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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Trigonometry & Radians (P1.6)

International A Level Mathematics · P1: Pure Mathematics 1 (WMA11)

46 questions · 346 marks · Mark schemes included

Past papers January 2019 - June 2025 · 3 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

46 questions · 346 marks · 3 sub-topics

SUB-TOPIC P1.6a

SINE RULE, COSINE RULE & AREA OF A TRIANGLE

Sub-topic P1.6a · 6 questions · 37 marks · lowest tariff first



4. A parallelogram $ABCD$ has area 40 cm^2

Given that AB has length 10 cm, BC has length 6 cm and angle DAB is obtuse, find

- (a) the size of angle DAB , in degrees, to 2 decimal places, (3)

- (b) the length of diagonal BD , in cm, to one decimal place. (2)

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Question 4 continued

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Q4

(Total 5 marks)



2. In the triangle ABC ,

- $AB = 21$ cm
- $BC = 13$ cm
- angle $BAC = 25^\circ$
- angle $ACB = x^\circ$

(a) Use the sine rule to find the value of $\sin x^\circ$, giving your answer to 4 decimal places. (2)

Given also that AB is the longest side of the triangle,

(b) find the value of x , giving your answer to 2 decimal places. (3)

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Question 2 continued

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(Total 5 marks)

Q2



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 5 marks)



Question 7 continued

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Q7

(Total 6 marks)



Question 5 continued



Question 5 continued



Question 5 continued

(Total for Question 5 is 7 marks)



Question 3 continued

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Question 3 continued

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(Total 9 marks)

Q3



SUB-TOPIC P1.6b

RADIANS, ARC LENGTH & SECTOR AREA

Sub-topic P1.6b · 21 questions · 193 marks · lowest tariff first



Question 1 continued

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(Total 5 marks)

Q1



Given that the area of the sector is 6 cm^2 and that the perimeter of the sector is 10 cm ,

$$3\theta^2 - 13\theta + 12 = 0$$

(b) Hence find possible values of r and θ .



Q10

Question 5 continued

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Question 5 continued

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Question 5 continued

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Lined area for writing the answer to Question 5.

(Total 7 marks)

Q5



9.

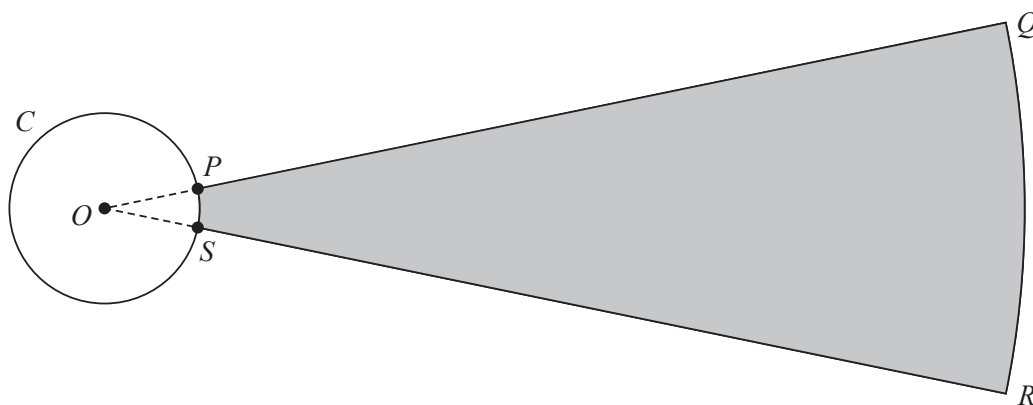
Diagram **NOT**
accurately drawn

Figure 3

Figure 3 shows the plan view of the area being used for a ball-throwing competition.

Competitors must stand within the circle C and throw a ball as far as possible into the target area, $PQRS$, shown shaded in Figure 3.

Given that

- circle C has centre O
- P and S are points on C
- $OPQRSO$ is a sector of a circle with centre O
- the length of arc PS is 0.72 m
- the size of angle POS is 0.6 radians

(a) show that $OP = 1.2\text{ m}$

(1)

Given also that

- the target area, $PQRS$, is 90 m^2
- length $PQ = x$ metres

(b) show that

$$5x^2 + 12x - 1500 = 0$$

(3)

(c) Hence calculate the total perimeter of the target area, $PQRS$, giving your answer to the nearest metre.

(3)





Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)



Question 5 continued

Lined area for writing the answer to Question 5 continued.





Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 7 marks)

Question 4 continued

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Question 4 continued

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Lined area for writing the answer to Question 4 continued.



Question 4 continued

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(Total 9 marks)

Q4



Question 5 continued

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Question 5 continued

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Handwriting practice area with horizontal lines.



Question 5 continued

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Q5

(Total 9 marks)



8.

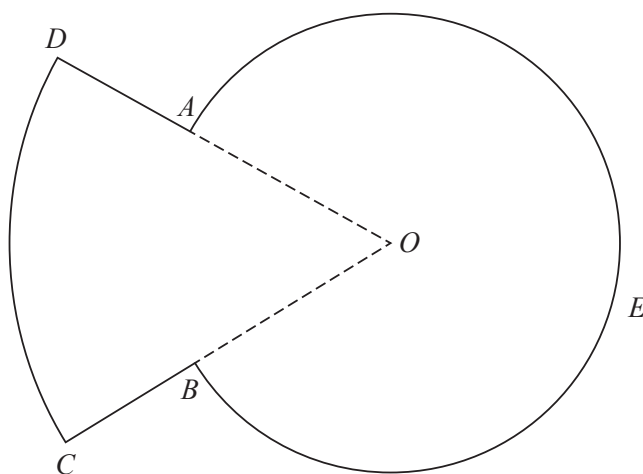


Figure 3

Figure 3 shows a sketch of the plan view of a platform.

The plan view of the platform consists of a sector DOC of a circle centre O joined to a sector $AOBEA$ of a different circle, also with centre O .

Given that

- angle $AOB = 0.8$ radians
- arc length $CD = 9$ m
- $DA : AO = 3 : 5$

(a) show that $AO = 7.03$ m to 3 significant figures.

(3)

(b) Find the perimeter of the platform, in m, to 3 significant figures.

(3)

(c) Find the total area of the platform, giving your answer in m^2 to the nearest whole number.

(3)

Question 8 continued





Question 8 continued

(Total for Question 8 is 9 marks)



Question 6 continued



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 9 marks)





Question 5 continued

(Total for Question 3 is 3 marks)



6.

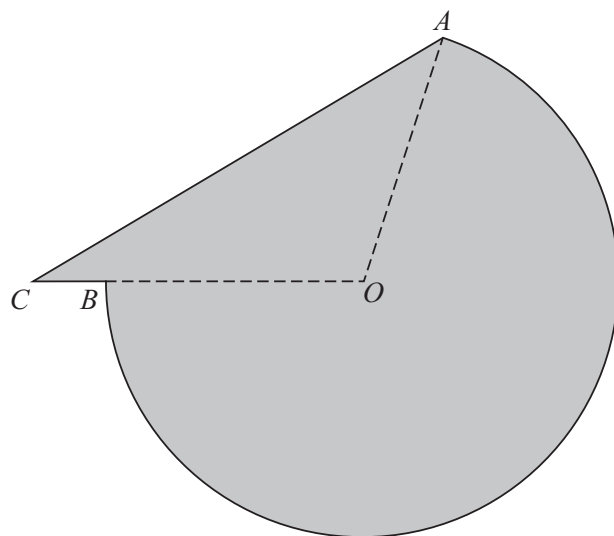


Figure 2

The shaded area in Figure 2 shows the plan view of a helicopter landing pad.

The area consists of the major sector AOB of a circle centre O joined to a triangle AOC .

Given that

- $AO = OB = 15$ m
- $BC = 2$ m
- CBO is a straight line
- angle $ACO = 0.6$ radians

- (a) show that angle COA is 1.847 radians to 3 decimal places. (3)
- (b) Find the total area of the helicopter landing pad.
Give your answer in m^2 to 3 significant figures. (3)
- (c) Find the perimeter of the helicopter landing pad.
Give your answer in metres to 3 significant figures. (3)

Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 9 marks)



9.

Diagram NOT to scale

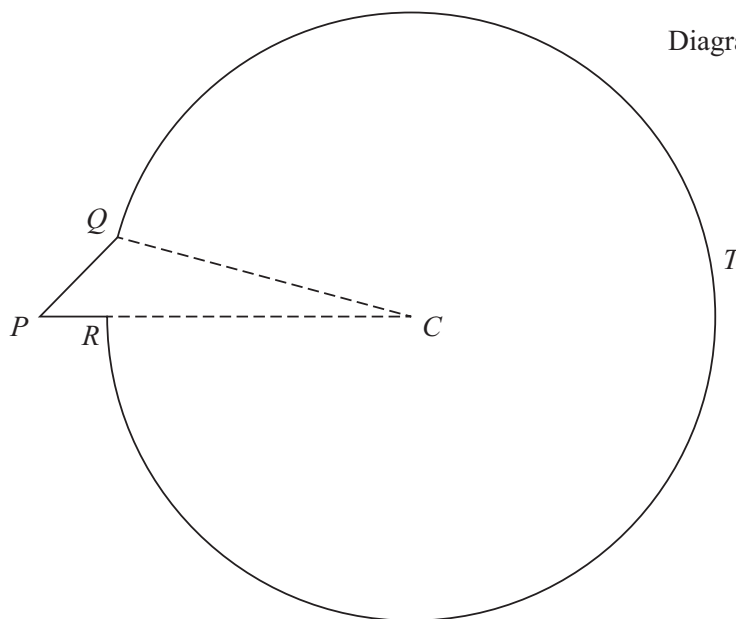


Figure 4

Figure 4 shows the outline of a sign that is used to advertise a bird sanctuary.

The sign is composed of a triangle CPQ joined to a sector $QCRTQ$ of a circle, centre C .

Given that

- angle $QPR = 0.8$ radians
- $PQ = 0.5$ m
- $PC = 1.84$ m
- PRC is a straight line

(a) find the radius, CQ , of the sector, in metres to 3 decimal places.

(2)

(b) Hence show that angle PCQ is 0.236 radians to 3 decimal places.

(2)

(c) Find the total area of the sign, giving your answer in m^2 to one decimal place.

(3)

(d) Find the total perimeter of the sign, giving your answer in metres to one decimal place.

(2)



Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 9 marks)



Question 7 continued

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Question 7 continued

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Question 7 continued

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Q7

(Total 10 marks)



3.

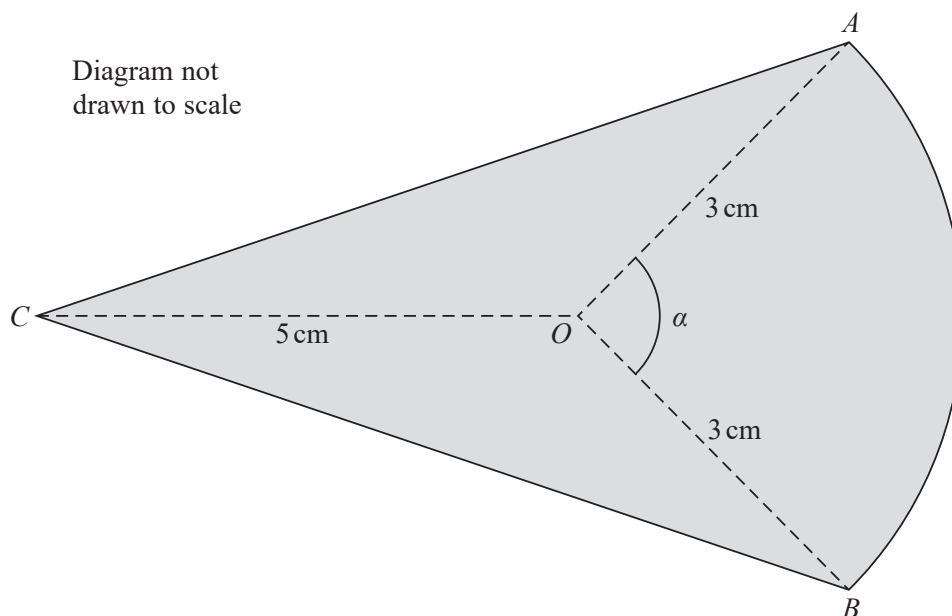
Diagram not
drawn to scale

Figure 1

Figure 1 shows the design for a badge.

The design consists of two congruent triangles, AOC and BOC , joined to a sector AOB of a circle centre O .

- Angle $AOB = \alpha$
- $AO = OB = 3$ cm
- $OC = 5$ cm

Given that the area of sector AOB is 7.2 cm^2 (a) show that $\alpha = 1.6$ radians.

(2)

(b) Hence find

- the area of the badge, giving your answer in cm^2 to 2 significant figures,
- the perimeter of the badge, giving your answer in cm to one decimal place.

(8)

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Question 3 continued

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Question 3 continued

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Q3

7.

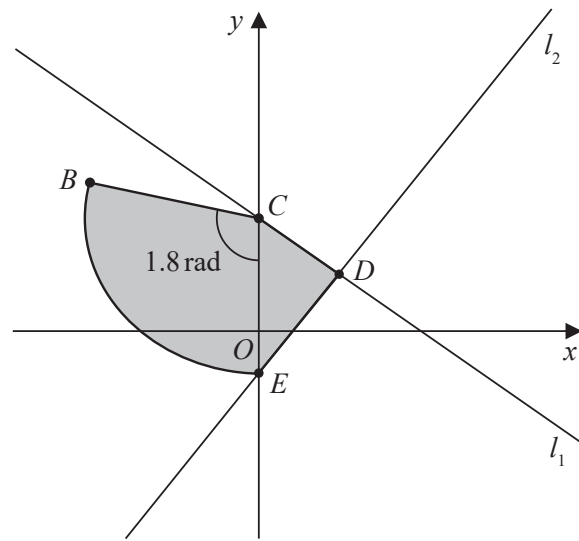


Figure 3

The line l_1 has equation $4y + 3x = 48$

The line l_1 cuts the y -axis at the point C , as shown in Figure 3.

(a) State the y coordinate of C .

(1)

The point $D(8, 6)$ lies on l_1

The line l_2 passes through D and is perpendicular to l_1

The line l_2 cuts the y -axis at the point E as shown in Figure 3.

(b) Show that the y coordinate of E is $-\frac{14}{3}$

(3)

A sector BCE of a circle with centre C is also shown in Figure 3.

Given that angle BCE is 1.8 radians,

(c) find the length of arc BE .

(3)

The region $CBED$, shown shaded in Figure 3, consists of the sector BCE joined to the triangle CDE .

(d) Calculate the exact area of the region $CBED$.

(3)

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Question 7 continued

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Question 7 continued

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Q7

Question 7 continued

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Question 7 continued

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Lined area for writing the answer to Question 7 continued.



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Q7

(Total 10 marks)



8.

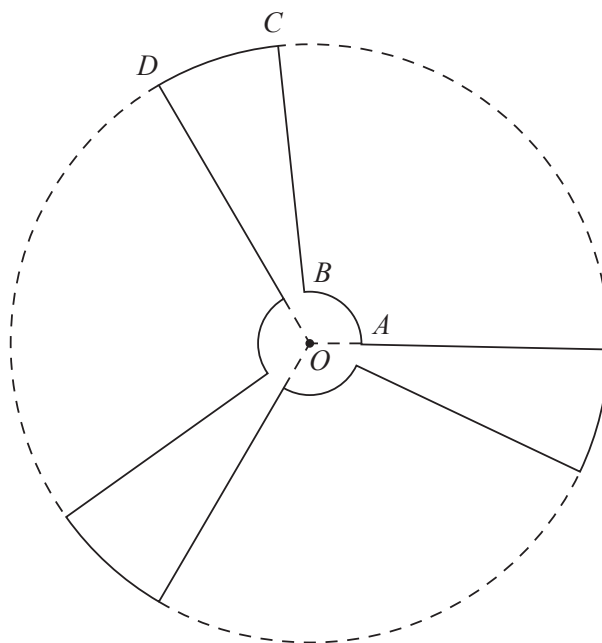
Leave
blank**Figure 3**

Figure 3 shows a sketch of the outline of the face of a ceiling fan viewed from below.

The fan consists of three identical sections congruent to $OABCD$, shown in Figure 3, where

- $OABO$ is a sector of a circle with centre O and radius 9 cm
- $OBCDO$ is a sector of a circle with centre O and radius 84 cm
- angle $AOD = \frac{2\pi}{3}$ radians

Given that the length of the arc AB is 15 cm,

- (a) show that the length of the arc CD is 35.9 cm to one decimal place.

(3)

The face of the fan is modelled to be a flat surface.

Find, according to the model,

- (b) the perimeter of the face of the fan, giving your answer to the nearest cm,

(2)

- (c) the surface area of the face of the fan.

Give your answer to 3 significant figures and make your units clear.

(5)

Question 8 continued

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Lined area for writing the answer to Question 8 continued.



Question 8 continued

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Question 8 continued

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(Total 10 marks)

Q8







Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 10 marks)



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 10 marks)



8.

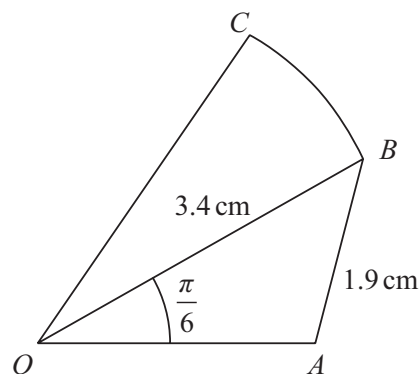


Figure 1

Figure 1 shows a sketch of a design for a badge.

The design consists of a triangle OAB joined to a sector OBC of a circle with centre O

In the design

- $OB = 3.4$ cm
- $AB = 1.9$ cm
- angle $AOB = \frac{\pi}{6}$ radians
- angle $OAB > \frac{\pi}{2}$ radians

Making your method clear,

(a) find the size of angle OAB , giving your answer in radians to 4 significant figures, (3)

(b) find the area of triangle OAB , in cm^2 , giving your answer to 3 significant figures. (2)

Given that the ratio of the area of sector OBC to the area of triangle OAB is 3 : 2

(c) show that angle BOC is 0.462 radians to 3 significant figures. (3)

(d) Hence find the perimeter of the badge, in cm, to the nearest integer. (5)





Question 8 continued

(Total for Question 8 is 13 marks)



8.

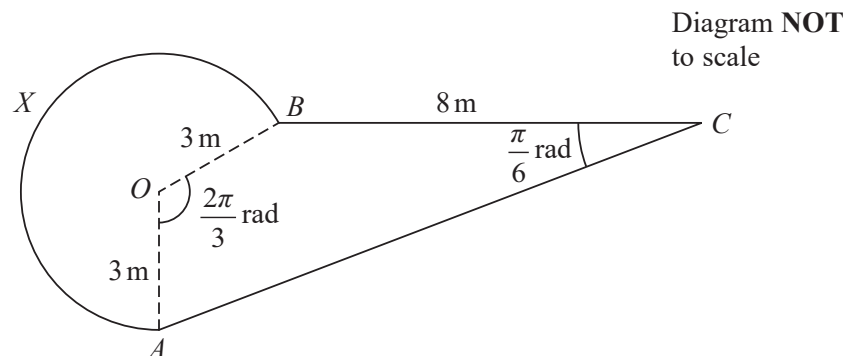


Figure 2

Figure 2 shows the plan view of a design for a pond.

The design consists of a sector $AOBX$ of a circle centre O joined to a quadrilateral $AOBC$.

- $BC = 8$ m
- $OA = OB = 3$ m
- angle AOB is $\frac{2\pi}{3}$ radians
- angle BCA is $\frac{\pi}{6}$ radians

(a) Calculate (i) the exact area of the sector $AOBX$,

(ii) the exact perimeter of the sector $AOBX$.

(5)

(b) Calculate the exact area of the triangle AOB .

(2)

(c) Show that the length AB is $3\sqrt{3}$ m.

(2)

(d) Find the total surface area of the pond. Give your answer in m^2 correct to 2 significant figures.

(5)

Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 14 marks)



SUB-TOPIC P1.6c

TRIGONOMETRIC GRAPHS

Sub-topic P1.6c · 19 questions · 116 marks · lowest tariff first



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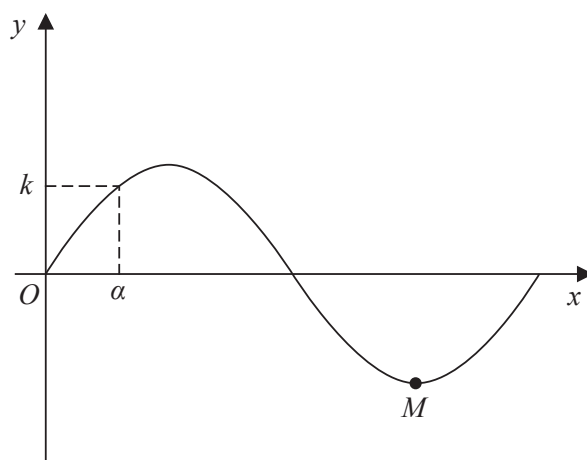


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = \sin\left(\frac{x}{12}\right)$, where x is measured in radians. The point M shown in Figure 5 is a minimum point on C .

- (a) State the period of C . **(1)**
- (b) State the coordinates of M . **(1)**

Question 9 continued

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(Total 4 marks)

Q9



9.

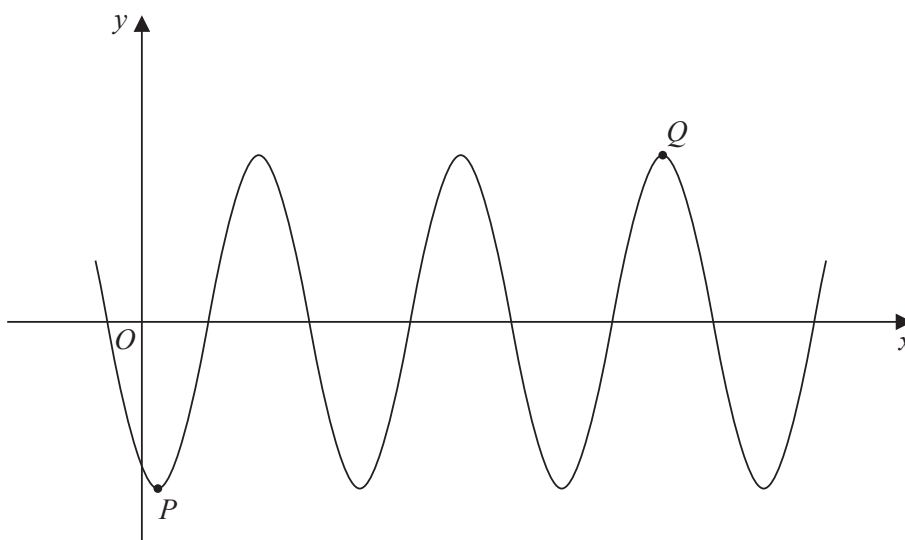
Leave
blank**Figure 4**

Figure 4 shows part of the curve with equation

$$y = A \cos(x - 30)^\circ$$

where A is a constant.

The point P is a minimum point on the curve and has coordinates $(30, -3)$ as shown in Figure 4.

- (a) Write down the value of A . (1)

The point Q is shown in Figure 4 and is a maximum point.

- (b) Find the coordinates of Q . (3)

Question 9 continued

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Handwriting practice area with horizontal lines.

(Total 4 marks)

Q9



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 4 marks)



Question 7 continued

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(Total 5 marks)

Q7



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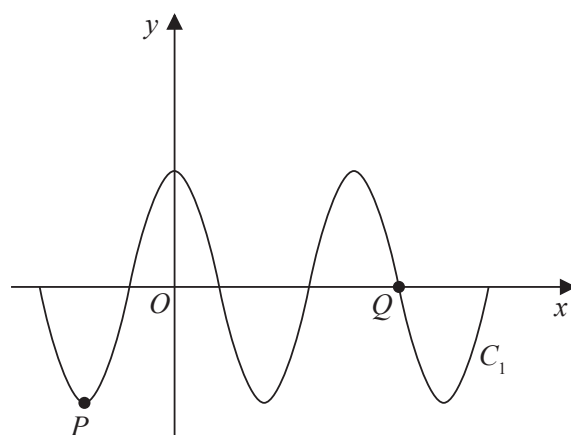


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation $y = 4 \cos x^\circ$

The point P and the point Q lie on C_1 and are shown in Figure 1.

(a) State

- (i) the coordinates of P ,
- (ii) the coordinates of Q .

(3)

The curve C_2 has equation $y = 4 \cos x^\circ + k$, where k is a constant.

Curve C_2 has a minimum y value of -1

The point R is the maximum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R .

(2)

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Question 3 continued

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Q3

(Total 5 marks)



4.

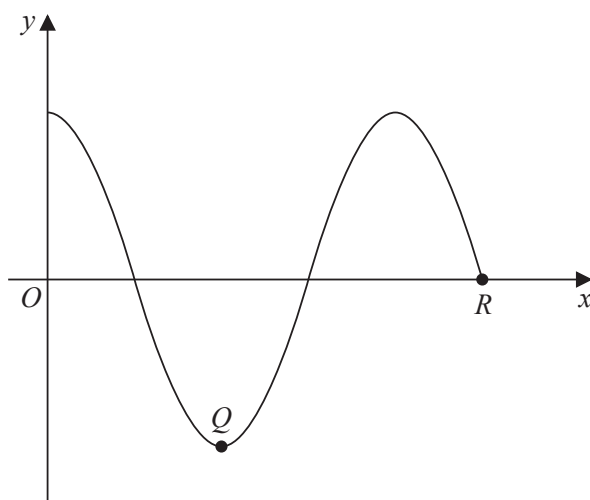


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \cos 2x^\circ \quad 0 \leq x \leq k$$

The point Q and the point $R(k, 0)$ lie on the curve and are shown in Figure 2.

(a) State

- (i) the coordinates of Q ,
- (ii) the value of k .

(3)

(b) Given that there are exactly two solutions to the equation

$$\cos 2x^0 = p \quad \text{in the region } 0 \leq x \leq k$$

find the range of possible values for p .

(2)

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Question 4 continued

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Lined area for writing the answer to Question 4.

(Total 5 marks)

Q4

Mark box for Q4.



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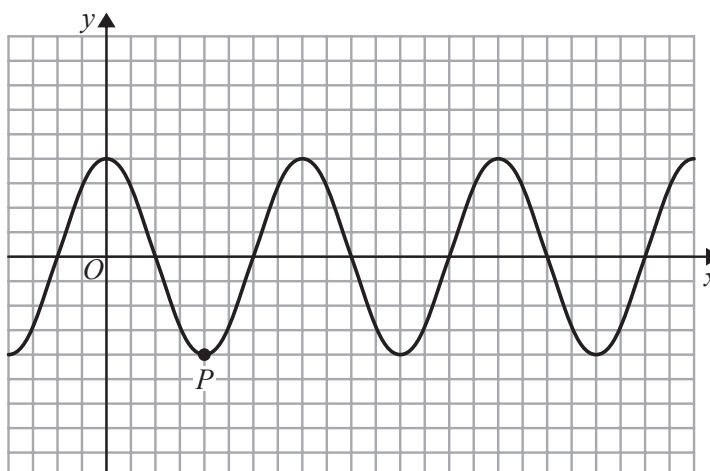


Figure 2

Figure 2 shows a plot of part of the curve with equation $y = \cos 2x$ with x being measured in radians.

The point P , shown on Figure 2, is a minimum point on the curve.

(a) State the coordinates of P .

(2)

A copy of Figure 2, called Diagram 1, is shown at the top of the next page.

(b) Sketch, on Diagram 1, the curve with equation $y = \sin x$

(2)

(c) Hence, or otherwise, deduce the number of solutions of the equation

(i) $\cos 2x = \sin x$ that lie in the region $0 \leq x \leq 20\pi$

(ii) $\cos 2x = \sin x$ that lie in the region $0 \leq x \leq 21\pi$

(2)

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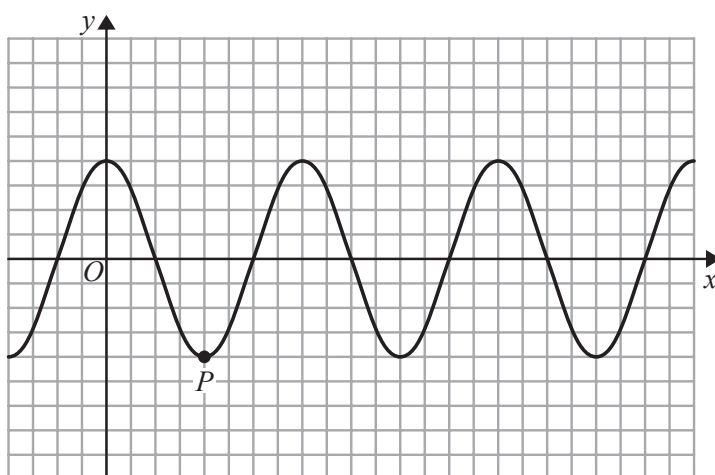


Diagram 1

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(Total 6 marks)

10.

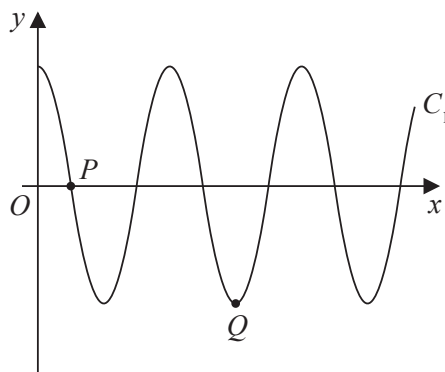


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 3 \cos\left(\frac{x}{n}\right)^\circ \quad x \geq 0$$

where n is a constant.

The curve C_1 cuts the positive x -axis for the first time at point $P(270, 0)$, as shown in Figure 4.

(a) (i) State the value of n

(ii) State the period of C_1

(2)

The point Q , shown in Figure 4, is a minimum point of C_1

(b) State the coordinates of Q .

(2)

The curve C_2 has equation $y = 2 \sin x^\circ + k$, where k is a constant.

The point $R\left(a, \frac{12}{5}\right)$ and the point $S\left(-a, -\frac{3}{5}\right)$, both lie on C_2

Given that a is a constant less than 90

(c) find the value of k .

(2)

(Total for Question 10 is 6 marks)



6.

