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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Exponentials & Logarithms (P2.3)

International A Level Mathematics · P2: Pure Mathematics 2 (WMA12)

25 questions · 189 marks · Mark schemes included

Past papers June 2019 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

25 questions · 189 marks · 2 sub-topics

SUB-TOPIC P2.3a

LAWS OF LOGARITHMS

Sub-topic P2.3a · 19 questions · 140 marks · lowest tariff first



4. Using the laws of logarithms, solve

$$\log_3(32 - 12x) = 2\log_3(1 - x) + 3$$

(5)

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Question 4 continued

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Lined area for writing the answer to Question 4.

(Total 5 marks)

Q4



5. Use the laws of logarithms to solve

$$\log_2(16x) + \log_2(x + 1) = 3 + \log_2(x + 6)$$

(5)



(Total for Question 3 is 3 marks)



(5)



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 5 marks)



4. In this question you must show all stages of your working.

Give your answers in fully simplified surd form.

Given that a and b are positive constants, solve the simultaneous equations

$$\begin{aligned}a - b &= 8 \\ \log_4 a + \log_4 b &= 3\end{aligned}$$

(6)

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Question 4 continued

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Q4

(Total 6 marks)



4. (i) Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7) \quad (3)$$

(ii) Given that

$$\sum_{r=1}^2 \log_a(y^r) = \sum_{r=1}^2 (\log_a y)^r \quad y > 1, a > 1, y \neq a$$

find y in terms of a , giving your answer in simplest form. (3)

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 6 marks)



(Total for Question 3 is 6 marks)



3.

In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

(i) Using the laws of logarithms, solve

$$2\log_2(2-x) = 4 + \log_2(x+10) \quad (5)$$

Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 6 marks)



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 6 marks)



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7. Given $\log_a b = k$, find, in simplest form in terms of k ,

$$(i) \quad \log_a \left(\frac{\sqrt{a}}{b} \right) \tag{2}$$

$$(ii) \quad \frac{\log_a a^2 b}{\log_a b^3} \quad (2)$$

$$(iii) \sum_{n=1}^{50} (k + \log_a b^n) \tag{3}$$

Question 7 continued

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Q7

(Total 7 marks)



10. Given $a = \log_2 3$

(i) write, in simplest form, in terms of a ,

(a) $\log_2 9$

(b) $\log_2\left(\frac{\sqrt{3}}{16}\right)$

(3)

(ii) Solve

$$3^x \times 2^{x+4} = 6$$

giving your answer, in simplest form, in terms of a .

(4)

Question 10 continued

Lined area for writing the answer to Question 10.



TOTAL FOR PAPER IS 75 MARKS



3. (i) Solve

$$7^{x+2} = 3$$

giving your answer in the form $x = \log_7 a$ where a is a rational number in its simplest form.

(3)

(ii) Using the laws of logarithms, solve

$$1 + \log_2 y + \log_2(y + 4) = \log_2(5 - y)$$

(5)

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Question 3 continued

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Q3

(Total 8 marks)



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 8 marks)







(Total for Question 6 is 8 marks)



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8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b \log_2 3$, where a and b are constants to be found. (4)

- (ii) Using the laws of logarithms, solve

$$\log_5(7-2y) = 2\log_5(y+1) - 1 \quad (5)$$



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Question 8 continued

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Question 8 continued

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Q8

(Total 9 marks)



Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 9 marks)



8. In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation $y = f(x)$ where $f(x)$ is a polynomial in x .

Given

$$f'(x) = 2(3x - 2)(x + 5)$$

- (a) deduce the range of values of x for which y is decreasing. (2)

Given further that C cuts the x -axis at $\frac{7}{2}$

- (b) state a factor of $f(x)$, giving the answer in the form $(Ax + B)$ where A and B are integers.

- (c) Hence find an expression for $f(x)$ in a fully factorised form. (6)

Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 9 marks)



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Question 9 continued

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(Total 10 marks)

Q9

END

TOTAL FOR PAPER IS 75 MARKS



7. (a) Given that

$$3\log_3(2x-1) = 2 + \log_3(14x-25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0 \quad (4)$$

(b) Show that -4 is a root of this cubic equation.

(c) Hence, using algebra and showing each step of your working, solve

$$3 \log_3(2x - 1) = 2 + \log_3(14x - 25) \quad (4)$$

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Question 7 continued

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Question 7 continued

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(Total 10 marks)

Q7



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 10 marks)



SUB-TOPIC P2.3b

EXPONENTIAL EQUATIONS

Sub-topic P2.3b · 6 questions · 49 marks · lowest tariff first



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5. (a) Given $0 < a < 1$, sketch the curve with equation

$$y = a^x$$

showing the coordinates of the point at which the curve crosses the y-axis.

(2)

x	2	2.5	3	3.5	4
y	4.25	6.427	9.125	12.34	16.06

The table above shows corresponding values of x and y for $y = x^2 + \left(\frac{1}{2}\right)^x$

The values of y are given to 4 significant figures as appropriate.

Using the trapezium rule with all the values of y in the given table,

- (b) obtain an estimate for $\int_2^4 \left(x^2 + \left(\frac{1}{2}\right)^x \right) dx$

(3)

Using your answer to part (b) and making your method clear, estimate

- (c) $\int_2^4 \left(x(x-3) + \left(\frac{1}{2}\right)^x \right) dx$

(2)



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Q5

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9. (a) Sketch the curve with equation

$$y = 3 \times 4^x$$

showing the coordinates of any points of intersection with the coordinate axes.

(2)

The curve with equation $y = 6^{1-x}$ meets the curve with equation $y = 3 \times 4^x$ at the point P .

- (b) Show that the x coordinate of P is $\frac{\log_{10} 2}{\log_{10} 24}$

(5)



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Question 9 continued

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(Total 7 marks)

Q9



2.

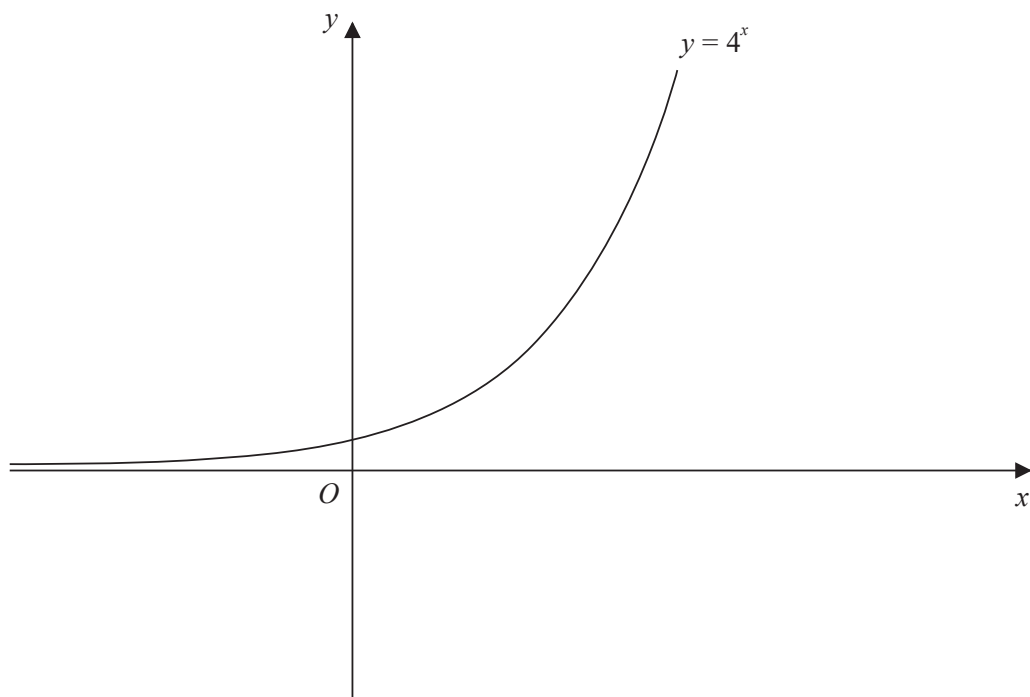
**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = 4^x$

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(a) On Diagram 1, sketch the curve with equation

(i) $y = 2^x$

(ii) $y = 4^x - 6$

Label clearly the coordinates of any points of intersection with the coordinate axes.

(4)

The curve with equation $y = 2^x$ meets the curve with equation $y = 4^x - 6$ at the point P .

(b) Using algebra, find the exact coordinates of P .

(4)

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Question 2 continued

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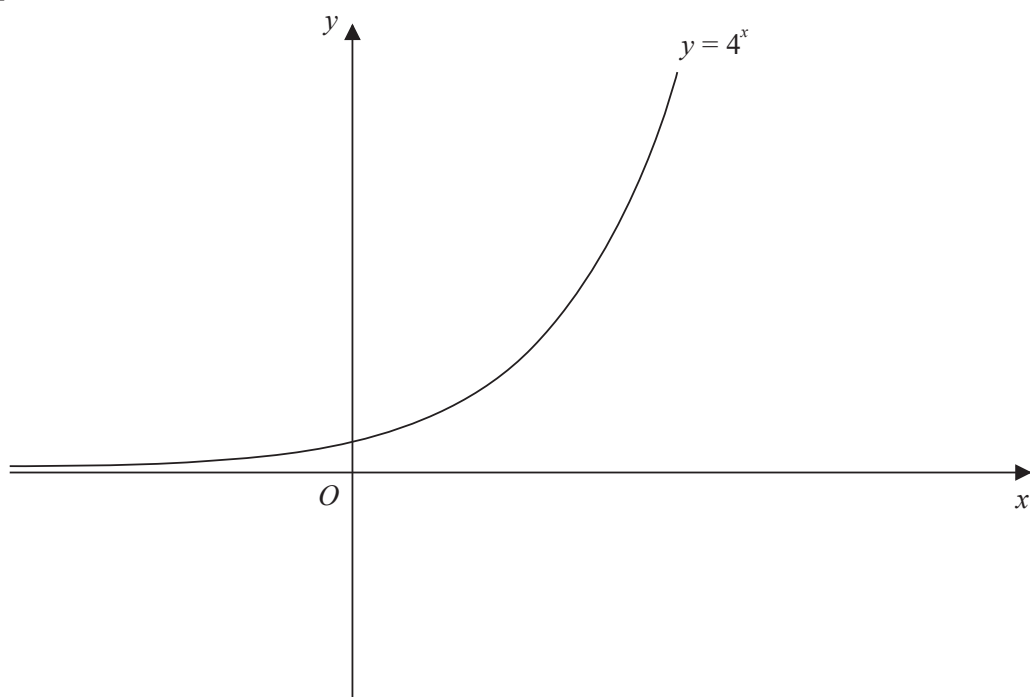


Diagram 1

Q2

(Total 8 marks)



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Question 9 continued

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Lined area for writing the answer to Question 9 continued.

4. (a) Sketch the curve with equation

$$y = a^{-x} + 4$$

where a is a constant and $a > 1$

On your sketch show

- the coordinates of the point of intersection of the curve with the y -axis
- the equation of the asymptote to the curve.

(3)

x	-4	-1.5	1	3.5	6	8.5
y	13	6.280	4.577	4.146	4.037	4.009

The table above shows corresponding values of x and y for $y = 3^{-\frac{1}{2}x} + 4$

The values of y are given to four significant figures, as appropriate.

Using the trapezium rule with all the values of y in the table,

- (b) find an approximate value for

$$\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4 \right) dx$$

giving your answer to two significant figures.

(3)

- (c) Using the answer to part (b), find an approximate value for

(i) $\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} \right) dx$

(ii) $\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4 \right) dx + \int_{-8.5}^4 \left(3^{\frac{1}{2}x} + 4 \right) dx$

(3)





(Total for Question 4 is 3 marks)



6. (a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y -axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^{3.5} (2^x - 2x) \, dx$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} (2^x + 2x) \, dx$

(ii) $\int_2^{3.5} (2^{x+1} - 4x) \, dx$

(3)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 9 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Notes	Marks
1(a)	$h = 0.5$	Correct strip width	B1
	$A \approx \frac{1}{2} \times \frac{1}{2} \{6.792 + 5.113 + 2(6.298 + 5.858 + 5.466)\}$ Correct application of the trapezium rule with their h		M1
	$= 11.79$	Cao	A1
			(3)
(b)(i)	$A \approx 2 \times "11.79"$	Multiplies their answer to (a) by 2	M1
	$= 23.58$	Correct answer or correct ft	A1ft
(b)(ii)	$A \approx "11.79" + 6$	Adds 6 to their answer to (a)	M1
	$= 17.79$	Correct answer or correct ft	A1ft
			(4)
			Total 7

Notes:**(a)**

B1: Correct value for h stated or implied by $\frac{1}{4}\{\dots\}$

M1: Attempts the trapezium rule with their h . Must include the $\frac{1}{2} \times "h"$ but this may be implied by any multiple of the bracket if h is not identified separately. Must have the correct inner bracket structure, though allow if all that is wrong is a miscopy of numerals. May be written as separate trapezia.

Bracketing error $\frac{1}{4}(6.792 + 5.113) + 2(6.298 + 5.858 + 5.466)$ is M0 unless recovered by their answer.

A1: cao 11.79. Must be to 2 d.p. (Note the actual value is 11.78 to 2 d.p. and score no marks if just this is seen)

(b)(i)

M1: For twice their answer to (a), allowing for rounding. No need to carry out the doubling, sight of $2 \times "their (a)"$ is sufficient.

A1ft: For 23.58 or awrt 23.57 following a correct part (a) rounded or truncated to 11.78 (if they use more than 2 d.p. then 23.5745 is the value they should get) **or** follow through their answer to (a) using the same principle. May be given to more than 2 d.p. and isw. (Allow answer to less than 2 d.p. if just trailing zeros are omitted.)

Note: accurate answer is 23.56 to 2 d.p., which is M0A0 if no method is shown.

(ii)

M1: For 6+ their (a) stated or implied (but see note). The 6 need not be simplified if evaluated from an integral (but the integral and substitution must be correct giving an evaluation that simplifies to 6).

A1ft: 17.79 or follow through their answer to (a) + 6 evaluated. May be given to more than 2 d.p. and isw

Note: accurate answer is 17.78 to 2 d.p., which is M0A0 if no method is shown.

Note: Repeated trapezium rule is M0 in (b).



Question Number	Scheme	Notes	Marks
2(a)	$y = 27x^{\frac{1}{2}} - x^{\frac{3}{2}} - 20$	$x^n \rightarrow x^{n-1}$	M1
	$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{27}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$	Correct simplified derivative	A1
			(2)
(b)	$\frac{27}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0 \Rightarrow x = \dots$	Sets their derivative = 0 attempts to solve for x	dM1
	$x = 9$	Correct x value	A1
	$x = 9 \Rightarrow y = \dots$	Uses their x value to find a value for y	M1
	$y = 34$	Correct y value	A1
			(4)
(c)	$\frac{d^2y}{dx^2} = \pm Ax^{-\frac{3}{2}} \pm Bx^{-\frac{1}{2}}$	Attempts second derivative and substitutes their x or considers the sign (for x > 0)	M1
	$\left(\frac{d^2y}{dx^2}\right)_{x=9} = -\frac{27}{4}(9)^{-\frac{3}{2}} - \frac{3}{4}(9)^{-\frac{1}{2}} = \left(-\frac{1}{2}\right)$		
	$\frac{d^2y}{dx^2} = -\frac{27}{4}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$ $\frac{d^2y}{dx^2} < 0$ for x > 0 / when x = 9 so maximum	Correct second derivative and conclusion with correct reason	A1
			(2)
			Total 8

Notes**(a)****M1:** Power decreased by 1 in at least one term in x**A1:** Correct, simplified derivative. Accept as shown or $\frac{27}{2\sqrt{x}} - \frac{3}{2}\sqrt{x}$ or with decimalequivalents for coefficients. No need to see the $\frac{dy}{dx}$, score for a correct expression.**(b)****Note: Answer only with no initial equation set up scores no marks.****dM1:** Depends on the M in (a). Sets their derivative equal to 0 and reaches a value x. The “=0” may be implied by a clear attempt to solve it. Do not be concerned about the method for solving as long as a value of x is reached. If substituting $a = \sqrt{x}$ (or similar) must return to x for the method.**A1:** Correct value for x from correct work. Ignore references to x = 0 or any negative values but A0 if extra positive values.**M1:** Substitutes their positive x value into the curve equation to find a value for y. May need to check if no substitution is shown.**A1:** Correct y value. Ignore reference to any point at x = 0 or negative values but A0 if other coordinates with positive x are given.**(c)** Ignore work relating to x = 0 in (c)**M1:** Attempts the second derivative achieving the form $\pm Ax^{-\frac{3}{2}} \pm Bx^{-\frac{1}{2}}$ and attempts to use it to classify the stationary point. Look for an attempt at substituting their value or a consideration of the sign.

A1: Correct second derivative and conclusion drawn with supporting evidence from which it is reasonable to deduce the nature. Accept "concave down" for maximum, but not just "concave".

For the evidence accept either evaluation to the correct value $-\frac{1}{2}$, or correct sign deduced from substitution of $x = 9$, or via **statement** it is negative for all $x > 0$ without substitution seen. Use of a value other than $x = 9$ scores A0.



