



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Proof & Algebraic Methods (P2.1)

International A Level Mathematics · P2: Pure Mathematics 2 (WMA12)

34 questions · 213 marks · Mark schemes included

Past papers June 2019 - June 2025 · 2 sub-topics

 guidemygrades.com

 [@Guidemygrades](https://www.instagram.com/Guidemygrades)

 [@Guidemygrades](https://www.facebook.com/Guidemygrades)

 [@Guidemygrades](https://www.youtube.com/Guidemygrades)

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
P2.1a PROOF (DEDUCTION, EXHAUSTION & COUNTER-EXAMPLE)		16 Qs · 74 marks	3
1. Q1	IAL P2 (WMA12) October 2022	3	4
2. Q1	IAL P2 (WMA12) October 2023	3	6
3. Q1	IAL P2 (WMA12) June 2025	3	8
4. Q3	IAL P2 (WMA12) June 2019	4	10
5. Q5	IAL P2 (WMA12) January 2021	4	12
6. Q9	IAL P2 (WMA12) October 2021	4	14
7. Q10	IAL P2 (WMA12) January 2023	4	16
8. Q10	IAL P2 (WMA12) June 2025	4	19
9. Q3	IAL P2 (WMA12) June 2021	5	22
10. Q10	IAL P2 (WMA12) January 2022	5	26
11. Q8	IAL P2 (WMA12) June 2023	5	29
12. Q8	IAL P2 (WMA12) January 2025	5	31
13. Q8	IAL P2 (WMA12) January 2024	6	33
14. Q5	IAL P2 (WMA12) June 2024	6	35
15. Q11	IAL P2 (WMA12) October 2024	6	37
16. Q3	IAL P2 (WMA12) June 2022	7	40
P2.1b ALGEBRAIC DIVISION, FACTOR & REMAINDER THEOREMS		18 Qs · 139 marks	44
17. Q1	IAL P2 (WMA12) January 2024	3	45
18. Q1	IAL P2 (WMA12) January 2021	6	47
19. Q2	IAL P2 (WMA12) June 2023	6	51
20. Q7	IAL P2 (WMA12) June 2022	7	53
21. Q3	IAL P2 (WMA12) October 2024	7	55
22. Q3	IAL P2 (WMA12) June 2025	7	59
23. Q6	IAL P2 (WMA12) June 2019	8	63
24. Q4	IAL P2 (WMA12) October 2019	8	67
25. Q3	IAL P2 (WMA12) January 2020	8	71
26. Q4	IAL P2 (WMA12) October 2021	8	73
27. Q5	IAL P2 (WMA12) January 2022	8	77
28. Q4	IAL P2 (WMA12) June 2024	8	81
29. Q5	IAL P2 (WMA12) January 2023	9	85
30. Q4	IAL P2 (WMA12) October 2023	9	89
31. Q6	IAL P2 (WMA12) June 2024	9	93
32. Q5	IAL P2 (WMA12) January 2025	9	97
33. Q4	IAL P2 (WMA12) June 2025	9	101



QUESTION	SOURCE	MARKS	PAGE
34. Q3	IAL P2 (WMA12) October 2020	10	105



QUESTIONS

34 questions · 213 marks · 2 sub-topics

SUB-TOPIC P2.1a

PROOF (DEDUCTION, EXHAUSTION & COUNTER-EXAMPLE)

Sub-topic P2.1a · 16 questions · 74 marks · lowest tariff first



Question 1 continued

(Total for Question 1 is 3 marks)



1. Given that a, b and c are integers greater than 0 such that

- $c = 3a + 1$
- $a + b + c = 15$

prove, by exhaustion, that the product abc is always a multiple of 4

You may use the table below to illustrate your answer.

(3)

You may not need to use all rows of this table.

[illegible]

Question 1 continued

(Total for Question 1 is 3 marks)



Question 1 continued

(Total for Question 1 is 3 marks)



3. (i) Use algebra to prove that for all real values of x

$$(x - 4)^2 \geq 2x - 9$$

(3)

- (ii) Show that the following statement is untrue.

$2^n + 1$ is a prime number for all values of n , $n \in \mathbb{N}$

(1)

Leave
blank



Leave
blank

Q3

(Total 4 marks)

Leave
blank

5. (i) Use algebra to prove that for all $x \geq 0$

$$3x + 1 \geq 2\sqrt{3x}$$

(3)

- (ii) Show that the following statement is not true.

“The sum of three consecutive prime numbers is always a multiple of 5”

(1)

Question 5 continued

Leave
blank

Q5

(Total 4 marks)



Leave
blank

9. (a) Prove that for all positive values of x and y ,

$$\frac{x+y}{2} \geq \sqrt{xy}$$

(3)

- (b) Prove by counter-example that this inequality does not hold when x and y are both negative.

(1)



Question 9 continued

Leave
blank

(Total 4 marks)

Q9



10. A student was asked to prove by exhaustion that

if n is an integer then $2n^2 + n + 1$ is **not** divisible by 3

The start of the student's proof is shown in the box below.

Consider the case when $n = 3k$

$$2n^2 + n + 1 = 18k^2 + 3k + 1 = 3(6k^2 + k) + 1$$

which is not divisible by 3

Complete this proof.

(4)

Question 10 continued

Question 10 continued

(Total for Question 10 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS



10.

In this question you must show detailed reasoning.

Use algebra to prove by exhaustion that,

for all positive integers m that are **not** multiples of 3, the value of

$$m^2 + 3m + 2$$

is always a multiple of 3

(4)

Question 10 continued

Lined area for writing the answer to Question 10.



Question 10 continued

(Total for Question 10 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS



Leave
blank

3. (i) Prove that for all single digit prime numbers, p ,

$p^3 + p$ is a multiple of 10

(2)

- (ii) Show, using algebra, that for $n \in \mathbb{N}$

$(n + 1)^3 - n^3$ is not a multiple of 3

(3)

Leave
blank

Leave
blank

Question 3 continued

Leave
blank

(Total 5 marks)

Q3



“if p is a prime number then $2p + 1$ is also a prime number”
is not true.

(1)

$$5n^2 + n + 12$$

(4)



Question 10 continued

Leave
blank



Question 10 continued

Leave
blank

Q10

(Total 5 marks)

END

TOTAL FOR PAPER IS 75 MARKS



- “When a and b are consecutive **prime** numbers, $a^2 + b^2$ is never a multiple of 10”

(2)

- $$1 < x^2 - \frac{xy}{4} < 15$$

(3)



(Total for Question 8 is 5 marks)



8. (i) A student states

“If x and y are irrational numbers, $x \neq y$, then xy is also irrational.”

Show, by counter example, that this statement is not always true.

(1)

(ii) Prove, using algebra, that for all odd integers n , the value of the expression

$$n^3 + 3n + 2$$

is always even but never a multiple of 4

(4)

Question 8 continued

(Total for Question 8 is 5 marks)



8. (i) Use a counter example to show that the following statement is **false**

“ $n^2 + 3n + 1$ is prime for all $n \in \mathbb{N}$ ”

(2)

- (ii) Use algebra to prove by exhaustion that for all $n \in \mathbb{N}$

" $n^2 - 2$ is not a multiple of 4"

(4)



Question 8 continued

(Total for Question 8 is 6 marks)



5.

In this question you must show detailed reasoning.

- (a) Given that x and y are positive numbers such that

$$(x-y)^3 > x^3 - y^3$$

prove that

$$y > x$$

(4)

- (b) Using a counter example, show that the result in part (a) is not true for all real numbers.

(2)

Question 5 continued

(Total for Question 5 is 6 marks)



11. (i) Prove by counter example that the statement

“If n is a prime number then $3^n + 2$ is also a prime number.”

is false.

(2)

(ii) Use proof by exhaustion to prove that if m is an integer that is **not** divisible by 3, then

$$m^2 - 1$$

is divisible by 3

(4)



Question 11 continued

Question 11 continued

(Total for Question 11 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS



- “(n + 1)³ − n³ is prime for all n ∈ ℕ”

(ii) Given that the points $A(1, 0)$, $B(3, -10)$ and $C(7, -6)$ lie on a circle, prove that AB is a diameter of this circle.

(5)



Question 3 continued

Leave
blank



Question 3 continued

Leave
blank

Lined area for writing the answer to Question 3 continued.



Leave
blank

Q3

(Total 7 marks)



SUB-TOPIC P2.1b

ALGEBRAIC DIVISION, FACTOR & REMAINDER THEOREMS

Sub-topic P2.1b · 18 questions · 139 marks · lowest tariff first



1. $f(x) = ax^3 + 3x^2 - 8x + 2$ where a is a constant

Given that when $f(x)$ is divided by $(x - 2)$ the remainder is 3, find the value of a .

(3)

Leave
blank

1. $f(x) = x^4 + ax^3 - 3x^2 + bx + 5$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b .

(4)

Question 1 continued

Leave
blank



Question 1 continued

Leave
blank



Question 1 continued

Leave
blank

Q1

(Total 6 marks)



Question 2 continued

(Total for Question 2 is 6 marks)



7.

$$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$

- $\int_3^5 f(x) dx = 176$

find the value of A and the value of B .

(7)

Leave
blank



Question 7 continued

Leave
blank

Lined area for writing the answer to Question 7.

(Total 7 marks)

Q7



Question 3 continued



Question 3 continued

Question 3 continued

(Total for Question 3 is 7 marks)



3.

$$f(x) = (3x^2 - 4x - 5)(x - k) - 5$$

where k is a constant.

- (a) Deduce the value of the remainder when $f(x)$ is divided by $(x - k)$.

(1)

Given that the remainder when $f(x)$ is divided by $(x + 2)$ is 25

- (b) show that the value of k is -4

(2)

- (c) Hence find the quotient and remainder when $f(x)$ is divided by $(3x - 1)$

(4)



Question 3 continued



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 6 continued

Leave
blank



Question 6 continued

Leave
blank



Question 6 continued

Leave
blank

(Total 8 marks)

Q6



Leave
blank

4. $f(x) = (x - 3)(3x^2 + x + a) - 35$ where a is a constant

- (a) State the remainder when $f(x)$ is divided by $(x - 3)$. (1)

Given $(3x - 2)$ is a factor of $f(x)$,

- (b) show that $a = -17$ (2)

- (c) Using algebra and showing each step of your working, fully factorise $f(x)$. (5)

Question 4 continued

Leave
blank

Question 4 continued

Leave
blank



Question 4 continued

Leave
blank

(Total 8 marks)

Q4



Leave
blank

3.

$$f(x) = 6x^3 + 17x^2 + 4x - 12$$

- (a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

- (b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

- (c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6 \tan^3 \theta + 17 \tan^2 \theta + 4 \tan \theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

Question 3 continued

Leave
blank

Q3

(Total 8 marks)



Leave
blank

4. $f(x) = (x^2 - 2)(2x - 3) - 21$

- (a) State the value of the remainder when $f(x)$ is divided by $(2x - 3)$ (1)
- (b) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$ (2)
- (c) Hence,
 - (i) factorise $f(x)$
 - (ii) show that the equation $f(x) = 0$ has only one real root. (5)

Question 4 continued

Leave
blank



Leave
blank

Question 4 continued

Leave
blank

(Total 8 marks)

Q4



Question 5 continued

Leave
blank



Question 5 continued

Leave
blank



Question 5 continued

Leave
blank

Q5

(Total 8 marks)





Question 4 continued

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 8 marks)

5.

$$f(x) = x^3 + (p + 3)x^2 - x + q$$

where p and q are constants and $p > 0$

Given that $(x - 3)$ is a factor of $f(x)$

(a) show that

$$9p + q = -51 \quad (2)$$

Given also that when $f(x)$ is divided by $(x + p)$ the remainder is 9

(b) show that

$$3p^2 + p + q - 9 = 0 \quad (2)$$

(c) Hence find the value of p and the value of q . (3)

(d) Hence find a quadratic expression $g(x)$ such that

$$f(x) = (x - 3)g(x) \tag{2}$$

Question 5 continued



Question 5 continued



Question 4 continued





Question 4 continued

(Total for Question 4 is 9 marks)



6.

$$f(x) = 4x^3 + px^2 + 8x + q$$

where p and q are constants.

Given that

- $(2x + 3)$ is a factor of $f(x)$
- $f(x)$ has a remainder of -5 when divided by $(x + 2)$

(a) (i) show that $p = 10$

(ii) find the value of q .

(5)

(b) Hence find the range of values of x for which $f(x)$ is decreasing.

(Solutions based entirely on calculator technology are not acceptable.)

(4)





Question 6 continued



Question 5 continued



Question 5 continued



Question 5 continued

(Total for Question 5 is 9 marks)







Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 9 marks)

Question 3 continued

Leave
blank



Question 3 continued

Leave
blank



Question 3 continued

Leave
blank

(Total 10 marks)

Q3



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$; $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Marks
1.(a)	(i) $a_2 = 1$	B1
	(ii) $a_{107} = 3$	B1
		(2)
(b)	$\sum_{n=1}^{200} (2a_n - 1) = 5 + 1 + 5 + 1 + \dots + 5 + 1 = 100 \times (5 + 1)$	M1
	$= 600$	A1
		(2)
		(4 marks)
Notes		
(a) (i) B1 $a_2 = 1$ Accept the sight of 1. Ignore incorrect working (a)(ii) B1 $a_{107} = 3$ Accept sight of just 3. Ignore incorrect working If there are lots of 1's and 3's without reference to any suffices they need to choose 3. (b) M1 Establishes an attempt to find the sum of a series with two distinct terms. Look for $100 \times a + 100 \times b$ or $200 \times a + 200 \times b$ where a and b are allowable terms. Examples of allowable terms are $a, b = 1, 5$ (which are correct) $a, b = 1, 3$ (which are the values for (a)) $a, b = 3, 7$ (which is using $2a_n + 1$) $a, b = 0, 5$ (which is a slip on the first value) Methods using AP (and GP) formulae are common and score 0 marks. A1 600. 600 should be awarded both marks as long as no incorrect working is seen		

Question Number	Scheme	Marks
2.(a)	Attempts $(x \pm 2)^2 + (y \pm 5)^2 \dots = 0$ (i) Centre $(-2, 5)$ (ii) Radius $\sqrt{50}$ or $5\sqrt{2}$	M1 A1 B1 (3)
(b)	Gradient of radius $= \frac{(5)-4}{(-2)-5} = -\frac{1}{7}$ which needs to be in simplest form Uses $m_2 = -\frac{1}{m_1}$ to find gradient of tangent Equation of tangent $y - 4 = -\frac{1}{7}(x - 5) \Rightarrow y = -\frac{1}{7}x + \frac{29}{7}$	B1ft M1 M1 A1 (4) (7 marks)

Notes

(a) **Note that the open set up here is M1 M1 B1**M1 Attempts to complete the square on both terms or states the centre as $(\pm 2, \pm 5)$ For completing the square look for $(x \pm 2)^2 + (y \pm 5)^2 \dots = \dots$ A1 Centre $(-2, 5)$ Allow $x = -2, y = 5$ This alone can score both marks even following incorrect lines eg $(x + 2)^2 \dots (y - 5)^2 = \dots$ where $\dots$ could be $+$, for example a minus sign or blankA1 Radius $\sqrt{50}$ or $5\sqrt{2}$ You may award after a correct answer.If a candidate attempts to use $x^2 + y^2 + 2fx + 2gy + c = 0$ then M1 may be awarded for a centre of $(\pm 2, \pm 5)$ (b) **Note that the open set up here is M1 M1 M1 A1**B1ft Correct answer for the gradient of the line joining $P(5, 4)$ to their centre.

You may award on their centre but the value must be fully simplified.

M1 Awarded for using $m_2 = -\frac{1}{m_1}$ to find gradient of tangent.

Do be aware that some good candidates may do the first two marks at once so you may need to look at what value they are using for the gradient of the tangent.

M1 For an attempt to find the equation of the tangent using $P(5, 4)$ and a changed gradient. Condone bracketing slips only.If the candidate uses the form $y = mx + c$ they must use x and y the correct way around and proceed as far as $c = \dots$ A1 $y = 7x - 31$ stated. It must be written in this form.(It cannot be awarded from $y = mx + c$ by just stating $c = -31$).....
Attempts at (b) using differentiation.B1 $x^2 + y^2 + 4x - 10y - 21 = 0 \rightarrow 2x + 2y \frac{dy}{dx} + 4 - 10 \frac{dy}{dx} = 0$.M1 Substitutes $P(5, 4)$ into an expression of the form $ax + by \frac{dy}{dx} + c + d \frac{dy}{dx} = 0$ AND finds the value of $\frac{dy}{dx} = (7)$. The values of a, b, c and d must be non-zero.

Question Number	Scheme	Marks
1(a)	$f(-1) = (-1)^4 + a(-1)^3 - 3(-1)^2 + b(-1) + 5 = 4$	M1
	$1 - a - 3 - b + 5 = 4 \Rightarrow a + b = -1$ *	A1*
		(2)
(b)	$f(2) = (2)^4 + a(2)^3 - 3(2)^2 + b(2) + 5 = -23$	M1
	$\Rightarrow 8a + 2b = -32$ oe (eg $4a + b = -16$)	A1
	$b = -1 - a \Rightarrow 4a - 1 - a = -16 \Rightarrow a = \dots$	dM1
	$a = -5, b = 4$	A1
		(4)
		(6 marks)

(a) NB Do not apply a misread on the equation $a + b = -1$ as it gains no extra marks. They can score a maximum of M1A0 M1A1M1A0 if they use an incorrect equation such as $a + b = 1$.

M1 Attempts to substitute ± 1 into $f(x)$ and set equal to 4 to obtain an equation in a and b . Powers of -1 need not be seen, accept e.g. $1 - a - 3 - b + 5 = 4$. Condone invisible brackets on powers for the M mark, but penalise in the A (if incorrect). Alternatively, attempts long division reaching a remainder which is then set equal to 4. Look for a quotient starting $x^3 + \dots$ and linear remainder in a and b . It may be seen in tabulated form.
Another alternative method: Writes $f(x) = (x+1)(x^3 + \alpha x^2 + \beta x + \gamma) + 4$ (allow with $\gamma = 1$), expands and equates coefficients, and proceeds to eliminate α, β and γ from the equations.

A1* Rearranges the equation with no errors and achieves the given answer with at least one intermediate line with power evaluated. FYI by long division the quotient is $x^3 + (a-1)x^2 + (-a-2)x + a+b+2$ and the remainder is $3-a-b$.
By equating coefficients look for $\gamma + 4 = 5$ (may be implied), $a = \alpha + 1$, $b = \beta + 1$, $-3 = \alpha + \beta$ being solved correctly.

(b)
M1 Attempts to substitute ± 2 into $f(x)$ and set equal to ± 23 to obtain an equation in a and b . Again, accept attempts at long division leading to a remainder linear in a and b (which is $8a + 2b + 9$), which is then set equal to ± 23 , or via equating coefficients (look for at least three equations formed). May have substituted for a or b already.

A1 $8a + 2b = -32$ oe need not be fully gathered but powers should be evaluated. So e.g. accept $16 + 8a - 12 + 2b + 5 = -23$ or accept $16 + 8a - 12 + 2(-a-1) + 5 = -23$ if substitution for b happens first (similar for a). Via equating coefficients, all equations should be correct.

dM1 Attempts to solve simultaneously and achieves a value for a or b . It is dependent on the previous method mark. Allow as long as values for a or b appears after writing out their two equations in both a and b (or solving a linear equation if substitution occurs before remainder theorem). Allow for slips in copying their initial equations.

A1 $a = -5, b = 4$ cao



Question Number	Scheme	Marks
2(a)	$\left(\frac{dy}{dx} = \right) 3x^2 - 2x - 16$	M1A1
	$3x^2 - 2x - 16 = 0 \Rightarrow x = \dots$	M1
	$x = \frac{8}{3}, -2$	A1
		(4)
(b)	$\left(\frac{d^2y}{dx^2} = \right) Cx + D$	M1
	$\Rightarrow \frac{d^2y}{dx^2} = C \times \dots + D (> 0) \Rightarrow \text{min or } (< 0) \Rightarrow \text{max}$	dM1
	All three components: $\left(\frac{d^2y}{dx^2} = \right) 6x - 2$ when $x = \frac{8}{3}$, $\frac{d^2y}{dx^2} = \dots (= 14) > 0 \Rightarrow \text{min}$ when $x = -2$, $\frac{d^2y}{dx^2} = \dots (= -14) < 0 \Rightarrow \text{max}$	A1
		(3)
Alt (b)	E.g. $x = -2.5, \frac{dy}{dx} = \frac{31}{4}, x = 0, \frac{dy}{dx} = -16$	M1
	E.g. $x = -2.5, \frac{dy}{dx} = \frac{31}{4} > 0, x = 0, \frac{dy}{dx} = -16 < 0$ so $x = -2$ is a maximum $\left(x = 0, \frac{dy}{dx} = -16 < 0\right), x = 3, \frac{dy}{dx} = 5 > 0$, so $x = \frac{8}{3}$ is a minimum	dM1 A1
		(3)
		(7 marks)

(a)

M1 Differentiates to achieve $\left(\frac{dy}{dx} = \right) Ax^2 + Bx + C$ where $A, B, C \neq 0$

A1 $\left(\frac{dy}{dx} = \right) 3x^2 - 2x - 16$

M1 Sets their $\frac{dy}{dx} = 0$ and attempts to solve their quadratic. Usual rules apply. Setting equal to zero may be implied by an attempt to solve the quadratic.

