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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Simple Harmonic Motion (M3.4)

International A Level Mathematics · M3: Mechanics 3 (WME03)

4 questions · 34 marks · Mark schemes included

Past papers October 2021 - June 2024 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.
Click any row below to jump straight to that sub-topic or question.
On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.
A gold bar in the left margin marks exactly where the question sits on the original page.
Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.
Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
M3.4a SIMPLE HARMONIC MOTION		4 Qs · 34 marks	2
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2. Q1	IAL M3 (WME03) June 2022	6	5
3. Q4	IAL M3 (WME03) June 2024	10	7
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QUESTIONS

4 questions · 34 marks · 1 sub-topic

SUB-TOPIC M3.4a

SIMPLE HARMONIC MOTION

Sub-topic M3.4a · 4 questions · 34 marks · lowest tariff first



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1. A particle P is moving in a straight line with simple harmonic motion of period 4 s. The centre of the motion is the point O

At time $t = 0$, P passes through O

At time $t = 0.5$ s, P is moving with speed 2 m s^{-1}

- (a) Show that the amplitude of the motion is $\frac{4\sqrt{2}}{\pi} \text{ m}$

(4)

- (b) Find the maximum speed of P

(2)



Question 1 continued

Leave
blank

Lined area for writing the answer to Question 1.

(Total 6 marks)

Q1



1. A particle P moves in a straight line with simple harmonic motion between two fixed points A and B . The particle performs 2 complete oscillations per second. The midpoint of AB is O and the midpoint of OA is C

The length of AB is 0.6 m.

- (a) Find the maximum speed of P

(4)

- (b) Find the time taken by P to move directly from O to C

(2)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 6 marks)

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 10 marks)







Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 12 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
1(a)	$\frac{2\pi}{\omega} = 4$	M1
	$\omega = \frac{\pi}{2}$	A1
	$2 = a \frac{\pi}{2} \cos\left(\frac{\pi}{2} \times 0.5\right) \Rightarrow 2 = a \frac{\pi}{2} \times \frac{1}{\sqrt{2}}$	M1
	$a = \frac{4\sqrt{2}}{\pi} \text{ m}^*$	A1* (4)
1(b)	$v_{MAX} = \frac{4\sqrt{2}}{\pi} \times \text{their } \omega$	M1
	$2\sqrt{2} \text{ (m s}^{-1}\text{)}$	A1 (2)
		(6)
	Notes for question 1	
1(a)	M1 Need to see this equation, as it's a 'show that'. Allow with 4 or T or in a rearranged form.	
	A1 seen	
	M1 Complete method to obtain an equation in a only Use of $x = a \sin \omega t$ to find x followed by $v^2 = \omega^2 (a^2 - x^2)$ may be seen. (Use of $v = \pm a\omega \sin \omega t$ scores M0 (this implies $t = 0$ at an end-point.)	
	A1* Correct answer correctly obtained	
1(b)	M1 Use of $a\omega$ with the given value of a	
	A1 Allow 2.8 or better. Ignore units but must be positive.	

Question Number	Scheme	Marks
1(a)	$\frac{2\pi}{\omega} = \frac{1}{2} \Rightarrow \omega = \dots$ $\omega = 4\pi$ $v = \omega \times 0.3$ $v = 1.2\pi, 3.8 \text{ or better (m s}^{-1}\text{)}$	M1 A1 M1 A1 (4)
(b)	$x = a \sin \omega t \Rightarrow 0.15 = 0.3 \sin 4\pi t \Rightarrow t = \dots$ $t = \frac{1}{4\pi} \times \frac{\pi}{6} = \frac{1}{24} \text{ (s) } 0.04166\dots = 0.042 \text{ or better}$	M1 A1 (2) [6]
Notes		
(a) M1 A1 M1 A1 (b) M1 A1	Use period = 1/frequency to find a value for ω . Must be correct way up. Correct value for ω Use of $v = a\omega$ or $v^2 = \omega^2(a^2 - x^2)$ with $x=0$. cao Use $0.15 = a \sin \omega t$ to obtain a value for t . Use <i>their</i> a and ω . Correct value, 0.042 or better	
ALT 1(b) MI A1	Using cos Complete method using $x = a \cos \omega t$ AND $\frac{T}{4}$ to obtain a value for t $x = a \cos \omega t \Rightarrow 0.15 = 0.3 \cos 4\pi t \Rightarrow t = \dots$ $\frac{T}{4} - t = \frac{0.5}{4} - t = \dots$ Correct value, 0.042 or better	