Question Number	Scheme	Notes	Marks
3(a)	$\left(2 - \frac{kx}{4}\right)^8 = 2^8 + \binom{8}{1}2^7\left(-\frac{kx}{4}\right) + \binom{8}{2}2^6\left(-\frac{kx}{4}\right)^2 + \binom{8}{3}2^5\left(-\frac{kx}{4}\right)^3 + \dots$ $\text{Or } \left(1 - \frac{kx}{8}\right)^8 = 1 + \binom{8}{1}\left(-\frac{kx}{8}\right) + \binom{8}{2}\left(-\frac{kx}{8}\right)^2 + \binom{8}{3}\left(-\frac{kx}{8}\right)^3 + \dots$		M1
	$= 256 - 256kx + 112k^2x^2 - 28k^3x^3 + \dots$	256 – 256kx	B1
		112k ² x ² or –28k ³ x ³ (unsimplified)	A1
		112k ² x ² and –28k ³ x ³ (simplified)	A1
			(4)
(b)	$f(x) = (5 - 3x)\left(2 - \frac{kx}{4}\right)^8 = (5 - 3x)(256 - 256kx + 112k^2x^2 - 28k^3x^3 + \dots)$ Coefficient of x is 5 × –256k – 3 × 256		M1
	$5 \times 256 = 3(-1280k - 768) \Rightarrow k = \dots$	Sets 5 × their constant term from (a) = 3 × their coefficient of x from f(x) and solves for k	M1
	$k = -\frac{14}{15}$	Correct value.	A1
			(3)
			Total 7

Notes**(a)**

M1: Attempts the binomial expansion on $\left(2 \pm \frac{kx}{4}\right)^8$ or $\left(1 \pm \frac{kx}{\beta}\right)^8$ up to at least the third (x^2)

term with an acceptable structure. Look for the correct binomial coefficient (accept alternative notation ${}^nC_r$) combined with the correct power of x but allow if powers of 2 are incorrect and if brackets are missing. M0 for descending powers.

B1: for $256 - 256kx$, may be listed, must be simplified. Allow for $256(1 - kx + \dots)$ if the 2^8 is taken out first.

A1: Correct third or fourth term, may be listed. Need not be simplified but the binomial coefficients must be numerical. Allow for one term from $256\left(\dots + \frac{28}{64}(kx)^2 - \frac{56}{512}(kx)^3 \dots\right)$ if the 2^8 is taken out first. May have powers as $(kx)^n$ for this mark. Allow for the correct x^2 term if the sign was incorrect in their bracket.

A1: Correct simplified third and fourth terms as shown in scheme, may be listed. Must have $k^n x^n$ terms.

Note: isw after correct terms are seen if they try to divide through.

(b)

M1: Correct strategy for the coefficient of x or the x term. E.g. $5 \times \text{their } -256k - 3 \times \text{their } 256$ or may be part of a full expansion – look for $(5 \times \text{their } -256k - 3 \times \text{their } 256)x$ but terms must have been combined.

M1: Sets $5 \times \text{their constant term from (a)} = 3 \times \text{their coefficient of x from } f(x)$ and solves for k. Should be an equation in k only, but allow recovery if they initially include the x but later cross it out to give a constant for the answer. The attempt at the x coefficient must have been an attempt at a sum of two terms from their expansion of f(x)

A1: Correct value, must be exact. Allow $-0.9\bar{3}$ but not a terminating decimal.



Question Number	Scheme	Marks
2a	$\left(f\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) + a = 0 \Rightarrow a = \dots\right.$	M1
	$\Rightarrow \frac{27}{2} - 18 + \frac{15}{2} + a = 0 \Rightarrow a = -3 \quad *$	A1*
		(2)
b	Example where $2x - 3$ is a linear factor: $\begin{array}{r} 2x^2 - x + 1 \\ 2x - 3 \overline{) 4x^3 - 8x^2 + 5x - 3} \\ \underline{4x^3 - 6x^2} \\ -2x^2 + 5x \\ \underline{-2x^2 + 3x} \\ +2x - 3 \\ \underline{+2x - 3} \\ 0 \end{array}$	M1A1
	$(-1)^2 - 4 \times 2 \times 1 = -7 < 0 \Rightarrow \text{no real roots so } x = \frac{3}{2} \text{ is the only one real root } *$	dM1A1*
		(4)
		(6 marks)

(a)

M1 Substitutes $x = \frac{3}{2}$ into the expression, sets equal to 0 (may be implied for this mark) and rearranges to find a . Condone invisible brackets for this mark only and condone arithmetical slips in their working.

Substituting $x = -\frac{3}{2}$ is M0 and attempts via long division score M0.

A1* $a = -3$ achieved with no errors seen including the omission of brackets.

There must be sight of $x = \frac{3}{2}$ which may be embedded in the expression or $f\left(\frac{3}{2}\right)$ followed

by at least one intermediate stage of working/manipulation (it may just be the left hand side of the equation), before proceeding to the given answer. The " $= 0$ " must be seen somewhere in their working before the given answer.

e.g.

$$4\left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) + a = 0 \Rightarrow a = -3 \text{ is M1A0}$$

$$f\left(\frac{3}{2}\right) = \frac{27}{2} - 18 + \frac{15}{2} + a = 0 \Rightarrow a = -3 \text{ is M1A1}$$

Note the trivial case $f\left(\frac{3}{2}\right) = 3 + a = 0 \Rightarrow a = -3$ is M1A0 (not enough shown)

(b) Work done in (a) must be made use of in (b) to gain credit.

M1 Attempts to find a quadratic factor of $4x^3 - 8x^2 + 5x - 3$ using $2x - 3$ as a factor

- score for $2x^2 \pm x \pm \dots$ if algebraic division is used or
- score for $2x^2 \pm \dots x \pm 1$ if equating coefficients/inspection used.



Alternatively, they may attempt to find a quadratic factor using $x - \frac{3}{2}$ as a factor so

- score for $4x^2 \pm 2x \pm \dots$ if algebraic division is used or
- score for $4x^2 \pm \dots x \pm 2$ if equating coefficients/inspection used.

A1 $2x^2 - x + 1$ (for $2x - 3$ as the linear factor) or $4x^2 - 2x + 2$ (for $x - \frac{3}{2}$ as the linear factor)

dM1 Attempts to show their quadratic factor has no real roots. It is dependent on the previous method mark. Factorisation attempts are M0. Accept via

- an attempt at $b^2 - 4ac$ for their quadratic factor so score for values correctly embedded in $b^2 - 4ac$ for their quadratic
- an attempt to solve their $2x^2 - x + 1$ or $4x^2 - 2x + 2$ using the quadratic formula with values embedded or via a calculator (you must check their roots are correct for their quadratic)
- an attempt to complete the square e.g. $2\left(x \pm \frac{1}{4}\right)^2 \pm \dots$

A1* Requires correct factorisation of the cubic, correct calculation for the quadratic, reason and conclusion

For example, after factorising their cubic to e.g. $(2x - 3)(2x^2 - x + 1)$ which does not need to be explicitly stated accept e.g.

- the discriminant of $2x^2 - x + 1$ is $-7 < 0$ so only has a root at $x = \frac{3}{2}$. If correct values are embedded which give -7 then this is fine. (For $4x^2 - 2x + 2$ the discriminant is -28)
- $2x^2 - x + 1 = 0 \Rightarrow x = \frac{1}{4} \pm i\frac{\sqrt{7}}{4}$, $(2x - 3) = 0 \Rightarrow x = \frac{3}{2}$ so only one real root. (The roots will be the same for $4x^2 - 2x + 2 = 0$)
- $2x^2 - x + 1 = 0 \Rightarrow 2\left(x - \frac{1}{4}\right)^2 + \frac{7}{8} > 0$, hence $x = \frac{3}{2}$ is the only one real root (The completed square form for $4x^2 - 2x + 2$ could be the same or a multiple of it depending on how they have rearranged their inequality)

There must be some reference to the root $x = \frac{3}{2}$ either by stating it or indicating the linear term has a root

(e.g. writing “root from here” next to it), but do not accept incorrect statements such as only real root is $(2x - 3)$. Condone “solution” for “root”

Do not allow statements such as “Math error” without interpretation of what this means.



Question Number	Scheme	Marks
3a(i)	$r = \sqrt{(8-2)^2 + (-3-5)^2} = 10$	M1A1
(ii)	$(x-2)^2 + (y-5)^2 = 100$	A1ft
		(3)
b	Gradient between centre and $P = -\frac{4}{3}$	B1
	Perpendicular gradient $= \frac{3}{4}$	M1
	$y+3 = \frac{3}{4}(x-8)$	M1
	$3x-4y-36=0$	A1
		(4)
		(7 marks)

(a)(i) **Mark (i) and (ii) together**

M1 Attempts to find the radius. Score for the values embedded in the expression e.g. $\sqrt{(8-2)^2 + (-3-5)^2}$ or $\sqrt{(2-8)^2 + (5+3)^2}$. They must be attempting to find the difference in the x coordinates and the difference in the y coordinates.

A1 ($r =$) 10 ignore any units ± 10 is A0 Do not accept $\sqrt{100}$ for this mark. isw once a correct answer is seen.

(ii) **Note that if M0 is scored in (a)(i) then this is A0 in (ii)**

A1ft $(x-2)^2 + (y-5)^2 = "100"$ o.e. e.g. $(x-2)^2 + (y-5)^2 = "10^2"$ or $x^2 + y^2 - 4x - 10y = "71"$
Follow through on their positive single value of r . isw once a correct equation is seen.

(b)

B1 Gradient between centre and P is $-\frac{4}{3}$ (may be implied by their gradient of the tangent)

M1 Attempts to find the negative reciprocal for their gradient. May be embedded within their equation of the tangent.

M1 Attempts to find the equation of the tangent at P using a changed gradient (cannot just be a stated gradient of the tangent which is incorrect and does not follow from any earlier work) and $(8, -3)$

Score for $y+3 = \frac{3}{4}(x-8)$. The coordinates $(8, -3)$ must be correctly placed.

If they use $y = mx + c$ they must proceed as far as $c = \dots$

A1 $3x-4y-36=0$ o.e. (provided all the coefficients are integers and all terms are on one side of the equation) isw once a correct equation is seen.



Alt (b)**Alternative method using differentiation**

B1 $\frac{dy}{dx} = \frac{4-2x}{2y-10}$ o.e. such as $2(x-2) + 2(y-5)\frac{dy}{dx} = 0$ or

e.g. $y = (100 - (x-2)^2)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = -(x-2)(100 - (x-2)^2)^{-\frac{1}{2}}$ (may be implied by $\frac{3}{4}$)

M1 Attempts to find the gradient of the tangent at P using their $\frac{dy}{dx}$ and the given coordinate(s).

Do not be concerned with poorly differentiated equations for this mark and just look for the x (and possibly y) coordinate(s) to be substituted into a differential equation leading to a value for the gradient. Condone sign slips substituting in.

M1 As above in main scheme notes

A1 As above in main scheme notes

Alternative method using formula to find the equation of the tangent at a point on a circle

Using the formula $(x-a)(x_1-a) + (y-b)(y_1-b) = r^2$ with centre (a,b) and point (x_1, y_1)

B1 $(x-2)(x_1-2) + (y-5)(y_1-5) \dots$ (may be implied by further work)

M1 Sets equal to their radius: $(x-2)(x_1-2) + (y-5)(y_1-5) = "100"$

M1 Uses the point P and sets equal to their radius: $(x-2)(8-2) + (y-5)(-3-5) \dots = "100"$

A1 $3x - 4y - 36 = 0$ oe (provided all the coefficients are integers and all terms are on one side of the equation)



Question Number	Scheme	Marks
4a	3^5 or 243	B1
		(1)
bi	$(B=)5 \times 3^4 p (=405p) \quad (D=)10 \times 3^2 p^3 (=90p^3)$ $B = 18D \Rightarrow 405p = 18 \times 90p^3 \Rightarrow p^2 = \frac{1}{4} \Rightarrow p = -\frac{1}{2}$	M1A1 M1A1
ii	$(C=)^5C_2 \times 3^3 \times \left(-\frac{1}{2}\right)^2 = \frac{135}{2}$	M1A1
		(6)
		(7 marks)

(a) **Mark (a) and (b) together**

B1 3^5 or 243 seen. Do not award if it is outside the bracket as $3^5 \left(1 + \frac{px}{3}\right)^5$, but allow if some of the terms of the binomial expansion are found and 243 is the constant term.
e.g. $243 + 405px + \dots$ Do not accept e.g. ${}^5C_0 \times 243 \times (px)^0$ for this mark.

(b)(i)

M1 $(B=)5 \times 3^4 p (=405p)$ or $(D=)10 \times 3^2 p^3 (=90p^3)$ (simplified or unsimplified including unprocessed indices).

The coefficients of B or D may be embedded within the binomial expansion such as

$\dots + 5 \times 3^4 px + \dots + 10 \times 3^2 (px)^3$. Also allow e.g. $\binom{5}{1} 3^4 p$ or $\binom{5}{3} 3^2 p^3$ or ${}^5C_1 3^4 p$ or ${}^5C_3 3^2 p^3$

A1 $(B=)5 \times 3^4 p (=405p)$ and $(D=)10 \times 3^2 p^3 (=90p^3)$ (simplified or unsimplified) Allow this mark if the coefficients have not been isolated from x and x^3 , but the brackets around (px) and $(px)^3$ must be correctly removed. Binomial notation such as ${}^5C_1$ or ${}^5C_3$ must have been evaluated.

M1 Uses " $B=18D$ " to form a cubic equation in p and attempts to take out a factor of p or divide by p leading to $p^2 = \dots$ (or may be implied by their value of p). " $B=D$ " is M0

Condone equations involving x and x^3 as long as they are removed as they proceed to $p^2 = \dots$

Note that equations may have been simplified before they are formed by cancelling factors

e.g. $3^2, 3, 5$ or p so look carefully at equations which are acceptable such as $5 \times 3^2 p = 180p^3$

or $9 = 36p^2 \Rightarrow p^2 = \dots$ (condone if negative)

Note $405p = 90p^3 \Rightarrow p = \pm \frac{3\sqrt{2}}{2}$ **scores M1A1M0A0 in (b)(i)** (did not use " $B=18D$ ")

They can proceed in (b)(ii) to score M1A0.

A1 $p = -\frac{1}{2}$ only following correct coefficients for B and D (the positive root and $p = 0$ must

be rejected if found / a clear choice of $p = -\frac{1}{2}$) Watch

out for notation rejecting the positive root such as:

$$\begin{array}{l}
 p^2 = \frac{1}{4} \\
 p_1 = \frac{1}{2}, p_2 = -\frac{1}{2} \\
 p < 0
 \end{array}$$

Can only be scored provided all previous marks have been awarded.

Note if the coefficients of B and D are incorrect leading to $p = -\frac{1}{2}$ then this is max M0A0M1A0 in (b) and M1A0 in (c)

Alternative method (b)(i)

If the expansion is written as $243 \left(1 + 5 \left(\frac{px}{3} \right) + \frac{5 \times 4}{2!} \left(\frac{px}{3} \right)^2 + \frac{5 \times 4 \times 3}{3!} \left(\frac{px}{3} \right)^3 \right)$ then 243 is not required to find p as this will cancel from both sides when setting " $B = 18D$ ".

M1 $(B =) (243 \times) 5 \left(\frac{p}{3} \right)$ or $(D =) (243 \times) \frac{5 \times 4 \times 3}{3!} \left(\frac{p}{3} \right)^3$ (unsimplified with or without the 243)

A1 $(B =) (243 \times) 5 \left(\frac{p}{3} \right)$ and $(D =) (243 \times) \frac{5 \times 4 \times 3}{3!} \left(\frac{p}{3} \right)^3$ (unsimplified either both with or both without the 243 included). Allow this mark if the coefficients have not been isolated from x and x^3 , but the brackets around $\left(\frac{px}{3} \right)$ and $\left(\frac{px}{3} \right)^3$ must be correctly removed. Any binomial notation would now have to be evaluated as in the main scheme notes.

M1 As above in main scheme notes

A1 As above in main scheme notes

(ii)

M1 Uses their value of p^2 , p or $|p|$ and either attempts ${}^5C_2 \times 3^3 p^2$ to find a value for C , or attempts to find ${}^5C_2 \times 3^3 p^2 x^2$ and then proceeds to identify the value for C .

Condone values for p where x had not been removed from (i) e.g. $\pm \frac{1}{2}x$ being substituted in provided a value without x is identified/extracted in (ii)

Condone a slip in calculating ${}^5C_2$ and allow this mark if they substitute both values of p into the expression and find two values for C .

The method mark may be implied by the value found using their p

A1 $\frac{135}{2}$ o.e. e.g. 67.5 or $67\frac{1}{2}$ or $\frac{270}{4}$ following a correctly found $p = \pm \frac{1}{2}$ or $p^2 = \frac{1}{4}$

Condone where both values of p are used provided they both give the correct answer.

Do not accept e.g. $\frac{135}{2}x^2$ unless the coefficient is clearly identified



Question Number	Scheme	Marks
5	$\log_2 16x + \log_2 (x+1) = 3 + \log_2 (x+6)$ $\log_2 a = b \Rightarrow 2^b = a$ $\log_2 16x(x+1) = \log_2 8(x+6)$ $2x^2 + x - 6 = 0$ $(2x-3)(x+2) = 0$ $x = \frac{3}{2} \text{ only}$	 B1 M1 A1 dM1 A1cso (5 marks)

Note a correct answer with no working is 0 marks

B1 Correct use of $\log_2 a = b \Rightarrow 2^b = a$ This will usually result from either $\log_2 8 = 3$ or $\log_2 16 = 4$ being stated or seen applied.

e.g. $\log_2 \left(\frac{16x(x+1)}{x+6} \right) = 3 \Rightarrow \frac{16x(x+1)}{x+6} = 8$

May be implied by e.g. $\log_2 16x(x+1) = \log_2 8(x+6)$ or $\log_2 \left(\frac{x(x+1)}{x+6} \right) = -1$ o.e.

