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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Integration (P1.8)

International A Level Mathematics · P1: Pure Mathematics 1 (WMA11)

43 questions · 320 marks · Mark schemes included

Past papers January 2019 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

43 questions · 320 marks · 2 sub-topics

SUB-TOPIC P1.8a

INDEFINITE INTEGRATION

Sub-topic P1.8a · 22 questions · 130 marks · lowest tariff first



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 3 marks)



Answer ALL questions. Write your answers in the spaces provided.

1. Find

$$\int \left(\frac{2}{3}x^3 - \frac{1}{2x^3} + 5 \right) dx$$

simplifying your answer.

(4)

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(Total 4 marks)

1. Find, in simplest form,

$$\int \left(\frac{8x^3}{3} - \frac{1}{2\sqrt{x}} - 5 \right) dx$$

(4)

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Q1

1. Find

$$\int \left(\frac{8x^3}{5} - \frac{2}{3x^4} - 1 \right) dx$$

giving each term in simplest form.

(4)

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Question 1 continued

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(Total 4 marks)

Q1



1. Find

$$\int \left(10x^5 + 6x^3 - \frac{3}{x^2} \right) dx$$

giving your answer in simplest form.

(4)

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Question 1 continued

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Lined area for writing the answer to Question 1.

(Total 4 marks)

Q1



1. Find

$$\int \left(\frac{1}{2}x^3 + \frac{3}{\sqrt{x}} - 4 \right) dx$$

writing your answer in simplest form.

(4)

Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 4 marks)



$$\int \left(8x^3 - 6\sqrt{x} - \frac{2}{5x^3} \right) dx$$

(4)



(Total for Question 1 is 4 marks)



4. Find

$$\int \frac{4x^2 + 1}{2\sqrt{x}} dx$$

giving the answer in its simplest form.

(5)

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Q4

1. Find

$$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4} dx$$

giving each term in simplest form.

(5)

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Question 1 continued

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Lined area for writing the answer to Question 1.

(Total 5 marks)

Q1



$$\int \frac{4x^5 + 3}{2x^2} dx$$

(5)



(Total for Question 3 is 5 marks)



1. Find

$$\int (2x - 5)(3x + 2)(2x + 5)dx$$

writing your answer in simplest form.

(5)



(Total for Question 1 is 5 marks)



3. Find

$$\int \frac{(x+3)^2}{3\sqrt{x}} dx$$

writing your answer in simplest form.

(5)

Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 5 marks)



4. Find

$$\int \frac{(3\sqrt{x} + 2)(x - 5)}{4\sqrt{x}} dx$$

writing each term in simplest form.

(6)

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Question 4 continued

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(Total 6 marks)

Q4



(Total for Question 5 is 6 marks)



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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$(\dot{\mathbf{i}})$$

$$f(x) = (x + \sqrt{2})^2 + (3x - 5\sqrt{8})^2$$

Express $f(x)$ in the form $ax^2 + bx\sqrt{2} + c$ where a , b and c are integers to be found.

(3)

(ii) Solve the equation

$$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are simplified fractions to be found.

(4)

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Question 3 continued

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Q3

(Total 7 marks)



2. Given that

$$(x-5)(2x+1)(x+3) \equiv ax^3 + bx^2 - 32x - 15$$

where a and b are constants,

(a) find the value of a and the value of b .

(2)

(b) Hence find

$$\int \frac{(x-5)(2x+1)(x+3)}{5\sqrt{x}} \, dx$$

writing each term in simplest form.

(5)

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



4. In this question you must show all stages of your working.

(a) Write

$$y = \frac{5x^2 + \sqrt{x^3}}{\sqrt[3]{8x}}$$

in the form

$$y = Ax^p + Bx^q$$

where A, B, p and q are constants to be found.

(4)

(b) Hence find $\frac{dy}{dx}$ giving each coefficient in simplest form.

(3)

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 7 marks)



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



3.

In this question you must show all stages of your working.

$$f(x) = \frac{(x+5)^2}{\sqrt{x}} \quad x > 0$$

(a) Find $\int f(x) \, dx$

(4)

(b) (i) Show that when $f'(x) = 0$

$$3x^2 + 10x - 25 = 0$$

(ii) Hence state the value of x for which

$$f'(x) = 0$$

(5)



Question 3 continued

Lined area for writing the answer to Question 3 continued.



(Total for Question 5 is 7 marks)





(Total for Question 4 is 9 marks)

Question 2 continued

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Question 2 continued

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Q2

Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



(Total for Question 3 is 10 marks)



SUB-TOPIC P1.8b

FINDING THE EQUATION OF A CURVE FROM $F'(x)$

Sub-topic P1.8b · 21 questions · 190 marks · lowest tariff first



5.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Fully factorise

$$9x^3 - 10x^2 + x$$

(2)

(b) Hence solve

$$9 \times 27^y - 10 \times 9^y + 3^y = 0$$

(3)



Question 5 continued

(Total for Question 5 is 5 marks)



9. A curve with equation $y = f(x)$ passes through the point (9, 10).

Given that

$$f'(x) = 27x^2 - \frac{21x^3 - 5x}{2\sqrt{x}} \quad x > 0$$

find $f(x)$, fully simplifying each term.

(6)

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Question 9 continued

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Question 9 continued

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Q9

(Total 6 marks)

TOTAL FOR PAPER IS 75 MARKS

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Question 10 continued

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Q10

(Total 7 marks)

END

TOTAL FOR PAPER IS 75 MARKS



8. A curve C has equation $y = f(x)$.

The point P with x coordinate 3 lies on C

Given

- $f'(x) = 4x^2 + kx + 3$ where k is a constant
- the normal to C at P has equation $y = -\frac{1}{24}x + 5$

(a) show that $k = -5$

(3)

(b) Hence find $f(x)$.

(4)

Question 8 continued

(Total for Question 8 is 7 marks)



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Question 6 continued

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Lined area for writing the answer to Question 6.

(Total 8 marks)

Q6

Mark box for Q6.



11. A curve C has equation $y = f(x)$, $x > 0$

Given that

- $f''(x) = 4x + \frac{1}{\sqrt{x}}$
- the point P has x coordinate 4 and lies on C
- the tangent to C at P has equation $y = 3x + 4$

(a) find an equation of the normal to C at P

(2)

(b) find $f(x)$, writing your answer in simplest form.

(6)



Question 11 continued

(Total for Question 11 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



12. The curve with equation $y = f(x)$, $x > 0$, passes through the point $P(4, -2)$.

Given that

$$\frac{dy}{dx} = 3x\sqrt{x} - 10x^{-\frac{1}{2}}$$

- (a) find the equation of the tangent to the curve at P , writing your answer in the form $y = mx + c$, where m and c are integers to be found.

(4)

- (b) Find $f(x)$.

(5)

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Question 12 continued

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Question 12 continued

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Q12

(Total 9 marks)

TOTAL FOR PAPER IS 75 MARKS



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8. The curve C with equation $y = f(x)$, $x > 0$, passes through the point $P(4, 1)$.

Given that $f'(x) = 4\sqrt{x} - 2 - \frac{8}{3x^2}$

- (a) find the equation of the normal to C at P . Write your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(4)

- (b) Find $f(x)$.

(5)



Question 8 continued

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Question 8 continued

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Question 8 continued

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Q8

(Total 9 marks)



Question 7 continued

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Question 7 continued

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(Total 9 marks)

Q7





Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)



11. A curve has equation $y = f(x)$.

The point $P\left(4, \frac{32}{3}\right)$ lies on the curve.

Given that

- $f''(x) = \frac{4}{\sqrt{x}} - 3$
- $f'(x) = 5$ at P

find

- (a) the equation of the tangent to the curve at P , writing your answer in the form $y = mx + c$, where m and c are constants to be found,

(2)

- (b) $f(x)$.

(8)

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Question 11 continued

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Q11

(Total 10 marks)

END

TOTAL FOR PAPER IS 75 MARKS



Question 8 continued

Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

(Total for Question 8 is 10 marks)



Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)



Question 10 continued



Question 10 continued



(Total for Question 10 is 10 marks)



Question 10 continued



Question 10 continued

(Total for Question 10 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 11 continued

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Question 11 continued

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Q11

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END



Question 9 continued

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Q9

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END



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Question 6 continued

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Question 6 continued

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(Total 11 marks)

Q6



8. A curve C with equation $y = f(x)$ passes through the point $R(4, 13)$.

Given that

$$f'(x) = 2(x - 3)(3x + 2)$$

- (a) use integration to find $f(x)$, giving your answer in simplest form.

(5)

- (b) Given that $f(x)$ can be written in the form

$$(x - 3)^2(px + q)$$

find the value of the constant p and the value of the constant q .

(2)

- (c) Sketch the graph of $y = f(2x)$, showing the coordinates of any points where the curve touches or crosses the coordinate axes.

(4)

Question 8 continued



Question 8 continued

Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 11 marks)



Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$ **Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

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Attempt to use correct formula (with values for a , b and c).

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Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

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General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

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General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

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Answers without working

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Question Number	Scheme	Marks
2.(a)	$\frac{1}{4-2\sqrt{2}} = \frac{1}{4-2\sqrt{2}} \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}}$	M1
	$= \frac{4+2\sqrt{2}}{16-8} = \frac{1}{2} + \frac{1}{4}\sqrt{2}$ oe	A1 (2)
(b)	$4x = 2\sqrt{2}x + 20\sqrt{2} \Rightarrow (4-2\sqrt{2})x = 20\sqrt{2}$	M1
	$\Rightarrow x = \frac{20\sqrt{2}}{(4-2\sqrt{2})} = 20\sqrt{2} \times (a)$	dM1
	$\Rightarrow x = 20\sqrt{2} \times \left(\frac{1}{2} + \frac{1}{4}\sqrt{2}\right) = 10 + 10\sqrt{2}$	A1 (3) (5 marks)

(a)

M1 For sight of $\frac{1}{4-2\sqrt{2}} \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}}$ oeA1 For achieving $\frac{1}{2} + \frac{1}{4}\sqrt{2}$ or exact equivalent such as $0.5 + \frac{\sqrt{2}}{4}$, $\frac{2}{4} + \frac{2}{8}\sqrt{2}$ or correct a and b .

Remember it does not have to be simplified and isw following a correct answer

(b) Hence

M1 For attempting to collect the terms in x on one side of the equation and the constant term on the other side. Condone slips but there must be an attempt to collect terms with a bracket or implied bracketdM1 For using part (a) and attempting to find $k\sqrt{2} \times (a)$ A1 $10\sqrt{2} + 10$ or $10 + 10\sqrt{2}$ but NOT $10(\sqrt{2} + 1)$. It cannot be awarded without sight of $k\sqrt{2} \times (a)$

Otherwise (1)- SQUARING APPROACH

M1 Squaring both sides $4x = 2\sqrt{2}x + 20\sqrt{2} \rightarrow 16x^2 = 8x^2 + 160x + 800$ Condone slips on coefficients
Cannot be scored by squaring each term. Look $ax^2 = px^2 + qx + r$

dM1 Re-arranging and attempting to solve their 3TQ usual rules

Eg $8x^2 - 160x - 800 = 0 \Rightarrow x^2 - 20x - 100 = 0 \Rightarrow x = \frac{20 \pm \sqrt{400 + 400}}{2}$

A1 $10\sqrt{2} + 10$ or $10 + 10\sqrt{2}$ following a correct solution of the quadratic equation seen above.

Otherwise (2)- REPEATING THE PROCESS OF PART (a)

M1 Rearranges $4x = 2\sqrt{2}x + 20\sqrt{2} \Rightarrow (4 \pm 2\sqrt{2})x = 20\sqrt{2}$ condoning slips. May even divide by 2 firstdM1 Then divide, rationalise and attempt to simplify. Eg $x = \frac{20\sqrt{2}}{(4-2\sqrt{2})} \times \frac{(4+2\sqrt{2})}{(4+2\sqrt{2})} = \frac{80\sqrt{2} + 40\sqrt{2} \times \sqrt{2}}{16-8}$ oe

A1 $10\sqrt{2} + 10$ or $10 + 10\sqrt{2}$ only. It cannot be awarded without sight of the correct intermediate line seen above

Question Number	Scheme	Marks
3.(a)	Attempts perimeter of garden = $2 \times 5x + 2 \times (6x - 2)$ Sets $2 \times 5x + 2 \times (6x - 2) > 29 \Rightarrow 22x > 33$ $\Rightarrow x > \frac{33}{22} \Rightarrow x > 1.5$ *	M1 dM1 A1* (3)
(b)	Attempts area of garden = $2x(2x - 1) + 3x(6x - 2)$ Sets $A < 72 \Rightarrow 22x^2 - 8x - 72 < 0$ Finds critical values $11x^2 - 4x - 36 \Rightarrow x = -\frac{18}{11}, 2$ Chooses inside region $-\frac{18}{11} < x < 2$	M1 A1 M1 ddM1 A1 (5)
(c)	$1.5 < x < 2$	B1 (1)
		(9 marks)

(a)

M1 An attempt at finding the perimeter of the garden.

Scored for sight of $5x + 2x - 1 + 2x + 6x - 2 +$ additional term(s) involving x

Individual lengths may not be seen so imply for sight of a total of $ax + b$, where $a > 15$

dM1 Sets their $P > 29$ and attempts to solve by proceeding to $ax > c$

You may condone an attempt in which $P = 29 \Rightarrow ax = c$

A1* cso with at least one correct intermediate (simplified) line $22x > 33$ or $x > \frac{33}{22}$ before $x > 1.5$ seen.

Condone an attempt in which you see $P = 29 \Rightarrow x = 1.5$ before $x > 1.5$ seen

Note that it is possible to start with $x > 1.5$ and prove $P > 29$ but for the A1* to be scored there must be a final statement of the type "hence $x > 1.5$ ". There is no requirement for any units

(b) **Mark part (b) and (c) together**

M1 For an attempt at finding the area of the garden. For this to be scored look for

The sum of two areas $2x(2x - 1) + \dots x(6x - 2)$ condoning slips

The sum of two areas $5x(2x - 1) + \dots x(\dots x \pm \dots)$ condoning slips

The difference between two areas $5x(6x - 2) - 2x(\dots x \pm \dots)$ condoning slips.

A1 A "correct and simplified" equality or inequality, condoning $< \leftrightarrow \leq \leftrightarrow =$ Eg. $22x^2 - 8x - 72 < 0$ oe

M1 A valid attempt to find the critical values of their 3TQ. Allow factorisation, formula, completion of square or use of calculator. If a calculator is used then the answer(s) must be correct for their 3TQ. Condone candidates who fail to state the negative root of their quadratic.

ddM1 Dependent upon both M's. For choosing the inside region for their critical values. Condone $< \leftrightarrow \leq$

Condone for this mark replacing a negative root with 0, 0.5 or 1.5. So accept for example one of either $1.5 < x < 2$, $0 < x < 2$ or $0.5 < x < 2$

A1 $-\frac{18}{11} < x < 2$ Allow $0 < x < 2$ or $0.5 < x < 2$ due to context. Allow alternative notation. See below

(c)

General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



but $\frac{5}{\sqrt{x}} - 3\sqrt{x} = \frac{5}{\sqrt{2}} - 3\sqrt{2} = \frac{5\sqrt{2}}{2} - 3\sqrt{2} = -\frac{\sqrt{2}}{2}$ or $-\frac{1}{\sqrt{2}}$ scores M1A1

Other examples of sufficient working:

$$\frac{5}{\sqrt{x}} - 3\sqrt{x} = \frac{5}{\sqrt{2}} - 3\sqrt{2} = \frac{5-6}{\sqrt{2}} = -\frac{1}{\sqrt{2}}, \quad \frac{5}{\sqrt{x}} - 3\sqrt{x} = \frac{5}{\sqrt{2}} - 3\sqrt{2} = \left(\frac{5}{2} - 3\right)\sqrt{2} = -\frac{\sqrt{2}}{2}$$

Apply isw once a correct exact answer is seen.

Question Number	Scheme	Marks
2(a)	$P(-3,7), Q(9,11)$ and $R(12,2)$	
Way 1	$\text{grad } PQ = \frac{11-7}{9-(-3)} = \frac{1}{3}, \text{ grad } QR = \frac{11-2}{9-12} = -3$	M1 A1
	$\frac{1}{3} \times -3 = -1$ so angle $PQR = 90^\circ$	A1
		(3)
Way 2	$PQ^2 = (9-(-3))^2 + (11-7)^2 = 160$ $QR^2 = (12-9)^2 + (2-11)^2 = 90$ $PR^2 = (12-(-3))^2 + (2-7)^2 = 250$	M1 A1
	$PQ^2 + QR^2 = PR^2$ (or e.g. $90 + 160 = 250$) so angle $PQR = 90^\circ$ or e.g. $\cos \theta = \frac{160+90-250}{2\sqrt{160}\sqrt{90}} = 0 \Rightarrow \theta = 90^\circ$	A1
		(3)
Way 3	$\overrightarrow{PQ} = \begin{pmatrix} 9-(-3) \\ 11-7 \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}, \overrightarrow{QR} = \begin{pmatrix} 12-9 \\ 2-11 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \end{pmatrix}$	M1A1
	$\overrightarrow{PQ} \cdot \overrightarrow{QR} = \begin{pmatrix} 12 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -9 \end{pmatrix} = 36 - 36 = 0$ so angle $PQR = 90^\circ$	A1
		(3)

General Guidance for part (a)

M1: Requires some correct work depending on the method

A1: Correct work for the chosen method

A1: Depends on both previous marks and requires an explanation and a conclusion.

The conclusion must refer to angle PQR being 90° or equivalent e.g. “ PQ and QR are perpendicular”,

“ Q is 90° ”, “ Q is a right angle” etc. but condone e.g. “ $A = 90^\circ$ ” or e.g. “ $\theta = 90^\circ$ ” if the solution is otherwise correct.

This may occur in a preamble e.g. If $\text{grad } PQ \times \text{Grad } QR = -1$ then $PQR = 90^\circ$,



$$\frac{11-7}{9-3} = \frac{1}{3}, \frac{11-2}{9-12} = -3, \frac{1}{3} \times -3 = -1 \text{ hence proven.}$$

Scores full marks.

(a) **Way 1**

M1: Attempts both gradients with an attempt at $\frac{\text{difference in } y}{\text{difference in } x}$ seen at least once.

A1: Achieves both correct gradients which may be left unsimplified e.g. $\text{grad } PQ = \frac{4}{12}$ and

$$\text{grad } QR = \frac{9}{-3}$$

A1: e.g. $\frac{1}{3} \times -3 = -1 \Rightarrow PQR = 90^\circ$ or e.g. $\frac{1}{3}$ is the negative reciprocal of -3 so $Q = 90$.

Do not allow ambiguous statements e.g. $\frac{1}{3}$ is the opposite inverse of -3 for the explanation.

NB: $\frac{11-7}{9-3} = \frac{1}{3}, \frac{11-2}{9-12} = -3, \frac{1}{3} \times -3 = -1 \Rightarrow PQR = 90^\circ$ would be a minimum for M1A1A1

(a) **Way 2**

M1: Attempts all three lengths with an attempt at “the difference between the coordinates” and “squaring” seen at least twice. Note that the “differences” may be implied by their answers or their lengths.

A1: All three lengths or lengths² correct as single exact terms or as decimals – allow one d.p.

Lengths are $\sqrt{160}, \sqrt{90}, \sqrt{250}$ or $4\sqrt{10}, 3\sqrt{10}, 5\sqrt{10}$ or e.g. 12.6, 9.5 (or 9.4 truncated), 15.8

A1: via Pythagoras or cosine rule e.g.

$$PQ^2 + QR^2 = PR^2 \text{ so } PQR = 90^\circ \text{ or e.g. } \cos \theta = \frac{160 + 90 - 250}{2\sqrt{160}\sqrt{90}} = 0 \Rightarrow \theta = 90^\circ$$

If via the cosine rule, all values must be in the correct positions with correct signs.

“160, 90, 250 is a Pythagorean triple so $PQR = 90^\circ$ ” is acceptable for the final mark.

Values must appear as exact for this mark. E.g. $\cos \theta = \frac{12.6^2 + 9.5^2 - 15.8^2}{2 \times 12.6 \times 9.5} = 0 \Rightarrow \theta = 90^\circ$ scores

M1A1A0

(a) **Way 3**

M1: Attempts $\pm \overrightarrow{PQ}$ and $\pm \overrightarrow{QR}$ with an attempt at “the difference between the coordinates” seen at least twice

A1: Correct vectors for $\pm \overrightarrow{PQ}$ and $\pm \overrightarrow{QR}$

A1: $\overrightarrow{PQ} \cdot \overrightarrow{QR} = 0$ so angle $PQR = 90^\circ$

Part (b)		
(b)	E.g. $(-3, 7) + (3, -9) = \dots$ or $(12, 2) - (12, 4) = \dots$	M1
	$(0, -2)$	A1
		(2)
ALT 1	$\text{grad } PQ = \frac{11-7}{9-3} = \frac{1}{3} \Rightarrow \text{eqn } RS \text{ is } y-2 = \frac{1}{3}(x-12)$ $\text{grad } QR = \frac{11-2}{9-12} = -3 \Rightarrow \text{eqn } PS \text{ is } y-7 = -3(x+3)$ $\Rightarrow x = \dots, y = \dots$	M1
	$(0, -2)$	A1
ALT 2	Midpoint PR is $\left(\frac{9}{2}, \frac{9}{2}\right) \Rightarrow \frac{9+x}{2} = \frac{9}{2}, \frac{11+y}{2} = \frac{9}{2} \Rightarrow x = \dots, y = \dots$	M1
	$(0, -2)$	A1
		(5 marks)

(b)

M1: Any suitable method of finding at least x or y for S . It can be implied by one correct coordinate which may be seen on a diagram.

e.g. Via vectors $(-3, 7) + (3, -9) = (0, -2)$ or $(12, 2) - (12, 4) = (0, -2)$

A1: Correct coordinates $(0, -2)$ which may be written separately e.g. $x = 0, y = -2$ or as $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Alt 1(b)

M1: Attempts the equation of line PS , the equation of line RS and solves simultaneously to find x or y

A1: Correct coordinates $(0, -2)$ which may be written separately e.g. $x = 0, y = -2$ or as $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Alt 2(b)

M1: Attempts the midpoint of PR and uses Q to find S for at least one of x or y

A1: Correct coordinates $(0, -2)$ which may be written separately e.g. $x = 0, y = -2$ or as $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$



Question Number	Scheme	Marks
3	$\int \frac{4x^5+3}{2x^2} dx = \int 2x^3 + \frac{3}{2}x^{-2} dx$	M1 A1
	$= \frac{1}{2}x^4 - \frac{3}{2}x^{-1} + c$	dM1 A1 A1
		(5)
		(5 marks)

M1: Attempts to write as a **sum of two terms** with one processed index correct.