A1 $x = \frac{8}{3}, -2$ or equivalent single fraction (do not accept 2.67 but $2.\dot{6}$ can be accepted.)

(b)

M1 Attempts to differentiate their $\frac{dy}{dx}$ to achieve $\left(\frac{d^2y}{dx^2} = \right) Cx + D$ where $C, D \neq 0$. Allow if found in part (a)

dM1 Substitutes in one of their x values for part (a) into their $\frac{d^2y}{dx^2}$ and draws a consistent conclusion with their value found. Accept with correct unsimplified values **OR** with correct reason, e.g. $\frac{d^2y}{dx^2} < 0$ hence maximum, if no substitution seen It is dependent on the previous method mark.
Accept “concave down” for a maximum and “concave up” for a minimum.

A1 Correct second derivative, correct values (need not be simplified but if simplified values seen they must be correct) for $\frac{d^2y}{dx^2}$ at $x = \frac{8}{3}, -2$ and draws a correct conclusion with reason (>0 or <0 , positive or negative) for both. It must be clear which point corresponds with each conclusion.
NB Allow M1dM1A0 if no substitution is seen as long as reason is given, e.g. When $x = -2$, $\frac{d^2y}{dx^2} < 0$ hence max.

Alt (b)

M1 Attempts to evaluate $\frac{dy}{dx}$ both side of at least one of their stationary points.

dM1 Evaluates $\frac{dy}{dx}$ between their two stationary points and the other side of at least one of them, considers signs and draws a consistent conclusion.

A1 Evaluates $\frac{dy}{dx}$ correctly either side of both of their stationary points (three points required, you may need to check the evaluation) and draws correct conclusions for both.

A method via calculus is required, so reasoning from the shape of a cubic scores no marks.

Question Number	Scheme	Marks
3(i)	$7^{x+2} = 3 \Rightarrow 7^2 \times 7^x = 3$	M1
	$\Rightarrow x = \log_7 \frac{3}{7^2}$	A1
	$x = \log_7 \frac{3}{49}$	A1
		(3)
Alt(i)	$7^{x+2} = 3$	
	$(x+2)\log_7 7 = \log_7 3$	M1
	$\Rightarrow x = \log_7 3 - 2$	A1
	$x = \log_7 \frac{3}{49}$	A1
		(3)
(ii)	$1 + \log_2 y + \log_2 (y+4) = \log_2 (5-y)$	
	E.g. $1 + \log_2 y(y+4) = \log_2 (5-y)$ or $1 \rightarrow \log_2 2$	M1
	$\log_2 \left(\frac{y(y+4)}{5-y} \right) = -1 \Rightarrow \frac{y(y+4)}{5-y} = \frac{1}{2}$ or $\log_2 (2y(y+4)) = \log_2 (5-y) \Rightarrow 2y(y+4) = 5-y$	dM1
	$2y^2 + 9y - 5 = 0$	A1
	$(2y-1)(y+5) = 0 \Rightarrow y = \dots$	ddM1
	$y = \frac{1}{2}$	A1
		(5)
		(8 marks)

(i) Note this appears as MMA on ePEN but is being marked MAA.

M1 Applies the addition law to write the equation in the form $7^{\dots} \times 7^x = 3$.
Alternatively takes logs on both sides and applies the power law (condone invisible brackets). Allow any base for the logarithm.

A1 Any correct exact expression for x , such as $x = \log_7 \frac{3}{7^2}$, $\log_7 3 - \log_7 49$, $\log_7 3 - 2$,
 $\frac{\log 3}{\log 7} - 2$ etc. Allow with any base for the logarithm.

A1 $\log_7 \frac{3}{49}$ Correct answer only.

(ii)

M1 Correctly applies the addition or subtraction law of logs at least once to combine two terms at some stage in their working (may be on incorrect terms) **or** correctly replaces 1 with $\log_2 2$ at some point (oe).

dM1 Attempts to rearrange the equation to the form $\log_2(\dots) = C$ or $\log_2(\dots) = \log_2(\dots)$ **with no incorrect log** work and removes the logs correctly. (There may be slips when rearranging terms but all log work should be correct.) It is dependent on the previous method mark. May see $\log_2 2y(y+4) = \log_2(5-y) \Rightarrow 2y(y+4) = 5-y$ (may be implied by a correct equation).

A1 $2y^2 + 9y - 5 = 0$ oe 3TQ

ddM1 Attempts to solve their quadratic. Usual rules apply. It is dependent on them achieving a 3TQ and both previous method marks.

A1 $y = \frac{1}{2}$ only (accept decimal equivalent). (The -5 must have been rejected if seen.)

SC Allow M1dM0A0 SC M1A1 (if the A is appropriate) for the solution

$$1 + \log_2 y + \log_2(y+4) = \log_2(5-y) \rightarrow \frac{\log_2 y(y+4)}{\log_2(5-y)} = -1 \rightarrow \log_2 \frac{y(y+4)}{5-y} = -1 \rightarrow \frac{y^2 + 4y}{5-y} = \frac{1}{2}$$

etc, allowing the recovery from the incorrect division of logs as a log for the last two marks.

Question Number	Scheme	Marks
9.(a)	(If x and y are positive) $(\sqrt{x} - \sqrt{y})^2 \dots 0 \Rightarrow x - \dots \sqrt{xy} + y \dots$ $\Rightarrow x - 2\sqrt{xy} + y \dots$ $\Rightarrow \frac{x+y}{2} \dots \sqrt{xy}$	M1 A1 A1* (3)
(b)	States for example when $x = -8, y = -2, \frac{x+y}{2} = -5, \sqrt{xy} = 4$ so $\frac{x+y}{2} \leq \sqrt{xy}$	B1 (1) (4 marks)

(a)

M1: Sets up a correct inequality and attempts to expand $(\sqrt{x} - \sqrt{y})^2$ leading to three terms.

A1: Correct expanded equation.

A1: Rearranges to the required equation with no errors seen.

If working in reverse allow the first M and A (if steps correct) but require also a minimal conclusion for the final A.

(b)

B1: Gives a suitable example with both sides evaluated correctly and a minimal conclusion.

There is no need to refer to x and y in the conclusion, so long as it has been shown the required inequality does not hold. E.g. $\frac{-8-2}{2} = -5 \not\leq 4 = \sqrt{16} = \sqrt{-8 \times -2}$ QED is fine.

A common response most likely to score 2 out of 3 marks

Question Number	Scheme	Marks
9.(a) Alt 1	$\frac{x+y}{2} \dots \sqrt{xy} \Rightarrow \frac{(x+y)^2}{4} \dots xy \Rightarrow \frac{x^2 + \dots xy + y^2}{4} \dots xy$ $\Rightarrow x^2 - 2xy + y^2 \dots 0 \Rightarrow (x-y)^2 \dots 0$ States both of the following o.e • $(x-y)^2 \dots 0$ as it is a square number • so $\frac{x+y}{2} \dots \sqrt{xy}$ is true	M1 A1 A1* (3)

M1: Assumes $\frac{x+y}{2} \dots \sqrt{xy}$ true and attempts to square obtaining at least three terms.

A1: Correct expansion and rearranges the inequality correctly to factorise to a perfect square.

A1: A complete conclusion given.

Question Number	Scheme	Marks
9.(a) Alt 2	States $(x-y)^2 \dots 0 \Rightarrow x^2 - 2xy + y^2 \dots 0$ Rearranges $\Rightarrow x^2 + 2xy + y^2 \dots 4xy \Rightarrow (x+y)^2 \dots 4xy$ States that as x, y positive, so $x+y > 0$ (and $xy > 0$) $\Rightarrow (x+y) \dots \sqrt{4xy} \Rightarrow \frac{x+y}{2} \dots \sqrt{xy}$	M1 A1 A1* (3)

M1: Sets up an inequality using an appropriate perfect square and expands to at least three terms.

A1: Makes a correct rearrangement and factors the left hand side to produce the equation shown.

A1: Makes a full conclusion justifying why the square root gives $x+y$.

Question Number	Scheme	Marks
10	Considers another relevant case for n e.g. $n = 3k - 1$ or $3k + 1$ or $3k + 2$ For example $n = 3k + 1$ $\Rightarrow 2n^2 + n + 1 = 2(3k + 1)^2 + (3k + 1) + 1 = 18k^2 + 15k + 4$	M1
	$= 3(6k^2 + 5k + 1) + 1$ which is not divisible by 3	A1
	Considers a third relevant case to complete the 3 cases for n e.g. one of $n = 3k - 1$ or $3k + 1$ or $3k + 2$ For example $n = 3k - 1$ $\Rightarrow 2n^2 + n + 1 = 2(3k - 1)^2 + (3k - 1) + 1 = 18k^2 - 9k + 2$	dM1
	$= 3(6k^2 - 3k) + 2$ which is not divisible by 3 Hence $2n^2 + n + 1$ is not divisible by 3	A1
		(4)
		Total 4

Condone the use of another letter or even n for the different cases for the first 3 marks.

Do not withhold accuracy marks for any errors which are not part of the main body of the work.

Look at the whole solution and score for a correct case first (which may appear later) so that the first two marks M1A1 can be awarded.

- M1 Considers another relevant case for n by attempting to substitute in their expression in k , multiplies out brackets and collects terms. Condone slips.
May attempt eg $3k + 4$ or any other multiples of those given which is acceptable.
- A1 Completes the process with no errors to show their expression is not divisible by 3. Usually this is by factorising part of the expression, but may identify a particular term which is not divisible by 3.
- dM1 Considers a third relevant case which is different to the given case and not an equivalent one to their first relevant case to complete the 3 cases for n by attempting to substitute the final different expression in k , multiplies out brackets and collects terms. Condone slips.
- A1 Completes the process with no errors to show their 3rd expression is not divisible by 3 (usually by factorising, but may identify a particular term which is not divisible by 3) and concludes the proof with a statement e.g. hence $(2n^2 + n + 1)$ not divisible by 3. Alternatively, they may write a statement/preamble at the beginning and the proof can be ended with a tick/QED oe
Must have been in terms of k for all cases.

The table below may help when checking for accuracy of quadratics

Case	$2n^2 + n + 1$	Possible factorisation of expression
$3k - 2$	$18k^2 - 21k + 7$	$= 3(6k^2 - 7k + 2) + 1$
$3k - 1$	$18k^2 - 9k + 2$	$= 3(6k^2 - 3k) + 2$
$3k + 1$	$18k^2 + 15k + 4$	$= 3(6k^2 + 5k + 1) + 1$
$3k + 2$	$18k^2 + 27k + 11$	$= 3(6k^2 + 9k + 3) + 2$
$3k + 4$	$18k^2 + 51k + 37$	$= 3(6k^2 + 17k + 12) + 1$

Question Number	Scheme	Marks
10	Sets $m = 3p + 1$ or $m = 3p + 2$ ($p > 0$) o.e. and attempts $m^2 + 3m + 2$	M1
	E.g. $m^2 + 3m + 2 = (3p + 1)^2 + 3(3p + 1) + 2 = 9p^2 + 15p + 6 = 3(3p^2 + 5p + 2)$	A1
	Sets $m = 3p + 1$ and $m = 3p + 2$ ($p > 0$) and attempts $m^2 + 3m + 2$	dM1
	E.g. $m^2 + 3m + 2 = (3p + 2)^2 + 3(3p + 2) + 2 = 9p^2 + 21p + 12 = 3(3p^2 + 7p + 4)$ And gives a minimal conclusion *	A1* (4 marks)

Main scheme algebraic method using e.g. $m = 3p + 1$ and $m = 3p + 2$

You will need to look at both cases and mark the one which is fully correct first.

Allow a different variable to p and may be different letters for the two cases.

Condone use of m as a variable for the first three marks.

There should be no errors in the algebra for the A marks including invisible brackets.

Note that there are other allowable pairs of combinations covering the two distinct cases

e.g. $m = 3p - 1$ and $m = 3p + 1$ or e.g. $m = 3p - 2$ and $m = 3p + 2$

but not e.g. $m = 3p - 1$ and $m = 3p + 2$

Note: An attempt substituting in values rather than using algebra will score 0 marks.

M1: Sets $m = 3p + 1$ o.e. **or** $m = 3p + 2$ o.e., expands $m^2 + 3m + 2$ and achieves a quadratic expression which may be unsimplified

A1: Correct quadratic expression with constant terms collected for $m^2 + 3m + 2$ for either of the two cases and shows or gives a reason why the expression is a multiple of 3. They must have fully multiplied out or the quadratic expression must have a factor of 3 taken out.
Do not isw if they simplify their quadratic incorrectly.

e.g. $9p^2 + 15p + 6$ and states each term is a multiple of 3 or alternatively, correctly divides by 3 and concludes that it is divisible by 3

e.g. $3(3p^2 + 5p + 2)$ or e.g. $3(3p^2 + 2p) + 3(3p + 1) + 3$

e.g. $9p^2 + 21p + 12$ and states each term is a multiple of 3 or alternatively correctly divides by 3 and concludes that it is divisible by 3

e.g. $3(3p^2 + 7p + 4)$

dM1: Sets $m = 3p + 1$ o.e. **and** $m = 3p + 2$ o.e. and expands $m^2 + 3m + 2$ and achieves two quadratic expressions which may be unsimplified. See first M1 for guidance. It is dependent on the previous method mark.

A1*: Requires

- correct quadratic expression for $m^2 + 3m + 2$ for both cases which is not in terms of m
- shows or gives a reason for why the expressions are multiples of 3 (see first A1 for guidance)
- makes a concluding overall statement. “Hence $m^2 + 3m + 2$ is always a multiple of 3 for all m that are not multiples of 3” (or equivalent) Allow generic conclusions e.g. “proven”, “hence true”

Withhold the final mark if they have a spurious $= 0$ on the end of their quadratic expressions.

Below is a table for the different possible cases and the quadratic expressions:

	$m^2 + 3m + 2$
$3p - 2$	$9p^2 - 3p = 3(3p^2 - p)$
$3p - 1$	$9p^2 + 3p = 3(3p^2 + p)$
$3p + 1$	$9p^2 + 15p + 6 = 3(3p^2 + 5p + 2)$
$3p + 2$	$9p^2 + 21p + 12 = 3(3p^2 + 7p + 4)$

Factorising first then using logic

Note the same marking principle can be applied as in the main scheme and notes

$$m^2 + 3m + 2 = (m+1)(m+2)$$

Sets e.g. $m = 3p + 1$ leading to $m^2 + 3m + 2 = (3p+1+1)(3p+1+2) = 3(3p+2)(p+1)$

Sets e.g. $m = 3p - 1$ leading to $m^2 + 3m + 2 = (3p-1+1)(3p-1+2) = 3p(3p+1)$

Alternative using algebra with logic

$$m^2 + 3m + 2 = (m+1)(m+2)$$

e.g. $(m+1)(m+2)$ represents two consecutive numbers and if m is not a multiple of 3 then one of $(m+1)$ or $(m+2)$ must be a multiple of 3 hence $m^2 + 3m + 2$ is a multiple of 3.

If you see this approach or other attempts that you think may be credit-worthy then send to review.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Summary of changes from Provisional Mark Scheme

A few minor changes were made to the Mark Scheme before marking on the marking service began.

Question Number	Summary of changes
Q03	Removed reference to ePEN marking traits. Added clarification in parenthesis on the M and first A mark how to mark cases where students split into cases for odd and even integers
Q04	Added “at least twice in the x , x^2 and x^3 terms” to the M1 notes
Q06	Added clarification that the 0 ordinate need not be seen, and the coordinates may be seen anywhere.
Q08	Added that part (ii) may be marked as a whole (ii) M1 added “may be seen in (b)”, A1 added “result must be clearly stated but may be seen in (b).”

Question Number	Scheme	Notes	Marks
10(i)	E.g. $p = 7 \Rightarrow 2p + 1 = 15$ Which is not a prime number (so the statement is not true)	Identifies a counter example and makes a conclusion/shows it is not prime.	B1
			(1)
(ii)	$n \text{ odd} \Rightarrow n = 2k + 1$ $\Rightarrow 5n^2 + n + 12 = 5(2k + 1)^2 + 2k + 1 + 12$ or $n \text{ even} \Rightarrow n = 2k$ $\Rightarrow 5n^2 + n + 12 = 5(2k)^2 + 2k + 12$	Starts the proof by considering n odd or n even and substituting into the expression (see notes for logical approach)	M1
	$n \text{ odd} \Rightarrow n = 2k + 1$ $\Rightarrow 5n^2 + n + 12 = 5(2k + 1)^2 + 2k + 1 + 12$ and $n \text{ even} \Rightarrow n = 2k$ $\Rightarrow 5n^2 + n + 12 = 5(2k)^2 + 2k + 12$	Considers n odd and n even (as above)	M1
	$n \text{ odd: } 20k^2 + 22k + 18 \text{ which is even}$ and $n \text{ even: } 20k^2 + 2k + 12 \text{ which is even}$	Attempts both with at least one correct expression that is stated to be even	A1
	$n \text{ odd: } 2(10k^2 + 11k + 9)$ and $n \text{ even: } 2(10k^2 + k + 6)$ These are both even so $5n^2 + n + 12$ must be even for all integers n	Fully correct proof that considers both n odd and n even, shows the resulting expressions are even and makes a suitable conclusion	A1
			(4)
			Total 5

Notes**(i)**

B1: For any correct counter example shown with calculation and suitable conclusion. Another common suitable example is $p = 13$, as $2 \times 13 + 1 = 27 = 3^3$ so not prime. Accept "not prime" or showing not prime by indicating a factor as conclusion. E.g. a minimal acceptable answer is " $2 \times 7 + 1 = 15$ not prime", or " $2 \times 7 + 1 = 15 = 5 \times 3$ "

(ii) Note No marks if all they do is try a few cases.

M1: Considers the case of n being odd or n being even by setting it up algebraic substituting $n = 2k$ or $n = 2k \pm 1$ into the quadratic expression. Allow with variables other than k – including n .

M1: Considers both odd and even cases, same criteria for each as in the first M.

A1: Both cases attempted with at least one correct expression (allow with n) simplified to a point where all coefficients are even integers with deduction made that the expression is even.

Note for $n = 2k - 1$ it is $20k^2 - 18k + 16 = 2(10k^2 - 9k + 8)$

A1: Complete proof with all work correct and with factor 2 seen extracted in both expressions or explanation that all terms are multiples of 2/even hence it is even, and conclusion that the given expression is even for all integers. **Must be using a variable other than n for this mark.**

For approaches via "logic" a maximum of M1M1A0A0 is possible unless all relevant "odd $\times$ odd = odd" properties are also proved (which is unlikely to occur).

M1: Attempts logic reasoning that builds by term and gets at least as far as explaining a product of two terms, so "if n is odd/even then n^2 is odd/even" or equivalent if they have



Question Number	Scheme	Marks
7a	$H = t^{\frac{1}{2}} + \frac{3t^{\frac{3}{2}}}{10} - \frac{t^{\frac{5}{2}}}{20} + 17 \Rightarrow \left(\frac{dH}{dt} = \right) \frac{1}{2}t^{-\frac{1}{2}} + \frac{9}{20}t^{\frac{1}{2}} - \frac{1}{8}t^{\frac{3}{2}}$	M1A1A1
	e.g. $\frac{1}{2}t^{-\frac{1}{2}} + \frac{9}{20}t^{\frac{1}{2}} - \frac{1}{8}t^{\frac{3}{2}} = 0 \Rightarrow \frac{1}{2} + \frac{9}{20}t - \frac{1}{8}t^2 = 0 \dots \Rightarrow 5\alpha^2 - 18\alpha - 20 = 0$ *	dM1A1*
		(5)
b	$\alpha = \frac{9 + \sqrt{181}}{5} = \text{awrt } 4.491$	B1
		(1)
c	$\left(\frac{d^2H}{dt^2} = \right) -\frac{1}{4}t^{-\frac{3}{2}} + \frac{9}{40}t^{-\frac{1}{2}} - \frac{3}{16}t^{\frac{1}{2}} \Rightarrow -\frac{("4.49")^{\frac{3}{2}}}{4} + \frac{9("4.49")^{\frac{1}{2}}}{40} - \frac{3("4.49")^{\frac{1}{2}}}{16}$	M1
	$\frac{d^2H}{dt^2} = (-0.3\dots) < 0 \Rightarrow \text{max} *$	A1*
		(2)
		(8 marks)

(a) If they change t to α before differentiating then score maximum M1A1A0dM1A0

M1 Multiplies out and attempts to differentiate. Award for one correct index from correct working (may be implied if they multiply out and differentiate at the same time) $t^{\frac{3}{2}}$ or $t^{\frac{1}{2}}$ or $t^{-\frac{1}{2}}$ (condone if these are $\alpha^{\frac{3}{2}}$ or $\alpha^{\frac{1}{2}}$ or $\alpha^{-\frac{1}{2}}$ or any other variable) The indices do not need to be processed for this mark.

e.g. $t^{\frac{5}{2}-1}$

Alternatively, they may attempt the product rule so look for a correct general expression such as

$\left(\frac{dH}{dt} = \right) \dots t^{-\frac{1}{2}}(20 + 6t - t^2) + \dots t^{\frac{1}{2}}(A + Bt)$ (condone if in terms of α or another variable instead)

A1 One of $\frac{1}{2}t^{-\frac{1}{2}}$, $\frac{9}{20}t^{\frac{1}{2}}$, $-\frac{1}{8}t^{\frac{3}{2}}$ (simplified or unsimplified) via the main scheme

If they differentiate using the product rule then this mark can only be awarded for one of

$\frac{1}{40}t^{-\frac{1}{2}}(20 + 6t - t^2) + \dots t^{\frac{1}{2}}(A + Bt)$ or $\dots t^{-\frac{1}{2}}(20 + 6t - t^2) + \frac{1}{20}t^{\frac{1}{2}}(6 - 2t)$ (unsimplified).

