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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Graphs & Transformations (P1.4)

International A Level Mathematics · P1: Pure Mathematics 1 (WMA11)

36 questions · 308 marks · Mark schemes included

Past papers January 2019 - June 2025 · 3 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

36 questions · 308 marks · 3 sub-topics

SUB-TOPIC P1.4a

SKETCHING CURVES (CUBICS, QUARTICS & RECIPROCALs)

Sub-topic P1.4a · 17 questions · 153 marks · lowest tariff first



6. (a) Given that k is a positive constant such that $0 < k < 4$ sketch, on **separate axes**, the graphs of

(i) $y = (2x - k)(x + 4)^2$

(ii) $y = \frac{k}{x^2}$

showing the coordinates of any points where the graphs cross or meet the coordinate axes, leaving coordinates in terms of k , where appropriate.

(5)

- (b) State, with a reason, the number of roots of the equation

$$(2x - k)(x + 4)^2 = \frac{k}{x^2}$$

(1)



Question 6 continued

(Total for Question 6 is 6 marks)



4.

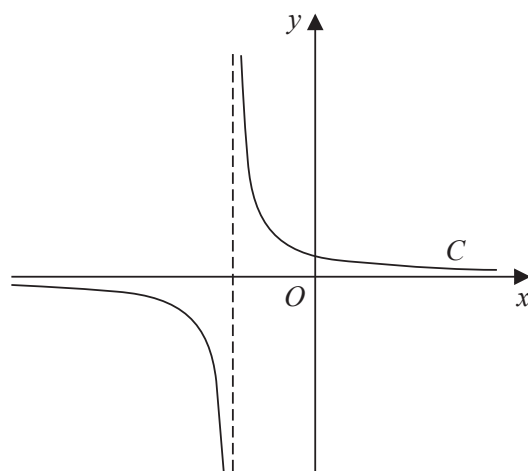


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \frac{1}{x+2}$

(a) State the equation of the asymptote of C that is parallel to the y -axis.

(1)

(b) Factorise fully $x^3 + 4x^2 + 4x$

(2)

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(c) On Diagram 1, add a sketch of the curve with equation

$$y = x^3 + 4x^2 + 4x$$

On your sketch, state clearly the coordinates of each point where this curve cuts or meets the coordinate axes.

(3)

(d) Hence state the number of real solutions of the equation

$$(x+2)(x^3 + 4x^2 + 4x) = 1$$

giving a reason for your answer.

(1)

Question 4 continued

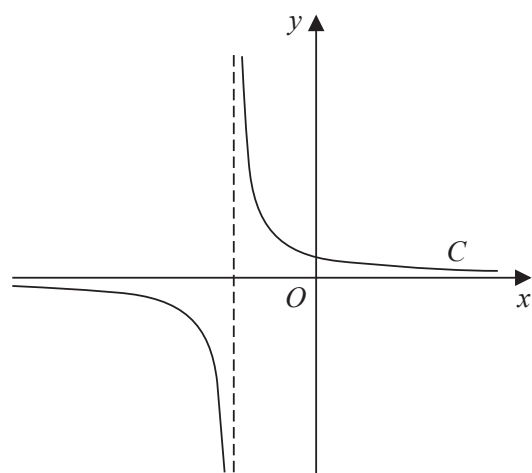
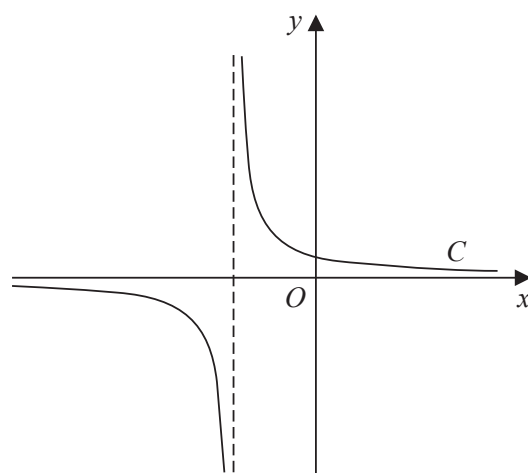


Diagram 1



copy of Diagram 1

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

(Total for Question 4 is 7 marks)

8. The curve C_1 has equation

$$y = x(4 - x^2)$$

- (a) Sketch the graph of C_1 showing the coordinates of any points of intersection with the coordinate axes.

(3)

The curve C_2 has equation $y = \frac{A}{x}$ where A is a constant.

- (b) Show that the x coordinates of the points of intersection of C_1 and C_2 satisfy the equation

$$x^4 - 4x^2 + A = 0$$

(1)

- (c) Hence find the range of possible values of A for which C_1 meets C_2 at 4 distinct points.

(3)

Question 8 continued

(Total for Question 8 is 7 marks)



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6. The curve C has equation $y = \frac{4}{x} + k$, where k is a positive constant.
- (a) Sketch a graph of C , stating the equation of the horizontal asymptote and the coordinates of the point of intersection with the x -axis.
- (3)**

The line with equation $y = 10 - 2x$ is a tangent to C .

- (b) Find the possible values for k . (5)

Question 6 continued

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Q6

(Total 8 marks)



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10. The curve C_1 has equation $y = f(x)$, where

$$f(x) = (4x - 3)(x - 5)^2$$

- (a) Sketch C_1 showing the coordinates of any point where the curve touches or crosses the coordinate axes.

(3)

- (b) Hence or otherwise

(i) find the values of x for which $f\left(\frac{1}{4}x\right) = 0$

- (ii) find the value of the constant p such that the curve with equation $y = f(x) + p$ passes through the origin.

(2)

A second curve C_2 has equation $y = g(x)$, where $g(x) = f(x + 1)$

- (c) (i) Find, in simplest form, $g(x)$. You may leave your answer in a factorised form.

- (ii) Hence, or otherwise, find the y intercept of curve C_2

(3)



Question 10 continued

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Q10

(Total 8 marks)



7.

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

The curve C has equation

$$y = \frac{2}{x} - k$$

where k is a **positive** constant.

(a) Sketch the graph of C .

Show on your sketch

- the coordinates of any points of intersection of C with the coordinate axes
- the equation of the horizontal asymptote to C

stating each in terms of k .

(3)

The line l has equation $y = -kx - 6$

Given that l intersects C at 2 distinct points,

(b) find the range of possible values of k .

(5)





Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)



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10. The curve C has equation

$$y = \frac{1}{x^2} - 9$$

(a) Sketch the graph of C .

On your sketch

- show the coordinates of any points of intersection with the coordinate axes
- state clearly the equations of any asymptotes

(4)

The curve D has equation $y = kx^2$ where k is a constant.

Given that C meets D at 4 distinct points,

(b) find the range of possible values for k .

(5)



Question 10 continued

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Question 10 continued

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Q10

(Total 9 marks)

END

TOTAL FOR PAPER: 75 MARKS

7. (a) Sketch the graph of the curve C with equation

$$y = \frac{4}{x - k}$$

where k is a positive constant.

Show on your sketch

- the coordinates of any points where C cuts the coordinate axes
- the equation of the vertical asymptote to C

(4)

Given that the straight line with equation $y = 9 - x$ does not cross or touch C

(b) find the range of values of k .

(5)

Question 7 continued

(Total for Question 7 is 9 marks)



6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Sketch the curve C with equation

$$y = \frac{1}{2-x} \quad x \neq 2$$

State on your sketch

- the equation of the vertical asymptote
- the coordinates of the intersection of C with the y -axis

(3)

The straight line l has equation $y = kx - 4$, where k is a constant.

Given that l cuts C at least once,

(b) (i) show that

$$k^2 - 5k + 4 \geq 0$$

(ii) find the range of possible values for k .

(6)



Question 6 continued



Question 6 continued

(Total for Question 6 is 5 marks)



7.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Sketch the curve C with equation

$$y = \frac{1}{x + 6}$$

State on your sketch

- the equation of the vertical asymptote
- the coordinates of the point of intersection of C with the y -axis

(3)

The straight line l has equation $y = mx - 4$, where m is a constant.

Given that l cuts C at least once,

(b) (i) show that

$$9m^2 + 13m + 4 \geq 0$$

(ii) find the range of values of m .

(6)



Question 7 continued

Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 9 marks)



10.

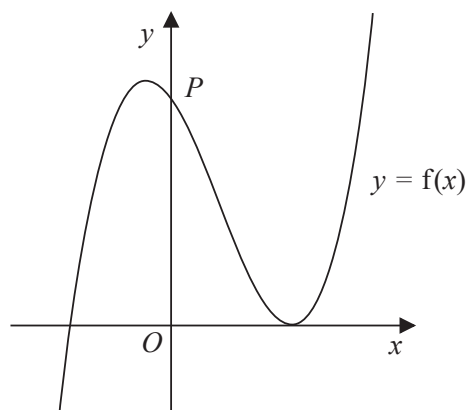


Figure 6

Figure 6 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x + 5)(x - 3)^2$$

- (a) Deduce the values of x for which $f(x) \leq 0$ (2)

The curve crosses the y -axis at the point P , as shown.

- (b) Expand $f(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a , b , c and d are integers to be found.

(3)

- (c) Hence, or otherwise, find

- (i) the coordinates of P ,
 (ii) the gradient of the curve at P . (2)

The curve with equation $y = f(x)$ is translated two units in the positive x direction to a curve with equation $y = g(x)$.

- (d) (i) Find $g(x)$, giving your answer in a simplified factorised form.
 (ii) Hence state the y intercept of the curve with equation $y = g(x)$. (3)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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(Total 10 marks)

Q10



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6. (a) Sketch the curve with equation

$$y = -\frac{k}{x} \quad k > 0 \quad x \neq 0 \quad (2)$$

- (b) On a separate diagram, sketch the curve with equation

$$y = -\frac{k}{x} + k \quad k > 0 \quad x \neq 0$$

stating the coordinates of the point of intersection with the x -axis and, in terms of k , the equation of the horizontal asymptote.

(3)

- (c) Find the range of possible values of k for which the curve with equation

$$y = -\frac{k}{x} + k \quad k > 0 \quad x \neq 0$$

does not touch or intersect the line with equation $y = 3x + 4$

(5)



Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total 10 marks)

Q6



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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = f(x)$ where

$$f(x) = 2(x + 1)(x - 3)^2$$

(a) Sketch a graph of C .

Show on your graph the coordinates of the points where C cuts or meets the coordinate axes.

(3)

(b) Write $f(x)$ in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are constants to be found.

(3)

(c) Hence, find the equation of the tangent to C at the point where $x = \frac{1}{3}$

(4)



Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total 10 marks)

Q6



9.

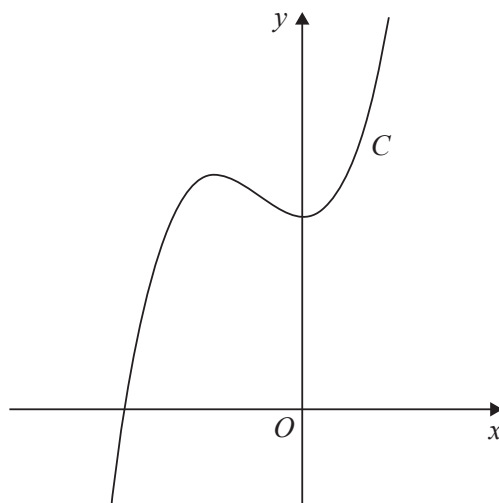


Figure 4

Figure 4 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = (x + 5)(3x^2 - 4x + 20)$$

- (a) Deduce the range of values of x for which $f(x) \geq 0$

(1)

- (b) Find $f'(x)$ giving your answer in simplest form.

(3)

The point $R(-4, 84)$ lies on C .

Given that the tangent to C at the point P is parallel to the tangent to C at the point R

- (c) find the x coordinate of P .

(4)

- (d) Find the point to which R is transformed when the curve with equation $y = f(x)$ is transformed to the curve with equation,

(i) $y = f(x - 3)$

(ii) $y = 4f(x)$

(2)



Question 7 continued

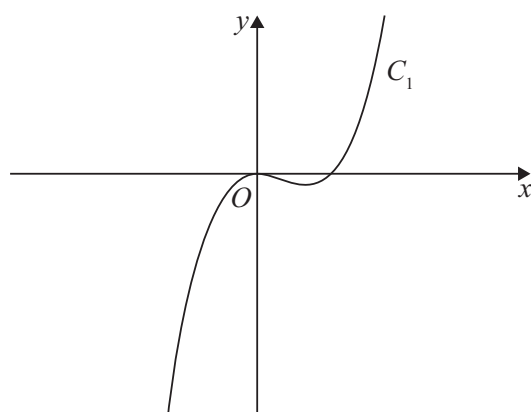


Diagram 1

Question 7 continued

Handwriting practice area with horizontal lines.



(Total for Question 7 is 10 marks)



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7. The curve C has equation

$$y = \frac{1}{2-x}$$

- (a) Sketch the graph of C . On your sketch you should show the coordinates of any points of intersection with the coordinate axes and state clearly the equations of any asymptotes.

(3)

The line l has equation $y = 4x + k$, where k is a constant.

Given that l meets C at two distinct points,

- (b) show that

$$k^2 + 16k + 48 > 0$$

(4)

- (c) Hence find the range of possible values for k .

(4)



Question 7 continued

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Question 7 continued

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(Total 11 marks)

Q7



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11. (a) On Diagram 1 sketch the graphs of

(i) $y = x(3 - x)$

(ii) $y = x(x - 2)(5 - x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(4)

- (b) Show that the x coordinates of the points of intersection of

$$y = x(3 - x) \text{ and } y = x(x - 2)(5 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 13) = 0$

(3)

The point P lies on both curves. Given that P lies in the first quadrant,

- (c) find, using algebra and showing your working, the exact coordinates of P .

(5)



Question 11 continued

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Question 11 continued

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Q11

(Total 12 marks)



SUB-TOPIC P1.4b

INTERSECTIONS OF GRAPHS

Sub-topic P1.4b · 5 questions · 35 marks · lowest tariff first



6.

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation $y = (x + 5)(3x + 2)(2x - 5)$

The curve C_2 has equation $y = 3x^2 - 33x - 50$

Use algebra to find the x coordinates of the points of intersection of C_1 and C_2

(5)



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 5 marks)



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 6 marks)



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6. The line with equation $y = 4x + c$, where c is a constant, meets the curve with equation $y = x(x - 3)$ at only one point.

(a) Find the value of c .

(4)

(b) Hence find the coordinates of the point of intersection.

(3)

Question 6 continued

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Lined area for writing the answer to Question 6.

(Total 7 marks)

Q6







Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 7 marks)



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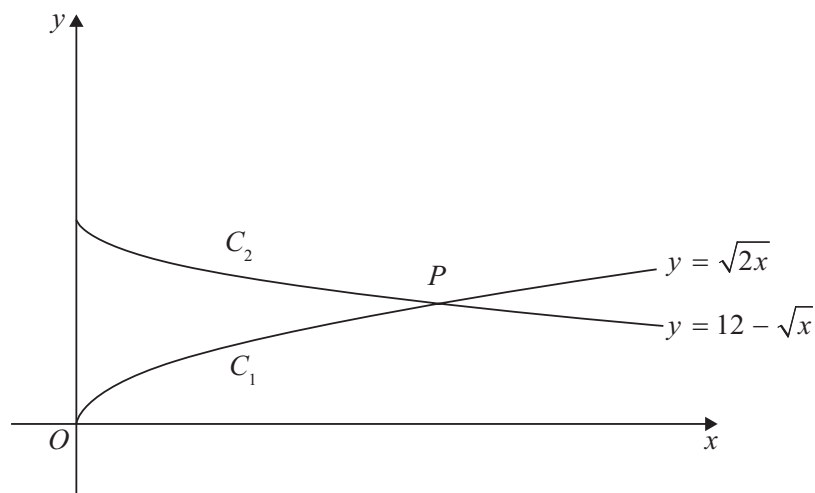


Figure 4

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 4 shows a sketch of

- the graph C_1 with equation $y = \sqrt{2x}$
- the graph C_2 with equation $y = 12 - \sqrt{x}$

(a) Describe fully the single transformation that would transform

- the graph with equation $y = \sqrt{x}$ onto C_1
- the graph with equation $y = -\sqrt{x}$ onto C_2

(4)

The graphs C_1 and C_2 meet at the point P , as shown in Figure 4.

(b) (i) Show that the x coordinate of P is a solution of

$$\sqrt{x} = 12(\sqrt{2} - 1)$$

(ii) Hence find, in simplest form, the exact coordinates of P .

(6)

Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 10 marks)



SUB-TOPIC P1.4c

TRANSFORMATIONS OF GRAPHS

Sub-topic P1.4c · 14 questions · 120 marks · lowest tariff first



1.

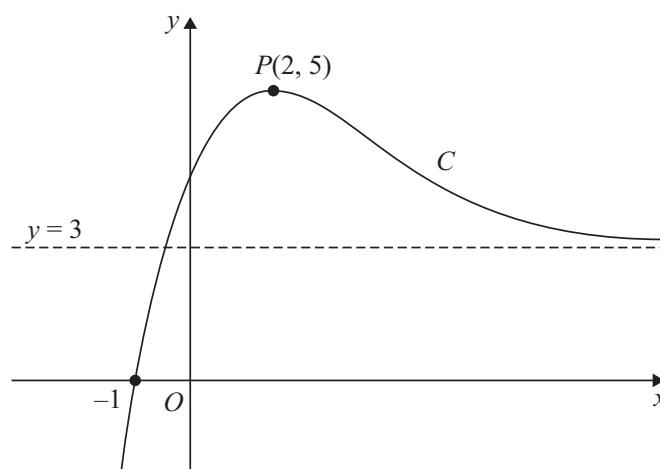


Figure 1

The curve C , shown in Figure 1,

- has equation $y = f(x)$, $x \in \mathbb{R}$
- cuts the x -axis at -1
- has a maximum turning point at $P(2, 5)$
- has a horizontal asymptote with equation $y = 3$

The curve C has no other turning points or asymptotes.

- (a) Find the coordinates of the point to which P is transformed when the curve with equation $y = f(x)$ is transformed to the curve with equation

(i) $y = f(x) + 7$

(ii) $y = 3f(x)$

(2)

Given that the line with equation $y = k$, where k is a constant, cuts or meets C exactly once,

- (b) state the range of possible values of k .

(2)

- (c) Write down the solution of the equation

$$f(x + 4) = 0$$

(1)

Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)



8.

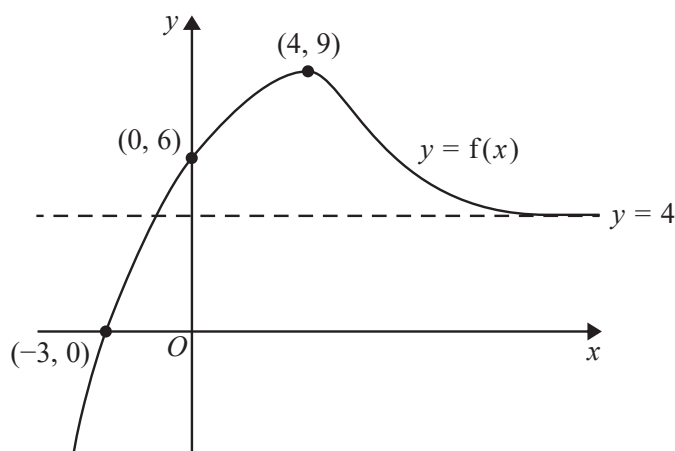
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Figure 4

The curve C with equation $y = f(x)$ is shown in Figure 4.

The curve C

- has a single turning point, a maximum at $(4, 9)$
- crosses the coordinate axes at only two places, $(-3, 0)$ and $(0, 6)$
- has a single asymptote with equation $y = 4$

as shown in Figure 4.

(a) State the equation of the asymptote to the curve with equation $y = f(-x)$. (1)

(b) State the coordinates of the turning point on the curve with equation $y = f\left(\frac{1}{4}x\right)$. (1)

Given that the line with equation $y = k$, where k is a constant, intersects C at exactly one point,

(c) state the possible values for k . (2)

The curve C is transformed to a new curve that passes through the origin.

(d) (i) Given that the new curve has equation $y = f(x) - a$, state the value of the constant a .

(ii) Write down an equation for another single transformation of C that also passes through the origin. (2)

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Q8

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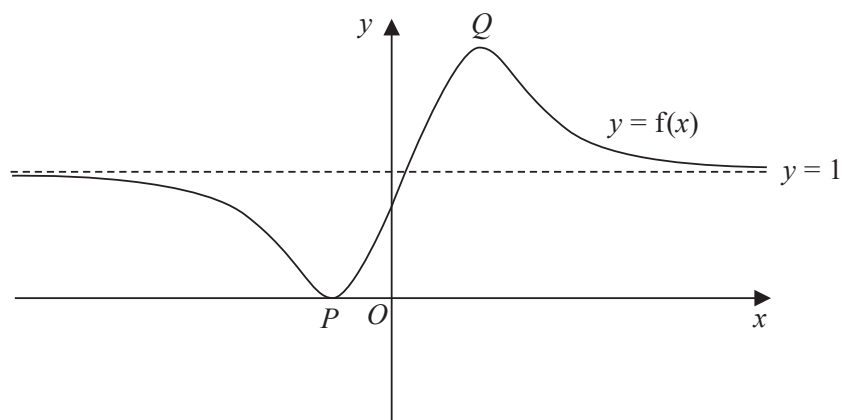
**Figure 1**

Figure 1 shows a sketch of a curve with equation $y = f(x)$

The curve has a minimum at $P(-1, 0)$ and a maximum at $Q\left(\frac{3}{2}, 2\right)$

The line with equation $y = 1$ is the only asymptote to the curve.

On separate diagrams sketch the curves with equation

(i) $y = f(x) - 2$

(3)

(ii) $y = f(-x)$

(3)

On each sketch you must clearly state

- the coordinates of the maximum and minimum points
- the equation of the asymptote

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Question 4 continued

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(Total 6 marks)

Q4



3.

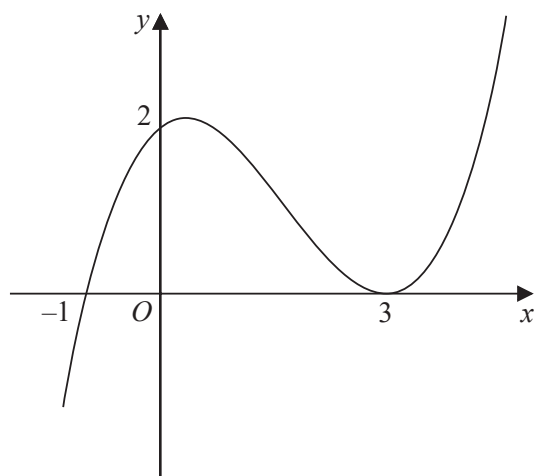
**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-1, 0)$ and $(0, 2)$ and touches the x -axis at the point $(3, 0)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(x + 3)$

(3)

(b) $y = f(-3x)$

(3)

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

Question 3 continued

(Total for Question 3 is 6 marks)



7.

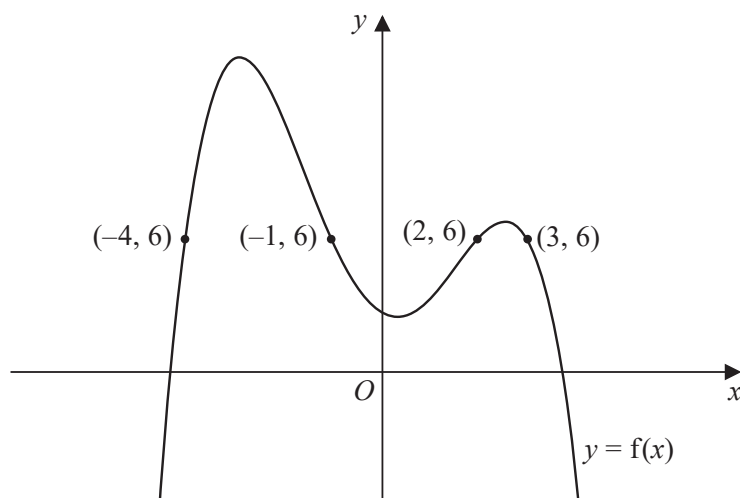


Figure 1

Figure 1 shows the curve with equation $y = f(x)$.

The points $P(-4, 6)$, $Q(-1, 6)$, $R(2, 6)$ and $S(3, 6)$ lie on the curve.

(a) Using Figure 1, find the range of values of x for which

$$f(x) < 6$$

(3)

(b) State the largest solution of the equation

$$f(2x) = 6$$

(1)

(c) (i) Sketch the curve with equation $y = f(-x)$.

On your sketch, state the coordinates of the points to which P , Q , R and S are transformed.

(ii) Hence find the set of values of x for which

$$f(-x) \geq 6 \text{ and } x < 0$$

(4)







(Total for Question 7 is 8 marks)



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5. (i)

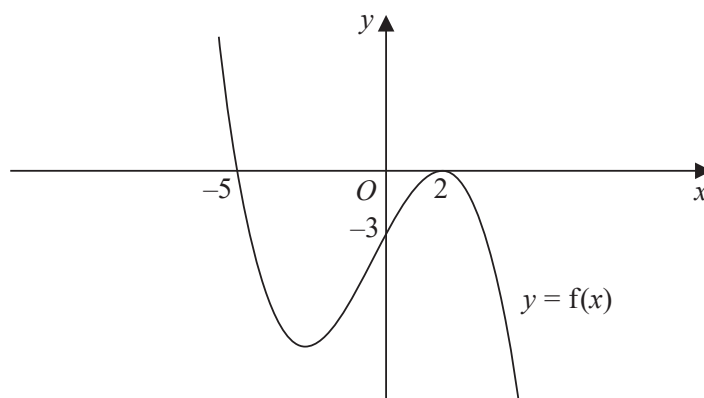
**Figure 2**

Figure 2 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-5, 0)$ and $(0, -3)$ and touches the x -axis at the point $(2, 0)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 2)$

(b) $y = f(-x)$

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

(6)

(ii)

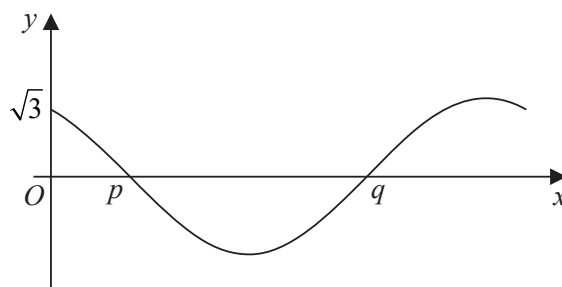
**Figure 3**

Figure 3 shows a sketch of the curve with equation

$$y = k \cos\left(x + \frac{\pi}{6}\right) \quad 0 \leq x \leq 2\pi$$

where k is a constant.

The curve meets the y -axis at the point $(0, \sqrt{3})$ and passes through the points $(p, 0)$ and $(q, 0)$.

Find

(a) the value of k ,

(b) the exact value of p and the exact value of q .

(3)

Question 5 continued

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Lined area for writing the answer to Question 5.

(Total 9 marks)

Q5



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9. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

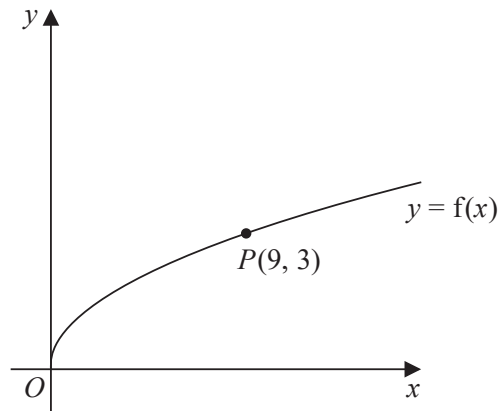


Figure 5

Figure 5 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \sqrt{x} \quad x > 0$$

The point $P(9, 3)$ lies on the curve and is shown in Figure 5.

On the next page there is a copy of Figure 5 called Diagram 1.

- (a) On Diagram 1, sketch and clearly label the graphs of

$$y = f(2x) \quad \text{and} \quad y = f(x) + 3$$

Show on each graph the coordinates of the point to which P is transformed.

(3)

The graph of $y = f(2x)$ meets the graph of $y = f(x) + 3$ at the point Q .

- (b) Show that the x coordinate of Q is the solution of

$$\sqrt{x} = 3(\sqrt{2} + 1)$$

(3)

- (c) Hence find, in simplest form, the coordinates of Q .

(3)

Question 9 continued

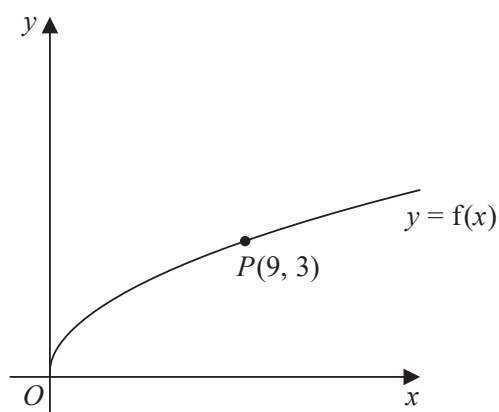


Diagram 1

Turn over for a copy of Diagram 1 if you need to redraw your graphs.

