 : T_2 = \frac{13}{4} : \frac{49}{16} = 52 : 49$	M1 M1 A1 M1 A1 A1 (7) [11]
(a) M1 A2 A1cso (b) M1 M1 A1 M1 M1 A1 A1cao	Energy equation from projection to reaching the ceiling. Must have at least one GPE term and 2 KE terms Correct equation. -1 for each error. Correct expression for v from fully correct work Energy equation from the point of projection to B. Must have all required terms Form equation of motion at B and eliminate w^2 to obtain an expression for T_1. Must have attempted a velocity at B. Condone $r = a$. Correct expression for T_1 Form energy equation from leaving the ceiling to reaching B. Must have attempted to use the coeff of restitution to find the initial speed for this equation. Condone $r = a$. Attempt an equation of motion at B and eliminate V^2 to obtain an expression for T_2. Must have attempted a velocity at B. Correct expression for T_2 Correct ratio. Question asks for simplest form, so must be 52:49 (Condone $\frac{52}{49}$)	



Question Number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 + mg(2a) = \frac{1}{2}m(3\sqrt{ag})^2 - mg(2a \cos 60^\circ)$	M1A1A1
	$(v^2 = 3ag)$	
	$T + mg = \frac{mv^2}{2a}$	M1A1
	$T = \frac{m(3ag)}{2a} - mg = \frac{mg}{2}$	dM1A1
	$T > 0$, therefore string remains taut and particle performs complete vertical circles.	A1
		(8)
(b)	From initial: $\frac{1}{2}mV^2 = \frac{1}{2}m(3\sqrt{ag})^2 + mg(2a - 2a \cos 60^\circ)$	M1A1
	Or from top: $\frac{1}{2}mV^2 = \frac{1}{2}m(3ag) + mg(4a)$	
	$(V^2 = 11ag)$	
	$T - mg = \frac{m(11ag)}{2a}$	M1A1
	$T = \frac{13mg}{2} < 7mg$. Tension less than critical value, so particle completes vertical circles.	A1
		(5)
ALT		[13]
(a)	$\frac{1}{2}mv^2 + mg(2a \cos 60^\circ - 2a \cos \theta) = \frac{1}{2}m(3\sqrt{ag})^2 \rightarrow v^2 = ag(7 + 4 \cos \theta)$	M1A1A1
	$T - mg \cos \theta = \frac{mv^2}{2a}$	M1A1
	$T - mg \cos \theta = \frac{mag}{2a}(7 + 4 \cos \theta)$ AND $\theta = \pi$ or $\cos \theta \geq -1$	dM1
	$T + mg = \frac{mg}{2}(3) \rightarrow T = \frac{mg}{2}$	A1
	$T > 0$, string stays taut and particle completes vertical circles.	A1
(b)	$\theta = 2\pi \rightarrow T - mg \cos 2\pi = \frac{mg}{2}(7 + 4 \cos 2\pi)$	M1, M1
	$T - mg = \frac{m(11ag)}{2a}$	A1, A1
	$T = \frac{13mg}{2} < 7mg$. Tension less than critical value, so particle completes vertical circles.	A1

(a)**M1** Energy equation from projection to top of the circle. Must have 2 KE terms and a difference in GPE.**A1** Equation with at most one error.**A1** Fully correct equation.**M1** Equation of motion towards centre of circle at top. Allow acceleration in either form.**A1** Correct equation. Acceleration must be in form $\frac{v^2}{r}$. Condone $2a = r$ if substituted later**dM1** Eliminate v to form equation for T . Dependent on the first two M marks**A1** Correct unsimplified equation for T .**A1** Correct inequality with a concluding statement.**SC** uses $T > 0$ without $T =$ For the last five marks:**M1** Finds resultant force at the top**A1** $F = \frac{m(3ag)}{2a} = \frac{3mg}{2}$ **dM1** Compares their resultant force to the weight. Dependent on the first two M marks**A1** $\frac{3mg}{2} - mg > 0$ or $\frac{3mg}{2} > mg$ **A1** Correct concluding statement that must include mention of their being tension in the string at the top**(b)****M1** Attempt energy equation from either initial position, or top, to the bottom of the circle.**A1** Correct equation.**M1** Equation of motion at the bottom of the circle. Allow in terms of V .**A1** Correct equation for tension, with V eliminated. Must have attempted to calculate V **A1** Correct tension and concluding statement.

ALT (a and b) using a general point on the circle

M1 Energy equation from point of projection to a general point. Must have 2 KE terms and a difference in GPE one of which is in θ

A1 Equation with at most one error.

A1 Fully correct equation.

M1 Equation of motion towards centre of circle at the general point. Allow acceleration in either form.

