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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Statics & Centres of Mass of Solids (M3.5)

International A Level Mathematics · M3: Mechanics 3 (WME03)

20 questions · 198 marks · Mark schemes included

Past papers January 2020 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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M3.5b TOPPLING & RIGID BODIES			

No questions on this sub-topic in the available past papers.



QUESTIONS

20 questions · 198 marks · 1 sub-topic

SUB-TOPIC M3.5a

CENTRES OF MASS OF SOLIDS (BY INTEGRATION)

Sub-topic M3.5a · 20 questions · 198 marks · lowest tariff first



1.

In this question you must show all stages in your working.

Solutions relying on calculator technology are not acceptable.

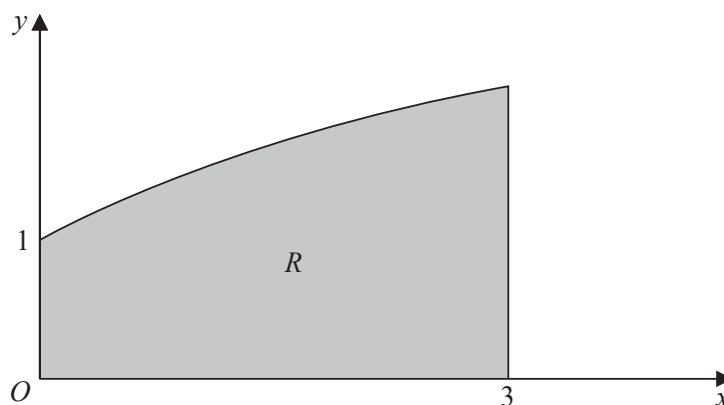


Figure 1

The finite region R , shown shaded in Figure 1, is bounded by the x -axis, the line with equation $x = 3$, the curve with equation $y = \sqrt{x + 1}$ and the y -axis.

Find the **y coordinate** of the centre of mass of a uniform lamina in the shape of R .

(5)



Question 1 continued

(Total for Question 1 is 5 marks)



1.

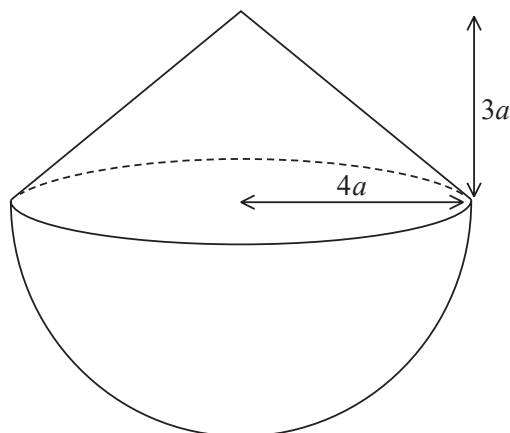


Figure 1

A hollow toy is formed by joining a uniform right circular conical shell C , with radius $4a$ and height $3a$, to a uniform hemispherical shell H , with radius $4a$. The circular edge of C coincides with the circular edge of H , as shown in Figure 1.

The mass per unit area of C is λ and the mass per unit area of H is $k\lambda$ where k is a constant.

Given that the centre of mass of the toy is a distance $4a$ from the vertex of the cone, find the value of k .

(6)

Question 1 continued

(Total for Question 1 is 6 marks)



1.

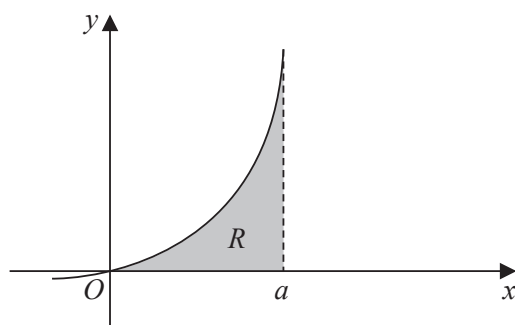


Figure 1

A uniform lamina is in the shape of the region R .

Region R is bounded by the curve with equation $y = x(x + a)$ where a is a positive constant, the positive x -axis and the line with equation $x = a$, as shown shaded in Figure 1.

Find the **y coordinate** of the centre of mass of the lamina.

(7)

Question 1 continued

(Total for Question 1 is 7 marks)



Question 1 continued

Leave
blank

(Total 8 marks)

Q1



Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)



Question 1 continued

(Total for Question 1 is 8 marks)



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 8 marks)



Question 3 continued



Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)



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4.

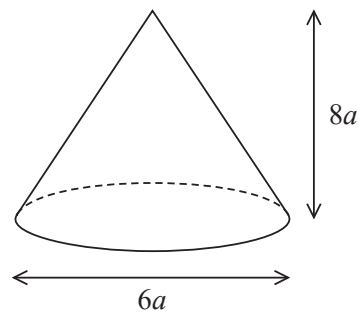


Figure 3

A uniform right solid cone C has diameter $6a$ and height $8a$, as shown in Figure 3.

The solid S is formed by removing a cone of height $4a$ from the top of C and then removing an identical, inverted cone. The vertex of the removed cone is at the point O in the centre of the base of C , as shown in Figure 4.

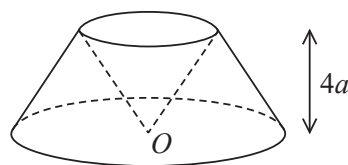


Figure 4

(a) Find the distance of the centre of mass of S from O .

(5)

The point A lies on the circumference of the base of S and the point B lies on the circumference of the top of S . The points O , A and B all lie in the same vertical plane, as shown in Figure 5.

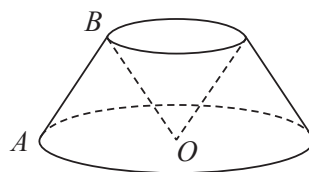


Figure 5

The solid S is freely suspended from the point B and hangs in equilibrium.

(b) Find the size of the angle that AB makes with the downward vertical.

(4)

Question 4 continued

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Question 4 continued

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Question 4 continued

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Q4

(Total 9 marks)



- Use algebraic integration to

- (b) find the x coordinate of the centre of mass of S .

- (5)

Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

(Total for Question 3 is 9 marks)



Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total 10 marks)

Q4



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 10 marks)



3.

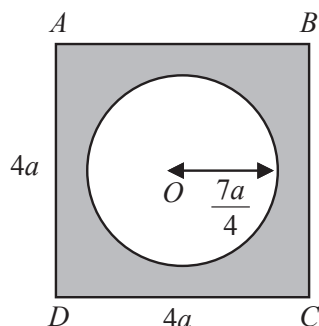


Figure 3

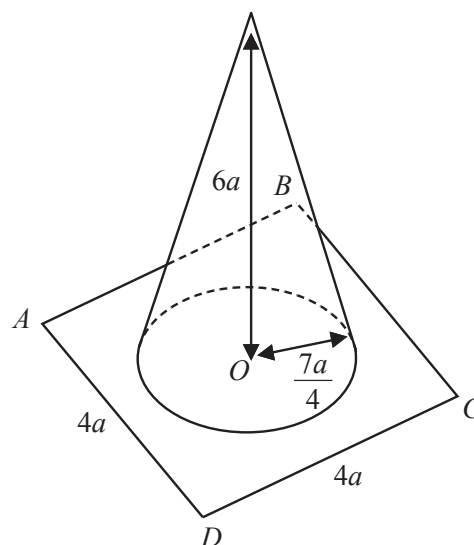


Figure 4

A square $ABCD$ of side $4a$ is made from thin uniform cardboard. The centre of the square is O . A circle with centre O and radius $\frac{7a}{4}$ is then removed from the square to form a template T , shown shaded in Figure 3.

A right conical shell, with no base, has radius $\frac{7a}{4}$ and perpendicular height $6a$.

The shell is made of the same thin uniform cardboard as T .

The shell is attached to T so that the circumference of the end of the shell coincides with the circumference of the circle centre O , to form the hat H , shown in Figure 4.

[The surface area of a right conical shell of radius r and slant height l is πrl .]

(a) Show that the exact distance of the centre of mass of H from O is

$$\frac{175\pi a}{(63\pi + 128)} \quad (8)$$

A fixed rough plane is inclined to the horizontal at an angle α . The hat H is placed on the plane, with $ABCD$ in contact with the plane, and AB parallel to a line of greatest slope of the plane. The plane is sufficiently rough to prevent the hat from sliding down the plane.

Given that the hat is on the point of toppling,

(b) find the exact value of $\tan \alpha$, giving your answer in simplest form.

(2)

[illegible]

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)



4. A uniform solid hemisphere H has radius r and centre O

- (a) Show that the centre of mass of H is $\frac{3r}{8}$ from O

$$\left[\text{You may assume that the volume of } H \text{ is } \frac{2\pi r^3}{3} \right]$$

(4)

A uniform solid S , shown below in Figure 3, is formed by attaching a uniform solid right circular cylinder of height h and radius r to H , so that one end of the cylinder coincides with the plane face of H .

The point A is the point on H such that $OA = r$ and OA is perpendicular to the plane face of H

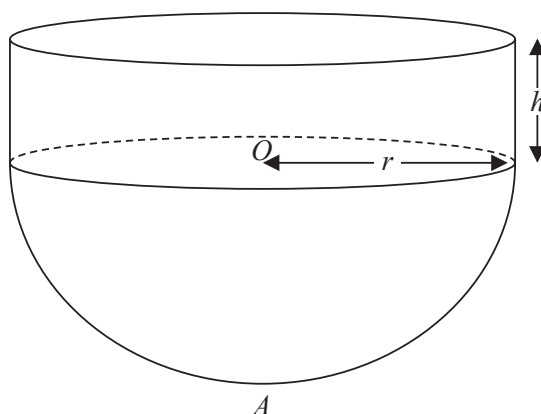


Figure 3

- (b) Show that the distance of the centre of mass of S from A is

$$\frac{5r^2 + 12rh + 6h^2}{8r + 12h}$$

(5)

The solid S can rest in equilibrium on a horizontal plane with any point of the curved surface of the hemisphere in contact with the plane.

- (c) Find r in terms of h .

