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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Coordinate Geometry of Circles (P2.2)

International A Level Mathematics · P2: Pure Mathematics 2 (WMA12)

20 questions · 160 marks · Mark schemes included

Past papers June 2019 - June 2025 · 2 sub-topics

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 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
P2.2a EQUATIONS OF CIRCLES		20 Qs · 160 marks	2
1. Q3	IAL P2 (WMA12) June 2024	5	3
2. Q9	IAL P2 (WMA12) June 2025	5	5
3. Q3	IAL P2 (WMA12) January 2024	6	7
4. Q7	IAL P2 (WMA12) June 2024	6	9
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6. Q6	IAL P2 (WMA12) October 2019	7	15
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9. Q2	IAL P2 (WMA12) June 2025	7	25
10. Q6	IAL P2 (WMA12) January 2020	8	27
11. Q6	IAL P2 (WMA12) October 2021	8	31
12. Q6	IAL P2 (WMA12) January 2022	8	35
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17. Q6	IAL P2 (WMA12) January 2023	10	55
18. Q10	IAL P2 (WMA12) October 2024	10	59
19. Q10	IAL P2 (WMA12) June 2022	12	63
20. Q9	IAL P2 (WMA12) October 2022	12	68
P2.2b TANGENTS, CHORDS & INTERSECTIONS WITH CIRCLES			

No questions on this sub-topic in the available past papers.



QUESTIONS

20 questions · 160 marks · 1 sub-topic

SUB-TOPIC P2.2a

EQUATIONS OF CIRCLES

Sub-topic P2.2a · 20 questions · 160 marks · lowest tariff first



3. A circle has equation

$$x^2 + y^2 + 8x - 14y - 79 = 0$$

(a) Find

- (i) the coordinates of the centre of the circle,
- (ii) the radius of the circle.

Given that P is the point on the circle that is nearest the origin O ,

(b) find the exact length of OP

(2)



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 5 marks)



9. The circle C

- has a centre which lies on the x -axis
- touches the y -axis
- passes through the point $(5, 6)$

(a) On Diagram 1, sketch a graph of C .

(1)

(b) Find an equation for C .

(4)

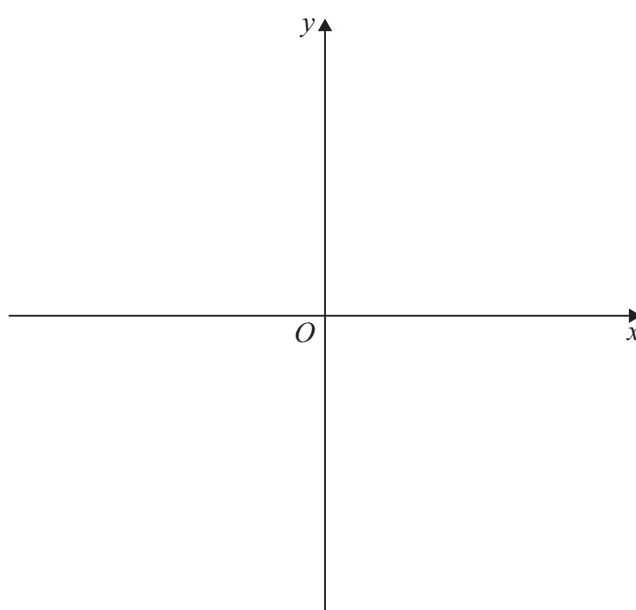


Diagram 1

Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 5 marks)

(Total for Question 3 is 6 marks)



Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 6 marks)



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Q2

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Question 6 continued

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Q6

(Total 7 marks)



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6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x -axis is a tangent to this circle,

(a) find the value of k .

(3)

The circle meets the coordinate axes at the points R , S and T .

(b) Find the exact area of the triangle RST .

(4)

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Question 6 continued

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(Total 7 marks)

Q6



3. A circle C has centre $(2, 5)$

Given that the point $P(8, -3)$ lies on C

- (a) (i) find the radius of C

- (ii) find an equation for C

(3)

- (b) Find the equation of the tangent to C at P giving your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found.

(4)



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 7 marks)



(2)

(Total for Question 2 is / marks)



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Question 6 continued

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Question 6 continued

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Q6

(Total 8 marks)







Question 6 continued

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Handwriting practice area with 30 horizontal lines.

(Total 8 marks)

Q6

Answer box for Q6



Question 6 continued

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Question 6 continued

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Lined area for writing the answer to Question 6.

(Total 8 marks)

Q6

Mark box for Q6.



7.

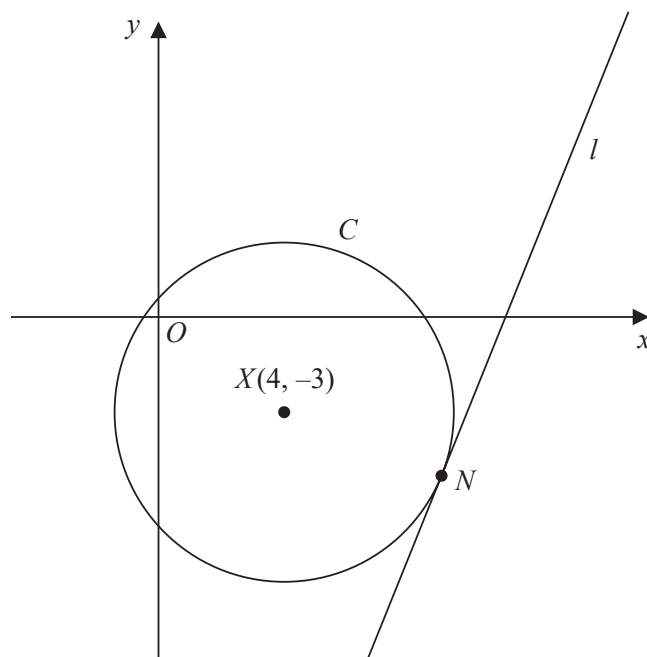


Figure 2

Figure 2 shows a sketch of

- the circle C with centre $X(4, -3)$
- the line l with equation $y = \frac{5}{2}x - \frac{55}{2}$

Given that l is the tangent to C at the point N ,

(a) show that an equation for the straight line passing through X and N is

$$2x + 5y + 7 = 0 \quad (3)$$

(b) Hence find

- the coordinates of N ,
- an equation for C .

(5)





(Total for Question 7 is 6 marks)



6.

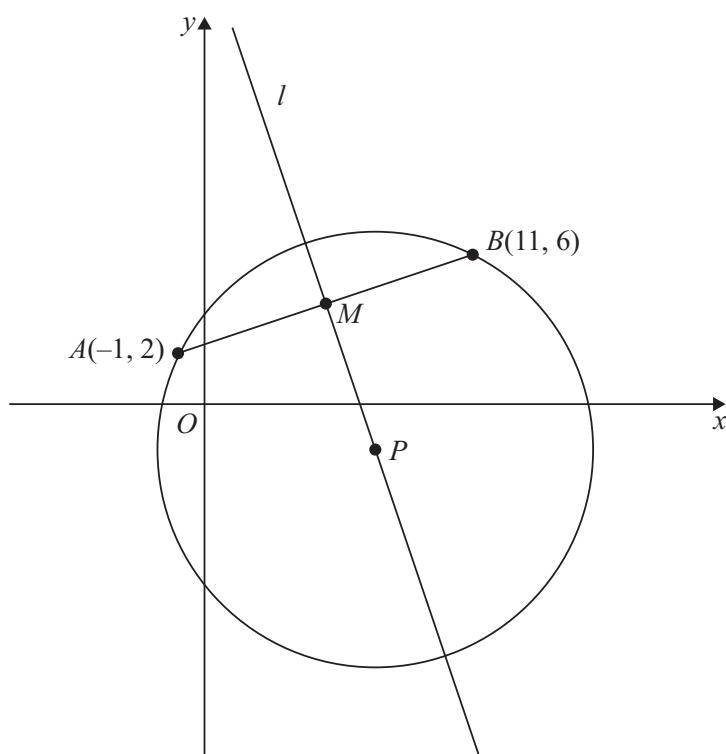


Figure 2

The point $A(-1, 2)$ and the point $B(11, 6)$ both lie on a circle with centre P .

The point M is the midpoint of AB .

Given that the line l passes through M and P , as shown in Figure 2,

- (a) find an equation for l , giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Given that P has coordinates $(7, k)$, where k is a constant,

- (b) find the value of k ,

(1)

- (c) find an equation for the circle.

(3)





(Total for Question 6 is 6 marks)



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4. The points P and Q have coordinates $(-11, 6)$ and $(-3, 12)$ respectively.

Given that PQ is a diameter of the circle C ,

- (a) (i) find the coordinates of the centre of C ,

- (ii) find the radius of C .

(4)

- (b) Hence find an equation of C .

(2)

- (c) Find an equation of the tangent to C at the point Q giving your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found.

(3)

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Q4

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Q9

Question 6 continued

Lined area for writing the answer to Question 6 continued.





Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 10 marks)







(Total for Question 10 is 10 marks)



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10. The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C ,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

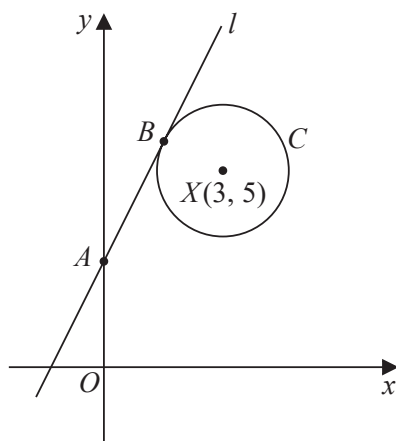


Figure 2

The line l

- cuts the y -axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

Question 10 continued

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Lined area for writing the answer to Question 10.



Question 10 continued

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Handwriting practice area with 30 horizontal lines.



Question 10 continued

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Q10

(Total 12 marks)

END

TOTAL FOR PAPER IS 75 MARKS



9.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

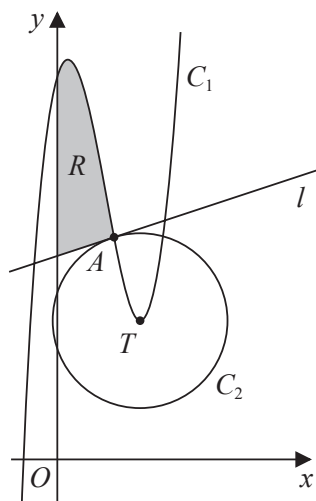


Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

- (a) find the coordinates of
- T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

- (b) Find an equation of the circle
- C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

- (c) Show that an equation of
- l
- is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R , shown shaded in Figure 3, is bounded by C_1 , l and the y -axis.

- (d) Find the exact area of
- R
- .

(3)





Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 12 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

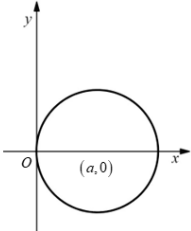
Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
9(a)		B1
(b)	$(x \pm a)^2 + y^2 = \dots$ $(x \pm a)^2 + y^2 = a^2$ Uses (5, 6) in $(x \pm a)^2 + y^2 = a^2$ to form and solve an equation in a E.g. $(5 - a)^2 + 36 = a^2 \Rightarrow 10a = 61 \Rightarrow a = 6.1$ $(x - 6.1)^2 + y^2 = 6.1^2$	M1 A1 dM1 A1 (4) (5 marks)

(a)

B1: A circle lying in quadrants 1 and 4 only which

- passes through (0, 0)
- is symmetrical about the x -axis

Condone slips of the pen. Mark this positively.