Award for $Px^3 + Qx^k$ or $Px^k + Qx^{-2}$ or $Px^k + \frac{Q}{x^2}$ where k could be 0 i.e. a constant term.

A1: Correct integrand written as a sum of two terms with indices processed. E.g. $2x^3 + \frac{3}{2}x^{-2}$ seen in one expression.

Award for any exact equivalent such as $\frac{1}{2}\left(4x^3 + \frac{3}{x^2}\right)$ or e.g. $\frac{4}{2}x^3 + \frac{3}{2x^2}$ or e.g. $\frac{4x^3 + 3x^{-2}}{2}$

dM1: Attempts to integrate an expression of the form $Px^m + Qx^n$, $m = 3$ or $n = -2$, raising one of the indices by one. Depends on the first M mark.

A1: Either term correct and simplified and from correct work: $\frac{1}{2}x^4 + Ax^n (+c)$ or $Bx^m - \frac{3}{2}x^{-1} (+c)$

A1: $\frac{1}{2}x^4 - \frac{3}{2}x^{-1} + c$. Allow simplified equivalents e.g. $\frac{1}{2}x^4 - \frac{3}{2x} + c$, $0.5x^4 - 1.5x^{-1} + c$ but **not** e.g.

$\frac{1}{2}x^4 - \frac{3}{2x} + c$ and **not** $\frac{1}{2}x^4 + -\frac{3}{2}x^{-1} + c$.

Condone poor notation such as $\int \frac{1}{2}x^4 - \frac{3}{2x} + c(dx)$, $y = \frac{1}{2}x^4 - \frac{3}{2x} + c$

You can ignore subsequent working if necessary e.g. award the marks once a correct answer is seen as a correct single expression.

Correct answer only scores full marks.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

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Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

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$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

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Question Number	Scheme	Marks
1(a)	$\left(\frac{dy}{dx}\right) \dots x^1 + \dots x^{\frac{3}{2}} + \dots x^{-2}$	M1
	$\left(\frac{dy}{dx}\right) \frac{2}{3}x - 2x^{\frac{3}{2}} - \frac{8}{3}x^{-2}$	A1A1A1
		(4)
(b)	$\frac{dy}{dx} = \frac{2}{3} \times 4 - 2 \times 4^{\frac{-3}{2}} - \frac{8}{3} \times 4^{-2} = \frac{9}{4}$	M1
	$\frac{9}{4} \rightarrow -\frac{4}{9}$	M1
	$y - 3 = -\frac{4}{9}(x - 4)$	dM1
	$4x + 9y - 43 = 0$	A1
		(4)
		(8 marks)

(a)

M1 Reduces the power by 1 on any of the following terms

$$\dots x^2 \rightarrow \dots x^1, \dots x^{\frac{1}{2}} \rightarrow \dots x^{-\frac{3}{2}}, \dots x^{-1} \rightarrow \dots x^{-2}, 5 \rightarrow 0$$

A1 One of $\frac{2}{3}x$, $-2x^{\frac{3}{2}}$, $-\frac{8}{3}x^{-2}$ or exact simplified equivalent terms eg $\frac{-2}{(\sqrt{x})^3}$ or $-\frac{8}{3x^2}$ or

$$-2.6x^{-2}$$

Condone $\frac{2}{3}x^1$ and condone double signs such as $+ - 2x^{\frac{3}{2}}$. The terms do not need to be on the same line for this mark.

A1 Two of $\frac{2}{3}x$, $-2x^{\frac{3}{2}}$, $-\frac{8}{3}x^{-2}$ or exact simplified equivalent terms eg $\frac{-2}{(\sqrt{x})^3}$ or $-\frac{8}{3x^2}$.

Condone $\frac{2}{3}x^1$ and condone double signs such as $+ - 2x^{\frac{3}{2}}$. The terms do not need to be on the same line for this mark.

A1 $\frac{2}{3}x - 2x^{\frac{3}{2}} - \frac{8}{3}x^{-2}$ or exact simplified equivalent all on one line eg $\frac{2}{3}x - \frac{2}{(\sqrt{x})^3} - \frac{8}{3x^2}$.

Do not allow $\frac{2}{3}x^1 - 2x^{\frac{3}{2}} - \frac{8}{3}x^{-2}$ or double signs and do not isw in this part (including rounding decimals).



(b)

M1 Substitutes $x = 4$ into their $\frac{dy}{dx}$ to find the numerical gradient of the tangent at P .

Condone slips in their working. They must proceed as far as finding a value.

It may be implied by their answer or embedded in further work such as finding the equation

of the perpendicular line. Do not be too concerned by the labelling of their $\frac{dy}{dx}$ (It may even

be labelled as $y = \dots$)

M1 For a correct attempt at using $m_N = -\frac{1}{m_T}$ or equivalent to find the gradient of the perpendicular.

dM1 It is for the method of finding a line passing through $(4, 3)$ with a changed gradient.

Eg $\frac{9}{4} \rightarrow \frac{4}{9}$ would be acceptable as a changed gradient.

Look for $(y - 3) = \text{changed } m_T (x - 4)$ Both brackets must be correct

Alternatively uses the form $y = mx + c$ AND proceeds as far as $c = \dots$

It is dependent only on the first method mark.

A1 $4x + 9y - 43 = 0$ or exact equivalent with all terms on one side = 0.

Accept $\pm A(4x + 9y - 43 = 0)$ where $A \in \mathbb{N}$

Question Number	Scheme	Marks
2(a)(i)		

(ii)	$-a + 6a + 8 + a^2 = 32 \Rightarrow a^2 + 5a - 24 = 0$ $(a + 8)(a - 3) = 0$ $a = 3 \text{ or } a = -8 \text{ and chooses } a = 3 \text{ with reason } *$	M1 dM1 A1* cso
		(3)
	$3x^3 + 26x^2 - 9x = 0 \Rightarrow x(3x^2 + 26x - 9) = 0$ $x(3x - 1)(x + 9)$ $(x =) 0, \frac{1}{3}, -9$	M1 A1
		(2)
(b)(i)	$(y =) 0$ $y^{\frac{1}{3}} = \frac{1}{3} \text{ or } y^{\frac{1}{3}} = -9 \Rightarrow y = \dots \quad (\text{or } (-9)^3 = \dots \text{ or } \left(\frac{1}{3}\right)^3 = \dots)$ $(y =) \frac{1}{27}, -729$	B1 M1 A1
		(3)
	$9^z = \frac{1}{3} \rightarrow z = \dots$ $(z =) -\frac{1}{2} \text{ only}$	M1 A1
(b)(ii)		(2)
		(10 marks)

(a)(i)

M1 Substitutes in $x = \pm 1, y = 32$ and proceeds to a 3TQ in terms of a with all terms on one side of the equation. Condones the lack of $= 0$ and condones slips in their rearrangement.

dM1 Attempts to solve their quadratic equation by either factorising, completing the square or the quadratic formula. They cannot just state the roots. It is dependent on the first method mark. In all cases they must show their working to score this mark so:

- Solving by factorising requires the factorised form of their $a^2 + 5a - 24 = 0$ to be stated before proceeding to the roots i.e. $(a + "8")(a - "3") (= 0)$
- Solving by using the quadratic formula requires the values for a, b and c to be stated in the formula before proceeding to the roots. They cannot just state the values of a, b and c .



- Solving by completing the square requires eg $\left(a \pm \frac{5}{2}\right)^2 \pm \dots$ before rearranging to find the roots.

A1* $a = 3$ or $a = -8$ and chooses $a = 3$ with a minimal reason. Eg “as a is a positive constant” or “Since $a > 0$ ”, “ a cannot be negative”.
The final mark cannot be scored without the previous method marks being scored and there cannot be any errors even if missing brackets are recovered. Just crossing out -8 without a reason is A0.

(a)(ii)

M1 Takes out a factor of x from the given cubic with $a = 3$ (or divides through by x) and attempts to solve the resulting quadratic equation. You must see at least one intermediate line of working before proceeding to the roots eg the factorised form, values in the quadratic formula or completed square form.

A1 $(x =) 0, \frac{1}{3}, -9$ provided M1 has been scored.

Solutions with no working in this part scores 0 marks.

(b)(i)

B1 $(y =) 0$

M1 Sets $y^{\frac{1}{3}}$ equal to any of their non-zero solutions from part (a) and attempts to cube their value to find a value for y . You must see at least one stage of working to score this mark.
 $(-9)^3 = \dots$ on its own scores M1.

A1 $(y =) \frac{1}{27}, -729$ and no others other than 0.

Solutions without any working will score a maximum of B1M0A0 in this part.

(ii)

M1 Sets 9^z equal to any of their positive solutions found in part (a) and proceeds to find a value for z . They may write 9^z as 3^{2z} before proceeding to find a value for z .
Alternatively, you may see attempts to link b(i) and b(ii) together:

Eg $y = 9^{3z} \Rightarrow \frac{1}{27} = 729^z \Rightarrow z = \dots$ which can score M1.

You must see at least one stage of working to score this mark.
The method may include log statements which is acceptable.

A1 $(z =) -\frac{1}{2}$ only (provided M1 has been scored).

Answer stated without working scores 0 marks. Evidence of calculator use will also be A0.

Question Number	Scheme	Marks
3(a)	$(2\sqrt{2})^2 = p^2 + q^2 - 2pq \cos 60^\circ \text{ oe}$ $p^2 + q^2 - pq = 8 \quad *$	M1 A1*
		(2)
(b)	$q = p + 2 \Rightarrow 8 = p^2 + (p + 2)^2 - p(p + 2)$ $p^2 + 2p - 4 = 0 \text{ or } q^2 - 2q - 4 = 0$ $p = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-4)}}{2} \text{ or } q = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-4)}}{2}$ $p = -1 + \sqrt{5} \text{ or } q = 1 + \sqrt{5}$ $p = -1 + \sqrt{5} \text{ and } q = 1 + \sqrt{5} \text{ only}$	M1 A1 M1 B1 (A1 on EPEN) A1cso
		(5)
(c)	$\text{Area} = \frac{1}{2} \times (-1 + \sqrt{5})(1 + \sqrt{5}) \times \sin 60^\circ$ $\text{Area} = \sqrt{3} \text{ (m}^2\text{)}$	M1 A1
		(2)
Alt(a)	Forming a line BX which is perpendicular to AC where X is on the line AC . $AX = p \cos 60 = \frac{p}{2}$ $BX = \sqrt{p^2 - \left(\frac{p}{2}\right)^2} = \frac{\sqrt{3}}{2}p \text{ or } BX = p \sin 60$ $\left(\frac{\sqrt{3}}{2}p\right)^2 + \left(q - \frac{p}{2}\right)^2 = (2\sqrt{2})^2 \text{ or } (p \sin 60)^2 + \left(q - \frac{p}{2}\right)^2 = (2\sqrt{2})^2$ $\frac{3p^2}{4} + q^2 - pq + \frac{p^2}{4} = 8$ $p^2 + q^2 - pq = 8 \quad *$	M1 A1*
		(9 marks)

(a)



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1a	$y = 5x^3 + \frac{3}{x^2} - 7x = 5x^3 + 3x^{-2} - 7x$ $\left(\frac{dy}{dx} = \right) 15x^2 - 6x^{-3} - 7$	M1A1A1
		(3)
b	$\left(\frac{d^2y}{dx^2} = \right) 30x + 18x^{-4}$	M1A1
		(2)
		(5 marks)

Do not be concerned by the labelling of their answers in part (a) or part (b)

(a)

M1: Decreases the power by 1 on one of the terms $\dots x^3 \rightarrow \dots x^2$ $\dots x^{-2} \rightarrow \dots x^{-3}$ $\pm 7x \rightarrow \pm 7$
The index does not have to be processed. e.g. $\dots x^3 \rightarrow \dots x^{3-1}$

A1: Two correct unsimplified terms (indices must be processed). May be seen on different lines or listed. Condone $7x^0$ as an unsimplified term. Do not withhold this mark if there is a $+c$ present.

A1: $15x^2 - 6x^{-3} - 7$ or simplified equivalent. Condone any spurious notation around the expression but withhold this mark if there is a constant of integration. Isw once a correct answer is seen. Do not accept $15x^2 - 6x^{-3} - 7x^0$ or e.g. $15x^2 + -6x^{-3} - 7$

(b)

M1: Decreases the power by 1 on one of the terms of their changed function. Do not allow this mark to be scored for a constant of integration in (a) $\rightarrow 0$ in (b)

A1: $30x + 18x^{-4}$ or simplified equivalent e.g. $30x + \frac{18}{x^4}$. Condone e.g. $30x^1 + \frac{18}{x^4}$. Isw once a correct answer is seen. Withhold this mark if there is a constant of integration and it has not already been penalised in (a).
Allow this mark to be scored following $15x^2 - 6x^{-3} \pm 7$ in (a). Also allow full marks to be scored if they have no constant.

Note: (a) $15x^2 - 6x^{-1} - 7$ and (b) $30x + 6$ will score M1A1A0 M1A0



Question Number	Scheme	Marks
2a	$\frac{1}{8}x$	B1
		(1)
b	$\frac{1}{256}x^{\frac{3}{2}}$	B1
		(1)
c	$\left(\frac{1}{2}\left(\frac{1}{64}x^2 \times \frac{16}{\sqrt{x}}\right)\right)^{-\frac{4}{3}} = \left(\frac{1}{8}x^{\frac{3}{2}}\right)^{-\frac{4}{3}} = 16x^{-2}$	M1A1
		(2)
		(4 marks)

(a)

B1: $\frac{1}{8}x$ or simplified equivalent e.g. $0.125x$ or $\frac{1}{8}x^1$ Accept e.g. $\frac{x}{8}$ but $\pm\frac{1}{8}x$ is B0.

Do not withhold this mark unless it is clear that they intend to write $\frac{1}{8x}$

(b)

B1: $\frac{1}{256}x^{\frac{3}{2}}$ or simplified equivalent e.g. $0.00390625x^{\frac{3}{2}}$ condone $\frac{x^{\frac{3}{2}}}{256}$. Isw after a correct answer is seen.

Do not accept e.g. $\frac{1}{256}\sqrt{x^3}$ or $\frac{1}{256}(\sqrt{x})^3$ or $\frac{1}{256}x\sqrt{x}$ as n is not a simplified constant.

Do not accept $x^{\frac{3}{2}} \div 256$ or $\frac{1}{256x^{\frac{3}{2}}}$ (not of the required form)

Do not withhold this mark unless it is clear that they intend to write $\frac{1}{256x^{\frac{3}{2}}}$

(c)

M1: Attempts to use the index laws and proceeds to either

- $\left(\dots x^{\frac{3}{2}}\right)^{-\frac{4}{3}}$ or $\left(\dots x^{-\frac{3}{2}}\right)^{\frac{4}{3}}$
- $\dots x^{-\frac{8}{3}} \times \dots x^{\frac{2}{3}}$ (or $\dots x^{-\frac{8}{3}} \div \dots x^{-\frac{2}{3}}$)
- $\dots x^{-2}$
- $16x^n$ where $n \neq -\frac{1}{2}$

Allow $\frac{16}{x^2}$ to score M1.

Be aware that incorrect understanding of indices such as $\frac{16}{\sqrt{x}} = 16x^{-2}$ is M0A0

A1: $16x^{-2}$ (from a correct method)

(Correct answer with no incorrect working seen can score M1A1). Isw after a correct answer is seen. $\frac{16}{x^2}$ is A0

Question Number	Scheme	Marks
3a	$\frac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}} \times \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{3}-\sqrt{5}} = \frac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{12-5}$	M1
	e.g. $\frac{21\sqrt{3}-14\sqrt{5}}{7}$	dM1
	$3\sqrt{3}-2\sqrt{5}$	A1
		(3)
b	$(x+5\sqrt{3})\sqrt{5} = 40-2x\sqrt{3} \Rightarrow x\sqrt{5}+2x\sqrt{3} = 40-5\sqrt{15}$	M1
	$(x=) \frac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$	A1
	$(x=) 15\sqrt{3}-10\sqrt{5}$	A1ft
		(3)
		(6 marks)

(a) Note that $3\sqrt{3}-2\sqrt{5}$ with no working scores M0dM0A0

M1: Attempts to rationalise the denominator by multiplying both numerator and denominator by $k(2\sqrt{3}-\sqrt{5})$, where

k is an integer usually 1, and proceeds to a fraction such as $\frac{\dots\sqrt{3}\pm\dots\sqrt{5}\pm\dots\sqrt{45}\pm\dots\sqrt{75}}{\dots}$ or $\frac{\dots}{12-5}$ or $\frac{\dots}{7}$

but not for e.g. $\frac{\dots}{12+2\sqrt{15}-2\sqrt{15}-5}$ or $\frac{\dots}{(2\sqrt{3})^2-(\sqrt{5})^2}$ as these still have surds on the denominator.

Allow $\sqrt{45}$ and $\sqrt{75}$ to be written in terms of $\sqrt{3}$ and $\sqrt{5}$ as well.

Condone slips in multiplying out as well as miscopying errors.

Note $\frac{21\sqrt{3}-14\sqrt{5}}{7}$ with no intermediate working can still score M1 for the denominator.

Attempting to multiply by a multiple of $\frac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$ is M0dM0A0

dM1: Attempts to simplify surds and may collect terms to achieve a fraction where

- the numerator is in terms of $\sqrt{3}$ and $\sqrt{5}$ only.
- the denominator is a multiple of 7 (which may be unsimplified)

It is dependent on the previous method mark.

If they have not fully multiplied out the numerator (or implied) then this mark cannot be scored. e.g.

$$\frac{(8-\sqrt{15})(2\sqrt{3}-\sqrt{5})}{\dots} = \frac{16\sqrt{3}+8\sqrt{5}}{\dots}$$
 scores a maximum of M1dM0A0.

Note $\frac{21\sqrt{3}-14\sqrt{5}}{7}$ with no previous working seen scores M1dM0A0 because they have not shown any multiplying out of the brackets on the numerator.

Note
$$\frac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}} = \frac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{7} = 3\sqrt{3}-2\sqrt{5}$$
 is M1dM0A0 because they have not collected terms

on the numerator (or changed them all to be in terms of $\sqrt{3}$ and $\sqrt{5}$ before simplifying to the final answer).



A1: $3\sqrt{3} - 2\sqrt{5}$. The answer does not imply the method marks. Do not withhold this mark for slips in working such as invisible brackets provided they are recovered/implied by further work.

Note that a number of candidates are misreading $\frac{8-\sqrt{15}}{2\sqrt{3}+5} \times \frac{2\sqrt{3}-5}{2\sqrt{3}-5} = \frac{16\sqrt{3}-40-2\sqrt{45}+5\sqrt{15}}{12-25}$ this can score SC100 condoning one sign slip in their multiplying out of the brackets in the numerator.

(b) **Note that $x = 15\sqrt{3} - 10\sqrt{5}$ with no working scores M0A0A0**

M1: Multiplies out the brackets and isolates the x terms on one side. Condone for this mark if the surds are converted to rounded decimals.

e.g. $(\sqrt{5} + 2\sqrt{3})x = 5(8 - \sqrt{15})$ scores M1

Alternatively divides both sides by $\sqrt{5}$ and isolates the x terms on one side.

e.g. $x + 2x\frac{\sqrt{3}}{\sqrt{5}} = \frac{40}{\sqrt{5}} - 5\sqrt{3}$

Condone slips in their working and invisible brackets, but there must be two terms on each side of the equation.

A1: $x = \frac{40 - 5\sqrt{15}}{2\sqrt{3} + \sqrt{5}}$ or exact equivalent (which cannot be $15\sqrt{3} - 10\sqrt{5}$).

In the alternative method it may be seen as $x = \frac{\frac{40}{\sqrt{5}} - 5\sqrt{3}}{1 + 2\frac{\sqrt{3}}{\sqrt{5}}}$ which can score this mark.

Do not accept proceeding from $(\sqrt{5} + 2\sqrt{3})x = 5(8 - \sqrt{15}) \Rightarrow x = 15\sqrt{3} - 10\sqrt{5}$ without any intermediate expression for x . This scores A0 as the use of calculator technology is not acceptable.

An expression in decimals only is A0.

A1ft: $x = 15\sqrt{3} - 10\sqrt{5}$ only (or exact simplified equivalent) e.g. $x = 5(3\sqrt{3} - 2\sqrt{5})$ This mark cannot be awarded without the previous A mark being scored.

Follow through on their part (a) answer of the form $a\sqrt{3} + b\sqrt{5}$ so award for $5a\sqrt{3} + 5b\sqrt{5}$ or including $5(a\sqrt{3} + b\sqrt{5})$ where a and b may be fractions. Isw once they have achieved the correct answer.

Note: If a candidate has an unsimplified answer in part (a) e.g. $\frac{21\sqrt{3} - 14\sqrt{5}}{7}$ then do not withhold this mark in

part (b) if they proceed to e.g. $\frac{105\sqrt{3} - 70\sqrt{5}}{7}$ as they have already been penalised once for not giving an answer in simplest form.

Alternative method – squaring both sides (send to review if unsure)

M1: Attempts to square both sides and proceeds to a three term quadratic in x with terms collected on one side of the equation (condone slips in multiplying out and collecting terms).

A1: As above in main scheme

A1ft: As above in main scheme

Question Number	Scheme	Marks
1.	$\int \left(\frac{8x^3}{5} - \frac{2}{3x^4} - 1 \right) dx = \frac{1}{4} \times \frac{8x^4}{5} - \frac{2}{3} \times \frac{1}{-3} x^{-3} - x$	M1 A1
	$\frac{2}{5}x^4 + \frac{2}{9}x^{-3} - x + c$	A1 A1
		(4)
		Total 4

M1: For $x^3 \rightarrow x^4$ or $x^{-4} \rightarrow x^{-3}$ or $1 \rightarrow x$. Also allow eg $x^3 \rightarrow x^{3+1}$

A1: Any 2 correct unsimplified or simplified terms, which may appear on separate lines. The indices must be processed but fractions within fractions are acceptable. The $+c$ does not count as a correct term here.

A1: For any 2 correct simplified terms of $\frac{2}{5}x^4 + \frac{2}{9}x^{-3} - x$, which may appear on separate lines. The $+c$ does not count as a correct term here.

Accept exact decimals including eg. 0.2 for coefficients. Condone $-1x$ $-1x^1$, $-\frac{x}{1}$, $+ -1x^1$ for this mark.