In both cases the indices must be processed. Condone if in terms of α

A1 $\left(\frac{dH}{dt} = \right) \frac{1}{2}t^{-\frac{1}{2}} + \frac{9}{20}t^{\frac{1}{2}} - \frac{1}{8}t^{\frac{3}{2}}$ or equivalent e.g. $\left(\frac{dH}{dt} = \right) \frac{1}{40}t^{-\frac{1}{2}}(20 + 18t - 5t^2)$ which may be unsimplified. Also allow

e.g. $\left(\frac{dH}{dt} = \right) \frac{1}{40}t^{-\frac{1}{2}}(20 + 6t - t^2) + \frac{1}{20}t^{\frac{1}{2}}(6 - 2t)$ (via the product rule)

dM1 Sets their derivative of the form $\dots t^{-\frac{1}{2}} \pm \dots t^{\frac{1}{2}} \pm \dots t^{\frac{3}{2}} = 0$ (may be implied) and proceeds via a correct method to a 3TQ in t or α (or any other variable) where the powers and the coefficients of the terms are integers. Condone sign slips only. It is dependent on the previous method mark.

Proceeding directly to the given answer is M0, unless they state how they have proceeded

from the derivative directly to the given answer e.g. set $t = \alpha$ and $\times -40t^{\frac{1}{2}}$



Question Number	Scheme	Marks
8(i)	Provides a counter example E.g. $\sqrt{2} \times \sqrt{8} = 4$	B1
(ii)	Knows that odd numbers are of the form $2k + 1$, ($k \in \mathbf{N}$)	B1
	Attempts $(2k + 1)^3 + 3(2k + 1) + 2 = \dots$	M1
	$= 8k^3 + 12k^2 + 12k + 6$	A1
	$= 4 \times (2k^3 + 3k^2 + 3k + 1) + 2$	A1*
	Which is even but not a multiple of 4. Hence proven	(4)
		(5 marks)

(i)

B1: Provides a counter example and shows that the statement is not always true.

The correct result must be written out showing that it is rational (that is, giving it in a rational form)

 So $\sqrt{2} \times \sqrt{8} = \sqrt{16}$ is insufficient but $\frac{\pi}{2} \times \frac{6}{\pi} = \frac{6}{2}$ is acceptable

(ii)

 A proof attempted by substituting odd numbers $n = 1, 3, 5$ etc into $n^3 + 3n + 2$ will not score any marks

 B1: Knows that odd numbers are of the form $2k \pm 1$ using any variable apart from n

 Other variations may be used, for example $2k \pm 3$, $2k \pm 5$, etc but will be much less common.

 Note that use of $4k + 1$ is flawed as it doesn't pick up all the odd numbers so on its own scores B0

 M1: Attempts to expand $(2k + 1)^3 + 3(2k + 1) + 2$ or $(2k - 1)^3 + 3(2k - 1) + 2$ using any variable including n .

 Generally, look for a minimum of $(2k + 1)^3 + 3(2k + 1) + 2 \rightarrow$ a cubic expression in k

 As with the B mark, other variations may be used, for example $2k \pm 3$ but will be much less common.

 The use of $n = 4k + 1$ in $n^3 + 3n + 2$ and expanded to a cubic form can be awarded this M mark only

 A1: A correct expansion for $(2k + 1)^3 + 3(2k + 1) + 2$ Allow the use of any variable including n

 Look for $8k^3 + 12k^2 + 12k + 6 \dots$

 but ...allow any correct expression that allows the problem to be solved e.g. $4(2k + 1)(k^2 + k + 1) + 2$

 FYI the correct expansion for $(2k - 1)^3 + 3(2k - 1) + 2$ is $8k^3 - 12k^2 + 12k - 2$

A1*: Requires the candidate to have fully correct algebra, scored all 3 previous marks and

1) prove/show that the result is even

2) prove/show that the result is not a multiple of 4

3) give a minimal conclusion

Examples of expressions, for $n = 2k + 1$, that show the result is even;

- $8k^3 + 12k^2 + 12k + 6$ with a statement saying that it is a sum of even numbers
- $(8k^3 + 12k^2 + 12k + 6) \div 2 = 4k^3 + 6k^2 + 6k + 3$
- $8k^3 + 12k^2 + 12k + 6 = 2(4k^3 + 6k^2 + 6k + 3)$

Examples of expressions, for $n = 2k + 1$, that show the result is not a multiple of 4

- $8k^3 + 12k^2 + 12k + 6 = 2(4k^3 + 6k^2 + 6k + 3)$ with some statement alluding to the fact that another factor of 2 cannot be taken out of the $(4k^3 + 6k^2 + 6k + 3)$
- $(8k^3 + 12k^2 + 12k + 6) \div 4 = 2k^3 + 3k^2 + 3k + \frac{3}{2}$
- $8k^3 + 12k^2 + 12k + 6 = 4(2k^3 + 3k^2 + 3k + 1.5)$

Examples of expressions, for $n = 2k + 1$, that show the result is both even but not a multiple of 4

- $8k^3 + 12k^2 + 12k + 6 = 4(2k^3 + 3k^2 + 3k + 1) + 2$
- $8k^3 + 12k^2 + 12k + 6 = 4 \times (2k^3 + 3k^2 + 3k) + 6$



(b)

M1: Attempts to use the GP formula in an attempt to find 'r'

Look for $4000 = 300r^{11} \Rightarrow r^{11} = \frac{4000}{300} \Rightarrow r = \sqrt[11]{\frac{4000}{300}}$ condoning slips. Implied by $r = \text{awrt } 1.3$ provided they do not start from an incorrect equation.

M1: A correct attempt to find the **second term** (do not allow a misread) by multiplying 300 by their "1.266..." . It is dependent on an attempt to find r from either $4000 = 300r^{11}$ or $4000 = 300r^{12}$ condoning slips. If no method is seen to find r then it must be correct. e.g. following $4000 = 300r^{12}$ or $\sqrt[12]{\frac{4000}{300}}$.

Condone slips.

A1: awrt 380 (to the nearest 10) (tonnes) isw once a correct answer is seen.

(c) **Condone use of their d and their r even if they have come from incorrect methods**

M1: A correct method to find the sum of either the AP or the GP

For the AP accept an attempt at either $\frac{12}{2}\{300 + 4000\}$ or $\frac{12}{2}\{2 \times 300 + 11 \times "336"\}$

For the GP accept an attempt at either $\frac{300("1.266..."^{12} - 1)}{"1.266..." - 1}$ or $\frac{300(1 - "1.266..."^{12})}{1 - "1.266..."}$

dM1: Both formulae must be attempted "correctly" (see above) and the difference taken

FYI if d and r are correct, the sums are 25 800 tonnes and 17 935A1: Difference = 7860 or 7870 (tonnes) **This is NOT AWRT.**(Note that use of $r = 1.266$ will lead to a difference of 7810 which is max M1dM1A0)**Alt (c) Listing values**

Accept listing the values

M1: Attempts to add the 12 terms for the arithmetic or geometric series. Must include 300 and 4000, but condone one omission of one of the middle terms. May be implied by calculating the difference between the AP and GP values for each year and adding those together.

dM1: Attempts to add the 12 terms for the arithmetic and geometric series. Must include 300 and 4000 for both sums, but condone one omission of one of the middle terms in each of the summations. May be implied.

A1: Difference = 7860 or 7870 **This is NOT AWRT.**

Year	AP	GP	Difference
1	300	300	0
2	636.3636	379.6544	256.7093
3	972.7273	480.4582	492.2691
4	1309.091	608.0268	701.0641
5	1645.455	769.4668	875.9877
6	1981.818	973.7715	1008.047
7	2318.182	1232.322	1085.86
8	2654.545	1559.521	1095.024
9	2990.909	1973.597	1017.312
10	3327.273	2497.616	829.6567
11	3663.636	3160.77	502.8668
12	4000	4000	0
Sum	25800	17935.2	7864.796



Question Number	Scheme	Marks
4.(a)	21	B1 (1)
(b)	Sets $f\left(\pm\frac{1}{2}\right) = 0 \rightarrow$ equation in k Eg. $\left(\frac{1}{2} - 2\right)\left(2 \times \frac{1}{4} + 5 \times \frac{1}{2} + k\right) + 21 = 0$ $\Rightarrow 3 + k = 14 \Rightarrow k = 11$ *	M1 A1* (2)
(c)	(i) $f(x) = (x-2)(2x^2 + 5x + 11) + 21 = 2x^3 + x^2 + x - 1$ $= (2x-1)(x^2 + x + 1)$ (ii) Attempts to find the number of roots of their $(x^2 + x + 1)$ - States $(x^2 + x + 1)$ has no roots with reason - Concludes $f(x) = 0$ has 1 root (at $x = \frac{1}{2}$)	M1 dM1, A1 M1 A1 (5) (8 marks)

(a)

B1: States 21

(b) If division is attempted in (b), mark (b) and (c) as a whole.

M1: Sets $f\left(\pm\frac{1}{2}\right) = 0 \rightarrow$ equation in k . The “= 0” may be implied. If division is used look for setting the remainder after dividing by $(2x - 1)$ equal to zero to form an equation in k .

A1*: Solves a correct equation in k showing at least one correct intermediate line (the “= 0” may be implied but must have been a correct equation preceding the answer).
Condone missing brackets if clearly recovered.

May see $k = 11$ embedded so $\left(-\frac{3}{2}\right)(3+11) + 21 = -21 + 21 = 0 \checkmark$ is acceptable.

(c) (i) Only allow work carried out in (b) to count in (c) if carried forward into (c).

M1: Attempts to multiply out $(x-2)(2x^2 + 5x + 11) + 21$ to achieve a cubic. If an incorrect value for k is used, allow two M’s for equivalent work.

dM1: Attempts to divide by or factor out $(2x-1)$ or $\left(x - \frac{1}{2}\right)$ from their attempt at expanding $(x-2)(2x^2 + 5x + 11) + 21$.

Long division – look for the $x^2 +$ extra terms and first subtraction

$$\begin{array}{r} x^2 + \dots \\ 2x-1 \overline{) 2x^3 + x^2 + x - 1} \\ \underline{2x^3 - x^2} \\ "2x^2" + \dots \\ \dots \end{array}$$

Inspection/comparing coefficients – look for correct magnitudes for first and last term in the brackets. $(2x-1)(x^2 \pm \dots \pm 1)$



A1: $(2x-1)(x^2+x+1)$ from correct work. Must be brought together in one expression.

Accept $2\left(x-\frac{1}{2}\right)(x^2+x+1)$ but not $\left(x-\frac{1}{2}\right)(2x^2+2x+2)$ Condone if set equal to zero.

Award A0 if they subsequently incorrectly further factorise the quadratic term.

(c)(ii)

M1: Must have attempted to take the correct factor out of their cubic. Attempts to find the number of roots of their (x^2+x+1) **not of** $2x^2+5x+11$ Allow for an attempt **any** of the following:

- an attempt at b^2-4ac
- an attempt at the formula or completing the square
- correct (imaginary) roots from a calculator (must be seen)

Do not accept “no roots” (oe) seen with no evidence for why. Do not accept “Math Error” without an explanation of the error.

A1: Requires all of the following (or equivalents)

- correct factorisation $f(x)=(2x-1)(x^2+x+1)$ or correct separate factors
- correct reason as to why $(x^2+x+1)=0$ hasn't any roots. E.g

$b^2-4ac=1-4(-3)<0$ hence no roots. There must be a (correct) calculation, reference to the sign of the discriminant and deduction/implication of no roots, or equivalent via other methods

- concludes $f(x)=0$ has one solution

Ignore erroneous extra statements after a correct reason has been given.

Some examples (following correct factorisation).

Scores M1A1:

$x^2+x+1 \rightarrow \Delta=1-4<0$ no real solutions therefore $x=\frac{1}{2}$ is only solution.

$b^2-4ac=-3<0$ so no solutions to x^2+x+1 hence $f(x)$ has only one real solution $(2x-1)$.
We ignore any reference to $(2x-1)$ being the solution.

Scores M1A0:

(ii) one real solution as x^2+x+1 has $b^2-4ac=-3$ so no real solution. *No reference to <0*

$b^2-4ac=1^2-4(1)(1)=-3\leq 0$ so there is only 1 real solution as $x^2+x+1\leq 0$ so the only real solution is $(2x-1)$ which is $x=\frac{1}{2}$ *Needs <0 to be stated*

$\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2}$ no solution, $2x-1$, $x=\frac{1}{2}$ is the only real solution. *Needs either complex roots stated (accepting $\sqrt{-3}$ for $3i$) or sign of discriminant considered.*

If you see an alternative approach that you think deserves credit, send to review.

Question Number	Scheme	Marks
5 (a)	$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$	M1
	$(x-y)^3 > x^3 - y^3 \Rightarrow -3x^2y + 3xy^2 > 0$	A1
	$\Rightarrow 3xy(y-x) > 0$	dM1
	As x and y are positive numbers $(y-x) > 0 \Rightarrow y > x$ *	A1*
		(4)
(b)	Chooses a suitable counter example. E.g. $x=3, y=-1$	M1
	Shows that $(3-1)^3 > (3)^3 - (-1)^3$ as $64 > 28$ (BUT $-1 < 3$)	A1
		(2)
		(6 marks)

(a)

M1: Attempts $(x-y)^3 = x^3 \pm Ax^2y \pm Bxy^2 \pm y^3$ but may be unsimplified. Condone at most one slip in an index as long as the middle terms have at least xy in them.

A1: Correct simplified inequality, cubed terms cancelled, other terms may not all be gathered.

E.g. $-3x^2y + 3xy^2 > 0$ o.e. Terms need not be all on one side, e.g. $3xy^2 > 3x^2y$

dM1: Takes out a common factor of kxy or divides by kxy . Must be clear division or stated as dividing, but dividing and rearranging to the given answer in one step from $-3x^2y + 3xy^2 > 0$ to $y > x$ is dM0 if method/intermediate step is not stated.

A1*: Requires

- correct working (no incorrect lines) before achieving $y > x$
- reasoning given in the correct/appropriate place,
 - e.g. as x and y are positive so $3xy(y-x) > 0 \Rightarrow (y-x) > 0$

Alt

M1: Attempts $(x-y)^3 = (x-y)(x^2 + axy + y^2)$ and $x^3 - y^3 = (x-y)(x^2 + bxy + y^2)$ but may be unsimplified

A1: Both expressions correct $(x-y)^3 = (x-y)(x^2 - 2xy + y^2)$ and $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

dM1: Takes to one side, takes out a common factor $(x-y)$ (dividing by this term is M0) and cancels square terms.

A1*: Requires

- correct working (no incorrect lines) before achieving $y > x$
- reasoning given in the correct/appropriate place,
 - e.g. as x and y are positive so $3xy(y-x) > 0 \Rightarrow (y-x) > 0$

(b)

M1: Chooses a suitable counter example. E.g. $x=3, y=-1$

Any positive x and negative y will work. Substitution is not needed for this mark. Note that if they choose x and y both positive it is M0.

A1: Shows that the result is not true for their values. As a minimum accept substitution into both sides of the inequality.



Question Number	Scheme	Marks
10(a) (i) (ii)	$x^2 + y^2 + 4x - 30y + 209 = 0$ $\Rightarrow (x \pm 2)^2 + (y \pm 15)^2 \dots = 0$	M1
	Centre $(-2, 15)$	A1
	Radius $\sqrt{20}$	A1
		(3)
(b)	$y = mx + 1 \Rightarrow (x + 2)^2 + (mx + 1 - 15)^2 = 20$ or $y = mx + 1 \Rightarrow x^2 + (mx + 1)^2 + 4x - 30(mx + 1) + 209 = 0$	M1
	$x^2 + m^2x^2 + 4x - 28mx + 180 = 0$ $b^2 - 4ac = 0 \Rightarrow (4 - 28m)^2 - 4(1 + m^2) \times 180 = 0$	dM1
	$(4 - 28m)^2 - 4(1 + m^2)180 = 0 \Rightarrow 16 - 224m + 784m^2 - 720 - 720m^2 = 0$ $\Rightarrow 2m^2 - 7m - 22 = 0^*$	A1*
		(3)
(c)	$2m^2 - 7m - 22 = 0 \Rightarrow m = \frac{11}{2}, -2$	M1
	$m = \frac{11}{2} \Rightarrow \frac{125}{4}x^2 - 150x + 180 = 0 \Rightarrow x = \frac{12}{5} \Rightarrow y = \frac{71}{5}$ or $m = -2 \Rightarrow 5x^2 + 60x + 180 = 0 \Rightarrow x = -6 \Rightarrow y = 13$	M1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ or $(-6, 13)$ oe	A1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ and $(-6, 13)$ oe	A1
		(4)
		Total 10

(a) Look out for answers to part (a) written in the body of the question.

M1: Attempts to complete the square for **both** variables or states a centre of $(\pm 2, \pm 15)$

For completing the square allow $(x \pm 2)^2 \dots (y \pm 15)^2 \dots = \dots$

A1: Centre $(-2, 15)$ or e.g. $x = -2, y = 15$. Condone e.g. $-2, 15$ or $[-2, 15]$

A1: For $\sqrt{20}$ or $2\sqrt{5}$ which may be scored following $(x \pm 2)^2 + (y \pm 15)^2 = 20$

Do **not** allow $\pm\sqrt{20}$ but apply isw if a correct radius is seen which is then e.g. converted to a decimal or simplified incorrectly.

(b) Mark (b) and (c) together.

M1: Attempts to substitute $y = mx + 1$ into the given equation or their rearranged equation to obtain an equation in m and x only. Condone slips as long as the intention is clear.

dM1: Attempts $b^2 - 4ac \dots 0$ where $\dots$ is “=” or “>” or “<” etc. or equivalent e.g. $b^2 = 4ac$ where $a = A + Bm^2, b = C + Dm, c = E$ where A, B, C, D and E are non-zero constants. Condone copying slips if the intention is clear if the above conditions are met. **Depends on the first M mark.**

A1*: Obtains the printed answer with no errors e.g. must have had $b^2 - 4ac = 0$ or equivalent throughout and before the final line e.g. not $b^2 - 4ac > 0$ and brackets expanded. **Note this is a given answer so must follow correct work.**



(b) Alternative: Perpendicular distance from a point to a line:

$$(-2, 15), \quad mx - y + 1 = 0 \quad \text{with} \quad d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

$$\sqrt{20} = \frac{|-2m - 15 + 1|}{\sqrt{m^2 + 1}} \Rightarrow 20(m^2 + 1) = (2m + 14)^2$$

$$\Rightarrow 16m^2 - 56m - 176 = 0 \Rightarrow 2m^2 - 7m - 22 = 0^*$$

M1: Substitutes into a correct distance formula with their centre and the given L and sets equal to their radius.

dM1: Squares, multiplies up to obtain $\alpha(Am^2 + B) = \beta(Cm + D)^2$ oe, where A, B, C and D are non-zero.

A1*: Obtains the printed answer with no errors

(c)

M1: Solves the given quadratic equation by any valid means, including calculator, to obtain at least one value for m . May be implied by their value(s).

One correct value **only** with no incorrect work is sufficient for M1

Condone $x = \dots$ for this mark.

M1: Uses at least one of their values of m to attempt one position for P .

This must be a **complete and correct method to find a position for P** i.e. finds at least one value for x **and** then the corresponding value for y correctly.

E.g. substitutes at least one value for x into their $(x + 2)^2 + (mx + 1 - 15)^2 = 20$, solves the resulting 3TQ by any method including a calculator and then uses the correct $y = mx + 1$ to find the y value. Condone slips when e.g. simplifying their 3TQ providing the complete method is correct.

