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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Trigonometric Identities & Equations (P2.6)

International A Level Mathematics · P2: Pure Mathematics 2 (WMA12)

20 questions · 175 marks · Mark schemes included

Past papers June 2019 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
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P2.6a TRIGONOMETRIC IDENTITIES

No questions on this sub-topic in the available past papers.

P2.6b SOLVING TRIGONOMETRIC EQUATIONS	20 Qs · 175 marks	2
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1. Q5	IAL P2 (WMA12) June 2022	6	3
2. Q7	IAL P2 (WMA12) January 2020	7	5
3. Q7	IAL P2 (WMA12) October 2020	7	7
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5. Q3	IAL P2 (WMA12) October 2023	7	13
6. Q7	IAL P2 (WMA12) June 2024	7	15
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9. Q6	IAL P2 (WMA12) January 2021	8	23
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13. Q8	IAL P2 (WMA12) June 2025	8	38
14. Q8	IAL P2 (WMA12) January 2023	9	42
15. Q8	IAL P2 (WMA12) June 2021	10	46
16. Q10	IAL P2 (WMA12) October 2021	10	50
17. Q9	IAL P2 (WMA12) October 2019	12	53
18. Q8	IAL P2 (WMA12) June 2024	12	57
19. Q6	IAL P2 (WMA12) June 2025	12	61
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QUESTIONS

20 questions · 175 marks · 1 sub-topic

SUB-TOPIC P2.6b

SOLVING TRIGONOMETRIC EQUATIONS

Sub-topic P2.6b · 20 questions · 175 marks · lowest tariff first



5. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $-180^\circ < \theta \leq 180^\circ$, the equation

$$3 \tan(\theta + 43^\circ) = 2 \cos(\theta + 43^\circ)$$

(6)

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Question 5 continued

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(Total 6 marks)

Q5



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7. (a) Show that the equation

$$8 \tan \theta = 3 \cos \theta$$

may be rewritten in the form

$$3 \sin^2 \theta + 8 \sin \theta - 3 = 0$$

(3)

- (b) Hence solve, for $0 \leq x \leq 90^\circ$, the equation

$$8 \tan 2x = 3 \cos 2x$$

giving your answers to 2 decimal places.

(4)

Question 7 continued

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Q7

(Total 7 marks)



7. (i) Show that

$$\tan \theta + \frac{1}{\tan \theta} \equiv \frac{1}{\sin \theta \cos \theta} \quad \theta \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (3)$$

(ii) Solve, for $0 \leq x < 90^\circ$, the equation

$$3 \cos^2(2x + 10^\circ) = 1$$

giving your answers in degrees to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

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(Total 7 marks)

Q7



In this question you must show all stages of your working.

(a) Show that

$$3 \cos \theta (\tan \theta \sin \theta + 3) = 11 - 5 \cos \theta$$

may be written as

$$3 \cos^2 \theta - 14 \cos \theta + 8 = 0 \quad (3)$$

(b) Hence solve, for $0 < x < 360^\circ$

$$3 \cos 2x (\tan 2x \sin 2x + 3) = 11 - 5 \cos 2x$$

giving your answers to one decimal place.

(4)



Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 7 marks)



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 7 marks)



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 7 marks)



9.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2 \tan \theta = 3 \cos \theta$$

can be written as

$$3 \sin^2 \theta + 2 \sin \theta - 3 = 0 \quad (3)$$

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2 \tan \left(2x + \frac{\pi}{3} \right) = 3 \cos \left(2x + \frac{\pi}{3} \right)$$

giving your answers to 3 significant figures. (4)

Question 9 continued

(Total for Question 9 is 7 marks)



Question 9 continued

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Question 9 continued

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Question 9 continued

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(Total 8 marks)

Q9



6. (a) Show that the equation

$$\frac{3 \sin \theta \cos \theta}{2 \sin \theta - 1} = 5 \tan \theta \quad \sin \theta \neq \frac{1}{2}$$

can be written in the form

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

(4)

(b) Hence solve, for $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$\frac{3 \sin 2x \cos 2x}{2 \sin 2x - 1} = 5 \tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

Question 6 continued

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7. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

- (i) Solve, for $-90^\circ < x < 90^\circ$, the equation

$$3 \sin(2x - 15^\circ) = \cos(2x - 15^\circ)$$

giving your answers to one decimal place.

(4)

- (ii) Solve, for $0 < \theta < 2\pi$, the equation

$$4 \sin^2 \theta + 8 \cos \theta = 3$$

giving your answers to 3 significant figures.

(4)

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Q7

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$(3 \cos \theta - \tan \theta) \cos \theta = 2$$

can be written as

$$3 \sin^2 \theta + \sin \theta - 1 = 0 \quad (3)$$

(b) Hence solve for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$$(3 \cos 2x - \tan 2x) \cos 2x = 2 \quad (5)$$



Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)





Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 8 marks)



8.

In this question you must show all stages of your working.

Solutions based entirely on calculator technology are not acceptable.

- (i) Solve, for $-\frac{\pi}{2} < x < \pi$, the equation

$$5 \sin(3x + 0.1) + 2 = 0$$

giving your answers, **in radians**, to 2 decimal places.

(4)

- (ii) Solve, for $0 < \theta < 360^\circ$, the equation

$$2 \tan \theta \sin \theta = 5 + \cos \theta$$

giving your answers, **in degrees**, to one decimal place.

(5)





(Total for Question 8 is 9 marks)



Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total 10 marks)

Q8



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Question 10 continued

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Q10

(Total 10 marks)

END

TOTAL FOR PAPER IS 75 MARKS



(i) Solve, for $0 \leq \theta < 180^\circ$, the equation

giving your answers to one decimal place.

(a) Show that $2 \cos \alpha = 3 \sin^2 \alpha$

(b) find, showing all working, the value of α to 3 decimal places.



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Question 9 continued

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(Total 12 marks)

Q9







Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 12 marks)



6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

(i) Given that θ is measured in degrees and

$$\bullet \cos \theta = \frac{1}{\sqrt{5}}$$

$$\bullet 180^\circ < \theta < 360^\circ$$

use trigonometric identities to find the exact value of

(a) $\sin \theta$

(b) $\tan \theta$

giving the answers as fully simplified surds where appropriate.

(4)

(ii)

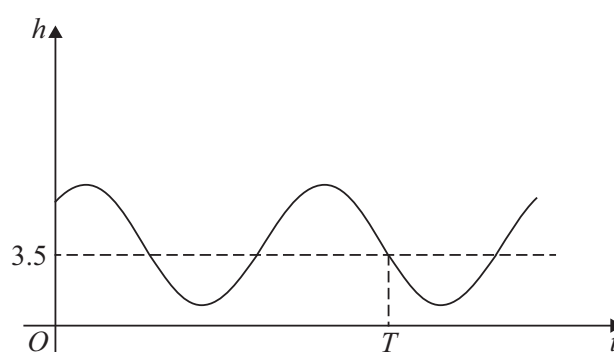


Figure 2

The height of sea water, h metres, on a harbour wall, t hours after midnight on a particular day is given by

$$h = 4 + 3 \cos(30t - 40)^\circ \quad 0 \leq t < 24$$

A sketch of h against t is shown in Figure 2.

(a) Find the minimum height of sea water on the harbour wall.

(1)

(b) Find the exact time of day when this minimum height **first** occurs.

(3)

When $t = T$, as shown in Figure 2, a boat enters the harbour when the height of sea water on the harbour wall is 3.5 m.

(c) Use Figure 2 and the given equation to find the value of T to 2 decimal places.

(4)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 12 marks)



9.

In this question you must show detailed reasoning.**Solutions relying entirely on calculator technology are not acceptable.**

- (i) Solve, for
- $0 \leq x < 360^\circ$
- , the equation

$$\sin x \tan x = 5$$

giving your answers to one decimal place.

(6)

- (ii)

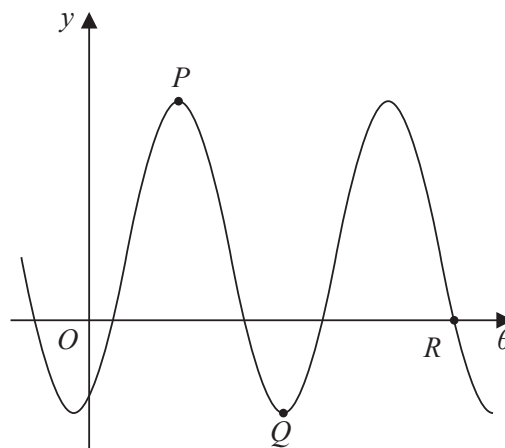
**Figure 1**

Figure 1 shows a sketch of part of the curve with equation

$$y = A \sin\left(2\theta - \frac{3\pi}{8}\right) + 2$$

where A is a constant and θ is measured in radians.The points P , Q and R lie on the curve and are shown in Figure 1.Given that the y coordinate of P is 7

- (a) state the value of
- A
- ,

(1)

- (b) find the exact coordinates of
- Q
- ,

(3)

- (c) find the value of
- θ
- at
- R
- , giving your answer to 3 significant figures.

(4)

Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Handwriting practice area with horizontal lines.



MARK SCHEMES

official mark schemes, in the same order as the questions



Question	Scheme	Marks
4	$\log_4 a + \log_4 b = \log_4 ab$ or $\log_4 "ab" = 3 \Rightarrow "ab" = 4^3$	M1
	$ab = K, a - b = 8 \Rightarrow a(a - 8) = K$ or $(b + 8)b = K$	dM1
	$a^2 - 8a - 64 = 0$ or $b^2 + 8b - 64 = 0$	A1
	$a = \frac{-(-8) + \sqrt{(-8)^2 - 4 \times 1 \times -64}}{2 \times 1} = \dots$ or $b = \frac{-8 + \sqrt{8^2 - 4 \times 1 \times -64}}{2 \times 1} = \dots$	M1
	$a = '4 + 4\sqrt{5}' \Rightarrow b = -4 + 4\sqrt{5}$	M1
	$a = 4 + 4\sqrt{5}$ and $b = -4 + 4\sqrt{5}$ and no other solutions.	A1
		(6)
(6 marks)		
Notes:		
M1: Correct addition law applied (may be implied) or undoes a log equation correctly, or replaces the 3 by $\log_4 4^3$ or The addition law may be seen after substitution from the first equation e.g. $a - b = 8 \Rightarrow a = b + 8 \Rightarrow \log_4 a + \log_4 b = \log_4 (b + 8) + \log_4 b = \log_4 b(b + 8)$ Condone $\log_4 (b + 8) + \log_4 b = \log_4 b^2 + 8b$ dM1: Removes logs correctly and proceeds from an equation of the form $ab = K$ and the given equation $a - b = 8$ to a quadratic equation in either a or b. A1: Correct quadratic equation. Brackets must be expanded but terms not necessarily all on one side. The “=0” may be implied by their attempt to solve. M1: Solves a 3TQ in a or b to obtain a positive solution. There is no need to see the negative solution in their working. M1: Uses their a to find b or vice versa. This depends on having solved a 3TQ earlier and obtained a root of the form $p + q\sqrt{r}$, $p, q, r \neq 0$ to obtain another root of a similar form i.e. not a decimal. The value of a or b does not necessarily have to be positive for this mark. Note that having found a or b they may repeat the process above to find the other value which is acceptable. A1: Both of $a = 4 + 4\sqrt{5}$ and $b = -4 + 4\sqrt{5}$ simplified, with no other solutions. Apply isw once correct answers are seen, e.g. if they subsequently go into decimals.		