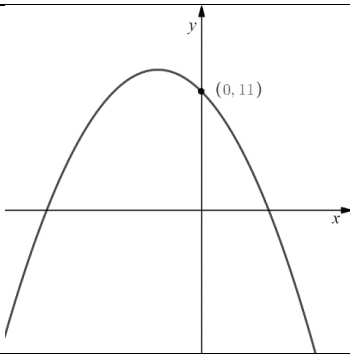
A1: $\frac{2}{5}x^4 + \frac{2}{9}x^{-3} - x + c$ or simplified equivalent. All correct and on the same line (in any order)

including a constant of integration. $\frac{2}{9x^3}$ is acceptable but not $\frac{2/9}{x^3}$.

Accept exact decimals including eg. 0.2 for coefficients. Condone $-1x$ only. Isw after a correct answer. Give the benefit of the doubt to terms such as $\frac{2}{9x^3}$ where it is unclear whether the x^3 term is on the denominator or not provided no incorrect working is seen.

Award once a correct expression is seen and isw but if there is any additional/incorrect notation and no correct expression has been seen on its own, withhold the final mark.

Eg. $\int \frac{2}{5}x^4 + \frac{2}{9}x^{-3} - x + c \, dx \quad \frac{2}{5}x^4 + \frac{2}{9}x^{-3} - x + c = 0$

Question Number	Scheme	Marks
2(a)	$f(x) = 11 - 4x - 2x^2$ $\Rightarrow \dots - 2(2x + x^2) \dots$ or $\Rightarrow \dots - 2(2x + x^2 \dots)$	B1
	$\dots(2x + x^2) \Rightarrow \dots((x+1)^2 \pm \dots)$	M1
	$(f(x) =) 13 - 2(x+1)^2$	A1
		(3)
(b)		M1
		A1
		(2)
(c)	$x = -1$	B1ft
		(1)
		Total 6
Alt(a)	$a + b(x^2 + 2cx + c^2) = 11 - 4x - 2x^2$	B1
	$b = -2$	
	$2bc = -4 \Rightarrow c = \dots (=1)$	M1
	$a + bc^2 = -4 \Rightarrow a = \dots (=13)$ $(f(x) =) 13 - 2(x+1)^2$	A1

(a)B1: $b = -2$ M1: Attempts to complete the square on $x^2 \pm 2x$ so score for $(x \pm 1)^2 \pm \dots$ or alternatively attempts to compare coefficients to find a value for c .Condone $11 - 4x - 2x^2 \Rightarrow \dots - 2(2x - x^2) \dots \Rightarrow -2(x \pm 1)^2 \pm \dots$ A1: $13 - 2(x+1)^2$. If a , b and c are stated and there is a contradiction then mark their final expression. Condone $13 + -2(x+1)^2$ or just the values of a , b and c being stated.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Apply isw if possible if there is no contradiction so e.g. correct work leading to $x \leq -\frac{1}{4}, x \geq 2$ followed by $-\frac{1}{4} \geq x \geq 2$ can score full marks. If several different answers are offered, mark the final answer.

Particular cases and how to mark them:

Case 1

$$4x^2 - 3x + 7 \geq 4x + 9 \Rightarrow 4x^2 - 7x - 2 \geq 0$$

$$x = -\frac{1}{4}, 2 \Rightarrow x \leq -\frac{1}{4}, x \geq 2$$

Scores M0A0 M1A0

Case 2

$$4x^2 - 3x + 7 \geq 4x + 9 \Rightarrow 4x^2 - 7x - 2 \geq 0$$

$$\Rightarrow x \leq -\frac{1}{4}, x \geq 2$$

Scores M0A0M0A0



Question	Scheme	Marks
2(a)	$2x + 2y = 350$	B1
		(1)
(b)	E.g. $xy = 7350$, $x \times y = 7350$	B1
		(1)
(c)	$x(175 - x) = 7350$ or $(175 - y)y = 7350$	M1
	E.g. $x^2 - 175x + 7350 = 0 \Rightarrow (x - 70)(x - 105) = 0 \Rightarrow x = \dots$	dM1
	$x = 70$ or 105	A1
	$(x > y \Rightarrow)x = 105, y = 70$	A1
		(4)
(6 marks)		

Notes:

Ignore labelling and mark all parts together.
Ignore any units given whether correct or incorrect.

(a)

B1: Correct equation. Accept any correct equation e.g. $2(x + y) = 350$, $x + y = 175$

(b)

B1: Correct equation. Accept any correct equation e.g. $y = \frac{7350}{x}$

(c)

M1: Substitutes for x or y into either of their equations to produce a 2 or 3 term quadratic equation in one variable. Note that terms do not need to be collected for this mark so e.g. $x(175 - x) = 7350$ or $(175 - y)y = 7350$ or is acceptable for this mark.

Condone slips when substituting if the intention is clear e.g. substituting for $2x$ rather than x :
 $x(350 - 2x) = 7350$

(Note, if substituting $x = \frac{7350}{y}$ into $2x + 2y = 350$ they will need to multiply through by y to score

this mark. Condone slips but must produce a 2 or 3 term quadratic equation.)

dM1: Depends on the first method mark. Solves their 3 term quadratic equation, any suitable means such as formula or by calculator (may be implied by correct values for their quadratic).

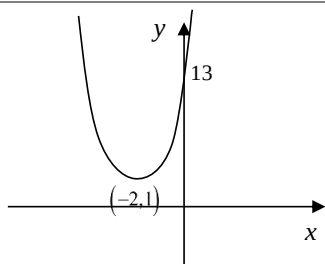
A1: Correct **simplified** roots of the quadratic. Do not be concerned about which is x and y for this mark so just look for the values 70 and 105. It is possible that candidates may only select the larger value for x (105) or the smaller value for y (70) **but** if both roots are found they must both be correct.

A1: Both x and y correct and correctly assigned and all previous marks scored.

Special Case:

Candidates who use Trial and Improvement in part (c) or guess the correct answers with no quadratic formed allow a special case of M1M1A0A0. If the values are the wrong way round or both sets of answers are offered then score special case of M1M0A0A0.



Question	Scheme		Marks
3(a)	$a = 3$		B1
	$b = \pm 2$		M1
	$3x^2 + 12x + 13 = 3(x + 2)^2 + 1$ or $a = 3, b = 2, c = 1$		A1
			(3)
(b)		Correct U shape with minimum in second quadrant	B1
		Intercept 13 on y-axis.	B1
		Vertex at $(-2, 1)$	B1ft
			(3)
(6 marks)			

Notes:**(a)****B1:** Correct value for a stated or shown by working.**M1:** Obtains $b = \pm 2$. Allow unsimplified e.g. $b = \pm \frac{4}{2}, \pm \frac{12}{6}$. May be implied by e.g. $3(x \pm 2)^2 + \dots$ **A1:** Fully correct expression or correct values.Note that there are various methods e.g. $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \Rightarrow 3x^2 + 12x + 13 = \dots$ or $a(x+b)^2 + c = ax^2 + 2abx + ab^2 + c \Rightarrow a = 3, 2ab = 12, ab^2 + c = 13 \Rightarrow b, c = \dots$ **(b)****There must be a sketch to score marks in (b)****Labelling on the sketch takes precedence****Treat part (b) as Hence, or otherwise, i.e. part (a) may be incorrect but full marks are available in part (b) for a correct sketch of $y = 3x^2 + 12x + 13$** **B1:** Correct U shape with minimum point in the second quadrant but **not** on the x-axis. Do not allow a clear V shape.**B1:** Correct y intercept labelled or stated. Allow as just 13 or (0, 13) or (13, 0) as long as it is in the correct position. If stated away from the sketch it must be as (0, 13) and correspond to the sketch.

Their curve must at least touch at (0, 13). Any other intercepts can be ignored.

B1ft: Correct vertex labelled in some way, or follow through their b and c i.e. $x = -b$ and $y = c$. Must be a turning point but could be a maximum or a minimum. Allow the labelling as shown or as $x = \dots, y = \dots$ or as the ordinates shown on the axes. It must correspond to the sketch. If stated away from the sketch it must be correct or correct follow through and correspond to the sketch.

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$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

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1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

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Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$ **Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$\int \left(8x^3 - 6\sqrt{x} - \frac{2}{5x^3} \right) dx = 2x^4 - 4x^{\frac{3}{2}} + \frac{1}{5x^2} + c$	M1 A1A1A1
		(4 marks)

M1 Increases the power by one on one of the terms in x

Must be for any one of $x^3 \rightarrow x^4$ $x^{\frac{1}{2}} \rightarrow x^{\frac{3}{2}}$ $x^{-3} \rightarrow x^{-2}$. Indices do not need to be processed for this mark. e.g. $x^3 \rightarrow x^{3+1}$ is fine.

A1 One term of $2x^4 - 4x^{\frac{3}{2}} + \frac{1}{5x^2}$ (may be unsimplified but indices must be processed)

A1 Any two terms of $2x^4 - 4x^{\frac{3}{2}} + \frac{1}{5x^2}$ (may be unsimplified but indices must be processed). Terms may appear as a list or be on different lines.

A1 $2x^4 - 4x^{\frac{3}{2}} + \frac{1}{5x^2} + c$ or simplified equivalent (including the constant of integration) all on one line.

e.g. accept $2x^4 - 4(\sqrt{x})^3 + 0.2x^{-2} + c$ but do not accept $\frac{0.2}{x^2}$ or $\frac{\frac{1}{5}}{x^2}$ for $\frac{1}{5x^2}$

isw once a correct answer is seen but withhold if there is any spurious notation e.g. an integral sign or dx

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

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Question Number	Scheme	Marks
	$y = \frac{1}{2}x^4 - 3 + \frac{10}{x^2}$	
5(a)	$\left(\int \frac{1}{2}x^4 - 3 + \frac{10}{x^2} dx\right) = \frac{1}{10}x^5 - 3x - \frac{10}{x} + c$	M1A1A1
		(3)
(b)(i)	$\left(\frac{dy}{dx}\right) = 2x^3 - \frac{20}{x^3}$	M1A1A1
		(3)
(b)(ii)	$2x^3 - \frac{20}{x^3} = 3 \Rightarrow 2x^6 - 20 = 3x^3$	M1
	$2x^6 - 3x^3 - 20 = 0 \Rightarrow (2x^3 + 5)(x^3 - 4) = 0 \Rightarrow x^3 = \dots \Rightarrow x = \dots$ or e.g. $x^3 = a \Rightarrow (2a + 5)(a - 4) = 0 \Rightarrow a = \dots \Rightarrow x^3 = \dots \Rightarrow x = \dots$	dM1
	$\Rightarrow x = -\sqrt[3]{\frac{5}{2}}, \sqrt[3]{4}$	A1A1
		(4)
		(10 marks)
Notes		
(a)		
M1: For increasing any power by one. E.g. $\frac{1}{2}x^4 \rightarrow \dots x^5$ or $-3 \rightarrow \dots x$ or $\frac{10}{x^2} \rightarrow \dots x^{-1}$		
Allow unprocessed indices for this mark e.g. $\frac{1}{2}x^4 \rightarrow \dots x^{4+1}$		
A1: For two correct and simplified terms of $\frac{1}{10}x^5 - 3x - \frac{10}{x}$.		
Allow $0.1x^5$ for $\frac{1}{10}x^5$ and $-10x^{-1}$ for $-\frac{10}{x}$.		
Condone $-3x^1$ for $-3x$ and $-\frac{10}{x^1}$ for $-\frac{10}{x}$ for <u>this</u> mark.		
A1: $\frac{1}{10}x^5 - 3x - \frac{10}{x} + c$. Must include a constant of integration.		
Ignore the lhs i.e. ignore what they call their integral.		
Allow $0.1x^5$ for $\frac{1}{10}x^5$ and $10x^{-1}$ for $\frac{10}{x}$.		
Condone spurious integral signs e.g. $\int \frac{1}{2}x^4 - 3 + \frac{10}{x^2} dx = \int \frac{1}{10}x^5 - 3x - \frac{10}{x} + c$		
Do not allow e.g. $\frac{1}{10}x^5 - 3x + -\frac{10}{x} + c$.		



Mark (b)(i) and (ii) together.**(b)(i)**

M1: For decreasing any power by one. E.g. $\frac{1}{2}x^4 \rightarrow \dots x^3$ or $-3 \rightarrow 0$ or $\frac{10}{x^2} \rightarrow \dots x^{-3}$

Allow unprocessed indices for this mark e.g. $\frac{1}{2}x^4 \rightarrow \dots x^{4-1}$

A1: For one correct and **simplified** term. $2x^3$ or $-\frac{20}{x^3}$. Allow $20x^{-3}$ for $\frac{20}{x^3}$.

A1: $2x^3 - \frac{20}{x^3}$. Allow $20x^{-3}$ for $\frac{20}{x^3}$.

(b)(ii)

M1: Sets their $2x^3 - \frac{20}{x^3}$ equal to 3 and multiplies to form a **polynomial equation** in x .

Allow copying slips but it must be their $\frac{dy}{dx}$ from part (b)(i) and it must be of the form

$Ax^m + \frac{B}{x^n}$ where m and n are integers and $m > 0$ and $n > 0$

It is for proceeding from $Ax^m + \frac{B}{x^n} = 3$ to $Ax^{m+n} + B = 3x^n$ or equivalent work.

dm1: Solves an equation of the form $px^6 + qx^3 + r = 0$, $p, q, r \neq 0$ via **non calculator** means to find at least one value for x . It is not for finding a value of x^3 so e.g.

$$t = x^3 \Rightarrow 2x^6 - 3x^3 - 20 = 2t^2 - 3t - 20, 2t^2 - 3t - 20 = 0 \Rightarrow (2t + 5)(t - 4) = 0 \Rightarrow t = -\frac{5}{2}, 4$$

on its own scores M0 unless a value of x is found by cube rooting. Note we will condone the “cube rooting” done on a calculator for this mark but **not** the solving of their equation.

A1: Either solution of $x = -\sqrt[3]{\frac{5}{2}}, \sqrt[3]{4}$ o.e. such as $-2.5^{\frac{1}{3}}, 4^{\frac{1}{3}}$

Allow $(-2.5)^{\frac{1}{3}}$ for $-2.5^{\frac{1}{3}}$ and $2^{\frac{2}{3}}$ for $4^{\frac{1}{3}}$

Must come from correct work i.e. from the correct equation $2x^6 - 3x^3 - 20 = 0$ oe
Apply isw once a correct answer is seen.

A1: $x = -\sqrt[3]{\frac{5}{2}}, \sqrt[3]{4}$ o.e. such as $-2.5^{\frac{1}{3}}, -\frac{\sqrt[3]{20}}{2}, 4^{\frac{1}{3}}$

Allow $(-2.5)^{\frac{1}{3}}$ for $-2.5^{\frac{1}{3}}$ and $2^{\frac{2}{3}}$ for $4^{\frac{1}{3}}$

Must come from correct work i.e. from the correct equation $2x^6 - 3x^3 - 20 = 0$ oe
Apply isw once correct answers are seen.

Notes on evidence of use of a calculator in (b)(ii):

$$\text{e.g. } 2x^6 - 3x^3 - 20 = 0 \Rightarrow (x^3 + 2.5)(x^3 - 4) \Rightarrow x^3 = -2.5, 4 \Rightarrow x = \dots$$

and

$$x^3 = a \Rightarrow 2a^2 - 3a - 20 = 0 \Rightarrow (a + 2.5)(a - 4) \Rightarrow a = -2.5, 4 \Rightarrow x^3 = \dots \Rightarrow x = \dots$$

Both score dM0A0A0



Question Number	Scheme	Marks
2. (a)	$5(x+3) > 4(2x-5) \Rightarrow 5x+15 > 8x-20 \Rightarrow ax > b \text{ or } px < q$ $\Rightarrow x < \frac{35}{3}$	M1 A1 (2)
(b) (i)	$x^2 - 6x + 1 = (x-3)^2 \pm \dots = (x-3)^2 - 8$	M1, A1
(ii)	$(x-3)^2 - 8 = 0 \Rightarrow x = 3 + \sqrt{8} \text{ or } 3 - \sqrt{8}$ $x^2 - 6x + 1 \geq 0 \Rightarrow x \leq 3 - \sqrt{8}, x \geq 3 + \sqrt{8}$	M1 A1 (4)
(c)	$x \leq 3 - \sqrt{8}, 3 + \sqrt{8} \leq x < \frac{35}{3}$	B1 (1) (7 marks)

This is effectively a non- calculator question and methods must be clearly shown.

(a)

M1: Attempts to expand brackets and then collect terms. Look for

- $5x \pm 15 > 8x \pm 20$
- followed by $ax > b$ or $px < q$

Also condone with an = sign in place of the inequality

A1: $x < \frac{35}{3}$ or exact equivalent such as $\frac{35}{3} > x$ **following the award of M1.**

Do not accept $x < 11.66$ or $x < 11.7$ but $x < 11.\dot{6}$ is acceptable and ISW after a correct answer

Mark part (b) as one question

(b) (i)

M1: Starts the process by writing $x^2 - 6x + 1$ as $(x-3)^2 \pm c$, where c is a constant (which may not be simplified)

A1: $x^2 - 6x + 1 = (x-3)^2 - 8$. It is acceptable to write this down for the two marks

(b) (ii) Note that this must follow on from their (b)(i) and CANNOT be restarted via the quadratic formula

M1: Correctly solves their $(x+a)^2 - b = 0$, $b > 0$ and finds at least one critical value.

Look for an intermediate line of working such as $(x+a)^2 = b$ or $x+a = \pm\sqrt{b}$

Do not allow decimals, i.e. awrt 5.83, 0.17 but isw after a correct (exact) answer

It can be implied by an attempt to solve their inequality as long as an intermediate line is seen.

E.g. $(x+a)^2 - b \geq 0$, $b > 0 \Rightarrow x+a \dots \sqrt{b} \Rightarrow x \dots -a + \sqrt{b}$ where ... can even be an incorrect inequality

Again, it is important to see an intermediate line of working.

A1: $x \leq 3 - \sqrt{8}, x \geq 3 + \sqrt{8}$ o.e following the award of M1

ISW if this is subsequently changed to decimals. Accept $\sqrt{8}$ as $2\sqrt{2}$



Question	Scheme	Marks
9	$\frac{21x^3 - 5x}{2\sqrt{x}} = \alpha x^{\frac{5}{2}} + \dots \text{ or } \frac{21x^3 - 5x}{2\sqrt{x}} = \dots + \beta x^{\frac{1}{2}}$ $f(x) = \frac{27}{3}x^3 - \frac{21}{2} \times \frac{2}{7}x^{\frac{7}{2}} + \frac{5}{2} \times \frac{2}{3}x^{\frac{3}{2}} (+c) \quad \left(= 9x^3 - 3x^{\frac{7}{2}} + \frac{5}{3}x^{\frac{3}{2}} (+c) \right)$ $f(9) = 10 \Rightarrow 9(9)^3 - 3(9)^{\frac{7}{2}} + \frac{5}{3}(9)^{\frac{3}{2}} + c = 10 \Rightarrow c = \dots$ $(f(x) =) 9x^3 - 3x^{\frac{7}{2}} + \frac{5}{3}x^{\frac{3}{2}} - 35$	M1 M1A1A1 dM1 A1
		(6 marks)

Notes

On EPEN it is B1M1A1A1dM1A1. We are marking this as M1M1A1A1dM1A1

M1 Uses correct index laws to obtain at least one correct index from splitting the fraction.

Award for $\alpha x^{\frac{5}{2}} + \dots$ or $\dots + \beta x^{\frac{1}{2}}$

M1 $x^n \rightarrow x^{n+1}$ correctly seen on one term (usually the $27x^2 \rightarrow \dots x^3$). The indices do not need to be processed. This mark can also be awarded for integrating terms from incorrect attempts to split the fraction. Eg $\frac{21x^3 - 5x}{2x^{\frac{1}{2}}} = 42x^{\frac{7}{2}} - 10x^{\frac{3}{2}} \quad 42x^{\frac{7}{2}} \rightarrow \dots x^{\frac{9}{2}} \text{ or } -10x^{\frac{3}{2}} \rightarrow \dots x^{\frac{5}{2}}$

This mark cannot be awarded for only seeing:

$$\frac{21x^3 - 5x}{2x^{\frac{1}{2}}} \rightarrow \frac{\dots x^4 \pm \dots x^2}{\dots x^{\frac{3}{2}}}$$

A1 Two correct terms simplified or unsimplified, but the indices must have been processed.

A1 All correct simplified or unsimplified (+ c not required)

dM1 Uses $f(9) = 10$ and attempts to find c. Do not be too concerned by the mechanics of their arrangement. It is dependent on the previous method mark.

A1 $9x^3 - 3x^{\frac{7}{2}} + \frac{5}{3}x^{\frac{3}{2}} - 35$ All correct and simplified. Accept other simplified equivalent expressions for $f(x)$ such as $\frac{1}{3}(27x^3 - 9x^{\frac{7}{2}} + 5x^{\frac{3}{2}} - 105)$



Question Number	Scheme	Marks
10(a)	$f'(x) = ax - 12x^{\frac{1}{3}} \Rightarrow f''(x) = a - 4x^{-\frac{2}{3}}$	B1
	Sets $f''(27) = 0 \Rightarrow 0 = a - 4 \times \frac{1}{9} \Rightarrow a = \frac{4}{9}$	M1 A1
		(3)
(b)	$f'(x) = ax - 12x^{\frac{1}{3}} \Rightarrow (f(x) =) \frac{1}{2}ax^2 - 9x^{\frac{4}{3}} + c$	M1 A1ft
	Substitutes $x = 1, f(x) = -8 \Rightarrow c = \dots$	dM1
	$(f(x) =) \frac{2}{9}x^2 - 9x^{\frac{4}{3}} + \frac{7}{9}$	A1
		(4)
		(7 marks)

Mark (a) and (b) together

Do not be too concerned with notation e.g. in (a) when they differentiate they may call it

$f'(x)$ or e.g. $\frac{dy}{dx}$

(a)

B1 States or uses $f''(x) = a - 4x^{-\frac{2}{3}}$ which may be unsimplified

M1 Sets their $f''(27) = 0$ and proceeds to a value for a .

It is dependent upon one correct term in $f''(x)$ e.g. a or $-4x^{-\frac{2}{3}}$ (oe)

A1 $a = \frac{4}{9}$

(b)

M1 Integrates $ax - 12x^{\frac{1}{3}}$ with one term correct e.g. $\frac{1}{2}ax^2$ or $-\frac{12x^{\frac{4}{3}}}{\frac{4}{3}}$ with the indices processed.