Question Number	Scheme	Marks
3(a)	$v \frac{dv}{dx} = \frac{3\sqrt{x+1}}{4}$	M1
	$\frac{1}{2}v^2 = \frac{1}{2}(x+1)^{\frac{3}{2}} (+C)$	M1A1
	$x = 15, v = 8 \Rightarrow C = 0$ so $v = (x+1)^{\frac{3}{4}} *$	A1* (4)
3(b)	$\frac{dx}{dt} = (x+1)^{\frac{3}{4}}$	M1
	$4(x+1)^{\frac{1}{4}} = t (+C)$	M1A1
	$x = 15, t = 0 \Rightarrow C = 8$ so $4v^{\frac{1}{3}} = t + 8$	M1
	$t = 4v^{\frac{1}{3}} - 8$	A1 (5)
	OR	
	$\frac{dv}{dt} = \frac{3}{4}v^{\frac{2}{3}}$	M1
	$3v^{\frac{1}{3}} = \frac{3}{4}t (+C)$	M1A1
	$t = 0, v = 8 \Rightarrow C = 6$ so $3v^{\frac{1}{3}} = \frac{3}{4}t + 6$	M1
	$t = 4v^{\frac{1}{3}} - 8$	A1 (5)
		(9)
	Notes for question 3	
3(a)		
M1	Set up a differential equation in v and x only M0 if acceleration is $\frac{dv}{dx}$ or $\frac{dv}{dt}$ M0 if there is no differential equation eg starting with $\frac{1}{2}v^2 = \int \frac{3\sqrt{x+1}}{4} dx$	
M1	Clear attempt to separate variables and integrate acceleration in terms of v and x . At least one of the powers must increase by 1.	
A1	Correct integration, condone missing + C	
A1*	Given answer obtained from complete and correct working. Must include use of the boundary conditions and the initial differential equation. A0 if +C is not dealt with correctly eg If +C is only considered <i>after</i> the square root.	
3(b)		
M1	Set up a differential equation in x and t only. Using the given answer in (a).	
M1	Clear attempt to separate the variables and integrate in terms of x and t . At least one of the powers must increase by 1	
A1	Correct integration, condone missing + C	
M1	Use of boundary conditions in an integrated equation and use of (a) to form an equation in v and t . M0 if boundary conditions are not used.	
A1	Correct answer	
	OR	
M1	Set up a differential equation in v and t only	



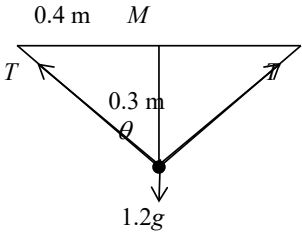
Question Number	Scheme	Marks
M1	Clear attempt to separate the variables and integrate in terms of v and t . At least one of the powers must increase by 1	
A1	Correct integration, condone missing $+ C$	
M1	Use of boundary conditions in an integrated equation to form an equation in v and t . M0 if boundary conditions are not used.	
A1	Correct answer	



Question Number	Scheme	Marks
4(a)	$a = 3 \text{ (m)}$	B1
	$\frac{38}{3} = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{3\pi}{19}$	M1A1
	$x = -a \cos \omega t \Rightarrow v = a\omega \sin \omega t$ or similar	M1
	$v = 3 \times \frac{3\pi}{19} \sin\left(\frac{3\pi}{19} \times \frac{95}{60}\right)$	M1
	$= \frac{9\pi\sqrt{2}}{38}, 1.1, 1.05, 1.052, \dots \text{ (m h}^{-1}\text{)}$	A1 (6)
4(b)	$-1.5 = 3 \cos \frac{3\pi t}{19}$	M1A1ft
	$t = \frac{38}{9} \text{ (h)}$	A1
	Time is 16:13 or 16:14	A1 (4)
		(10)
Notes for question 4		
4(a)		
B1	$a = 3$ seen or implied	
M1	For use of $T = \frac{2\pi}{\omega}$ to give an equation in ω where $T = 2 \times \frac{19}{3}$ (double the time between 12:00 and 18:20). Condone use of $T = 760$ min or 45600 seconds.	
A1	A correct equation using hrs, min or seconds.	
M1	Form a relevant equation in v and t using their a and ω Eg $x = -a \cos(\omega t) \Rightarrow v = a\omega \sin(\omega t)$ $x = a \cos(\omega t) \Rightarrow v = -a\omega \sin(\omega t)$ $x = a \cos(\omega t) = (2.12 \dots) \Rightarrow v^2 = \omega^2(a^2 - x^2)$	
M1	Use correct equation with an appropriate value of t	
A1	Correct answer, must be positive and must be in metres per hour.	
4(b)		
M1	A complete method to find the required time eg - Use of $-1.5 = a \cos \omega t$ to find required time is $\frac{1}{\omega} \cos^{-1}\left(\frac{x}{a}\right)$ - Use of $1.5 = a \sin \omega t$ to find required time is $\frac{1}{4} \text{ Period} + \frac{1}{\omega} \sin^{-1}\left(\frac{x}{a}\right)$ - Use of $1.5 = a \cos \omega t$ to find required time is $\frac{1}{\omega} \cos^{-1}\left(\frac{x}{a}\right)$ subtracted from 18:20	
A1ft	A correct equation, ft on their a and ω $\frac{1}{4}\left(\frac{38}{3}\right) + \frac{1}{\omega} \sin^{-1}\left(\frac{x}{a}\right)$	
A1	A correct t value in hours or minutes or seconds $t = \frac{38}{9} \text{ (h)}$, $t = \frac{760}{3} \text{ (min)}$ $t = 15200 \text{ (s)}$	
A1	For the correct time . Accept 4.13pm or 4.14 pm or 16:13 or 16:14	