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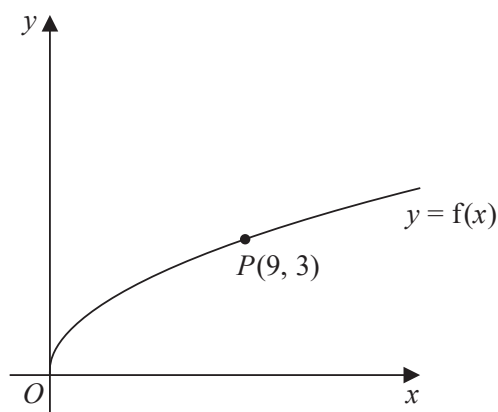
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Question 9 continued

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Only use this copy if you need to redraw your graphs.



Copy of Diagram 1

(Total 9 marks)

Q9



10. A curve has equation $y = f(x)$, where

$$f(x) = (x - 4)(2x + 1)^2$$

The curve touches the x -axis at the point P and crosses the x -axis at the point Q .

- (a) State the coordinates of the point P . (1)
- (b) Find $f'(x)$. (4)
- (c) Hence show that the equation of the tangent to the curve at the point where $x = \frac{5}{2}$ can be expressed in the form $y = k$, where k is a constant to be found. (3)

The curve with equation $y = f(x + a)$, where a is a constant, passes through the origin O .

- (d) State the possible values of a . (2)

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Q10

TOTAL FOR PAPER IS 75 MARKS



7. (a) On Diagram 1, sketch a graph of the curve C with equation

$$y = \frac{6}{x} \quad x \neq 0 \quad (2)$$

The curve C is transformed onto the curve with equation $y = \frac{6}{x-2} \quad x \neq 2$

- (b) Fully describe this transformation. (2)

The curve with equation

$$y = \frac{6}{x-2} \quad x \neq 2$$

and the line with equation

$$y = kx + 7 \quad \text{where } k \text{ is a constant}$$

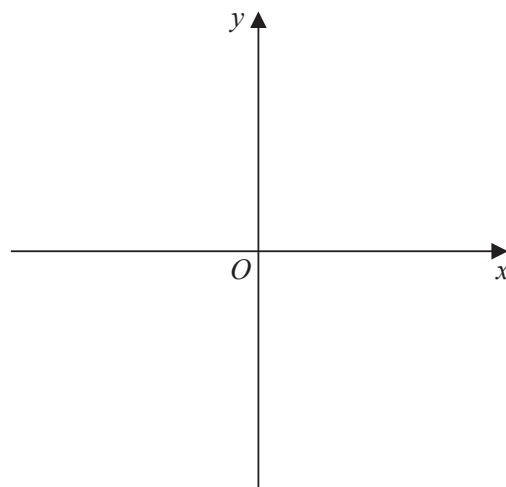
intersect at exactly two points, P and Q .

Given that the x coordinate of point P is -4

- (c) find the value of k , (2)
- (d) find, using algebra, the coordinates of point Q .

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

Question 7 continued**Diagram 1**

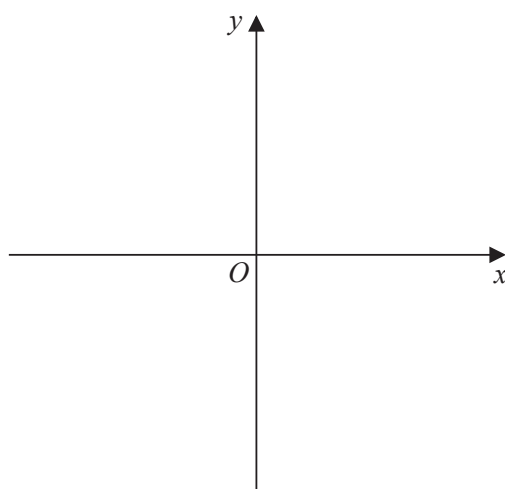
Turn over for a copy of Diagram 1 if you need to redraw your graph.





Question 7 continued

Only use this copy of Diagram 1 if you need to redraw your graph.



Copy of Diagram 1

(Total for Question 7 is 10 marks)



10.

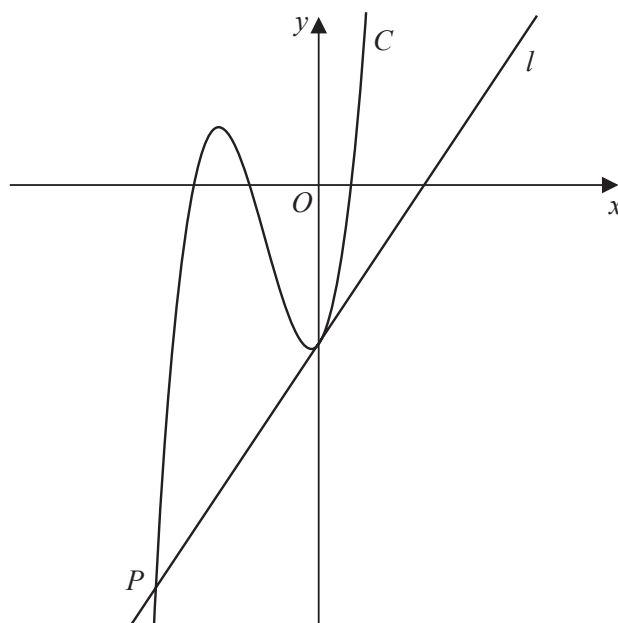
**Figure 4**

Figure 4 shows a sketch of part of the curve C with equation $y = f(x)$, where

$$f(x) = (3x + 20)(x + 6)(2x - 3)$$

- (a) Use the given information to state the values of x for which

$$f(x) > 0$$

(2)

- (b) Expand $(3x + 20)(x + 6)(2x - 3)$, writing your answer as a polynomial in simplest form.

(3)

The straight line l is the tangent to C at the point where C cuts the y -axis.

Given that l cuts C at the point P , as shown in Figure 4,

- (c) find, using algebra, the x coordinate of P

(Solutions based on calculator technology are not acceptable.)

(5)







(Total for Question 10 is 10 marks)



9. The curve C_1 has equation $y = f(x)$.

Given that

- $f(x)$ is a quadratic expression
- C_1 has a maximum turning point at $(2, 20)$
- C_1 passes through the origin

(a) sketch a graph of C_1 showing the coordinates of any points where C_1 cuts the coordinate axes,

(2)

(b) find an expression for $f(x)$.

(3)

The curve C_2 has equation $y = x(x^2 - 4)$

Curve C_1 and C_2 meet at the origin, and at the points P and Q

Given that the x coordinate of the point P is negative,

(c) using algebra and showing all stages of your working, find the coordinates of P

(5)



Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 10 marks)



4.

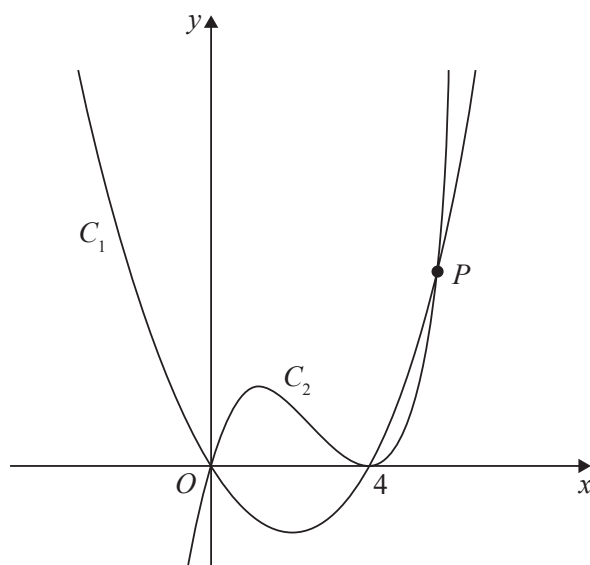


Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2

Given that C_1

- has equation $y = f(x)$ where $f(x)$ is a quadratic function
- cuts the x -axis at the origin and at $x = 4$
- has a minimum turning point at $(2, -4.8)$

(a) find $f(x)$

(3)

Given that C_2

- has equation $y = g(x)$ where $g(x)$ is a cubic function
- cuts the x -axis at the origin and meets the x -axis at $x = 4$
- passes through the point $(6, 7.2)$

(b) find $g(x)$

(3)

The curves C_1 and C_2 meet in the first quadrant at the point P , shown in Figure 1.

(c) Use algebra to find the coordinates of P .

(4)

Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4 continued.



(Total for Question 4 is 10 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.





(Total for Question 6 is 10 marks)



7.

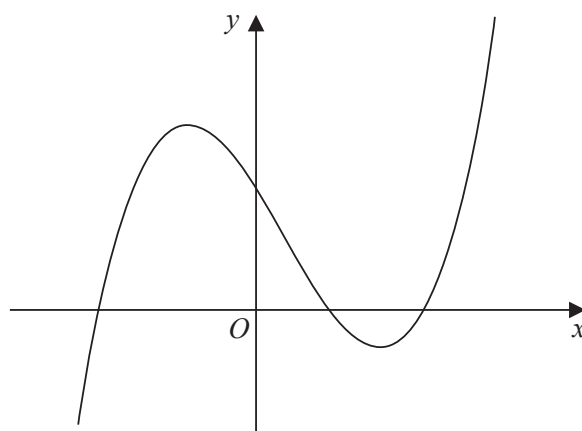
Leave
blank**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x + 4)(x - 2)(2x - 9)$$

Given that the curve with equation $y = f(x) - p$ passes through the point with coordinates $(0, 50)$

- (a) find the value of the constant p . (2)

Given that the curve with equation $y = f(x + q)$ passes through the origin,

- (b) write down the possible values of the constant q . (2)

- (c) Find $f'(x)$. (4)

- (d) Hence find the range of values of x for which the gradient of the curve with equation $y = f(x)$ is less than -18 (3)

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Question 7 continued

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Handwriting practice area with 25 horizontal lines.

(Total 11 marks)

Q7



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
2(a)	$a = 2$	B1
	$b = -3$	B1
		(2)
(b)	Any two term of $\int \frac{2x^3 - 3x^2 - 32x - 15}{5\sqrt{x}} dx = \int \frac{2}{5}x^{\frac{5}{2}} - \frac{3}{5}x^{\frac{3}{2}} - \frac{32}{5}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} dx$	M1A1
	$x^n \rightarrow x^{n+1}$	M1
	$\frac{4}{35}x^{\frac{7}{2}} - \frac{6}{25}x^{\frac{5}{2}} - \frac{64}{15}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$	A1A1
		(5)
		(7 marks)

- (a) If there is a contradiction between their stated values and the embedded values in a cubic expression then the embedded values take precedence.

B1 $a = 2$ which may be embedded

B1 $b = -3$ which may be embedded

(b)

M1 Attempts to write as a sum of terms. Award for any term with a correct index from correct working. The index does not need to be processed.

A1 Any **two** correct unsimplified or simplified terms of the expression

$$\frac{2}{5}x^{\frac{5}{2}} - \frac{3}{5}x^{\frac{3}{2}} - \frac{32}{5}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} \quad \text{oe eg } 0.4x^{\frac{5}{2}} - 0.6x^{\frac{3}{2}} - 6.4x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} \quad (\text{indices must be processed})$$

M1 Increases the power of any of their non-integer terms by 1. ($x^n \rightarrow x^{n+1}$) This mark cannot be awarded for just increasing the power by 1 on a numerator or denominator term. The index does not need to be processed.

A1 Any two terms correct unsimplified or simplified (see below). May appear in a list or on separate lines.

A1 $\frac{4}{35}x^{\frac{7}{2}} - \frac{6}{25}x^{\frac{5}{2}} - \frac{64}{15}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$ all on one line (including the constant c and all simplified)

Allow exact equivalents but not rounded decimals so $\frac{4}{35}$ must be written as a fraction.

Accept $\frac{4}{35}x^{\frac{7}{2}} - 0.24x^{\frac{5}{2}} - 4.26x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$. Also allow terms written as eg $\frac{4}{35}x^3\sqrt{x}$ Withhold this mark if the final answer has the integral sign and dx around it or any other spurious notation.

Question Number	Scheme	Marks
3(a)	$1.05 = 3p + q$ $1.65 = 5p + q$ e.g. $\Rightarrow 2p = 0.6$ or $\Rightarrow 1.05 = 3p + (1.65 - 5p)$ or $\Rightarrow 1.65 = 5p + (1.05 - 3p)$ $p = 0.3 \quad q = 0.15$	M1 A1A1
		(3)
(b)	$2.5 = "0.3"T + "0.15" \Rightarrow T =$ $T = 7.8$	M1 A1
		(2)
		(5 marks)

(a)

M1 Forms two simultaneous equations (which may be in pence) and proceeds to find a value for p or q

May be implied by a correct p or q (allow sight of 30 or 15).

Also score for an attempt to calculate $\frac{1.65 - 1.05}{5 - 3}$ or equivalent.

A1 $p = 0.3$ or $q = 0.15$ oe

A1 $p = 0.3$ and $q = 0.15$ oe

(b)

M1 Uses their values for p and q with $V = 2.5$ and rearranges to find a value for T . It must come from $\frac{2.5 \pm q}{p}$ which you may need to check on your calculator, allowing for truncation or

rounding (eg 8 following correct values for V , p and q)

A1 7.8 cao isw after sight of the correct answer.



Question Number	Scheme	Marks
4(a)	$x^2(2x+1)-15x \Rightarrow 2x^3+x^2-15x = x(2x^2+x-15)$	M1
	$x(2x-5)(x+3) = 0 \Rightarrow x = \dots$	dM1
	Two of $x = 0, \frac{5}{2}, -3$	B1
	$x = 0, \frac{5}{2}, -3$	A1
		(4)
(b)	$y^{\frac{2}{3}} = \frac{5}{2} \Rightarrow y = \left(\frac{5}{2}\right)^{\frac{3}{2}}$	M1
	$\frac{5}{4}\sqrt{10}$	A1cso
		(2)
		(6 marks)

The question says “In this question you must show detailed reasoning. Solutions relying on calculator technology are not acceptable”. Correct answers do not imply full marks.

(a)

M1 Multiplies out the bracket to achieve a cubic, takes out a linear factor or cancels the x leading to a quadratic factor (usually $2x^2 + x - 15$). Award for $\dots x(\dots x^2 \pm \dots x \pm \dots)$ or $\dots x(\dots x \pm \dots)(\dots x \pm \dots)$ May be implied by eg $x(2x-5)(x+3)$ May see $= 0$ or may be implied.

dM1 Attempts to solve their quadratic either by factorising or using the quadratic formula or completing the square. They cannot just state the roots and the fully factorised version or values of a , b and c must match their quadratic. It is dependent on the previous method mark. The $= 0$ can be implied and the factorised quadratic may appear under their solutions.

B1 Two of $x = 0, \frac{5}{2}, -3$

A1 $x = 0, \frac{5}{2}, -3$ provided all previous marks have been scored. Check for $x = 0$ in earlier work

Eg1 $2x^3 + x^2 - 15x = x(2x-5)(x+3) \Rightarrow 0, \frac{5}{2}, -3$ M1dM1B1A1

Eg2 $2x^3 + x^2 - 15x = 0 \Rightarrow x^3 + \frac{1}{2}x^2 - \frac{15}{2}x = 0 \Rightarrow x\left(x - \frac{5}{2}\right)(x+3) = 0 \Rightarrow 0, \frac{5}{2}, -3$ M1dM1B1A1

Eg3 $2x^3 + x^2 - 15x = x\left(x - \frac{5}{2}\right)(x+3) \Rightarrow 0, \frac{5}{2}, -3$ M1dM0B1A0 (the cubic and factorised version are not equal to each other)

Eg4 $2x^3 + x^2 - 15x = x(2x^2 + x - 15) \Rightarrow 0, \frac{5}{2}, -3$ M1dM0B1A0 (no method seen to solve the quadratic)

Eg5 $0, \frac{5}{2}, -3$ M0dM0B1A0 (no method seen at all)



This response below can score full marks.

$$\begin{aligned} \text{ca). } f(x) &= x^2(2x+1) - 15x. \\ 0 &= 2x^3 + x^2 - 15x \\ &\text{[scribbled out]} \\ (2x-5)(x+3)(x+0). \\ x &= \frac{5}{2} \quad x = -3 \quad x = 0. \end{aligned}$$

(b)

M1 $y^{\frac{2}{3}} = \frac{5}{2} \Rightarrow y = \left(\frac{5}{2}\right)^{\frac{3}{2}}$ (or one of their positive solutions from part (a)). The question states a calculator cannot be used so do not award for an implied method if they have decimal solutions or the exact answer without the method to deal with the fractional power seen. No marks can be scored without a positive solution from (a) or they restart in (b).

Allow notation such as $y = \sqrt[3]{\frac{5}{2}}$ and you do not need to see $y = \dots$

A1 $\frac{5}{4}\sqrt{10}$ or $1.25\sqrt{10}$ or $1\frac{1}{4}\sqrt{10}$ **only and no other solutions** cso

Eg1 $y^{\frac{2}{3}} = \frac{5}{2} \Rightarrow y = 3.95\dots$ is M0A0

Eg2 $y^{\frac{2}{3}} = \frac{5}{2} \Rightarrow y = \frac{5}{4}\sqrt{10}$ is M0A0

Eg3 $y^{\frac{2}{3}} = \frac{5}{2} \Rightarrow y^2 = \frac{125}{8} \Rightarrow y = \frac{5}{4}\sqrt{10}$ can score M1A1 as they have shown a method to deal with the fractional power.

Question Number	Scheme	Marks
5(a)	$f'(x) = 12x^{-\frac{1}{2}} + \frac{x}{3} - 4$	
	One of $x^{-\frac{1}{2}} \rightarrow x^{\frac{1}{2}}, -4 \rightarrow -4x, x \rightarrow x^2$	M1
	$f(x) = \int 12x^{-\frac{1}{2}} + \frac{x}{3} - 4 dx = 24x^{\frac{1}{2}} + \frac{x^2}{6} - 4x + c$	A1A1
	$8 = 24(9)^{\frac{1}{2}} + \frac{(9)^2}{6} - 4(9) + c \Rightarrow c = \dots$	dM1
	$(f(x) =) 24x^{\frac{1}{2}} + \frac{x^2}{6} - 4x - \frac{83}{2}$	A1
		(5)
(b)	$f'(9) = \frac{12}{\sqrt{9}} + \frac{9}{3} - 4 \quad (=3)$	M1
	$3 \rightarrow -\frac{1}{3}$	dM1
	$y - 8 = -\frac{1}{3}(0 - 9)$	M1
	$(0, 11)$	A1
		(4)
		(9 marks)

(a)

M1 Integrates by raising the power on one of the terms (ie $x^{-\frac{1}{2}} \rightarrow x^{\frac{1}{2}}, -4 \rightarrow -4x^1, x \rightarrow x^2$). The index does not need to be processed.

A1 Two terms correct of $24x^{\frac{1}{2}} + \frac{x^2}{6} - 4x$ or unsimplified equivalent which may appear as a list.
The indices must be processed. Allow x^1

A1 $24x^{\frac{1}{2}} + \frac{x^2}{6} - 4x (+c)$ or unsimplified equivalent. Condone the lack of $+c$ for this mark.
Allow x^1 and ignore any spurious notation including $= 0$

dM1 Attempts to substitute $x = 9, y = 8$ into their $f(x)$ and proceeds to find c . It is dependent on the previous method mark. If they have no $+c$ then this mark cannot be scored.

A1 $(f(x) =) 24x^{\frac{1}{2}} + \frac{x^2}{6} - 4x - \frac{83}{2}$ or simplified equivalent. Withhold this mark if they attempt to make the coefficients integers. Do not accept rounded decimals such as $0.166x^2$. Also withhold this mark if spurious notation around the answer is seen such as the integral and dx or $an = 0$

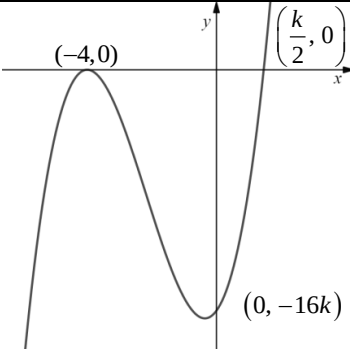
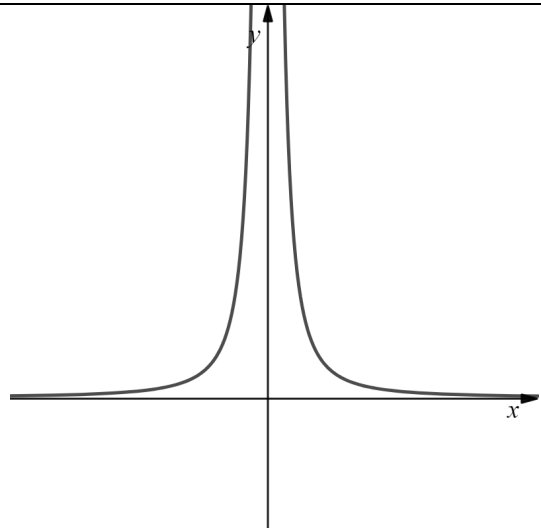
(b)

M1 Attempts to substitute $x = 9$ to find a value for $f'(9)$

dM1 Attempts to find the negative reciprocal to find the gradient of the normal. It is dependent on the previous method mark.

M1 It is for the attempt to find the normal line passing through $(9, 8)$, using a changed gradient from $f'(9)$ and substituting $x = 0$. Alternatively, they may just use their gradient of the normal to determine where the line crosses the y -axis knowing that for every “3 units to the left it is 1 up”. Condone a changed gradient from finding $f''(9)$ instead of from $f'(9)$ for this mark.

A1 $(0, 11)$ or allow it to be written as $x = 0, y = 11$

Question Number	Scheme	Marks
6(a)(i)		B1B1B1
		(3)
(a)(ii)		B1B1
		(2)
(b)	One root because the two graphs intersect each other once	B1
		(1)
		(6 marks)

(a)(i) If multiple attempts are made then mark the last attempt. Condone if (i) and (ii) are drawn on the same set of axes

B1 A positive cubic shape with a local maximum and a local minimum drawn anywhere on a set of axes. Do not be concerned regarding the location of the minimum. Mark of the intention to draw a cubic so condone aspects which may appear linear or slips of the pen.

B1 Intersects (not just meet) the x-axis at $x = \frac{k}{2}$ to the right of the y-axis and has a local maximum (or minimum) on $x = -4$ which must be to the left of the y-axis. If there is a contradiction between the labelling on the graph and values stated separately then the graph labels take precedence. Condone for example $(-4, 0)$ labelled as $(0, -4)$ as long as it is in the correct position on the axis

B1 y-intercept is $-16k$ (which must be below the x-axis) If there is a contradiction between the labelling on the graph and the y-intercept stated separately then the graph label takes precedence. Condone for example $(0, -16k)$ labelled as $(-16k, 0)$ as long as it is in the correct position on the axis

(a)(ii)

B1 Correct shaped curve in the first **or** second quadrant. It must not cross either axis. Do not be concerned by any labelled asymptotes or extra “branches” appearing in these quadrants. Mark the intention to draw a graph which does not have a clear turning point.

B1 Correct curves in the first **and** second quadrants and no other curves appearing in quadrants 3 and 4 (unless the cubic from (i) has been drawn on the same axes). Mark the intention to draw a graph tending towards the axes and does not have a clear turning point. If any asymptotes are labelled or indicated with dashed lines which are not the coordinate axes then withhold this mark.

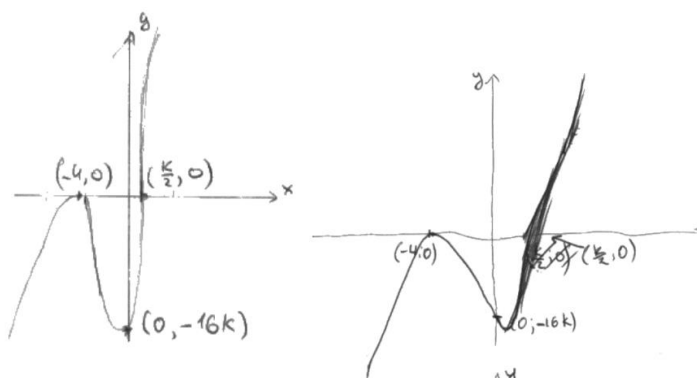
(b) This mark can only be awarded provided both graphs are the correct shape and position in part (a) (the intercepts with the axes are not needed for this mark)

B1 One root because the two graphs intersect each other once (only). Condone alternative wording which implies they “meet” once. Do not allow references to intersecting the axes.

Examples for (a)(i)

B1B1B1

B1B1B1

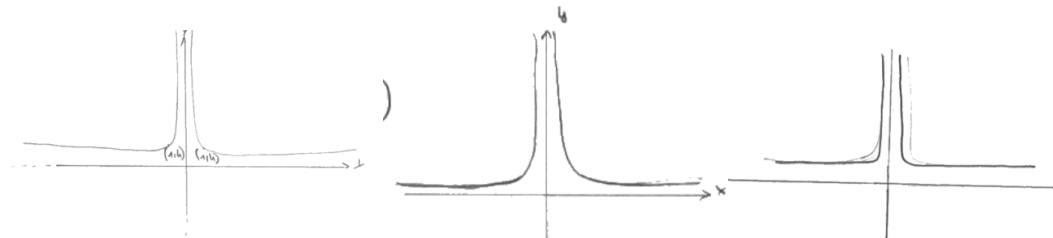


Examples for (a)(ii)

B1B1

B1B1

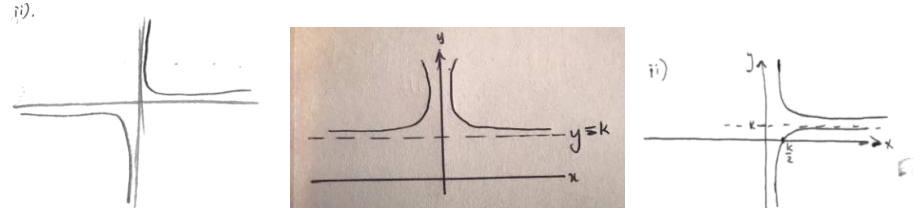
B1B1

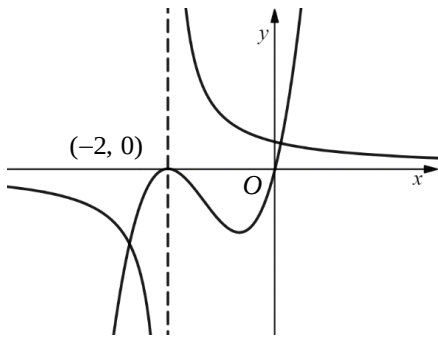


B1B0
ii)

B1B0

B1B0



Question Number	Scheme	Marks
4a	$x = -2$	B1
		(1)
b	$x^3 + 4x^2 + 4x = x(x+2)^2$	M1A1
		(2)
c		B1B1B1
		(3)
d	2 as the graphs intersect (each other) twice (since $(x+2)(x^3 + 4x^2 + 4x) = 1$ is the same as $x^3 + 4x^2 + 4x = \frac{1}{x+2}$)	B1
		(1)
		(7 marks)

Note: Check for answers by the questions or on the Figure (rather than the diagram). If there is a contradiction between the answers by the question and the main body of the solution then the main body of the solution takes precedence.

(a)

B1: $x = -2$ only. Accept labelled on the diagram but do not accept just -2 . If other equations are given (other than $y = 0$) then B0.

Beware that $x = -2$ is one of the solutions to the cubic so the equation of the asymptote must be seen in (a), on the diagram or by the question.

(b)

M1: Attempts to take out a linear factor eg $x(x^2 \pm 4x \pm 4)$ or $(x+2)(x^2 \pm 2x)$. May be implied by the correct answer.

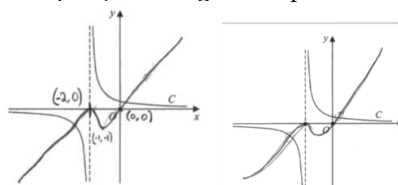
A1: $x(x+2)^2$ or $x(x+2)(x+2)$. Accept $(x+2)(x+2)x$ and condone a missing trailing bracket on the final linear factor e.g. $x(x+2)(x+2$ but $x(x+2^2$ is A0.

Condone $(x+0)(x+2)^2$. Isw once a correct expression is seen and ignore a spurious $= 0$

(c) **Note: If there are multiple attempts then score the highest one**

B1: Sketches a positive cubic anywhere on a set of axes. Do not penalise poor curvature provided the intention is clear and ignore if the curvature looks asymptotic. The cubic does not need to have two turning points for this mark.

Examples of acceptable cubic shapes (including where poor curvature would be condoned):



(If you are unsure then send to review)

B1: A cubic which

- has a turning point where the x -axis and vertical asymptote intersect
- passes through the origin (from quadrants 3 to 1 or from quadrants 2 to 4 – it cannot start or stop at the origin) It also cannot be a turning point.

Do not penalise poor curvature provided the intention is clear and ignore if the curvature looks asymptotic.

B1: $(-2, 0)$ **indicated on the graph** where their graph crosses or turns on the **negative x -axis** Do not be concerned regarding any other points where the graph crosses or turns on the x -axis including the origin.

Condone -2 labelled on the x -axis or the coordinates the wrong way round as $(0, -2)$ or with missing brackets.

The asymptote labelled $x = -2$ does not score this mark and do not be concerned with the point of intersection relative to the asymptote.