A1ft $f'(x) = ax - 12x^{\frac{1}{3}} \Rightarrow (f(x) =) \frac{1}{2}ax^2 - 9x^{\frac{4}{3}} + c$ follow through on a or a numerical a or a “made up” a but must include $+ c$

Allow simplified or unsimplified so allow e.g. $\frac{\frac{4}{9}x^2}{2} - \frac{12x^{\frac{4}{3}}}{\frac{4}{3}} + c$

dM1 Substitutes $x = 1$ and $f(x) = -8$ to obtain a value for c . Must have numerical a now.
Depends on the first M mark.

A1 $(f(x) =) \frac{2}{9}x^2 - 9x^{\frac{4}{3}} + \frac{7}{9}$. Allow equivalent correct fractions for $\frac{2}{9}, \frac{7}{9}$ or recurring decimals

e.g. $0.\dot{2}$, $0.\dot{7}$ with clear dots over the 2 and 7.

Allow e.g. $\sqrt[3]{x^4}$ for $x^{\frac{4}{3}}$

Look for a correct expression so no need to see $f(x) = \dots$ and isw if necessary.

Note that a fairly common error is to obtain $a = -\frac{4}{9}$ in part (a) leading to $c = \frac{11}{9}$ in part (b)

Question Number	Scheme	Marks
4 (a)	States or implies that $[f(x) =]kx(x-4)$	M1
	Attempts to find k . E.g. $-4.8 = k \times 2 \times (2-4) \Rightarrow k = \dots$	dM1
	$[f(x) =]1.2x(x-4)$	A1
(b)	States or implies that $[g(x) =]\lambda x(x-4)^2$	(3)
	Attempts to find λ . E.g. $7.2 = \lambda \times 6 \times (6-4)^2 \Rightarrow \lambda = \dots$	M1
	$[g(x) =]0.3x(x-4)^2$	dM1
(c)	Sets their $1.2x(x-4) = 0.3x(x-4)^2$	A1
	Valid attempt to solve $1.2\cancel{x(x-4)} = 0.3\cancel{x(x-4)}^{\cancel{2}} \Rightarrow x = 4 + \frac{1.2}{0.3}$	(3)
	$x = 8$	B1ft
	$(8, 38.4)$	M1
		A1
		A1
		(4)
		(10 marks)

(a)

M1: States or implies that $f(x) = kx(x-4)$. Allow with $k = 1$. The $f(x)$ need not be seen – assume they are working with $f(x)$ in part (a)

Alternatives include $f(x) = k(x-2)^2 - 4.8$ Allow with $k = 1$ and $f(x) = ax^2 + bx$ with at least one of $4a + b = 0$ and/or $4a + 2b = -4.8$

dM1: Attempts to find all the constants in their equation for $f(x)$ using $(2, -4.8)$. E.g.

$$-4.8 = k \times 2 \times (2-4) \Rightarrow k = \dots$$

A1: $[f(x) =]1.2x(x-4)$ o.e. such as $[f(x) =]1.2x^2 - 4.8x$ or $[f(x) =]1.2(x-2)^2 - 4.8$ and isw. Accept with $(x-0)$. The $f(x)$ may be missing, it can be assumed (but use of $g(x)$ is A0). Allow if the full expression is given in latter parts if all the work was correct in (a) to find the constants.

(b)

M1: States or implies that $g(x) = \lambda x(x-4)^2$ Allow with $\lambda = 1$. The $g(x)$ need not be seen – assume they are working with $g(x)$ in part (b).

Alternatives include $g(x) = ax^3 + bx^2 + cx$ with at least two of $64a + 16b + 4c = 0$, $48a + 8b + c = 0$, and/or $216a + 36b + 6c = 7.2$ (allow minor slips).

dM1: Attempts a full equation for $g(x)$ using $(6, 7.2)$

$$\text{E.g. } 7.2 = \lambda \times 6 \times (6-4)^2 \Rightarrow \lambda = \dots$$

A1: $[g(x) =]0.3x(x-4)^2$ o.e. such as $[g(x) =]0.3x^3 - 2.4x^2 + 4.8x$ and isw. Accept with $(x-0)$. The $g(x)$ may be missing, it can be assumed (but use of $f(x)$ is A0). Allow if the full expression is used in (c) if all the work was correct in (b) to find the constants.

(c)

B1ft: Sets their $1.2x(x-4) = 0.3x(x-4)^2$. It is for a "correct" ft equation on their constants, but the form (quadratic = cubic) must be correct.

M1: Valid attempt to solve their equations to find at least one non-zero solution as long as they have a cubic and quadratic equated. E.g. $1.2\cancel{x}(x-4) = 0.3\cancel{x}(x-4)^2 \Rightarrow x = 4 + \frac{1.2}{0.3}$.

Alternatively, they may expand, factor out the x and solve the quadratic (allowing for slips), or even expand to a cubic (allowing slips) and solve via calculator (may need to check, and you may ignore "incorrect factorisation" if the answers are correct for their equation).

A1: $x = 8$ identified as the x coordinate of P . A0 if they give 0,4 and 8 and do not select the correct answer.

A1: $(8, 38.4)$ or exact equivalent for 38.4

Question Number	Scheme	Marks
5 (a)	Attempts $r\theta = 5 \times 1.2$ Perimeter $= 5 + 5 + 6 = 16$ (km)	M1 A1 (2)
(b)	Attempts $\frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times 1.2$ Area $AOP = \frac{1}{4} \times \left(\frac{1}{2} \times 5^2 \times 1.2 \right) = 3.75 \text{ km}^2$ *	M1 M1, A1* (3)
(c)	Sets $\frac{1}{2} \times 5 \times OP \times \sin 1.2 = 3.75 \Rightarrow OP = \dots$ $OP = 1.6\dots$ $AP^2 = 5^2 + "1.6\dots" - 2 \times 5 \times "1.6\dots" \times \cos 1.2$ $AP = 4.7 \text{ km}$ or 4700m	M1 A1 M1 A1cso (4) (9 marks)

(a)

M1: Attempts $r\theta = 5 \times 1.2$

A1: Achieves 16 (km)

(b)

M1: Attempts $\frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times 1.2$ (or $\frac{1}{2}rl = \frac{1}{2} \times 5 \times 6$) May be implied by Area of sector = 15 seen.M1: Attempts $\frac{1}{4} \times \left(\frac{1}{2} \times 5^2 \times 1.2 \right)$ or $\frac{3}{4} \times \left(\frac{1}{2} \times 5^2 \times 1.2 \right)$ or other full method to find the area for one of the two regions.

A1*: Achieves given answer, including units, showing all steps with no errors and full accuracy kept throughout. Score A0 if rounded values were used during working.

Approaches by verification are acceptable. The first M1 will apply the same way, a clear attempt at the sector area formula will be needed to determine the area.

The second M may then be gained for attempting
Area Sector $AOB = \text{Area } R_1 + \text{Area } R_2 = 3 \times 3.75 + 3.75 = \dots$ Then for the A mark the calculations need to both be correct (giving 15), and a conclusion made: E.g. (so) Area $R_2 = 3.75 \text{ km}^2$.

(c)

M1: Sets $\frac{1}{2} \times 5 \times OP \times \sin 1.2 = 3.75 \Rightarrow OP = \dots$ There are some longer methods to find OP , so this may be scored for any complete method. E.g finds area of triangle APB from difference of area triangle $OAB \left(= \frac{1}{2} 5^2 \sin 1.2 \right)$ and 3.75, the uses ratio $OP:PB$ is same

as ratio of these areas to form and solve an equation for OP . Other method may be possible. Or $\frac{1}{2}OP \cdot h = 3.75$, $h = 5\sin 1.2 \Rightarrow OP = \dots$

A1: Achieves distance $OP = \text{awrt } 1.6$ (allow if called AP). May be implied by subsequent work.

M1: Attempts the cosine rule, or other full method, to find at least AP^2 using angle 1.2, $OA = 5$ and their OP . Condone if AP^2 is missing or labelled AP .

Again there are longer winded ways, e.g. using right angle triangle OAX where X is the base of perpendicular to OB through A , then $AX = 5\sin 1.2$, $PX = \frac{AX}{\tan 1.2} - OP$ and

$AP^2 = AX^2 + PX^2$ can be used to find AP .

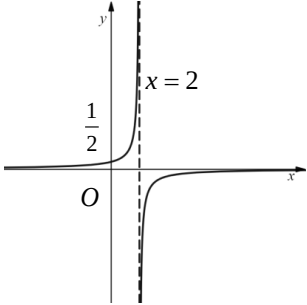
A1cso: $AP = 4.7\text{km}$ or 4700m (isw once seen if further rounding occurs). Must include the correct units and be given to nearest 100m.

Watch out for using of $\frac{1}{2} \times 5 \times OP \times 1.2 = 3.75 \Rightarrow OP = 1.25$ which also leads to 4.7km. This scores at most M0A0M1A0 despite giving the same answer.

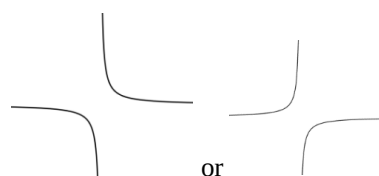
SC If R_1 and R_2 are mixed up in part (c) then score as a misread if the work is otherwise correct – so M1 for attempting $\frac{1}{2} \times 5 \times OP \times \sin 1.2 = 11.25 \Rightarrow OP = \dots$ and dM1 for a correct use of the cosine rule to find AP .

Some methods do not find OP , but a different intermediate length for a triangle containing AP . Such methods score the M1 for the attempt to find two other side lengths for a triangle with side AP and the A1 for a correct length, then second M for full method to find at least AP^2 . For example

Question Number	Scheme	Marks
5(c) Alt	Let X be base of perpendicular to OA through P , then $\frac{1}{2} \times 5 \times XP = 3.75 \Rightarrow XP = 1.5 \Rightarrow OX = \frac{1.5}{\tan 1.2}$ so	M1
	$AX = 5 - \frac{1.5}{\tan 1.2} = \dots$	A1
	$AX = 4.416\dots$	M1
	$AP^2 = 4.416\dots^2 + 1.5^2 = \dots$	A1
	$AP = 4.7\text{km}$ or 4700m	(4) (9 marks)

Question Number	Scheme	Marks
6.(a)	 1/x type shape Fully correct Correct equation of vertical asymptote and y intercept	M1 A1 B1
(b) (i)	Sets $kx - 4 = \frac{1}{2-x} \Rightarrow (kx - 4)(2 - x) = 1$ $kx^2 + (-4 - 2k)x + 9 = 0$ Attempts use $b^2 - 4ac \dots 0 \Rightarrow (-4 - 2k)^2 - 4k \times 9 \dots 0$ $\Rightarrow 4k^2 - 20k + 16 \dots 0 \Rightarrow k^2 - 5k + 4 \dots 0$ *	(3) M1 A1 dM1 A1*
(ii)	$(k - 1)(k - 4) \dots 0 \Rightarrow k = 1, k = 4$	M1, A1 (6) (9 marks)

(a)

M1: Correct shape for a $\frac{1}{x}$ type graph. Look for

or

Must have two branches, be generous with curves that bend back on themselves if the intent is clear.

A1: Fully correct shape and position. Do not accept curves that clearly bend back on themselves, but “pen flicks” at the end of a curve can be condoned.

B1: Correct equation (must be an equation) of asymptote and y intercept. The graph must be asymptotic here and in the correct places (e.g. the equation must match the placement), though it need not be fully correct – as long as these features are shown. Information stated on the graph takes precedence but if coordinates or equation are missing, allow if they are stated in the text. Accept $\frac{1}{2}$ labelled on the y-axis for the intercept. Condone $(\frac{1}{2}, 0)$ if it is in the correct position on the axis.

(b)(i)

M1: Sets $kx - 4 = \frac{1}{2-x} \Rightarrow (kx - 4)(2 - x) = 1$

Question Number	Scheme	Marks
6(a)	$f'(8) = \frac{32}{3 \times 8^2} + 3 - 2\sqrt[3]{8} \quad \left(= -\frac{5}{6} \right)$	M1
	$y - 2 = -\frac{5}{6}(x - 8)$	dM1
	$y = -\frac{5}{6}x + \frac{26}{3}$	A1
		(3)
(b)	$f'(x) = \frac{32}{3x^2} + 3 - 2\sqrt[3]{x} = \dots x^{-2} + 3 + \dots x^{\frac{1}{3}}$	
	$x^{-2} \rightarrow x^{-1}, \quad 3 \rightarrow 3x, \quad x^{\frac{1}{3}} \rightarrow x^{\frac{4}{3}}$	M1
	$f(x) = \int \frac{32}{3}x^{-2} + 3 - 2x^{\frac{1}{3}} dx = -\frac{32}{3}x^{-1} + 3x - \frac{3}{2}x^{\frac{4}{3}} + c$	A1A1
	$2 = -\frac{32}{3} \times 8^{-1} + 3 \times 8 - \frac{3}{2} \times 8^{\frac{4}{3}} + c \Rightarrow c = \dots$	dM1
	$(f(x) =) -\frac{32}{3}x^{-1} + 3x - \frac{3}{2}x^{\frac{4}{3}} + \frac{10}{3}$	A1
		(5)
		(8 marks)

Ignore labelling of parts (a) and (b)

(a)

M1 Substitutes $x = 8$ to find a value for $f'(8)$. Condone slips in their substitution and $-\frac{5}{6}$ seen will imply this mark.

dM1 It is for the method of finding a line passing through $(8, 2)$ using their value for $f'(8)$. Score for $(y - 2) = -\frac{5}{6}(x - 8)$ with both brackets correct. If they use $y = mx + c$ they must proceed as far as $c = \dots$ It is dependent on the previous method mark.

A1 $y = -\frac{5}{6}x + \frac{26}{3}$

(b)



M1 Integrates by raising the power on one of the terms (ie $x^{-2} \rightarrow x^{-1}$, $3 \rightarrow 3x$, $x^{\frac{1}{3}} \rightarrow x^{\frac{4}{3}}$)

A1 Two terms correct of $-\frac{32}{3}x^{-1}$, $+3x$ or $-\frac{3}{2}x^{\frac{4}{3}}$ seen (or unsimplified equivalents). The indices must be processed.

A1 $-\frac{32}{3}x^{-1} + 3x - \frac{3}{2}x^{\frac{4}{3}} (+c)$ seen or unsimplified equivalent. Condone the lack of $+c$ for this mark. $-10.7x^{-1}$ is not a correct term but allow $-10.\dot{6}x^{-1}$.

dM1 Substitutes $x=8$, $y=2$ into their $f(x)$ and proceeds to find c . It is dependent on the previous method mark and condone slips in their rearrangement to find c .

A1 $(f(x) =) -\frac{32}{3}x^{-1} + 3x - \frac{3}{2}x^{\frac{4}{3}} + \frac{10}{3}$ or simplified equivalent. isw after a correct answer

Eg $-\frac{32}{3x} + 3x - \frac{3}{2}x^{\frac{4}{3}} + \frac{10}{3}$ or $-\frac{10.\dot{6}}{x} + 3x - 1.5x^{\frac{4}{3}} + 3.\dot{3}$ but do not accept rounded decimals for the coefficients.

Question Number	Scheme	Marks
11(a)	States or implies that the gradient of the normal is $-\frac{1}{3}$	M1
	Correct equation of normal e.g. $y-16 = -\frac{1}{3}(x-4)$	A1
		(2)
(b)	$f''(x) = 4x + x^{-\frac{1}{2}} \Rightarrow f'(x) = 2x^2 + 2x^{\frac{1}{2}} + c$	M1
	$x = 4, f'(x) = 3 \Rightarrow 3 = 32 + 4 + c \Rightarrow c = \dots(-33)$	dM1
	$f'(x) = 2x^2 + 2x^{\frac{1}{2}} - 33$	A1
	$f'(x) = 2x^2 + 2x^{\frac{1}{2}} - 33 \Rightarrow f(x) = \frac{2}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}} - 33x + d$	dM1
	$x = 4, f(x) = 16 \Rightarrow (f(x) =) \frac{2}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}} - 33x + \frac{284}{3}$	ddM1A1
		(6)
		(8 marks)

(a)

M1: States or implies that the gradient of the normal is $-\frac{1}{3}$ **A1:** Finds the equation of the normal $y-16 = -\frac{1}{3}(x-4)$ o.e. e.g. $y = -\frac{1}{3}x + \frac{52}{3}$, $3y + x - 52 = 0$ But **not** $\frac{y-16}{x-4} = -\frac{1}{3}$.

Requires a full correct equation. Apply isw once a correct equation is seen.

(b) Award marks if work in (b) is seen in (a).

M1: Attempts to integrate $f''(x)$ once with one index correct. E.g. $4x \rightarrow \dots x^2$ or $\frac{1}{\sqrt{x}} \rightarrow \dots x^{\frac{1}{2}}$ **dM1:** Applies $f'(4) = 3$ and solves to find a constant of integration. **Depends on first M1.****A1:** $(f'(x) =) 2x^2 + 2x^{\frac{1}{2}} - 33$ or obtains $(f'(x) =) 2x^2 + 2x^{\frac{1}{2}} + c$ with c correctly calculated as -33 Ignore labelling of the function e.g. they may call it $f(x)$ **ddM1: Dependent upon first M1 only.** For an attempt to integrate $f''(x)$ twice and achieve a form $(f(x) =) ax^3 + bx^{\frac{3}{2}} + \dots$ where $\dots$ could be 0. Ignore labelling of the function e.g. they may call it $f(x)$ **dddM1: Dependent upon all previous M's.** It is for using $f(4) = 16$ (may be implied) to find the constant of integration.**A1:** $(f(x) =) \frac{2}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}} - 33x + \frac{284}{3}$ or exact equivalent. Apply isw once a correct **expression** is seen.

The “ $f(x)=$ ” is not required and ignore any label they may have given it.

Also allow $(f(x)=)\frac{2}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}} - 33x + c$ with c correctly calculated as $\frac{284}{3}$.



(b)

B1ft: Correct follow through normal equation for their $y = -\frac{1}{4}x + \frac{17}{2}$. Implied by $y = 4x + 0$ following correct (a). So $y = -\frac{1}{m}x$ for their $y = mx + c$. It may be implied by a solution of $-\frac{1}{m}x = mx + c$

M1: Attempts to solve their $y = 4x$ and their $y = -\frac{1}{4}x + \frac{17}{2}$ simultaneously via a non-calculator route.

Look for a value for x or a value for y following an attempt at solving $\pm \frac{1}{m}x = mx + c$

There should be some working, allow as a minimum " $4x = -0.25x + 8.5 \Rightarrow \dots x = \dots \Rightarrow x = \dots$ "

Don't be too concerned with accuracy here.....it is a method mark

A1: One correct coordinate, usually $x = 2$ but could be for $y = 8$ if solved differently

A1: $R = (2, 8)$. May be written separately

Solutions with no working, insufficient working and/ or via use of a calculator

(b)

For the M mark you should expect to see some working. As a minimum look for the two highlighted equations (or their equivalent) where the x terms have been collected

$$y = 4x \text{ and } y = -\frac{1}{4}x + \frac{17}{2} \Rightarrow 4x = -\frac{1}{4}x + \frac{17}{2} \Rightarrow 4.25x = 8.5 \text{ followed by } (2, 8)$$

If $(2, 8)$ follows the two correct equations without the highlighted working (o.e.) score SC 1011

(c)

M1: States or attempts $\sqrt{2^2 + 8^2}$ using their coordinates for R or $\sqrt{3^2 + 12^2}$ using the given P and Q . It can be implied by an exact answer or an answer to 3sf for their coordinates
If two right angled triangles are used to find the area, it would be for attempts at OR , or PR and RQ .

dM1: Full attempt at area of triangle OPQ using $\frac{1}{2} \times \text{their } OR \times \text{their } PQ$ o.e.

Both PQ and OR must be attempted via a correct method

This can be awarded for decimal work. It is dependent on the previous M

A1: Correct answer via a correct method with sufficient working to suggest a non-calculator method

Look for $\frac{1}{2} \times 2\sqrt{17} \times 3\sqrt{17}$ (or other correct and relevant work) before you see 51.

Don't allow this to be scored via decimals or from just $\frac{1}{2} \times \sqrt{153} \times \sqrt{68} = 51$ unless extra relevant lines are seen as above.

Alternatives to part (c) exist via the cosine rule, the shoelace method

ALT 1: the cosine rule.

One example shown using angle POQ

$$\cos POQ = \frac{136 + 85 - 153}{2\sqrt{85}\sqrt{136}} = \frac{1}{\sqrt{10}} \quad \text{Area triangle } POQ = \frac{1}{2} \sqrt{85}\sqrt{136} \sin POQ = \frac{1}{2} \sqrt{85}\sqrt{136} \times \frac{3}{\sqrt{10}} = 51$$

M1: As main scheme for length PQ

dM1: Full method. Likely to involve decimals here and the angle 71.6° . If decimals are used it will be followed by A0



ALT 2: the shoelace method.

Note that the example below starts at $(0, 0)$. It could start at any of the coordinates

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 0 & -2 & 10 & 0 \\ 0 & 9 & 6 & 0 \end{vmatrix} = \frac{1}{2} |(0 \times 9) + (-2 \times 6) + (10 \times 0) - (-2 \times 0) - (10 \times 9) - (0 \times 6)| = \frac{1}{2} \times 102 = 51$$

M2: For a full attempt at the area. Score for $\frac{1}{2} |(0 \times 9) + (-2 \times 6) + (10 \times 0) - (-2 \times 0) - (10 \times 9) - (0 \times 6)|$

A1: 51

ALT 3: Trapezium minus two right-angled triangles

$$\frac{12}{2}(9+6) - \frac{1}{2} \times 10 \times 6 - \frac{1}{2} \times 2 \times 9 = 90 - 30 - 9 = 51$$

M2: For a full attempt at area $\frac{12}{2}(9+6) - \frac{1}{2} \times 10 \times 6 - \frac{1}{2} \times 2 \times 9$

A1: 51

ALT 4: Two scalene triangles, OPC & OCQ (where C is where intercept of l_1 with the y -axis)

$$\text{When } x=0, y=\frac{17}{2} \quad \text{Area} = \frac{1}{2} \times \frac{17}{2} \times 10 + \frac{1}{2} \times \frac{17}{2} \times 2 = \frac{85}{2} + \frac{17}{2} = 51$$

M1: Attempts the y intercept of l_1

dM1: For a full attempt at area

A1: 51

Question Number	Scheme	Marks
6. (a)	$(540^\circ, -5)$	B1, B1 (2)
(b) (i)	$(360^\circ, 3)$	B1, B1 (2)
(ii)	$(180^\circ, 5)$	B1, B1 (2)
		(6 marks)

Notes applying to all parts:

Note 1

Score B1 for one correct coordinate, so B0 B1 is NOT possible

So for example (a) $(720^\circ, -5)$ scores B1 B0

Note 2

Coordinates can be given separately

E.g. (a) $x = 540^\circ, y = -5$

Note 3

Coordinates may appear within the question. Solutions to (b)(i) for example may well contain two sets of coordinates. E.g. $(360^\circ, 5)$ followed by $(360^\circ, 3)$. This will most likely be their working so in almost all cases you will mark their final set of coordinates

Note 4

Score B1 B0 for a **correct** solution but with the coordinates "flipped".

