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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Sequences & Series (P2.5)

International A Level Mathematics · P2: Pure Mathematics 2 (WMA12)

35 questions · 261 marks · Mark schemes included

Past papers June 2019 - June 2025 · 3 sub-topics

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 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

35 questions · 261 marks · 3 sub-topics

SUB-TOPIC P2.5a

ARITHMETIC SEQUENCES & SERIES

Sub-topic P2.5a · 11 questions · 88 marks · lowest tariff first



1. The arithmetic series S is given by

$$S = 2 + 5 + 8 + 11 + ... + 254$$

Find

- (a) the number of terms in the series,

(2)

- (b) the sum of the series.

(2)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 4 marks)

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5. A company that owned a silver mine

- extracted 480 tonnes of silver from the mine in year 1
- extracted 465 tonnes of silver from the mine in year 2
- extracted 450 tonnes of silver from the mine in year 3

and so on, forming an arithmetic sequence.

- (a) Find the mass of silver extracted in year 14

(2)

After a total of 7770 tonnes of silver was extracted, the company stopped mining.

Given that this occurred at the end of year N ,

- (b) show that

$$N^2 - 65N + 1036 = 0$$

(3)

- (c) Hence, state the value of N .

(1)

Question 5 continued

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Lined area for writing the answer to Question 5.

(Total 6 marks)

Q5



7. Jem pays money into a savings scheme, A , over a period of 300 months.

Jem pays £20 into scheme *A* in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

- (a) Show that Jem pays £69.50 into scheme A in month 100

(1)

- (b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B , over the same period of 300 months.

In a model, the amounts Kim pays into scheme *B* increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

- (c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme *A* and the total amount paid into scheme *B* over the period of 300 months.

(3)







(Total for Question 7 is 6 marks)



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8. (i) An arithmetic series has first term a and common difference d .

Prove that the sum to n terms of this series is

$$\frac{n}{2}\{2a + (n-1)d\} \quad (3)$$

- (ii) A sequence $u_1, u_2, u_3, \dots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

- $$(a) \quad u_5 \tag{1}$$

$$(b) \sum_{n=1}^{59} u_n \tag{3}$$

Question 8 continued

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Question 8 continued

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Question 8 continued

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Q8

(Total 7 marks)



Question 1 continued

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(Total 7 marks)

Q1







Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 7 marks)



2.

In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

In an arithmetic series,

- the sixth term is 2
- the sum of the first ten terms is -80

For this series,

- (a) find the value of the first term and the value of the common difference.

(4)

- (b) Hence find the smallest value of n for which

$$S_n > 8000$$

(3)



(Total for Question 2 is / marks)



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7. Kim starts working for a company.
 - In year 1 her annual salary will be £16 200
 - In year 10 her annual salary is predicted to be £31 500

Model *A* assumes that her annual salary will increase by the same amount each year.

- (a) According to model A , determine Kim's annual salary in year 2. (3)

Model *B* assumes that her annual salary will increase by the same percentage each year.

- (b) According to model *B*, determine Kim's annual salary in year 2. Give your answer to the nearest £10
- (3)**

- (c) Calculate, according to the two models, the difference between the total amounts that Kim is predicted to earn from year 1 to year 10 inclusive. Give your answer to the nearest £10
- (3)**



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Question 7 continued

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(Total 9 marks)

Q7



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(Total 11 marks)

Q5



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10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A , he will increase the distance he runs each Sunday by the same amount.

- (a) Calculate the distance he will run on Sunday in week 5 under training plan A . (3)

In training plan B , he will increase the distance he runs each Sunday by the same percentage.

- (b) Calculate the distance he will run on Sunday in week 5 under training plan *B*.
Give your answer in km to one decimal place.

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
 - the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km
- (c) (i) find the maximum value of x , if he uses training plan A ,
- (ii) find the maximum value of x , if he uses training plan B .
- (5)**



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Question 10 continued

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Q10

(Total 11 marks)

END

TOTAL FOR PAPER IS 75 MARKS



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



(Total for Question 8 is 13 marks)



SUB-TOPIC P2.5b

GEOMETRIC SEQUENCES & SERIES

Sub-topic P2.5b · 14 questions · 110 marks · lowest tariff first



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2. The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

- (a) Find, according to the model, the adult population of the town at the start of 2032 (3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

- (b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000
- (3)**



Question 2 continued

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(Total 6 marks)

Q2



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 6 marks)



2. A geometric series has first term a and common ratio r .

Given that

- the third term in the series is 64
- the sixth term in the series is -8

(a) show that $r = -\frac{1}{2}$

(2)

(b) find the sum to infinity of the series.

(4)

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 6 marks)



(Total for Question 8 is 7 marks)



Given that

- the 3rd term is 20
- the 5th term is 12.8

(a) show that $r = 0.8$

(1)

(b) Hence find the value of a .

(2)

(c) find the smallest possible value of n .

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)



(Total for Question 7 is / marks)



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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A colony of bees is being studied.

The number of bees in the colony at the start of the study was 30 000

Three years after the start of the study, the number of bees in the colony is 34 000

A model predicts that the number of bees in the colony will increase by $p\%$ each year, so that the number of bees in the colony at the end of each year of study forms a geometric sequence.

Assuming the model,

(a) find the value of p , giving your answer to 2 decimal places.

(3)

According to the model, at the end of N years of study the number of bees in the colony exceeds 75 000

(b) Find, showing all steps in your working, the smallest integer value of N .

(5)



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Q5

Question 8 continued

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Question 8 continued

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Q8

- the common ratio is r
- $u_2 + u_3 = 6$
- $u_4 = 8$

$$3r^2 - 4r - 4 = 0$$

Given that the geometric sequence has a sum to infinity,

(3)

(2)



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Q6

Question 10 continued

Lined area for writing the answer to Question 10 continued.



Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62mm into the ground with the 20th hit. (1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits. (2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r , giving your answer to 3 decimal places. (2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N . (4)

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Question 8 continued

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Q8

(Total 9 marks)



6.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \quad k \quad k + 4$$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

- (a) show that $k^2 - 7k - 60 = 0$

(2)

- (b) predict the number of downloads in the 4th month.

(4)

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the N th month.

- (c) Calculate the value of N according to the model.

(3)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 9 marks)







(Total for Question 7 is 2 marks)



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 9 marks)



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7. (i) A geometric sequence has first term 4 and common ratio 6
Given that the n^{th} term is greater than 10^{100} , find the minimum possible value of n . (3)

- (ii) A different geometric sequence has first term a and common ratio r .

Given that

- the second term of the sequence is -6
- the sum to infinity of the series is 25

- (a) show that

$$25r^2 - 25r - 6 = 0 \quad (3)$$

- (b) Write down the solutions of

$$25r^2 - 25r - 6 = 0 \quad (1)$$

Hence,

- (c) state the value of r , giving a reason for your answer, (1)
- (d) find the sum of the first 4 terms of the series. (2)

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Question 7 continued

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Lined area for writing the answer to Question 7 continued.



Question 7 continued

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Lined area for writing the answer to Question 7.

(Total 10 marks)

Q7



SUB-TOPIC P2.5c

RECURRENCE RELATIONS & SIGMA NOTATION

Sub-topic P2.5c · 10 questions · 63 marks · lowest tariff first



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Answer all questions. Write your answers in the spaces provided.

1. A sequence $a_1, a_2, a_3, \dots$ is defined by

$$\begin{aligned} a_{n+1} &= 4 - a_n \\ a_1 &= 3 \end{aligned}$$

Find the value of

- (a) (i) a_2

- $$(ii) \ a_{107}$$

(2)

- $$(b) \sum_{n=1}^{200} (2a_n - 1)$$

(2)

Question 1 continued

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Lined area for writing the answer to Question 1.

(Total 4 marks)

Q1

Mark box for Q1.



Question 2 continued

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Lined area for writing the answer to Question 2.

(Total 5 marks)

Q2

Box for marking the question.



Question 2 continued

(Total for Question 2 is 5 marks)



1. The sequence $u_1, u_2, u_3, \dots$ satisfies

$$u_{n+2} = 3u_{n+1} - 2u_n$$

Given that

- $u_1 = 7$

- $u_3 = 4$

(a) find the value of u_2

(2)

(b) find $\sum_{r=1}^4 (u_r + 2r)$

(3)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)



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Question 8 continued

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Question 8 continued

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(Total 7 marks)

Q8



(Total for Question 3 is 7 marks)



(Total for Question 2 is / marks)



Question 5 continued

(Total for Question 5 is 7 marks)



Question 11 continued

Lined area for writing the answer to Question 11.



Question 11 continued

Lined area for writing the answer to Question 11.

(Total for Question 11 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS







(Total for Question 3 is 6 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
1 (a)	$254 = 2 + 3(n-1) \Rightarrow n = \dots$	M1
	85	A1
		(2)
(b)	$\frac{"85"}{2}(2 \times 2 + ("85"-1) \times 3)$	M1
	10 880	A1
		(2)
		4 marks

(a)

M1: Attempts to find the number of terms.

Look for $254 = 2 + 3(n-1) \Rightarrow n = \dots$, $3n-1 = 254 \Rightarrow n = \dots$ or $\left(\frac{254-2}{3}\right) + 1$ o.e. condoning slips such as writing 254 as 245. The values of a and d must be correct.

Sight of either $\frac{254-2}{3}$, $\frac{252}{3}$ or 84 without any incorrect working can score this mark.

Note that $\frac{254}{3} = 84$ or similar calculations that give non-integer solutions (with no further working e.g. trial and error attempts) score M0 even if rounded

A1: 85.

An answer of 85 **with no incorrect working** or without any working scores both marks

.....
Alt (a) Trial and error attempts

You may see candidates using $2 + (n-1) \times 3$ with various values for n

M1: Attempts $2 + (n-1) \times 3$ with $n = 85$ A1: Achieves $2 + 84 \times 3 = 254$ **and** concludes 85 (terms)

.....
(b)

M1: Attempts the sum formula $\frac{n}{2}(2a + (n-1)d)$ or $\frac{n}{2}(a + l)$ following through on their value for n .

Look for $\frac{"85"}{2}(2 \times 2 + ("85"-1) \times 3)$, $\frac{"85"}{2}(2 \times 254 + ("85"-1) \times -3)$ or $\frac{"85"}{2}(2 + 254)$

The values of a , d and l if used must be correct (condoning a slip such as 245 for 254) but follow through on their value of n from part (a)

A1: 10 880.

This may be awarded even following an 85 in (a) that was found using an incorrect method

10 880 without working or following listing scores both marks.

Listing leading to an incorrect answer scores M0 A0

Question Number	Scheme	Marks
4.(a)	States -21	B1 (1)
(b)	Attempts $f(3) = (3^2 - 2)(2 \times 3 - 3) - 21$ Achieves $f(3) = 0 \Rightarrow (x - 3)$ is a factor of $f(x)$ *	M1 A1* (2)
(c) (i)	$f(x) = 2x^3 - 3x^2 - 4x - 15 = (x - 3)(2x^2 \dots \pm 5)$ $= (x - 3)(2x^2 + 3x + 5)$	B1 M1 A1
(ii)	Attempts $b^2 - 4ac$ for their $2x^2 + 3x + 5$ Achieves $b^2 - 4ac < 0$ and states that only root is $x = 3$ o.e. *	M1 A1* (5) (8 marks)

(a)

B1: -21

(b)

M1: Attempts to substitute $x = 3$ into $f(x) = (x^2 - 2)(2x - 3) - 21$ or its expanded form.Condone slips but don't accept just $f(3) = 0$. Attempts via long division score M0.A1*: Achieves $f(3) = 0$ and states that $(x - 3)$ is a factor of $f(x)$. If this latter is given in a preamble accept a minimal conclusion such as // or QED

(c)(i)

B1: Expands $f(x) = (x^2 - 2)(2x - 3) - 21$ to reach correct 4 term cubic. Allow if the cubic is seen anywhere.M1: Correct attempt to find quadratic factor using either division by $(x - 3)$ or inspection.

To score the M mark look for e.g. correct first and last terms by inspection, or first two terms by division. (Two correct terms will imply the mark.) Allow for work seen in (b) as long as it is referred to in (c).

A1: $(f(x) = (x - 3)(2x^2 + 3x + 5))$ following a correct cubic. Must be seen together on one line.

(c) (ii)

M1: Attempts to show their quadratic factor has no real roots. Factorisation attempts are M0.
Accept via

- an attempt at $b^2 - 4ac$ for their quadratic factor
- an attempt to solve their $2x^2 + 3x + 5$ using the quadratic formula or calculator
- an attempt to complete the square

A1*: Requires correct factorisation, correct calculation, reason and conclusion

For example after (c)(i) $f(x) = (x-3)(2x^2 + 3x + 5)$ accept e.g.

- $2x^2 + 3x + 5$ has no roots as $3^2 - 4 \times 2 \times 5 < 0$ so $f(x) = 0$ only has root at $x = 3$
- $2x^2 + 3x + 5 = 0 \Rightarrow x = -\frac{3}{4} \pm i\frac{\sqrt{31}}{4}$, $(x-3) = 0 \Rightarrow x = 3$. So only one real root.
- $2x^2 + 3x + 5 = 0 \Rightarrow 2\left(x + \frac{3}{4}\right)^2 - \frac{9}{8} + 5 \dots \frac{31}{8} > 0$, so no roots hence 3 is the only root

There must be some reference to the root either by stating it or indicating the linear term has a root (e.g. writing “root” next to it), but do not accept incorrect statements such as only real root is $(x-3)$

Do not allow statements such as “Math error” without interpretation of what this means.



Question Number	Scheme	Marks
5.(a)	Attempts $480 + 13 \times -15$ $= 285$ tonnes	M1 A1 (2)
(b)	Sets $\frac{N}{2} \{2 \times 480 + (N-1) \times -15\} = 7770$ $\frac{N}{2} \{960 - 15N + 15\} = 7770 \Rightarrow N^2 - 65N + 1036 = 0^*$	M1A1 A1* (3)
(c)	States 28 only	B1 (1) (6 marks)

(a)

M1: Attempts to use $a + (n-1)d$ with at least two of a , d and n correct. (So allow with $a + nd$ if both a and d are correct, bod.) They may have $d > 0$ for this mark.

A1: 285 tonnes. Condone the omission of the units

(b)

Note: Use of n instead of N is fine throughout part (b).

M1: Attempts to use a correct sum formula with $S = 7770$ using their a and d , condoning slips.

A1: A correct equation for N . E.g. $\frac{N}{2} \{2 \times 480 + (N-1) \times -15\} = 7770$

A1*: Achieves $N^2 - 65N + 1036 = 0$ (including the “=0”) following a correct intermediate line (see scheme) and with no errors, although “=0” may be implied in intermediate steps.

(c)

B1: States 28 only. The 37 should be rejected if found. 29 is B0.

Question Number	Scheme	Marks
3(a)	$2(-3)^3 - (-3)^2 + A(-3) + B = 55$ or e.g. $-54 - 9 - 3A + B = 55$	M1
	$-54 - 9 - 3A + B = 55$ $\Rightarrow 3A - B = -118^*$	A1*
		(2)

(a)

M1: Attempts $f(-3) = 55$. The -3 embedded in the expression set = 55 is sufficient.Condone missing brackets e.g. $2 - 3^3 - -3^2 + A(-3) + B = 55$ Note that $2(-3)^3 - (3)^2 + A(-3) + B = 55$ is acceptable.May be implied by further work. If -3 embedded in the expression is not seen, condone slips in their evaluation provided there is still evidence that the intention was to substitute in -3 and set = 55 but $f(3) = 55$ is M0**A1*:** Completes proof with at least one intermediate simplified and correct line that is not the final line such as $-54 - 9 - 3A + B = 55$ or $118 = -3A + B$. Any incorrect lines written after the start of their 'proof' should be scored A0*Must be the given equation and not e.g. $B = 3A + 118$ unless the correct equation is seen previously but condone $-B + 3A = -118$ $2(-3)^3 - (-3)^2 + A(-3) + B = 55 \Rightarrow 3A - B = -118$ scores M1A0

Tabular or division methods may be seen. The question does not demand use of the factor or remainder theorem so these methods should be rewarded.

The M1 would be scored for a full attempt and A1* for a correct proof.

If you cannot see what they are doing and find it hard to award, then use review.

e.g. long division:

$$\begin{array}{r}
 2x^2 - 7x + A + 21 \\
 x + 3 \overline{) 2x^3 - x^2 + Ax + B} \\
 \underline{2x^3 + 6x^2} \\
 -7x^2 + Ax + B \\
 \underline{-7x^2 - 21x} \\
 (A + 21)x + B \\
 \underline{(A + 21)x + 3(A + 21)} \\
 B - 3A - 63 = 55
 \end{array}$$

Score M1 for obtaining a quotient $\dots x^2 \pm \dots x \pm \dots A + \dots$ and a remainder $\dots A \pm \dots B \pm \dots$ set = 55 where “...” are non-zero constants. Then A1* for a fully correct proof.

e.g. tabular method:

	$2x^2$	$-7x$	$21 + A$
x	$2x^3$	$-7x^2$	$(21 + A)x$
3	$6x^2$	$-21x$	$63 + 3A$

So $B - (63 + 3A) = 55$ etc.Score M1 for obtaining a constant $\dots A + \dots$ where ... are non-zero constants and then sets $B - \text{their constant} = 55$. Then A1* for a fully correct proof.