M1 Applies the addition (or subtraction) laws of logs at least once correctly. Condone invisible brackets. e.g. $\log_2 16x^2 + 16x$
This mark can be scored for expressions where the law has been applied correctly and does not need to be a full equation e.g. $\log_2 \left(\frac{x+6}{x+1} \right)$ scores M1. Do not withhold this mark if they subsequently apply a law incorrectly.

e.g. $\log_2 16x(x+1) = 3 + \log_2 x + \log_2 6$ still scores M1

Also allow the mark to be scored despite an error dealing with the 3 when writing it as a log e.g. $3 + \log_2 (x+6) = \log_2 6 + \log_2 (x+6) = \log_2 6(x+6)$

A1 $2x^2 + x - 6 = 0$ or equivalent 3TQ (quite often $16x^2 + 8x - 48 = 0$) Terms must be collected but do not need to be collected on the same side. Condone the omission of $= 0$. Can only be scored from correct working.

dM1 Attempts to solve their 3TQ via any valid method including via a calculator (you may need to check this). Condone complex roots. It is dependent on the previous method mark.

A1cso $\frac{3}{2}$ o.e. only (-2 must be rejected if found)

This mark can only be scored if a correct quadratic is achieved (following correct log work)



Question Number	Scheme	Marks
3(a)	$2(-3)^3 - (-3)^2 + A(-3) + B = 55$ or e.g. $-54 - 9 - 3A + B = 55$	M1
	$-54 - 9 - 3A + B = 55$ $\Rightarrow 3A - B = -118^*$	A1*
		(2)

(a)

M1: Attempts $f(-3) = 55$. The -3 embedded in the expression set = 55 is sufficient.Condone missing brackets e.g. $2 - 3^3 - -3^2 + A(-3) + B = 55$ Note that $2(-3)^3 - (3)^2 + A(-3) + B = 55$ is acceptable.May be implied by further work. If -3 embedded in the expression is not seen, condone slips in their evaluation provided there is still evidence that the intention was to substitute in -3 and set = 55 but $f(3) = 55$ is M0**A1*:** Completes proof with at least one intermediate simplified and correct line that is not the final line such as $-54 - 9 - 3A + B = 55$ or $118 = -3A + B$. Any incorrect lines written after the start of their 'proof' should be scored A0*Must be the given equation and not e.g. $B = 3A + 118$ unless the correct equation is seen previously but condone $-B + 3A = -118$ $2(-3)^3 - (-3)^2 + A(-3) + B = 55 \Rightarrow 3A - B = -118$ scores M1A0

Tabular or division methods may be seen. The question does not demand use of the factor or remainder theorem so these methods should be rewarded.

The M1 would be scored for a full attempt and A1* for a correct proof.

If you cannot see what they are doing and find it hard to award, then use review.

e.g. long division:

$$\begin{array}{r}
 2x^2 - 7x + A + 21 \\
 x + 3 \overline{) 2x^3 - x^2 + Ax + B} \\
 \underline{2x^3 + 6x^2} \\
 -7x^2 + Ax + B \\
 \underline{-7x^2 - 21x} \\
 (A + 21)x + B \\
 \underline{(A + 21)x + 3(A + 21)} \\
 B - 3A - 63 = 55
 \end{array}$$

Score M1 for obtaining a quotient $\dots x^2 \pm \dots x \pm \dots A + \dots$ and a remainder $\dots A \pm \dots B \pm \dots$ set = 55 where “...” are non-zero constants. Then A1* for a fully correct proof.

e.g. tabular method:

	$2x^2$	$-7x$	$21 + A$
x	$2x^3$	$-7x^2$	$(21 + A)x$
3	$6x^2$	$-21x$	$63 + 3A$

So $B - (63 + 3A) = 55$ etc.Score M1 for obtaining a constant $\dots A + \dots$ where ... are non-zero constants and then sets $B - \text{their constant} = 55$. Then A1* for a fully correct proof.

Question	Scheme	Marks
4	$\log_4 a + \log_4 b = \log_4 ab$ or $\log_4 "ab" = 3 \Rightarrow "ab" = 4^3$	M1
	$ab = K, a - b = 8 \Rightarrow a(a - 8) = K$ or $(b + 8)b = K$	dM1
	$a^2 - 8a - 64 = 0$ or $b^2 + 8b - 64 = 0$	A1
	$a = \frac{-(-8) + \sqrt{(-8)^2 - 4 \times 1 \times -64}}{2 \times 1} = \dots$ or $b = \frac{-8 + \sqrt{8^2 - 4 \times 1 \times -64}}{2 \times 1} = \dots$	M1
	$a = '4 + 4\sqrt{5}' \Rightarrow b = -4 + 4\sqrt{5}$	M1
	$a = 4 + 4\sqrt{5}$ and $b = -4 + 4\sqrt{5}$ and no other solutions.	A1
		(6)
(6 marks)		
Notes:		
M1: Correct addition law applied (may be implied) or undoes a log equation correctly, or replaces the 3 by $\log_4 4^3$ or The addition law may be seen after substitution from the first equation e.g. $a - b = 8 \Rightarrow a = b + 8 \Rightarrow \log_4 a + \log_4 b = \log_4 (b + 8) + \log_4 b = \log_4 b(b + 8)$ Condone $\log_4 (b + 8) + \log_4 b = \log_4 b^2 + 8b$ dM1: Removes logs correctly and proceeds from an equation of the form $ab = K$ and the given equation $a - b = 8$ to a quadratic equation in either a or b. A1: Correct quadratic equation. Brackets must be expanded but terms not necessarily all on one side. The “=0” may be implied by their attempt to solve. M1: Solves a 3TQ in a or b to obtain a positive solution. There is no need to see the negative solution in their working. M1: Uses their a to find b or vice versa. This depends on having solved a 3TQ earlier and obtained a root of the form $p + q\sqrt{r}$, $p, q, r \neq 0$ to obtain another root of a similar form i.e. not a decimal. The value of a or b does not necessarily have to be positive for this mark. Note that having found a or b they may repeat the process above to find the other value which is acceptable. A1: Both of $a = 4 + 4\sqrt{5}$ and $b = -4 + 4\sqrt{5}$ simplified, with no other solutions. Apply isw once correct answers are seen, e.g. if they subsequently go into decimals.		

Question Number	Scheme	Marks
4(i)	E.g. $2 = \log_3 9$ Eg $\log_3(4x) - \log_3(5x+7) = \log_3 \frac{4x}{5x+7}$	M1
	E.g. $36x = 5x+7$ $\frac{4x}{5x+7} = \frac{1}{9}$	A1
	$x = \frac{7}{31}$	A1
		(3)
(ii)	$\left(\sum_{r=1}^2 \log_a y^r\right) \log_a y + \log_a y^2$ or $\left(\sum_{r=1}^2 (\log_a y)^r\right) \log_a y + (\log_a y)^2$	B1
	$\log_a y + \log_a y^2 = \log_a y + (\log_a y)^2 \Rightarrow 2 \log_a y - (\log_a y)^2 = 0$ $\Rightarrow \log_a y(2 - \log_a y) = 0 \Rightarrow \log_a y = 2$	M1
	$y = a^2$	A1
		(3)
		Total 6

(i) Answer on its own with no working score 0 marks.

M1 Demonstrates at least one correct law of logarithms.

$$\text{e.g. } 2 = \log_3 9 \text{ or } \log_3(4x) = \log_3 4 + \log_3 x \text{ or } \log_3(4x) - \log_3(5x+7) = \log_3 \frac{4x}{5x+7}$$

$$\log_3(5x+7) - \log_3(4x) = \log_3 \frac{5x+7}{4x}$$

(Do not be concerned with slips on the 2 if manipulating the equation)

$$\text{e.g. } \log_3(4x) + 2 = \log_3(5x+7) \Rightarrow \log_3 \frac{4x}{5x+7} = 2 \text{ scores M1}$$

A1 For a correct equation in any form with no logs eg $4x = \frac{5x+7}{9}$. Must have come from correct log work.A1 $\frac{7}{31}$ only and follows a correct equation with no logs.

Note $\frac{\log_3 5x+7}{\log_3 4x} = -2 \Rightarrow x = \frac{7}{31}$ will only score a maximum of M1A0A0 if they are able to demonstrate at least one correct law of logarithms.

(ii) Answer on its own with no working score 0 marks.B1 For either (or both) summation correct $\log_a y + \log_a y^2$ or $\log_a y + (\log_a y)^2$ or equivalent

Can also be awarded for a correct equation:

$$\text{e.g. } \log_a y^2 = (\log_a y)^2 \text{ or } 2 \log_a y = (\log_a y)^2 \text{ or } 3 \log_a y = \log_a y + (\log_a y)^2$$



Note that there are many parallel solutions in part (a) that can be marked similarly.

Alt I

Using $\sin\theta = \tan\theta\cos\theta$ o.e

$$2\tan\theta + 3\tan\theta\cos\theta = 0 \Rightarrow \tan\theta(2 + 3\cos\theta) = 0$$

In this case the B1 mark is for achieving $\tan\theta = 0 \Rightarrow \theta = 180^\circ, 360^\circ$ Condone 0 appearing as a solution.

Alt II

Multiplying throughout by $\sin\theta$ and using $\sin^2\theta + \cos^2\theta = 1$ eventually leads to a cubic $\cos\theta$ which has roots of $\cos\theta = -\frac{2}{3}$ and $\cos\theta = 1$. Allow use of a calculator to solve cubic equations, etc.

$$2\frac{\sin\theta}{\cos\theta} + 3\sin\theta = 0 \xrightarrow{\times \sin\theta} 2\frac{\sin^2\theta}{\cos\theta} + 3\sin^2\theta = 0 \rightarrow 2\frac{(1-\cos^2\theta)}{\cos\theta} + 3(1-\cos^2\theta) = 0$$

$$\rightarrow 3\cos^3\theta + 2\cos^2\theta - 3\cos\theta - 2 = 0 \rightarrow \cos\theta = -\frac{2}{3}, \cos\theta = \pm 1$$

M1: For $\tan\theta = \frac{\sin\theta}{\cos\theta}$

dM1: for $\cos\theta = k, |k| < 1, k \neq 0$

A1: For $\cos\theta = -\frac{2}{3}$ and a solution as in the main scheme

A1: : For $\cos\theta = -\frac{2}{3}$ and both solutions as in the main scheme

B1 Achieves $\cos\theta = \pm 1$ followed by $\theta = 180^\circ, 360^\circ$. Condone 0 appearing as a solution

Alt III

Multiplying throughout by $\cos\theta$ and using $\sin^2\theta + \cos^2\theta = 1$ eventually leads to a quartic in $\sin\theta$ which has roots $\sin\theta = \pm\frac{\sqrt{5}}{3}$ and $\sin\theta = 0$

$$2\sin\theta = -3\sin\theta\cos\theta \xrightarrow{\text{SQUARING}} 4\sin^2\theta = 9\sin^2\theta\cos^2\theta$$

$$\rightarrow 4\sin^2\theta = 9\sin^2\theta(1-\sin^2\theta) \rightarrow 9\sin^4\theta - 5\sin^2\theta = 0$$

$$\rightarrow \sin^2\theta = \frac{5}{9}, \sin\theta = 0$$

M1: For $\tan\theta = \frac{\sin\theta}{\cos\theta}$

dM1: For $\sin^2\theta = k$ or $\sin\theta = k, |k| < 1, k \neq 0$

A1: For $\sin\theta = \frac{\sqrt{5}}{3}, \sin\theta = -\frac{\sqrt{5}}{3}$ or $\sin^2\theta = \frac{5}{9}$ and a correct solution as in the main scheme

A1: For $\sin\theta = \frac{\sqrt{5}}{3}, \sin\theta = -\frac{\sqrt{5}}{3}$ or $\sin^2\theta = \frac{5}{9}$ and both solutions (only) in the main scheme

B1: Achieves $\sin\theta = 0$ followed by $\theta = 180^\circ, 360^\circ$ Condone 0 appearing as a solution

.....
The solution must be obtained from correct working.
Calculator solutions like $2\tan\theta + 3\sin\theta = 0 \Rightarrow \theta = 131.8^\circ$ and 228.2° score no marks in (a)
Even the solution $\theta = 180^\circ, 360^\circ$ cannot score the B1 mark if it is not preceded by $\sin\theta = 0$ or $\tan\theta = 0$
(b) is hence or otherwise so must follow their solution in (a) or be a restart.



Question Number	Scheme	Marks
4.(a)	$f(x) = 4x^3 + ax^2 - 29x + b$ Sets $f\left(-\frac{1}{2}\right) = 0 \rightarrow 4 \times \left(-\frac{1}{2}\right)^3 + a \times \left(-\frac{1}{2}\right)^2 - 29 \times \left(-\frac{1}{2}\right) + b = 0$ $\Rightarrow \frac{1}{4}a + b + 14 = 0 \Rightarrow a + 4b = -56^*$	M1 A1*
		(2)
(b)	Sets $f(2) = -25 \rightarrow 4 \times 2^3 + a \times 2^2 - 29 \times 2 + b = -25$ $4a + b = 1$	M1 A1
		(2)
(c)	(i) Solves $a + 4b = -56$ with their $4a + b = 1 \Rightarrow a = \dots, b = \dots$ $a = 4, b = -15$	M1 A1
	(ii) $4x^3 + 4x^2 - 29x - 15 = (2x+1)(2x^2 + x - 15)$ $= (2x+1)(2x-5)(x+3)$ cs0	M1, A1 A1
		(5) (9 marks)

(a)

M1: Attempts to set $f\left(-\frac{1}{2}\right) = 0 \rightarrow$ equation in a and b Condone slips (e.g. on signs) but $f\left(\frac{1}{2}\right) = 0$ is M0.

Score when you see embedded values within the equation or two correct terms on the lhs of the equation.

A1*: Completes proof with at least one intermediate simplified and correct line such as

$\frac{1}{4}a + b + 14 = 0$. Simplifying each term is sufficient $-\frac{1}{2} + \frac{1}{4}a + \frac{29}{2} + b = 0$. Any incorrect lines written after the start of their 'proof' should be scored A0*

Solutions where the $= 0$ is implied but not stated should be awarded M1 A0*

E.g. $f\left(-\frac{1}{2}\right) = 4 \times \left(-\frac{1}{2}\right)^3 + a \times \left(-\frac{1}{2}\right)^2 - 29 \times \left(-\frac{1}{2}\right) + b = \frac{1}{4}a + b + 14$. So $\frac{1}{4}a + b = -14 \Rightarrow a + 4b = -56$

(b)

M1: Attempts to set $f(2) = -25 \rightarrow$ equation in a and b Condone slips, e.g. sign slips or setting $f(2) = 25$

Setting $f(-2) = -25$ is not a slip and should be marked M0.

A1: $4a + b = 1$ or other simplified equivalent such as $a + \frac{1}{4}b - \frac{1}{4} = 0$. The constant terms must be collected.

Note that this is not a given answer so allow recovery.

(c)(i)

M1: Attempts to solve $a + 4b = -56$ (allowing for slips) with their $4a + b = 1 \Rightarrow a = \dots, b = \dots$

Award as long as values appear for both a and b , so allow these just written down (say from a calculator)
There is no need their calculations

A1: Both values correct: $a = 4, b = -15$. Both just written down without working scores M1 A1 provided (b) is correct



(c)(ii)

M1: Attempt to divide or factorise out $(2x+1)$ from their $4x^3 + 4x^2 - 29x - 15$ or multiple thereofFor factorisation look for correct first and last terms E.g. $4x^3 + "a"x^2 - 29x + "b" = (2x+1)(2x^2 + kx \pm "b")$ For division look for a quadratic quotient of the form $2x^2 + px + q$ with some "correct" work to find the term in x for example. Condone sign slip when attempting to find p A1: Correct quadratic factor $2x^2 + x - 15$ A1: cso $(2x+1)(2x-5)(x+3)$. Note that M1, A1 must have been awarded and the full product must be seen. It is not acceptable to see just the factors.....
The question states "Hence" so there is an expectation that they use the factor of $(2x+1)$ which is given.Solutions that just state $4x^3 + 4x^2 - 29x - 15 = (2x+1)(2x-5)(x+3)$ score M0 and therefore A0 A0

If they wish to start with another factor they must prove it is a factor first.

E.g. $f(-3) = 4 \times -27 + 4 \times 9 - 29 \times -3 - 15 = 0 \Rightarrow (x+3)$ is a factor followed by division by $(x+3)$
Tabular or methods of division may be seen in (a) and (b). The question does not demand use of the factor or remainder theorem so these methods should be rewarded.

The M1 would be scored for a full attempt and A1 for a correct answer or proof.

If you cannot see what they are doing and find it hard to award, then put in review please

Example of a tabular version for (a):

 $(2x+1)$ is a factor of $f(x) = 4x^3 + ax^2 - 29x + b$

	$2x^2$	$(-29-2b)x$	b
$2x$	$4x^3$	$2(-29-2b)x^2$	$2bx$
1	$2x^2$	$(-29-2b)x$	b

Then states that $2(-29-2b) + 2 = a \Rightarrow$

Example of a long division method for (b).

When $f(x) = 4x^3 + ax^2 - 29x + b$ is divided by $(x-2)$ the remainder is -25

$$\begin{array}{r}
 4x^2 + (a+8)x + (2a-13) \\
 x-2 \overline{) 4x^3 + ax^2 - 29x + b} \\
 \underline{4x^3 - 8x^2} \\
 (a+8)x^2 - 29x \\
 \underline{(a+8)x^2 - (2a+16)x} \\
 (2a-13)x + b \\
 \underline{(2a-13)x + (-4a+26)} \\
 b+4a-26
 \end{array}$$

Remainder is -25 so $b+4a-26 = -25 \Rightarrow b+4a = 1$ 

Question Number	Scheme	Marks
5. (i)	$3^a = 70 \Rightarrow a \log 3 = \log 70$ $\Rightarrow a = \frac{\log 70}{\log 3} = 3.867$	M1 A1 (2)
(ii)	$4 = \log_3 81$ or $3 \log_3 b = \log_3 b^3$ or $\log_3 5b = \log_3 5 + \log_3 b$ $4 + 3 \log_3 b = \log_3 5b \Rightarrow \log_3 81b^3 = \log_3 5b$ $\Rightarrow 81b^3 = 5b \Rightarrow b = \dots$ $\Rightarrow b = \sqrt{\frac{5}{81}}$	B1 M1 dM1 A1 (4) (6 marks)

This question demands working from the candidates so answers cannot appear without appropriate methods

(i)

M1: Takes (the same) log of both sides and uses the power law.