E.g. (b) (i) $(3, 360^\circ)$ instead of $(360^\circ, 3)$

Note 5

Condone omission of degrees

Note 6

If solutions are given in radians then withhold a mark the first time it happens

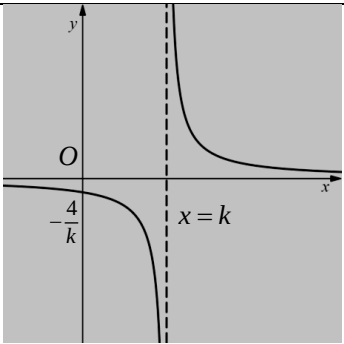
E.g. (a) $(3\pi, -5)$ (b)(i) $(2\pi, 3)$ (b)(ii) $(\pi, 5)$ scores (a) B1 B0 (b)(i) B1 B1 (b)(ii) B1 B1

Note 7

If solutions are not in brackets (and not stated as $x = y =$) then withhold a mark the first time it happens

E.g. (a) $540^\circ, -5$ (b)(i) $360^\circ, 3$ (b)(ii) $180^\circ, 5$ scores (a) B1 B0 (b)(i) B1 B1 (b)(ii) B1 B1



Question Number	Scheme	Marks
7. (a)	 Shape in quadrant One Fully correct shape and position C cuts the y-axis at $-\frac{4}{k}$ C has a vertical asymptote at $x = k$	M1 A1 B1 B1 (4)
(b)	$\frac{4}{x-k} = 9-x \Rightarrow x^2 - (9+k)x + 9k + 4 = 0$ Uses $b^2 - 4ac < 0 \Rightarrow (9+k)^2 - 4 \times 1 \times (9k+4) < 0$ $k^2 - 18k + 65 < 0 \Rightarrow (k-13)(k-5) < 0 \Rightarrow 5 < k < 13$	M1, A1 dM1 ddM1, A1 (5) (9 marks)

(a) Marks for intercept and asymptote must be shown on the graph and hence cannot be scored without a sketch

M1: For a monotonically decreasing function in quadrant one. It must not cross either axis but be tolerant of functions that (a) don't go down as far as the x-axis and (b) extend as far left as the y-axis

A1: Correct shape and position (for whole curve). Be tolerant of slips of the pen but the intension should be clear. Look for the curve in quadrants 1, 3 and 4 with a vertical asymptote in quadrant 1 and 4. The left-hand branch should not intentionally dip below the intercept away from the x-axis

B1: C cuts the y-axis once at $-\frac{4}{k}$ which must be on the negative y-axis as $-\frac{4}{k}, \left(0, -\frac{4}{k}\right)$ but not $\left(-\frac{4}{k}, 0\right)$

B1: C must have one vertical asymptote to the right of the y-axis marked $x = k$

(b)

M1: Equates curve with given line and attempts to form a quadratic. The terms do not need to be collected

Look for $\frac{4}{x-k} = 9-x$ (condoning slips), cross multiplies and proceeds to a quadratic expression

A1: Achieves a simplified quadratic with collected terms. Look for $x^2 - (9+k)x + 9k + 4 = 0$ o.e. or else gives the correct values of a , b and c which may be implied by values embedded within $b^2 - 4ac$.

dM1: Attempts to use the discriminant $b^2 - 4ac \dots 0$ for their $ax^2 + bx + c = 0$ with both b and c expressions in k

ddM1: Solves their quadratic in k resulting from $b^2 - 4ac = 0$ and chooses the inside region for their critical values. Allow calculator solutions here so you may need to check if the method is not apparent. Condone the boundaries being included in the inequality for this mark.

It is dependent upon both previous M's

A1: CSO $5 < k < 13$ o.e. The variable must be k .

Accept alternatives such as $k > 5$ and $k < 13$ $k > 5, k < 13$ (5,13) or $k \in (5,13)$

BUT NOT $k > 5$ or $k < 13$



Question Number	Scheme	Marks
8 (a)	Attempts to use $S = r\theta \Rightarrow 9 = OD \times 0.8 \Rightarrow OD = 11.25$ or $\frac{45}{4}$	M1, A1
	$AO = \frac{5}{8} \times 11.25 = 7.03 \text{ m}$ *	A1*
		(3)
(b)	Attempts $A = 7.03 \times (2\pi - 0.8) = (38.55)$	M1
	Attempts $9 + 2 \times (11.25 - 7.03) + 7.03 \times \theta$	M1
	Perimeter = awrt 56.0 m	A1
(c)		(3)
	Attempts $\frac{1}{2} \times 11.25^2 \times 0.8 = (50.625)$ OR $\frac{1}{2} \times 7.03^2 \times (2\pi - 0.8) = (135.4)$	M1
	Full method for area of platform = $\frac{1}{2} \times 11.25^2 \times 0.8 + \frac{1}{2} \times 7.03^2 \times (2\pi - 0.8)$	M1
	=awrt 186 m ²	A1
		(3)
		(9 marks)

(a) This is a show that question and it is important that the method is shown

M1: Attempts to use $S = r\theta \Rightarrow 9 = r \times 0.8 \Rightarrow r = \dots$ Implied by $(r) = \frac{9}{0.8}$

A1: Achieves $OD = 11.25$ or $OC = 11.25$ but condone " r " = 11.25. May be implied by $AO = \frac{5}{8} \times \left(\frac{9}{0.8}\right)$ o.e

A1*: Shows that AO is $7.03(m)$ via $\frac{5}{8} \times 11.25$, $11.25 - \frac{3}{8} \times 11.25$ o.e. Units are not required.

You do not need to see AO specifically mentioned as long as there was an OD , OC or r stated earlier.

You do not need to see 7.03125 before it being rounded to 7.03.

You can also condone leaving as a more accurate value, e.g. $AO = 7.03125$

Allow a more accurate value of 7.03 to be used in part (b) and (c). E.g. 7.03125

(b)

M1: Attempts arc length AEB via $7.03 \times (2\pi - 0.8)$ o.e such as $2\pi(7.03) - 7.03 \times (0.8)$

Allow the angle calculation to be implied by sight of awrt 5.48 so score for 7.03×5.48

If the calculation $(2\pi - 0.8)$ has been seen, allow 5.5 to be used for the angle

M1: Attempts to add the correct parts together E.g. $9 + 2 \times (11.25 - 7.03) + 7.03 \times \theta$

The major arc must have been attempted by a correct formula but allow for this mark an incorrect angle to be used for candidates who don't equate 2π to 360° . So allow for this mark calculations with e.g.

$7.03(\pi - 0.8)$

You may see $9 + 2 \times \left(\frac{3}{8} \times 11.25\right) + 7.03 \times \theta$. Allow accuracy to 1 dp so $9 + 2 \times 4.2 + 38.6$

A1: Perimeter = awrt 56.0m. ISW after a correct answer. Condone 56m following a correct calculation. The units are not necessary, they can be ignored



(c)

 M1: Attempts to find the value of $\frac{1}{2}r^2\theta$ with their $r = 11.25$ and $\theta = 0.8$

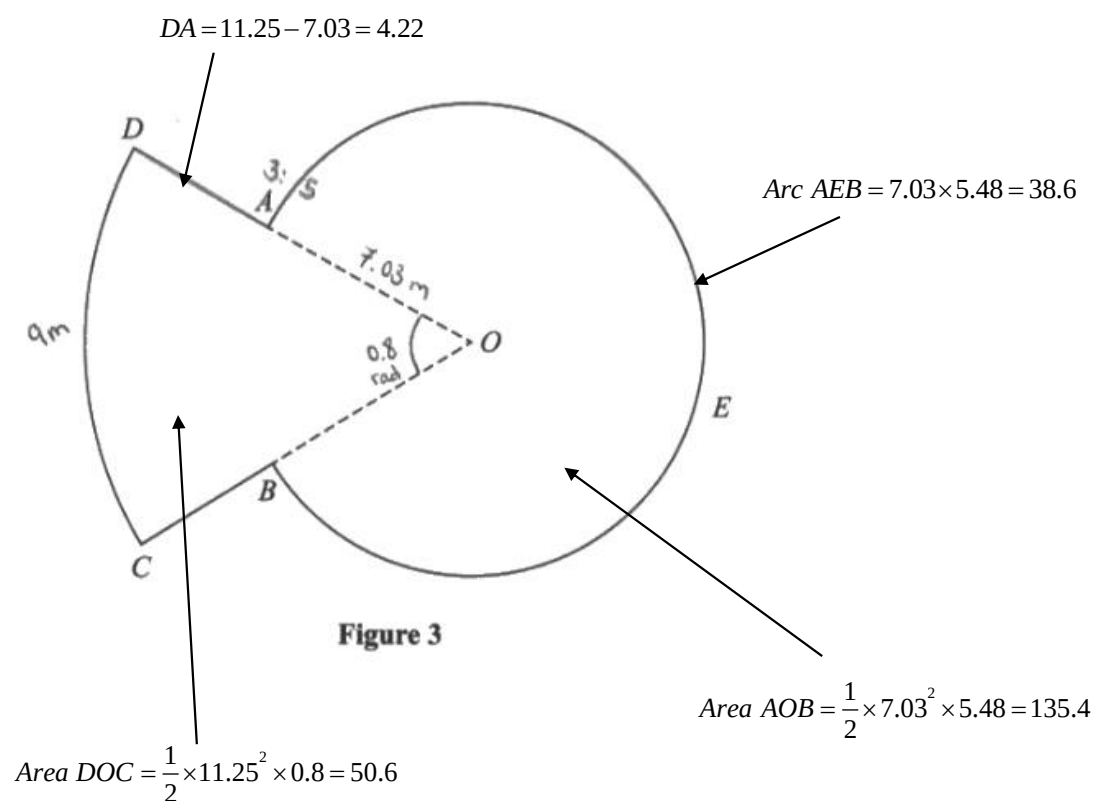
 or the value of $\frac{1}{2}r^2\theta$ with $r = 7.03$ and $\theta = 2\pi - 0.8$ or awrt 5.48 or 5.5 following sight of $2\pi - 0.8$

 An alt for the second sector is $\pi \times 7.03^2 - \frac{1}{2} \times 7.03^2 \times 0.8$

dM1: Adds two sectors. See above for how to apply the method for each sector

A1: Awrt 186. The units are not necessary, they can be ignored

Useful diagram

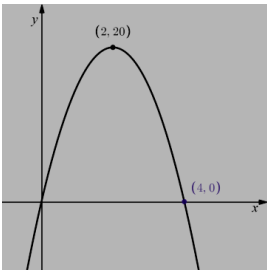

 Note: It is possible to do this question in degrees and use the formulae $\frac{\theta}{360} \times 2\pi r$ in (b) and $\frac{\theta}{360} \times \pi r^2$ in (c)

Only allow such a method if the conversion has been attempted by a correct method. Allow for M's awrt 46

 (a) $0.8 \text{ rad} = 0.8 \times \frac{180}{\pi} = 45.8^\circ$ so solve $9 = \frac{45.8}{360} \times 2\pi r$ for the first M1

 (b) Arc AEB = $\frac{360 - 45.8}{360} \times 2\pi \times 7.03$

 (c) Sector DOC = $\frac{45.8}{360} \times \pi \times 11.25^2$ and Sector AOB = $\frac{360 - 45.8}{360} \times \pi \times 7.03^2$

Question Number	Scheme	Marks
9 (a)		B1
		Intersection at (4, 0) B1
		(2)
	(b) Attempts form of equation. E.g. $y = Ax(x-4)$ or $y = 20 \pm C(x-2)^2$ Full attempt to find equation. E.g. $20 = A \times 2(2-4) \Rightarrow A = \dots$ Or $0 = 20 + C(4-2)^2 \Rightarrow C = \dots$ $y = -5x(x-4)$, $y = 20 - 5(x-2)^2$ o.e.	M1 dM1 A1
	(c) Sets $x(x^2 - 4) = -5x(x-4)$ $x^3 + 5x^2 - 24x = 0 \Rightarrow x(x^2 + 5x - 24) = 0$ $(x+8)(x-3) = 0 \Rightarrow x$ coordinate of P is -8 $P = (-8, -480)$	M1 dM1 ddM1, A1 A1
		(5) (10 marks)

(a)

B1: $\cap$ shaped quadratic passing through the origin with maximum on the rhs of the y-axis and an intercept on the +ve x-axis. Be tolerant of slips of the pen. It must appear in quadrants 3, 1 and 4. Ignore extra graphs (e.g. cubics) if they are superimposed on top of this one.

Score if the intention seems correct but do penalise $\wedge$ shaped "curves"

B1: Intersection at (4, 0). Allow 4 marked on the positive x-axis but (0, 4) is B0. Condone graphs that just meet the x-axis at 4 for ones that seem to "sit" on the x-axis and only appear in quadrant 1.

(b)

M1: Attempts form of equation. E.g. $y = \pm Ax(x-4)$ or $y = 20 \pm C(x-2)^2$. Condone with $A, C = 1$

It is possible they could try $y = ax^2 + bx + c$ with an attempt to use all three coordinates.

Look for setting up 3 different equations using the given information

- Uses (0, 0) in $y = ax^2 + bx + c \Rightarrow c = 0$
- Uses (2, 20) in $y = ax^2 + bx + c \Rightarrow 20 = 4a + 2b$
- Uses (4, 0) $y = ax^2 + bx + c \Rightarrow 0 = 16a + 4b$ OR $\left. \frac{dy}{dx} \right|_{x=2} = 2ax + b = 0 \Rightarrow 4a + b = 0$

It is acceptable just to state the rhs of the equation. E.g. $Ax(x-4)$

dM1: Full attempt at equation for $f(x)$ with an attempt at finding values for A or C or a, b, c

A1: $y = -5x(x-4)$, $y = 20 - 5(x-2)^2$ o.e. such as $y = -5x^2 + 20x$ Allow $f(x) \leftrightarrow y$

The question asks for an expression for $f(x)$ so allow just $5x(4-x)$ o.e.

Allow this to be written down for all 3 marks. ISW after a correct answer.

Allow this to be scored in part (c) if you are certain that this is meant for (b). Use review if unsure.



(c) **All stages of working should be shown in this part so any omissions will be penalised**

M1: Sets $x(x^2 - 4) = -5x(x - 4)$ condoning slips

The form of the quadratic must be correct. That is it must pass through (0, 0)

dM1: Multiplies out to reach an equation of the form $px^3 + qx^2 + rx = 0$ from which they factorise (or cancel) out a factor of x . This may be implied as the x may be cancelled out previously, say on their first line

ddM1: Solves the resulting quadratic via an appropriate method.

Condone the use of a calculator for this mark. The solutions must be correct for their quadratic to score this mark via the calculator.

A1: **Chooses** -8 for x coordinate of P . If $x = 0, 3$ are seen they must be rejected or -8 chosen

Can be awarded **choosing** $x = -8$ following sight of a correct quadratic equation $x^2 + 5x - 24 = 0$
or for **choosing** $x = -8$ following sight of a correct cubic equation $x^3 + 5x^2 - 24x = 0$

A1: $P = (-8, -480)$ which may be given separately $x = \dots, y = \dots$

Can be awarded for $(-8, -480)$ following sight of a correct quadratic equation $x^2 + 5x - 24 = 0$

or for $(-8, -480)$ following sight of a correct cubic equation $x^3 + 5x^2 - 24x = 0$

Examples of applying the scheme:

Example 1: M1, M0, M0, A0, A0 for 1 out of 5 marks

$$x(x^2 - 4) = -5x(x - 4) \Rightarrow x = -8 \Rightarrow P = (-8, -480)$$

Example 2: M1, M0, M0, A1, A1 for 3 out of 5 marks

$$x(x^2 - 4) = -5x(x - 4) \Rightarrow x^3 + 5x^2 - 24x = 0 \Rightarrow x = (0, 3) - 8 \Rightarrow P = (-8, -480)$$

Example 3: M1, M1, M1, A1, A1 for 5 out of 5 marks. The solution of the quadratic is implied (calculator)

$$x(x^2 - 4) = -5x(x - 4) \Rightarrow x^2 + 5x - 24 = 0 \Rightarrow x = (3) - 8 \Rightarrow P = (-8, -480)$$

Example 4: M1, M1, M1, A1, A1 for 5 out of 5 marks

$$x(x^2 - 4) = -5x(x - 4) \Rightarrow x^3 + 5x^2 - 24x = 0 \Rightarrow x(x^2 + 5x - 24) = 0 \Rightarrow x = (0, 3) - 8 \Rightarrow P = (-8, -480)$$

Question Number	Scheme	Marks
10 (a)	$f'(x) = 4\sqrt{x^3} + \frac{k}{x^2} = 4x^{\frac{3}{2}} + kx^{-2}$ $f''(x) = 6x^{\frac{1}{2}} - 2kx^{-3}$ $f''(2) = 6\sqrt{2} - 2k \times \frac{1}{8} = 0 \Rightarrow k = 24\sqrt{2}$	M1, A1 dM1, A1 (4)
(b)	$f'(x) = 4x^{\frac{3}{2}} + kx^{-2} \Rightarrow f(x) = 4 \times \frac{2}{5} x^{\frac{5}{2}} - kx^{-1} (+c)$ Uses $P(2, 8\sqrt{2}) \Rightarrow 8\sqrt{2} = 4 \times \frac{2}{5} \times 2^{\frac{5}{2}} - \frac{k}{2} + c \Rightarrow c = p\sqrt{2}$ $f(x) = \frac{8}{5} x^{\frac{5}{2}} - \frac{24\sqrt{2}}{x} + \frac{68}{5} \sqrt{2}$	M1, A1 ft dM1 A1 (4) (8 marks)

Relevant work must be done in the correct part of the question.

If they are clearly labelled and they just integrate in (a) and they just differentiate in (b) award no marks but if they use their integration that was in part (a) in part (b) for example they can pick up marks in part (b).

(a)

M1: Differentiates $4\sqrt{x^3} + \frac{k}{x^2}$ and achieves at least one index correct. Look for $px^{\frac{1}{2}} - qx^{\dots}$ or $px^{\dots} - qx^{-3}$

A1: Correct differentiation, which may be left unsimplified. E.g. $4 \times \frac{3}{2} \times x^{\frac{1}{2}} - 2k \times x^{-3}$ or $6x^{\frac{1}{2}} - 2kx^{-3}$

dM1: Sets $f''(2) = 0$ and proceeds to a value for k . It is dependent upon the previous M mark. Allow even if called something else

A1: $k = 24\sqrt{2}$ or exact equivalent. ISW after a correct exact value

(b)

M1: Integrates $4\sqrt{x^3} + \frac{k}{x^2}$ and achieves at least one index correct. Look for $px^{\frac{5}{2}} - qx^{\dots}$ or $px^{\dots} - qx^{-1}$

A1ft: Correct integration to $4 \times \frac{2}{5} x^{\frac{5}{2}} - kx^{-1}$. There is no requirement to simplify or have $+c$

Follow through on their value of k but also allow with " k " as seen above

dM1: Uses $P(2, 8\sqrt{2})$ and their $k = \dots\sqrt{2}$ to find c as a multiple of $\sqrt{2}$.

It is dependent upon the previous M mark. Both indices must now be correct

The $\dots x^{\frac{5}{2}}$ term must produce a term in $\sqrt{2}$

A1: Achieves $\frac{8}{5} x^{\frac{5}{2}} - \frac{24\sqrt{2}}{x} + \frac{68}{5} \sqrt{2}$ for $f(x)$ Accept any other simplified equivalent.

There is no requirement for $f(x) =$

Question Number	Scheme	Marks
12.(a)	Substitutes $x = 4$ in $\frac{dy}{dx} = 3x\sqrt{x} - 10x^{-\frac{1}{2}} = 3 \times 4 \times 2 - \frac{10}{2} = 19$ Attempts $(y - (-2)) = "19" \times (x - 4) \Rightarrow y = 19x - 78$	M1A1 M1A1 cao (4)
(b)	$f'(x) = 3x^{\frac{3}{2}} - 10x^{-\frac{1}{2}} \Rightarrow f(x) = \frac{6}{5}x^{\frac{5}{2}} - 20x^{\frac{1}{2}} + c$ $x = 4, f(x) = -2 \Rightarrow$ $-2 = 38.4 - 40 + c \Rightarrow c = \dots(-0.4)$ $[f(x) =] \frac{6}{5}x^{\frac{5}{2}} - 20x^{\frac{1}{2}} - 0.4$	M1 A1 A1 M1 A1 cso (5) (9 marks)

(a)

M1 Substitutes $x = 4$ into $\frac{dy}{dx} = 3x\sqrt{x} - 10x^{-\frac{1}{2}}$. Do not award this mark if they attempt to differentiate the expression first (look at the $-10x^{-\frac{1}{2}}$ for evidence of the power decreasing) but do condone an error made on the power of the first x term if they try to write it as a single power of x .

A1 Gradient = 19

M1 Attempts an equation of a tangent using their $f'(4)$ and $(4, -2)$. If they attempt $(y + 2) = "19"(x - 4)$ at least one of the brackets must be correct. If the form $y = mx + c$ is used they must proceed as far as finding c . They must either have shown their gradient and $(4, -2)$ substituted into $y = mx + c$ and rearrange (maybe with errors) to find c or if they show no working then their c must be correct.

A1 $y = 19x - 78$ cao

(b)

M1 Raises the power of any term by one $x^{-\frac{1}{2}} \rightarrow x^{\frac{1}{2}}, x^{\frac{3}{2}} \rightarrow x^{\frac{5}{2}}$ Accept eg $x^{\frac{3}{2}} \rightarrow x^{\frac{3}{2}+1}$

A1 Any term correct (may be un-simplified) with or without $+c$

A1 Both terms correct (may be un-simplified) with or without $+c$

M1 Substitutes $x = 4, y = -2$ into their $f(x)$ containing $+c$ to obtain c . Condone errors in evaluating and rearranging

A1 $[f(x) =] \frac{6}{5}x^{\frac{5}{2}} - 20x^{\frac{1}{2}} - 0.4$ or equivalent including $(y =) \dots$ cso



Question Number	Scheme	Marks
8.(a)	Substitutes $x=4$ in $f'(4) = 4 \times 2 - 2 - \frac{8}{3 \times 4^2} = \left(\frac{35}{6}\right)$ Attempts to find the gradient of the perpendicular $= -\frac{6}{35}$ Attempts the normal $y-1 = -\frac{6}{35} \times (x-4) \Rightarrow 6x + 35y - 59 = 0$	M1 dM1 M1A1 (4)
(b)	$f'(x) = 4x^{\frac{1}{2}} - 2 - \frac{8}{3x^2} \Rightarrow f(x) = \frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3x} (+c)$ $x=4, f(x)=1 \Rightarrow 1 = \frac{8}{3} \times 8 - 8 + \frac{2}{3} + c \Rightarrow c = \dots (-13)$ $f(x) = \frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3x} - 13$	M1 A1 A1 dM1 A1 (5) (9 marks)

Notes

Points for marking:

- If all a candidate does is integrate, then mark as though they are attempting (b) even though their solution may have part (a) marked in the margin
- If a candidate differentiates $f'(x)$ in (a) and integrates $f''(x)$ in (b) getting back to the same or similar expression it is 0 marks in (b)
- See Practice items. Read annotations carefully to understand how to apply these principles.
- If in any doubt ask your TL for advice

(a)

M1 Attempts $f'(4) = 4 \times 2 - 2 - \frac{8}{3 \times 4^2}$.