A1 Correct equation. Acceleration must be in form $\frac{v^2}{r}$. Condone $2a = r$ if substituted later

dM1 Eliminate v to form equation for T in m, a, g, θ AND set θ or $\cos \theta$ to evaluate T at the top. Dependent on the first two M marks.

A1 Correct unsimplified equation for T with θ now substituted.

A1 Correct inequality with a concluding statement.

(b)

M1, M1 Substitute for θ or $\cos \theta$ to evaluate T at the bottom

A1, A1 Correct unsimplified equation for T

A1 Correct tension and concluding statement.



Question Number	Scheme	Marks
5(a)	$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mga(1 + \cos \theta)$	M1A2,1,0
	$mg \cos \theta = \frac{mv^2}{a}$	M1A1
	Eliminate θ	M1
	$3v^2 = u^2 - 2ag^*$	A1*
		(7)
5(b)	Vertical motion: $-\frac{a\sqrt{3}}{2} = (v \sin 30^\circ)T - \frac{1}{2}gT^2$	M1A1
	Solve for T : $T = \frac{v \pm \sqrt{v^2 + 4ag\sqrt{3}}}{2g}$	M1
	Use $v^2 = ag \frac{\sqrt{3}}{2}$ and $T > 0$ to show the given answer: $T = \frac{2v}{g}$	A1*
		(4)
5(c)	Horizontal motion: $x = v \cos 30^\circ \times \frac{2v}{g}$	M1
	$= \frac{3a}{2}$ and hence taut ($= a + a \sin 30^\circ$)*	A1*
		(2)
		(13)
	Notes for question 5	
5(a)	M1 for an energy equation with the correct terms	
	A2 for a correct equation, A1 for an equation with at most one error	
	M1 for an equation of motion towards O with correct terms, condone sign errors and sin/cos confusion (R may appear)	
	A1 for a correct equation ($R = 0$ must be used at some point)	
	M1 for eliminating θ	
	A1* for given answer correctly obtained	
5(b)	M1 for equation for vertical motion, with correct terms, condone sign errors and sin/cos confusion	
	A1 for a correct equation	
	M1 for solving for T	
	A1* for correctly obtaining the given answer	
5(c)	M1 for equation for horizontal motion, with correct terms, condone sign errors and sin/cos confusion	
	A1* for given answer correctly justified	



Question Number	Scheme	Marks
6(a)	$S - mg \cos \theta = \frac{mv^2}{a}$ $\frac{1}{2} \times mv^2 - \frac{1}{2} \times m \times \frac{9ag}{5} = mga \cos \theta$ $mv^2 = 2mga \cos \theta + \frac{9}{5} mga$ $S = mg \cos \theta + 2mg \cos \theta + \frac{9}{5} mg$ $S = \frac{3}{5} mg (5 \cos \theta + 3) \quad *$	M1A1 M1A1 DM1 A1* cso (6)
(b)	$S = 0 \quad \cos \theta = -\frac{3}{5}$ $v^2 = \frac{3ag}{5} \quad v = \sqrt{\frac{3ag}{5}} \quad *$	B1 M1A1* (3)
(c)	$\text{vert comp} = \sqrt{\frac{3ag}{5}} \times \frac{4}{5}$ $\text{Vert distance to highest point: } 0 = \frac{16}{25} \times \frac{3ag}{5} - 2gs$ $s = \frac{24}{125} a$ $\text{Total distance above } O = \frac{24}{125} a + \frac{3}{5} a = \frac{99}{125} a, \quad 0.79a \text{ or better}$	M1 M1 A1 A1ft (4)
Notes		[13]
(a) M1	Equation of motion along the radius. Must have 3 terms with weight resolved. Acceleration in either form.	
A1	Fully correct equation with acceleration v^2/r	
M1	Energy equation from A to general position. Difference of 2 KE terms and loss of PE (one or two terms) required. M0 for $v^2 = u^2 + 2as$	
A1	Fully correct equation	
DM1	Eliminate v^2 between the 2 equations. Depends on both preceding M marks	
A1 *cso	Obtain the given result from fully correct working.	
(b)		
B1	$\cos \theta = -\frac{3}{5}$ seen explicitly or used	
M1	Use their value of $\cos \theta$ to obtain the value of v^2 or v	
A1*	Correct answer from correct working	
(c)		
M1	Use their values for θ and v to obtain the vertical comp of velocity (allow sin/cos confusion)	
M1	Correct method to find the vertical distance to highest point using their vertical comp of vel	
A1	Correct expression for this vertical distance (may be implied)	
A1ft	Find the total distance above O by adding $\frac{3a}{5}$ to their previous answer. Both M marks needed.	