Ignore any reference to the point (5, 6)

Ignore any reference to the coordinates of the centre

(b) Way One: Candidates start from an appropriate equation involving x^2 and y^2 then use (5, 6)The following equations are mostly written with a constant a . Any suitable constant may be used.The sign of the constant is also not important so accept $(x + a)^2 + y^2 = \dots$ or $(x - a)^2 + y^2 = \dots$ as well as $(a - x)^2 + y^2 = \dots$ M1: Allowable left-hand side for the equation of C For example $(x \pm a)^2 + y^2 = \dots$ or $(a - x)^2 + y^2 = \dots$ This must involve a constant and not be $(x - 5)^2 + y^2 = \dots$ for exampleA1: Correct form for the equation involving a **single** constant.Examples of correct equations are $(x - a)^2 + y^2 = a^2$, $(x + k)^2 + y^2 = k^2$ and $x^2 + y^2 = \pm 2px$ If they write $(x - a)^2 + y^2 = r^2$ and $r^2 = a^2$ this can be implieddM1: Uses (5, 6) in $(x \pm a)^2 + y^2 = a^2$ o.e. to form and solve an equation in a .E.g. $(5 - a)^2 + 36 = a^2 \Rightarrow \pm 10a = 61 \Rightarrow a = \dots$ $5^2 + 6^2 = \pm 10p \Rightarrow p = \dots$ A1: $(x - 6.1)^2 + y^2 = 6.1^2$ o.e. such as $x^2 + y^2 = 12.2x$

.....

General Principles for Pure Mathematics Marking

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Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

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Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1	$f(x) = ax^3 + 3x^2 - 8x + 2$	
	Sets $f(2) = 3 \Rightarrow 8a + 12 - 16 + 2 = 3$	M1
	$\Rightarrow 8a = 5 \Rightarrow a = \frac{5}{8}$	M1A1
		(3)
		(3 marks)

M1: Attempts to set $f(2) = 3$. The values embedded and the expression set equal to 3 is sufficient. May be implied by further work. If 2 embedded in the expression is not seen, condone slips in their evaluation provided there is still evidence that the intention was to substitute in 2.

May also be seen as $(f(2) - 3 =) a(2)^3 + 3(2)^2 - 8(2) - 1 = 0$

M1: Solves a linear equation in a arising from setting $f(\pm 2) = \pm 3$. Condone slips in their rearrangement proceeding to $a = \dots$ (May be implied by further work).

A1: $a = \frac{5}{8}$ or exact equivalent

Answers of $a = \frac{5}{8}$ may appear with very little or no working, possibly via trial and improvement. If so, then marks can only be allocated if evidence is shown.

e.g. $f(x) = \frac{5}{8}x^3 + 3x^2 - 8x + 2 \Rightarrow f(2) = 5 + 3(2)^2 - 8(2) + 2 = 3 \Rightarrow$ dividing by $(x - 2)$ gives a remainder of 3

More difficult alternative methods may be seen:

Alt I: e.g. algebraic division (note they may also divide $ax^3 + 3x^2 - 8x - 1$ by $(x - 2)$ and set the remainder equal to 0)

$$\begin{array}{r}
 \overline{ax^2 + (3+2a)x + (-2+4a)} \\
 x-2 \overline{) ax^3 - 8x } \\
 \underline{ax^3 - 2ax^2} \\
 (3+2a)x^2 \\
 \underline{(3+2a)x^2 + (-6-4a)x} \\
 (-2+4a)x \\
 \underline{(-2+4a)x + 4 - 8a} \\
 -2 + 8a \Rightarrow -2 + 8a = 3 \Rightarrow a = \frac{5}{8}
 \end{array}$$

M1: Divides the cubic by $(x - 2)$, leading to a quadratic quotient where both the coefficient of x and the constant term are in terms of a . They should then have a linear remainder in a which is then set equal to 3

M1: Solves an equation resulting from setting a linear remainder in a equal to ± 3 . It is dependent on the first method mark via this route.

A1: Completely correct with $a = \frac{5}{8}$

Question Number	Scheme	Marks
7 (a)	$x^2 + y^2 + 8x - 10y = 29$	
	Attempts $(x+4)^2 + (y-5)^2 \dots = \dots$	M1
	Correct centre $(-4, 5)$	A1
(b)	Exact radius $\sqrt{70}$	A1
	Attempts distance (or distance ²) from $(-4, 5)$ to $(5, -8)$	M1
	Full method. E.g. compares $\sqrt{9^2 + 13^2}$ to $(\sqrt{70} + \sqrt{52})$	dM1
	States that - distance between centres $= \sqrt{250} = 15.81$ - sum of radii $= \sqrt{70} + \sqrt{52} = 15.58$ Concludes as sum of radii < distance between centres, result proven *	A1*
		(3) (6 marks)

(a)

M1: Attempts to complete the square for **both** variables. Look for $(x \pm 4)^2, (y \pm 5)^2 \dots = \dots$ or may be implied by $(\pm 4, \pm 5)$ by inspection from the equation.

A1: $(-4, 5)$

A1: $\sqrt{70}$ which may be scored following $(x \pm 4)^2 + (y \pm 5)^2 = 70$

(b)

M1: Attempts distance (or distance ²) from $(-4, 5)$ to $(5, -8)$ (condone sign errors)

dM1: Full method. E.g. compares their $\sqrt{9^2 + 13^2}$ to their $(\sqrt{70} + \sqrt{52})$

A1*: Requires correct calculation(s), correct reason and minimal conclusion. Must be evidence to support the inequality – e.g. decimal values – not just the inequality stated. May use the difference between sum of radii and distance between centres.

(b) Alt

M1: Attempts to solve $x^2 + y^2 + 8x - 10y = 29$ and $x^2 + y^2 - 10x + 16y = -37$ forming a linear equation (need not be simplified for this mark)

FYI: Linear equation is $18x - 26y = 66$ o.e. if the calculations are correct

dM1: Attempts to solve their $18x - 26y = 66$ simultaneously with either circle equation reaching a 3 term quadratic for either x or y.

A1*: Requires

- correct equations
- correct "solution" stating no roots and concluding no points of intersection and minimal conclusion

E.g. Substitute $y = \frac{9x-33}{13}$ into $x^2 + y^2 + 8x - 10y = 29$ gives $250x^2 - 412x + 478 = 0$

and $"b^2 - 4ac" = -308256 < 0$ so no solutions

FYI the equation in y (if correct) is $250y^2 + 984y + 1116 = 0$



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

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Exact answers

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Answers without working

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n	$x1.02^{(n-1)}$	x5
1	25000	125000
2	25500	127500
3	26010	130050
4	26530	132651
5	27061	135304
6	27602	138010
7	28154	140770
8	28717	143586
9	29291	146457
10	29877	149387
11	30475	152374
12	31084	155422
13	31706	158530
14	32340	161701
	399348	1996742



Question	Scheme	Marks
2(a)		(i) Correct shape and position for $y = 2^x$ Crosses y axis with same intersection as $y = 4^x$ but with gentler slope. B1
		(ii) Graph of same shape as $y = 4^x$ but translated down B1
		y intercept at -5 B1
		x intercept at $\log_4 6$ B1
		(4)
(b)	$2^x = 4^x - 6$ or $y = (2^x)^2 - 6 = y^2 - 6$	M1
	$\Rightarrow 2^{2x} - 2^x - 6 = 0 \Rightarrow (2^x - 3)(2^x + 2) = 0 \Rightarrow 2^x = \dots$ or $(y - 3)(y + 2) = 0 \Rightarrow y = \dots$	M1
	$y = 2^x = 3$	A1
	$x = \log_2 3$ oe	A1
		(4)
(8 marks)		
Notes:		
(a)(i) Note: Be tolerant of “wobbles” in the graph if it is clear the correct shapes are meant, but the graph must not clearly bend away from the asymptotes. B1: Correct sketch for $y = 2^x$. This should be correct shape, but shallower gradient – look for crossing at same y intercept as $y = 4^x$, above $y = 4^x$ to the left of the y axis and below $y = 4^x$ to the right of the y axis.		
(ii) B1: Graph of $y = 4^x$ translated down by 6 units. Look for the same shape, always below $y = 4^x$ though be tolerant as the graph increases (accept if the graphs don’t “narrow” as shown). B1: Correct y intercepts of -5 B1: Correct x intercept of $\log_4 6$. Accept equivalents such as $\log 6 / \log 4$ or awrt 1.29 Ignore any references to asymptotes for these marks.		
(b) M1: Sets the equations equal to form an equation in x only, or writes 4^x in terms of 2^x and substitutes to achieve an equation in y only. M1: Solves the equation to find a value for 2^x or y A1: Correct y (or 2^x) value of intersection. (May have second solution) A1: Correct x value of intersection. Must have rejected second “solution”. Accept exact equivalents, such as $\log 3 / \log 2$ or $2 \log_4 3$		



Question	Scheme	Marks
3(i)	At least three of: For $p = 2$: $2^3 + 2 = 8 + 2 = 10$; For $p = 3$: $3^3 + 3 = 27 + 3 = 30$ For $p = 5$: $5^3 + 5 = 125 + 5 = 130$; For $p = 7$: $7^3 + 7 = 343 + 7 = 350$	M1
	Each case gives a multiple of 10. As 2,3,5 and 7 are the only single digit primes, the result has been proved for all single digit primes.	A1
		(2)
(ii)	$(n+1)^3 - n^3 = n^3 + 3n^2 + 3n + 1 - n^3 = 3n^2 + 3n + 1$	M1A1
	$= 3(n^2 + n) + 1$ which is one more than a multiple of 3, so is not divisible by 3 for any $n \in \mathbb{N}$	A1
		(3)
(5 marks)		
Notes:		
(i) M1: Checks result for at least three of the four single digit primes (2,3,5 and 7) – attaining a multiple of 10 is enough, no need to see the product. Allow if there are slips. A1: All four cases correctly checked, with minimal conclusion that the result is true. Ignore checks on non-prime values such as $p = 8$ which gives $8^3 + 8 = 520$, but award A0 if the case $p = 1$, leads to a conclusion that the result is not true. (ii) M1: Expands to a four term cubic (may have incorrect coefficients) and then cancels the n^3 terms (if $n = 2k$ and $n = 2k + 1$ are used, both must be attempted.) A1: Correct quadratic in n. (Both correct if $n = 2k$ and $n = 2k + 1$ are used) A1: Correct explanation and conclusion given. For the explanation accept e.g factors out 3 from the relevant terms to achieve a form $3 \times (n^2 + n) + 1$ which is one more than a multiple of 3, or explains each term other than 1 is divisible by 3		

Question	Scheme	Marks
4(a)	$\left(2 + \frac{x}{8}\right)^{13} = 8192 + \dots$	B1
	$+ \binom{13}{1} 2^{12} \left(\frac{x}{8}\right)^1 + \binom{13}{2} 2^{11} \left(\frac{x}{8}\right)^2 + \binom{13}{3} 2^{10} \left(\frac{x}{8}\right)^3 + \dots$	M1
	$\left(2 + \frac{x}{8}\right)^{13} = (8192) + 6656x + 2496x^2 + 572x^3 + \dots$	A1A1
		(4)
(b)	$\frac{x}{8} = 0.0125 \Rightarrow x = 0.1$	B1
	$2.0125^{13} \approx 8192 + 665.6 + 24.96 + 0.572$	M1
	$= 8883.132 \text{ cao}$	A1
		(3)
(c)	As all the terms in the expansion are positive, the truncated series will give an underestimate of the actual value.	B1
		(1)
(8 marks)		

Notes:**(a)****B1:** For the correct constant term of 8192 in their expression. Do not accept 2^{13} **M1:** Correct binomial coefficients linked with correct powers of x at least twice in the x , x^2 and x^3 terms in their expansion. Accept alternative notation, such as ${}^{13}C_1$ or just 13 etc, for the coefficients. (The power 1 need not be seen.) The powers of 2 and the 8's may be missing or incorrect.**A1:** Any two of the final three terms correct. (See also note below.)**A1:** All three of the final terms correct.