(b)	$2\left(\frac{5}{2}\right)^3 - \left(\frac{5}{2}\right)^2 + A\left(\frac{5}{2}\right) + B = 0$	M1
	$3A - B = -118, 5A + 2B = -50$ $\Rightarrow A = \dots, \text{ or } B = \dots$	M1
	$A = -26, B = 40$	A1
		(3)

(b) Do not allow mis-reads of $(2x - 5)$ in part (b).

M1: Attempts $f\left(\frac{5}{2}\right) = 0$. The $\frac{5}{2}$ embedded in the expression set = 0 is sufficient.

$f\left(-\frac{5}{2}\right) = 0$ scores M0.

May be implied by further work. Again, tabular or division methods may be seen.
e.g. long division:

$$\begin{array}{r}
 x^2 + 2x + \frac{1}{2}A + 5 \\
 2x - 5 \overline{) 2x^3 - x^2 + Ax + B} \\
 \underline{2x^3 - 5x^2} \\
 4x^2 + Ax + B \\
 \underline{4x^2 - 10x} \\
 (A + 10)x + B \\
 \underline{(A + 10)x - 5\left(\frac{1}{2}A + 5\right)} \\
 B + \frac{5}{2}A + 25 = 0
 \end{array}$$

Score M1 for obtaining a quotient $\dots x^2 \pm \dots x \pm \dots A + \dots$ and a remainder $\dots A \pm \dots B \pm \dots$ set = 0 where “...” are non-zero constants.

e.g. tabular method:

	x^2	$2x$	$5 + \frac{A}{2}$
$2x$	$2x^3$	$4x^2$	$(10 + A)x$
-5	$-5x^2$	$-10x$	$-25 - \frac{5A}{2}$

So $B - \left(-25 - \frac{5A}{2}\right) = 0$. Score M1 for obtaining a constant $\dots A + \dots$ where “...” are

non-zero constants, and then sets $B - \text{their constant} = 0$

M1: Solves $3A - B = -118$ simultaneously with their equation in A and B from having attempted $f\left(\pm\frac{5}{2}\right) = 0$ or long division/tabular method using the correct factor of

$(2x - 5)$ and obtains a value for A or B .

May be via a calculator and may be implied by their values. You do not need to check but the above conditions must be satisfied.

For reference, use of $f\left(-\frac{5}{2}\right) = 0$ gives $A = -161, B = -365$

A1: Both correct values

(c)	$f(x) = (x-7)(2x^2 + \dots x + \dots) + \dots$	M1
	$2x^2 + 13x + 65$	A1
		(2)
		Total 7

(c) Do not allow mis-reads of $(x-7)$ in part (c).

M1: Uses any appropriate method e.g. long division/inspection/table with numeric non-zero A and B to obtain $2x^2 + px + q$ where p and q are non-zero.
There is no requirement to find the remainder but assuming $(x-7)$ is a **factor** is M0 (see example below)

A1: Must follow correct values of A and B .

Correct expression $2x^2 + 13x + 65$ which is clearly identified e.g. by circling/underlining or e.g. Quotient = $2x^2 + 13x + 65$, $Q = 2x^2 + 13x + 65$
Any remainder, correct or incorrect, can be ignored. (NB correct remainder is 495)

Long division for reference:

$$\begin{array}{r}
 2x^2 + 13x + 65 \\
 x-7 \overline{) 2x^3 - x^2 - 26x + 40} \\
 \underline{2x^3 - 14x^2} \\
 13x^2 - 26x + 40 \\
 \underline{13x^2 - 91x} \\
 65x + 40 \\
 \underline{65x - 455} \\
 495
 \end{array}$$

Tabular method for reference:

	$2x^2$	$13x$	65
x	$2x^3$	$13x^2$	$65x$
-7	$-14x^2$	$-91x$	-455

But note that **just** the above score M1A0 with no indication that the quotient is $2x^2 + 13x + 65$

Some candidates assume that $(x-7)$ is a **factor** of $f(x)$ and try to find the quadratic factor:

$$\begin{array}{l}
 c) \quad 2x^3 - x^2 - 26x + 40 = (x-7)(2x^2 + kx - \frac{40}{7}) \div (x-7) \\
 -7k - \frac{40}{7} = -26 \Rightarrow k = \frac{142}{49} \\
 (2x^2 + \frac{142}{49}x - \frac{40}{7}) \text{ is the quotient}
 \end{array}$$

This scores M0 in part (c)

But work that doesn't assume $(x-7)$ is a factor is fine e.g.:

$$\begin{array}{l}
 c) \quad 2x^3 - x^2 - 26x + 40 \\
 (2x-7)(ax^2 + bx + c) \\
 2x^3 = 2ax^3 \quad a=2 \\
 -x^2 = -14x^2 + 7bx^2 \\
 13x^2 = 7bx^2 \quad b=13 \\
 -26x = -91x + 7cx \\
 65x = 7cx \quad c=65 \\
 2x^3 + 13x^2 + 65x
 \end{array}$$

Scores M1A1



Question Number	Scheme	Marks
4(a)	$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3$ $\Rightarrow \left(\frac{dy}{dx}\right) = 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}}$	M1A1
		(2)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow 4x - 9 = 0 \Rightarrow x = \dots$	M1
	$x = \frac{9}{4} \text{ oe e.g. } 2.25$	A1
		(2)

(a)**M1:** For $x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$ or $x^{-\frac{1}{2}} \rightarrow x^{-\frac{3}{2}}$. Allow unprocessed e.g. $x^{\frac{1}{2}} \rightarrow x^{\frac{1}{2}-1}$ or $x^{-\frac{1}{2}} \rightarrow x^{-\frac{1}{2}-1}$.**A1:** Correct simplified derivative. $\frac{dy}{dx}$ = is not required. Isw once a correct answer is seen.Allow equivalent simplified expressions e.g. $2x^{-\frac{1}{2}} - 4.5x^{-\frac{3}{2}}$ $2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} + c$ scores A0 as does $2x^{-\frac{1}{2}} + -\frac{9}{2}x^{-\frac{3}{2}}$ **(b)****M1:** Starts from a derivative of the form $Ax^{-\frac{1}{2}} - Bx^{-\frac{3}{2}}$ where $A \times B > 0$, sets = 0 (seen or implied) and solves via a correct method to obtain an expression or value of $x = \frac{B}{A}$.**A1:** $x = \frac{9}{4}$ oe with no incorrect working seen in their method.Condone if e.g. $\frac{dy}{dx} > 0$ is seen as long as they reach the correct value for x .There must be no other values. **Must come from a correct derivative but see * note below.**The minimum acceptable for both marks is $2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow x = \frac{9}{4}$

The following is condoned:

$$2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow \frac{2}{x^{\frac{1}{2}}} - \frac{9}{2x^{\frac{3}{2}}} = 0 \Rightarrow \frac{4}{x} - \frac{81}{4x^3} = 0 \Rightarrow x = \frac{9}{4}$$

Ignore any attempts to find the y coordinate.***Allow full recovery in (b) if the working is correct in (a) but isw has been applied e.g.**

$$(a) \left(\frac{dy}{dx}\right) = 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}} \text{ M1A1 (isw)}$$

$$(b) 4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}} = 0 \Rightarrow x = \frac{9}{4} \text{ M1A1}$$



4(c)(i)	$\left(\frac{d^2y}{dx^2}\right) = -x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}}$ oe e.g. $-x^{-\frac{3}{2}} + 6.75x^{-\frac{5}{2}}$	B1ft
(ii)	$\left(\left(\frac{d^2y}{dx^2}\right)_{x=\frac{9}{4}}\right) = -\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}} \left(= \frac{16}{27}(0.5925\dots)\right)$ $\left(\frac{d^2y}{dx^2}\right) > 0 \text{ so (local) minimum}$	B1
		(2)
(d)	$0 < x < \frac{9}{4}$	B1ft
		(1)
		Total 7

(c)(i)**B1ft:** Correct simplified second derivative.Follow through their first derivative provided there are 2 terms with **different** fractional (non-integer) powers. $\frac{d^2y}{dx^2} =$ is not required.Condone $-1x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}}$ for $-x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}}$

Inclusion of “+ c” scores B0ft as with the first derivative.

(ii)**B1:** Fully correct working, reasoning and conclusion. This requires:

- a correct second derivative
- substitutes $x = \frac{9}{4}$ to obtain $\frac{16}{27}$ or awrt 0.6 or truncated 0.5
- reference to the sign (> 0 or positive). Must see the evaluation as it is not obviously positive but allow if expression is not fully evaluated as long as it is clear it is positive (e.g. $\frac{8}{9} - \frac{8}{27} > 0$).
- reference to minimum

Condone work that doesn't specifically reference $\frac{d^2y}{dx^2}$ e.g.
$$-\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}} = \frac{16}{27} > 0 \text{ so minimum is fine for this mark.}$$
(d)**B1ft:** For $0 < x < \frac{9}{4}$. Allow $0 < x \leq \frac{9}{4}$ and allow equivalent statements e.g.
$$\left(0, \frac{9}{4}\right), \left[0, \frac{9}{4}\right].$$
Follow through their positive x value from part (b). Condone an answer of $x < \frac{9}{4}$ or
$$x \leq \frac{9}{4} \quad \text{Condone } 0 \leq x \leq \frac{9}{4}$$
Do not allow $-\infty < x < \frac{9}{4}$

Question Number	Scheme	Marks
7 (a)	$8\tan\theta = 3\cos\theta$	
	Uses $\tan\theta = \frac{\sin\theta}{\cos\theta} \rightarrow 8\frac{\sin\theta}{\cos\theta} = 3\cos\theta$	M1
	$8\sin\theta = 3\cos^2\theta$ oe	
	Uses $\cos^2\theta = 1 - \sin^2\theta \rightarrow 8\sin\theta = 3(1 - \sin^2\theta)$	M1
(b)	$3\sin^2\theta + 8\sin\theta - 3 = 0$ *	A1 *
	$(3\sin 2x - 1)(\sin 2x + 3) = 0$	M1
	Critical value(s) of $\frac{1}{3}, (-3)$	A1
	Correct method to find x from their $\sin 2x = \frac{1}{3}$	dM1
	$x = \text{awrt } 9.74^\circ, 80.26^\circ$	A1
		(4)
		(7 marks)

(a)

M1 Uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (oe) in $8\tan\theta = 3\cos\theta$. Condone slips in coefficients.

M1 Multiplies by $\cos\theta$ and uses $\cos^2\theta = 1 - \sin^2\theta$ (oe) to produce an equation in just $\sin\theta$

A1* Proceeds to $3\sin^2\theta + 8\sin\theta - 3 = 0$ with no arithmetical or notational errors.
No mixed variables within the lines of the "proof"

An example of a notational error is $\cos\theta^2$ for $\cos^2\theta$ (Note that this would not lose the M1)

(b)

M1 Attempts to find the critical values of the given quadratic. Implied by correct values given if no working shown. Allow if just $\frac{1}{3}$ is given.

A1 Critical value of $\frac{1}{3}$ and no incorrect second value. The -3 need not be stated. Accept awrt 0.333 for $\frac{1}{3}$

dM1 A correct method to find one value of x from their $\sin 2x = \frac{1}{3}$

It requires both arcsin and $\div 2$ and may be implied by one answer to one decimal place rounded or truncated eg awrt 9.7 or awrt 80.3 (or for 2sf radian answer awrt 0.17, or awrt 1.4)

A1 $x = \text{awrt } 9.74^\circ, 80.26^\circ$. A0 if extra solutions are given in the range but ignore solutions outside.

Note that M1A0M1A1 is possible if correct answers come from a critical value of $\frac{1}{3}$ if a second incorrect C.V. was also found but does not lead to extra solutions in the range.



Question Number	Scheme	Marks
8.(i)	States $(S =) a + (a + d) + \dots \{a + (n - 2)d\} + \{a + (n - 1)d\}$	B1
	$(S =) \{a + (n - 1)d\} + \{a + (n - 2)d\} + \dots (a + d) + a$ and adds	M1
	$2S = n(2a + (n - 1)d) \Rightarrow S = \frac{n}{2}\{2a + (n - 1)d\}$ *	A1*
		(3)
(ii)	(a) $u_5 = 22$	B1
	(b) $\sum_{n=1}^{59} u_n = (5 + 10 + 15 + \dots) + (-3 + 3 - 3 + \dots)$ $= \frac{59}{2}\{10 + 58 \times 5\} + (-3) = 8850 - 3 = 8847$	M1 B1 A1
		(4)
		(7 marks)

(i)

B1 Writes down an expression for S in a minimum of 3 in a and d terms including the first and last terms.
 Eg. States that $S = a + (a + d) + (a + 2d) + \dots + a + (n - 2)d + a + (n - 1)d$

$S = a + (a + d) + \dots + l$ scores B1 only if $l = a + (n - 1)d$ is later identified as only two terms in a and d .

M1 Attempts to reverse their sum and add terms. Must include at least two pairs of matching terms to be enough to establish the pattern (allow if second sum misses last terms).

A1* Correctly achieves the given result including the intermediate line $2S = n\{2a + (n - 1)d\}$ There must be no errors and at least 3 terms should have been shown for the sum and its reverse.

If $S = a_1 + a_2 + \dots + a_n$ is used allow the B and final A only if $a_m = a + (m - 1)d$ (oe) is clearly identified in the working, or other clear reasoning why each term gives $2a + (n - 1)d$, but the M may be gained.

If commas used instead of + in the summation, eg $S = a, (a + d), \dots, a + (n - 2)d, a + (n - 1)d$ the score

B0 as no correct sum, but allow M1A1 if the sum is implied by working and all else is correct.

If you see other attempts you feel are worthy of credit then consult your team leader.

(ii) (a)

B1 22

(ii) (b)

M1 Attempts to use the sum of an AP with $n = 59, a = 5, d = 5$ Also allow $\frac{n}{2}(a + l) = \frac{59}{2}(5 + 59 \times 5)$

B1 For $\sum_{n=1}^{59} 3 \times (-1)^n = -3$

A1 8847

Listing terms can score 3/3 as it is a correct method and can be marked per main scheme.

Answers with no incorrect working can score 3/3 If the correct answer appears from incorrect working then apply SC M0B1A0.

There are other variations on how to do this part (b)(ii) such as

Alt 1: Splits to odd and even terms:

$$S_{\text{odd}} + S_{\text{even}} = \frac{1}{2}(2 + 292) \times 30 + \frac{1}{2}(13 + 293) \times 29 = 4410 + 4437 = 8847$$

M1 separates into the two sequences and applies summation formula to at least one

B1 Correct expression for both summations (must have correct number of terms for each)

A1 8847

Alt 2: Pairs terms (or can be first term + pairs)

$$S = (2 + 13) + (12 + 23) + \dots + (282 + 293) + 292$$

$$= 15 + 35 + \dots + 575 + 292 = \frac{29}{2}(15 + 575) + 292 = 8847$$

M1 pairs terms appropriately, may use $1^{\text{st}} + (2^{\text{nd}} + 3^{\text{rd}}) + \dots + (58^{\text{th}} + 59^{\text{th}})$ and applies summation formula to the terms

B1 For the extra term correctly shown $2 + \dots$ or $\dots + 292$

A1 8847

In general apply

M1 For a correct overall strategy that includes a summation

B1 For dealing with the $\sum (-1)^n$ correctly within the strategy (which may be for a correct overall expression in many cases).

A1 8847

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
7(a)	States or implies that gradient $XN = -\frac{2}{5}$ Uses $(4, -3)$ with a gradient of $\pm\frac{2}{5}$ to form equation of XN $y+3 = -\frac{2}{5}(x-4)$ $5y+15 = -2x+8 \Rightarrow 2x+5y+7 = 0$ *	B1 M1 A1* (3)
(b)	(i) Solves $2x+5y+7=0$ and $y = \frac{5}{2}x - \frac{55}{2} \Rightarrow x = \dots, y = \dots$ $N = (9, -5)$ (ii) Attempts $(9-4)^2 + (-5+3)^2$ Correct form of equation E.g. $(x-4)^2 + (y+3)^2 = (9-4)^2 + (-5+3)^2$ $(x-4)^2 + (y+3)^2 = 29$	M1 A1 M1 M1 A1 (5) (8 marks)

(a) **This is a given answer so be careful of students who work backwards.**

B1: States or implies that gradient $XN = -\frac{2}{5}$ o.e such as -0.4

This cannot be scored from candidates who show you they are just working back from the given answer.

M1: Uses $(4, -3)$ with a gradient of $\pm\frac{2}{5}$ o.e. to form equation of XN

E.g $\pm\frac{2}{5} = \frac{y-(-3)}{x-4}$. If the form $y = mx + c$ is used look for $-3 = \left(\pm\frac{2}{5}\right) \times 4 + c$ leading to $c = \dots$

A1*: Shows sufficient working and proceeds to $2x+5y+7=0$. There should be one correct line

showing working between $y+3 = -\frac{2}{5}(x-4)$ and $2x+5y+7=0$, for example $5y+15 = -2x+8$

If $y = mx + c$ is used expect to see $c = -\frac{7}{5}$ and $y = -\frac{2}{5}x - \frac{7}{5}$ before $2x+5y+7=0$.

Note that this is a given answer so there cannot be incorrect work used leading to this answer.