ALT 1	Conservation of Energy from <u>slack</u> to find vertical height
M1	Uses their value of θ and v to obtain the horizontal component at the highest point $\sqrt{\frac{3ag}{5}} \cos \theta$
M1	Forms an energy equation. Must have 2 KE terms and gain in PE $\frac{1}{2} m \frac{3ag}{5} - \frac{1}{2} m \frac{3ag}{5} \left(\frac{3}{5}\right)^2 = mgs$
A1	Correct expression for this vertical distance $s = \frac{24}{125} a$
A1ft	Find the total distance above O by adding $\frac{3a}{5}$ to their previous answer. Both M marks needed. $\frac{99}{125} a$, $0.79a$ or better
ALT 2	Conservation of Energy from <u>initial position</u> (A) to find vertical height
M1	Uses their value of θ and v to obtain the horizontal component at the highest point $\sqrt{\frac{3ag}{5}} \cos \theta$
M1	Forms an energy equation. Must have 2 KE terms and gain in PE
A1	$\frac{1}{2} m \frac{9ag}{5} - \frac{1}{2} m \frac{3ag}{5} \left(\frac{3}{5}\right)^2 = mgh$
A1	Gives the total distance above O as $h = \frac{99}{125} a$ (do not isw)

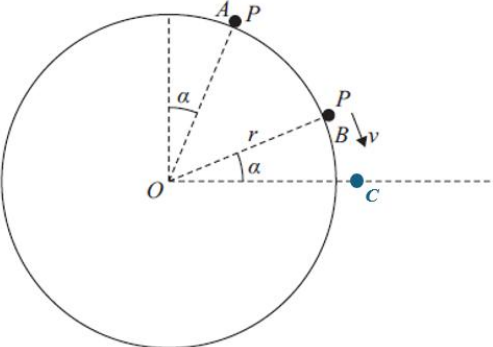
7a	Conservation of mechanical energy:	M1	All terms required. Dimensionally correct $\cos \theta = \frac{5}{13}, \sin \theta = \frac{12}{13}$
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(r + r \cos \theta)$	A1	Correct unsimplified equation
	$v^2 = u^2 - \frac{36}{13}gr$ *	A1*	Obtain given answer from correct working
		[3]	
7b	Equation of motion	M1	All terms required. Dimensionally correct. Condone sign errors and sin/cos confusion. Condone use of $R = 0$
	$R + mg \cos \theta = \frac{mv^2}{r}$	A1	Correct unsimplified equation. Condone (strict) inequality the right way round.
	Use $R \dots 0$ and solve for u^2	M1	Complete method to obtain u^2 Condone use of $R = 0$ or $R > 0$
	$\frac{mv^2}{r} - mg \cos \theta \dots 0$ $\Rightarrow u^2 - \frac{36}{13}gr \dots \frac{5}{13}gr, u^2 \dots \frac{41}{13}gr$ *	A1*	Obtain given answer from correct working. Must have stated the inequality $R \geq 0$ If there is no reference to R , the max mark in (b) is M1A1M1A0*
		[4]	
7c	$BC = 2r \sin \theta = \frac{24}{13}r$	B1	Or equivalent $BC = 1.846 \dots r$
	Relevant vertical motion Eg time to return to the level of BC	M1	Complete method vertically using <i>suvat</i>
	$t = \frac{2v \sin \theta}{g} = \frac{24v}{13g}$	A1	Correct unsimplified expression for time Accept $\frac{24}{13g} \times 4\sqrt{\frac{gr}{13}}, \frac{24}{13}\sqrt{\frac{16r}{13g}}$ $\frac{96}{13}\sqrt{\frac{r}{13g}}, 0.65\sqrt{r}$
	Relevant horizontal motion Eg distance travelled by P	M1	Complete method horizontally
	$= (v \cos \theta)t = v^2 \times \frac{120}{169g}$	A1	Correct unsimplified expression for distance $0.87r, \frac{1920}{2197}r, 0.0892gr$
	$= \frac{16gr}{13} \times \frac{120}{169g} = \frac{160r}{169} \times \frac{12}{13} < 2r \times \frac{12}{13}$	A1*	Obtain given conclusion from correct working

	hence falls into the bowl *		
ALT 1 for last 3 marks	Horizontal: time, T , required to travel the length BC	M1	Complete method horizontally
	$2r \sin \theta = v \cos \theta \times T$ $T = \frac{2r \frac{12}{13}}{4\sqrt{\frac{gr}{13}} \times \frac{5}{13}} = 1.38\sqrt{r}$	A1	Correct unsimplified expression for T
	$t < T$ since $0.654\sqrt{r} < 1.38\sqrt{r}$ hence falls into the bowl *	A1*	Obtain given conclusion from correct working
ALT 2 for last 3 marks	Horizontal: speed, V , required to reach C	M1	Complete method horizontally
	$-V \sin \theta = V \sin \theta - g \frac{2r \sin \theta}{V \cos \theta}$ $\Rightarrow V = \sqrt{\frac{gr}{\cos \theta}} = \sqrt{\frac{13gr}{5}}$	A1	Correct unsimplified expression for V
	$v < V$ since $\sqrt{\frac{13gr}{5}} < \sqrt{\frac{16gr}{13}}$ hence falls into the bowl *	A1*	Obtain given conclusion from correct working
	SC: If range formula is quoted correctly award M1A1M1A1. Range = $\frac{2v^2 \sin \theta \cos \theta}{g}$		
		[6]	
		(13)	