Alternatively writes $a = \log_3 70$ with the correct base. But $a = \log 70$ is M0

A1: CSO. 3.867 following a suitable intermediate line. This is not awrt but isw after sight of 3.867

Cannot be achieved via Trial and Improvement

(ii) Note that this was marked **M1 A1 dM1 A1**. It is now being marked **B1 M1 dM1 A1**

B1: One correct log law used on a term **of the initial equation**.

Can be achieved without the bases being present

Look for $4 = \log_3 81$ or $3 \log_3 b = \log_3 b^3$ or $\log_3 5b = \log_3 5 + \log_3 b$

M1: Correctly combines two terms **of the initial equation** to form a log equation of the correct form

$\log_3 \dots = \log_3 \dots$ or $\log_3 \dots = \pm 4$.

Possible answers are; $\log_3 81b^3 = \log_3 5b$, $\log_3 81b^2 = \log_3 5$, $\log_3 \left(\frac{b^3}{5b} \right) = -4$

For this mark you can condone a slip on the sign of the 4.

E.g. $4 + 3 \log_3 b = \log_3 5b \Rightarrow \log_3 b^3 - \log_3 5b = 4 \Rightarrow \log_3 \frac{b^3}{5b} = 4$

dM1: Removes the logs correctly and proceeds to a value for b using a correct method.

Must follow M1 and an equation of the form $\log_3 \dots = \log_3 \dots$ or $\log_3 \dots = \pm 4$

A1: $b = \sqrt{\frac{5}{81}}$ or exact equivalent such as $b = \frac{1}{9}\sqrt{5}$. The negative (and 0) solution, if found, must be discarded.

.....
Correct answer from incorrect working: Watch for solutions like the following

$$4 + 3 \log_3 b = \log_3 5b \Rightarrow 4 = \log_3 5b - \log_3 b^3 \Rightarrow 4 = \frac{\log_3 5b}{\log_3 b^3} \Rightarrow 81 = \frac{5b}{b^3} \Rightarrow b = \sqrt{\frac{5}{81}}$$

Score B1 M0 dM0 A0



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1. (a)	$\left(1 - \frac{1}{6}x\right)^9 = 1 + 9\left(-\frac{1}{6}x\right) + \frac{9 \times 8}{2}\left(-\frac{1}{6}x\right)^2 + \frac{9 \times 8 \times 7}{3!}\left(-\frac{1}{6}x\right)^3 + \dots$ $= 1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3$	M1 A1, A1 (3)
(b)	$(10x + 3)\left(1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3\right)$ Coefficient of $x^3 = 10 \times 1 + 3 \times -\frac{7}{18} = \frac{53}{6}$	M1 A1 (2) (5 marks)

(a)

M1: For an attempt at the binomial expansion. Score for a correct attempt at any of term 2, 3 or 4.

Accept sight of ${}^9C_1\left(\pm\frac{1}{6}x\right)^1$ or ${}^9C_2\left(\pm\frac{1}{6}x\right)^2$ or ${}^9C_3\left(\pm\frac{1}{6}x\right)^3$ condoning the omission of brackets.

Coefficients may appear from Pascal's triangle but should be correct for the term.

A1: For any two simplified terms of $-\frac{3}{2}x + x^2 - \frac{7}{18}x^3$

A1: For $1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3$ ignoring terms with greater powers. Allow as a list, allow $+\frac{3}{2}x$ etc but the fractions must be simplified. Accept $1x^2$. ISW following a correct expression (e.g. if multiply through by 2). If given as decimals they must be exact (recurring if appropriate).

(b)

M1: For attempting to find $10b \pm 3c$ combined to a single term for their $(10x + 3)\left(1 + ax + bx^2 + cx^3 + \dots\right)$

A1: cao for $\frac{53}{6}$ but condone $\frac{53}{6}x^3$ and isw. Accept embedded in an expansion with other terms as long as simplified to correct single x^3 term. If given as a decimal must be exact recurring.

Question Number	Scheme	Marks
2.(a)	$a + 5d = 2$ and $\frac{10}{2}\{2a + 9d\} = -80$ Solves simultaneously $\Rightarrow d = 20, a = -98$	B1 B1 M1 A1 (4)
(b)	Attempts $\frac{n}{2}\{2 \times -98 + (n-1) \times 20\} \dots 8000$ $5n^2 - 54n - 4000 \dots 0 \Rightarrow n = (34.2)$ $n = 35$	M1 dM1 A1 (3) (7 marks)

(a)

B1: One of $a + 5d = 2$ or $\frac{10}{2}\{2a + 9d\} = -80$

B1: Both equations correct. May be un-simplified (isw after correct equations seen).

M1: Solves their two equations in a and d (one of which must be correct) and finds at least a or d . There may be slips, but score for any attempt that reaches a value provided at least one equation was correct and both attempted. Implied by correct answers following correct equations stated.

A1: For $d = 20, a = -98$

Alt

B1: Correct formula in d only using 6th and 10th term $\frac{10}{2}((2 - 5d) + (2 + 4d)) = -80$

B1: $d = 20$

M1: Attempts to find a using $a + 5d = 2$

A1: Correct a

(b)

Note: allow all marks in (b) with any equality or inequality used throughout.

M1: Attempts to use $\frac{n}{2}\{2 \times -98 + (n-1) \times 20\} \dots 8000$ with their a and d . Accept if set equal to 8001. NB If 800 or 80000 is used, allow the M (misread).

dM1: Sets up and **solves** a 3 term quadratic equation in n . Accept if just the positive root is given, but at least one positive root must be obtained.

Allow use of calculator /formulae (may need to check values if no method shown).

May be implied by a rounded or truncated positive integer value following a suitable equation having been set up.

FYI the correct equation is $5n^2 - 54n - 4000 \dots 0$

A1: $(n =) 35$ stated following a correct equation formed (allowing for incorrect inequality).

Alt by trial and improvement

M1: Attempts to use $\frac{n}{2}\{2 \times -98 + (n-1) \times 20\}$ with their a and d and integer values of n either side of the sum being 8000

dM1: Two consecutive values for n giving results either side of 8000 attempted.

A1: $n = 35$ stated following correct working with both $n=34$ and $n= 35$ having been seen.



Question Number	Scheme	Marks
5 (a)	$D = 8 + 5 \sin\left(\frac{\pi \times 2}{6} + 3\right) = 4.07$	B1
		(1)
(b)	(b) $6 = 8 + 5 \sin\left(\frac{\pi t}{6} + 3\right) \Rightarrow \sin\left(\frac{\pi t}{6} + 3\right) = -\frac{2}{5}$ $\Rightarrow \left(\frac{\pi t}{6} + 3\right) = \arcsin\left(-\frac{2}{5}\right) = \text{Any of } 3.55, 5.87, 9.84, 12.2$ $\Rightarrow t = \text{Any of } \frac{6(3.55-3)}{\pi}, \frac{6(5.87-3)}{\pi}, \frac{6(9.84-3)}{\pi}, \frac{6(12.2-3)}{\pi}$ 13:04 or 1:04 pm	M1 , A1 dM1 ddM1 A1
		(5)
		Total 6

(a)

B1: Scored for sight of $8 + 5 \sin\left(\frac{\pi \times 2}{6} + 3\right)$ o.e followed by 4.06, 4.07 or awrt 4.07. Units can be omitted

(b) It is acceptable to set $x = \frac{\pi t}{6} + 3$ which allows access to the first three marks

M1: Substitutes $D = 6$ and proceeds to $\sin\left(\frac{\pi t}{6} + 3\right) = k$

A1: $\sin\left(\frac{\pi t}{6} + 3\right) = -\frac{2}{5}$. o.e

dM1: Takes arcsink where $k < 0$ and achieves any **correct positive value** for their arcsink

Any of 3.55, 5.87, 9.84, 12.2 are acceptable where $k = -\frac{2}{5}$. Follow through on their $-1 < k < 0$

ddM1: Full and complete method to find a positive value of t

It is dependent upon both previous M's **AND** $k = -\frac{2}{5}$

Score for $t = \text{Any of } \frac{6(3.55-3)}{\pi}, \frac{6(5.87-3)}{\pi}, \frac{6(9.84-3)}{\pi}, \frac{6(12.2-3)}{\pi}$

It can be implied for any t value to 3sf,

Score for any t value awrt 1.05/1.06, 5.48/5.49, 13.0/13.1, 17.5/17.6 following M1 A1dM1

A1: 13:04 or 1:04 pm. Condone/allow an answer of 13.03 or 1.03pm.

Condone one hour and 4 minutes after midday

Requires $\arcsin(-0.4) = \text{awrt } 9.84$, $t = \text{awrt } 13.1$ and a time of 13:03 o.e.



n	$x1.02^{(n-1)}$	x5
1	25000	125000
2	25500	127500
3	26010	130050
4	26530	132651
5	27061	135304
6	27602	138010
7	28154	140770
8	28717	143586
9	29291	146457
10	29877	149387
11	30475	152374
12	31084	155422
13	31706	158530
14	32340	161701
	399348	1996742



Question Number	Scheme	Marks
9(a)	$y = x^3 - 5x^2 + 3x + 14 \Rightarrow \frac{dy}{dx} = 3x^2 - 10x + 3 = 0$	M1
	Roots are $3, \frac{1}{3} \Rightarrow$ when $x = 3, y = 3^3 - 5 \times 3^2 + 3 \times 3 + 14 = \dots$	dM1
	Centre is (3, 5)	A1
		(3)
(b)	At A $y = 8$	B1
	$r^2 = (2 - 3)^2 + (8 - 5)^2 (=10)$	M1
	$(x - 3)^2 + (y - 5)^2 = 10$	A1
		(3)
(c)	$\frac{8 - 5}{2 - 3} = \dots (-3)$	M1
	$y - 8 = \frac{1}{3}(x - 2)$	M1
	$y = \frac{1}{3}x + \frac{22}{3} \quad *$	A1*
		(3)
(d)	$\int_0^2 x^3 - 5x^2 + 3x + 14 \, dx = \dots \left(\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 14x \right)$	M1
	Area = $\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 14x \right]_0^2 - \left(\frac{1}{2} \times \left(\frac{22}{3} + 8 \right) \times 2 \right) = \dots$	dM1
	$\frac{1}{4} \times 16 - \frac{5}{3} \times 8 + \frac{3}{2} \times 4 + 14 \times 2 - \frac{46}{3}$	
	$\frac{74}{3} - \frac{46}{3} = \frac{28}{3}$	A1
		(3)
		(12 marks)

(a)

M1: Differentiates the equation of the curve and sets equal to 0 which may be implied by their attempt to solve below. Do not credit this differentiation in any other parts of the question.

For the differentiation, look for at least 2 of $x^3 \rightarrow \dots x^2, -5x^2 \rightarrow \dots x, 3x \rightarrow 3$

dM1: Solves a 3 term quadratic equation by any valid means (including a calculator) and substitutes one of their roots in the original equation to find the y coordinate. **Depends on the first mark.**

A1: Correct coordinates (3, 5) or e.g. $x = 3, y = 5$ (Ignore any work with e.g. $x = \frac{1}{3}$ and ignore any work attempting to prove maximum/minimum points – just look for correct coordinates identified)

(b)

B1: y coordinate at A is 8 (Allow this to score anywhere)

M1: Attempts to find the radius of the circle (or radius²) using (2, 8) and their minimum point.

Score for $(2 - 3)^2 + (8 - 5)^2$ or equivalent correct work for their coordinates of T and A.

You can ignore what they call it e.g. condone $\text{radius} = (2 - 3)^2 + (8 - 5)^2$

A1: $(x - 3)^2 + (y - 5)^2 = 10$ oe e.g. $(x - 3)^2 + (y - 5)^2 = (\sqrt{10})^2, \sqrt{(x - 3)^2 + (y - 5)^2} = \sqrt{10}$



(c)

M1: Attempts to find the gradient between A and T using their coordinates. Note that as the equation of the tangent is given we do not accept -3 just written down.

M1: Attempts to find the equation of the straight line using $x = 2$, their y value at A and the negative reciprocal of what they think is the gradient of AT .

A1*: $y = \frac{1}{3}x + \frac{22}{3}$ with no errors and both previous method marks scored.

Alternative for first **M**: $(x-3)^2 + (y-5)^2 = 10 \Rightarrow 2(x-3) + 2(y-5)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{3-x}{y-5} = \frac{3-2}{8-5} = \frac{1}{3}$

(d)

M1: Attempts to integrate the curve. Award for a power increasing by 1 on one of the terms.

dM1: A correct method to find the shaded area:

- Substitutes in 2 (and 0) into the integrated function and subtracts either way round. May not see 0 substituted in.
- Attempts to find the shaded area by subtracting the area of the trapezium

$\frac{1}{2} \times \left(\frac{22}{3} + "8" \right) \times 2$ from their area under the curve. It is dependent on the previous method mark.

It must be a correct method for the area of the trapezium e.g. $\frac{1}{2} \times \left(\frac{22}{3} + \text{their } y \text{ at } A \right) \times 2$

Or e.g. rectangle + triangle: $2 \times \frac{22}{3} + \frac{1}{2} \times 2 \left(\text{their } y \text{ at } A - \frac{22}{3} \right)$

Or by integration $\int_0^2 \left(\frac{1}{3}x + \frac{22}{3} \right) dx = \left[\frac{1}{6}x^2 + \frac{22}{3}x \right]_0^2 = \frac{2}{3} + \frac{44}{3} = \frac{46}{3}$

A1: $\frac{28}{3}$ or exact equivalent cso

Alt(d)

M1: Attempts to integrate (curve – line) or (line – curve).

Award for the power increasing by 1 on one of the terms.

dM1: A correct method to find the shaded area.

Substitutes 2 (and 0) into the integrated function and subtracts either way round:

$$\int_0^2 x^3 - 5x^2 + \frac{8}{3}x + \frac{20}{3} dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{4x^2}{3} + \frac{20x}{3} \right]_0^2 = \frac{2^4}{4} - \frac{5 \times 2^3}{3} + \frac{4 \times 2^2}{3} + \frac{20 \times 2}{3} = \dots$$

$$\begin{aligned} \text{Or e.g. } \int_0^2 x^3 - 5x^2 + 3x + 14 - \frac{1}{3}x - \frac{22}{3} dx &= \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 14x - \frac{x^2}{6} - \frac{22x}{3} \right]_0^2 = \\ &= \frac{2^4}{4} - \frac{5 \times 2^3}{3} + \frac{3 \times 2^2}{2} + 14 \times 2 - \frac{2^2}{6} - \frac{22 \times 2}{3} = \dots \end{aligned}$$

You can condone slips when collecting terms or for a slip with brackets e.g. attempting:

$$\int_0^2 x^3 - 5x^2 + 3x + 14 - \frac{1}{3}x + \frac{22}{3} dx$$

A1: $\frac{28}{3}$ or exact equivalent cso

With otherwise correct work leading to $-\frac{28}{3}$ allow full marks if then made positive.

Question Number	Scheme	Marks
10(i)(a)	$2a$	B1
(b)	$\log_2 \left(\frac{\sqrt{3}}{16} \right) = \log_2 \sqrt{3} - \log_2 16, = \frac{1}{2}a - 4$	M1A1
		(3)

(i)(a)**B1:** $2a$ **(b)**

M1: Uses the subtraction rule for logs to write $\log_2 \left(\frac{\sqrt{3}}{16} \right) = \log_2 \sqrt{3} - \log_2 16$ seen or implied.

A1: $\frac{1}{2}a - 4$

Do not isw here so if they reach $\frac{1}{2}a - 4$ correctly and then say $= a - 8$ score M1A0

(ii) This general guidance will apply to most cases you will come across:

M1: Takes logs (same base) of **both** sides and applies the addition rule for logs on the lhs

See possible rearrangements below which may be seen before the addition rule is applied.

dM1: Correct method to make x the subject. Condone slips in rearrangement but there must be more than one term in x . Award for collecting terms in x on one side and non x terms on the other side, factorising and then dividing. **Dependent on the first method mark.**

B1: The correct power law seen for logs. Allow this mark to score independently so sight of e.g. $\log_2 2^x = x \log_2 2$, $\log_2 3^x = x \log_2 3$, $\log_2 6^x = x \log_2 6$, $\log_2 2^{x+4} = (x+4) \log_2 2$ etc. can score this mark. (Condone e.g. $\log_2 2^{x+4} = x + 4 \log_2 2$ for this mark)

Note that this may follow incorrect work e.g.

$$3^x \times 2^{x+4} = 6 \Rightarrow \log_2 3^x \times \log_2 2^{x+4} = \log_2 6 \Rightarrow x \log_2 3 \times (x+4) \log_2 2^{x+4} = \log_2 6$$

A1: $\frac{a-3}{a+1}$ or e.g. $\frac{3-a}{-a-1}$

Note that candidates are unlikely to be working in anything other than base 2 so you can condone the omission of the base throughout but see 3rd below the main scheme below.

Note that for reference, $\frac{a-3}{a+1} = -0.54741122...$ which may be useful in some circumstances.