A1: At least one correct point. Allow as a coordinate pair or as $x = \dots, y = \dots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \dots, y = \dots$



(c) Alternative 1:

Finds the intersections of the possible tangents with the perpendiculars passing through the centre of the circle:

M1: $\left(m = \frac{11}{2}, -2\right)$ As above

For $m = \frac{11}{2}$ perpendicular is $y - 15 = -\frac{2}{11}(x + 2) \left(y = -\frac{2}{11}x + \frac{161}{11} \right)$

$$-\frac{2}{11}x + \frac{161}{11} = \frac{11}{2}x + 1 \Rightarrow x = \frac{12}{5}, y = \frac{71}{5}$$

or

For $m = -2$ perpendicular is $y - 15 = \frac{1}{2}(x + 2) \left(y = \frac{1}{2}x + 16 \right)$

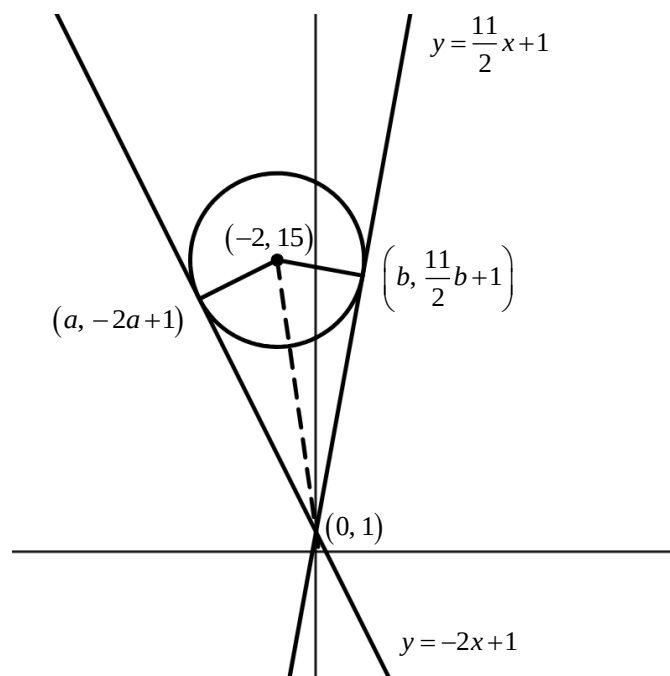
$$\frac{1}{2}x + 16 = -2x + 1 \Rightarrow x = -6, y = 13$$

M1: For a complete method by

- forming the equation of at least one of the perpendiculars, with the negative reciprocal gradient and the coordinates of their centre correctly placed
- solving simultaneously with the corresponding tangent to find x or y
- finding the corresponding x or y coordinate

A1: At least one correct point. Allow as a coordinate pair or as $x = \dots, y = \dots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \dots, y = \dots$

(c) Alternative 2:

$$(15-1)^2 + 2^2 = a^2 + (2a)^2 + (\sqrt{20})^2$$

or

$$(15-1)^2 + 2^2 = b^2 + \left(\frac{11}{2}b\right)^2 + (\sqrt{20})^2$$

Then

$$a^2 = 36 \Rightarrow a = -6 \Rightarrow -2a + 1 = 13$$

or

$$b^2 = \frac{144}{25} \Rightarrow b = \frac{12}{5} \Rightarrow \frac{11}{2}b + 1 = \frac{71}{5}$$

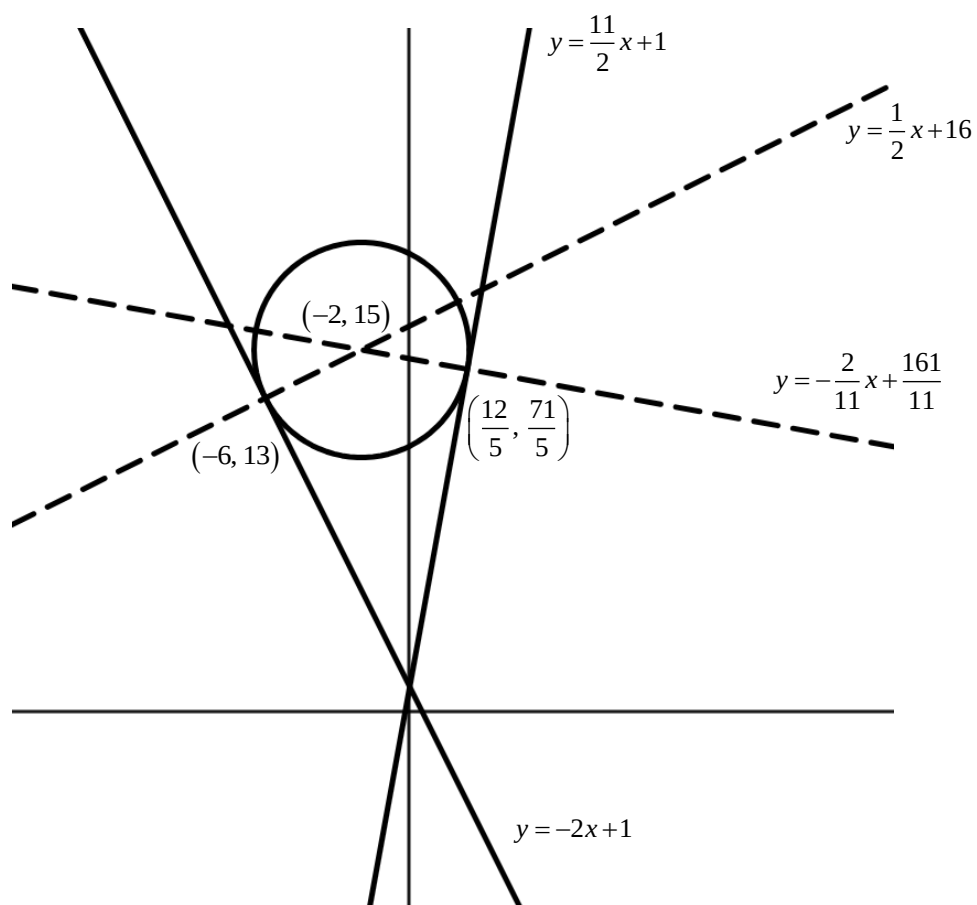
Score as:

M1: As above**M1:** For a complete method by

- using Pythagoras correctly to find the distance or distance² between (0, 1) and their centre
- applying Pythagoras correctly with a general point on either tangent and their radius to obtain an equation in one variable and solves
- finding the corresponding x or y coordinate

A1: At least one correct point. Allow as a coordinate pair or as $x = \dots, y = \dots$ and allow equivalent values e.g. (2.4, 14.2) etc.**A1:** Both points correct and no others. Allow as coordinate pairs or as $x = \dots, y = \dots$ 

General diagram for reference:



Question Number	Scheme	Marks
11(i)	E.g. $n = 5 \Rightarrow 3^5 + 2 = 245$	M1
	245 is not a prime number or e.g. 245 is divisible by 5 so not true	A1
		(2)

M1: Attempts to evaluate $3^n + 2$ with n a prime number.

May be seen as a list of attempts with various numbers even if some are non-prime e.g. $n = 1$ as long as **at least one prime number** is attempted.

Condone a slip in evaluating provided the intention was to substitute in a valid value for n .

A1: A correct calculation for the value and a conclusion.

There must be some reference to it not being prime **or** they show that the number is divisible by e.g. 5 and state that it is false/not true.

The value they have chosen must be clearly identified as their counter example and other considerations of n can be ignored.

Examples:

$$3^5 + 2 = 245 \text{ which is not prime is M1A1}$$

$$\text{when } n=11 \text{ then } 3^{11} + 2 = 177149 \text{ and } 177149 \div 7 = 25307 \text{ so false is M1A1}$$

$$3^5 + 2 = 245 \text{ so false is M1A0}$$

$$3^5 + 2 = 245 \text{ which is divisible by 25 so not prime is M1A0}$$

Most candidates are likely to choose $n = 5$ as their counter example but other numbers are possible e.g.:

n	$3^n + 2$
7	2189
11	177149
13	1594325
17	129140165

$$\begin{array}{l}
 n=1 \Rightarrow 3^1 + 2 = 5 \quad \checkmark \\
 n=3 \Rightarrow 3^3 + 2 = 29 \quad \checkmark \\
 n=5 \Rightarrow 3^5 + 2 = 245 \quad \times \quad \begin{array}{l} 245 : 5 = 49 \\ 245 : 1 = 245 \\ 245 : 245 = 1 \end{array}
 \end{array}$$

Scores: M1A0 (we would need e.g. “not prime” or $245/5 = 49$ so false) but note that we would have ignored the consideration of any non-primes e.g. $n = 1$

(ii)	$m = 3k + 1 \Rightarrow m^2 - 1 = (3k + 1)^2 - 1 = 9k^2 + 6k + 1 - 1$ or $m = 3k + 2 \Rightarrow m^2 - 1 = (3k + 2)^2 - 1 = 9k^2 + 12k + 4 - 1$	M1
	$m^2 - 1 = 9k^2 + 6k = 3(3k^2 + 2k)$ or $m^2 - 1 = 9k^2 + 12k + 3 = 3(3k^2 + 4k + 1)$	A1
	$m = 3k + 1 \Rightarrow m^2 - 1 = (3k + 1)^2 - 1 = 9k^2 + 6k + 1 - 1$ and $m = 3k + 2 \Rightarrow m^2 - 1 = (3k + 2)^2 - 1 = 9k^2 + 12k + 4 - 1$	dM1
	$3(3k^2 + 2k)$ and $3(3k^2 + 4k + 1)$ are both multiples of 3 so $m^2 - 1$ must be divisible by 3 when m is not divisible by 3	A1
		(4)
		Total 6

For the accuracy marks there should be no errors in the algebra but allow e.g. invisible brackets to be “recovered”.

Withhold the final mark if m is used instead of their k if the work is otherwise correct.

M1: Starts the proof by considering at least one algebraic expression not divisible by 3 e.g. $m = 3k + 1$ and attempts $m^2 - 1$ by expanding the brackets.
Alternatively, writes $m^2 - 1 = (m - 1)(m + 1)$ and substitutes at least one algebraic expression not divisible by 3.
Allow equivalent representations of numbers not divisible by 3 e.g. $m = 3k + 2$ or $m = 3k - 1$

Condone arithmetical slips and condone the use of e.g. $m = 3m + 2$ or $m = 3m - 1$

A1: They must

- obtain at least one correct expression
- show that their expression is a multiple of 3 by factoring out 3 or e.g. $3k$ e.g. $3(3k^2 + 2k)$ or e.g. $3k(3k + 2)$ or e.g. $3(k + 1)(3k + 1)$

or

show that their expression is divisible by 3 e.g. $\frac{9k^2 + 6k}{3} = 3k^2 + 2k$.

or

make a reasoned argument as to why the expression is divisible by 3 e.g. obtains $9k^2 + 6k$ and states e.g. is divisible by 3 as both 9 and 6 are divisible by 3 (or equivalent reasoning)

Note that this may be seen in the final conclusion.

Condone a spurious “= 0” e.g. $3(3k^2 + 2k) = 0$ **for this mark.**

dM1: Considers both **distinct** cases where m is not divisible by 3 (see above)

See table below – they must have one of case *A* and one of case *B*.

A1: Fully correct proof. They must

- achieve correct expressions for $m^2 - 1$ for both cases
- show that each expression is a multiple of 3 as above
- make a concluding overall statement. “Hence is a multiple of 3”. Accept “hence proven”, “statement proved”, “QED” if they have shown for each separate case that the expression is a multiple of 3. Ignore any reference to what m is e.g. “for all integers m ”

Do **not** condone any spurious “= 0” e.g. $3(3k^2 + 2k) = 0$ **for this mark**.

You can ignore any cases considered where m is divisible by 3 e.g. $m = 3k$ provided this does not clearly form part of their proof.

For reference:

Case	m	$m^2 - 1$
<i>A</i>	$3k + 1$	$9k^2 + 6k$
<i>B</i>	$3k + 2$	$9k^2 + 12k + 3$
<i>B</i>	$3k - 1$	$9k^2 - 6k$
<i>A</i>	$3k - 2$	$9k^2 - 12k + 3$



General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question	Scheme	Marks
1	1024 + ...	B1
	$\dots + {}^{10}C_1 \times \dots x + {}^{10}C_2 \times \dots x^2 + {}^{10}C_3 \times \dots x^3 + \dots$	M1
	$\dots + 10 \times 2^9 \times \frac{3}{8}x + 45 \times 2^8 \times \left(\frac{3}{8}x\right)^2 + 120 \times 2^7 \times \left(\frac{3}{8}x\right)^3$ Or two of $\dots + 1920x + 1620x^2 + 810x^3$	A1
	$\dots + 1920x + 1620x^2 + 810x^3$	A1
		(4)
(4 marks)		

Notes:

B1: Correct constant term of 1024 as an integer.

M1: Correct binomial coefficient multiplied by the correct powers of x for **at least 2 terms** in x , x^2 or x^3 .

Allow e.g. ${}^{10}C_1$, ${}^{10}C_2$, ${}^{10}C_3$ or $\binom{10}{1}$, $\binom{10}{2}$, $\binom{10}{3}$ or evaluated coefficients and condone missing

brackets e.g. $\frac{3}{8}x^2$ for $\left(\frac{3}{8}x\right)^2$ or $\frac{3}{8}x^3$ for $\left(\frac{3}{8}x\right)^3$

May take out a common factor of 2^{10} first, but again look for correct binomial coefficients multiplied by the correct powers of x for **at least 2 terms** in x , x^2 or x^3 .

E.g. $\left(2 + \frac{3}{8}x\right)^{10} = 2^{10} \left(1 + \frac{3}{16}x\right)^{10} = 2^{10} \left(1 + {}^{10}C_1 \dots x + {}^{10}C_2 \dots x^2 + {}^{10}C_3 \dots x^3 + \dots\right)$

A1: Allow for a fully correct unsimplified expression (ignoring the constant term) with **evaluated binomial coefficients**, must be expanded if a common factor of 2^{10} is taken out first
OR two of the three terms in x , x^2 and x^3 correct and simplified.

E.g. $\dots + 10 \times 2^9 \times \frac{3}{8}x + 45 \times 2^8 \times \left(\frac{3}{8}x\right)^2 + 120 \times 2^7 \times \left(\frac{3}{8}x\right)^3$

Or $\dots + 10 \times 2^{10} \times \frac{3}{16}x + 45 \times 2^{10} \times \left(\frac{3}{16}x\right)^2 + 120 \times 2^{10} \times \left(\frac{3}{16}x\right)^3$

The brackets must be present unless they are implied by subsequent work.

OR two of $\dots + 1920x + 1620x^2 + 810x^3$. Allow terms to be “listed”.

A1: Final three terms fully correct and simplified. Allow terms to be “listed”.

Once a correct expansion (or list of terms) is seen then isw. E.g. some candidates think they have to list the coefficients separately but apply isw.

Ignore any extra terms if found.

For reference incorrect bracketing:

$$\dots + 45 \times 2^8 \times \frac{3}{8} x^2 + 120 \times 2^7 \times \frac{3}{8} x^3 \text{ gives } \dots + 4320x^2 + 5760x^3$$

And usually scores B1M1A0A0 if the 1024 is correct

Special case: Some candidates are just finding the coefficients $1024 + 1920 + 1620 + 810$ and this scores B1 only if the x 's never make an appearance.



Question	Scheme							Marks
2(a)	x	0.5	1	1.5	2	2.5	3	B1
	y	1.319	1.075	1.001	1.041	1.223	1.850	B1
								(2)
(b)	h = 0.5 seen or implied							B1
	$\text{Area} \approx \left\{ \frac{0.5}{2} \right\} \times (1.319 + 2("1.075"+1.001+"1.041"+1.223)+1.850)$							M1
	= awrt 2.96							A1
								(3)
(c)	$3 + \log_{10}(\sin(x)) = 4 - (1 - \log_{10}(\sin(x)))$							B1
	$\int_{0.5}^3 3 + \log_{10}(\sin(x)) \, dx = [4x]_{0.5}^3 - "2.96" = 12 - 2 - "2.96"$							M1
	= 7.04 follow through 10 – their 2.96							A1ft
								(3)
Alt (c)	$\int_{0.5}^3 \log_{10}(\sin(x)) \, dx = \int_{0.5}^3 1 \, dx - "2.96" = 2.5 - "2.96"$							B1
	$\int_{0.5}^3 3 + \log_{10}(\sin(x)) \, dx = [3x]_{0.5}^3 + (2.5 - "2.96") = 9 - 1.5 + 2.5 - "2.96"$							M1
	= 7.04 follow through 10 – their 2.96							A1ft
								(3)
(8 marks)								
Notes:								
(a)								
B1: Either of awrt 1.075 or awrt.1.041 correct								
B1: Both of awrt 1.075 and awrt.1.041 correct								
Remember to check the body of the script for the values if the table has not been completed.								
Note that the incorrect values of 2.758 and 2.457 come from the use of degrees.								
(Which gives 4.51 in (b))								
(b)								
B1: Correct interval width seen or implied by working.								
M1: Correct application of the trapezium rule. Must all be y values, and should be first + last + 2(sum of middle values), but if one middle value is omitted allow as a slip. A repeated value is M0. Follow through their values from (a). Allow obvious copying slips.								
Note that $\left\{ \frac{"0.5"}{2} \right\} \times 1.319 + 2("1.075"+1.001+"1.041"+1.223)+1.850$ scores M0 unless the missing brackets are implied by subsequent work.								

Allow this mark if they add the areas of individual trapezia e.g.

$$\left\{\frac{"0.5"}{2}\right\} \times (1.319 + "1.075") + \left\{\frac{"0.5"}{2}\right\} \times ("1.075" + 1.001) + \dots$$

A1: For awrt 2.96. Allow to come from answers in (a) incorrectly rounded or wrong accuracy.

FYI: Use of rounded figure gives 2.96225, use of calculated figures gives 2.9625726... Actual value is 2.89 to 2 decimal places.

(c) Condone all work in (c) written without the "10" in $\log_{10}(\sin(x))$

B1: Writes $3 + \log_{10}(\sin(x))$ correctly in terms of $1 - \log_{10}(\sin(x))$. May be implied.

M1: Integrates the K in $K \pm (1 - \log_{10}(\sin(x)))$ to Kx , $K \neq 3$, substitutes in the limits 3 and 0.5 and subtracts oe e.g. $4 \times (3 - 0.5)$ and uses the answer to (b).

A1ft: Correct answer awrt 7.04, follow through their 2.96 if given to 3sf or better.

Alt:

B1: Rearranges integral to find $\int_{0.5}^3 \log_{10}(\sin(x)) dx = 2.5 - "2.96"$. May be implied.

M1: Integrates the 3 and substitutes in given limits and subtracts oe e.g. $3 \times (3 - 0.5)$ and adds their attempt at

$\int_{0.5}^3 \log_{10}(\sin(x)) dx$ found using the answer to (b).

A1ft: Correct answer awrt 7.04, follow through their 2.96 if given to 3sf or better.

A common incorrect method in (c):

$$\int_{0.5}^3 3 + \log_{10}(\sin(x)) dx = [3x]_{0.5}^3 + "2.96" = "10.46"$$

Scores no marks as they have not used the integral in part (b)

Attempts to use the trapezium rule again in (c) score no marks

Question	Scheme	Marks
3(i)	E.g. $n = 1 : 2^3 - 1^3 = 7, n = 2 : 3^3 - 2^3 = 19, n = 3 : 4^3 - 3^3 = \dots$ Or identifies counterexample directly.	M1
	e.g. $6^3 - 5^3 = 91 = 7 \times 13$ so not true for $n = 5$, hence statement is not true.	A1
		(2)
	Notes for part (i) M1: Shows evidence of trying to find a counter example for a positive integer (at least one attempt). $2^3 - 1^3$ is prime is sufficient. A1: Gives a correct counter example with reason (shows factorisation) and concludes e.g. “which is not prime”. Ignore any previous “incorrect” attempts e.g. $6^3 - 5^3 = 91$ which is prime. Note $n = 7$ ($169 = 13 \times 13$) and $n = 8$ ($217 = 7 \times 31$) and $n = 12$ ($469 = 7 \times 67$) are the next few counter examples. (Bigger examples are not likely to be seen!) Allow equivalent reasons for not being prime e.g. $169/13 = 13$ or 169 is divisible by 13 (condone “can be divided by 13”) Generally algebraic approaches score no marks unless they substitute numbers as indicated above.	
3(ii)	The majority of methods here will follow ways 1, 2 or 3 below In these cases the general guidance is as follows: M1: Attempts to find - the gradient of any relevant line, e.g. AC or BC or - the length of any relevant line, e.g. AB/AB^2 or BC/BC^2 or AC/AC^2 or - the mid-point M of line AB A1: Correct relevant calculation of - gradients AC and BC - lengths of lines AB/AB^2, BC/BC^2 and AC/AC^2 - mid-point of line AB dM1: Full attempt at combining all relevant information required to solve the problem - attempts product of gradients or equivalent - attempts to show Pythagoras $AB^2 = AC^2 + BC^2$ - attempts to show $MA^2 = MC^2$ A1: Correct calculations or equivalent providing required evidence for the above A1: Provides correct reason and conclusion with all previous marks scored.	