Note: Students may attempt to take out a common factor of 2^{13} first. In such cases the **B** mark is for correct constant term in their final expansion, **M** follows as above, correct coefficients linked to correct powers of x . The first **A** can be awarded as above or SC $8192 \left(1 + \frac{13}{16}x + \frac{39}{128}x^2 + \frac{143}{2048}x^3\right)$

(oe), final **A** as scheme**(b)****B1:** Identifies $x = 0.1$ needs to be substituted into the expression.**M1:** Substitutes their $x = 0.1$ into their answer to (a)**A1:** Correct answer only, must be to 3 dp.**(c)****B1:** States or implies underestimate with correct explanation of why the answer found is an underestimate. Must refer to terms in the expansion being positive or equivalent wording. Do not accept just "as adding more terms" without reference to the terms being non-negative.

If all that is done is a comparison with the true value, then B0.



Question	Scheme	Marks
5(a)	$h = \frac{1}{4}$	B1
	$A \approx \left(\frac{1}{2} \times \frac{1}{4}\right) \{5.453 + 5 + 2(7.764 + \dots + 7.626)\}$	M1
	$A \approx \left(\frac{1}{2} \times \frac{1}{4}\right) \{5.453 + 5 + 2(7.764 + 9.375 + 9.964 + 9.367 + 7.626)\}$	A1
	= awrt 12.33	A1
		(4)
(b)	$\int x^{\frac{3}{2}} - 3x + 9 \, dx = kx^{\frac{5}{2}} - lx^2 + mx$ (at least two powers correct)	M1
	$\int x^{\frac{3}{2}} - 3x + 9 \, dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{3}{2}x^2 + 9x(+c)$	A1A1
		(3)
(c)	$\left[\frac{2}{5}x^{\frac{5}{2}} - \frac{3}{2}x^2 + 9x \right]_{\frac{5}{2}}^4 = \left(\frac{2}{5} \times 4^{\frac{5}{2}} - \frac{3}{2} \times 4^2 + 9 \times 4 \right) - \left(\frac{2}{5} \left(\frac{5}{2} \right)^{\frac{5}{2}} - \frac{3}{2} \left(\frac{5}{2} \right)^2 + 9 \left(\frac{5}{2} \right) \right) = \dots$ (= 24.8 – 17.077... = 7.722...)	M1
	Area $R =$ “12.33” – “7.722” = ...	M1
	= awrt 4.6	A1
		(3)
(10 marks)		

Notes:**(a)**

B1: For using $h = 0.25$ or equivalent. Score for sight of $h = 0.25$ or $\frac{1}{2} \times \frac{1}{4}$ or just $\frac{1}{8}$ outside the brackets.

M1: Correct bracket structure for the trapezium rule. Look for “ $h/2$ ” (first + last y values + 2(attempt at sum of remaining values)). If one value is missing, allow the M as a slip, but M0 if 1st or last value repeated. Must be y values used. Incorrect/invisible external bracketing may be recovered by implication.

A1: Correct bracket “ $h/2$ ”{...} with all 7 y values used, though allow a minor miscopy of a term if it is clear the correct value is meant. May be implied by the correct answer.

A1: awrt 12.33

(b)

M1: Attempts to integrate. Power to increase by 1 in at least two terms.

A1: Any two terms correct.

A1: Fully correct. No need for constant of integration. Need not be simplified.

(c)

M1: Applies limits of 2.5 and 4 to their answer to (b) to get the area under C_2 . Limits may be either way around for this mark. May be implied by awrt 7.72 and allow if scored from a calculator.

M1: Subtracts their area under C_2 from the answer to (a). Must be subtracting a positive number from “12.33”, so if the limits were the wrong way round (or a negative answer was found in error), the answer must be made positive before subtracting.

A1: for awrt 4.6 following a correct answer to (a).



Question	Scheme	Marks
6(a)	Way 1: Eqn is $(x-3)^2 + (y+4)^2 - 9 - 16 + k = 0$	M1
	So radius must be 4 $\Rightarrow 25 - k = 16$	M1
	$\Rightarrow k = 9$	A1
		(3)
	Way 2: $y = 0 \Rightarrow x^2 - 6x + k = 0$ has one solution	M1
	$\Rightarrow 6^2 - 4 \times 1 \times k = 0$	M1
	$\Rightarrow k = 9$	A1
		(3)
(b)	C intersects x axis at (3,0)	B1
	Intersects y axis when $y^2 + 8y + 9 = 0 \Rightarrow y = \dots$ or $y = -4 \pm \sqrt{16 - 9}$ Or uses base of triangle is $2\sqrt{16 - 9}$	M1
	Area $RST = \frac{1}{2} \times 2\sqrt{7} \times 3$	M1
	$= 3\sqrt{7}$	A1
		(4)
(7 marks)		

Notes:**(a) Way 1**

M1: Attempts to complete square on both terms. May be implied by a centre of $(\pm 3, \pm 4)$

M1: Uses a valid method to find k , e.g. y coordinate of centre and sets $9 + 16 - k = (\text{their "4"})^2$.

Can be implied by sight of $k = (\text{their } x \text{ coordinate of centre})^2$

A1: Correct value for k

Way 2

M1: Substitutes equation for x axis ($y = 0$) into the circle equation.

M1: Sets the discriminant of the resulting quadratic to 0 (since as a tangent there is only one point of intersection).

A1: Correct value for k

(b)

B1: Identifies or implies the coordinates where circle touches the x axis. Probably from having completed the square on the equation. The 0 ordinate need not be seen and may be seen anywhere.

M1: Attempts the y coordinates of the intersections with the y axis, or the distance between these two points. E.g. may set $x = 0$ in the circle equation, or may use centre is 3 units from y axis etc.

M1: Uses a correct method for the area of the triangle RST . Longer methods are possible, such as use of cosine rule to find an angle in the triangle, followed by $\frac{1}{2}ab \sin C$, or may use the shoelace

method, e.g. $\frac{1}{2} \begin{vmatrix} 3 & 0 & 0 \\ 0 & -4 + \sqrt{7} & -4 - \sqrt{7} \end{vmatrix} = \frac{1}{2} \left| 3(-4 + \sqrt{7}) + 0 + 0 - 0 - 3(-4 - \sqrt{7}) \right|$

A1: Correct answer.



Question Number	Scheme	Marks
2a	$\left(f\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) + a = 0 \Rightarrow a = \dots\right.$	M1
	$\Rightarrow \frac{27}{2} - 18 + \frac{15}{2} + a = 0 \Rightarrow a = -3 \quad *$	A1*
		(2)
b	Example where $2x - 3$ is a linear factor: $\begin{array}{r} 2x^2 - x + 1 \\ 2x - 3 \overline{) 4x^3 - 8x^2 + 5x - 3} \\ \underline{4x^3 - 6x^2} \\ -2x^2 + 5x \\ \underline{-2x^2 + 3x} \\ +2x - 3 \\ \underline{+2x - 3} \\ 0 \end{array}$	M1A1
	$(-1)^2 - 4 \times 2 \times 1 = -7 < 0 \Rightarrow \text{no real roots so } x = \frac{3}{2} \text{ is the only one real root } *$	dM1A1*
		(4)
		(6 marks)

(a)

M1 Substitutes $x = \frac{3}{2}$ into the expression, sets equal to 0 (may be implied for this mark) and rearranges to find a . Condone invisible brackets for this mark only and condone arithmetical slips in their working.

Substituting $x = -\frac{3}{2}$ is M0 and attempts via long division score M0.

A1* $a = -3$ achieved with no errors seen including the omission of brackets.

There must be sight of $x = \frac{3}{2}$ which may be embedded in the expression or $f\left(\frac{3}{2}\right)$ followed

by at least one intermediate stage of working/manipulation (it may just be the left hand side of the equation), before proceeding to the given answer. The " $= 0$ " must be seen somewhere in their working before the given answer.

e.g.

$$4\left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) + a = 0 \Rightarrow a = -3 \text{ is M1A0}$$

$$f\left(\frac{3}{2}\right) = \frac{27}{2} - 18 + \frac{15}{2} + a = 0 \Rightarrow a = -3 \text{ is M1A1}$$

Note the trivial case $f\left(\frac{3}{2}\right) = 3 + a = 0 \Rightarrow a = -3$ is M1A0 (not enough shown)

(b) Work done in (a) must be made use of in (b) to gain credit.

M1 Attempts to find a quadratic factor of $4x^3 - 8x^2 + 5x - 3$ using $2x - 3$ as a factor

- score for $2x^2 \pm x \pm \dots$ if algebraic division is used or
- score for $2x^2 \pm \dots x \pm 1$ if equating coefficients/inspection used.



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
2. (a)	For correct term ${}^6C_4 3^2 (ax)^4$ Sets $15 \times 3^2 a^4 = 540$ $a^4 = 4 \Rightarrow a = \pm\sqrt{2}$	M1 A1 dM1 A1 (4)
(b)	$\frac{1}{81} \times 3^6 + a^6$ $= 17$	M1 A1 A1 (3) (7 marks)

(a)

M1 For an attempt at the correct term of the binomial expansion which may be embedded within a full expansion.

Accept ${}^6C_4 3^2 (ax)^4$, $\frac{6!}{4!2!} 3^2 (ax)^4$, $\binom{6}{4} 3^2 (ax)^4$, $135(ax)^4$ etc but must have correct coefficient and

correct powers of 3 and x . Allow equivalents such as ${}^6C_2 3^2 (ax)^4$ and condone a missing bracket for this mark ie. $135ax^4$.

You may condone a slip on the 3 if the rest of an expansion is correct.

A1 For a correct equation in a . Allow $15 \times 3^2 a^4 = 540$ or $15 \times 3^2 (ax)^4 = 540x^4$ The coefficient must be evaluated for this mark

dM1 For " 135 " $a^4 = 540 \Rightarrow a = \dots$ Condone slips on the 540 or errors simplifying an initially correct coefficient, but must be attempting a fourth root. This is implied by $a = 1.41$

A1 $a = \pm\sqrt{2}$ must be exact. Accept $\pm\sqrt[4]{4}$ oe

(b)

M1 For identifying two terms independent of x . Allow if the power of 3 is incorrect but it is clear the correct terms are meant, so $\frac{1}{81} \times \text{constant term from } (3 + ax)^6$ and $a^6 x^6 \times \frac{1}{x^6} = a^6$ extracted.

A1 Both terms correct and added, ie. $\frac{1}{81} \times 3^6 + a^6$ with a or following through on their value of a but allow

if terms are initially correct individually but slips when evaluating.

A1 17 Allow recovery to 17 from use of decimals but 17.0 is A0.



Question Number	Scheme	Marks
3. (a)	Attempts $f\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^3 + 17\left(-\frac{3}{2}\right)^2 + 4\left(-\frac{3}{2}\right) - 12$ $= 0 \Rightarrow (2x+3) \text{ is a factor} \quad *$	M1 A1* (2)
(b)	$6x^3 + 17x^2 + 4x - 12 = (2x+3)(3x^2 + 4x - 4)$ $= (2x+3)(3x-2)(x+2)$	M1 A1 dM1 A1 (4)
(c)	Solves $\tan \theta = -\frac{3}{2}$ or “-2” or “ $\frac{2}{3}$ ” $\theta = \text{awrt } 2.03, 2.16$	M1 A1 (2) (8 marks)

(a)

M1 Allow for an attempt at finding a value of $f\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^3 + 17\left(-\frac{3}{2}\right)^2 + 4\left(-\frac{3}{2}\right) - 12$

Sight of embedded values is sufficient. Attempted division is M0

A1* Correctly shows $f\left(-\frac{3}{2}\right) = 0$ and states "hence factor" (or substantial equivalent conclusion).

They may state “factor if $f\left(-\frac{3}{2}\right) = 0$ ” in a preamble, in which case accept a minimal conclusion such as //

The M must be scored so there needs to be evidence of either embedded $\left(-\frac{3}{2}\right)$'s or calculations.

(b) Allow for work done in part (a) if referred to in part (b).