(ii)	Examples: $3^x \times 2^{x+4} = 6 \Rightarrow \log_2 3^x + \log_2 2^{x+4} = \log_2 6$ or $3^x \times 2^{x+4} = 6 \Rightarrow 3^x \times 2^x \times 2^4 = 6 \Rightarrow \log_2 3^x + \log_2 2^x + \log_2 2^4 = \log_2 6$ or $3^x \times 2^{x+4} = 6 \Rightarrow 3^x \times 2^{x+3} = 3 \Rightarrow \log_2 3^x + \log_2 2^{x+3} = \log_2 3$ or $3^x \times 2^{x+4} = 6 \Rightarrow 3^x \times 2^x = \frac{3}{8} \Rightarrow \log_2 3^x + \log_2 2^x = \log_2 \frac{3}{8}$ or $3^x \times 2^{x+4} = 6 \Rightarrow 3^{x-1} \times 2^{x+3} = 1 \Rightarrow \log_2 3^{x-1} + \log_2 2^{x+3} = \log_2 1$	M1
	Examples: $x \log_2 3 + (x+4) \log_2 2 = \log_2 6 \Rightarrow x(\log_2 3 + \log_2 2) = \log_2 6 - 4 \Rightarrow x = \dots$ or $x \log_2 3 + x \log_2 2 + 4 = \log_2 6 \Rightarrow x(\log_2 3 + \log_2 2) = \log_2 6 - 4 \Rightarrow x = \dots$ or $x \log_2 3 + (x+3) \log_2 2 = \log_2 3 \Rightarrow x(\log_2 3 + \log_2 2) = \log_2 3 - 3 \log_2 2 \Rightarrow x = \dots$ or $x \log_2 3 + x \log_2 2 = \log_2 \frac{3}{8} \Rightarrow x(\log_2 3 + \log_2 2) = \log_2 \frac{3}{8} \Rightarrow x = \dots$ or $(x-1) \log_2 3 + (x+3) \log_2 2 = 0 \Rightarrow ax + x = a - 3 \Rightarrow x = \dots$	dM1
	$\log_2 a^b = b \log_2 a$	B1
	$x = \frac{a-3}{a+1}$	A1
		(4)
		(7 marks)

Some alternatives you may come across are below:

Alternative not requiring the addition law:

$$3^x \times 2^{x+4} = 6 \Rightarrow 3^x \times 2^x = \frac{3}{8} \Rightarrow \log_2 3^x \times 2^x = \log_2 \frac{3}{8} \Rightarrow \log 6^x = \log_2 \frac{3}{8}$$

$$\log 6^x = \log_2 \frac{3}{8} \Rightarrow x \log_2 6 = \log_2 \frac{3}{8} \Rightarrow x = \dots$$

$$x = \frac{\log_2 \frac{3}{8}}{\log_2 6} = \frac{\log_2 3 - \log_2 8}{\log_2 6} = \frac{a-3}{a+1}$$

Score as:

M1: Divides by 2^4 , takes logs of **both** sides and writes $3^x \times 2^x$ as 6^x

B1: $\log_2 6^x = x \log_2 6$

dM1: Makes x the subject

A1: $\frac{a-3}{a+1}$ or e.g. $\frac{3-a}{-a-1}$



Alternative not requiring logs:

$$a = \log_2 3 \Rightarrow 3 = 2^a$$

$$2^{ax} \times 2^{x+4} = 3 \times 2 = 2^a \times 2$$

$$2^{ax+x+4} = 2^{a+1} \Rightarrow ax + x + 4 = a + 1 \Rightarrow x(a+1) = a - 3 \Rightarrow x = \dots$$

$$x = \frac{a-3}{a+1}$$

Score as:

$$\mathbf{B1:} \quad a = \log_2 3 \Rightarrow 3 = 2^a$$

M1: Attempts to write all terms as powers of 2**dM1:** Combines and equates powers and makes x the subject as in main scheme

$$\mathbf{A1:} \quad \frac{a-3}{a+1} \text{ or e.g. } \frac{3-a}{-a-1}$$

Alternative using change of base:

$$3^x \times 2^{x+4} = 6 \Rightarrow 2^{x+3} = 3^{1-x} \Rightarrow \log 2^{x+3} = \log 3^{1-x}$$

$$\Rightarrow (x+3)\log 2 = (1-x)\log 3$$

$$\Rightarrow (x+3)\frac{\log_2 2}{\log_2 10} = (1-x)\frac{\log_2 3}{\log_2 10} \Rightarrow x+3 = a(1-x) \Rightarrow x(a+1) = a-3 \Rightarrow x = \dots$$

Score as:**M1:** Divides by 2 and 3^x and takes logs of **both** sides

$$\mathbf{B1:} \text{ E.g. } \log 2^{x+3} = (x+3)\log 2 \text{ or } \log 3^{1-x} = (1-x)\log 3$$

dM1: Changes to base 2 correctly and makes x the subject as main scheme

$$\mathbf{A1:} \quad \frac{a-3}{a+1} \text{ or e.g. } \frac{3-a}{-a-1}$$

Alternative using logs base 6:

$$3^x \times 2^{x+4} = 6 \Rightarrow 3^x \times 2^x = \frac{3}{8} \Rightarrow 6^x = \frac{3}{8} \Rightarrow \log_6 6^x = \log_6 \frac{3}{8}$$

$$\log_6 6^x = \log_6 \frac{3}{8} \Rightarrow x \log_6 6 = \log_6 \frac{3}{8}$$

$$x = \log_6 \frac{3}{8} = \frac{\log_2 \frac{3}{8}}{\log_2 6}$$

$$\frac{\log_2 \frac{3}{8}}{\log_2 6} = \frac{\log_2 3 - \log_2 8}{\log_2 3 + \log_2 2} = \frac{a-3}{a+1}$$

M1: Divides by 2^4 , writes $3^x \times 2^x$ as 6^x and takes logs base 6 of both sides

$$\text{which may be implied by e.g. } 6^x = \frac{3}{8} \Rightarrow x = \log_6 \frac{3}{8}$$

B1: For $\log_6 6^x = x \log_6 6$ (may be implied)**dM1:** Changes to log base 2 correctly (and makes x the subject)**A1:** Correct expression

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1(a)	$f(-1) = (-1)^4 + a(-1)^3 - 3(-1)^2 + b(-1) + 5 = 4$	M1
	$1 - a - 3 - b + 5 = 4 \Rightarrow a + b = -1$ *	A1*
		(2)
(b)	$f(2) = (2)^4 + a(2)^3 - 3(2)^2 + b(2) + 5 = -23$	M1
	$\Rightarrow 8a + 2b = -32$ oe (eg $4a + b = -16$)	A1
	$b = -1 - a \Rightarrow 4a - 1 - a = -16 \Rightarrow a = \dots$	dM1
	$a = -5, b = 4$	A1
		(4)
		(6 marks)

(a) NB Do not apply a misread on the equation $a + b = -1$ as it gains no extra marks. They can score a maximum of M1A0 M1A1M1A0 if they use an incorrect equation such as $a + b = 1$.

M1 Attempts to substitute ± 1 into $f(x)$ and set equal to 4 to obtain an equation in a and b . Powers of -1 need not be seen, accept e.g. $1 - a - 3 - b + 5 = 4$. Condone invisible brackets on powers for the M mark, but penalise in the A (if incorrect). Alternatively, attempts long division reaching a remainder which is then set equal to 4. Look for a quotient starting $x^3 + \dots$ and linear remainder in a and b . It may be seen in tabulated form.
Another alternative method: Writes $f(x) = (x+1)(x^3 + \alpha x^2 + \beta x + \gamma) + 4$ (allow with $\gamma = 1$), expands and equates coefficients, and proceeds to eliminate α, β and γ from the equations.

A1* Rearranges the equation with no errors and achieves the given answer with at least one intermediate line with power evaluated. FYI by long division the quotient is $x^3 + (a-1)x^2 + (-a-2)x + a+b+2$ and the remainder is $3-a-b$.
By equating coefficients look for $\gamma + 4 = 5$ (may be implied), $a = \alpha + 1$, $b = \beta + 1$, $-3 = \alpha + \beta$ being solved correctly.

(b)
M1 Attempts to substitute ± 2 into $f(x)$ and set equal to ± 23 to obtain an equation in a and b . Again, accept attempts at long division leading to a remainder linear in a and b (which is $8a + 2b + 9$), which is then set equal to ± 23 , or via equating coefficients (look for at least three equations formed). May have substituted for a or b already.

A1 $8a + 2b = -32$ oe need not be fully gathered but powers should be evaluated. So e.g. accept $16 + 8a - 12 + 2b + 5 = -23$ or accept $16 + 8a - 12 + 2(-a-1) + 5 = -23$ if substitution for b happens first (similar for a). Via equating coefficients, all equations should be correct.

dM1 Attempts to solve simultaneously and achieves a value for a or b . It is dependent on the previous method mark. Allow as long as values for a or b appears after writing out their two equations in both a and b (or solving a linear equation if substitution occurs before remainder theorem). Allow for slips in copying their initial equations.

A1 $a = -5, b = 4$ cao



Question Number	Scheme	Marks																
1	<table><tr><td>a</td><td>b</td><td>c</td><td>(abc)</td></tr><tr><td>6</td><td>1</td><td>3</td><td>(18)</td></tr><tr><td>4</td><td>2</td><td>4</td><td>(32)</td></tr><tr><td>2</td><td>3</td><td>5</td><td>(30)</td></tr></table>	a	b	c	(abc)	6	1	3	(18)	4	2	4	(32)	2	3	5	(30)	B1
	a	b	c	(abc)														
	6	1	3	(18)														
	4	2	4	(32)														
2	3	5	(30)															
Any one correct row for $b = 1$, $b = 2$ or $b = 3$. Products do not need to be found for this mark.																		
Attempts the product abc for at least 2 valid combinations.		M1																
Finds all three valid combinations with correct products seen and somewhere/shows why this is exhaustive and concludes. *		A1*																
		(3 marks)																

Note that in most cases the M1 can only follow B1 but there may be some exceptions.

Numerical approach using the table:

B1: Any one correct row for $b = 1$, $b = 2$ or $b = 3$. Products do not need to be found for this mark.

M1: Attempts the product abc for at least 2 valid combinations.

A1*: Requires:

- All three valid combinations with correct products
- No other combinations shown unless they are crossed out or e.g. have a cross at the end of the row or are discounted in some way
- A (minimal) conclusion e.g. the product of a , b and c is even, hence proven, QED, hence it is even, each product stated as even, etc.

Algebraic/logic approach:

B1: Uses the information to obtain a correct equation connecting a and b e.g. $a + 2b = 8$, $a = 8 - 2b$

M1: States a must be even and considers the product abc in some way

A1*: States e.g. abc is even with a reason e.g. “even $\times$ anything is even”

Pure Algebraic approach:

B1: Uses the information to obtain a correct equation connecting a and b e.g. $a + 2b = 8$, $a = 8 - 2b$

M1: $abc = (8 - 2b)b(b + 2)$

Attempts the product of a , b and c in terms of b (or some other letter)

A1*: $abc = 2(4 - b)b(b + 2)$ which is even, hence proven, QED etc.

Concludes abc is even and makes a (minimal) conclusion. There must be no algebraic errors.

NB using this approach “ $abc = -2b^3 + 4b^2 + 16b$ which is even hence proven” is not sufficient – they would need to say e.g. which is even + even + even or factor out the 2.

Question Number	Scheme	Marks
3(a)	Attempts $(8-3)^2 + (5-7)^2 = \dots$	M1
	Writes $(x-3)^2 + (y-5)^2 = k$	M1
	$(x-3)^2 + (y-5)^2 = 169$	A1
		(3)
(b)	Attempts $d^2 + (2\sqrt{22})^2 = 169 \Rightarrow d = \dots$	M1
	States or uses $(y=) 5 + d$	dM1
	$y = 14$	A1
		(3)
		(6 marks)

(a)

M1: Attempts to find the radius or radius². Must proceed to find a value. Condone a sign slip if attempting $5 - -7$ if this is seen as $5 - 7$ o.e. May be implied by 13 or 169. Do not be concerned with the labelling of the expression or value as r or r^2

M1: Writes the equation of the circle in the form $(x-3)^2 + (y-5)^2 = k$, where $k > 0$, o.e.

$$\text{e.g. } x^2 + y^2 - 6x - 10y + c = 0 \text{ where } c < 0$$

Invisible brackets may be implied by further work. Condone $(x-3)^2 + (y-5)^2 = r^2$ where no attempt has been made to find the radius or radius²

A1: $(x-3)^2 + (y-5)^2 = 169$ o.e. e.g. $x^2 + y^2 - 6x - 10y - 135 = 0$ isw once a correct unsimplified equation

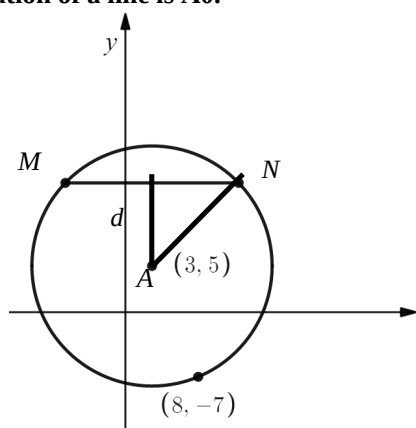
for C is found. Condone $(x-3)^2 + (y-5)^2 = 13^2$. A correct equation scores M1M1A1 but it must be seen in (a).

(b) Note that $(3 + 2\sqrt{22}, 14)$ as the final answer is A0

M1: Attempts Pythagoras' theorem with "13" (as the hypotenuse) and $2\sqrt{22}$ to find d . Not just d^2 (See diagram)

dM1: States or uses the equation for MN , $(y=) 5 + d$ but also condone stating/using $(y=) 5 - d$ (may be implied by their value for y)

A1: $y = 14$ only (Must be exactly 14 e.g. $13.99\dots = 14$ is A0). **Further work leading to a different equation of a line is A0.**



Alternative using their centre and the x coordinates of M and N are equidistant from the centre

M1: Attempts to find the x coordinates of M or N e.g. $3 \pm 2\sqrt{22}$, uses their equation of the circle with one of $x = 3 \pm 2\sqrt{22}$ and forms a three term quadratic in y (e.g. $y^2 - 10y - 56 = 0$)

dM1: Attempts to solve their quadratic proceeding to a value for y. Usual rules for solving a quadratic. Accept via a calculator. It is dependent on the previous method mark.

A1: $y = 14$ only (Must be exactly 14 e.g. $13.99... = 14$ is A0) **Further work leading to a different equation of a line is A0.**

Alternative finding an angle in the triangle MNA, then using the angle to find d

M1: e.g. attempts the cosine rule to find the angle MNA, and then trigonometry to find d

$$\cos(\angle MNA) = \frac{13^2 + (4\sqrt{22})^2 - 13^2}{2 \times 13 \times 4\sqrt{22}} \Rightarrow \angle MNA = 43.8...^\circ$$

$$d = \sin 43.8... \times 13 = ...$$

dM1: Same as main scheme

A1: $y = 14$ only (Must be exactly 14 e.g. $13.99... = 14$ is A0) **Further work leading to a different equation of a line is A0.**

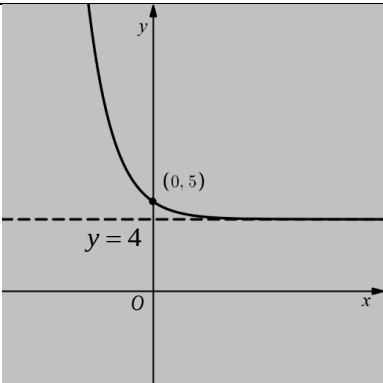
Note: There may be other more complex attempts (including use of area) Send to review if unsure.

e.g. Attempting to find the line which passes through AN $\left(y = \frac{9}{2\sqrt{22}}x + \frac{220 - 27\sqrt{22}}{44} \right)$ and solving

simultaneously with the equation of the circle $\Rightarrow y = 14$

This typically will score M1 for attempting Pythagoras' theorem, but can score for attempting the gradient of the line AN.

The dM1 mark would be for the full attempt to find where the line through AN intersects the equation of the circle.

Question Number	Scheme	Marks
4(a)	 Correct shape Correct intercept Correct equation for asymptote	B1 B1 B1
		(3)
(b)	Strip width = 2.5	B1
	$\int_{-4}^{8.5} \left(3^{\frac{1}{2}x} + 4 \right) dx \approx \frac{5}{4} \{ 13 + 4.009 + 2 \times (6.280 + 4.577 + 4.146 + 4.037) \}$ $= 68.86125 (= \text{awrt } 69)$	M1 A1
		(3)
(c)	(i) $\int_{-4}^{8.5} \left(3^{\frac{1}{2}x} \right) dx = 69 - 4 \times (8.5 - -4) = \text{awrt } 19$	M1, A1ft
	(ii) $\int_{-4}^{8.5} \left(3^{\frac{1}{2}x} + 4 \right) dx + \int_{-8.5}^4 \left(3^{\frac{1}{2}x} + 4 \right) dx = \text{awrt } 138$	B1ft
		(3)
		(9 marks)

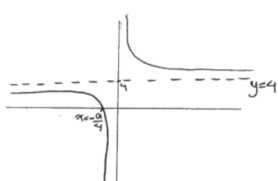
(a) If there are multiple attempts then mark the most complete attempt. If coordinates/the equation of the asymptote are stated by the question and on their diagram, then the sketch takes precedence.

B1: Correct shape in quadrants 1 and 2 and does not appear in quadrants 3 or 4 or touch the x-axis. Do not be concerned about the y intercept or asymptote. Mark the intention to draw an exponentially decreasing graph – do not penalise parts of the curve which may appear linear.

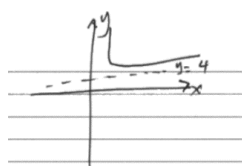
B1: Intercept at (0, 5). Condone invisible brackets, or may just have 5 marked on the y-axis, or $x = 0$, $y = 5$. If the point is labelled as (5, 0) then condone provided it is a y intercept which is above the x-axis.

B1: Graph with asymptote at $y = 4$ (asymptote does not need to be drawn) Must be an equation. There must be a graph with the intention of a horizontal asymptote above the x-axis to score the mark. If more than one horizontal asymptote is given, then B0. Where the asymptote is not drawn (e.g. with dashed lines) it must be clear that $y = 4$ refers to the asymptote of the graph and not the y-intercept.