Way 1	$m_{AC} = \frac{-6-0}{7-1} = \dots$ or $m_{BC} = \frac{-6-(-10)}{7-3} = \dots$	M1
	$m_{AC} = -1$ and $m_{BC} = 1$	A1
	So $m_{AC} \times m_{BC} = -1 \times 1 = -1$ or e.g. m_{AC} is negative reciprocal of m_{BC}	dM1A1
	So e.g. angle (at C) is a right angle hence AB is a diameter (Or equivalent)	A1
		(5)
M1: Attempts the gradients of AC or BC. Allow slips but score M0 if both attempts are clearly incorrect. A1: Correct gradients from correct formulae dM1: Applies perpendicular condition. May be seen as shown but allow equivalent work. A1: Correct calculations or equivalent A1: Suitable explanation and conclusion given with no errors and all previous marks awarded with no incorrect statements seen.		
Way 2	$AB = \sqrt{(3-1)^2 + (-10-0)^2} = \dots$ or $AC = \sqrt{(7-1)^2 + (-6-0)^2} = \dots$ or $BC = \sqrt{(7-3)^2 + (-6+10)^2} = \dots$	M1
	$AB = \sqrt{104} (2\sqrt{26}), AC = \sqrt{72} (6\sqrt{2}), BC = \sqrt{32} (4\sqrt{2})$	A1
	$AB^2 = 104 = 72 + 32 = AC^2 + BC^2$	dM1A1
	Hence ABC is a right-angle triangle with hypotenuse AB hence AB is a diameter. (Or equivalent)	A1
		(5)
M1: Attempts length of AB or AC or BC or their squares. Allow slips but score M0 if attempts are clearly incorrect. A1: Correct values for AB, AC and BC or their squares. dM1: Applies Pythagoras' theorem with their values. (May see cosine rule used.) A1: All calculations correct for this approach. A1: Suitable explanation and conclusion given with no errors and all previous marks awarded with no incorrect statements seen.		

Way 3	If AB is diameter centre must be midpoint of AB ie $M\left(\frac{1+3}{2}, \frac{0-10}{2}\right)$	M1
	$= (2, -5)$	A1
	$MA = \sqrt{(2-1)^2 + (-5-0)^2} = \sqrt{26}$, $MC = \sqrt{(2-7)^2 + (-5-(-6))^2} = \sqrt{26}$	dM1A1
	$MA = \sqrt{26}$, $MC = \sqrt{26}$ so $MA=MC(=MB)$ As the length from M to each of A and C is the same M is the centre of the circle hence AB is a diameter. (Or equivalent)	A1
		(5)
M1: Attempts midpoint of AB. If no method is shown accept one correct coordinate as evidence. A1: Correct midpoint dM1: Attempts length of MC and at least one of MA or MB, or AB. As M is midpoint of AB there is no need to find both MA and MB, these may be assumed to be the same. If they find AB then they must halve it to find the radius. A1: All required calculations correct for this approach. A1: Suitable explanation made which may be in a preamble and conclusion given with no errors and all previous marks awarded.		

The following approach is less common and should be marked as shown:

Way 4	If AB is diameter centre must be midpoint of AB ie $M\left(\frac{1+3}{2}, \frac{0-10}{2}\right)$	M1
	$= (2, -5)$	A1
	$MA = r = \sqrt{(2-1)^2 + (-5-0)^2} = \sqrt{26}$ $\Rightarrow (x-2)^2 + (y+5)^2 = 26$ $C(7, -6) \Rightarrow (7-2)^2 + (-6+5)^2 = 5^2 + 1^2 = 26$	dM1A1
	As C also satisfies the equation of the circle then AB must be the diameter (or equivalent) There must be some further justification as above rather than just “ AB is a diameter” which may be in a preamble e.g. If C lies on the circle...	A1
		(5)
M1: Attempts midpoint of AB. If no method is shown accept one correct coordinate as evidence. A1: Correct midpoint dM1: Attempts length of MA or MB to find r or r^2, forms equation of the circle and substitutes the coordinates of C. A1: All required calculations correct for this approach. A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.		

SEE END OF MARKSCHEME FOR SOME OTHER MORE UNUSUAL METHODS THAT MAY BE SEEN DURING MARKING WITH SUGGESTED MARKING GUIDELINES.

There may be other methods. Choose the way that best fits the overall response.
If you are in any doubt if a particular response deserves credit then use Review.



General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$ **Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$h = 0.2$ $\frac{1}{2} \times 0.2 \times [9.2 + 8.6 + 2(8.4556 + 3.8512 + 5.0342 + 7.8297)]$ 6.814	B1 M1 A1
		(3 marks)

B1 $h = 0.2$ o.e. (including $\frac{5-4}{5}$) seen or implied by sight of e.g. $\frac{0.2}{2}$ or $\frac{1}{10}$ in front of the bracket.
 May also be implied by a correct answer if no incorrect working seen. $h = -0.2$ is B0

M1 Correct application of the trapezium rule with their h (which may be 1 or h).
 Look for a correct inner bracket structure $9.2 + 8.6 + 2(8.4556 + 3.8512 + 5.0342 + 7.8297)$
 condoning slips copying values from the table or the omission of the final brackets on the rhs

e.g. $\frac{1}{2} \times 0.2 \times (9.2 + 8.6 + 2(8.4556 + 3.8512 + 5.0342 + 7.8297))$ is M1

but $\frac{1}{2} \times 0.2 \times 9.2 + 8.6 + 2(8.4556 + 3.8512 + 5.0342 + 7.8297)$ is M0 unless the brackets are recovered or implied by the correct answer (you may need to check this)

Also allow for a correct method adding individual trapezia using their h condoning copying errors but the brackets must be correct (or recovered or implied by later work)

e.g. $\frac{1}{2} \times 0.2 \times (9.2 + 8.4556) + \frac{1}{2} \times 0.2 \times (8.4556 + 3.8512) + \frac{1}{2} \times 0.2 \times (3.8512 + 5.0342) +$
 $\frac{1}{2} \times 0.2 \times (5.0342 + 7.8297) + \frac{1}{2} \times 0.2 \times (7.8297 + 8.6)$

A1 awrt 6.814 isw once a correct answer is seen. Correct answer with no working scores B1M1A1 but if there is evidence of using $h = -0.2$ then maximum awarded is B0M1A0
 The A mark cannot be awarded without both B1M1 being awarded on this question.



Question	Scheme	Marks
7	$(x+2)$ a factor $\Rightarrow f(-2) = 0 \Rightarrow -8A + 24 + 8 + B = 0$	M1A1
	$\int f(x)dx = \frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$	M1 A1
	$\int_3^5 f(x)dx = 176 \Rightarrow \left[\frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx \right]_3^5 = 176$ $\Rightarrow \left(\frac{A}{4}5^4 + 2(5^3) - 2(5^2) + 5B \right) - \left(\frac{A}{4}3^4 + 2(3^3) - 2(3^2) + 3B \right) = 176$	dM1
	$\left. \begin{array}{l} 8A - B = 32 \\ 136A + 2B = 12 \end{array} \right\} \Rightarrow A = \dots, B = \dots$	dM1
	$A = \frac{1}{2}, B = -28$	A1
		(7)
(7 marks)		

Notes:

M1: Uses the factor theorem or long division (see below) to find one linear **equation** relating A and B. The “= 0” must be seen or implied by later work.

A1: Correct equation. Award once a correct equation is seen unsimplified or simplified.

So allow e.g. $A(-2)^3 + 6(-2)^2 - 4(-2) + B = 0$ but not e.g. $A \times -2^3 + 6 \times -2^2 - 4 \times -2 + B = 0$

unless the indices are processed correctly subsequently to e.g. $-8A + 24 + 8 + B = 0$

The “= 0” must be seen or implied by later work.

M1: Attempts to integrate the given expression, look for at least one power increased by 1.

This can also be evidenced by $B \rightarrow Bx$

A1: Correct integration. Allow unsimplified e.g. $\frac{6x^{2+1}}{2+1}$ for $2x^3$

Condone poor notation e.g. $\int f(x)dx = \int \frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$ and ignore any “+ c”

dM1: Applies limits 5 and 3 either way round and subtracts and equates to 176 to obtain an equation in A and B. **Depends on previous M.**

Condone poor use of brackets when subtracting e.g. allow

$$\frac{A}{4}5^4 + 2(5^3) - 2(5^2) + 5B - \frac{A}{4}3^4 + 2(3^3) - 2(3^2) + 3B = 176$$

dM1: Solves their two equations in A and B to find values for both constants.

Depends on all previous M marks but allow this mark if the only error was to attempt $f(2) = 0$ at the beginning.

A1: Correct values.

Note that some candidates eliminate A or B by rearranging the first equation and substituting into the integral. This essentially follows the main scheme:



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1(a)	$h = 1.25$	B1
	$A \approx \frac{1}{2} \times 1.25 \{3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)\}$	M1
	$= 30.9$	A1
		(3)
(b)	$\int_{0.5}^{5.5} (f(x) + 4x) dx = 30.9 + \left[2x^2 \right]_{0.5}^{5.5} = 30.9 + 2 \times 5.5^2 - 2 \times 0.5^2$ or $\int_{0.5}^{5.5} (f(x) + 4x) dx = \int_{0.5}^{5.5} f(x) dx + \frac{1}{2}(4 \times 0.5 + 4 \times 5.5) \times 5 = \dots$	M1
	$= 30.9 + 60 = 90.9$	A1ft
		(2)
		Total 5

(a)

B1: Correct strip width of 1.25. Allow equivalent numerical expressions e.g. $\frac{5.5 - 0.5}{4}$.

May be implied by sight of e.g. $\frac{1.25}{2}$ or $\frac{5}{8}$ or 0.625 in front of the bracket. May also be implied by a correct answer if no incorrect working seen. $h = -1.25$ is B0 unless they recover and subsequently use $h = +1.25$

M1: Correct application of the trapezium rule with their h (which may be 1).

This requires a correct inner bracket structure $3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)$

multiplied by $\frac{1}{2}h$. Condone slips copying values from the table or the omission of the

final bracket on the rhs e.g. $\frac{1}{2} \times "1.25"(3.479 + 5.182 + 2(6.101 + 7.448 + 6.823))$ is M1

but $\frac{1}{2} \times "1.25" \times 3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)$ is M0 unless the missing

brackets are recovered or implied by the correct answer (you may need to check). Also allow for a correct method adding individual trapezia using their h condoning copying errors but the brackets must be correct (or recovered or implied by later work)

e.g. $\frac{1}{2} \times "1.25"(3.479 + 6.101) + \frac{1}{2} \times "1.25"(6.101 + 7.448) + \frac{1}{2} \times "1.25"(7.448 + 6.823) +$

$\frac{1}{2} \times "1.25"(6.823 + 5.182)$

A1: For awrt 30.9. isw once a correct answer is seen. Correct answer with no working scores B1M1A1 but if there is evidence of using $h = -1.25$ then maximum awarded is B0M1A0. The A mark cannot be awarded without both B1M1 being awarded in this question. The exact answer is $\frac{9881}{320}$ and scores A0 unless followed by awrt 30.9

For reference, note that use of $h = 1$ (from using 5 strips) gives 24.7... or $\frac{9881}{400}$

(a), $\int_{0.5}^{5.5} 1.25 (3.479 + 6.101 + 7.448 + 6.823 + 5.182) = 36.3$ Scores B1M0A0 (BOD)



Question Number	Scheme					Marks
1.	<i>a</i>	<i>b</i>	<i>c</i>	<i>abc</i>	One correct set <i>a</i> , <i>b</i> & <i>c</i>	B1
	2	15	3	90		
	3	9	8	216	Two correct rows	M1
	4	1	15	60		
	All values of <i>abc</i> are multiples of 6				Fully correct + statement	A1
						(3 marks)

Main method: Numerical answer finding all known solutions

B1: Any correct set of values for *a*, *b* and *c*. You can ignore any attempt at the product *abc*

M1: Any two fully correct rows including the correct calculations for the products.
Condone any extra or incorrect rows

A1: Fully correct. The solution must have

- the three correct rows with correct calculations for *abc*
- any extra (or incorrect) rows that may be present must be deleted or somehow excluded
- a minimal statement showing that they are aware that 90, 216 and 60 are multiples of 6.

E.g they are all divisible by 6, use of a tick ✓, proven, QED, calculations such as $\frac{90}{6} = 15$. They can be divided by 6 (on its own) is insufficient without further clarification such as "to give whole numbers"

- no incorrect calculations. E.g if their attempt to prove that 90 being a multiple of 6 includes $\frac{90}{6} = 10$ then it is A0

Algebraic solution or solutions that rely on explanations: These will be rare and unlikely to lead to completely correct solutions.

Example of an attempt that has some merit based upon odds and evens

If *a* is even

Then $a = 2n$, leading to $c = 4n^2 - 1 = (2n+1)(2n-1)$ and $b = 21 - 4n^2 - 2n$

giving $abc = 2n(21 - 4n^2 - 2n)(2n+1)(2n-1)$

If *a* is odd.

Then $a = 2n+1$, leading to $c = 4n^2 + 4n = 4n(n+1)$ and $b = 19 - 4n^2 - 6n$

giving $abc = (2n+1)(19 - 4n^2 - 6n)4n(n+1)$

B1: Attempts both odd and even and achieves correct algebraic expressions for *a*, *b* and *c* in both cases.

M1: Correct products with attempts when $n = 1, 2$ and achieves two fully correct rows

A1: Fully correct with no additional rows

Question Number	Scheme	Marks
	Accept sight of $6\left(\pm\frac{1}{8}x\right)^1 - \frac{6 \times 5}{2}\left(\pm\frac{1}{8}x\right)^2 + \frac{6 \times 5 \times 4}{3!}\left(\pm\frac{1}{8}x\right)^3$ condoning omission of brackets	
A1	For any two terms of $64 - 48x + 15x^2 - 2.5x^3$	
A1	For all four terms $64 - 48x + 15x^2 - 2.5x^3$ ignoring terms with greater powers	
		
	Attempts to multiply out	
B1	For 64	
M1	Multiplies out to form $a + bx + cx^2 + dx^3 + \dots$ and gets b, c or d correct.	
A1A1	As main scheme	

Question Number	Scheme	Marks
5.(a)	$\frac{dP}{dx} = 12 - \frac{3}{2}x^{\frac{1}{2}}$ Sets $\frac{dP}{dx} = 0 \rightarrow 12 - \frac{3}{2}x^{\frac{1}{2}} = 0 \rightarrow x^n = \dots$ $x = 64$ When $x = 64 \Rightarrow P = 12 \times 64 - 64^{\frac{3}{2}} - 120 = \dots$ Profit = (£) 136 000	M1A1 dM1 A1 M1 A1 (6)
(b)	$\left(\frac{d^2P}{dx^2}\right) = -\frac{3}{4}x^{-\frac{1}{2}}$ and substitutes in their $x = 64$ to find its value or state its sign At $x = 64$ $\frac{d^2P}{dx^2} = -0.09375 < 0 \Rightarrow$ maximum	M1 A1 (2) (8 marks)

Notes

You should mark parts a and b together. You may see work in (a) from (b)

(a)

M1 Attempts to differentiate $x^n \rightarrow x^{n-1}$ seen at least once. It must be an x term and **not** the $120 \rightarrow 0$

A1 $\frac{dP}{dx} = 12 - \frac{3}{2}x^{\frac{1}{2}}$ with no need to see the lhs. Condone $\frac{dy}{dx}$ all of the way through part (a).

dM1 Sets their $\frac{dP}{dx} = 0$ and proceeds to $x^n = k, k > 0$. Dependent upon the previous M. Don't be too

concerned with the mechanics of process. Condone an attempted solution of $\frac{dP}{dx} \dots 0$ where ...

could be an inequality

A1 $x = 64$. Condone $x = \pm 64$ here

M1 Substitutes their solution for $\frac{dP}{dx} = 0$ into P and attempts to find the value of P .

The value of x must be positive. If two values of x are found, allow this mark for any attempt using a positive value.

A1 CSO. Profit = (£) 136 000 or 136 thousand but not 136 or $P = 136$.

This cannot follow two values for x , eg $x = \pm 64$ Condone a lack of units or incorrect units such as \$

(b)

M1 Achieves $\frac{d^2P}{dx^2} = kx^{-\frac{1}{2}}$ and attempts to find its value at $x = "64"$

Alternatively achieves $\frac{d^2P}{dx^2} = kx^{-\frac{1}{2}}$ and attempts to state its sign. Eg $\frac{d^2P}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}} < 0$

Allow $\frac{d^2P}{dx^2}$ appearing as $\frac{d^2y}{dx^2}$ for the both marks.

A1 Achieves $x = 64$, $\frac{d^2P}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$ and states $\frac{d^2P}{dx^2} = -\frac{3}{32} < 0$ (at $x = 64$) then the profit is maximised.



This requires the correct value of x , the correct value of the second derivative (allowing for awrt -0.09) a reason + conclusion.

Alt: Achieves $x = 64$, $\frac{d^2P}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$ and states as $x > 0$ or $\sqrt{x} > 0$ means that $\frac{d^2P}{dx^2} < 0$ then the profit is maximised.

Part (b) merely requires the use of calculus so allow

M1 Attempting to find the value of $\frac{dP}{dx}$ at two values either side, but close to their 64. Eg. For 64 , allow the lower value to be $63.5 \leq x < 64$ and the upper value to be $64 < x \leq 64.5$

A1 Requires correct values, correct calculations with reason and conclusion



Question Number	Scheme	Marks
6.(a)	Sets $f(3)=0 \rightarrow$ equation in k Eg. $27k-135-96-12=0$ $\Rightarrow 27k=243 \Rightarrow k=9$ * ($=0$ must be seen)	M1 A1* (2)
(b)	$9x^3-15x^2-32x-12=(x-3)(9x^2+12x+4)$ $= (x-3)(3x+2)^2$	M1 A1 dM1 A1 (4)
(c)	Attempts $\cos \theta = -\frac{2}{3}$ $\theta = 131.8^\circ, 228.2^\circ$ (awrt)	M1 A1 (2) (8 marks)

Notes

(a)

M1 Attempts to set $f(3)=0 \rightarrow$ equation in k Eg. $27k-135-96-12=0$. Condone slips.Score when you see embedded values within the equation or two correct terms on the lhs of the equation. It is implied by sight of $27k-243=0$ or $27k=135+96+12$.A1* Completes proof with at least one intermediate "solvable" line namely $27k=243 \Rightarrow k=9$ or $27k-243=0 \Rightarrow k=9$. This is a given answer so there should be no errors.

It is a "show that" question so expect to see

(i) Either $f(3)=0$ explicitly stated or implied by sight of $27k-135-96-12=0$ or $27k-243=0$ (ii) One solvable intermediate line followed by $k=9$ A candidate could use $k=9$ and start with $f(x)=9x^3-15x^2-32x-12$ M1 For attempting $f(3)=9 \times 3^3-15 \times 3^2-32 \times 3-12$.Alt attempts to divide $f(x)$ by $(x-3)$. See below on how to score such an attemptA1* Shows that $f(3)=0$ and makes a minimal statement to the effect that "so $k=9$ "If division is attempted it must be correct and a statement is required to the effect that there is no remainder, "so $k=9$ "If candidates have divided (correctly) in part (a) they can be awarded the first two marks in (b) when they start factoring the $9x^2+12x+4$ term.

(b)

M1 Attempt to divide or factorise out $(x-3)$. Condone students who use a different value of k .For factorisation look for first and last terms $9x^3-15x^2-32x-12=(x-3)(\pm 9x^2 \dots \dots \pm 4)$

$$\begin{array}{r} 9x^2 + \dots \dots \dots \\ x-3 \overline{) 9x^3 - 15x^2 - 32x - 12} \\ \underline{9x^3 - 27x^2} \end{array}$$
A1 Correct quadratic factor $9x^2+12x+4$.

You may condone division attempts that don't quite work as long as the correct factor is seen.



Question Number	Scheme	Marks
dM1 A1	Attempt at factorising their $9x^2 + 12x + 4$ Apply the usual rules for factorising $(x-3)(3x+2)^2$ or $(x-3)(3x+2)(3x+2)$ on one line. Accept $9(x-3)\left(x+\frac{2}{3}\right)^2$ oe. It must be seen as a product Remember to isw for candidates who go on to give roots $f(x) = (x-3)(3x+2)^2 \Rightarrow x = \dots$ Note: Part (b) is "Hence" so take care when students write down the answer to (b) without method If candidates state $x = -\frac{2}{3}, 3 \Rightarrow f(x) = \left(x + \frac{2}{3}\right)\left(x + \frac{2}{3}\right)(x-3)$ score 0 0 0 0 If candidates state $x = -\frac{2}{3}, 3 \Rightarrow f(x) = (3x+2)(3x+2)(x-3)$ they score SC 1010. If candidates state $x = -\frac{2}{3}, 3 \Rightarrow f(x) = 9\left(x + \frac{2}{3}\right)\left(x + \frac{2}{3}\right)(x-3)$ they score SC 1010. If candidate writes down $f(x) = (3x+2)(3x+2)(x-3)$ with no working they score SC 1010. If a candidate writes down $(x-3)(3x+2)$ are factors it is 0000 (c) M1 A correct attempt to find one value of θ in the given range for their $\cos \theta = -\frac{2}{3}$ (You may have to use a calculator). So if (b) is factorised correctly the mark is for one of the values. This can be implied by sight of awrt 132 or 228 in degrees or awrt 2.3 which is the radian solution. A1 CSO awrt $\theta = 131.8^\circ, 228.2^\circ$ with no additional solutions within the range $0 \leq \theta < 360^\circ$ Watch for correct solutions appearing from $3\cos \theta - 2 = 0 \Rightarrow \cos \theta = \frac{2}{3}$. This is M0 A0 Answers without working are acceptable. M1 For one correct answer M1 A1 For two correct answers with no additional solutions within the range.	