Question	Scheme	Marks
5	$3 \tan(\theta + 43^\circ) = 2 \cos(\theta + 43^\circ) \Rightarrow 3 \frac{\sin(\theta + 43^\circ)}{\cos(\theta + 43^\circ)} = 2 \cos(\theta + 43^\circ)$ $\Rightarrow 3 \sin(\theta + 43^\circ) = 2 \cos^2(\theta + 43^\circ)$	M1
	$\Rightarrow 3 \sin(\theta + 43^\circ) = 2(1 - \sin^2(\theta + 43^\circ))$	M1
	$\Rightarrow 2 \sin^2(\theta + 43^\circ) + 3 \sin(\theta + 43^\circ) - 2 = 0$ $\Rightarrow (2 \sin(\theta + 43^\circ) - 1)(\sin(\theta + 43^\circ) + 2) = 0 \Rightarrow \sin(\theta + 43^\circ) = \dots$	M1
	$\sin(\theta + 43^\circ) = \frac{1}{2}$	A1
	$\theta = \arcsin \frac{1}{2} - 43^\circ$	M1
	$\theta = -13^\circ, 107^\circ$	A1
		(6)
(6 marks)		

Notes:

M1: Uses $\tan \dots = \frac{\sin \dots}{\cos \dots}$ and multiplies through to form an equation of the form $A \sin \dots = B \cos^2 \dots$

Condone poor notation e.g.:

$$3 \tan(\theta + 43^\circ) = 2 \cos(\theta + 43^\circ) \Rightarrow 3 \frac{\sin}{\cos}(\theta + 43^\circ) = 2 \cos(\theta + 43^\circ)$$

$$\Rightarrow 3 \sin(\theta + 43^\circ) = 2 \cos^2(\theta + 43^\circ) \text{ (with or without brackets)}$$

M1: Applies Pythagorean identity to obtain a 3 term quadratic equation in $\sin$.

Allow use of $\cos^2 \dots = \pm 1 \pm \sin^2 \dots$

M1: Solves a 3 term quadratic in $\sin(\theta + 43^\circ)$ by any valid means.

This may be implied by at least one correct root for their quadratic.

Allow if they have $\sin(\theta + 43^\circ) = x$ or another variable or e.g. $\sin \alpha$ where $\alpha = \theta + 43^\circ$

A1: Correct value of $\sin(\theta + 43^\circ)$. If $\sin(\theta + 43^\circ) = x$ is used, it must be clear they mean

$\sin(\theta + 43^\circ)$ but this may be implied if they have e.g. $\sin \alpha = \frac{1}{2}$ where $\alpha = \theta + 43^\circ$

If $x = \frac{1}{2}$ is left as a final answer it is A0.

M1: Correct method for solving $\sin(\theta + 43^\circ) = k, |k| < 1$, look for use of inverse sine followed by

subtraction of 43 from $\sin^{-1}(\text{their } k)$. Implied by one correct solution for their k

Do **not** allow mixing of degrees and radians for this mark.

A1: Correct solutions and no others in the range.



Question Number	Scheme	Marks
4	$\int (2x^2 + 7) dx = \frac{2}{3}x^3 + 7x \quad \text{or} \quad \int (10 - 2x^2) dx = 10x - \frac{2}{3}x^3$ Achieves/uses a limit of $\sqrt{5}$ $\text{Area} = 17\sqrt{5} - \int_0^{\sqrt{5}} (2x^2 + 7) dx \quad \left \quad \text{Area} = \int_0^{\sqrt{5}} (10 - 2x^2) dx \right.$ $= 17\sqrt{5} - \frac{2}{3} \times 5\sqrt{5} - 7\sqrt{5} \quad \left \quad = 10\sqrt{5} - \frac{2}{3} \times 5\sqrt{5} \right.$ $= \frac{20}{3}\sqrt{5}$	M1 A1 B1 M1 M1 A1 (6) (6 marks)

Line and curve separate

M1 Correct attempt at integration on $2x^2 + 7$ ie raises a power by one

A1 $\frac{2}{3}x^3 + 7x$ (need not be simplified)

B1 Achieves a limit of $\sqrt{5}$ either attached to an integral or used to find the area of the rectangle.

M1 For applying Area of $R = \pm(\text{area of rectangle} - \text{area under curve}) = \pm \left(17\sqrt{5} - \int (2x^2 + 7) dx \right)$

M1 Area under curve = $\left[\frac{2}{3}x^3 + 7x \right]_0^{\sqrt{5}} = \frac{2}{3}\sqrt{5}^3 + 7\sqrt{5}$. This is for applying the limits of 0 and their $\sqrt{5}$

(must be an x value) to the integral which must be a changed function. The application of the 0 may be implied. Can be scored if the rectangle is not considered. May be awarded on decimals correct to 1d.p. (allow if found by calculator (23.1)).

A1 $\frac{20}{3}\sqrt{5}$ or exact (single term) equivalent and can be recovered from a negative made positive

Line and curve together

M1 Attempts $\pm$ (line – curve) with correct attempt at integration on their $\pm 10 \pm 2x^2$ (raises a power by one)

A1 $\pm \left(10x - \frac{2}{3}x^3 \right)$ (may be either way round)

B1 Achieves a limit of $\sqrt{5}$

M1 For Area = $\pm \int_0^{\sqrt{5}}$ their "line – curve" dx with limits 0 and their $\sqrt{5}$ which must be an x value.

M1 $\left[10x - \frac{2}{3}x^3 \right]_0^{\sqrt{5}} = 10\sqrt{5} - \frac{2}{3}\sqrt{5}^3$ This is for applying the limits of 0 (may be implied) and their $\sqrt{5}$ (must be an x value) to the integral. May be awarded on decimals (1d.p., calculator use ie 14.9 if correct integral).

A1 $\frac{20}{3}\sqrt{5}$ or exact (single term) equivalent and can be recovered from a negative made positive

.....
Note that Area of $R = \int_7^{17} \frac{(y-7)^{\frac{1}{2}}}{\sqrt{2}} dy = \left[\frac{2(y-7)^{\frac{3}{2}}}{3\sqrt{2}} \right]_7^{17}$ is not on specification but allowable and the above

scheme can be applied having made x the subject. M1A1 correct integration B1 uses limit 7 M1 attempts integral w.r.t. y with limits their 7 and 17 (need not be correct integration) M1 applies limits A1 answer.



Question Number	Scheme	Marks
5 (a)	Attempts $30\,000 \times r^3 = 34\,000$ $r^3 = \frac{17}{15} \Rightarrow r = 1.0426 \text{ Hence } p = 4.26$	M1 A1 A1ft [3]
(b)	Attempts $30\,000 \times (1.0426)^{N^*} = 75\,000$ or with $>$ or $<$ etc throughout. $(1.0426)^{N^*} = \frac{5}{2}$ $\log\left(\frac{5}{2}\right)$ Takes logs $N^* = \frac{\log\left(\frac{5}{2}\right)}{\log 1.0426}$ (= awrt 21.96) $N = 22$	M1 A1 M1 A1 B1 [5]
		(8 marks)

(a)

M1 Attempts to find the common ratio by use of $30\,000 \times r^3 = 34\,000$ or $30\,000 \times r^2 = 34\,000$. Must involve taking a root. It may be called p . Condone slips on the 30000 or 34000.

A1 For awrt 1.04. It may be called p

A1ft $p = 4.26$ or follow through on a correct percentage to 2 d.p. for their r if the method has been earned (usually 6.46). Note it must be 2d.p. for this mark.

(b)

M1 States/uses $30\,000 \times ("r")^{N^*} = 75\,000$ or $30\,000 \times ("r")^{N-1} = 75\,000$ with their ratio r , but not if their percentage is used. E.g. follow through on their 1.0426 but not 4.26

A1 "Correct" intermediate statement $("r")^{N^*} = \frac{5}{2}$ or $("r")^{N-1} = \frac{5}{2}$

Accept $r = \text{awrt } 1.04$ or $r = \text{awrt } 1.06$ if r^2 was used in (a)

M1 Uses logs correctly $N^* = \frac{\log\left(\frac{5}{2}\right)}{\log "1.0426"}$ or $N-1 = \frac{\log\left(\frac{5}{2}\right)}{\log "1.0426"}$ or $N^* = \log_{"1.0426"}\left(\frac{5}{2}\right) =$

Award for the correct solution of any index equation using logs (correctly). The equation may be from incorrect work, e.g. having used S_n but must be a solvable equation (ie positive base and operand)

A1 Correct expression for N or $N = \text{awrt } 21.96$ following correct work

B1 Depends on $r = \text{awrt } 1.04$ and ar^N used. It is for 22 or f.t. their " N " rounded up to nearest integer providing the only errors have been due to rounding. E.g. For $r = 1.04$ this will be for $N = 24$

Note: Accept answers in (b) using inequalities and don't be concerned about the inequalities.

S.C. If power $N-1$ is used then $N = 22.962$ will be reached if all is correct. If they then round down to 22 then the final A and B can be recovered (it is correct work).

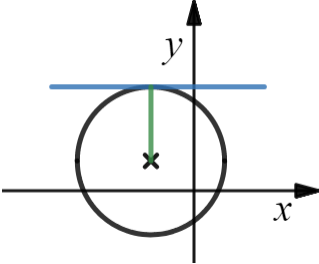
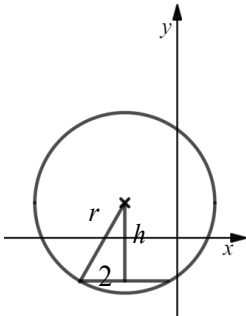
SC For trial and improvement M1 Tries $30\,000 \times ("r")^N$ with N either side of 22 or appropriate value for their r (eg 15 for 1.0646) with A1 correct values found either side of 75000 for $r = \text{awrt } 1.04$ or 1.06

M1 Values for N directly either side of 75000 found for their r . A1 Must be using $r = 1.0426$. Finds

$30\,000 \times (1.0426)^{21} = 72043$ and $30\,000 \times (1.0426)^{22} = 75112$.

B1 Deduces 22 or f.t. $N = 24$ for $r = 1.04$.