(b)(i)

M1: Attempts to solve $2x+5y+7=0$ and $y = \frac{5}{2}x - \frac{55}{2}$ leading to $x = \dots, y = \dots$

Allow the values to be just written down, for example via use of a calculator

A1: $N = (9, -5)$ which may be written $x=9, y=-5$



(b)(ii)

M1: Attempts radius or radius². Condone slips on lhs, for example r or $d = ("9"-4)^2 + ("-5"+3)^2$

You must see an attempt to take the difference for both components before each are squared.

M1: Correct form of equation E.g. $(x-4)^2 + (y+3)^2 = "r" ^2$

Look for $(x-4)^2 + (y+3)^2 = ("9"-4)^2 + ("-5"+3)^2$

Condone for this mark a slip on one of the components when finding their r^2

A1: $(x-4)^2 + (y+3)^2 = 29$ or exact equivalent. The constant terms must be collected and not $25 + 4$

ISW after a correct answer.

Alt (b)(ii)

Note that an alternative version is $x^2 + y^2 - 8x + 6y - 4 = 0$. It is possible to proceed using this alternative version of the formula for a circle

M1: States equation of circle is $x^2 + y^2 - 8x + 6y + c = 0$

M1: Substitutes their $N = (9, -5)$ into $x^2 + y^2 - 8x + 6y + c = 0$ o.e. and finds "c"

A1: $x^2 + y^2 - 8x + 6y - 4 = 0$ o.e.

Question Number	Scheme	Marks
8. (a)	$\text{Uses } S_{10} = 360 \Rightarrow \frac{10}{2} \{2 \times a + 9 \times 4\} = 360$ $\Rightarrow 2a + 36 = 72 \Rightarrow a = 18$	M1 A1 (2)
(b)	$\text{Uses } S_n = 2146 \Rightarrow \frac{n}{2} \{2 \times "18" + (n-1) \times 4\} = 2146$ $\Rightarrow n \{16 + 2n\} = 2146$ $\Rightarrow n^2 + 8n - 1073 = 0 \quad *$	M1 A1* (2)
(c)	(i) States 29 (ii) Attempts $"18" + ("29" - 1) \times 4 = 130$	B1 M1, A1 (3)
		(7 marks)

(a)

M1: Uses $S_{10} = 360$ with a correct formula to set up a linear equation in " a ". E.g. $\frac{10}{2} \{2 \times a + 9 \times 4\} = 360$

Alternatively uses the sum of all 10 terms

$$a + (a + 4) + (a + 8) + (a + 12) + (a + 16) + (a + 20) + (a + 24) + (a + 28) + (a + 32) + (a + 36) = 360$$

A1: $a = 18$ following a correct equation

(b)

M1: Attempts to use $S_n = 2146 \Rightarrow \frac{n}{2} \{2 \times "18" + (n-1) \times 4\} = 2146$ following through on their value of " a "

A1*: Proceeds to $n^2 + 8n - 1073 = 0$ showing sufficient working, which in most circumstances should be regarded as one correct simplified intermediate line. (See main scheme)

(c)(i)

B1: States 29.

The -37 if written down must be deleted/crossed out, or the 29 chosen by being "ringed" or underlined

(c)(ii)

M1: Attempts $a + (n-1) \times 4$ with their values of a and n . E.g. $"18" + ("29" - 1) \times 4$

$$\text{Alternatively uses } 2146 \Rightarrow \frac{29}{2} \{ "18" + l \} \Rightarrow l = \dots$$

A1: Maximum number of seats = 130 which may be labelled $l = 130$

This alone scores both marks (but only if they had $a = 18$ and $n = 29$ earlier)



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
7.(a)	Attempts to use $31\,500 = 16\,200 + 9d$ to find 'd' For $16\,200 +$ their $d = (1700)$ where d has been found by an allowable method Year 2 salary is (£)17 900	M1 M1 A1 (3)
(b)	Attempts to use $31\,500 = 16\,200r^9$ to find 'r' For $16\,200 \times$ their $r = (1.077)$ where r has been found by an allowable method Year 2 salary in the range $17440 \leq S \leq 17450$	M1 M1 A1 (3)
(c)	Attempts $\frac{10}{2}\{16200 + 31500\}$ or $\frac{16200(1.077^{10} - 1)}{1.077 - 1}$ Finds $\pm \left(\frac{10}{2}\{16200 + 31500\} - \frac{16200(1.077^{10} - 1)}{1.077 - 1} \right)$ Difference = £7480 cao	M1 dM1 A1 (3) (9 marks)
Notes		
(a)		
M1	Attempts to use the AP formula in an attempt to find 'd' Accept an attempt at $31\,500 = 16\,200 + 9d$ resulting in a value for d . Accept the calculation $\frac{31500 - 16200}{9}$ condoning slips on the 31500 and 16200	
M1	A correct attempt to find the second term by adding 16 200 to their 'd' which must have been found via an allowable method. Allow d to be found from an "incorrect" AP formula with $10d$ being used instead of $9d$. Eg $31\,500 = 16\,200 + 10d$ or more likely $\frac{31500 - 16200}{10} = 1530$ usually leading to an answer of 17730	
A1	Year 2 salary is (£) 17 900	
(b)		
M1	Attempts to use the GP formula in an attempt to find 'r' Accept an attempt at $31\,500 = 16\,200r^9 \Rightarrow r^9 = \frac{31500}{16200} \Rightarrow r = \dots$ condoning numerical slips. Accept the calculation $\sqrt[9]{\frac{31500}{16200}}$ or $\sqrt[9]{\frac{35}{18}}$ condoning slips on the 31500 and 16200. It will most likely be implied by a value of r rounding to 1.08 Accept an attempt at $31\,500 = 16\,200r^9$ via logs condoning slips but correct log work must be seen	

Question Number	Scheme	Marks
M1	A correct attempt to find the second term by multiplying 16 200 by their 'r' which must have been found via an allowable method. Allow r to be found from an "incorrect" GP formula with 10 being used instead of 9. Eg following $31500 = 16\,200r^{10}$ or $\sqrt[10]{\frac{31500}{16200}}$. You may also award, condoning slips, for an attempt at $16200 \times r$ where r is their solution of $31500 = 16\,200r^n$ where $n = 9$ or 10	
A1	For an answer in the range $\pounds 17440 \leq S \leq 17450$ Note that $r = 1.077 \Rightarrow 17447.40$	
(c)		
M1	A correct method to find the sum of either the AP or the GP For the AP accept an attempt at either $\frac{10}{2}\{16200 + 31500\}$ or $\frac{10}{2}\{2 \times 16200 + 9 \times 'd'\}$ For the GP accept an attempt at either $\frac{16200(r^{10} - 1)}{r - 1}$ or $\frac{16200(1 - r^{10})}{1 - r}$	
dM1	Both formulae must be attempted "correctly" (see above) and the difference taken (either way around) FYI if d and r are correct, the sums are $\pounds 238\,500$ and $\pounds 231\,019.(24)$	
A1	Difference = $\pounds 7480$ CAO. Note that this answer is found using the unrounded value for r. Note that using the rounded value will give $\pounds 7130$ which is A0	
If the solutions for (a) and (b) are reversed, eg GP in (a) and AP in (b) then please send to review.		
(i)	General approach to marking part (i) This is now marked M1 A1 M1 A1 on open	
M1	Takes log of both sides and uses the power law. Accept any base. Condone missing brackets	
A1	For a correct linear equation in x which only involve logs of base 2 usually $\log_2 6$, $\log_2 2$ or $\log_2 8$ but sometimes $\log_2 \frac{3}{4}$ and others so read each solution carefully	
M1	Attempts to use a log law to create a linear equation in $\log_2 3$ Eg. $\log_2 6 = \log_2 2 + \log_2 3$ which is implied by $\log_2 6 = 1 + \log_2 3$ Eg. $\log_2 \frac{3}{4} = \log_2 3 - \log_2 4$ which may be implied by $\log_2 3 - 2$	
A1	For $x = -\frac{1}{3} + \frac{\log_2 3}{6}$ or in the form required by the question. Note that $x = \frac{\log_2 3 - 2}{6}$ is A0	

Question Number	Scheme					Notes			Marks
2(a)		x	-0.25	0	0.25	0.5	0.75		B1
		y	0.462	0.577	0.653	0.686	0.698		
	Allow awrt these values and look for the values appearing in the body of the script or within their calculation in part (b). Also allow exact value for the 0.577 e.g. $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$								
									(1)
(b)	$h = 0.25$					Correct strip width. May be implied by $\frac{1}{8}$ or $\frac{1}{2} \times 0.25$			B1
	$A \approx \frac{1}{2} \times "0.25" \{ 0.462 + 0.698 + 2 ("0.577" + "0.653" + "0.686") \}$ Correct application of the trapezium rule with their h Must be a correct application of the rule so e.g. $A \approx \frac{1}{2} \times "0.25" \times 0.462 + 0.698 + 2 ("0.577" + "0.653" + "0.686")$ Scores M0 unless any missing brackets are implied by subsequent work. $A \approx \frac{1}{2} \times "0.25" \{ 0.462 + 0.698 + 2 ("0.577" + "0.653" + "0.686") \}$ Would also score M0 unless the closing bracket was implied by subsequent work Condone copying slips e.g. 0.426 instead of 0.462. Must use all the y -values. Repeated or missing y -values scores M0. Allow separate trapezia e.g. $A \approx \frac{1}{2} \times "0.25" (0.462 + "0.577") + \frac{1}{2} \times "0.25" ("0.577" + 0.653) + \frac{1}{2} \times "0.25" (0.653 + "0.686") + \frac{1}{2} \times "0.25" ("0.686" + 0.698)$ Allow use of the function e.g. $A \approx \frac{1}{2} \times 0.25 \left\{ \frac{2^{-0.25}}{\sqrt{5(-0.25)^2 + 3}} + \frac{2^{0.75}}{\sqrt{5(0.75)^2 + 3}} + 2 \left(\frac{2^0}{\sqrt{5(0)^2 + 3}} + \frac{2^{0.25}}{\sqrt{5(0.25)^2 + 3}} + \frac{2^{0.5}}{\sqrt{5(0.5)^2 + 3}} \right) \right\}$								M1
	$=$ awrt 0.624 or $\frac{78}{125}$ oe e.g. $\frac{312}{500}$					accept awrt 0.624 or exact fraction but isw if necessary			A1
	Note that the calculator answer for the integral is 0.6265569683...								
	(3)								
Total 4									

Question Number	Scheme	Notes	Marks
3(a)	$a(-4)^3 - (-4)^2 + b(-4) + 4 = -108$ Attempts to set $f(-4) = -108$ to obtain an equation in a and b. Score when you see “-4” embedded in the equation or 2 correct terms (excluding the “+ 4”) on lhs. May be implied by e.g. $-64a - 16 - 4b + 4 = -108$ Condone minor slips on the lhs e.g. one sign error between terms but must use - 108		M1
	As an alternative for the first mark we will condone an attempt at long division. This requires a complete method to divide $(ax^3 - x^2 + bx + 4)$ by $(x + 4)$ to obtain a remainder in terms of a and b which is then equated to -108 For reference, the quotient is $ax^2 - (1+4a)x + 16a + b + 4$ and the remainder is $-4b - 64a - 12$		
	$-64a - 16 - 4b + 4 = -108$ $\Rightarrow 16a + b = 24 *$	Correct equation obtained with no errors and at least one line of intermediate working if starting with e.g. $a(-4)^3 - (-4)^2 + b(-4) + 4 = -108$	A1*
			(2)
(b)	$a\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 4 = 0$ Attempts to set $f\left(\frac{1}{2}\right) = 0$ to obtain an equation in a and b. Condone slips. Score when you see “$\frac{1}{2}$” embedded in the equation or 2 correct terms (excluding the “+ 4”) on lhs. May be implied by e.g. $\frac{a}{8} - \frac{1}{4} + \frac{b}{2} + 4 = 0$ The “= 0” may be implied when they attempt to solve simultaneously below		M1
	An alternative for the first mark is to attempt long division. This requires a complete method to divide $(ax^3 - x^2 + bx + 4)$ by $(2x - 1)$ to obtain a remainder in a and b which is then equated to 0 For reference, the quotient is $\frac{a}{2}x^2 + \left(\frac{a}{4} - \frac{1}{2}\right)x + \left(\frac{b}{2} - \frac{1}{4} + \frac{a}{8}\right)$ and the remainder is $\frac{15}{4} + \frac{b}{2} + \frac{a}{8}$		
	$16a + b = 24, a + 4b = -30$ $\Rightarrow a = \dots, b = \dots$	Attempts to solve $16a + b = 24$ simultaneously with their equation in a and b. This may be implied if values of a and b are obtained (e.g. calculator)	M1
	$a = 2, b = -8$	Correct values	A1
			(3)
(c)	$f(x) = 2x^3 - x^2 - 8x + 4$ $\Rightarrow f'(x) = 6x^2 - 2x - 8$	Correct derivative (follow through their a and b). Allow unsimplified and apply isw if necessary. Allow with the letters “a” and “b” and a “made up” “a” and “b”.	B1ft
			(1)

Question Number	Scheme	Marks
10(a)	$d = \frac{37-15}{11} (= 2)$	M1
	$u_5 = 15 + 4 \times "2" = \dots$	M1
	Week 5 Sunday run = 23 (km)	A1
		(3)
(b)	$r^{11} = \frac{37}{15} \Rightarrow r = \sqrt[11]{\frac{37}{15}} = \dots$	M1
	$u_5 = 15 \times "1.0855..."^4 = \dots$	M1
	Week 5 Sunday run = awrt 20.8 (km)	A1
		(3)
(c)	$S_{12} = \frac{12}{2} [2 \times 15 + (12-1) \times "2"]$ or $\frac{12}{2} (15+37) (= 312)$	M1
	$S_{12} = \frac{15(1 - "1.08554..."^{12})}{1 - "1.08554..."}$ (= 294.185..)	M1
	Difference = $\left(\frac{360-312}{12} \right)$ or $\left(\frac{360-294.185...}{12} \right) = \dots$	dM1
	Either Training model A: $x \leq 4$ km, Training model B: $x \leq 5.4 \dots$ km with = or $\leq$	A1
	Training model A: $x = 4$ km, Training model B: $x = 5$ km	A1cso
		(5)
		(11 marks)



(a)

M1 Attempts to find the common difference. Allow for $d = \frac{37-15}{k}$ where $k = 11$ or 12 . Must be using 37 and 15.

M1 Use their common difference to find the distance run on Sunday of week 5. Must be correct formula. They may work out the first 5 terms, in which case look for there being 5 terms.

A1 23 (km)

(b)

M1 A correct method to find r award for $r^{11} = \frac{37}{15}$ oe and proceeding to find a value for r . May be implied by awrt 1.09. Allow if $r^{11} = \frac{37}{15}$ is followed by stating awrt 8.6% increase if no value is given for r .

M1 Uses their value of r to find the distance run on Sunday of week 5. Must be correct formula. They may work out the first 5 terms, in which case look for there being 5 terms.

A1 awrt 20.8 (km)

(c)

M1 Attempts to find the total distance run on Sundays over the 12 weeks using training model A with $n = 12$ and **either** $a = 15$ and their d or $a = 15$ and $l = 37$. The formula must be correct with the values substituted in the correct places. Listing and adding requires all 12 terms

M1 Attempts to find the total distance run on Sundays over the 12 weeks using training model B with $a = 15$ and their r . The formula must be correct with the values substituted in the correct places., or listing and adding requires all 12 terms

dM1 Attempts to find the weekly amount run on a Wednesday for **either** model A **or** model B .
Look for $\frac{360 - \text{their total}}{12}$. They may set up an inequality in x such as
their total $+ 12x \leq 360$ and attempt to solve, which is equivalent working.
Allow with $=, <$ instead of $\leq$ or with inequalities in the wrong direction.

It is dependent on having scored the previous method mark appropriate for their attempt.

A1 For identifying **either** 4 for model A **or** 5 for model B as the critical values. May be part of an inequality such as $x \leq 4$ for model A or $x \leq 5.4...$ for model B etc. (Note that use of $r = 1.09$ will give 4.8... instead of 5.4... and will not be eligible for the A mark if model A is also incorrect.)
Allow with $=, <$ instead of $\leq$ or with inequalities in the wrong direction.