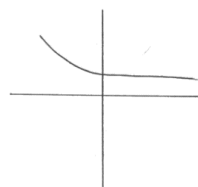
Examples:



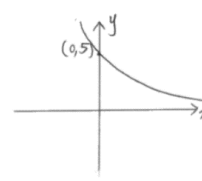
B0B0B1



B0B0B1



B1B0B0



B1B1B0

(b)

B1: For a correct strip width of $h = 2.5$ o.e. May be implied by 1.25 or equivalent in front of the bracketM1: Correct application of the trapezium rule with all the y values and their h (which may be 1 and condone if negative)Condone a missing trailing bracket. Condone other bracketing errors only if followed by a correct answer for their values. Condone miscopying of the y values from the table but the intended values must be used in the correct places.

Also award for attempts at forming individual trapezia and adding these together.

e.g.

$$2.5\left(\frac{13+6.280}{2}\right) + 2.5\left(\frac{6.280+4.577}{2}\right) + 2.5\left(\frac{4.577+4.146}{2}\right) + 2.5\left(\frac{4.146+4.037}{2}\right) + 2.5\left(\frac{4.037+4.009}{2}\right)$$

A1: awrt 69 (exact answer is 68.86125) isw once a correct answer is seen. awrt 69 following a correct calculation.

Note that the calculator answer for $\int_{-4}^{8.5} \left(3^{\frac{-1}{2}x} + 4\right) dx$ is 66.367... which scores 0 marks.

(c) They may use their 69 or a more accurate value of their 69 which is acceptable.

(i) **Attempts at redoing the trapezium rule with new values is M0A0**M1: Attempts " $69 \pm 4 \times (8.5 - -4)$ " or " $69 \pm \int_{-4}^{8.5} 4 dx$ " or (b) ± 50 May also be implied by just stating the value of (b) ± 50 A1ft: awrt 19 but follow through on (b) -50 .

(c)(ii)

B1ft: awrt 138 but follow through on $2 \times (b)$ **Part (c) when the answer to part (b) is awrt 66 (where (b) has been done using the calculator)**(c)(i) **If a method is shown** then M1A1ft is possible e.g. $66 \pm 4 \times (8.5 - -4)$ or e.g. $66 - 50$ for awrt 16. awrt 16 with no working is M0A0ft (as this could have been done using the calculator)

(c)(ii) awrt 132 scores B1ft (condoned)



Question Number	Scheme	Marks
5(i)	$r = 0.25$	B1
	$S_{\infty} = \frac{1.5}{1-0.25} = 2$	M1 A1
		(3)
(ii)	(a) $(u_1 = 3), u_2 = 0, u_3 = \frac{3}{2}, u_4 = 3, \dots$	M1
	$(u_1 = 3), u_2 = 0, u_3 = \frac{3}{2}, u_4 = 3, \dots$ and e.g. states that $u_4 = u_1$ so sequence will repeat (is periodic)	A1
		(2)
	(b) (Order) 3	A1
		(1)
	(c) $\sum_{n=1}^{70} u_n = 23 \times \left(3 + 0 + \frac{3}{2} \right) + 3 = 106.5$	M1, A1
		(2)
		(8 marks)

(i)

B1: States or uses $r = 0.25$ (may be implied by using $r = 0.25$ in the sum to infinity formula). 6×0.25 or any other term in the sequence is not sufficient evidence that this is a geometric sequence.
Alternatively, shows an understanding of the sigma notation by writing at least the first three terms in

the sequence e.g. $6 \times \frac{1}{4} + 6 \times \left(\frac{1}{4} \right)^2 + 6 \times \left(\frac{1}{4} \right)^3 + \dots$

Condone invisible brackets and may be written as a list.

M1: Attempts $S_{\infty} = \frac{a}{1-r}$ with $a = 1.5$ (or 6×0.25), $r = 0.25$ or $a = 6$, $r = 0.25$

You may also see $S_{\infty} = \frac{a}{1-r} - a$ with $a = 6$, $r = 0.25$ which is acceptable

Beware of other variants such as taking a factor of 6 out and using $a = 0.25$, $r = 0.25$

The values embedded in the expression is sufficient to score this mark.

A1: 2 cao **following evidence of the use of the sum to infinity formula**

Note $6 \times \frac{1}{4} + 6 \times \left(\frac{1}{4} \right)^2 + 6 \times \left(\frac{1}{4} \right)^3 + \dots = 2$ is B1M0A0 (no evidence of using the sum to infinity formula)

(ii)(a)

M1: Uses the recurrence relation correctly at least once. If they do not achieve a correct $u_2 = 0$ then check e.g. their u_3 is a correct follow through for their u_2

A1: $(u_1 = 3), u_2 = 0, u_3 = \frac{3}{2}, u_4 = 3, \dots$ with a statement or conclusion such as $u_4 = u_1$ /repeats/cycles (is periodic) As a minimum accept $(3), 0, \frac{3}{2}, 3$ (hence) periodic

(ii)(b) **This mark can only be scored following correct values in (ii)(a)** (or a restart in (ii)(b))

A1: (Order is) 3 (do not accept (repeats every) third). Cannot be for just listing the terms of the sequence.



(ii)(c)

M1: Establishes a correct method of finding $\sum_{n=1}^{70} u_n$ for their periodic sequence of order 3 found in (a).

If they did not have a periodic sequence of order 3 in (a) and they do not restart then this mark cannot be scored.

Seeing the values embedded in the expression is sufficient to score this mark.

Attempts to use the sum of a geometric or an arithmetic series is M0A0

A1: 106.5 or exact equivalent e.g. $\frac{213}{2}$ ignore any subsequent incorrect attempts to round.



Question Number	Scheme	Marks
6 (a)	$2\log_4(x+3) = \log_4(x+3)^2$ or $\frac{1}{2} = \log_4 2$ o.e	B1
	Combines two terms e.g. $2\log_4(x+3) + \log_4 x = \log_4 x(x+3)^2$	M1
	e.g. $x(x+3)^2 = 2(4x+2)$	A1
	e.g. $x(x^2+6x+9) = 8x+4 \Rightarrow x^3+6x^2+x-4=0^*$	A1*
		(4)
(b) (i)	$x^3+6x^2+x-4 = (x+1)(x^2+5x-4)$	M1
	$x = \frac{-5 \pm \sqrt{25+16}}{2} \Rightarrow x = \frac{-5 \pm \sqrt{41}}{2}$	dM1 A1
		(3)
	$x = \frac{-5 + \sqrt{41}}{2}$	B1
		(1)
		(8 marks)

(a) Do not penalise the omission of base 4 in their working provided the terms and values are consistent with base 4.

B1: Writes (or may be implied by further work)

- $2\log_4(x+3)$ as $\log_4(x+3)^2$ or
- $\frac{1}{2}$ as $\log_4 2$

M1: Correctly combines (at least) **two of the original terms**. e.g.

- $2\log_4(x+3) + \log_4 x = \log_4 x(x+3)^2$ is M1 but $2\log_4(x+3) + \log_4 x = 2\log_4 x(x+3)$ is M0
- $\log_4(4x+2) - \log_4 x = \log_4\left(\frac{4x+2}{x}\right)$ is M1
- $\log_4(4x+2) + \frac{1}{2} = \log_4(8x+4)$ is M1

Do not penalise if they subsequently make errors or apply laws incorrectly. May be implied if they proceed to an equation not involving logs e.g. $x(x+3)^2 = 2(4x+2)$ (but not the given answer)

Beware that $2\log(x+3) + \log x = 2\log x(x+3) = \log x(x+3)^2$ is B1M0A0A0

A1: A correct intermediate equation not involving logs (but not the given answer).

Must follow correct log work. e.g. $\frac{\log_4 x(x+3)^2}{\log_4(4x+2)} = \frac{1}{2} \Rightarrow \frac{x(x+3)^2}{(4x+2)} = 2$ is A0

A1*: Correct proof. Expect to see

- a correct equation not involving logs which is not the final answer
- the brackets multiplied out $(x+3)^2$ or $(x^2+3x)(x+3)$ before proceeding to the given answer.

Do not allow this mark for recovery from incorrect log work but allow invisible brackets to be “recovered” or implied if the intention is clear.

(b) (i) **Answers with no working score 0 marks in (b)(i)**

M1: Divides by $(x+1)$. May be implied by a correct quadratic. May be seen as the quotient if attempting algebraic division. If via division look for $(x+1)\left(x^2 \pm 5x \dots\right)$ or via inspection $(x+1)\left(x^2 \pm \dots x \pm 4\right)$

dM1: Attempts to find the roots of their quadratic factor (usual rules for solving a quadratic). Accept solving the quadratic via a calculator (you may need to check this). Condone decimals to 2sf for this mark. e.g. may see 0.70 or -5.7

Note that if there are multiple attempts to solve the quadratic (including via a calculator so roots may be stated) then mark the one which scores the most marks).

A1: $\left(x = \right) \frac{-5 \pm \sqrt{41}}{2}$ or exact equivalent e.g. $-2.5 \pm \sqrt{10.25}$ (Do not accept decimal answers but isw if seen after an exact answer is seen)
Ignore the presence or absence of $x = -1$

(b)(ii) **This mark can only be scored provided $\left(x = \right) \frac{-5 + \sqrt{41}}{2}$ (or awrt 0.702) is found in (i), even if this is via solving the cubic on a calculator**

B1: $x = \frac{-5 + \sqrt{41}}{2}$ or exact equivalent ONLY (accept awrt 0.702 if given in (b)(i)) If both answers are present then they must indicate they have chosen $x = \frac{-5 + \sqrt{41}}{2}$ e.g. circling or ticking (or putting a cross by the other one)

Question Number	Please read notes for 8(i) before looking at scheme		Marks
8.(i)	$8^{2x+1} = 6 \Rightarrow 2x+1 = \log_8 6$ M1 $\Rightarrow 2x+1 = \frac{\log_2 6}{\log_2 8}$ A1 $\Rightarrow 2x+1 = \frac{\log_2 2 + \log_2 3}{3}$ M1 $\Rightarrow x = \frac{\log_2 3}{6} - \frac{1}{3}$ A1	$2^{6x+3} = 6$ $\Rightarrow (6x+3)\log_2 2 = \log_2 6$ M1 A1 $\Rightarrow (6x+3) = \log_2 2 + \log_2 3$ M1 $\Rightarrow x = \frac{\log_2 3}{6} - \frac{1}{3}$ A1	(4)
	(ii) $\log_5 (7-2y) = 2\log_5 (y+1) - 1$ $\log_5 (7-2y) = \log_5 (y+1)^2 - 1$ $\log_5 (7-2y) = \log_5 (y+1)^2 - \log_5 5$ $(7-2y) = \frac{(y+1)^2}{5}$ $y^2 + 12y - 34 = 0 \Rightarrow y =$ $y = -6 + \sqrt{70}$ oe only	$2\log_5 (y+1) - \log_5 (7-2y) = 1$ $\log_5 (y+1)^2 - \log_5 (7-2y) = 1$ $\log_5 \frac{(y+1)^2}{(7-2y)} = 1$ $\frac{(y+1)^2}{(7-2y)} = 5$ $y^2 + 12y - 34 = 0 \Rightarrow y =$ $y = -6 + \sqrt{70}$ oe only	
			(5) (9 marks)

Notes

There are many different ways to attempt this but essentially can be marked in a similar way.

If index work is used marks are not scored until the log work is seen

$$\text{Eg 1: } 8^{2x+1} = 6 \Rightarrow 8^{2x} \times 8 = 6 \Rightarrow 8^{2x} = \frac{3}{4}.$$

1ST M1 is scored for $2x = \log_8 \frac{3}{4}$ and then 1ST A1 for $2x = \frac{\log_2 \frac{3}{4}}{\log_2 8}$

but BOTH of these marks would be scored for $2x \log_2 8 = \log_2 \frac{3}{4}$

2nd M1 would then be awarded for $\log_2 \frac{3}{4} = \log_2 3 - \log_2 4$ which may be implied by $\log_2 3 - 2$

Two more examples where the candidate initially uses index work.

$8^{2x+1} = 6 \Rightarrow 2^{3(2x+1)} = 6$ $3(2x+1) = \log_2 6$ is M1 A1 as it is a correct linear equation in x involving a $\log_2$ term	$8^{2x+1} = 6 \Rightarrow 64^x = \frac{3}{4}$ $\Rightarrow x = \log_{64} \frac{3}{4}$ is M1 But $\Rightarrow x \log_2 64 = \log_2 \frac{3}{4}$ is M1 A1
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Question Number	Please read notes for 8(i) before looking at scheme	Marks
(ii)		
M1	Attempts a correct log law. This may include $2\log_5(y+1) \rightarrow \log_5(y+1)^2$ $1 \rightarrow \log_5 5$ You may award this following incorrect work. Eg $1 = 2\log_5(y+1) - \log_5(7-2y) \Rightarrow 1 = \log_5 2(y+1) - \log_5(7-2y) \Rightarrow 1 = \log_5 \frac{2(y+1)}{(7-2y)}$	
dm1	Uses two correct log laws. It may not be awarded following errors (see above) It is awarded for $2\log_5(y+1) - 1 = \log_5 \frac{(y+1)^2}{5}$, $2\log_5(y+1) - \log_5(7-2y) = \log_5 \frac{(y+1)^2}{(7-2y)}$ $1 + \log_5(7-2y) = \log_5 5(7-2y)$ or $2\log_5(y+1) - 1 = \log_5(y+1)^2 - \log_5 5$	
A1	A correct equation in 'y' not involving logs	
ddM1	A correct attempt at finding at least one value of y from a 3TQ in y All previous M's must have been awarded. It can be awarded for decimal answer(s), 2.4 and –	
14.4		
A1	$y = -6 + \sqrt{70}$ or exact equivalent only. It cannot be the decimal equivalent but award if the candidate chooses 2.4 following the exact answer. If $y = -6 \pm \sqrt{70}$ then the final A mark is withheld	
.....		
	Special case: Candidates who write $\log_5(y+1)^2 - \log_5(7-2y) = 1 \Rightarrow \frac{\log_5(y+1)^2}{\log_5(7-2y)} = 1 \Rightarrow \frac{(y+1)^2}{(7-2y)} = 5$ can score M1 dm0 A0 ddM1 A1 if they find the correct answer.	

Question Number	Scheme	Marks
9 (a)	Substitute $t = 4$ in $3\log_2(t+4) - 2\log_2(t-2) \Rightarrow 3\log_2 8 - 2\log_2 2$ $= 3 \times 3 - 2 \times 1 = 7 \quad \checkmark$	M1 A1
		(2)
(b)	One correct log law applied. E.g. $3\log_2(t+4) = \log_2(t+4)^3$	M1
	Correctly removes logs $\frac{(t+4)^3}{(t-2)^2} = 2^7$	A1
	$t^3 + 12t^2 + 48t + 64 = 128(t^2 - 4t + 4) \Rightarrow t^3 - 116t^2 + 560t - 448 = 0$	A1*
		(3)
(c)	$t^3 - 116t^2 + 560t - 448 = (t-4)(t^2 + \dots \pm 112)$	M1
	$t^3 - 116t^2 + 560t - 448 = (t-4)(t^2 - 112t + 112)$	A1
	Solves their $t^2 - 112t + 112$ and finds at least the value of t greater than 2	dM1
	$t = 4, 56 + 12\sqrt{21}$	A1
		(4)
		Total 9

(a) Note the demand of the question.

Candidates cannot substitute $t = 4$ into $t^3 - 116t^2 + 560t - 448 = 0$ unless all the work in part (b) is done in part (a)

M1: Substitute $t = 4$ in $3\log_2(t+4) - 2\log_2(t-2)$ and simplifies to achieve

$$\text{either } 3\log_2 8 - 2\log_2 2 \quad \text{or } 3\log_2(4+4) - 2\log_2(4-2) = 9 - 2$$

There are other possible ways including substituting $t = 4$ into $\log_2 \frac{(t+4)^3}{(t-2)^2} \rightarrow \log_2 \frac{8^3}{2^2}$

If the equation is adapted it must be correct

A1: Correctly verifies with a minimal conclusion seen. E.g. $= 3 \times 3 - 2 \times 1 = 7 \quad \checkmark$

Condone sight of the calculation partially completed if deemed reasonable, E.g. $= 9 - 2 = 7 \quad \checkmark$

There are other possible ways including

- $3\log_2 8 - 2\log_2 2 = 9\log_2 2 - 2\log_2 2 = 7\log_2 2 = 7$ Hence true

- $3\log_2(t+4) - 2\log_2(t-2) = \log_2 \frac{(t+4)^3}{(t-2)^2} = \log_2 \frac{8^3}{2^2} = \log_2 128 = 7$ QED

The calculation of the logs must be something that could reasonably be done in your head.

So $\log_2 \frac{8^3}{2^2} = 7$ is insufficient

(b) Work from part (a) can only count in part (b) if it used in part (b). See example

M1: One correct log law seen and applied to the given equation.

In almost all cases this will be a power law but it can be awarded for $7 = \log_2 128$

A1: Removes logs and forms a correct un-simplified equation. E.g. $\frac{(t+4)^3}{(t-2)^2} = 2^7$ o.e.

A1*: Proceeds to the given answer with sufficient intermediate work shown.

Any incorrect working (including missing brackets) within the body of their solution should be penalised.

See main scheme for a minimal acceptable solution

(c)

M1: Uses the fact that $t = 4$ is a solution so $(t - 4)$ must be a factor.Score for attempting to divide or factor out $(t - 4)$ For factorisation look for $t^3 - 116t^2 + 560t - 448 = (t - 4)(t^2 \pm \dots t \pm 112)$ For division look for $t - 4 \overline{) t^3 - 116t^2 + 560t - 448}$ or $t - 4 \overline{) t^3 - 116t^2 + 560t - 448}$ A1: Correct quadratic factor $(t^2 - 112t + 112)$ dM1: Solves their $t^2 - 112t + 112$ and finds at least the value of t greater than 2

It is dependent upon the previous method. Allow use of a calculator to solve the quadratic.