(2)



Question 4 continued

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 11 marks)



5. A uniform right solid circular cone C has radius r and height $4r$.

- (a) Show, using algebraic integration, that the distance of the centre of mass of C from its vertex is $3r$.

[You may assume that the volume of C is $\frac{4}{3}\pi r^3$]

(4)

A uniform solid S , shown below in Figure 3, is formed by **removing** from C a uniform solid right circular cylinder of height r and radius $\frac{1}{2}r$, where the centre of one end of the cylinder coincides with the centre of the plane face of C and the axis of the cylinder coincides with the axis of C .

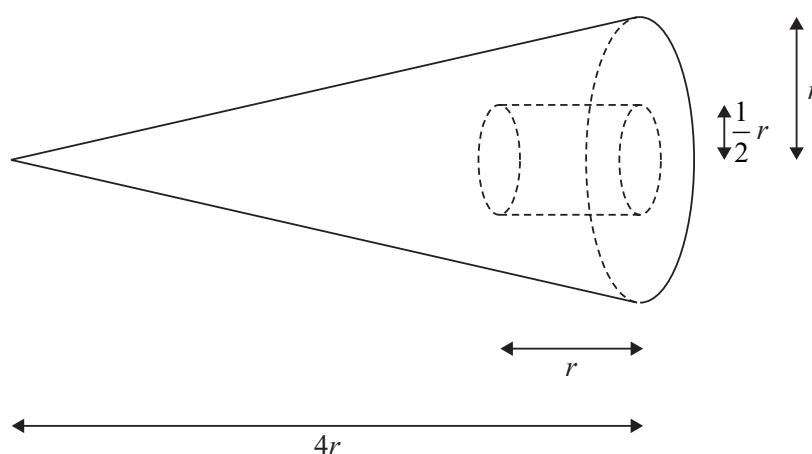


Figure 3

- (b) Show that the distance of the centre of mass of S from the vertex of C is $\frac{75}{26}r$

(5)

A rough plane is inclined at an angle α to the horizontal.

The solid S rests in equilibrium with its plane face in contact with the inclined plane.

Given that S is on the point of toppling,

- (c) find the exact value of $\tan \alpha$

(3)

Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



4.

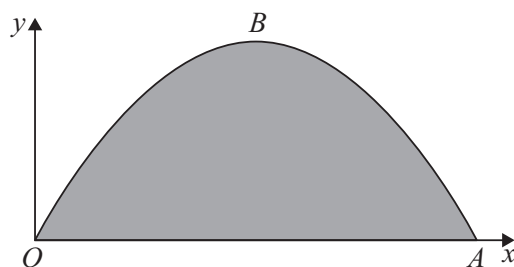


Figure 3

Figure 3 shows a shaded uniform lamina OAB which is in the shape of the region enclosed by the x -axis and the curve with equation $y = \frac{1}{a}(ax - x^2)$, where a is a positive constant. The centre of mass of the lamina is at G .

(a) Find, in terms of a , the y coordinate of G .

(6)

The lamina is now freely pivoted in a vertical plane about a smooth horizontal axis which passes through O and is perpendicular to the plane of the lamina.

The lamina is held in equilibrium by a horizontal force which is applied to the lamina at A . The line OA makes an angle θ with the downward vertical through O , where

$\tan \theta = \frac{3}{4}$, as shown in Figure 4.

The line of action of the force lies in the plane of the lamina.

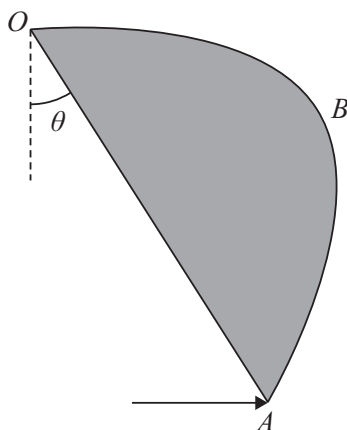


Figure 4

The mass of the lamina is M and the magnitude of the applied force is F .

(b) Find F in terms of M and g .

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 12 marks)



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7.

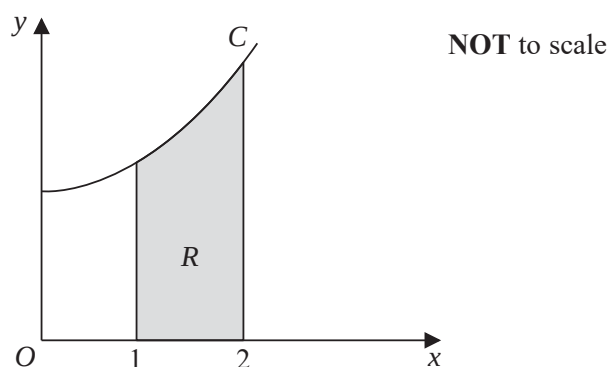


Figure 3

Figure 3 shows part of the curve C with equation $y = x^2 + 4$. The shaded region R is bounded by C , the line with equation $x = 1$, the x -axis and the line with equation $x = 2$.

The unit of length on each axis is one centimetre.

A uniform wooden solid, S , is made in the shape formed by rotating the region R through 360° about the x -axis.

(a) Using algebraic integration,

(i) show that the volume of S is $\frac{613\pi}{15} \text{ cm}^3$

(ii) find, to 3 significant figures, the distance of the centre of mass of S from O .

(8)

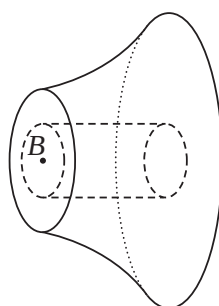


Figure 4

A solid, S_1 , is formed by removing a solid cylinder of radius 3 cm and length 1 cm from S . A metal cylinder, of radius 3 cm and length 1 cm is placed in the resulting hole to form a new solid T , as shown in Figure 4. The axis of the metal cylinder coincides with the axis of symmetry of S_1 . The point B is the centre of the smaller plane face of T . The mass per unit volume of S_1 is M and the mass per unit volume of the metal cylinder is $5M$.

(b) Find the distance of the centre of mass of T from B .

(5)

Question 7 continued

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Question 7 continued

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Question 7 continued

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Q7

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

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7. [You may assume that the volume of a cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$.]

A uniform solid right circular cone C , with vertex V , has base radius r and height h .

- (a) Show that the centre of mass of C is $\frac{3}{4}h$ from V (4)

A solid F , shown below in Figure 4, is formed by removing the solid right circular cone C' from C , where cone C' has height $\frac{1}{3}h$ and vertex V

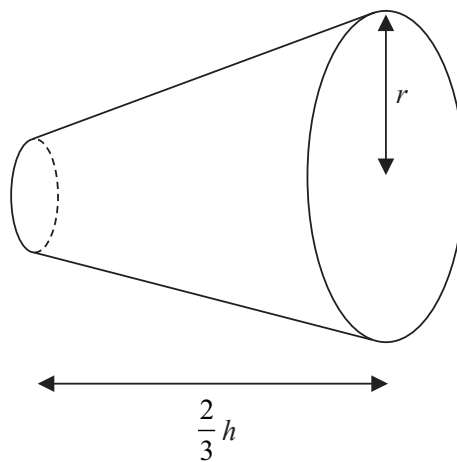


Figure 4

- (b) Show that the distance of the centre of mass of F from its larger plane face is $\frac{3}{13}h$ (5)

The solid F rests in equilibrium with its curved surface in contact with a horizontal plane.

- (c) Show that $13r^2 \leq 17h^2$ (5)

Question 7 continued

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Question 7 continued

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Lined area for writing the answer to Question 7 continued.



Question 7 continued

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Question 7 continued

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Lined area for writing the answer to Question 7.

(Total 14 marks)

TOTAL FOR PAPER = 75 MARKS

END

Q7

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6. (a) Show, using algebraic integration, that the centre of mass of a uniform **solid** hemisphere H of radius a is a distance $\frac{3}{8}a$ from O , the centre of its plane face.

[You may assume that the volume of the hemisphere is $\frac{2}{3}\pi a^3$]

(5)

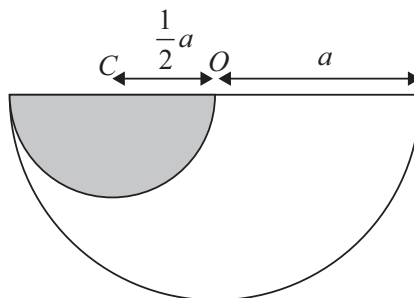


Figure 5

A uniform **solid** S is formed by removing a smaller **solid** hemisphere of radius $\frac{1}{2}a$ from H such that

- the plane face of the smaller hemisphere has centre C and is part of the plane face of H
- $OC = \frac{1}{2}a$

Figure 5 shows a cross section of S , where S is the **unshaded** part.

- (b) Show that the centre of mass of S is $\frac{45}{112}a$ from the line through O and C .

(4)

The solid S rests in equilibrium with its curved surface in contact with a rough horizontal plane, as shown in Figure 6.

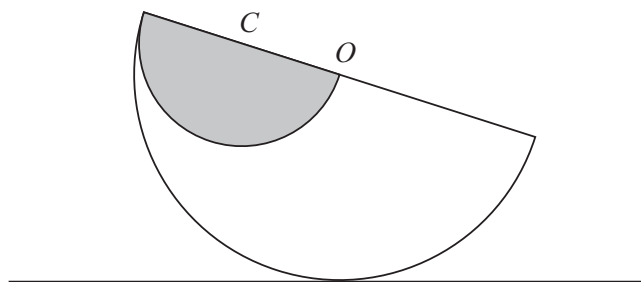


Figure 6

The angle between CO and the horizontal is θ .

- (c) Find the exact value of $\tan \theta$.