A1cso Training model A $x = 4$ km and training model B $x = 5$ km
(It must be clear the correct values are linked to the correct models, but they need not be labelled “model A ” and “model B ”)



Question Number	Scheme	Marks
8 (i)	(a) $S_n = a + (a + d) + \dots + (a + (n-1)d)$ (1)	B1
	$S_n = (a + (n-1)d) + \dots + (a + d) + a$ (2)	M1
	(1) + (2) $2S_n = (2a + (n-1)d) + (2a + (n-1)d) + \dots + (2a + (n-1)d)$	
	$2S_n = n(2a + (n-1)d) \Rightarrow S_n = \frac{n}{2}(2a + (n-1)d)$ *	A1*
		(3)
(ii)	(b) States or uses two of $a = 900$, $d = -8$ and $n = 51$ Or alternatively states or uses two of $a = 500$, $d = 8$ and $n = 51$	B1
	Complete method E.g. $n = \frac{900-500}{8} + 1$ and $S = \frac{51}{2}\{2 \times 900 + 50 \times -8\}$	M1
	35700	A1
		(3)
	(a) $\frac{11-k}{k-2} = \frac{k-2}{k+4}$ $44 - k^2 + 7k = k^2 - 4k + 4$ $2k^2 - 11k - 40 = 0$ *	M1 dM1 A1*
		(3)
	(b) $2k^2 - 11k - 40 = 0 \Rightarrow k = \left(-\frac{5}{2}\right), 8$ Uses either k value and attempts to find both a and r . E.g. with $k = 8 \Rightarrow a = 8 + 4, r = \frac{8-2}{8+4}$ Uses $S_\infty = \frac{a}{1-r} = \frac{12}{1-\frac{1}{2}} = 24$	B1 M1 dM1, A1
		(4)
		Total 13

(i)(a)

B1: A correct expression for S_n (but allow S) with a minimum of three terms including the first and last terms **WITH** no incorrect terms. Note that an incorrect order does not imply an incorrect term
So, you may see $S_n = a + (a + d) + \dots + (a + (n-1)d) + (a + (n-2)d)$ which is B1

M1: A correct method to find an expression for S_n in terms of a , n and d . Look for

- a sum with correct first and last terms
- the sum reversed (with at least first and last terms) and an attempt made to add the two series

This mark is usually scored when the candidate works with the sum $S_n = a + \dots + a + (n-1)d$

Condone attempts with incorrect terms. (See below).

E.g. $S_n = a + (a + d) + \dots + a + nd + (a + (n-1)d)$

$$S_n = (a + (n-1)d) + a + nd + \dots + (a + d) + a$$

$$2S_n = 2a + (n-1)d + \dots + 2a + (n-1)d$$

A1*: Correct proof that has previous lines scoring both B1 and M1 as well as a line equivalent to

$$2S_n = n(2a + (n-1)d) \text{ or equivalent before the given answer.}$$

You may see alternative proofs which define $l = a + (n-1)d$.

This is acceptable and the same marking scheme can be applied.



A minimal acceptable proof would be

$$\begin{aligned}
 S_n &= a + (a + d) + \dots + (a + (n-1)d) \\
 S_n &= (a + (n-1)d) + \dots + (a + d) + a \\
 \hline
 2S_n &= n\{2a + (n-1)d\} \\
 S_n &= \frac{n}{2}\{2a + (n-1)d\}
 \end{aligned}$$

(i) (b)

B1: States or uses at least two correct values of (i) the first term (ii) the common difference (iii) the number of terms.

Accept two of $a = 900$, $d = -8$ and $n = 51$ but it may be implied by embedded values within a formula.

Alternatively allow the sum to be reversed so $a = 500$, $d = 8$ and $n = 51$

M1: Full method of finding $900 + 892 + 884 + \dots + 500$

This must include both

- an attempt to find the value of n . E.g. $500 = 900 + (n-1) \times -8 \Rightarrow n = \dots$ but condone $n = \frac{900-500}{8}$ which is implied by $n = 50$
- with an attempt at using the correct formula.

Either $S_n = \frac{n}{2}(2a + (n-1)d)$ with $a = 900$ or 500 **and** $d = \pm 8$ **and** $n = 50$ or 51

or $S_n = \frac{n}{2}(a + l)$ with $a + l = 900 + 500$ **and** $n = 50$ or 51

A1: 35700

(ii)

The two parts in (ii) may be marked as a single question

(a)

M1: Correct attempt to use the fact that these are consecutive terms in a GP to set up an equation in k .

Look for $\frac{11-k}{k-2} = \frac{k-2}{k+4}$ or $11-k = \left(\frac{k-2}{k+4}\right) \times (k-2)$ or equivalent condoning slips

dM1: Cross multiplies, expands and starts to collect terms. It is dependent upon the previous M

See main scheme which has terms collected on either side of the $=$ sign.

A1*: Correct working with sufficient intermediate work to show the given answer of $2k^2 - 11k - 40 = 0$.

An intermediate line of $44 - k^2 + 7k = k^2 - 4k + 4$ o.e. is sufficient. The $= 0$ must be present

(ii)(b)

B1: Solves $2k^2 - 11k - 40 = 0$ and achieves at least 8. The other value if found can be ignored

M1: Uses either of their k values and attempts to find both a and r . May be embedded within a formula.

The value of a must be found using $k + 4$ and r must be attempted using the ratio $\frac{11-k}{k-2}$ or $\frac{k-2}{k+4}$

$$\text{FYI when with } k = -\frac{5}{2} \Rightarrow a = -\frac{5}{2} + 4 = \left(\frac{3}{2}\right), \quad r = \frac{-\frac{5}{2} - 2}{-\frac{5}{2} + 4} = (-3)$$

dM1: Uses $S_\infty = \frac{a}{1-r}$ with their $|r| < 1$. It is dependent on the previous M but also a value of r where

$|r| < 1$

A1: 24 only. If two values are found it must be made clear that only this one is acceptable.



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
4 (a)	$\frac{a(1-r^3)}{1-r} = 70.2, \quad \frac{a}{1-r} = 75$ Sub (2) in (1) $75(1-r^3) = 70.2 \Rightarrow r^3 = (0.064)$ $r = 0.4$	B1, B1 M1 A1 (4)
(b)	Substitutes $r = 0.4$ into $\frac{a}{1-r} = 75$ and finds a $a = 45$	M1 A1 (2) (6 marks)

(a)

B1: One correct equation. Most of the time it is one of

- $\frac{a}{1-r} = 75$
- $\frac{a(1-r^3)}{1-r} = 70.2, \quad a + ar + ar^2 = 70.2 \quad \text{o.e.}$

B1: Two different equations that allow the problem to be solved.

Another more complicated option that can be used (with either of the above bullets) is

$$\frac{a}{1-r} - \frac{a(1-r^3)}{1-r} = 4.8$$

M1: Combines both necessary equations (condoning numerical slips e.g. 72.0 for 70.2) and proceeds to an equation of the form $\delta r^3 = \beta$. Also condone the equations of the same form with sign slips

$$\frac{a}{r-1} = 75$$

Working is necessary here so candidates cannot just write down the equations and the solution without sight of the line $\delta r^3 = \beta$ or $\delta r^3 - \beta = 0$.

A more complicated method involves substituting $a = 75 - 75r$ into $a - ar^3 = 70.2(1-r)$ to produce a simplified quartic equation in r . FYI the correct simplified equation is $75r^4 - 75r^3 - 4.8r + 4.8 (= 0)$.

If they achieve a simplified quartic equation in r and write down a value for r ($r \neq 1, r \in \mathbb{R}$) that solves their equation they can score M1. So, if they write down the correct simplified equation followed by $r = 0.4$ from their calculator they can score M1, A1.

If you spot a method that does not proceed via one of these two methods and you feel deserves credit, then please send to review

A1: $r = 0.4$ only, following award of the method mark.

(b)

M1: Substitutes their value of $r, |r| < 1$, found in part (a) into one of their correct equations and find a value for a .

Candidates cannot just make up a value. It must follow a value in r found from an equation in just r .

A1: $a = 45$ Can only be awarded if two correct equations are written down and $r = 0.4$



Part (b) can be done via substituting $r = 1 - \frac{a}{75}$ into $a + ar + ar^2 = 70.2$ for example, to produce an equation in just a , FYI $\frac{a^3}{5625} - \frac{a^2}{25} + 3a - 70.2 = 0$ This can score M1 for setting up and solving a cubic with A1 for the correct cubic with $a = 45$



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice



(c)

M1: Uses $x-1$ is a factor to establish the quadratic factor. May be by inspection or long division leading to an expression of the form $2x^2 + \alpha x + \beta$

A1: Obtains the correct quadratic factor $2x^2 - 8x + 5$

A1: $x = \frac{4 \pm \sqrt{6}}{2}$ or exact simplified equivalents e.g. $x = 2 + \frac{1}{2}\sqrt{6}$, $2 - \frac{1}{2}\sqrt{6}$ or $\frac{8 \pm \sqrt{24}}{4}$.

(The roots may have been found using a calculator but the **M1A1** must have been awarded)

Some candidates are doing the work for part (c) in part (b) so allow the marks for (c) to score in (b) i.e. mark parts (b) and (c) together.

Question Number	Scheme	Marks
8(a)	$3a = \frac{a}{1-r} \Rightarrow r = \dots$	M1
	$r = \frac{2}{3}^*$	A1*
		(2)
(b)	$ar - ar^3 = 16$	B1
	$\frac{10}{27}a = 16 \Rightarrow a = \dots$	M1
	$a = 43.2$	A1
	$S_{10} = \frac{43.2(1 - (\frac{2}{3})^{10})}{1 - \frac{2}{3}} = 127.4$	dM1A1
		(5)
		(7 marks)

(a)

M1: Sets $3a = \frac{a}{1-r}$, cancels all the a 's and attempts to rearrange to find a numerical value for r

A1*: $r = \frac{2}{3}$ with no errors and at least one intermediate step after $3a = \frac{a}{1-r}$

Alternative by verification:

$$r = \frac{2}{3}, \Rightarrow \frac{a}{1-r} = \frac{a}{1-\frac{2}{3}} = 3a \quad \text{Hence } r = \frac{2}{3}$$

Score **M1** for substituting $r = \frac{2}{3}$ into a correct sum to infinity formula and obtaining $3a$

Score **A1** for correct work followed by a (minimal) conclusion e.g. QED, proven, etc.

(b)

B1: $ar - ar^3 = 16$ seen or implied.

M1: Proceeds to a value for a from a linear equation in a using $r = \frac{2}{3}$ and $ar - ar^3 = 16$

But condone use of $r = \frac{2}{3}$ and $ar^2 - ar^4 = 16$

A1: $a = 43.2$ or any equivalent correct numerical expression e.g. $\frac{16 \times 27}{10}$, $\frac{216}{5}$

dM1: Substitutes their a , $r = \frac{2}{3}$ and $n = 10$ into the correct sum formula.

(Also allow a correct and full attempt to calculate the sum of all 10 terms separately)

It is dependent on the previous method mark.

A1: awrt 127.4 For reference the exact answer is $\frac{92840}{729}$

If candidates use $ar + ar^3 = 16$ rather than $ar - ar^3 = 16$ we will treat this as a copying slip and allow B0M1A0dM1A0 if the subsequent work merits it.



Question Number	Scheme	Marks
7(a)	$r = \sqrt{\frac{12.8}{20}} = 0.8^*$ or e.g. $20 \times 0.8 \times 0.8 = 12.8$ so $r = 0.8^*$	B1*
		(1)
(b)	$a = 20 \div 0.8^2$	M1
	$= 31.25$	A1
		(2)
(c)	$\frac{31.25(1-0.8^n)}{1-0.8} > 156$	M1
	Eg $1-0.8^n > 0.9984$ $\Rightarrow 0.8^n < 0.0016$	dM1
	$0.8^n < 0.0016 \Rightarrow n > \frac{\log(0.0016)}{\log(0.8)}$ or $0.8^n < 0.0016 \Rightarrow n > \log_{0.8} 0.0016$	M1
	$n = 29$	A1
		(4)
		Total 7

(a)

B1* Correctly demonstrates $r = 0.8$ oe with no incorrect working seen. If they use 0.8 within their solution there must be either some preamble or concludes $r = 0.8$

Minimum acceptable is an expression for r which is not 0.8 or an equation involving r with all values substituted in. At some point we must see r being linked with 0.8 either as part of an equation or e.g. with a conclusion $r = 0.8$

May all be in fractions instead eg $20 \times \frac{4}{5} \times \frac{4}{5} = \frac{64}{5} \Rightarrow r = \frac{4}{5}$ is B1

Note that $ar^5 = 12.8$, $ar^3 = 20 \Rightarrow r = \sqrt{\frac{12.8}{20}} = 0.8$ is B0

(b) Work seen in (a) must be used or stated in (b) to score.

M1 Correct method for the first term. Score for the expression $12.8 \div 0.8^4$ or $20 \div 0.8^2$ oe

May be seen as $\frac{64}{5} \div \left(\frac{4}{5}\right)^4$ (Do not penalise invisible brackets for this mark).

If a candidate has stated incorrectly $ar^5 = 12.8$ or $ar^3 = 20$ in (a) then allow M1 if they use this correctly to find a for their equation.

A1 31.25 or $\frac{125}{4}$ or $31\frac{1}{4}$ with no incorrect working seen



(c) Note on EPEN this is M1A1M1A1 but we are marking this M1M1M1A1

M1 Attempts to set up an equation or inequality using their first term and $r = 0.8$ in the sum formula.

Score for $\frac{31.25(1-0.8^n)}{1-0.8} \dots 156$ (Allow any inequality or =)

May be seen as $\frac{125 \left(1 - \left(\frac{4}{5}\right)^n\right)}{1 - \frac{4}{5}} \dots 156$

dM1 Rearranges to $A \times 0.8^n \dots B$ where A could be 1 and B could be unsimplified (Allow any inequality or =). Do not be concerned by the mechanics of their rearrangement. It is dependent on the first method mark. May be implied by further work if logarithms are used correctly.

M1 Attempts to find n by solving $0.8^n \dots$ their 0.0016 **using logarithms** correctly. (May appear as

$0.8^n \dots$ their $\frac{1}{625}$ or even $0.8^{-n} \dots$ their 625. Cannot be scored from an unsolvable equation.

(Allow any inequality or =).

This can also be scored for solving an equation or inequality involving a term rather than the sum such as $0.8^{n-1} = \dots$

A1 29 cso (It must have come from a correct equation (although allow any inequality)).

All three Ms must have been earned.

Examples for the last two marks in (c)

e.g. 1 $0.8^n < 0.0016 \Rightarrow n = 29$ scores M0A0 (no method seen)

e.g. 2 $0.8^n < 0.0016 \Rightarrow n = 28.85 = 29$ scores M0A0 (no method seen)

e.g. 3 $0.8^n < 0.0016 \Rightarrow n < \frac{\log(0.0016)}{\log(0.8)} \Rightarrow n = 29$ scores M1A1 (condoning the incorrect inequality sign in intermediate work)

Note: Methods relying on trial and improvement or the equation solver will usually only score a maximum M1dM1M0A0 for setting up the equation or inequality to $A \times 0.8^n \dots B$

Question Number	Scheme	Marks
1 (a)	$h = 3$ $\text{Area} \approx \frac{3}{2} \{2 + 4.81 + 2 \times (3.32 + 4 + 4.46)\}$ $= \text{awrt } 45.6$	B1 M1 A1 (3)
(b)(i)	States or uses $\log_2 4x^2 = 2 \log_2 2x$ Hence $\int_2^{14} \frac{\log_2 4x^2}{5} dx = \frac{2}{5} \times (a) = \text{awrt } 18.2$	M1 A1 ft
(ii)	Attempts to identify $\log_2 2x$ e.g. $\log_2 \frac{2}{x} = \log_2 4 - \log_2 2x = 2 - \log_2 2x$ Hence $\int_2^{14} \log_2 \frac{2}{x} dx = \int_2^{14} (2 - \log_2 2x) dx = 24 - 45.6 = \text{awrt } -21.6$	M1 A1 ft (4) (7 marks)

(a)

B1 For $h = 3$ This is implied by sight of $\frac{3}{2}$ in front of the bracketM1 For a correct bracket condoning slips (e.g. 4.18 instead of 4.81). Look for {first + last + 2(sum of three terms)}, but repeated terms scores M0. Must all be y values. Condone “invisible” outside brackets for the M1.

A1 awrt 45.6 An answer of 45.6 can imply the B and M mark if no contrary working is seen and the M may be recovered from “incorrect bracketing” if the answer implies correct bracketing is used..

(b) (i)

M1 States or uses $\log_2 4x^2 = 2 \log_2 2x$ (oe)A1ft For calculating $\frac{2}{5} \times (a) = \text{awrt } 18.2$ but following through on their answer for (a)

(b)(ii)

M1 Attempts to write equation in terms of $\log_2 2x$ using the subtraction rule work to be able to apply (a).Should be $\log_2 \frac{2}{x} = \log_2 2 - \log_2 x (= 2 - \log_2 2x)$ but allow eg. $\log_2 \frac{2}{x} = \log_2 \frac{2x}{x^2} = \log_2 2x - \log_2 x^2$ for M1.A1ft For calculating $24 -$ their (a) = awrt -21.6 but following through on their answer for (a) and isw if later made positive.Do not allow for answers that evaluate an integral by calculator (so use of $\log_2 2x - \log_2 x^2$ will likely score M1A0).

Note that trapezium rule used again in (b) is M0 in both parts unless the relevant log work has been seen. Watch out for these as they give the same answers as those in the scheme to 1 dp.

Working must be seen in part (b) to gain credit.

$$\text{Alt for (b)(ii)} \quad \int_2^{14} \log_2 \frac{2}{x} dx = \int_2^{14} (\log_2 2 - \log_2 x) dx = 12 - \int_2^{14} \log_2 x dx \quad \text{and} \quad \int_2^{14} \log_2 2x dx = \int_2^{14} (\log_2 2 + \log_2 x) dx = 12 + \int_2^{14} \log_2 x dx$$

solved simultaneously scores the M when the $\log_2 x$ terms are eliminated.