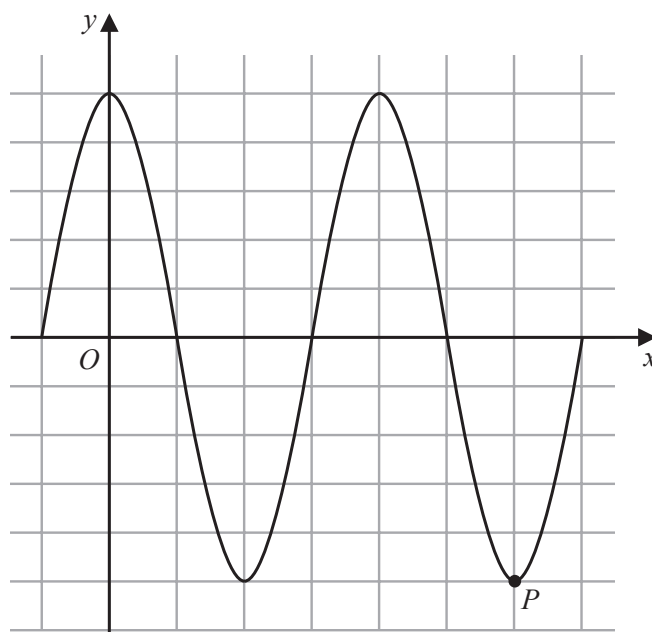


Figure 2

Figure 2 shows a plot of part of the curve C_1 with equation

$$y = 5 \cos x$$

with x being measured in degrees.

The point P , shown in Figure 2, is a minimum point on C_1

(a) State the coordinates of P

(2)

The point Q lies on a different curve C_2

Given that point Q

- is a maximum point on the curve
- is the maximum point with the **smallest** x coordinate, $x > 0$

(b) find the coordinates of Q when

(i) C_2 has equation $y = 5 \cos x - 2$

(ii) C_2 has equation $y = -5 \cos x$

(4)

(Total for Question 6 is 6 marks)



11.

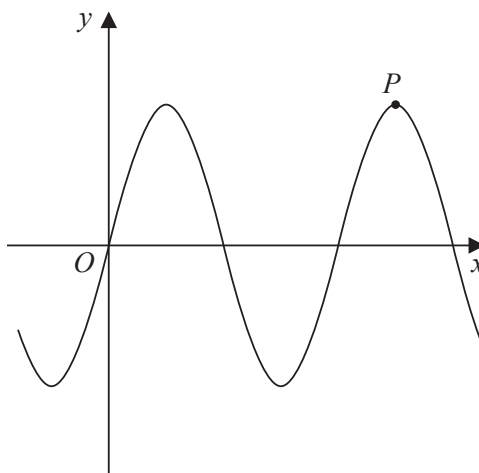


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 12 \sin x$$

where x is measured in radians.

The point P shown in Figure 4 is a maximum point on C_1

(a) Find the coordinates of P .

(2)

The curve C_2 has equation

$$y = 12 \sin x + k$$

where k is a constant.

Given that the **maximum** value of y on C_2 is 3

(b) find the coordinates of the **minimum** point on C_2 which has the **smallest** positive x coordinate.

(2)

The curve C_3 has equation

$$y = 12 \sin(x + B)$$

where B is a positive constant.

Given that $\left(\frac{\pi}{4}, A\right)$, where A is a constant, is the **minimum** point on C_3 which has the **smallest** positive x coordinate,

(c) find

(i) the value of A ,

(ii) the smallest possible value of B .

(2)



Question 11 continued

Lined area for writing the answer to Question 11.



Question 11 continued

Lined area for writing the answer to Question 11.

(Total for Question 11 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS



4.

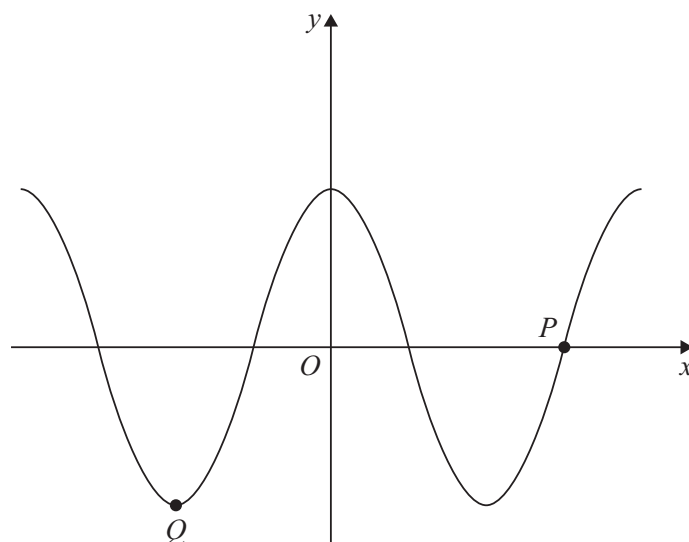


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 4 \cos x$$

where x is measured in degrees.

The points P and Q lie on the curve and are shown in Figure 1.

(a) State the coordinates of P .

(1)

(b) State the coordinates of Q .

(2)

(c) State the **number** of solutions of the equation

(i) $4 \cos x = 3$ in the interval $0 < x \leq 18\,000^\circ$

(ii) $5 + 4 \cos x = 1$ in the interval $-720^\circ < x \leq 720^\circ$

(iii) $4 \cos x - 3 = 1$ in the interval $-1080^\circ \leq x \leq 1080^\circ$

(3)

(Total for Question 4 is 6 marks)



9.

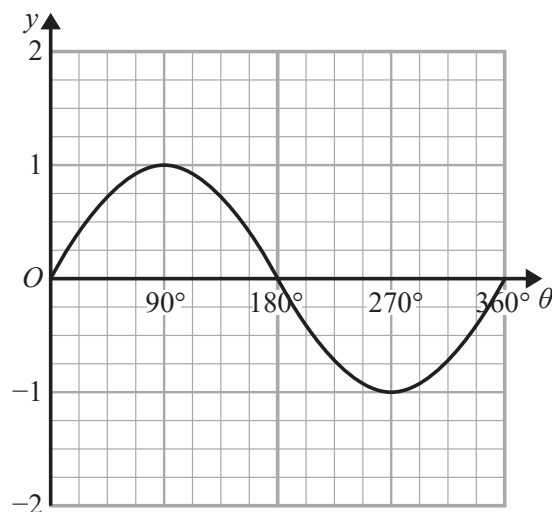
Leave
blank**Figure 3**

Figure 3 shows a plot of the curve with equation $y = \sin \theta$, $0 \leq \theta \leq 360^\circ$

- (a) State the coordinates of the minimum point on the curve with equation

$$y = 4 \sin \theta, \quad 0 \leq \theta \leq 360^\circ$$

(2)

A copy of Figure 3, called Diagram 1, is shown on the next page.

- (b) On Diagram 1, sketch and label the curves

(i) $y = 1 + \sin \theta$, $0 \leq \theta \leq 360^\circ$

(ii) $y = \tan \theta$, $0 \leq \theta \leq 360^\circ$

(2)

- (c) Hence find the number of solutions of the equation

(i) $\tan \theta = 1 + \sin \theta$ that lie in the region $0 \leq \theta \leq 2160^\circ$

(ii) $\tan \theta = 1 + \sin \theta$ that lie in the region $0 \leq \theta \leq 1980^\circ$

(3)

Question 9 continued

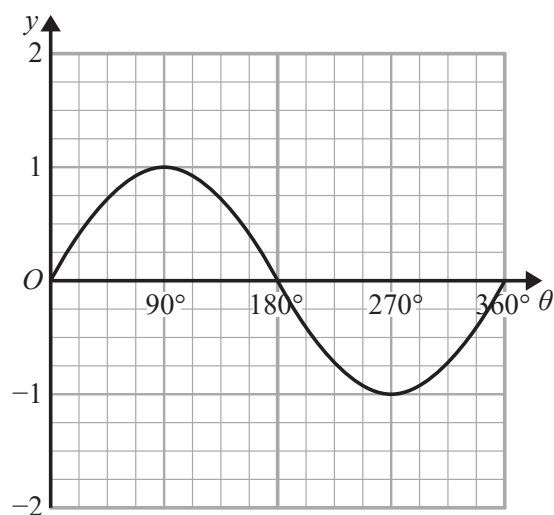
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Diagram 1

(Total 7 marks)

Q9



9.

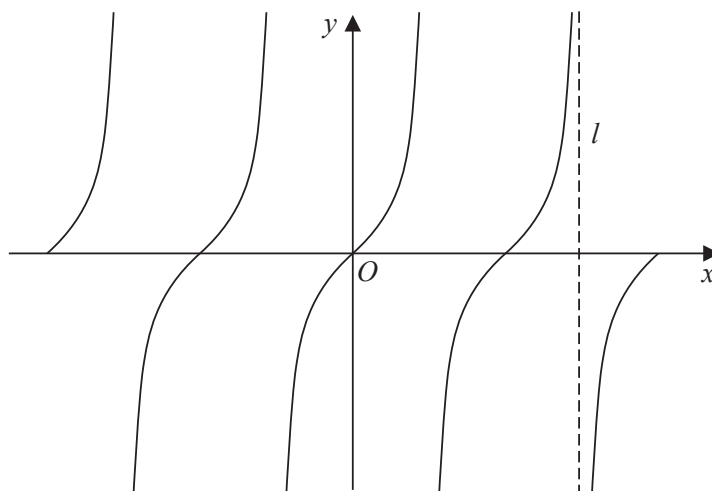


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \quad -2\pi \leq x \leq 2\pi$$

The line l , shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l .

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \quad -2\pi \leq x \leq 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leq x \leq 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leq x \leq 40\pi$

(ii) $-10\pi \leq x \leq \frac{5}{2}\pi$

(2)

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Question 9 continued

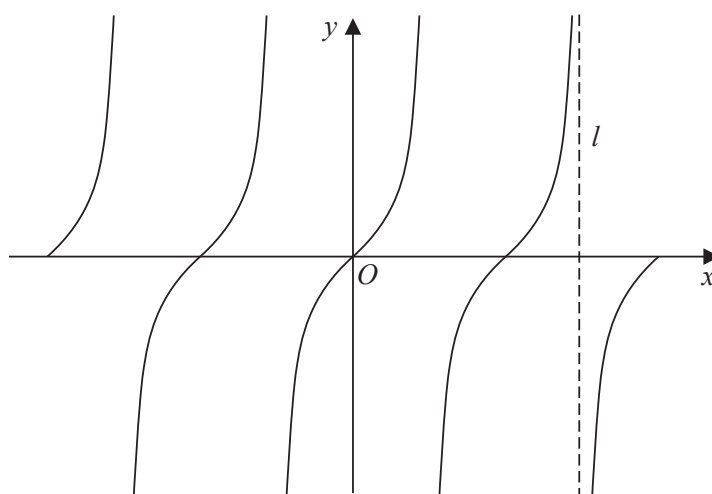
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Diagram 1

Question 9 continued

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Handwriting area with horizontal lines.

(Total 7 marks)

Q9

TOTAL FOR PAPER IS 75 MARKS

END



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5.

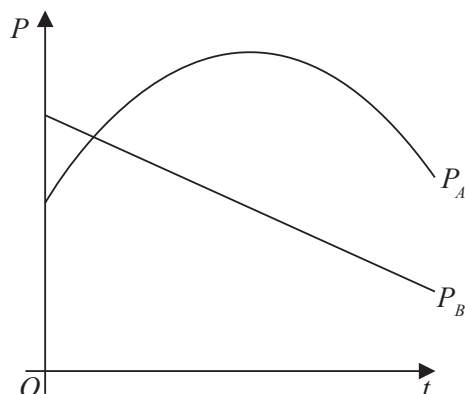


Figure 2

The share value of two companies, company *A* and company *B*, has been monitored over a 15-year period.

The share value P_A of **company A**, in millions of pounds, is modelled by the equation

$$P_A = 53 - 0.4(t - 8)^2 \quad t \geq 0$$

where t is the number of years after monitoring began.

The share value P_B of **company B**, in millions of pounds, is modelled by the equation

$$P_B = -1.6t + 44.2 \quad t \geq 0$$

where t is the number of years after monitoring began.

Figure 2 shows a graph of both models.

Use the equations of one or both models to answer parts (a) to (d).

- (a) Find the difference between the share value of **company A** and the share value of **company B** at the point monitoring began. (2)
- (b) State the maximum share value of **company A** during the 15-year period. (1)
- (c) Find, using algebra and showing your working, the times during this 15-year period when the share value of **company A** was greater than the share value of **company B**. (4)
- (d) Explain why the model for **company A** should not be used to predict its share value when $t = 20$ (1)

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Question 5 continued

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Lined area for writing the answer to Question 5.

(Total 8 marks)

Q5



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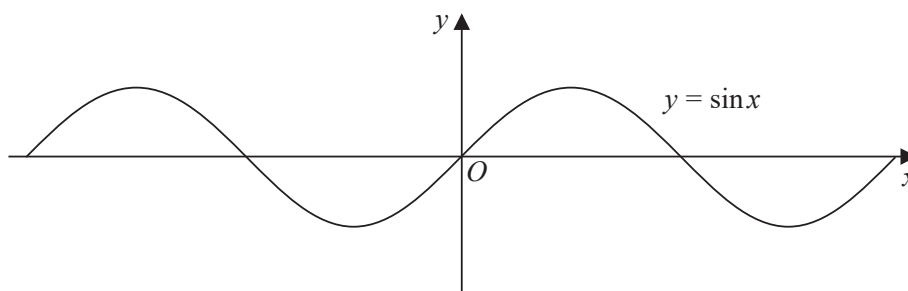
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Figure 4

Figure 4 shows part of the graph of the curve with equation $y = \sin x$

Given that $\sin \alpha = p$, where $0 < \alpha < 90^\circ$

(a) state, in terms of p , the value of

(i) $2 \sin(180^\circ - \alpha)$

(ii) $\sin(\alpha - 180^\circ)$

(iii) $3 + \sin(180^\circ + \alpha)$

(3)

A copy of Figure 4, labelled Diagram 1, is shown on page 27.

On Diagram 1,

(b) sketch the graph of $y = \sin 2x$

(2)

(c) Hence find, in terms of α , the x coordinates of any points in the interval $0 < x < 180^\circ$ where

$$\sin 2x = p$$

(3)

Question 9 continued

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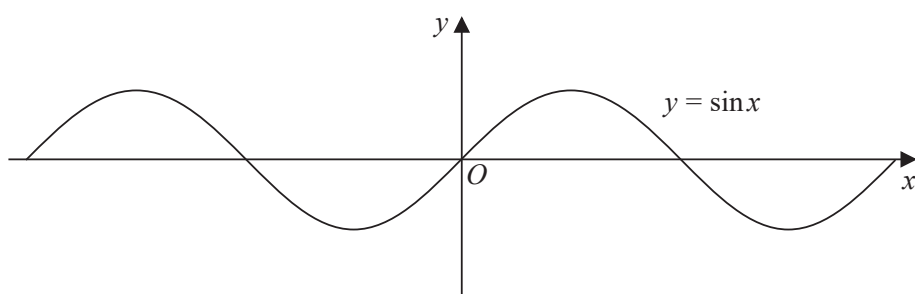


Diagram 1

(Total 8 marks)

Q9



9.

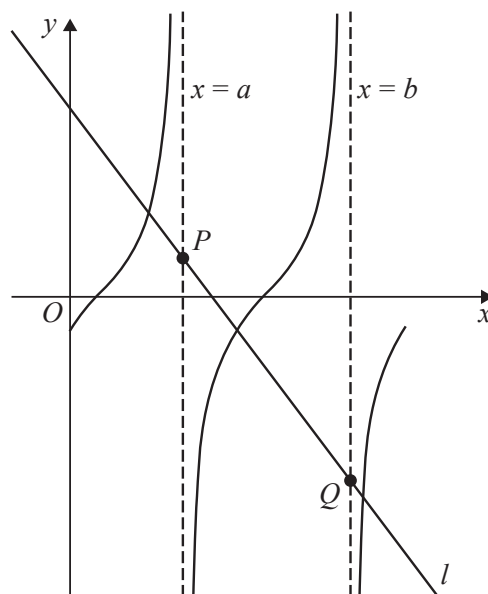


Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan\left(x - \frac{\pi}{6}\right)$ for $0 \leq x \leq 2\pi$
- part of the straight line l with equation $y = \pi - x$

(a) State the number of solutions of the equation

(i) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leq x \leq 2\pi$

(ii) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leq x \leq 100\pi$

(iii) $\tan\left(x - \frac{\pi}{6}\right) = \pi + x$ in the interval $0 \leq x \leq 2\pi$

(3)

The line with equation $x = a$, shown in Figure 3, is the asymptote to the curve with the smallest positive x coordinate.

(b) State the value of a

(1)

The line with equation $x = b$, also shown in Figure 3, is the asymptote to the curve with the second smallest positive x coordinate.

The line l meets $x = a$ at point P and meets $x = b$ at point Q as shown in Figure 3.

(c) Find the midpoint of the line segment PQ .

(4)



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 8 marks)

7.

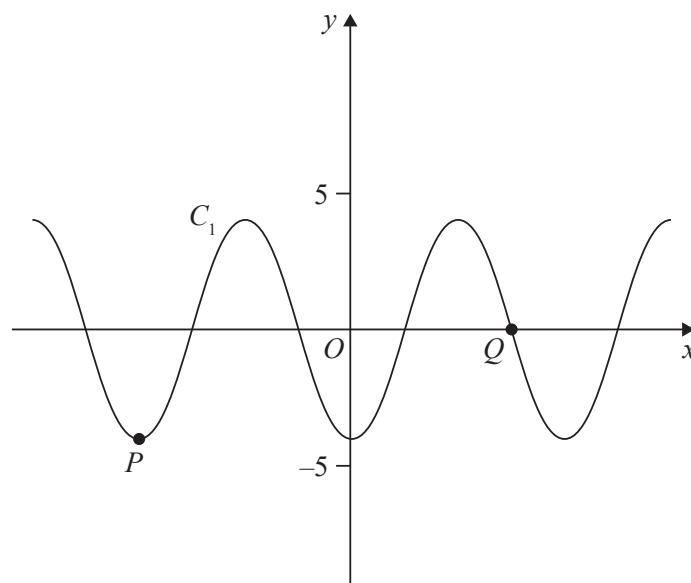
**Figure 3**

Figure 3 shows a plot of part of the curve C_1 with equation

$$y = -4 \cos x$$

where x is measured in radians.

Points P and Q lie on the curve and are shown in Figure 3.

(a) State

- (i) the coordinates of P
- (ii) the coordinates of Q

(3)

The curve C_2 has equation $y = -4 \cos x + k$ where x is measured in radians and k is a constant.

Given that C_2 has a maximum y value of 11

- (b) (i) state the value of k
- (ii) state the coordinates of the minimum point on C_2 with the smallest positive x coordinate.

(3)

On the opposite page there is a copy of Figure 3 labelled Diagram 1.

(c) Using Diagram 1, state the number of solutions of the equation

$$-4 \cos x = 5 - \frac{10}{\pi} x$$

giving a reason for your answer.

(2)

Question 7 continued

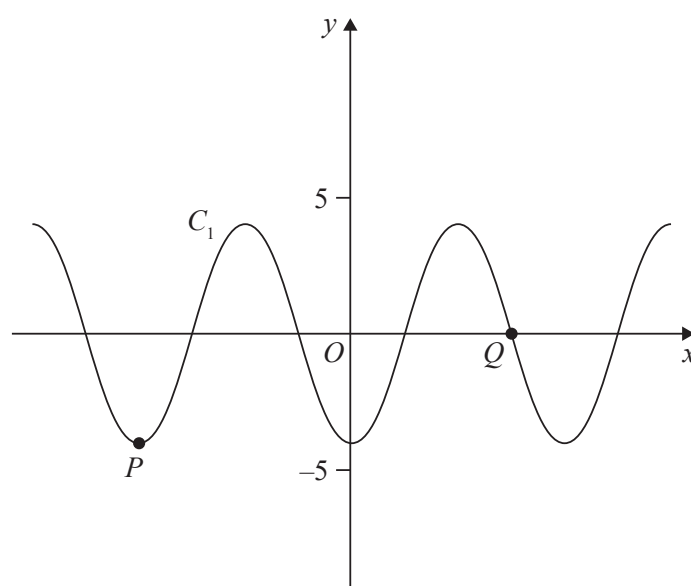


Diagram 1

(Total for Question 7 is 8 marks)



9. (i)

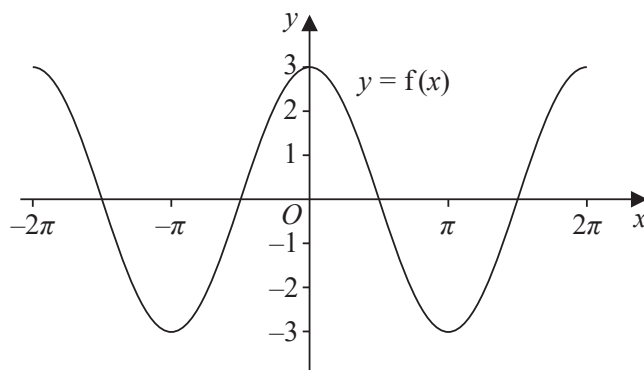


Figure 3

Figure 3 shows part of the graph of the trigonometric function with equation $y = f(x)$

(a) Write down an expression for $f(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = f\left(x + \frac{\pi}{4}\right)$

Show clearly the coordinates of all the points where the curve intersects the coordinate axes.

(3)

(ii)

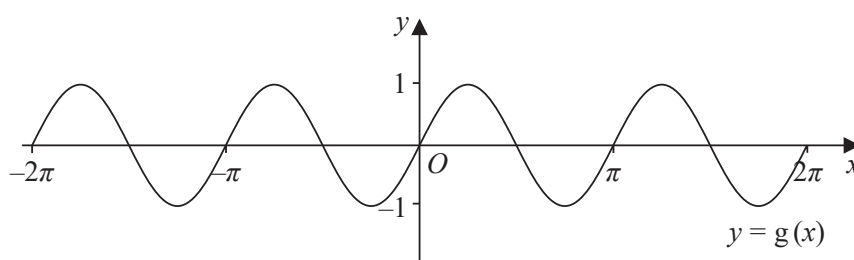


Figure 4

Figure 4 shows part of the graph of the trigonometric function with equation $y = g(x)$

(a) Write down an expression for $g(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = g(x) - 2$

Show clearly the coordinates of the y intercept.

(2)



Question 9 continued



Question 9 continued



Question 9 continued

(Total for Question 9 is 9 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
3. (a)	$x^2 - 5x + 13 = (x - 2.5)^2 - 2.5^2 + 13 = (x - 2.5)^2 + 6.75$ Coordinates $M = (2.5, 6.75)$	M1 A1
(b)	Attempts the equation of l using their M $y = \frac{6.75}{2.5}x$ ($y = 2.7x$) Attempts to solve their $y = 2.7x$ with $y = x^2 - 5x + 13$ $\Rightarrow 2.7x = x^2 - 5x + 13 \Rightarrow x^2 - 7.7x + 13 = 0 \Rightarrow (x - 2.5)(x - 5.2) = 0$ $x = 5.2$ oe Coordinates $N = (5.2, 14.04)$	M1 M1 A1 dM1 A1
(c)	States two of $y < x^2 - 5x + 13$, $y > 2.7x$, $0 \leq x < 2.5$ States all three of $y < x^2 - 5x + 13$, $y > 2.7x$, $0 \leq x < 2.5$	M1 A1ft
		(10 marks)

(a)

M1 For attempting to complete the square. Look for $(x - 2.5)^2$ A1 $(x - 2.5)^2 + 6.75$ or for correctly extracting $x = 2.5$ as the x coordinate.A1 $M = (2.5, 6.75)$ or $\left(\frac{5}{2}, \frac{27}{4}\right)$

(b)

M1 Uses their M to find an equation for l . Look for a correct attempt at the gradient, so $y = \frac{6.75}{2.5}x$ or $y = "6.75" = \frac{6.75}{2.5}(x - "2.5")$ M1 Depends on having made an attempt (not necessarily correct) to use O and M to find the equation of l . Attempts to solve their equation for l with $y = x^2 - 5x + 13$ Look for a full attempt leading to $x = \dots$ *Correct answers following a correct simplified quadratic is fine for the method.*A1 $x = 5.2$ or equivalent such as $x = \frac{26}{5}$ dM1 Depends on second M mark. Substitutes their $x = 5.2$ into either equation to find $y = \dots$ If a y value follows an x value with no clearly incorrect working, then allow as an attempt.A1 Coordinates $N = (5.2, 14.04)$ oe such as $\left(\frac{26}{5}, \frac{351}{25}\right)$. Allow this to be written separately.

(c)

M1 States any two of $y < x^2 - 5x + 13$, $y > "2.7"x$, $0 \leq x < b$ where $"2.5" \leq b \leq "5.2"$ Also allow the first two combined, or with loose inequalities, e.g $"2.7"x \leq y \leq x^2 - 5x + 13$ Also allow either $\leq$ or $<$ on the left hand end of $0 \leq x < 2.5$ due to the y -axis.**Use of R instead of y is M0**A1ft States all three inequalities. Same conditions as above, follow through on their $2.7x$ from (b) and 2.5.....
Allow part (a) to be attempted via finding minimum pointM1 For setting $\frac{dy}{dx} = ax + b = 0$ and finding a value for x , or for stating $x_{min} = -b/2a = \dots$ A1 $x = 2.5$ A1 $M = (2.5, 6.75)$

Note: Answer with no working score M0A0A0

General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**1. Factorisation $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ 2. FormulaAttempt to use correct formula (with values for a , b and c).3. Completing the squareSolving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:**1. DifferentiationPower of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)2. IntegrationPower of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
7.(a)	$\text{Attempts } \frac{\sin \angle ACB}{6.5} = \frac{\sin 35}{4.7}$ $\angle ACB = \text{awrt}(52 \text{ or } 53)^\circ \quad \text{or} \quad \text{awrt}(127 \text{ or } 128)^\circ$ $\angle ACB = 127.5^\circ$	M1 A1 A1
(b)	$\text{Eg } \frac{(AC)}{\sin 17.5^\circ} = \frac{6.5}{\sin 127.5^\circ} \quad \text{or} \quad \frac{4.7}{\sin 35^\circ}$ $\left[\frac{(CD)}{\sin 75^\circ} = \frac{4.7}{\sin 127.5^\circ} \Rightarrow (CD) = \dots \Rightarrow (AC) + (CD) \right] = \text{awrt } 8.2$ $\text{Total length of wood} = 8.1 + 6.5 + 4.7 + 4.7 = \text{awrt } 24.1$	M1 A1 A1 (3)
Alt1(a)	$\cos 35 = \frac{AC^2 + 6.5^2 - 4.7^2}{2 \times 6.5 \times AC} \Rightarrow AC^2 - 13 \cos(35)AC + 20.16 = 0 \Rightarrow AC = \dots$ $\cos \angle ACB = \frac{AC^2 + 4.7^2 - 6.5^2}{2 \times AC \times 4.7} \text{ oe}$	M1 (6 marks)

(a)

M1 Uses the sine rule with the angles and sides in the correct positions.

Alternatively they may use the cosine rule on ACB and then solve the subsequent quadratic to find AC and then use the cosine rule again

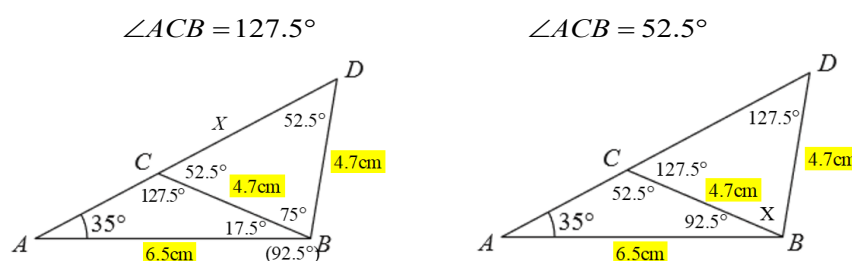
A1 $\angle ACB = \text{awrt}(52 \text{ or } 53)^\circ \quad \text{or} \quad \text{awrt}(127 \text{ or } 128)^\circ$ A1 $\angle ACB = 127.5^\circ$ only

(b) Working for (b) may be found in (a) which is acceptable

M1 Uses a formula that finds **part or all of the length AD** (eg AC , CD , AX , XD , AD).

The minimum required for this mark is the use of angle(s) and lengths in the correct places in the formula (which may have been rearranged to an alternative form). Condone mislabelling of the unknown length. This is usually the sine or cosine rule but they could split the triangle into two right angled triangles. See WAYS for additional guidance on methods. Sight of awrt8.2, awrt2.46 or awrt5.72 would imply this mark.

Condone angles in their triangles which do not add up to 180° and condone angles found with no working shown. For reference below these are the angles that would be found with $\angle ACB = 127.5^\circ$ and $\angle ACB = 52.5^\circ$ although they may “restart” so check their diagram as this may help.

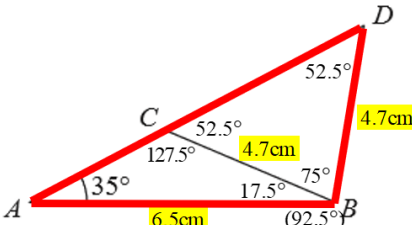
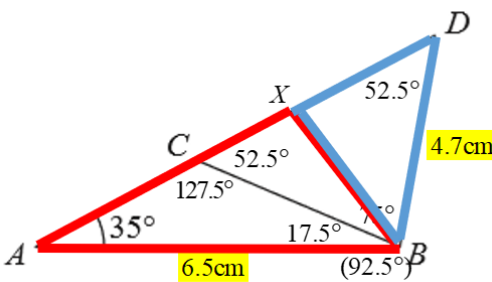


A1 awrt 8.2 Sight of awrt8.2 implies the length AD has been found. Ignore any labelling of lengths in their intermediate working and ignore any reference to AC (Accept "... = 8.2"). May be implied by a sum that totals awrt8.2 (eg awrt2.46+awrt5.72)

A1 awrt 24.1 ISW. Do not accept 24 or 25 (the length to the nearest metre) without seeing awrt24.1 or a calculation that totals awrt24.1 (eg $4.7 + 4.7 + 6.5 + 8.2 (= 24.1) \Rightarrow 24$)

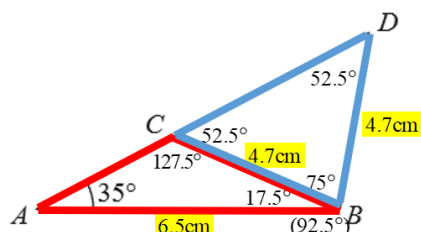
Candidates who assumed $\angle ACB = 52.5^\circ$ (acute) in (a):

Full marks can still be achieved as candidates may have restarted in (b) or not used the acute angle in their calculation which is often unclear. We are condoning any reference to $AC = 8.2$ so ignore any labelling of the lengths they are finding.