Question Number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mgr(1 - \cos \theta)$	M1A2
	$mg \cos \theta = \frac{mv^2}{r}$	M1A1
	Eliminate v^2 and solve for $\cos \theta$	M1
	$\cos \theta = \frac{2gr + u^2}{3gr} *$	A1*
		(7)
6(b)	$\cos \theta = \frac{4}{5}$	B1
	$v^2 = rg \cos \theta \quad \left(v = \sqrt{\frac{4rg}{5}} \right)$	M1
	Horiz cpt at C: $H = v \cos \theta$ $\left(H = \frac{4}{5} \sqrt{\frac{4rg}{5}} = \sqrt{\frac{64rg}{125}} \right)$	M1 M1
	Vert cpt at C: $V = \sqrt{(v \sin \theta)^2 + 2gr \cos \theta}$ $\left(V = \sqrt{\frac{236rg}{125}} \right)$	
	Speed at C: W where $A \text{ to } C: \frac{1}{2}m \left(W^2 - \frac{2gr}{5} \right) = mgr$ OR $B \text{ to } C: \frac{1}{2}m (W^2 - v^2) = mgr \cos \theta$ $\left(W = \sqrt{\frac{12rg}{5}} \right)$	
	$\tan \alpha = \frac{V}{H} = \frac{\sqrt{W^2 - H^2}}{H} = \frac{V}{\sqrt{W^2 - V^2}}$	DM1
	$= \frac{\sqrt{59}}{4}$	A1
		(6)
		(13)
	Notes for question 6	
6(a)		
M1	Use conservation of energy to form a dimensionally correct equation. All terms present and no extras. Condone sign errors and sin/cos confusion. Anything that should be resolved has been resolved.	
A1	An unsimplified equation with at most one error.	
A1	A correct unsimplified equation.	
M1	Use N2L to form an equation of motion towards O . Equation must be dimensionally correct. All terms present and no extras. Condone sign errors and sin/cos confusion. Anything that should be resolved has been resolved Allow this mark with or without R .	

Question Number	Scheme	Marks
	Condone $\pm R + mg \cos \theta = \frac{mv^2}{r}$	
A1	Correct equation ($R = 0$ must be used now at some point) A0 if R never becomes 0	
M1	Solve to find an expression for $\cos \theta$ in terms g, r and u	
A1*	Given answer obtained from correct and complete working. Working should include a line with v^2 eliminated before reaching the given answer.	
6(b)		
B1	For $\cos \theta = \frac{4}{5}$ seen or implied	
M1	Solve to find v in terms of g, r and θ	
M1	Correct method to find at least one of H, V or W in terms of g, r and θ Condone finding H^2, V^2 or W^2 Equation must be dimensionally correct. All terms present and no extras. Condone sign errors and sin/cos confusion. Anything that should be resolved has been resolved. M0: If they use the speed from (a) instead of v	
M1	Correct method to find any two of H, V or W in terms of g, r and θ Condone finding H^2, V^2 or W^2 Equation must be dimensionally correct. All terms present and no extras. Condone sign errors and sin/cos confusion. Anything that should be resolved has been resolved. M0: If they use the speed from (a) instead of v	
DM1	Dependent on previous two M's. Complete method to find $\tan \alpha$ Condone if they go straight to $\alpha = \tan^{-1}(\dots)$ and never state $\tan \alpha = \dots$	
A1	A correct value for $\tan \alpha = \frac{\sqrt{59}}{4}$ Accept any equivalent surd, eg $\sqrt{\frac{59}{16}}$ but must be exact. A0 if they go straight to α and never find $\tan \alpha$	