M1 Attempt to divide or factorise out $(2x+3)$

By factorisation look for $6x^3 + 17x^2 + 4x - 12 = (2x+3)(3x^2 + kx \pm 4)$

By division look for $\begin{array}{r} 3x^2 \pm 4x \dots\dots\dots \\ 2x+3 \overline{) 6x^3 + 17x^2 + 4x - 12} \\ \underline{6x^3 + 9x^2} \end{array}$

By comparing coefficients look for $6x^3 + 17x^2 + 4x - 12 = (2x+3)(ax^2 + bx + c) \Rightarrow a=3, c=\pm 4, b=...$

A1 Correct quadratic factor $(3x^2 + 4x - 4)$ found. May be within the long division.

dM1 Attempts to factorise their $(3x^2 + 4x - 4)$ usual rules.

A1 $(2x+3)(3x-2)(x+2)$ written out as a product of factors (not as a list) and isw.

Allow $3(2x+3)\left(x-\frac{2}{3}\right)(x+2)$ oe as long as there are three linear factors.

Factors given but no working shown scores M0A0dM0A0



(c)

M1 Solves $\tan \theta = k$ where k is any of their roots to their cubic. Allow if $\tan^{-1}$ seen followed by an answer, or it can be implied by awrt -0.98 or -1.1 or 0.59 or 0.58 truncated (or awrt -56.3 or -63.4 or 33.7 (degrees))

A1 $\theta =$ awrt $2.03, 2.16$ and no other solutions inside the range.

Accept 0.6476π and 0.687π as these give correct answers to 3s.f. but not 0.648π as this is not correct to 3s.f.



Question Number	Scheme	Marks
6 (i)	$x^2 + y^2 + 10x - 12y = k$	
(a)	Attempts $(x \pm 5)^2 + (y \pm 6)^2 \dots = 0$ Centre $(-5, 6)$	M1 A1 (2)
(b)	Sets $k + (\pm 5)^2 + (\pm 6)^2 > 0$ $k > -61$	M1 A1 (2)
(ii)	Centre $\left(\frac{-2+8}{2}, \frac{10+(-14)}{2}\right) = \dots$; radius $\frac{1}{2}\sqrt{(-2-8)^2 + (10-(-14))^2} = \dots$ Centre is $(3, -2)$; radius is 13 $\Rightarrow C_2 : (x-3)^2 + (y+2)^2 = 13^2 \Rightarrow (p-3)^2 + 4 = 169 \Rightarrow p = \dots$ Or $PX = r \Rightarrow (p-3)^2 + (0-(-2))^2 = 169 \Rightarrow p = \dots$ $p = 3 + \sqrt{165}$ only	M1 A1 M1 A1 (4)
		(8 marks)

(i) (a)

M1: Attempts to complete the square on both x and y or states the centre as $(\pm 5, \pm 6)$

For completing the square look for $(x \pm 5)^2, (y \pm 6)^2 \dots = \dots$

A1: Centre $(-5, 6)$ This written down can score M1 A1

Allow written as separate coordinates ie. $x = -5, y = 6$

(i)(b)

M1: $k + (\pm 5)^2 + (\pm 6)^2 > 0$. Follow through on their $(-5, 6)$. Allow $k + (\pm 5)^2 + (\pm 6)^2 \dots 0$
They must have an inequality for this mark.

A1: $k > -61$ but allow $k \dots -61$

(ii)

M1: Attempts centre and radius (or radius squared) of C_2 using a correct method. (If only the diameter is found it is M0)

NB the attempt at the radius may arise from an attempt at QX or RX where X is the centre.



A1: Correct centre and radius. Centre at $(3, -2)$ and radius = 13

M1: Uses their centre and radius (allow if they think the diameter is the radius) in a correct method to find p (or x when $y = 0$). E.g.

- finds the equation of C_2 , sets $y = 0$ and solves to find p (or x).
- uses the fact that $PX = r$ or $PX = QX$ or $PX = RX$ (where X is the centre) to form and solve an equation in p

For this mark allow if their centre and radius of the circle from (i) are used.

A1: $3 + \sqrt{165}$ ONLY

Question Number	Scheme	Marks
Alt 6 (ii)	Uses either $PQ^2 + PR^2 = RQ^2$ or $\text{grad } PQ \times \text{grad } PR = -1$	M1
	Correct equation. E.g. $(p+2)^2 + 10^2 + (p-8)^2 + 14^2 = (-2-8)^2 + (10+14)^2$	A1
	Alternatively $\frac{-10}{p+2} \times \frac{14}{p-8} = -1$	
	Correct method to set up and solve 3TQ. FYI $p^2 - 6p - 156 = 0$ $p = 3 + \sqrt{165}$ only	dM1 A1
		(4)

M1: Attempts

- either $PQ^2 + PR^2 = QR^2$ to set up an equation in p .
- or $\text{grad } PR \times \text{grad } PQ = -1$ to set up an equation in p

Expect to see a correct attempt at the lengths or the gradients but condone slips.

A1: Correct equation which may be unsimplified. See scheme

dM1: Attempts to set up and solve a 3TQ resulting in a value for p .

A1: $3 + \sqrt{165}$ ONLY

Question Number	Scheme	Notes	Marks
6(a)	Examples: $m_{PQ} = \frac{14+30}{23-15}, m_{QR} = \frac{-30+26}{15+7} \Rightarrow m_{PQ} \times m_{QR} = ..$ or $PQ^2 = 8^2 + 44^2, QR^2 = 22^2 + 4^2, PR^2 = 30^2 + 40^2$ $PQ^2 + QR^2 = ..$	Correct strategy to show that $\angle PQR = 90^\circ$. E.g. attempts gradient of PQ and gradient of QR and attempts product or finds side lengths and attempts Pythagoras.	M1
	$m_{PQ} \times m_{QR} = \frac{11}{2} \times \left(-\frac{2}{11}\right) = -1 \Rightarrow \angle PQR = 90^\circ$ Or $PQ^2 = 2000, QR^2 = 500, PR^2 = 2500$ $2000 + 500 = 2500 \Rightarrow \angle PQR = 90^\circ$	Correct proof and conclusion	A1
			(2)
(b)(i)	Centre is $(8, -6)$	Correct coordinates	B1
(b)(ii)	$r = \sqrt{(23-8)^2 + (14+6)^2}$ or e.g. $r = \frac{1}{2} \sqrt{(23+7)^2 + (14+26)^2}$	Fully correct method for the radius	M1
	$r = 25$	Cao	A1
			(3)
(c)	S is $(1, 18)$ or $m_T = \frac{7}{24}$	Correct coordinates for S or correct gradient for the tangent.	B1
	$m_N = \frac{18+6}{1-8} \Rightarrow m_T = \frac{8-1}{18+6} \Rightarrow y-18 = \frac{7}{24}(x-1)$ Uses a correct straight line method for the tangent using their S and the negative reciprocal of the radius gradient		M1
	$7x - 24y + 425 = 0$	Allow any integer multiple	A1
			(3)
			Total 8

Notes**(a)**

M1: For a correct strategy to show that angle $PQR = 90^\circ$. Possible methods are:
 Find gradients of PQ and RQ (in order to check the perpendicularity condition), getting at least as far as both gradients.
 Finding the lengths of all three sides and attempting to show $PQ^2 + QR^2 = PR^2$ attempting substitution into at least $PQ^2 + QR^2 = ..$ When finding lengths, accept at least two correct lengths stated to imply method for finding lengths if no method is shown.
 Finding the lengths of all three sides and applying the cosine rule to find the angle, reaching at least substitution into the cosine rule. May find the two acute angles and subtract from 180° .
 Finding the midpoint of PR and showing it is equidistant from P , Q and R to deduce PR is a diameter and applying circle theorem. Must reach at least the calculations of lengths.
 Finding the length of all three sides and calculating angle at P from both sin and cosine ratios to show both agree. Must reach angle from both ratios.
 Other methods are possible, if you see a method you are unsure of then send to review.



Question Number	Scheme	Marks
7(a)	States or implies that gradient $XN = -\frac{2}{5}$ Uses $(4, -3)$ with a gradient of $\pm\frac{2}{5}$ to form equation of XN $y+3 = -\frac{2}{5}(x-4)$ $5y+15 = -2x+8 \Rightarrow 2x+5y+7=0$ *	B1 M1 A1* (3)
(b)	(i) Solves $2x+5y+7=0$ and $y = \frac{5}{2}x - \frac{55}{2} \Rightarrow x = \dots, y = \dots$ $N = (9, -5)$ (ii) Attempts $(9-4)^2 + (-5+3)^2$ Correct form of equation E.g. $(x-4)^2 + (y+3)^2 = (9-4)^2 + (-5+3)^2$ $(x-4)^2 + (y+3)^2 = 29$	M1 A1 M1 M1 A1 (5) (8 marks)

(a) **This is a given answer so be careful of students who work backwards.**

B1: States or implies that gradient $XN = -\frac{2}{5}$ o.e such as -0.4

This cannot be scored from candidates who show you they are just working back from the given answer.

M1: Uses $(4, -3)$ with a gradient of $\pm\frac{2}{5}$ o.e. to form equation of XN

E.g $\pm\frac{2}{5} = \frac{y-(-3)}{x-4}$. If the form $y = mx + c$ is used look for $-3 = \left(\pm\frac{2}{5}\right) \times 4 + c$ leading to $c = \dots$

A1*: Shows sufficient working and proceeds to $2x+5y+7=0$. There should be one correct line

showing working between $y+3 = -\frac{2}{5}(x-4)$ and $2x+5y+7=0$, for example $5y+15 = -2x+8$

If $y = mx + c$ is used expect to see $c = -\frac{7}{5}$ and $y = -\frac{2}{5}x - \frac{7}{5}$ before $2x+5y+7=0$.

Note that this is a given answer so there cannot be incorrect work used leading to this answer.

(b)(i)

M1: Attempts to solve $2x+5y+7=0$ and $y = \frac{5}{2}x - \frac{55}{2}$ leading to $x = \dots, y = \dots$

Allow the values to be just written down, for example via use of a calculator

A1: $N = (9, -5)$ which may be written $x=9, y=-5$



Question Number	Scheme	Marks
6(a)	States or uses $M = (5, 4)$ or $\text{grad } AB = \frac{1}{3}$ o.e.	B1
	States or uses $M = (5, 4)$ and $\text{grad } AB = \frac{1}{3}$ o.e.	B1
	Equation of l is $y - 4 = -3(x - 5)$	M1
	$y = -3x + 19$	A1
(b)	$k = -2$	B1 ft
(c)	Attempts radius or radius ² E.g. $(11 - 7)^2 + (6 - -2)^2$	M1
	States $(x - 7)^2 + (y + 2)^2 = (11 - 7)^2 + (6 - -2)^2$	dM1
	$(x - 7)^2 + (y + 2)^2 = 80$	A1
		(3)
		(8 marks)

(a)

B1: States or uses $M = (5, 4)$ **or** $\text{grad } AB = \frac{1}{3}$ o.e. such as $\frac{4}{12}$ Note that gradient AB may be implied by use of a gradient for l of -3 o.e.B1: States or uses $M = (5, 4)$ **and** $\text{grad } AB = \frac{1}{3}$ o.e. such as $\frac{4}{12}$ Note that gradient AB may be implied by use of a gradient for l of -3 o.e.M1: Attempts equation of l using their M and their correctly adapted gradient.The gradient used must be the negative reciprocal of their gradient of AB . If it just appears without any working it must be correct.If the form $y = mx + c$ is used this mark is only scored when they proceed as far as $c = \dots$ A1: CAO $y = -3x + 19$

(b)

B1ft: $k = -2$ or follow through on the value of their $-3 \times 7 + 19$ Allow this to be scored if the coordinates of P are given.For example, as $(7, -2)$ or even $y = -2$ following use of 7

(c)

M1: Attempts either the radius² or the radius using their $(7, k)$ and one of the given points

$$\text{E.g. } (11 - 7)^2 + (6 - -2)^2 \text{ or } \sqrt{(7 - (-1))^2 + (-2 - 2)^2}$$

It does not need to be correctly identified as r or r^2 dM1: Attempts equation of circle using their radius² and their $(7, k)$ Note that $(x - 7)^2 + (y - k)^2 = (11 - 7)^2 + (6 - k)^2$ would score M1 dM1A1: $(x - 7)^2 + (y + 2)^2 = 80$ The rhs of the equation must be simplified and ISW after a correct answer.