Question Number	Scheme	Marks
2. (a)	For correct term ${}^6C_4 3^2 (ax)^4$ Sets $15 \times 3^2 a^4 = 540$ $a^4 = 4 \Rightarrow a = \pm\sqrt{2}$	M1 A1 dM1 A1 (4)
(b)	$\frac{1}{81} \times 3^6 + a^6$ $= 17$	M1 A1 A1 (3) (7 marks)

(a)

M1 For an attempt at the correct term of the binomial expansion which may be embedded within a full expansion.

Accept ${}^6C_4 3^2 (ax)^4$, $\frac{6!}{4!2!} 3^2 (ax)^4$, $\binom{6}{4} 3^2 (ax)^4$, $135(ax)^4$ etc but must have correct coefficient and

correct powers of 3 and x . Allow equivalents such as ${}^6C_2 3^2 (ax)^4$ and condone a missing bracket for this mark ie. $135ax^4$.

You may condone a slip on the 3 if the rest of an expansion is correct.

A1 For a correct equation in a . Allow $15 \times 3^2 a^4 = 540$ or $15 \times 3^2 (ax)^4 = 540x^4$ The coefficient must be evaluated for this mark

dM1 For " 135 " $a^4 = 540 \Rightarrow a = \dots$ Condone slips on the 540 or errors simplifying an initially correct coefficient, but must be attempting a fourth root. This is implied by $a = 1.41$

A1 $a = \pm\sqrt{2}$ must be exact. Accept $\pm\sqrt[4]{4}$ oe

(b)

M1 For identifying two terms independent of x . Allow if the power of 3 is incorrect but it is clear the correct terms are meant, so $\frac{1}{81} \times \text{constant term from } (3+ax)^6$ and $a^6 x^6 \times \frac{1}{x^6} = a^6$ extracted.

A1 Both terms correct and added, ie. $\frac{1}{81} \times 3^6 + a^6$ with a or following through on their value of a but allow if terms are initially correct individually but slips when evaluating.

A1 17 Allow recovery to 17 from use of decimals but 17.0 is A0.



Question Number	Scheme	Notes	Marks
8(a)	$S_n = a + ar + \dots + ar^{n-1}$ $rS_n = ar + ar^2 + \dots + ar^n$ Writes down at least 3 correct terms of a geometric series and multiplies their sequence by r. There may be extra incorrect terms but allow this mark if there are 3 correct terms in both sequences and at least one “+” in both sequences but see special case below		M1
	$S_n - rS_n = a - ar^n \text{ or } rS_n - S_n = ar^n - a$ Obtains either equation where both S_n and rS_n had the correct first and last terms and at least one other correct term but no incorrect terms. Both sides must be seen unfactorised.		A1(M1 on EPEN)
	$(1-r)S_n = a(1-r^n) \Rightarrow S_n = \frac{a(1-r^n)}{1-r} *$ Factorises both sides and divides by $1-r$ to obtain the printed answer Should be as printed but allow e.g. $S_n = \frac{a(1-r^n)}{(1-r)}$ but not $S_n = \frac{a(r^n-1)}{(r-1)}$ unless followed by correct version		A1*
	Special case: If terms are listed rather than added and the working is otherwise correct score 110 See next page for proof by induction.		
			(3)
	Alternative for (a): $S_n = a + ar + \dots + ar^{n-1}$ $(1-r)S_n = (1-r)(a + ar + \dots + ar^{n-1}) \text{ or } S_n = \frac{(1-r)(a + ar + \dots + ar^{n-1})}{(1-r)}$ Writes down at least 3 correct terms of a geometric series and multiplies both sides by $1-r$ or multiplies the right hand side by $\frac{1-r}{1-r}$ There may be extra incorrect terms but allow this mark if there are 3 correct terms		M1
	$(1-r)S_n = a - ar^n \text{ or } S_n = \frac{a - ar^n}{1-r}$ Obtains the above equation where S_n had the correct first and last terms and at least one other correct term and no incorrect terms. Right hand side must be seen unfactorised unless the “a” was factored out earlier		A1 (M1 on EPEN)
	$(1-r)S_n = a - ar^n = a(1-r^n) \Rightarrow S_n = \frac{a(1-r^n)}{1-r} *$ or $S_n = \frac{a - ar^n}{1-r} \Rightarrow S_n = \frac{a(1-r^n)}{1-r} *$ Should be as printed but allow e.g. $S_n = \frac{a(1-r^n)}{(1-r)}$ but not $S_n = \frac{a(r^n-1)}{(r-1)}$ unless followed by correct version		A1*

(b)	Mark (b) and (c) together		
	$r^3 = -\frac{20.48}{320} \Rightarrow r = \sqrt[3]{-\frac{20.48}{320}}$	Correct strategy for r . Allow for dividing the 2 given terms either way round and attempting to cube root.	M1
	$= -0.4$	Correct value (and no others) but allow equivalents e.g. $-2/5$. Correct answer only scores both marks.	A1
	Note that some candidates take $ar^2 = -320$ and $ar^5 = \frac{512}{25}$ and use these correctly to give $r^3 = -\frac{20.48}{320} \Rightarrow r = \sqrt[3]{-\frac{20.48}{320}} = -0.4$ In such cases you can allow full marks for (b) but see note * in (c)		
			(2)
(c)	$r = -0.4 \Rightarrow a = \frac{-320}{-0.4} (= 800)$ or $r = -0.4 \Rightarrow a = \frac{512}{25} \div \left(-\frac{2}{5}\right)^4 (= 800)$	Correct attempt at the first term using $\pm$ their r and the -320 or the $\frac{512}{25}$. May be implied by their a but must be using e.g. $ar = -320$ or $ar^4 = \frac{512}{25}$ not $ar^2 = -320$ or $ar^5 = \frac{512}{25}$ *	M1
	$S_{13} = \frac{"800"(1 - "-0.4"^{13})}{1 - "-0.4"}$ Correct attempt at the sum using their a and their r and $n = 13$ to find a value for S_{13}. <u>Must be a fully correct attempt at the sum here using $n = 13$, their a and their r.</u> Note that $\frac{800(1 + 0.4^{13})}{1 + 0.4}$ is equivalent to $\frac{800(1 - (-0.4)^{13})}{1 - (-0.4)}$ and is acceptable for this mark.		M1
	$= 571.43$	Correct value. Note that S_{∞} is also 571.43 so working must be seen i.e. correct answer only scores no marks.	A1
			Total 8

Proof by induction for part (a):

$$n = 1 \Rightarrow S_1 = \frac{a(1-r^1)}{1-r} = a \text{ so true for } n = 1$$

$$\text{Assume true for } n = k \text{ so } S_k = \frac{a(1-r^k)}{1-r}$$

$$\begin{aligned} \text{Add } (k+1)^{\text{th}} \text{ term } S_{k+1} &= \frac{a(1-r^k)}{1-r} + ar^k = \frac{1-ar^k + ar^k - ar^{k+1}}{1-r} \\ &= \frac{a-ar^{k+1}}{1-r} = \frac{a(1-r^{k+1})}{1-r} \end{aligned}$$

So if true for $n = k$ it has been shown true for $n = k + 1$ and as it is true for $n = 1$ it is true for (for all n)

Mark as follows:

M1: Shows true for $n = 1$ and assumes true for $n = k$ and adds the $(k + 1)^{\text{th}}$ term

A1(M1 on EPEN): Finds common denominator obtains $\frac{a-ar^{k+1}}{1-r}$ using correct algebra

A1: Fully correct proof reaching $\frac{a(1-r^{k+1})}{1-r}$ with all steps shown and conclusion

If you are in any doubt about awarding marks in this case or any other cases that you think deserve credit, send to your Team Leader using Review

Question	Scheme	Marks
6(a)	$u_n = ar^{n-1} \Rightarrow ar + ar^2 = 6 \text{ and } ar^3 = 8$	M1
	$\Rightarrow \frac{ar + ar^2}{ar^3} = \frac{6}{8} \Rightarrow 1 + r = \frac{3}{4}r^2$	M1
	$\Rightarrow 3r^2 - 4r - 4 = 0^*$	A1*
		(3)
(a) Way 2	$u_4 = 8 \Rightarrow u_3 = \frac{8}{r}, u_2 = \frac{8}{r^2}$	M1
	$\frac{8}{r} + \frac{8}{r^2} = 6$	M1
	$\Rightarrow 3r^2 - 4r - 4 = 0^*$	A1*
		(3)
(b)	$r = -\frac{2}{3}$	B1
	$ar^3 = 8 \Rightarrow a = \frac{8}{\left(-\frac{2}{3}\right)^3} = \dots$	M1
	$u_1 = -27$	A1ft
		(3)
(c)	$S_\infty = \frac{-27}{1 - \left(-\frac{2}{3}\right)} = \dots$	M1
	$= -\frac{81}{5}$	A1
		(2)
(8 marks)		
Notes:		
(a) Ignore labelling and mark (a), (b) and (c) together M1: Uses the correct n-th term formula for a GP to set up two equations in a and r May also be in terms of u_1 and r or u_2 and r or u_3 and r e.g. $u_1r + u_1r^2 = 6, u_1r^3 = 8$ or $u_2 + u_2r = 6, u_2r^2 = 8$ or $\frac{u_3}{r} + u_3 = 6, ru_3 = 8$ Must be using the correct term formula so e.g. $ar^2 + ar^3 = 6$ and $ar^4 = 8$ is M0 Alternatively may use the correct sum formula for the first equation, $a \frac{1-r^3}{1-r} - a = 6$ oe. M1: Attempts to solve their two equations to get an equation in r. Look for an attempt to divide the two equations, or an attempt to find a in terms of r from one and substitute into the other. Allow slips but the algebra should essentially be correct so do not allow use of e.g. $ar^3 = (ar)^3$		

Alternatively, attempts to eliminate r from the equation, $r = \frac{2}{\sqrt[3]{a}} \Rightarrow \frac{2a}{\sqrt[3]{a}} + \frac{4a}{\sqrt[3]{a^2}} = 6$

$\Rightarrow 2a^{\frac{2}{3}} + 4a^{\frac{1}{3}} = 6$. Award when they reach a quadratic in $a^{\frac{1}{3}}$ in this case.

A1*: CSO Note that as we are marking (a), (b) and (c) together, allow the printed answer to appear anywhere as long as it follows correct work but the “=0” must be seen not implied.

Way 2:

M1: Uses $u_4 = 8$ to write u_2 and u_3 in terms of r

M1: Uses $u_2 + u_3 = 6$ to get an equation in r

A1*: CSO

(b)

B1: Correct value of r seen or used in their working even if subsequently rejected and ignore any other value offered e.g. $r = 2$

M1: Uses a value for r from solving the equation given in (a) where $|r| < 1$ in one of their equations from (a) to find a value for u_1 or a . Allow slips but the algebra should essentially be correct

A1ft: Correct value and no other values. Follow through on $\frac{8}{(\text{their } r)^3}$ for a value of r with $|r| < 1$

(c)

M1: Uses the correct sum formula with their a (u_1) obtained from $|r| < 1$ and r where $|r| < 1$, to find the sum to infinity.

A1: Correct answer and no other values. Allow equivalents e.g. -16.2

Question Number	Scheme	Marks
7 (a)	$x^2 + y^2 + 8x - 10y = 29$	
	Attempts $(x+4)^2 + (y-5)^2 \dots = \dots$	M1
	Correct centre $(-4, 5)$	A1
(b)	Exact radius $\sqrt{70}$	A1
	Attempts distance (or distance ²) from $(-4, 5)$ to $(5, -8)$	M1
	Full method. E.g. compares $\sqrt{9^2 + 13^2}$ to $(\sqrt{70} + \sqrt{52})$	dM1
	States that - distance between centres $= \sqrt{250} = 15.81$ - sum of radii $= \sqrt{70} + \sqrt{52} = 15.58$ Concludes as sum of radii < distance between centres, result proven *	A1*
		(3) (6 marks)

(a)

M1: Attempts to complete the square for **both** variables. Look for $(x \pm 4)^2, (y \pm 5)^2 \dots = \dots$ or may be implied by $(\pm 4, \pm 5)$ by inspection from the equation.

A1: $(-4, 5)$

A1: $\sqrt{70}$ which may be scored following $(x \pm 4)^2 + (y \pm 5)^2 = 70$

(b)

M1: Attempts distance (or distance ²) from $(-4, 5)$ to $(5, -8)$ (condone sign errors)

dM1: Full method. E.g. compares their $\sqrt{9^2 + 13^2}$ to their $(\sqrt{70} + \sqrt{52})$

A1*: Requires correct calculation(s), correct reason and minimal conclusion. Must be evidence to support the inequality – e.g. decimal values – not just the inequality stated. May use the difference between sum of radii and distance between centres.

(b) Alt

M1: Attempts to solve $x^2 + y^2 + 8x - 10y = 29$ and $x^2 + y^2 - 10x + 16y = -37$ forming a linear equation (need not be simplified for this mark)

FYI: Linear equation is $18x - 26y = 66$ o.e. if the calculations are correct

dM1: Attempts to solve their $18x - 26y = 66$ simultaneously with either circle equation reaching a 3 term quadratic for either x or y.

A1*: Requires

- correct equations
- correct "solution" stating no roots and concluding no points of intersection and minimal conclusion

E.g. Substitute $y = \frac{9x-33}{13}$ into $x^2 + y^2 + 8x - 10y = 29$ gives $250x^2 - 412x + 478 = 0$

and $"b^2 - 4ac" = -308256 < 0$ so no solutions

FYI the equation in y (if correct) is $250y^2 + 984y + 1116 = 0$



Question Number	Scheme	Marks
8 (i)	States or uses $\tan x = \frac{\sin x}{\cos x} \Rightarrow 5 \sin x \times \frac{\sin x}{\cos x} + 13 = \cos x$ $5 \sin^2 x + 13 \cos x = \cos^2 x \Rightarrow 5(1 - \cos^2 x) + 13 \cos x = \cos^2 x$ $\Rightarrow 6 \cos^2 x - 13 \cos x - 5 = 0$ $\Rightarrow (3 \cos x + 1)(2 \cos x - 5) = 0 \Rightarrow \cos x = -\frac{1}{3}$ $\Rightarrow x = 1.91$	B1 M1 A1 M1 A1 (5)
(ii) (a)	$20 = 10 + 12 \sin(6k + 18)^\circ \Rightarrow \sin(6k + 18)^\circ = \frac{5}{6}$ $\Rightarrow (6k + 18) = 56.4, 123.6$ $\Rightarrow k = 6.41, 17.59$	M1 dM1 A1, A1 (4)
(b)	22°C	B1 (1)
(c)	"6.41" $t + 18 = 90 \Rightarrow t = 11.23$ Time of day = 11:14	M1, A1 (2) (12 marks)

(i)

B1: States or uses $\tan x = \frac{\sin x}{\cos x}$ E.g. $5 \sin x \times \frac{\sin x}{\cos x} + 13 = \cos x$ M1: Attempts to use $\tan x = \frac{\sin x}{\cos x}$, $\sin^2 x + \cos^2 x = 1$ and multiply by $\cos x$ to form a quadratic equation in $\cos x$ (allow if there are slips in coefficients but the trig terms must be correct)A1: Correct simplified quadratic $6 \cos^2 x - 13 \cos x - 5 (= 0)$ The " $=0$ " may be implied.M1: Solves a 3TQ in $\cos x$ leading to at least one value for $\cos x$ A1: awrt $x = 1.91$ (following $\cos x = -\frac{1}{3}$) and no other values in the range.Note: Answers only with no working score no marks. From $6 \cos^2 x - 13 \cos x - 5 (= 0)$ to 1.91 directly, score final M0A0.

(ii) (a)

M1: Attempts to use the given information and proceeds to $\sin(6k + 18)^\circ = c$, or may be implied by

$$A \sin(6k + 18)^\circ = B \rightarrow 6k + 18 = \arcsin \frac{B}{A} \text{ (evaluated) (which scores M1dM1)}$$

dM1: Takes arcsin leading to a value for $6k + 18$. Accept radian values seen for this mark. (0.985, 2.16).
Allow awrt 2 s.f. answers for evidence.A1: One value for k : 6.41 or 17.59 Must be in degrees.

SC Allow for 6.4 and 17.6 both given if no more accurate answers are stated.

A1: awrt $k = 6.41, 17.59$ and no other values

Note answers only scores no marks. – the requirements of the M marks must be seen to be able to score them.

(ii)(b)

B1: cao 22°C Condone just 22.

(ii)(c)

M1: Sets their "6.41" $t + 18 = 90 \Rightarrow t = \dots$ May be implied by 11.23.

$$\text{Note "6.41" } t + 18 = \frac{\pi}{2} \text{ is M0.}$$

A1: cao Time of day = 11:14 o.e. (e.g. 11h 14m is acceptable).



Question Number	Scheme	Marks
9.(a)	$y = 2x^{\frac{3}{2}}(4-x) = 8x^{\frac{3}{2}} - 2x^{\frac{5}{2}}$ $\frac{dy}{dx} = 12x^{\frac{1}{2}} - 5x^{\frac{3}{2}}$ Stationary point $12x^{\frac{1}{2}} - 5x^{\frac{3}{2}} = 0 \Rightarrow x = \frac{12}{5}$	M1, A1 dM1, A1 (4)
(b)	$\int 8x^{\frac{3}{2}} - 2x^{\frac{5}{2}} dx = \frac{16}{5}x^{\frac{5}{2}} - \frac{4}{7}x^{\frac{7}{2}}$ $\frac{16}{5}k^{\frac{5}{2}} - \frac{4}{7}k^{\frac{7}{2}} = 0 \Rightarrow k = \frac{16 \times 7}{4 \times 5} = \frac{28}{5}$	M1, A1 dM1, A1 (4) (8 marks)

(a)

M1: Complete attempt to differentiate.