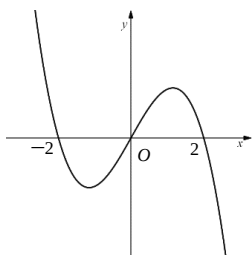
(d) **This mark can only be scored provided B1B1B0 or B1B1B1 is scored in (c)**

B1: **2 (real) roots and a valid reason** e.g. (graphs/curves/functions/equations/they) **intersect/meet/cross/touch** (each other) **twice**. e.g. “intersect twice” or “2 intersections” can score this mark.

May also refer to an intersection in the first and third quadrants.

Only withhold the mark if it is clear that they are referring to intersections between the cubic graph and the x -axis.

Do not accept responses relating to the roots of the equation, use of the discriminant or other algebraic methods which do not use the graphs.

Question	Scheme	Marks
8(a)		B1B1B1
		(3)
(b)	$x(4 - x^2) = \frac{A}{x} \Rightarrow 4x^2 - x^4 = A$ $\Rightarrow x^4 - 4x^2 + A = 0^*$	B1*
		(1)
(c)	$A > 0$	B1
	$b^2 = 4ac \Rightarrow 16 = 4A \Rightarrow A = \dots$	M1
	$0 < A < 4$	A1
		(3)
		Total 7

Notes:

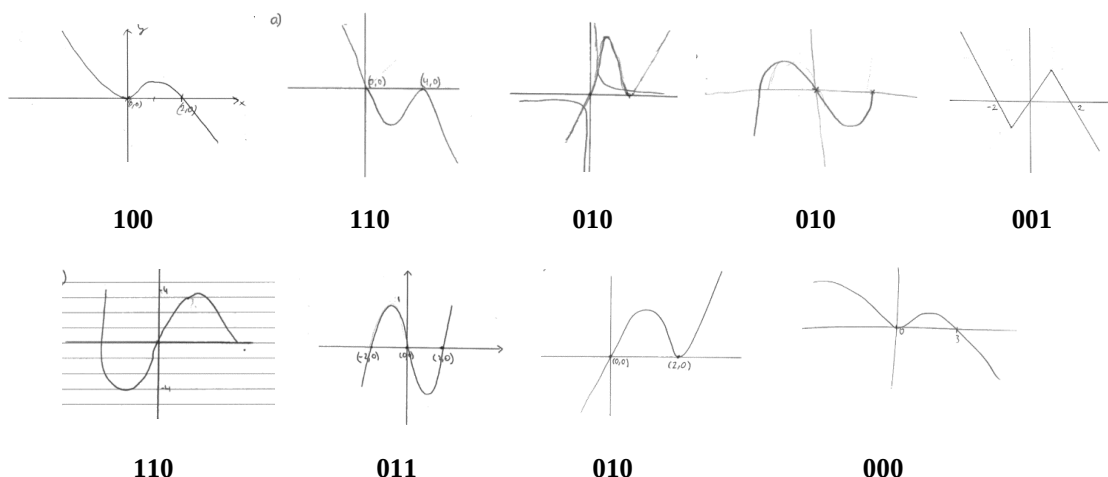
(a)

B1: Negative cubic curve in any position. Must have one maximum and one minimum.**B1:** Positive or negative cubic curve **passing through** (not touching) the origin.

The origin does not have to be labelled.

B1: Cuts the x-axis at $x = 2$ and $x = -2$. May be as shown or as $(2, 0)$ and $(-2, 0)$.Must cut so extends beyond the x-axis from above or below but **not** a turning point.Condone $(0, 2)$ and $(0, -2)$ as long as they are in the correct positions.Allow away from the sketch but must be $(2, 0)$ and $(-2, 0)$.

If there is any ambiguity, the sketch takes precedence.

Examples:

(b)

B1*: Equates the curves and shows sufficient working to obtain the printed answer with no errors.Note that $x(4-x^2)$ may be expanded first before equating which is fine.There must be at least one intermediate step between $x(4-x^2) = \frac{A}{x}$ and the printed answer.As a minimum accept $x(4-x^2) = \frac{A}{x} \Rightarrow 4x - x^3 = \frac{A}{x} \Rightarrow x^4 - 4x^2 + A = 0$

(c)

B1: For $A > 0$ as one boundary. May be seen embedded in their final answer. $x > 0$ is B0.**M1:** Attempts $b^2 = 4ac$ or equivalent e.g. $b^2 - 4ac = 0$ with $a = 1$, $b = -4$ and $c = A$ to obtain a value for A .Condone use of an inequality rather than “=” to obtain a value or inequality for A e.g. $b^2 - 4ac > 0$ or $b^2 - 4ac < 0$ leading to $A > \dots$ or $A < \dots$ May be implied by e.g. $A = 4$, $A < 4$, $A > 4$ etc.**A1:** Correct range for A . Allow equivalents e.g. “ $A > 0$ and $A < 4$ ”, “ $(0, 4)$ ”but not “ $A > 0$ or $A < 4$ ”, “ $A > 0, A < 4$ ”**Correct answer with no working scores 3/3****Partially correct answer with no working e.g. $0 < A \leq 4$ can score B1 M1 (implied) A0**

Question Number	Scheme	Marks
4. (a)	Area $ABCD$ is $40 \text{ cm}^2 \Rightarrow 40 = 6 \times 10 \times \sin \theta$ oe $\sin \theta = \frac{2}{3} \Rightarrow \theta = 180^\circ - 41.8^\circ$ $\angle DAB = \text{awrt } 138.19^\circ$	M1 M1 A1 (3)
(b)	Attempts $DB^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \cos 138.19^\circ$ $DB = \text{awrt } 15.0 \text{ (cm)}$	M1 A1 (2) (5 marks)

(a)

M1 Scored for a correct attempt at using the area of $ABCD$ is 40 cm^2 Score for $40 = 6 \times 10 \times \sin \theta$ or $20 = \frac{1}{2} \times 6 \times 10 \times \sin \theta$ where θ is one of the corner angles.M1 Score for $\sin \theta = k \Rightarrow \theta = 180^\circ - \arcsin k$ A1 $\angle DAB = \text{awrt } 138.19^\circ$

(b)

M1 Attempts $DB^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \cos 138.19^\circ$ - allow if the angle used is acute as long as it is clearly their attempt at angle DAB . So allow use of 41.8° unless they have correctly found angle DAB and chosen the wrong one here.A1 $DB = \text{awrt } 15.0 \text{ (cm)}$ Accept 15 in place of 15.0. Allow from attempts using awrt 138°
Alt for (a)M1 Area $ABCD$ is $40 \text{ cm}^2 \Rightarrow h = \frac{40}{10} = \dots \Rightarrow \sin \angle ABC = \frac{4}{6}$ OR $\cos \angle ABC = \frac{4}{6}$ oeEssentially this mark is for using the area together with an appropriate trig identity to form an equation in the sine or cosine of one of the angles of the parallelogram. Attempts finding “ DX ” where X is where the perpendicular to DC through A meets DC are possible.M1 $\angle DAB = 180^\circ - \arcsin\left(\frac{4}{6}\right) = \dots$ or may see $\angle DAB = 90^\circ + \arccos\left(\frac{4}{6}\right) = \dots$ This is for a complete correct method to find the angle DAB A1 $\theta = \text{awrt } 138.19^\circ$ 

Question Number	Scheme	Marks
5.(a)	$\frac{dy}{dx} = \frac{1}{2}x^2 + 2x^{-\frac{1}{2}}$	M1A1 A1 (3)
(b)	$\left. \frac{dy}{dx} \right _{x=4} = \frac{1}{2} \times 4^2 + 2 \times \frac{1}{\sqrt{4}} = (9)$ Gradient of normal is $-\frac{1}{9}$ $y - \frac{11}{3} = -\frac{1}{9}(x - 4) \Rightarrow x + 9y - 37 = 0$	M1 dM1 M1 A1 (4) (7 marks)

(a)

M1 For reducing the power by one on any x termA1 Two correct terms which may be unsimplified (but ignore spurious extra terms like -15 for this mark)A1 Fully correct and simplified. Accept exact simplified equivalents. Eg $\frac{x^2}{2} + \frac{2}{\sqrt{x}}$

Withhold the final A mark if “+c” is included.

(b)

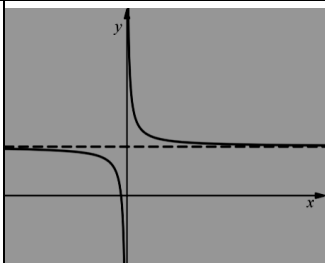
M1 For substituting $x = 4$ into their $\frac{dy}{dx}$

dM1 For the correct method of using the negative reciprocal to find the gradient of the normal.

M1 For an attempt at finding the equation of the normal. It is for using a changed gradient and the point $\left(4, \frac{11}{3}\right)$ If the form $y = mx + c$ is used they must proceed to $c = \dots$ A1 $x + 9y - 37 = 0$ or any (positive or negative) integer multiple thereof.

Accept with terms in a different order, but must include “=0”. ISW after a correct answer.



Question Number	Scheme	Marks
6. (a)	 Shape States asymptote as $y = k$ States intercept as $-\frac{4}{k}$	B1 B1 B1 (3)
(b)	$10 - 2x = \frac{4}{x} + k \Rightarrow 10x - 2x^2 = 4 + kx$ $\Rightarrow 2x^2 + (k - 10)x + 4 = 0$ Attempts "$b^2 - 4ac$" = $0 \Rightarrow (k - 10)^2 - 4 \times 2 \times 4 = 0$ $k = 10 \pm 4\sqrt{2} \text{ oe}$	M1 A1 M1 M1 A1 (5) (8 marks)

(a)

B1 Correct shape for graph. Look for a $y = \frac{1}{x}$ curve translated in any direction. Be tolerant with slips of pen how close the approach to asymptotes are, but the curve must not bend back on itself.

B1 Curve has horizontal asymptote above the x -axis with asymptote **stated** as $y = k$ on diagram or in text. (Do not accept just k marked on the axis for this mark.)

B1 Curve crosses (not just touches) the negative x -axis, with intercept marked or stated as $-\frac{4}{k}$

(b)

M1 Equates and attempts to multiply by x obtaining terms in x^2 , x and constant(s).

A1 $2x^2 + (k - 10)x + 4 = 0$ This may be implied by a correct a , b and c

M1 Attempts " $b^2 - 4ac$ " = $0 \Rightarrow (k - 10)^2 - 4 \times 2 \times 4 = 0$ Withhold this mark if an inequality is applied.

M1 For a correct method of finding at least one value for k from this attempt at the discriminant which is quadratic in k . Allow this mark for attempts using inequalities rather than equality, e.g. $b^2 - 4ac < 0$

A1 $k = 10 \pm 4\sqrt{2}$ oe (Do not accept decimal approximations and do not isw if an inequality is later stated.)

Alt for 6(b)

M1 Attempts to set $\frac{dy}{dx} = -2$ (usual rules for differentiation)

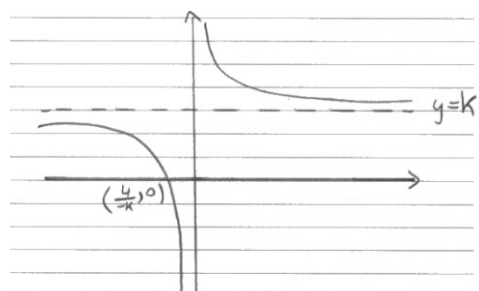
A1 Finds at least one value on the curve at which $\frac{dy}{dx} = -2 \Rightarrow -\frac{4}{x^2} = -2 \Rightarrow x = \pm\sqrt{2}$ (accept awrt ± 1.41)

dM1 Substitutes one of their x values into $y = 10 - 2x$ to find at least one y value. Eg $x = \sqrt{2}$, $y = 10 - 2\sqrt{2}$

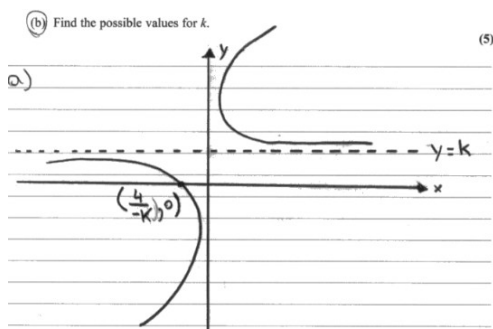
ddM1 Substitutes one of their (x, y) values into $y = \frac{4}{x} + k$ and proceeds to find one value for k .

A1 $k = 10 \pm 4\sqrt{2}$

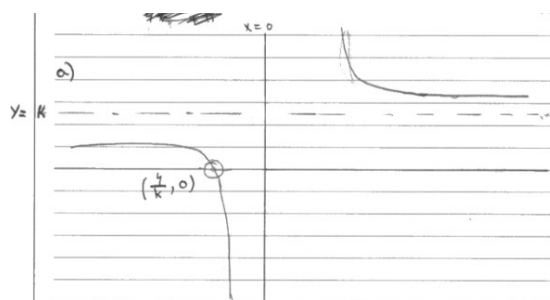
Examples for part (a)



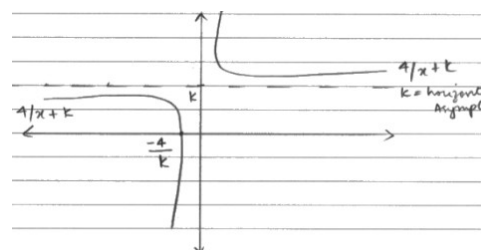
B1B1B1 Pen slip rather than turning back



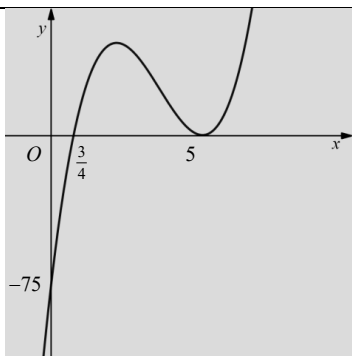
B0B1B1 Turning back on itself



B1B1B0 Generous with a bit of gap between curve and asymptote



B0B0B1 All four approaches bend away from the asymptote

Question Number	Scheme	Marks
10.(a)	 Shape for +ve x^3 Cuts x-axis at $\left(\frac{3}{4}, 0\right)$ and meets at $(5, 0)$ Crosses y-axis at $(0, -75)$	B1 B1 B1 (3)
(b)(i)	$(x =) 3, 20$	B1ft
(ii)	$(p =) 75$	B1ft
		(2)
(c) (i)	$(g(x) =) (4(x+1)-3)(x+1-5)^2 = (4x+1)(x-4)^2$	M1 A1
(ii)	16	B1
		(3)
		(8 marks)

(a)

For any of the coordinates if there are contradictions between the graph and the text then the graph takes precedence.

B1 Shape for a positive cubic, any position, with one maximum and one minimum. Condone no axes for this mark. Condone cubic curves which have a cusp like appearance at the minimum point.

B1 For a graph crossing at $\left(\frac{3}{4}, 0\right)$ and meeting at $(5, 0)$. The graph should not stop or cross at $(5, 0)$.

Allow just the x values instead of the full coordinates marked on the axes or written in the text and condone a slip of x and y the wrong way round as long as the sketch would give the correct coordinates. **Only allow this mark if a graph is drawn.**

B1 For a graph crossing the y -axis at $(0, -75)$

Allow the y value to be marked instead of the full coordinate or written in the text and condone a slip of x and y the wrong way round as long as the sketch would give the correct coordinates. **Only allow this mark if a graph is drawn.** Do not condone 75 marked on the negative y -axis.



Note: in both (b)(i) and (ii) for the follow through the graph takes precedence.

(b)(i)

B1ft $(x =) 3, 20$ and no others Follow through on $4x$ their x intercepts.

Allow $(3, 0)$ $(20, 0)$ ignore $(0, -75)$

(b)(ii)

B1ft $(p =) 75$ Follow through on their y intercept. Allow if $(y =) f(x) + 75$ seen. Do not allow $y = 75$

Mark (i) and (ii) together

(c)(i)

M1 Attempts $g(x) = (4(x+1) - 3)(x+1-5)^2$ condoning slips. Either award for sight of $x+1$ embedded in the equation or award for sight of an expression of the form $(4x+a)(x-4)^2$

Alternatively they expand $(4x-3)(x-5)^2 = 4x^3 + Px^2 + Qx + R$, where P, Q and $R \neq 0$, and then replace x with $x+1$ to achieve $4(x+1)^3 + P(x+1)^2 + Q(x+1) + R$ (condoning slips).

A1 $(g(x) =) (4x+1)(x-4)^2$ or simplified equivalent so accept $4x^3 - 31x^2 + 56x + 16$

(c)(ii)

B1 16 but accept $(0, 16)$.

Note that they may attempt $f(1) = (4 \times 1 - 3)(1 - 5)^2 = 16$



Question Number	Scheme	Marks
6(a)	$\frac{2(4)^2 + a \times 4 + b}{4\sqrt{4}} = 7 \Rightarrow 32 + 4a + b = 56 \Rightarrow 4a + b = 24$ *	M1A1*
		(2)
(b)	$4a + b = 24, a + b = -9 \Rightarrow a = 11, b = -20$	M1A1
	$\frac{x^{\frac{3}{2}}}{2} + \frac{11x^{\frac{1}{2}}}{4} - 5x^{-\frac{1}{2}}$	M1
	$\int \frac{x^{\frac{3}{2}}}{2} + \frac{11x^{\frac{1}{2}}}{4} - 5x^{-\frac{1}{2}} dx \Rightarrow \text{Two of } \frac{x^{\frac{5}{2}}}{5}, \frac{11x^{\frac{3}{2}}}{6}, \frac{-20x^{\frac{1}{2}}}{2}$	dM1A1ft
	$\frac{(4)^{\frac{5}{2}}}{5} + \frac{11(4)^{\frac{3}{2}}}{6} - 10(4)^{\frac{1}{2}} + c = -5 \Rightarrow c = \dots$	M1
(c)	$(f(x) =) \frac{1}{5}x^{\frac{5}{2}} + \frac{11}{6}x^{\frac{3}{2}} - 10x^{\frac{1}{2}} - \frac{91}{15}$	A1
		(7)
	$(7, -5)$	B1
		(1)
		(10 marks)

Mark (a) and (b) together**(a)**

M1 Attempts to substitute $x = 4$ into $f'(x)$ and sets equal to 7. Condone slips or incorrect manipulation prior to substituting in $x = 4$ provided the intention is clear.

A1* Rearranges and achieves the given answer with no errors and at least one intermediate stage of working seen.

(b)

M1 Attempts to solve the two equations in a and b simultaneously to find a value for a or b . You do not need to be concerned with the mechanics of the rearrangement. Implied by any value for a or b

A1 $a = 11, b = -20$ (correct values for a and b scores M1A1)

M1 Attempts to split the fraction and achieves at least one term with a correct index i.e. $x^{\frac{3}{2}}, x^{\frac{1}{2}}$ or $x^{-\frac{1}{2}}$

(for the relevant term). Do not allow for e.g. $\frac{\dots}{x^{\frac{1}{2}}}$ but the required index $x^{-\frac{1}{2}}$ may be implied by

further work. You do not need to be concerned by the coefficient of the term, which may be in terms of a or b .

Do not allow to be scored for e.g. $\frac{2x^2}{4\sqrt{x}} \rightarrow \dots x^{\frac{1}{2}}$

dM1 Increases the power by one on one of the terms in x with a correct index. It is dependent on the previous method mark. $x^{\frac{3}{2}} \rightarrow x^{\frac{5}{2}}, x^{\frac{1}{2}} \rightarrow x^{\frac{3}{2}}$ or $x^{-\frac{1}{2}} \rightarrow x^{\frac{1}{2}}$. The index does not need to be processed for this mark e.g. $x^{\frac{3}{2}} \rightarrow x^{\frac{3}{2}+1}$.

Do not award this mark for increasing the power by one on the denominator term.



If either a or b are not numerical then A0ftM0A0 is scored for the remaining marks in part (b)

Condone using numerical values for both a and b which appear with no evidence of working.

A1ft Two correct terms of $\frac{1}{5}x^{\frac{5}{2}}$ or $\frac{11}{6}x^{\frac{3}{2}}$ or $\frac{-20}{2}x^{\frac{1}{2}}$ The coefficients of the terms do not need to be simplified, but the index must be processed (follow through their a and/or b which must be numerical but both must have been found)

M1 Either states e.g. $x = 4, y = -5$ and states a value for c or attempts to substitute $(4, -5)$ into their changed expression with a constant of integration and proceeds to find a value for c . They must have a constant of integration in their expression to score this mark.
You do not need to check the mechanics of their rearrangement.

A1 $(f(x) =) \frac{1}{5}x^{\frac{5}{2}} + \frac{11}{6}x^{\frac{3}{2}} - 10x^{\frac{1}{2}} - \frac{91}{15}$ or exact simplified equivalent. isw once a correct answer is seen.

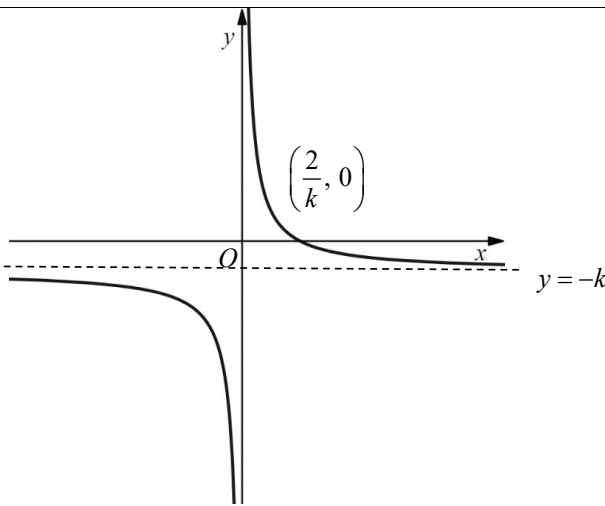
Must have the correct value for c substituted in.

Condone $-6.0\bar{6}$ for $-\frac{91}{15}$ but not e.g. -6.07 or $-6.066\dots$

Accept $(f(x) =) \frac{1}{30} \left(6x^{\frac{5}{2}} + 55x^{\frac{3}{2}} - 300x^{\frac{1}{2}} - 182 \right)$

(c) Check by the question or at the start of their work in the main body of the text

B1 $(7, -5)$ May be written as $x = 7, y = -5$. Condone missing brackets.

Question Number	Scheme	Marks
7(a)		B1B1B1
		(3)
(b)	$-kx - 6 = \frac{2}{x} - k \Rightarrow -kx^2 - 6x = 2 - kx \Rightarrow kx^2 + (6 - k)x + 2 = 0$ $(6 - k)^2 - 4 \times k \times 2 \Rightarrow k^2 - 20k + 36$ $\Rightarrow \text{CVs} = 2, 18 \Rightarrow k < "2" \text{ or } k > "18"$ $(0 <) k < 2 \text{ or } k > 18$	M1 dM1A1 M1 A1
		(5)
		(8 marks)

(a)

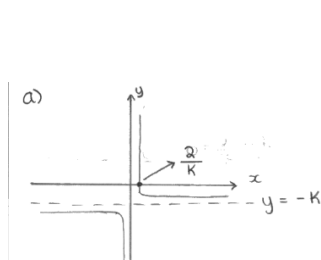
B1 Correct positive reciprocal shape graph in the correct position:

- The left branch must appear in quadrant 3 only
- The right branch must appear in both quadrants 1 and 4 only
- It must be drawn on a set of axes
- Condone slips provided a maximum or minimum is not intended

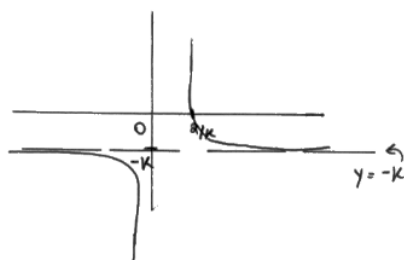
 B1 Intersects the x -axis at $\left(\frac{2}{k}, 0\right)$ only. Condone $\left(0, \frac{2}{k}\right)$ provided it is marked in the correct place. May just see $\frac{2}{k}$ labelled which is acceptable. Do not be concerned by missing brackets.

 B1 $y = -k$ Dashed lines are not needed on the sketch but if this asymptote is indicated on the sketch with dashed/solid lines then it must be a horizontal asymptote below the x -axis. Ignore any reference to a vertical asymptote. $y = -k$ must be labelled on the diagram (not just $-k$) or if labelled as $-k$ then $y = -k$ must be clearly identified as the asymptote in their working. Do not withhold this mark if they appear to have two horizontal asymptotes i.e. if the right branch is only in quadrant 1.

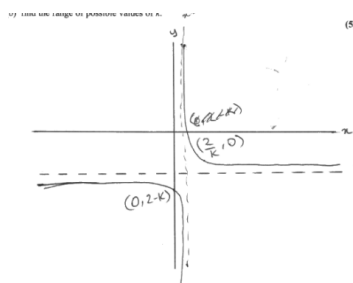
Examples



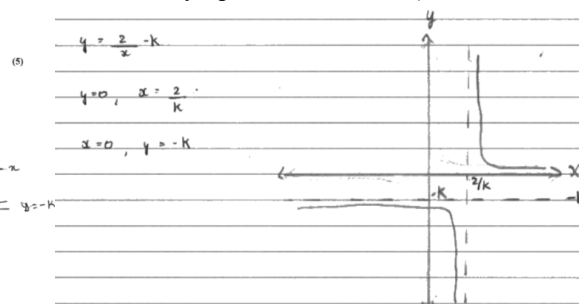
B1B1B1 (condone linearity)



B1B1B1 (condone the right arm as long as not intending to curve back on itself/two vertical asymptotes not labelled)



B0B1B1

B0B0B0 ($-k$ marked is insufficient for third B1 and $y = -k$ is not clearly identified as an asymptote)

(b)

M1 Sets the equation of the curve equal to the equation of the line, multiplies both sides by x and collects terms on one side of the equation. They do not need to simplify to a 3 term quadratic in x for this mark but it requires an equation which would simplify to $\dots kx^2 \pm (\dots k \pm \dots)x \pm \dots = 0$. Condone slips and $= 0$ may be implied by attempting the discriminant.

dM1 Attempts the discriminant (ignore $>$ or $= 0$) for their 3TQ in x and rearranges to a 3TQ in k . Condone slips and do not be concerned by the mechanics of the rearrangement. It is dependent on the first method mark.

A1 $\pm(k^2 - 20k + 36)$ (ignore $>$ or $= 0$)

M1 Finds the critical values (which may be directly from a calculator) from their quadratic in k and attempts the outside regions (usual rules apply for solving a quadratic). May just appear as their final answer.

Do not be concerned with „ or ... instead of $<$ or $>$. If just the roots are stated for their quadratic with no method shown you may need to check this for their quadratic and look for both values to be correct to 2sf.

Condone e.g. " $18 < k < 2$ " or e.g. $(-\infty, 2) \cup (18, \infty)$ to indicate the outside regions for this mark

May be in terms of x

A1 $(0 <) k < 2$ or $k > 18$ **Must be in terms of k**

Accept alternatives representing the same set of inequalities. e.g. $\{(0 <) k < 2 \cup k > 18\}$ or

$(0, 2) \cup (18, \infty)$ " $k > 18, k < 2$ " Ignore $k \neq 0$ if stated. $18 < k < 2$ is A0

Withhold this mark for use of AND o.e. (i.e. $\cap$) or if they have inequalities which would include 2 and/or 18.

If a lower limit is given then it must be $k > 0$ e.g. $0, k < 2, k > 18$ is A0.

Question Number	Scheme	Marks
10(a)		B1B1B1B1
		(4)
(b)	$k < 0$	B1
	$\frac{1}{x^2} - 9 = kx^2 \Rightarrow 1 - 9x^2 = kx^4$	M1
	$1 - 9x^2 = kx^4 \Rightarrow kx^4 + 9x^2 - 1 = 0$ Require $b^2 - 4ac \Rightarrow 81 - 4 \times k \times -1$	M1
	Critical value ($k =$) $-\frac{81}{4}$	A1
	$-\frac{81}{4} < k < 0$	A1cso
		(5)
		Total 9

(a)

B1: shape anywhere on a set of axes. Condone slips of the pen towards the asymptotes as long as they are not clear turning points.

B1: x intercepts at $\left(-\frac{1}{3}, 0\right)$ and $\left(\frac{1}{3}, 0\right)$. May be marked as $-\frac{1}{3}$ and $\frac{1}{3}$. Do not allow $\left(0, -\frac{1}{3}\right)$ or $\left(0, \frac{1}{3}\right)$ but condone lack of brackets around the coordinates. Only accept $0.\dot{3}$ and $-0.\dot{3}$ or equivalent. May be stated underneath.

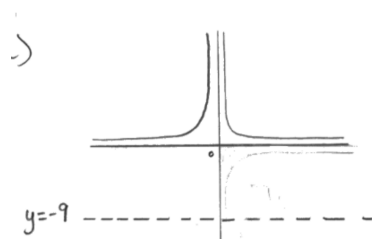
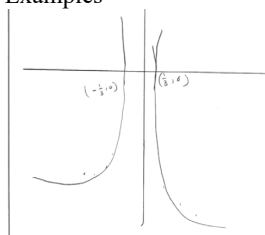
Cannot be scored without a corresponding sketch.

B1: States the asymptote $x = 0$ or $y = -9$ as long as it is an asymptote for their sketch

B1: States both asymptotes $x = 0$ and $y = -9$ as long as they are asymptotes for their sketch

For the coordinates or asymptotes if they are written on the sketch and then stated underneath then the sketch takes precedence.