Question Number	Scheme	Marks
6.(a)	$x^2 + y^2 + 6x - 4y - 14 = 0$ Attempts $(x \pm 3)^2 + (y \pm 2)^2 \dots (= 0)$ (i) Centre $(-3, 2)$ (ii) Radius $\sqrt{27}$ or $3\sqrt{3}$	M1 A1 A1 (3)
(b)	 Attempts $y/k = "2 \pm 3\sqrt{3}"$ Both $k = 2 + 3\sqrt{3}$ and $k = 2 - 3\sqrt{3}$	M1 A1 (2)
(c)	Alt: $y = k \Rightarrow x^2 + 6x + k^2 - 4k - 14 = 0$ $\Rightarrow b^2 - 4ac = 0 \Rightarrow 6^2 - 4 \times 1 \times (k^2 - 4k - 14) = 0$ $\Rightarrow 4k^2 - 16k - 92 = 0 \Rightarrow k = \dots$ $k = 2 \pm 3\sqrt{3}$ (oe)	M1 A1 (2)
	 Attempts Pythagoras to find h $h^2 = r^2 - 2^2$ Attempts $p = 2 - h$ $p = 2 - \sqrt{23}$ (oe)	M1 dM1 A1 (3)
	Alt1: chord touches circle when $x = -1$ (or -5) or need $(x + 3) = \pm 2$ $\Rightarrow 2^2 + (y - 2)^2 = 27 \Rightarrow y = \dots$ or $(-1)^2 + y^2 + 6(-1) - 4y - 14 = 0 \Rightarrow y = \dots$ $p = 2 - \sqrt{23}$	M1 dM1 A1 (3)
	Alt2: Sets $y = p$ and applies difference of roots to be 4 $\Rightarrow x^2 + 6x + p^2 - 4p - 14 = 0 \Rightarrow x = \frac{-6 \pm \sqrt{36 - 4(p^2 - 4p - 14)}}{2}$ $\Rightarrow 2\sqrt{23 + 4p - p^2} = 4$ $\Rightarrow p^2 - 4p - 19 = 0 \Rightarrow p = \dots$ $p = 2 - \sqrt{23}$	M1 dM1 A1 (3) (8 marks)

(a)

M1 Attempts to complete the square. Look for $(x \pm 3)^2 + (y \pm 2)^2 \dots = 0$ M1 may be awarded for a centre of $(\pm 3, \pm 2)$ (from no, or from incorrect, working)A1 Centre $(-3, 2)$ A1 Radius $\sqrt{27}$ or $3\sqrt{3}$

(b)

M1 Attempts one value of k by " $2 \pm r$ ". In the alternative it is for setting $y = k$ in the circle equation, applying $b^2 - 4ac = 0$ and then solving the resulting quadratic in k , or equivalent method. But substitution of $x=0$ is M0.A1 For both $k = 2 + 3\sqrt{3}$ and $k = 2 - 3\sqrt{3}$ Allow $y \leftrightarrow k$ Accept exact equivalents, e.g. with $\sqrt{27}$

Alt by differentiation requires

M1 Achieves $ax + by \frac{dy}{dx} + cx + d \frac{dy}{dx} = 0$ followed by setting $\frac{dy}{dx} = 0$, finding x and substituting into the circle equation then attempting to solve the quadratic. A1 Correct answers.

(c)

M1 For an attempt to use Pythagoras's theorem with 2 and their ' r ' to find ' h ' or ' h^2 'dM1 Attempts $p = "2 \pm h"$ Condone $p \leftrightarrow y$ Follow through their y coordinate of the centre.
Can be implied by the correct decimal answer (awrt -2.8)A1 $p = 2 - \sqrt{23}$ only Condone $p \leftrightarrow y$ Accept equivalent exact forms e.g. $-(\sqrt{23} - 2)$

Alternatives are possible:

Alt 1

M1 Identifies an x coordinate where the chord touches the circle, $x = -3 \pm 2$, or applies $x + 3 = \pm 2$

dM1 Substitutes into the circle equation (either form) and attempts to solve the quadratic (usual rules)

A1 $p = 2 - \sqrt{23}$ only Condone $p \leftrightarrow y$

Alt2

M1 Substitutes $y = p$ into C and applies the difference between roots is 4dM1 Solves the resulting quadratic to find p .A1 $p = 2 - \sqrt{23}$ only Condone $p \leftrightarrow y$ 

Question Number	Scheme	Notes	Marks
4(a)(i)	$(-7, 9)$ or e.g. $x = -7, y = 9$	$x = -7$ or $y = 9$	B1
		$x = -7$ and $y = 9$	B1
	Award the marks in (a) once correct answers are seen. Special case: If all you see is $(9, -7)$ award B1B0		
(a)(ii)	<u>Examples:</u> $r = \sqrt{(-3 - ("7"))^2 + (12 - "9")^2}$ or $r = \sqrt{(-11 - ("7"))^2 + (6 - "9")^2}$ or $r = \frac{1}{2}\sqrt{(-3 + 11)^2 + (12 - 6)^2}$	Correct strategy for the radius. Must be a correct method for their centre (if used) but allow 1 sign slip within one of the brackets . A correct answer scores both marks. Must see the $\frac{1}{2}$ if finding the length of the diameter.	M1
	$r = 5$	Correct radius	A1
			(4)
(b)	$(x + 7)^2 + (y - 9)^2 = 5^2$ or e.g. $x^2 + y^2 + 2 \times 7x - 2 \times 9y + 7^2 + 9^2 - 5^2 = 0$ M1: Correct attempt at circle equation using their values. Allow for $(x \pm (\text{their } 7))^2 + (y \pm (\text{their } 9))^2 = (\text{their numerical } r)^2$ or e.g. $x^2 + y^2 \pm 2 \times (\text{their } 7)x \pm 2 \times (\text{their } 9)y + (\text{their } 7)^2 + (\text{their } 9)^2 - (\text{their numerical } r)^2 = 0$ A1: Correct equation in any form		M1A1
			(2)
(c)	$m_{\text{radius}} = \frac{12 - 9}{-3 + 7} \left(= \frac{3}{4} \right) \text{ or }$ $m_{\text{tangent}} = -1 \div \left(\frac{12 - 9}{-3 + 7} \right) \left(= -\frac{4}{3} \right) \text{ or }$ $m_{\text{tangent}} = -\left(\frac{-3 + 7}{12 - 9} \right) \left(= -\frac{4}{3} \right)$	This mark is for an attempt to find the radius gradient or the tangent gradient. If the method is not clear allow one sign error in the numerator or denominator.	M1
	Alternative for the first M: $(x + 7)^2 + (y - 9)^2 = 5^2 \Rightarrow 2(x + 7) + 2(y - 9) \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{x + 7}{9 - y} = \frac{-3 + 7}{9 - 12} \left(= -\frac{4}{3} \right)$ Allow for $(x + 7)^2 + (y - 9)^2 = 5^2 \Rightarrow \alpha(x + 7) + \beta(y - 9) \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \dots$		
	$y - 12 = -\frac{4}{3}(x + 3)$ Uses a correct straight line method for the tangent using the point Q. Must be fully correct work here so must be a clear attempt at the tangent not the radius. So if the radius gradient is found previously, must apply negative reciprocal rule to their radius gradient. If using $y = mx + c$ must reach as far as $c = \dots$		M1
	$4x + 3y - 24 = 0$	Allow any integer multiple	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$t_{40} = 100 + (40 - 1) \times 5$	Uses $a + (n - 1)d$ with $a = 100$, $d = 5$ and $n = 40$. This may be implied by a correct expression e.g. $100 + 39 \times 5$	M1
	$= (£)295$	Cao. Correct answer with no working scores both marks.	A1
			(2)
(b)	$S_{60} = \frac{1}{2}(60)(2 \times 100 + (60 - 1) \times 5)$ or $l = 100 + (60 - 1) \times 5 = 395$ $S_{60} = \frac{1}{2}(60)(100 + 395)$	Uses a correct sum formula with $a = 100$, $d = 5$ and $n = 60$ or $n = 40$. May be implied by a correct numerical expression. If using $\frac{1}{2}n(a + l)$ with $n = 40$ you may see $\frac{1}{2}(40)(100 + 295)$ using their result from (a) and this scores M1 also.	M1
		Correct numerical expression with $n = 60$. If there are any missing brackets then this mark should be withheld unless the correct expression is implied by their answer.	A1
	$= (£)14\ 850$	Cao. Correct answer with no working scores 3 marks. Apply isw if necessary and award this mark once a correct answer is seen.	A1
			(3)
(c)	$\frac{1}{2}n(2 \times 600 + (n - 1) \times -10) = 18200$	Attempts to use a correct sum formula with $a = 600$, $d = -10$ and sets = 18 200. Condone poor use of brackets.	M1
		Correct equation which may be implied by subsequent work.	A1
	$600n - 5n^2 + 5n = 18200$ $5n^2 - 605n + 18200 = 0$ $n^2 - 121n + 3640 = 0^*$	Obtains the printed answer with at least one intermediate line and no errors. Allow other variables to be used for n but the final answer must be as printed including “= 0”	A1*
			(3)
(d)	$(n - 56)(n - 65) = 0$ $\Rightarrow (n =) 56, 65$	Attempts to solve the given quadratic. This may be implied by correct answers. Apply general guidance if necessary but must reach at least one value for n . (Allow them to use x rather than n)	M1
		Correct values (ignore how they are labelled e.g. allow $x = \dots$)	A1
			(2)
(e)	E.g. $(n =) 65$ because e.g. the money has already been saved after 56 months	States $(n =) 65$ and gives a suitable reason – see below for examples of acceptable comments. There must be no contradictory statements and any calculations must be correct.	B1
			(1)
			Total 11

Acceptable comments for 5(e):

$n = 65$ means $t = 600 - 10 \times 64 = -40$ which is not possible/doesn't make sense/etc.