Alt (c)

M1: Writes equation of circle as $x^2 + y^2 - 14x - 2k'y = \lambda$ following through on their k dM1: Substitutes $(-1, 2)$ or $(11, 6)$ into the equation of the circle and finds a value for λ 

Question Number	Scheme	Notes	Marks
1(a)	$\left(2 - \frac{x}{4}\right)^{10} = 2^{10} + \binom{10}{1}2^9\left(-\frac{x}{4}\right) + \binom{10}{2}2^8\left(-\frac{x}{4}\right)^2 + \binom{10}{3}2^7\left(-\frac{x}{4}\right)^3 + \dots$ Attempts the binomial expansion to get the third and/or fourth term with an acceptable structure. The correct binomial coefficient must be combined with the correct power of $\frac{x}{4}$ and the correct power of 2 but condone omission of brackets. You can ignore the signs between the terms and allow the terms to be listed. Allow for e.g. $\pm \binom{10}{2}2^8\left(\pm\frac{x}{4}\right)^2$ or $\pm^{10}C_32^7\left(\pm\frac{x}{4}\right)^3$ but condone omission of brackets. NB $^{10}C_2 = 45$, $^{10}C_3 = 120$ NB $^{10}C_2 = ^{10}C_8$ and $^{10}C_3 = ^{10}C_7$ Alternative: $\left(2 - \frac{x}{4}\right)^{10} = 2^{10}\left(1 - \frac{x}{8}\right)^{10} = 2^{10}\left(1 - \frac{10x}{8} + \frac{10 \times 9}{2}\left(-\frac{x}{8}\right)^2 + \frac{10 \times 9 \times 8}{3!}\left(-\frac{x}{8}\right)^3 + \dots\right)$ Score M1 for $2^{10}\left(\dots \pm \frac{10 \times 9}{2}\left(-\frac{x}{8}\right)^2 + \dots\right)$ or $2^{10}\left(\dots \pm \frac{10 \times 9 \times 8}{3!}\left(-\frac{x}{8}\right)^3 + \dots\right)$		M1
	$= 1024 - 1280x + 720x^2 - 240x^3$	1024 – 1280x	B1
		720x ² or –240x ³	A1
		720x ² and –240x ³	A1
	Note that if any of the “–”s are “+ –”s then penalise once on the first occurrence		
	Allow the terms to be listed e.g. 1024, –1280x, 720x², –240x³ Apply isw once a correct answer is seen. Ignore any extra terms		
			(4)
(b)	$\left(3 - \frac{1}{x}\right)^2 = 9 - \frac{6}{x} + \frac{1}{x^2} \text{ or } 9 - \frac{3}{x} - \frac{3}{x} + \frac{1}{x^2}$	Correct expansion. May be implied by their work to find the constant.	B1
	$\left(3 - \frac{1}{x}\right)^2 \left(2 - \frac{x}{4}\right)^{10} = \left(9 - \frac{6}{x} + \frac{1}{x^2}\right)(1024 - 1280x + 720x^2 - 240x^3 + \dots)$ constant term = $9 \times 1024 - \frac{6}{x}(-1280x) + \frac{1}{x^2}720x^2$ This mark depends on having obtained an expression of the form $A + \frac{B}{x} + \frac{C}{x^2}$ for $\left(3 - \frac{1}{x}\right)^2$ and at least a 3-term quadratic expression from part (a) so award for: $A \times "1024" + B \times "-1280" + C \times "720"$ A, B, C non-zero. Allow 1 sign error on their values. May be seen as part of a complete expansion but there must be an attempt to calculate the value of the constant term with the above conditions. For reference, true value calculation is: 9216 + 7680 + 720		M1
	$= 17616$	Correct value. Must be “extracted” if a complete expansion is found above.	A1
			(3)
			Total 7



Question Number	Scheme					Notes			Marks												
2(a)	<table><tr><td>x</td><td>- 0.25</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td></tr><tr><td>y</td><td>0.462</td><td>0.577</td><td>0.653</td><td>0.686</td><td>0.698</td></tr></table>					x	- 0.25	0	0.25	0.5	0.75	y	0.462	0.577	0.653	0.686	0.698				B1
	x	- 0.25	0	0.25	0.5	0.75															
	y	0.462	0.577	0.653	0.686	0.698															
Allow awrt these values and look for the values appearing in the body of the script or within their calculation in part (b). Also allow exact value for the 0.577 e.g. $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$																					
									(1)												
(b)	$h = 0.25$					Correct strip width. May be implied by $\frac{1}{8}$ or $\frac{1}{2} \times 0.25$			B1												
	$A \approx \frac{1}{2} \times "0.25" \{ 0.462 + 0.698 + 2 ("0.577" + "0.653" + "0.686") \}$ Correct application of the trapezium rule with their h Must be a correct application of the rule so e.g. $A \approx \frac{1}{2} \times "0.25" \times 0.462 + 0.698 + 2 ("0.577" + "0.653" + "0.686")$ Scores M0 unless any missing brackets are implied by subsequent work. $A \approx \frac{1}{2} \times "0.25" \{ 0.462 + 0.698 + 2 ("0.577" + "0.653" + "0.686") \}$ Would also score M0 unless the closing bracket was implied by subsequent work Condone copying slips e.g. 0.426 instead of 0.462. Must use all the y -values. Repeated or missing y -values scores M0. Allow separate trapezia e.g. $A \approx \frac{1}{2} \times "0.25" (0.462 + "0.577") + \frac{1}{2} \times "0.25" ("0.577" + 0.653) + \frac{1}{2} \times "0.25" (0.653 + "0.686") + \frac{1}{2} \times "0.25" ("0.686" + 0.698)$ Allow use of the function e.g. $A \approx \frac{1}{2} \times 0.25 \left\{ \frac{2^{-0.25}}{\sqrt{5(-0.25)^2 + 3}} + \frac{2^{0.75}}{\sqrt{5(0.75)^2 + 3}} + 2 \left(\frac{2^0}{\sqrt{5(0)^2 + 3}} + \frac{2^{0.25}}{\sqrt{5(0.25)^2 + 3}} + \frac{2^{0.5}}{\sqrt{5(0.5)^2 + 3}} \right) \right\}$					M1															
	$= \text{awrt } 0.624 \text{ or } \frac{78}{125} \text{ oe e.g. } \frac{312}{500}$					accept awrt 0.624 or exact fraction but isw if necessary			A1												
	Note that the calculator answer for the integral is 0.6265569683...																				
									(3)												
									Total 4												

Question Number	Scheme	Marks
9(a) (i)	Centre = $(k, 2k)$	B1
	Radius = $\sqrt{k+7}$	B1
		(2)
(b)(i)	$(2-k)^2 + (3-2k)^2 = k+7 \Rightarrow 4-4k+k^2+9-12k+4k^2 = k+7$	M1
	$5k^2 - 17k + 6 = 0$ *	A1*
	$(k =) \frac{2}{5}, 3$	B1
		(3)
(c)		
	Centre is $\left(\frac{2}{5}, \frac{4}{5}\right)$	B1ft
	Gradient of tangent $\pm \frac{2 - \frac{2}{5}}{3 - \frac{4}{5}} = \left(-\frac{8}{11}\right)$	M1
	$y - 3 = -\frac{8}{11}(x - 2) \Rightarrow \text{sets } y = 0 \Rightarrow x = \dots$ Alternatively, $\tan \angle PTO = \frac{8}{11} \Rightarrow XT = \frac{3}{8/11} = \dots$ where X is $(2, 0)$	M1
	Area $OPT = \frac{1}{2} \times \frac{49}{8} \times 3 = \dots$ Alternatively, Area $OPT = \frac{1}{2} \times 2 \times 3 + \frac{1}{2} \times \frac{33}{8} \times 3 = \dots$	dM1
	$= \frac{147}{16}$ oe	A1
		(5)
		(10 marks)

(a)
(i)

B1 $(k, 2k)$

(ii)

B1 $\sqrt{k+7}$ and isw after correct answer seen.



(b)

(i)

M1 Substitutes $(2, 3)$ into the equation of the circle and attempts to multiply out.A1* Proceeds to $5k^2 - 17k + 6 = 0$ with no errors

(ii)

B1 $(k = \frac{2}{5}, 3)$ only. Do not allow if they go on to go to give a range of values for k .(c) **NB The method marks may be scored with any k .** If a candidate has solved incorrectly and has both values less than 2, they can score for use of either (best attempt) in solving the problem. (If neither value is less than 2 allow the methods for any value of k used.)B1ft Uses the correct value for k and finds the centre of the circle. Follow through their $(k, 2k)$ with $k = \frac{2}{5}$. Accept decimal equivalents. Must be clearly identified or used as the centre, not just k substituted into the original equation.M1 Attempts the gradient of the tangent. Look for an attempt at $\pm(\text{change in } x / \text{change in } y)$ using their centre and P . May find the gradient from centre to P first and then apply the (negative) reciprocal.M1 Attempts to find the point T by finding the equation of the tangent at P using their gradient and $(2, 3)$, setting $y = 0$ and proceed to find a value for x or equivalent processing. The gradient need not be correct but must be from an attempt with their centre and P . Alternatively, the tangent may be used to find angle PTO and attempt the length XT where X is $(2, 0)$. (Note they do not need to find T in this method.)dM1 Correct method to find the area of the triangle. It is dependent on the previous method mark. There may be attempts at finding the lengths of each side, the cosine rule to find an angle, and the area formula $\frac{1}{2}ab \sin C$. It must be a complete method with valid attempts to find the appropriate lengths.A1 $\frac{147}{16}$ oe (decimal is 9.1875 and must be exact)

SC for the dM1

O is defined in the question and some may use O as the centre of the circle. These still need to attempt the first three marks as in the scheme. Also allow the final M mark for a correct attempt at the area OPT with O as the centre of the circle but must be a complete method. Mostly likely finding the lengths $TP = \frac{3\sqrt{185}}{8}$ and $r = \sqrt{k+7}$ (oe) and using

$\frac{1}{2}rTP$.



(b)

M1 Attempts the remainder theorem by setting $f(\pm p) = 9$ oe Award for e.g.

$$(-p)^3 + (p+3)(-p)^2 + p + q = 9 \text{ condoning sign slips and invisible brackets.}$$

Alternatively, they may attempt to divide algebraically by $x + p$ leading to a remainder in terms of p^2 , p and q which is equated to 9. Condone slips in their working.

$$\begin{array}{r} x^2 \quad + 3x \quad -1-3p \\ x+p \overline{) x^3 + (p+3)x^2 - x + q} \\ \underline{x^3 \quad \quad px^2} \quad \quad \quad \\ \quad 3x^2 \quad - x \quad \quad \quad \\ \underline{\quad 3x^2 \quad + 3px} \quad \quad \quad \\ \quad \quad (-1-3p)x \quad + q \quad \quad \quad \\ \underline{\quad \quad (-1-3p)x + (-1-3p)p} \end{array}$$

With alternative methods look for a correct method, leading to a remainder in terms of p^2 , p and q which is equated to 9

A1* Correct proof with no errors including brackets. There must be at least one intermediate stage of working before proceeding to the final answer

$$\text{e.g. } (-p)^3 + (p+3)(-p)^2 - (-p) + q = 9 \Rightarrow 3p^2 + p + q - 9 = 0 \text{ scores M1A0 (no intermediate stage)}$$

$$(-p)^3 + (p+3)(-p)^2 - (-p) + q = 9 \Rightarrow 3p^2 + p + q = 9 \Rightarrow 3p^2 + p + q - 9 = 0 \text{ scores M1A1}$$

(c)

M1 Attempts to use both given equations to form a 3TQ equation in p (or q). (terms do not need to be all on one side and condone the omission of $= 0$)

Do not be too concerned by slips in their substitution/rearrangements or miscopying of the given equations. May be implied by either a correct value for p or a correct value for q

A1 $p = 6$ (ignore any reference to $-\frac{10}{3}$)

A1 $q = -105$ only

(d)

M1 Uses their values for p and q and a correct strategy (inspection or long division) to obtain the quadratic factor.