Question Number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 - mgl$ or $mgl - \frac{1}{2}mv^2$ seen or implied	B1
	Use of EPE	M1
	$\frac{mg}{2l}l^2$	A1
	$\frac{mg}{2l}(l\sqrt{2}-l)^2$	A1
	$\frac{1}{2}mv^2 + \frac{mg}{2l}(l\sqrt{2}-l)^2 = mgl + \frac{mg}{2l}l^2$	M1
	Solve for v^2	dM1
	$v^2 = 2gl\sqrt{2}$ *	A1*
		(7)
(b)	$T = \frac{mg(l\sqrt{2}-l)}{l} = mg(\sqrt{2}-1)$	M1 A1
	$\pm N + T \cos 45^\circ = \frac{mv^2}{l}$	M1A1A1
	$\pm N + mg(\sqrt{2}-1) \times \frac{\sqrt{2}}{2} = \frac{m}{l} \times 2gl\sqrt{2}$	dM1
	$N = \frac{1}{2}mg(5\sqrt{2}-2)$ *	A1*
		(7)
		(14)
Notes		
(a)		
B1	Difference between KE and GPE, seen either way round.	
M1	Use of EPE formula at top or at B	
A1	Correct EPE at top	
A1	Correct EPE at B	
M1	Use of conservation of energy, with 1 GPE, 1 KE and 2 EPE terms, condone sign errors	
dM1	Solve for v^2 , dependent on previous M	
A1*	Exact given answer correctly obtained	
(b)		
M1	Use of Hooke's Law at B – this may appear in an attempted equation of motion	
A1	Correct unsimplified tension at B	
M1	Equation of motion at B horizontally with correct terms, condone sign errors	
A1	Correct equation with at most one error	
A1	Correct equation	
dM1	Sub for T and v^2 . Dependent on both previous M marks	
A1*	Given answer correctly obtained (exactly). If $N = -\frac{1}{2}mg(5\sqrt{2}-2)$ then clear justification is required to reach the given answer eg use of 'magnitude' or modulus signs.	

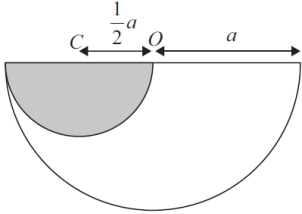
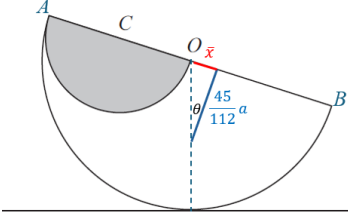
Question Number	Scheme	Marks
		
5(a)	Form an energy equation from A to B	M1
	$\frac{1}{2}mv^2 = mgr \cos \alpha - mgr \sin \alpha$	A1
	$v^2 = 2gr(\cos \alpha - \sin \alpha)^*$	A1*
		(3)
5(b)	N2L towards O with $R = 0$	M1
	$mg \sin \alpha = \frac{mv^2}{r}$	A1
	Eliminate v^2	
	$\frac{1}{2}mgr \sin \alpha = mgr(\cos \alpha - \sin \alpha)$	M1
	$(3 \sin \alpha = 2 \cos \alpha \Rightarrow) \tan \alpha = \frac{2}{3}^*$	A1*
		(4)
5(c)	Expression for the horizontal component of the speed of P at C. Condone cos/sin confusion.	M1
	For example, $W \cos \theta \text{ or } W_{\text{horiz}} = v \sin \alpha \left(= \sqrt{\frac{8gr}{13\sqrt{13}}} \right)$	A1
	Method to form relevant equation for the motion from A to C or B to C. Condone cos/sin confusion, consistent with their horizontal component.	M1
	[1] Energy from A to C $\frac{1}{2}mW^2 = mgr \cos \alpha$	A1
	[2] Energy from B to C $\frac{1}{2}mW^2 - \frac{1}{2}mv^2 = mgr \sin \alpha$	
	[3] Vertical motion under gravity from B to C $(w \sin \theta)^2 = (v \cos \alpha)^2 + 2gr \sin \alpha$	
	Dependent on the previous M only. Follow their notation. Use method [1] or [2] to find an expression for the speed of P at C in terms of g and r only.	dM1



	$\left(W = \sqrt{\frac{6gr}{\sqrt{13}}} \right)$ Or Use method [3] to find the vertical component of the speed of P at C in terms of g and r only. For example, $W \sin \theta$ or $W_{\text{vert}} = \sqrt{\frac{70gr}{13\sqrt{13}}}$	
	Dependent on the first two M marks. Use the motion of P at C to form an expression in θ only.	dM1
	Obtain given answer from correct working $\cos \theta = \frac{2}{\sqrt{39}}^*$	A1*
		(7)
		(14)
Notes for question 5		
(a)		
M1	Complete method to form a dimensionally correct conservation of energy equation in (m) , g , r and α (m must be seen at some stage). All required terms present and no extra. Condone sign errors and sin/cos confusion on GPE terms.	
A1	A correct equation	
A1*	Given answer correctly obtained and written exactly as printed.	
(b)		
M1	Use N2L to form a dimensionally correct equation of motion towards O . All required terms present with no extras. If R is present, must use $R = 0$ at some point. M0 if R is never zero. Condone sin/cos confusion.	
A1	A correct equation	
M1	Eliminate v^2 using the given answer in (a) to form an equation in α (m and g)	
A1*	Given answer correctly obtained. A line of working with terms collected must be seen before reaching the given answer.	
(c)		
	Note: $\cos \alpha = \frac{3}{\sqrt{13}}$, $\sin \alpha = \frac{2}{\sqrt{13}}$, $v = \sqrt{\frac{2gr}{\sqrt{13}}}$ $W_{\text{horiz}} = \sqrt{\frac{8gr}{13\sqrt{13}}}$, $W_{\text{vert}} = \sqrt{\frac{70gr}{13\sqrt{13}}}$, $W = \sqrt{\frac{6gr}{\sqrt{13}}}$	
M1	Use of constant horizontal velocity component at C , condone sin/cos confusion.	
A1	Correct equation (follow their notation)	
M1	Energy equation with the correct terms or use of $v^2 = u^2 + 2as$ vertically. Condone sin/cos confusion consistent with their horizontal component. If t is used in a $suvat$ equation, a second $suvat$ equation is also required to eliminate t correctly.	
A1	A correct equation (follow their notation)	
dM1	Dependent on the previous M mark only. Accept expressions for the speed or velocity squared. Follow their notation.	