Allow a decimal answer here, accuracy awrt 3sf.

A1: States $t = 4, 56 \pm 12\sqrt{21}$ and no other solutions following the award of M1 A1 dM1.....
Candidates who simply write $t^3 - 116t^2 + 560t - 448 = 0$ followed by $t = 4, 56 \pm 12\sqrt{21}$ score 0 marks.All previous M's and A must have been scored before the final A1 mark is given.
.....

Question Number	Scheme	Marks
8. (a)	$-5 < x < \frac{2}{3}$	M1, A1 (2)
(b)	$(2x-7)$	B1 (1)
(c)	$f'(x) = 2(3x-2)(x+5) = 6x^2 + 26x - 20$ $f(x) = 2x^3 + 13x^2 - 20x + c$ $x = \frac{7}{2}, y/f(x) = 0 \Rightarrow c = (-175)$ $f(x) = 2x^3 + 13x^2 - 20x - 175 = (2x-7)(x^2 + 10x + 25)$ $= (2x-7)(x+5)^2$	M1, A1 dM1, A1 ddM1 A1 (6) (9 marks)

(a)

M1: Achieves the correct critical values. Condone for this mark -5 and awrt 0.67

They may be seen as part of an incorrect inequality

A1: $-5 < x < \frac{2}{3}$ or $-5, x, \frac{2}{3}$. Either of these on their own (with consistent inequality use) scores both marksAccept other valid alternatives such as $x \in \left(-5, \frac{2}{3}\right)$ and $\{x : x > -5\} \cap \left\{x : x < \frac{2}{3}\right\}$ but not $x < \frac{2}{3}$ or $x > -5$

(b)

B1: States $(2x-7)$ or $A = 2, B = -7$ which must be seen in (b).Condone candidates who write $\left(x - \frac{7}{2}\right) = (2x-7)$ Note that stating $\left(x - \frac{7}{2}\right)$ on its own is B0(c) Some candidates integrate in part (b). Allow these candidates to score the marks in (c) as long as they re-write $f(x)$ in (c) or do some extra work involving $f(x)$ in (c)M1: Multiplies out $f'(x)$ to form a quadratic expression and attempts to integrate.Look for $f'(x) = 2(3x-2)(x+5) = ax^2 + bx + c \rightarrow f(x) = px^3 + qx^2 + rx...(+c)$ with or without the + cA1: $f(x) = 2x^3 + 13x^2 - 20x + c$ with or without the + cdM1: Substitutes $x = \frac{7}{2}, y/f(x) = 0 \Rightarrow c = ...$ Alternatives exist via division using the fact that $(2x-7)$ is a factor of $2x^3 + 13x^2 - 20x + c$ A1: $f(x) = 2x^3 + 13x^2 - 20x - 175$ but allow $f(x) = 2x^3 + 13x^2 - 20x + c$ followed by $c = -175$

ddM1: Attempts to divide or factorise out a factor of $(2x-7)$ or $\left(x-\frac{7}{2}\right)$ from their

$$f(x) = 2x^3 + 13x^2 - 20x - 175$$

Awarded for 'correct' first and last terms or correct first two terms for their $f(x)$ condoning sign slips

$$\text{E.g. } 2x^3 + 13x^2 - 20x - 175 = (2x-7)(x^2 + \dots x \pm 25) \text{ or}$$

$$2x^3 + 13x^2 - 20x - 175 = \left(x - \frac{7}{2}\right)(2x^2 + \dots x \pm 50)$$

It is dependent upon both previous M's so cannot be awarded if they don't have a value for c
Mark their final response here as there may be working before the final answer

A1: CSO Requires working to be seen (see bold sentences in question) so look for

- $f(x) = (2x-7)(x+5)^2$ or $f(x) = (2x-7)(x+5)(x+5)$ following correct $f(x)$
- AND correct intermediate factorisation $(2x-7)(x^2 + 10x + 25)$

Going directly from $2x^3 + 13x^2 - 20x - 175$ to $(2x-7)(x+5)^2$ without sight of working would result in the loss of the last two marks

Note that the last 4 marks can be scored without finding the value of c BUT the constant c must be present.

We know that $f(x) = 2x^3 + 13x^2 - 20x + c$ and that $(2x-7)$ is a factor

So $2x^3 + 13x^2 - 20x + c = (2x-7)(Ax^2 + Bx + C)$ which may be attempted via division

$$2x^3 + 13x^2 - 20x + c = (2x-7)(1x^2 + 10x + 25) = (2x-7)(x+5)^2 \text{ o.e.}$$

The 6 marks in part (c) are awarded as follows

M1, A1: For $f(x) = 2x^3 + 13x^2 - 20x + c$ with or without the $+c$

dM1: For setting their $f(x) = '2'x^3 + '13'x^2 - '20'x + c = (2x-7)(Ax^2 + Bx + C)$ and correctly deduces the value for A for their $f(x)$. You may see a division attempted here which is equivalent work

A1: Correctly deduces $2x^3 + 13x^2 - 20x + c = (2x-7)(x^2 + 10x + C)$

Via division, the $(x^2 + 10x + \dots)$ can be awarded implied from within the calculation

ddM1: Correctly deduces $2x^3 + 13x^2 - 20x + c = (2x-7)(x^2 + 10x + 25)$

A1: $f(x) = (2x-7)(x+5)^2$ o.e.

Question Number	Scheme	Notes	Marks
9(i)	$4 = \log_3 81$ or $4 = \log_3 3^4$ May be implied by e.g. $\log_3 \frac{x+5}{2x-1} = 4 \Rightarrow \frac{x+5}{2x-1} = 3^4$ (or 81)		B1
	Examples: $\log_3 (x+5) - \log_3 81 = \log_3 \frac{x+5}{81}$ or $\log_3 (x+5) - \log_3 (2x-1) = \log_3 \frac{x+5}{2x-1}$ or $\log_3 (2x-1) + \log_3 81 = \log_3 81(2x-1)$ This mark is for combining 2 log terms correctly and can be awarded following an incorrect rearrangement e.g. $\log_3 (x+5) - 4 = \log_3 (2x-1) \Rightarrow \log_3 (x+5) + \log_3 (2x-1) = 4$ $\Rightarrow \log_3 (2x-1)(x+5) = \dots$		M1
	$\frac{x+5}{81} = 2x-1$	Obtains this equation in any form e.g. $\frac{x+5}{2x-1} = 3^4$	A1
	$x = \frac{86}{161}$	Cao	A1
	Condone the omission of the base throughout		
			(4)
	Alternative for first 3 marks: $\log_3 (x+5) - 4 = \log_3 (2x-1) \Rightarrow 3^{\log_3 (x+5) - 4} = 3^{\log_3 (2x-1)}$ $\Rightarrow 3^{\log_3 (x+5)} \times 3^{-4} = 2x-1 \Rightarrow \frac{x+5}{81} = 2x-1$ Score B1 for sight of 3^{-4} and M1 for applying $3^{a \pm b} = 3^a \times 3^{\pm b}$ and A1 as above		
	(a) Special Case $\log_3 (x+5) - \log_3 (2x-1) = 4 \Rightarrow \frac{\log_3 (x+5)}{\log_3 (2x-1)} = 4 \Rightarrow \frac{x+5}{2x-1} = 81 \Rightarrow x = \frac{86}{161}$ Scores B1(implied) M0 A0 A1		
(ii)(a)	$3^{y+3} = 3^y \times 3^3$ or $2^{1-2y} = 2 \times 2^{-2y}$ or $\frac{2}{2^{2y}}$ or 2×4^{-y} or $\frac{2}{4^y}$	One correct index law seen or implied anywhere in their working	B1
	$3^{y+3} \times 2^{1-2y} = 27 \times 3^y \times 2 \times 2^{-2y} = \dots$	Applies both correct index laws to the lhs of the equation	M1(B1 on EPEN)
	$3^y \times 2^{-2y} = \frac{108}{27 \times 2}$ or $\frac{3^y}{4^y} = \frac{108}{27 \times 2}$ or $\frac{3^y}{2^{2y}} = \frac{108}{54}$ Isolates the terms in y (as powers of 3 and 2(or 4)) on the lhs and the constants on the rhs. There must be no incorrect work to combine terms e.g. $3^y \times 3^3 = 27^y$ etc.		M1
	$(0.75)^y = 2 *$	Cso. Reaches the given answer with no errors and all steps shown with 2^{2y} appearing as 4^y at some point.	A1*
			(4)

Alternative 1 for (ii)(a) using logs:		
$\log(3^{y+3} \times 2^{1-2y}) = \log 3^{y+3} + \log 2^{1-2y}$ Or $\log 3^{y+3} = (y+3)\log 3$ Or $\log 2^{1-2y} = (1-2y)\log 2$	One correct log law seen or implied anywhere in their working. No bracketing errors allowed for this mark.	B1
$\log 3^{y+3} + \log 2^{1-2y} = (y+3)\log 3 + (1-2y)\log 2$ Applies the correct log laws to the lhs. You can condone missing brackets around the $y+3$ and/or $1-2y$		M1(B1 on EPEN)
$(y+3)\log 3 + (1-2y)\log 2 = \log 108 \Rightarrow \log 3^y - \log 2^{2y} = \log 108 - 3\log 3 - \log 2$ $\Rightarrow \log \frac{3^y}{2^{2y}} = \log \frac{108}{3^3 \times 2}$ Proceeds to isolate the terms in y on the lhs and combines the constants on the rhs or e.g. $(y+3)\log 3 + (1-2y)\log 2 = \log 108 \Rightarrow y(\log 3 - 2\log 2) = \log \frac{108}{3^3 \times 2}$ Proceeds to isolate the terms in y on the lhs and combines the constants on the rhs		M1
$(0.75)^y = 2 *$	Cso. With e.g. $2\log 2$ seen as $\log 4$ or $\log 2^2$ or implied at some point.	A1*
Alternative 2 for (ii)(a) using factors of 108:		
$3^{y+3} \times 2^{1-2y} = 108 = 2^2 \times 3^3$ $\Rightarrow \frac{3^{y+3} \times 2^{1-2y}}{2^2 \times 3^3} = \dots$ $\Rightarrow 3^y \times 2^{-1-2y} = \dots$	One correct index law seen or implied anywhere in their working e.g. $\frac{3^{y+3}}{3^3} = 3^y$ or $\frac{2^{1-2y}}{2^2} = 2^{-1-2y}$	B1
$\Rightarrow 3^y \times 2^{-1-2y} = \dots$	Applies both correct index laws to the lhs of the equation	M1(B1 on EPEN)
$\Rightarrow 3^y \times 2^{-2y} = 2$	Proceeds to isolate the terms in y (as powers of 3 and 2(or 4)) on the lhs and the constants on the rhs	M1
$(0.75)^y = 2 *$	Cso. Reaches the given answer with no errors and all steps shown with 2^{2y} appearing as 4^y at some point.	A1*
(b)	$(0.75)^y = 2 \Rightarrow y = \frac{\log 2}{\log 0.75}$ or $(0.75)^y = 2 \Rightarrow y = \log_{0.75} 2$	Correct processing to obtain a value for y May be implied by awrt – 2.4
	$y = -2.409$	Awrt –2.409 A correct answer implies both marks
		(2)
		Total 10

Question	Scheme	Marks
6(a)	Way 1: Eqn is $(x-3)^2 + (y+4)^2 - 9 - 16 + k = 0$	M1
	So radius must be 4 $\Rightarrow 25 - k = 16$	M1
	$\Rightarrow k = 9$	A1
		(3)
	Way 2: $y = 0 \Rightarrow x^2 - 6x + k = 0$ has one solution	M1
	$\Rightarrow 6^2 - 4 \times 1 \times k = 0$	M1
	$\Rightarrow k = 9$	A1
		(3)
(b)	C intersects x axis at (3,0)	B1
	Intersects y axis when $y^2 + 8y + 9 = 0 \Rightarrow y = \dots$ or $y = -4 \pm \sqrt{16 - 9}$ Or uses base of triangle is $2\sqrt{16 - 9}$	M1
	Area $RST = \frac{1}{2} \times 2\sqrt{7} \times 3$	M1
	$= 3\sqrt{7}$	A1
		(4)
(7 marks)		

Notes:**(a) Way 1**

M1: Attempts to complete square on both terms. May be implied by a centre of $(\pm 3, \pm 4)$

M1: Uses a valid method to find k , e.g. y coordinate of centre and sets $9 + 16 - k = (\text{their "4"})^2$.

Can be implied by sight of $k = (\text{their } x \text{ coordinate of centre})^2$

A1: Correct value for k

Way 2

M1: Substitutes equation for x axis ($y = 0$) into the circle equation.

M1: Sets the discriminant of the resulting quadratic to 0 (since as a tangent there is only one point of intersection).

A1: Correct value for k

(b)

B1: Identifies or implies the coordinates where circle touches the x axis. Probably from having completed the square on the equation. The 0 ordinate need not be seen and may be seen anywhere.

M1: Attempts the y coordinates of the intersections with the y axis, or the distance between these two points. E.g. may set $x = 0$ in the circle equation, or may use centre is 3 units from y axis etc.

M1: Uses a correct method for the area of the triangle RST . Longer methods are possible, such as use of cosine rule to find an angle in the triangle, followed by $\frac{1}{2}ab \sin C$, or may use the shoelace

method, e.g. $\frac{1}{2} \begin{vmatrix} 3 & 0 & 0 \\ 0 & -4 + \sqrt{7} & -4 - \sqrt{7} \end{vmatrix} = \frac{1}{2} \left| 3(-4 + \sqrt{7}) + 0 + 0 - 0 - 3(-4 - \sqrt{7}) \right|$

A1: Correct answer.



Question	Scheme	Marks
7(a)	$2 = \log_3 3^2$ oe seen or implied by working	B1
	$3 \log_3 (2x-1) = \log_3 (2x-1)^3$ or $\log_3 "3^2" + \log_3 (14x-25) = \log_3 ("3^2"(14x-25))$	M1
	$\Rightarrow (2x-1)^3 = 9(14x-25)$	dM1
	$\Rightarrow 8x^3 - 12x^2 + 6x - 1 = 126x - 225$ $\Rightarrow 8x^3 - 12x^2 - 120x + 224 = 0 \Rightarrow 2x^3 - 3x^2 - 30x + 56 = 0^*$	A1*
		(4)
(b)	$2(\pm 4)^3 - 3(\pm 4)^2 - 30(\pm 4) + 56 = \dots$	M1
	$2(-4)^3 - 3(-4)^2 - 30(-4) + 56 = -128 - 48 + 120 + 56 = 0$ Hence -4 is a root of the equation.	A1
		(2)
Alt (b)	$2x^3 - 3x^2 - 30x + 56 = (x \pm 4)(2x^2 + \dots \pm 14)$	M1
	$2x^3 - 3x^2 - 30x + 56 = (x+4)(2x^2 - 11x + 14)$ so -4 is a root of the equation.	A1
		(2)
(c)	$2x^3 - 3x^2 - 30x + 56 = 0 \Rightarrow (x+4)(2x^2 + \dots \pm 14) = 0$	M1
	$(x+4)(2x^2 - 11x + 14) = 0$	A1
	$(x+4)(2x-7)(x-2) = 0 \Rightarrow x = \dots$	dM1
	(Equation not defined for $x = -4$ so) solutions are 2 and $\frac{7}{2}$	A1
		(4)
(10 marks)		

Notes:**(a)****B1:** States or implies $2 = \log_3 3^2$ or equivalent. May be gained via $\log_3 f(x) = 2 \Rightarrow f(x) = 9$ **M1:** Uses a correct log law, either power law on $3 \log_3 (2x-1) = \log_3 (2x-1)^3$ or sum law on $\log_3 "3^2" + \log_3 (14x-25) = \log_3 ("3^2"(14x-25))$ or equivalent work (e.g. difference law if combining the two log terms on one side first).**dM1:** Uses correct log work to remove the logs. Allow slips, but all log work must have been correct.**A1*:** Expands and simplifies to the given cubic. Must see an intermediate step between the factorised sides and final answer.

Question Number	Scheme	Marks
6. (i)	$2\log_2(4-x) = 3 + \log_2\left(\frac{x+11}{2}\right)$ One correct law applied – examples (not exhaustive): $2\log_2(4-x) = \log_2(4-x)^2, \log_2\left(\frac{x+11}{2}\right) = \log_2(x+11) - \log_2 2$ $3 = \log_2 8$ Correctly combines two original terms e.g. $2\log_2(4-x) = \log_2(4x+44)$ Correct equation not involving logs $(4-x)^2 = 4x+44$ $x^2 - 12x - 28 = 0 \Rightarrow (x-14)(x+2) = 0 \Rightarrow x = \dots$ $\Rightarrow x = -2 \text{ only}$	B1 M1 A1 dM1 A1 (5)
(ii)	Combines $y = 6 \times 3^x$ and $y = 3^{2x+1} \Rightarrow 3^{2x+1} = 6 \times 3^x$ $\Rightarrow 3^x \times 3 = 6$ $\Rightarrow 3^x = 2 \Rightarrow x = \log_3 2$ $y = 6 \times 3^{\log_3 2} = 6 \times "2"$ coordinates of P are: $(\log_3 2, 12)$	M1 dM1, A1 ddM1 A1 (5) (10 marks)

(i)

B1: Sight of one correct log law applied. See scheme. There will be other equivalent examples e.g. $\log_2\left(\frac{x+11}{2}\right) = \log_2(x+11) - 1$. This mark can still be awarded if they have made errors before applying the law and do not withhold this mark for any subsequent errors.