Via inspection score for $x^2 + \dots x \pm \frac{q}{3}$

Via long division score for proceeding as far as $x^2 \pm (p+6)x$. If they attempt algebraic division in (a) you may need to check their quotient with their value for p to see if the method mark can be scored. They may also restart which is acceptable.

Condone the use of a negative value for p for this mark (eg even if $p = -6 \Rightarrow 0x$)

A1 $x^2 + 12x + 35$ oe eg $(x+5)(x+7)$ Allow embedded as $(x-3)(x^2 + 12x + 35)$ and condone poor notation such as $f(x) = x^2 + 12x + 35$. Also allow this mark to be scored if seen within their long division.

Question Number	Scheme	Marks
6(a)(i)	Centre is $(-4, 2)$	B1
(ii)	$x^2 + y^2 + 8x - 4y = 0$ $\Rightarrow (x+4)^2 + (y-2)^2 - 16 - 4 = 0 \Rightarrow r = \dots$	M1
	$r = 2\sqrt{5}$ oe	A1
		(3)
(b)	$x^2 + y^2 + 8x - 4y = 0, x + 2y + 10 = 0$ $\Rightarrow (-2y - 10)^2 + y^2 + 8(-2y - 10) - 4y = 0$ or $x^2 + \left(\frac{-x-10}{2}\right)^2 + 8x - 4\left(\frac{-x-10}{2}\right) = 0$	M1
	$y^2 + 4y + 4 = 0$ or e.g. $x^2 + 12x + 36 = 0$	A1
	$(y+2)^2 = 0 \Rightarrow y = \dots$ or e.g. $(x+6)^2 = 0 \Rightarrow x = \dots$	dM1
	$(-6, -2)$	A1
		(4)
(c)	$x + 2y + 10 = 0 \Rightarrow m_T = -\frac{1}{2} \Rightarrow m_N = 2$ or $m_N = \frac{"2"-("2")}{"-4"-("6")}$	M1
	Usually either $y - 2 = 2(x + 4)$ or $y + 2 = 2(x + 6)$	dM1
	$y = 2x + 10$	A1
		(3)
		Total 10

(a)(i) Mark (i) and (ii) together.

B1 Correct centre. Allow written as $x = -4, y = 2$

(ii)

M1 Correct method for the radius. Award for $(x \pm a)^2 + (y \pm b)^2 \dots \Rightarrow r = \sqrt{"a^2" + "b^2"} = \dots$ or maybe seen using the general equation of a circle e.g. $x^2 + 2gx + y^2 + 2fy + c = 0 \Rightarrow r = \sqrt{f^2 + g^2 - c}$

May be implied by a correct radius for their centre if no method is shown.

A1ft $r = 2\sqrt{5}$ or exact equivalent eg $\sqrt{20}$. Do not accept $\pm$.

Only follow through their centre with coordinates $(\pm 4, \pm 2)$

(b) Note that answer only scores 0 marks.

M1 Uses both equations to eliminate one variable. Condone slips in substituting in. Also allow use of the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ oe allowing for slips in manipulation



- A1 Correct 3TQ or any multiple of this (allow any equivalent 3 term equations) which do not need all the terms on one side of an equation. Condone lack of $= 0$
- dM1 Solves their 3TQ to obtain the x or y coordinate of P . Usual rules apply. (May need to check if via a calculator). It is dependent on the first method mark.
- A1 Correct coordinates. Allow written as $x = -6$, $y = -2$.

Alt (b) Geometric approach example

- M1 Forms a right angled triangle using the radius as the hypotenuse and letting the two shorter sides be $2a$ and a (gradient of the radius from the centre to tangent is 2) such that $(2a)^2 + a^2 = (2\sqrt{5})^2 \Rightarrow a = \dots$
- A1 $a = 2$
- dM1 A correct method to find x or y : $x = -4 - 2 = \dots$ and $y = 2 - 2(2) = \dots$
- A1 $(-6, -2)$

Alt (b) Finding the point of intersection between the tangent and the normal

- M1 Attempts to find the equation of the normal using their centre and a gradient of 2
- A1 $y = 2x + 10$
- dM1 Attempts to solve simultaneously to find x or y
- A1 $(-6, -2)$

Alt (b) Implicit differentiation

- M1 Attempts to differentiate implicitly, substituting in $\frac{dy}{dx} = -\frac{1}{2}$ and using the equation of the tangent to C at P to eliminate one variable. $x^2 + y^2 + 8x - 4y = 0 \Rightarrow 2x + 2y \frac{dy}{dx} + 8 - 4 \frac{dy}{dx} = 0$
- $\Rightarrow 2x + 2y \left(-\frac{1}{2}\right) + 8 - 4 \left(-\frac{1}{2}\right) = 0 \Rightarrow 2x - y + 10 = 0$
- $\Rightarrow 2(-2y - 10) - y + 10 = 0$ or $x + 2(2x + 10) + 10 = 0$ (using the equation of the tangent to C at P)
- A1 A correct simplified linear equation in x or y
e.g. $5y = -10$ or $5x = -30$ which may be implied by the correct x or y coordinate.
- dM1 Attempts to solve their linear equation to find x or y
- A1 $(-6, -2)$
- There may be other credit worthy methods so look carefully at their solution and send to review if unsure**

(c) Credit worthy work may be seen in (b) but must be used in (c)

- M1 Attempts to find the gradient of the line between their centre and their P by either using
- the given tangent equation which may be implied by sight or use of gradient $= 2$
 - their centre and their P . Score for a correct expression to find the gradient using their centre and their P eg $\frac{2 - (-2)}{-4 - (-6)}$. Also allow with other valid coordinates which would lie on the normal e.g. $(-2, 6)$ which may have come from solving simultaneously the equation of the normal with the equation of the circle.
- Allow this mark if they then subsequently find the negative reciprocal of this.
- dM1 Correct straight line method using their normal gradient (via a correct method) and their centre or their P (or any other valid pair of coordinates which would lie on the normal
e.g. using their gradient using a geometric approach 1 to the right, 2 up

If they use $(y - y_1) = m(x - x_1)$ then both brackets including signs must be correct for their coordinates.

If they use $y = mx + c$ they must proceed as far as $c = \dots$

It is dependent on the previous method mark.

A1 $y = 2x + 10$ cao



(c)

M1: Correct strategy to find r e.g. $\sqrt[299]{\frac{250}{20}}$. Award for e.g. $(r =) \sqrt[299]{\frac{250}{20}}$ or e.g.

$(r =) \left(\frac{250}{20}\right)^{\frac{1}{299}}$. May be implied by awrt 1.01 if no incorrect work is seen e.g. clear

use of $\sqrt[300]{\frac{250}{20}}$ which gives 1.008454636... is M0

May be done via logs e.g. $r = 10^{\frac{1}{299} \log 12.5}$

M1: Uses their r with a correct GP sum formula with $a = 20$ and $n = 300$ and subtracts from the answer to part (b) either way round.

Depends on having attempted the **sum** of an **AP** in (b).

May be implied by their working/value.

Note this has been seen more than once for the GP sum following $\sqrt[299]{\frac{250}{20}}$:

$$S_{300} = \frac{20(1-12.5)}{1-1.008483032}$$

and scores M0 as it suggests $S_n = \frac{a(1-r^{n-1})}{1-r}$ has been used.

A1: Cao **not awrt** with or without the “£” symbol.

Note that the accuracy being used for “ r ” varies considerably.

Some rounded values for reference are:

r	S_{300}	Correct (b) – S_{300}
1.01	37576.93252...	9151.932524...
1.008	24796.03972...	3628.960276...
1.0085	27458.42195...	966.5780531...
1.00848	27345.9295...	1079.070499...



Question Number	Scheme	Marks
8(a)	$x^2 + 3 = 13 - \frac{9}{x^2} \Rightarrow x^4 + 3x^2 = 13x^2 - 9$ $\Rightarrow x^4 - 10x^2 + 9 = 0$	M1A1
	$x^4 - 10x^2 + 9 = 0 \Rightarrow (x^2 - 1)(x^2 - 9) = 0 \Rightarrow x^2 = \dots$ $\Rightarrow x = \dots$	M1
	$x = 1, \quad x = 3$	A1
		(4)

(a) Note we are now marking part (a) as M1A1M1A1 not M1M1A1A1

M1: Equates the 2 curves and multiplies through by x^2 to obtain a quadratic equation in x^2

A1(M1 on EPEN): Correct 3TQ in x^2 with terms collected and all on one side.

Note that the “= 0” may be implied by their attempt to solve.

M1(A1 on EPEN): Solves a 3TQ in x^2 by a suitable method e.g. factorising, completing the square or quadratic formula **and takes the square root** to obtain at least one value for x . They may use a calculator, **but** the solution must be via x^2 .

E.g. $x^4 - 10x^2 + 9 = 0 \Rightarrow x^2 = 1, 9 \Rightarrow x = 1, 3$ is acceptable for this mark as is

$$x^4 - 10x^2 + 9 = 0 \Rightarrow (x^2 - 1)(x^2 - 9) = 0 \Rightarrow x = 1, 3$$

They cannot go from e.g. $x^4 - 10x^2 + 9 = 0$ to $x = 1, 3$

Note $x^4 - 10x^2 + 9 = 0 \Rightarrow (x-1)(x+1)(x-3)(x+3) = 0 \Rightarrow x = 1, 3$ scores M0

A1: Both correct and no other values. Condone any confusion with which is P and which is Q and condone e.g. $P = 1, Q = 3$

Examples:

$$x^2 + 3 = 13 - \frac{9}{x^2} \Rightarrow x^4 + 3x^2 = 13x^2 - 9$$

$$\Rightarrow x^4 - 10x^2 + 9 = 0 \Rightarrow x = 1, 3$$

Scores M1A1M0A0

$$x^2 + 3 = 13 - \frac{9}{x^2} \Rightarrow x^4 + 3x^2 = 13x^2 - 9$$

$$\Rightarrow x^4 - 10x^2 + 9 = 0 \Rightarrow x^2 = 1, 9 \Rightarrow x = 1, 9$$

Scores M1A1M1A0 (BOD)

Attempts to solve $x^4 - 10x^2 + 9 = 0$ by identifying roots/factors and then using e.g. long division should be sent to review.