Question Number	Scheme	Marks
1 (a)	$y = 2x^2(x-5) = 2x^3 - 10x^2$ $\frac{dy}{dx} = 6x^2 - 20x$ Sets $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 20x = 0 \Rightarrow x = 0, \frac{10}{3}$ oe	B1 M1 dM1 A1 (4)
(b)	$x \leq 0, \quad x \geq \frac{10}{3}$	M1 A1 (2) (6 marks)
Alt (a)	$\left(\frac{d(uv)}{dx} = uv' + vu' \right) \quad u = 2x^2, u' = 4x, v = x-5, v' = 1$ $\left(\frac{dy}{dx} = \right) Ax(x-5) + Bx^2$ Sets $\frac{dy}{dx} = 0 \Rightarrow 4x(x-5) + 2x^2 = 0 \Rightarrow x = 0, \frac{10}{3}$	B1 M1 dM1 A1

(a)

 B1 Expands to a correct form $(y =) 2x^2(x-5) = 2x^3 - 10x^2$

 (may be implied by sight of $\frac{dy}{dx} = 6x^2 - 20x$)

 If a candidate uses the product rule then award this mark for correct terms for u, u', v and v' stated or implied.

M1 Differentiated reducing a power by one on one of their terms.

 If a candidate uses the product rule then look for $\left(\frac{dy}{dx} = \right) Ax(x-5) + Bx^2$. (Do not award for just differentiating the factor in front of the bracket and the $x-5$)

 dM1 $\frac{dy}{dx} = 0$ and solves a quadratic equation to find at least one solution. (Usual rules for solving a quadratic). Values such as 0 and 3.33 to imply this mark.

 It is dependent on the previous method mark. Dividing by x to find one of the solutions is also acceptable for this mark.

Allow answers to be just written down but you may need to check these on your calculator.

A1 $x = 0, \frac{10}{3}$ oe (Must be seen in (a))

Allow if embedded within pairs of coordinates for the stationary points of C

SC If no calculus work shown then 0001 for achieving the correct x coordinates.

SC If no credit worthy work seen in (a) but they use calculus in (b) then B1M1M1 can be awarded but withhold the A mark.

(b) Note we are now marking this on open as M1A1

M1 One of $x \leq 0$ or $x \geq \frac{10}{3}$. Allow for $x < 0$ or $x > \frac{10}{3}$.

They must have only achieved a maximum of two x coordinates in (a).

Do not award for inequalities using solutions from $y = 0$ or if they have integrated instead.

A1 Both $x \leq 0, x \geq \frac{10}{3}$. Allow both $x < 0, x > \frac{10}{3}$. Ignore any joining statements such as “and”, “or” or any

set notation eg. $\cup$ or $\cap$ and allow statements such as $(-\infty, 0], \left[\frac{10}{3}, \infty\right)$. $\frac{10}{3}$ does not need to be in its

lowest terms. Isw after two correct inequality statements unless they contradict. (Eg ignore after two

correct inequalities $x < 0, x > \frac{10}{3}$ a statement such as $0 > x > \frac{10}{3}$ but withhold the final mark if they

subsequently write $0 < x < \frac{10}{3}$)

Question Number	Scheme	Marks
2 (a)	States or uses $r = 1.02$ Attempts $25000 \times 1.02^{13} = 32340$ or 32341 or 32300	B1 M1 A1 (3)
(b)	Attempts $\frac{a(r^n - 1)}{(r - 1)} = \frac{25000(1.02^{14} - 1)}{1.02 - 1}$ or $\frac{125000(1.02^{14} - 1)}{1.02 - 1}$ £1 997 000	M1 A1 A1 (3) (6 marks)

(a)

B1 States or uses $r = 1.02$, 102%, $(1+2\%)$ oe

M1 Attempts $25000 \times r^{n12}$ or $25000 \times r^{n13}$. Allow eg $r = 2$ for this mark. They may show the increase each year which is acceptable. Expect to see at least one correct increase for their r and attempting 12 or 13 increases by the same percentage. Allow a misread if 2500 or 250000 is used instead.
Awr 31706 which is 25000×1.02^{12} is usually sufficient evidence for this mark.

A1 32340 (or 32341). Must be an integer. Allow 32300 which is the answer rounded to 3sf.

(b)

M1 Attempts $\frac{a(r^n - 1)}{r - 1}$ or $\frac{a(1 - r^n)}{1 - r}$ with $a = 25000$ or 125000 , $n = 13$ or 14 and $r = 1.02$ or their r from part (a).

A1 For $\frac{25000(1.02^{14} - 1)}{1.02 - 1}$ or $\frac{125000(1.02^{14} - 1)}{1.02 - 1}$ or equivalent. Awrt 1 997 000 implies this mark.

A1 1 997 000 only

.....

Alt(b)

M1 If they attempt to add year by year then look for 13 or 14 values being added with the first year 25000 and at least one increase year to year of 2% or their r .

A1 All 14 values seen or implied from either column 2 or 3 of the table on the next page. Allow values to be truncated or rounded to 3 significant figures. Awrt 1 997 000 also implies this mark.

A1 1 997 000 only



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \dots$$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0, \quad \text{leading to } x = \dots$$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1.(a)	States/uses either $16k = -4$ or $\frac{16 \times 15}{2} k^2 = p$	M1
	(i) $k = -\frac{1}{4}$	A1
	(ii) $p = \frac{15}{2}$	A1
		(3)
(b)	$g(x) = \left(2 + \frac{16}{x}\right)(1 + kx)^{16}$	
	Attempts either $2^{\text{nd}} p^{\text{th}}$ or $16 \times \frac{16 \times 15 \times 14}{3!} \times k^3$	M1
	Attempts sum of $2^{\text{nd}} p^{\text{th}}$ and $16 \times \frac{16 \times 15 \times 14}{3!} \times k^3$	dM1
	Term in $x^2 = (15 - 140)x^2 = -125x^2$	A1
		(3)
		(6 marks)

(a)

M1: States or uses either $16k = -4$ or $\frac{16 \times 15}{2} k^2 = p$. It may be implied by a correct value for k or p if no incorrect working is seen.

(i)

A1: $k = -\frac{1}{4}$ or -0.25

(ii)

A1: $p = \frac{15}{2}$ or 7.5 Allow if this follows from $k = \pm \frac{1}{4}$

(b)

M1: Attempts one of the two relevant terms or coefficients. Either $2p(x^2)$ or

$16 \times \frac{16 \times 15 \times 14}{3!} \times k^3 (x^2)$ or exact equivalent. May be part of an attempt at a full expansion. May be in terms of p and k

dM1: Must have attempted at least one of the terms correctly. Attempts sum of their

$2 \times p(x^2)$ and their $16 \times \frac{16 \times 15 \times 14}{3!} \times k^3 (x^2)$ which must be arising from correct combination of powers from the expanded brackets though the coefficients need not be correct. Must be numerical terms. May be part of an attempt at a full expansion.



FYI If the k is not cubed in the second of these then you may see $2240x^2$ and can score the dM.

A1: Term in $-125x^2$. Must include the x^2 . Do not allow as part of an expansion for this mark.



Question Number	Scheme	Marks
2.(a)	Attempts to substitute $u_2 = 6k + 3$ in $u_3 (= ku_2 + 3)$ $u_3 = k(6k + 3) + 3$	M1 A1 (2)
(b)	Uses $\sum_{n=1}^3 u_n = 117 \Rightarrow 6 + 6k + 3 + k(6k + 3) + 3 = 117$ $6k^2 + 9k - 105 = 0 \Rightarrow k = \dots$ $k = \frac{7}{2}$	M1 dM1 A1 (3) (5 marks)

(a)

M1: Attempts a full method of finding u_3 . Allow if the “+3” is missing once only. E.g. score for an attempt at substituting $u_2 = 6k + 3$ into $u_3 = ku_2 (+3)$ or their u_2 into $u_3 = 6u_2 + 3$
 May be implied by a correct answer, but an incorrect answer with no substitution is M0.

A1: $u_3 = k(6k + 3) + 3$ OR $u_3 = 6k^2 + 3k + 3$ but isw after a correct answer is seen

(b)

M1: Sets their $u_1 + u_2 + u_3 = 117$ to produce an equation in just k .

dM1: Solves a 3TQ by any valid method to find at least one value for k .

A1: $k = \frac{7}{2}$ ONLY



Question Number	Scheme	Marks
3. (a)	States or uses $h = 3$	B1
	Attempts $\frac{3}{2}\{\log_{10} 2 + \log_{10} 14 + 2 \times (\dots)\}$	M1
	$= \frac{3}{2}\{\log_{10} 2 + \log_{10} 14 + 2 \times (\log_{10} 5 + \log_{10} 8 + \log_{10} 11)\} = 10.10$ (2dp)*	A1*
		(3)
		(1)
(b)	Increase the number of strips	B1
(c) (i)	$\int_2^{14} \log_{10} \sqrt{x} \, dx = \frac{1}{2} \times 10.10 = 5.05$	B1
(ii)	$\log_{10} 100x^3 = 2 + 3 \log_{10} x$	B1
	$\int_2^{14} \log_{10} 100x^3 \, dx = [2x]_2^{14} + 3 \times 10.10 = 54.30$	M1 A1
		(4)
		(8 marks)

(a)

B1: States or uses $h = 3$. (If there is a conflict between what is stated and used award bod if one is correct.)

M1: An attempt at the trapezium rule with their value of h . Look for

$\frac{3}{2}\{\log_{10} 2 + \log_{10} 14 + 2 \times (\dots)\}$ where the $\dots$ is an attempt at at least two intermediate terms and does not repeat the end points. The bracket structure must be correct or implied. (Use of $h = 4$ is likely to have two terms in this inner bracket.) Allow with values correct to 2 s.f. for the first and last terms.

A1*: Reaches 10.10 (or allow 10.1) following at least one correct intermediate line and no incorrect lines that would give a significantly different answer.

Example are $\frac{3}{2}\{\log_{10} 2 + \log_{10} 14 + 2 \times (\log_{10} 5 + \log_{10} 8 + \log_{10} 11)\}$ or possibly

$\frac{3}{2}\{\log_{10} 28 + 2 \log_{10} 440\}$ or $\frac{3}{2}\{0.301 + 1.146 + 2 \times (0.699 + 0.903 + 1.041)\}$ with values correct to at least 2d.p.

(Note that integration via the calculator gives 10.23)



(b)

B1: States "increase the number of strips", "decrease the width of the strips", "use more intervals" o.e.

(c)(i)

B1: (awrt) 5.05 or allow $\frac{1}{2} \times (a)$ if they make a slip miscopying the 10.10

(This is a B mark, so allow even if this answer comes from a repeat at using the trapezium rule.)

(c)(ii)

B1: States or implies that $\log_{10} 100x^3 = 2 + 3\log_{10} x$

M1: $\int_2^{14} \log_{10} 100x^3 \, dx = [ax]_2^{14} + 3 \times 10.10$ or equivalent work for finding the area of the rectangle.

Use of the trapezium rule again here is M0. Must see use of (a).

A1: 54.30 Accept 54.3.



Question Number	Scheme	Marks
4.(a)	States -21	B1 (1)
(b)	Attempts $f(3) = (3^2 - 2)(2 \times 3 - 3) - 21$ Achieves $f(3) = 0 \Rightarrow (x - 3)$ is a factor of $f(x)$ *	M1 A1* (2)
(c) (i)	$f(x) = 2x^3 - 3x^2 - 4x - 15 = (x - 3)(2x^2 \dots \pm 5)$ $= (x - 3)(2x^2 + 3x + 5)$	B1 M1 A1
(ii)	Attempts $b^2 - 4ac$ for their $2x^2 + 3x + 5$ Achieves $b^2 - 4ac < 0$ and states that only root is $x = 3$ o.e. *	M1 A1* (5) (8 marks)

(a)

B1: -21

(b)

M1: Attempts to substitute $x = 3$ into $f(x) = (x^2 - 2)(2x - 3) - 21$ or its expanded form.Condone slips but don't accept just $f(3) = 0$. Attempts via long division score M0.A1*: Achieves $f(3) = 0$ and states that $(x - 3)$ is a factor of $f(x)$. If this latter is given in a preamble accept a minimal conclusion such as // or QED

(c)(i)

B1: Expands $f(x) = (x^2 - 2)(2x - 3) - 21$ to reach correct 4 term cubic. Allow if the cubic is seen anywhere.M1: Correct attempt to find quadratic factor using either division by $(x - 3)$ or inspection.

To score the M mark look for e.g. correct first and last terms by inspection, or first two terms by division. (Two correct terms will imply the mark.) Allow for work seen in (b) as long as it is referred to in (c).

A1: $(f(x) = (x - 3)(2x^2 + 3x + 5))$ following a correct cubic. Must be seen together on one line.

Question Number	Scheme	Notes	Marks
3(a)	$\left(2 - \frac{kx}{4}\right)^8 = 2^8 + \binom{8}{1}2^7\left(-\frac{kx}{4}\right) + \binom{8}{2}2^6\left(-\frac{kx}{4}\right)^2 + \binom{8}{3}2^5\left(-\frac{kx}{4}\right)^3 + \dots$ $\text{Or } \left(1 - \frac{kx}{8}\right)^8 = 1 + \binom{8}{1}\left(-\frac{kx}{8}\right) + \binom{8}{2}\left(-\frac{kx}{8}\right)^2 + \binom{8}{3}\left(-\frac{kx}{8}\right)^3 + \dots$		M1
	$= 256 - 256kx + 112k^2x^2 - 28k^3x^3 + \dots$	$256 - 256kx$	B1
		$112k^2x^2$ or $-28k^3x^3$ (unsimplified)	A1
		$112k^2x^2$ and $-28k^3x^3$ (simplified)	A1
			(4)
(b)	$f(x) = (5 - 3x)\left(2 - \frac{kx}{4}\right)^8 = (5 - 3x)(256 - 256kx + 112k^2x^2 - 28k^3x^3 + \dots)$ Coefficient of x is $5 \times -256k - 3 \times 256$		M1
	$5 \times 256 = 3(-1280k - 768) \Rightarrow k = \dots$	Sets $5 \times$ their constant term from (a) = $3 \times$ their coefficient of x from $f(x)$ and solves for k	M1
	$k = -\frac{14}{15}$	Correct value.	A1
			(3)
			Total 7

Notes**(a)**

M1: Attempts the binomial expansion on $\left(2 \pm \frac{kx}{4}\right)^8$ or $\left(1 \pm \frac{kx}{\beta}\right)^8$ up to at least the third (x^2)

term with an acceptable structure. Look for the correct binomial coefficient (accept alternative notation ${}^nC_r$) combined with the correct power of x but allow if powers of 2 are incorrect and if brackets are missing. M0 for descending powers.

B1: for $256 - 256kx$, may be listed, must be simplified. Allow for $256(1 - kx + \dots)$ if the 2^8 is taken out first.

A1: Correct third or fourth term, may be listed. Need not be simplified but the binomial coefficients must be numerical. Allow for one term from $256\left(\dots + \frac{28}{64}(kx)^2 - \frac{56}{512}(kx)^3 \dots\right)$ if the 2^8 is taken out first. May have powers as $(kx)^n$ for this mark. Allow for the correct x^2 term if the sign was incorrect in their bracket.

A1: Correct simplified third and fourth terms as shown in scheme, may be listed. Must have $k^n x^n$ terms.

Note: isw after correct terms are seen if they try to divide through.

(b)

M1: Correct strategy for the coefficient of x or the x term. E.g. $5 \times$ their $-256k - 3 \times$ their 256 or may be part of a full expansion – look for $(5 \times$ their $-256k - 3 \times$ their 256) x but terms must have been combined.

M1: Sets $5 \times$ their constant term from (a) = $3 \times$ their coefficient of x from $f(x)$ and solves for k . Should be an equation in k only, but allow recovery if they initially include the x but later cross it out to give a constant for the answer. The attempt at the x coefficient must have been an attempt at a sum of two terms from their expansion of $f(x)$

A1: Correct value, must be exact. Allow $-0.9\bar{3}$ but not a terminating decimal.



Question Number	Scheme	Notes	Marks
4	$2\log_3(1-x) = \log_3(1-x)^2$ or $3 = \log_3 3^3$	Correct power law used or implied	B1
	$\log_3(32-12x) - \log_3(1-x)^2 = \log_3 \frac{32-12x}{(1-x)^2}$ Combines 2 log terms correctly		M1
	$\frac{32-12x}{(1-x)^2} = 27$	Obtains this equation in any form	A1
	$\Rightarrow 27x^2 - 42x - 5 = 0 \Rightarrow x = \dots$	Solves 3TQ	M1
	$x = -\frac{1}{9}$	This value only i.e. the $\frac{5}{3}$ must clearly be discarded if seen.	A1
			(5)
			Total 5

Notes**(a)**

- B1:** Correct use of implication of the power law. Award for sight any of $\log_3(1-x)^2$, $\log_3 3^3$, $\log_3 27$ or for $3 \rightarrow 27$ moving from $\log_3(\dots) = 3$ or $\dots = 27$ (oe)
Allow for $\log_3((1-x)+3)^2$ as a misread (the M will be lost)
Allow B1 for $2\log_3(1-x)+3 \rightarrow \log_3 3(1-x)^2$ without first seeing the $\log_3(1-x)^2$ but again the M will be lost without further sufficient work seen.
- M1:** For correctly combining two of the terms $\log_3(32 \pm 12x)$, $\log_3(1 \pm x)^2$ or $\log_3 A$ (where the latter is their attempt at writing 3 as a log term) into one log term. E.g. as shown in the scheme or may see $\log_3(1-x)^2 + \log_3 "27" \rightarrow \log_3("27"(1-x)^2)$ Allow for slips copying terms but must be combining terms of the correct form. $\frac{\log_3(32-12x)}{\log_3(1-x)^2}$ is M0
- A1:** For a correct equation with logarithms removed, any form. Allow with 3^3 in place of 27.
- M1:** For solving a quadratic equation that has come from a valid attempt to remove logarithms. A valid attempt is one that moves from $\log_3(f(x)) = \log_3(g(x))$ to $f(x) = g(x)$ or from $\log_3(f(x)) = A$ to $f(x) = 3^A$, although allow "recovery" from $\frac{\log_3(32-12x)}{\log_3(1-x)^2} = 3 \rightarrow \frac{32-12x}{(1-x)^2} = 3^3$ for this mark and the next A mark.
If no method of solution of the quadratic is shown then the method is implied by at least one correct solution for their quadratic.
- A1:** For $x = -\frac{1}{9}$ only. The $\frac{5}{3}$ must clearly be discarded if seen, e.g. by "reject" stated, or crossed out or $x = -\frac{1}{9}$ underlined or boxed to in some way indicate it is the only solution.

Question Number	Scheme	Notes	Marks
5(a)	$3(1)^3 + A(1)^2 + B(1) - 10 = k$ or $3(-1)^3 + A(-1)^2 + B(-1) - 10 = -10k$	Attempts $f(\pm 1) = k$ or $f(\pm 1) = -10k$	M1
	$A + B - 7 = k, A - B - 13 = -10k$ $\Rightarrow -10A - 10B + 70 = A - B - 13$	Uses $f(\pm 1) = k$ and $f(\mp 1) = -10k$ to eliminate k and obtains an equation in A and B only	M1
	$\Rightarrow 11A + 9B = 83^*$	Correct proof with no errors	A1*
			(3)
(b)	$3\left(\frac{2}{3}\right)^3 + A\left(\frac{2}{3}\right)^2 + B\left(\frac{2}{3}\right) - 10 = 0$	Attempts $f\left(\frac{2}{3}\right) = 0$	M1
	$11A + 9B = 83, 12A + 18B = 246$ $\Rightarrow A = \dots, B = \dots$	Solves $11A + 9B = 83$ simultaneously with <i>their</i> equation in A and B	M1
	$A = -8, B = 19$	Correct values	A1
			(3)
(c)	$f(x) = (3x - 2)(x^2 + \dots x + \dots)$	Uses any appropriate method e.g. long division/inspection to obtain $x^2 + px + q$ where p and q are non-zero.	M1
	$g(x) = x^2 - 2x + 5$	Correct expression	A1
			(2)
			Total 8

Notes
(a)

M1: Attempts to apply the remainder theorem for either term – so substitutes ± 1 into f and equates to k or $-10k$. The powers of ± 1 need not be seen. For attempts via long division look for reaching a quotient $3x^2 + (A \pm 3)x + \dots$ and remainder $\pm A \pm B - \alpha$ before setting equal to the remainder.

M1: Must have attempted remainder theorem on both terms with opposite sign (ie both $f(1)$ and $f(-1)$ attempted, but may be set to the wrong remainders). Uses their two equations to eliminate the k and form an equation in A and B only. Note: finding a value for k (incorrectly) and substituting is M0.

A1*: Correct equation reached from fully correct work.

(b)

M1: Applies the factor theorem with $f\left(\frac{2}{3}\right) = 0$. Must be correct sign here, but no need to evaluate powers for the method mark. Allow if all that is wrong is a slip in one term. The “=0” may be implied by working. For attempts via long division look for quotient $x^2 + \frac{1}{3}(A \pm 2)x + \dots$ and then their remainder (in A and B) set to zero.

M1: Solves their equation and the one from (a) simultaneously. Not dependent so may be scored from an attempt at $f\left(-\frac{2}{3}\right) = 0$. Must have produced a second equation in A and B . Look for an attempt to match coefficients and add/subtract, or an attempt to substitute for a variable, reaching values for A and B . If no method is shown, values reached must match their equations.

A1: Correct values found.