(5)

Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 14 marks)





Question 5 continued

MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
1.	$\int_0^3 \sqrt{x+1} dx$	M1
	$= \frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_0^3$	A1
	$\frac{\int_0^3 \frac{1}{2} (\sqrt{x+1})^2 dx}{\int_0^3 \sqrt{x+1} dx}$ or $\frac{\int_0^3 \frac{1}{2} (x+1) dx}{\int_0^3 \sqrt{x+1} dx}$	M1
	$= \frac{\frac{1}{2} \left[\frac{1}{2} x^2 + x \right]_0^3}{\frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_0^3}$ or $\frac{\frac{1}{2} \left[\frac{1}{2} (x+1)^2 \right]_0^3}{\frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_0^3}$	A1
	$= \frac{45}{56}$ (0.80 or better)	A1 (5)
		(5)
	Notes	
M1	Use of $\int_0^3 \sqrt{x+1} dx$. Limits not needed. Accept $k \times \int_0^3 \sqrt{x+1} dx$ where k is a constant. To give the mark for 'use' we must see an attempt at integration. An attempt at integration is seen when the powers increase by 1.	
A1	Correct integrated expression with correct limits	
M1	Use of $\frac{\int_0^3 \frac{1}{2} (\sqrt{x+1})^2 dx}{\int_0^3 \sqrt{x+1} dx}$. Limits not needed. The formula must be correct but allow a constant multiple if it appears on both numerator and denominator. We must see the correct formula and an attempt at integrating the numerator	
A1	Correct integrated expression for the numerator in the correct formula with correct limits.	
A1	Correct answer. This question comes with a calculator warning: the correct answer must come from integrated expressions ie both previous A's must have been awarded. Numerical substitution does not need to be seen.	

Question Number	Scheme				Marks
1	Slant height = $5a$				B1
		Hemisphere	Cone	Total	
	Mass	$2\pi(4a)^2 k\lambda$	$\pi \times 4a \times 5a \times \lambda$	$32\pi a^2 k\lambda + 20\pi a^2 \lambda$	B1
	Mass ratio(m)	$8k$	5	$8k + 5$	
	Distance from vertex (y)	$5a$	$2a$	$4a$	B1
	my	$40ka$	$10a$	$4a(8k + 5)$	
	$40ka + 10a = 4a(8k + 5)$				M1A1
	$k = 1.25$ oe				A1
					[6]
	<i>Alternatives for the moments equation:</i>				
ALT 1	Moments about the centre of mass (G) of the toy:				
	Distances: $(-)a$ $2a$ 0 (Accept a provided the minus appears in the equation)				B1
	Equation: $-8ka + 10a = 0$				M1A1
ALT 2	Moments about the lowest point of the hemisphere:				
	Distances: $2a$ $5a$ $3a$				B1
	Equation: $16ka - 5a = a(8k + 5)$				M1A1
ALT 3	Moments about the centre of the circular base of the conical shell/hemispherical shell				
	Distances: $2a$ $(-)a$ a				B1
	Equation: $16ka + 25a = 3a(8k + 5)$				M1A1

B1	Slant height = $5a$ seen anywhere (could be on diagram)	
B1	Correct masses/mass ratio for hemisphere, cone and combined shape.	
B1	Correct distances seen. If using Alts 1 or 3 minus signs not needed here.	
M1	Moments equation attempted with all 3 terms (2 terms if about G). Condone inconsistent mass dimensions, if they are clearly using what they consider to be a mass. The equation can be formed using actual masses or a ratio of masses.	
A1	Correct moments equation, with actual masses or a ratio of masses and all signs correct.	
A1	Correct value for k . 1.25 , $\frac{5}{4}$, $1\frac{1}{4}$ or any other equivalent fraction.	

Question Number	Scheme	Marks
1.	$\text{Area} = \int_0^a (x^2 + ax) dx = \left[\frac{1}{3}x^3 + \frac{1}{2}ax^2 \right]_0^a = \frac{5a^3}{6}$ $\int \frac{1}{2}y^2 dx = \int_0^a \frac{1}{2}(x^4 + 2ax^3 + a^2x^2) dx$ $= \frac{1}{2} \left[\frac{1}{5}x^5 + \frac{a}{2}x^4 + \frac{a^2}{3}x^3 \right]_0^a \left(= \frac{31a^5}{60} \right)$ $\bar{y} = \frac{\int \frac{1}{2}y^2 dx}{\int y dx} = \frac{31a^5}{60} \div \frac{5a^3}{6} = \frac{31a^2}{50}$	M1A1 M1 DM1A1 M1A1 (7) [7]
M1 A1 M1 DM1 A1 M1 A1	Attempt the area by integration. Powers of both terms to increase by 1. Correct area. Use $\int \frac{1}{2}y^2 dx$ to give $\int_0^a \frac{1}{2}(x^4 + 2ax^3 + a^2x^2) dx$. Limits not needed. Squaring to be correct. For method mark, condone missing $\frac{1}{2}$ or any multiple. Attempt the integration (powers of at least 2 terms to increase by 1). Depends on second M mark. Correct integration and correct limits shown. Limits needed but substitution does not need to be seen. Use $\bar{y} = \frac{\int \frac{1}{2}y^2 dx}{\int y dx}$ Note: This independent method mark is for use of the correct formula. Correct answer.	

Question Number	Scheme	Marks
1(a)	$V = \pi \int_1^a y^2 dx$	
	$V = (\pi) \int_1^a \frac{1}{x^2} dx = (\pi) \left[-\frac{1}{x} \right]_1^a$	M1A1
	$V = \pi \left(1 - \frac{1}{a} \right)^*$	A1*
		(3)
(b)	$(\pi) \int xy^2 dx$	M1
	$(\pi) \int_1^a \frac{1}{x} dx = (\pi) [\ln x]_1^a = (\pi) \ln a$	dM1A1
	$(\pi) \left(1 - \frac{1}{a} \right) \bar{x} = (\pi) \ln a$	
	$\bar{x} = \frac{a \ln a}{a - 1}$	M1A1
		(5)
		[8]



(a)

M1 Use of $\int y^2 dx$ AND an attempt at algebraic integration (power increasing by one)

A1 Correct integration. π not needed.

A1* Given result reached from fully correct working. If π not included from the start, its inclusion must now be justified.

(b)

M1 Use of $\int xy^2 dx$, must have substituted for y . π not needed.

dM1 Attempt at algebraic integration ($\ln x$ neds to be seen)

A1 Correct result after substitution of limits. π not needed.

M1 Use of $\frac{\int xy^2 dx}{\int y^2 dx}$. If π and/or ρ appear, they must appear consistently.

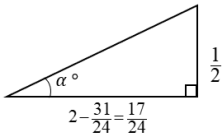
A1 Correct final answer. They lose this mark if they leave $1 - \frac{1}{a}$ in the denominator.



Question Number	Scheme	Marks
4(a)	Ratio of masses: $4\pi a^2$ $4\pi a \times ka$ $8\pi a^2$ $12\pi a^2 + 4k\pi a^2$	B1
	Distances: (0) $\frac{k}{2}a$ $(1+k)a$ $\bar{y}$	B1
	$(0+)k \times \frac{k}{2}a + 2(1+k)a = (k+3)\bar{y}$	M1A1ft
	$\left(\frac{k^2}{2} + 2 + 2k\right)a = (k+3)\bar{y}$	
	$\bar{y} = \frac{(k^2 + 4k + 4)}{2(k+3)}a$ *	A1 * (5)
(b)	$\tan 60^\circ = \frac{(k^2 + 4k + 4)}{2(k+3)}a \div 2a$	M1
	$k^2 + 4k(1 - \sqrt{3}) + (4 - 12\sqrt{3}) = 0$	A1
	$k > 0 \Rightarrow k = 5.8147... = 5.8$ or 5.81 or better	A1 (3)
[8]		
(a) B1 B1 M1 A1ft A1* (b) M1 A1 A1	Correct ratio of masses – any equivalent to that shown Correct distances from O or a parallel axis. Attempt a moments equation. Must be dimensionally correct (not using volumes) and have no extra terms. Correct equation, follow through their ratio of masses and distances Correct given expression with sufficient working Use $\tan 60 = \frac{\bar{y}}{2a}$ or $\frac{2a}{\bar{y}}$ May also use $\tan 30$ Obtain the correct 3TQ Correct value for k. Note for (a): The distance from O for the combined cylinder and base is $\frac{ak^2}{2(1+k)}$.	

Question Number	Scheme	Marks
1(a)	$\pi \int_0^1 (1 + \sqrt{x})^2 dx$	M1
	$= \pi \left[x + \frac{4}{3} x^{\frac{3}{2}} + \frac{1}{2} x^2 \right]_0^1$	A1
	$= \frac{17\pi}{6} \text{ m}^3 \text{ * including units}$	A1*
		(3)
(b)	$\pi \int_0^1 x(1 + \sqrt{x})^2 dx$	M1
	$= \pi \left[\frac{1}{2} x^2 + \frac{4}{5} x^{\frac{5}{2}} + \frac{1}{3} x^3 \right]_0^1$	A1
	$= \frac{49\pi}{30}$	A1
	$\bar{x} = \frac{\frac{49\pi}{30}}{\frac{17\pi}{6}}$	dM1
	$= \frac{49}{85} \text{ m} \text{ * including units}$	A1 *
		(5)
		(8)
Notes		
NB: Penalise missing units maximum of once per question.		
(a)		
M1	Use of $\pi \int_0^1 (1 + \sqrt{x})^2 dx$. Limits not needed. π is required.	
A1	Correct integration – limits not needed	
A1*	Correct given answer correctly obtained. Must include units. Limits must be seen (sight of substitution is not required). Accept $\frac{17}{6} \pi \text{ m}^3$	
(b)		
M1	Use of $\pi \int_0^1 x(1 + \sqrt{x})^2 dx$. Limits not needed (π 's will cancel so it may not be seen)	
A1	Correct integration – limits not needed	
A1	Correct unsimplified with or without π (may see $\frac{1}{2} + \frac{4}{5} + \frac{1}{3} - 0$)	
dM1	Correct expression with their numerator (consistent π - seen in neither or both)	
A1*	Correct given answer correctly obtained. Must include units.	