Part (a) may also be done without obtaining a quartic:

$$\begin{aligned}
 y = x^2 + 3, y = 13 - \frac{9}{x^2} &\Rightarrow x^2 + 3 = 13 - \frac{9}{x^2} \Rightarrow x^2 - 10 + \frac{9}{x^2} = 0 \\
 &\Rightarrow \left(x - \frac{9}{x}\right)\left(x - \frac{1}{x}\right) = 0 \\
 x - \frac{9}{x} = 0 &\Rightarrow x^2 = 9 \Rightarrow x = 3 \quad \text{or} \quad x - \frac{1}{x} = 0 \Rightarrow x^2 = 1 \Rightarrow x = 1
 \end{aligned}$$

M1: Solves simultaneously and attempts to factorise to $\left(x - \frac{\alpha}{x}\right)\left(x - \frac{\beta}{x}\right) = 0$

A1: Correct factorisation

M1: Attempts to solve via x^2

A1: $x = 1$ and $x = 3$. Both correct and no other values. Condone any confusion with which is P and which is Q and condone e.g. $P = 1, Q = 3$

Part (a) may also be done via y e.g.:

$$\begin{aligned}
 y = x^2 + 3, y = 13 - \frac{9}{x^2} &\Rightarrow y = 13 - \frac{9}{y-3} \Rightarrow y^2 - 16y + 48 = 0 \\
 y^2 - 16y + 48 &= 0 \Rightarrow y = 4, 12 \\
 y = 4 &\Rightarrow x = 1, y = 12 \Rightarrow x = 3
 \end{aligned}$$

M1: Solves simultaneously to obtain a 3TQ in y

A1: Correct 3TQ in y

M1: Solves their 3TQ in y and uses at least one value of y to find a value for x

A1: $x = 1$ and $x = 3$. Both correct and no other values. Condone any confusion with which is P and which is Q and condone e.g. $P = 1, Q = 3$



(b)	$\int \left\{ 13 - \frac{9}{x^2} - (x^2 + 3) \right\} dx = 13x + \frac{9}{x} - \frac{x^3}{3} - 3x(+c)$ or $\int \left(13 - \frac{9}{x^2} \right) dx = 13x + \frac{9}{x} (+c), \quad \int (x^2 + 3) dx = \frac{x^3}{3} + 3x(+c)$	M1A1
	$\left[10x + \frac{9}{x} - \frac{x^3}{3} \right]_1^3 = 10(3) + \frac{9}{3} - \frac{3^3}{3} - \left(10 + 9 - \frac{1}{3} \right) = \dots$ or $\left[13x + \frac{9}{x} \right]_1^3 - \left[\frac{x^3}{3} + 3x \right]_1^3 = 39 + 3 - (13 + 9) - \left\{ 9 + 9 - \left(\frac{1}{3} + 3 \right) \right\} = \dots$	dM1
	$= \frac{16}{3}$	A1
		(4)
		Total 8

Note that this question has the calculator warning so attempts that do not use algebraic integration score no marks.

(b)

M1: Evidence of integration $x^n \rightarrow x^{n+1}$ for at least one term for either **curve** or for the difference between the **2 curves** e.g. it is not for integrating $x^4 - 10x^2 + 9$

A1: Correct integration for both curves either as a difference (either way round) or

separately. E.g. $\pm \left(13x + \frac{9}{x} - \frac{x^3}{3} - 3x(+c) \right)$ or $13x + \frac{9}{x}$ and $\frac{x^3}{3} + 3x$

Allow simplified or unsimplified but indices must be processed.

dM1: Depends on the first method mark.

Substitutes their positive values from part (a) and subtracts if attempted as a difference or substitutes their values into the separate integrations, subtracts and then subtracts the 2 results either way round to obtain a value.

Condone poor bracketing as long as the intention is clear.

May be implied by the correct final answer following correct integration.

If the integration and/or limits are incorrect and the substitution is not shown explicitly then you may need to check.

A1: $\frac{16}{3}$ or **exact** equivalent. $-\frac{16}{3}$ scores A0 unless it gets corrected to $+\frac{16}{3}$



Question Number	Scheme	Marks
9(a)	$(2 \tan \theta = 3 \cos \theta \Rightarrow) \frac{2 \sin \theta}{\cos \theta} = 3 \cos \theta$	M1
	$\frac{2 \sin \theta}{\cos \theta} = 3 \cos \theta \Rightarrow 2 \sin \theta = 3 \cos^2 \theta = 3(1 - \sin^2 \theta)$	M1
	$2 \sin \theta = 3(1 - \sin^2 \theta) \Rightarrow 3 \sin^2 \theta + 2 \sin \theta - 3 = 0^*$	A1*
		(3)
(b)	$\left(\sin \left(2x + \frac{\pi}{3} \right) = \right) \frac{-1 \pm \sqrt{10}}{3}$ (May only see positive root) NB decimal roots are: $-1.387\dots, 0.7207\dots$	M1
	$2x + \frac{\pi}{3} = \sin^{-1}(0.7207\dots) \Rightarrow x = \dots$	M1
	$-0.121, -2.50, 0.645, 3.02$	A1A1
		(4)
		Total 7

(a)**For full marks condone a complete proof entirely in x (or another variable) instead of θ** **M1:** Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ to write the equation in terms of sine and cosine only.Must be the correct identity so e.g. $2 \tan \theta = \frac{2 \sin \theta}{2 \cos \theta}$ is M0May be implied by e.g. $2 \tan \theta = 3 \cos \theta \Rightarrow 2 \sin \theta = 3 \cos^2 \theta$ **M1:** Uses $\cos^2 \theta = 1 - \sin^2 \theta$ to obtain a quadratic equation in sine only.Must be the correct identity so e.g. $3 \cos^2 \theta = 1 - 3 \sin^2 \theta$ is M0**A1*:** Correct work with all necessary steps shown leading to the given answer.Condone e.g. $2 \sin \theta + 3 \sin^2 \theta - 3 = 0$ There should be **no notational or bracketing errors and no mixed or missing variables**. E.g. we would consider

- $\cos^2 \theta$ written as $\cos \theta^2$ a notational error
- $\tan \theta = \frac{\sin}{\cos} \theta$ as a missing variable

Working backwards:

$$\begin{aligned}
 (3 \sin^2 \theta + 2 \sin \theta - 3 = 0 \Rightarrow) & 3(1 - \cos^2 \theta) + 2 \sin \theta - 3 = 0 \\
 \Rightarrow & 2 \sin \theta - 3 \cos^2 \theta = 0 \\
 \Rightarrow & 2 \sin \theta = 3 \cos^2 \theta \\
 \Rightarrow & 2 \tan \theta = 3 \cos \theta^* \text{ Hence proven}
 \end{aligned}$$

M1: Uses $\sin^2 \theta = 1 - \cos^2 \theta$ to obtain an equation in sine and cosine² only.**M1:** Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ which may be implied.**A1*:** Reaches $2 \tan \theta = 3 \cos \theta$ with the same conditions as above but with a (minimal) conclusion.**(b)**

- M1:** Attempts to solve the quadratic $3\sin^2 x + 2\sin x - 3 = 0$ to obtain a value for $\sin x$ where x is any variable. Usual rules apply for solving a quadratic (via a calculator is also acceptable and may imply this mark). If no working is shown then the root(s) must be correct but condone premature rounding e.g. 0.72, -1.3
- M1:** Attempts to find one angle within the range by finding the inverse sine of one of their roots, subtracting $\frac{\pi}{3}$ and dividing by 2.
- For this mark allow to work in degrees if done correctly. E.g. they would need to change $\frac{\pi}{3}$ to 60° and then find the inverse sine of one of their roots in degrees, subtract 60° and divide by 2.
- May be implied by a correct value of x in degrees or radians.
- NB the answers in degrees are: -143, -6.94, 36.9, 173 (3sf)
- Do **not** allow the mixing of degrees and radians for this mark e.g. $x = \frac{46.1... - \frac{\pi}{3}}{2}$
- A1:** Any two of awrt -0.12, -2.5, 0.64 or 0.65, 3.0 (Must be in radians)
- A1:** All four of awrt -0.121, -2.50, 0.645, 3.02 and no others in the range.
(Must be in radians)
- Condone -2.5 for -2.50 but the others must be awrt as shown for this final mark.

Question Number	Scheme	Marks
10(a)	$x^2 + y^2 + 4x - 30y + 209 = 0$ $\Rightarrow (x \pm 2)^2 + (y \pm 15)^2 \dots = 0$	M1
(i)	Centre $(-2, 15)$	A1
(ii)	Radius $\sqrt{20}$	A1
		(3)
(b)	$y = mx + 1 \Rightarrow (x + 2)^2 + (mx + 1 - 15)^2 = 20$ or $y = mx + 1 \Rightarrow x^2 + (mx + 1)^2 + 4x - 30(mx + 1) + 209 = 0$	M1
	$x^2 + m^2x^2 + 4x - 28mx + 180 = 0$ $b^2 - 4ac = 0 \Rightarrow (4 - 28m)^2 - 4(1 + m^2) \times 180 = 0$	dM1
	$(4 - 28m)^2 - 4(1 + m^2)180 = 0 \Rightarrow 16 - 224m + 784m^2 - 720 - 720m^2 = 0$ $\Rightarrow 2m^2 - 7m - 22 = 0^*$	A1*
		(3)
(c)	$2m^2 - 7m - 22 = 0 \Rightarrow m = \frac{11}{2}, -2$	M1
	$m = \frac{11}{2} \Rightarrow \frac{125}{4}x^2 - 150x + 180 = 0 \Rightarrow x = \frac{12}{5} \Rightarrow y = \frac{71}{5}$ or $m = -2 \Rightarrow 5x^2 + 60x + 180 = 0 \Rightarrow x = -6 \Rightarrow y = 13$	M1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ or $(-6, 13)$ oe	A1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ and $(-6, 13)$ oe	A1
		(4)
		Total 10

(a) Look out for answers to part (a) written in the body of the question.

M1: Attempts to complete the square for **both** variables or states a centre of $(\pm 2, \pm 15)$

For completing the square allow $(x \pm 2)^2 \dots (y \pm 15)^2 \dots = \dots$

A1: Centre $(-2, 15)$ or e.g. $x = -2, y = 15$. Condone e.g. $-2, 15$ or $[-2, 15]$

A1: For $\sqrt{20}$ or $2\sqrt{5}$ which may be scored following $(x \pm 2)^2 + (y \pm 15)^2 = 20$

Do **not** allow $\pm\sqrt{20}$ but apply isw if a correct radius is seen which is then e.g. converted to a decimal or simplified incorrectly.

(b) Mark (b) and (c) together.

M1: Attempts to substitute $y = mx + 1$ into the given equation or their rearranged equation to obtain an equation in m and x only. Condone slips as long as the intention is clear.

dM1: Attempts $b^2 - 4ac \dots 0$ where $\dots$ is “=” or “>” or “<” etc. or equivalent e.g. $b^2 = 4ac$ where $a = A + Bm^2, b = C + Dm, c = E$ where A, B, C, D and E are non-zero constants. Condone copying slips if the intention is clear if the above conditions are met. **Depends on the first M mark.**

A1*: Obtains the printed answer with no errors e.g. must have had $b^2 - 4ac = 0$ or equivalent throughout and before the final line e.g. not $b^2 - 4ac > 0$ and brackets expanded. **Note this is a given answer so must follow correct work.**

Question	Scheme	Marks
10(a)	Equation of circle is $(x-3)^2 + (y-5)^2 = r^2$ and line is $y = 2x + k$ So intersect when $(x-3)^2 + (2x+k-5)^2 = r^2$	M1
	$\Rightarrow x^2 - 6x + 9 + 4x^2 + 4(k-5)x + (k-5)^2 = r^2$ $\Rightarrow 5x^2 + (-6+4k-20)x + 9+k^2-10k+25-r^2 = 0$	dM1
	$\Rightarrow 5x^2 + (4k-26)x + k^2 - 10k + 34 - r^2 = 0^*$	A1*
		(3)
(b)	Tangent to C $\Rightarrow b^2 - 4ac = 0 \Rightarrow (4k-26)^2 - 4 \times 5 \times (k^2 - 10k + 34 - r^2) = 0$	M1
	$\Rightarrow 16k^2 - 208k + 676 - 20k^2 + 200k - 680 + 20r^2 = 0$ $\Rightarrow 5r^2 = \dots$	M1
	$\Rightarrow 5r^2 = k^2 + 2k + 1 = (k+1)^2$	A1
		(3)
(b) Way 2	Gradient of BX is $-\frac{1}{2}$ so equation of BX is $y-5 = -\frac{1}{2}(x-3)$ $y-5 = -\frac{1}{2}(x-3), y = 2x+k \Rightarrow x = \dots, y = \dots \left(\frac{13-2k}{5}, \frac{26+k}{5} \right)$	M1
	$\left(\frac{13-2k}{5} - 3 \right)^2 + \left(\frac{26+k}{5} - 5 \right)^2 = r^2$	dM1
	$\Rightarrow 5r^2 = k^2 + 2k + 1 = (k+1)^2$	A1
		(3)
(c)	Triangle AXB is right angled so $AB^2 + r^2 = XA^2 = (3-0)^2 + (5-k)^2$	M1
	$AB^2 = 4r^2$ so $AB^2 + r^2 = 5r^2$	M1
	$\Rightarrow 5r^2 = 9 + (5-k)^2$	A1
	$\Rightarrow k^2 + 2k + 1 = 9 + 25 - 10k + k^2$	M1
	$\Rightarrow 12k = 33 \Rightarrow k = \dots$	dM1
	$k = \frac{11}{4}$	A1
		(6)
(12 marks)		
Notes:		
(a)		
M1: Forms equation of circle and substitutes in equation of line. The circle equation must be of the form $(x \pm 3)^2 + (y \pm 5)^2 = r^2$		
dM1: Expands both sets of brackets and collects terms in x^2 and x .		