Question Number	Scheme	Marks
5(a)	$x = 4 \cos\left(\frac{1}{5}\pi t\right) \quad \dot{x} = -4 \times \frac{\pi}{5} \sin\left(\frac{1}{5}\pi t\right)$ $\ddot{x} = -4 \times \left(\frac{\pi}{5}\right)^2 \cos\left(\frac{1}{5}\pi t\right)$ $\ddot{x} = -\left(\frac{\pi}{5}\right)^2 x \quad \therefore \text{SHM}$	M1A1 A1 (3)
(b)	period = $\frac{2\pi}{\frac{\pi}{5}} = 10$ (s)	M1A1 (2)
(c)	amplitude = 4 (m)	B1 (1)
(d)	$\dot{x} = -4 \times \frac{\pi}{5} \sin\left(\frac{1}{5}\pi t\right) \quad \text{or} \quad \dot{x}_{\max} = a\omega$ $\text{Max speed} = 4 \times \frac{\pi}{5} = \frac{4\pi}{5} \quad \text{or} \quad 0.8\pi \text{ (ms}^{-1}\text{)}$	M1 A1 (2)
(e)	At A $x = 1.5$ $1.5 = 4 \cos\left(\frac{1}{5}\pi t\right) \Rightarrow t_A = \frac{5}{\pi} \cos^{-1}\left(\frac{1.5}{4}\right)$ At B $x = -2.5$ $-2.5 = 4 \cos\left(\frac{1}{5}\pi t\right) \Rightarrow t_B = \frac{5}{\pi} \cos^{-1}\left(\frac{-2.5}{4}\right)$ Time A to B $= t_B - t_A = \frac{5}{\pi} \cos^{-1}\left(\frac{-2.5}{4}\right) - \frac{5}{\pi} \cos^{-1}\left(\frac{1.5}{4}\right) = 1.6862... = 1.7 \text{ or better (s)}$	M1A1 A1 A1 (4)
[12]		
(a) M1 A1 A1 (b) M1 A1 (c) B1 (d) M1 A1 (e) M1 A1 A1 A1	Differentiate the given expression for x twice (Both derivatives must be shown) Need to see: cos to sin to cos (ignore signs) Both derivatives correct Rewrite in the standard form for SHM and give the conclusion. Correct method Correct period Correct amplitude Use either method to obtain the max speed Correct max speed Find the time from the start to either A or B One correct time Second relevant time Correct time from A to B. 1.7 (s) or better	



Question Number	Scheme	Marks
ALT (e)	$1.5 = 4 \sin\left(\frac{1}{5}\pi t\right) \Rightarrow t_A = \frac{5}{\pi} \sin^{-1}\left(\frac{1.5}{4}\right)$ $2.5 = 4 \sin\left(\frac{1}{5}\pi t\right) \Rightarrow t_B = \frac{5}{\pi} \sin^{-1}\left(\frac{2.5}{4}\right)$ ${}_A t_B = t_B + t_A = \frac{5}{\pi} \sin^{-1}\left(\frac{2.5}{4}\right) + \frac{5}{\pi} \sin^{-1}\left(\frac{1.5}{4}\right) = 1.6862.. \text{ 1.7 or better}$	M1A1 A1 A1
6(a)	$T = \frac{\lambda x}{l} \Rightarrow 30 = \frac{\lambda \times 0.3}{0.5}$ $\lambda = 50 *$	M1A1 A1* (3)
(b)	 Extension = 0.5 m (used in (b) or (c)) $T = \frac{50 \times 0.5}{0.5} = (50)$ $R(\uparrow) \quad 2T \cos \theta - 1.2g = 1.2a$ $100 \times \frac{3}{5} - 1.2 \times 9.8 = 1.2a$ $a = 40.2 \quad a = 40 \text{ or } 40.2 \text{ m s}^{-2} \text{ (positive)}$	B1 M1A1ft M1 A1ft A1 (6)
(c)	$\text{E.P.E.} = \frac{1}{2} \times 50 \times \frac{0.5^2}{0.5}$ $1.2g \times 0.3 + \frac{1}{2} \times 1.2v^2 = \frac{1}{2} \times 50 \times \frac{0.5^2}{0.5} - \frac{1}{2} \times 50 \times \frac{0.3^2}{0.5}$ $v^2 = \frac{1}{0.6} \left(25 \times \frac{0.5^2}{0.5} - 25 \times \frac{0.3^2}{0.5} - 1.2g \times 0.3 \right) (= 7.452)$ $v = 2.730... = 2.7 \text{ or } 2.73 \text{ ms}^{-1}$	B1ft (any correct EPE) M1A1A1 DM1 A1 (6)