Writes as a sum of two terms and reduces the power of at least one term by 1

 Alternatively, by the product rule look for $Ax^{\frac{1}{2}}(4-x) - Bx^{\frac{3}{2}}$

 A1: $\left(\frac{dy}{dx} = \right) 12x^{\frac{1}{2}} - 5x^{\frac{3}{2}}$ or $3x^{\frac{1}{2}}(4-x) - 2x^{\frac{3}{2}}$. May be left unsimplified

 dM1: Sets $\frac{dy}{dx} = 0$ and attempts to solve to find a non-zero value for x . Do not be concerned about the method of solving for this mark.

 A1cso: $x = \frac{12}{5}$ from correct work. Ignore reference to $x = 0$. Watch for incorrect methods of

 solving, such as $\left(12x^{\frac{1}{2}} - 5x^{\frac{3}{2}}\right)^2 = 0 \Rightarrow 144x - 25x^3 = 0 \Rightarrow x = \frac{12}{5}$, which score A0.

(b)

M1: Complete attempt to integrate. Writes as a sum of two terms and increases the power of at least one term by 1. For attempts at integrating by parts they must proceed through the second integration to score the mark.

 A1: $\frac{16}{5}x^{\frac{5}{2}} - \frac{4}{7}x^{\frac{7}{2}}$ which may be unsimplified

 dM1: Substitutes $x = k$ in their $\frac{16}{5}x^{\frac{5}{2}} - \frac{4}{7}x^{\frac{7}{2}} = 0$ and solves. Alternatively, applies limits 0 and 4 to find the area $R_1 \left(= \frac{1024}{35} \right)$ and sets equal to the negative of the expression from limits 4 to k (or reverses the limits and sets equal) and proceeds to reach a value for k .

 A1: $k = \frac{28}{5}$ (ignore references to $k = 0$) Accept as decimal 5.6.

Alt for the dM:

$$\text{Area } R_1 = \left[\frac{16}{5} x^{\frac{5}{2}} - \frac{4}{7} x^{\frac{7}{2}} \right]_0^4 = \frac{1024}{35} \Rightarrow \text{Area } R_2 = \left[\frac{16}{5} x^{\frac{5}{2}} - \frac{4}{7} x^{\frac{7}{2}} \right]_4^k = -\frac{1024}{35} \Rightarrow k = \dots$$

Note if $\text{Area } R_1 = \text{Area } R_2 = \left[\frac{16}{5} x^{\frac{5}{2}} - \frac{4}{7} x^{\frac{7}{2}} \right]_0^4 = \frac{1024}{35} = \left[\frac{16}{5} x^{\frac{5}{2}} - \frac{4}{7} x^{\frac{7}{2}} \right]_4^k \Rightarrow k = \dots$ is used then

this scores dM0. However, if a student realises the areas cancel and effectively achieves the correct equation after stating areas equal then allow recovery for dM1A1.

Question Number	Scheme	Marks
10 (a)	$2000 \times 1.03^6 = \text{awrt } 2390$	M1, A1 (2)
(b)	$3690 = ab^4, 3470 = ab^7 \Rightarrow b^3 = \frac{3470}{3690} = (0.94\dots)$ Hence $b = \sqrt[3]{\frac{3470}{3690}} = 0.9797\dots$ $a = \frac{3690}{0.9797\dots^4}$ $N = 4000 \times 0.98^t$	M1 dM1 A1 (3)
(c)	Sets $2000 \times 1.03^T = 4000 \times 0.98^T \Rightarrow \left(\frac{1.03}{0.98}\right)^T = \frac{4000}{2000}$ $\Rightarrow T \log\left(\frac{1.03}{0.98}\right) = \log 2 \Rightarrow T = \dots$ $\Rightarrow T = 13.9$	M1 dM1 A1 (3) (8 marks)

(a)

M1: Attempts 2000×1.03^5 or 2000×1.03^6 . May use successive iterations (finding each year) but end up at one of these in effect.

A1: Awrt 2390

(b)

M1: Makes progress in using the model and attempting to find one of the variables a or b^3 (or b). Allow if r is used instead of b for the M's.

Allow from the indexing error $3690 = ab^3, 3470 = ab^6 \Rightarrow b^3 = \frac{3470}{3690}$

dM1: Full method to find values for a and b .

Condone this being scored from $3690 = ab^3, 3470 = ab^6$

A1: $N = 4000 \times 0.98^t$ Accept awrt 4000 to 2sf, awrt 0.98. Must be seen as an equation not just values of a and b but allow if the equation is seen (c) (including the $N =$).
Condone n or T used the variables.

(c)

M1: Makes progress towards solving the problem by forming a single term with single power T . Follow through on their equations provided that they are of the correct form.

Alternatively takes logs of both sides $2000 \times 1.03^T = 4000 \times 0.98^T$ and uses addition law and power law of logs, e.g. $\ln 2000 + T \ln 1.03 = \ln 4000 + T \ln 0.98$

Award from an attempt on $2000 \times 1.03^{T-1} = 4000 \times 0.98^T \Rightarrow \left(\frac{1.03}{0.98}\right)^T = \frac{4000 \times 1.03}{2000}$ or

similar index error.

dM1: Full and complete attempt to find T . Dependent upon previous method but may be implied for the correct value for their $a = b^T$

A1: cao $T = 13.9$



Question Number	Scheme	Notes	Marks
6(a)	Examples: $m_{PQ} = \frac{14+30}{23-15}, m_{QR} = \frac{-30+26}{15+7} \Rightarrow m_{PQ} \times m_{QR} = ..$ or $PQ^2 = 8^2 + 44^2, QR^2 = 22^2 + 4^2, PR^2 = 30^2 + 40^2$ $PQ^2 + QR^2 = ..$	Correct strategy to show that $\angle PQR = 90^\circ$. E.g. attempts gradient of PQ and gradient of QR and attempts product or finds side lengths and attempts Pythagoras.	M1
	$m_{PQ} \times m_{QR} = \frac{11}{2} \times \left(-\frac{2}{11}\right) = -1 \Rightarrow \angle PQR = 90^\circ$ Or $PQ^2 = 2000, QR^2 = 500, PR^2 = 2500$ $2000 + 500 = 2500 \Rightarrow \angle PQR = 90^\circ$	Correct proof and conclusion	A1
			(2)
(b)(i)	Centre is $(8, -6)$	Correct coordinates	B1
(b)(ii)	$r = \sqrt{(23-8)^2 + (14+6)^2}$ or e.g. $r = \frac{1}{2} \sqrt{(23+7)^2 + (14+26)^2}$	Fully correct method for the radius	M1
	$r = 25$	Cao	A1
			(3)
(c)	S is $(1, 18)$ or $m_T = \frac{7}{24}$	Correct coordinates for S or correct gradient for the tangent.	B1
	$m_N = \frac{18+6}{1-8} \Rightarrow m_T = \frac{8-1}{18+6} \Rightarrow y-18 = \frac{7}{24}(x-1)$ Uses a correct straight line method for the tangent using their S and the negative reciprocal of the radius gradient		M1
	$7x - 24y + 425 = 0$	Allow any integer multiple	A1
			(3)
			Total 8

Notes**(a)**

M1: For a correct strategy to show that angle $PQR = 90^\circ$. Possible methods are:
 Find gradients of PQ and RQ (in order to check the perpendicularity condition), getting at least as far as both gradients.
 Finding the lengths of all three sides and attempting to show $PQ^2 + QR^2 = PR^2$ attempting substitution into at least $PQ^2 + QR^2 = ..$ When finding lengths, accept at least two correct lengths stated to imply method for finding lengths if no method is shown.
 Finding the lengths of all three sides and applying the cosine rule to find the angle, reaching at least substitution into the cosine rule. May find the two acute angles and subtract from 180° .
 Finding the midpoint of PR and showing it is equidistant from P , Q and R to deduce PR is a diameter and applying circle theorem. Must reach at least the calculations of lengths.
 Finding the length of all three sides and calculating angle at P from both sin and cosine ratios to show both agree. Must reach angle from both ratios.
 Other methods are possible, if you see a method you are unsure of then send to review.



A1: For a fully correct proof with all necessary details shown and some kind of concluding statement made (need not be the final statement). Must include the relevant correct calculations (e.g. showing product equals -1 for gradient approach, evaluation of the squares for Pythagoras etc) and deduce the right angle. If details/calculations are missing, then A0. If rounded values (e.g. when finding values) are used, then A0. Conclusion should refer to the angle, not just PQ and QR being perpendicular.

(b)

(i)

B1: Correct coordinates for the centre, seen anywhere – may be on the diagram. May have been found in part (a).

(ii)

M1: Correct method for the radius. May have been found in part (a). Implied by correct answer if no method is shown. May be seen on the diagram.

A1: $r = 25$ (seen anywhere, e.g. on diagram, as long as it is clearly the radius). Isw after a correct radius is found.

(c)

B1: Either identifies the correct point S , or finds the correct gradient for the tangent. Look for the gradient they use in their equation.

M1: For a full method to find the equation of the line. E.g. as in scheme, attempts to find the diametrically opposite point to Q , finds radius gradient and takes negative reciprocal (oe, may be using centre and either S or Q) and uses these to form the equation. They must be using an attempt at S and not Q (but see alt) but accept any attempt that gives a point above the x -axis for S .

Alternatively, may find the gradient of tangent using centre and Q and attempt a translation of the line. Look for

$$y = \frac{7}{24}(x - 15) - 30 \rightarrow y = \frac{7}{24}((x \pm 2 \times "7") - 15) - 30 \pm 2 \times "24" \text{ (or may find the vertical}$$

translation first via trig or Pythagoras).

For attempts at the gradient look for an attempt at the radius followed by negative reciprocal, or an attempt at change in x over change in y directly. Allow if there are sign slips if the intent is clear. Some may attempt differentiation, look for evidence of implicit differentiation giving a $y \frac{dy}{dx}$ term before attempting to substitute x and y values.

A1: $7x - 24y + 425 = 0$ or any (non-zero) integer multiple of this. Accept terms in any order, but have the $"=0"$.

Question Number	Scheme	Notes	Marks
7(i)	$3 \sin(2x - 15^\circ) = \cos(2x - 15^\circ)$ $\Rightarrow \tan(2x - 15^\circ) = \frac{1}{3}$	Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and reaches $\tan(2x - 15^\circ) = \dots$	M1
	$\Rightarrow x = \frac{\tan^{-1}\left(\frac{1}{3}\right) \pm 15^\circ}{2}$	Correct strategy for finding x	M1
	$x = 16.7^\circ, -73.3^\circ$	One of awrt 16.7 or -73.3	A1
		Awrt 16.7 and awrt -73.3 and no extras in range	A1
			(4)
(ii)	$4 \sin^2 \theta + 8 \cos \theta = 3$ $\Rightarrow 4(1 - \cos^2 \theta) + 8 \cos \theta = 3$ $\Rightarrow 4 \cos^2 \theta - 8 \cos \theta - 1 = 0$	Applies $\sin^2 \theta = 1 - \cos^2 \theta$ and collects terms to obtain a 3TQ in $\cos \theta$	M1
	$\cos \theta = \frac{8 \pm \sqrt{64 + 4 \times 4}}{2 \times 4} \Rightarrow \theta = \cos^{-1}\left(1 - \frac{\sqrt{5}}{2}\right) = \dots$	Solves their 3TQ and takes inverse cos to obtain at least one value for θ	M1
	$\theta = 1.69, 4.59$	Awrt 1.69 or 4.59	A1
		Awrt 1.69 and awrt 4.59 and no extras in range	A1
			(4)
			Total 8

Notes (i)

M1: Evidence of correct identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$ seen, with attempt made to reach an equation

$\tan(2x - 15^\circ) = k$. Note $\tan(2x - 15^\circ) = 3$ with no correct identity stated is M0, but if correct identity stated first allow M1 as a slip. Allow recovery for all relevant marks for

$3 \tan(2x - 15^\circ) = 0 \rightarrow \tan(2x - 15^\circ) = \frac{1}{3}$ but allow only the first M if $\tan(2x - 15^\circ) = 0$ follows.

Allow with e.g. $\tan y = \dots$ as long as y has been clearly defined as $2x - 15^\circ$.

Alternatively, may square both sides (including the 3) and apply $\sin^2 \theta + \cos^2 \theta = 1$ to reach an

equation in sin or cos. They should get $\sin(2x - 15^\circ) = \pm \frac{1}{\sqrt{10}}$ or $\cos(2x - 15^\circ) = \pm \frac{3}{\sqrt{10}}$ if correct but

allow the method if slips in rearranging are made.

M1: Correct order of operations from $\tan(2x - 15^\circ) = k$ (any non-zero k) to produce a value of x, so applies arctan, then attempts to move the 15° across before dividing by 2. Must be working in degrees OR entirely in radians (if the 15° is converted first).

A1: One of awrt 16.7 **or** -73.3. Must have come from a correct equation – answer from calculators score A0 (they will not have score the M).

A1: Both of awrt 16.7 **and** -73.3 and no other solutions in the range. Must have come from a correct equation – answer from calculators score A0 (they will not have scored the M).

(ii)

M1: Applies $\sin^2 \theta = 1 - \cos^2 \theta$ and collects terms to obtain a 3TQ in $\cos \theta$ or rearranges suitably to be able to solve via completing the square (or method). Allow the method marks if there are notation errors (e.g. missing θ).



- M1:** Solves their 3TQ and takes inverse cos to obtain at least one value for θ (may be implied by answers to 1d.p.). Usual rules for solving. Answers in degrees can score this method mark for find one angle.
- A1:** One correct answer, awrt 1.69 or awrt 4.59. Must be in radians.
- A1:** Both correct, awrt 1.69 and awrt 4.59, and no other solutions in the range.



Question Number	Scheme	Notes	Marks
8(a)	$u_{20} = 100 + 19(-2) = 62^*$	Correct method shown	B1*
			(1)
(b)	$S_{20} = \frac{1}{2}(20)\{2 \times 100 + 19(-2)\} = \dots$ or $S_{20} = \frac{1}{2}(20)\{100 + 62\} = \dots$	Applies a correct AP sum formula with $n = 20, a = 100$ and $d = -2$ or $n = 20, a = 100$ and $l = 62$	M1
	$= 1620 \text{ (mm)}$	Correct value	A1
			(2)
(c)	$62 \times r^2 = 60 \Rightarrow r^2 = \dots$	Correct strategy to find r	M1
	$r^2 = \frac{60}{62} \Rightarrow r = \sqrt{\frac{60}{62}}$ $= 0.983738\dots$	$r = \text{awrt } 0.984$	A1
			(2)
(d)	Total distance from GS hits = $\frac{62 \times 0.983\dots(1 - 0.983\dots^n)}{1 - 0.983\dots}$		M1
	$1620 + \frac{62 \times 0.983\dots(1 - 0.983\dots^n)}{1 - 0.983\dots} > 3000$	Correct equation set up with their r and suitable a	M1
	$0.983\dots^n < 0.63207\dots \Rightarrow n = \frac{\log(0.63207\dots)}{\log(0.983\dots)}$	Fully correct processing to find n from an equation of suitable form,	M1
	$n = 27.98\dots \Rightarrow N = 20 + 28 = 48$	$N = 48$ only	A1cso
			(4)
	(d) Alternative taking 20 th hit as first term of GP		
	Total distance from GS hits = $\frac{62(1 - 0.983\dots^n)}{1 - 0.983\dots}$		M1
	$1620 - 62 + \frac{62(1 - 0.983\dots^n)}{1 - 0.983\dots} > 3000$	Correct equation set up with their r and suitable a	M1
	$0.983\dots^n < 0.62179\dots \Rightarrow n = \frac{\log(0.62179\dots)}{\log(0.983\dots)}$	Fully correct processing to find n from an equation of suitable form,	M1
	$n = 28.98\dots \Rightarrow N = 19 + 29 = 48$	$N = 48$ only	A1cso
			Total 9

Notes**(a)****B1*:** Correct method shown, identifies the common difference and attempts the 20th term.Accept as minimum seeing $100 + 19(-2) = 62$.Alternatively, allow for setting up an equation $62 = 100 + (n-1)(-2)$ and solving to find $n = 20$

If listing, there must be the correct 20 terms with final term 62.

(b)**M1:** Applies a correct AP sum formula with $n = 20, a = 100$ and $d = -2$ or with $n = 20, a = 100$ and $l = 62$. Alternatively, by listing look for 20 terms listed with attempt to sum, or implied by answer.**A1:** Correct value, units not needed.**(c)**

M1: Correct strategy, attempts to set up and solve the equation $62 \times r^2 = 60$

A1: $r = \text{awrt } 0.984$. Accept the exact answer $\sqrt{\frac{30}{31}}$ (simplified to this)

(d)

This is now being marked as MMAA (on ePen it is MMAA).