	Use method [1] or [2] to find an expression for the speed of P at C in terms of g and r only. or Use method [3] to find the vertical component of the speed of P at C in terms of g and r only.	
dM1	Dependent on the first 2 method marks. Use the motion of P at C to form an expression in θ only. Accept the trig expression squared.	
A1*	Correctly obtain the given answer. A0* if the final answer comes from working with a decimal value of α	



Question Number	Scheme	Marks
	 	
6(a)	Use of $\int xy^2 dx$ with circle equation	M1
	$\int_0^a x(a^2 - x^2) dx$	A1
	$\left[\frac{a^2 x^2}{2} - \frac{x^4}{4} \right]_0^a = \frac{a^4}{4}$	A1
	Use of $\bar{x} = \frac{\pi \int_0^a x(a^2 - x^2) dx}{\frac{2}{3} \pi a^3}$	dM1
	$\left\{ \frac{\pi \frac{a^4}{4}}{\frac{2}{3} \pi a^3} \Rightarrow \right\} = \frac{3}{8} a^*$	A1*
		(5)
6(b)	Mass ratios: $\frac{8}{3}a$ 1 $\frac{7}{16}a$	B1
	Distances: $\frac{3}{8}a$ $\frac{3}{16}a$ $\bar{y}$	B1
	$8 \times \frac{3a}{8} - 1 \times \frac{3}{16}a = 7\bar{y}$	M1
	$(\bar{y}) = \frac{45}{112}a^*$	A1*
		(4)
6(c)	Form a moments equation about an axis perpendicular to OC - Axis through O: $(8 \times 0 +) 1 \times \frac{a}{2} = 7\bar{x}$ - Axis through A: $8 \times a - 1 \times \frac{a}{2} = 7\bar{x}$ - Axis through B: $8 \times a - 1 \times \frac{3a}{2} = 7\bar{x}$ If equation is seen in earlier working, it must be used in (c) to earn the marks in (c)	M1
	Correct $\bar{x}$ for their axis through $O: (\bar{x}) = \frac{a}{14} \quad A: (\bar{x}) = \frac{15a}{14} \quad B: (\bar{x}) = \frac{13a}{14}$	A1

Question number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga \sin \theta$	M1A1A1
	$T - mg \sin \theta = \frac{mv^2}{a}$	M1A1A1
	$T = \frac{mu^2}{a} + 3mg \sin \theta^*$	A1*
		(7)
(b)	$0 = \frac{m(\frac{12ag}{5})}{a} + 3mg \sin \theta$	M1
	$\sin \theta = -\frac{4}{5}$	A1
	$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{12ag}{5}\right) = mga \times -\frac{4}{5}$ OR $0 - mg \times -\frac{4}{5} = \frac{mv^2}{a}$	M1
	$v = 2\sqrt{\frac{ag}{5}}, 0.89\sqrt{ag}$ or better	A1
		(4)
(c)	Vertical motion $0 = \left(2\sqrt{\frac{ag}{5}} \times \frac{3}{5}\right)^2 - 2gh$ OR Energy $mgh = \frac{1}{2}m\left(2\sqrt{\frac{ag}{5}}\right)^2 - \frac{1}{2}m\left(2\sqrt{\frac{ag}{5}} \times \frac{4}{5}\right)^2$	M1A1ft
	$h = \frac{18a}{125}$	A1
	$H, \text{ height above } O = h - a \sin \theta = \frac{18a}{125} + \frac{4a}{5}$	dM1
	$= \frac{118a}{125}, 0.94a, 0.944a$	A1
		(5)
	OR using energy from start to top	
	$mgH = \frac{1}{2}m\left\{\frac{12ag}{5} - \left(2\sqrt{\frac{ag}{5}} \times \frac{4}{5}\right)^2\right\}$	M2A1ft A1
	$H = \frac{118a}{125}, 0.94a, 0.944a$	A1
		(5)
		(16)
	Notes	
6(a)		
M1	Energy equation with correct no. of terms, dim correct. May use h instead of $a \sin \theta$	
A1	Correct equation with at most one error	