M1: Correctly combines two of the three original terms of the equation. e.g. $\log_2\left(\frac{2(4-x)^2}{x+11}\right) = 3$

May be implied by a correct equation in any form not involving logs provided it has not come from incorrect log work. e.g. $\frac{\log_2(4-x)^2}{\log_2\left(\frac{x+11}{2}\right)} = 3 \Rightarrow \frac{(4-x)^2}{\frac{x+11}{2}} = 8$ is M0

A1: Correct equation in any form not involving logs which has not come from incorrect log work.

dM1: Solves a quadratic equation via factorisation, the formula or completing the square (usual rules apply). They are not allowed to just state the roots. Dependent on the previous method mark.

A1: $x = -2$ only (the 14 if found must be rejected or not included in their final answer)



- (ii) **Note that if logarithms are written as decimals instead of keeping their values exact then**
 - if they find x first the maximum score is M1dM0A0ddM0A0
 - if they find y first the maximum score is M1dM1A1ddM0A0

M1: Sets up a correct equation in x or y . Most likely in x but it is possible to form an equation in y

$$\text{e.g. } y = 3^{2x+1} = 3 \times (3^x)^2 = 3 \times \left(\frac{y}{6}\right)^2$$

dM1: A correct method of solving the equation in x (or y) to find either

- an unsimplified expression or value for x in terms of logs, or
- an expression or value for y not involving logs

It is dependent on the previous method mark. Condone arithmetical slips but any log/index work must be correct.

There may be some alternative approaches here to solving:

e.g. taking logarithms of base 3:

$$\log_3(3^{2x+1}) = \log_3(6 \times 3^x) \Rightarrow 2x+1 = \log_3 6 + \log_3 3^x \Rightarrow 2x+1 = 1 + \log_3 2 + x \Rightarrow x = \dots$$

e.g. forming a quadratic in 3^x (or another variable)

$$3 \times 3^{2x} - 6 \times 3^x = 0 \Rightarrow 3^x(3^x - 2) = 0 \Rightarrow 3^x = 2 \Rightarrow x = \dots$$

$$\text{If solving an equation in } y \text{ then e.g. } y = 3 \times \left(\frac{y}{6}\right)^2 \Rightarrow y = 3 \times \frac{y^2}{36} \Rightarrow y =$$

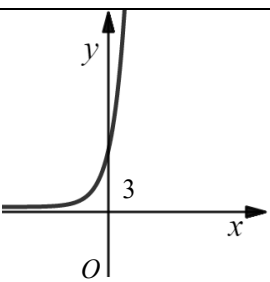
A1: Correct x coordinate ($\log_3 2$ or equivalent expressions such as $\log_3 6 - 1$ or $\frac{\log 2}{\log 3}$ o.e.) **or** y coordinate which must be 12

ddM1: Uses a correct method to find values for both x **and** y . Condone arithmetical slips but any log/index work must be correct.

It is dependent on the previous two method marks.

May be implied by a correct coordinate for their x or y . Do not allow logarithms to be written as decimals.

A1: $(\log_3 2, 12)$ Accept if written separately e.g. $x = \log_3 2$, $y = 12$. isw if they incorrectly state the values of a and b after correct coordinates are seen.
 Condone missing brackets but do not allow the coordinates to be the wrong way round.

Question Number	Scheme	Marks
9.(a)		Shape B1 (0,3) B1 (2)
(b)	$6^{1-x} = 3 \times 4^x$ $(1-x)\log 6 = \log 3 + x \log 4$ $x(\log 4 + \log 6) = \log 6 - \log 3$ $\Rightarrow x = \frac{\log\left(\frac{6}{3}\right)}{\log(4 \times 6)} \Rightarrow x = \frac{\log 2}{\log 24}$ Alt Method: $6^{1-x} = 3 \times 4^x \Rightarrow \frac{6}{6^x} = 3 \times 4^x$ $\Rightarrow \frac{6}{3} = 6^x 4^x \Rightarrow 2 = 24^x$ $\Rightarrow \log_{10} 2 = x \log_{10} 24 \Rightarrow \frac{\log_{10} 2}{\log_{10} 24}$	M1 dM1 A1 ddM1 A1* (5) M1 dM1A1 ddM1A1* (5) (7 marks)

(a)

B1 Correct shape and position. Should be increasing, always above the x axis and must be in both quadrants 1 and 2. Be tolerant with the asymptote as long as it does not cross the axis or clearly bend away (but don't be concerned if there is a gap between curve and axis).

B1 Intercept at $(0,3)$ Accept 3 or $(3,0)$ marked on the axis, but $(3,0)$ away from the graph is B0. Their graph must have crossed the positive y axis to score this mark. Ignore any x intercepts

(b)

M1 Attempt to takes logs (any base, including 6) and attempts either the addition law or the power law
Eg. $\log 6^{1-x} \rightarrow (1-x)\log 6$ or $\log 3 \times 4^x \rightarrow \log 3 + \log 4^x$ (Condone invisible brackets for Ms)

dM1 Takes logs of both sides and attempts **both** the addition law and the power law to achieve a linear equation in x (can still be in any base at this stage)

A1 A correct linear equation in x (any base).

ddM1 Attempts to make x the subject of the formula with no incorrect log work. Allow with just log, but any other base must be changed to base 10 to allow this mark.

A1* Proceeds to the given answer showing a correct intermediate step with no incorrect working seen.

For example $x(\log 4 + \log 6) = \log 6 - \log 3 \Rightarrow x = \frac{\log\left(\frac{6}{3}\right)}{\log(4 \times 6)}$ or $\dots \Rightarrow x \log 24 = \log 2 \Rightarrow x = \frac{\log 2}{\log 24}$

Alt (b): M1: applies index law to 6^{1-x} ie $6/6^x$

dM1: rearranges to equation of form $a^x = b$ using correct power laws (ie $p^x q^x = (pq)^x$) A1: Correct equation

ddM1: solve equation of the form shown using logs

A1: correctly achieved.



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

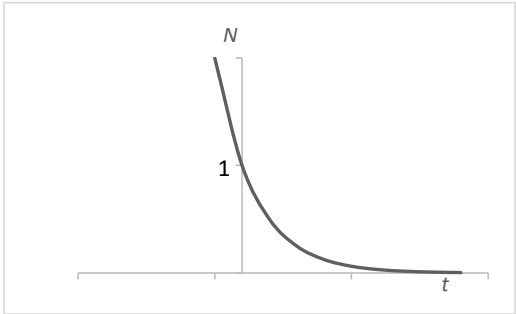
Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question	Scheme	Marks
9(a)		
	Correct shape from or passing through a point on positive vertical axis. May extend to the left of the vertical axis and allow to pass into quadrant 4. There must be no (obvious) turning points. Labels not required on axes and ignore any that are given.	M1
	Shape and position correct, accept 1 or k as intercept on the positive vertical axis and allow to extend to the left of the vertical axis as shown. Condone (1, 0) or (k, 0) as long as it is in the correct position. The curve should approach a horizontal asymptote that is half-way between the horizontal axis and the intercept or below. Be tolerant with “wobbles” as it approaches the asymptote. May just “touch” the horizontal axis but not go below it. Labels not required on axes and ignore any that are given.	A1
		(2)
(b)	$\frac{1}{2}k = k\lambda^{5700} \Rightarrow \lambda^{5700} = \frac{1}{2}$ (see notes for method via substitution)	M1
	$\Rightarrow \lambda = \left(\frac{1}{2}\right)^{\frac{1}{5700}} = 0.999878 \text{ to 6 d.p.*}$	A1*
		(2)
(c)	When $t = 3250$, $N = 15 \times 0.999878^{3250} = \dots$	M1
	$= \text{awrt } 10.1 \text{ (grams)}$	A1
		(2)
(d)	$18 = 25 \times 0.999878^t$	B1
	$\Rightarrow 0.999878^t = \frac{18}{25} \Rightarrow t = \frac{\log \frac{18}{25}}{\log 0.999878} = \dots$	M1
	$t = 2692.49\dots$ so item is 2700 years old	A1
		(3)
(9 marks)		
Notes:		
(a)		
M1: See scheme.		
A1: See scheme.		

(b)

M1: Uses the information to set up a correct equation and rearranges to form $\lambda^a = b$ where a and b are constants. Allow $\lambda^{5700} = \frac{1}{2}$ to be written down directly.

A1*: Evidence of taking fractional root seen, leading to the given answer. Alternatively, may see

logs used, $5700 \log \lambda = \log \frac{1}{2} \Rightarrow \log \lambda = \frac{-\log 2}{5700} \Rightarrow \lambda = 10^{\frac{-\log 2}{5700}} = \dots$ Look for at least one correct

intermediate step (and no incorrect ones). Allow greater accuracy e.g. 0.9998784026...

Note that the use of 5699 instead of 5700 scores M1A0

Alt:

M1: Substitutes values 0.9998775 and 0.9998785 or a tighter range containing the root (e.g. 0.999878 and 0.9998785 will do) to calculate N or just λ^t at $t = 5700$.

A1*: Correct values, with suitable conclusion that as half value occurs between these $\lambda = 0.999878$ to 6 d.p.

(c)

M1: Substitutes given values (or more accurate λ) into the equation and evaluates. Implied by a correct answer, but an incorrect answer with no evidence is M0. Allow 3249 for 3250.

Condone the use of a less accurate λ e.g. 0.9999 if the intention is clear.

A1: Awrt 10.1. Units not required. Answer only can score both marks.

(d)

B1: Sets up the correct equation from the information given. It must be clear they are using the given λ or better accuracy so clear use of e.g. 0.9999 scores B0 but allow the M1 below.

This may be implied if they write e.g. $18 = 25 \times \lambda^t$ and then go on to use the given value of λ .

Allow this mark if a different letter is used for t as long as the equation is correct.

M1: Solves an equation of the form $a = b\lambda^t$ for t to obtain a value

Must be correct log work so look for e.g. $a = b\lambda^t \Rightarrow \frac{a}{b} = \lambda^t \Rightarrow t = \log_{\lambda} \frac{a}{b}$

Or e.g. $a = b\lambda^t \Rightarrow \frac{a}{b} = \lambda^t \Rightarrow t = \log \frac{a}{b} = \log \lambda^t = t \log \lambda \Rightarrow t = \frac{\log \frac{a}{b}}{\log \lambda}$

Allow a mis-copied λ or a less accurate λ for this mark.

A1: Correct answer of 2700 (not awrt). Accept “27 hundred” or even “2692.49... so 27”

FYI use of $\lambda = 2^{\frac{1}{5700}}$ gives 2701.4 years.



Question Number	Scheme	Marks
1	$f(x) = ax^3 + 3x^2 - 8x + 2$	
	Sets $f(2) = 3 \Rightarrow 8a + 12 - 16 + 2 = 3$	M1
	$\Rightarrow 8a = 5 \Rightarrow a = \frac{5}{8}$	M1A1
		(3)
		(3 marks)

M1: Attempts to set $f(2) = 3$. The values embedded and the expression set equal to 3 is sufficient. May be implied by further work. If 2 embedded in the expression is not seen, condone slips in their evaluation provided there is still evidence that the intention was to substitute in 2.

May also be seen as $(f(2) - 3 =) a(2)^3 + 3(2)^2 - 8(2) - 1 = 0$

M1: Solves a linear equation in a arising from setting $f(\pm 2) = \pm 3$. Condone slips in their rearrangement proceeding to $a = \dots$ (May be implied by further work).

A1: $a = \frac{5}{8}$ or exact equivalent

Answers of $a = \frac{5}{8}$ may appear with very little or no working, possibly via trial and improvement. If so, then marks can only be allocated if evidence is shown.

e.g. $f(x) = \frac{5}{8}x^3 + 3x^2 - 8x + 2 \Rightarrow f(2) = 5 + 3(2)^2 - 8(2) + 2 = 3 \Rightarrow$ dividing by $(x - 2)$ gives a remainder of 3

More difficult alternative methods may be seen:

Alt I: e.g. algebraic division (note they may also divide $ax^3 + 3x^2 - 8x - 1$ by $(x - 2)$ and set the remainder equal to 0)

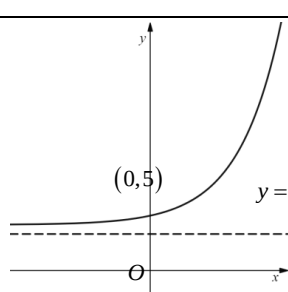
$$\begin{array}{r}
 \overline{ax^2 + (3+2a)x + (-2+4a)} \\
 x-2 \overline{) ax^3 - 8x } \\
 \underline{ax^3 - 2ax^2} \\
 (3+2a)x^2 - 8x \\
 \underline{(3+2a)x^2 + (-6-4a)x} \\
 (-2+4a)x + 2 \\
 \underline{(-2+4a)x + 4-8a} \\
 -2+8a \Rightarrow -2+8a = 3 \Rightarrow a = \frac{5}{8}
 \end{array}$$

M1: Divides the cubic by $(x - 2)$, leading to a quadratic quotient where both the coefficient of x and the constant term are in terms of a . They should then have a linear remainder in a which is then set equal to 3

M1: Solves an equation resulting from setting a linear remainder in a equal to ± 3 . It is dependent on the first method mark via this route.

A1: Completely correct with $a = \frac{5}{8}$



Question Number	Scheme	Marks
6.(a)	 Shape or asymptote Intercept Fully correct $h = 0.3$	M1 B1 A1 (3)
(b)	Area $\approx \frac{0.3}{2} \{0 + 4.3137 + 2 \times (0.3246 + 0.8629 + 1.6643 + 2.7896)\}$ = awrt 2.34	M1 A1 (3)
(c) (i)	$\int_2^{3.5} (2^x + 2x) dx = \int_2^{3.5} (2^x - 2x + 4x) dx = 2.34 + \left[2x^2 \right]_2^{3.5}$ $= 2.34 + 16.5 = 18.84$	M1 A1ft
(ii)	$\int_2^{3.5} (2^{x+1} - 4x) dx = \int_2^{3.5} 2(2^x - 2x) dx = 2 \times 2.34 = 4.68$	B1 ft (3)
		(9 marks)

(a)

M1: For either the correct shape, an increasing curve in any position in quadrants 1 and 2 **or** for the correct asymptote (labelled in some way at height 4) but must have a curve approaching as an asymptote, not just the line drawn. Be tolerant on “pen slips” approaching the asymptote.

B1: Correct intercept (touching or crossing). Accept 5 on axis, or stated as (0,5). Condone (5,0) on the graph if it is in the right place.

A1: Fully correct. Shape in quadrants 1 and 2 only with the y intercept at 5 and asymptote of $y = 4$ clearly drawn on the graph and labelled or stated separately. Be tolerant on “pen slips” approaching the asymptote. If there is conflicting information, what is on the graph takes precedence.

(b)

B1: For $h = 0.3$ This is implied by sight of $\frac{0.3}{2}$ in front of the bracket.

M1: Applies the trapezium rule with correct bracket condoning slips in copying (or even truncating) values. May use separate trapezia. Allow a missing final bracket, but otherwise bracketing must be correct or recovered by correct answer.

A1: awrt 2.34 Note that the calculator answer for this integral is 2.30

(c) (i)

M1: For realising that $\int_2^{3.5} (2^x + 2x) dx = \int_2^{3.5} (2^x - 2x + 4x) dx = 2.34 + \left[kx^2 \right]_2^{3.5}$ The 4x should

be seen with an attempt to integrate (increase in power) made (though may be implied).

Could also use answer to (b) and add the area under the trapezium $\frac{1.5}{2}(4 \times 2 + 4 \times 3.5)$ - may be implied by correct answer after seeing the integral correctly split.

Another alternative is to isolate the $\int_2^{3.5} 2^x dx = \left[x^2 \right]_2^{3.5} + "2.34"$ and use this to find a value for the integral required.

A1ft: "2.34" + 16.5 = 18.84 but follow through on their 2.34 and allow awrt to 3s.f..

There must be a suitable method for this mark to be awarded.

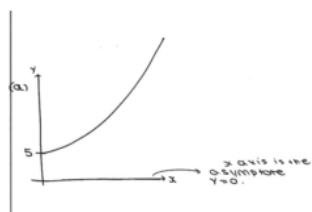
Answer via repeated trapezium rule scores M0A0

(c)(ii)

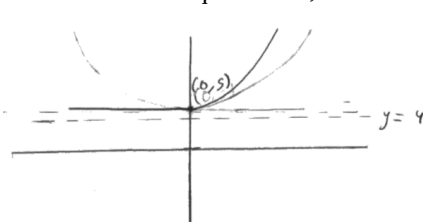
B1 ft: For $2 \times$ their "2.34" = 4.68 ((awrt to 3s.f.) must be evaluated). Follow through on their answer to (b).

Note: Answer via repeated trapezium rule is permitted to score the B1 as long the answer is twice their (b).

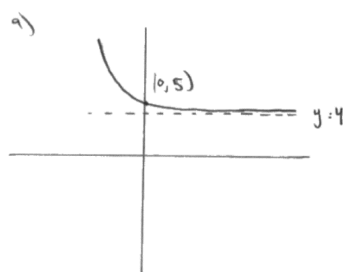
Some examples of graphs for (a):



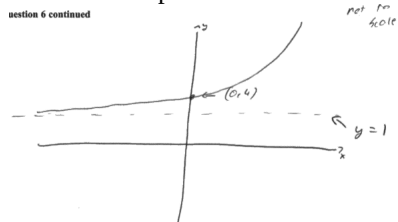
M0B1A0 Not in quadrant 2, but does have correct intercept.



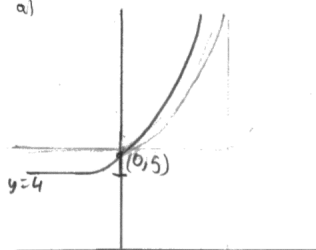
M1B1A0 Does not approach asymptote



M1B1A0 Correct asymptote and intercept



M1B0A0 Incorrect intercept and asymptote



M1B1A0 asymptote not drawn.