A1*: Reaches the answer given with no errors seen.

Note that some candidates expand the brackets first before substitution e.g.:

$$(x-3)^2 + (y-5)^2 = r^2 \Rightarrow x^2 - 6x + 9 + y^2 - 10y + 25 = r^2 \Rightarrow x^2 - 6x + 9 + (2x+k)^2 - 10(2x+k) + 25 = r^2$$

This implies the first M and then the second M will score when terms in x^2 and x are collected.

Note about poor squaring e.g. $(x-3)^2 = x^2 + 9$: The first M is available in both cases above but the second M requires at least two x^2 terms and at least two x terms from the expansions.

Note that it is acceptable to go from a completely correct full expansion to the printed answer e.g.

$$(x-3)^2 + (y-5)^2 = r^2 \Rightarrow x^2 - 6x + 9 + y^2 - 10y + 25 = r^2 \Rightarrow x^2 - 6x + 9 + (2x+k)^2 - 10(2x+k) + 25 = r^2 \\ \Rightarrow x^2 - 6x + 9 + 4x^2 + 4kx + k^2 - 20x - 10k + 25 = r^2 \Rightarrow 5x^2 + (4k-26)x + k^2 - 10k + 34 - r^2 = 0$$

Scores full marks in (a)

(b)

M1: Uses the discriminant is zero to form an equation in k and r

dM1: Expands and rearranges to make ar^2 the subject

A1: Correct answer

Way 2

M1: Attempts the equation of BX and solves simultaneously with l to find the coordinates of B

Alternatively uses $x = -\frac{b}{2a} = \frac{13-2k}{5}$ at B and uses this to find y at B

dM1: Correct use of Pythagoras for BX and sets $= r^2$

A1: Correct answer

(c)

M1: Attempts XA^2 **correctly** in terms of k (the k may appear as y but must be replaced by k later) and uses it in Pythagoras theorem for triangle AXB .

May be implied.

M1: Applies $AB = 2r$ to get $AB^2 + r^2$ in terms of r . Condone with $AB^2 = 2r^2$ used.

A1: Correct equation.

M1: Substitutes the result from (b) and expands brackets.

dM1: Solves a linear equation in k . **Depends on the previous M.**

A1: Correct value.

Alt (c)	At B $x = -\frac{b}{2a} = \frac{13-2k}{5}$, $y = 2\left(\frac{13-2k}{5}\right) + k$	M1
	$AB^2 = \left(\frac{13-2k}{5} - 0\right)^2 + \left(\frac{26-4k}{5} + k - k\right)^2$	M1 A1
	$\Rightarrow 25 \times 4r^2 = (13-2k)^2 + (26-4k)^2 = \dots$	M1
	$\Rightarrow 20 \times (k+1)^2 = (13-2k)^2 + (26-4k)^2 = 20k^2 - 260k + 845$ $\Rightarrow k^2 + 2k + 1 = k^2 - 13k + \frac{169}{4} \Rightarrow k = \dots$	dM1
	$\Rightarrow k = \frac{11}{4}$	A1
		(6)

Notes:**(c)****M1:** Uses the result in (a) to find the x coordinate where the line and circle meet and then finds y An alternative is to find the equation of BX as in (b) way 2 and solve with l to find x and y at B (May have already found the coordinates of B in (a) but must be re-stated or used in (c) to score this mark)**M1:** Uses distance formula to find an expression in k for AB or AB^2 **A1:** Correct expression for AB or AB^2 . Need not be simplified.**M1:** Applies $AB = 2r$ to the equation. Condone with $AB^2 = 2r^2$ used.**dM1:** Substitutes the result from (b) and solves a linear equation in k . **Depends on the previous M.****A1:** Correct value.**Some more unusual methods for 3ii**

Via perpendicular bisectors:

Way 5	If AB is diameter centre must be midpoint of AB ie $M\left(\frac{1+3}{2}, \frac{0-10}{2}\right)$	M1
	$= (2, -5)$	A1
	Attempts 2 of: $m_{BC} = \frac{-6+10}{7-3} = 1$ and midpoint is $\left(\frac{7+3}{2}, \frac{-6-10}{2}\right) = (5, -8)$ so perpendicular bisector is $y + "8" = -\frac{1}{"1"}(x - "5")$ or $m_{AC} = \frac{7-1}{-6-0} = -1$ and midpoint is $\left(\frac{7+1}{2}, \frac{-6}{2}\right) = (4, -3)$ so perpendicular bisector is $y + "3" = -\frac{1}{"-1"}(x - "4")$ or $m_{AB} = \frac{-10-0}{3-1} = -5$ and midpoint is $\left(\frac{3+1}{2}, \frac{-10}{2}\right) = (2, -5)$ so perpendicular bisector is $y + "5" = -\frac{1}{"-5"}(x - "2")$ $y + 8 = -(x - 5)$ oe or $y + 3 = x - 4$ oe or $y + 5 = \frac{1}{5}(x - 2)$ oe And solves simultaneously: E.g. $y + 3 = x - 4, y + 8 = 5 - x \Rightarrow 5 - x - 5 = x - 4 \Rightarrow x = 2, y = -5$	dM1
	Hence centre of circle is $(2, -5)$	A1
	E.g. Midpoint of AB is the centre of the circle so AB is a diameter (or equivalent)	A1
		(5)



M1: Attempts midpoint of AB . If no method is shown accept one correct coordinate as evidence.

A1: Correct midpoint

dM1: Attempts 2 perpendicular bisectors, and solves simultaneously

A1: Obtains $(2, -5)$

A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.

Via circle equation:

Way 6	Uses $(x-a)^2 + (y-b)^2 = r^2$ With $(1, 0)$, $(7, -6)$ and $(3, -10)$ To find $(a, b) = \dots$ or $r/r^2 = \dots$	M1
	Centre $(2, -5)$ or radius $\sqrt{26}$	A1
	E.g. Equation of AB is $y = -5(x-1)$ and $-5(2-1) = -5$ or $AB = \sqrt{(3-1)^2 + (-10-0)^2} = \sqrt{104} = 2\sqrt{26}$ or midpoint of AB is $\left(\frac{1+3}{2}, \frac{0-10}{2}\right) = (2, -5)$	dM1A1
	So centre is on AB or AB is twice the radius or midpoint is the centre hence AB is a diameter of the circle. (or equivalent)	A1
		(5)

M1: Uses all three points in circle equation to set up three equations in three unknowns to find centre or radius.
A1: Correct centre or correct radius
dM1: Finds e.g. equation of AB , distance AB or midpoint of AB
A1: Correct equation of AB , distance AB or midpoint of AB
A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.

Via intersecting circles:

(ii) Way 7	If AB is diameter centre must be midpoint of AB ie $M\left(\frac{1+3}{2}, \frac{0-10}{2}\right)$	M1
	$= (2, -5)$	A1
	Circle centre C radius r is $(x-7)^2 + (y+6)^2 = r^2$ Circle centre B radius r is $(x-3)^2 + (y+10)^2 = r^2$ These intersect when $(x-7)^2 + (y+6)^2 = (x-3)^2 + (y+10)^2$ $\Rightarrow x + y = -3$ Circle centre A radius r is $(x-1)^2 + y^2 = r^2$ $(x-3)^2 + (y+10)^2 = (x-1)^2 + y^2 \Rightarrow x - 5y = 27$	dM1A1

	Solves simultaneously: $x + y = -3, x - 5y = 27 \Rightarrow x = 2, y = -5$	
	E.g. Midpoint of AB is the centre of the circle so AB is a diameter (or equivalent)	A1
		(5)
M1: Attempts midpoint of AB . If no method is shown accept one correct coordinate as evidence. A1: Correct midpoint dM1: Attempts equations of 2 circles with A, B or C as centre with radius r , repeats the process for 2 different circles and finds the intersection of both straight lines and solves simultaneously A1: Correct coordinates of centre A1: Suitable explanation and conclusion given with no errors and all previous marks awarded.		

Question Number	Scheme	Marks
9(a)	$y = x^3 - 5x^2 + 3x + 14 \Rightarrow \frac{dy}{dx} = 3x^2 - 10x + 3 = 0$	M1
	Roots are $3, \frac{1}{3} \Rightarrow$ when $x = 3, y = 3^3 - 5 \times 3^2 + 3 \times 3 + 14 = \dots$	dM1
	Centre is (3, 5)	A1
		(3)
(b)	At A $y = 8$	B1
	$r^2 = (2 - 3)^2 + (8 - 5)^2 (=10)$	M1
	$(x - 3)^2 + (y - 5)^2 = 10$	A1
		(3)
(c)	$\frac{8 - 5}{2 - 3} = \dots (-3)$	M1
	$y - 8 = \frac{1}{3}(x - 2)$	M1
	$y = \frac{1}{3}x + \frac{22}{3} \quad *$	A1*
		(3)
(d)	$\int_0^2 x^3 - 5x^2 + 3x + 14 \, dx = \dots \left(\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 14x \right)$	M1
	Area = $\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 14x \right]_0^2 - \left(\frac{1}{2} \times \left(\frac{22}{3} + 8 \right) \times 2 \right) = \dots$	dM1
	$\frac{1}{4} \times 16 - \frac{5}{3} \times 8 + \frac{3}{2} \times 4 + 14 \times 2 - \frac{46}{3}$	
	$\frac{74}{3} - \frac{46}{3} = \frac{28}{3}$	A1
		(3)
		(12 marks)

(a)

M1: Differentiates the equation of the curve and sets equal to 0 which may be implied by their attempt to solve below. Do not credit this differentiation in any other parts of the question.

For the differentiation, look for at least 2 of $x^3 \rightarrow \dots x^2, -5x^2 \rightarrow \dots x, 3x \rightarrow 3$

dM1: Solves a 3 term quadratic equation by any valid means (including a calculator) and substitutes one of their roots in the original equation to find the y coordinate. **Depends on the first mark.**

A1: Correct coordinates (3, 5) or e.g. $x = 3, y = 5$ (Ignore any work with e.g. $x = \frac{1}{3}$ and ignore any

work attempting to prove maximum/minimum points – just look for correct coordinates identified)

(b)

B1: y coordinate at A is 8 (Allow this to score anywhere)

M1: Attempts to find the radius of the circle (or radius²) using (2, 8) and their minimum point.

Score for $(2 - 3)^2 + (8 - 5)^2$ or equivalent correct work for their coordinates of T and A.

You can ignore what they call it e.g. condone $\text{radius} = (2 - 3)^2 + (8 - 5)^2$

A1: $(x - 3)^2 + (y - 5)^2 = 10$ oe e.g. $(x - 3)^2 + (y - 5)^2 = (\sqrt{10})^2, \sqrt{(x - 3)^2 + (y - 5)^2} = \sqrt{10}$