If you do not see embedded values

1) award if the candidate has two of the three terms correct. Eg two of $8-2-$ "0.166"

2) award for sight of $\frac{35}{6}$ or awrt 5.83 if the candidate just offers a value

dM1 Attempts perpendicular gradient using the correct rule. The M1 must have been awarded

M1 Attempts to find the equation of the normal using (4,1) and a **changed** gradient.

If the candidate differentiates $f'(x)$ they can be awarded this mark

Condone one sign slip when using the (4,1)

A1 $6x + 35y - 59 = 0$

Accept any $\pm k(6x + 35y - 59) = 0$ where $k \in \mathbb{N}$



(b)

M1 Raises any correct index by one $x^{\frac{1}{2}} \rightarrow x^{\frac{3}{2}}$, $2 \rightarrow 2x$, $\frac{1}{x^2} \rightarrow \frac{1}{x}$ The indices must be processed

A1 Any two terms correct (may be un-simplified) with or without $+c$

Do not allow the indices or the coefficients to be left un processed Eg $4x^{\frac{1}{2}} \rightarrow \frac{4}{1+\frac{1}{2}} x^{\frac{1}{2}+1}$

A1 All three terms correct and simplified with or without $+c$. Condone spurious notation.

Look for $\frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3x}$ or exact simplified equivalent such as $\frac{8x^{\frac{3}{2}}}{3} - 2x + \frac{8}{3}x^{-1}$

This may be implied by a final line where c has been found.

dM1 Substitute $x = 4$, $y = 1$ into their $f(x)$ containing $+c$ to obtain c

A1 $f(x) = \frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3x} - 13$ or exact simplified equivalent such as $f(x) = \frac{8}{3}x\sqrt{x} - 2x + \frac{8}{3}x^{-1} - 13$

Condone $f(x) = y$ or allow if there is no left- hand side at all.

Isw after a correct answer.

Do NOT allow solutions that rely on huge jumps. The M marks must be awarded as above.

For example

$$\frac{\sqrt{180} - \sqrt{80}}{\sqrt{5}} = \frac{(\sqrt{180} - \sqrt{80}) \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{30 - 20}{5} = 2 \text{ scores M0 A0}$$

without sight of $\sqrt{180} \times \sqrt{5}$ as $\sqrt{900}$ or $\sqrt{80} \times \sqrt{5}$ as $\sqrt{400}$ o.e.

Or

$$\frac{\sqrt{180} - \sqrt{80}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}} = 2 \text{ scores M0 A0 without sight of } \sqrt{180} \text{ as } 6\sqrt{5} \text{ or } \sqrt{80} \text{ as } 4\sqrt{5}$$

(ii)

Main method

M1: Correct attempt to rationalise. Look for attempt at multiplying numerator and denominator by $7 + 3\sqrt{5}$
The multiplications need not be carried out for this mark.

dM1: It is dependent upon the previous mark. Scored for

- Correct expression for denominator (without surds) Accept as shown, or as $7^2 - 3^2 \times 5$ or just 4
- and attempts to multiply out the numerator with the sight of 4 terms . Condone slips here. See ***

A1: Correct answer any way around. Allow $6.25 + 3.25\sqrt{5}$

Do NOT allow in the form $\frac{25 + 13\sqrt{5}}{4}$ but accept if followed by "hence $a = \frac{25}{4}, b = \frac{13}{4}$ "

(ii) Alt method

M1: Sets $\frac{4\sqrt{5} - 5}{7 - 3\sqrt{5}} = a + b\sqrt{5} \Rightarrow 4\sqrt{5} - 5 = (a + b\sqrt{5})(7 - 3\sqrt{5})$ and attempts to set up simultaneous

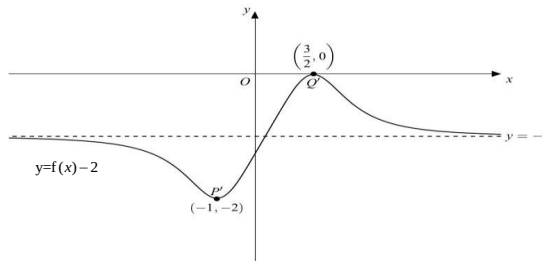
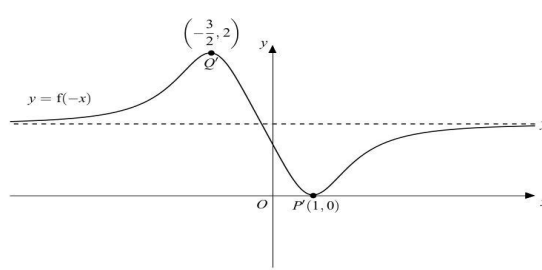
equations in a and b . FYI the correct equations are $7a - 15b = -5, 7b - 3a = 4$

dM1: Attempts to solve their equations. They must have at least one correct equation

A1: Correct answer. Must be $\frac{25}{4} + \frac{13}{4}\sqrt{5}$ and not **just** values of a and b

The following solution is a minimally acceptable solution for 3 marks, the first M1 being implied.

$$\frac{4\sqrt{5} - 5}{7 - 3\sqrt{5}} = \frac{(4\sqrt{5} - 5) \times (7 + 3\sqrt{5})}{4} = \frac{28\sqrt{5} - 35 + 60 - 15\sqrt{5}}{4} = \frac{13}{4}\sqrt{5} + \frac{25}{4} \quad ***$$

Question	Scheme	Marks	
4(i)		Correct shape, translated down.	B1
		Correct horizontal asymptote labelled	B1
		Correct maximum and minimum points labelled	B1
		(3)	
(ii)		Correct shape, reflected in y axis	B1
		Correct horizontal asymptote labelled	B1
		Correct maximum and minimum points labelled.	B1
		(3)	
(6 marks)			
Notes:			

In both parts be tolerant with “pen slips” for the asymptotes. Judge the intent of the shape.

(i) There is no MR for sketching $y = f(x - 2)$. These marks are independent of each other

B1: Same shape as the original graph but translated vertically downwards.

Do not consider any coordinates here.

Look for a minimum to the left of the y axis and a maximum to the right

Be tolerant of the asymptote not being **exactly** at the same level at either end of the curve but withhold the mark if it is intentionally different. Condone slight upturns at either end

B1: For $y = -1$ labelled as the horizontal asymptote. It must be below the x axis and the intention must be for ends to be at the same height/level. The intention must be for the curve to be asymptotic here (at both ends)

B1: For the coordinates of the maximum and minimum points in the correct positions. E.g. $\left(\frac{3}{2}, 0\right)$ must be

on the positive x-axis and $(-1, -2)$ is in quadrant 3. Note that P and Q don't need to be seen, just their coordinates. Coordinates may be given in the body of the script but must match with maximum and minimum points marked in some way on the graph

(ii) There is no MR for sketching $y = -f(x)$ These marks are independent of each other

B1: For the correct shape, a reflection in the y-axis. The intention should be for the asymptotes to be at the same level but be tolerant of slips. Look for the image Q' a maximum being to the left of y-axis in quadrant two and P' a minimum on the positive x- axis.

B1: For $y = 1$ labelled as the horizontal asymptote. It must be above the x axis and the intention must be for ends to be at the same height/level. The curve must be asymptotic here (at both ends)



B1: For the coordinates of the maximum and minimum points in correct positions. E.g. P' must be on the x -axis although P and Q don't need to be seen, just their coordinates. Coordinates may be given in the body of the script but must match with maximum and minimum points marked in some way on the graph.

Question	Scheme	Marks
5(a)	E.g. $12 - a(x+2)^2$ or $a(x-1)(x+5)$ or $y = ax^2 + bx + c \Rightarrow 4a - 2b + c = 12$ and $25a - 5b + c = 0$	M1A1
	E.g. $0 = 12 - a(-5+2)^2 \Rightarrow a = \dots$ or $12 = a(-2-1)(-2+5) \Rightarrow a = \dots$ or $2a(-2) + b = 0, 25a - 5b + c = 0, 4a - 2b + c = 12 \Rightarrow a = \dots, b = \dots, c = \dots$	dM1
	$12 - \frac{4}{3}(x+2)^2$ or $-\frac{4}{3}(x-1)(x+5)$ or $-\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$ oe	A1
		(4)
(b)	Gradient of l_2 is $-\frac{5}{4}$ o.e.	M1
	Equation of l_2 is $y = -\frac{5}{4}(x+5)$	M1
	$y = -\frac{5}{4}x - \frac{25}{4}$	A1
		(3)
(c)	For two of $y \geq -\frac{5}{4}x - \frac{25}{4}$; $y \geq \frac{4}{5}x$; $y \leq -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$ Or with strict inequalities	M1
	For all three of $y \geq -\frac{5}{4}x - \frac{25}{4}$; $y \geq \frac{4}{5}x$; $y \leq -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$ Or with strict inequalities	A1ft
		(2)
(9 marks)		
Notes:		

(a) Just the expression for $f(x)$ is required but you may see $y = \dots$ or $f(x) = \dots$ which is fine

M1: For knowing a quadratic form and using two correct pieces of information. This can be scored by

- using the maximum value occurs at $(-2, 12)$ to state the form $12 \pm a(x \pm 2)^2$
- use symmetry, usually with roots of -5 and 1 to state the form $a(x \pm 1)(x \pm 5)$
- using the general form $y = ax^2 + bx + c$ and substituting in two points, usually $(-2, 12)$ and $(-5, 0)$. An alternative is using just one point and setting $\frac{dy}{dx} = 0$ at $x = -2 \Rightarrow -4a + b = 0$

Condone for each of these: (1) slips (e.g incorrect squaring) or (2) the use of $a = \pm 1$

A1: For a **correct equation/expression set up** with the correct unknown constants included. It is for getting in a position where only one more equation or constant, usually the value of a , needs to be found. If a is set $= \pm 1$ then this mark cannot be awarded

Usually scored for one of

- using the maximum of $(-2, 12)$ to state the form $(y) = 12 \pm a(x+2)^2$
- using the symmetry to deduce roots, usually -5 and 1 to state the form $(y) = a(x-1)(x+5)$
- using the general form $(y) = ax^2 + bx + c$ and substituting in two points, usually $(-2, 12)$ and $(-5, 0)$ to form two different equations in a , b and c . Award for example for $4a - 2b + c = 12$ and $25a - 5b + c = 0$

dM1: For a full method of finding $f(x)$ in any allowable form.

- using the form $y = 12 \pm a(x+2)^2$ with usually $(-5, 0)$ to find a
- using the form $y = a(x-1)(x+5)$ with usually $(-2, 12)$ to find a
- using the general form $y = ax^2 + bx + c$ scored for forming three equations and solving to find values for a , b and c . E.g Uses $(-2, 12)$ and $(-5, 0)$ in $y = f(x)$ and then uses $2ax + b = 0$ at $x = -2$

A1: Correct equation or expression for $f(x)$ or y . Accept in any equivalent form

- $(f(x)) = 12 - \frac{4}{3}(x+2)^2$
- $(y) = -\frac{4}{3}(x-1)(x+5)$
- $(y) = -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$

(b)

M1: Applies perpendicular condition to find gradient of l_2

Look for $-\frac{5}{4}$, -1.25 or it may be implied by an equation for l_2 of $y = -\frac{5}{4}x + \dots$

M1: Uses their changed gradient with $(-5, 0)$ to find the equation of the line. Look for $y = "m_n"(x+5)$

If they use the form $y = mx + c$ with $(-5, 0)$ they must proceed as far as $c = \dots$

A1: Correct equation $y = -\frac{5}{4}x - \frac{25}{4}$ o.e. Allow exact equivalents but must be in the form $y = mx + c$

(c)

M1: For identifying any two of the three restrictions given in the scheme. FT on their answers to (a) and (b) where (a) is a quadratic and (b) is linear. Allow for this mark only the inequality for the quadratic to be $y \leq ax^2 + bx + c$ or $y \leq f(x)$ if that is what they identify as the curve. They may combine two of these

as one statement, e.g. $\frac{4}{5}x \leq y \leq -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$, which is fine (and scores the M1). Use of R instead of y is M0 A0

A1ft: For all three of the restrictions given in the scheme, follow through their answers to (a) and (b) Accept with strict inequalities, but should be consistent in all inequalities.



The use of set notation is fine $\left\{(x, y) : \frac{4}{5}x \leq y \leq -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}\right\} \cap \left\{(x, y) : y \geq -\frac{5}{4}x - \frac{25}{4}\right\}$

Spurious additional restrictions such as $x \geq -5$ may be seen and should not be marked as incorrect unless the additional restriction gives further restrictions on the region R , e.g. $y \geq 0$

Do not ISW here. Mark their final answer.

Question	Scheme	Marks
6(a)	$2xy - 3x^2 = 50; y - x^3 + 6x = 0$	
	$\Rightarrow 2x(x^3 - 6x) - 3x^2 = 50$	M1
	$\Rightarrow 2x^4 - 12x^2 - 3x^2 - 50 = 0 \Rightarrow 2x^4 - 15x^2 - 50 = 0^*$ CSO	A1*
		(2)
(b)	$\Rightarrow (2x^2 + 5)(x^2 - 10) = 0 \Rightarrow x^2 = \dots$	M1
	So $x^2 = 10$	A1
	$\Rightarrow y = (\sqrt{10})^3 - 6\sqrt{10} = \dots$	M1
	one solution pair is $x = \sqrt{10}, y = 4\sqrt{10}$	A1
	Solutions are $x = \sqrt{10}, y = 4\sqrt{10}$ and $x = -\sqrt{10}, y = -4\sqrt{10}$ CSO	A1
		(5)
		(7 marks)

(a)

M1: Attempts to substitute their expression for y from rearranging the second equation into the first equation. More difficult approaches are possible, but all should lead to an equation in just x

E.g 1 substituting their expression for y from rearranging the first equation into the second equation

E.g. 2 multiplying the second equation by $2x$ and subtracting

A1*: Correct work leading to the given answer. Expect to see correct bracketing etc and at least one intermediate line with the two x^2 terms initially uncollected. See scheme

(b) This is a non calculator question so look for evidence of calculations

M1: Uses a non calculator method to solve the given quadratic in x^2 .

Allow factorisation, the quadratic formula or completing the square.

You may see a substitution, say $u = x^2$. In this case it is for solving to find u

Condone substitutions $y = x^2$ or $x = x^2$ if it is clear what is meant.

$$\text{E.g. } 2x^4 - 15x^2 - 50 = 0 \Rightarrow x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4 \times 2 \times -50}}{4} = 10, -\frac{5}{2} \text{ scores 0 unless recovered to } x^2 = 10 \text{ or}$$

$$x = \sqrt{10}$$

A1: A correct solution for x or x^2 e.g. $x^2 = 10$, $u = 10$ $x = \pm\sqrt{10}$, $x = \sqrt{10}$ etc but not $x = 10$

Candidates cannot just write down $2x^4 - 15x^2 - 50 = 0 \Rightarrow x = \pm\sqrt{10}$



M1: Uses at least one x value of the form $p\sqrt{a}$, $a > 0$ and uses a non calculator method to find a value for $y = q\sqrt{a}$. Must use x correctly, not x^2 in place of x . It is not a dependent mark so if $x = \sqrt{10}$ followed by a correct non calculator method you may award the mark. Some calculations must be seen, not just a written down answer. So $x = \sqrt{10} \Rightarrow y = (\sqrt{10})^3 - 6\sqrt{10} = 4\sqrt{10}$ is acceptable

A1: At least one correct pair of solutions. Need not be fully simplified but should be single terms, so accept e.g. $\left(\sqrt{10}, \frac{40}{\sqrt{10}}\right)$ if they use the other equation to find y .

So a mark of 00110 is possible for candidates who show evidence for y but not for x .

A1: CSO. All previous marks must have been awarded in part (b). Both pairs of solutions correct and simplified with no incorrect extra answers given. Must be clearly paired so do not accept

$x = \pm\sqrt{10}, y = \pm 4\sqrt{10}$ unless clear pairings have been given elsewhere.

Question	Scheme	Marks
7(a)	$f'(x) = 2x^{-\frac{1}{2}} + Ax^{-2} + 3 \Rightarrow f''(x) = \dots x^{-\frac{1}{2}-1} + \dots x^{-2-1}$	M1
	$\Rightarrow f''(x) = 2 \times -\frac{1}{2} x^{-\frac{3}{2}} + -2Ax^{-3} = -x^{-\frac{3}{2}} - 2Ax^{-3}$	A1
	$f''(4) = 0 \Rightarrow -4^{-\frac{3}{2}} - 2A \times 4^{-3} = 0 \Rightarrow A = \dots$	dM1
	$-\frac{1}{8} - \frac{2A}{64} = 0 \Rightarrow A = -4$	A1
		(4)
(b)	$f(x) = \int 2x^{-\frac{1}{2}} + Ax^{-2} + 3 \, dx = \frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{Ax^{-2+1}}{-2+1} + 3x(+c)$	M1
	$= 4x^{\frac{1}{2}} - \frac{A}{x} + 3x(+c)$	A1ft
	$f(12) = 8\sqrt{3} \Rightarrow 4\sqrt{12} - \frac{A}{12} + 36 + c = 8\sqrt{3} \Rightarrow c = \dots$	dM1
	$c = 8\sqrt{3} - 4\sqrt{12} - 36 - \frac{4}{12} = -\frac{109}{3}$ or follow through $c = \frac{A}{12} - 36$	A1ft
	So $f(x) = 4x^{\frac{1}{2}} + \frac{4}{x} + 3x - \frac{109}{3}$ oe	A1
		(5)
(9 marks)		
Notes: Mark parts (a) and (b) together		

Question Number	Scheme	Marks
2(a)	$a = 2$	B1
	$b = -3$	B1
		(2)
(b)	Any two term of $\int \frac{2x^3 - 3x^2 - 32x - 15}{5\sqrt{x}} dx = \int \frac{2}{5}x^{\frac{5}{2}} - \frac{3}{5}x^{\frac{3}{2}} - \frac{32}{5}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} dx$	M1A1
	$x^n \rightarrow x^{n+1}$	M1
	$\frac{4}{35}x^{\frac{7}{2}} - \frac{6}{25}x^{\frac{5}{2}} - \frac{64}{15}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$	A1A1
		(5)
		(7 marks)

- (a) If there is a contradiction between their stated values and the embedded values in a cubic expression then the embedded values take precedence.

B1 $a = 2$ which may be embedded

B1 $b = -3$ which may be embedded

(b)

M1 Attempts to write as a sum of terms. Award for any term with a correct index from correct working. The index does not need to be processed.

A1 Any **two** correct unsimplified or simplified terms of the expression

$$\frac{2}{5}x^{\frac{5}{2}} - \frac{3}{5}x^{\frac{3}{2}} - \frac{32}{5}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} \quad \text{oe eg } 0.4x^{\frac{5}{2}} - 0.6x^{\frac{3}{2}} - 6.4x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} \quad (\text{indices must be processed})$$

M1 Increases the power of any of their non-integer terms by 1. ($x^n \rightarrow x^{n+1}$) This mark cannot be awarded for just increasing the power by 1 on a numerator or denominator term. The index does not need to be processed.

A1 Any two terms correct unsimplified or simplified (see below). May appear in a list or on separate lines.

A1 $\frac{4}{35}x^{\frac{7}{2}} - \frac{6}{25}x^{\frac{5}{2}} - \frac{64}{15}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$ all on one line (including the constant c and all simplified)

Allow exact equivalents but not rounded decimals so $\frac{4}{35}$ must be written as a fraction.

Accept $\frac{4}{35}x^{\frac{7}{2}} - 0.24x^{\frac{5}{2}} - 4.2\dot{6}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$. Also allow terms written as eg $\frac{4}{35}x^3\sqrt{x}$ Withhold this mark if the final answer has the integral sign and dx around it or any other spurious notation.

11. (a)	Attempts $y - \frac{32}{3} = 5(x - 4) \Rightarrow y = 5x - \frac{28}{3}$	M1 A1	(2)
(b)	$f''(x) = \frac{4}{\sqrt{x}} - 3 \Rightarrow f'(x) = 8x^{\frac{1}{2}} - 3x + k$ Substitutes $x = 4, f'(x) = 5 \Rightarrow k = 1$ $f'(x) = 8x^{\frac{1}{2}} - 3x + 1 \Rightarrow f(x) = \frac{16}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 + x + d$ Substitutes $x = 4, f(x) = \frac{32}{3} \Rightarrow d = -12$ $f(x) = \frac{16}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 + x - 12$	M1 A1 dM1 A1 dM1 A1 ddM1 A1	(8) (10 marks)

(a)

M1 Uses gradient of 5 at point $P\left(4, \frac{32}{3}\right)$ to form tangent. For example, $y - \frac{32}{3} = 5(x - 4)$

A1 $y = 5x - \frac{28}{3}$ Accept recurring decimal, but $y = 5x - 9.33$ is A0.