WAY 1 (b)	Uses triangle ABD:  Option 1: Cosine rule $4.7^2 + 6.5^2 - 2 \times 4.7 \times 6.5 \cos(180^\circ - 35^\circ - 52.5^\circ)$ Option 2: Sine rule $\frac{(AD)}{\sin 92.5^\circ} = \frac{4.7}{\sin 35^\circ} \text{ or } = \frac{6.5}{\sin 52.5^\circ}$ Option 3: Cosine rule using 35° to form a quadratic $\text{Eg. } \cos 35^\circ = \frac{6.5^2 + (AD)^2 - 4.7^2}{2 \times 6.5 \times (AD)}$	M1 M1 M1
WAY 2 (b)	Forms two right angled triangles ABX and BDX:  Either of $6.5 \cos 35^\circ$ or $4.7 \cos 52.5^\circ$	M1

WAY 3
(b)

Uses the triangles ABC and BCD :



Length AC

Option 1: Cosine rule

$$(AC)^2 = 4.7^2 + 6.5^2 - 2 \times 4.7 \times 6.5 \cos(180^\circ - 35^\circ - 127.5^\circ)$$

M1

Option 2: Sine rule

$$\frac{(AC)}{\sin 17.5^\circ} = \frac{6.5}{\sin 127.5^\circ} \text{ or } \frac{4.7}{\sin 35^\circ}$$

M1

Option 3: Cosine rule using 35° to form a quadratic in AC

$$\cos 35^\circ = \frac{6.5^2 + (AC)^2 - 4.7^2}{2 \times 6.5 \times (AC)}$$

M1

Length CD

Option 1: Cosine rule

$$(CD)^2 = 4.7^2 + 4.7^2 - 2 \times 4.7 \times 4.7 \cos(180^\circ - 2 \times 52.5^\circ)$$

Option 2: Sine rule

$$\frac{(CD)}{\sin 75^\circ} = \frac{4.7}{\sin 127.5^\circ}$$

M1

Option 3: Cosine rule using 52.5° to produce a quadratic in CD

$$\cos 52.5^\circ = \frac{4.7^2 + (CD)^2 - 4.7^2}{2 \times 4.7 \times (CD)}$$

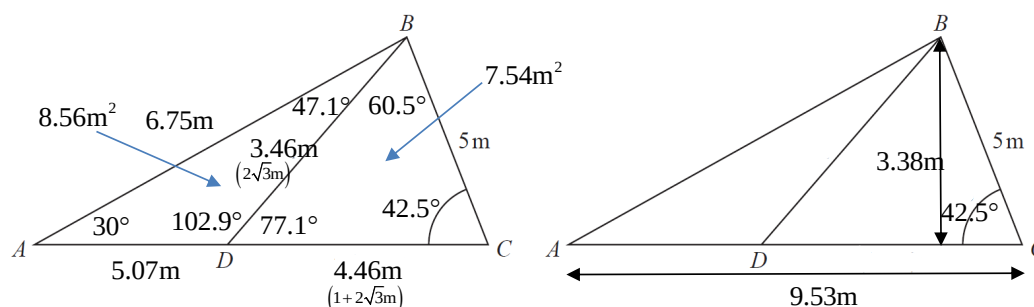
M1

Option 4: Using isosceles triangle property on BCD to find half of CD

$$4.7 \cos 52.5^\circ \text{ (they do not have to double to find } CD \text{ for M1)}$$

M1

(b) Note there are a variety of different methods to finding the area of triangle ABC



B1: $\theta = \text{awrt } 42^\circ$ seen or implied. Accept 42.5. May work in radians (awrt 0.73/0.74 radians)

M1: Attempts to find AB , AD or AC using a correct method. Angles and lengths must be in the correct positions in the relevant formula or formulae. Condone slips in any rearrangement, calculations and substituting in $x = 2\sqrt{3}$ provided the method is correct. Condone working in radians provided the angles are consistently in degrees or radians within the expression or formula.

A1: awrt 6.75/6.76 or awrt 9.53/9.54 or awrt 5.07/5.08 (may be implied by further work).

dM1: Correct full method to find the total area (the expression is sufficient. It is dependent on the previous method mark. Condone use of incorrectly rounded angles/slips and may work in radians.

$$\text{e.g. Area} = \frac{1}{2} \times 5 \times 6.75 \times \sin\left(\pi - \frac{\pi}{6} - 0.74\right)$$

May find the areas of the two separate triangles ABD and BDC and add them together.

$$\text{e.g. Area} = \frac{1}{2} \times 5.07 \times 6.75 \times \sin 30 + \frac{1}{2} \times 4.46 \times 5 \times \sin 42.5$$

Use the diagrams above to help with the various methods. Invisible brackets may be implied by further work or their answer.

Note that if, as part of their method to find the total area, they find angle ADB but incorrectly deduce this as an acute angle then this is dM0

A1: awrt 16.1 m² (condone lack of units)

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

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Question Number	Scheme	Marks
1(a)	$\left(\frac{dy}{dx}\right) \dots x^1 + \dots x^{\frac{3}{2}} + \dots x^{-2}$	M1
	$\left(\frac{dy}{dx}\right) \frac{2}{3}x - 2x^{\frac{3}{2}} - \frac{8}{3}x^{-2}$	A1A1A1
		(4)
(b)	$\frac{dy}{dx} = \frac{2}{3} \times 4 - 2 \times 4^{\frac{3}{2}} - \frac{8}{3} \times 4^{-2} = \frac{9}{4}$	M1
	$\frac{9}{4} \rightarrow -\frac{4}{9}$	M1
	$y - 3 = -\frac{4}{9}(x - 4)$	dM1
	$4x + 9y - 43 = 0$	A1
		(4)
		(8 marks)

(a)

M1 Reduces the power by 1 on any of the following terms

$$\dots x^2 \rightarrow \dots x^1, \dots x^{\frac{1}{2}} \rightarrow \dots x^{-\frac{3}{2}}, \dots x^{-1} \rightarrow \dots x^{-2}, 5 \rightarrow 0$$

A1 One of $\frac{2}{3}x$, $-2x^{\frac{3}{2}}$, $-\frac{8}{3}x^{-2}$ or exact simplified equivalent terms eg $\frac{-2}{(\sqrt{x})^3}$ or $-\frac{8}{3x^2}$ or

$$-2.6x^{-2}$$

Condone $\frac{2}{3}x^1$ and condone double signs such as $+ - 2x^{\frac{3}{2}}$. The terms do not need to be on the same line for this mark.

A1 Two of $\frac{2}{3}x$, $-2x^{\frac{3}{2}}$, $-\frac{8}{3}x^{-2}$ or exact simplified equivalent terms eg $\frac{-2}{(\sqrt{x})^3}$ or $-\frac{8}{3x^2}$.

Condone $\frac{2}{3}x^1$ and condone double signs such as $+ - 2x^{\frac{3}{2}}$. The terms do not need to be on the same line for this mark.

A1 $\frac{2}{3}x - 2x^{\frac{3}{2}} - \frac{8}{3}x^{-2}$ or exact simplified equivalent all on one line eg $\frac{2}{3}x - \frac{2}{(\sqrt{x})^3} - \frac{8}{3x^2}$.

Do not allow $\frac{2}{3}x^1 - 2x^{\frac{3}{2}} - \frac{8}{3}x^{-2}$ or double signs and do not isw in this part (including rounding decimals).



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
10. (a)	Correct equations $\frac{1}{2}r^2\theta = 6, \quad 2r + r\theta = 10$	B1 B1
	Eliminates $r = \frac{10}{2+\theta} \Rightarrow \frac{1}{2}\left(\frac{10}{2+\theta}\right)^2 \theta = 6$	M1
	$\Rightarrow 50\theta = 6(4 + 4\theta + \theta^2) \Rightarrow 3\theta^2 - 13\theta + 12 = 0 \quad *$	A1*
		(4)
(b)	$(3\theta - 4)(\theta - 3) = 0 \Rightarrow \theta = \frac{4}{3}, 3$	B1
	$\theta = \frac{4}{3}, r = 3 \quad \theta = 3, r = 2$	M1 A1
		(3)
		(7 marks)

(a)

B1 One correct equation $\frac{1}{2}r^2\theta = 6$ or $2r + r\theta = 10$ (may use φ instead of θ which is fine)

B1 Two correct equations $\frac{1}{2}r^2\theta = 6$ and $2r + r\theta = 10$ ($r + r + r\theta = 10$ is acceptable)

Note you may see one of both of these equations in (b) which you can award the marks retrospectively. They may be implied from later work:

Eg. $r^2 = \frac{12}{\theta} \Rightarrow 2\sqrt{\frac{12}{\theta}} + \sqrt{\frac{12}{\theta}}\theta = 10$ implies B1B1

M1 Scored for eliminating r and reaching an equation in θ only. The initial equations must be of a similar form to the area of the sector and the perimeter but possibly with errors (look for $\dots r^2\theta = 6$ and $2r + \dots r\theta = 10$). Condone errors when rearranging but their subsequent substitution must be correct for their rearrangement. Eg. $10 = r\theta + 2r \Rightarrow \frac{10}{\theta} = 3r$

A1* Reaches the given answer with no errors. There must be at least one intermediate line of manipulation following the elimination stage and $(\theta + 2)^2$ must be multiplied out correctly before achieving the final answer to score full marks.



(b)

B1 $\theta = \frac{4}{3}, 3$ (They may just appear from a calculator)

M1 Substitutes one of their values of θ into one of the previous equations of the allowable form and proceeds as far as $r = \dots$

A1 Both results $\theta = \frac{4}{3}, r = 3$ $\theta = 3, r = 2$

Values must be exact. Withhold the final mark if they do not rule out $r = -3 / r = -2$ from using the area of a sector equation and withhold the final mark if the corresponding values of r are incorrectly paired to the other values of θ ($\theta = \frac{4}{3}, r = 2$ and $\theta = 3, r = 3$)



Eg $x^2 + 2x\sqrt{2} + 2 + 9x^2 - 30x\sqrt{8} + 25 \times 8 \Rightarrow 10x^2 - 58x\sqrt{2} + 202$ scores B1M1A1
as $\sqrt{8} = 2\sqrt{2}$ is correctly seen by the correct collection of the terms.

Eg $x^2 + 2x\sqrt{2} + 2 + 9x^2 - 30x\sqrt{8} + 25 \times 8 \Rightarrow 10x^2 - 60x\sqrt{2} + 202$ scores B0M1A0
as $\sqrt{8} = 2\sqrt{2}$ is not correctly seen or implied as the coefficient of x is incorrect

(ii) On EPEN this is M1A1M1A1 we are marking this M1M1M1A1

M1: Attempts to multiply out and isolates the two y terms on one side of the equation.
Condone sign slips only in their rearrangement.

dM1: Attempts to make y or ky the subject (where k is an integer or fraction which would simplify to an integer). Score for $y = f(x)$ or $ky = g(x)$ where the function includes $\sqrt{3}$ but it cannot be scored for directly proceeding to $y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ or $y = \frac{57 + 41\sqrt{3}}{23}$.

It is dependent on the previous method mark.

Note $y(4\sqrt{3} - 5) = 9 + \sqrt{3} \Rightarrow y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ is M1dM0ddM0A0

ddM1: Proceeds to $y = \dots$ with a rational denominator. The denominator does not need to be simplified. It is dependent on the previous method mark. They must have shown the step of either rationalising the denominator or showing how they achieve $ky = \dots$ and proceeds to $y = \dots$

A1: Correct answer in the correct form with all stages of working seen

- Collects y terms on one side of the equation
- Makes y the subject
- Rationalises and proceeds to the correct answer. Somewhere in their solution they will have had to have multiplied two brackets involving surds together and they must have shown this being multiplied out (calculators are not allowed)

eg $\frac{9 + \sqrt{3}}{4\sqrt{3} - 5} \times \frac{4\sqrt{3} + 5}{4\sqrt{3} + 5} = \frac{36\sqrt{3} + 45 + 12 + 5\sqrt{3}}{23}$

Condone solutions with invisible brackets to score full marks, provided the general method is sound and the answer has not just come from a calculator or incorrect working.

Do not accept $y = \frac{57 + 41\sqrt{3}}{23}$

Alt(ii)1 (main scheme alternative)

M1: Substitutes $y = p + q\sqrt{3}$ expands, collects terms. Condone sign slips.

dM1: Compares rational/irrational parts to form two equations. It is dependent on the previous method mark.

ddM1: Solves 2 linear equations in p and q using an acceptable method. They cannot just state the values. Condone slips in their working. It is dependent on the previous method mark.

A1: $y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ (or $y = 2\frac{11}{23} + 1\frac{18}{23}\sqrt{3}$) with full working shown.

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Alt(ii)2 Squaring and completing the square

M1: Squares both sides and multiplies out brackets (condone slips)
eg. $(48y^2 - 72\sqrt{3}y + 81 = 25y^2 + 3 + 10\sqrt{3}y)$

dM1: Rearranges to form a 3TQ = 0 ($23y^2 - 82\sqrt{3}y + 78 = 0$). It is dependent on the previous method mark.

ddM1: Attempts to complete the square and proceeds to make y the subject. It is dependent on the previous method mark.

A1: $y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ (or $y = 2\frac{11}{23} + 1\frac{18}{23}\sqrt{3}$)

Alt(ii)3 Dividing by $\sqrt{3}$, then rationalising the denominator and collecting terms:

$$\begin{aligned}
 \text{ii) } 4y - 3\sqrt{3} &= \frac{54 + \sqrt{3}(15\sqrt{3})}{\sqrt{3}} \\
 4y - 3\sqrt{3} &= \frac{\sqrt{3}(15 + \sqrt{3})}{3} \\
 4y - 3\sqrt{3} &= \frac{5\sqrt{3}y + 3}{3} \\
 4y - 3\sqrt{3} &= \frac{5\sqrt{3}y + 3}{3} \\
 4y - \frac{5\sqrt{3}y}{3} &= 1 + 3\sqrt{3} \\
 y(4 - \frac{5\sqrt{3}}{3}) &= 1 + 3\sqrt{3} \\
 y &= \frac{1 + 3\sqrt{3}}{4 - \frac{5\sqrt{3}}{3}} \times \frac{4 + \frac{5\sqrt{3}}{3}}{4 + \frac{5\sqrt{3}}{3}} \\
 y &= \frac{4 + 12\sqrt{3} + \frac{5\sqrt{3}}{3} + 15}{16 - \frac{25}{3}} \\
 y &= \frac{4 + 12\sqrt{3} + \frac{5\sqrt{3}}{3} + 15}{\frac{48 - 25}{3}} \\
 y &= \frac{19 + \frac{41}{3}\sqrt{3}}{\frac{23}{3}} \\
 y &= \frac{57 + 41\sqrt{3}}{23}
 \end{aligned}$$

M1 for isolating y terms on one side of the equation

dM1 for making the y the subject

ddM1 for proceeds to $y = \dots$ with a rational denominator

A1 Correct answer with full working shown

Question Number	Scheme	Marks
9a	$0.72 = OP \times 0.6 \Rightarrow OP = 1.2$ or $\frac{0.72}{0.6} = 1.2$ or $0.72 = 1.2 \times 0.6 \Rightarrow OP = 1.2$ *	B1*
		(1)
b	$\frac{1}{2} \times (x+1.2)^2 \times 0.6$	B1
	$\frac{1}{2} \times (x+1.2)^2 \times 0.6 - \frac{1}{2} \times 1.2^2 \times 0.6 = 90 \Rightarrow 5x^2 + 12x - 1500 = 0$ *	M1A1*
		(3)
c	$x = \text{awrt } 16.2 \left(= \frac{-6 + 4\sqrt{471}}{5} \right)$	B1
	Perimeter = $0.72 + "16.2" + "16.2" + 0.6 \times ("16.2" + 1.2) = \text{awrt } 43 / \text{awrt } 44 \text{ m}$ (see notes)	M1A1
		(3)
		(7 marks)

(a)

B1*: Either

- forms an equation using $l = r\theta$ with $l = 0.72$ and $\theta = 0.6$ and concludes $OP = 1.2$
e.g. $0.72 = r \times 0.6 \Rightarrow r = 1.2$
- states $\frac{0.72}{0.6} = 1.2$
- forms the equation using $0.72 = 1.2 \times 0.6$ and concludes that $OP = 1.2$

Condone the use of r or any other variable instead of OP . Condone lack of units or incorrect units.

Allow the angle to be in degrees instead or attempts to find the diameter first, but they must not round the value at any point in their working.

e.g. $0.72 = \frac{0.6}{360} \times 180 \pi d \Rightarrow d = 2.4 \Rightarrow r = 1.2$ scores B1 (exact angle used)

e.g. $\frac{0.72}{\left(\frac{34.4}{360}\right)} = 2\pi r \Rightarrow r = 1.2$ scores B0 (34.4 is a rounded angle)

(b)

B1: Correct expression for the area of sector $OPQRSO$

$\frac{1}{2} \times (x+1.2)^2 \times 0.6$ Missing brackets may be implied by further work.

May appear within an equation e.g. $\frac{1}{2} \times 0.6 [(x+1.2)^2 - 1.2^2] = 90$

Condone incorrect equations i.e. $\frac{1}{2} \times (x+1.2)^2 \times 0.6 = 90$

M1: Proceeds from an equation of the form $\dots(x+1.2)^2 \times 0.6 - \dots \times 1.2^2 \times 0.6 = 90$ or equivalent, (i.e. both should be positive but may be missing e.g. $\frac{1}{2}$) and attempts to multiply out the brackets. Condone slips and invisible brackets.



Question	Scheme	Marks
4(a)	$x^2 + kx - 9 = -3x^2 - 5x + k \Rightarrow 4x^2 + kx + 5x - 9 - k (= 0)$	M1
	$b^2 - 4ac = 0 \Rightarrow (k+5)^2 - 4 \times 4(-9-k) = 0$	M1
	$k^2 + 26k + 169 = 0^*$	A1*
		(3)
(b)	$k^2 + 26k + 169 = 0 \Rightarrow k = -13$	B1
	$k = -13 \Rightarrow 4x^2 - 8x + 4 = 0 \Rightarrow x = \dots$	M1
	$(1, -21)$	A1
		(3)
		Total 6

Notes:**Mark (a) and (b) together****(a)****M1:** Equates both curves and collects terms to one side. Allow slips but there must be some attempt to collect terms to one side. The “= 0” may be implied by their attempt at the discriminant.**M1:** Attempts to find the discriminant “ $b^2 - 4ac$ ” of a 3TQ. May be seen as an attempt at e.g. $b^2 - 4ac = 0$ or equivalent e.g. $b^2 = 4ac$ or may be seen embedded in an attempt at the quadratic formula. It requires b and c both of the form $pk + q$, $p, q \neq 0$ and requires a as a constant.If clearly a wrong formula/expression is used e.g. “ $b^2 + 4ac$ ” this scores M0**A1*:** Achieves the printed answer with no errors and sufficient working shown.The “= 0” must appear at least once before the printed answer unless they start with $b^2 = 4ac$.**(b)****B1:** Correct value of k . Condone $x = -13$ if it is subsequently used as a value for k .**M1:** Either:

- substitutes their value of k into their equation obtained by equating the curves and collecting terms from part (a) and solves a 3TQ for x by any correct method including a calculator **or**
- substitutes their k into the 2 given equations, equates, collects terms to one side and solves a 3TQ for x by any correct method including a calculator

A1: Correct coordinates. Allow as e.g. $x = 1$, $y = -21$ **Correct answer only in (b) scores 3/3**

Question Number	Scheme	Marks
1.	$\int \left(\frac{8}{3}x^3 - \frac{1}{2}x^{\frac{1}{2}} - 5 \right) dx = \frac{8}{3} \times \frac{x^4}{4} - \frac{1}{2} \times 2x^{\frac{1}{2}} - 5x + c$ $= \frac{2}{3}x^4 - x^{\frac{1}{2}} - 5x + c$	M1 A1 A1 A1 (4 marks)

- M1 For raising a power of x by 1 seen at least once (i.e. $x^n \rightarrow x^{n+1}$) or $-5 \rightarrow -5x$.
The index does not have to be processed for this mark.

Award following incorrect manipulations eg $-\frac{1}{2\sqrt{x}} \rightarrow \dots x^{\frac{1}{2}} \rightarrow \dots x^{\frac{3}{2}}$

- A1 For two of $\frac{8}{3} \times \frac{x^4}{4}$, $-\frac{1}{2} \times 2x^{\frac{1}{2}}$, $-5x^1$ correct (unsimplified).

This may be implied by a correct simplified answer. May be seen on different lines. Indices must be processed. Ignore any spurious notation for any of the A marks.

- A1 For two of $\frac{2}{3}x^4$, $-x^{\frac{1}{2}}$, $-5x$ correct and in simplest form. Accept forms such as $\frac{2x^4}{3}$ and $-\sqrt{x}$

Condone $-5x^1$, $-1x^{\frac{1}{2}}$ but not $-\frac{5x}{1}$. Correct terms may be seen on different lines. Allow $\frac{2}{3}$ to be written as $0.\dot{6}$ or $0.6666\dots$ only (must be exact)

- A1 Fully correct **on one line**, and simplified with $+c$. Accept simplified equivalents (see above)

Question Number	Scheme	Marks
4.(a)	Gradient $PQ = -3$ Attempts to find equation of l Eg. $y - 13 = -3(x + 2)$ $y = -3x + 7$	B1 M1 A1 (3)
(b)	Attempts to use minimum is $(4, -5)$ Eg $y = \dots(x - 4)^2 - 5$ Attempts to use $(-2, 13)$ with $y = a(x - 4)^2 - 5 \Rightarrow a = \dots$ $\Rightarrow y = \frac{1}{2}(x - 4)^2 - 5$ or $y = \frac{1}{2}x^2 - 4x + 3$ oe	M1 dM1 A1 (3)
(c)	Two of $y > -3x + 7$, $y < \frac{1}{2}(x - 4)^2 - 5$ $x < -2$ All three of $y > -3x + 7$, $y < \frac{1}{2}(x - 4)^2 - 5$ $x < -2$	M1 A1 (2)
		(8 marks)

(a)B1 Finds the gradient of l to be -3 If simultaneous equations are used then it would be scored for $m = -3$

M1 Attempts to find the equation of the line using their gradient for PQ and one of the points. Sight of embedded values eg $(y - -5) = "-3"(x - 4)$ is sufficient (allow one sign slip on one of the brackets) or if they use $y = mx + c$ they must proceed as far as $c = \dots$

If the perpendicular gradient is used it is M0.

A simultaneous method, eg solving $13 = -2m + c$ and $-5 = 4m + c$ must reach values for m and c .A1 $y = -3x + 7$ cso Condone $m = -3$, $c = 7$ **(b)**M1 Attempts to use the **minimum point** $(4, -5)$ to form a valid equation**Way One:** e.g. $(y =) A(x - 4)^2 - 5$ (condoning A to be 1). Condone**Way Two:** Letting $y = ax^2 + bx + c$ and either:

- finding $\frac{dy}{dx}$ and setting $2a \times 4 + b = 0$
- uses the minimum point and symmetry starting with $-\frac{b}{2a} = 4$ ($\Rightarrow -b = 8a \Rightarrow 8a + b = 0$)
- uses symmetry about the minimum point to identify the point $(10, 13)$ and form the equation $13 = 100a + 10b + c$

Do not award for only substituting the minimum point into a linear equation.

dM1 **Way One:** Attempts to use $(-2, 13)$ with $y = a(x-4)^2 - 5 \Rightarrow a = \dots$

Way Two: Attempts to use $(-2, 13)$ and $(4, -5)$ in $y = ax^2 + bx + c$ and using these two equations, $4a - 2b + c = 13$ and $16a + 4b + c = -5$, with $2a \times 4 + b = 0$ to find a , b and c . Condone slips in their working.

This may also be done using a calculator or use matrices

$$\begin{pmatrix} 16 & 4 & 1 \\ 4 & -2 & 1 \\ 8 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -5 \\ 13 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \dots$$

Candidates who used symmetry with the point $(-2, 13)$ and the point $(10, 13)$ to form the equation $13 = 100a + 10b + c$ attempt to solve simultaneously with two other equations from either $2a \times 4 + b = 0$ or $4a - 2b + c = 13$ and $16a + 4b + c = -5$

A1 $y = \frac{1}{2}(x-4)^2 - 5$ or equivalent such as $y = \frac{1}{2}x^2 - 4x + 3$ or $2y = x^2 - 8x + 6$ (cannot be $C = \dots$)

(c)

M1 Two of $y > -3x + 7$, $y < \frac{1}{2}(x-4)^2 - 5$, $x < -2$ (ignore any others for this mark)

Their line from part (a) must have a negative gradient and their curve from part (b) must be a positive quadratic.

(Allow two of $y > -3x + 7$, $y < \frac{1}{2}(x-4)^2 - 5$, $x < k$ where k is any value between -2 and 4)

$-3x + 7 < y < \frac{1}{2}(x-4)^2 - 5$ also scores M1 for having two of the inequalities.

BUT $-3x + 7 < x < \frac{1}{2}(x-4)^2 - 5$ with one other inequality is M0 as this is insufficient to define the inequality $x < -2$

Ignore any use of set notation for this mark.

A1 All three of $y > -3x + 7$, $y < \frac{1}{2}(x-4)^2 - 5$, $x < -2$ (and no others)

(Allow all three of $y > -3x + 7$, $y < \frac{1}{2}(x-4)^2 - 5$, $x < k$ where k is any value between -2 and 4)

$-3x + 7 < y < \frac{1}{2}(x-4)^2 - 5$, $x < -2$ (or $x < k$ where k is any value between -2 and 4) is also acceptable.

If set notation is used, then they must use " $\cap$ " between any of their inequalities rather than " $\cup$ ". Condone attempts as long as the intention is clear.

Eg $\left\{ x, y \in \mathbb{R} : y > -3x + 7 \cap y < \frac{1}{2}(x-4)^2 - 5 \cap x < -2 \right\}$

$\left\{ x, y \in \mathbb{R} : y > -3x + 7, y < \frac{1}{2}(x-4)^2 - 5, x < -2 \right\}$ would be acceptable

Allow consistent use of $> \leftrightarrow \geq$ for all of their inequalities including the inequality $x < -2$

If $x < k$ or $x \leq k$ is stated where $(-2 < k \leq 4)$ then only the inequalities for their " $-3x + 7$ " and the positive quadratic needs to be consistent with each other.



Alt (a)

Some may have done WMA13 and may attempt the quotient rule

M1, dM1 for an attempt at the quotient rule to obtain $\frac{2x^2 \times ax^2 - (5x^3 - 8) \times bx}{(2x^2)^2}$ where $a > 0, b > 0$

condoning lack of brackets

So allow, for example, if you see $2x^4$ on the denominator.

These two marks are scored together so 1000 is not possible via this route

Then A1 for $\frac{2x^2 \times 15x^2 - (5x^3 - 8) \times 4x}{(2x^2)^2}$ and finally A1 for $\frac{5x^3 + 16}{2x^3}$ or other simplified equivalent

Alt (b)

Some may use the $\frac{dy}{dx}$ function of their calculator to find the gradient of the tangent at $x = 2$

So if the gradient of $\frac{7}{2}$ appears without any working **AND** following an incorrect or missing part (a) score as follows

M0: No sight of using $x = 2$ in their $\frac{dy}{dx}$

M1: For sight of $y - 4 = \frac{7}{2}(x - 2)$ or equivalent

A1: For $7x - 2y - 6 = 0$ or integer multiple

Question Number	Scheme	Marks
4 (a)	$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$ Uses an index law and states or implies any of $4^x = p^2, \quad 2^{x+3} = 8p \quad \text{or} \quad 2^{x-1} = \frac{p}{2}$ Writes the given equation in terms of p $2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4 \Rightarrow 2p^2 - 2^3 \times p = \frac{17p}{2} - 4$ Proceeds to $4p^2 - 33p + 8 = 0$ via $2p^2 - 8p = \frac{17p}{2} - 4$ * CSO	B1 M1 A1* (3)
(b)	$4p^2 - 33p + 8 = 0 \Rightarrow (4p-1)(p-8) = 0 \Rightarrow p = \dots, \dots$ Sets $2^x = \frac{1}{4}, 8 \Rightarrow x = \dots$ $x = -2, 3$	M1 M1 A1 (3) (6 marks)

(a) Watch out here as this is a given answer. All stages of working are required to score the 3 marks

B1: Uses an index law and states or implies any of $4^x = p^2$, $2^{x+3} = 8p$ or $2^{x-1} = \frac{p}{2}$

Allow equivalentents such as $4^x = p \times p$, $2^{x+3} = 8 \times p$, $p \times 8$ or $2^{x-1} = \frac{1}{2}p$, $p \div 2$, $0.5p$

M1: Attempts to write the given equation in x to a quadratic equation in terms of p

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4 \Rightarrow 2p^2 - 2^3 \times p = \frac{17p}{2} - 4 \quad \text{or} \quad 2p^2 - 2^3 \times p = 17p \times 2^{-1} - 4$$

All three index laws must be seen but condone slips on signs or on the 2^3 if there was an incorrect attempt to process.

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4 \Rightarrow 2p^2 - 6 \times p = \frac{17p}{2} - 4 \quad \text{would be fine for the M1}$$

Allow a recovery for this M1, e.g. $2^{x+3} = 2^x + 2^3 = 8p$ but not the A1*

Watch for candidates who manipulate the given equation first. This is acceptable.

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4 \Rightarrow 4^x - 2^{x+2} = 17 \times 2^{x-2} - 2$$

So

$$\Rightarrow p^2 - 2^2 \times p = \frac{17p}{2} - 2$$

A1*: CSO Proceeds to the given answer of $4p^2 - 33p + 8 = 0$ with no errors or omissions.

Condone working such as $4^x = 2^{x^2}$ leading to p^2 . It is often hard to decipher the relative heights of the indices.

An intermediate line of $2p^2 - 8p = \frac{17p}{2} - 4$ o.e. must be seen.



(b) **This is a non-calculator part so the use of a calculator is penalised**

M1: Valid **non calculator** attempt at solving $4p^2 - 33p + 8 = 0$. Allow slips/miscopies on $4p^2 - 33p + 8 = 0$ for example, $4p^2 - 33p - 8 = 0$. The roots cannot just appear.