Question Number	Scheme	Marks
2.	$T = mg \cos \theta$	M1A1
	$T = \frac{2mg \left(\frac{21}{10}a - ka \right)}{ka}$	M1A1
	$\frac{4}{5}mg = \frac{2mg \left(\frac{21}{10}a - ka \right)}{ka}$	dM1
	$k = \frac{3}{2}$ or 1.5	A1
		(6)
	Notes	
M1	Resolve parallel to the string, correct no. of terms, condone sign errors and sin/cos confusion (or resolve in two directions and eliminate the unknown force or use trig on a right-angled triangle of forces) to give an equation in T , mg and θ only	
A1	Correct equation. Trig does not need to be substituted.	
M1	Use Hooke's Law with correct structure.	
A1	Correct equation	
dM1	Substitute trig and eliminate T to produce equation in k only, dependent on previous M's. If x is used for extension, should see $x = \frac{2k}{5}a \rightarrow ka + \frac{2k}{5}a = \frac{21}{10}a$	
A1	cao	
ALT 1	First M1A1	
M1A1	Complete method to form an equation in T and θ Vert: $mg = T \cos \theta + F \sin \theta$ Horiz: $F \cos \theta = T \sin \theta$ Eliminate F eg $\frac{\sin \theta}{\cos \theta} = \frac{mg - T \cos \theta}{T \sin \theta}$	

3a	Moment of S about the y-axis	M1	Use of formula $(\pi)(\rho) \int xy^2 dx$ No need to see the correct limits here. The curve equation must be substituted correctly and an attempt to integrate seen (at least one term must have a power of x raised by 1) Note the correct expression for integrating is $x \left(\frac{1}{4} x(3-x) \right)^2 = \frac{1}{16} x^3 (3-x)^2$ $= \frac{1}{16} (9x^3 - 6x^4 + x^5)$
	$= (\pi)(\rho) \frac{1}{16} \left[\frac{9}{4} x^4 - \frac{6}{5} x^5 + \frac{1}{6} x^6 \right]$	A1	Correct integrated expression.
	$= \frac{31}{60} (\pi)(\rho)$	A1	Correct use of correct limits (0 and 2). No need to see a line of working showing substitution of limits. However, must see $\frac{31}{60}$ or equivalent numerical evaluation of integral.
	$\bar{x} = \frac{\frac{31}{60} (\pi)(\rho)}{\frac{2}{5} (\pi)(\rho)}$	M1	Complete method to find the distance. Formula must be the right way up $\bar{x} = \frac{(\pi)(\rho) \int xy^2 dx}{M}$ Must have consistent use of π and of ρ .
	$= \frac{31}{24} *$	A1*	Obtain given answer from correct working
		[5]	
3b	Correct use of trig 	M1	Correct trig ratio to find a relevant angle, α° or $(90 - \alpha)^\circ$ Must use curve equation with $x = 2$ and $\left(2 - \frac{31}{24} \right)$
	$\tan \alpha^\circ = \frac{1}{2} \div \frac{17}{24} \left(= \frac{12}{17} \right)$	A1	Or equivalent. Condone reciprocal.
	$\alpha = 35$	A1	2 sf or better (35.2175...) A0 for use of radians.
		[3]	
		(8)	

Question Number	Scheme					Marks
4(a)		Top cone	inside cone	C	S	
	Mass ratio	(-) 1	(-) 1	8	6	B1
	y distance	$5a$	$3a$	$2a$	$\bar{y}$	B1
	My	$5a$	$3a$	$16a$	$6\bar{y}$	
	$8 \times 2a - 1 \times 5a - 1 \times 3a = 6\bar{y}$					M1A1 ft
	$6\bar{y} = 8a \rightarrow \bar{y} = \frac{4}{3}a$					A1
						(5)
(b)	$\tan \alpha = \frac{3}{8} \quad (\alpha = 20.556 \dots \text{or } 69.44 \dots)$					B1
	$\tan \beta = \frac{\frac{3a}{2}}{4a - \frac{4a}{3}} = \frac{9}{16} \quad (\beta = 29.357 \dots \text{or } 60.642 \dots)$					M1A1ft
	$\alpha + \beta = \theta = 50^\circ$ (or better 49.91379...)					A1
	or $180 - 69.44 \dots - 60.64 \dots = 50^\circ$					(4)
						[9]
ALT (b)	$\cos \theta = \frac{AB^2 + BG^2 - AG^2}{2 \times AB \times BG}$					B1
	$BG^2 = (1.5a)^2 + (4a - \bar{y})^2 \quad (= \frac{337a^2}{36})$ $AG^2 = (3a)^2 + (\bar{y})^2 \quad (= \frac{97a^2}{9})$ $\{AB^2 = (1.5a)^2 + (4a)^2 \quad (= \frac{73a^2}{4})\}$					M1
	$\cos \theta = \dots \dots \dots = 0.6439 \dots \dots$					A1ft
	$\theta = 50^\circ$ (or better 49.91379...)					A1



(a)**B1** Correct mass ratio seen for 3 cones and S. Allow consistent (-)**B1** Correct distances for the 3 cones. (Allow distances from vertex ($3a$, $5a$, $6a$) or small plane face ($-a$, a , $2a$).) Condone missing (-)**M1** Dimensionally correct moments equation about any parallel axis. Must include 4 terms.**A1ft** Correct moments equation follow through their distances.**A1** $\bar{y} = \frac{4}{3}a$ o.e.**SC** if a 's are missing B1B0M1A1ftA0 is the maximum available**(b)****B1** Correct expression for $\tan \alpha$ or α seen (either way round)**M1** Correct attempt to use their $\bar{y}$ to find $\tan \beta$ (either way round)**A1ft** Correct expression for $\tan \beta$ or β (either way round). Ft their $\bar{y}$ **A1** 50° or better (0.87 rad or better 0.87116.....)**ALT (b)** uses the cosine rule with triangle ABG **B1** Correct expression for $\cos \theta$ in terms of AB , AG and BG **M1** Correct attempt to use their $\bar{y}$ to find AG and BG **A1ft** Correct expression for $\cos \theta$. Ft their $\bar{y}$ **A1** 50° or better

Question Number	Scheme	Marks
2a)	$\sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}$	
	$T \cos \theta = mg$	M1A1
	$\frac{4}{5}T = mg \rightarrow T = \frac{5mg}{4}$	A1
		(3)
2b)	$r = a + \frac{5a}{4} \times \frac{3}{5} = \frac{7a}{4}$	B1
	$T \sin \theta = m\omega^2 \left(\frac{7a}{4} \right)$	M1A1A1
	$\frac{3}{5} \left(\frac{5mg}{4} \right) = \frac{7}{4} m\omega^2$	DM1
	$\omega = \sqrt{\frac{3g}{7a}}$	A1
		(6)
		[9]
a)		
M1	Resolving vertically. $T \cos \theta$ or $T \sin \theta$ accepted	
A1	Correct equation	
A1	Correct tension	
b)		
B1	Correct radius of motion seen explicitly or used in N2L	
M1	Attempt at horizontal equation of motion. Allow either form of acceleration. $T \cos \theta$ or $T \sin \theta$ accepted. Allow this mark if $r = \frac{3a}{4}$ used.	
A1	Correct LHS	
A1	Correct RHS. Acceleration must now be in $r\omega^2$ form.	
DM1	Substitute in trig and eliminate T to find value for ω or ω^2 . Depends on first M mark in (b)	
A1	Correct value for ω . (Square root sign must cover all terms)	

Question Number	Scheme	Marks
3a)	$\int_0^9$	B1
	$V = (\pi) \int_0^9 (3 - \sqrt{x})^2 dx$	M1
	$V = (\pi) \int_0^9 (9 - 6\sqrt{x} + x) dx$	
	$V = (\pi) \left[9x - 4x^{\frac{3}{2}} + \frac{x^2}{2} \right]_0^9$	A1
	$V = (\pi) \left[81 - 108 + \frac{81}{2} \right] \quad (-\pi[0])$	
	$V = \frac{27}{2} \pi *$	A1 *
		(4)
3b)	$(\pi) \int xy^2 dx = (\pi) \int \left(9x - 6x^{\frac{3}{2}} + x^2 \right) dx$	M1
	$= (\pi) \left[\frac{9}{2} x^2 - \frac{12}{5} x^{\frac{5}{2}} + \frac{x^3}{3} \right]_0^9$	A1
	$= (\pi) \left(\left[\frac{729}{2} - \frac{2916}{5} + \frac{729}{3} \right] - [0] \right)$	
	$= \frac{243}{10} (\pi)$	A1
	$\bar{x} = \frac{(\pi) \int xy^2 dx}{(\pi) \int y^2 dx}$	DM1
	$\bar{x} = \frac{\left(\frac{243}{10} \right)}{\left(\frac{27}{2} \right)} = 1.8$	A1
		(5)
		[9]



(a)		
B1	Identifying correct limits.	
M1	Attempt at $(\pi) \int_0^9 (3 - \sqrt{x})^2 dx$. π not needed. Limits may be missing. Minimum accepted for the squaring is $9 \pm k\sqrt{x} \pm x$. At least one term must be integrated (power increased) and none to be differentiated (power decreased)	
A1	Correct integration. π not needed but (correct) limits now needed.	
A1*	Given result reached from fully correct working. π must not just appear on final line without justification (as this is a “show that” question).	
(b)		
M1	$(\pi) \int xy^2 dx = (\pi) \int \left(9x - 6x^{\frac{3}{2}} + x^2 \right) dx$. π not needed.	
A1	Correct integration. π not needed.	
A1	Correct result from substitution of correct upper limit. (Lower limit is 0 and substitution of 0 gives 0)	
DM1	Use of $\bar{x} = \frac{(\pi) \int xy^2 dx}{(\pi) \int y^2 dx}$. π must appear in both or neither. Depend on both previous M marks	
A1	1.8. Accept any exact equivalent eg $\frac{18}{10}, \frac{9}{5}, 1\frac{4}{5}$ etc.	