(c)

M1: Attempts inspection, factorisation or long division to try and find the factor. Must produce a three-term quadratic. Implied by the correct answer if no incorrect method seen. For inspection look



for an attempt to set up and solve at least one equation using coefficients, for factorisation look for correct first and last term, for long division look for correct x term for their A and a constant (do not be concerned about a remainder for this mark).

A1: Correct $g(x)$. May be stated separately or accept if seen in a factorised cubic form $f(x) = (3x - 2)(x^2 - 2x + 5)$ or the correct factor from long division – as long as there is no remainder.



Question Number	Scheme	Marks
3 (i)	$2 \log_2 (2-x) = 4 + \log_2 (x+10)$ One correct log law $2 \log_2 (2-x) \rightarrow \log_2 (2-x)^2$ or $4 \rightarrow \log_2 16$ Correct attempt to combine two terms Correct equation (not involving logs) E.g. $(2-x)^2 = 16(x+10)$ $x^2 - 20x - 156 = 0$ $(x+6)(x-26) = 0$ $x = -6$ only	B1 M1 A1 M1 A1 (5)
(ii)	12	B1 (1) (6 marks)

(i)

B1: One correct log law seen or used in relation to the equation e.g. $2 \log_2 \dots \rightarrow \log_2 \dots^2$ or $4 \rightarrow \log_2 16$ (which may appear as $\log_2 2^4$ or $\log_2 4^2$). May be seen anywhere and allow if on incorrect terms.

M1: **Correct** attempt to combine two of the **original** terms, condoning a slip with the 16. E.g. $4 + \log_2 (x+10) = \log_2 16 + \log_2 (x+10) = \log_2 16(x+10)$

Do not allow if e.g. they incorrectly divide by 2 first and attempt to combine $\log_2 (2-x) - \log_2 (x+10)$

A1: Correct equation (not involving logs) from fully correct log work E.g.

$$(2-x)^2 = 16(x+10) \text{ or } \frac{(2-x)^2}{x+10} = 16$$

M1: Correct attempt to solve a 3TQ in x achieving a real root (may be by calculator). B1 must have been scored but not all log work need be correct.

A1: $x = -6$ only following the award of all previous marks (all log work must have been correct). The $x = 26$ solution must be rejected if seen.

SC Allow max B1M0A0M1A0 for e.g. $\frac{\log_2(2-x)^2}{\log_2(x+10)} = 4 \rightarrow (2-x)^2 = 16(x+10)$ etc.

Allow the SC if no combination of logs is shown at all, but the correct quadratic if produced.

(ii)

B1: 12 clearly identified as the answer. Condone $a = 12$ as long as 12 is clearly the answer being offered.



Question Number	Scheme	Marks
4(i)	E.g. $2 = \log_3 9$ Eg $\log_3(4x) - \log_3(5x+7) = \log_3 \frac{4x}{5x+7}$	M1
	E.g. $36x = 5x+7$ $\frac{4x}{5x+7} = \frac{1}{9}$	A1
	$x = \frac{7}{31}$	A1
		(3)
(ii)	$\left(\sum_{r=1}^2 \log_a y^r\right) \log_a y + \log_a y^2$ or $\left(\sum_{r=1}^2 (\log_a y)^r\right) \log_a y + (\log_a y)^2$	B1
	$\log_a y + \log_a y^2 = \log_a y + (\log_a y)^2 \Rightarrow 2 \log_a y - (\log_a y)^2 = 0$ $\Rightarrow \log_a y(2 - \log_a y) = 0 \Rightarrow \log_a y = 2$	M1
	$y = a^2$	A1
		(3)
		Total 6

(i) Answer on its own with no working score 0 marks.

M1 Demonstrates at least one correct law of logarithms.

$$\text{e.g. } 2 = \log_3 9 \text{ or } \log_3(4x) = \log_3 4 + \log_3 x \text{ or } \log_3(4x) - \log_3(5x+7) = \log_3 \frac{4x}{5x+7}$$

$$\log_3(5x+7) - \log_3(4x) = \log_3 \frac{5x+7}{4x}$$

(Do not be concerned with slips on the 2 if manipulating the equation)

$$\text{e.g. } \log_3(4x) + 2 = \log_3(5x+7) \Rightarrow \log_3 \frac{4x}{5x+7} = 2 \text{ scores M1}$$

A1 For a correct equation in any form with no logs eg $4x = \frac{5x+7}{9}$. Must have come from correct log work.A1 $\frac{7}{31}$ only and follows a correct equation with no logs.

Note $\frac{\log_3 5x+7}{\log_3 4x} = -2 \Rightarrow x = \frac{7}{31}$ will only score a maximum of M1A0A0 if they are able to demonstrate at least one correct law of logarithms.

(ii) Answer on its own with no working score 0 marks.B1 For either (or both) summation correct $\log_a y + \log_a y^2$ or $\log_a y + (\log_a y)^2$ or equivalent

Can also be awarded for a correct equation:

$$\text{e.g. } \log_a y^2 = (\log_a y)^2 \text{ or } 2 \log_a y = (\log_a y)^2 \text{ or } 3 \log_a y = \log_a y + (\log_a y)^2$$

Sight of either expression or the equation scores even if they make subsequent errors.

Poor bracketing may be recovered or implied by later work. Do not penalise the absence of base a on some or all of the log terms for this mark.

- M1 Proceeds from their quadratic equation in $\log_a y$, collects terms, factorises or cancels $\log_a y$ and obtains $\log_a y = k$. They may define $\log_a y$ in terms of another variable e.g. x such that $x + 2x = x + x^2 \Rightarrow x = k$ which is acceptable

Alternatively, they may change the base and obtain $\log_a y = k$

$$\text{e.g. } \log_a y^2 = (\log_a y)^2 \Rightarrow \frac{\log_a y^2}{\log_a y} = \log_a y \Rightarrow \log_y y^2 = \log_a y \Rightarrow \log_a y = 2$$



Question Number	Scheme	Marks
5(a)	$3^3 + 3^2(p+3) - 3 + q = 0$	M1
	eg $\Rightarrow 27 + 9p + 27 - 3 + q = 0 \Rightarrow 9p + q = -51^*$	A1*
		(2)
(b)	$(-p)^3 + (p+3)(-p)^2 - (-p) + q = 9$	M1
	eg $-p^3 + p^3 + 3p^2 + p + q = 9 \Rightarrow 3p^2 + p + q - 9 = 0^*$	A1*
		(2)
(c)	$3p^2 + p + q - 9 = 0$ $\Rightarrow 3p^2 + p - 51 - 9p - 9 = 0$ $\Rightarrow 3p^2 - 8p - 60 = 0$	M1
	$p = 6$	A1
	$q = -51 - 9p = -105$	A1
		(3)
(d)	$f(x) = x^3 + 9x^2 - x - 105$ $f(x) = (x-3)(\dots x^2 + \dots x + \dots)$	M1
	$g(x) = x^2 + 12x + 35$	A1
		(2)
		Total 9

(a)

M1 Attempts to use the factor theorem by setting $f(\pm 3) = 0$. Score for the values embedded in the expression leading to an equation in p and q . The $= 0$ may be implied by later work for this mark. Alternatively, there may be other attempts eg to divide algebraically by $x - 3$

$$\begin{array}{r}
 x^2 + (p+6)x + 3p+17 \\
 x-3 \overline{) x^3 + (p+3)x^2 - x + q} \\
 \underline{x^3 - 3x^2} \\
 (p+6)x^2 - x + q \\
 \underline{(p+6)x^2 + (-3p-18)x} \\
 (3p+17)x + q \\
 \underline{(3p+17)x - 9p - 51} \\
 \Rightarrow -9p - 51 = q \Rightarrow 9p + q = -51
 \end{array}$$

To score the method mark they would need to proceed as far as equating q with their " $-9p - 51$ " to achieve an equation in p and q .

With alternative methods look for a correct method, condoning slips and invisible brackets leading to an equation in p and q .

A1* Correct proof with no errors including brackets. There must be at least one intermediate stage of working and $= 0$ must be seen at some point in their solution via the factor theorem method.

e.g. $3^3 + (p+3) \times 3^2 - 3 + q = 0 \Rightarrow 9p + q = -51$ scores M1A0 (no intermediate stage seen)

$27 + (p+3) \times 9 - 3 + q = 0 \Rightarrow 51 + 9p + q = 0 \Rightarrow 9p + q = -51$ scores M1A1



(b)M1 Attempts the remainder theorem by setting $f(\pm p) = 9$ oe Award for e.g.

$$(-p)^3 + (p+3)(-p)^2 + p + q = 9 \text{ condoning sign slips and invisible brackets.}$$

Alternatively, they may attempt to divide algebraically by $x + p$ leading to a remainder in terms of p^2 , p and q which is equated to 9. Condone slips in their working.

$$\begin{array}{r}
 x^2 \quad + 3x \quad -1-3p \\
 x+p \overline{) x^3 + (p+3)x^2 \quad -x \quad + q} \\
 \underline{x^3 \quad px^2} \\
 3x^2 \quad -x \\
 \underline{3x^2 \quad + 3px} \\
 (-1-3p)x \quad + q \\
 \underline{(-1-3p)x + (-1-3p)p}
 \end{array}$$

With alternative methods look for a correct method, leading to a remainder in terms of p^2 , p and q which is equated to 9

A1* Correct proof with no errors including brackets. There must be at least one intermediate stage of working before proceeding to the final answer

$$\text{e.g. } (-p)^3 + (p+3)(-p)^2 - (-p) + q = 9 \Rightarrow 3p^2 + p + q - 9 = 0 \text{ scores M1A0 (no intermediate stage)}$$

$$(-p)^3 + (p+3)(-p)^2 - (-p) + q = 9 \Rightarrow 3p^2 + p + q = 9 \Rightarrow 3p^2 + p + q - 9 = 0 \text{ scores M1A1}$$

(c)M1 Attempts to use both given equations to form a 3TQ equation in p (or q). (terms do not need to be all on one side and condone the omission of $= 0$)

Do not be too concerned by slips in their substitution/rearrangements or miscopying of the given equations. May be implied by either a correct value for p or a correct value for q

A1 $p = 6$ (ignore any reference to $-\frac{10}{3}$)

A1 $q = -105$ only

(d)M1 Uses their values for p and q and a correct strategy (inspection or long division) to obtain the quadratic factor.

Via inspection score for $x^2 + \dots x \pm \frac{q}{3}$

Via long division score for proceeding as far as $x^2 \pm (p+6)x$. If they attempt algebraic division in (a) you may need to check their quotient with their value for p to see if the method mark can be scored. They may also restart which is acceptable.

Condone the use of a negative value for p for this mark (eg even if $p = -6 \Rightarrow 0x$)

A1 $x^2 + 12x + 35$ oe eg $(x+5)(x+7)$ Allow embedded as $(x-3)(x^2 + 12x + 35)$ and condone poor notation such as $f(x) = x^2 + 12x + 35$. Also allow this mark to be scored if seen within their long division.

Question Number	Scheme					Marks																				
1.	<table><tr><td>a</td><td>b</td><td>c</td><td>abc</td></tr><tr><td>1</td><td>10</td><td>4</td><td>40</td></tr><tr><td>2</td><td>6</td><td>7</td><td>84</td></tr><tr><td>3</td><td>2</td><td>10</td><td>60</td></tr><tr><td colspan="4">All values of abc are multiples of 4</td></tr></table>				a	b	c	abc	1	10	4	40	2	6	7	84	3	2	10	60	All values of abc are multiples of 4				One correct set a, b & c	B1
	a	b	c	abc																						
	1	10	4	40																						
	2	6	7	84																						
	3	2	10	60																						
	All values of abc are multiples of 4																									
				Two correct rows	M1																					
				Fully correct + statement	A1																					
(3 marks)																										

Main method: Numerical solution finding all known solutions

B1: Any correct set of values for a, b and c . You can ignore any attempt at the product abc

M1: Any two fully correct rows including the correct calculations for the products.
Condone extra/ incorrect rows

A1: Fully correct. The solution must have

- the three correct rows with correct calculations for abc
- any extra (or incorrect) rows must be deleted or somehow excluded
- a minimal statement showing that they are aware that 40, 84 and 60 are multiples of 4. E.g they are all divisible by 4, use of a tick ✓, prove, QED, calculations such as $\frac{40}{4} = 10$. They can be divided by 4 (on its own) is insufficient without further clarification such as "to give whole numbers"
- no incorrect calculations. E.g if their attempt to prove that 60 being a multiple of 4 includes $\frac{60}{4} = 16$ then it is A0

Alt 1: Via deduction

B1: Either

Solves $a + b + c = 15$ with $c = 3a + 1$ to conclude that $4a + b = 14$ o.e. **and** states that b is even

Or

Uses $c = 3a + 1$ to state that if a is odd then c is even and if a is even then c is odd

M1: Attempts to generalise and offers one of the two possibilities for the product abc .

Either

states then when a is odd, b is even and c is even, therefore $abc = \text{odd} \times \text{even} \times \text{even} = \text{multiple of 4}$

Or

states then when a is even, b is even and c is odd, therefore $abc = \text{even} \times \text{even} \times \text{odd} = \text{multiple of 4}$

A1: Fully correct. The solution must have

- the two possible "calculations" for abc .
- no incorrect "calculations". E.g. if they state that $\text{even} \times \text{even}$ is a multiple of 4 because $2n \times 2n = 4n^2$ it is A0. It must be along the lines of $2m \times 2n = 4mn$ with different variables.

You may see a hybrid solution that can be marked along similar lines. Mark positively.

If you see a tabular solution and one via deduction, award marks for the better one.

Please use review if you are uncertain.



Question Number	Scheme	Marks
2.(a)	$u_2 = \frac{2}{3}, u_3 = -4, u_4 = 3$	M1, A1, A1 (3)
(b)	$\sum_{r=1}^{100} u_r = 33 \times \left(3 + \frac{2}{3} + -4 \right) + 3$ $= -8$	M1 A1 (2) (5 marks)

(a)

M1: Attempts to use the iteration formula seen at least once. E.g. Award for $u_{i+1} = 2 - \frac{4}{3} = \dots$

May be implied by

- either a correct value assigned to the correct term. Condone $\frac{2}{3}$ as 0.67 for u_2
- or a correct follow through value. So if $u_2 = -\frac{2}{3}$ then $u_3 = 2 - \frac{4}{-\frac{2}{3}}$ or 8 would score the mark

It must be a correct method. Watch for correct values attached to the wrong term

E.g. $u_2 = 2 - \frac{4}{2}, u_3 = 2 - \frac{4}{3}, u_4 = 2 - \frac{4}{4}$ does achieve $\frac{2}{3}$ but it is M0. It is assigned to u_3

A1: Any correct value assigned to the correct term. Either $u_2 = \frac{2}{3}$ (but condone as 0.67) $u_3 = -4$ or $u_4 = 3$

A1: All 3 values correct and labelled: $u_2 = \frac{2}{3}, u_3 = -4$ AND $u_4 = 3$. May be awarded from part (b)

(b)

M1: Establishes a correct method to find the sum of 100 terms.

Amongst other correct methods are;

$$33 \times \left(3 + \frac{2}{3} + -4 \right) + 3$$

$$34 \times \left(3 + \frac{2}{3} + -4 \right) - \frac{2}{3} - -4$$

You may see listing but this cannot score the marks unless all terms are present or the correct result is given.

An incorrect part (a) is unlikely to score any marks as it is unlikely to be repeating every 3 terms

A1: -8 . This alone scores both marks following a correct (a), as long as there is no incorrect method.



Question Number	Scheme	Marks
3 (a)	$2 \tan \theta + 3 \sin \theta = 0$	
	States or uses $\tan \theta = \frac{\sin \theta}{\cos \theta} \rightarrow 2 \frac{\sin \theta}{\cos \theta} + 3 \sin \theta = 0$	M1
	$\sin \theta (2 + 3 \cos \theta) = 0$	
	$2 + 3 \cos \theta = 0$	dM1
	$\cos \theta = -\frac{2}{3} \Rightarrow \theta = \text{awrt } 132^\circ \text{ or awrt } 228^\circ$	A1
(b)	$\cos \theta = -\frac{2}{3} \Rightarrow \theta = \text{awrt } 131.8^\circ \text{ and awrt } 228.2^\circ$	A1
	$\sin \theta = 0 \Rightarrow \theta = 180^\circ, 360^\circ$	B1
		(5)
	Sets $2x + 40^\circ = \text{their } 131.8^\circ \Rightarrow x = \dots$	M1
	$x = \text{awrt } 45.9^\circ$	A1
		(2)
		(7 marks)

(a) Condone a lack of a degrees symbol throughout.

Condone solutions that drop θ 's in parts of the working

M1: States $\tan \theta = \frac{\sin \theta}{\cos \theta}$ or attempts to use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ in the given equation. Condone slips in coefficients.

dM1: Attempts to use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ o.e., cross multiplies and either factorises or cancels to form a linear equation in $\cos \theta$

A1: Achieves $\cos \theta = -\frac{2}{3}$ and finds one correct θ value to the nearest degree.

Condone radian solutions for this mark so allow awrt 2.3(0) or awrt 3.98 following $\cos \theta = -\frac{2}{3}$

A1: Achieves $\cos \theta = -\frac{2}{3}$ and finds both θ values in degrees correct to one dp.

Any extra values in the range (for $\cos \theta = -\frac{2}{3}$) is A0

B1: Achieves $\sin \theta = 0$ followed by $\theta = 180^\circ, 360^\circ$ Condone 0 appearing as a solution

(b) Condone a lack of a degrees symbol

M1: Sets $2x + 40^\circ = \text{their } 131.8^\circ$ and proceeds to a value for x .

Look for correct order of operations so $2x + 40^\circ = "131.8^\circ" \Rightarrow x = \frac{"131.8^\circ" \pm 40^\circ}{2}$

There may be multiple attempts, which can be ignored for this mark

Follow through on their smallest value (must be greater than 40°), from part (a)

A1: $x = \text{awrt } 45.9^\circ$ following a correct equation. Do not accept multiple solutions here



Note that there are many parallel solutions in part (a) that can be marked similarly.

Alt I

Using $\sin\theta = \tan\theta\cos\theta$ o.e

$$2\tan\theta + 3\tan\theta\cos\theta = 0 \Rightarrow \tan\theta(2 + 3\cos\theta) = 0$$

In this case the B1 mark is for achieving $\tan\theta = 0 \Rightarrow \theta = 180^\circ, 360^\circ$ Condone 0 appearing as a solution.

Alt II

Multiplying throughout by $\sin\theta$ and using $\sin^2\theta + \cos^2\theta = 1$ eventually leads to a cubic $\cos\theta$ which has

roots of $\cos\theta = -\frac{2}{3}$ and $\cos\theta = 1$. Allow use of a calculator to solve cubic equations, etc.

$$2\frac{\sin\theta}{\cos\theta} + 3\sin\theta = 0 \xrightarrow{\times \sin\theta} 2\frac{\sin^2\theta}{\cos\theta} + 3\sin^2\theta = 0 \rightarrow 2\frac{(1-\cos^2\theta)}{\cos\theta} + 3(1-\cos^2\theta) = 0$$

$$\rightarrow 3\cos^3\theta + 2\cos^2\theta - 3\cos\theta - 2 = 0 \rightarrow \cos\theta = -\frac{2}{3}, \cos\theta = \pm 1$$

M1: For $\tan\theta = \frac{\sin\theta}{\cos\theta}$

dM1: for $\cos\theta = k, |k| < 1, k \neq 0$

A1: For $\cos\theta = -\frac{2}{3}$ and a solution as in the main scheme

A1: : For $\cos\theta = -\frac{2}{3}$ and both solutions as in the main scheme

B1 Achieves $\cos\theta = \pm 1$ followed by $\theta = 180^\circ, 360^\circ$. Condone 0 appearing as a solution

Alt III

Multiplying throughout by $\cos\theta$ and using $\sin^2\theta + \cos^2\theta = 1$ eventually leads to a quartic in $\sin\theta$ which has

roots $\sin\theta = \pm \frac{\sqrt{5}}{3}$ and $\sin\theta = 0$

$$2\sin\theta = -3\sin\theta\cos\theta \xrightarrow{\text{SQUARING}} 4\sin^2\theta = 9\sin^2\theta\cos^2\theta$$

$$\rightarrow 4\sin^2\theta = 9\sin^2\theta(1-\sin^2\theta) \rightarrow 9\sin^4\theta - 5\sin^2\theta = 0$$

$$\rightarrow \sin^2\theta = \frac{5}{9}, \sin\theta = 0$$

M1: For $\tan\theta = \frac{\sin\theta}{\cos\theta}$

dM1: For $\sin^2\theta = k$ or $\sin\theta = k, |k| < 1, k \neq 0$

A1: For $\sin\theta = \frac{\sqrt{5}}{3}, \sin\theta = -\frac{\sqrt{5}}{3}$ or $\sin^2\theta = \frac{5}{9}$ and a correct solution as in the main scheme

A1: For $\sin\theta = \frac{\sqrt{5}}{3}, \sin\theta = -\frac{\sqrt{5}}{3}$ or $\sin^2\theta = \frac{5}{9}$ and both solutions (only) in the main scheme

B1: Achieves $\sin\theta = 0$ followed by $\theta = 180^\circ, 360^\circ$ Condone 0 appearing as a solution

.....
The solution must be obtained from correct working.