Examples such as $4p^2 - 33p + 8 = (p-8)\left(p - \frac{1}{4}\right) = 0 \Rightarrow p = 8, \frac{1}{4}$ is obvious calculator work and scores M0

Award for an attempt to

- factorise (usual rules) leading to values for (p)
- use the quadratic formula condoning this calculation $\frac{33 \pm \sqrt{(-33)^2 - 4(4)(8)}}{2(4)}$ followed by $\frac{1}{4}, 8$
- complete the square leading to values for (p)

M1: Valid non calculator attempt at solving an equation of the form $2^x = k$, $k > 0$

This can be implied for a correct solution for either root.

If the value of k is not a power of 2 then score for $2^x = k \Rightarrow x = \log_2 k$

A1: Both solutions $x = -2, 3$ following the correct quadratic equation in p . There is no need to state $x =$

If they then go on to reject $x = -2$ say then A0

Note that 011 is possible in part (b) for candidates who don't show a non-calculator solution of

$$4p^2 - 33p + 8 = 0$$

Question Number	Scheme	Marks
2. (a)	$5(x+3) > 4(2x-5) \Rightarrow 5x+15 > 8x-20 \Rightarrow ax > b \text{ or } px < q$ $\Rightarrow x < \frac{35}{3}$	M1 A1 (2)
(b) (i)	$x^2 - 6x + 1 = (x-3)^2 \pm \dots = (x-3)^2 - 8$	M1, A1
(ii)	$(x-3)^2 - 8 = 0 \Rightarrow x = 3 + \sqrt{8} \text{ or } 3 - \sqrt{8}$ $x^2 - 6x + 1 \geq 0 \Rightarrow x \leq 3 - \sqrt{8}, x \geq 3 + \sqrt{8}$	M1 A1 (4)
(c)	$x \leq 3 - \sqrt{8}, 3 + \sqrt{8} \leq x < \frac{35}{3}$	B1 (1) (7 marks)

This is effectively a non- calculator question and methods must be clearly shown.

(a)

M1: Attempts to expand brackets and then collect terms. Look for

- $5x \pm 15 > 8x \pm 20$
- followed by $ax > b$ or $px < q$

Also condone with an = sign in place of the inequality

A1: $x < \frac{35}{3}$ or exact equivalent such as $\frac{35}{3} > x$ **following the award of M1.**

Do not accept $x < 11.66$ or $x < 11.7$ but $x < 11.\dot{6}$ is acceptable and ISW after a correct answer

Mark part (b) as one question

(b) (i)

M1: Starts the process by writing $x^2 - 6x + 1$ as $(x-3)^2 \pm c$, where c is a constant (which may not be simplified)

A1: $x^2 - 6x + 1 = (x-3)^2 - 8$. It is acceptable to write this down for the two marks

(b) (ii) Note that this must follow on from their (b)(i) and CANNOT be restarted via the quadratic formula

M1: Correctly solves their $(x+a)^2 - b = 0$, $b > 0$ and finds at least one critical value.

Look for an intermediate line of working such as $(x+a)^2 = b$ or $x+a = \pm\sqrt{b}$

Do not allow decimals, i.e. awrt 5.83, 0.17 but isw after a correct (exact) answer

It can be implied by an attempt to solve their inequality as long as an intermediate line is seen.

E.g. $(x+a)^2 - b \geq 0$, $b > 0 \Rightarrow x+a \dots \sqrt{b} \Rightarrow x \dots -a + \sqrt{b}$ where ... can even be an incorrect inequality

Again, it is important to see an intermediate line of working.

A1: $x \leq 3 - \sqrt{8}, x \geq 3 + \sqrt{8}$ o.e following the award of M1

ISW if this is subsequently changed to decimals. Accept $\sqrt{8}$ as $2\sqrt{2}$

(c)

B1: CAO $x \leq 3 - \sqrt{8}$, $3 + \sqrt{8} \leq x < \frac{35}{3}$ o.e. It must be given as two separate inequalities.

Allow set notation $\{x \in \mathbb{R} : x \leq 3 - \sqrt{8}\} \cup \{x \in \mathbb{R} : 3 + \sqrt{8} \leq x < \frac{35}{3}\}$

A list $x < \frac{35}{3}$, $x \leq 3 - \sqrt{8}$, $x \geq 3 + \sqrt{8}$ is B0.

There must be a correct attempt to state the two separate regions.

Condone a solution with 'and' between. E.g. $x \leq 3 - \sqrt{8}$ and $3 + \sqrt{8} \leq x < \frac{35}{3}$

Handwritten student work for a quadratic inequality problem:

(b)(i): $x^2 - 6x + 1$
 $= x^2 - 6x + 9 - 8$
 $= (x - 3)^2 - 8$
 $a = -3 \quad b = -8$

(ii): $x^2 - 6x + 1 \geq 0$
 $x \leq 3 - 2\sqrt{2}$ or $x \geq 3 + 2\sqrt{2}$
 so $x \in (-\infty, 3 - 2\sqrt{2}] \cup [3 + 2\sqrt{2}, +\infty)$

(c): $x \leq 3 - 2\sqrt{2}$ or $3 + 2\sqrt{2} \leq x < \frac{35}{3}$

Example of how to mark:

(b)(i) M1, A1: Completely correct

(ii) M0, A0: No intermediate line.

This could have been done on a calculator

(c) B1: Completely correct answer

Question Number	Scheme	Marks
3.(a)	$\left(\frac{dy}{dx}\right) = 3x^2 + \frac{48}{\sqrt{x}}$	M1, A1, A1 (3)
(b)	$\left(\frac{d^2y}{dx^2}\right) = 6x - \frac{24}{x^{\frac{3}{2}}}$ $\Rightarrow x^{\frac{5}{2}} = \dots \quad x = 2^{\frac{4}{5}}$	M1 dM1 A1 (3) (6 marks)

(a)

M1: For reducing a correct power by one any term. So, award for $x^3 \rightarrow x^2$, $x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$ or even $5 \rightarrow 0$
 As in question 1, the indices must be processed

A1: One of the two terms, correct **and** simplified, either $3x^2$ or $+\frac{48}{\sqrt{x}}$ or exact simplified equivalent

A1: CAO $3x^2 + \frac{48}{\sqrt{x}}$ or exact simplified equivalent such as $3x^2 + 48x^{-\frac{1}{2}}$

(b)

M1: For reaching $\left(\frac{d^2y}{dx^2}\right) = Ax \pm \frac{B}{x^{\frac{3}{2}}}$ o.e where A and B are non-zero constants

dM1: Sets their $\frac{d^2y}{dx^2} = Ax \pm \frac{B}{x^{\frac{3}{2}}} = 0$ and proceeds to $x^{\frac{5}{2}} = C$ (or equivalent working) where C is a constant.

Example of equivalent working $x = \frac{4}{x^{\frac{3}{2}}} \Rightarrow x^2 = \frac{16}{x^3} \Rightarrow x^5 = 16$

A1: CAO $x = 2^{\frac{4}{5}}$ following correct $\frac{d^2y}{dx^2}$ with dM1 also being scored. If it is achieved via decimals it is A0

Condone sight of correct work with correct k . E.g. $x^{\frac{5}{2}} = 4 \Rightarrow k = \frac{4}{5}$



Question Number	Scheme	Marks
4.(a)	Sets $kx + 2 = \frac{2}{x} + 3x - 4$ and attempts to collect terms or multiply through by x $(k-3)x + 6 - \frac{2}{x} = 0 \Rightarrow (k-3)x^2 + 6x - 2 = 0 \quad *$	M1 A1* (2)
(b)	Attempts $b^2 - 4ac$ for $(k-3)x^2 + 6x - 2 = 0$ Solves $b^2 - 4ac = 0$ for $(k-3)x^2 + 6x - 2 = 0 \Rightarrow 36 + 8(k-3) = 0 \Rightarrow k = \dots$ $k = -\frac{3}{2}$	M1 dM1 A1 (3) (5 marks)

Ignore labels here. Mark (a) and (b) as one complete question.

(a)

M1: Sets $kx + 2 = \frac{2}{x} + 3x - 4$ condoning slips and either attempts to collect terms or else multiply through by x
Allow for either of

- $kx + 2 = \frac{2}{x} + 3x - 4 \Rightarrow (k \pm 3)x + 6 = \frac{2}{x}$ o.e condoning slips on the 6. E.g. condone $6 \leftrightarrow -2$
- $kx + 2 = \frac{2}{x} + 3x - 4 \Rightarrow kx^2 + 2x = 2 + 3x^2 - 4x$ with correct multiplication seen on at least 3 terms

A1*: Proceeds to given answer showing at least the required/ necessary steps with no incorrect lines.
Examples of correct work gaining M1 A1 showing a minimum number of necessary steps (three)

- By initially collecting terms: $kx + 2 = \frac{2}{x} + 3x - 4 \Rightarrow (k-3)x + 6 - \frac{2}{x} = 0 \Rightarrow (k-3)x^2 + 6x - 2 = 0$
- By multiplying initially: $kx + 2 = \frac{2}{x} + 3x - 4 \Rightarrow kx^2 + 2x = 2 + 3x^2 - 4x \Rightarrow (k-3)x^2 + 6x - 2 = 0$
- Bracketing must be correct. There may be extra lines, which is fine, as long as they are correct.

(b)

M1: Attempts $b^2 - 4ac$ for $(k-3)x^2 + 6x - 2 = 0$ with **correct** values for a and b with c condoned as ± 2

This is implied by setting $b^2 = 4ac$ but also $b^2 - 4ac > 0$ and $b^2 - 4ac < 0$ as well as non-strict versions. Note that stating and attempting $b^2 + 4ac$ is M0 followed by dM0, A0

This cannot be scored if $b^2 - 4ac$ can only be seen embedded within a quadratic formula.

dM1: Solves $b^2 - 4ac = 0$ o.e. for $(k-3)x^2 + 6x - 2 = 0 \Rightarrow 6^2 - 4(k-3)(-2) = 0 \Rightarrow k = \dots$

$$\text{Also } (k-3)x^2 + 6x - 2 = 0 \Rightarrow 36 + 8(k-3) = 0 \Rightarrow k = \dots$$

It is dependent upon the previous M1 being an **equality** AND **correct** a , b and c leading to a value for k

Don't be too concerned by the processing of the equation leading to the value for k

A1: $k = -\frac{3}{2}$ o.e.

.....
 Alt (b) via gradients.

M1: Sets $\frac{dy}{dx} = \frac{-2}{x^2} + 3 = k$. Look for a $\frac{dy}{dx}$ in the form $\frac{\alpha}{x^2} + \beta$

dM1: Attempts to find $x^2 = \left(\frac{2}{3-k}\right)$, substitutes into $(k-3)x^2 + 6x - 2 = 0$ and solves to find $k = \dots$

Alternatively substitutes $k = \frac{-2}{x^2} + 3$ into $(k-3)x^2 + 6x - 2 = 0$ and solves to find x and then k .

A1: $k = -\frac{3}{2}$ o.e such as $-\frac{12}{8}$ or -1.5

.....

Question Number	Scheme	Marks
5 (a)	$9x^3 - 10x^2 + x = x(9x^2 - 10x + 1) = x(ax \pm 1)(bx \pm 1)$ with $ab = 9$ $= x(9x - 1)(x - 1)$	M1 A1 (2)
(b)	States or implies that $x = 3^y$ AND sets = to their 1 or $\frac{1}{9}$ Solves their $3^y = "1"$ or $3^y = "\frac{1}{9}"$ $y = 0, -2$	M1 dM1 A1 (3) (5 marks)

THIS IS A NON- CALCULATOR QUESTION.

(a)

M1: Takes out (or divides by) a common factor of x and attempts to factorise the quadratic factor.
In most cases this will be

- either $9x^3 - 10x^2 + x = x(9x^2 - 10x + 1) = x(ax \pm 1)(bx \pm 1)$ with $ab = 9$
- or $9x^2 - 10x + 1 = (ax \pm 1)(bx \pm 1)$ with $ab = 9$

Other methods exist such as $9x^3 - 10x^2 + x = (x - 1)(9x^2 - x)$ but the method is only scored when the factor of x is taken from the second bracket

It would be very difficult to score following an attempt to find the roots of $9x^2 - 10x + 1 = 0$

A1: $x(9x - 1)(x - 1)$ o.e such as $(9x - 1)(x - 1)(x - 0)$, $x(-9x + 1)(-x + 1)$ or $9x\left(x - \frac{1}{9}\right)(x - 1)$ following the award of the M1 mark. **This must be done in part (a) but use review if unsure**

.....
Watch out for these.

Example 1: If you see $9x^3 - 10x^2 + x = 0 \Rightarrow x = 0, 1, \frac{1}{9}$ followed by $x(9x - 1)(x - 1)$ it is M0 A0
(calculator use)

Example 2: If you see $9x^3 - 10x^2 + x = x\left(x - \frac{1}{9}\right)(x - 1)$ it is M0 A0
(Incorrect and probable calculator use)

Special case:

If you see $9x^3 - 10x^2 + x = x(9x - 1)(x - 1)$ or $9x\left(x - \frac{1}{9}\right)(x - 1)$ without any other working it is SC M1 A0
.....



- (b) If a candidate has just written down the roots of part (a) $=0$ without having factorised it, then allow all marks to be scored here. So, 00 111 is possible.

M1: This is awarded for linking the equation in y with the roots of the part (a)

Score for stating that $x = 3^y$ **AND** setting x or $3^y =$ one of the non-zero roots of their $x(9x-1)(x-1) = 0$

It is important to note that it is not just for the non-zero roots of their $x(9x-1)(x-1) = 0$

This can be implied by setting $3^y = 1$ OR $\frac{1}{9}$ if they write down the roots of part (a) as 0, 1 and $\frac{1}{9}$

Can be implied by $x = 1$ OR $\frac{1}{9}$ followed by a correct y value.

It is also implied by this sort of work as $a = 3^y$

Handwritten work showing the substitution $a = 3^y$ and solving the cubic equation:

$$\text{let } 3^y = a \quad 9^y = 3^{2y} = a^2 \quad 27^y = 3^{3y} = a^3$$

$$9a^3 - 10a^2 + a = 0$$

$$a(9a-1)(a-1) = 0$$

$$a = 0, \frac{1}{9}, 1$$

dM1: Solves their $3^y = "1"$ or $3^y = "\frac{1}{9}"$.

To score this, the root must be either correct or else a power of 3

A1: CSO $y = 0, -2$ only.

The solutions must follow M1, dM1. Candidates cannot just state $y = 0$ without any working and score marks.

Condone use of 'or' between the values.

Condone the use of logs in proceeding to an answer. E.g. $3^y = \frac{1}{9} \Rightarrow y = \log_3 \frac{1}{9} = -2$

Question Number	Scheme	Marks
6 (a)	Angle $AFB = \frac{\pi - 2.275}{2} = 0.433$ *	B1*
(b)	Attempts $r\theta = 6.2 \times 2.275 = (14.105)$	M1
	Attempts $(x^2) = 6.4^2 + 6.2^2 - 2 \times 6.4 \times 6.2 \cos(0.433) = (7.36)$ or $(x = 2.71)$	M1
	Correct attempt $= 2x + r\theta + 12.8 = 2 \times 2.714 + 14.105 + 12.8 = 32.3$ (m)	dM1, A1
(c)	Attempts $\frac{1}{2}r^2\theta = \frac{1}{2} \times 6.2^2 \times 2.275 = (43.7255)$	M1
	Attempts $\frac{1}{2}ab \sin C = \frac{1}{2} \times 6.2 \times 6.4 \times \sin 0.433 = (8.325)$	M1
	Correct attempt $= 2 \times 8.325 + 43.7255 = 60.4$ (m ²)	dM1, A1
		(9 marks)

(a)

B1*: Shows that angle $AFB = 0.433$ radians.You may see the angle labelled θ or BFA . This is not an issue, just look for the calculationIt is a "show that" so it is important to see either $\frac{\pi - 2.275}{2}$ or decimal equivalent with at least thisaccuracy $\frac{3.1416 - 2.275}{2}$ before we see 0.433 written down. The bracketing if required must becorrect. So $(\pi - 2.275) \div 2$ is correct but $\pi - 2.275 \div 2$ is not. Condone a more accurate answerbeing written down and left without rounding to 3 decimal places. E.g. $\frac{\pi - 2.275}{2} = 0.433296327$

(b)

M1: Attempts $r\theta = 6.2 \times 2.275$. Implied by sight of awrt 14.1Sight of $2 \times 6.2 \times 2.275$ would be M0 as this would imply an incorrect formula. There are not two arc lengths within the perimeter of the tunnel.M1: Attempts to apply the cosine rule to find AB or DE .Can be achieved for an attempt at $6.4^2 + 6.2^2 - 2 \times 6.4 \times 6.2 \cos(0.433)$ irrespective of the LHS.Condone a larger calculation that includes $2 \times \left(\sqrt{6.4^2 + 6.2^2 - 2 \times 6.4 \times 6.2 \cos(0.433)} \right)$ or even $2 \times \left(6.4^2 + 6.2^2 - 2 \times 6.4 \times 6.2 \cos(0.433) \right)$ as both AB and DE are part of the perimeter.

The angle used must be 0.433 or a more accurate version of this

This can be implied by, for example, by sight of $AB^2 = 7.36...$ OR $AB = 2.71...$ Condone $\cos 0.433 = \frac{6.2^2 + 6.4^2 - AB^2}{2 \times 6.2 \times 6.4}$ leading to a value for AB^2 or AB Attempts via right angled triangles and incorrect cosine rules, e.g. $(c^2) = a^2 + b^2 - ab \cos C$ are M0dM1: Full attempt to find the perimeter of the tunnel. $2 \times "2.714" + "14.105" + 12.8$ It is dependent upon both M's and a correct attempt at the cosine rule by square rooting their answer to $6.4^2 + 6.2^2 - 2 \times 6.4 \times 6.2 \cos(0.433)$ o.e.

A1: awrt 32.3(m)



(c)

M1: Attempts $\frac{1}{2}r^2\theta = \frac{1}{2} \times 6.2^2 \times 2.275$. Implied by awrt 43.7M1: Attempts either $\frac{1}{2}ab\sin C = \frac{1}{2} \times 6.2 \times 6.4 \times \sin 0.433$ or $ab\sin C = 6.2 \times 6.4 \times \sin 0.433$

The angle used must be 0.433 or a more accurate version of this.

dM1: Full attempt to find the cross-sectional area of the tunnel.

It is dependent upon both M's and an attempt to add a sector and two triangles

A1: awrt $60.4 \text{ (m}^2\text{)}$
Answers using degrees can still be awarded ALL of the marks.

In part (a) it would be difficult to score the mark but you should expect to see the following calculations

$$\text{angle } BFD = 2.275 \times \frac{180}{\pi} = \text{awrt } 130.3^\circ$$

$$\text{angle } AFB = \frac{180^\circ - 130.34^\circ}{2} = \text{awrt } 24.8^\circ = 24.8 \times \frac{\pi}{180} = 0.433$$

For parts (b) and (c) the calculations must be attempted using angles with (at least) the following accuracy.

$$\text{angle } BFD = \text{awrt } 130^\circ \text{ and angle } AFB = \text{awrt } 25^\circ$$

For (b)

M1: Attempts $\frac{\theta}{360} \times 2\pi r = \frac{130}{360} \times 2\pi \times 6.2$ M1: Attempts $(x^2) = 6.4^2 + 6.2^2 - 2 \times 6.4 \times 6.2 \cos 25^\circ$

For (c)

M1: Attempts $\frac{\theta}{360} \times \pi r^2 = \frac{130}{360} \times \pi \times 6.2^2$ M1: Attempts $\frac{1}{2}ab\sin C = \frac{1}{2} \times 6.2 \times 6.4 \times \sin 25$ or $ab\sin C = 6.2 \times 6.4 \times \sin 25$

Question Number	Scheme	Marks
1. (a)	Attempts gradient = $\frac{20 - -4}{-5 - 3} = (-3)$ Attempts equation of line $y - 20 = "-3"(x + 5)$ or $y + 4 = "-3"(x - 3)$ $y = -3x + 5$	M1 dM1 A1 (3)
(b)	Gradient $\frac{1}{3}$ or midpoint $(-1, 8)$ $y - 8 = \frac{1}{3}(x + 1)$ $x - 3y + 25 = 0$	B1ft M1 A1 (3) (6 marks)

(a)

M1: Attempts gradient = $\frac{\delta y}{\delta x}$ condoning slips. Look for an attempt at a difference in y coordinates over a difference in x coordinates. There is no requirement to simplify. They may use $20 - -4 = m(-5 - 3)$ to find m .

dM1: Full method for equation of line using gradient and either point – as shown or may use $y = "-3"x + c$ proceeding as far as finding c .

A1: $y = -3x + 5$ The coefficients must be simplified, award A0 for $y = \frac{24}{-8}x + 5$

Alt (a)

M1: Sets up two simultaneous equations using both points E.g $20 = -5m + c$ and $-4 = 3m + c$ (allow one slip).

dM1: Solves the pair of simultaneous equations to find values for m and c (accept any values following two suitable equations having been set up).

A1: $y = -3x + 5$

(b)

B1ft: Correct mid point or gradient, allowing follow through on the negative reciprocal of their gradient in (a)

M1: Attempts the perpendicular bisector (need not be a simplified form). Must be using $\pm$ the reciprocal of their gradient from (a) with a valid attempt at the midpoint – at least one correct coordinate (or calculation for) must be seen for the midpoint.

A1: $x - 3y + 25 = 0$ but allow any integer multiple and term in a different order (but must be on one side). Ignore if they give a label $l_2 = x - 3y + 25 = 0$



Question Number	Scheme	Marks
3.	$\int \frac{(x+3)^2}{3\sqrt{x}} dx$ $\frac{(x+3)^2}{3\sqrt{x}} = \frac{x^2+6x+9}{3\sqrt{x}} = \frac{1}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$ $\int \frac{(x+3)^2}{3\sqrt{x}} dx = \frac{1}{3} \times \frac{2}{5}x^{\frac{5}{2}} + 2 \times \frac{2}{3}x^{\frac{3}{2}} + 3 \times \frac{2}{1}x^{\frac{1}{2}} + c$ $= \frac{2}{15}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + c$	M1 dM1 A1, A1, A1 (5 marks) (5)

M1: For an attempt to multiply out the numerator and divide at least two terms by the denominator with at least one power correct. Allow if there are only two terms from squaring.

dM1: For correctly achieving two correct powers following integration (need not be simplified).

A1: Previous dM must have been gained (so two correct powers after integrating), at least **one** correct algebraic term, allow unsimplified. E.g, allow $\frac{1}{3}x^{\frac{5}{2}}$ Allow $\frac{1}{3}(\dots)$ with one correct term inside the bracket.

A1: Two correct **simplified** algebraic terms. May have the $\frac{1}{3}$ factored out.

A1: Fully correct and simplified including the + c, accepting equivalent simplified forms. Ignore spurious extra integral signs or dx's if they have the correct expression. May have the $\frac{1}{3}$ factored out.

ISW if they multiply through their answer by a constant after the integration.

Note: If you see attempts at parts or substitution that look worthy of merit then send to review.



Question Number	Scheme	Marks
4. (a)	$(270^\circ, 0)$	B1 (1)
(b)	$(-180^\circ, -4)$	B1, B1 (2)
(c)(i)	100	B1
(ii)	4	B1
(iii)	7	B1
		(3) (6 marks)

Note: Watch for answers next to the question itself rather than in line. If different answers are given next to a question and in the answer space then score the response in the answer space.

(a)

B1: $(270^\circ, 0)$ but allow $x = 270, y = 0$ (degrees symbol may be missing) Just 270 without reference to coordinates is B0. Condone missing brackets, but the order must be correct.

(b)

B1: Either $(-180^\circ, y)$ or $(x, -4)$ and allow if coordinates are the wrong way round for this mark, but they must be giving a coordinate pair. E.g. just -4 on its own is B0, but $(-4, p)$ or $(q, -180^\circ)$ can score B1.

SC: Allow $(-\pi, y)$ (if y is incorrect – they will score B1 regardless of if y is correct).

B1: $(-180^\circ, -4)$ but allow $x = -180, y = -4$

(degrees symbol may be missing). Condone missing brackets, but the order must be correct.

(c)

B1: 100

B1: 4

B1: 7

Note : if there is no labelling for (i),(ii) and (iii) then score in the order listed.

Question Number	Scheme	Marks
5. (a)	$x - 2y + 25 = 0$	
	$y = \frac{1}{2}x + \frac{25}{2} \Rightarrow y = -2x + \dots$	M1
	$y = -2x$	A1
(b)	Substitutes $y = -2x$ into $x - 2y + 25 = 0 \Rightarrow x + 4x + 25 = 0$ $\Rightarrow x = -5, y = 10$	M1 A1, A1 (3)
(c)	Shortest distance = $\sqrt{5^2 + 10^2} = 5\sqrt{5}$	M1, A1cso (2)
		(7 marks)

(a)

M1: Attempts the normal gradient. E.g. writes $x - 2y + 25 = 0$ in the form $y = px + q$ AND changes the gradient (need not be correctly changed). Alternatively, may switch the x and y coordinates and change a sign – e.g. perpendicular line is $2x + y + k = 0$. Another approach is to find the gradient of l_1 by identifying two points on the line and attempting the gradient (must be change in y over change in x) between these two points.

May be implied by the correct answer.

A1: $y = -2x$ o.e. Score for the correct answer even if an incorrect “ q ” was seen.

(b)

M1: Attempts to solve their $y = -2x$ (must be of form kx) with $x - 2y + 25 = 0$ (or their rearrangement of this).

Score for setting up an equation in a single variable.

A1: $x = -5$ OR $y = 10$ from correct work.

A1: Both $x = -5, y = 10$ from correct work

(c)

M1: Attempts to use Pythagoras theorem to find the distance or distance squared between $(0, 0)$ and their $(-5, 10)$ or other full method to find the shortest distance.

A1cso: $5\sqrt{5}$ Must be simplified surd form. Must have come from correct coordinates in (b).



Question Number	Scheme	Marks
6 (a)	$\frac{\sin CAO}{17} = \frac{\sin 0.6}{15} \Rightarrow CAO = 0.6944\dots$ $\text{Angle } COA = \pi - 0.6 - 0.6944 = 1.847 \quad *$	M1 dM1, A1* (3)
(b)	Attempts $\frac{1}{2}r^2\theta = \frac{1}{2} \times 15^2 \times \theta$ where $\theta = (2\pi - 1.847)$ or just 1.847 OR attempts $\frac{1}{2}ab \sin C = \frac{1}{2} \times 15 \times 17 \sin(1.847)$ Attempts $\frac{1}{2}r^2\theta = \frac{1}{2} \times 15^2 \times (2\pi - 1.847) (\approx 499)$ AND $\frac{1}{2}ab \sin C = \frac{1}{2} \times 15 \times 17 \sin(1.847) (\approx 122.7)$ AND adds (awrt) 622 m²	M1 dM1 A1 (3)
(c)	$r\theta = 15 \times \theta$ where $\theta = (2\pi - 1.847)$ or just 1.847 OR $(AC^2 =) 15^2 + 17^2 - 2 \times 15 \times 17 \cos(1.847) (\approx 653)$ Attempts $r\theta = 15 \times (2\pi - 1.847) (= 66.54)$ AND $\{AC = \} \sqrt{15^2 + 17^2 - 2 \times 15 \times 17 \cos(1.847)}$ 92.1 + 2 = (awrt) 94.1 m	M1 dM1 A1 (3) (9 marks)

Note: Work must be completed in the appropriate parts to gain credit, but if no labelling is given, apply benefit of doubt that area work is for (b) and perimeter work is for (c).

Question Number	Scheme	Marks
9(a)	$(CQ^2 =) 0.5^2 + 1.84^2 - 2 \times 0.5 \times 1.84 \cos 0.8$	M1
	(Radius =) $CQ = 1.534 \text{ m}$	A1
		(2)
(b)	$\frac{\sin PCQ}{0.5} = \frac{\sin 0.8}{1.534}$ or $0.5^2 = 1.534^2 + 1.84^2 - 2 \times 1.534 \times 1.84 \cos PCQ$	M1
	$\Rightarrow \sin PCQ = \frac{0.5 \sin 0.8}{1.534} (= 0.233 \dots) \Rightarrow PCQ = 0.236^*$ or $\cos PCQ = \frac{1.534^2 + 1.84^2 - 0.5^2}{2 \times 1.534 \times 1.84} (= 0.972 \dots) \Rightarrow PCQ = 0.236^*$	A1*
		(2)
(c)	$\frac{1}{2} r^2 \theta = \frac{1}{2} \times 1.534^2 \times (2\pi - 0.236)$ oe e.g. $\pi 1.534^2 - \frac{1}{2} \times 1.534^2 \times 0.236$ (= 7.114...) OR $\frac{1}{2} ab \sin C = \frac{1}{2} \times 0.5 \times 1.84 \sin(0.8)$ or $\frac{1}{2} \times 1.534 \times 1.84 \sin(0.236)$ (= 0.3299...)	M1
	Attempts $\frac{1}{2} r^2 \theta = \frac{1}{2} \times 1.534^2 \times (2\pi - 0.236)$ AND $\frac{1}{2} ab \sin C = \frac{1}{2} \times 0.5 \times 1.84 \sin(0.8)$ AND adds	dM1
	(awrt) $7.4 \text{ (m}^2\text{)}$	A1
		(3)
(d)	Attempts $r\theta = 1.534 \times (2\pi - 0.236)$ (= 9.276...)	M1
	Perimeter = $1.534 \times (2\pi - 0.236) + 0.5 + (1.84 - 1.534) = 10.1 \text{ (m)}$	A1
		(2)
		(9 marks)

Notes

It is acceptable in this question to work in degrees and convert if necessary.

NB 0.8 radians is 45.8366...° and angle PCQ is 13.5205...°

Note that there will be other valid methods – if in doubt use review.

See right angled triangle methods for (a) and (b) at the end of the MS

(a)

M1: Writes down the correct rhs for the cosine rule.

Allow e.g. $\cos 0.8 = \frac{0.5^2 + 1.84^2 - CQ^2}{2 \times 0.5 \times 1.84}$

A1: $CQ = 1.534$ This value only i.e. not awrt.

(b)

M1: A correct sine rule statement following through on their CQ which may appear as

$\sin PCQ = \frac{0.5 \sin 0.8}{1.534}$ and would also count as the intermediate step below.



A1*: Correct proof with an intermediate line such as $\sin PCQ = \frac{0.5 \sin 0.8}{1.534}$ or $\sin PCQ = 0.233...$

If awarding for the value of $\sin PCQ$ then allow $\sin PCQ = \text{awrt } 0.23$

There should be no incorrect statements in the proof e.g. $PCQ = \frac{0.5 \sin 0.8}{1.534} = 0.236$

and no obvious incorrect work. E.g. although using CQ as 1.53 gives $PCQ = 0.237$ to 3 dp we will condone PCQ given as 0.236 if no incorrect work is seen as we assume greater accuracy for CQ has been used. If CQ is used to even less accuracy in their working e.g. 1.5 score M1A0 for otherwise correct work.