Examples



(b)

B1: $k < 0$ (which may be part of their final answer).M1: Sets $C = D$ and multiplies through by x^2 to achieve a quartic (does not need to be a 3TQ in x^2 for this mark). Terms do not need to be collected on one side of the equation.M1: Considers the discriminant of their **3TQ** in x^2 to produce an expression in terms of k . Condones sign slips in their rearrangement before attempting the discriminant.A1: Identifies $(k =) -\frac{81}{4}$ as a critical value from correct working which may be seen within their solution. Ignore use of equals or an inequality sign for this mark or x used instead of k .Note $1 - 9x^2 = kx^4 \Rightarrow kx^4 - 9x^2 - 1 = 0 \Rightarrow -\frac{81}{4}$ is A0A0 (sign error when rearranging)A1 also: $-\frac{81}{4} < k < 0$ or other equivalent expressions such as $k > -\frac{81}{4} \cap k < 0$ or $k > -\frac{81}{4}$ and $k < 0$ $0 > k > -\frac{81}{4}$. **Must be in terms of k** Do not accept " $k > -\frac{81}{4}$ or $k < 0$ " " $k > -\frac{81}{4}, k < 0$ " " $k > -\frac{81}{4} \cup k < 0$ "

Question Number	Scheme	Marks
3.(a)	$y = \frac{5x^3 - 8}{2x^2} = \frac{5}{2}x - 4x^{-2}$ $\frac{dy}{dx} = \frac{5}{2} + 8x^{-3}$	M1 dM1 A1 A1 (4)
(b)	Substitutes $x = 2$ into their $\frac{dy}{dx} = \frac{5}{2} + \frac{8}{x^3} \Rightarrow \frac{dy}{dx} = \frac{7}{2}$ Uses their $\frac{7}{2}$ and $(2, 4) \Rightarrow y - 4 = \frac{7}{2}(x - 2)$ $\Rightarrow 7x - 2y - 6 = 0$	M1 M1 A1 (3) (7 marks)

(a)

M1: Attempts to write y as **a sum of two terms** and achieves at least one term with the correct index.Award for $Px + Qx^k$ or $Px^k + Qx^{-2}$. Condone $Qx^{-2} \leftrightarrow \frac{Q}{x^2}$

dM1: Requires both

- the given expression to be written as a sum of two terms with both indices correct
- correct differentiation applied to the indices $Px + Qx^{-2} \rightarrow A + Bx^{-3}$ o.e

A1: One correct term which need not be simplified. Condone $\frac{5}{2}x^0$

Note that 1010 is possible where candidates only form one of the two correct terms and differentiate that correctly

A1: $\left(\frac{dy}{dx}\right) = \frac{5}{2} + 8x^{-3}$ or simplified equivalent such as $\frac{1}{2}\left(5 + \frac{16}{x^3}\right)$. There is no need to have the $\frac{dy}{dx}$

(b)

M1: Substitutes $x = 2$ into their $\frac{dy}{dx}$ and attempts its value. Their $\frac{dy}{dx}$ cannot be the same as their "y"Score for sight of embedded 2's in their $\frac{dy}{dx}$ followed by a value or it may be implied for a correctvalue for their $\frac{dy}{dx}$

M1: Correct method for finding the equation of a tangent.

Look for a correct use of their $\frac{dy}{dx}\bigg|_{x=2}$ and the point $(2, 4)$ to form equation $y - 4 = \frac{7}{2}(x - 2)$.If the form $y = mx + c$ is used it must proceed as far as $c = \dots$ Award this mark if they achieve $\frac{dy}{dx} = \frac{5}{2}$ in part (a) and correctly form the equation using $m = \frac{5}{2}$ and $(2, 4)$. They cannot score the previous mark.A1: $7x - 2y - 6 = 0$ or any multiple thereof

.....



Alt (a)

Some may have done WMA13 and may attempt the quotient rule

M1, dM1 for an attempt at the quotient rule to obtain $\frac{2x^2 \times ax^2 - (5x^3 - 8) \times bx}{(2x^2)^2}$ where $a > 0, b > 0$

condoning lack of brackets

So allow, for example, if you see $2x^4$ on the denominator.

These two marks are scored together so 1000 is not possible via this route

Then A1 for $\frac{2x^2 \times 15x^2 - (5x^3 - 8) \times 4x}{(2x^2)^2}$ and finally A1 for $\frac{5x^3 + 16}{2x^3}$ or other simplified equivalent

Alt (b)

Some may use the $\frac{dy}{dx}$ function of their calculator to find the gradient of the tangent at $x = 2$

So if the gradient of $\frac{7}{2}$ appears without any working **AND** following an incorrect or missing part (a) score as follows

M0: No sight of using $x = 2$ in their $\frac{dy}{dx}$

M1: For sight of $y - 4 = \frac{7}{2}(x - 2)$ or equivalent

A1: For $7x - 2y - 6 = 0$ or integer multiple

Question Number	Scheme	Marks
2. (i)	$\frac{3y^3 \left(2x^4\right)^3}{4x^2 y^4} = \frac{6x^{10}}{y}$	B1, B1, B1 (3)
(ii)	$\frac{16}{\sqrt{3}+1} = a\sqrt{27} + 4$ States or uses $\sqrt{27} = 3\sqrt{3}$ Correct attempt at rationalising seen e.g. $\frac{16}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$ Correct attempt to make "a" the subject and make progress towards the form $p\sqrt{3} + q$ e.g. $8\sqrt{3} - 8 = 3a\sqrt{3} + 4 \Rightarrow 3a\sqrt{3} = 8\sqrt{3} - 12 \Rightarrow a = \frac{8\sqrt{3}}{3\sqrt{3}} - \frac{12}{3\sqrt{3}}$ $a = -\frac{4}{3}\sqrt{3} + \frac{8}{3}$	B1 M1 M1 A1cso (4) (7 marks)

(i)

B1: For any one of 6 , x^{10} or $\frac{1}{y}$. Allow y^{-1} for $\frac{1}{y}$ - may be seen separately, and allow for one correctly simplified term even if a spurious extra term in the same variable is present.

B1: For any two of the above as the only term in that variable within a single expression.

E.g. $2x^{10} \times \frac{3}{y}$ is fine for B1B1 but $2x^{10} + \frac{3}{y}$ is B1B0.

B1: For $\frac{6x^{10}}{y}$ or $6x^{10}y^{-1}$ - must be one expression. Allow with y^1 in the denominator.

(ii)

B1: States or uses $\sqrt{27} = 3\sqrt{3}$. May be implied by working as long as it is seen before the final answer.

M1: Correct attempt at rationalisation seen anywhere. This can be on any expression with a two term surd expression in a denominator during their working. The denominator may be simplified without the multiplication being seen (and need not be correct), but the evidence of the process must be clear.

M1: Attempts to make "a" the subject and make progress towards the form $p\sqrt{3} + q$

($p, q \neq 0$) - must reach a two term expression for a (but may be over common denominator), though the $\sqrt{27}$ may not have been simplified. E.g. accept for

$a = \frac{8\sqrt{3}-12}{\sqrt{27}}$. Can be scored if a calculator was used to rationalise the surd expression

but there must be an intermediate step before their final answer to show the method.

A1cso: $a = -\frac{4}{3}\sqrt{3} + \frac{8}{3}$ but isw after a correct expression. Must have score all previous marks, showing the relevant steps as outline above before the final answer is given but condone if “invisible brackets” are recovered. Accept with terms in the other order, and allow $\frac{1}{3}(8 - 4\sqrt{3})$

Note that there are many ways of attempting (b). The above scheme can still be applied
A common alternative method is;

$$\begin{aligned}\frac{16}{\sqrt{3}+1} &= a\sqrt{27} + 4 \Rightarrow 16 = (a\sqrt{27} + 4)(\sqrt{3}+1) = 9a + 4 + 4\sqrt{3} + 3\sqrt{3}a \quad \text{for B1} \\ \Rightarrow 12 - 4\sqrt{3} &= (9 + 3\sqrt{3})a \Rightarrow a = \frac{12 - 4\sqrt{3}}{9 + 3\sqrt{3}} \times \frac{9 - 3\sqrt{3}}{9 - 3\sqrt{3}} \quad \text{for M1} \\ \Rightarrow a &= \frac{144 - 72\sqrt{3}}{54} = -\frac{4}{3}\sqrt{3} + \frac{8}{3} \quad \text{for M1 A1}\end{aligned}$$

Notes: The B mark can be implied by working as long as it is seen before the final answer.

So, for example, $\frac{8\sqrt{3}+4}{\sqrt{27}} \times \frac{\sqrt{27}}{\sqrt{27}} = \frac{72+12\sqrt{3}}{27} = \dots$ is sufficient to score the B mark.

An acceptable equivalent of rationalising the denominator of an expression for the first M is rationalising the coefficient of a in a rearranged expression, e.g.

$$(9 + 3\sqrt{3})a = 12 - 4\sqrt{3} \rightarrow (81 - 27)a = (9 - 3\sqrt{3})(12 - 4\sqrt{3})$$

The second M mark requires an attempt to make a the subject and make progress towards the form $p\sqrt{3} + q$, which means the denominator must have been reduced to a single term with

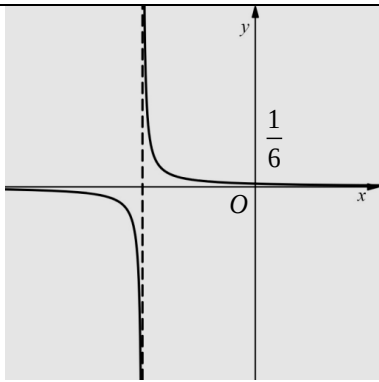
two terms in the numerator. So $a = \frac{12 - 4\sqrt{3}}{9 + 3\sqrt{3}}$ is not sufficient progress. In the example above

the M is not awarded until the final line where the denominator is 54.

However, once the first B and M are scored (having seen evidence before the final answer) reaching the correct answer will then be sufficient for the M (and A if all correct).

But from $\frac{12 - 4\sqrt{3}}{9 + 3\sqrt{3}}$ to $\frac{8 - 4\sqrt{3}}{3}$ with no further work shown is 2nd M0 (calculator use with

no further progress). However, if a correct attempt to rearrange this to $-\frac{4\sqrt{3}}{3} + \frac{8}{3}$ (or with terms reversed) follows (or similar if there were errors) then the 2nd M becomes eligible as there is further work to support the method.

Question Number	Scheme		Marks
7(a)	 $x = -6$	Correct shape and position	B1
		Correct equation of vertical asymptote. E.g. $x = -6$ oe e.g. $x + 6 = 0$	B1
		Correct y intercept $\left(0, \frac{1}{6}\right)$	B1
			(3)

Notes

There must be a sketch to score marks in (a)

Allow the equation of the asymptote and the y intersection to be identified away from the sketch but they must clearly be identified as the asymptote and/or the intersection and correspond to their sketch. If there is any ambiguity, the sketch takes precedence.

(a)

B1: Correct shape and position.

The right hand branch should

- be in quadrants 1 and 2 only (this includes just touching the y-axis)
- approach the x-axis from above on the right
- approach the vertical upwards on the left in quadrant 2

The left hand branch should

- be in quadrant 3 only
- approach the x-axis from below on the left
- approach the vertical downwards on the right

The branches must not clearly overlap.

The curve must not clearly turn back on itself at the vertical or horizontal asymptotes but condone “slips of the pen”.

B1: Correct **equation** for vertical asymptote. The curve must be asymptotic here for their curve and for both branches if drawn.

The asymptote does not need to be “drawn” e.g. dotted or solid line.

Must be an equation (not just -6 marked on the x-axis)

B1: Correct y intercept. The curve must only cross the y-axis at this point.

Allow as $\frac{1}{6}$ marked on the y-axis or as $\left(0, \frac{1}{6}\right)$ and condone $\left(\frac{1}{6}, 0\right)$ as long as it is in the correct position on the positive y-axis.

Also allow $y = \frac{1}{6}$ if it is clearly indicating the point of intersection and not the equation of an asymptote.

See supplementary document for some examples.

(b)(i)	$mx - 4 = \frac{1}{x+6} \Rightarrow (mx-4)(x+6) = 1$	M1
	$\Rightarrow mx^2 + (6m-4)x - 25 = 0$ or $mx^2 + 6mx - 4x - 25 = 0$	A1
	" $b^2 - 4ac$ " = $(6m-4)^2 - 4 \times m \times (-25)$	dM1
	$\Rightarrow (6m-4)^2 + 100m \geq 0$ $\Rightarrow 36m^2 + 52m + 16 \geq 0 \Rightarrow 9m^2 + 13m + 4 \geq 0$ *	A1*
(ii)	$(m+1)(9m+4) \geq 0 \Rightarrow m \leq -1, m \geq -\frac{4}{9}$	M1 A1
		(6)

Mark (b)(i) and (ii) together.

(b)(i)

M1: Sets $mx - 4 = \frac{1}{x+6}$ and attempts to cross multiply

A1: Correct simplified quadratic equation with terms on one side which may be implied by subsequent work. The " $= 0$ " may be implied.
The $(6m-4)x$ may be expanded.

dM1: Attempts the discriminant " $b^2 - 4ac$ " for their 3TQ. There must be no x 's in their a , b and c .
May be seen embedded in an attempt to use the quadratic formula.

A1*: cso. Correct proof with an intermediate line after e.g. $(6m-4)^2 + 100m \geq 0$
The ≥ 0 should appear before the final line and there must have been no contradictory statements such as $(6m-4)^2 + 100m = 0$ **but** if the discriminant is found and a justification that it must be ≥ 0 is given, this mark can be awarded.

(b)(ii)

M1: Uses a correct method to find the critical values of $9m^2 + 13m + 4 = 0$ **and** finds the outside range for their critical values. Allow any method of solving $9m^2 + 13m + 4 = 0$ including use of a calculator. Condone strict inequalities for this mark and condone use of x instead of m .

A1: $m \leq -1, m \geq -\frac{4}{9}$ or e.g. $m \leq -1$ or $m \geq -\frac{4}{9}$ Condone $m \leq -1$ and $m \geq -\frac{4}{9}$ but **not**
 $-\frac{4}{9} \leq m \leq -1$

Allow equivalents e.g. $(-\infty, -1], [-\frac{4}{9}, \infty)$ and allow exact equivalents for $-\frac{4}{9}$ including $-0.\dot{4}$

Note that they cannot go straight from $9m^2 + 13m + 4 \geq 0$ to $m \leq -1, m \geq -\frac{4}{9}$ without

finding or stating the roots first so that $9m^2 + 13m + 4 \geq 0 \Rightarrow m \leq -1, m \geq -\frac{4}{9}$ scores M0A0

But $9m^2 + 13m + 4 \geq 0 \Rightarrow m = -1, m = -\frac{4}{9} \Rightarrow m \leq -1, m \geq -\frac{4}{9}$ scores M1A1

$9m^2 + 13m + 4 \geq 0 \Rightarrow (9m+4)(m+1) \geq 0 \Rightarrow m \leq -1, m \geq -\frac{4}{9}$ scores M1A1

Or e.g. a sketch of the quadratic with -1 and $-\frac{4}{9}$ indicated followed by $m \leq -1, m \geq -\frac{4}{9}$

scores M1A1

Generally, there must be some intermediate work seen before the inequalities are written down.

Question Number	Scheme	Marks
9. (a)	24π	B1 (1)
(b)	$(18\pi, -1)$	B1ft (1)
(c)(i)	$-12\pi - \alpha$	B1 ft
(ii)	$6\pi - \alpha$	B1 ft (2) (4 marks)

(a) Do not allow the mark if an inequality or coordinates are given, but you may assume if $0 < x < p$ or $(p, 0)$ is given that their period is p for the purposes of the remaining marks.

See above

Note that (b) and (c) are for (correct or) follow through and hence

(b) is scored for $\left(\frac{3p}{4}, -1\right)$ -- this may be seen on the graph but take clearly labelled part (b) as precedent.

Follow through on $p \neq 2\pi$

(c) (i) is scored for $-\frac{1}{2}p - \alpha$

(ii) is scored for $\frac{1}{4}p - \alpha$

where p is their period stated in (a) (which may be 2π)

For answers in degrees penalise the first mark due only, but allow if degree symbol is missing.

For reference (a) 4320° (b) $(3240^\circ, -1)$ (c)(i) $-2160^\circ - \alpha$ (ii) $1080^\circ - \alpha$

Note for example that (a) $0 < x < 4320^\circ$ (b) $(3240^\circ, -1)$ (c)(i) $-2160^\circ - \alpha$ (ii) $1080^\circ - \alpha$ would score B0 B0 B1ft B1ft

Question Number	Scheme	Marks
10 (a)	$f(x) \leq 0 \Rightarrow x \leq -\frac{5}{2}, \quad x = 3$	M1 A1



Question Number	Scheme	Marks
(b)	$f(x) = (2x+5)(x-3)^2 = (2x+5)(x^2 - 6x + 9)$ $= 2x^3 - 12x^2 + 18x + 5x^2 - 30x + 45$ $= 2x^3 - 7x^2 - 12x + 45$	(2) M1 M1 A1
(c)	(i) $P(0,45)$ (ii) Gradient = -12	(3) B1ft B1ft
(d)	(i) $g(x) = (2(x-2)+5)(x-2-3)^2 = (2x+1)(x-5)^2$ (ii) 25	(2) M1 A1 B1 (3) (10 marks)

(a)

M1 For either $x \leq -\frac{5}{2}$ or $x = 3$ Condone for this mark $x < -\frac{5}{2}$

A1 For both $x \leq -\frac{5}{2}$ and $x = 3$ Accept answers given in set notation. Accept with “and” or “or” between – it is for both correct inequalities.

Note: Mark the final answer. Answers such as $-\frac{5}{2} \leq x \leq 3$ or $3 \leq x \leq -\frac{5}{2}$ are M0A0.

(b)

M1 Attempts to multiply two of the brackets together, achieving x^2 , x and constant terms.

M1 Multiplies the result by the third bracket to reach a four term cubic expression (not necessarily simplified).

A1 $2x^3 - 7x^2 - 12x + 45$ Ignore any reference to “=0” or “<0” etc for this mark.

(c)(i)

B1ft $P(0,45)$ following through on their 'd'. Do not accept e.g. (45,0) or just 45.

(c)(ii)

B1 ft Gradient = -12 following through on their 'c'

(d)(i)

M1 Attempts to replace x with $(x-2)$ in $(2x+5)(x-3)^2$ or in their expansion of this.

Allow M1 for one correct bracket if no incorrect working is seen (bod). Condone invisible brackets

A1 $g(x) = (2x+1)(x-5)^2$ If substituting into the expand form, must factorise correctly to achieve A1.

(d)(ii)

B1 25 Accept (0,25).

Question Number	Scheme	Marks

Question Number	Scheme	Marks
5 (a)	Attempts the sine rule $\frac{\sin \alpha}{14} = \frac{\sin 0.43}{6}$ $\Rightarrow \alpha = 1.337$ (radians) Accept awrt 1.33/1.34 or awrt 76.6/76.7 ($^{\circ}$) angle $AOD = \pi - 1.337 = \text{awrt } 1.805$ (radians)	M1 A1 A1 (3)
	(b) Attempts $s = r\theta$ with $r = 6$ and an allowable θ Arc length $ABC = \text{awrt } 26.9$ m	M1 A1 (2)
	(c) Attempts $\frac{1}{2}r^2\theta$ with $r = 6$ and an allowable θ in radians ($= 80.6$) Attempts area $AOD = \frac{1}{2} \times 6 \times 14 \times \sin(0.91)$ oe ($= 33.1$) Attempts sector + triangle with correct attempt at angles $= 113.7$ (m^2)	M1 M1 dM1 A1 (4)
		(9 marks)

(a) **Note on EPEN it is M1M1A1 but we are marking this M1A1A1**

M1 Attempts the sine rule $\frac{\sin \alpha}{14} = \frac{\sin 0.43}{6}$. Sight of the values embedded in the equation or awrt 1.33/1.34 implies this mark. They may also work in degrees so sight of awrt 76.6/76.7 also implies this mark.

A1 awrt 1.33/1.34 (radians) or awrt 76.6/76.7 (degrees)

A1 awrt 1.805 (radians)

(b) **Note method marks can still be awarded even if their angle from part (a) is rounded.**

M1 Attempts $s = r\theta$ with $r = 6$ and an allowable angle.
Accept as an allowable their angle " (a) ", " $\pi - (a)$ " or " $2\pi - (a)$ "
Note if their answer was 1.805 in (a):

Angle used:	$r\theta$
1.805	10.8
$\pi - 1.805$ (1.34)	8.02
$2\pi - 1.805$ (4.48)	26.9

They may also work in degrees using an allowable angle so look for $\frac{\dots}{360} \times 12\pi$ oe

A1 Awrt 26.9 (metres) Must come from angle $AOD = \text{awrt } 1.8$ in (a).

Note if they use the acute angle for AOD then arc length is 29.7 (m) which scores M1A0



(c) **Beware of different methods to find the required area. Send to review if unsure.**

M1 Attempts $\frac{1}{2}r^2\theta$ with $r = 6$ and an allowable θ ($\frac{1}{2}r \times \text{arc length}$ is also acceptable)

Accept as an allowable angle their "(a)", $\pi - "(a)"$ or $2\pi - "(a)"$

Note if their answer was 1.805 in (a):

Angle used:	$\frac{1}{2}r^2\theta$
1.805	32.5
$\pi - 1.805$ (1.34)	24.1
$2\pi - 1.805$ (4.48)	80.6

They may also work in degrees using an allowable angle so look for $\frac{\dots}{360} \times 36\pi$ or

Condone use of $\frac{1}{2}r^2(\theta - \sin \theta)$ with $r = 6$ and an allowable θ to score this mark.

M1 Correct method to find area of triangle AOD .

Look for the correct combination of sides and inclusive angle.

Angle OAD must be found by a correct method $\pi - 0.43 - "(a)"$ (or in degrees: $180 - 24.6 - "(a)"$)

Alternatively, they may form two right angled triangles. This must be a correct method to find the area of both triangles and add them together.

Another method is to find OD eg using the sine rule ($= 11.3\dots$) and calculate $\frac{1}{2} \times 14 \times OD \times \sin(0.43)$

dM1 Full method to find the correct area

Both areas must be found using a correct method.

Look for eg $\frac{1}{2} \times 6^2 \times (2\pi - "(a)") + \frac{1}{2} \times 6 \times 14 \times \sin(\pi - 0.43 - "(a)")$

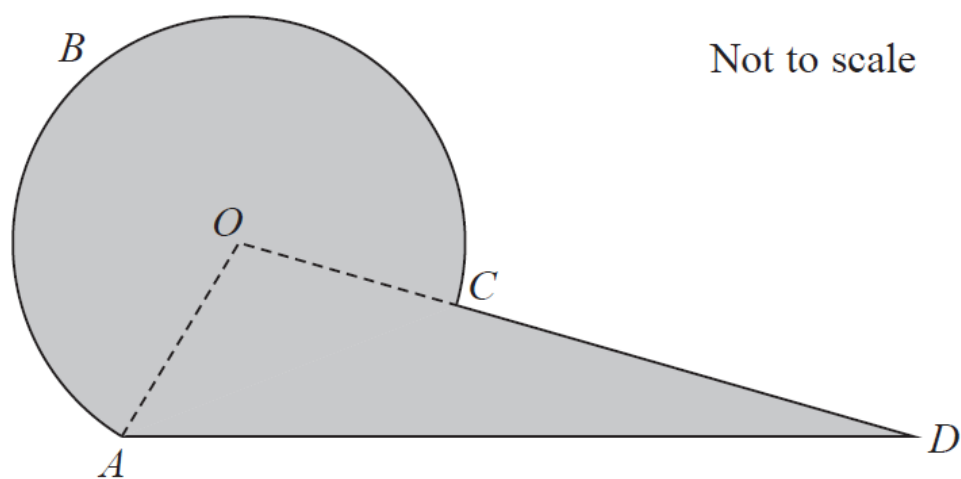
It is dependent on both of the previous method marks.

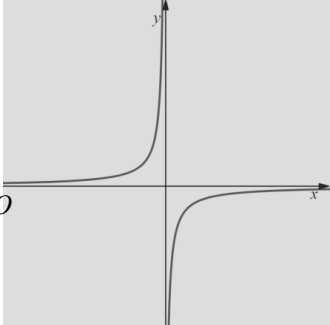
A1 awrt 113.7 (m^2)

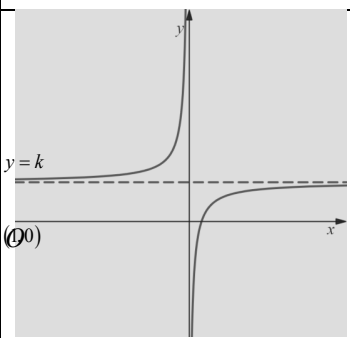
Note if they use the acute angle for AOD then total area is 130 (m^2) which scores M1M1dM1A0

If the acute angle is used then as a guide see the diagram below:

Max score is (a) M1A1A0, (b) M1A0, (c) M1M1dM1A0



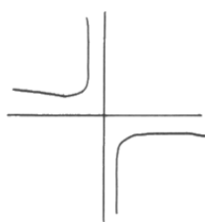
Question Number	Scheme	Marks
6.(a)		M1
	Fully correct	A1
		(2)

Question Number	Scheme	Marks
(b)	 Graph is part (a) translated $\uparrow$ Correct asymptote or intercept Correct asymptote and intercept	B1ft B1 B1 (3)
(c)	Sets $3x + 4 = -\frac{k}{x} + k \Rightarrow 3x^2 + (4-k)x + k = 0$ Attempts use $b^2 - 4ac = 0$ to find the critical values Uses $b^2 - 4ac < 0$ and selects inside region for critical values $10 - 2\sqrt{21} < k < 10 + 2\sqrt{21}$	M1, A1 M1 dM1 A1 (5) (10 marks)

(a)

M1 Negative reciprocal shape (top left/bottom right sections) in any position on a set of axes with no clear vertical or horizontal 'overlaps'.

A1 Correct sketch appearing in quadrants 2 and 4 only. Condone slips of the pen at the ends as long as the graph does not curve back on itself. Ignore any scaling on the axes.



This scores M1A1. We condone these as they are not clear turning points and are regarded as slips of the pen.

(b) Note on EPEN it is M1B1A1. We are marking this B1ftB1B1

B1ft For attempting to translate their graph from part (a) up (or a correct graph if (a) is incorrect)

B1 For a correct intercept stated (or allow $1, \frac{k}{k}$ or $(1,0)$ marked on the correct axis with the curve passing through this) OR $y = k$ stated. Do not allow just k to be marked on the line of the asymptote.

B1 For a correct intercept stated (or allow $1, \frac{k}{k}$ or $(1,0)$ marked on the correct axis with the curve passing through this) AND $y = k$ stated. Do not allow just k to be marked on the line of the asymptote.

Note if they have a contradiction between what is stated and what is written on the graph then the graph takes precedence.

(c)

M1 Attempts to set $3x + 4 = -\frac{k}{x} + k$ and proceeds to a 3TQ in x on one side of the equation but terms do not need to be collected ($=0$ may be omitted).

A1 Correct quadratic seen $3x^2 + (4 - k)x + k = 0$ with the x terms collected together or implied by their values for a , b and c

M1 Attempts to use their values in $b^2 - 4ac \dots 0$ ($= k^2 - 20k + 16$) to find the critical values where ... is = or an inequality.

Dependent upon having $a = 3$ with both b and c expressions in k .

The solution of the quadratic in k ($= 10 \pm 2\sqrt{21}$) must be by allowable methods but allow decimal answers. As a minimum a quadratic in k must have been produced but it is acceptable to then state the critical values from their calculator. awrt19.2 / awrt0.8. You may need to check these on your calculator using their values of a , b and c .

dM1 Uses $b^2 - 4ac < 0$ OR $b^2 - 4ac \leq 0$ and selects inside region for their critical values. It is dependent upon both previous M's. Condone if a different variable to k is used.

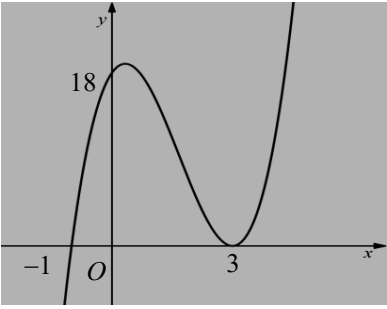
A1 Accept any of $10 - 2\sqrt{21} < k < 10 + 2\sqrt{21}$ or exact equivalent expressions such as $10 - \sqrt{84} < k < 10 + \sqrt{84}$ or Allow $10 + 2\sqrt{21} > k > 10 - 2\sqrt{21}$ or expressions such as $\{k : 10 - 2\sqrt{21} < k < 10 + 2\sqrt{21}\}$,

Must be in terms of k (Not x or any other variable)

ALLOW ' $10 - 2\sqrt{21} < k$ AND $k < 10 + 2\sqrt{21}$ ' (or equivalent) but

DO NOT ALLOW ' $10 - 2\sqrt{21} < k$ OR $k < 10 + 2\sqrt{21}$ '

Question Number	Scheme	Marks
7. (a)	$2x - 3\sqrt{x} - 5 = 9 \Rightarrow 2x - 3\sqrt{x} - 14 = 0$ and treats as quadratic equation $\Rightarrow (2\sqrt{x} - 7)(\sqrt{x} + 2) = 0 \Rightarrow (\sqrt{x} =) \frac{7}{2}, (-2)$ $\Rightarrow x = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$	M1 A1 dM1 A1 (4)
(b)	$(f'(x) =) 2 - \frac{3}{2}x^{-\frac{1}{2}}$	B1

Question Number	Scheme	Marks
6.(a)		M1
	Positive cubic shape anywhere with 1 maximum and 1 minimum	
	Positive cubic shape that at least reaches the x-axis at $x = -1$ and with a minimum on the x-axis at $x = 3$	A1
	y intercept at 18. Must correspond with their sketch	B1
	For the intercepts allow as numbers as above or allow as coordinates e.g. (18, 0), (0, -1), (0, 3) as long as they are marked in the correct place.	
		(3)
(b)	E.g. $(2x+2)(x^2-6x+9) = \dots$	M1
	$= 2x^3 - 10x^2 + 6x + 18$	A1 A1
		(3)
(c)	$(f'(x) =) 6x^2 - 20x + 6$	B1ft
	$f'\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right)^2 - 20\left(\frac{1}{3}\right) + 6$	M1
	$f'\left(\frac{1}{3}\right) = 0$	A1
	$y = \frac{512}{27}$	A1
		(4)
		(10 marks)

(a)

M1 Correct shape for a $y = +x^3$ graph. Do not be too concerned if the “ends” become vertical or even go beyond the vertical slightly. Condone with no axes and condone cusp like appearance for the turning points.