$n = 65$ because Lina will have saved the money after 56 months

$n = 65$ because Lina will have saved the money before then

$600 + (n - 1) \times -10 = 0 \Rightarrow n = 61$ so she will have paid off the loan before $n = 65$

Condone “because $65 > 60$ ” or equivalent e.g. it is only over 60 months (or 5 years)

$n = 65$ means $t = 600 - 10 \times 64 = -40$ so reject (but not just “it is negative”)



Question Number	Scheme	Notes	Marks
6(a)	$x^3 - 6x + 9 = -2x^2 + 7x - 1$ $\Rightarrow \dots$	Sets $C_1 = C_2$, <u>and collects terms</u>	M1
	$\Rightarrow \pm(x^3 + 2x^2 - 13x + 10) = 0$	Correct cubic equation. The “= 0” may be implied by their attempt to solve.	A1
	<u>Examples:</u> $x^3 + 2x^2 - 13x + 10 = (x - 1)(x^2 + \dots x + \dots) = (x - 1)(x + \dots)(x + \dots) \Rightarrow x = \dots$ Attempts to factorise using $(x - 1)$ as a factor or uses long division by $(x - 1)$ to obtain a quadratic factor and proceeds to solve quadratic or factorise and solve $\text{NB } x^3 + 2x^2 - 13x + 10 = (x - 1)(x^2 + 3x - 10)$ or $x^3 + 2x^2 - 13x + 10 = (x - 1)(x + \dots)(x + \dots) \Rightarrow x = \dots$ Attempts 3 factors directly (by considering roots) or $x^3 + 2x^2 - 13x + 10 = 0 \Rightarrow x = \dots$ Solves (using calculator) to obtain 3 roots (may need to check if cubic incorrect)		M1
	$x = 2, y = 5 \text{ or } (2, 5)$ Correct values <u>from a correct cubic.</u> Allow as a coordinate pair or written separately. If there are any errors in the algebra e.g. wrong factors, wrong working etc. this mark should be withheld even if they have $(2, 5)$ and score as M1A1B1(Second M on EPEN)A0		A1
	<u>Special Case</u> If you see: $x^3 - 6x + 9 = -2x^2 + 7x - 1 \Rightarrow x^3 + 2x^2 - 13x + 10 = 0$ $\Rightarrow x = 2, y = 5 \text{ or } (2, 5)$ Score M1A1B1(Second M on EPEN)A0		
			(4)

(b)	$x^n \rightarrow x^{n+1}$	For increasing any power of x by 1 for C_1 or C_2 or for $\pm (C_1 - C_2)$	M1
	$\pm \int \left\{ -2x^2 + 7x - 1 - (x^3 - 6x + 9) \right\} dx = \pm \int (-x^3 - 2x^2 + 13x - 10) dx$ $= \pm \left(-\frac{x^4}{4} - \frac{2x^3}{3} + \frac{13x^2}{2} - 10x \right)$ or $\pm \left\{ \int (-2x^2 + 7x - 1) dx - \int (x^3 - 6x + 9) dx \right\}$ $= \pm \left(-\frac{2x^3}{3} + \frac{7x^2}{2} - x - \left(\frac{x^4}{4} - \frac{6x^2}{2} + 9x \right) \right)$ or $\int (-2x^2 + 7x - 1) dx = -\frac{2x^3}{3} + \frac{7x^2}{2} - x, \int (x^3 - 6x + 9) dx = \frac{x^4}{4} - \frac{6x^2}{2} + 9x$		dM1A1
	dM1: For correct integration of 1 term for C_1 and one term for C_2 or for correct integration for 2 terms of <u>their</u> $\pm (C_1 - C_2)$ A1: Fully correct integration of both C_1 and C_2 or for $\pm (C_1 - C_2)$. Award this mark as soon as fully correct integration is seen and ignore subsequent work.		
	$= -\frac{2^4}{4} - \frac{2(2)^3}{3} + \frac{13(2)^2}{2} - 10(2) - \left(-\frac{1^4}{4} - \frac{2(1)^3}{3} + \frac{13(1)^2}{2} - 10(1) \right)$ Fully correct strategy for the area. Depends on both previous M marks. Uses the limits “2” and 1 in their “changed” expression(s) and subtracts either way round.		ddM1
	$= \frac{13}{12}$ If the attempt is correct apart from subtracting the wrong way round (for limits or functions) and $-\frac{13}{12}$ is obtained, allow recovery if they then make their answer positive.		A1
			(5)
			Total 9

Some values for reference:

$$\left[\frac{-2x^3}{3} + \frac{7x^2}{2} - x \right]_1^2 = \frac{20}{3} - \frac{11}{6} = \frac{29}{6} \quad \left[\frac{x^4}{4} - \frac{6x^2}{2} + 9x \right]_1^2 = 10 - \frac{25}{4} = \frac{15}{4}$$

$$= -\frac{2^4}{4} - \frac{2(2)^3}{3} + \frac{13(2)^2}{2} - 10(2) - \left(-\frac{1^4}{4} - \frac{2(1)^3}{3} + \frac{13(1)^2}{2} - 10(1) \right) = -\frac{10}{3} - \left(-\frac{53}{12} \right)$$

Question Number	Scheme	Notes	Marks
7(i)	$\tan \theta + \frac{1}{\tan \theta} = \frac{\sin \theta}{\cos \theta} + \frac{1}{\frac{\sin \theta}{\cos \theta}}$ or $\tan \theta + \frac{1}{\tan \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$	Uses $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ on both terms	M1
	$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \quad \text{or} \quad \frac{\sin^2 \theta}{\sin \theta \cos \theta} + \frac{\cos^2 \theta}{\sin \theta \cos \theta}$ Uses $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ and $\frac{1}{\tan \theta} \equiv \frac{\cos \theta}{\sin \theta}$ and attempts common denominator of $\sin \theta \cos \theta$ with a 2 term numerator one of which is correct. Or attempts 2 separate fractions with a denominator of $\sin \theta \cos \theta$ one of which is correct. Depends on the first mark.		dM1
	$= \frac{1}{\sin \theta \cos \theta} *$ or $\frac{1}{\cos \theta \sin \theta}$	Correct proof with no notation errors or missing variables but allow “ $\equiv$ ” instead of “ $=$ ”. If there are any spurious “ $= 0$ ”s alongside the proof score A0.	A1*
			(3)
Alternative 1 for (i)			
	$\tan \theta + \frac{1}{\tan \theta} = \frac{\tan^2 \theta + 1}{\tan \theta} \left(\text{or } \frac{\tan^2 \theta}{\tan \theta} + \frac{1}{\tan \theta} \right)$	Attempts common denominator of $\tan \theta$	M1
	$= \frac{\sec^2 \theta}{\tan \theta} = \frac{1}{\cos^2 \theta} \times \frac{\cos \theta}{\sin \theta}$ Or $= \frac{\frac{\sin^2 \theta}{\cos^2 \theta} + 1}{\frac{\sin \theta}{\cos \theta}} = \frac{1}{\cos^2 \theta} \times \frac{\cos \theta}{\sin \theta}$	Applies appropriate and correct identities to obtain in terms of $\sin \theta$ and $\cos \theta$ only and eliminates “double decker” fractions if necessary	dM1
	$= \frac{1}{\sin \theta \cos \theta} *$ or $\frac{1}{\cos \theta \sin \theta}$	Correct proof with no notation errors or missing variables but allow “ $\equiv$ ” instead of “ $=$ ”. If there are any spurious “ $= 0$ ”s alongside the proof score A0.	A1*
Alternative 2 for (i)			
	$\tan \theta + \frac{1}{\tan \theta} = \frac{1}{\sin \theta \cos \theta} \Rightarrow \frac{\sin^2 \theta}{\cos \theta} + \cos \theta = \frac{1}{\cos \theta}$ Uses $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ and multiplies through by $\sin \theta$ or $\cos \theta$		M1
	$\Rightarrow \sin^2 \theta + \cos^2 \theta = 1$ Uses $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ and multiplies through by $\sin \theta$ and $\cos \theta$		dM1
	$\sin^2 \theta + \cos^2 \theta = 1$ is true hence proved	Fully correct work reaching a correct identity with a conclusion. If there are any spurious “ $= 0$ ”s alongside the proof score A0.	A1*

(ii)	$3 \cos^2(2x+10^\circ)=1 \Rightarrow \cos^2(2x+10^\circ)=\frac{1}{3} \Rightarrow \cos(2x+10^\circ)=(\pm)\sqrt{\frac{1}{3}}$ Divides by 3 and takes square root of both sides . The “±” is not required.		M1
	$2x+10^\circ = \cos^{-1}\left(\left(\pm\right)\sqrt{\frac{1}{3}}\right)$ $\cos^{-1}\left(\left(\pm\right)\sqrt{\frac{1}{3}}\right)-10^\circ$ $\Rightarrow x = \frac{\hspace{10em}}{2}$	$\cos^{-1}\left(\left(\pm\right)\sqrt{\frac{1}{3}}\right) \pm 10^\circ$ Applies $x = \frac{\hspace{10em}}{2}$ You may need to check their values if no working is shown.	M1
	For reference $2x+10^\circ = 54.735\dots, 125.264\dots$		
	$x = 22.4^\circ$ or $x = 57.6^\circ$	Awrt one of these	A1
	$x = 22.4^\circ$ and $x = 57.6^\circ$	Awrt both with no extras in range	A1
	If mixing degrees and radians allow the method marks.		
			(4)
	Alternative 1 for part (b)		
	$3 \cos^2(2x+10^\circ)=1 \Rightarrow 3(1-\sin^2(2x+10^\circ))=1 \Rightarrow$ $\Rightarrow \sin^2(2x+10^\circ)=\frac{2}{3} \Rightarrow \sin(2x+10^\circ)=(\pm)\sqrt{\frac{2}{3}}$ Uses a correct identity, rearranges and takes square root of both sides . The “±” is not required.		M1
	$2x+10^\circ = \sin^{-1}\left(\left(\pm\right)\sqrt{\frac{2}{3}}\right)$ $\sin^{-1}\left(\left(\pm\right)\sqrt{\frac{2}{3}}\right)-10^\circ$ $\Rightarrow x = \frac{\hspace{10em}}{2}$	$\sin^{-1}\left(\left(\pm\right)\sqrt{\frac{2}{3}}\right) \pm 10^\circ$ Applies $x = \frac{\hspace{10em}}{2}$ You may need to check their values if no working is shown.	M1
	$x = 22.4^\circ$ or $x = 57.6^\circ$	Awrt one of these	A1
	$x = 22.4^\circ$ and $x = 57.6^\circ$	Awrt both with no extras in range	A1
	Alternative 2 for part (b)		
	$3 \cos^2(2x+10^\circ)=3\left(\frac{1+\cos(4x+20^\circ)}{2}\right) \Rightarrow \cos(4x+20^\circ)=-\frac{1}{3}$ Uses a correct identity, rearranges to make $\cos(4x+20^\circ)$ the subject		M1
	$2x+10^\circ = \cos^{-1}\left(\left(-\frac{1}{3}\right)\right)$ $\cos^{-1}\left(\left(-\frac{1}{3}\right)\right)-20^\circ$ $\Rightarrow x = \frac{\hspace{10em}}{4}$	$\cos^{-1}\left(\left(-\frac{1}{3}\right)\right)-20^\circ$ Applies $\Rightarrow x = \frac{\hspace{10em}}{4}$ You may need to check their values if no working is shown.	M1
	For reference $4x+20^\circ = 109.47\dots, 250.52\dots$		
	$x = 22.4^\circ$ or $x = 57.6^\circ$	Awrt one of these	A1
	$x = 22.4^\circ$ and $x = 57.6^\circ$	Awrt both with no extras in range	A1
			Total 7