Must be 0.236 and not e.g. 0.2360

Alternative:

M1: A correct cosine rule statement following through on their CQ which may appear as

$$\cos PCQ = \frac{1.534^2 + 1.84^2 - 0.5^2}{2 \times 1.534 \times 1.84} \text{ and would also count as the intermediate step below.}$$

A1*: Correct proof with an intermediate line such as $\cos PCQ = \frac{1.534^2 + 1.84^2 - 0.5^2}{2 \times 1.534 \times 1.84}$ or

$$\cos PCQ = 0.972...$$

If awarding for the value of $\sin PCQ$ then allow $\cos PCQ = \text{awrt } 0.97$

There should be no incorrect statements in the proof.

If they work in degrees and reach e.g. $PCQ = 13.5^\circ$ it is acceptable to then state “= 0.236”

Must be 0.236 and not e.g. 0.2360

(c)

M1: Correct method for the area of major sector CQR **or** correct method for area of triangle PQC
May be seen embedded in a calculation. Condone the use of 1.8 for 1.84 as long as the intention is clear.

dM1: Fully correct method for the total area. E.g. finds area of major sector CQR and area triangle PQC and adds **or** finds (area of circle – minor sector CQR + area triangle PQC) or (area of circle + (area triangle PQC - minor sector CQR))

A1: (awrt) 7.4 (m²)

Beware: The area of just the circle is 7.39... i.e. awrt 7.4 but on its own scores no marks.

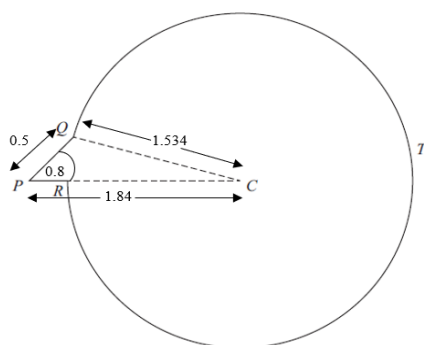
There are also other incorrect methods that give awrt 7.4 so you need to check their working carefully!

(d)

M1: Attempts $r\theta = "1.534" \times (2\pi - 0.236)$ or e.g. $2\pi \times "1.534" - "1.534" \times 0.236$

A1: awrt 10.1 (m)

For reference:



Major sector area 7.114...

Minor sector area 0.277...

Triangle area 0.3299...

Angle $PQC = \text{awrt } 2.11$

Question Number	Scheme	Marks
5.(a)	$2x^3 + 3x^2 - 35x = 0 \Rightarrow x(2x^2 + 3x - 35) = 0$ $(2x-7)(x+5) = 0 \Rightarrow x = \dots$ $x = -5, 0, \frac{7}{2}$	M1 dM1 A1 (3)
(b)	$2(y-5)^6 + 3(y-5)^4 - 35(y-5)^2 = 0$ States that $y = 5$ is a solution $(y-5)^2 = \frac{7}{2} \Rightarrow y = \dots$ $y = 5 + \sqrt{\frac{7}{2}}$ or $y = 5 - \sqrt{\frac{7}{2}}$ or exact equivalent Both $y = 5 + \sqrt{\frac{7}{2}}$ and $y = 5 - \sqrt{\frac{7}{2}}$ or exact equivalent.	B1 M1 A1ft A1 (4) (7 marks)

(a)

M1 Takes out a common factor of x . Score if each term is divided by x .dM1 Attempts to solve the resulting quadratic **via algebra** (usual rules). Allow factorisation, formula or completion of square. They cannot just write down answers from their calculator for this mark.A1 $x = -5, 0, \frac{7}{2}$

Note 1: Some candidates will just write down their answers from a calculator. This scores 0,0,0

Note 2: Some students will attempt to solve the cubic by the quadratic formula Eg.

$$2x^3 + 3x^2 - 35x = 0 \Rightarrow a = 2, b = 3, c = -35 \text{ and use } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = -5, \frac{7}{2}$$

This scores 0,0,0 as the method used is incorrect

(b)

B1 States that 5 is a solution of the given equation in (b)

M1 Realises that $x = (y-5)^2$ and proceeds to find a value for y using $(y-5)^2 = \frac{7}{2} \Rightarrow y = \dots$ Follow through on any positive value from (a). Allow decimal answers here. Don't be overly concerned by the mechanics of their solution.A1ft A solution of $5 + \sqrt{\frac{7}{2}}$ or $5 - \sqrt{\frac{7}{2}}$ You should follow through on their positive root.
Allow decimals for this mark only. So accept awrt 6.87 or awrt 3.13A1 Both $5 + \sqrt{\frac{7}{2}}$ and $5 - \sqrt{\frac{7}{2}}$ with no other solutions for part (b) apart from 5. Do not allow decimal equivalents. Don't allow complex solutions.

Question Number	Scheme	Marks
6.(a)	Sets $4x + c = x(x - 3)$ and attempts to write as a 3TQ Uses $b^2 = 4ac$ for their $x^2 - 7x - c = 0$ Correct equation $49 = -4c$ or $49 + 4c = 0$ $c = -12.25$ oe	M1 dM1 A1 A1 (4)
(b)	Attempt to solve $x^2 - 7x - c = 0$ with their c Attempt to find the y coordinate for their x coordinate $\left(\frac{7}{2}, \frac{7}{4}\right)$ oe	M1 dM1 A1 (3) (7 marks)

- (a)
- M1 Sets $4x + c = x(x - 3)$ and attempts to write as a 3TQ. All terms don't need to be on the same side of the equation and you should condone slips
- dM1 Attempts to use $b^2 = 4ac$ or $b^2 - 4ac = 0$ for their 3TQ. This may be implied by later work.
For a correct 3TQ, errors on "c" will be common, so condone $\pm 49 \pm 4c = 0$ oe
- A1 Correct equation formed in 'c'
- A1 Correct solution $c = -12.25$ oe
- (b)
- M1 Attempts to solve their $x^2 - 7x - c = 0$ with their c ($c \neq 0$)
Allow usual methods. If a calculator is used then the answer(s) must be correct for their 3TQ
Incorrect values of c may produce two values of x
- dM1 Attempt to find the y coordinate from their x coordinate. It is dependent upon the previous M
If there are two coordinates they only need to find the y value for one of their x values to score this mark.
- A1 Correct coordinate given $\left(\frac{7}{2}, \frac{7}{4}\right)$ or exact equivalent. Allow written separately $x = 3.5, y = 1.75$
If c is correct allow the answer to score all 3 marks as long as no incorrect working is seen.

.....
Alternative to (a) by using gradients

- M1 Attempts to set $\frac{d}{dx}(x^2 - 3x) = 4 \Rightarrow 2x - 3 = 4 \Rightarrow x = \dots$
- dM1 Attempts to find the y coordinate by substituting this answer in $y = x(x - 3)$
- A1 Correct equation in c Eg $\frac{7}{4} = 4 \times \frac{7}{2} + c$
- A1 Correct solution $c = -12.25$ oe

Note: Using this method the answer to (b) is found before (a). Some candidates will not realise this however. For the marks in (b) to be awarded there must be a statement somewhere in the solution to the effect that they know that they have solved part (b). This could be simply

- (b) (3.5, 1.75) or coordinates of point of intersection is $\left(\frac{7}{2}, \frac{7}{4}\right)$



Question Number	Scheme	Marks
7.(a)	Attempts to use $\frac{1}{2}r^2\theta$ with $r = 6$ and any allowable angle θ	M1
	Full method to find area $\frac{1}{2} \times 6^2 \times (2\pi - 0.7)$ or $\pi \times 6^2 - \frac{1}{2} \times 6^2 \times 0.7$ = 100.5 cm ² (awrt)	M1 A1 (3)
(b)	Attempts $\frac{\sin \angle ADO}{6} = \frac{\sin 0.7}{5} \Rightarrow \sin \angle ADO = 0.77...$ $\angle ADO = 2.258$ (awrt)	M1 A1 A1 (3)
(c)	Attempts arc length $ABC = 6 \times (2\pi - 0.7)$ 33.50	M1
	Attempts length OD $\frac{\sin(\pi - 0.7 - "2.258")}{OD} = \frac{\sin 0.7}{5} \Rightarrow OD = ...$ 1.42	M1
	Full method to find perimeter = "33.50" + 5 + 6 – "1.42" = 43.1 cm	ddM1 A1 (4)
	(10 marks)	
Alt (c)	Alternative for arc length $ABC = 12\pi - 6 \times 0.7$	M1
	Alternative for finding OD using the cosine rule $OD^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \cos(\pi - 0.7 - "2.258") \Rightarrow OD$	M1
	Solutions where candidate changes to degrees Look for angle $AOD =$ awrt 40° to score M marks	
7.(a)	Attempts to use $\frac{\theta}{360} \pi r^2$ with $r = 6$ and angle $\theta =$ awrt 40 or 320	M1
	Full method to find area $\frac{(360 - \text{awrt } 40)}{360} \times \pi 6^2$ or $\pi \times 6^2 - \frac{\text{awrt } 40}{360} \times \pi 6^2$ = 100.5 cm ² (awrt)	M1 A1 (3)
(b)	Attempts $\frac{\sin \angle ADO}{6} = \frac{\sin 40^\circ}{5} \Rightarrow \sin \angle ADO = 0.77...$ $\angle ADO = 129.4^\circ$ (awrt)	M1 A1 A1 (3)
(c)	Attempts arc length $ABC = \frac{(360 - 40)}{360} \times 2\pi 6$ 33.50	M1
	Attempts length OD $\frac{\sin(180 - 40 - "129.4")}{OD} = \frac{\sin 40}{5} \Rightarrow OD = ...$ 1.42	M1
	Full method to find perimeter = "33.50" + 5 + 6 – "1.42" = 43.1 cm	ddM1 A1 (4)
	(10 marks)	



Notes

(a)

 M1 Attempts to use $\frac{1}{2}r^2\theta$ with $r = 6$ and any angle allowable angle θ

 Allowable angles are; 0.7 $\pi - 0.7 =$ allow awrt 2.4 $2\pi - 0.7 =$ allow awrt 5.6

 M1 A correct attempt to find the area of sector $ABCOA$. See scheme Accept awrt 100 or 101 for this mark

 A1 awrt 100.5(cm²) The units are not required

(b)

 M1 Attempts the sine rule with the lengths and angles in the correct positions $\frac{\sin \angle ADO}{6} = \frac{\sin 0.7}{5}$

 A1 Correct value for $\sin \angle ADO = 0.77\dots$ Be careful here! $\angle ADO = 0.77\dots$ is A0

May be implied by either a correct answer or awrt 0.88

 A1 awrt $\angle ADO = 2.258$ or 129.4°

(c)

 M1 A correct method to find arc length ABC May be implied by sight of $6 \times$ awrt 5.6 or awrt 33.5 or 33.6

 M1 A correct method to find length OD using either the sine rule or cosine rule. The angle OAD must be attempted using a correct method ($\pi - 0.7 - "2.258"$).

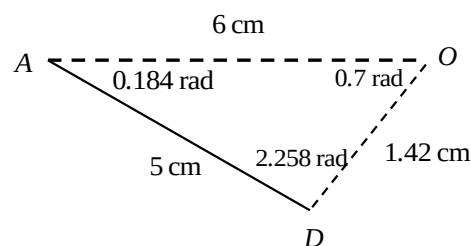
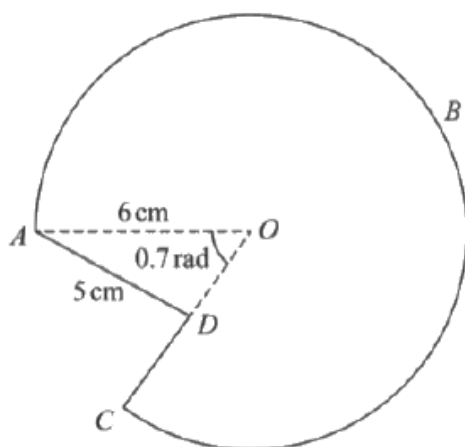
 Eg. For the sine rule $\frac{OD}{\sin(\pi - 0.7 - "2.258")} = \frac{5}{\sin 0.7} = \frac{6}{\sin "2.258"} \Rightarrow OD = \dots$

 For cosine rule it could be $5^2 = 6^2 + x^2 - 2 \times 6 \times x \times \cos(0.7) \Rightarrow 3\text{TQ in } x$ which must be solved by correct methods

 ddM1 Both previous M's must have been scored. It is for a correct method to find the perimeter of the shape. Look for " 33.5 " + $5 + 6$ – " 1.42 ". It is implied by awrt 43

A1 cso and cao 43.1 cm. Units are not required

Handy Diagrams



General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



WMA11 October 2020 Mark Scheme

Question	Scheme	Marks
1	$a = 162$	B1
	$b = 5$	B1
	$c = 12$	B1
		(3 marks)

Notes

Make sure you mark in this order on open.

B1 $a = 162$

B1 $b = 5$ condone $p = 5$

B1 $c = 12$ condone $q = 12$

Note: The values may be implied by their expression. If there is a contradiction between what appears to be their final expression and values for a , b or c which are incorrectly stated afterwards then treat this as isw.



Question	Scheme	Marks
2(a)	$b = 2$ $\dots \pm \dots (x \pm 3)^2$ $(f(x) =) 21 - 2(x - 3)^2$	B1 M1 A1
		(3)
(b)	R is $(0, -4)$ or " h " = 4 $f(x) - 7 = 14 - 2(x - 3)^2 \Rightarrow x = \dots$ or $f(x) - 7 = -4 + 12x - 2x^2 \Rightarrow x = \dots$ (NB $x = 3 \pm \sqrt{7}$) $\text{Area} = \frac{1}{2} \times ("3 + \sqrt{7}" - ("3 - \sqrt{7}")) \times "4"$ $= 4\sqrt{7}$	B1 M1 dM1 A1
		(4)
		(7 marks)

Notes

(a)

B1 $b = 2$ implied from their expression or stated. Eg look for $-2(x \pm \dots)^2$

 Beware of expressions such as $21 + 2(x - 3)^2$ which would be B0

M1 Sight of $(x \pm 3)^2$
A1 $21 - 2(x - 3)^2$. Condone $21 - 2(x + -3)^2$ or they may state the values for a, b and c . If there is a contradiction between what appears to be their final expression and values for a, b or c which are incorrectly stated afterwards then treat this as isw.

(b)

B1 R is $(0, -4)$ or " h " = 4 (The coordinate may be seen on a diagram as a point of their triangle or quadratic so allow just -4 rather than the full coordinate to be indicated on the y -axis). It may also be implied by their working.

M1 Attempts to solve $f(x) - 7 = 0$

Score for an attempt to solve a 3TQ of the form:

 $-2x^2 + 12x + C = 0$ where $C \neq 3$ or $A - 2(x - 3)^2 = 0$ where $A \neq 21$

 or if they use their part (a) " a " - " b " $(x + "c")^2 = 0 \Rightarrow d - "b"(x + "c")^2 = 0$ where $d \neq a$


Question Number	Scheme	Marks
7(a)	$y \text{ coordinate} = 12$	B1
		(1)
(b)	$\text{Gradient of } l_1 = -\frac{3}{4}$ $\Rightarrow \text{Gradient of } l_2 = \frac{4}{3} \Rightarrow (y-6) = \frac{4}{3}(x-8)$ $y \text{ coordinate} = -\frac{14}{3} \quad *$	B1 M1 A1* cso
		(3)
(c)	$\text{Radius} = 12 + \frac{14}{3} = \frac{50}{3}$ $\text{Length of arc} = \frac{50}{3} \times 1.8 = 30$	B1ft M1A1cao
		(3)
(d)	$\text{Area of sector} = \frac{1}{2} \times \left(\frac{50}{3}\right)^2 \times 1.8 \quad (= 250)$ $250 + \frac{1}{2} \times \frac{50}{3} \times 8 = 250 + \frac{200}{3}$ $= \frac{950}{3} \quad (\text{units}^2)$	M1 M1 A1cao
		(3)
		(10 marks)

Mark all parts together. May work in degrees.

(a)

B1 12 (Check by the question and also on the diagram). If there is a contradiction then their answer in the main solution takes precedence.

(b)



B1 States gradient of l_1 is $-\frac{3}{4}$ but can be implied by further work. Eg sight of a gradient of $\frac{4}{3}$ in their equation for l_2 can also score this mark.
The value must be identified or used so it cannot just be awarded from a rearranged equation for l_1 . Circling the coefficient is acceptable but stating $-\frac{3}{4}x$ with no further work is B0.

M1 Attempts to find the gradient of the perpendicular line " $-\frac{3}{4}$ " $\rightarrow \frac{4}{3}$ and attempts to find the equation of l_2 . Look for $(y-6) = \frac{4}{3}(x-8)$ with both of the brackets correct. If they attempt using $y = mx + c$ then they must proceed as far as $c = \dots$

A1* $-\frac{14}{3}$ cso must be clearly stated as the y coordinate with no errors seen after achieving a correct equation for l_2 .

(c)

B1ft Finds the radius of the circle following through on their answer to (a). " $12 + \frac{14}{3}$ " is acceptable for this mark or it may be implied by their length of the arc. May be seen on the diagram or in other parts.

M1 Attempts to find the length of the arc with $\theta = 1.8$ and their $r = "12" + \frac{14}{3}$

A1 30 cao

(d)

M1 Attempts to find the area of the sector with $\theta = 1.8$ and their $r = "12" + \frac{14}{3}$

M1 **Adds the area of their sector with a correct method to find the area of the triangle.**

There are various ways to find the area of the triangle. They may find the lengths CD and DE using Pythagoras and proceed to find the area of the triangle:

$$\text{Eg } CD = \sqrt{6^2 + 8^2} = 10 \text{ and } DE = \sqrt{8^2 + \left(\frac{32}{3}\right)^2} = \frac{40}{3} \Rightarrow \text{Area} = \frac{1}{2} \times 10 \times \frac{40}{3} = \frac{200}{3}$$

Alternatively, via the shoelace method:

$$\text{Eg } \frac{1}{2} \begin{vmatrix} 0 & 12 \\ 8 & 6 \\ 0 & -\frac{14}{3} \\ 0 & 12 \end{vmatrix} = \frac{1}{2} \times \left| \left(8 \times -\frac{14}{3} \right) - (12 \times 8) \right| = \frac{200}{3}$$

A1 $\frac{950}{3}$ cao (accept $316\frac{2}{3}$ or $316.\dot{6}$ but not 316.7)

Question Number	Scheme	Marks
7.(a)	$OB^2 = 0.6^2 + 1.4^2 - 2 \times 0.6 \times 1.4 \cos 2 \Rightarrow OB^2 = \dots$ or $OB = \dots$	M1
	$OB = 1.738$	A1
	$\frac{\sin AOB}{1.4} = \frac{\sin 2}{1.738} \Rightarrow AOB = 0.822$ or e.g. $\frac{\sin ABO}{0.6} = \frac{\sin 2}{1.738} \Rightarrow ABO = 0.319 \dots \Rightarrow AOB = \pi - 2 - 0.319 \dots$	dM1
	$\theta = 2 \times AOB = 2 \times 0.822 = 1.64^*$	A1*
		(4)
(b)	Attempts $0.6 \times \alpha$ with $\alpha = 2\pi - 1.64$ or $\alpha = \pi - 1.64$ or Attempts $2 \times \pi \times 0.6 - 0.6 \times 1.64$	M1
	$0.6 \times (2\pi - 1.64) + 2.8 = 5.6 \text{ m}$	A1
		(2)
(c)	Attempts $\frac{1}{2} \times 0.6^2 \times \alpha$ with $\alpha = 2\pi - 1.64$ or $\alpha = \pi - 1.64$ or Attempts $\pi \times 0.6^2 - \frac{1}{2} \times 0.6^2 \times 1.64$	M1
	Attempts $0.6 \times 1.4 \sin 2$	M1
	Full method $\frac{1}{2} \times 0.6^2 \times (2\pi - 1.64) + 0.6 \times 1.4 \sin 2 = 1.6 \text{ m}^2$	ddM1 A1
		(4)
		(10 marks)

In part (a), candidates may not be careful with the use of θ . E.g. in their working, their θ may be angle AOB which they correctly double to get 1.64 – condone this poor notation and give credit if the intention is clear.

Example:

$$OB^2 = 0.6^2 + 1.4^2 - 2 \times 0.6 \times 1.4 \cos 2 \Rightarrow OB = 1.73756 \dots$$

$$\cos \theta = \frac{0.6^2 + 1.738^2 - 1.4^2}{2 \times 0.6 \times 1.738} \Rightarrow \theta = 0.822$$

$$\theta = 2 \times 0.822 = 1.64$$

Is acceptable for full marks in (a)

(a)

M1 Attempts the cosine rule to get OB or OB^2 **seen in part (a) only**

A1 $OB =$ awrt 1.74 or truncated as 1.7 or e.g. 1.73 (may be implied)

dM1 Attempts the sine rule with the sides and angles in the correct positions in an attempt to find AOB or $1/2 AOB$. **This must be a full attempt including using inverse sin to find the angle.**



This may be achieved by attempting the sine rule to find angle ABO first and then using the angle sum. This requires use of the sine rule with the sides and angles in the correct positions in an attempt to find ABO followed by $\pi - 2 - "0.319..."$ or e.g.
 $2\pi - 4 - 2 \times "0.319..."$

Alternatively uses the cosine rule to find angle AOB

$$\text{E.g. } 1.4^2 = 0.6^2 + 1.738^2 - 2 \times 0.6 \times 1.738 \cos \frac{\theta}{2} \Rightarrow \frac{\theta}{2} = \dots$$

Depends on the first method mark.

- A1* Fully correct work leading to the given answer with no obvious rounding errors.
 E.g. if they obtain angle $AOB = 0.827$ following correct a method then state $2 \times 0.827 = 1.64$ this mark should be withheld.

Alternatives working backwards:

$$\theta = 1.64 \Rightarrow \frac{\theta}{2} = 0.82, \frac{BC}{\sin 0.82} = \frac{0.6}{\sin(\pi - 2 - 0.82)} \Rightarrow BC = 1.4 \text{ so } \theta = 1.64$$

$$\text{Or } \theta = 1.64 \Rightarrow \frac{\theta}{2} = 0.82, \frac{1.4}{\sin 0.82} = \frac{OC}{\sin(\pi - 2 - 0.82)} \Rightarrow OC = 0.6 \text{ so } \theta = 1.64$$

M1: Finds $\frac{\theta}{2}$ and uses angle sum of triangle and sine rule

A1: Correct sine rule statement

dM1: Rearranges for BC or OC . **Depends on first method mark.**

A1: Fully correct work to obtain $BC = 1.4$ or $OC = 0.6$ with minimal conclusion e.g. tick, QED etc.

- (b)
 M1 Attempts $0.6 \times \alpha$ with an allowable α
 For an allowable angle accept $2\pi - 1.64$ (awrt 4.64) or $\pi - 1.64$ (awrt 1.50)
 An alternative is to find the circumference and subtract the minor arc AC
 For reference the correct value is 2.78... which may imply the method (3.7699... - 0.984)
 A1 $0.6 \times (2\pi - 1.64) + 2.8 = \text{awrt } 5.6 \text{ m.}$ Condone lack of units

(c) In general the marks in part (c) are M1: Attempting the major sector, M2 attempting the kite area or half of it e.g. area of triangle AOB but not e.g. area of triangle OCA (they would need to find the area of triangle ABC as well, or half of both of these), dM3: A complete and correct method for the total area, A1: awrt 1.6

M1 **This mark is for an attempt at the sector area $OCXA$:**

$$\text{E.g. Attempts } \frac{1}{2} \times 0.6^2 \times \alpha \text{ with } \alpha = 2\pi - 1.64 \text{ or } \alpha = \pi - 1.64$$

An alternative is to find the area of the circle and subtract the area of the minor sector
 For reference the correct value is 0.835... which may imply the method (1.130... - 0.2952)

M1 **This mark is for an attempt at the kite $ABCO$ (or half of it):**

$$\text{Examples: Attempts } 0.6 \times 1.4 \sin 2 \text{ which may be part of } \frac{1}{2} \times 0.6 \times 1.4 \sin 2$$

$$\text{Attempts } OCA + ABC \text{ e.g. } \frac{1}{2} \times 0.6^2 \sin 1.64 + \frac{1}{2} \times 1.4^2 \sin(2\pi - 4 - 1.64)$$

Attempts " $1.74 \times 0.6 \sin \frac{\theta}{2}$ " which may be part of $\frac{1}{2} \times 1.74 \times 0.6 \sin \frac{\theta}{2}$

ddM1 **This mark is for a complete and correct attempt at the total area and depends on both previous method marks:**

e.g. $\frac{1}{2} \times 0.6^2 \times (2\pi - 1.64) + 0.6 \times 1.4 \sin 2$ o.e

A1 awrt 1.6 m^2 Condone a lack of units

Alternative which doesn't follow the above but is equivalent:

$$\text{Area} = \pi \times 0.6^2 - \frac{1}{2} \times 0.6^2 (1.64 - \sin 1.64) + \frac{1}{2} \times 1.4^2 \sin (2\pi - 4 - 1.64)$$

Award M1 for the attempt at the major segment and M1 for the attempt at triangle ABC (or half of it) then ddM1A1 as above.



Some lengths and angles for reference:

$$\begin{aligned}
 OB^2 &= 3.019... \\
 OB &= 1.7375... \\
 AC^2 &= 0.7697... \\
 AC &= 0.8773... \\
 \text{Angle } ABO &= 0.322 \\
 \text{Angle } COA &= 4.64...
 \end{aligned}$$

Solutions where candidates change to degrees		
NB 2 radians is 114.591559...°		
(a)	$OB^2 = 0.6^2 + 1.4^2 - 2 \times 0.6 \times 1.4 \cos 114.59... \Rightarrow OB^2 = ... \text{ or } OB = ...$	M1
	$OB = 1.738$	A1
	$\frac{\sin AOB}{1.4} = \frac{\sin 114.59...}{1.738} \Rightarrow AOB = 0.822$ or e.g.	M1
	$\frac{\sin ABO}{0.6} = \frac{\sin 114.59...}{1.738} \Rightarrow ABO = 18.3 \Rightarrow AOB = 180 - 114.59... - 18.3$	
	$\theta = 2 \times AOB = 2 \times 47.1 = 94.2^\circ = 1.64^*$	A1*
(4)		
(b)	Attempts $\frac{\alpha}{360} \times 2 \times \pi \times 0.6$ with $\alpha = 360 - \text{awrt } 94^\circ$ or Attempts $2 \times \pi \times 0.6 - \frac{\text{awrt } 94^\circ}{360} \times 2 \times \pi \times 0.6$	M1
	$\frac{360 - \text{awrt } 94}{360} \times 2 \times \pi \times 0.6 + 2.8 = 5.6 \text{ m}$	A1
		(2)
(c)	Attempts $\frac{\alpha}{360} \times \pi \times 0.6^2$ with $\alpha = 360 - \text{awrt } 94^\circ$ or Attempts $\pi \times 0.6^2 - \frac{\text{awrt } 94^\circ}{360} \times \pi \times 0.6^2$	M1
	Attempts e.g. $0.6 \times 1.4 \sin 114.59...$	M1
	Full method e.g. $\frac{360 - \text{awrt } 94^\circ}{360} \times \pi \times 0.6^2 + 0.6 \times 1.4 \sin 114.59... = 1.6 \text{ m}^2$	dM1 A1
		(4)
(10 marks)		

M1: Uses at least one x value of the form $p\sqrt{a}$, $a > 0$ and uses a non calculator method to find a value for $y = q\sqrt{a}$. Must use x correctly, not x^2 in place of x . It is not a dependent mark so if $x = \sqrt{10}$ followed by a correct non calculator method you may award the mark. Some calculations must be seen, not just a written down answer. So $x = \sqrt{10} \Rightarrow y = (\sqrt{10})^3 - 6\sqrt{10} = 4\sqrt{10}$ is acceptable

A1: At least one correct pair of solutions. Need not be fully simplified but should be single terms, so accept e.g. $\left(\sqrt{10}, \frac{40}{\sqrt{10}}\right)$ if they use the other equation to find y .

So a mark of 00110 is possible for candidates who show evidence for y but not for x .

A1: CSO. All previous marks must have been awarded in part (b). Both pairs of solutions correct and simplified with no incorrect extra answers given. Must be clearly paired so do not accept

$x = \pm\sqrt{10}, y = \pm 4\sqrt{10}$ unless clear pairings have been given elsewhere.

Question	Scheme	Marks
7(a)	$f'(x) = 2x^{-\frac{1}{2}} + Ax^{-2} + 3 \Rightarrow f''(x) = \dots x^{-\frac{1}{2}-1} + \dots x^{-2-1}$	M1
	$\Rightarrow f''(x) = 2 \times -\frac{1}{2} x^{-\frac{3}{2}} + -2Ax^{-3} = -x^{-\frac{3}{2}} - 2Ax^{-3}$	A1
	$f''(4) = 0 \Rightarrow -4^{-\frac{3}{2}} - 2A \times 4^{-3} = 0 \Rightarrow A = \dots$	dM1
	$-\frac{1}{8} - \frac{2A}{64} = 0 \Rightarrow A = -4$	A1
		(4)
(b)	$f(x) = \int 2x^{-\frac{1}{2}} + Ax^{-2} + 3 \, dx = \frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{Ax^{-2+1}}{-2+1} + 3x(+c)$	M1
	$= 4x^{\frac{1}{2}} - \frac{A}{x} + 3x(+c)$	A1ft
	$f(12) = 8\sqrt{3} \Rightarrow 4\sqrt{12} - \frac{A}{12} + 36 + c = 8\sqrt{3} \Rightarrow c = \dots$	dM1
	$c = 8\sqrt{3} - 4\sqrt{12} - 36 - \frac{4}{12} = -\frac{109}{3}$ or follow through $c = \frac{A}{12} - 36$	A1ft
	So $f(x) = 4x^{\frac{1}{2}} + \frac{4}{x} + 3x - \frac{109}{3}$ oe	A1
		(5)
(9 marks)		
Notes: Mark parts (a) and (b) together		

(a)**M1:** Correct method of differentiation, at least one **correct** power reduced by 1.

The indices or coefficients do not have to be processed/simplified.