M1: Applies a correct GS summation with their r , $a = 62$ or $62 \times$ their r and n or $n - 1$ or $n - 19$ or $n - 20$ as part of their solution in (d). Special case, allow with $a = 100$ for this mark if they have used this in part (c) as a misunderstanding of the question.

M1: States or uses $1620 + \frac{62 \times 0.983... (1 - 0.983...^n)}{1 - 0.983...} > 3000$ with their 1620 from part (b) and their r with n or $n - 1$ or $n - 19$ or $n - 20$. Allow " $>$ ", " $=$ ", " $<$ " etc. but the starting term of the GS for the combination of series must be correct.

Alternatively States or uses $1620 - 62 + \frac{62(1 - 0.983...^n)}{1 - 0.983...} > 3000$ with their 1620 from part (b) and their r with n or $n - 1$ or $n - 19$ or $n - 20$. Allow " $>$ ", " $=$ ", " $<$ " etc. but the starting term of the GS for the combination of series must be correct.

M1: Correct processing to solve for n in their equation which must be an attempt at combining 3000, their (b) and the sum of a GS with their r and $a = 62$ or $62 \times$ their r and n or $n - 1$ or $n - 19$ or $n - 20$. Look for reaching $(\text{their } r)^n = \dots$ before applying logs appropriately to proceed to $n = \dots$ (Note $n = 27.98$ or 28.98 are possible correct values at this stage.)

Allow for any of " $>$ ", " $=$ ", " $<$ " used in the equations.

A1cso: For $N = 48$ only and must have come from a correct equation (although allow for any of " $>$ ", " $=$ ", " $<$ ") – all three M's must have been earned. Look for the correct values of n as a clue. If using power and $n - 19$ or $n - 20$ they may get directly to the answer.

Note: It is possible to get the correct answer from incorrect methods and you may see M1M0M1A0 often. Make sure they are working from a correct equation. The values 27.98 and 28.98 may help (if full accuracy is kept). A value between 27.2 and 27.8 (depending on degree of rounding used) before adding 20 likely implies a mismatch between starting term of GS and ending of AS so check carefully if the second M was earned.



Question Number	Scheme	Marks
5	$\log_2 16x + \log_2 (x+1) = 3 + \log_2 (x+6)$ $\log_2 a = b \Rightarrow 2^b = a$ $\log_2 16x(x+1) = \log_2 8(x+6)$ $2x^2 + x - 6 = 0$ $(2x-3)(x+2) = 0$ $x = \frac{3}{2} \text{ only}$	 B1 M1 A1 dM1 A1cso (5 marks)

Note a correct answer with no working is 0 marks

B1 Correct use of $\log_2 a = b \Rightarrow 2^b = a$ This will usually result from either $\log_2 8 = 3$ or $\log_2 16 = 4$ being stated or seen applied.

e.g. $\log_2 \left(\frac{16x(x+1)}{x+6} \right) = 3 \Rightarrow \frac{16x(x+1)}{x+6} = 8$

May be implied by e.g. $\log_2 16x(x+1) = \log_2 8(x+6)$ or $\log_2 \left(\frac{x(x+1)}{x+6} \right) = -1$ o.e.

M1 Applies the addition (or subtraction) laws of logs at least once correctly. Condone invisible brackets. e.g. $\log_2 16x^2 + 16x$

This mark can be scored for expressions where the law has been applied correctly and does not need to be a full equation e.g. $\log_2 \left(\frac{x+6}{x+1} \right)$ scores M1. Do not withhold this mark if they subsequently apply a law incorrectly.

e.g. $\log_2 16x(x+1) = 3 + \log_2 x + \log_2 6$ still scores M1

Also allow the mark to be scored despite an error dealing with the 3 when writing it as a log e.g. $3 + \log_2 (x+6) = \log_2 6 + \log_2 (x+6) = \log_2 6(x+6)$

A1 $2x^2 + x - 6 = 0$ or equivalent 3TQ (quite often $16x^2 + 8x - 48 = 0$) Terms must be collected but do not need to be collected on the same side. Condone the omission of $= 0$. Can only be scored from correct working.

dM1 Attempts to solve their 3TQ via any valid method including via a calculator (you may need to check this). Condone complex roots. It is dependent on the previous method mark.

A1cso $\frac{3}{2}$ o.e. only (-2 must be rejected if found)

This mark can only be scored if a correct quadratic is achieved (following correct log work)



Question Number	Scheme	Marks
6a	$\frac{k}{2k-15} = \frac{k+4}{k} \Rightarrow k^2 = 2k^2 + 8k - 15k - 60 \Rightarrow k^2 - 7k - 60 = 0$ *	M1A1*
		(2)
b	$k = 12$ $\Rightarrow \frac{"12"}{2 \times "12" - 15} = \frac{4}{3}$ e.g. $("12" + 4) \times \frac{4}{3} = \dots$ or $(2 \times "12" - 15) \times \left(\frac{4}{3}\right)^3 = \dots$ $= 21300$	B1 M1 M1 A1
		(4)
c	$\text{e.g. } \frac{9 \left(1 - \left(\frac{4}{3}\right)^N\right)}{1 - \frac{4}{3}} > 3000 \Rightarrow \left(\frac{4}{3}\right)^N > \frac{1009}{9}$ $\log\left(\frac{4}{3}\right)^N > \log\left(\frac{1009}{9}\right) \Rightarrow N > \frac{\log\left(\frac{1009}{9}\right)}{\log\left(\frac{4}{3}\right)}$ $N > 16.405 \dots \Rightarrow N = 17$	M1 dM1 A1
		(3)
		(9 marks)

(a)

- M1 Forms a correct equation linking the three terms and attempts to rearrange to the given quadratic (it cannot be awarded for just proceeding in one step from the starting equation to the given answer and usually will involve attempting to multiply out brackets or dealing with any fractions). Must proceed as far as a quadratic with terms collected. Condone slips in their rearrangement and invisible brackets. Possible starting equations will involve using the n th term formula or the sum of a geometric series. The ones below are not exhaustive.

$$\frac{k}{2k-15} = \frac{k+4}{k} \quad \text{or} \quad k^2 = (k+4)(2k-15) \quad \left((u_2)^2 = u_1 \times u_3\right) \quad \text{or} \quad (2k-15)\left(\frac{k+4}{k}\right) = k \quad \text{or}$$

$$k\left(\frac{k}{2k-15}\right) = k+4 \quad \text{or} \quad 2k-15 + k + k+4 = \frac{(2k-15)\left(1 - \left(\frac{k}{2k-15}\right)^3\right)}{1 - \frac{k}{2k-15}} \quad (\text{sum of 3 terms})$$

- A1* Rearranges to achieve $k^2 - 7k - 60 = 0$ with no errors seen including brackets. There must be at least some intermediate working (e.g. brackets multiplied out or multiplying to remove any fractions) between their starting equation and the given answer.

(b) **Note a candidate who uses k as r can only score maximum B1 in (b) and no marks in (c)**

- B1 12 (ignore -5) seen or used in (b) (may be implied by the value of the second term)

- M1 Substitutes their 12 (must be positive) into the expressions for two of the terms and attempts to find a value for r . Condone arithmetical slips but they the division the correct way round.



Question Number	Scheme	Marks
7.(a)	Attempts to use $4000 = 300 + 11d$ to find 'd'	M1
	Uses $300 + 3"d$	M1
	Wheat production in year 4 is awrt 1310 (to the nearest 10) (tonnes)	A1
		(3)
(b)	Attempts to use $4000 = 300r^{11}$ to find 'r'	M1
	Finds $r = (1.266\dots)$ and MULTIPLIES this by 300	M1
	Wheat production in year 2 is awrt 380 (to the nearest 10) (tonnes)	A1
		(3)
(c)	Attempts $\frac{12}{2}\{300 + 4000\}$ or $\frac{300("1.266\dots"^{12} - 1)}{"1.266\dots" - 1}$	M1
	Finds $\frac{12}{2}\{300 + 4000\} - \frac{300("1.266\dots"^{12} - 1)}{"1.266\dots" - 1} = (25800 - 17935)$	dM1
	Difference = 7860 but allow 7870 (tonnes) (not AWR T)	A1
		(3)
		(9 marks)

Condone slips copying 300 and 4000 if they lose or gain an extra 0 in all parts

(a)

M1: Attempts to use the AP formula in an attempt to find 'd'

Accept an attempt at $4000 = 300 + 11d$ resulting in a value for d .

Accept the calculation $\frac{4000 - 300}{11}$ proceeding to a value (condoning slips)

May be implied by $\frac{3700}{11}$ or awrt 336

M1: Attempts to find the **fourth term** (do not allow a misread) by attempting $300 + 3 \times "336"$.

You may award this following an "incorrect" AP formula with $12d$ being used instead of $11d$. e.g.

$4000 = 300 + 12d$ or more likely $\frac{4000 - 300}{12}$ usually leading to an answer of 1225.

If no method seen to find d then it must be correct.

A1: awrt 1310 (**to the nearest 10**) (tonnes) isw once a correct answer is seen.

Alt (a)

M1: Forms the equation $\frac{x - 300}{4000 - 300} = \frac{4 - 1}{12 - 1}$ or may be implied by further work

M1: Rearranges the equation to $x = 300 + \frac{4 - 1}{12 - 1} \times (4000 - 300)$ or allow this mark for $x = 300 + \frac{4}{12} \times (4000 - 300)$

A1: awrt 1310 (**to the nearest 10**) (tonnes) isw once a correct answer is seen.



(b)

M1: Attempts to use the GP formula in an attempt to find 'r'

Look for $4000 = 300r^{11} \Rightarrow r^{11} = \frac{4000}{300} \Rightarrow r = \sqrt[11]{\frac{4000}{300}}$ condoning slips. Implied by $r = \text{awrt } 1.3$ provided they do not start from an incorrect equation.

M1: A correct attempt to find the **second term** (do not allow a misread) by multiplying 300 by their "1.266..." . It is dependent on an attempt to find r from either $4000 = 300r^{11}$ or $4000 = 300r^{12}$ condoning slips. If no method is seen to find r then it must be correct. e.g. following $4000 = 300r^{12}$ or $\sqrt[12]{\frac{4000}{300}}$.

Condone slips.

A1: awrt 380 (to the nearest 10) (tonnes) isw once a correct answer is seen.

(c) **Condone use of their d and their r even if they have come from incorrect methods**

M1: A correct method to find the sum of either the AP or the GP

For the AP accept an attempt at either $\frac{12}{2}\{300 + 4000\}$ or $\frac{12}{2}\{2 \times 300 + 11 \times "336"\}$

For the GP accept an attempt at either $\frac{300("1.266..."^{12} - 1)}{"1.266..." - 1}$ or $\frac{300(1 - "1.266..."^{12})}{1 - "1.266..."}$

dM1: Both formulae must be attempted "correctly" (see above) and the difference taken

FYI if d and r are correct, the sums are 25 800 tonnes and 17 935A1: Difference = 7860 or 7870 (tonnes) **This is NOT AWRT.**(Note that use of $r = 1.266$ will lead to a difference of 7810 which is max M1dM1A0)**Alt (c) Listing values**

Accept listing the values

M1: Attempts to add the 12 terms for the arithmetic or geometric series. Must include 300 and 4000, but condone one omission of one of the middle terms. May be implied by calculating the difference between the AP and GP values for each year and adding those together.

dM1: Attempts to add the 12 terms for the arithmetic and geometric series. Must include 300 and 4000 for both sums, but condone one omission of one of the middle terms in each of the summations. May be implied.

A1: Difference = 7860 or 7870 **This is NOT AWRT.**

Year	AP	GP	Difference
1	300	300	0
2	636.3636	379.6544	256.7093
3	972.7273	480.4582	492.2691
4	1309.091	608.0268	701.0641
5	1645.455	769.4668	875.9877
6	1981.818	973.7715	1008.047
7	2318.182	1232.322	1085.86
8	2654.545	1559.521	1095.024
9	2990.909	1973.597	1017.312
10	3327.273	2497.616	829.6567
11	3663.636	3160.77	502.8668
12	4000	4000	0
Sum	25800	17935.2	7864.796

Question Number	Scheme	Marks
7. (i)	10+8+6.4+...	
(a)	Attempts $S_{\infty} = \frac{a}{1-r}$ with $a=10, r = \frac{8}{10}$ $S_{\infty} = 50$	M1 A1 (2)
(b)	Sets $10 \times 0.8^{k-1} < 0.0005 \Rightarrow 0.8^{k-1} < 0.00005$ e.g. $\Rightarrow k-1 > \frac{\log(0.00005)}{\log(0.8)}$ or e.g. $k-1 > \log_{0.8} 0.00005$ (see notes) $\Rightarrow k-1 > 44.38 \Rightarrow (k =) 46$	M1 M1 A1 (3)
(ii)	Sets $850 + (n-1) \times -7 = 0 \Rightarrow n = 122.4$ States or implies that $n = 122$ $S = \frac{122}{2} \{2 \times 850 + 121 \times -7\} = 52033$	M1 A1 dM1, A1 (4)
		(9 marks)

(i) (a)

M1: Attempts $S_{\infty} = \frac{a}{1-r}$ with $a=10, r = \frac{8}{10}$ but condone slips on an incorrectly evaluated "r" provided the method to finding it is correct and $-1 < r < 1$. The expression with the values substituted in is sufficient.

A1: $(S_{\infty} =) 50$

(b) Note that candidates may misread the number of 0s in 0.0005 but can still score M1M1A0

M1: Sets $10 \times 0.8^{k-1} < 0.0005$ and proceeds to $0.8^{k-1} < 0.00005$ (or they may manipulate correctly to $0.8^k < 0.00005 \times 0.8$). If they take logs before this stage then they will need to proceed to e.g.
 $\log 0.8^{k-1} < \log 0.0005 - \log 10$
 Condone other signs being used e.g. = or >

M1: Uses a correct method to solve an equation or inequality of the type $p^k \dots q$ or $p^{k-1} \dots q$
 It cannot be for just stating the value for k. They only need to proceed as far as $k-1$ on one side of an equation or inequality. Evidence of using logs is required which cannot be e.g. 44.38... otherwise M0A0
 Do not be concerned with the direction of inequalities, if used, in their solution for this mark.

A1: $(k =) 46$ from correct working.

If inequalities are used in their method then the direction must be correct.



Question Number	Scheme	Marks
7. (i)	States or implies that $4 \times 6^{n-1} > 10^{100}$ Takes logs correctly to produce an equation without powers $4 \times 6^{n-1} > 10^{100} \Rightarrow \log 4 + (n-1)\log 6 > 100 \log 10$ $n = 129$	B1 M1 A1 (3)
(ii) (a)	States $ar = -6$ and $\frac{a}{1-r} = 25$ Combines to form an equation in $r \Rightarrow \frac{-6}{r(1-r)} = 25$ $\Rightarrow -6 = 25r(1-r) \Rightarrow 25r^2 - 25r - 6 = 0$ *	B1 M1 A1* (3)
(b)	$r = \frac{6}{5}, -\frac{1}{5}$	B1 (1)
(c)	$r = -\frac{1}{5}$ as $ r < 1$ (for S_∞ to exist)	B1 (1)
(d)	Attempts $S_4 = \frac{a(1-r^4)}{1-r}$ with $n=4, r = \text{their}(c)$ and $a = \frac{-6}{\text{their}(c)}$ $S_4 = \frac{30 \left(1 - \left(-\frac{1}{5} \right)^4 \right)}{1 - \left(-\frac{1}{5} \right)} = 24.96 \text{ o.e.}$	M1 A1 (2)
		(10 marks)

(i)

B1: States or implies the solution can be found by solving $4 \times 6^{n-1} \dots 10^{100}$ where ... can be = or any inequality

M1: Shows a correct method of solving an equation of the form $4 \times 6^N \dots 10^{100}$ by correctly taking logs to produce an equation without powers. The log and index work must be correct, but allow slips in rearranging terms.

A1: $n = 129$

(ii)(a)

B1: States or implies the correct two equations in a and r . $ar = -6$ and $\frac{a}{1-r} = 25$

M1: Combines $ar = -6$ and $\frac{a}{1-r} = 25$ to form a single equation in r

A1*: Proceeds to $25r^2 - 25r - 6 = 0$ showing at least one correct simplified intermediate line and no errors

(ii)(b)

B1: $r = \frac{6}{5}, -\frac{1}{5}$ or exact equivalent – award when seen even if not in part (b).

(ii)(c)

B1: $r = -\frac{1}{5}$ as $|r| < 1$ (for S_∞ to exist). Requires a minimal reason, but accept reasons that reject $\frac{6}{5}$ since it would mean all the terms are negative so cannot give a positive sum. Do not accept just “as the GS is convergent”.

(ii)(d)

M1: Attempts $S_4 = \frac{a(1-r^4)}{1-r}$ with $n=4, r = \text{their}(c)$ and $a = \frac{-6}{\text{their}(c)}$. If there was no attempt to answer (c) accept with either value from (b) for r .

If the correct formula is quoted then you can allow slips in substitution, but if the correct formula is not quoted then the equation should be correct for their r and a .

Alternatively, they may find and add the first 4 terms.