Question Number	Scheme	Marks
7a	$H = t^{\frac{1}{2}} + \frac{3t^{\frac{3}{2}}}{10} - \frac{t^{\frac{5}{2}}}{20} + 17 \Rightarrow \left(\frac{dH}{dt} = \right) \frac{1}{2}t^{-\frac{1}{2}} + \frac{9}{20}t^{\frac{1}{2}} - \frac{1}{8}t^{\frac{3}{2}}$	M1A1A1
	e.g. $\frac{1}{2}t^{-\frac{1}{2}} + \frac{9}{20}t^{\frac{1}{2}} - \frac{1}{8}t^{\frac{3}{2}} = 0 \Rightarrow \frac{1}{2} + \frac{9}{20}t - \frac{1}{8}t^2 = 0 \dots \Rightarrow 5\alpha^2 - 18\alpha - 20 = 0$ *	dM1A1*
		(5)
b	$\alpha = \frac{9 + \sqrt{181}}{5} = \text{awrt } 4.491$	B1
		(1)
c	$\left(\frac{d^2H}{dt^2} = \right) -\frac{1}{4}t^{-\frac{3}{2}} + \frac{9}{40}t^{-\frac{1}{2}} - \frac{3}{16}t^{\frac{1}{2}} \Rightarrow -\frac{("4.49")^{\frac{3}{2}}}{4} + \frac{9("4.49")^{\frac{1}{2}}}{40} - \frac{3("4.49")^{\frac{1}{2}}}{16}$	M1
	$\frac{d^2H}{dt^2} = (-0.3\dots) < 0 \Rightarrow \text{max} *$	A1*
		(2)
		(8 marks)

(a) If they change t to α before differentiating then score maximum M1A1A0dM1A0

M1 Multiplies out and attempts to differentiate. Award for one correct index from correct working (may be implied if they multiply out and differentiate at the same time) $t^{\frac{3}{2}}$ or $t^{\frac{1}{2}}$ or $t^{-\frac{1}{2}}$ (condone if these are $\alpha^{\frac{3}{2}}$ or $\alpha^{\frac{1}{2}}$ or $\alpha^{-\frac{1}{2}}$ or any other variable) The indices do not need to be processed for this mark.

e.g. $t^{\frac{5}{2}-1}$

Alternatively, they may attempt the product rule so look for a correct general expression such as

$\left(\frac{dH}{dt} = \right) \dots t^{-\frac{1}{2}}(20 + 6t - t^2) + \dots t^{\frac{1}{2}}(A + Bt)$ (condone if in terms of α or another variable instead)

A1 One of $\frac{1}{2}t^{-\frac{1}{2}}$, $\frac{9}{20}t^{\frac{1}{2}}$, $-\frac{1}{8}t^{\frac{3}{2}}$ (simplified or unsimplified) via the main scheme

If they differentiate using the product rule then this mark can only be awarded for one of

$\frac{1}{40}t^{-\frac{1}{2}}(20 + 6t - t^2) + \dots t^{\frac{1}{2}}(A + Bt)$ or $\dots t^{-\frac{1}{2}}(20 + 6t - t^2) + \frac{1}{20}t^{\frac{1}{2}}(6 - 2t)$ (unsimplified).

In both cases the indices must be processed. Condone if in terms of α

A1 $\left(\frac{dH}{dt} = \right) \frac{1}{2}t^{-\frac{1}{2}} + \frac{9}{20}t^{\frac{1}{2}} - \frac{1}{8}t^{\frac{3}{2}}$ or equivalent e.g. $\left(\frac{dH}{dt} = \right) \frac{1}{40}t^{-\frac{1}{2}}(20 + 18t - 5t^2)$ which may be unsimplified. Also allow

e.g. $\left(\frac{dH}{dt} = \right) \frac{1}{40}t^{-\frac{1}{2}}(20 + 6t - t^2) + \frac{1}{20}t^{\frac{1}{2}}(6 - 2t)$ (via the product rule)

dM1 Sets their derivative of the form $\dots t^{-\frac{1}{2}} \pm \dots t^{\frac{1}{2}} \pm \dots t^{\frac{3}{2}} = 0$ (may be implied) and proceeds via a correct method to a 3TQ in t or α (or any other variable) where the powers and the coefficients of the terms are integers. Condone sign slips only. It is dependent on the previous method mark.

Proceeding directly to the given answer is M0, unless they state how they have proceeded

from the derivative directly to the given answer e.g. set $t = \alpha$ and $\times -40t^{\frac{1}{2}}$



Derivatives which are already factorised before being set = 0 and have the quadratic factor $20 + 18t - 5t^2$ can score this mark for

$$\text{e.g. } \left(\frac{dH}{dt} = \right) \frac{1}{40} t^{-\frac{1}{2}} ("20 + 18t - 5t^2") = 0 \Rightarrow "5t^2 - 18t - 20" = 0$$

- A1* Achieves $5\alpha^2 - 18\alpha - 20 = 0$ with no errors seen and all previous marks scored. Must be in terms of α (do not be concerned if it looks like a). The $= 0$ must have been seen before achieving the given answer.

Do not be concerned with the labelling of the derivative if seen e.g. $\frac{dy}{dx}$ instead

Do not penalise invisible brackets as long as they are recovered and the intention is clear through their subsequent lines of working.

Alternative method: They may attempt to multiply by e.g. 20 or take out a factor and then differentiate. This can score full marks, provided for A1* they explain at some point that this does not affect the given answer. You will need to check their terms are correct for the first A mark and withhold the A marks if e.g. they do not multiply all terms including the +17 by 20

(b) Mark (b) and (c) together

- B1 $\alpha = \text{awrt } 4.491$ only. Accept the exact answer. If both roots are stated then awrt 4.491 must be the only one used within their $\frac{d^2H}{dt^2}$, or if both values are used then the other is rejected. If awrt 4.491 is seen then allow for it to be subsequently rounded to 2 or 3sf when used as evidence of selecting awrt 4.491

(c)

- M1 Attempts to differentiate their $\frac{dH}{dt}$ to achieve a second derivative of the form $\dots t^{-\frac{3}{2}} + \dots t^{-\frac{1}{2}} + \dots t^{\frac{1}{2}}$ (may be in terms of α or another variable) and either attempts to substitute in their value of α which must be between 0 and 7 or considers the sign of their second derivative. Condone their α to have been subsequently rounded before substituted in

$$\text{May be seen as } \frac{1}{40} t^{-\frac{1}{2}} (20 + 18t - 5t^2) \rightarrow -\frac{1}{80} t^{-\frac{3}{2}} (20 - 18t + 15t^2)$$

If differentiation in part (a) is via the product rule then look for an attempt to apply the product rule

on their $\left(\frac{dH}{dt} = \right) \dots t^{-\frac{1}{2}} (20 + 6t - t^2) + \dots t^{\frac{1}{2}} (A + Bt)$ leading to

$$\dots t^{-\frac{3}{2}} (20 + 6t - t^2) + \dots t^{\frac{1}{2}} (C + Dt) + \dots t^{\frac{1}{2}}$$

Alternatively, attempts to show that the second derivative is always negative. e.g. attempts to complete the square on $20 - 18t + 15t^2$ and considers its sign.

- A1* $\frac{d^2H}{dt^2} (-0.3) < 0 \Rightarrow \text{max when } \alpha = \text{awrt } 4.491$. If evaluated, it must be -0.3 or better

$(-0.3174\dots)$. The second derivative must be correct but condone in terms of α

In the alternative method look for correct working and valid argument explaining that eg

$20 - 18t + 15t^2$ is always greater than zero so $-\frac{1}{80} t^{-\frac{3}{2}} (20 - 18t + 15t^2)$ is always negative for any value of t

Condone poor phrasing as part of the explanation e.g. " $t = 4.491$ is a maximum" instead of " $t = 4.491$ is the value at which H is at a maximum"

Condone poor notation for $\frac{d^2H}{dt^2}$ and condone e.g. y''



Question Number	Scheme	Marks															
8(i)	Eg $7^2 + 11^2 = 170 \Rightarrow 170$ is a multiple of 10 so the statement is untrue *	M1A1*															
		(2)															
(ii)	Check first if they have multiplied the inequality e.g. by 4 to give $4 < 4x^2 - xy < 60$ <table border="1"> <thead> <tr> <th>x</th><th>y</th><th>$x^2 - \frac{xy}{4}$ or $4x^2 - xy$</th></tr> </thead> <tbody> <tr> <td>2</td><td>2</td><td>3 12</td></tr> <tr> <td>2</td><td>4</td><td>2 8</td></tr> <tr> <td>4</td><td>2</td><td>14 56</td></tr> <tr> <td>4</td><td>4</td><td>12 48</td></tr> </tbody> </table> Concludes that $1 < x^2 - \frac{xy}{4} < 15$ for all x and y that are positive even integers less than 6 *	x	y	$x^2 - \frac{xy}{4}$ or $4x^2 - xy$	2	2	3 12	2	4	2 8	4	2	14 56	4	4	12 48	M1A1 A1*
x	y	$x^2 - \frac{xy}{4}$ or $4x^2 - xy$															
2	2	3 12															
2	4	2 8															
4	2	14 56															
4	4	12 48															
		(3)															
		(5 marks)															

(i)

M1 Attempts to find $a^2 + b^2$ using two consecutive prime numbers a and b . It does not matter whether this combination of prime numbers results in a multiple of 10 to score M1. Condone slips in their evaluation of the sum of the squares of these two numbers. If a candidate has several combinations, you only need to look for one calculation using two consecutive prime numbers.

Note $1^2 + 2^2 = \dots$ as the only calculation is M0A0

A1* Correctly:

- evaluates the sum of two consecutive prime numbers squared
- either
 - makes a comment that the answer is a multiple of 10
 - makes a comment that 10 is a factor of their answer
 - makes a comment that their answer is divisible by 10 (do not accept can be divided by 10)
 - shows that the answer is a multiple of 10 (such as splitting the number into two factors, one of which is 10, or showing the number is divisible by 10 e.g. $\frac{170}{10} = 17$)
- concludes not true o.e. (condone e.g. “not correct”, “not always true”)

If a candidate has several combinations, you do not need to check all of them. Just look for one which would lead to correct counter example.

Below are a list of common combinations which provide the required counter example but there will be others

- $7^2 + 11^2 = 170$
- $11^2 + 13^2 = 290$
- $17^2 + 19^2 = 650$
- $19^2 + 23^2 = 890$
- $23^2 + 29^2 = 1370$



(ii)

M1 Attempts at least two valid combinations of x and y and evaluates $x^2 - \frac{xy}{4}$ condone slips in the evaluation of the combinations.

Alternatively, attempts at least two valid combinations of x and y and evaluates $4x^2 - xy$

A1 All four valid combinations of x and y correctly evaluated for $x^2 - \frac{xy}{4}$ (or in the alternative method $4x^2 - xy$)

A1* A minimal conclusion that $1 < x^2 - \frac{xy}{4} < 15$ (or condone $4 < 4x^2 - xy < 60$) for all x and y that are positive even integers less than 6.
e.g. “hence statement is true”, “they are all between 1 and 15” (or 4 and 60 in the alternative), “proven”, “QED” or they may have a preamble which can be concluded with a tick or similar.