A1: Correct differentiation, need not be simplified, but the indices and coefficients must be processed correctly.**dM1:** Sets $f''(4) = 0$ and proceeds to find a value for A . It is dependent upon the previous M1

Don't be overly concerned with the mechanics of the solution here

A1: $A = -4$ **(b)****M1:** Attempts to integrate $f'(x)$. Look for a **correct** power increased by 1 on at least one term. The indices and coefficients do not have to be processed/simplified. Constant of integration is not needed for this mark.**A1ft:** Correct integration, either with A or follow through their value of A . The indices and coefficients must be processed correctly. Constant of integration is not needed for this mark.**dM1:** Sets $f(12) = 8\sqrt{3}$ and proceeds to find a value for a constant of integration " c " using their value of A . It is dependent upon the previous M1**A1ft:** Correct value for c found. Follow through their value of A , so $c = \frac{A}{12} - 36$.

Accept awrt 1 dp

A1: Correct answer, $4x^2 + \frac{4}{x} + 3x - \frac{109}{3}$, not follow through.

If they go on to multiply by 3 etc it is A0



Question	Scheme	Marks
8(a)	With θ being the angle subtended by arc AB and ϕ being the angle subtended by arc CD	
	$15 = 9 \times \theta \Rightarrow \theta = \frac{5}{3} = (1.67)$	M1
	Therefore $\phi = \frac{2\pi}{3} - \frac{5}{3} = (0.4277\dots)$	dM1
	So length of arc $CD = 84 \times \left(\frac{2\pi}{3} - \frac{5}{3} \right) = 35.929\dots = 35.9 \text{ cm (1 d.p.)}^*$ CSO	A1*
		(3)
(b)	Perimeter $= 3 \times (15 + 35.9\dots) + 6 \times (84 - 9)$	M1
	$= \text{awrt } 603 \text{ cm (602.787\dots)}$	A1
		(2)
(c)	FOR EXAMPLE Area of a “blade” is $\frac{1}{2} \times 84^2 \times \left(\frac{2\pi - 5}{3} \right) = \text{awrt (1510)}$	M1
	Area of sector of inner circle between “blades” is $\frac{1}{2} \times 9^2 \times \frac{5}{3} = (67.5)$	dM1 A1
	Total area is $3 \left(\frac{1}{2} \times 84^2 \times \left(\frac{2\pi - 5}{3} \right) + \frac{1}{2} \times 9^2 \times \frac{5}{3} \right) = \dots (4729.577764 \text{ cm}^2)$	ddM1
	So area is awrt 0.473 m^2 or awrt 4730 cm^2	A1
		(5)
		(10 marks)
Notes:		

(a)

M1: Correct use of the arc length formula to find the angle subtended by arc AB .

Attempts $15 = 9 \times \theta \Rightarrow \theta = \dots$ Don't be concerned by what the angle is called

dM1: Correct method to find the angle subtended by arc CD using their angle for arc AB .

Note that $\phi = \frac{1}{3} \left(2\pi - 3 \times \frac{5}{3} \right)$ is also correct. It is dependent upon the previous M

A1*: CSO Arrives at 35.9 with a correct value to at least 2 d.p. (rounded or truncated) seen first.

Alternatively sight of $84 \times \left(\frac{2\pi}{3} - \frac{5}{3} \right)$ or $84 \times \text{awrt } 0.4277$ followed by $35.9(\text{cm})$ is fine

Note that there are equivalent methods such as $84 \times \frac{2\pi}{3} - 84 \times \frac{5}{3} = 35.9$ or $\frac{2\pi}{3} \times 84 - 140 = 35.9$

(b)

M1: Correct method to find the perimeter, it should include all six arcs and radial edges.

Look for $3 \times 15 + 3 \times 35.9 + 6 \times \dots$ If no method is seen it is implied by awrt 603



Question Number	Scheme	Marks
6(a)	$A = \frac{1}{2} r^2 \theta \Rightarrow 40 = \frac{1}{2} r^2 \times 2.4 \Rightarrow r = \dots$	M1
	$r = \sqrt{\frac{80}{2.4}} \Rightarrow r = 5.77 \text{ (m)}$	A1
		(2)
(b)	States or uses that angle $AOB = \text{awrt } 0.37$	B1
		(1)
(c)	$\frac{1}{2} \times "5.77" \times 6.25 \times \sin "0.37" (= 6.5377\dots)$	M1
	Full method for area of stage = $40 + 2 \times \frac{1}{2} \times 5.77 \times 6.25 \times \sin 0.37$	dM1
	$= 53.1 \text{ m}^2$	A1
		(3)
(d)	$r\theta = "5.77" \times 2.4 = (13.848)$	M1
	$x^2 = 6.25^2 + "5.77"^2 - 2 \times 6.25 \times "5.77" \cos "0.37" \quad (x = 2.26)$	M1
	Full method for perimeter of stage = $12.5 + 2 \times "2.26" + "5.77" \times 2.4$	ddM1
	$= 30.9 \text{ m}$	A1
		(4)
		(10 marks)

Allow equivalent correct work in degrees but part (b) must be in radians.

(a)

M1: Attempts to use $A = \frac{1}{2} r^2 \theta$ with $A = 40$ and $\theta = 2.4$ to find a value for r

A1: Achieves awrt 5.77 (m). Apply isw if necessary e.g. if they subsequently write 5.77 as 5.8.

(b)

B1: Angle $AOB = \text{awrt } 0.37$. Allow this to score anywhere in their answer.

(c)

M1: Attempts area of triangle AOB (or triangle DOC) $\frac{1}{2} \times "a" \times 6.25 \times \sin "b"$ (May be seen "doubled")

NB: "doubled" is 13.075...

dM1: Attempts area of stage $40 + 2 \times \frac{1}{2} \times "a" \times 6.25 \times \sin "b"$

or e.g. $\frac{1}{2} \times "5.77"^2 \times 2.4 + 2 \times \frac{1}{2} \times "a" \times 6.25 \times \sin "b"$

A1: Awrt 53.1 (m²). Condone awrt 53.0 but not just 53 unless awrt 53.1 or awrt 53.0 is seen earlier.



(d)

M1: Attempts $r\theta = "a" \times 2.4$

M1: Attempts to use the cosine rule to find length AB or AB^2 (or CD/CD^2) (Allow anywhere in their solution)

e.g. $x^2 = 6.25^2 + "a"^2 - 2 \times 6.25 \times "a" \cos "b"$

ddM1: Full method to find perimeter of stage $12.5 + 2 \times AB + "a" \times 2.4$

It must be clear they are using AB not AB^2

Depends on both previous M marks.

A1: Awrt 30.9 (m)

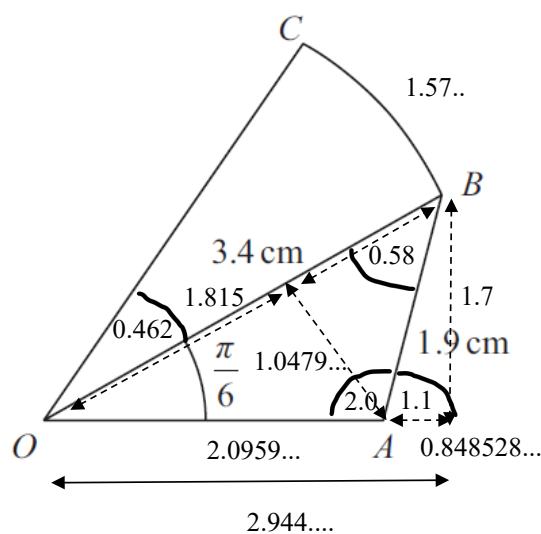
Beware in (d): $AB = 6.25 \sin(0.37) = 2.26...$ erroneously and will lead to the correct answer 30.9

Generally, this will score M1M0ddM0A0 if the arc length calculation is correct.



Question	Scheme	Marks
4(a)	$y = \frac{5x^2 + \sqrt{x^3}}{\sqrt[3]{8x}} = \frac{5x^2 + x^{\frac{3}{2}}}{\dots x^{\frac{1}{3}}}$	M1
	$= \frac{5x^{2-\frac{1}{3}}}{2} + \frac{x^{\frac{3}{2}-\frac{1}{3}}}{2}$	M1
	$= \frac{5}{2}x^{\frac{5}{3}} + \frac{1}{2}x^{\frac{7}{6}}$	A1A1
		(4)
(b)	$\left(\frac{dy}{dx} = \dots x^{\frac{2}{3}} + \dots x^{\frac{1}{6}}\right)$	M1
	$\left(\frac{dy}{dx} = \right) \frac{25}{6}x^{\frac{2}{3}} + \frac{7}{12}x^{\frac{1}{6}}$	A1ftA1
		(3)
(7 marks)		
Notes:		
(a)		
M1: Converts the radicals into powers of x , with at least one power correct. E.g. $\sqrt{x^3} \rightarrow x^{\frac{3}{2}}$ or $\sqrt[3]{8x} \rightarrow \dots x^{\frac{1}{3}}$		
M1: Correct subtraction index law seen used at least once. May be implied. It is for $\frac{5x^2}{\sqrt[3]{8x}} \rightarrow \frac{\dots x^2}{\dots x^p} \rightarrow \dots x^{2-p}$ or $\frac{\sqrt{x^3}}{\sqrt[3]{8x}} \rightarrow \frac{\dots x^q}{\dots x^p} \rightarrow \dots x^{q-p}$ Indices do not need to be processed for this mark.		
A1: One term correct (coefficient and index) or both indices correct. E.g. $\frac{5}{2}x^{\frac{5}{3}}$ or $\frac{1}{2}x^{\frac{7}{6}}$ or $\dots x^{\frac{5}{3}} + \dots x^{\frac{7}{6}}$. Depends on both M marks.		
A1: Fully correct expression in the required form. Depends on all previous marks. $y = \dots$ is not required. Apply isw once a correct expression is seen.		
(b) The $\frac{dy}{dx} =$ is not required throughout in part (b)		
M1: $x^n \rightarrow \dots x^{n-1}$ for at least one term where $n \neq 1$.		
A1ft: Correct differentiation of one of their terms where $n \neq 1$. Follow through their part (a). But not for $kx \rightarrow k$ May be unsimplified. Depends on both M marks in part (a).		
A1: Correct derivative, with terms in simplest form. Apply isw once a correct expression is seen. Score A0 if they include “+ c”		

Question Number	Scheme	Marks
8(a)	$\frac{\sin OAB}{3.4} = \frac{\sin\left(\frac{\pi}{6}\right)}{1.9} \Rightarrow OAB = 2.034$	M1A1A1
		(3)
(b)	$\text{Area} = \frac{1}{2} \times 3.4 \times 1.9 \times \sin\left(\pi - \frac{\pi}{6} - "2.034"\right) = \text{awrt } 1.78 \text{ (cm}^2\text{)}$	M1A1
		(2)
(c)	Area of sector $OBC = "1.78" \times \frac{3}{2} (= "2.67")$	M1
	$\frac{1}{2} \times 3.4^2 \times \theta = \frac{1.78 \times 3}{2}$	M1
	$\theta = \text{awrt } 0.462 *$	A1*
		(3)
(d)	$OA = \sqrt{3.4^2 + 1.9^2 - 2 \times 3.4 \times 1.9 \times \cos\left(\pi - \frac{\pi}{6} - "2.034"\right)}$	M1
	$OA = \text{awrt } 2.1 \text{ (cm)}$	A1
	Length of arc $BC = 0.462 \times 3.4 (= 1.57...)$	B1
	Perimeter = $3.4 + 1.9 + "2.1" + 0.462 \times 3.4$	dM1
	$= \text{awrt } 9 \text{ (cm)}$	A1
		(5)
		(13 marks)



Note that in all parts they may work in degrees and convert back where appropriate which is acceptable. If they do not then the method marks are still available. We do not need to see the conversion, but in calculations they cannot have a mix of radians and degrees in the same expression/equation

- (a) Note the correct angle of awrt 2.034 scores full marks
 M1 Correct equation using the sine rule with the sides and angles in the correct places.
 A1 awrt 1.11 (principal angle may be implied by final answer)
 A1 awrt 2.034 (calc 2.033751055)

Alt(a) – Using the cosine rule twice – must be seen in (a) to score

- M1 Correct equation using the cosine rule with the sides and angles in the correct places i.e.

$$1.9^2 = 3.4^2 + OA^2 - 2 \times 3.4 \times OA \times \cos\left(\frac{\pi}{6}\right) \text{ o.e.}$$

- A1 $OA =$ awrt 2.1

- A1 awrt 2.034

(b) Note that those who find and use OAB as “1.108” instead of 2.034 can score max M1A0

- M1 Correct expression to find the area of the triangle using their angle OAB .

Do not be concerned by the size (or sign) of the angle found by attempting $\pi - \frac{\pi}{6}$ – their OAB . Do

look out for alternative ways of finding the area of the triangle.

Use the diagram on the previous page to help you. Condone slips in the rearrangement of any expressions provided they were seen to be correct initially.

- A1 awrt 1.78 (condone lack of units). (calc 1.781564499)

(c) Note that those who find and use OAB as “1.108” instead of 2.034 can score max M1M1A0

- M1 Attempts to find the area of the sector using the given ratio by dividing their area of the triangle from (b) by 2 and multiplying by 3. May be implied by their area of the sector – you may need to check this on your calculator.

- M1 Sets $\frac{1}{2} \times 3.4^2 \times \theta =$ their area for the sector (or rearranged equivalent)

Condone equating to incorrect areas, provided it is not their area of triangle OAB , which may have been found from an incorrect method or may appear with little or no working.

- A1* Achieves the given answer (in radians) with no errors seen following a correct equation

$$\text{i.e. } \frac{1}{2} \times 3.4^2 \times \theta = \text{awrt } 2.6 \text{ or awrt } 2.7 \Rightarrow \theta = \text{awrt } 0.462$$

Condone premature earlier rounding provided the method is sound and they achieve awrt 0.462 or state 0.462 following a correct method. (calc 0.4623437281)

(d) Note that those who find and use OAB as “1.108” instead of 2.034 to find length OA can score max M1A0B1dM0A0

- M1 Attempts to find the length OA . There are various methods so score for the overall method condoning slips. Use the diagram on the previous page to help you. The expression for OA is sufficient. May be implied by awrt 2.1. If found in an earlier part it must be used in (d) to score. **Condone use of their angle OAB even if it is not obtuse for this mark.**

- A1 awrt 2.1 (or may be implied by later work/the final answer) (calc 2.095958235)

- B1 Correct expression or value for the length of the arc BC .
 Allow to be implied by sight of 1.462×3.4 (if they have combined OC and arc BC)

- dM1 A correct method to find the total perimeter. It is dependent on the previous M and B marks. Condone slips, but the overall method/strategy must be correct. The expression is sufficient.

If their angle OAB is acute instead of obtuse and used to find OA (or other lengths) then this mark cannot be scored. (Note it is possible to find OA using the cosine rule and not use their answer to part (a))

- A1 awrt 9 (cm) following a correct method



Question Number	Scheme	Marks
8(a)(i)	$2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$	B1
	Area of sector = $\frac{1}{2} \times 3^2 \times \frac{4\pi}{3} = 6\pi \text{ (m}^2\text{)}$	M1A1
(ii)	Length of arc = $3 \times \frac{4\pi}{3} \Rightarrow$ Perimeter = $4\pi + 6 \text{ (m)}$	M1A1
		(5)
(b)	$\frac{1}{2} \times 3^2 \times \sin\left(\frac{2\pi}{3}\right) = \frac{9\sqrt{3}}{4} \text{ (m}^2\text{)}$	M1A1
		(2)
(c)	Eg $AB^2 = 3^2 + 3^2 - 2 \times 3 \times 3 \times \cos\left(\frac{2\pi}{3}\right) \Rightarrow AB^2 = 27 \Rightarrow AB = 3\sqrt{3} \text{ (m)}^*$	M1A1*
		(2)
(d)	$\frac{\sin BAC}{8} = \frac{\sin\left(\frac{\pi}{6}\right)}{3\sqrt{3}} \Rightarrow \sin BAC = \dots \text{ or } BAC = \dots$ $\sin BAC = \text{awrt } \frac{4\sqrt{3}}{9} \text{ or } BAC = \text{awrt } 0.88 \text{ (0.8785...)}$ Area $ABC = \frac{1}{2} \times 3\sqrt{3} \times 8 \times \sin\left(\pi - \frac{\pi}{6} - 0.88\right) (= 20.4896\dots)$ Total area = $18.8 + 3.90 + 20.5$ = awrt 43 (m ²)	M1 A1 M1 dM1 A1
		(5)
		(14 marks)

If lengths or areas are found in other parts then credit can be awarded for these as long as they are referred to or used in the relevant part.

(a)(i)

B1 Finds the correct angle for the sector $AOBX$. They may find the minor sector first and then subtract from the area of the whole circle so may be implied from later work. They may also work in degrees. Sight of $\theta = \frac{4\pi}{3}$ on the diagram or within their working scores this mark.

M1 States or uses $\frac{1}{2}r^2\theta$ with $r = 3$ and $\theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$ or the equivalent method working in degrees. May be implied by the correct answer, expression or awrt 9.42 or awrt 18.8
A1 $6\pi \text{ (m}^2\text{)}$ cao must be exact. Isw after a correct answer.

(ii)

M1 States or uses $r\theta$ with $r = 3$ and $\theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$ or the equivalent method working in degrees. The addition of two radii to find the perimeter is not required for this mark. May be implied by the correct answer, expression or awrt 6.28 or awrt 12.6

A1 $4\pi + 6 \text{ (m)}$ cao must be exact. Isw after a correct answer.

(b)

M1 States or uses $\frac{1}{2}ab\sin C$ with $a = b = 3, \theta = \frac{2\pi}{3}$ (or may work in degrees). Alternatively, they may split the isosceles triangle into two right angled triangles including finding AB . Score for the overall method or awrt 3.90

A1 $\frac{9\sqrt{3}}{4} \text{ (m}^2\text{)}$ or exact equivalent. Isw after a correct answer



Question Number	Scheme	Marks
9. (a)	24π	B1 (1)
(b)	$(18\pi, -1)$	B1ft (1)
(c)(i)	$-12\pi - \alpha$	B1 ft
(ii)	$6\pi - \alpha$	B1 ft (2) (4 marks)

(a) Do not allow the mark if an inequality or coordinates are given, but you may assume if $0 < x < p$ or $(p, 0)$ is given that their period is p for the purposes of the remaining marks.

See above

Note that (b) and (c) are for (correct or) follow through and hence

(b) is scored for $\left(\frac{3p}{4}, -1\right)$ -- this may be seen on the graph but take clearly labelled part (b) as precedent.

Follow through on $p \neq 2\pi$

(c) (i) is scored for $-\frac{1}{2}p - \alpha$

(ii) is scored for $\frac{1}{4}p - \alpha$

where p is their period stated in (a) (which may be 2π)

For answers in degrees penalise the first mark due only, but allow if degree symbol is missing.

For reference (a) 4320° (b) $(3240^\circ, -1)$ (c)(i) $-2160^\circ - \alpha$ (ii) $1080^\circ - \alpha$

Note for example that (a) $0 < x < 4320^\circ$ (b) $(3240^\circ, -1)$ (c)(i) $-2160^\circ - \alpha$ (ii) $1080^\circ - \alpha$ would score B0 B0 B1ft B1ft

Question Number	Scheme	Marks
10 (a)	$f(x) \leq 0 \Rightarrow x \leq -\frac{5}{2}, \quad x = 3$	M1 A1

Question Number	Scheme	Marks
9(a)	$(A=)-3$	B1
		(1)
(b)	$y = 3$	B1
	eg $x = 30 + 5 \times 180$ or $x = 210 + 720$ or $x = 180 + 2 \times 360 + 30$	M1
	$x = 930$	A1
		(3)
		Total 4

Check for answers next to the questions and on the graph. If there are contradictions then the answers given in the main body of the work takes precedence

(a)

B1: $(A=)-3$

(b)

B1: Correct y coordinate only (others must be discarded)

M1: Correct strategy for the x coordinate. See scheme for examples.
Values embedded is sufficient for the mark.

A1: Correct x coordinate only (others must be discarded). Isw. Note (930, 3) with no incorrect working and no other coordinates scores full marks.

Special case: If they give (3, 930) or $\left(\frac{31}{6}\pi, 3\right)$ rather than (930, 3) score B1M1A0



A1: Fully defines region correctly (**not ft here**).

E.g. $3x - 12 < y < 2(x - 4)(x - 10)$, $0 < x < 4$ (or $x > 0, x < 4$)

Or e.g. $y > 3x - 12$, $y < 2(x - 4)(x - 10)$, $x > 0$, $x < 4$

The right hand side of the inequality $0 < x < 4$ may be larger. Accept $0 < x < p$ as long as

$$4 \leq p \leq \frac{23}{2}$$

Allow **consistent** use of $> \leftrightarrow \geq$ for all of their inequalities.

Question Number	Scheme	Marks
9(a)	π	B1
		(1)
(b)(i)	3	B1
		(1)
(ii)	5	B1
		(1)
(iii)	201	B1
		(1)
		(4 marks)

(a)

B1: Period is π (radians) but condone 180° or just 180

(b)(i)

B1: 3

(ii)

B1: 5

(iii)

B1: 201



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)(i)	$(2, 12)$	B1
(ii)	$(2, 15)$	B1
		(2)
(b)	$k \leq 3, k = 5$	M1 A1
		(2)
(c)	$(x =) - 5$	B1
		(1)
		(5 marks)

Notes**Check for answers given in the body of the question. (There are lots!)****In the absence of any labelling for (a)(i) and (ii) the first answer is to be taken as (i)****(a)(i)****B1:** $(2, 12)$ but allow coordinates given separately e.g. $x = 2, y = 12$ Condone missing brackets e.g. 2, 12 and condone $\begin{pmatrix} 2 \\ 12 \end{pmatrix}$ **(a)(ii)****B1:** $(2, 15)$ but allow coordinates given separately e.g. $x = 2, y = 15$ Condone missing brackets e.g. 2, 15 and condone $\begin{pmatrix} 2 \\ 15 \end{pmatrix}$ **(b)****M1:** One of $k \leq 3, k < 3$ or $k = 5$. Condone use of y for k but not x for k .**A1:** $k \leq 3, k = 5$ or e.g. $k \leq 3$ or $k = 5$ Condone $k \leq 3$ and $k = 5$ Must be in terms of k . Allow $(-\infty, 3]$ for $k \leq 3$ An answer of $5 \leq k \leq 3$ scores M1A0**(c)****B1:** -5 only and no other values.But beware values of x that form part of their working which are clearly not part of their final answer e.g. $f(x) = 0$ when $x = -1$ so $f(x+4) = 0$ when $x = -5$ scores B1 $x = \dots$ is not required so just look for the correct value. $(-5, 0)$ is B0

Question Number	Scheme	Marks
2(a)	$6 = 5a + b, 3.3 = 8a + b$	M1 A1
	$a = -0.9, b = 10.5$	dM1 A1
		(4)
(b)(i)	10.5 m^3	B1ft
(ii)	$-0.9\sqrt{t} + 10.5 = 0 \Rightarrow 0.9\sqrt{t} = 10.5 \Rightarrow t = \dots$	M1
	$t = 136 \text{ (min)}$	A1
		(3)
		(7 marks)

Notes**(a)****M1:** Sets up the two simultaneous equations, one of which must be correct.Allow if square roots are present for this mark e.g. $6 = \sqrt{25a + b}, 3.3 = \sqrt{64a + b}$ Condone e.g. $6 = \pm 5a + b$ **A1:** Two correct equations with the square roots processed. May be implied.Allow e.g. $6 = \pm 5a + b$ and $3.3 = \pm 8a + b$ here.**dM1:** Solves their equations by any means including calculator to obtain values for a and b .**A1:** Correct values $a = -0.9, b = 10.5$ or the equivalents e.g. $a = -\frac{9}{10}, b = \frac{21}{2}$ only.If they solve e.g. $6 = \pm 5a + b$ and $3.3 = \pm 8a + b$ and offer extra values score A0.**(b)(i)****B1ft:** 10.5 m^3 or e.g. $\frac{21}{2} \text{ m}^3$ including the units, ft on their b provided $b > 0$ **(b)(ii)****M1:** Solves their $-0.9\sqrt{t} + 10.5 = 0 \Rightarrow t = \dots$ which must involve squaring, not square rooting.You may need to check but may be implied by e.g. $t = \frac{1225}{9}$ (if a and b are correct)Note that if a and b are correct, square rooting rather than squaring leads to $3.41565\dots$ and scores M0**A1:** Awrt 136Units are **not** required but if any are given they must be correct.

Condone “m” for “min”

Apply isw if necessary e.g. score A1 for $t = 136.1 = 137$ to the nearest minute.Correct answer only of awrt 136 following a correct a and b scores M1A1

Question Number	Scheme	Marks
3(a)	$(f(x)=)-3\cos x$ or $(f(x)=)3\sin(x-90^\circ)$	M1 A1
		(2)
(b)(i)	8	B1
(ii)	5	B1
		(2)
		(4 marks)

Notes**Check for answers given in the body of the question.****(a)**

M1: $(f(x)=)\pm 3\cos x$ or $(f(x)=)\pm 3\cos(-x)$ or $(f(x)=)\pm 3\sin(x\pm 90^\circ)$ or equivalent e.g.
 $(f(x)=)\pm 3\cos(x\pm 180^\circ)$

For this mark, their equation must be correct or be the equation of the given curve reflected in the x-axis.

Brackets are required if needed e.g. $(f(x)=)3\sin x - 90^\circ$ scores M0

The “ $f(x)=$ ” is **not** required.

The degrees symbol is **not** required.

Condone use of another variable for this mark e.g. θ .

A1: $(f(x)=)-3\cos x$ or $(f(x)=)3\sin(x-90^\circ)$ or $(f(x)=)-3\cos(-x)$ or equivalent.

If correct, the equations are likely to be one of the above but there are an infinite number of possibilities e.g. $(f(x)=)3\cos(x-180^\circ)$, $(f(x)=)3\cos(x+180^\circ)$, $(f(x)=)-3\sin(x-270^\circ)$ etc.

The “ $f(x)=$ ” is **not** required so ignore how they reference their expression.

The degrees symbol is **not** required.

Must be in terms of x not e.g. θ .

Note that just $(f(x)=)3\sin(x\pm 90^\circ)$ with no choice made scores M1A0

(b)(i)

B1: 8 cao. Must be 8 not just a list of roots.

(b)(ii)

B1: 5 cao. Must be 5 not just a list of roots.



Question Number	Scheme	Marks
7. (a)(i)	$P = (90^\circ, 3)$	M1, A1
(ii)	$Q = (540^\circ, 0)$	B1
		(3)
(b)	$(270^\circ, 4)$	M1, A1
		(2)
		(5 marks)

Note we condone the absence of the degree symbol in this question.

(a)(i)

M1 For one value in the coordinate pair of $(90^\circ, 3)$. Condone lack of brackets and do not be concerned with x or y correctly paired for this mark eg condone $(\dots, 90^\circ)$ or $(3, \dots)$ Allow in radians.

A1 For both of $(90^\circ, 3)$. Allow $x = 90$, $y = 3$

(a)(ii)

B1 $Q = (540^\circ, 0)$ Allow $x = 540$, $y = 0$

(b)

M1 For one value in the coordinate pair of $(270^\circ, 4)$. Condone lack of brackets and do not be concerned with x or y correctly paired for this mark eg condone $(\dots, 270^\circ)$ or $(4, \dots)$. Allow in radians.

A1 For both of $(270^\circ, 4)$. Allow $x = 270$, $y = 4$

For answers missing the brackets or radians(or both) in all parts then penalise the first time the A or B mark is due

FYI (a)(i) $\left(\frac{\pi}{2}, 3\right)$ (a)(ii) $Q = (3\pi, 0)$, (b) $\left(\frac{3\pi}{2}, 4\right)$ would score M1 A0(first error) B1 M1 A1

For all correct values the wrong way round $(3, 90^\circ), (0, 540^\circ), (4, 270^\circ)$ SC M1A0B0M1A1



General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1. (a)	$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7 \Rightarrow \frac{dy}{dx} = 6x^2 - 10x + \frac{3}{2x^2}$	M1 A1 A1 (3)
(b)	$x = \frac{1}{2} \Rightarrow y = 3$	B1
	Substitutes $x = \frac{1}{2}$ into their $\frac{dy}{dx} = 6x^2 - 10x + \frac{3}{2x^2} = \dots \left(= \frac{5}{2} \right)$	M1
	Uses the perpendicular gradient rule Eg. $\frac{5}{2} \rightarrow -\frac{2}{5}$	dM1
	Attempts the equation of the normal at P $y - 3 = -\frac{2}{5} \left(x - \frac{1}{2} \right)$	M1
	$2x + 5y - 16 = 0$ oe	A1
		(5) (8 marks)

(a)

M1 For reducing any power by 1 seen at least once $x^n \rightarrow x^{n-1}$ including $7 \rightarrow 0$ and from an incorrect index when manipulating $\frac{3}{2x}$ eg $\frac{3}{2x} \rightarrow \dots x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$. Allow eg $\dots x^2 \rightarrow \dots x^1$ or $\dots x^3 \rightarrow \dots x^{3-1}$ for this mark.

A1 For two of three correct terms (simplified) $6x^2 - 10x + \frac{3}{2x^2}$ which may appear on different lines of working. Do not allow $-10x^1$

A1 $6x^2 - 10x + \frac{3}{2x^2}$ or $6x^2 - 10x + \frac{3}{2}x^{-2}$ oe on one line (in any order) Do not accept $6x^2 + -10x + \frac{3}{2}x^{-2}$ and do not allow $-10x^1$

(b) Be careful to check whether they are attempting to find y or $\frac{dy}{dx}$. Send to review if unsure.

B1 Correct coordinates for P seen or used $x = \frac{1}{2} \Rightarrow y = 3$. Ignore their labelling of y .

M1 Attempts to find the value of $\frac{dy}{dx}$ when $x = \frac{1}{2}$ by substituting into their part (a) answer. They must proceed as far as finding a value for this mark. Ignore their labelling of $\frac{dy}{dx}$.

dM1 Finds or uses their gradient of the normal using the correct rule. Implied by $m \rightarrow -\frac{1}{m}$. It is dependent on the previous method mark.

- M1 Attempts to find the equation of the normal using a changed gradient and their $\left(\frac{1}{2}, 3\right)$. Sight of embedded values eg $y - 3 = -\frac{2}{5}\left(x - \frac{1}{2}\right)$ is sufficient (the bracket must be correct) or if they use $y = mx + c$ they must proceed as far as $c = \dots$

This mark can only be scored if they have attempted to find y and $\frac{dy}{dx}$ when $x = \frac{1}{2}$. If only the values of these are stated or seen within the equation of the normal then they must be $y = 3$ and gradient of the normal $= -\frac{2}{5}$ (or the correct negative reciprocal value for their $\frac{dy}{dx}$ when $x = \frac{1}{2}$).