Question Number	Scheme	Marks
4(a)	$M\bar{y} = (\rho)\pi \int_0^a y(a^2 - y^2)dy$	M1 A1
	$M\bar{y} = (\rho)\pi \left[\frac{a^2 y^2}{2} - \frac{y^4}{4} \right]_0^a$	A1
	$\left(M\bar{y} = (\rho)\pi \frac{a^4}{4} \right)$	
	$\frac{\bar{y}}{\bar{y}} = \frac{(\pi\rho)\frac{a^4}{4}}{(\pi\rho)\frac{2a^3}{3}} = \frac{3a}{8} *$	M1A1 *
	S.C. Clear use of $r = 1$, with a substituted at end can score max M1A0A0 M1A0	(5)
(b)	$(k+1)m\bar{x} = kma \left(\Rightarrow \bar{x} = \frac{ka}{k+1} \right)$	M1A1
	$(k+1)m\bar{y} = m \times \frac{3}{8}a \left(\Rightarrow \bar{y} = \frac{3a}{8(k+1)} \right)$	A1
	$\tan \alpha = \frac{\bar{x}}{\bar{y}}$	M1
	$k = \frac{3\sqrt{3}}{8}$	A1
		(5)
		[10]
Alt b	Moments about O $m \times \frac{3a}{8} \sin \alpha = kma \times \cos \alpha$	M1A2
	$k = \frac{3\sqrt{3}}{8}$	M1A1



(a)

M1 Using $(\pi) \int_0^r x^2 y \, dy$ with or without π . Must be dimensionally correct with integrand of the form $y(a^2 - y^2)$. Limits not needed.

A1 $\int_0^a y(a^2 - y^2) \, dx$ Correct integral. Limits not needed.

A1 Correct integration. Limits not needed.

M1 Using $\bar{y} = \frac{\int_0^a \pi x^2 y \, dy}{\frac{2}{3} \pi a^3}$. π in numerator and denominator or in neither.

A1 Correct **given** result with no errors seen. **Cso**

S.C. If they clearly find CoM of a hemisphere with a radius other than a , they can gain the M marks for a correct method for their hemisphere. If they work with an algebraic radius, they can gain the first 2 A marks and if they replace their radius with a at the end, they can gain the final mark.

(b) First 3 marks can be awarded for equations only seen as part of a vector equation.

M1 Dimensionally correct moments equation for $\bar{x}$ or $\bar{y}$

A1 Correct equation in $\bar{x}$ or $\bar{y}$ (Give BOD on use of x and/or y).

A1 Correct equations for $\bar{x}$ and $\bar{y}$

M1 Dividing to form equation in $\tan \alpha$ and solve for k . Trig must be right

A1 $k = \frac{3\sqrt{3}}{8}$ any correct exact form.

ALT b) - Taking moments about O .

M1 Dimensionally correct moments equation about O .

A1 Moments equation with at most one error.

A1 Fully correct equation.

M1 Solve for k .

A1 c.a.o.



Question Number	Scheme	Marks									
5(a)	$(\pi\rho)\int_0^r xy^2 dx$ $= (\pi\rho)\int_0^r x(r^2 - x^2) dx$ $= (\pi\rho)\left[\frac{1}{2}x^2r^2 - \frac{x^4}{4}\right]_0^r$ $= (\pi\rho)\frac{r^4}{4}$ $\frac{2\pi\rho r^3}{3}\bar{x} = \pi\rho\int xy^2 dx$ $\bar{x} = \frac{\pi\rho r^4}{4} \div \frac{2\pi\rho r^3}{3} = \frac{3}{8}r \quad *$	M1 A1 A1 M1 A1* (5)									
(b)	<table border="0"> <tr> <td></td><td>Hemisphere</td><td>Cone</td></tr> <tr> <td>Mass</td><td>$\frac{2}{3}\pi r^3$</td><td>$\frac{1}{3}\pi k r^3$</td></tr> <tr> <td>Dist of c of m from centre of common plane</td><td>$\frac{3}{8}r$</td><td>$\frac{1}{4}kr$</td></tr> </table> $\frac{2}{3} \times \frac{3}{8}r = \frac{k}{3} \times \frac{1}{4}kr$ $k^2 = 3 \quad k = \sqrt{3}$		Hemisphere	Cone	Mass	$\frac{2}{3}\pi r^3$	$\frac{1}{3}\pi k r^3$	Dist of c of m from centre of common plane	$\frac{3}{8}r$	$\frac{1}{4}kr$	B1 B1 M1A1ft A1 (5) [10]
	Hemisphere	Cone									
Mass	$\frac{2}{3}\pi r^3$	$\frac{1}{3}\pi k r^3$									
Dist of c of m from centre of common plane	$\frac{3}{8}r$	$\frac{1}{4}kr$									
(a)	M1 Use of $(\pi\rho)\int_0^r xy^2 dx$ with $y^2 = r^2 - x^2$ and attempt the integration. Limits not needed. A1 Correct integration – limits not needed A1 Sub correct (upper) limit. (Sub of 0 not needed) M1 Use of $V\rho\bar{x} = \pi\rho\int xy^2 dx$ with their result to obtain $\bar{x} = \dots$ where V is the volume of the hemisphere or sphere (π, p must be on both sides or neither) A1* $\bar{x} = \frac{3}{8}r$ (b) B1 Correct mass ratio for hemisphere and cone. Total mass not needed for this mark. B1 Correct distances of c of m for cone and hemisphere from centre of common plane (or another point). Both can be positive or one can be negative. Distances from vertex of cone (H) $kr + \frac{3}{8}r$ (C) $\frac{3}{4}kr$ Distances from peak of hemisphere (H) $\frac{5}{8}r$ (C) $r + \frac{1}{4}kr$ M1 Form a dimensionally correct moments equation with the correct value for $\bar{x}$ depending on where they have taken moments. (0 from plane face, kr from vertex of cone, r from peak of hemisphere) Allow even if formula for sphere is used. Ignore signs.										

A1ft A1	Correct equation, follow through their masses and distances, signs to be correct here. Correct exact result.
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Question Number	Scheme	Marks	
2.	$F \cos \alpha = mg$	M1	A1
	$F \sin \alpha = T$		A1
	$T = \frac{2mgx}{l}$ or $T = \frac{2mg(AB-l)}{l}$	M1	
	$\frac{3}{4}mg = \frac{2mgx}{l}$	dM1	
	$AB = \frac{11l}{8}$	A1	
		(6)	
Notes			
M1	Resolve vertically or horizontally, correct no. of terms, condone sign errors and sin/cos confusion (or use trig on a right-angled triangle of forces)		
A1	Correct vertical equation		
A1	Correct horizontal equation (A2 for $T = mg \tan \alpha$ from triangle of forces)		
M1	Hooke's Law. Must clearly be an extension and not AB. Since x is not defined in the question, other extensions may be used including $(AB - l)$ or xl where x is found to be the constant $\frac{3}{8}$.		
dM1	Substitute trig (not necessarily correctly) to produce an equation in 'x' (and l) only, dependent on previous M's and on having two equations.		
A1	Cao Accept 1.375l , 1.4l, 1.38l		

Question Number	Scheme	Marks
	M1 complete the square	
	A1* given answer correctly obtained, with at least one line of working and justification of positive root e.g. $x > 0$	
4(a)	$\bar{x} = \frac{\pi \int_0^r x(r^2 - x^2) dx}{\frac{2\pi r^3}{3}}$	M1A1
	$= \frac{\left[r^2 \frac{x^2}{2} - \frac{x^4}{4} \right]_0^r}{\frac{2r^3}{3}}$	A1
	$= \frac{3r}{8} *$	A1*
		(4)
4(b)	Mass ratios: $\frac{2\pi r^3}{3}$ $\pi r^2 h$ $\left(\frac{2\pi r^3}{3} + \pi r^2 h \right)$ Distances: $\frac{5r}{8}$ $\left(r + \frac{1}{2}h \right)$ $\bar{y}$	B1 B1
	$\left(\frac{2\pi r^3}{3} \times \frac{5r}{8} \right) + \pi r^2 h \left(r + \frac{1}{2}h \right) = \left(\frac{2\pi r^3}{3} + \pi r^2 h \right) \bar{y}$	M1A1
	$\bar{y} = \frac{5r^2 + 12rh + 6h^2}{8r + 12h} *$	A1*
		(5)
4(c)	$r = \frac{5r^2 + 12rh + 6h^2}{8r + 12h}$	M1
	$r = \sqrt{2}h$	A1
		(2)
		(11)
	Notes for question 4	



Question Number	Scheme	Marks
4(a)	M1 for use of $\bar{x} = \frac{\pi \int_0^r xy^2 dx}{\frac{2\pi r^3}{3}}$	
	A1 for $\bar{x} = \frac{\pi \int_0^r x(r^2 - x^2) dx}{\frac{2\pi r^3}{3}}$	
	M1 for integrating with powers increasing by 1	
	A1* for given answer correctly obtained	
4(b)	B1 correct mass ratios	
	B1 correct distances (for their parallel axis)	
	M1 for use of a moments equation with correct terms (Allow about a parallel axis)	
	A1 for correct unsimplified equation (for their parallel axis)	
	A1* for given answer correctly obtained	
4(c)	M1 for equating given answer to r oe	
	A1 cao	