Question Number	Scheme	Marks
9a	$3\cos\theta(\tan\theta\sin\theta+3)=11-5\cos\theta$	
	$3\sin^2\theta+9\cos\theta=11-5\cos\theta$ or $3\cos\theta\left(\frac{\sin\theta}{\cos\theta}\sin\theta+3\right)=11-5\cos\theta$	M1
	$3(1-\cos^2\theta)+9\cos\theta=11-5\cos\theta$	dM1
	$3\cos^2\theta-14\cos\theta+8=0$ *	A1*
		(3)
b	$\cos 2x = \frac{2}{3}$	B1
	$\cos^{-1}\left(\frac{2}{3}\right)$	M1
	$x = \frac{\dots}{2}$ $x = \text{awrt } 24.1, \text{ awrt } 155.9, \text{ awrt } 204.1, \text{ awrt } 335.9$	A1A1
		(4)
		(7 marks)

(a)

M1 Substitutes or uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to achieve an equation in sine and cosine only.

May be in another variable and condone if there is a mix of variables in the same equation for this mark.

dM1 Attempts to use $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ to achieve an equation in cosine only. It is dependent on the previous method mark. Condone slips in their manipulation of the equation.
May be in another variable and condone if there is a mix of variables in the same equation for this mark.

A1* Achieves $3\cos^2\theta - 14\cos\theta + 8 = 0$ with no errors seen including brackets. Withhold this mark for poor notation used such as $3\cos\theta^2 - 14\cos\theta + 8 = 0$
Allow a candidate to work consistently in one variable and then change to θ but withhold this final mark if there is a mixture of variables within the same equation/line of working.

(b) **Note that answers only scores 0 marks. May work in any variable and do not be concerned as to whether they are writing e.g. x or 2x in their working**

B1 $(\cos\dots) = \frac{2}{3}$ (ignore any reference to 4)

M1 Solves using the correct order of operations to find at least one value for x. You may need to check this on your calculator. May be implied by a correct value for $\cos 2x = \frac{2}{3}$

A1 Two of awrt 24, awrt 156, awrt 204, awrt 336. (Condone in radians two of awrt 0.42, awrt 2.7, awrt 3.6, awrt 5.9)

A1 awrt 24.1, awrt 155.9, awrt 204.1, awrt 335.9 and no others in the range
Minimum acceptable to be able to score full marks:

e.g. $\cos\dots = \frac{2}{3} \Rightarrow \text{answers}$ or $(\dots) = \cos^{-1}\left(\frac{2}{3}\right) \Rightarrow \text{answers}$



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$; $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
7 (i)	States or uses $\tan x = \frac{\sin x}{\cos x}$	M1
	$3\sin x \tan x = 11 + \cos x \Rightarrow 3\sin^2 x = 11\cos x + \cos^2 x$	
	$\Rightarrow 3(1 - \cos^2 x) = 11\cos x + \cos^2 x \Rightarrow 4\cos^2 x + 11\cos x - 3 = 0$	dM1, A1
	$\Rightarrow (4\cos x - 1)(\cos x + 3) = 0 \Rightarrow \cos x = \frac{1}{4} \Rightarrow x = 1.318, 4.965$	dM1 A1
		(5)
(ii)	$\cos \theta = \frac{1}{3} \Rightarrow \sin^2 \theta = 1 - \frac{1}{3^2} \Rightarrow \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{1 - \frac{1}{3^2}}}{\frac{1}{3}}$	M1
	$\Rightarrow \tan \theta = 2\sqrt{2}$	A1
		(2)
		Total 7

(i)

M1: States or uses $\tan x = \frac{\sin x}{\cos x}$ o.e.

dM1: Uses $\tan x = \frac{\sin x}{\cos x}$, attempts to multiply by $\cos x$ and then uses $\sin^2 x = \pm 1 \pm \cos^2 x$ to set up a 3 TQ equation in $\cos x$

A1: Correct simplified quadratic equation in $\cos x$. Terms can be on either side of the equation. The '=' can be implied by a subsequent attempt to solve.

dM1: Solves a quadratic equation in $\cos x$ by any allowable method (including a calculator) to find $\cos x$ and proceeds to a value for x (in degrees or radians) via arccos. Dependent upon the first M only
Minimum working required would be $4\cos^2 x + 11\cos x - 3 = 0 \Rightarrow \cos x = 0.25 \Rightarrow x = 1.3$
Note that

- $\cos x = \frac{1}{4} \Rightarrow x = 0.2527$ is dM0 as it is $\arcsin\left(\frac{1}{4}\right)$ not $\arccos\left(\frac{1}{4}\right)$
- $\cos x = \frac{1}{4} \Rightarrow x = 0.9689$ is dM0 as it is $\cos\left(\frac{1}{4}\right)$ not $\arccos\left(\frac{1}{4}\right)$

A1: $x = \text{awrt } 1.318, 4.965$ following the award of all previous marks.

Ignore any extra solutions outside the range, but withhold this mark for any extra solutions within it.

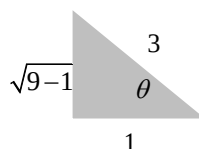
Notes: Condone notation such as $\tan = \frac{\sin}{\cos}$ and $\cos x^2$ for $\cos^2 x$. This is not a proof.



(ii)

M1: Full method of finding a value for $\tan \theta$. The value for $\sin \theta$ cannot just appear.Other methods will be seen but most will include Pythagoras at some point. ($1 + \tan^2 \theta = \sec^2 \theta = \frac{1}{\cos^2 \theta}$)

E.g.

Allow sight of $\tan \theta = \sqrt{9-1}$ o.e without any other supporting evidenceA1: $\tan \theta = 2\sqrt{2}$ following the award of M1.....
Answers without working:

In both parts candidates are able to use calculators to solve the whole of the question. This does not score any marks due to the demand of the question. All method marks must be awarded before accuracy marks are given.

If in doubt then please send to review.

4(c)(i)	$\left(\frac{d^2y}{dx^2}\right) = -x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}}$ oe e.g. $-x^{-\frac{3}{2}} + 6.75x^{-\frac{5}{2}}$	B1ft
(ii)	$\left(\left(\frac{d^2y}{dx^2}\right)_{x=\frac{9}{4}}\right) = -\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}} \left(= \frac{16}{27}(0.5925\dots)\right)$ $\left(\frac{d^2y}{dx^2}\right) > 0 \text{ so (local) minimum}$	B1
		(2)
(d)	$0 < x < \frac{9}{4}$	B1ft
		(1)
		Total 7

(c)(i)**B1ft:** Correct simplified second derivative.Follow through their first derivative provided there are 2 terms with **different** fractional (non-integer) powers. $\frac{d^2y}{dx^2} =$ is not required.Condone $-1x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}}$ for $-x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}}$

Inclusion of “+ c” scores B0ft as with the first derivative.

(ii)**B1:** Fully correct working, reasoning and conclusion. This requires:

- a correct second derivative
- substitutes $x = \frac{9}{4}$ to obtain $\frac{16}{27}$ or awrt 0.6 or truncated 0.5
- reference to the sign (> 0 or positive). Must see the evaluation as it is not obviously positive but allow if expression is not fully evaluated as long as it is clear it is positive (e.g. $\frac{8}{9} - \frac{8}{27} > 0$).
- reference to minimum

Condone work that doesn't specifically reference $\frac{d^2y}{dx^2}$ e.g.
$$-\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}} = \frac{16}{27} > 0 \text{ so minimum is fine for this mark.}$$
(d)**B1ft:** For $0 < x < \frac{9}{4}$. Allow $0 < x \leq \frac{9}{4}$ and allow equivalent statements e.g.
$$\left(0, \frac{9}{4}\right), \left[0, \frac{9}{4}\right].$$
Follow through their positive x value from part (b). Condone an answer of $x < \frac{9}{4}$ or $x \leq \frac{9}{4}$ Condone $0 \leq x \leq \frac{9}{4}$ Do not allow $-\infty < x < \frac{9}{4}$

Question Number	Scheme	Marks
5(a)	$(2+ax)^6 = 2^6 + \binom{6}{1}2^5(ax) + \binom{6}{2}2^4(ax)^2 + \dots$	M1
	$= 64 + 192ax + 240a^2x^2 + \dots$	A1A1
		(3)
(b)	$\left(3 + \frac{1}{x}\right)^2 = 9 + \frac{6}{x} + \frac{1}{x^2}$ or $9 + \frac{3}{x} + \frac{3}{x} + \frac{1}{x^2}$	B1
	$f(x) = \left(9 + \frac{6}{x} + \frac{1}{x^2}\right)(64 + 192ax + 240a^2x^2 + \dots) = \dots$ Constant term is $9 \times 64 + 6 \times 192a + 240a^2$	M1
	$576 + 1152a + 240a^2 = 576 \Rightarrow 1152a + 240a^2 = 0$ $\Rightarrow 1152 + 240a = 0 \Rightarrow a = \dots$	dM1
	$a = -\frac{24}{5}$	A1
		(4)
		Total 7

(a)

M1: Attempts the binomial expansion up to at least the second term to obtain the correct structure for the **2nd or 3rd term** i.e. the correct binomial coefficient (allow alternative notation) with the correct power of 2 and the correct power of ax .

The binomial coefficients do not have to be evaluated but must be correct if they are.

If awarding this mark for the x^2 term you can condone missing brackets e.g.

$${}^6C_2 \times 2^4 \times ax^2$$

Alternatively writes e.g. $(2+ax)^6 = 2^6 \left(1 + \frac{ax}{2}\right)^6 = 2^6 \left(1 + \frac{6}{2}ax + \frac{6 \times 5}{2} \left(\frac{ax}{2}\right)^2 + \dots\right)$

which can also score M1 for the expansion up to at least the second term with an acceptable structure for either the **2nd or 3rd term**. They do not have to multiply out the brackets for this mark but the 2^6 cannot be omitted unless it is later recovered.

Condone missing brackets e.g. $\frac{6 \times 5}{2} \times \frac{ax^2}{2}$

A1: For any 2 correct simplified terms in any order (Allow a^2x^2 or $(ax)^2$ for the third term **for this mark**)

Allow terms to be listed.

Do **not** allow $64x^0$ for 64 or $192ax^1$ for $192ax$

A1: All correct and in any order. Allow terms to be listed. Must be a^2x^2 not $(ax)^2$ for the 3rd term but allow (ax) for ax .

Apply isw if they divide through by e.g. 2 or 4 or 16 or set their expression = 0

Ignore any extra terms if attempted.



(b)**B1:** Correct expansion of $\left(3 + \frac{1}{x}\right)^2$ unsimplified or simplified.

Note that this may be implied by the omission of the “9” as $9 \times 64 = 576$ and so the “9” is not required.