(b)

M1 Attempts to integrate $\frac{4}{\sqrt{x}} - 3$ with one **index** correct

A1 $(f'(x)) = \frac{4}{\sqrt{x}} - 3 \rightarrow 8x^{\frac{1}{2}} - 3x + k$ with or without the $+ k$

dM1 Substitutes $x = 4, f'(x) = 5$ into an integrated form (with $+ k$) and proceeds to find the value of k

A1 $f'(x) = 8x^{\frac{1}{2}} - 3x + 1$ which may be implied (allow if $k = 1$ is found following a correct integral with k)

dM1 Dependent upon the first M. It is for integrating 'again' with one **term** correct

A1 $f(x) = \frac{16}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 + kx + d$ following through on their $k \neq 0$ (which may be a letter or number) and with or without d (Both constants may be called c for this mark.)

ddM1 Dependent upon 1st and third M's (ie having attempted to integrate twice), **and** both " k " and " d " must have been added and processed correctly. Note " $cx + c$ " will score M0 as this is not a correct process for both constant

This mark is scored for using $x = 4, f(x) = \frac{32}{3}$ in an attempt to find ' d '

A1 $f(x) = \frac{16}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 + x - 12$ oe $(x\sqrt{x}$ instead of $x^{\frac{3}{2}}$ is fine)



Question	Scheme	Marks
7(a)	$0 = 10 - 2x \Rightarrow x = 5$ or $y = 2, y = 10 - 2x \Rightarrow x = 4$	B1
	Examples: $\frac{1}{2} \times 2(5 + 4 - a) = \frac{27}{4} \text{ or } \frac{1}{2} \times 2\left(5 + 4 - \frac{2}{k}\right) = \frac{27}{4}$ Trapezium or $\frac{1}{2} \times 2a + \frac{1}{2} \times 2(5 - a + 4 - a) = \frac{27}{4} \text{ or } \frac{1}{2} \times 2 \times \frac{2}{k} + \frac{1}{2} \times 2\left(5 - \frac{2}{k} + 4 - \frac{2}{k}\right) = \frac{27}{4}$ Triangle + Trapezium or $\frac{1}{2} \times 2a + 2(4 - a) + \frac{1}{2} \times 1 \times 2 = \frac{27}{4} \text{ or } \frac{1}{2} \times 2 \times \frac{2}{k} + 2\left(4 - \frac{2}{k}\right) + \frac{1}{2} \times 1 \times 2 = \frac{27}{4}$ Triangle + Rectangle + Triangle or $\frac{1}{2} \times 5 \times 2 + \frac{1}{2}(4 - a) \times 2 = \frac{27}{4} \text{ or } \frac{1}{2} \times 5 \times 2 + \frac{1}{2}\left(4 - \frac{2}{k}\right) \times 2 = \frac{27}{4}$ 2 Triangles	M1
	$\Rightarrow k = \frac{8}{9}, a = \frac{9}{4}$	A1 A1ft
		(4)
(b)	Two of $y \geq \frac{8}{9}x, y \leq 10 - 2x, x > \frac{9}{4}$	M1
	All three of $y \geq \frac{8}{9}x, y \leq 10 - 2x, x > \frac{9}{4}$	A1
		(2)
(6 marks)		
Notes:		
(a) B1: Correct x intercept for $y = 10 - 2x$ or correct x coordinate for intersection of $y = 2$ with $y = 10 - 2x$. May be seen on the diagram or in a sketch or implied by their working. M1: Correct method for area and sets equal to $\frac{27}{4}$ to form an equation in a or k. The $2 \times \frac{1}{2}$ may be implied. May use e.g. trapezium, triangle + trapezium, triangle + rectangle + triangle or 2 triangles. See above for some examples. Note that with a correct “5” and “4” the area is $9 - a$ or $9 - \frac{2}{k}$. A1: Correct value for a or k as a single value not a calculation. A1ft: Correct values for a and k. Correct values or follow through using $ak = 2$ from their first value to the second provided the M1 is scored. Allow exact equivalents or exact ft equivalents so if correct allow e.g. $k = 0.\dot{8}$, $a = 2.25$ or $2\frac{1}{4}$		



Question Number	Scheme	Marks
7ai	$f'(4) = \frac{4(4)^2 + 10 - 7(4)^{\frac{1}{2}}}{4(4)^{\frac{1}{2}}} = \frac{15}{2}$	B1
ii	$-\frac{15}{2} \rightarrow -\frac{2}{15}$	M1
	$y + 1 = -\frac{2}{15}(x - 4)$	M1
	$2x + 15y + 7 = 0$	A1
		(4)
b	$\frac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}} = \pm \dots x^{\frac{3}{2}} \pm \dots x^{-\frac{1}{2}} \pm \dots$	M1
	Two of the terms of $x^{\frac{3}{2}} + \frac{5}{2}x^{-\frac{1}{2}} - \frac{7}{4}$	A1
	$\int \left(x^{\frac{3}{2}} + \frac{5}{2}x^{-\frac{1}{2}} - \frac{7}{4} \right) dx = \frac{2}{5}x^{\frac{5}{2}} + 5x^{\frac{1}{2}} - \frac{7}{4}x (+c)$	dM1A1ft
	$\frac{2}{5}(4)^{\frac{5}{2}} + 5(4)^{\frac{1}{2}} - \frac{7}{4}(4) + c = -1 \Rightarrow c = \dots$	ddM1
	$(f(x) =) \frac{2}{5}x^{\frac{5}{2}} + 5x^{\frac{1}{2}} - \frac{7}{4}x - \frac{84}{5}$	A1
		(6)
		(10 marks)

Mark (a) and (b) together so do not be concerned with labelling of the parts

(a)

(i)

B1: $\frac{15}{2}$ oe stated (as the gradient of $f(x)$ at P).

(ii)

M1 Finds the negative reciprocal of their gradient in part (i). If they do not have a gradient in (i) then only allow $-\frac{2}{15}$ or an attempt at $-\frac{1}{f'(4)}$

M1 Attempts to find the equation of the normal using a changed gradient to that found in (i) and $(4, -1)$ with the coordinates in the correct positions. If they do not have a gradient in (i) then allow any gradient $\neq \frac{15}{2}$. If they use $y = mx + c$ they must proceed as far as $c = \dots$

A1: $2x + 15y + 7 = 0$ or any multiple of this where all the coefficients are integers and all terms are on the same side of the equation. e.g. $30y + 14 + 4x = 0$ scores A1



(b)

M1: Splits into **three** separate terms with at least one term with the correct index. The index does not need to be processed. e.g. $x^{2-\frac{1}{2}}$

A1: Two of $x^{\frac{3}{2}} + \frac{5}{2}x^{\frac{1}{2}} - \frac{7}{4}$ (unsimplified but the indices must be processed). May appear as a list of terms on different lines. May be implied by a correctly integrated expression.

dM1: Attempts to increase the power by one on at least one term. It is dependent on the previous method mark. The index does not need to be processed for this mark. It cannot be scored for attempting to integrate individual terms on the numerator or denominator.

A1ft: $\frac{2}{5}x^{\frac{5}{2}} + 5x^{\frac{1}{2}} - \frac{7}{4}x$ or exact unsimplified equivalent (indices processed). e.g. $\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{\frac{5}{2}x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{7}{4}x$

Follow through on an expression of the form $Ax^{\frac{3}{2}} + Bx^{\frac{1}{2}} + C \rightarrow \frac{Ax^{\frac{5}{2}}}{\frac{5}{2}} + \frac{Bx^{\frac{3}{2}}}{\frac{1}{2}} + Cx$ where A ,

B and C are all non-zero.

Do not be concerned with the presence or omission of the constant of integration.

Accept x^1 for x .

Ignore any spurious notation including the presence of the integral sign.

ddM1: Attempts to substitute $x=4$ into their changed expression (condone one slip in substituting in), sets equal to -1 and attempts to find c .

This mark cannot be scored if they do not have a constant of integration.

It is dependent on both of the previous method marks.

Do not be concerned by the mechanics of the rearrangement and condone arithmetical slips, but they must achieve a value.

The substitution may be implied by a correct value for c for their integrated expression if no working is shown (you may need to check this), or allow for $\pm \frac{84}{5}$

A1: $(f(x) =) \frac{2}{5}x^{\frac{5}{2}} + 5x^{\frac{1}{2}} - \frac{7}{4}x - \frac{84}{5}$ or any equivalent expression e.g. $0.4x^{\frac{5}{2}} + 5x^{\frac{1}{2}} - 1.75x - 16.8$

Accept x^1 for x . Condone equivalent fractions for the coefficients provided both numerator

and denominator are integers. e.g. $(f(x) =) \frac{x^{\frac{5}{2}}}{2.5} + 5x^{\frac{1}{2}} - \frac{7}{4}x - \frac{84}{5}$ is A0

Withhold the final mark if there is still integration notation around the answer or if it is set equal to 0

Question	Scheme	Marks
10(a)	$f'(4) = 6(4) - \frac{7 \times 14}{4} = -\frac{1}{2}$	B1
	$m_T = -\frac{1}{2} \Rightarrow m_N = \frac{-1}{-1/2}$	M1
	$y - 12 = 2(x - 4)$ or $y = mx + c \Rightarrow y = 2x + c \Rightarrow 12 = 2(4) + c \Rightarrow c = \dots$	M1 (A1 on ePen)
	$y = 2x + 4$	A1
		(4)
(b)	$\dots \frac{(2x-1)(3x+2)}{2\sqrt{x}} = \dots \frac{6x^2+x-2}{2\sqrt{x}} = \dots 3x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}}$	M1
	$f(x) = \frac{6x^2}{2} - \frac{6}{5}x^{\frac{5}{2}} - \frac{1}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} (+c)$ or e.g. $f(x) = \frac{6x^2}{2} - \frac{3}{5/2}x^{\frac{5}{2}} - \frac{1/2}{3/2}x^{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{1/2} (+c)$ or e.g. $f(x) = \frac{6x^2}{2} - \left(\frac{3}{5/2}x^{\frac{5}{2}} + \frac{1/2}{3/2}x^{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{1/2} \right) (+c)$	M1A1A1
	$12 = \frac{6(4)^2}{2} - \frac{6}{5}(4)^{\frac{5}{2}} - \frac{1}{3}(4)^{\frac{3}{2}} + 2(4)^{\frac{1}{2}} + c \Rightarrow c = \dots$	M1
	$(f(x)) = 3x^2 - \frac{6}{5}x^{\frac{5}{2}} - \frac{1}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + \frac{16}{15}$	A1
		(6)
		Total 10

Notes:

(a)

B1: Correct gradient at P (may be implied)

M1: Attempts to use the perpendicular gradient rule to find the normal gradient.

 Look for e.g. $m_N = \frac{-1}{m_T}$ or e.g. $m_T \times m_N = -1 \Rightarrow -\frac{1}{2}m_N = -1 \Rightarrow m_N = \dots$

May be implied by their normal gradient.

M1(A1 on Epen): Attempts the equation of the normal using a “changed” gradient with $x = 4$ and $y = 12$ correctly placed.

 Alternatively uses $y = mx + c$ with a “changed” gradient with $x = 4$ and $y = 12$ to find a value for c .

A1: Correct equation in the required form


(b) **Allow work for part (b) seen in part (a) to score in (b) provided it is seen or used in (b)**

M1: Expands the numerator of the fraction and attempts to split.

Score for one correct index achieved from correct work e.g. $\frac{...x^2}{\sqrt{x}} \rightarrow ...x^{\frac{3}{2}}$ or $\frac{...x}{\sqrt{x}} \rightarrow ...x^{\frac{1}{2}}$ or $\frac{...}{\sqrt{x}} \rightarrow ...x^{-\frac{1}{2}}$

M1: Attempts to integrate a fractional power. E.g. $...x^{\frac{3}{2}} \rightarrow ...x^{\frac{5}{2}}$ or $x^{\frac{1}{2}} \rightarrow ...x^{\frac{3}{2}}$ or $x^{-\frac{1}{2}} \rightarrow ...x^{\frac{1}{2}}$ etc.

Do not allow this mark for an attempt to integrate the $\sqrt{x}$ in the denominator.

A1: Any 2 correct terms simplified or unsimplified.

A1: All correct simplified or unsimplified. The “+ c” is not required here.

M1: Uses $x = 4$ and $y = 12$ following an attempt to increase at least one power x by 1 to find a value for their c .

Their c may not be fully evaluated but must be a numerical expression.

A1: Correct simplified form. The “ $f(x) =$ ” is not required just look for the correct expression.

Accept equivalent simplified forms and allow $(f(x) =) 3x^2 - \left(\frac{6}{5}x^{\frac{5}{2}} + \frac{1}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} \right) + \frac{16}{15}$

Apply isw if applicable unless they multiply all terms by e.g. 15

Question Number	Scheme	Marks
10(a)	$m = \frac{4}{5}$	B1
	$y + 3 = \frac{4}{5}(x - 4) \text{ or } -3 = \frac{4}{5} \times 4 + c \Rightarrow c = \dots$	M1
	$y = \frac{4}{5}x - \frac{31}{5}$	A1
		(3)
(b)	$f'(4) = \frac{4}{5} \Rightarrow \frac{k\sqrt{4}(4-3)}{5} = \frac{4}{5} \Rightarrow k = \dots$	M1
	$k = 2$	A1
		(2)
(c)	$(f'(x) =) \frac{2\sqrt{x}(x-3)}{5} = \frac{2x^{\frac{3}{2}}}{5} - \frac{6x^{\frac{1}{2}}}{5}$	M1
	$(f(x) =) \frac{4x^{\frac{5}{2}}}{25} - \frac{4x^{\frac{3}{2}}}{5} (+c)$	M1 A1ft
	$x = 4, y = -3 \Rightarrow -3 = \frac{4(4)^{\frac{5}{2}}}{25} - \frac{4(4)^{\frac{3}{2}}}{5} + c \Rightarrow c = \left(-\frac{43}{25}\right)$	ddM1
	$(f(x) =) \frac{4x^{\frac{5}{2}}}{25} - \frac{4x^{\frac{3}{2}}}{5} - \frac{43}{25}$	A1
		(5)
		(10 marks)

Notes**Mark (a), (b) and (c) together****(a)****B1:** States or implies that gradient of tangent (to the curve at P) is $\frac{4}{5}$ **M1:** Uses their **changed** gradient (i.e. not $-\frac{5}{4}$) with $x = 4$ and $y = -3$ where the -3 is their attempt atfinding y from $y = -\frac{5}{4}x + 2 = -\frac{5}{4} \times 4 + 2$ to form the equation of tangent.E.g. $y + 3 = \frac{4}{5}(x - 4) \text{ or } y = \frac{4}{5}x + c \Rightarrow -3 = \frac{4}{5} \times 4 + c \Rightarrow c = \dots$ Note that some candidates are assuming P lies on the x -axis and use $(4, 0)$ and this scores M0**A1:** $y = \frac{4}{5}x - \frac{31}{5}$ or exact equivalent in this form e.g. $y = 0.8x - 6.2$ but not $y = \frac{(4x - 31)}{5}$ unless the correct form was seen previously then apply isw.

(b)

M1: Uses $f'(4)$ = their **changed** gradient (i.e. not $-\frac{5}{4}$) and solves for k .Allow equivalent work e.g. uses $f'(4) \times -\frac{5}{4} = -1$ and solves for k .**A1:** $k = 2$

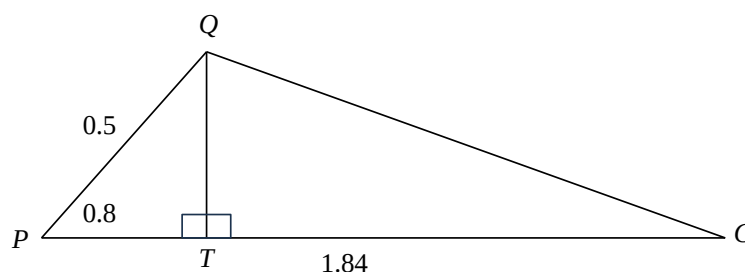
Correct answer with no working scores both marks.

(c)

M1: Attempts to expand and writes $f'(x)$ in the form $\alpha x^{\frac{3}{2}} + \beta x^{\frac{1}{2}}$ **M1:** Integrates with at least one index correct i.e. $\dots x^{\frac{3}{2}} \rightarrow \dots x^{\frac{5}{2}}$ or $\dots x^{\frac{1}{2}} \rightarrow \dots x^{\frac{3}{2}}$ which must come from correct work but indices may be unprocessed at this stage.**A1ft:** Correct integration with or without the $+c$. Allow with constant k or follow through on anincorrect k only i.e. the expression should be otherwise correct e.g. allow $\frac{2kx^{\frac{5}{2}}}{25} - \frac{2kx^{\frac{3}{2}}}{5} (+c)$ orthis expression with their k substituted. Need not be simplified at this stage so indices may be unprocessed.The “ $f(x) =$ ” is **not** required. Condone spurious integral signs.**ddM1:** Must have a constant of integration now. It is for using $x = 4$ and $y = -3$ in their $(f(x) =) \frac{4x^{\frac{5}{2}}}{25} - \frac{4x^{\frac{3}{2}}}{5} + c$ in an attempt to find their c , where the -3 is their attempt at finding y from $y = -\frac{5}{4}x + 2$ Many candidates are assuming P lies on the x -axis and use $(4, 0)$ and this scores ddM0**Depends on both previous M marks.****A1:** $(f(x) =) \frac{4x^{\frac{5}{2}}}{25} - \frac{4x^{\frac{3}{2}}}{5} - \frac{43}{25}$ Correct simplified expression. The “ $f(x) =$ ” is **not** required.

Condone spurious integral signs.

Apply isw if necessary once a correct expression is seen.

Question 9(a) and (b) using right angled triangles:**(a)**

e.g. $QT = 0.5 \sin 0.8 (= 0.3586\dots)$, $PT = 0.5 \cos 0.8 (= 0.3483\dots)$

$$QC = \sqrt{QT^2 + TC^2} = \sqrt{0.3586\dots^2 + (1.84 - 0.3483\dots)^2} = 1.534$$

Score M1 for a complete and correct method leading to a value for the length of QC and A1 as MS

(b)

e.g. $\sin PCQ = \frac{QT}{CQ} = \frac{0.5 \sin 0.8}{1.534} = 0.23379\dots \Rightarrow PCQ = 0.236^*$

Score M1 for a complete and correct method leading to an expression for e.g. $\sin PCQ$ and A1 for a correct proof.

Question Number	Scheme	Marks
11 (a)	Gradient of normal $= \frac{1}{4}$ Equation of normal $(y + 50) = \frac{1}{4}(x - 4) \Rightarrow y = \frac{1}{4}x - 51$	B1 M1 A1 (3)
(b)	$(f''(x) =) \frac{6}{\sqrt{x^3}} + x = 6x^{-\frac{3}{2}} + x \Rightarrow f'(x) = -12x^{-\frac{1}{2}} + \frac{1}{2}x^2 + k$ Substitutes $x = 4, f'(x) = -4 \Rightarrow k = -6$ $(f'(x) =) -12x^{-\frac{1}{2}} + \frac{1}{2}x^2 - 6 \Rightarrow (f(x) =) -24x^{\frac{1}{2}} + \frac{1}{6}x^3 - 6x + d$ Substitutes $x = 4, f(x) = -50 \Rightarrow d = \frac{34}{3}$ $(f(x) =) -24x^{\frac{1}{2}} + \frac{1}{6}x^3 - 6x + \frac{34}{3}$	M1 A1 dM1 A1 dM1 A1ft dddM1 A1 (8) (11 marks)

Mark (a) and (b) together

(a)

B1 Deduces that the gradient of the normal is $\frac{1}{4}$ M1 Attempts to find the equation of a line passing through $P(4, -50)$ with a **changed** gradient. Allow one sign slip on a coordinate so either $(y + 50)$ or $(x - 4)$ must be correct. If they use $y = mx + c$ then at least one of the coordinates must be correctly substituted in and they must proceed as far as $c = \dots$ A1 $y = \frac{1}{4}x - 51$

(b)

M1 Attempts to integrate $\frac{6}{\sqrt{x^3}} + x$ with one index correct. Either $\dots x^{-\frac{1}{2}}$ or $\dots x^2$ A1 $(f'(x) =) -12x^{-\frac{1}{2}} + \frac{1}{2}x^2 + k$ (unsimplified) with or without the $+k$ dM1 Substitutes $x = 4, f'(x) = -4$ into an integrated form (with $+k$) and proceeds to find the value of k
This is dependent on the first M1.A1 $(f'(x) =) -12x^{-\frac{1}{2}} + \frac{1}{2}x^2 - 6$ (unsimplified) which may be implieddM1 Dependent upon the first M. It is for integrating 'again' with one index correct. Either $\dots x^{\frac{1}{2}}$ or $\dots x^3$ A1ft $(f(x) =) -24x^{\frac{1}{2}} + \frac{1}{6}x^3 - 6x + d$ (unsimplified) following through ONLY on their k (allow kx) and with or without d 

dddM1 Dependent upon all three previous Ms.

Both " k " and " d " must have been added although condone calling them both c .

This mark is scored for using $x = 4$, $f(x) = -50$ in an attempt to find ' d '. Do not be concerned by the mechanics of their rearrangement.

A1 $(f(x) =) -24x^{\frac{1}{2}} + \frac{1}{6}x^3 - 6x + \frac{34}{3}$ or **exact equivalent expressions**. Eg Do not allow $\frac{1}{6}$ to be written as 0.167 but condone $-6x^1$. The indices must have been processed and the terms must all be on one line including $\frac{34}{3}$. isw after a correct expression.



Question Number	Scheme	Marks
	$(f''(x) =) \frac{3}{4}x^{-\frac{3}{2}}$ Attempts $\frac{3}{4}x^{-\frac{3}{2}} = 6 \Rightarrow x^{-\frac{3}{2}} = 8 \Rightarrow x = \frac{1}{4}$	M1 A1 dM1 A1 (5) (9 marks)

- (a) **Note calculators in this question are not acceptable so answers on their own or roots without working score 0 marks.**

M1 **Way One:** Sets $2x - 3\sqrt{x} - 5 = 9 \Rightarrow 2x - 3\sqrt{x} - 14 = 0$ and attempts to solve a 3TQ quadratic in $\sqrt{x}$ or sets eg $u = \sqrt{x}$ and attempts to solve a 3TQ in u ($2u^2 - 3u - 14 = 0$). See general guidance for solving a quadratic. Condone use of other variables including $x = \sqrt{x}$.

Condone slips in their rearrangement to achieving a 3TQ quadratic and proceeding to find a value but the method must be sound. To score they must either

- show the factorised form of their quadratic eg $(2u - 7)(u + 2)$
- show embedded values in the quadratic formula
- show their method completing the square

Way Two: Sets $2x - 14 = 3\sqrt{x}$ oe and attempts to square leading to a 3TQ quadratic on one side of an equation. Condone slips when multiplying out eg $(2x - 14)^2$ and rearranging their equation but their method must be sound.

Note: $2x - 14 = 3\sqrt{x} \Rightarrow 4x^2 - 196 = 9x$ would be M0.