- A1 $2x + 5y - 16 = 0$ oe allowing any integer multiples of this. Must have $= 0$. Accept $2x + 5y + -16 = 0$

Question Number	Scheme	Marks
2 .(a)	Attempts to use the given model at least once. Eg $2^3 = p \times 3^2 + q$ Two correct simplified equations $9p + q = 8$ $25p + q = 13.8(24)$ Solves simultaneously to get at least one of p or q $p = 0.364, q = 4.72(4)$	M1 A1 dM1 A1 (4)
(b)	Attempts to find T when $H = 5$ Eg. Calculates $\sqrt{\frac{125 - "q"}{"p"}}$ $(T =) 18.2$	M1 A1 (2) (6 marks)

(a)

M1 For an attempt to use the model at least once. Eg. either $2^3 = p \times 3^2 + q$ or $2.4^3 = p \times 5^2 + q$

A1 Two correct simplified equations $9p + q = 8$ oe $25p + q = 13.8(24)$ oe
 (may be implied by later work) $25p + q = 13\frac{103}{125} \left(= \frac{1728}{125} \right)$

dM1 Solves simultaneously to get a value for p or a value for q . Condone slips in their working and sight of $p = \dots$ or $q = \dots$ is sufficient for this mark.

A1 $p = 0.364, q = 4.72(4)$ allow fractions eg $p = \frac{91}{250}$ and $q = 4\frac{181}{250} \left(= \frac{1181}{250} \right)$. Correct answers with no working scores full marks. Isw after correct values for p and q .

(b)

M1 Makes $T^2 = \frac{125 - "q"}{"p"}$ and proceeds to $T = \dots$ when $H = 5$ using their p and their q or values taken to be their p and their q . This can only be scored if $\frac{125 - "q"}{"p"} > 0$

Eg. $(T =) \sqrt{\frac{125 - "q"}{"p"}}$ is sufficient or $T^2 = \frac{125 - "q"}{"p"} \Rightarrow T = \dots$

If only a value is stated you will need to check this on your calculator.

A1 $(T =) 18.2$ cao (allow $T = 18.2$ years or 18 years 2.4 months)

Question Number	Scheme	Marks
3. (a)(i)	$P(-180, -4)$	B1, B1
(ii)	$Q(450, 0)$	B1
(b)	$R(360, 7)$	(3)
		B1, B1
		(2)
		(5 marks)

For all parts condone missing brackets and check the graph/next to the question for answers.

Condone use of the degree symbol for their x values eg $(-180^\circ, \dots)$ instead of $(-180, \dots)$

(a)(i)

B1 $(-180, \dots)$ or $(\dots, -4)$ or $x = -180$ or $y = -4$ condone x in radians

B1 $(-180, -4)$ or $x = -180, y = -4$ Must be in degrees

SC1 $(-4, -180)$ (on EPEN this would be scored B1B0)

(a)(ii)

B1 $(450, 0)$ or $x = 450, y = 0$ condone $\left(\frac{5}{2}\pi, 0\right)$

(b)

B1 $(360, \dots)$ or $(\dots, 7)$ $x = 360$ or $y = 7$ condone x in radians

B1 $(360, 7)$ or $x = 360, y = 7$ Must be in degrees. Ignore any reference to $(0, 7)$

SC1 $(7, 360)$ (on EPEN this would be scored B1B0)

Note if radians used throughout then max score:

(a)(i) $(-\pi, -4)$ B1B0 **(a)(ii)** $\left(\frac{5}{2}\pi, 0\right)$ B1 **(b)** $(2\pi, 7)$ B1B0

Question Number	Scheme	Marks
1.	$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4} dx = 12 \times \frac{x^4}{4} + \frac{1}{6} \times 2x^{\frac{1}{2}} - \frac{3}{2} \times \frac{x^{-3}}{-3}$	M1
	$= 3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + c$	A1A1A1A1
		(5)
		(5 marks)

M1 Applies $\int x^n dx \rightarrow x^{n+1}$ for at least one index.

The index must be processed so allow for $x^3 \rightarrow x^4$ or $\frac{1}{\sqrt{x}} \rightarrow x^{\frac{1}{2}}$ or $\frac{1}{x^4} \rightarrow x^{-3}$

A1 One correct term simplified or $+c$. Look for one of $3x^4$, $+\frac{1}{3}x^{\frac{1}{2}}$, $+\frac{1}{2}x^{-3}$ or the $+c$.

A1 Two correct terms simplified or one correct simplified with $+c$.

Look for two of $3x^4$, $+\frac{1}{3}x^{\frac{1}{2}}$, $+\frac{1}{2}x^{-3}$, $+c$

A1 Three correct terms simplified or two correct simplified with $+c$.

Look for three of $3x^4$, $+\frac{1}{3}x^{\frac{1}{2}}$, $+\frac{1}{2}x^{-3}$, $+c$

A1 $3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + c$ all correct and simplified and on one line.

Allow simplified equivalents such as $\frac{1}{3}x^{\frac{1}{2}} \leftrightarrow \frac{1}{3}\sqrt{x}$ and $\frac{1}{2}x^{-3} \leftrightarrow \frac{1}{2x^3}$

Award once a correct expression is seen and isw but if there is any additional/incorrect notation and no correct expression has been seen on its own, withhold the final mark.

E.g. $\int 3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + c dx$, $3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + c = 0$

Question Number	Scheme	Marks
3.(a)	Attempts to make y the subject States $-\frac{3}{5}$ or exact equivalent	M1 A1 (2)
(b)	Uses perpendicular gradients rule $\Rightarrow$ gradient $l_2 = \frac{5}{3}$ Forms equation of l_2 using (6,-2) $y + 2 = \frac{5}{3}(x - 6)$ $y = \frac{5}{3}x - 12$	M1 M1 A1 (3) (5 marks)
Alt1(a)	Eg Coordinates of two points on the line (0,1.4) and (1,0.8) Gradient = $\frac{0.8 - 1.4}{1 - 0}$ Gradient = -0.6	M1 A1

(a)

M1 For an attempt to rearrange $3x + 5y - 7 = 0$ and make y the subject.Expect to see $\pm 5y = \dots$ followed by $y = \dots$ or equivalent. $\frac{3}{5}$ on its own is M0.

Alternatively they may find two pairs of coordinates and find the gradient between those two points.

Allow one slip in calculating the coordinates and it must be clear that they are attempting $\frac{y_2 - y_1}{x_2 - x_1}$ A1 For stating $-\frac{3}{5}$ or exact equivalent in (a). A correct answer implies both marks and isw after a correct gradient is stated.. (The value of c does not need to be correct).Do not allow $y = -\frac{3}{5}x + \frac{7}{5}$ without some statement for ' m '. Do not allow $-\frac{3}{5}x$.

A1*: Achieves the given answer with no errors seen including invisible brackets. They must have multiplied out any brackets before proceeding to the given answer to score this mark.

e.g. $\frac{54}{125} + \frac{18}{25}x + \frac{3}{10}x^2 - 0.432 = 90 \Rightarrow 5x^2 + 12x - 1500 = 0$ is acceptable as a final step to score A1*

(c)

B1: $x = \text{awrt } 16.2 \left(= \frac{-6 + 4\sqrt{471}}{5} \right)$ (ignore the negative root) May be seen on the diagram or next to the quadratic in the question. It may also be implied in their working to find the perimeter. It cannot be for the values embedded in the quadratic formula.

M1: A correct method to find the perimeter using their value for x which must be positive. They must proceed as far as a value for the perimeter to score this mark. Do not condone the lengths 0.72 and 1.2 to be rounded, but condone for example their “16.2” to be “16”.

May see exact values used in the expression e.g. $2\left(\frac{-6 + 4\sqrt{471}}{5}\right) + 0.72 + \left(\frac{12\sqrt{471}}{25}\right) = \dots$

A1: awrt 43 m or awrt 44 m, **including units, following a correct method. Typically look for 43.4...m or 43.5...m**

Note some incorrect methods lead to awrt 43 which is maximum B1M0A0.

e.g. $"16.2" \times 0.6 + "16.2" + "16.2" + 0.72 = 42.7$ which is B1M0A0 (uses 16.2 instead of $16.2 + 1.2$)

e.g. $"16.2 + 1.2" \times 0.6 + "16.2" + "16.2" = 42.8$ which is B1M0A0 (missing arc PS)

Question Number	Scheme	Marks
10ai	$(n =) 3$	B1
ii	1080	B1
		(2)
b	$(1620, -3)$	B1B1
		(2)
c	e.g. $k = \frac{1}{2} \left(\frac{12}{5} - \frac{3}{5} \right) = \frac{9}{10}$	M1A1
		(2)
		(6 marks)

Note: answers may appear next to questions or on the diagram.

If there is a contradiction between the answer by the question/on the diagram and the main body of the work then the answer in the main body of the work takes precedence.

(a)

(i)

B1: $(n =) 3$

(ii)

B1: 1080 (must be in degrees). Do not accept e.g. (1080, 3)

(b)

B1: One of the two coordinates $(1620, -3)$ Condone missing brackets. Allow $x = \dots$, $y = \dots$

Condone 9π instead of 1620 for this mark and condone the coordinates to be the wrong way round e.g. $(-3, 1620)$ scores B1B0

B1: Both coordinates $(1620, -3)$ Condone missing brackets. Allow $x = \dots$, $y = \dots$

(c)

M1: Attempts to find a value for k by either

- attempting to find $k = \frac{1}{2} \left(\frac{12}{5} - \frac{3}{5} \right) = \dots$
- attempting to find $k = \frac{12}{5} - \frac{1}{2} \left(\frac{12}{5} + \frac{3}{5} \right) = \dots$ or $k = -\frac{3}{5} + \frac{1}{2} \left(\frac{12}{5} + \frac{3}{5} \right) = \dots$
- forming two **correct** simultaneous equations and eliminating $\sin a$ (may be labelled as a different variable, condone a for $\sin a$)

$$2 \sin(a) + k = \frac{12}{5}, \quad -2 \sin(a) + k = -\frac{3}{5} \Rightarrow k = \dots \text{ scores M1}$$

They must have correctly used $\sin(-a) = -\sin(a)$. If they eliminate k instead, they must proceed to find a or $\sin a$ and then substitute back into one of the equations to find a value for k

$$\text{i.e. } 4 \sin a = 3 \Rightarrow a = 48.59 \dots \Rightarrow 2 \sin(48.59 \dots) + k = \frac{12}{5} \Rightarrow k = \dots$$

A1: $\frac{9}{10}$ or 0.9 (this is not awrt 0.9 so if they achieve a rounded value e.g. 0.899... and round to 0.9 then M1A0)



Question Number	Scheme	Marks
3.(a)	$y = \frac{5x^3 - 8}{2x^2} = \frac{5}{2}x - 4x^{-2}$ $\frac{dy}{dx} = \frac{5}{2} + 8x^{-3}$	M1 dM1 A1 A1 (4)
(b)	Substitutes $x = 2$ into their $\frac{dy}{dx} = \frac{5}{2} + \frac{8}{x^3} \Rightarrow \frac{dy}{dx} = \frac{7}{2}$ Uses their $\frac{7}{2}$ and $(2, 4) \Rightarrow y - 4 = \frac{7}{2}(x - 2)$ $\Rightarrow 7x - 2y - 6 = 0$	M1 M1 A1 (3) (7 marks)

(a)

M1: Attempts to write y as **a sum of two terms** and achieves at least one term with the correct index.Award for $Px + Qx^k$ or $Px^k + Qx^{-2}$. Condone $Qx^{-2} \leftrightarrow \frac{Q}{x^2}$

dM1: Requires both

- the given expression to be written as a sum of two terms with both indices correct
- correct differentiation applied to the indices $Px + Qx^{-2} \rightarrow A + Bx^{-3}$ o.e

A1: One correct term which need not be simplified. Condone $\frac{5}{2}x^0$

Note that 1010 is possible where candidates only form one of the two correct terms and differentiate that correctly

A1: $\left(\frac{dy}{dx}\right) = \frac{5}{2} + 8x^{-3}$ or simplified equivalent such as $\frac{1}{2}\left(5 + \frac{16}{x^3}\right)$. There is no need to have the $\frac{dy}{dx}$

(b)

M1: Substitutes $x = 2$ into their $\frac{dy}{dx}$ and attempts its value. Their $\frac{dy}{dx}$ cannot be the same as their "y"Score for sight of embedded 2's in their $\frac{dy}{dx}$ followed by a value or it may be implied for a correctvalue for their $\frac{dy}{dx}$

M1: Correct method for finding the equation of a tangent.

Look for a correct use of their $\frac{dy}{dx}\bigg|_{x=2}$ and the point $(2, 4)$ to form equation $y - 4 = \frac{7}{2}(x - 2)$.If the form $y = mx + c$ is used it must proceed as far as $c = \dots$ Award this mark if they achieve $\frac{dy}{dx} = \frac{5}{2}$ in part (a) and correctly form the equation using $m = \frac{5}{2}$ and $(2, 4)$. They cannot score the previous mark.A1: $7x - 2y - 6 = 0$ or any multiple thereof

.....

Question	Scheme	Marks
11(a)	$x = \frac{5\pi}{2}$ or $y = 12$	B1
	$x = \frac{5\pi}{2}$ and $y = 12$	B1
		(2)
(b)	$x = \frac{3\pi}{2}$ or $y = -21$	B1
	$x = \frac{3\pi}{2}$ and $y = -21$	B1
		(2)
(c)(i)	$(A =) -12$	B1
(ii)	$(B =) \frac{5\pi}{4}$	B1
		(2)
		Total 6

Notes:**Look for answers written in the body of the question or on the sketch.****In all cases, answers written in the body of the script take precedence over answers written on the diagram.****Angles and numbers must be written as single terms e.g. not $2\pi + \frac{\pi}{2}$ for $\frac{5\pi}{2}$** **Penalise the use of degrees once the first time it occurs**e.g. (a) $(450^\circ, 12)$ (b) $(270^\circ, -21)$ (c) $A = -12, B = 225^\circ$ scores (a) B1B0 (b) B1B1 (c) B1B1**The degrees symbol is not required****If both coordinates in (a) and (b) are correct but the wrong way round score as B1B0 each time**e.g. (a) $\left(12, \frac{5\pi}{2}\right)$ (b) $\left(-21, \frac{3\pi}{2}\right)$ scores (a) B1B0 (b) B1B0**If the coordinates in (a) or (b) are the wrong way round and only one is correct score as B0B0 each time****If the coordinates are the wrong way round and correct in degrees in (a) and (b)**e.g. (a) $(12, 450^\circ)$ (b) $(-21, 270^\circ)$ award (a) B0B0 (b) B1B0**The degrees symbol is not required**

(a)

B1: One coordinate correct.

May be seen as $x = \dots$ or $y \dots$ or embedded in a coordinate pair e.g. $\left(\frac{5\pi}{2}, 12\right)$ or in a vector $\begin{pmatrix} \frac{5\pi}{2} \\ 12 \end{pmatrix}$

B1: Both coordinates correct.

May be seen as $x = \dots$ and $y \dots$ or as a coordinate pair e.g. $\left(\frac{5\pi}{2}, 12\right)$ or as a vector $\begin{pmatrix} \frac{5\pi}{2} \\ 12 \end{pmatrix}$

(b)

B1: One coordinate correct.

May be seen as $x = \dots$ or $y \dots$ or embedded in a coordinate pair e.g. $\left(\frac{3\pi}{2}, -21\right)$ or in a vector $\begin{pmatrix} \frac{3\pi}{2} \\ -21 \end{pmatrix}$

B1: Both coordinates correct.

May be seen as $x = \dots$ and $y \dots$ or as a coordinate pair e.g. $\left(\frac{3\pi}{2}, -21\right)$ or as a vector $\begin{pmatrix} \frac{3\pi}{2} \\ -21 \end{pmatrix}$

(c)(i)

B1: Correct value for A. The “A =” is not required but it must be clear it is the answer to (i) or is the value of A.

(ii)

B1: Correct value for B. The “B =” is not required but it must be clear it is the answer to (ii) or is the value of B.

If candidates write the values as a coordinate pair e.g. $\left(\frac{5\pi}{4}, -12\right)$ or $\left(-12, \frac{5\pi}{4}\right)$ so it is not clear which value is which, score B0B0

Question Number	Scheme	Marks
1. (a)	$p^{\frac{1}{2}} = \left(\frac{1}{16}x^4\right)^{\frac{1}{2}} = \frac{1}{4}x^2$	B1 (1)
(b)	$(pq)^{-1} = \left(\frac{1}{16}x^4 \times \frac{40}{x^3}\right)^{-1} = \left(\frac{5}{2}x\right)^{-1} = \frac{2}{5}x^{-1}$	M1, A1 (2)
(c)	$p q^2 = \frac{1}{16}x^4 \times \left(\frac{40}{x^3}\right)^2 = \frac{1600}{16} \times \frac{x^4}{x^6} = 100x^{-2}$	M1, A1 (2) (5 marks)

(a)

B1: For $\frac{1}{4}x^2$. Allow $\frac{x^2}{4}$ or $0.25x^2$ Condone $\pm \frac{1}{4}x^2$

Note: **final** answer below for (b) and (c) does not include going on to give alternative correct forms for the same answer. If further incorrect work follows a correct answer then do not isw but take the final expression as their answer.

(b)

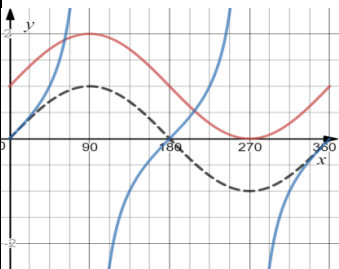
M1: A **final** answer with a correct simplified coefficient or index. E.g $\frac{2}{5}x^n$, $0.4x^n$, kx^{-1} or $\frac{k}{x^1}$ (or equivalent variations) Also allow $(kx)^{-1}$

A1: A **final** answer of $\frac{2}{5}x^{-1}$ or $0.4x^{-1}$ but isw $\frac{2}{5x}$ following this. Incorrect “simplifications” will be A0, though.

(c)

M1: A **final** answer with a correct simplified coefficient or index. E.g $100x^n$, kx^{-2} or $\frac{k}{x^2}$

A1: A **final** answer of $100x^{-2}$ but isw $\frac{100}{x^2}$ following this. Incorrect “simplifications” will be A0, though.

Question Number	Scheme	Marks
9. (a)	(270°, -4)	B1 B1 (2)
(b)	 For $y = 1 + \sin \theta$ $y = \tan \theta$	B1 B1 (2)
(c)	(i) $6 \times 2 = 12$ (ii) 11	M1 A1 B1 ft (3) (7 marks)

(a)

B1 Either coordinate correct. Look for either 270° or -4 in the correct position within (,).

Alternatively look for either $x = 270$ or $y = -4$ Condone $\frac{3\pi}{2} = 270^\circ$

Do not accept multiple answers unless one point is chosen or it is clearly part of their thought process. There is no need for the degrees symbol. Condone swapped coordinates, ie $(-4, 270)$ for this mark

B1 For correct coordinates.

$(270^\circ, -4)$ with or without degrees symbol. Condone $x = 270^\circ, y = -4$

(b) These may appear on Figure 3 rather than Diagram 1

B1 For $y = 1 + \sin \theta$ Score for a curve passing through $(0, 1), (90^\circ, 2), (180^\circ, 1), (270^\circ, 0), (360^\circ, 1)$ with acceptable curvature. Do not accept straight lines

B1 For $y = \tan \theta$ with acceptable curvature. Must go beyond $y = 1$ and -1

Score for the general shape of the curve rather than specific coordinates. See practice and qualification items for clarification.

First quadrant from $(0, 0) \rightarrow (90^\circ, \infty)$

Second and third quadrants from $(90^\circ, -\infty) \rightarrow (270^\circ, \infty)$ passing through $(180^\circ, 0)$

Fourth quadrant from $(270^\circ, -\infty) \rightarrow (0, 0)$

(c)(i) The question states hence so it is expected the results come from graphs.

If neither or only one graph is drawn then score for 12 in (i) for M1 A1 and 11 in (ii) B1

M1 For the calculation $\frac{2160}{360} = 6$ or $\frac{2160}{180} = 12$ or multiplying the number of intersections in their (b) by 6
Sight of 6 or 12 will imply this mark.

A1 12. 12 will score both marks.

(c) (ii)

B1 ft For either 11 (correct answer)

or follow through on n less than their answer to (c) (i) where n is their number of solutions in the range $180^\circ < \theta \leq 360^\circ$

Question Number	Scheme	Marks
8(a)	$3x^2 + 6x + 9 = 3(x \pm \dots)^2 \pm \dots \quad a = 3$	B1
	$3x^2 + 6x + 9 = 3(x+1)^2 \pm \dots \quad a = 3 \text{ \& } b = 1$	M1
	$3x^2 + 6x + 9 = 3(x+1)^2 + 6$	A1
		(3)
(b)	$(-1, 6)$	B1ft
		(1)
(c)	$y = \alpha(x+4)(x+2)(x-3)$	B1
	$6 = \alpha(-1+4)(-1+2)(-1-3)$	M1
	$\alpha = -\frac{1}{2}$	A1
	$y = -\frac{1}{2}(x+4)(x+2)(x-3) \Rightarrow y = \dots x^3 + \dots x^2 + \dots x + \dots$	M1
	$A = -\frac{1}{2}, B = -\frac{3}{2}, C = 5, D = 12$	A1
		(5)
Alt (c)	$-64A + 16B - 4C + D = 0$	
	$-8A + 4B - 2C + D = 0$	B1
	$27A + 9B + 3C + D = 0$	
	$-A + B - C + D = 6$	M1
	One of $A = -\frac{1}{2}, B = -\frac{3}{2}, C = 5, D = 12$	A1
	Fully solves their simultaneous equations	M1
	$A = -\frac{1}{2}, B = -\frac{3}{2}, C = 5, D = 12$	A1
		(9 marks)

(a)

B1 Achieves $3x^2 + 6x + 9 = 3(x \pm \dots)^2 \pm \dots$ or states that $a = 3$

M1 Deals correctly with the first two terms of $3x^2 + 6x + 9$
 Scored for $3x^2 + 6x + 9 = 3(x+1)^2 \pm \dots$ or states that $a = 3$ & $b = 1$

A1 $3x^2 + 6x + 9 = 3(x+1)^2 + 6$

This may be done by equating coefficients using the expanded form
 $a(x+b)^2 + c = ax^2 + 2abx + ab^2 + c$

(b)

B1ft $(-1, 6)$ or follow through their $(-b, c)$ from (a). Condone lack of brackets and accept
 eg $x = -1, y = 6$

(c)

B1 Identifies that three factors of the cubic equation are $(x+4)(x+2)(x-3)$

M1 A correct method to find the scale factor by using the minimum point found in part (c). Look
 for the minimum point to be substituted into their equation for the cubic

A1 Scale factor = $-\frac{1}{2}$

M1 Attempts to multiply $(x+4)(x+2)(x+3)$ to achieve $x^3 + \dots \pm 24$. This may have been
 multiplied by their scale factor so look for $\alpha(x^3 + \dots \pm 24)$

A1 $A = -\frac{1}{2}, B = -\frac{3}{2}, C = 5, D = 12$. Accept $y = -\frac{1}{2}x^3 - \frac{3}{2}x^2 + 5x + 12$

In the alternative method using simultaneous equations

B1 Three correct equations formed using $x = -4, -2$ and 3 with $y = 0$ in each case.

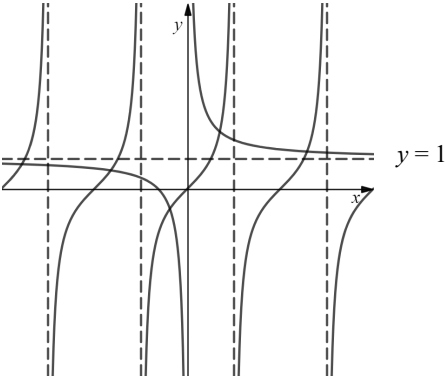
M1 Forms four simultaneous equations using the three intercepts and their point P

A1 One of $A = -\frac{1}{2}, B = -\frac{3}{2}, C = 5, D = 12$

M1 Fully solves their simultaneous equations either using matrices or elimination. This may be
 done on a calculator which is acceptable.

Eg. Using matrices
$$\begin{pmatrix} -64 & 16 & -4 & 1 \\ -8 & 4 & -2 & 1 \\ 27 & 9 & 3 & 1 \\ -1 & 1 & -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 6 \end{pmatrix} = \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \Rightarrow \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \dots$$

A1 $A = -\frac{1}{2}, B = -\frac{3}{2}, C = 5, D = 12$. Accept $y = -\frac{1}{2}x^3 - \frac{3}{2}x^2 + 5x + 12$

Question Number	Scheme	Marks
9(a)	$x = \frac{3\pi}{2}$ oe	B1
		(1)
(b)(i)		B1B1
(ii)	5 (solutions)	B1
	Number of solutions are the number of points of intersections between the graphs	B1
		(4)
(c)		
(i)	(Number of solutions) = 40	B1ft
(ii)	(Number of solutions) = 14	B1
		(2)
		(7 marks)

(a)

B1 $x = \frac{3\pi}{2}$ oe and no others. Do not accept in degrees. It may be labelled on the graph, but it must be an equation. If multiple answers are given then $x = \frac{3\pi}{2}$ oe must be identified (eg may be circled)

(b)(i)

B1 For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature – do not penalise candidates unless it is clear that a minimum point was intended.

B1 Correct shape and position for both branches with an asymptote in the correct position **and** labelled as $y = 1$ or stated in their work. Again, do not penalise the sketch unless it is clear that turning points are intended. The asymptote line/dashed line does not need to be drawn on the sketch.

(ii)

B1 5 only

B1 Number of solutions are the number of points of intersections between the graphs. (Do not allow if they mention where they cross the x -axis).

(c)

(i)

B1ft 40 Follow through from their sketch
(eg number of intersections in first quadrant $\times 20$)

(ii)

B1 14

Question Number	Scheme	Marks
3(a)	$(2\sqrt{2})^2 = p^2 + q^2 - 2pq \cos 60^\circ \text{ oe}$ $p^2 + q^2 - pq = 8 \quad *$	M1 A1*
		(2)
(b)	$q = p + 2 \Rightarrow 8 = p^2 + (p + 2)^2 - p(p + 2)$ $p^2 + 2p - 4 = 0 \text{ or } q^2 - 2q - 4 = 0$ $p = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-4)}}{2} \text{ or } q = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-4)}}{2}$ $p = -1 + \sqrt{5} \text{ or } q = 1 + \sqrt{5}$ $p = -1 + \sqrt{5} \text{ and } q = 1 + \sqrt{5} \text{ only}$	M1 A1 M1 B1 (A1 on EPEN) A1cso
		(5)
(c)	$\text{Area} = \frac{1}{2} \times (-1 + \sqrt{5})(1 + \sqrt{5}) \times \sin 60^\circ$ $\text{Area} = \sqrt{3} \text{ (m}^2\text{)}$	M1 A1
		(2)
Alt(a)	Forming a line BX which is perpendicular to AC where X is on the line AC . $AX = p \cos 60 = \frac{p}{2}$ $BX = \sqrt{p^2 - \left(\frac{p}{2}\right)^2} = \frac{\sqrt{3}}{2}p \text{ or } BX = p \sin 60$ $\left(\frac{\sqrt{3}}{2}p\right)^2 + \left(q - \frac{p}{2}\right)^2 = (2\sqrt{2})^2 \text{ or } (p \sin 60)^2 + \left(q - \frac{p}{2}\right)^2 = (2\sqrt{2})^2$ $\frac{3p^2}{4} + q^2 - pq + \frac{p^2}{4} = 8$ $p^2 + q^2 - pq = 8 \quad *$	M1 A1*
		(9 marks)

(a)

M1 $(2\sqrt{2})^2 = p^2 + q^2 - 2pq \cos 60^\circ$ or $(2\sqrt{2})^2 = p^2 + q^2 - 2pq \times \frac{1}{2}$ or $8 = p^2 + q^2 - 2pq \cos 60^\circ$

They may carry this out in two stages by forming two right angled triangles with BX being perpendicular to AC (see Alt(a)). To score this mark they must proceed as far as

$$\left(\frac{\sqrt{3}}{2}p\right)^2 + \left(q - \frac{p}{2}\right)^2 = (2\sqrt{2})^2 \quad \text{or} \quad (p \sin 60^\circ)^2 + \left(q - \frac{p}{2}\right)^2 = (2\sqrt{2})^2$$

Condone missing brackets for M1.

A1* Achieves $p^2 + q^2 - pq = 8$ with no errors including omission of brackets. One of the lines above must have been seen for M1A1. If they state $= 8$ without showing any working then A1 cannot be scored.

(b)

M1 Substitutes $q = p + 2$ (oe) into the given equation

A1 $p^2 + 2p - 4 = 0$ or the equivalent equation in q ($q^2 - 2q - 4 = 0$)

M1 Attempts to solve to find p using the formula or completing the square using their values. Alternatively, they achieve a quadratic equation in q and attempt to solve to find q using their values. Usual rules for solving quadratics apply. If they state the roots or factorise then M0. If they use the quadratic formula then the values must be embedded.

B1 $p = -1 + \sqrt{5}$ or $q = 1 + \sqrt{5}$ (ignore any other solutions) Must be exact. This is independent of the previous method mark so if the roots are just stated this mark can be scored. (Note this is A1 on EPEN)

A1 $p = -1 + \sqrt{5}$ and $q = 1 + \sqrt{5}$ only cso (all other marks must have been scored to award A1)

(c)

M1 Attempts to find the area of the triangle using $\frac{1}{2} \times (-1 + \sqrt{5})(1 + \sqrt{5}) \times \sin 60^\circ$. Must see at least one stage of working using their p and their q

A1 $\sqrt{3}$ (m^2) condone lack of units. Do not accept rounded answers.

Question Number	Scheme	Marks
4	$\int \frac{3x^{\frac{3}{2}} - 15x^{\frac{1}{2}} + 2x - 10}{4\sqrt{x}} dx = \int \frac{3}{4}x - \frac{15}{4} + \frac{1}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{-\frac{1}{2}} dx$	M1A1A1
	$x^n \rightarrow x^{n+1}$	dM1
	$\frac{3}{8}x^2 - \frac{15}{4}x + \frac{1}{3}x^{\frac{3}{2}} - 5x^{\frac{1}{2}} + C$	A1A1
		(6 marks)

M1 Attempts to write as a sum of terms using correct index laws at least once.
Award for any term with a correct index.