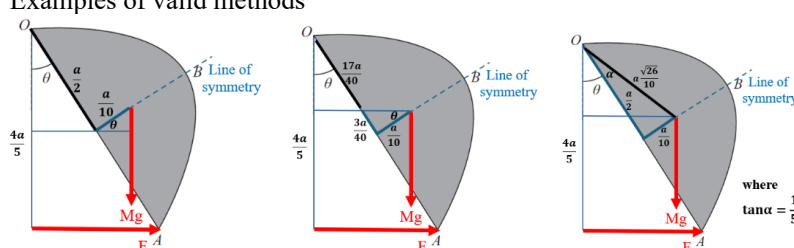
Question Number	Scheme	Marks																			
5(a)	$\bar{x} = \frac{\pi \int_0^{4r} x \left(\frac{1}{4}x\right)^2 dx}{\frac{4\pi r^3}{3}} \quad \text{or} \quad \bar{x} = \frac{\pi \int_0^{4r} x \left(r - \frac{1}{4}x\right)^2 dx}{\frac{4\pi r^3}{3}}$	M1A1																			
	$= \frac{3}{256r^3} \left[x^4 \right]_0^{4r}$	A1																			
	$= 3r^*$	A1*																			
		(4)																			
5(b)	<table><tr><td></td><td>Cone</td><td>Cylinder</td><td>S</td></tr><tr><td rowspan="2">Mass ratio</td><td>$\frac{4\pi r^3}{3}$</td><td>$\pi \left(\frac{1}{2}r\right)^2 \times r$</td><td>$\left(\frac{4\pi r^3}{3} - \pi \left(\frac{1}{2}r\right)^2 \times r\right)$</td></tr><tr><td>$\frac{4}{3}$</td><td>$\frac{1}{4}$</td><td>$\frac{13}{12}$</td></tr><tr><td>Distance from vertex</td><td>$3r$</td><td>$\left(4r - \frac{1}{2}r\right)$</td><td>$\bar{y}$</td></tr><tr><td>Distance from plane face</td><td>r</td><td>$\frac{1}{2}r$</td><td>$\bar{y}$</td></tr></table>		Cone	Cylinder	S	Mass ratio	$\frac{4\pi r^3}{3}$	$\pi \left(\frac{1}{2}r\right)^2 \times r$	$\left(\frac{4\pi r^3}{3} - \pi \left(\frac{1}{2}r\right)^2 \times r\right)$	$\frac{4}{3}$	$\frac{1}{4}$	$\frac{13}{12}$	Distance from vertex	$3r$	$\left(4r - \frac{1}{2}r\right)$	$\bar{y}$	Distance from plane face	r	$\frac{1}{2}r$	$\bar{y}$	B1 B1
	Cone	Cylinder	S																		
Mass ratio	$\frac{4\pi r^3}{3}$	$\pi \left(\frac{1}{2}r\right)^2 \times r$	$\left(\frac{4\pi r^3}{3} - \pi \left(\frac{1}{2}r\right)^2 \times r\right)$																		
	$\frac{4}{3}$	$\frac{1}{4}$	$\frac{13}{12}$																		
Distance from vertex	$3r$	$\left(4r - \frac{1}{2}r\right)$	$\bar{y}$																		
Distance from plane face	r	$\frac{1}{2}r$	$\bar{y}$																		
	$\left(\frac{4\pi r^3}{3} \times 3r\right) - \left(\pi \left(\frac{1}{2}r\right)^2 \times r\right) \left(4r - \frac{1}{2}r\right) = \left(\frac{4\pi r^3}{3} - \pi \left(\frac{1}{2}r\right)^2 \times r\right) \bar{y}$	M1A1																			
	$\bar{y} = \frac{75}{26} r^*$	A1*																			
		(5)																			
5(c)	$\tan \alpha = \frac{r}{4r - \frac{75}{26}r}$	M1A1																			
	$\tan \alpha = \frac{26}{29}$	A1																			
		(3)																			
		(12)																			
	Notes for question 5																				
5(a)																					
M1	Correct method to find the distance of the centre of mass from vertex or plane face, using $\bar{x} = \frac{\pi \int_0^{4r} xy^2 dx}{\frac{4\pi r^3}{3}}$. The formula must be correct but allow a constant multiple if it appears in both numerator and denominator or cancelled π . The v																				

Question Number	Scheme	Marks
	must be replaced with $y = \frac{1}{4}x$ or $y = r - \frac{1}{4}x$. Condone a gradient of $\pm \frac{r}{h}$ if h is later replaced with $4r$. There must be an attempt to integrate the numerator i.e. the power of x must increase by 1. The denominator of $\frac{4\pi r^3}{3}$ is given in the question. Condone sight of $\text{vol} = \pi \int_0^{4r} \left(\frac{1}{4}x\right)^2 dx$ as denominator. Ignore limits for the method mark.	
A1	Correct equation for the distance of the centre of mass from vertex or plane face. $\bar{x} = \frac{\pi \int_0^{4r} x \left(\frac{1}{4}x\right)^2 dx}{\frac{4\pi r^3}{3}} \quad \text{or} \quad \bar{x} = \frac{\pi \int_0^{4r} x \left(r - \frac{1}{4}x\right)^2 dx}{\frac{4\pi r^3}{3}} \quad \text{Ignore limits.}$	
A1	A correct expression for the distance of the centre of mass from vertex or plane face following integration and division by $\frac{4\pi r^3}{3}$. Limits must be correct at this point. $\frac{3}{256r^3} [x^4]_0^{4r}$ or $\frac{3}{4r^3} \left[\frac{r^2 x^2}{2} - \frac{rx^3}{6} + \frac{x^4}{64} \right]_0^{4r}$	
A1*	Given answer obtained from complete and correct working. If distance is found from plane face this must be subtracted to find required distance.	
5(b)		
B1	Correct mass ratios	
B1	Correct distances (for their parallel axis) Ignore signs.	
M1	Form a moments equation with correct number of terms (allow about a parallel axis). Equation must be dimensionally correct (mass ratio $\times$ distance).	
A1	Correct unsimplified equation	
A1*	Given answer obtained from complete and correct working. Working should include a line of simplification. The simplification could occur between the moments equation and the given answer or in the initial stage eg in a table.	
5(c)		
M1	Use of tan to obtain an equation for a relevant angle, allow reciprocal $\frac{r}{4r - \frac{75}{26}r}$	
A1	For a correct equation, condone reciprocal.	
A1	Correct answer, $\tan \alpha = \frac{26}{29}$ o.e. Must be an exact value for $\tan \alpha$. A0 if they got straight to α .	

Question Number	Scheme	Marks
	Gain = Loss $\frac{1}{2}mV^2 - \frac{1}{2}mag + \frac{mg}{2a}\left(\frac{3a}{2}\right)^2 - \frac{mg}{2a}\left(\frac{a}{2}\right)^2 = mg \times 2a$	
A1	Correct unsimplified equation	
A1	Correct answer in terms of a and g , accept $1.3\sqrt{ag}$ or better	

Question Number	Scheme	Marks
4(a)	Use of $\frac{1}{2} \int y^2 dx$ with $y = \frac{1}{a}(ax - x^2)$ to integrate the expression $\left(\frac{1}{2}\right)\left(\frac{1}{a}(ax - x^2)\right)^2$ oe	M1
	Correct integration $\left(\frac{1}{2}\right)\frac{1}{a^2}\left[\frac{a^2x^3}{3} - \frac{2ax^4}{4} + \frac{x^5}{5}\right]$	A1
	Use of $\int y dx$ with $y = \frac{1}{a}(ax - x^2)$ to integrate the expression $\frac{1}{a}(ax - x^2)$ oe	M1
	Correct integration $\frac{1}{a}\left[\frac{ax^2}{2} - \frac{x^3}{3}\right]$	A1
	Correct use of centre of mass formula for $\bar{y}$ with $y = \frac{1}{a}(ax - x^2)$ $\bar{y} = \frac{\frac{1}{2} \int_0^a \frac{1}{a^2}(ax - x^2)^2 dx}{\int_0^a \frac{1}{a}(ax - x^2) dx} \quad \left(= \frac{\frac{a^3}{60}}{\frac{a^2}{6}} \right)$	M1
	$(\bar{y}) = \frac{1}{10}a$	A1



Question Number	Scheme	Marks
		(6)
4(b)	$\bar{x} = \frac{1}{2}a$	B1
	Horizontal distance of G from the vertical at O Examples of valid methods  where $\tan \alpha = \frac{1}{5}$ $\frac{a}{2} \sin \theta + \frac{a}{10} \cos \theta$ $\frac{17a}{40} \sin \theta + \frac{a}{8}$ $\frac{a\sqrt{26}}{10} \sin(\theta + \alpha)$	M1
	Correct horizontal distance $\frac{19a}{50}$	A1
	Moments equation about O to form an equation in F, M, θ (and g) $Fa \cos \theta = Mg \times d$	DM1
	$F = \frac{19Mg}{40}$ oe	A1
		(6)
		(12)
	Notes for question 4	
4(a)		
M1	Attempt to integrate $\frac{1}{2}y^2$ with $y = \frac{1}{a}(ax - x^2)$. Must see at least two powers of x increasing by 1. Ignore any limits and condone missing $\frac{1}{2}$. The correct expansion to integrate is $\left(\frac{1}{2}\right) \frac{1}{a^2}(a^2x^2 - 2ax^3 + x^4)$ oe. Condone a slip when expanding the brackets before integration.	
A1	Correct integrated expression (ignore limits, condone missing $\frac{1}{2}$)	
M1	Attempt to integrate y with $y = \frac{1}{a}(ax - x^2)$. Must see both powers of x increase by 1. Ignore any limits.	
A1	Correct integrated expression (ignore limits)	

Question Number	Scheme	Marks
M1	Complete method to find $\bar{y}$. Use of the correct formula (up the right way). If ρ appears it must appear in both numerator and denominator. Limits must be correct and $\frac{1}{2}$ must be present. $\bar{y} = \frac{\frac{1}{2} \int_0^a \frac{1}{a^2} (ax - x^2)^2 dx}{\int_0^a \frac{1}{a} (ax - x^2) dx}$	
A1	Correct answer, $\frac{1}{10}a$ or $0.1a$ oe	
4(b)		
B1	$\bar{x} = \frac{1}{2}a$ seen or implied. May be seen on diagram, embedded in a length eg $\frac{1}{2}a \sin \theta$ or found from first principles using $\bar{x} \int_0^a y dx = \int_0^a xy dx$	
M1	A complete method to obtain the required horizontal distance from G to the vertical at O . For example, method to find the lengths of two right-angled triangles that sum to the required distance. Condone sin/cos confusion.	
A1	Correct horizontal distance	
DM1	Dependent on first method mark. Moments about O , using the horizontal distance, to obtain an equation in F , M and θ (and a). Dimensionally correct with all required terms of the correct structure and no extras. Condone sin/cos confusion. Missing g is an accuracy error.	
A1	Correct unsimplified equation.	
A1	Accept $0.48Mg$ or $0.475Mg$ (must be in terms of M and g)	