A1 $y = +x^3$ shape, intersects (or at least reaches the x-axis) at -1 , minimum at $x = 3$ but must not stop or cross at $x = 3$

B1 y intercept at 18

You can ignore the position of the maximum i.e. it may be to the left of or right of or on the y-axis.

(b) Mark (b) and (c) together.

M1 Attempts to multiply out.

E.g. Look for an attempt to square $(x-3)$ to obtain $x^2 \pm 6x \pm 9$ and then an attempt to multiply by $(x+1)$ or $(2x+2)$ or an attempt to multiply $(x+1)$ or $(2x+2)$ by $(x-3)$ and then multiply the result by $(x-3)$
 Condone slips e.g. attempting $(2x+1)(x-3)^2$ but expect to see an expression of the required form



A1 Two correct terms of $2x^3 - 10x^2 + 6x + 18$

A1 Fully correct $2x^3 - 10x^2 + 6x + 18$. (Ignore any spurious “= 0”)

Special case: if they obtain $2x^3 - 10x^2 + 6x + 18$ but then attempt to “simplify” as e.g.

$f(x) = x^3 - 5x^2 + 3x + 9$ then score A1A0 **but note that all marks are available in (c) in such cases.**

(c)

B1ft Correctly differentiates their $2x^3 - 10x^2 + 6x + 18$. Allow follow through but only from a 4 term cubic.

Allow use of product rule e.g.

$$f(x) = 2(x+1)(x-3)^2 \rightarrow f'(x) = 2(x-3)^2 + 4(x+1)(x-3)$$

You can condone poor notation so just look for the correct or correct ft expression.

M1 Attempts $f'\left(\frac{1}{3}\right)$

A1 Correctly achieves $f'\left(\frac{1}{3}\right) = 0$. Must follow a correct derivative but allow this mark if

they have e.g. divided their derivative by 2 before substituting or e.g. if they have divided their expanded $f(x)$ by 2 before differentiating so they have

$f'(x) = 3x^2 - 10x + 3$ or if only their constant in their expansion in (b) is incorrect

The working may be minimal so allow e.g. $f'\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right)^2 - 20\left(\frac{1}{3}\right) + 6 = 0$ or even

$f'\left(\frac{1}{3}\right) = 0$ on its own following a correct derivative. Just look for the answer of 0 (e.g. they may call it y).

A1 $y = \frac{512}{27}$ or exact equivalent. All previous marks must have been scored in (c).

Acceptable alternative to show $f'\left(\frac{1}{3}\right) = 0$:

$$f'(x) = 6x^2 - 20x + 6 = 0 \Rightarrow (6x-2)(x-3) = 0 \Rightarrow f'(x) = 0 \text{ at } x = \frac{1}{3} \Rightarrow f'\left(\frac{1}{3}\right) = 0$$

Score M1 for attempting to solve quadratic (usual rules) and A1 for deducing

$$f'\left(\frac{1}{3}\right) = 0$$

A1: $kx^2 + (-4 - 2k)x + 9 = 0$ with terms collected. This may be implied by later work, e.g. $a = \dots$, $b = \dots$, $c = \dots$ listed or used. The “=0” may be implied.

dM1: Attempts use $b^2 - 4ac \dots 0 \Rightarrow (-4 - 2k)^2 - 4k \times 9 \dots 0$ with their coefficients to set up a quadratic inequality in k . Must be the correct direction of inequality but allow $>$ or $\dots$. Use of $b^2 - 4ac = 0$ will score dM0 unless clearly recovered before stating that answer.

A1*: Achieves the given answer of $k^2 - 5k + 4 \dots 0$ with no errors and sufficient working shown – the bracket should be expanded before the final answer is reached. A0 if x used in place of k .

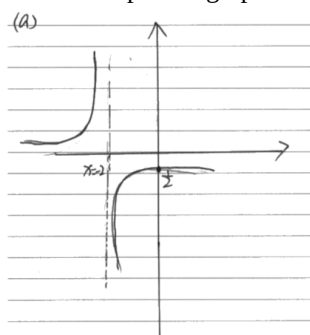
(b)(ii)

M1: Solves to find the critical values – may be implied by sight of 1 and 4 used in their answer but must have two values from a valid attempt to solve. Allow from attempts at their “ $b^2 - 4ac$ ”...0 with any inequality or equality.

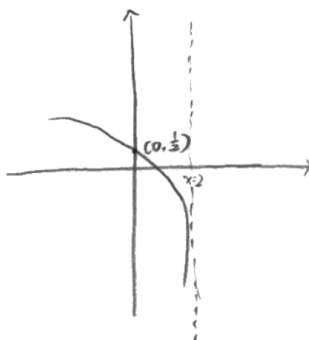
A1: $k, 1, k \dots 4$ Accept with “and” or “or” used. Accept interval notation

$(k \in) (-\infty, 1] \cup [4, \infty)$ - must be correct union if set notation used. Must be the final answer, but allow if recovery from “incorrect” inequalities when solving for the critical values. A0 if x used instead of k . Note 4,, k ,, 1 is A0.

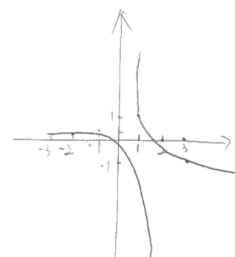
Some examples of graphs:



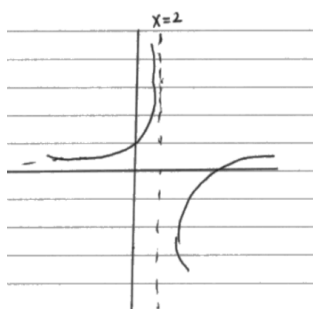
M1A0B0 intent clear for first M. Wrong position, A0, incorrect asymptote and intercept, B0.



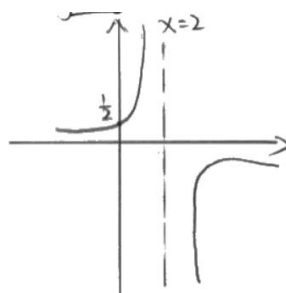
M0A0B1 Only one branch, but asymptote and intercept correct.



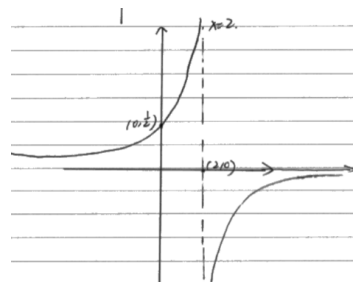
M1A0B0 this intent is acceptable for the shape, but not fully correct as “overlapping” so A0. Intercept/asymptotes incorrect.



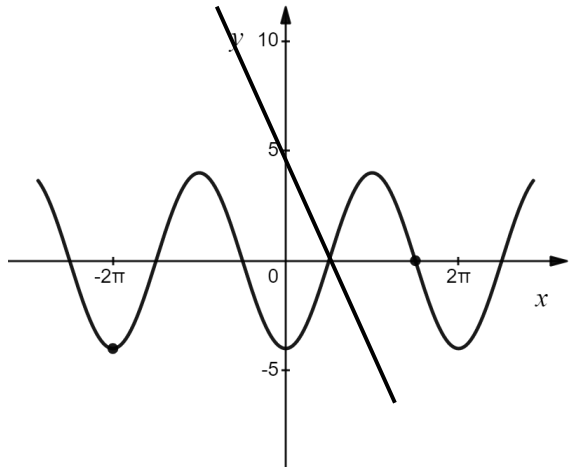
M1A0B0 Intent clear for the First M, but clearly bending back on itself, so A0. No intercept given.



M1A0B1 Intent clear for first M, clearly bends down in fourth quadrant, A0. Asymptote and intercept correct.



M1A1B1 Shape and position correct, the slight “pen flick” is condoned. Intercept and vertical asymptote correctly labelled.

Question Number	Scheme	Marks
7 (a)(i)	$P = (-2\pi, -4)$	B1, B1
(ii)	$Q = \left(\frac{3\pi}{2}, 0\right)$	B1
(b) (i)	$k = 7$	(3) B1
(ii)	$(2\pi, 3)$	M1 A1 (3)
(c)	 One root as there is one point of intersection	M1A1 (2) (8 marks)

NB: Watch for the answers being stated next to the question rather than in the answer space.

(a)(i) Allow all marks in (a) if P and Q are mislabelled but the coordinates are correct.

B1: For one of $(-2\pi, -4)$ Condone/allow one of $(-360^\circ, -4)$ Condone for this B mark if the coordinates are reversed.

B1: For both of $(-2\pi, -4)$ (in correct order).

(a)(ii)

B1: $(Q =) \left(\frac{3\pi}{2}, 0\right)$

(b)(i)

B1: For $k = 7$

(b)(ii)

M1: For one of $(2\pi, 3)$ **or** follow through on the negative of their x coordinate of P or their $-4 +$ their k Condone/allow one of $(360^\circ, 3)$ and condone for the M if the coordinates are given in the wrong order.

A1: $(2\pi, 3)$ Allow $(360^\circ, 3)$ if B marks in (a) were lost due solely due to working in degrees.

(c)

M1: Draws a line with negative gradient passing through (0, 5) and cutting the graph at least once, with a consistent number of solutions for their sketch stated.

A1: Correct line drawn (intersecting at graph on the x axis) with correct deduction (one root) and reason (intersects only once).

Note use of degrees only will score a maximum of B1B0 B0 B1 M1A1 M1A0, though if the

$\frac{10}{\pi}$ is converted to degrees resulting in a “correct” graph, send to review for consideration.



Question Number	Scheme	Marks
8 (a)	States or implies that gradient of tangent is 24 Solves $f'(3) = 24$ to find k . E.g. $4 \times 3^2 + k \times 3 + 3 = 24 \Rightarrow k = \dots$ $3k + 39 = 24 \Rightarrow k = \frac{24-39}{3} = -5$ *	B1 M1 A1* (3)
(b)	$f'(x) = 4x^2 - 5x + 3 \Rightarrow f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x + c$ Substitutes $x=3, y = -\frac{3}{24} + 5$ into $y = f(x)$ to find "c" $f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x - \frac{141}{8}$	M1 A1 dM1 A1 (4) (7 marks)

Mark this question as a whole, some may try finding $f(x)$ first in part (a), so award positively.

(a)

B1: States or implies that gradient of tangent (to the curve at P) is 24. This may occur within the use of the perpendicular condition, e.g. $f'(3) \times -\frac{1}{24} = -1$ implies B1.

M1: Attempts to solve $f'(3) = \pm 24$ or $\frac{1}{24}$ (or equivalent equations) to find k . Accept as minimum at least one unsimplified equation in k before the given answer is stated. Alternatively, allow for substitution of $k = -5$ and $x = 3$ into $f'(x)$ and simplifying to a value to compare with the gradient (verification method).

A1*: Shows that $k = -5$ with at least one correct equation with simplified terms (not necessarily all gathered) and no incorrect work shown before stating the printed answer. By verification all steps must be correct with conclusion given.

(b)

M1: Integrates to a form $px^3 + qx^2 + rx$, with at least $p, q \neq 0$ with no requirement for $+c$. Allow with k used in place of -5 for this mark.

A1: $f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x$ with no requirement for $+c$

dM1: Substitutes $x=3, y = -\frac{3}{24} + 5$ into $y = f(x)$ and finds a value for "c". Must have had a constant of integration.

A1: $\left(f(x) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 3x - \frac{141}{8}\right)$ Accept with $y = \dots$ instead of $f(x)$, or even with no label at all.

Question Number	Scheme	Marks
9 (a)	$x \dots -5$	B1 (1)
(b)	$f(x) = (x+5)(3x^2 - 4x + 20) = 3x^3 + 11x^2 + 100$ $f'(x) = 9x^2 + 22x$	M1 M1 A1cso (3)
(c)	Finds $f'(-4) = 9 \times (-4)^2 + 22 \times -4 = (56)$ Sets $f'(x) = "9x^2 + 22x" = "56"$ $9x^2 + 22x - 56 = 0 \Rightarrow x = \frac{14}{9}, (-4)$	M1 dM1 ddM1 A1cso (4)
(d)(i)	$(-1, 84)$	B1
(ii)	$(-4, 336)$	B1
		(2) (10 marks)

Note: mark the question as a whole – do not be concerned if there is incorrect part labelling.

(a)

B1: Correct range of values given, $x \dots -5$ or exact equivalent in set notation. Must be clearly identified as the answer, do not isw if a different range is given later but allow if $x = -5$ is stated as well as the range.

(b)

M1: Attempts to multiply out to form a cubic (which may be unsimplified), achieving at least $3x^3 + \dots \pm 100$ with at least one x or x^2 term between, and allow if there are extra constant terms between too.

M1: Differentiates to form a quadratic. Power decreased by 1 at least twice.

A1cso: $f'(x) = 9x^2 + 22x$ Must have come from a correct expansion.

The product rule may be attempted:

M1: For $f'(x) = (1)(3x^2 - 4x + 20) + (x+5)(Ax + B)$

M1: Expands and simplifies to a quadratic as long as a sum of terms was attempted.

A1: $f'(x) = 9x^2 + 22x$

(c)

M1: Finds $f'(-4)$ for their $f'(x)$

dM1: Sets $f'(x) = "9x^2 + 22x" = "56"$ with their derivative and value.

ddM1: Solves their $"9x^2 + 22x = 56"$ which must be a 3 term quadratic. May be implied by the correct answer for their equation.



A1cso: For $x = \frac{14}{9}$ from fully correct work. Ignore references to $x = -4$ if left in but A0 if any erroneous extra solutions are given. The derivative must have been correct. Note the same answer will follow if they have a constant term in their (otherwise correct) derivative.

(d)

(i) B1: $(-1, 84)$ (ii) B1: $(-4, 336)$

(a)

M1: Writes down a correct sine rule and attempts to find angle CAO . Alternatively, writes down a correct cosine rule and attempts to find AC :

$$15^2 = 17^2 + AC^2 - 2 \times 17 \times AC \cos 0.6 \Rightarrow AC = \frac{34 \cos 0.6 \pm \sqrt{(34 \cos 0.6)^2 - 4 \times 64}}{2} = \dots$$

Allow if degrees mode is used. Here angle $CAO = 0.680$

Watch for cases where the inverse sine is not taken, which give 0.6399. These will score M0dM0A0.

dM1: Full method to find angle COA . As scheme or in the Alt they may proceed to use the sine rule with their AC (note they should get 1.293... from this method).

A1*: Complete and correct proof. In the Alt the selection of the positive root must be justified (from diagram angle COA is obtuse so...) and justification of the correct angle seen (ie $\pi - 1.293...$ before the 1.847...). Allow as long as all values are to at least 3dp throughout (if seen).

There may be attempts via verification. Allow

M1: For a correct attempt at the sine (or cosine) rule as a part of their check.

dM1: For a full process to check the angle, e.g.

$$\text{angle } CAO = \pi - 0.6 - 1.847 = 0.69459... \Rightarrow \angle ACO = \arcsin\left(\frac{15 \sin 0.69459...}{17}\right) = 0.600 \checkmark$$

A1*: If all is correct with a minimal conclusion.

Note: for (b) and (c) if they use their 1.847 rather than the given value in the question, allow the M's for correct method with their value.

(b)

M1: Correct attempt at one relevant area, allowing the minor sector as relevant. See scheme.

dM1: Complete method to find both areas and adds.

A1: Awrt 622 m² including the units

(c)

M1: Attempts major or minor arc length AB OR length AC or its square, e.g. $r\theta = 15 \times \theta$ where

$$\theta = (2\pi - 1.847) \text{ or just } 1.847 \text{ OR attempts } 15^2 + 17^2 - 2 \times 15 \times 17 \cos(1.847) \text{ condoning if it is called } AC.$$

$$\text{There may be alternative approaches to these (e.g. sine rule } AC = \frac{15 \sin 1.847}{\sin 0.6}).$$

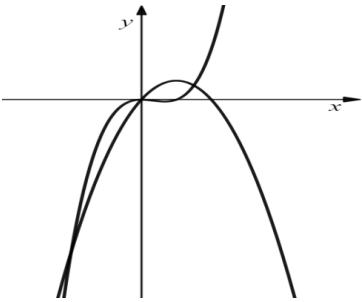
For each approach allow if correct formula is quoted but they subsequently forget to apply the cosine or sine to the angle.

dM1: Attempts both major arc length and length AC ie $r\theta = 15 \times (2\pi - 1.847)$ AND

$$AC = \sqrt{15^2 + 17^2 - 2 \times 15 \times 17 \cos(1.847)} \text{ or equivalent approaches. Must correctly apply the trig ratios for this mark.}$$

A1: Awrt 94.1 m. Should include units unless the A mark in (b) was penalised only for lack of units (ie penalise only the first instance of incorrect or missing units).



Question Number	Scheme	Marks
7 (a)	States or implies that $f(x) = \lambda x^2(x-4)$ Attempts to find λ . E.g. $120 = \lambda 10^2(10-4) \Rightarrow \lambda = \dots$ $\{f(x) = \} 0.2x^2(x-4)$	M1 dM1 A1 (3)
(b)	 "Upside down" parabola Passing through O and +ve x axis > 4	M1 A1 (2)
(c)	Sets $1.2x(8-x) = \text{their } 0.2x^2(x-4)$ $1.2x(8-x) = 0.2x^2(x-4) \Rightarrow x(x^2 + 2x - 48) = 0$ or $\Rightarrow x^3 + 2x^2 - 48x = 0$ $\Rightarrow x = \dots, \dots, (0)$ For $x = -8, 0, 6$ OR one of $(-8, -153.6), (6, 14.4)$ All of $(-8, -153.6), (6, 14.4), (0, 0)$ $\left\{ \left(-8, -\frac{768}{5}\right), \left(6, \frac{72}{5}\right), (0, 0) \right\}$	B1ft M1 dM1 A1 A1 (5) (10 marks)

(a)

M1: States or implies that $f(x) = \lambda x^2(x-4)$. Allow with $\lambda = 1$ (or any other constant). Credited when first seen, but must use this expression to access the dM mark.

dM1: Attempts to find λ . E.g. $120 = \lambda 10^2(10-4) \Rightarrow \lambda = \dots$

A1: $\{f(x) = \} 0.2x^2(x-4)$ o.e. Allow with $(x-0)^2$

Alt: May see attempts at using $f(x) = ax^3 + bx^2 + cx + d$

M1: States or implies $f(x) = ax^3 + bx^2 + cx + d$ AND uses $(0,0)$ on curve to deduce $d = 0$

dM1: Full method to find the remaining constants, so $f'(0) = 0 \Rightarrow c = 0$ and

$f(4) = 0 \Rightarrow 64a + 16b = 0 \Rightarrow b = -4a$ then $f(10) = 120 \Rightarrow 1000a + 100b = 120 \Rightarrow 50a + 5b = 6 \Rightarrow a = \dots, b = \dots$

They must set up sufficient equations to be able to solve for all the constants.

A1: For $f(x) = 0.2x^3 - 0.8x^2$ o.e.

Note, do not be concerned if the $f(x) = \dots$ is missing, mark the expression.



(b)

Note: If there are multiple attempts with no clearly designated answer, Diagram 1 takes precedence if not crossed out. If Diagram 1 is not used, or is crossed out and no clear statement of answer is given, then an answer in the answer space takes precedence over an answer on Figure 3.

M1: Correct shape, a $\cap$ shape parabola in any position. Be tolerant with the shape, must at least touch the x -axis.

A1: Correct shape and position, a $\cap$ shape parabola passing through O and the +ve x axis to the right hand side of $x = 4$. The graph must **clearly** extend into quadrants 3 and 4, not stop at the x -axis.

Ignore any coordinates on the graph, and don't be concerned if the intersection in quadrant 3 is not shown.

(c)

B1ft: Sets $1.2x(8-x) = \text{their } 0.2x^2(x-4)$. It is for a "correct" ft equation on their part (a) as long as their answer to part (a) is a cubic expression.

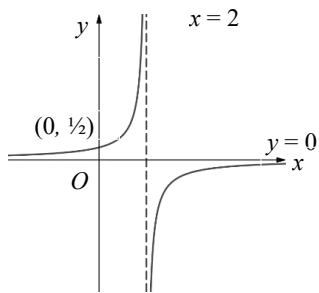
M1: Valid attempt to solve an equation of the form $1.2x(8-x) = \text{cubic}$ with their cubic (and their quadratic $1.2x(8-x)$ if they incorrectly try to simplify this) to reach at least a gathered cubic with no constant, or a quadratic in x . The x may be cancelled, e.g. $1.2x(8-x) = 0.2x^2(x-4) \Rightarrow \text{Quadratic equation in } x$. Do not allow solutions which reach a cubic with non-zero constant and simply state answers from a calculator. This is for progressing via algebra to a point at which a quadratic (possibly tacit in a cubic with no constant) can be identified.

dM1: Attempts to solve the resulting (possibly tacit) 3 term quadratic equation, usual rules. Allow solutions from calculator as long as a quadratic can be identified in their equation. You may need to check their values (both must be correct).

A1: Either all three x coordinates stated as answers (the 0 must be present but may have been given earlier), or one correct coordinate pair from $(-8, -153.6), (6, 14.4)$

A1: All three of $(-8, -153.6), (6, 14.4), (0, 0)$, accept as $x = \dots, y = \dots$ for each as long as they are paired correctly. The $(0,0)$ may be stated earlier, so check the whole solution if it is not listed at the end.



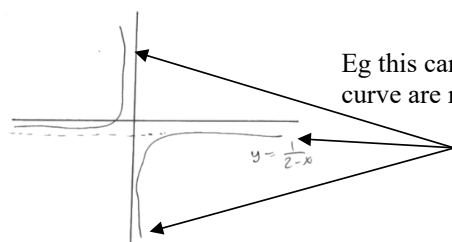
Question	Scheme	Marks
7(a)	 B1 Negative reciprocal shape B1 Intercept at $\left(0, \frac{1}{2}\right)$ B1 $x=2, y=0$	B1B1B1
		(3)
(b)	$4x+k = \frac{1}{2-x} \Rightarrow (4x+k)(2-x)=1 \Rightarrow 8x+2k-4x^2-kx-1(=0) \text{ oe}$ $4x^2 + (k-8)x + 1 - 2k = 0$ $a=4, b=k-8, c=1-2k \quad \text{or} \quad a=-4, b=8-k, c=2k-1$ $(k-8)^2 - 4 \times 4(1-2k)(>0) \text{ oe}$ $k^2 - 16k + 64 - 16 + 32k > 0 \Rightarrow k^2 + 16k + 48 > 0^*$	M1 A1 M1 A1*
		(4)
(c)	$k^2 + 16k + 48 = 0 \Rightarrow (k+12)(k+4) = 0 \Rightarrow k = \dots$ $k = -12, -4$ $k < -12 \text{ or } k > -4$	M1 A1 M1A1
		(4)
		(11 marks)

Notes

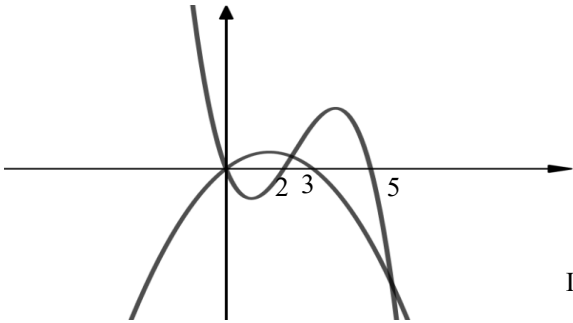
(a)

B1 Negative reciprocal shape (top left/bottom right sections) in any position on a set of axes with no clear vertical or horizontal ‘overlaps’.

Condone slips of the pen as long as there is not a clear turning point, but it should not curve back on itself. If there are several attempts, then mark the last one. If there are contradictions between points stated in the text and any on the graph, then the graph takes precedence.



Eg this can score B1 for the correct shape. The ends of their curve are regarded as a slip of the pen

Question Number	Scheme	Marks
11. (a)	 $\cap$ shaped quadratic Intercepts at O and 3 -ve cubic Intercepts at O, 2 and 5	B1 B1 B1 B1 (4)
(b)	Sets $x(x-2)(5-x) = x(3-x)$ $3x - x^2 = -x^3 + 7x^2 - 10x \Rightarrow \pm(x^3 - 8x^2 + 13x) (=0)$ OR $\pm x\{(x-2)(5-x) - (3-x)\} (=0)$ Proceeds to $x(x^2 - 8x + 13) = 0$ *	M1 dM1 A1* (3)
(c)	Solves $x^2 - 8x + 13 = 0 \Rightarrow x = 4 \pm \sqrt{3}$ Substitutes $x = "4 - \sqrt{3}"$ into $y = x(3-x)$ oe $y = (4 - \sqrt{3})(-1 + \sqrt{3}) = -4 + 4\sqrt{3} + \sqrt{3} - 3 = \dots$ $y = -7 + 5\sqrt{3}$	M1 A1 M1 M1 A1 (5) (12 marks)

(a) For both parts they must have a graph

(i)

B1 $\cap$ shaped quadratic appearing anywhere on the graph. This is for the general shape so do not be concerned with any parts which appear linear.

B1 A quadratic which crosses the x -axis at O and 3. Accept a mark of 3 on the x -axis. The origin does not need to be labelled as a point of intersection.

(ii)

B1 -ve cubic appearing anywhere on the graph with a maximum and a minimum. This is for the general shape so do not be concerned with any parts which appear linear.

B1 A cubic which crosses the x -axis at O , 2 and 5. Accept 2 and 5 marked on the x -axis. The origin does not need to be labelled as a point of intersection.

It is not a requirement that the curves meet in quadrant 4. The relative heights of the maximum points are not important either. Points of intersection with the coordinate axes may be listed separately. Condone the lack of brackets but the x and y coordinates must be the correct way round. They cannot simply state eg $x = 2, x = 5$

(b)

M1 Sets the equations equal to each other

dM1 Multiplies out and collects terms on one side (unsimplified). Condone errors in multiplying out and slips in collecting like terms but x must be a factor of each term. Condone invisible brackets for this mark and condone the absence of “=0”

Alternatively takes the factorised forms to one side and factors out the x term.

A1* Proceeds to $x(x^2 - 8x + 13) = 0$ with no errors including brackets. As a minimum you must see $(x - 2)(5 - x)$ being multiplied out, terms being collected on one side and a factor of x being taken out.

(c)

M1 Solves $x^2 - 8x + 13 = 0$ by completing the square or formula. Apply general marking principles for quadratics for this mark. Their solutions do not need to be exact for this mark.

A1 Either (or both) of $(x =) 4 \pm \sqrt{3}$ (oe but must be of the form $\frac{a \pm \sqrt{b}}{c}$ or simplified further)

M1 Substitutes $x = 4 - \sqrt{3}$ or their **lower** value of x into either equation to find y . This mark can be awarded if they had rounded decimal solutions to their quadratic. If they substitute both values in then this mark can still be awarded. You may need to check their y value on a calculator to imply this method mark if their x value is incorrect.

M1 Evidence of using the rules of surds to form a y coordinate that is exact and simplified. They must show evidence of working with surds before simplifying to two terms of the form $d + f\sqrt{g}$. Eg one of the bold terms from $y = (a - \sqrt{b})(c + \sqrt{b}) = ac + a\sqrt{b} - c\sqrt{b} - b$ would be sufficient. If no working shown then M0 and A0 will follow this.

A1 $(4 - \sqrt{3}, -7 + 5\sqrt{3})$ or exact equivalent. This must be the only coordinate stated as their final answer.

Note that the question stated using algebra and showing your working. If they simply state the solutions of the quadratic then they can only get M0A0M1M1A0.

If they simply state the solutions of the quadratic and show no surd work then the maximum they may be able to get is M0A0M1M0A0.