Note that $3^2 \left(1 + 2\left(\frac{1}{3x}\right) + \frac{1}{(3x)^2}\right)$ is correct.

Condone missing brackets if recovered e.g. $\left(3 + \frac{1}{x}\right)^2 = 9 + \frac{6}{x} + \frac{1}{x^2} = 9 + \frac{6}{x} + \frac{1}{x^2}$

M1: Uses their expansion in part (a) and their expansion of $\left(3 + \frac{1}{x}\right)^2$ to extract the constant

term. This depends on having obtained $\left(3 + \frac{1}{x}\right)^2 = \alpha + \frac{\beta}{x} + \frac{\gamma}{x^2}$ oe with α, β, γ non-

zero and an attempt at $\alpha \times "64" + \frac{\beta}{x} \times "192ax" + \frac{\gamma}{x^2} \times "240a^2x^2"$ oe

An expression of this form is sufficient i.e. with the x 's still included.

Note that the 9×64 may not be seen as this cancels with the 576 so this mark may be implied.

dM1: Sets their constant term = 576 and proceeds to obtain a non-zero value for a .

You do not need to be concerned about the processing for this mark.

This may be implied by e.g. $6 \times 192a + 240a^2 = 0 \Rightarrow a = \dots$ as $9 \times 64 = 576$

May be implied by their value(s).

Condone $x = \dots$ here.

Depends on the previous method mark.

A1: Correct value. Allow equivalents e.g. -4.8 , $-\frac{48}{10}$

The questions asks for the value of a so just look for the correct value e.g. “ $a = \dots$ ” is

not required but $x = -\frac{24}{5}$ scores A0.

If $a = 0$ is also given and not rejected, score A0



Question Number	Scheme	Marks
6	$2 \log_4 (x+1) = \log_4 (x+1)^2$	B1
	e.g. $\log_4 (12-2x) - \log_4 (x+1)^2 = \log_4 \frac{12-2x}{(x+1)^2}$	M1
	$\frac{12-2x}{(x+1)^2} = 16$	A1
	$\Rightarrow 8x^2 + 17x + 2 = 0 \Rightarrow x = \dots$	M1
	$x = -\frac{1}{8}$ oe e.g. -0.125	A1
		(5)
		Total 5

B1: For $2 \log_4 (x+1) = \log_4 (x+1)^2$ seen or implied.

M1: **Correct** attempt to combine two log terms.

$$\text{E.g. } 2 + \log_4 (x+1)^2 = \log_4 16 + \log_4 (x+1)^2 = \log_4 16(x+1)^2$$

Condone if they make a slip **before** combining as long as they are combining two log terms correctly.

e.g.

$$\log_4 (12-2x) = 2 + \log_4 (x+1)^2 \Rightarrow \log_4 (12-2x) + \log_4 (x+1)^2 = 2 \Rightarrow \log_4 (12-2x)(x+1)^2 = \dots$$

or e.g.

$$\log_4 (12-2x) = 2 + \log_4 (x+1)^2 = \log_4 2 + \log_4 (x+1)^2 = \log_4 2(x+1)^2$$

A1: Obtains this equation in any form not involving logs **from correct work so must follow B1M1**.

M1: Solves a 3TQ by any acceptable method including a calculator to obtain at least one **real** value for x . May be implied by their value(s) and may follow incorrect log work **but the B1 must have been scored**. You may need to check.

Note that we are allowing the use of a calculator to solve the 3TQ here. This means that correct work leading to e.g.:

$$\frac{12-2x}{(x+1)^2} = 16 \Rightarrow 16x^2 + 34x + 4 = 0$$

Followed by

$$\Rightarrow (8x+1)(x+2) = 0 \Rightarrow x = -\frac{1}{8}$$

Is acceptable for full marks as long as it is preceded by correct log work.

A1: This value only from correct work and all previous marks scored.
The other value (-2) must clearly be rejected or omitted.



Special Case: Beware incorrect log work which leads to the correct answer:

$$2\log_4(x+1) = \log_4(x+1)^2$$

$$\log_4(12-2x) - \log_4(x+1)^2 = \frac{\log_4(12-2x)}{\log_4(x+1)^2}$$

$$\frac{12-2x}{(x+1)^2} = 16$$

$$\Rightarrow 8x^2 + 17x + 2 = 0 \Rightarrow x = \dots$$

$$x = -\frac{1}{8}$$

This scores B1M0A0M1A0

Allow this SC if no combination of logs is shown at all, but the correct quadratic is produced.

$$\text{e.g. } \log_4(12-2x) = 2 + 2\log_4(x+1) \Rightarrow \frac{12-2x}{(x+1)^2} = 16 \text{ etc.}$$

or e.g.

$$\log_4(12-2x) = \log_4 16 + \log_4(x+1)^2 \Rightarrow 12-2x = 16(x+1)^2 \text{ etc.}$$

Could both score B1M0A0M1A0

Special Case: Beware incorrect work removing logs which leads to the correct answer:

$$2\log_4(x+1) = \log_4(x+1)^2$$

$$\log_4(12-2x) - \log_4(x+1)^2 = \log_4 \frac{12-2x}{(x+1)^2} = 2$$

$$\frac{12-2x}{(x+1)^2} = 2^4 = 16$$

$$\Rightarrow 8x^2 + 17x + 2 = 0 \Rightarrow x = \dots$$

$$x = -\frac{1}{8}$$

This scores B1M1A0M1A0

Alternative:

$$\log_4(12-2x) = 2 + 2\log_4(x+1)$$

$$\Rightarrow 4^{\log_4(12-2x)} = 4^{2+2\log_4(x+1)} = 4^{2+\log_4(x+1)^2}$$

$$= 4^2 \times 4^{\log_4(x+1)^2}$$

$$\Rightarrow 12-2x = 16(x+1)^2 \text{ etc.}$$

Score as:

$$\text{B1: } 2\log_4(x+1) = \log_4(x+1)^2$$

$$\text{M1: Uses correct index law e.g. } 4^{a+b} = 4^a \times 4^b$$

Then follow main scheme.



Question Number	Scheme	Marks
7(a)	$(u_{100} =) 20 + 99(0.5) = (£) 69.50^*$	B1*
		(1)
(b)	$S_{300} = \frac{1}{2}(300)\{2 \times 20 + 299(0.5)\} = \dots$ or $S_{300} = \frac{1}{2}(300)\{20 + 169.50\} = \dots$	M1
	$= (£) 28425$	A1
		(2)
(c)	$20 \times r^{299} = 250 \Rightarrow r = \sqrt[299]{\frac{250}{20}} (= 1.008483032\dots)$	M1
	$S_{300} = \frac{20(1 - r^{300})}{1 - r} = (27362.948\dots)$	M1
	$28425 - 27362.948\dots$	
	$(£) 1060$	A1
		(3)
		Total 6

(a)**B1*:** Correct method shown e.g. $20 + 99(0.5) = 69.50$ is sufficient.Note that the £ symbol is not required but $20 + 99(0.5) = 69.5$ scores B0

Condone attempts to work backwards e.g.

$$69.5 = 20 + (n-1) \times 0.5 \Rightarrow 49.5 = (n-1) \times 0.5 \Rightarrow n = 100 \text{ hence true}$$

Note that 69.5 can be used in this case but a (minimal) conclusion is required.

(b)**M1:** Attempts $\frac{1}{2}n(2 \times a + (n-1)d)$ with $n = 300$, $a = 20$ and $d = 0.5$ or e.g. $\frac{1}{2}n(a + l)$ with $n = 300$, $a = 20$ and $l = 169.5(0)$ (or an attempt at $20 + 299 \times 0.5$)**A1:** Correct value. The £ symbol is not required.

(c)

M1: Correct strategy to find r e.g. $\sqrt[299]{\frac{250}{20}}$. Award for e.g. $(r =) \sqrt[299]{\frac{250}{20}}$ or e.g.

$(r =) \left(\frac{250}{20}\right)^{\frac{1}{299}}$. May be implied by awrt 1.01 if no incorrect work is seen e.g. clear

use of $\sqrt[300]{\frac{250}{20}}$ which gives 1.008454636... is M0

May be done via logs e.g. $r = 10^{\frac{1}{299} \log 12.5}$

M1: Uses their r with a correct GP sum formula with $a = 20$ and $n = 300$ and subtracts from the answer to part (b) either way round.

Depends on having attempted the **sum** of an **AP** in (b).

May be implied by their working/value.

Note this has been seen more than once for the GP sum following $\sqrt[299]{\frac{250}{20}}$:

$$S_{300} = \frac{20(1-12.5)}{1-1.008483032}$$

and scores M0 as it suggests $S_n = \frac{a(1-r^{n-1})}{1-r}$ has been used.

A1: Cao **not awrt** with or without the “£” symbol.

Note that the accuracy being used for “ r ” varies considerably.

Some rounded values for reference are:

r	S_{300}	Correct (b) – S_{300}
1.01	37576.93252...	9151.932524...
1.008	24796.03972...	3628.960276...
1.0085	27458.42195...	966.5780531...
1.00848	27345.9295...	1079.070499...



Question Number	Scheme	Marks
9 (a)	Uses $\tan \theta = \frac{\sin \theta}{\cos \theta} \rightarrow \cos \theta - 1 = 4 \sin \theta \frac{\sin \theta}{\cos \theta}$ $\cos^2 \theta - \cos \theta = 4 \sin^2 \theta$ oe Uses $\sin^2 \theta = 1 - \cos^2 \theta \rightarrow \cos^2 \theta - \cos \theta = 4(1 - \cos^2 \theta)$ $5 \cos^2 \theta - \cos \theta - 4 = 0$ *	M1 A1 M1 A1 * (4)
(b)	$(5 \cos 2x + 4)(\cos 2x - 1) = 0$ Critical values of $-\frac{4}{5}, 1$ Correct method to find x from their $\cos 2x = -\frac{4}{5}$ $x = 0, 1.25$	M1 A1 dM1 A1 (4) (8 marks)
Notes		
(a) M1 Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ oe in their $\cos \theta - 1 = 4 \sin \theta \tan \theta$. Condone slips in coefficients and the equation may have been adapted. This may be implied by candidates who multiply by $\cos \theta$ and reach $\cos \theta - 1 = 4 \sin \theta \tan \theta \Rightarrow \cos^2 \theta - \cos \theta = 4 \sin^2 \theta$. This would be M1 A1 A1 Correct equation, without any fractional terms, in $\sin \theta$ and $\cos \theta$ If the identity $\sin^2 \theta = 1 - \cos^2 \theta$ is used before the multiplication by $\cos \theta$ then it will be for a correct equation, without any fractional terms, in $\cos \theta$ Condone incorrect notation $\cos \theta^2$ for $\cos^2 \theta$ M1 Uses $\sin^2 \theta = 1 - \cos^2 \theta$ to produce an equation in just $\cos \theta$ A1* Proceeds to $5 \cos^2 \theta - \cos \theta - 4 = 0$ with no arithmetical or notational errors. Both identities must be seen to have been applied. Candidates cannot just go from $\cos^2 \theta - \cos \theta = 4 \sin^2 \theta$ to the answer without any evidence of the appropriate identity. No mixed variables within the lines of the "proof" Condone incomplete lines if it is part of their working. $\cos^2 \theta - \cos \theta = 4 \sin^2 \theta$ $\text{Eg.} \quad = 4(1 - \cos^2 \theta)$ An example of a notational error is $\cos \theta^2$ for $\cos^2 \theta$ (Note that this would only lose the A1*) (b) M1 Attempts to find the critical values of the given quadratic by a correct method. A1 Critical values of $-\frac{4}{5}, 1$. Allow this to be scored even if written as $\cos x = \dots$ or even x. Allow these to be written down (from a calculator)		

Question Number	Scheme	Marks
dM1	A correct method to find one value of x from their $\cos 2x = -\frac{4}{5}$ Look for correct order of operations. It is dependent upon the previous mark. This can be implied by awrt 1.5/71.6° or awrt 1.24/1.25 (rads)	
A1	Both $x = 0$ and awrt 1.25 with no other values in the range $0 \leq x < \frac{\pi}{2}$. Condone 1.25 written as 0.398π . Condone if written as $\theta = ..$	
.....		
Answers without working can score all marks: Score M1 for one value and M1 A1 M1 A1 for both values and no others in the range.		