A1	Correct equation
M1	Equation of motion towards O , correct no. of terms, condone sign errors and sin/cos confusion. Accept acceleration in either circular form but do not accept ' a '. The radius may be given as r .
A1	Equation with at most one error
A1	Correct equation
A1*	Given answer correctly obtained and written exactly as printed.
6(b)	
M1	Put $T = 0$ and $u = 2\sqrt{\frac{3ag}{5}}$
A1	Correct value of $\sin \theta$
M1	Put $u = 2\sqrt{\frac{3ag}{5}}$ and their $\sin \theta$ into energy equation OR put $T = 0$ and their $\sin \theta$ into equation of motion
A1	Correct answer
6(c)	If an energy approach is used in (c) the equation must have 2 KE terms, one of which must have a sin/cos component included.
M1	Use vertical motion or energy to obtain an equation in h only. A component of speed must be used for either approach.
A1ft	Correct equation ft on their answer to (b).
A1	Correct value of h
dM1	Correct method to find H . Dependent on previous M.
A1	cao
	OR
M2	Complete method to obtain an equation in H only (must be using horizontal cpt of velocity at the top)
A1ft	Correct equation with at most one error
A1	Correct equation
A1	Correct answer

Question Number	Scheme	Marks
7(a)	$\frac{1}{2}m(8ag) + mg(8a) = \frac{1}{2}mv^2 + mg(8a \cos \theta)$	M1A1A1
	$(v^2 = 24ga - 16ga \cos \theta)$	
	$T + mg \cos \theta = \frac{mv^2}{8a}$	M1A1
	$T + mg \cos \theta = \frac{m(24ga - 16ga \cos \theta)}{8a}$	DM1
	$T + mg \cos \theta = 3mg - 2mg \cos \theta$	
	$T = 3mg - 3mg \cos \theta = 3mg(1 - \cos \theta)$ *	A1*
		(7)
(b)	At B $v_B^2 = 24ga$	B1
	$T_1 = \frac{m(24ag)}{8a} = 3mg$ or $T_2 = \frac{m(24ag)}{3a} = 8mg$	B1
	$\Delta T = 5mg$	B1
		(3)
(c)	$\frac{1}{2}mv_1^2 = \frac{1}{2}m(8ag) + mg(11a)$	M1
	$v_1^2 = 30ag$	
	After impact $v_2^2 = 20ag$	A1
	$\frac{1}{2}m(20ag) - mg(3a) = \frac{1}{2}mv_2^2 + mg(8a \cos \alpha)$	M1A1
	$(v_2^2 = 14ga - 16ga \cos \alpha)$	
	$mg \cos \alpha = \frac{m(14ga - 16ga \cos \alpha)}{8a}$	M1A1
	$mg \cos \alpha = \frac{7mg}{4} - 2mg \cos \alpha$	
	$\cos \alpha = \frac{7}{12}$ *	A1*

Question Number	Scheme	Marks
		(7)
		[17]

(a)

M1 Attempt at energy equation at a general point. Must be dimensionally correct and contain two KE terms and a change in GPE.

A1, A1 Correct unsimplified equation. -1 each error.

M1 Attempt to resolve radially. Acceleration can be in either circular form.

A1 Correct equation. Must be $\frac{mv^2}{r}$

DM1 Eliminate v to produce equation in T, m, g, θ . Dependent of the previous 2 M marks.

A1* Reach given result with no errors seen.

(b)

B1 $v_B^2 = 24ga$. Correct expression for speed (or speed squared) at B . This mark will **not** be implied by a correct tension if they simply use the final result in (a).

B1 Correct expression for Tension at B , for either radius. Can be found using the result from (a).

B1 Correct expression for change in tension.

(c)

M1 Attempt at energy equation at wall. Must include 2 KE terms and a change in GPE.

A1 Correct speed (or speed squared, or KE) after impact.

M1 Attempt at Energy equation to α . Must include 2 KE terms and a change in GPE.

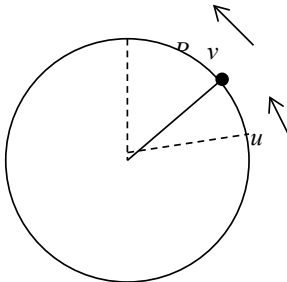
A1 Correct energy equation.