Question Number	Scheme	Marks
7 (a)(i)	$V = \pi \int_1^2 (x^2 + 4)^2 dx = \pi \int_1^2 (x^4 + 8x^2 + 16) dx$ $= \pi \left[\frac{1}{5}x^5 + \frac{8}{3}x^3 + 16x \right]_1^2 = \frac{613\pi}{15} \text{ (cm}^3\text{)} \quad *$	M1A1 A1cso
(ii)	$(\pi) \int_1^2 x(x^2 + 4)^2 dx = (\pi) \int_1^2 (x^5 + 8x^3 + 16x) dx$ $= (\pi) \left[\frac{1}{6}x^6 + 2x^4 + 8x^2 \right]_1^2 \quad \text{alt}(\pi) \left[\frac{(x^2 + 4)^3}{6} \right]_1^2$	M1A1
(b)	$\bar{x} = \frac{(\pi) \left[\frac{1}{6}x^6 + 2x^4 + 8x^2 \right]_1^2}{\frac{613}{15}(\pi)} = \frac{\frac{129}{2}}{\frac{613}{15}} = 1.578... = 1.58 \text{ (cm)}$ Mass $\frac{613\pi}{15}M$ $9\pi M$ $45\pi M$ $\left(36\pi + \frac{613\pi}{15} \right)M = \frac{1153\pi}{15}M$ Dist from B 0.578 0.5 0.5 $\bar{y}$ $\frac{613\pi}{15} \times 0.578 - 9\pi \times 0.5 + 45\pi \times 0.5 = \left(36\pi + \frac{613\pi}{15} \right) \bar{y}$ $\bar{y} = \frac{1249}{2306} = 0.5416... = 0.54 \text{ (cm)}$	M1dM1A1 (8) B1 B1ft M1A1ft A1 (5) [13]

(a)(i)M1	Attempt the squaring and integrating (at least one power going up). Allow w/o π
A1	Correct integration allow w/o π
A1*cs0	Correct volume, with no errors seen. (Must include π and no $V = \dots$ w/o π must have been seen.)
(ii)M1	Attempt $\int x(x^2 + 4)^2 dx$. Must either expand or obtain $k(x^2 + 4)^3$. π not needed. Limits not needed
A1	Correct algebraic integration, π not needed. Limits not needed
M1	Substitute the (correct) limits in their integrated function. Independent, but must have been attempting $\int xy^2 dx$
M1	Divide the two integrals (correct way up). Depends on the 1st and 2nd M marks. π and ρ in both or neither.
A1	Correct final result. Must be 3 sf.
	(SC Correct answer with no algebraic integration shown can score M0A0 M1 M0A0)
(b)	
B1	Correct masses seen explicitly or in an equation.
B1ft	Correct distances from B (or any vertical axis). Follow through distance from (a).
M1	Form a moments equation, with lighter cylinder subtracted and the heavier one added
A1ft	Correct equation, follow through their distance from (a).
A1	Correct distance from B , 2 sf or better

Alt (b) Find mass and CoM of S_1 first $\text{Mass} = \frac{478\pi}{15}M$ $\text{CoM} = \frac{287}{478} \approx 0.6004$

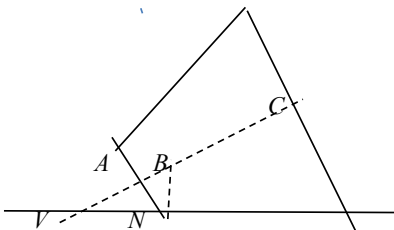
Award B1B1 when all component masses and distances are seen. Complete method needed for M1.

Award first A1 for correct masses/distances initially used in forming both equations.

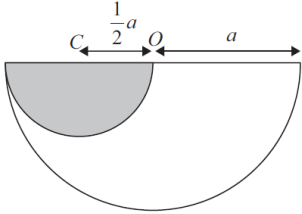
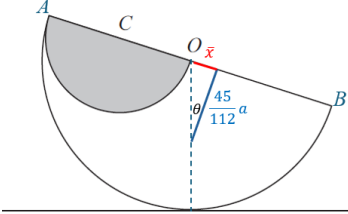
(Note: Use of 0.58 leads to $\bar{x} = 0.603$ (cm) for S_1 . This gives a final answer 0.543. If they give 0.54, award full marks, as premature approximation does not affect final answer, but penalise 0.543)

SC – If the use M and $5M$ for the masses, award max B0B1 M1A0A0



Question Number	Scheme	Marks															
7(a)	$\bar{x} = \frac{\int_0^h x \left(\frac{rx}{h} \right)^2 dx}{\int_0^h \left(\frac{rx}{h} \right)^2 dx}$ (Allow volume of cone formula quoted with π in the numerator)	M1DM1															
	$= \frac{\left[\frac{x^4}{4} \right]_0^h}{\left[\frac{x^3}{3} \right]_0^h} \text{ oe}$	A1															
	$= \frac{3h}{4} *$	A1* (4)															
7(b)	<table><tr><td></td><td>F</td><td>C</td><td>C'</td><td></td></tr><tr><td>Distance</td><td>$\bar{y}$</td><td>$\frac{1}{4}h$</td><td>$\frac{2h}{3} + \left(\frac{1}{4} \times \frac{h}{3} \right)$</td><td>$\left(= \frac{3h}{4} \right)$</td></tr><tr><td>Mass ratio</td><td>26</td><td>27</td><td>1</td><td>oe</td></tr></table>		F	C	C'		Distance	$\bar{y}$	$\frac{1}{4}h$	$\frac{2h}{3} + \left(\frac{1}{4} \times \frac{h}{3} \right)$	$\left(= \frac{3h}{4} \right)$	Mass ratio	26	27	1	oe	B1 B1
	F	C	C'														
Distance	$\bar{y}$	$\frac{1}{4}h$	$\frac{2h}{3} + \left(\frac{1}{4} \times \frac{h}{3} \right)$	$\left(= \frac{3h}{4} \right)$													
Mass ratio	26	27	1	oe													
	$26\bar{y} = \frac{1}{4}h \times 27 - \left[\frac{2h}{3} + \left(\frac{1}{4} \times \frac{h}{3} \right) \right] \times 1$	M1A1ft															
	$\bar{y} = \frac{3}{13}h *$	A1* (5)															
7(c)																	
	For equilibrium, $\bar{y} = \frac{3}{13}h \leq CB$ oe	M1															
	$\frac{AB}{AN} = \frac{AN}{AV} \Rightarrow AB = \frac{1}{3}r \times \frac{r}{h} = \frac{r^2}{3h} \text{ oe}$	M1A1															
	So, for equilibrium, $\frac{3}{13}h \leq \frac{2}{3}h - \frac{r^2}{3h}$	M1															
	$13r^2 \leq 17h^2 *$	A1* (5)															
		(14)															

Question Number	Scheme	Marks
	Notes for question 7	
7(a)	M1 for use of $\int_0^h xy^2 dx$ (Attempt at integration required)	
	DM1 for use of $\bar{x} = \frac{\int_0^h x \left(\frac{rx}{h}\right)^2 dx}{\int_0^h \left(\frac{rx}{h}\right)^2 dx}$ Depends on M mark above	
	A1 for $= \frac{\left[\frac{x^4}{4}\right]_0^h}{\left[\frac{x^3}{3}\right]_0^h}$	
	A1* for given answer correctly obtained. Upper limit(s) must be substituted.	
7(b)	B1 for distances from larger plane face or any parallel axis	
	B1 for mass (volume) ratios	
	M1 for moments about larger plane face or any parallel axis	
	A1ft for a correct equation, follow through their distances and masses	
	A1* for given answer correctly obtained. At least one step in the working from the equation must be seen.	
7(c)	M1 for overall method using a suitable inequality – may be comparing lengths or angles. If the limiting case is used this mark (and the final A mark) can only be awarded if a reason for the direction of the inequality is seen (eg $\bar{y} \ll CB$)	
	M1 for finding a length appropriate for their method	
	A1 for a correct relevant distance in terms of r and h	
	M1 for producing an inequality in r and h , must be right way round	
	A1* for correctly showing given inequality. At least one step in the working from their previous inequality must be seen.	

Question Number	Scheme	Marks
	 	
6(a)	Use of $\int xy^2 dx$ with circle equation	M1
	$\int_0^a x(a^2 - x^2) dx$	A1
	$\left[\frac{a^2 x^2}{2} - \frac{x^4}{4} \right]_0^a = \frac{a^4}{4}$	A1
	Use of $\bar{x} = \frac{\pi \int_0^a x(a^2 - x^2) dx}{\frac{2}{3} \pi a^3}$	dM1
	$\left\{ \frac{\pi \frac{a^4}{4}}{\frac{2}{3} \pi a^3} \Rightarrow \right\} = \frac{3}{8} a^*$	A1*
		(5)
6(b)	Mass ratios: $\frac{8}{3}a$ $\frac{1}{16}a$ $\frac{7}{y}$	B1
	Distances: $\frac{3}{8}a$ $\frac{3}{16}a$ $\bar{y}$	B1
	$8 \times \frac{3a}{8} - 1 \times \frac{3}{16}a = 7\bar{y}$	M1
	$(\bar{y}) = \frac{45}{112}a^*$	A1*
		(4)
6(c)	Form a moments equation about an axis perpendicular to OC - Axis through O: $(8 \times 0 +) 1 \times \frac{a}{2} = 7\bar{x}$ - Axis through A: $8 \times a - 1 \times \frac{a}{2} = 7\bar{x}$ - Axis through B: $8 \times a - 1 \times \frac{3a}{2} = 7\bar{x}$ If equation is seen in earlier working, it must be used in (c) to earn the marks in (c)	M1
	Correct $\bar{x}$ for their axis through $O: (\bar{x}) = \frac{a}{14} \quad A: (\bar{x}) = \frac{15a}{14} \quad B: (\bar{x}) = \frac{13a}{14}$	A1