Calculator solutions like $2\tan\theta + 3\sin\theta = 0 \Rightarrow \theta = 131.8^\circ$ and 228.2° score no marks in (a)

Even the solution $\theta = 180^\circ, 360^\circ$ cannot score the B1 mark if it is not preceded by $\sin\theta = 0$ or $\tan\theta = 0$

(b) is hence or otherwise so must follow their solution in (a) or be a restart.



Question Number	Scheme	Marks
5 (a)	$D = 8 + 5 \sin\left(\frac{\pi \times 2}{6} + 3\right) = 4.07$	B1
		(1)
(b)	(b) $6 = 8 + 5 \sin\left(\frac{\pi t}{6} + 3\right) \Rightarrow \sin\left(\frac{\pi t}{6} + 3\right) = -\frac{2}{5}$ $\Rightarrow \left(\frac{\pi t}{6} + 3\right) = \arcsin\left(-\frac{2}{5}\right) = \text{Any of } 3.55, 5.87, 9.84, 12.2$ $\Rightarrow t = \text{Any of } \frac{6(3.55-3)}{\pi}, \frac{6(5.87-3)}{\pi}, \frac{6(9.84-3)}{\pi}, \frac{6(12.2-3)}{\pi}$ 13:04 or 1:04 pm	M1 , A1 dM1 ddM1 A1
		(5)
		Total 6

(a)

B1: Scored for sight of $8 + 5 \sin\left(\frac{\pi \times 2}{6} + 3\right)$ o.e followed by 4.06, 4.07 or awrt 4.07. Units can be omitted

(b) It is acceptable to set $x = \frac{\pi t}{6} + 3$ which allows access to the first three marks

M1: Substitutes $D = 6$ and proceeds to $\sin\left(\frac{\pi t}{6} + 3\right) = k$

A1: $\sin\left(\frac{\pi t}{6} + 3\right) = -\frac{2}{5}$. o.e

dM1: Takes arcsink where $k < 0$ and achieves any **correct positive value** for their arcsink

Any of 3.55, 5.87, 9.84, 12.2 are acceptable where $k = -\frac{2}{5}$. Follow through on their $-1 < k < 0$

ddM1: Full and complete method to find a positive value of t

It is dependent upon both previous M's **AND** $k = -\frac{2}{5}$

Score for $t = \text{Any of } \frac{6(3.55-3)}{\pi}, \frac{6(5.87-3)}{\pi}, \frac{6(9.84-3)}{\pi}, \frac{6(12.2-3)}{\pi}$

It can be implied for any t value to 3sf,

Score for any t value awrt 1.05/1.06, 5.48/5.49, 13.0/13.1, 17.5/17.6 following M1 A1dM1

A1: 13:04 or 1:04 pm. Condone/allow an answer of 13.03 or 1.03pm.

Condone one hour and 4 minutes after midday

Requires $\arcsin(-0.4) = \text{awrt } 9.84$, $t = \text{awrt } 13.1$ and a time of 13:03 o.e.



Note that the steps may not occur in exactly the same way but it is equivalent work.

$$\begin{aligned}
 5 \sin\left(\frac{\pi t}{6} + 3\right) &= -2 \\
 \sin\left(\frac{\pi t}{6} + 3\right) &= -\frac{2}{5} \\
 \sin\left(\frac{\pi t}{6} + 3\right) &= -0.4 \\
 \frac{\pi t}{6} + 3 &= -0.41 \\
 \frac{\pi t}{6} &= -3.41 \\
 \pi t &= -20.47 \\
 t &= -6.52 \\
 t &= -6.52 + 4\pi \\
 t &= 17.49
 \end{aligned}$$

dM1 scored here

Adding on 4π at this stage is equivalent

work to $\frac{\pi t}{6} + 3 = -0.41 + 4\pi = \text{awrt } 12.2$

ddM1 scored on lhs for 17.49

Note that all the above steps must be seen to score full/part marks here

Example1

Using num-solv on TI-36XPro: $6 = 8 + 5 \sin\left(\frac{\pi t}{6} + 3\right) \Rightarrow t = 1.056$ scores 00000

Example2

Using num-solv on TI-36XPro: $6 = 8 + 5 \sin\left(\frac{\pi t}{6} + 3\right) \Rightarrow \sin\left(\frac{\pi t}{6} + 3\right) = -\frac{2}{5} \Rightarrow t = 1.056$ scores 11000

Example 3

Using 24 hours in a day:

$6 = 8 + 5 \sin\left(\frac{\pi t}{6} + 3\right) \Rightarrow \sin\left(\frac{\pi t}{6} + 3\right) = -\frac{2}{5} \Rightarrow \frac{\pi t}{6} + 3 = -0.4115 \Rightarrow t = -6.515 + 24 \Rightarrow t = 17.485$ scores

11000

Question Number	Scheme	Marks
6(a)	Sets $f\left(-\frac{3}{2}\right) = 0 \Rightarrow (9p + 4q = 102)$	M1
	Sets $f(-2) = -5 \Rightarrow (4p + q = 43)$	M1
	Solves to get values for p and q	dM1
	(i) $p = 10$ * (ii) $q = 3$ following two correct equations	A1*, A1
		(5)
(b)	$f'(x) = 12x^2 + 20x + 8$	B1
	Solves $f'(x) = 0 \Rightarrow 4(3x + 2)(x + 1) = 0 \Rightarrow x = -\frac{2}{3}, -1$	M1, A1
	$-1 < x < -\frac{2}{3}$	A1
		(4)
		Total 9

(a)

M1: Attempts to set $f\left(-\frac{3}{2}\right) = 0$ **to form an equation in p and q** . Setting $f\left(\frac{3}{2}\right) = 0$ is M0

Award for an attempt such as $4\left(\frac{-3}{2}\right)^3 + p\left(\frac{-3}{2}\right)^2 + 8\left(\frac{-3}{2}\right) + q = 0$ condoning slips (e.g. lack of bracketing)

Note that the partially simplified correct equation is $-\frac{27}{2} + \frac{9p}{4} - 12 + q = 0$

You may see an attempt via division. Look for division by $(2x + 3)$ leading to a remainder in p and q which should be set equal to 0

M1: Attempts to set $f(-2) = -5$ **to form an equation in p and q** . Setting $f(2) = -5$ or $f(-2) = 5$ is M0

Award for $4(-2)^3 + p(-2)^2 + 8(-2) + q = -5$ condoning slips (e.g. lack of bracketing).

Note that the partially simplified correct equation is $-32 + 4p - 16 + q = -5$

You may see an attempt via division. Look for division by $(x + 2)$ leading to a remainder in p and q which should be set equal to -5

dM1: Solves a pair of equations in both p and q resulting from setting $f\left(\pm\frac{3}{2}\right) = 0$ and $f(\pm 2) = \pm 5$

Just look for some work (either substitution or elimination) which may or may not be correct leading to values for p and q . They know that p should be 10 so there will be a certain amount of fudging. Be generous here and give BOD if they show **some** correct work leading to both values. Allow the solution to be done via a calculator but the solution must fit at least one of the equations if done this way.

So if both equations are correct, or only one equation is correct, writing down $p = 10, q = 3$ would score this mark as it is a solution to both.

Examples of sufficient work

Ex 1 (Correct equations/elimination): $\frac{9}{4}p + q = \frac{51}{2}$ (1),

$4p + q = 43$ (2), then $(2) - (1) \frac{7}{4}p = \dots \Rightarrow p = \dots, q = \dots$

Ex 2: (Incorrect first equation $f(3/2) = 0$, substitution)

$\frac{9}{4}p + q = -\frac{51}{2}, 4p + q = 43 \Rightarrow \frac{9}{4}p + 43 - 4p = \frac{51}{2} \Rightarrow p = \dots, q = \dots$



Ex 3: (Incorrect second equation $f(2) = -5$: Calculator solution)

Either $9p + 4q = 102, 4p + q = -53 \Rightarrow p = 10, q = -93$ (solution to 2nd equation)

Or $9p + 4q = 102, 4p + q = -53 \Rightarrow p = 10, q = 3$ (solution to 1st equation)

A1*: Achieves $p = 10$ from solving a correct pair of simultaneous equations.

There must have been some attempt to justify this as it is a given answer.

So, a correct intermediate line **must be seen**. E.g. $\frac{9p}{4} + q = \frac{51}{2}, 4p + q = 43 \Rightarrow \frac{7}{4}p = \frac{35}{2} \Rightarrow p = 10$

Allow this to be scored from a correct intermediate line of the form $ap + b = c$

A1: $q = 3$. **This can only be awarded if the two simultaneous that were 'solved' were both correct.**

There is no requirement to see an intermediate line here.

(b)

B1: Differentiates to achieve $12x^2 + 20x + 8$

M1: Attempts to find the critical values of the equation " $f'(x) = 0$ ".

Allow any valid method including via a calculator. Their $f'(x)$ must be of the form $ax^2 + bx + c$

A1: Achieves the correct exact critical values. Do not accept -0.67 but allow recurring decimals $-0.\dot{6}$

A1: $-1 < x < -\frac{2}{3}$ OR $-1 \leq x \leq -\frac{2}{3}$ but the inequality must be used consistently.

Allow versions such as $\left(-1, -\frac{2}{3}\right)$ and $\left[-1, -\frac{2}{3}\right]$

.....
Note that candidates cannot just use their calculators to solve part (b).

So $f(x) = 4x^3 + 10x^2 + 8x + 3$ followed by $-1 < x < -\frac{2}{3}$ is 0 marks.

The minimum working required would be sight of $f'(x)$ followed by the solution.

.....

Question Number	Scheme	Marks
5 (a)	$f(x) = 3x^3 + ax^2 - 10x + b$ $f\left(\frac{4}{3}\right) = 0 \Rightarrow 3 \times \left(\frac{4}{3}\right)^3 + a \left(\frac{4}{3}\right)^2 - 10 \left(\frac{4}{3}\right) + b = 0$ $\frac{64}{9} + \frac{16}{9}a - \frac{40}{3} + b = 0 \Rightarrow 64 + 16a - 120 + 9b = 0 \Rightarrow 16a + 9b = 56 *$	M1 A1* (2)
(b)	$f(2) = b \Rightarrow 3 \times (2)^3 + a(2)^2 - 10(2) + b = b \text{ leading to an equation in just } a$ $4a = -4 \Rightarrow a = -1$ Substitutes $a = -1$ in $-16 + 9b = 56 \Rightarrow b = \dots$ $\Rightarrow b = 8$	M1 A1 M1 A1 (4)
(c)	$f(x) = 3x^3 - x^2 - 10x + 8 = (3x - 4)(x^2 + x - 2)$ $= (3x - 4)(x + 2)(x - 1)$	M1, A1 A1 (3) (9 marks)

(a)

M1: Sets $f\left(\frac{4}{3}\right) = 0$ producing an equation in a and b . Condone slips but the intention must be clear.

For example, $3 \times \frac{4^3}{3} + a \times \frac{4^2}{3} - 10 \times \frac{4}{3} + b = 0$ is incorrect but the intention is clear and scores M1, A0

Finding a simplified $f\left(\frac{4}{3}\right) = \frac{64}{9} + \frac{16}{9}a - \frac{40}{3} + b = \frac{16}{9}a - \frac{56}{9} + b$ before setting it equal to 0 is acceptable

Candidates who find $f\left(\frac{4}{3}\right)$ and proceed to a correct equation without ever stating $f\left(\frac{4}{3}\right) = 0$ or setting their expression for $f\left(\frac{4}{3}\right)$ equal to 0 can be awarded M1, A0 by implication.

$$3\left(\frac{4}{3}\right)^3 + a\left(\frac{4}{3}\right)^2 - 10\left(\frac{4}{3}\right) + b$$

Example of M1 A0*:

$$\frac{64}{9} + \frac{16}{9}a - \frac{40}{3} + b$$

Candidate attempts $f\left(\frac{4}{3}\right)$ which they then simplify (and then multiply by 9)

$$64 + 16a - 120 + 9b$$

$$16a + 9b = 56$$

The final line implies $9 \times f\left(\frac{4}{3}\right) = 0$ and so implies $f\left(\frac{4}{3}\right) = 0$

Setting $f\left(-\frac{4}{3}\right) = 0$ is M0

A1*: Shows that $16a + 9b = 56$. This is a given answer so there must be no incorrect lines (no recovery) or working. You do not explicitly need to see the ' $f\left(\frac{4}{3}\right)$ ' just its result, see below.

It is acceptable to have just one correct intermediate line with either collected terms or non-fractional coefficients between $3 \times \left(\frac{4}{3}\right)^3 + a \left(\frac{4}{3}\right)^2 - 10 \left(\frac{4}{3}\right) + b = 0$ and $16a + 9b = 56$

For example: $3 \times \left(\frac{4}{3}\right)^3 + a \left(\frac{4}{3}\right)^2 - 10 \left(\frac{4}{3}\right) + b = 0 \Rightarrow \frac{16}{9}a + b - \frac{56}{9} = 0 \Rightarrow 16a + 9b = 56$ scores M1 A1



$$\text{as does } 3 \times \left(\frac{4}{3}\right)^3 + a \left(\frac{4}{3}\right)^2 - 10 \left(\frac{4}{3}\right) + b = 0 \Rightarrow 64 + 16a - 120 + 9b = 0 \Rightarrow 16a + 9b = 56$$

(b)

M1: Attempts to set $f(2) = b$ and proceeds to form an equation in just a .

Condone slips but setting $f(-2) = b$ is M0

A1: $a = -1$

M1: Substitutes their value of a found from the equation $f(\pm 2) = b$ into $16a + 9b = 56$ and finds a value for b

A1: $b = 8$

(c)

M1: Attempts to divide $f(x)$ by $(3x - 4)$. Alternatively attempts to factorise out $(3x - 4)$

Look for two 'correct' terms via either method.

$$\text{E.g. via factorisation look at first and last terms } 3x^3 + ax^2 - 10x + b = (3x - 4) \left(x^2 + \dots x \pm \frac{b}{4} \right)$$

$$\begin{array}{r} x^2 + \left(\frac{a+4}{3}\right)x \dots\dots\dots \\ 3x-4 \overline{) 3x^3 + ax^2 - 10x + b} \end{array}$$

via division look for first two terms $3x - 4$

$$\text{for their numerical values of } a \text{ and } b$$

The demand of the question is 'Hence', so the candidates must use the information given, that is

$(3x - 4)$

is a factor. Do not accept division or factorisation by other factors.

A1: Achieves the correct quadratic factor of $(x^2 + x - 2)$

A1: Fully factorised form required $(3x - 4)(x + 2)(x - 1)$ all together on one line.

It can only be scored following M1. ISW if you see the addition of $= 0$ and perhaps roots

You will see attempts that factorise attempt to $f(x)$ via factors formed via calculator methods. These should be scored M0, A0, A0

.....
Alt (c).

M1: Candidates can attempt to divide by $(x - 1)$ or $(x + 2)$ only if they have firstly proven/shown that

$f(1) = 0$ or $f(-2) = 0$. This can be marked in a similar way to the main scheme and can only really follow a correct $f(x)$. So expect to see $f(x) = 3x^3 - x^2 - 10x + 8$ with, for example,

$f(1) = 3 - 1 - 10 + 8 = 0 \Rightarrow (x - 1)$ is a factor. Then division by $(x - 1)$ with similar conditions as the main scheme.

.....
Inevitably we will see attempts at parts (a) and (b) via a table or by division. This is acceptable.

For example:

In part (a), award M1 for an attempt at

$$\begin{array}{r} x^2 \dots\dots\dots \\ 3x-4 \overline{) 3x^3 + ax^2 - 10x + b} \end{array}$$

that achieves a quadratic quotient and a remainder which is set $= 0$

$$\text{Then award A1 if it is correct. For example, } b + 4 \left(\frac{4}{9}a - \frac{14}{9} \right) = 0 \Rightarrow 16a + 9b = 56$$

In part (b), award M1 for an attempt at

$$x-2 \overline{) 3x^3 + ax^2 - 10x + b} \quad \begin{array}{l} 3x^2 + \dots \end{array}$$

that achieves a quadratic quotient and a remainder which is set = b

Then award A1 if it is correct. For example, $4a + 4 + b = b \Rightarrow a = -1$
If in doubt then please send to review



Question Number	Scheme	Marks
1(a)	$(1-4x)^7$ Correct form for term 3 or term 4: Either $\frac{7 \times 6}{2}(\dots x)^2$ or $\frac{7 \times 6 \times 5}{6}(\dots x)^3$ o.e. $\frac{7 \times 6}{2}(-4x)^2$ or $\frac{7 \times 6 \times 5}{6}(-4x)^3$ o.e. $= 1 - 28x + 336x^2 - 2240x^3$	M1 A1 <u>B1</u> , A1 (4)
(b)	Sets " $336 \times 5 - 28k = 1316$ $(k =) 13$	M1 A1 (2) (6 marks)

(a) Note that a correct expression scores 4/4

M1: Correct form for x^2 or x^3 . Look for the correct binomial coefficient with the correct power of x . You do not need to be concerned with the -4 which may have been omitted.

Allow alternative equivalent notation for the binomial coefficient for this mark e.g. ${}^7C_2$ or ${}^7C_5$ or $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$

or $\frac{7!}{2!5!}$ etc

Condone invisible brackets for this mark e.g. ${}^7C_2 1^5 - 4x^2$

A1: Correct x^2 **or** x^3 term (which may be unsimplified). They must have a numerical expression or value for their binomial coefficient for this mark. i.e. do not allow e.g. ${}^7C_3$

Do not condone poor bracketing for this mark but may be implied by further work.

B1: For the series starting $1 - 28x$ (terms may be in any order or as a list)

Condone $1 - 28x^1$ but not e.g. $1 + -28x$

isw if they subsequently incorrectly attempt to simplify their answer.

A1: $336x^2 - 2240x^3$ Do not accept e.g. $336x^2 + -2240x^3$ unless already penalised for B1.

Allow the terms to be written as a list.

isw once the correct simplified terms have been found

(b)

M1: Adds the correct two terms using their coefficients from (a) and sets equal to 1316. (Allow misread)

Look for " $336 \times 5 \pm 28k = 1316$ " i.e. they do not need to proceed to a value for k for this mark

If they still have x and x^2 present in the equation then these must be removed or implied by further work to score the mark. Condone transcription errors from their part (a).

If just a value is stated following an incorrect part (a), then you will need to check this.

A1: $(k =) 13$



Question Number	Scheme	Marks
2(a)	Midpoint of $(-2, 5)$ and $(4, 15)$ is $\left(\frac{-2+4}{2}, \frac{5+15}{2}\right) = (1, 10)$ Attempts radius ² or diameter ² : e.g. $D^2 = (4 - -2)^2 + (15 - 5)^2 = 136$ Radius ² = 34 $(x-1)^2 + (y-10)^2 = 34$	B1 M1 A1 M1, A1 (5)
(b)	$(1, 10 - \sqrt{34})$	B1, dB1 (2) (7 marks)

(a)

B1: Finds the midpoint of the two given points. Must be simplified to score this mark.
Can be implied by circle formula. Allow e.g. $x = 1$, $y = 10$ or may be without brackets. May be seen on a diagram.

M1: Attempts to find the radius² or diameter². Can be implied by the circle formula e.g.

$$(x \pm \dots)^2 + (y \pm \dots)^2 = 34$$

It does not need to be correctly labelled but they must be attempting to find the difference between the x coordinates and the difference between the y coordinates before squaring.

Condone square rooting to find the radius or diameter (which may be all that is seen)

e.g. $2\sqrt{34}$, $\sqrt{136}$ (=11.66..), $\sqrt{34}$ (=5.83...). Allow decimals for this mark.

May be seen embedded within $x^2 + y^2 + 2fx + 2gy + c = 0$

A1: Calculates the radius² = 34 o.e. (or allow to be implied by radius = $\sqrt{34}$)

Can be implied by circle formula or a value of $c = 67$ in $x^2 + y^2 + 2fx + 2gy + c = 0$

M1: Attempts correct form for the circle $(x \pm "1")^2 + (y \pm "10")^2 = "34"$ or $(x \pm "1")^2 + (y \pm "10")^2 = \frac{"136"}{4}$

Do not allow this mark if they have clearly attempted to use their radius instead of their radius²

If they have found the length of the diameter or diameter² however they have labelled it, then they must have attempted to divide by 4.

They may alternatively attempt $x^2 + y^2 + 2fx + 2gy + c = 0$ where $f = "1"$, $g = "10"$ and $c = "67"$

If a centre has not been found or attempted with just values appearing in the equation then they must be correct.

A1: $(x-1)^2 + (y-10)^2 = 34$ but not $(x-1)^2 + (y-10)^2 = (\sqrt{34})^2$ isw after a correct answer

Also allow equivalent expressions e.g. $x^2 + y^2 - 2x - 20y + 67 = 0$

(b) Note coordinates may be seen on a diagram. Do not allow $x = 1$ if just the midpoint from (a) is stated.

B1: Either $x = 1$ or $y = 10 - \sqrt{34}$ Condone lack of labelling for this mark.

Isw after an exact coordinate is seen.

dB1: Both $x = 1$ and $y = 10 - \sqrt{34}$ Isw after both exact coordinates are seen. **Note B0B1 is not possible.**



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