Question Number	Scheme	Marks
6.	$(x+5)(3x+2)(2x-5) = (3x^2+17x+10)(2x-5) = \dots$	M1
	$= 6x^3 - 15x^2 + 34x^2 - 85x + 20x - 50$ $\left(= 6x^3 + 19x^2 - 65x - 50 \right)$	A1
	$6x^3 + 19x^2 - 65x - 50 = 3x^2 - 33x - 50$ $\Rightarrow 6x^3 + 16x^2 - 32x = 0$	dM1
	$2x(3x-4)(x+4) = 0$	ddM1
	$x = -4, 0, \frac{4}{3}$	A1
		(5)
		(5 marks)
Notes		
M1: For an attempt to multiply out the $(x+5)(3x+2)(2x-5)$. Must obtain terms in x^3, x^2, x and a constant. A1: $6x^3 + 19x^2 - 65x - 50$ but may be left unsimplified e.g. $6x^3 - 15x^2 + 34x^2 - 85x + 20x - 50$ Award this mark if a correct unsimplified expansion is seen and errors are subsequently made when collecting terms. dM1: Sets their $6x^3 + 19x^2 - 65x - 50 = 3x^2 - 33x - 50$, collects terms and proceeds to a cubic expression $= 0$. This may be implied if they cancel the 50's and then divide through by x to obtain a quadratic expression $= 0$. The "$= 0$" may be implied by subsequent work. Depends on the previous method mark. ddM1: Attempts to solve a cubic of the form $px^3 + qx^2 + rx = 0$, $p, q, r \neq 0$ by non-calculator methods This must involve dividing or factorising out x (or e.g. $2x$) followed by an attempt to solve a quadratic of the form $px^2 + qx + r = 0$, $p, q, r \neq 0$ by non-calculator methods – see General Guidance. Just quoting the quadratic formula followed by roots written down is M0 Depends on both previous method marks. A1: $x = -4, 0, \frac{4}{3}$. Allow exact equivalents for $\frac{4}{3}$ including 1.3 Note that the $x = 0$ may appear earlier in their solution so look out for this. Guidance for the ddM1A1 marks: $6x^3 + 16x^2 - 32x = 0 \Rightarrow 6x^2 + 16x - 32 = 0 \Rightarrow (3x-4)(x+4) = 0 \Rightarrow x = \frac{4}{3}, -4$ Scores ddM0A0 $6x^3 + 16x^2 - 32x = 0 \Rightarrow 6x^2 + 16x - 32 = 0 \Rightarrow 2(3x-4)(x+4) = 0 \Rightarrow x = \frac{4}{3}, -4$ Scores ddM1A0 $6x^3 + 16x^2 - 32x = 0 \Rightarrow x(6x^2 + 16x - 32) = 0 \Rightarrow x\left(x - \frac{4}{3}\right)(x+4) = 0 \Rightarrow x = 0, \frac{4}{3}, -4$ Scores ddM0A0 $3x^3 + 8x^2 - 16x = 0 \Rightarrow (3x-4)(x+4) = 0 \Rightarrow x = \frac{4}{3}, -4$ Scores ddM0A0		

Question	Scheme	Marks
1	$\int \left(10x^4 - \frac{3}{2x^3} - 7 \right) dx = 2x^5 + \frac{3}{4}x^{-2} - 7x + c$	M1A1A1
		(3)
		Total 3

Notes:

M1: For increasing one of the powers of x by 1 e.g. $x^4 \rightarrow \dots x^5$ or $x^{-3} \rightarrow \dots x^{-2}$ or $7 \rightarrow \dots 7x$

A1: Any **two** correctly integrated terms, unsimplified or simplified.

Allow e.g. $\frac{10}{4+1}x^{4+1}$, $-\frac{3}{-4}x^{-3+1}$, $-7x^1$

A1: $2x^5 + \frac{3}{4}x^{-2} - 7x + c$ all correct and simplified in one expression and including “+ c ”.

Allow correct simplified equivalents for $\frac{3}{4}x^{-2}$ e.g. $0.75x^{-2}$, $\frac{3}{4x^2}$ but do **not** allow $-7x^1$ for $-7x$ or $\frac{2}{1}$ for 2

Award this mark once a correct simplified expression is seen and apply isw if necessary.

Condone poor notation e.g. spurious integral signs, “dx”, $\frac{dy}{dx} = \dots$ and look for the correct expression.

Question	Scheme	Marks
2(i)(a)	$2^{n+3} = 2^n \times 2^3 = 8m$	B1
		(1)
(b)	$16^{3n} = (2^4)^{3n}$	M1
	$= 2^{12n} = (2^n)^{12} = m^{12}$	A1
		(2)
Notes: (i)(a) B1: For $8m$ (Condone 8M). Do not allow 2^3m . (b) M1: For writing 16^{3n} correctly as an expression involving a power of 2. Examples: $(2^4)^{3n}$, $(2^2)^{6n}$, $(2^{3n})^4$, $(2^n)^{12}$, $(2^{12})^n$, 2^{12n} A1: For m^{12} (Condone M^{12}) Correct answer only scores both marks.		

(ii)	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow x\sqrt{3}-x=3+\sqrt{3}\Rightarrow x(\sqrt{3}-1)=3+\sqrt{3}\Rightarrow x=...$	M1
	$=\frac{3+\sqrt{3}}{\sqrt{3}-1}\times\frac{\pm(\sqrt{3}+1)}{\pm(\sqrt{3}+1)}$	M1
	$=\frac{\pm(4\sqrt{3}+6)}{\pm(3-1)}=3+2\sqrt{3}$	A1
		(3)

Notes:**In this part, the A1 depends on both M marks**

(ii)

M1: Collects x terms to one side, factorises and makes x the subjectIt is for processing the given equation and reaching $x = \frac{\alpha + \beta\sqrt{3}}{\gamma + \delta\sqrt{3}}$, $\alpha, \beta, \gamma, \delta \neq 0$ **M1:** Correct attempt to rationalise their denominator.Not formally dependent but requires $\frac{\dots}{\gamma + \delta\sqrt{3}} = \frac{\dots}{\gamma + \delta\sqrt{3}} \times \frac{\gamma - \delta\sqrt{3}}{\gamma - \delta\sqrt{3}}$ or equivalent.Sight of e.g. $\frac{3+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ is sufficient for this mark **but may be implied** by e.g.

$$\frac{3+\sqrt{3}}{\sqrt{3}-1} = \frac{3\sqrt{3}+3+3+\sqrt{3}}{3-1}$$

A1: For $3+2\sqrt{3}$ with at least one intermediate step e.g.

$$\frac{3+\sqrt{3}}{\sqrt{3}-1} = \frac{3\sqrt{3}+3+3+\sqrt{3}}{3-1} \text{ or } \frac{3+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4\sqrt{3}+6}{2} \text{ or } \frac{3+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{3\sqrt{3}+3+3+\sqrt{3}}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

Allow $2\sqrt{3}+3$. It is for this expression and not for just $p=3$, $q=2$ unless $3+2\sqrt{3}$ is seen.**Example of insufficient work:**

$$\begin{aligned} x\sqrt{3}-3=x+\sqrt{3}\Rightarrow x\sqrt{3}-x=3+\sqrt{3}\Rightarrow x(\sqrt{3}-1)=3+\sqrt{3}\Rightarrow x &= \frac{3+\sqrt{3}}{\sqrt{3}-1} \\ &= \frac{3+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 3+2\sqrt{3} \end{aligned}$$

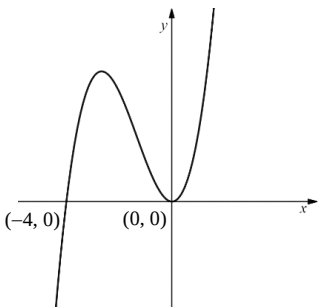
Scores M1M1A0**Note that:**

$$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow x\sqrt{3}-x=3+\sqrt{3}\Rightarrow x(\sqrt{3}-1)=3+\sqrt{3}\Rightarrow x = \frac{3+\sqrt{3}}{\sqrt{3}-1} = 3+2\sqrt{3}$$

Scores M1M0A0

Alternatives for part (ii)

(ii) ALT 1	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow(p+q\sqrt{3})\sqrt{3}-3=p+q\sqrt{3}+\sqrt{3}\Rightarrow p\sqrt{3}+3q-3=p+\sqrt{3}+q\sqrt{3}$ $\Rightarrow p=q+1, 3q-3=p$	M1
	$\Rightarrow p=q+1, 3q-3=p\Rightarrow p=..., q=...$	M1
	$x=3+2\sqrt{3}$	A1
Notes: M1: Substitutes $x=p+q\sqrt{3}$, collects terms, compares coefficients and forms 2 equations in p and q It is for obtaining 2 equations of the form $\alpha p + \beta q = \gamma$, $\alpha, \beta, \gamma \neq 0$ M1: Solves simultaneously with evidence of algebra i.e. answers not just written down from a calculator. Not formally dependent but requires the solution of 2 equations of the form $\alpha p + \beta q = \gamma$, $\alpha, \beta, \gamma \neq 0$ A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just $p=3, q=2$ unless $3+2\sqrt{3}$ is seen.		
(ii) ALT 2	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow 3x^2-6\sqrt{3}x+9=x^2+2\sqrt{3}x+3$ $\Rightarrow 2x^2-8\sqrt{3}x+6=0$	M1
	$x^2-4\sqrt{3}x+3=0\Rightarrow x=\frac{4\sqrt{3}\pm\sqrt{(4\sqrt{3})^2-4\times1\times3}}{2}$	M1
	$\Rightarrow x=\frac{4\sqrt{3}\pm\sqrt{36}}{2}\Rightarrow x=2\sqrt{3}\pm3$ $x=2\sqrt{3}+3$	A1
Notes: M1: Squares both sides. Must obtain a 3 term quadratic expression on both sides and collect terms to one side. M1: Solves a 3TQ of the form $px^2+q\sqrt{3}x+r=0$ by a correct non-calculator method. A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just $p=3, q=2$ unless $3+2\sqrt{3}$ is seen. If any other answers are offered and clearly not rejected score A0.		
(ii) ALT 3	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow(x\sqrt{3}-3)(x-\sqrt{3})=(x+\sqrt{3})(x-\sqrt{3})$ $\Rightarrow\sqrt{3}x^2-6x+3\sqrt{3}=x^2-3\Rightarrow(\sqrt{3}-1)x^2-6x+3\sqrt{3}+3=0$	M1
	$(\sqrt{3}-1)x^2-6x+3\sqrt{3}+3=0\Rightarrow x=\frac{6\pm\sqrt{36-4(\sqrt{3}-1)(3\sqrt{3}+3)}}{2(\sqrt{3}-1)}$ $=\frac{6\pm\sqrt{36-24}}{2(\sqrt{3}-1)}=\frac{6\pm\sqrt{12}}{2(\sqrt{3}-1)}=\frac{3\pm\sqrt{3}}{\sqrt{3}-1}=\frac{3\pm\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$	M1
	$=\frac{3\sqrt{3}+3\pm3\pm\sqrt{3}}{2}=2\sqrt{3}+3, \sqrt{3}$ $x=2\sqrt{3}+3$	A1
Notes: M1: Multiplies both sides by e.g. $(x-\sqrt{3})$ and collects terms to one side. Must obtain a 3 term quadratic expression on lhs and a 2 or 3 term quadratic expression on rhs. M1: Solves a 3TQ of the form $(A+B\sqrt{3})x^2+Cx+D+E\sqrt{3}=0$, $A, B, C, D, E \neq 0$ by a correct non-calculator method and then attempts to rationalise the denominator as in the main scheme. A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just $p=3, q=2$ unless $3+2\sqrt{3}$ is seen. If any other answers are offered and clearly not rejected score A0.		
		Total 6

Question	Scheme	Marks
3(a)		B1B1B1
		(3)

Notes:

If there is any ambiguity regarding coordinates, the sketch takes precedence.

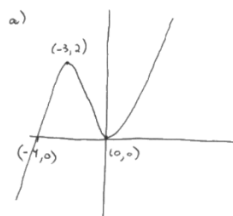
B1: Same shape translated parallel to the x -axis with no evidence **from the x -intercepts** (if labelled) that the transformation is anything other than a translation. So the x intercepts (if labelled) must be 4 units apart. Ignore any coordinates on or near the maximum.

B1: Minimum at the origin. The origin does not have to be labelled.

B1: Passes through $(-4, 0)$ or at least “reaches” this point from above or below.

Allow just -4 marked in the correct place and condone $(0, -4)$ as long as it is marked in the correct place.

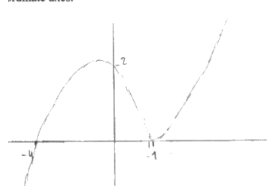
Allow to score away from the sketch as long as their curve passes through (or reaches) this point but must be identified correctly as $(-4, 0)$ and correspond to the sketch.

See examples below:

B1: Translation (the maximum at $(-3, 2)$ can be ignored)

B1: Minimum at the origin

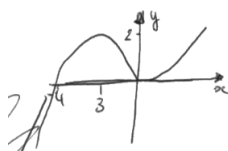
B1: Passes through $(-4, 0)$



B0: The x intercepts are contradictory for a translation (not 4 units apart)

B0: No minimum at the origin

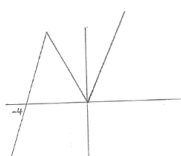
B1: Passes through $(-4, 0)$



B1: Translation

B1: Minimum at the origin (the intention is clear)

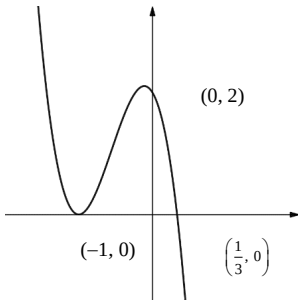
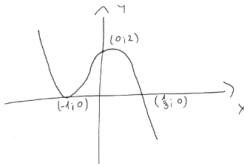
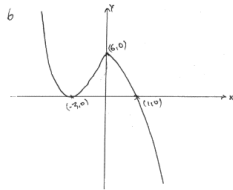
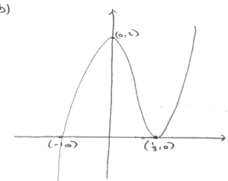
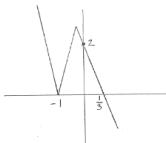
B1: Passes through $(-4, 0)$



B0: Incorrect shape

B1: Minimum at the origin

B1: Passes through $(-4, 0)$

(b)		B1B1B1
		(3)
		Total 6
Notes: If there is any ambiguity regarding coordinates, the sketch takes precedence. B1: Reflection in the y-axis. This requires a minimum on the negative x-axis and a maximum to the left of the y-axis in quadrant 2. If the maximum is on the y-axis score B0. The curve must continue upwards to the left of the y-axis to reach a maximum in quadrant 2. The curve must extend into quadrant 4 as shown. B1: Minimum at $(-1, 0)$. Allow just -1 marked in the correct place and condone $(0, -1)$ as long as it is marked in the correct place. Allow to score away from the sketch but must be identified correctly as $(-1, 0)$ and correspond to the sketch. B1: Passes through $(0, 2)$ and passes through or touches $\left(\frac{1}{3}, 0\right)$. For both points, allow just the intercepts marked in the correct place and condone $(2, 0)$ and/or $\left(0, \frac{1}{3}\right)$ as long as they are marked in the correct place. Allow to score away from the sketch but must be identified correctly as $(0, 2)$ and $\left(\frac{1}{3}, 0\right)$ and correspond to the sketch. See examples below:  B0: Maximum in quadrant 2 B1: Minimum at $(-1, 0)$ B1: Passes through $(0, 2)$ and $\left(\frac{1}{3}, 0\right)$  B0: Maximum on the y-axis (we would have tolerated the slight “pointy” maximum) B0: Minimum at $(-2, 0)$ B0: Does not pass through $(0, 2)$ and $\left(\frac{1}{3}, 0\right)$  B0: Not a reflection in the y-axis B0: No minimum at $(-1, 0)$ B1: Passes through $(0, 2)$ and touches $\left(\frac{1}{3}, 0\right)$  B0: Must be a curve – a reflection in the y-axis with the same shape. B1: Minimum at $(-1, 0)$ B1: Passes through $(0, 2)$ and $\left(\frac{1}{3}, 0\right)$		

A1 $10\sqrt{2} + 10$ or $10 + 10\sqrt{2}$ only. It cannot be awarded without sight of the correct intermediate line seen above

Question Number	Scheme	Marks
3.(a)	Attempts perimeter of garden = $2 \times 5x + 2 \times (6x - 2)$ Sets $2 \times 5x + 2 \times (6x - 2) > 29 \Rightarrow 22x > 33$ $\Rightarrow x > \frac{33}{22} \Rightarrow x > 1.5$ *	M1 dM1 A1* (3)
(b)	Attempts area of garden = $2x(2x - 1) + 3x(6x - 2)$ Sets $A < 72 \Rightarrow 22x^2 - 8x - 72 < 0$ Finds critical values $11x^2 - 4x - 36 \Rightarrow x = -\frac{18}{11}, 2$ Chooses inside region $-\frac{18}{11} < x < 2$	M1 A1 M1 ddM1 A1 (5)
(c)	$1.5 < x < 2$	B1 (1)
		(9 marks)

(a)

M1 An attempt at finding the perimeter of the garden.

Scored for sight of $5x + 2x - 1 + 2x + 6x - 2 +$ additional term(s) involving x

Individual lengths may not be seen so imply for sight of a total of $ax + b$, where $a > 15$

dM1 Sets their $P > 29$ and attempts to solve by proceeding to $ax > c$

You may condone an attempt in which $P = 29 \Rightarrow ax = c$

A1* cso with at least one correct intermediate (simplified) line $22x > 33$ or $x > \frac{33}{22}$ before $x > 1.5$ seen.

Condone an attempt in which you see $P = 29 \Rightarrow x = 1.5$ before $x > 1.5$ seen

Note that it is possible to start with $x > 1.5$ and prove $P > 29$ but for the A1* to be scored there must be a final statement of the type "hence $x > 1.5$ ". There is no requirement for any units

(b) **Mark part (b) and (c) together**

M1 For an attempt at finding the area of the garden. For this to be scored look for

The sum of two areas $2x(2x - 1) + \dots x(6x - 2)$ condoning slips

The sum of two areas $5x(2x - 1) + \dots x(\dots x \pm \dots)$ condoning slips

The difference between two areas $5x(6x - 2) - 2x(\dots x \pm \dots)$ condoning slips.

A1 A "correct and simplified" equality or inequality, condoning $< \leftrightarrow \leq \leftrightarrow =$ Eg. $22x^2 - 8x - 72 < 0$ oe

M1 A valid attempt to find the critical values of their 3TQ. Allow factorisation, formula, completion of square or use of calculator. If a calculator is used then the answer(s) must be correct for their 3TQ. Condone candidates who fail to state the negative root of their quadratic.

ddM1 Dependent upon both M's. For choosing the inside region for their critical values. Condone $< \leftrightarrow \leq$

Condone for this mark replacing a negative root with 0, 0.5 or 1.5. So accept for example one of either $1.5 < x < 2$, $0 < x < 2$ or $0.5 < x < 2$

A1 $-\frac{18}{11} < x < 2$ Allow $0 < x < 2$ or $0.5 < x < 2$ due to context. Allow alternative notation. See below

(c)

- B1 $1.5 < x < 2$. Accept versions such as $(1.5, 2)$, $x > 1.5$ **and** $x < 2$, $x > 1.5 \cap x < 2$
 Do not allow $x > 1.5$ or $x < 2$ $x > 1.5, x < 2$

Question Number	Scheme	Marks
4.	$\frac{4x^2 + 1}{2\sqrt{x}} = \frac{4x^2}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} = 2x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$ $\int \frac{4x^2 + 1}{2\sqrt{x}} dx = \frac{4}{5}x^{\frac{5}{2}} + x^{\frac{1}{2}} + c$	M1 A1 M1 A1 A1 (5 marks)

- M1 Attempts to write $\frac{4x^2 + 1}{2\sqrt{x}}$ as a **sum of two terms**.

Award if any index is correct and processed for a form $Px^m + Qx^n$

Do not allow if the indices are unprocessed Eg. $\frac{x^2}{\sqrt{x}} = x^{2-\frac{1}{2}}$

- A1 $2x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$ oe. The coefficients do not have to be simplified. Condone $\frac{4x^{\frac{3}{2}} + x^{-\frac{1}{2}}}{2}$ for M1 A1

- M1 Raises the power of one **correct** index by one.

The index must be processed correctly so award for sight of $\rightarrow \dots x^{\frac{5}{2}}$ or $\dots x^{\frac{1}{2}}$

- A1 For one correct term and in simplest form. Either $+\frac{4}{5}x^{\frac{5}{2}}$ or $+x^{\frac{1}{2}}$

- A1 Fully correct $\frac{4}{5}x^{\frac{5}{2}} + x^{\frac{1}{2}} + c$. Accept exact equivalent simplified answers such as $0.8x^2\sqrt{x} + \sqrt{x} + c$

Condone spurious notation such as $\int$ or a dx

Condone $\frac{4}{5}x^{\frac{5}{2}} + 1x^{\frac{1}{2}} + c$

Question Number	Scheme	Marks
8a	$x\left(x^3 - \frac{7}{2}x - 5\right) = \frac{15}{2} - 5x$ $\Rightarrow 2x^4 - 7x^2 - 15 = 0 \quad *$	M1 A1*
		(2)
b	$(2x^2 + 3)(x^2 - 5) = 0 \Rightarrow (x^2 =) \dots$ $(x =) (\pm)\sqrt{5}$	M1 B1
	$y = \frac{15}{2 \times (\pm)\sqrt{5}} - 5 = \dots \quad \text{or} \quad y = \left(\pm\sqrt{5}\right)^3 - \frac{7}{2}\left(\pm\sqrt{5}\right) - 5$	M1
	$\left(-\sqrt{5}, -\frac{3}{2}\sqrt{5} - 5\right) \quad \left(\sqrt{5}, \frac{3}{2}\sqrt{5} - 5\right)$ $PQ = \sqrt{(\pm 2\sqrt{5})^2 + (\pm 3\sqrt{5})^2} = \dots$	dM1
	$PQ = \sqrt{65}$	A1cao
		(5)
		(7 marks)

(a)

M1: Sets up a correct equation in x . Invisible brackets may be implied by further work.

e.g. $x^4 - \frac{7}{2}x^2 - 5x = \frac{15}{2} - 5x$

A1*: Multiplies out and rearranges the equation to achieve the given answer with no errors seen including invisible brackets. Condone the implied $= 0$ in their working, provided it is present in their final answer. There must be at least one stage of intermediate working between their starting equation and proceeding to the given answer.

$x^4 - \frac{7}{2}x^2 - 5x = \frac{15}{2} - 5x \Rightarrow 2x^4 - 7x^2 - 15 = 0$ is M1A0*

(b) **Work for (b) may be seen in (a) which can score**M1: Attempts to solve the quadratic equation in x^2 by either

- factorising
- completing the square
- quadratic formula

Usual rules apply. Do not be too concerned by the labelling e.g. it is acceptable to use a different variable as well as condoning $y = x^2$ and $x = x^2$

This mark cannot be awarded from directly using a calculator and stating the roots.

B1: At least one of $\sqrt{5}$ and $-\sqrt{5}$ (and no others)

M1: **Note that if they do not square root their solution to the quadratic in x^2 then this mark cannot be scored (and no further marks).**

Attempts to substitute in $\pm\sqrt{5}$ (which must be of the form $\pm k\sqrt{b}$ and $-k\sqrt{b}$ where k may be 1) into either of the equations of the curves and proceeds to find irrational expressions for the y coordinates of P and Q of the form $\pm 5 \pm n\sqrt{5}$ (which may be the same for both). They cannot be decimals.

May be seen as e.g. $\left(-\sqrt{5}, \frac{-3\sqrt{5}-10}{2}\right) \quad \left(\sqrt{5}, \frac{3\sqrt{5}-10}{2}\right)$



dM1: A full attempt to find the length PQ . It is dependent on the previous method mark.

- They must be using $(k\sqrt{b}, \pm 5 \pm n\sqrt{b})$, $(-k\sqrt{b}, \pm 5 \pm n\sqrt{b})$ or equivalent.
- Look for an expression for PQ of the form e.g.

$$\sqrt{\left((k\sqrt{b} - -k\sqrt{b})\right)^2 + \left((\pm 5 \pm n\sqrt{b}) - (\pm 5 \pm n\sqrt{b})\right)^2}$$

- They must be attempting to find $(x_2 - x_1)^2$ for their x coordinates which would lead to $\pm 2k\sqrt{b}$ but condone ambiguity in the location of negatives when calculating $(y_2 - y_1)^2$.

e.g. $x = \pm\sqrt{5}$: $\sqrt{\left((\sqrt{5} - -\sqrt{5})\right)^2 + \left(-\frac{10+3\sqrt{5}}{2} - \frac{-10+3\sqrt{5}}{2}\right)^2}$ scores B1M1dM1

e.g. $x = \pm\sqrt{11}$: $\sqrt{\left((-\sqrt{11} - \sqrt{11})\right)^2 + \left(\frac{110+15\sqrt{11}}{22} - \frac{-110-15\sqrt{11}}{22}\right)^2}$ scores B0M1dM1

(as their $(x_2 - x_1)^2$ is correct for their $\pm\sqrt{5}$ and we condone sign errors when finding $(y_2 - y_1)^2$ using their y coordinates of P and Q)

- If the subtractions are not shown then they must achieve either of the following:
 - $\sqrt{(2k\sqrt{b})^2 + (2n\sqrt{b})^2}$ (n does not have to be an integer) or equivalent
 - $\sqrt{(2k\sqrt{b})^2 + 0^2}$ (where the y coordinates were the same)
(They must have still found the y coordinates to score this mark)

Do not condone invisible brackets unless it does not affect the processing. This mark may be implied by their answer.

A1: $\sqrt{65}$ cao (this cannot be scored if the first method mark has not been scored)



Question Number	Scheme	Marks
9 (a) (i)	<u>Stretch</u> parallel to the x -axis $\times \frac{1}{2}$ or <u>stretch</u> parallel to the y -axis $\times \sqrt{2}$	<u>M1</u> , A1
(ii)	<u>Translate</u> by the vector $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$ (or translate up by 12 (units))	<u>M1</u> , A1
(b) (i)	$12 - \sqrt{x} = \sqrt{2}\sqrt{x}$ $12 = (\sqrt{2} + 1)\sqrt{x}$ $\Rightarrow \sqrt{x} = \frac{12}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = 12(\sqrt{2} - 1) *$	M1 dM1, A1 *
Alt (i)	$12 - \sqrt{x} = \sqrt{2}\sqrt{x} \Rightarrow (12 - \sqrt{x})^2 = 2x \Rightarrow x + 24\sqrt{x} - 144 = 0$ $\Rightarrow (\sqrt{x}) = \frac{-24 \pm \sqrt{24^2 - 4 \times -144}}{2} = -12 \pm \frac{12}{2} \sqrt{4 + 8} = -12 \pm 12\sqrt{2}$ $\sqrt{x} > 0 \Rightarrow \sqrt{x} = -12 + 12\sqrt{2} = 12(\sqrt{2} - 1) *$	M1 dM1 A1
(ii)	$\Rightarrow x = 12^2 (\sqrt{2} - 1)^2 = 144(2 + 1 - 2\sqrt{2}) = 144(3 - 2\sqrt{2})$ $y \{ = 12 - \sqrt{x} = 12 - 12(\sqrt{2} - 1) \} = 12(2 - \sqrt{2})$ Or common acceptable alt forms: $P(432 - 288\sqrt{2}, 24 - 12\sqrt{2})$	M1, A1 B1
		(6) (10 marks)

(a)(i)

M1: Correct type of transformation identified. Score for any of

- “Stretch”, x/y -“scaling” or “compression”
- Condone “contraction”, “enlargement”, “magnify”, “squish”, “shrink” and “elongate”. There may be other acceptable words, use discretion.

Do not accept such as “multiply each x value by 2”A1: Completely correct using correct terminology. The direction (x or y or horizontal or vertical), type (must be scaling or stretch or compression) and the correct scale factor must be given. If there are any contradictory statements, then it is A0.

Accept e.g.

- Stretch or scaling or compression in the x direction with (scale) factor $\frac{1}{2}$
 - Stretch or scaling in the y direction with (scale) factor $\sqrt{2}$
- Note a compression in the x direction by factor 2 is A0.

(a)(ii)

M1: Correct type of transformation identified. Score for e.g.

- Translate, shift
 - Move Up/down
 - Allow translation vector given as indicating translation
- Do not** accept such as “add 12 to each y value”.



A1: Completely correct using correct terminology. E.g. “move up by 12” is minimal acceptable answer. The type (translation or “move” or accept “shift”) direction (up or translation vector or positive y direction), and the correct distance must be given, but accept the translation vector to represent the direction and distance. If there are any contradictory statements, then it is A0.

(b)(i)

M1: Sets up equation and attempts to take out a common factor of $\sqrt{x}$

dM1: Correct attempt to rationalise. Must see evidence that the numerator and denominator both

multiplied by an appropriate term e.g. seeing the denominator as just $2-1$, ie $\frac{12}{\sqrt{2}+1} = \frac{12(\sqrt{2}-1)}{2-1}$ is fine.

A1*: Complete proof showing all necessary steps. Condone invisible brackets when rationalising. There may be variations where they do things in a slightly different order, but the scheme can still apply. E.g.

$$\sqrt{2x} = 12 - \sqrt{x} \Rightarrow \sqrt{x} + \sqrt{2x} = 12 \Rightarrow (\sqrt{2}-1)(\sqrt{x} + \sqrt{2x}) = 12(\sqrt{2}-1)$$

$$\Rightarrow \sqrt{x}(\sqrt{2}+2-1-\sqrt{2}) = 12(\sqrt{2}-1) \text{ scores M1 when } \sqrt{x} \text{ factored out}$$

$$\Rightarrow \sqrt{x} = 12(\sqrt{2}-1) \text{ scores dM1A1 when achieving } \sqrt{x} = \dots$$

Alt:

M1: Sets up equation and squares both sides, then rearranges to a quadratic in $\sqrt{x}$.

dM1: Solves the quadratic (allow if it is called $x = \dots$)

A1: Gives correct justification for the choice of roots and completes to the given answer.