	Method to find an expression for tan of a relevant angle	dM1
	$\tan \theta = \frac{\bar{x}}{\frac{45}{112}a}$	A1ft
	$\tan \theta = \frac{8}{45}$	A1
		(5)
		(14)
Notes for question 6		
(a)	N.B. M0 for a lamina	
M1	Use of $\int xy^2 dx$ with their attempt at a circle equation. The equation for y must be substituted into the formula in terms of x . Allow use of alternative forms if their circle equation is substituted in correctly to give an equation with one variable (including the 'dx') For example use of $\int yx^2 dy$ would need the circle equation substituted in for x^2 to give an integral in terms of y . Condone consistent use of r instead of a .	
A1	Correct expression with correct limits seen at some stage $\int_0^a x(a^2 - x^2) dx$	
A1	Correct expression $\frac{a^4}{4}$. If $\frac{a^4}{4}$ is not explicitly stated, accept correct integrated expression with correct limits.	
dM1	Dependent on previous M mark. A complete method to find the required distance using the equation of a circle. They may use $\frac{2}{3}\pi a^3$ or integrate correctly to find the volume. $\bar{x} = \frac{\pi \int_0^a xy^2 dx}{\frac{2}{3}\pi a^3} \quad \text{or} \quad \bar{x} = \frac{\pi \int_0^a xy^2 dx}{\pi \int_0^a y^2 dx}$ If seen, ρ and/or π must be present in both numerator and denominator.	
A1*	Given answer obtained from complete and correct working. Must be positive and in terms of a . Condone r replaced with a at the final stage.	
(b)	N.B. For a lamina, max score 0100	
B1	Correct mass ratios seen or implied.	
B1	Correct distances, allow if measured from a parallel axis. Ignore signs.	
M1	Method to form a dimensionally correct moments equation (the small hemisphere must be subtracted from the large hemisphere). All required terms present.	
A1*	Given answer obtained from a fully correct moments equation.	
(c)	N.B. M0 for a lamina	
M1	Method to form a dimensionally correct moments equation about an axis perpendicular to OC (the small hemisphere must be subtracted from the	

	large hemisphere). All required terms present. Use of correct mass ratios. Both distances must be measured from the same axis.	
A1	Correct distance for their axis, accept $\pm$	
dM1	Dependent on previous M. Find an expression for $\tan$ of a relevant angle (θ or $(90^\circ - \theta)$). Must be dimensionally correct. Must use the given answer from (b) and $\bar{x}$ as the distance to the axis through O . The correct addition or subtraction is required if an alternative axis is used. Condone if expressed immediately as $\theta = \tan^{-1}\left(\frac{\bar{x}}{\frac{45}{112}a}\right)$ and condone the reciprocal.	
A1ft	Correct equation. Follow through their calculated $ \bar{x} $ Do not condone the reciprocal. Must be $\tan\theta = \dots$	
A1	Correct exact answer. Must be $\tan\theta = \dots$ ISW Accept recurring decimal with correct notation. $0.1\dot{7}$	

Question Number	Scheme	Marks
7(a)		
	Equation of motion for P at a general position $T_{PB} - T_{PA} = m\ddot{x}$	M1
	Use HL in the equation of motion	M1
	$\frac{2mg}{2L}(0.5L - x) - \frac{mg}{3L}(1.5L + x) = m\ddot{x}$	A1
		A1
	Obtain SHM equation of the correct form and conclude $-\frac{4g}{3L}x = \ddot{x} \quad \text{and} \quad \text{SHM}$	A1*
	Use of $\omega^2 = \frac{4g}{3L}$ with the formula period = $\frac{2\pi}{\omega}$	dM1
7(b)	Must include working that leads to the given answer for the period. For example	A1*
	Clearly identify ω within the period formula to reach the answer period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{4g}{3L}}} = \pi\sqrt{\frac{3L}{g}}$	
	Clearly identify ω and state the period formula to reach the answer $\omega = \sqrt{\frac{4g}{3L}}$, period = $\frac{2\pi}{\omega} = \pi\sqrt{\frac{3L}{g}}$ $\Rightarrow (\text{period} =) \pi\sqrt{\frac{3L}{g}}$	
7(b)	amplitude = $0.5L$	B1
	$v_{\text{MAX}} = a \times \sqrt{\frac{4g}{3L}}$	M1
	$= \sqrt{\frac{gL}{3}}$	A1
		(3)
7(c)	Method for use of SHM equation for displacement with their amplitude. [1] $x = a \cos \omega t$ [2] $x = a \sin \omega t$ [3] $x = -a \cos \omega t$ [4] $x = -a \sin \omega t$ [5] Any of the above with their x found after use of $v^2 = \omega^2(a^2 - x^2)$	M1

5			Curve equation $x^2 + y^2 = r^2$
5a	Using x-axis $(\rho) \int x \times 2\sqrt{r^2 - x^2} dx$ or Using y-axis $(\rho) \frac{1}{2} \int 2(\sqrt{r^2 - x^2})^2 dx$	M1	Use of correct integral. Limits not needed here. Accept an integral of the form: x-axis: $k \int x\sqrt{r^2 - x^2} dx$ y-axis: $k \int r^2 - x^2 dx$
	x-axis $= -\frac{2}{3}(\rho)(r^2 - x^2)^{\frac{3}{2}}$ y-axis $= (\rho) \left(xr^2 - \frac{x^3}{3} \right)$	A1	Correct integration, ignore limits. Correct expression.
	$= \frac{2}{3}(\rho)r^3$	A1	Correct use of limits, 0 and r or $-r$ and r .
	Using x-axis $\frac{1}{2} \pi r^2 \rho \bar{x} = \rho \int_0^r 2xy dx$ Using y-axis $\frac{1}{2} \pi r^2 \rho \bar{y} = \rho \frac{1}{2} \int_{-r}^r y^2 dx$ or $\frac{1}{2} \pi r^2 \rho \bar{y} = \rho \int_0^r y^2 dx$	M1	Complete method to obtain distance. Use of a correct formula, consistent with the axis and limits used, to find centre of mass with curve equation. ρ must appear on both sides or neither.
	$\bar{x} = \frac{\frac{2}{3}r^3}{\frac{1}{2}\pi r^2} = \frac{4r}{3\pi} \quad *$	A1*	Obtain given answer from correct working
ALT 1 5(a)	Parametric approach $x = r \cos \theta, y = r \sin \theta$		Curve equation $x^2 + y^2 = r^2$
	Using x-axis $2r^3 \int_0^{\frac{\pi}{2}} \sin^2 \theta \cos \theta d\theta$	M1	Use of correct integral. Limits not needed here. Accept an integral of the form: $kr^3 \int \sin^2 \theta \cos \theta d\theta$

	$= 2r^3 \left[\frac{\sin^3 \theta}{3} \right]_0^{\frac{\pi}{2}}$	A1	Correct integration, ignore limits. Correct expression.												
	$= \frac{2}{3} r^3$	A1	Correct use of limits												
	$\bar{x} = \frac{\frac{2}{3} r^3}{\frac{1}{2} \pi r^2}$	M1	Complete method to obtain distance. Use of correct formula. ρ must appear on both sides or neither.												
	$= \frac{4r}{3\pi} *$	A1*	Obtain given answer from correct working												
		[5]													
ALT 2 5(a)	Using y-axis $r^3 \int_0^{\frac{\pi}{2}} \sin^3 \theta \, d\theta$	M1	Use of correct integral. Limits not needed here. Accept an integral of the form: $kr^3 \int \sin^3 \theta \, d\theta$												
	$r^3 \int_0^{\frac{\pi}{2}} (1 - \cos^2 \theta) \sin \theta \, d\theta$ $= r^3 \left[-\cos \theta + \frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}}$	A1	Correct integration, ignore limits. Correct expression.												
	$= \frac{2}{3} r^3$	A1	Correct use of limits												
	$\bar{x} = \frac{\frac{2}{3} r^3}{\frac{1}{2} \pi r^2}$	M1	Complete method to obtain distance. Use of correct formula. ρ must appear on both sides or neither.												
	$= \frac{4r}{3\pi} *$	A1*	Obtain given answer from correct working												
		[5]													
5b	<table border="1"> <thead> <tr> <th></th><th>large</th><th>Small removed</th><th>Small added</th></tr> </thead> <tbody> <tr> <td>mass</td><td>$8\pi a^2$</td><td>$2\pi a^2$</td><td>$2\pi a^2$</td></tr> <tr> <td>From AC</td><td>$\frac{16a}{3\pi}$</td><td>$\frac{8a}{3\pi}$</td><td>$(-)\frac{8a}{3\pi}$</td></tr> </tbody> </table>		large	Small removed	Small added	mass	$8\pi a^2$	$2\pi a^2$	$2\pi a^2$	From AC	$\frac{16a}{3\pi}$	$\frac{8a}{3\pi}$	$(-)\frac{8a}{3\pi}$	B1 B1	Correct mass ratios Correct distances
	large	Small removed	Small added												
mass	$8\pi a^2$	$2\pi a^2$	$2\pi a^2$												
From AC	$\frac{16a}{3\pi}$	$\frac{8a}{3\pi}$	$(-)\frac{8a}{3\pi}$												
	Moments about AC	M1	All terms required. Dimensionally correct or equivalent for a parallel axis. Condone sign errors. If column vectors are used, this mark is awarded once the equation is written separate to the column vectors.												

	$8\pi a^2 \times \frac{16a}{3\pi} - 2\pi a^2 \times \frac{8a}{3\pi} - 2\pi a^2 \times \frac{8a}{3\pi}$ $= 8\pi a^2 d$	A1	Correct unsimplified equation.
	$\frac{96a}{3\pi} = 8d \Rightarrow d = \frac{4a}{\pi} *$	A1*	Obtain given value from correct working. Need to see at least some simplification.
		[5]	
5c	Moments about perpendicular axis through A	M1	Dimensionally correct. Need all terms. Or equivalent for a parallel axis
	From A $4a \times 8\pi a^2 - 2a \times 2\pi a^2 + 6a \times 2\pi a^2 = 8\pi a^2 \bar{x}$	A1ft A1ft	Unsimplified equation with at most one error. Correct unsimplified equation Follow their mass ratio
	$\Rightarrow \bar{x} = 5a$	A1	Correct only. If measured from B, distance is a
	Correct use of trig to find an expression for $\tan \theta$	M1	$\tan \theta = \frac{d}{\bar{x}}$ or $\tan \theta = \frac{\bar{x}}{d}$ where $\bar{x}$ is distance from A.
	$\tan \theta = \frac{4}{5\pi}$	A1	Only
		[6]	
		(16)	