M1 Attempt at radial equation. If T included, it must be set to zero before this mark is awarded. Condone use of $3a$ for this mark?

A1 Correct equation in $\cos \alpha$ only oe.

A1* Solve to reach given result.



Question Number	Scheme	Marks
7	 (a) $mg l (\cos \theta - \cos \alpha) = \frac{1}{2} m u^2 - \frac{1}{2} m v^2$ $v^2 = u^2 - 2gl (\cos \theta - \cos \alpha)$ * (b) $\cos \alpha = \frac{2}{5}$ $v^2 = 3gl - 2gl \left(\cos \theta - \frac{2}{5} \right)$ At top $\theta = 0^\circ$ $v^2 = 3gl - 2gl \times \frac{3}{5}$ $v^2 = \frac{9gl}{5}$ $v^2 > 0 \Rightarrow$ complete circle * (c) Equation of motion along radius at lowest point: $kT - mg = \frac{mw^2}{l}$ $\theta = 180$ $w^2 = 3gl - 2gl \left(-1 - \frac{2}{5} \right) = \frac{29gl}{5}$ $kT = \frac{m}{l} \times \frac{29gl}{5} + mg = \frac{34mg}{5}$ At highest point: $T_2 + mg = \frac{mv^2}{l}$ $\theta = 0$ $T = \frac{9mg}{5} - mg = \frac{4mg}{5}$ $k \frac{4mg}{5} = \frac{34mg}{5} \Rightarrow k = \frac{17}{2}$	M1A1A1 A1* (4) M1 A1 M1 A1* (4) M1A1 M1 M1A1 M1 M1 A1 A1
		[17]



Question Number	Scheme	Marks
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(a)	
M1	Attempt energy equation from A to general position. Must have a difference of 2 PE terms and a difference of 2 KE terms.
A1	Correct gain in PE or loss of KE
A1	Fully correct equation
A1*	Reach the given result from fully correct working
(b)	
M1	Sub $u = \sqrt{3gl}$ and $\cos\alpha = \frac{2}{5}$ in the result in (a)
A1	Correct equation
M1	Put $\theta = 0$ to find an expression for v^2 at the top (maybe finding KE)
A1*	Fully correct working and conclusion with reason eg reference to v^2 , v , KE
(c)	
M1	Form an equation of motion along the radius at the lowest point. Acceleration in either form.
A1	Correct equation with acceleration in v^2/r form
M1	Use $\theta = 180$ in result from (a) to obtain an expression for w^2
M1	Eliminate w^2 and obtain an expression for kT
A1	Correct expression for kT
M1	Form an equation of motion along the radius at the highest point. Acceleration in either form.
M1	Sub $\theta = 0$ and obtain an expression for T
A1	Correct expression for T
A1	Correct value of k . Must be exact.
NB	The equation of motion at the top may be seen first. Award M1A1 for either equation correct and M1 for the second.

Question Number	Scheme	Marks
ALT1 7(c)	Better equation of motion at top or bottom: $T - mg = \frac{mv^2}{l}$ $T + mg = \frac{mv^2}{l}$	M1 A1
	Other equation of motion – see above	M1
	Finding speed at the bottom: $\theta = 180 \quad w^2 = 3gl - 2gl\left(-1 - \frac{2}{5}\right) = \frac{29gl}{5}$	M1
	Finding maximum Tension (lowest point) $\theta = 180, T = \frac{m}{l} \times \frac{29gl}{5} + mg = \frac{34mg}{5}$	M1 A1
	Finding minimum Tension (highest point) $\theta = 0 \quad T = \frac{9mg}{5} - mg = \frac{4mg}{5}$	M1 A1
	Dividing Tensions to reach the correct answer $k \frac{4mg}{5} = \frac{34mg}{5} \Rightarrow k = \frac{17}{2}$	A1



Question Number	Scheme	Marks
ALT 2 7 (c)	General equation of motion: $T + mg\cos\theta = \frac{mv^2}{l}$	M1 A1
	Use of $u = \sqrt{3gl}$ and $\cos\alpha = \frac{2}{5}$ to replace v^2 in their equation of motion	M1
	Finding speed at the lowest point: $\theta = 180 \quad w^2 = 3gl - 2gl\left(-1 - \frac{2}{5}\right) = \frac{29gl}{5}$	M1
	Finding maximum Tension (lowest point) $\theta = 180, T = \frac{m}{l} \times \frac{29gl}{5} + mg = \frac{34mg}{5}$	M1 A1
	Finding minimum Tension (highest point) $\theta = 0, T = \frac{9mg}{5} - mg = \frac{4mg}{5}$	M1 A1
	Dividing Tensions to reach the correct answer $k \frac{4mg}{5} = \frac{34mg}{5} \Rightarrow k = \frac{17}{2}$	A1