A1 **Way One:** $(\sqrt{x} =) \frac{7}{2}$ or eg $(u =) \frac{7}{2}$ Ignore any reference to the -2 . Condone $x = \frac{7}{2}$

Note the roots do not imply M1A1

Way Two: $4x^2 - 65x + 196 = 0$ oe (the terms should be collected on one side of the equation, but condone lack of $= 0$)

dM1 **Way One:** Attempts to find one value for x . Condone 4 or squaring -2

Way Two: Attempts to find one value for x by solving their quadratic (see general guidance for solving a quadratic). To score they must either

- show the factorised form of their quadratic eg $(4x - 49)(x - 4)$
- show embedded values in the quadratic formula
- show their method completing the square

A1 $x = \frac{49}{4}$ or 12.25 or $12\frac{1}{4}$ only.

If 4 is found it must be rejected

(b)

B1 $2 - \frac{3}{2}x^{-\frac{1}{2}}$ Correct differentiation. Look for two correct **terms** but may be unsimplified. The index must be processed.

M1 Differentiates again. Look for any index of their $f'(x)$ being reduced by one which may also be a "2" $\rightarrow 0$

A1 $(f''(x) =) \frac{3}{4}x^{-\frac{3}{2}}$ or exact equivalent

dM1 Proceeds to make x the subject by:

- Setting their $f''(x) = 6$ which must be of the form $Bx^k = 6$ (where k cannot be an integer)
- Achieving $x^m = A$ (where $m \neq 1$) and proceeding to $x = \dots$ condoning slips in their rearrangement
or proceeds from $Bx^k = 6$ to $x = \dots$ with at least one intermediate line of working (they cannot just state the answer)

A1 $\frac{1}{4}$ cso preceded by correct working shown by an intermediate line of working (see below)

Examples:

$$\Rightarrow x^{-\frac{3}{2}} = 8 \Rightarrow x = \frac{1}{4} \text{ is A1}$$

$$\Rightarrow x = \sqrt[3]{\frac{1}{8}} = \frac{1}{4} \text{ is A1}$$

$$\Rightarrow x = \sqrt[3]{\frac{1}{64}} = \frac{1}{4} \text{ is A1}$$

$$\Rightarrow x = \sqrt[3]{\frac{1}{8}} = \frac{1}{4} \text{ is A1}$$

$$\Rightarrow x^{-\frac{3}{2}} = 8 \Rightarrow x = (\sqrt{8})^{-3} = \frac{1}{4} \text{ M1A0 (incorrect inverse index laws shown)}$$

Question Number	Scheme	Marks
8 (a)	$x > 4$	B1 (1)
(b)	$(3x-2)^2(x-4) = (9x^2-12x+4)(x-4)$ $= 9x^3 - 48x^2 + 52x - 16$	M1 A1 A1 (3)

Question Number	Scheme	Marks
(c)	Sets $9x^3 - 48x^2 + 52x - 16 = -16 \Rightarrow 9x^3 - 48x^2 + 52x (= 0)$ $\Rightarrow x(9x^2 - 48x + 52) = 0 \Rightarrow x = \frac{48 \pm \sqrt{48^2 - 4 \times 9 \times 52}}{18} = \frac{16 \pm 4\sqrt{3}}{6}$ $\text{Distance } PQ = \frac{16 + 4\sqrt{3}}{6} - \frac{16 - 4\sqrt{3}}{6} = \frac{4}{3}\sqrt{3}$	B1ft M1 B1 M1 A1 (5) (9 marks)

Ignore labelling of parts in this question

(a)

B1 $x > 4$ only

(b)

M1 Attempts to multiply two of the brackets and then multiplies the result by the third. This may be seen in (a)

Accept $(3x-2)^2(x-4) = (9x^2 \pm \dots x \pm 4)(x-4) = \dots x^3 + \dots$

or $(3x-2)^2(x-4) = (3x^2 \pm \dots x \pm 8)(3x-2) = \dots x^3 + \dots$

Condone invisible brackets.

Note eg $(3x-2)^2(x-4) = (9x^2 \pm 4)(x-4) = \dots x^3 + \dots$ is M0. They must have x terms from multiplying two brackets together.

A1 Any two correct and simplified terms of $9x^3 - 48x^2 + 52x - 16$. (Be careful to check that M1 has been earned.)

A1 $9x^3 - 48x^2 + 52x - 16$ (ignore any reference to $= 0$)

(c) **Note in EPEN it is B1ftM1A1dM1A1 we are marking this B1ftM1B1M1A1**

B1ft Proceeds to $9x^3 - 48x^2 + 52x (= 0)$ but follow through on their a , b and c .

M1 **The quadratic must be seen or implied by working shown**

- **Either** factorises/cancels out the x from a cubic of the form $\dots x^3 \pm \dots x^2 \pm \dots x = 0$ to produce a 3TQ. As the 3TQ is seen then using a calculator is allowed so they can proceed to just stating the roots (or they may use the quadratic formula/completing the square methods).
- **Or** states $x = 0$, shows some working (eg completed square form or embedded values in the quadratic formula for their invisible quadratic) and proceeds towards at least one value for x .

If their quadratic factorises then this mark cannot be awarded.

c) y-intercept of $c = (0, -16)$

$$9x^3 - 48x^2 + 52x - 16 = -16$$

$$9x^3 - 48x^2 + 52x = 0$$

$$\frac{-(-48) \pm \sqrt{(-48)^2 - 4(9)(52)}}{2(9)}$$

$$\frac{8 \pm 2\sqrt{3}}{3}$$

Eg this scores M0 because they do not state $x = 0$ Send to review if unsure.

- B1 Correct roots which may be unsimplified $\frac{16 \pm 4\sqrt{3}}{6}$ ($b^2 - 4ac$ should be evaluated though)
Condone decimal answers here. Allow awrt 1.51, 3.82

- M1 Subtracts their two non-zero roots from setting their cubic $= 0$ either way round (cannot be from differentiating the cubic and setting the resulting $3TQ = 0$) which may be implied by their answer. They may also use the distance formula (embedded values is sufficient, condone a sign error in the second bracket involving their " d " from part (b)).

$$\sqrt{\left(\frac{16 + 4\sqrt{3}}{6} - \frac{16 - 4\sqrt{3}}{6}\right)^2 + ("d" - "d")^2}$$

- A1 $\frac{4}{3}\sqrt{3}$ cso (or exact equivalent such as $\frac{16}{12}\sqrt{3}$ or $1.\dot{3}\sqrt{3}$) It must have $...\sqrt{3}$ Do not allow $1.33\sqrt{3}$
This mark can only be scored if all previous marks have been awarded.
Note candidates who solve $f(x) = 0$ will not score any marks in (c)

$$\begin{aligned} -16 - (9x^3 - 48x^2 + 52x - 16) &= 0 \\ -16 = 9x^3 - 48x^2 + 52x - 16 &\Rightarrow 9x^3 - 48x^2 + 52x = 0 \\ x_p = \frac{8 - 2\sqrt{3}}{3} = 1.51 & \quad y_p = -16 \quad x = \frac{8 + 2\sqrt{3}}{3} \\ x_q = \frac{8 + 2\sqrt{3}}{3} = 3.82 & \quad y_q = -16 \\ \text{Coordinates of Point P} &= (1.51, -16) \\ \text{Coordinates of Point Q} &= (3.82, -16) \\ \text{Distance between P and Q} &= \sqrt{(1.51 - 3.82)^2 + (-16 + 16)^2} \\ &= \sqrt{(-2.3)^2 + (0)^2} \\ &= \sqrt{5.29} = 2.31 \end{aligned}$$

This example would score in (c)
B1ftM0B1M1A0

Question Number	Scheme	Marks

9. (i)	$\frac{(3x+2)^2}{4\sqrt{x}} = \frac{9x^2 + 12x + 4}{4\sqrt{x}} = \frac{9}{4}x^{\frac{3}{2}} + 3x^{\frac{1}{2}} + x^{-\frac{1}{2}}$ $\int \frac{(3x+2)^2}{4\sqrt{x}} dx = \frac{2}{5} \times \frac{9}{4} x^{\frac{5}{2}} + \frac{2}{3} \times 3x^{\frac{3}{2}} + 2 \times x^{\frac{1}{2}} (+c) = \frac{9}{10}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$	M1 dM1 A1 A1 A1
(ii)	$f'(x) = x^2 + ax + b$ Attempts to use $f'(3) = 2 \Rightarrow 2 = 9 + 3a + b$ Attempts to integrate $(f(x) =) \frac{1}{3}x^3 + \frac{1}{2}ax^2 + bx + c$ Attempts to use y intercept $= -8$ and $(3, -2)$ in $f(x) = \frac{1}{3}x^3 + \frac{1}{2}ax^2 + bx + c$ Correct equation in a and b $-2 = 9 + \frac{9}{2}a + 3b - 8$ Solves simultaneously to get values for a and b $a = -4, b = 5 \Rightarrow (f(x) =) \frac{1}{3}x^3 - 2x^2 + 5x - 8$	(5) M1 M1 dM1 A1 ddM1 A1 (6) (11 marks)

(i)

M1 Attempts to multiply out the numerator and divide (any term) by $(4)\sqrt{x}$

Award for one correct index coming from correct working (which may be implied):

$\frac{\dots x^2}{\dots \sqrt{x}} \rightarrow \dots x^{\frac{3}{2}}, \frac{\dots x}{\dots \sqrt{x}} \rightarrow \dots x^{\frac{1}{2}}, \frac{\dots}{\dots \sqrt{x}} \rightarrow \dots x^{-\frac{1}{2}}$. Do not award this mark for $\frac{\dots}{x^{\frac{1}{2}}}$ unless implied by further work.

dM1 Raises the power of any correct index by one $\dots x^{\frac{3}{2}} \rightarrow \dots x^{\frac{5}{2}}, \dots x^{\frac{1}{2}} \rightarrow \dots x^{\frac{3}{2}}, \dots x^{-\frac{1}{2}} \rightarrow \dots x^{\frac{1}{2}}$. Indices must be processed.

A1 One correct simplified term from $\frac{9}{10}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}}$. The term may be seen within intermediate working. Allow 0.9 instead of $\frac{9}{10}$

A1 Two correct simplified terms from $\frac{9}{10}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}}$ which do not have to be on one line. Allow 0.9 instead of $\frac{9}{10}$

A1 $\frac{9}{10}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$ all on one line (or simplified equivalent including the $+ c$). Allow 0.9 instead of $\frac{9}{10}$

Accept other simplified expressions such as $\frac{9}{10}(\sqrt{x})^5 + 2(\sqrt{x})^3 + 2\sqrt{x} + c$

Ignore any spurious notation including the integral sign or a dx .

(ii)

M1 Attempts to use $f'(3) = 2 \Rightarrow 2 = 9 + 3a + b$ oe. The expression does not need to be simplified so embedded values scores this mark. Condone slips when squaring but do not allow $f'(3) = -2$

M1 Attempts to integrate and achieves $(f(x) =) \dots x^3 + \dots ax^2 + bx + (c)$ with or without the $+ c$. $a, b \neq 0$

dM1 Uses y intercept $= -8$ and $(3, -2)$ in their $(f(x) =) \dots x^3 + \dots ax^2 + bx + c$. The values embedded in the expression are sufficient. **It is dependent on the previous method mark only.**

Alternatively uses $(3, -2)$ in $(f(x) =) \dots x^3 + \dots ax^2 + bx - 8$

Beware: they may also substitute in $b = -7 - 3a$ at some point to achieve an equation in a only.

A1 Correct unsimplified equation in a and b $-2 = 9 + \frac{9}{2}a + 3b - 8$.

Note: simplified becomes $9a + 6b = -6$

Alternatively they may have a correct unsimplified equation in a only: $-2 = 9 + \frac{9}{2}a + 3(-7 - 3a) - 8$ oe
eg $4.5a = -18$

ddM1 Dependent upon all previous M's.

It is for solving their two equations with $c = -8$ to find values for a and b or solving their equation in a and then substituting in to $b = -7 - 3a$ to find a value for b

Don't be too concerned by the process. A calculator method is acceptable.

A1 $\frac{1}{3}x^3 - 2x^2 + 5x - 8$ as their final answer. Do not isw eg $\frac{1}{3}x^3 - 2x^2 + 5x - 8 \rightarrow x^3 - 6x^2 + 15x - 24$
 $y = \dots$ or $f(x) = \dots$ are not required. Do not allow $5x^1$.

Question Number	Scheme	Marks
4(a)	$x + y = 6, y = 6x - 2x^2 + 1$ $\Rightarrow 6 - x = 6x - 2x^2 + 1$ $\Rightarrow 2x^2 - 7x + 5 = 0$ oe	M1
	$2x^2 - 7x + 5 = 0 \Rightarrow (2x - 5)(x - 1) = 0$ $\Rightarrow x = \frac{5}{2}, 1$	M1
	$x = \frac{5}{2} \Rightarrow y = \frac{7}{2}$ or $x = 1 \Rightarrow y = 5$	dM1
	(1, 5) and (2.5, 3.5)	A1
		(4)
(b)	$y \geq 6x - 2x^2 + 1$ oe $x + y \leq 6$ oe $x \geq a$ where $1 \leq a \leq 2.5$ (or $a \leq x \leq b$ where $1 \leq a \leq 2.5, b \geq 6$) $y \geq 0$ (or $0 \leq y \leq c$ where $c \geq 3.5$) Allow strict or non-strict inequalities	M1
		A1
		A1
		(3)
		Total 7

Note solutions relying on calculator technology are not acceptable so a complete method must be shown

(a) Answers only 0/4

M1: Uses the line and the curve to obtain a 3TQ in x or y equal to zero (may be implied by solving their 3TQ). Condone slips in their rearrangement.

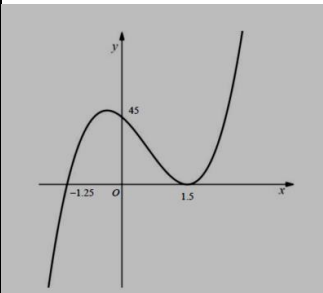
M1: Solves their 3TQ which must be different to $6x - 2x^2 + 1 = 0$ by an acceptable method (factorising, formula or completing the square). They cannot state the roots without seeing a correct line of intermediate working. Allow recovery and condone lack of 3TQ = 0. If they use the quadratic formula, the values must be embedded in the formula before proceeding to the roots.

If they factorise then do not accept $2x^2 - 7x + 5 = 0 \Rightarrow (x - \frac{5}{2})(x - 1) = 0 \Rightarrow x = \dots$

Condone $-2x^2 + 7x - 5 = 0 \Rightarrow (2x - 5)(x - 1) = 0 \Rightarrow x = \dots$

dM1: Solves for y for at least one of their values of x or alternatively solves for x for at least one of their values of y . It is dependent on the previous method mark.

A1: (1, 5) and (2.5, 3.5) which do not need to be paired. May be seen as $x = \dots, y = \dots$. But cannot be awarded if they are incorrectly paired in their final answer. Condone omission of brackets around the coordinates. Ignore if they have assigned coordinates as P and Q . Cannot be awarded if x and y are the wrong way round or if given as (1, 3.5) and (2.5, 5)

Question Number	Scheme	Marks
8 (a)	$f'(x) = 2(x-3)(3x+2) = 6x^2 - 14x - 12$ $f(x) = 2x^3 - 7x^2 - 12x + k$ Uses $P(4, 13) \Rightarrow 13 = 2 \times 64 - 7 \times 16 - 12 \times 4 + k \Rightarrow k = \dots$ $(f(x)) = 2x^3 - 7x^2 - 12x + 45$	B1 M1, A1 M1 A1 (5)
(b)	$2x^3 - 7x^2 - 12x + 45 \equiv (x^2 - 6x + 9)(px + q) \Rightarrow p = "2" \text{ or } q = \frac{"45"}{9}$ States either $2x^3 - 7x^2 - 12x + 45 \equiv (x-3)^2(2x+5)$ or $p = 2 \ \& \ q = 5$ following a fully correct (a) or correct but incomplete part (a) (See notes)	B1 ft B1 (2)
(c)	 + ve cubic curve "Correct" y intercept for their part (b) equation. So 9q or "45" Turning point at (1.5, 0) x intercept at (-1.25, 0)	B1 B1ft B1 B1 (4) (11 marks)

Mark parts (a) and (b) together but $f(x)$ must be obtained via integration AND the use of (4, 13)

(a)

B1: $2(x-3)(3x+2) = 6x^2 - 14x - 12$ which may be left unsimplified BUT the brackets must be expanded

M1: Attempts to multiply out $2(x-3)(3x+2)$ and integrate each term.

Look for an expanded form which would simplify to $ax^2 + bx + c$ and **then each term** being integrated with each of the powers being correctly dealt with. So $ax^2 \rightarrow \dots x^3$, $bx \rightarrow \dots x^2$ and $+c \rightarrow cx$

There is no requirement to have a constant of integration for this mark

A1: $f(x) = 2x^3 - 7x^2 - 12x + k$ including the $+k$. (Any constant is acceptable)

The expression must now be in simplest form

M1: Uses $P(4, 13)$ and finds a value for k . It is dependent upon some 'correct' integration.

So, look for substituting $x = 4, y = 13$ into their $y = f(x)$ and finding a value for k .

A1: $f(x) = 2x^3 - 7x^2 - 12x + 45$. There is no requirement to see the $f(x)$

(b)

Note: If there are multiple attempts with no clearly designated answer, Diagram 1 takes precedence if not crossed out. If Diagram 1 is not used, or is crossed out and no clear statement of answer is given, then an answer in the answer space takes precedence over an answer on Figure 3.

M1: Correct shape, a $\cap$ shape parabola in any position. Be tolerant with the shape, must at least touch the x -axis.

A1: Correct shape and position, a $\cap$ shape parabola passing through O and the $+ve$ x axis to the right hand side of $x = 4$. The graph must **clearly** extend into quadrants 3 and 4, not stop at the x -axis.

Ignore any coordinates on the graph, and don't be concerned if the intersection in quadrant 3 is not shown.

(c)

B1ft: Sets $1.2x(8-x) = \text{their } 0.2x^2(x-4)$. It is for a "correct" ft equation on their part (a) as long as their answer to part (a) is a cubic expression.

M1: Valid attempt to solve an equation of the form $1.2x(8-x) = \text{cubic}$ with their cubic (and their quadratic $1.2x(8-x)$ if they incorrectly try to simplify this) to reach at least a gathered cubic with no constant, or a quadratic in x . The x may be cancelled, e.g. $1.2x(8-x) = 0.2x^2(x-4) \Rightarrow \text{Quadratic equation in } x$. Do not allow solutions which reach a cubic with non-zero constant and simply state answers from a calculator. This is for progressing via algebra to a point at which a quadratic (possibly tacit in a cubic with no constant) can be identified.

dM1: Attempts to solve the resulting (possibly tacit) 3 term quadratic equation, usual rules. Allow solutions from calculator as long as a quadratic can be identified in their equation. You may need to check their values (both must be correct).

A1: Either all three x coordinates stated as answers (the 0 must be present but may have been given earlier), or one correct coordinate pair from $(-8, -153.6), (6, 14.4)$

A1: All three of $(-8, -153.6), (6, 14.4), (0, 0)$, accept as $x = \dots, y = \dots$ for each as long as they are paired correctly. The $(0,0)$ may be stated earlier, so check the whole solution if it is not listed at the end.



Question Number	Scheme	Marks
8 (a) (i)	$x = 4, f'(x) = 10, f'(x) = 3\sqrt{x} + kx^2 \Rightarrow 10 = 3\sqrt{4} + 4^2k \Rightarrow k = \dots$	M1
	$10 = 3 \times 2 + k \times 16 \Rightarrow k = \frac{1}{4} *$	A1*
(ii)	$x = 4, y = 12$ on $y = 10x + c \Rightarrow 12 = 10 \times 4 + c$ $\Rightarrow c = -28$	M1 A1 (4)
(b)	$f''(x) = \frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{2}x$ $\{ \Rightarrow f''(4) \} = \frac{11}{4}$	M1 A1ft A1 (3)
(c)	$f(x) = 2x^{\frac{3}{2}} + \frac{1}{12}x^3 + d$ Uses $P(4, 12) \Rightarrow 12 = 2 \times 8 + \frac{1}{12} \times 4^3 + d \Rightarrow d = \dots$ $\{f(x)\} = 2x^{\frac{3}{2}} + \frac{1}{12}x^3 - \frac{28}{3}$	M1, A1ft dM1 A1 (4) (11 marks)

Note: If work is done out of order (e.g. if an attempt to find $f(x)$ is made in part (a) but no attempt is made at (c)) then allow the M1A1 marks for differentiation in (b) and integration in (c) for the work in part (a) as long as it is clear it is an attempt at the correct process. However, e.g. an attempt at integration with a decrease in power should not be awarded the marks for (b).

(a)(i)

M1: Attempts to solve $f'(4) = 10$ to find k . Alternatively, substitutes $x = 4$ and $k = \frac{1}{4}$ into $f'(x)$ and evaluates.

A1*: Shows that $k = \frac{1}{4}$ with a correct intermediate line where powers/roots have been evaluated, and

with no incorrect steps. Alternatively, correctly reaches $f'(4) = 10$ and conclude $k = \frac{1}{4}$

(ii)

M1: Substitutes $x = 4, y = 12$ in $y = 10x + c$

Alternatively, substitutes $x = 4, y = 12$ and $m = 10$ into $y - y_1 = m(x - x_1)$

A1: Correct value for c which may be embedded in the equation, e.g. $y = 10x - 28$, as long as no incorrect value for c is subsequently given.

(b)

M1: Differentiates with attempt to decrease power by 1 in each term. This should be to a form

$Ax^{-\frac{1}{2}} + Bx$ but allow if a slip in one power has occurred before differentiating.

A1ft: $\frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{2}x$ Allow for $\frac{3}{2}x^{-\frac{1}{2}} + 2kx$ without k replaced or follow through their k if they use a different value than the one given. Need not be simplified.

A1: $\{f''(4)\} = \frac{11}{4}$ o.e.

(c)

M1: Integrates with attempt to increase each power by 1. This should be to a form $px^{\frac{3}{2}} + qx^3$ but allow if a slip in one power has occurred before integrating. There is no requirement for $+d$ for this mark. NB If they try starting from $f'(x)$ then ignore any spurious constant term in $f'(x)$ for this mark.

A1ft: $f(x) = 2x^{\frac{3}{2}} + \frac{1}{12}x^3 (+d)$ with no requirement for $+d$ for this mark. Allow with k for this mark or follow through their value of k if they use a different value from the one given. Need not be simplified.

dM1: Substitutes $x=4, y=12$ into $y=f(x)$ to find " d ". (Must have had a constant of integration but need not be called d .) They must be using a value for k at this stage.

A1: $\{f(x)\} = 2x^{\frac{3}{2}} + \frac{1}{12}x^3 - \frac{28}{3}$ Allow $y = \dots$ or just the expression with no $f(x)$.