(b)(ii)

M1: Attempts to square $12(\sqrt{2}-1)$ achieving a not necessarily simplified form $a\sqrt{2}+b$. Allow slips if a coefficient is miscopied. E.g. $x = (12\sqrt{2}-1)^2 = 144 \times 2 - 2\sqrt{2} + 1$ can score M1.

$$\text{A1: } x = 144(3-2\sqrt{2}) \text{ or } 432-288\sqrt{2}$$

$$\text{B1: } y = 24-12\sqrt{2} \text{ or } 12(2-\sqrt{2})$$

Note: It is possible some will find y before x and attempt a longer method to find x . Score the M at the point at which they attempt to square out the relevant bracket for their method.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
8 (a)	States $y = 4$	B1 (1)
(b)	States (16,9) only	B1 (1)
(c)	$k \leq 4, k = 9$	B1, B1 (2)
(d) (i)	$a = 6$	B1
(ii)	$y = f(x - 3)$	B1 (2)
		(6 marks)

(a)

B1 $y = 4$ only. May be written on a new graph

(b)

B1 (16,9) condone lack of brackets and must be the only answer (or clearly their final answer). May be written on a graph again.

(c)

B1 Sight of either one of $k \leq 4, k = 9$ Must be in terms of k . Ignore any others for this markB1 Both of $k \leq 4, k = 9$ ONLY Where several inequalities or equations are given only mark what appears to be their final answer for the $k = \dots$ and their final answer for $k \leq \dots$ Ignore any inequalities which are within $k \leq 4$ SC $y \leq 4, y = 9$ (They never write in terms of k so score B1B0)

(d)(i)

B1 $a = 6$ (6 on its own is sufficient). Also allow $(y =) f(x) - 6$ and isw if they proceed to state $a = -6$ B1 $y = f(x - 3)$ Also accept $y = -f(x) + 6$, $y = f(x + 4) - 9$ or even rearrangements such as $f(x) = 6 - y$ Do not accept combinations of different transformations as the question asked for a single transformation.

Question	Scheme	Marks
3(i)	Writes $\sqrt{180}$ as $6\sqrt{5}$ or $\sqrt{80}$ as $4\sqrt{5}$	M1
	Concludes working $\frac{6\sqrt{5} - 4\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}} = 2$	A1
		(2)
(ii)	$\frac{4\sqrt{5} - 5}{7 - 3\sqrt{5}} = \frac{4\sqrt{5} - 5}{7 - 3\sqrt{5}} \times \frac{7 + 3\sqrt{5}}{7 + 3\sqrt{5}} = \dots$	M1
	$= \frac{28\sqrt{5} - 35 + 12(\sqrt{5})^2 - 15\sqrt{5}}{49 - 9 \times 5}$	dM1
	$= \frac{25}{4} + \frac{13}{4}\sqrt{5}$	A1
		(3)
(5 marks)		
Notes: This is a non calculator question and all stages of working must be shown		

(i) Main method

M1: Writes $\sqrt{180}$ as $6\sqrt{5}$ or $\sqrt{80}$ as $4\sqrt{5}$.

Additional lines of working may be seen but they are not required for this mark

A1: Correct work leading to the answer 2. The M mark must have been awarded.

The solution shown in the main mark scheme is the minimum evidence required.

A valid alternative is $\frac{\sqrt{180} - \sqrt{80}}{\sqrt{5}} = \frac{6\sqrt{5} - 4\sqrt{5}}{\sqrt{5}} = \left(\frac{6\sqrt{5}}{\sqrt{5}} - \frac{4\sqrt{5}}{\sqrt{5}} \right) = 6 - 4 = 2$

(i) Alt method I $\frac{\sqrt{180} - \sqrt{80}}{\sqrt{5}} = \frac{\sqrt{180}}{\sqrt{5}} - \frac{\sqrt{80}}{\sqrt{5}} = \sqrt{36} - \sqrt{16} = 6 - 4 = 2$

M1: Writes $\frac{\sqrt{180}}{\sqrt{5}}$ as $\sqrt{36}$ or $\frac{\sqrt{80}}{\sqrt{5}}$ as $\sqrt{16}$

Additional lines of working may be seen but they are not required

A1: Correct work leading to the answer 2. The solution shown below is the minimum evidence required

E.g. $\frac{\sqrt{180}}{\sqrt{5}} - \frac{\sqrt{80}}{\sqrt{5}} = \sqrt{36} - \sqrt{16} = 6 - 4 = 2$

(i) Alt method II $\frac{\sqrt{180} - \sqrt{80}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{900} - \sqrt{400}}{5} = \frac{30 - 20}{5} = 2$

M1: Multiplies numerator and denominator by $\sqrt{5}$ and writes $\sqrt{180} \times \sqrt{5}$ as $\sqrt{900}$ or $\sqrt{80} \times \sqrt{5}$ as $\sqrt{400}$

A1: Correct work leading to the answer 2

General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$ **Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

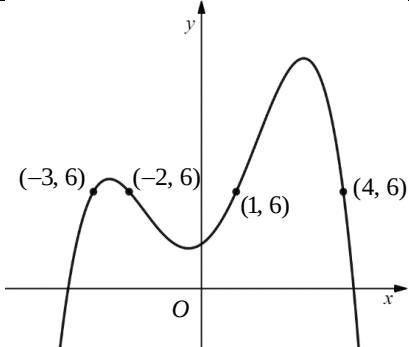
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
7(a)	$-1 < x < 2$	M1A1
	$x < -4, x > 3$	B1
		(3)
(b)	$(x =) 1.5$	B1
		(1)
(c)(i)		B1B1B1
(ii)	$-3, x, -2$	B1
		(4)
		(8 marks)

- (a) Ignore any references to OR or AND (or equivalent use of set notation) between $-1 < x < 2, x > 3, x < -4$

M1 Requires one end of the inside region between -1 and 2 . Score for $x > -1$ or $x < 2$, condoning use of „ ... Must be in terms of x

A1 $-1 < x < 2$ or any equivalent notation such as $-1 < x \cap x < 2, (-1, 2)$ and also score for $2 > x > -1$. Do not allow just two separate inequalities for this mark without AND or equivalent.

B1 $x > 3, x < -4$

(b)

B1 1.5 or $\frac{3}{2}$ Condone $(1.5, 6)$ or $x = 1.5, y = 6$ only (the y -coordinate must be correct if stated)

(c)(i) **Check the figure at the start of the question. If there is a contradiction, then the sketch with labelled coordinates in the main body of working takes precedence. There must be a sketch to score any marks.**

B1 Sketch reflected in the y -axis with the intention to have the higher maximum turning point in quadrant 1 and the lower maximum turning point in quadrant 2. Do not be concerned which side of the y -axis the minimum turning point is located.

B1 Either two correct pairs of coordinates **or** all x coordinates correct **or** all y coordinates correct. Do not be concerned with the relative locations of the coordinates in relation to each



other. The points must be in the correct quadrants, but may be stated separately. Do not be concerned with the labelling of P , Q , R and S and the associated coordinates if they are stated separately.

- B1 All four correct coordinates (Do not penalise poor notation and may be listed as $x = \dots$, $y = \dots$) Do not be concerned with the relative locations of the coordinates in relation to each other. The points must be in the correct quadrants, but may be stated separately. Do not be concerned with the labelling of P , Q , R and S and the associated coordinates if they are stated separately.

(ii)

- B1 $-3 \leq x \leq -2$ only or any equivalent notation such as $-3 \leq x \cap x \leq -2$, $-3 \leq x$ AND $x \leq -2$. $[-3, 2]$ but do not accept use of OR or $\cup$ if set notation is used. Do not accept $(-3, -2)$ or $-3 < x < -2$ Must be in terms of x
There may be several inequalities. Mark what appears to be their final answer.

Question	Scheme	Marks
3(a)	$\frac{1}{2} \times 3^2 \times \alpha = 7.2 \Rightarrow \alpha = \dots \text{ or } \frac{1}{2} \times 3^2 \times 1.6 = 7.2 \Rightarrow \alpha = 1.6$ $\alpha = 1.6^*$	M1 A1*
		(2)
(b)(i)	$\text{Angle } COA = \frac{1}{2}(2\pi - 1.6) (= 2.34\dots) \quad (\approx 134^\circ)$ $\text{Area } COA = \frac{1}{2} \times 5 \times 3 \sin("2.34") \quad (= 5.38\dots)$ $\text{Total Area} = 2 \times \frac{1}{2} \times 5 \times 3 \sin("2.34") + 7.2$ $= 18 \text{ (cm}^2\text{)} \quad \text{Awrt } 18 \text{ (cm}^2\text{)} \text{ (Ans = 17.96)}$	M1 M1 dM1 A1
(ii)	$\text{Arc } AB = 3 \times 1.6 (= 4.8)$ $(AC^2 =) 5^2 + 3^2 - 2 \times 5 \times 3 \cos("2.34")$ $\text{Total perimeter} = 2 \times \sqrt{5^2 + 3^2 - 2 \times 5 \times 3 \cos("2.34")} + 3 \times 1.6$ $= \text{Awrt } 19.6 \text{ (cm)}$	B1 M1 dM1 A1
		(8)
Alt (b)(i)	$AB = 2 \times 3 \sin 0.8$ $ON = 3 \cos 0.8$ $\text{Total Area} = \frac{1}{2}(5 + ON) \times AB + 7.2 - \frac{1}{2} \times 3 \cos 0.8 \times 2 \times 3 \sin 0.8$ $= 18 \text{ (cm}^2\text{)} \quad \text{Awrt } 18 \text{ (cm}^2\text{)} \text{ (Ans = 17.96)}$	M1 M1 dM1 A1
		(10 marks)

Notes

(a)

M1 Uses a correct sector area formula and 7.2 to find the value for α . They should show the values embedded in the equation and proceed to find a value for α .
 Alternatively, substitutes in $\alpha = 1.6$ into the area of a sector formula and achieves 7.2.

A1* Correct proof starting with $\frac{1}{2} \times 3^2 \times \alpha = 7.2$ and at least one intermediate line of working and no

errors. Eg $\frac{1}{2} \times 3^2 \times \alpha = 7.2 \Rightarrow \alpha = \frac{7.2}{4.5} = 1.6$ scores M1A1

Alternatively, they must conclude that $\alpha = 1.6$ or if there is a preamble then there should be some form of completion which could be a tick, QED etc.

If they use a different variable such as θ they must state/link somewhere that $\alpha = 1.6$



(b)(i) **Mark both (i) and (ii) together. If no angle calculation is seen then use what they think is their angle COA in bi and bii. Beware of values on the diagram that may imply a method.**

M1 $\frac{1}{2}(2\pi - 1.6)$ Correct method for angle COA . Sight of awrt 2.34 is sufficient to score this mark and may be on the diagram. (May also be implied by 134°)

M1 Uses a correct method for the area of triangle COA or COB . It is sufficient to see the values embedded in the expression such as $\frac{1}{2} \times 5 \times 3 \sin("2.34")$ (= awrt 5.38 / 5.39). Angle may be in degrees. If they state $\frac{1}{2}ab \sin C$ or but embed values as $\frac{1}{2}abC$ condone as a slip for M1.

dM1 Fully correct strategy for the area. It is dependent on the previous method mark so allow if their angle COA is incorrect. Look for $2 \times \text{area of triangle } COA + 7.2$. Embedded values are sufficient.

A1 awrt 18 (cm^2) **Must come from a correct method**

.....
Alt b(i)

M1 Find the length $AB = 2 \times 3 \sin 0.8$ (awrt 4.30)

M1 Finds the length ON where N is the midpoint of AB (awrt 2.09)

dM1 Fully correct strategy for the area. Look for $7.2 + \text{area of triangle } ABC - \text{area of triangle } AOB$

A1 awrt 18 (cm^2) **Must come from a correct method**

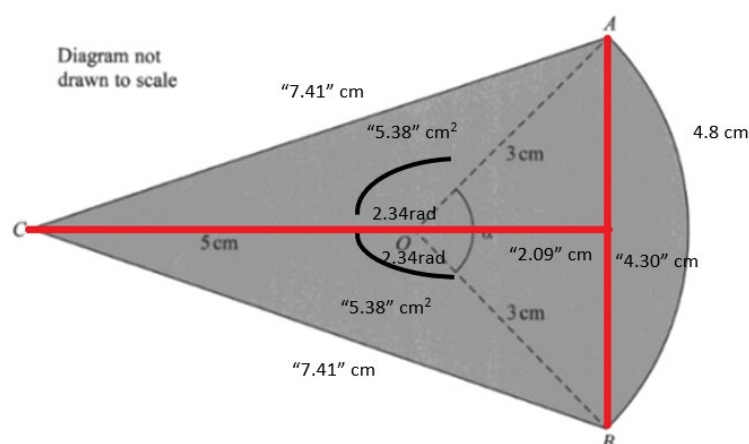
.....
(ii)

B1 Correct expression or value for the arc length (4.8)

M1 Uses a correct method for AC^2 , AC , CB^2 or CB . Embedded values in the associated formula is sufficient or sight of awrt 54.9 or awrt 7.41 would imply this mark. (Angle in degrees $\approx 134^\circ$) For this mark condone candidates confusing AC^2/CB^2 and AC/CB .

dM1 Total perimeter = $2 \times \sqrt{5^2 + 3^2} - 2 \times 5 \times 3 \cos("2.34") + 3 \times 1.6$. It is dependent on the B and the M marks and they must have remembered to square root AC^2 or CB^2 .

A1 awrt 19.6 (cm) (Ans = 19.619) **Must come from a correct method**



Question	Scheme	Marks
4	$y = 3x + 4 \Rightarrow x^2 + (3x + 4)^2 + 6x - 4(3x + 4) = 4$ or $x = \frac{y-4}{3} \Rightarrow \left(\frac{y-4}{3}\right)^2 + y^2 + 6\left(\frac{y-4}{3}\right) - 4y = 4$ $5x^2 + 9x - 2 (=0) \text{ or } 5y^2 - 13y - 46 (=0)$ $(5x-1)(x+2) = 0 \Rightarrow x = \dots \text{ or } (5y-23)(y+2) = 0 \Rightarrow y = \dots$ $x = 0.2, x = -2 \text{ or } y = 4.6, y = -2$ Substitutes their x into their $y = 3x + 4$ / Substitutes their y into their $x = \frac{y-4}{3}$ $x = 0.2 \left(\text{or } \frac{1}{5} \right), y = 4.6 \left(\text{or } 4\frac{3}{5} \text{ or } \frac{23}{5} \right)$ and $x = -2, y = -2$	M1 M1A1 dM1 B1 M1 A1 (7 marks)

Notes

- M1** rearrange the linear equation to $y = \dots$ or $x = \dots$ and attempts to fully substitute into the second equation.
- M1** Collect terms together to produce a 2 or 3 term quadratic expression $= 0$. The ' $= 0$ ' may be implied by later work. Condone slips in their rearrangement of the equation.
- A1** Correct quadratic equation in x or y . Condone the absence of " $= 0$ "
They may be multiples of the main scheme. Eg $10x^2 + 18x - 4 = 0$ or $10y^2 - 26y - 92 = 0$
- dM1** Attempt to factorise and solve or complete the square and solve or uses a correct quadratic formula for a 3 term quadratic and obtains at least one value of x or y . (see general guidance for solving a quadratic). If factorising then their **seen** factorised expression must equal their **seen** 3TQ quadratic expression. If they use the quadratic formula, we must see the values embedded in a correct formula.

Dependent on both previous method marks.

They cannot just write down their calculator values for this mark

Guidance on use of calculators for factorising quadratics and how to score:

$$\dots \Rightarrow 10x^2 + 18x - 4 = 0 \Rightarrow (5x-1)(x+2) = 0 \Rightarrow x = \frac{1}{5}, -2 \Rightarrow \dots \text{ max score M1M1A1 dM0 B1M1 A0}$$

$$\dots \Rightarrow 10x^2 + 18x - 4 = 0 \Rightarrow (x - \frac{1}{5})(x+2) = 0 \Rightarrow x = \frac{1}{5}, -2 \Rightarrow \dots \text{ max score M1M1A1 dM0 B1M1 A0}$$

$$\dots \Rightarrow 10x^2 + 18x - 4 = 0 \Rightarrow x = \frac{1}{5}, -2 \Rightarrow \dots \text{ max score M1M1A1 dM0 B1M1 A0}$$

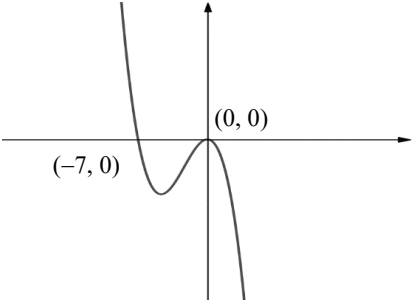
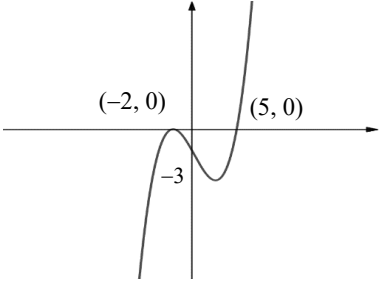
B1 Correct answers for either both values of x or both values of y (possibly unsimplified) which can only be scored if they have achieved a correct 3TQ.

M1 Substitute at least one value of x to find y or vice versa. This may be implied by their final answers.

A1 Fully correct solution **with all previous marks awarded** such that

- working is shown to produce a correct quadratic equation
- working is shown to solve the quadratic equation
- both pairs of coordinates found and simplified. Only withhold if the wrong x coordinate is clearly paired with the other y coordinate.

Condone recovery of invisible brackets.

Question	Scheme		Marks
5(i)(a)		B1 Horizontal translation ← B1 Maximum at origin B1 (-7, 0)	B1B1B1
			(3)
(b)		B1 Reflection in y-axis B1 Touches at (-2, 0) and passes through (5, 0) B1 Passes through (0, -3)	B1B1B1
			(3)
(ii)a	$x = 0 \Rightarrow y = k \cos\left(\frac{\pi}{6}\right) = \sqrt{3}$ $k \frac{\sqrt{3}}{2} = \sqrt{3} \Rightarrow k = 2$		B1
(b)	$(p =) \frac{\pi}{3} \text{ or } (q =) \frac{4\pi}{3}$ $(p =) \frac{\pi}{3} \text{ and } (q =) \frac{4\pi}{3}$		B1 B1
			(3)
			(9 marks)

Notes

(i)(a)

B1 Horizontal translation ←

The negative cubic should appear in quadrants 2, 3 and 4 only and the local maximum should be on the x axis where $x \leq 0$. The local minimum must be in the third quadrant. Condone any part of the graph which may look linear but withhold this mark if the graph curves back on itself.

B1 Local maximum at the origin (does not need to be labelled)

B1 Passes through $(-7, 0)$. Allow just the x value instead of both coordinates marked on the axis or written in the text and condone a slip of x and y the wrong way round as long as the sketch would give the correct coordinates. May be listed but cannot be awarded without a sketch. Condone lack of brackets. Do not allow 7 instead of -7 but condone transcription errors if the correct coordinate is stated in the text.

(b)

B1 Reflection in the y -axis. A positive cubic should be drawn appearing in quadrants 1, 3 and 4 only. The local maximum point should be on the x axis where $x \leq 0$ and the local minimum point should be in quadrant 4. Do not accept the local minimum on the y -axis.

B1 Touches at $(-2, 0)$ and passes through $(5, 0)$. Allow just the x values instead of both coordinates marked on the axis or written in the text and condone a slip of x and y the wrong way round as long as the sketch would give the correct coordinates. May be listed but cannot be awarded without a sketch. Condone lack of brackets. Do not allow 2 instead of -2 or -5 instead of 5 but condone transcription errors if the correct coordinates are stated in the text.

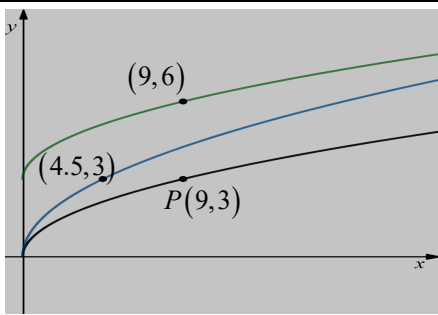
B1 Passes through $(0, -3)$. Allow just -3 instead of both coordinates marked on the axis or written in the text and condone a slip of x and y the wrong way round as long as the sketch would give the correct coordinates. May be listed but cannot be awarded without a sketch. Condone lack of brackets. Do not allow 3 instead of -3 but condone transcription errors if the correct coordinate is stated in the text.

(ii)(a)

B1 $k = 2$

B1 $(p =) \frac{\pi}{3}$ or $(q =) \frac{4\pi}{3}$ Award for sight of either of these values or equivalent. Ignore labelling. Allow awrt 1.05 or awrt 4.19 for this mark and allow in degrees (60 or 240).

B1 $(p =) \frac{\pi}{3}$ and $(q =) \frac{4\pi}{3}$ ignore labelling and condone $\left(0, \frac{\pi}{3}\right)$, $\left(0, \frac{4\pi}{3}\right)$. Allow exact equivalents.

Question Number	Scheme	Marks
9(a)		
	One correct sketch drawn and labelled correctly	M1
	One correct sketch drawn and labelled and with correct point	A1
	Completely correct sketches with both points	A1
		(3)
(b)	Sets $\sqrt{x} + 3 = \sqrt{2x}$	B1
	$3 = (\sqrt{2} - 1)\sqrt{x}$	M1
	$\sqrt{x} = \frac{3}{(\sqrt{2} - 1)} \times \frac{(\sqrt{2} + 1)}{(\sqrt{2} + 1)} = 3(\sqrt{2} + 1) \quad *$	A1*
		(3)
(c)	$\sqrt{x} = 3(\sqrt{2} + 1) \Rightarrow x = 9(\sqrt{2} + 1)^2 = \dots$	M1
	$\Rightarrow x = 9(3 + 2\sqrt{2}), y = 3\sqrt{2} + 6$	A1, B1
		(3)
		(9 marks)

- (a) Check **all 3** diagrams and score the best **single** diagram unless the candidate clearly indicates which one they want marked by e.g. crossing out the other(s).
- M1 One correct curve drawn and labelled correctly:
 For $f(2x)$ the curve should start at O and be above and remain above $f(x)$ and not head back towards it significantly i.e. at least maintain the same gap.
 For $f(x) + 3$ the curve should start on the positive y -axis and be approximately the same shape as $f(x)$
- A1 One correct curve drawn as above and **labelled** and with **correct point** for that curve.
 The point does **not** have to be in the correct relative position – just look for values.
- A1 Completely correct sketches with both points correct and at least one correctly labelled – you can assume the other is the other. Allow $f(2x)$ to cross $f(x) + 3$ as long as it is beyond $(9, 6)$ but with no other intersections for $x > 0$

The coordinates for the transformed P must be indicated on the sketch or if they are away from the sketch it must be clear which curve they relate to.

For examples see below.

If you are in any doubt use review.



(b)

B1 Correct equation $\sqrt{x} + 3 = \sqrt{2x}$ M1 Writes $\sqrt{2x}$ as $\sqrt{2}\sqrt{x}$ and proceeds to collect terms in $\sqrt{x}$

Note that this may be achieved via e.g.

$$\sqrt{x} + 3 = \sqrt{2x} \Rightarrow \sqrt{x} + 3 = \sqrt{2}\sqrt{x} \Rightarrow 1 + \frac{3}{\sqrt{x}} = \sqrt{2}$$

$$\text{Or e.g. } \sqrt{x} + 3 = \sqrt{2x} \Rightarrow \sqrt{x} + 3 = \sqrt{2}\sqrt{x} \Rightarrow \frac{1}{3} + \frac{1}{\sqrt{x}} = \frac{\sqrt{2}}{3}$$

A1* Proceeds to the given answer showing at least the steps

$$\sqrt{x} = \frac{3}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = 3(\sqrt{2}+1)$$

$$\text{or e.g. } 3 = (\sqrt{2}-1)\sqrt{x} \Rightarrow 3(\sqrt{2}+1) = (\sqrt{2}+1)(\sqrt{2}-1)\sqrt{x} = \sqrt{x}$$

Attempts using e.g. $\sqrt{x} + 3 = \frac{\sqrt{x}}{2}$ score no marks in part (b)**Alternative:**B1 Correct equation $\sqrt{x} + 3 = \sqrt{2x}$ M1 $\sqrt{x} + 3 = \sqrt{2x} \Rightarrow x + 6\sqrt{x} + 9 = 2x \Rightarrow x - 6\sqrt{x} - 9 = 0$

$$x - 6\sqrt{x} - 9 = 0 \Rightarrow \sqrt{x} = \frac{6 \pm \sqrt{36+36}}{2}$$

Squares both sides and collects terms to obtain a 3TQ in $\sqrt{x}$ and attempts to solve for $\sqrt{x}$

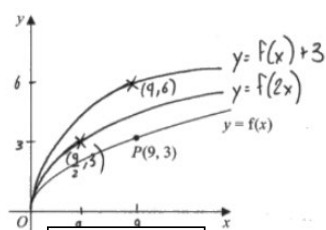
e.g. using quadratic formula

$$\text{A1* } \frac{6 \pm \sqrt{36+36}}{2} = 3 \pm 3\sqrt{2} = 3(\sqrt{2} \pm 1) \Rightarrow \sqrt{x} = 3(\sqrt{2} + 1)$$

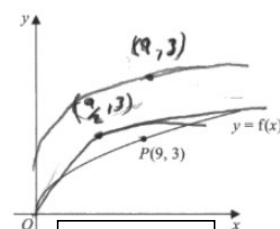
Simplifies and reaches the printed answer. If they give both answers score A0 but there is no requirement to explain why the other answer is rejected.

(c)

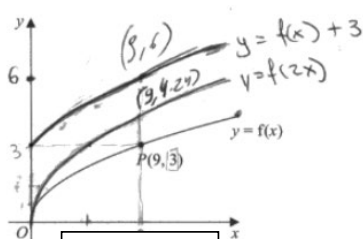
M1 Attempts to square the given expression to find x . Condone a slip on the 3 (it may remain 3) but must result in an expression of the form $\alpha + \beta\sqrt{2}$. (Working need not be shown as long as this condition is met)A1 $x = 9(3 + 2\sqrt{2})$ or such as $x = 27 + 18\sqrt{2}$ B1 $y = 3\sqrt{2} + 6$ Note that $y = \sqrt{18\sqrt{2} + 27} + 3$ is correct but is not simplified so scores B0**Note that working for (c) must be seen in (c) i.e. do not allow working for (c) to be credited in parts (a) and (b) unless the answers are copied into (c)**

Example marking for sketches for 9(a):

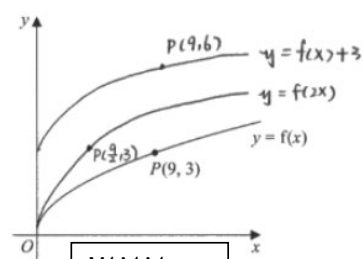
M1A1A0



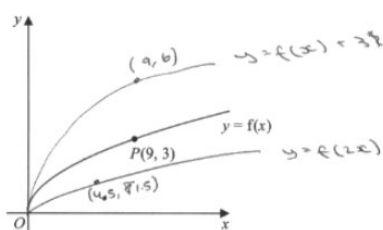
M0A0A0



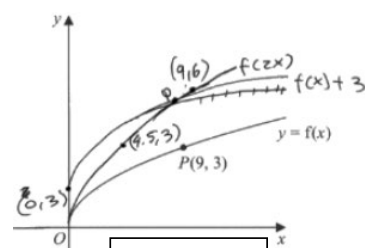
M1A1A0



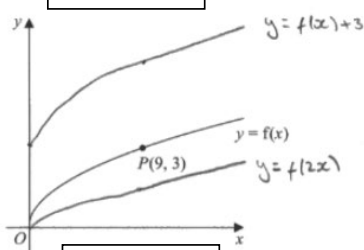
M1A1A1



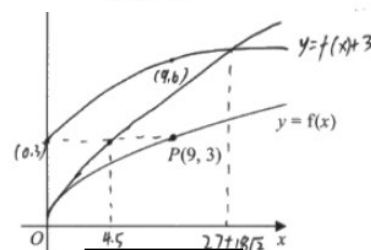
M0A0A0



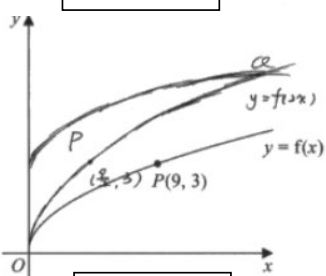
M1A1A0



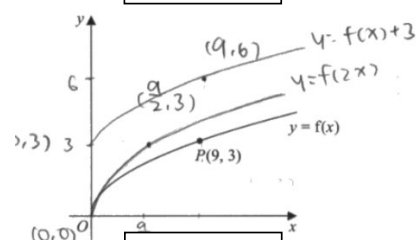
M1A0A0



M1A1A1



M1A1A0



M1A1A1

Question Number	Scheme	Marks
10 (a)	$P = \left(-\frac{1}{2}, 0\right)$	B1 (1)
(b)	$f(x) = (x-4)(2x+1)^2 \Rightarrow f(x) = ax^3 + bx^2 + cx + d$ $= 4x^3 - 12x^2 - 15x - 4$ oe $f'(x) = 12x^2 - 24x - 15$	M1 A1 dM1 A1 (4)
(c)	Attempts $f'(2.5) = 12 \times 2.5^2 - 24 \times 2.5 - 15 = 0$ Finds y coordinate for $x = 2.5$ $y = -54$	M1A1 A1 (3)
(d)	$a = -\frac{1}{2}, (+) 4$	B1, B1 (2) (10 marks)

(a)

B1 $P = \left(-\frac{1}{2}, 0\right)$ or exact equivalent. Accept coordinates written separately.