A1: 24.96 or equivalent such as $\frac{624}{25}$

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

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Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

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Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

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General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

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Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

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Question Number	Scheme	Marks
8(a)	$a_2 = 2p^2 - 7$	B1
		(1)
(b)	$a_3 = 2(2p^2 - 7 + 3)^2 - 7$	M1
	$p - 3 + 2p^2 - 7 + 8p^4 - 32p^2 + 25 = p + 15$ $\Rightarrow 8p^4 - 30p^2 = 0$	M1 A1
	Correct values of $p^2 = 0$ and $\frac{15}{4}$ (may be implied by $p = 0$ and $\pm\sqrt{\frac{15}{4}}$)	A1
	Uses their values of p to find a_2 using a correct method	dM1
	Possible values for a_2 are -7 and $\frac{1}{2}$	A1
		(6)
Alt (b)	$\sum_{n=1}^3 a_n = p + 15 \Rightarrow p - 3 + a_2 + 2(a_2 + 3)^2 - 7 = p + 15$	M1
	$\Rightarrow a_2 + 2(a_2 + 3)^2 - 25 = 0$ $\Rightarrow 2a_2^2 + 13a_2 - 7 = 0$	M1A1 A1
	$\Rightarrow (2a_2 - 1)(a_2 + 7) = 0 \Rightarrow a_2 = \dots$	dM1
	Possible values for a_2 are -7 and $\frac{1}{2}$	A1
		(6)
		(7 marks)

(a)

B1 $(a_2 =) 2p^2 - 7$

(b)

M1 An attempt to find a_3 by substituting their a_2 into the formula. Look for $2(\dots)^2 - 7$ where ... is a recognisable attempt at their $a_2 + 3$ (M0 if the -7 is missing).

M1 Attempts to add their first 3 terms and equate to $p + 15$. It is not dependent, and the attempts at the three terms may be poor, as long as they are attempting to add three terms.

A1 $8p^4 - 30p^2 = 0$ oe with terms gathered according to power (need not be all on one side).

A1 $(p^2 =) \frac{15}{4}, 0$. May be implied by $p = (\pm)\sqrt{\frac{15}{4}}$ and 0. Accept equivalent fractions or decimal answer for $\frac{15}{4}$, or awrt 1.94 for p . May be called by another letter, or even allow if p^2 is mistaken as p .

dM1 Attempts to find at least one value for a_2 using a value for p which has come from an attempt (however poor) at solving their equation. Must be using their formula from part (a). Depends on the second M mark.

A1cso Identifies both -7 and $\frac{1}{2}$ as the possible values for a_2 and no others.

Note that if they have $a_2 = 2p - 7$ by error in (a) they can score M1M1A0A1dM1A0cso in (b).

Alt

M1 Attempts sum of first three terms with a_3 in terms of a_2 and sets equal to $p + 15$

M1 Cancels p 's to reach an equation in just a_2

A1 Correct unsimplified equation in a_2

A1 Correct simplified quadratic in a_2

dM1 Solves the quadratic in a_2

A1cso Correct values for a_2

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Question Number	Scheme	Marks																
1	<table><tr><td>a</td><td>b</td><td>c</td><td>(abc)</td></tr><tr><td>6</td><td>1</td><td>3</td><td>(18)</td></tr><tr><td>4</td><td>2</td><td>4</td><td>(32)</td></tr><tr><td>2</td><td>3</td><td>5</td><td>(30)</td></tr></table>	a	b	c	(abc)	6	1	3	(18)	4	2	4	(32)	2	3	5	(30)	B1
	a	b	c	(abc)														
	6	1	3	(18)														
	4	2	4	(32)														
2	3	5	(30)															
Any one correct row for $b = 1$, $b = 2$ or $b = 3$. Products do not need to be found for this mark.																		
Attempts the product abc for at least 2 valid combinations.		M1																
Finds all three valid combinations with correct products seen and somewhere/shows why this is exhaustive and concludes. *		A1*																
		(3 marks)																

Note that in most cases the M1 can only follow B1 but there may be some exceptions.

Numerical approach using the table:

B1: Any one correct row for $b = 1$, $b = 2$ or $b = 3$. Products do not need to be found for this mark.

M1: Attempts the product abc for at least 2 valid combinations.

A1*: Requires:

- All three valid combinations with correct products
- No other combinations shown unless they are crossed out or e.g. have a cross at the end of the row or are discounted in some way
- A (minimal) conclusion e.g. the product of a , b and c is even, hence proven, QED, hence it is even, each product stated as even, etc.

Algebraic/logic approach:

B1: Uses the information to obtain a correct equation connecting a and b e.g. $a + 2b = 8$, $a = 8 - 2b$

M1: States a must be even and considers the product abc in some way

A1*: States e.g. abc is even with a reason e.g. “even $\times$ anything is even”

Pure Algebraic approach:

B1: Uses the information to obtain a correct equation connecting a and b e.g. $a + 2b = 8$, $a = 8 - 2b$

M1: $abc = (8 - 2b)b(b + 2)$

Attempts the product of a , b and c in terms of b (or some other letter)

A1*: $abc = 2(4 - b)b(b + 2)$ which is even, hence proven, QED etc.

Concludes abc is even and makes a (minimal) conclusion. There must be no algebraic errors.

NB using this approach “ $abc = -2b^3 + 4b^2 + 16b$ which is even hence proven” is not sufficient – they would need to say e.g. which is even + even + even or factor out the 2.

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Attempt to use correct formula (with values for a, b and c)

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Question Number	Scheme	Marks
3 (a)	$f(x) = (3x^2 - 4x - 5)(x - k) - 5$ States -5	B1 (1)
(b)	Sets $f(-2) = 25 \rightarrow (3 \times 4 - 4 \times -2 - 5)(-2 - k) - 5 = 25$ $15(-2 - k) = 30 \Rightarrow -2 - k = 2 \Rightarrow k = -4$ *	M1 A1* (2)
(c)	$(3x^2 - 4x - 5)(x + 4) - 5 = 3x^3 + 8x^2 - 21x - 25$ Attempts $3x - 1 \overline{) 3x^3 + 8x^2 - 21x - 25}$ to achieve quotient of $\dots x^2 + \dots x + \dots$ and a remainder that is a constant Quotient = $x^2 + 3x - 6$ OR Remainder = -31 Quotient = $x^2 + 3x - 6$ AND Remainder = -31	B1 M1 A1ft A1 (4) (7 marks)

(a)

 B1: States -5

 You may see $f(k)$ or a division sum attempted, which is fine. You are just awarding the sight of -5

 Don't be too concerned about what -5 is called. You may see $f(x)$, $f(k)$ or even x

(b)

 M1: Sets $f(-2) = 25$ to form an equation in k Condone sign slips within the calculations but setting

 $f(-2) = -25$ or $f(2) = 25$ is M0. Some may attempt to substitute -2 into the expanded form of

 $f(x) = 3x^3 - (4 + 3k)x^2 + (4k - 5)x + 5k - 5$ which is fine. Condone slips but expect to see linear coefficients for k in each of x^2 , x and the constant terms.

 Don't be concerned about poor attempts to solve the equation in k for this mark

 A1*: Shows sufficient working with no errors to prove that $k = -4$.

The answer must follow a simplified and (easily) solvable equation.

 (See scheme for possible route but allow simplified linear equations like $-30 - 15k = 30$)

 M1: Alternatively substitutes $k = -4$ into $f(x)$ to get $(3x^2 - 4x - 5)(x + 4) - 5$ and attempt the value of $f(-2)$

 A1: Shows that $f(-2) = (12 + 8 - 5)(-2 + 4) - 5 = 15 \times 2 - 5 = 25$ and makes a minimal conclusion e.g. ✓

Question Number	Scheme	Marks
11a	$u_2 = b - 3a$ $u_3 = b - a(b - 3a) \quad (= b - ab + 3a^2)$	B1 B1
		(2)
b	$3 + b - 3a + b - a(b - 3a) = 153$ oe $3 + (a + 9) - 3a + (a + 9) - a((a + 9) - 3a) = 153 \Rightarrow 2a^2 - 10a - 132 = 0$ $\Rightarrow a^2 - 5a - 66 = 0$ *	M1 dM1A1*
		(3)
c	$a = -6$ $b = 9 - 6 = 3 \Rightarrow u_2 = 3 - 3(-6) = 21$ or $u_2 = 9 - 2a = 9 - 2 \times -6 = 21$	B1 M1A1
		(3)
		(8 marks)

(a) Check next to the question. If there is a contradiction between the answer next to the question and the answer in the main body of the work then the answer in the main body of the work takes precedence.

B1 $(u_2 =) b - 3a$ oe

B1 $(u_3 =) b - a(b - 3a)$ oe isw

(b)

M1 Attempts to add 3, their u_2 and their u_3 and sets equal to 153. Condone slips in their copying of their expressions for u_2 and u_3 . Invisible brackets may be implied by further work.

May be in terms of a or b only if they substitute in $b = a + 9$ at the same time.

e.g. $3 + a + 9 - 3a + a + 9 - a(a + 9 - 3a) = 153$

dM1 Attempts to substitute $b = a + 9$ into their equation and then attempts to collect terms to achieve a 3TQ in a (condone the omission of $= 0$). The three terms do not all need to be on the same side of the equation. It is dependent on them having a u_3 term which would lead to an a^2 term. Do not be too concerned by the mechanics of their rearrangement for this mark. It is dependent on the previous method mark.

A1* Achieves the given answer with no arithmetical errors seen. Condone recovery of missing brackets, provided the recovery is before the final answer. Must see at least one intermediate stage of working following an equation in a only, before proceeding to the given quadratic equation.

(c)

B1 $a = -6$ seen or used in (c). Ignore the positive root for this mark.

M1 Attempts to substitute their negative root into $b = a + 9$ to find a value for b and then uses these in their expression for u_2 to find a value for u_2

Alternatively, attempts to substitute both their negative root and $b = a + 9$ into their expression for u_2 and proceeds to find a value for u_2

Ignore working relating to where they repeat the calculations to find u_2 using their positive root

May be implied by their answer (you may need to check this)

Note: For candidates who have two roots of the same sign then this mark can be awarded for using their smaller value (i.e. most negative/least positive) and proceeding to find a value for u_2

A1 21 only cao (if two answers are reached they must select 21 or reject the other answer)
Correct answer on its own scores full marks.



Question Number	Scheme	Marks
1	$f(x) = ax^3 + 3x^2 - 8x + 2$	
	Sets $f(2) = 3 \Rightarrow 8a + 12 - 16 + 2 = 3$	M1
	$\Rightarrow 8a = 5 \Rightarrow a = \frac{5}{8}$	M1A1
		(3)
		(3 marks)

M1: Attempts to set $f(2) = 3$. The values embedded and the expression set equal to 3 is sufficient. May be implied by further work. If 2 embedded in the expression is not seen, condone slips in their evaluation provided there is still evidence that the intention was to substitute in 2.

May also be seen as $(f(2) - 3) = a(2)^3 + 3(2)^2 - 8(2) - 1 = 0$

M1: Solves a linear equation in a arising from setting $f(\pm 2) = \pm 3$. Condone slips in their rearrangement proceeding to $a = \dots$ (May be implied by further work).

A1: $a = \frac{5}{8}$ or exact equivalent

Answers of $a = \frac{5}{8}$ may appear with very little or no working, possibly via trial and improvement. If so, then marks can only be allocated if evidence is shown.

e.g. $f(x) = \frac{5}{8}x^3 + 3x^2 - 8x + 2 \Rightarrow f(2) = 5 + 3(2)^2 - 8(2) + 2 = 3 \Rightarrow$ dividing by $(x - 2)$ gives a remainder of 3

More difficult alternative methods may be seen:

Alt I: e.g. algebraic division (note they may also divide $ax^3 + 3x^2 - 8x - 1$ by $(x - 2)$ and set the remainder equal to 0)

$$\begin{array}{r}
 \frac{ax^2 + (3+2a)x + (-2+4a)}{x-2} \overline{) \begin{array}{r} ax^3 - 8x \\ ax^3 - 2ax^2 \\ (3+2a)x^2 \\ (3+2a)x^2 + (-6-4a)x \\ (-2+4a)x \\ (-2+4a)x + 4 - 8a \end{array} \\
 \hline
 -2 + 8a \Rightarrow -2 + 8a = 3 \Rightarrow a = \frac{5}{8}
 \end{array}$$

M1: Divides the cubic by $(x - 2)$, leading to a quadratic quotient where both the coefficient of x and the constant term are in terms of a . They should then have a linear remainder in a which is then set equal to 3

M1: Solves an equation resulting from setting a linear remainder in a equal to ± 3 . It is dependent on the first method mark via this route.

A1: Completely correct with $a = \frac{5}{8}$

Alt II: You may also see a grid or an attempt at factorisation of $ax^3 + 3x^2 - 8x - 1$ via inspection

M1: For an attempt at factorising e.g. $ax^3 + 3x^2 - 8x - 1 = (x - 2)\left(ax^2 + bx + \frac{1}{2}\right)$

M1: Forms two correct simultaneous equations and proceeds to find a value for a . Condoning slips in their solving.

$$x^2: 3 = -2a + b$$

$$x: -8 = \frac{1}{2} - 2b \Rightarrow b = \frac{17}{4} \Rightarrow a = \frac{3 - \frac{17}{4}}{-2} = \frac{5}{8}$$

A1: Completely correct with $a = \frac{5}{8}$



Question Number	Scheme	Marks
2	$\left(\frac{3}{8} + 4x\right)^{12}$	
	Term in x^7 is ${}^{12}C_7\left(\frac{3}{8}\right)^5(4x)^7$ or coefficient of x^7 is ${}^{12}C_7\left(\frac{3}{8}\right)^5 4^7$	M1 A1
	Coefficient is 96228	A1
		(3)
		(3 marks)

Note that you do not need to see x^7 in their solution

M1: Combines a correct binomial coefficient with $\left(\frac{3}{8}\right)^5$ and either $(4x)^7$, 4^7 or x^7

Look for the binomial coefficient of the form e.g. $\binom{12}{7}$, ${}^{12}C_7$ or 792 o.e. Condone $\binom{12}{5}$ or ${}^{12}C_5$.

May be implied by further work.

$$\text{Alternatively } \left(\frac{3}{8} + 4x\right)^{12} = \left(\frac{3}{8}\right)^{12} \left(1 + \frac{32}{3}x\right)^{12} \Rightarrow \left(\frac{3}{8}\right)^{12} \times \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6}{7!} \times \left(\frac{32}{3}x\right)^7$$

Condone invisible brackets for this mark. e.g. ${}^{12}C_7 \times \frac{3^5}{8} \times 4x^7$

A1: Correct unsimplified term or coefficient e.g. ${}^{12}C_7\left(\frac{3}{8}\right)^5 \times 4^7$ or e.g. $\frac{12!}{5!7!}\left(\frac{3}{8}\right)^5 \times 4^7$. Invisible brackets may

be implied by further work. Do not be concerned by the presence or absence of x^7

Note ${}^{12}C_7\left(\frac{3}{8}\right)^5(4x)^7$ or ${}^{12}C_5\left(\frac{3}{8}\right)^5(4x)^7$ scores M1A1 (may be seen as part of a list or several terms found in the expansion)

A1: For 96228 but condone 96 228 x^7 isw once a correct answer is seen. The term or coefficient must be identified if they have more than one term. A correct answer on its own with no incorrect working seen can score full marks.

Note: $\left({}^{12}C_7\left(\frac{3}{8}\right)^5 \times 4x^7 = 23.493...\right) \approx 23.5$ will score M1A0A0 (where they multiply by 4 instead of 4^7)



Question Number	Scheme	Marks
3(a)	Attempts $(8-3)^2 + (5-7)^2 = \dots$	M1
	Writes $(x-3)^2 + (y-5)^2 = k$	M1
	$(x-3)^2 + (y-5)^2 = 169$	A1
		(3)
(b)	Attempts $d^2 + (2\sqrt{22})^2 = 169 \Rightarrow d = \dots$	M1
	States or uses $(y=) 5 + d$	dM1
	$y = 14$	A1
		(3)
		(6 marks)

(a)

M1: Attempts to find the radius or radius². Must proceed to find a value. Condone a sign slip if attempting $5 - -7$ if this is seen as $5 - 7$ o.e. May be implied by 13 or 169. Do not be concerned with the labelling of the expression or value as r or r^2

M1: Writes the equation of the circle in the form $(x-3)^2 + (y-5)^2 = k$, where $k > 0$, o.e.

$$\text{e.g. } x^2 + y^2 - 6x - 10y + c = 0 \text{ where } c < 0$$

Invisible brackets may be implied by further work. Condone $(x-3)^2 + (y-5)^2 = r^2$ where no attempt has been made to find the radius or radius²

A1: $(x-3)^2 + (y-5)^2 = 169$ o.e. e.g. $x^2 + y^2 - 6x - 10y - 135 = 0$ isw once a correct unsimplified equation

for C is found. Condone $(x-3)^2 + (y-5)^2 = 13^2$. A correct equation scores M1M1A1 but it must be seen in (a).

(b) Note that $(3 + 2\sqrt{22}, 14)$ as the final answer is A0

M1: Attempts Pythagoras' theorem with "13" (as the hypotenuse) and $2\sqrt{22}$ to find d . Not just d^2 (See diagram)

dM1: States or uses the equation for MN , $(y=) 5 + d$ but also condone stating/using $(y=) 5 - d$ (may be implied by their value for y)

A1: $y = 14$ only (Must be exactly 14 e.g. $13.99\dots = 14$ is A0). **Further work leading to a different equation of a line is A0.**