Score for any one of:

$$\frac{3x^{\frac{3}{2}}}{4\sqrt{x}} \rightarrow \dots x \quad \frac{-15x^{\frac{1}{2}}}{4\sqrt{x}} \rightarrow \text{const} \quad \frac{+2x}{4\sqrt{x}} \rightarrow \dots x^{\frac{1}{2}} \quad \frac{-10}{4\sqrt{x}} \rightarrow \dots x^{-\frac{1}{2}}$$

Cannot be scored for a term with a correct index arising from incorrect work

eg $\frac{-10}{4\sqrt{x}} \rightarrow \dots x^{\frac{1}{2}}$

A1 Two correct terms of $\frac{3}{4}x - \frac{15}{4} + \frac{1}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{-\frac{1}{2}}$ (oe). They do not need to be seen on

the same line for this mark and they do not need to be simplified. Eg $\frac{3}{4}x^1, \frac{-10x^{-\frac{1}{2}}}{4}$ are acceptable.

Indices must be processed though. Eg $\frac{3x^{\frac{3}{2}}}{4\sqrt{x}} \rightarrow \frac{3}{4}x^{1.5-0.5}$ is not acceptable.

A1 All four correct terms of $\frac{3}{4}x - \frac{15}{4} + \frac{1}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{-\frac{1}{2}}$ (oe) Terms do not need to be simplified for this mark and do not need to be on one line.

dM1 Increases the power of any of their terms by 1. ($x^n \rightarrow x^{n+1}$). It is dependent on the first method mark.

A1 Any two terms correct unsimplified (see below) but the indices must be processed. Condone $-\frac{15}{4}x^1$ as a correct term for this mark only.

A1 $\frac{3}{8}x^2 - \frac{15}{4}x + \frac{1}{3}x^{\frac{3}{2}} - 5x^{\frac{1}{2}} + C$ o.e. all on one line (including the constant C and all simplified).

Do not accept $-\frac{15}{4}x^1$ as a correct simplified term.

Accept alternative correct simplified forms such as $\frac{3}{8}x^2 - \frac{15}{4}x + \frac{1}{3}x^{\frac{3}{2}} - 5\sqrt{x} + C$ or

$$\frac{3}{8}x(x-10) + \frac{1}{3}x^{\frac{1}{2}}(x-15) + C$$

Question Number	Scheme	Marks
5(a)	$P_B - P_A = 44.2 - (53 - 0.4 \times 8^2) = \dots$	M1
	awrt (£) 16.8 million	A1
		(2)
(b)	(£) 53 (million)	B1
		(1)
(c)	$-1.6t + 44.2 = 53 - 0.4(t - 8)^2$	M1
	$\Rightarrow 0.4t^2 - 8t + \frac{84}{5} = 0 \Rightarrow t = \dots$	M1
	$t = 10 - \sqrt{58} = \text{awrt } 2.38 \text{ (years)}$	A1
	"2.38" < t (≤ 15)	A1ft
		(4)
(d)	"The share value would be negative" / "the model is known to hold for 15 years only (and 20 years is more than 15)."	B1
		(1)
		(8 marks)

(a)M1 Substitutes $t = 0$ into the equation for P_A and finds the difference between 44.2 and P_A .

A1 awrt (£) 16.8 million.

(b)

B1 (£) 53 million. Condone 53

(c)M1 Sets $-1.6t + 44.2 = 53 - 0.4(t - 8)^2$ They may have already attempted to multiply out $(t - 8)^2$ so condone errors that may occur before setting $P_A = P_B$. Also may score for use of any inequality sign.M1 Rearranges the equation to form a 3TQ on one side of the equation ($= 0$) or inequality and attempts to solve the quadratic. Condone slips in their working. See general rules for solving a quadratic. They may use a calculator to state the root(s) which is acceptable.

A1 awrt 2.38 (years) o.e. isw after a correct answer

A1ft "2.38" < t (≤ 15) oe from correct working. You may follow through on their value of t provided $t < 15$. Ignore any reference to an upper limit provided it is ≥ 15 

You may see eg $10 - \sqrt{58} < t < 10 + \sqrt{58}$ which can score this mark.

(d)

B1 “The **share value (or P)** would be negative” or “the **model** is known to hold for 15 years only (and 20 is greater than 15)” or equivalent explanations.
Eg “ P must be greater than (or equal to) zero”.

Note “ $-4.6 < 0$ ” is insufficient (it does not refer to the **model** or the **share value of P**)



A1: For awrt 603 (cm). The units need not be given.

(c) This part is now being marked M1 dM1 A1 ddM1 A1

Please look through all of the solution first. The marks can be awarded in the following way.

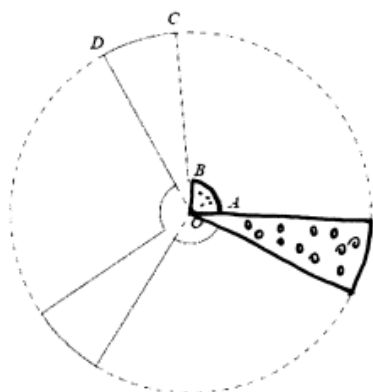
M1: A correct attempt at any relevant area

dM1: A correct attempt at a corresponding area that can be combined with the first area in some way to find the area of the fan. FT on angles found in part (a). Dependent upon previous mark

A1: Both areas correct. They do not need to be calculated but the angles must be correct to 3sf

ddM1: A correct combination of areas to find the area of the fan

A1: awrt 0.473 m² or awrt 4730 cm². Must include the units. ISW after a correct answer



Main method

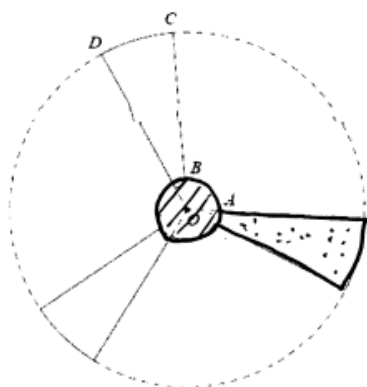
∴ M1: One relevant area eg $\frac{1}{2} \times 9^2 \times \frac{5^\circ}{3}$

∴ dM1: Corresponding area $\frac{1}{2} \times 84^2 \times \left(\frac{2\pi}{3} - \frac{5^\circ}{3}\right)$

A1: Both correct

ddM1: 3x  + 3x 

A1: Awrt 0.473 m² or 4730 cm²



Alt I

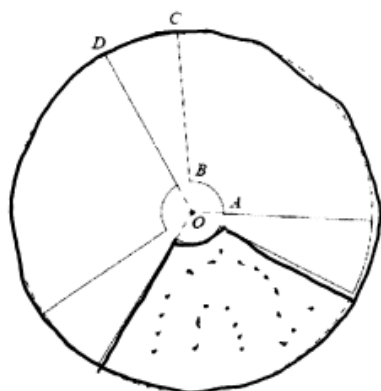
⊗ M1: One relevant area eg $\pi \times 9^2$

▢ dM1: Corresponding area $\frac{1}{2} \times 84^2 \left(\frac{2\pi}{3} - \frac{5^\circ}{3}\right) - \frac{1}{2} \times 9^2 \times \left(\frac{\pi}{3} - \frac{5^\circ}{3}\right)$

A1: Both correct

ddM1: ⊗ + 3x ▢

A1: Awrt 0.473 m² or 4730 cm²



Alt II



M1: one relevant area Eg $\pi \times 84^2$

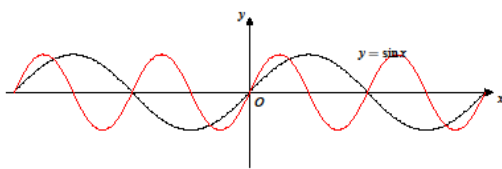
dm1: Corresponding area $\frac{1}{2} \times 84^2 \times \frac{5}{3} - \frac{1}{2} \times 9^2 \times \frac{5}{3}$

A1: Both correct

ddm1: - 3x

A1: Award 0.473 m² or 4730 cm²

Variations are possible, e.g. $3 \times$ area of blades (inc. circle) + area circle – area of blades within the circle, but these can be marked according to the scheme.

Question	Scheme	Marks
9(a)	(i) $2p$	B1
	(ii) $-p$	B1
	(iii) $3-p$	B1
		(3)
(b)		Correct shape, same height starting at O, scaling may be incorrect. Two repeats of the sinx graph each side
		(2)
(c)	For $x = \frac{\alpha}{2}$	B1
	Attempt at second root E.g. $x = \frac{180^\circ - \alpha}{2}$	M1
	$x = 90^\circ - \frac{\alpha}{2}$	A1
		(3)
(8 marks)		
Notes:		

(a)

(i) B1: For $2p$. Condone $p + p$. Award when $2p$ is the y coordinate of a coordinate pair. E.g. $(180 - \alpha, 2p)$

(ii) B1: For $-p$. Award when $-p$ is the y coordinate of a coordinate pair. E.g. $(\alpha - 180, -p)$

(iii) **B1:** For $3 - p$. Award when $3 - p$ is the y coordinate of a coordinate pair. E.g $(180 + \alpha, 3 - p)$

(b)

M1: Same shape, starting at O , and same height as original graph, but the scaling may be incorrect.

Mark the intention. For example their graph may be slightly taller or shorter than the given curve.

One complete sine curve on just one side of the y -axis is sufficient for the method mark.

Also allow this mark when candidates use Diagram 1 and mark the x - intercepts as below.

If they ignore the given Diagram and use their own axes you must be convinced their sketch for $y = \sin 2x$ is the same eight / amplitude as $y = \sin x$ but a different scaling / frequency. So you must be able to see some numbers marked on the axes or graph

A1: Correct sketch. Look for intersections at $(\pm 360^\circ, 0)$ and $(\pm 180^\circ, 0)$ in addition to the above

(c)

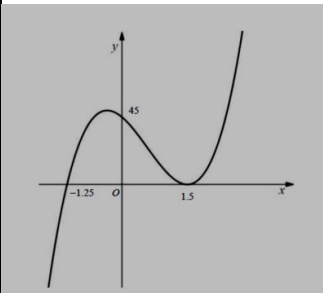
B1: $x = \frac{\alpha}{2}$ given as one solution.

M1: A second solution which may not be in the given range. Any of $\frac{180^\circ - \alpha}{2}$, $90^\circ - \frac{\alpha}{2}$, $180^\circ + \frac{\alpha}{2}$,

$-90^\circ - \frac{\alpha}{2}$, $-180^\circ + \frac{\alpha}{2}$, $360^\circ + \frac{\alpha}{2}$ are examples of using the symmetry to find a second solution.

A1: $x = 90^\circ - \frac{\alpha}{2}$ or $\frac{180^\circ - \alpha}{2}$ as second solution with no additional solutions within the given range.



Question Number	Scheme	Marks
8 (a)	$f'(x) = 2(x-3)(3x+2) = 6x^2 - 14x - 12$ $f(x) = 2x^3 - 7x^2 - 12x + k$ Uses $P(4, 13) \Rightarrow 13 = 2 \times 64 - 7 \times 16 - 12 \times 4 + k \Rightarrow k = \dots$ $(f(x)) = 2x^3 - 7x^2 - 12x + 45$	B1 M1, A1 M1 A1 (5)
(b)	$2x^3 - 7x^2 - 12x + 45 \equiv (x^2 - 6x + 9)(px + q) \Rightarrow p = "2" \text{ or } q = \frac{"45"}{9}$ States either $2x^3 - 7x^2 - 12x + 45 \equiv (x-3)^2(2x+5)$ or $p = 2 \ \& \ q = 5$ following a fully correct (a) or correct but incomplete part (a) (See notes)	B1 ft B1 (2)
(c)	 + ve cubic curve "Correct" y intercept for their part (b) equation. So 9q or "45" Turning point at (1.5, 0) x intercept at (-1.25, 0)	B1 B1ft B1 B1 (4) (11 marks)

Mark parts (a) and (b) together but f(x) must be obtained via integration AND the use of (4, 13)

(a)

B1: $2(x-3)(3x+2) = 6x^2 - 14x - 12$ which may be left unsimplified BUT the brackets must be expanded

M1: Attempts to multiply out $2(x-3)(3x+2)$ and integrate each term.

Look for an expanded form which would simplify to $ax^2 + bx + c$ and **then each term** being integrated with each of the powers being correctly dealt with. So $ax^2 \rightarrow \dots x^3$, $bx \rightarrow \dots x^2$ and $+c \rightarrow cx$

There is no requirement to have a constant of integration for this mark

A1: $f(x) = 2x^3 - 7x^2 - 12x + k$ including the $+k$. (Any constant is acceptable)

The expression must now be in simplest form

M1: Uses $P(4, 13)$ and finds a value for k . It is dependent upon some 'correct' integration.

So, look for substituting $x = 4, y = 13$ into their $y = f(x)$ and finding a value for k .

A1: $f(x) = 2x^3 - 7x^2 - 12x + 45$. There is no requirement to see the $f(x)$

(b)

B1ft: Achieves either value following through on their $2x^3 - 7x^2 - 12x + 45$.

So score for either of

- $p = 2$ or the coefficient of their x^3 term
- $q = 5$ or their constant term divided by 9

B1: $(x-3)^2(2x+5)$ OR $p = 2$ or $q = 5$.**Must follow a fully correct part (a) = (1,1,1,1,1) or a correct but incomplete part (a) = (1,1,1,0,0)**

Cannot be scored from multiple answers or implied from an attempt at division

Note that finding the two correct values is considered sufficient as long as $f(x) = 2x^3 - 7x^2 - 12x + 45$

Useful examples

(a) $f(x) = (2x-6)(3x+2)$
 $= 6x^2 + 4x - 18x - 12$
 $= 6x^2 - 14x - 12$
 $\int f'(x) dx = \int (6x^2 - 14x - 12) dx$
 $= 2x^3 - 7x^2 - 12x + C$

(b) $(x-3)^2(2x+5) = (x^2 - 6x + 9)(2x+5)$
 $= 2x^3 + 5x^2 - 6px^2 - 6qx + 9px + 9q$
 $= 2x^3 - 7x^2 - 12x + 9q$
 $\begin{cases} p=2 \\ 9-6p=-7 \\ -6q+9p=-12 \\ 9q=C \end{cases} \Rightarrow \begin{cases} p=2 \\ q=5 \\ C=45 \end{cases}$

E.g I

(a) 1, 1, 1, 0, 0

No use of (4, 13) at all in the question so the final two marks cannot be awarded. (Note that they do find $c = 45$ in part (b) from expanding $(x-3)^2(2x+5)$ but this does not score those marks.)

(b) 1, 1

Correct values in (b) following a correct but incomplete part (a).

Note that $2x^3 - 7x^2 - 12x + c = (x-3)^2(2x+5)$ when $c = 45$.

Note that they could go on to score all marks in part (c)

(a) $f'(x) = 2(3x^2 + 2x - 9x - 6)$
 $= 6x^2 + 4x - 18x - 12$
 $= 6x^2 - 14x - 6$
 $\int f'(x) dx = \int (6x^2 - 14x - 6) dx = 6 \cdot \frac{1}{3}x^3 - 14 \cdot \frac{1}{2}x^2 - 6x + C$
 $= 2x^3 - 7x^2 - 6x + C$
 $2x^3 - 7x^2 - 6x + C = 2x^3 - 7x^2 - 6x + 21$
 $-8 + C = 21$
 $C = 29$
 $\therefore f(x) = 2x^3 - 7x^2 - 6x + 21$

(b) $(x-3)^2(2x+5)$
 $\therefore p=2, q=5$

E.g II

(a) 0, 1, 0, 1, 0

Incorrect expansion in (a) but uses (4, 13) to find c

(b) 1, 0

Not a fully correct (a)

$$2x^3 - 7x^2 - 6x - 21 \neq (x-3)^2(2x+5)$$

Note that they could go on to score all marks in part (c) using the answer to part (b)

(c)

B1: For a positive cubic shaped curve with, from left to right, a maximum and then a minimum.

Do not be concerned by the position of the curve or the turning points. Be generous on graphs that appear linear in places but penalise three straight lines. Use review if unclear.

B1ft: For a correct y intercept for their $y = f(2x)$. Don't be concerned if this is their maximum turning point.FT on their answer to (a) for their ' $2x^3 - 7x^2 - 12x + \underline{45}$ ' provided that the constant term is not zero or their answer to (b) for their $9q$ provided $q \neq 0$ B1: Turning point tangential to the x -axis at 1.5. Allow for maximum or minimum point
Do not allow if marked (0, 1.5)B1: x intercept at -1.25 . The graph must pass through (not stop at) the axis here. Allow if there are other intercepts. Do not allow if marked (0, -1.25).

Question Number	Scheme	Marks
9. (a)	(i) 3 (ii) 101 (iii) 2	B1 B1 B1 (3)
(b)	$a = \frac{2\pi}{3}$ o.e.	B1 (1)
(c)	Attempts x coordinate of Q: $x = \frac{2\pi}{3} + \pi = \frac{5\pi}{3}$ Attempts both y coordinates $P\left(\frac{2\pi}{3}, \frac{\pi}{3}\right)$ $Q\left(\frac{5\pi}{3}, -\frac{2\pi}{3}\right)$ Attempts mid-point $PQ = \left(\frac{1}{2}\left(\frac{2\pi}{3} + \frac{5\pi}{3}\right), \frac{1}{2}\left(\frac{\pi}{3} - \frac{2\pi}{3}\right)\right)$ $= \left(\frac{7\pi}{6}, -\frac{\pi}{6}\right)$	M1 dM1 ddM1 A1 (4) (8 marks)

The answers to parts (a) and (b) may very well appear in the question. Be aware of this when marking.

- (a)(i) B1: 3
(a)(ii) B1: 101
(a)(iii) B1: 2
(b)

B1: $a = \frac{2\pi}{3}$ o.e. so accept $a = \frac{4\pi}{6}$. Condone $x \leftrightarrow a$

Do not accept/condone 120°

(c)
M1: For

- either the correct x coordinate of Q which can be scored for $\left(\frac{3\pi}{2} + \frac{\pi}{6}\right)$

- or an attempt at the x coordinate of Q by adding on π to the x coordinate of P

The following marks are dependent upon this, so if (c) starts M0, it is automatically dM0, ddM0, A0.

dM1: Attempts coordinates for **BOTH** P and Q using the equation $y = \pi - x$

It is dependent upon the previous method mark.

ddM1: Correct attempt at mid-point.

There must be an attempt to add the pairs of coordinates before halving the sum.

It is dependent upon BOTH the previous method marks **and the value of a lying in the region**

$\frac{\pi}{2} < a < \pi$. Look for $\left(\frac{a + (a + \pi)}{2}, \frac{(\pi - a) + (\pi - (a + \pi))}{2}\right)$

A1: $\left(\frac{7\pi}{6}, -\frac{\pi}{6}\right)$ but may be written as $x = \dots, y = \dots$

A1cso: $a = -\frac{4}{3}\sqrt{3} + \frac{8}{3}$ but isw after a correct expression. Must have score all previous marks, showing the relevant steps as outline above before the final answer is given but condone if “invisible brackets” are recovered. Accept with terms in the other order, and allow $\frac{1}{3}(8 - 4\sqrt{3})$

Note that there are many ways of attempting (b). The above scheme can still be applied. A common alternative method is;

$$\begin{aligned}\frac{16}{\sqrt{3}+1} &= a\sqrt{27} + 4 \Rightarrow 16 = (a\sqrt{27} + 4)(\sqrt{3}+1) = 9a + 4 + 4\sqrt{3} + 3\sqrt{3}a \quad \text{for B1} \\ \Rightarrow 12 - 4\sqrt{3} &= (9 + 3\sqrt{3})a \Rightarrow a = \frac{12 - 4\sqrt{3}}{9 + 3\sqrt{3}} \times \frac{9 - 3\sqrt{3}}{9 - 3\sqrt{3}} \quad \text{for M1} \\ \Rightarrow a &= \frac{144 - 72\sqrt{3}}{54} = -\frac{4}{3}\sqrt{3} + \frac{8}{3} \quad \text{for M1 A1}\end{aligned}$$

Notes: The B mark can be implied by working as long as it is seen before the final answer.

So, for example, $\frac{8\sqrt{3}+4}{\sqrt{27}} \times \frac{\sqrt{27}}{\sqrt{27}} = \frac{72+12\sqrt{3}}{27} = \dots$ is sufficient to score the B mark.

An acceptable equivalent of rationalising the denominator of an expression for the first M is rationalising the coefficient of a in a rearranged expression, e.g.

$$(9 + 3\sqrt{3})a = 12 - 4\sqrt{3} \rightarrow (81 - 27)a = (9 - 3\sqrt{3})(12 - 4\sqrt{3})$$

The second M mark requires an attempt to make a the subject and make progress towards the form $p\sqrt{3} + q$, which means the denominator must have been reduced to a single term with

two terms in the numerator. So $a = \frac{12 - 4\sqrt{3}}{9 + 3\sqrt{3}}$ is not sufficient progress. In the example above

the M is not awarded until the final line where the denominator is 54.

However, once the first B and M are scored (having seen evidence before the final answer) reaching the correct answer will then be sufficient for the M (and A if all correct).

But from $\frac{12 - 4\sqrt{3}}{9 + 3\sqrt{3}}$ to $\frac{8 - 4\sqrt{3}}{3}$ with no further work shown is 2nd M0 (calculator use with

no further progress). However, if a correct attempt to rearrange this to $-\frac{4\sqrt{3}}{3} + \frac{8}{3}$ (or with terms reversed) follows (or similar if there were errors) then the 2nd M becomes eligible as there is further work to support the method.

Question Number	Scheme	Marks
3. (a)	$\int f(x) dx = \int x^{\frac{3}{2}} + 10x^{\frac{1}{2}} + 25x^{-\frac{1}{2}} dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{20}{3}x^{\frac{3}{2}} + 50x^{\frac{1}{2}} + c$	M1 A1 A1 A1 (4)
(b) (i)	$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}}$ $f'(x) = 0 \Rightarrow \frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}} = 0$ AND $\times x^{\frac{3}{2}}$	M1 A1 dM1
(ii)	$\frac{3}{2}x^2 + 5x^1 - \frac{25}{2} = 0 \Rightarrow 3x^2 + 10x - 25 = 0$ * $(x = \frac{5}{3})$ only	A1* B1 (5) (9 marks)

(a)

M1: For attempting to divide by $x^{\frac{1}{2}}$ and increasing a correct power by 1 (ie must have expanded). Must be seen in part (a)

A1: For one correct term (allow un simplified)

A1: For two correct terms simplified.

A1: $\frac{2}{5}x^{\frac{5}{2}} + \frac{20}{3}x^{\frac{3}{2}} + 50x^{\frac{1}{2}} + c$ or exact equivalent. Must include the constant of integration.

Condone if they write “y=” and ignore any spurious integral signs or extra dx

(b)(i)

M1: For attempting to divide by $x^{\frac{1}{2}}$ and decreasing a correct power by one. Must be seen or used in part (b). For attempts at the quotient or product rule look for correct

denominator and one correct numerator term in $\frac{A(x+5)\sqrt{x} - B(x+5)^2 x^{-1/2}}{(\sqrt{x})^2}$ (oe for

product rule)

A1: For $f'(x) = \frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}}$ (oe e.g from quotient rule) which may be left un simplified.

dM1: Sets $\alpha x^{\frac{1}{2}} + \beta x^{-\frac{1}{2}} + \gamma x^{-\frac{3}{2}} = 0$ and attempts to multiply by $x^{\frac{3}{2}}$ seen in at least two terms or explicitly shown in a step before the final answer.

This may be derived in stages, first multiplying by $x^{\frac{1}{2}}$ and then by x . This is permissible for the M (and A) as evidence of the correct method has been shown by the stages. The dM can be scored if the index work is correct in at least two indices, and A1 if all is correct.

So $\frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}} = 0 \Rightarrow \frac{3}{2}x + 5 - \frac{25}{2}x^{-1} = 0 \Rightarrow \frac{3}{2}x^2 + 5x - \frac{25}{2} = 0$ is sufficient for the

dM1 as long as the middle step is seen with two correct powers (and the A1 if they then multiply through be 2).

Note also that dividing by $x^{-\frac{3}{2}}$ is an equivalent to multiplying by $x^{\frac{3}{2}}$.

A1*: Reaches $3x^2 + 10x - 25 = 0$ showing all steps with no errors. Note that they must work correctly with the equation, if they state $f'(x) = 3x^2 + 10x - 25$ before setting equal to 0 then allow the M (if suitable method shown) but it is A0.

(b)(ii)

B1: $\left(x = \right)\frac{5}{3}$ only. The other root need not be seen, but if shown it must be subsequently rejected in some way. Note that 1.6 (1.6 recurring) is acceptable as the answer for B1, but 1.67 is B0.



Question Number	Scheme	Marks
4 (a)	States or implies that $[f(x) =]kx(x-4)$	M1
	Attempts to find k . E.g. $-4.8 = k \times 2 \times (2-4) \Rightarrow k = \dots$	dM1
	$[f(x) =]1.2x(x-4)$	A1 (3)
(b)	States or implies that $[g(x) =]\lambda x(x-4)^2$	M1
	Attempts to find λ . E.g. $7.2 = \lambda \times 6 \times (6-4)^2 \Rightarrow \lambda = \dots$	dM1
	$[g(x) =]0.3x(x-4)^2$	A1 (3)
(c)	Sets their $1.2x(x-4) = 0.3x(x-4)^2$	B1ft
	Valid attempt to solve $1.2\cancel{x(x-4)} = 0.3\cancel{x(x-4)}^{\cancel{2}} \Rightarrow x = 4 + \frac{1.2}{0.3}$	M1
	$x = 8$	A1
	$(8, 38.4)$	A1
		(4) (10 marks)

(a)

M1: States or implies that $f(x) = kx(x-4)$. Allow with $k = 1$. The $f(x)$ need not be seen – assume they are working with $f(x)$ in part (a)

Alternatives include $f(x) = k(x-2)^2 - 4.8$ Allow with $k = 1$ and $f(x) = ax^2 + bx$ with at least one of $4a + b = 0$ and/or $4a + 2b = -4.8$

dM1: Attempts to find all the constants in their equation for $f(x)$ using $(2, -4.8)$. E.g.

$$-4.8 = k \times 2 \times (2-4) \Rightarrow k = \dots$$

A1: $[f(x) =]1.2x(x-4)$ o.e. such as $[f(x) =]1.2x^2 - 4.8x$ or $[f(x) =]1.2(x-2)^2 - 4.8$ and isw. Accept with $(x-0)$. The $f(x)$ may be missing, it can be assumed (but use of $g(x)$ is A0). Allow if the full expression is given in latter parts if all the work was correct in (a) to find the constants.

(b)

M1: States or implies that $g(x) = \lambda x(x-4)^2$ Allow with $\lambda = 1$. The $g(x)$ need not be seen – assume they are working with $g(x)$ in part (b).

Alternatives include $g(x) = ax^3 + bx^2 + cx$ with at least two of $64a + 16b + 4c = 0$, $48a + 8b + c = 0$, and/or $216a + 36b + 6c = 7.2$ (allow minor slips).

dM1: Attempts a full equation for $g(x)$ using $(6, 7.2)$

$$\text{E.g. } 7.2 = \lambda \times 6 \times (6-4)^2 \Rightarrow \lambda = \dots$$

A1: $[g(x) =]0.3x(x-4)^2$ o.e. such as $[g(x) =]0.3x^3 - 2.4x^2 + 4.8x$ and isw. Accept with $(x-0)$. The $g(x)$ may be missing, it can be assumed (but use of $f(x)$ is A0). Allow if the full expression is used in (c) if all the work was correct in (b) to find the constants.

Question	Scheme	Marks
7(a)	$0 = 10 - 2x \Rightarrow x = 5$ or $y = 2, y = 10 - 2x \Rightarrow x = 4$	B1
	Examples: $\frac{1}{2} \times 2(5 + 4 - a) = \frac{27}{4} \text{ or } \frac{1}{2} \times 2\left(5 + 4 - \frac{2}{k}\right) = \frac{27}{4}$ Trapezium or $\frac{1}{2} \times 2a + \frac{1}{2} \times 2(5 - a + 4 - a) = \frac{27}{4} \text{ or } \frac{1}{2} \times 2 \times \frac{2}{k} + \frac{1}{2} \times 2\left(5 - \frac{2}{k} + 4 - \frac{2}{k}\right) = \frac{27}{4}$ Triangle + Trapezium or $\frac{1}{2} \times 2a + 2(4 - a) + \frac{1}{2} \times 1 \times 2 = \frac{27}{4} \text{ or } \frac{1}{2} \times 2 \times \frac{2}{k} + 2\left(4 - \frac{2}{k}\right) + \frac{1}{2} \times 1 \times 2 = \frac{27}{4}$ Triangle + Rectangle + Triangle or $\frac{1}{2} \times 5 \times 2 + \frac{1}{2}(4 - a) \times 2 = \frac{27}{4} \text{ or } \frac{1}{2} \times 5 \times 2 + \frac{1}{2}\left(4 - \frac{2}{k}\right) \times 2 = \frac{27}{4}$ 2 Triangles	M1
	$\Rightarrow k = \frac{8}{9}, a = \frac{9}{4}$	A1 A1ft
		(4)
(b)	Two of $y \geq \frac{8}{9}x, y \leq 10 - 2x, x > \frac{9}{4}$	M1
	All three of $y \geq \frac{8}{9}x, y \leq 10 - 2x, x > \frac{9}{4}$	A1
		(2)
(6 marks)		
Notes:		
(a) B1: Correct x intercept for $y = 10 - 2x$ or correct x coordinate for intersection of $y = 2$ with $y = 10 - 2x$. May be seen on the diagram or in a sketch or implied by their working. M1: Correct method for area and sets equal to $\frac{27}{4}$ to form an equation in a or k . The $2 \times \frac{1}{2}$ may be implied. May use e.g. trapezium, triangle + trapezium, triangle + rectangle + triangle or 2 triangles. See above for some examples. Note that with a correct “5” and “4” the area is $9 - a$ or $9 - \frac{2}{k}$. A1: Correct value for a or k as a single value not a calculation. A1ft: Correct values for a and k . Correct values or follow through using $ak = 2$ from their first value to the second provided the M1 is scored . Allow exact equivalents or exact ft equivalents so if correct allow e.g. $k = 0.\dot{8}$, $a = 2.25$ or $2\frac{1}{4}$		