If both $(4, 0)$ and $\left(-\frac{1}{2}, 0\right)$ are given, then P must be identified as $\left(-\frac{1}{2}, 0\right)$

(b)

M1 Attempts to multiply out to form a 4 - term cubic. The terms do not have to be collected.

Look for a multiplied-out expression that would simplify to $ax^3 + bx^2 + cx + d$

A1 $4x^3 - 12x^2 - 15x - 4$. The terms do not have to be collected for this mark.

dM1 Reduces the power by one in all terms. The indices must be processed.

Look for $ax^3 + bx^2 + cx + d \rightarrow px^2 + qx + r$ with no zero constants

A1 cso $f'(x) = 12x^2 - 24x - 15$ which must be fully simplified.

Cannot be awarded, for instance, from incorrect e.g. $f(x) = 4x^3 - 12x^2 - 15x + 4$

Note: May be attempted by the product rule.

M1A1: $f(x) = (x-4)(2x+1)^2 \Rightarrow f'(x) = 1 \times (2x+1)^2 + A(x-4)(2x+1)$

All 4 marks $f(x) = (x-4)(2x+1)^2 \Rightarrow f'(x) = 1 \times (2x+1)^2 + 4(x-4)(2x+1)$

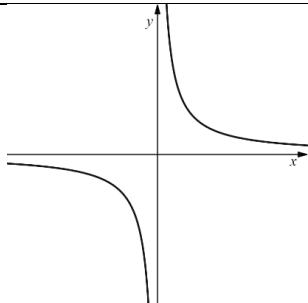
Note that the terms do not need to be multiplied out under this method

(c)

M1 Attempts $f'(2.5)$ for their $f'(x)$ A1 Shows $f'(2.5) = 12 \times 2.5^2 - 24 \times 2.5 - 15 = 0$ with either embedded values shown or $f'(2.5) = 75 - 60 - 15 = 0$ For this to be scored $f'(x)$ must be correctA1 CSO Finds y coordinate for $x = 2.5 \Rightarrow$ Equation of tangent $y = -54$ but allow $k = -54$
An alternative method would be toM1 Attempts to solve their $f'(x) = 0$ A1 For $f'(x) = 0 \Rightarrow x = (-0.5), 2.5$ For this to be scored $f'(x)$ must be correctA1 CSO Finds y coordinate for $x = 2.5 \Rightarrow$ Equation of tangent $y = -54$ but allow $k = -54$

(d)

B1 For one of $-\frac{1}{2}, (+) 4$. Alternatively score for both $a = +\frac{1}{2}, -4$ Implied by $y = f\left(x - \frac{1}{2}\right)$ or $y = f(x + 4)$ for this mark onlyB1 For both $a = -\frac{1}{2}, (+) 4$ and no others. Cannot be $x = \dots$ but allow just the values $-\frac{1}{2}, (+) 4$

Question Number	Scheme		Marks
7(a)		Correct Shape in quadrant 1 or 3	M1
		Fully correct shape and position	A1
			(2)
(b)	E.g. Translate 2 (units) to the right (parallel to the x-axis)		B1, B1
			(2)
(c)	When $x = -4 \Rightarrow \frac{6}{x-2} = kx + 7 \Rightarrow \frac{6}{-6} = -4k + 7 \Rightarrow k = \dots$		M1
	$k = 2$		A1
			(2)
(d)	$\frac{6}{x-2} = 2x + 7 \Rightarrow 6 = 2x^2 + 3x - 14$		M1
	$0 = 2x^2 + 3x - 20 \Rightarrow (2x - 5)(x + 4) = 0$		dM1
	$Q = \left(\frac{5}{2}, 12\right)$		A1, A1
			(4)
			(10 marks)

(a)

M1: For a monotonically decreasing function in quadrant 1 or 3 with no incorrect asymptotes in that quadrant.

It must not cross either axis but be tolerant of functions that don't go up or down as far as the x-axis and as far left or right as the y-axis.

The “ends” should not come back away from the axes significantly but be tolerant of “wobbles”.

A1: Correct shape and position with no incorrect asymptotes.

Remember to check both diagrams and score the best single attempt if both are used.

(b)

B1: Partial description that implies at least one of the two components but is not fully correct

E.g. "Translates 2 units left", "Shifts/moves 2 units right"

B1: Requires (1): Translate/translation **and** (2): $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ or “2 (units to the) right” or e.g. “+2 in the x direction”

A minimum could be e.g.: “Translate +2 on x”

(c)

M1: Substitutes $x = \pm 4$ into $\frac{6}{x-2} = kx + 7$ and solves for k .Allow equivalent work e.g. $x = -4 \Rightarrow y = -1 \Rightarrow -1 = k(-4) + 7 \Rightarrow k = \dots$

Note that some will rearrange before substitution which is fine.

E.g. $\frac{6}{x-2} = kx + 7 \Rightarrow 6 = (x-2)(kx+7) = kx^2 + (7-2k)x - 14, 6 = 24k - 42 \Rightarrow k = \dots$ **A1:** $k = 2$ oe e.g. $k = \frac{4}{2}$

(d)

M1: Equates $\frac{6}{x-2}$ with $kx + 7$ using their **value** for k , cross multiplies to obtain a quadratic equation in x with terms not necessarily collected. Note that the rearrangement may have already been done in part (c).**dM1:** Solves 3TQ by any acceptable method including via a calculator.**A1:** $x = \frac{5}{2}$. Condone $Q = \frac{5}{2}$ **A1:** $Q = \left(\frac{5}{2}, 12\right)$. Must be as coordinates or $x = \dots, y = \dots$

Question Number	Scheme	Marks
10(a)	One of $-\frac{20}{3} < x < -6, \quad x > \frac{3}{2}$	M1
	Both $-\frac{20}{3} < x < -6, \quad x > \frac{3}{2}$	A1
		(2)
(b)	$(3x+20)(x+6)(2x-3) = (3x+20)(2x^2+9x-18) =$	M1
	$= 6x^3 + 67x^2 + 126x - 360$	A1 A1
		(3)
(c)	$\frac{dy}{dx} = 18x^2 + 134x + 126 \Rightarrow$ Gradient at $x = 0$ is 126	M1
	Equation of l is $y = 126x - 360$	A1ft
	l cuts C again when $6x^3 + 67x^2 + 126x - 360 = 126x - 360$	dM1
	$6x^3 + 67x^2 = 0 \Rightarrow x^2(6x+67) = 0$	ddM1
	$x = -\frac{67}{6}$	A1
		(5)
		(10 marks)

(a)

M1: One of $-\frac{20}{3} < x < -6, \quad x > \frac{3}{2}$. Condone with $<\leftrightarrow\leq$ for this mark and allow equivalent notation

e.g. $\left(-\frac{20}{3}, -6\right), \left(\frac{3}{2}, \infty\right), \left[-\frac{20}{3}, -6\right], \left[\frac{3}{2}, \infty\right)$

Allow the first inequality to be written separately as e.g. $x > -\frac{20}{3}, \quad x < -6,$

$x > -\frac{20}{3}$ and $x < -6$

Condone incorrect notation e.g. $-\frac{20}{3} < f(x) < -6, \quad f(x) > \frac{3}{2}$

or e.g. $-\frac{20}{3} < y < -6, \quad y > \frac{3}{2}$

A1: Both $-\frac{20}{3} < x < -6, \quad x > \frac{3}{2}$.

Allow the first inequality to be written separately as e.g. $x > -\frac{20}{3}, \quad x < -6$ and allow equivalent

notation e.g. $\left(-\frac{20}{3}, -6\right), \left[\frac{3}{2}, \infty\right)$ but **not** e.g. $-\frac{20}{3} < f(x) < -6, \quad f(x) > \frac{3}{2}$

(b) Allow (b) marks to score anywhere in their solution.

M1: Attempts to multiply two brackets to create a quadratic before multiplying by the third to form a cubic

A1: $6x^3 + \alpha x^2 + \beta x \pm 360$ where α and β are not both zero.

A1: $6x^3 + 67x^2 + 126x - 360$. Condone a spurious “= 0” e.g. $6x^3 + 67x^2 + 126x - 360 = 0$ but do not isw
e.g. $x^3 + \frac{67}{6}x^2 + 21x - 60$ is A0 but see note below.

(c)

M1: Attempts the gradient of l by differentiating and substituting $x = 0$

For the differentiation look for one of $\dots x^3 \rightarrow \dots x^2$, $\dots x^2 \rightarrow \dots x$, $kx \rightarrow k$

A1ft: Equation of l is $y = 126x - 360$. Follow through on their $y = ax^3 + bx^2 + cx + d \Rightarrow y = cx + d$

Allow equivalent correct equations e.g. $y + 360 = 126(x - 0)$

dM1: Sets the equation of their l to their answer for (b). **Depends on the first M mark.**

ddM1: Attempts to solve cubic of the form $ax^3 + bx^2 = 0$ by taking out a factor of x^2 or e.g. by division by x^2 leading to a value for x . **Depends on both previous M marks.**

May be implied by e.g. $ax^3 + bx^2 = 0 \Rightarrow x = -\frac{b}{a}$

A1: $x = -\frac{67}{6}$ and no other solutions apart from $x = 0$ which can be ignored. Ignore any attempts to find y .

Note that **just** an incorrect x coefficient in the expansion in (b) will result in $x = -\frac{67}{6}$ in (c) if the subsequent work is correct. In such cases allow full recovery in (c).

Similarly, if the expansion in part (b) is e.g. divided by 6 to give $x^3 + \frac{67}{6}x^2 + 21x - 60$, allow full

recovery in (c) as correct work should lead to the correct answer of $x = -\frac{67}{6}$

Question Number	Scheme	Marks
5 (a)	Attempts the gradient $\frac{\Delta y}{\Delta x} = \frac{9-6}{-2-10} = \left(-\frac{1}{4}\right)$	M1
	Uses the gradient and a point to form an eqn for l_1 $y-9 = -\frac{1}{4}(x+2)$	dM1
	$y = -\frac{1}{4}x + \frac{17}{2}$	A1
(b)	Eqn of l_2 is $y = 4x$	B1ft
	Attempts to solve their $y = 4x$ and their $y = -\frac{1}{4}x + \frac{17}{2}$ simultaneously	M1
	$R = (2, 8)$	A1, A1
(c)	Attempts (OR) $= \sqrt{2^2 + 8^2}$ or $(PQ) = \sqrt{3^2 + 12^2}$	M1
	Full attempt at area $OPQ = \frac{1}{2} \times \sqrt{3^2 + 12^2} \times \sqrt{2^2 + 8^2}$	dM1
	$= \frac{1}{2} \times 3\sqrt{17} \times 2\sqrt{17} = 51$	A1
		(3)
		(10 marks)

This is a non-calculator question so the use of a calculator is penalised

(a)

M1: Attempts gradient. It must be the "correct way up" with $\frac{\Delta y}{\Delta x}$ and an attempt at differences (seen or implied at least once on either the numerator or denominator). It is implied by a correct answer.

dM1: Uses gradient and one of the points to form a straight line. The coordinates must be in the correct place in the unsimplified equation. It is dependent upon the previous M

A1: $y = -\frac{1}{4}x + \frac{17}{2}$ o.e. but it must be in the form $y = mx + c$. ISW after sight of a correct answer.

Alt (a) via simultaneous equations

M1: Attempts to form two simultaneous equations using both points and either $y = mx + c$ **or** $ax + by + c = 0$

dM1: Solves the two simultaneous equations **via a non calculator method** to find both unknowns. It is important to see some working but it can be minimal. For example, $9 = -2m + c$ and $6 = 10m + c$ must be followed by $12m = -3$ o.e before you see $m = -0.25$ oe.

A1: $y = -0.25x + 8.5$ o.e. but it must be in the form $y = mx + c$.

Note that 101 is possible for candidates who don't show any working in solving their simultaneous equations but produce the correct equation (in the correct form)



(b)

B1ft: Correct follow through normal equation for their $y = -\frac{1}{4}x + \frac{17}{2}$. Implied by $y = 4x + 0$ following correct (a). So $y = -\frac{1}{m}x$ for their $y = mx + c$. It may be implied by a solution of $-\frac{1}{m}x = mx + c$

M1: Attempts to solve their $y = 4x$ and their $y = -\frac{1}{4}x + \frac{17}{2}$ simultaneously via a non-calculator route.

Look for a value for x or a value for y following an attempt at solving $\pm \frac{1}{m}x = mx + c$

There should be some working, allow as a minimum " $4x = -0.25x + 8.5 \Rightarrow \dots x = \dots \Rightarrow x = \dots$ "

Don't be too concerned with accuracy here.....it is a method mark

A1: One correct coordinate, usually $x = 2$ but could be for $y = 8$ if solved differently

A1: $R = (2, 8)$. May be written separately

Solutions with no working, insufficient working and/ or via use of a calculator

(b)

For the M mark you should expect to see some working. As a minimum look for the two highlighted equations (or their equivalent) where the x terms have been collected

$$y = 4x \text{ and } y = -\frac{1}{4}x + \frac{17}{2} \Rightarrow 4x = -\frac{1}{4}x + \frac{17}{2} \Rightarrow 4.25x = 8.5 \text{ followed by } (2, 8)$$

If $(2, 8)$ follows the two correct equations without the highlighted working (o.e.) score SC 1011

(c)

M1: States or attempts $\sqrt{2^2 + 8^2}$ using their coordinates for R or $\sqrt{3^2 + 12^2}$ using the given P and Q . It can be implied by an exact answer or an answer to 3sf for their coordinates
If two right angled triangles are used to find the area, it would be for attempts at OR , or PR and RQ .

dM1: Full attempt at area of triangle OPQ using $\frac{1}{2} \times \text{their } OR \times \text{their } PQ$ o.e.

Both PQ and OR must be attempted via a correct method

This can be awarded for decimal work. It is dependent on the previous M

A1: Correct answer via a correct method with sufficient working to suggest a non-calculator method

Look for $\frac{1}{2} \times 2\sqrt{17} \times 3\sqrt{17}$ (or other correct and relevant work) before you see 51.

Don't allow this to be scored via decimals or from just $\frac{1}{2} \times \sqrt{153} \times \sqrt{68} = 51$ unless extra relevant lines are seen as above.

Alternatives to part (c) exist via the cosine rule, the shoelace method

ALT 1: the cosine rule.

One example shown using angle POQ

$$\cos POQ = \frac{136 + 85 - 153}{2\sqrt{85}\sqrt{136}} = \frac{1}{\sqrt{10}} \quad \text{Area triangle } POQ = \frac{1}{2} \sqrt{85}\sqrt{136} \sin POQ = \frac{1}{2} \sqrt{85}\sqrt{136} \times \frac{3}{\sqrt{10}} = 51$$

M1: As main scheme for length PQ

dM1: Full method. Likely to involve decimals here and the angle 71.6° . If decimals are used it will be followed by A0



Question Number	Scheme	Marks
1. (a)	Attempts gradient = $\frac{20 - -4}{-5 - 3} = (-3)$ Attempts equation of line $y - 20 = "-3"(x + 5)$ or $y + 4 = "-3"(x - 3)$ $y = -3x + 5$	M1 dM1 A1 (3)
(b)	Gradient $\frac{1}{3}$ or midpoint $(-1, 8)$ $y - 8 = \frac{1}{3}(x + 1)$ $x - 3y + 25 = 0$	B1ft M1 A1 (3) (6 marks)

(a)

M1: Attempts gradient = $\frac{\delta y}{\delta x}$ condoning slips. Look for an attempt at a difference in y coordinates over a difference in x coordinates. There is no requirement to simplify. They may use $20 - -4 = m(-5 - 3)$ to find m .

dM1: Full method for equation of line using gradient and either point – as shown or may use $y = "-3"x + c$ proceeding as far as finding c .

A1: $y = -3x + 5$ The coefficients must be simplified, award A0 for $y = \frac{24}{-8}x + 5$

Alt (a)

M1: Sets up two simultaneous equations using both points E.g $20 = -5m + c$ and $-4 = 3m + c$ (allow one slip).

dM1: Solves the pair of simultaneous equations to find values for m and c (accept any values following two suitable equations having been set up).

A1: $y = -3x + 5$

(b)

B1ft: Correct mid point or gradient, allowing follow through on the negative reciprocal of their gradient in (a)

M1: Attempts the perpendicular bisector (need not be a simplified form). Must be using $\pm$ the reciprocal of their gradient from (a) with a valid attempt at the midpoint – at least one correct coordinate (or calculation for) must be seen for the midpoint.

A1: $x - 3y + 25 = 0$ but allow any integer multiple and term in a different order (but must be on one side). Ignore if they give a label $l_2 = x - 3y + 25 = 0$

Question Number	Scheme	Marks
4(a)	$x + y = 6, y = 6x - 2x^2 + 1$ $\Rightarrow 6 - x = 6x - 2x^2 + 1$ $\Rightarrow 2x^2 - 7x + 5 = 0$ oe	M1
	$2x^2 - 7x + 5 = 0 \Rightarrow (2x - 5)(x - 1) = 0$ $\Rightarrow x = \frac{5}{2}, 1$	M1
	$x = \frac{5}{2} \Rightarrow y = \frac{7}{2}$ or $x = 1 \Rightarrow y = 5$	dM1
	(1, 5) and (2.5, 3.5)	A1
		(4)
(b)	$y \geq 6x - 2x^2 + 1$ oe $x + y \leq 6$ oe $x \geq a$ where $1 \leq a \leq 2.5$ (or $a \leq x \leq b$ where $1 \leq a \leq 2.5, b \geq 6$) $y \geq 0$ (or $0 \leq y \leq c$ where $c \geq 3.5$) Allow strict or non-strict inequalities	M1
		A1
		A1
		(3)
		Total 7

Note solutions relying on calculator technology are not acceptable so a complete method must be shown

(a) Answers only 0/4

M1: Uses the line and the curve to obtain a 3TQ in x or y equal to zero (may be implied by solving their 3TQ). Condone slips in their rearrangement.

M1: Solves their 3TQ which must be different to $6x - 2x^2 + 1 = 0$ by an acceptable method (factorising, formula or completing the square). They cannot state the roots without seeing a correct line of intermediate working. Allow recovery and condone lack of 3TQ = 0. If they use the quadratic formula, the values must be embedded in the formula before proceeding to the roots.

If they factorise then do not accept $2x^2 - 7x + 5 = 0 \Rightarrow (x - \frac{5}{2})(x - 1) = 0 \Rightarrow x = \dots$

Condone $-2x^2 + 7x - 5 = 0 \Rightarrow (2x - 5)(x - 1) = 0 \Rightarrow x = \dots$

dM1: Solves for y for at least one of their values of x or alternatively solves for x for at least one of their values of y . It is dependent on the previous method mark.

A1: (1, 5) and (2.5, 3.5) which do not need to be paired. May be seen as $x = \dots, y = \dots$. But cannot be awarded if they are incorrectly paired in their final answer. Condone omission of brackets around the coordinates. Ignore if they have assigned coordinates as P and Q . Cannot be awarded if x and y are the wrong way round or if given as (1, 3.5) and (2.5, 5)

(b) **Allow strict or non-strict inequalities and does not need to be consistent.**

Use of R instead of x or y will score 0 marks.

Note that $6x - 2x^2 + 1 \leq y \leq 6 - x$ counts as two inequalities.

M1: Any 2 of the inequalities.

A1: Any 3 of the inequalities. Ignore any reference to and/or or equivalent.

A1: All 4 correct. Withhold the final mark for use of “or” or $\cup$ is used between the different inequalities.



Question Number	Scheme	Marks
5(a)	$\angle BOD = \pi - 2 \times 0.7 = 1.742^*$	B1*
		(1)
(b)	Area of $BOD = \frac{1}{2} \times 3^2 \sin 1.742$ (= awrt 4.43)	M1
	Area of R is: $\frac{1}{2} \times 3^2 \times 1.742 - \frac{1}{2} \times 3^2 \sin 1.742$ or $\frac{1}{2} \times \pi \times 3^2 - \frac{1}{2} \times 3^2 \sin 1.742 - 2 \times \frac{1}{2} \times 3^2 \times 0.7$	dM1
	= awrt 3.4 (m ²)	A1
		(3)
(c)	$BD = \sqrt{3^2 + 3^2 - 2 \times 3 \times 3 \cos 1.742}$ (= awrt 4.59) or $BD = 2 \times 3 \sin \left(\frac{1.742}{2} \right)$ or $BD = 2 \times 3 \cos 0.7$ or $BD = \frac{3 \sin 1.742}{\sin \left(\frac{\pi - 1.742}{2} \right)}$ or arc $BCD = 3 \times 1.742$ (= 5.226)	M1
	Perimeter of R is: $3 \times 1.742 + "BD"$	dM1
	= awrt 9.8 (m)	A1
		(3)
		Total 7

They may work in degrees which is acceptable

(a)

B1*: Correct working to achieve 1.742 (or better). Alternatively, they may use $\angle BOD$ and add this to 2×0.7 to achieve π . They must write a minimal conclusion that $\angle BOD = 1.742$

May work in degrees: $180 - 2 \times \frac{0.7}{\pi} \times 180 = \text{awrt } 99.8^\circ \Rightarrow \frac{\text{awrt } 99.8}{180} \times \pi = 1.742$

(b)

M1: Correct strategy for the area of triangle BOD using $\angle BOD = 1.742$. May be implied by awrt 4.43

May work in degrees (1.742 radians as awrt 99.8)

eg. $\frac{1}{2} \times 3^2 \sin 99.8$



dM1: Applies a correct method for the area of R . The values embedded is sufficient. May also work in degrees correctly. It is dependent on the previous method mark.

$$\text{eg. } \pi \times 3^2 \times \frac{99.8}{360} - \frac{1}{2} \times 3^2 \sin 99.8$$

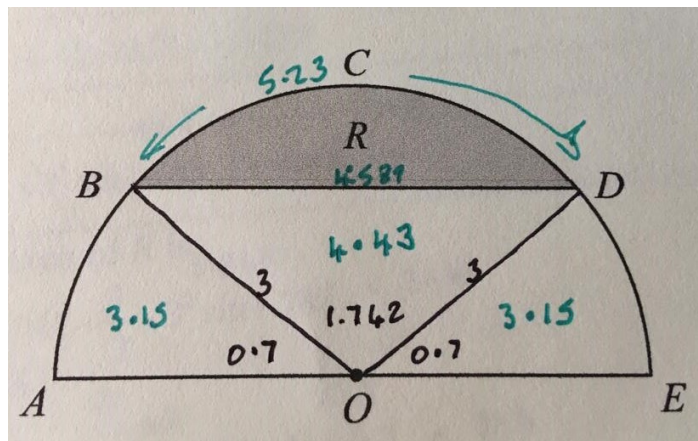
A1: awrt 3.4 (m²) Do not isw if they add or subtract other areas.

(c)

M1: Correct method for the length of BD which may be implied by awrt 4.59
OR a correct method for the length of arc BCD (= 5.226)
May work in degrees (Take 1.742 in radians as awrt 99.8).

dM1: Applies a fully correct method to find the perimeter of R by adding the length of arc BCD to their BD . **The methods to find both of these must be correct.**
It is dependent on the previous method mark.

A1: awrt 9.8 (m) Do not isw.



Question Number	Scheme	Marks
6(a)(i)	$x = 4 \Rightarrow f'(4) = \frac{(4+3)^2}{4\sqrt{4}} = \frac{49}{8} \quad (6.125)$	B1
(ii)	$y - 20 = \frac{49}{8}(x - 4)$	M1A1ft
	$49x - 8y - 36 = 0$	A1
		(4)
(b)	$f'(x) = \frac{(x+3)^2}{x\sqrt{x}} = \frac{x^2 + 6x + 9}{x\sqrt{x}}$ $= \frac{x^2}{x\sqrt{x}} + \frac{6x}{x\sqrt{x}} + \frac{9}{x\sqrt{x}} = \dots$	M1
	$= x^{\frac{1}{2}} + 6x^{-\frac{1}{2}} + 9x^{-\frac{3}{2}}$	A1
	$(f(x) =) \frac{2}{3}x^{\frac{3}{2}} + 12x^{\frac{1}{2}} - 18x^{-\frac{1}{2}} + c$	dM1 A1A1
	$20 = \frac{2}{3}(4)^{\frac{3}{2}} + 12(4)^{\frac{1}{2}} - 18(4)^{-\frac{1}{2}} + c \Rightarrow c = \dots$	M1
	$(f(x) =) \frac{2}{3}x^{\frac{3}{2}} + 12x^{\frac{1}{2}} - 18x^{-\frac{1}{2}} - \frac{1}{3}$	A1
		(7)
		Total 11

(a)(i)

B1: Correct value

(ii)M1: Correct straight line method using $y = 20$ and $x = 4$ with their $f'(4)$ as $(y - 20) = \frac{49}{8}(x - 4)$

allowing a sign error in one of the brackets.

If they use $y = mx + c$ they must proceed as far as $c = \dots$

Using a perpendicular gradient is M0.

A1ft: Correct equation in any form following through on their $f'(4)$

A1: Correct equation in the required form where all terms are on one side of an equation (allow any integer multiple)



(b)

M1: Squares the numerator (condone if only $x^2 + 9$ terms are found) and attempts to split the fraction. If this is done in (a) it must be used in (b). Score for one correct index being achieved from correct working: $\dots x^{\frac{1}{2}}$ or $\dots x^{-\frac{1}{2}}$ or $\dots x^{-\frac{3}{2}}$

A1: $x^{\frac{1}{2}} + 6x^{-\frac{1}{2}} + 9x^{-\frac{3}{2}}$ These terms may appear on different lines.

dM1: $\dots x^{\frac{1}{2}} \rightarrow \dots x^{\frac{3}{2}}$ or $\dots x^{-\frac{1}{2}} \rightarrow \dots x^{\frac{1}{2}}$ or $\dots x^{-\frac{3}{2}} \rightarrow \dots x^{-\frac{1}{2}}$
It is dependent on the previous method mark.

A1: Any 2 correct terms of $\frac{2}{3}x^{\frac{3}{2}} + 12x^{\frac{1}{2}} - 18x^{-\frac{1}{2}}$ which may be on different lines of their work (unsimplified or simplified)

A1: $(f(x) =) \frac{2}{3}x^{\frac{3}{2}} + 12x^{\frac{1}{2}} - 18x^{-\frac{1}{2}} (+c)$ (unsimplified or simplified) which may appear on different lines of their work (condone lack of $+c$ for this mark). Condone spurious notation.

M1: Uses $y = 20$ and $x = 4$ in their integrated function to find a value for “ c ”. (Not differentiated function or the original function)

A1: $\frac{2}{3}x^{\frac{3}{2}} + 12x^{\frac{1}{2}} - 18x^{-\frac{1}{2}} - \frac{1}{3}$ (accept equivalent simplified forms with exact coefficients and allow $y =$). Isw after a simplified correct answer, unless they attempt to multiply through by a value. Condone spurious notation.

Question Number	Scheme	Marks
7(a)	$4 \times -2 \times -9 = 72$ $p = "72" - 50$	M1
	$(p =) 22$	A1
		(2)
(b)	$(q =) -4, 2, 4.5$	B1B1
		(2)
(c)	$f(x) = (x+4)(x-2)(2x-9)$ $f(x) = (x^2 + 2x - 8)(2x - 9) = \dots x^3 \pm \dots x^2 \pm \dots x (\pm \dots)$	M1
	$= 2x^3 - 5x^2 - 34x (+72)$	A1
	$(f'(x) =) 6x^2 - 10x - 34$	M1A1
		(4)
(d)	$"6x^2 - 10x - 34" = -18$ $"6x^2 - 10x - 16" = 0 \Rightarrow x = \dots \left(-1, \frac{8}{3}\right)$	M1
	$-1 < x < \frac{8}{3}$	dM1A1
		(3)
		Total 11

(a)

M1: Correct method for p . Condone sign slips in finding the product of $4 \times -2 \times -9$
Alternatively multiplies out to achieve a cubic (eg ± 72) and subtracts 50 from their constant. $50 - 72 (= -22)$ is M0.

A1: 22 cao (22 with no working is M1A1) In the alternative method they must identify p as 22.
Check for answer next to question. If there is a contradiction then the answer in the main body of working takes precedence.
Accept $y = f(x) - 22$ but do not accept contradictions such as $f(x) - 22 \Rightarrow p = -22$ (M1A0)

(b) Check next to the question (ignore any references to x)

B1: Any 1 correct value of $-4, 2, 4.5$ in their final answer. If they proceed to changing the sign on the value then B0.

B1: $-4, 2, 4.5$ cao but if they exclude any of these then withhold this mark. If there is a contradiction, then the answer in the main body of working takes precedence and if they state the correct values followed by $4, -2, -4.5$ then withhold this mark.