Question Number	Scheme	Marks
4(a)	$(2 + px)^6 = (2^6 +) 6 \times 2^5 (px) + \frac{6 \times 5}{2} \times 2^4 (px)^2 + \dots$ or $\left(1 + \frac{p}{2}x\right)^6 = 1 + 6 \times \left(\frac{p}{2}x\right) + \frac{6 \times 5}{2} \times \left(\frac{p}{2}x\right)^2 + \dots$	M1
	2^6 or 64	B1
	$+192px$ or $+240p^2x^2$	A1
	$(64 +)192px + 240p^2x^2$	A1
		(4)
(b)	$\left(3 - \frac{1}{2}x\right)(2 + px)^6 \Rightarrow \left(3 - \frac{1}{2}x\right)(64 + 192px + 240p^2x^2)$	
	Attempts $3 \times "240p^2"$ and $\left(-\frac{1}{2}\right) \times "192p"$	M1
	$3 \times "240p^2" + \left(-\frac{1}{2}\right) \times "192p" = -\frac{3}{4}$	dM1
	$2880p^2 - 384p + 3 = 0 \Rightarrow p = \dots$	ddM1
	$(p =) \frac{1}{8}, \frac{1}{120}$	A1
		(4)
		(8 marks)



(a)

M1 Attempts to expand using the binomial theorem in an attempt to find term 2 and term 3. Look for correct binomial coefficients linked with correct powers of x , condoning invisible brackets and the p not being squared on the third term. Allow slips on powers of 2, or if 2^6 has been taken out. The binomial coefficient must have been evaluated but need not be simplified and at least one must be correct. Terms may be found separately.

B1 Sight of 2^6 or 64 as the first term or extracted from the bracket.

A1 $+192px$ or $+240p^2x^2$ having gained the M. Allow also for $2^6\left(1+3px+\frac{15}{4}p^2x^2\right)$ and accept with $(px)^2$

A1 $+192px$ and $+240p^2x^2$ correct x and x^2 terms, may be as part of a list. Accept with $(px)^2$ ISW after a correct solution if they go on to try and divide through.

(b)

M1 Identifies the two x^2 terms either separately or may be seen as part of a full expansion – look for two x^2 terms in their expansion. There may be slips in the coefficients but they should be coming from the correct combination of terms in the brackets.

dM1 A complete method to find the term in x^2 and set the coefficient equal to $-\frac{3}{4}$ to obtain an equation in p . The correct two terms in x^2 for their expansion in (a) must have been identified. It is dependent on the previous method mark.

ddM1 Attempts to solve a 3TQ in p . Usual rules apply for solving a quadratic but it is dependent on the previous two method marks

A1 $(p =) \frac{1}{8}, \frac{1}{120}$ Accept decimal equivalents, 0.125 and awrt 0.00833



Question Number	Scheme	Marks
5(i)	(As $x \geq 0$ so $\sqrt{3x}$ exists and) $(\sqrt{3x}-1)^2 \geq 0$	M1
	Hence $3x - 2\sqrt{3x} + 1 \geq 0$	M1
	$\Rightarrow 3x + 1 \geq 2\sqrt{3x}$ *	A1*
		(3)
Alt 1	$3x + 1 \geq 2\sqrt{3x} \Leftrightarrow (3x + 1)^2 \geq 12x \Leftrightarrow 9x^2 - 6x + 1 \geq 0$	M1
	$9x^2 - 6x + 1 \geq 0 \Leftrightarrow (3x - 1)^2 \geq 0$	M1
	Square numbers are greater than or equal to zero so $(3x - 1)^2 \geq 0$ is true hence $3x + 1 \geq 2\sqrt{3x}$ *	A1*
		(3)
Alt 2	If $3x + 1 < 2\sqrt{3x}$ then $3x - 2\sqrt{3x} + 1 < 0$	M1
	So $(\dots\sqrt{3x} \pm \dots)^2 < 0$ or $(\sqrt{x} \pm \dots)^2 < 0$	M1
	But $(\sqrt{3x} - 1)^2 \geq 0$ for all $x \geq 0$ so $3x + 1 \geq 2\sqrt{3x}$	A1
(ii)	Shows that it is not true for three consecutive prime numbers Eg $5 + 7 + 11 = 23$ which is not divisible by 5 (so not true)	B1
		(1)
		(4 marks)



Question Number	Scheme	Notes	Marks
6(a)	Examples: $m_{PQ} = \frac{14+30}{23-15}, m_{QR} = \frac{-30+26}{15+7} \Rightarrow m_{PQ} \times m_{QR} = ..$ or $PQ^2 = 8^2 + 44^2, QR^2 = 22^2 + 4^2, PR^2 = 30^2 + 40^2$ $PQ^2 + QR^2 = ..$	Correct strategy to show that $\angle PQR = 90^\circ$. E.g. attempts gradient of PQ and gradient of QR and attempts product or finds side lengths and attempts Pythagoras.	M1
	$m_{PQ} \times m_{QR} = \frac{11}{2} \times \left(-\frac{2}{11}\right) = -1 \Rightarrow \angle PQR = 90^\circ$ Or $PQ^2 = 2000, QR^2 = 500, PR^2 = 2500$ $2000 + 500 = 2500 \Rightarrow \angle PQR = 90^\circ$	Correct proof and conclusion	A1
			(2)
(b)(i)	Centre is $(8, -6)$	Correct coordinates	B1
(b)(ii)	$r = \sqrt{(23-8)^2 + (14+6)^2}$ or e.g. $r = \frac{1}{2} \sqrt{(23+7)^2 + (14+26)^2}$	Fully correct method for the radius	M1
	$r = 25$	Cao	A1
			(3)
(c)	S is $(1, 18)$ or $m_T = \frac{7}{24}$	Correct coordinates for S or correct gradient for the tangent.	B1
	$m_N = \frac{18+6}{1-8} \Rightarrow m_T = \frac{8-1}{18+6} \Rightarrow y-18 = \frac{7}{24}(x-1)$ Uses a correct straight line method for the tangent using their S and the negative reciprocal of the radius gradient		M1
	$7x - 24y + 425 = 0$	Allow any integer multiple	A1
			(3)
			Total 8

Notes**(a)**

- M1:** For a correct strategy to show that angle $PQR = 90^\circ$. Possible methods are:
 Find gradients of PQ and RQ (in order to check the perpendicularity condition), getting at least as far as both gradients.
 Finding the lengths of all three sides and attempting to show $PQ^2 + QR^2 = PR^2$ attempting substitution into at least $PQ^2 + QR^2 = ..$ When finding lengths, accept at least two correct lengths stated to imply method for finding lengths if no method is shown.
 Finding the lengths of all three sides and applying the cosine rule to find the angle, reaching at least substitution into the cosine rule. May find the two acute angles and subtract from 180° .
 Finding the midpoint of PR and showing it is equidistant from P , Q and R to deduce PR is a diameter and applying circle theorem. Must reach at least the calculations of lengths.
 Finding the length of all three sides and calculating angle at P from both sin and cosine ratios to show both agree. Must reach angle from both ratios.
 Other methods are possible, if you see a method you are unsure of then send to review.



Question Number	Scheme	Marks
2(a)	$f\left(\frac{5}{4}\right) = \left(2 - k \times \frac{5}{4}\right)^5 = \frac{243}{32} \Rightarrow \left(2 - k \times \frac{5}{4}\right) = \sqrt[5]{\frac{243}{32}} \Rightarrow k = \dots$	M1
	$\frac{3}{2} \Rightarrow \frac{5k}{4} = \frac{1}{2} \Rightarrow k = \frac{2}{5} \quad *$	A1*
		(2)
(b)	$\pm {}^5C_1 \times 2^4 \times \left(\pm \frac{2}{5}x\right) \quad \text{or} \quad \pm {}^5C_2 \times 2^3 \times \left(\pm \frac{2}{5}x\right)^2$	M1
	$32 - 32x + \frac{64}{5}x^2$	A1A1
		(3)
(c)	$f'(x) = -32 + \frac{128}{5}x + \dots \Rightarrow f'(0) = \dots$	M1
	$f'(0) = -32$	A1ft
		(2)
		(7 marks)

(a)

M1: Substitutes $x = \frac{5}{4}$ into $f(x)$, equates to $\frac{243}{32}$ and attempts to make k the subject by taking the 5th root of both sides.

A1*: $k = \frac{2}{5}$ with no errors and sufficient working shown.

Accept as a minimum $\left(2 - k \times \frac{5}{4}\right)^5 = \frac{243}{32} \Rightarrow \left(2 - k \times \frac{5}{4}\right) = \frac{3}{2} \Rightarrow k = \frac{2}{5}$

Note that **just** $\left(2 - k \times \frac{5}{4}\right)^5 = \frac{243}{32} \Rightarrow k = \frac{2}{5}$ scores no marks as the minimum for the M mark requires taking the 5th root of both sides.

Alternative by verification:

M1: $k = \frac{2}{5}, x = \frac{5}{4} \Rightarrow \left(2 - \frac{2}{5} \times \frac{5}{4}\right)^5 = \left(\frac{3}{2}\right)^5 = \frac{243}{32}$

Substitutes $k = \frac{2}{5}$ and $x = \frac{5}{4}$ and attempts to raise **an evaluated** $2 - kx$ to the power of 5

A1: Hence $k = \frac{2}{5}$

Fully correct work and makes a (minimal) conclusion e.g. Hence proven, QED, Therefore true, etc.

Note that **just** $\left(2 - \frac{2}{5} \times \frac{5}{4}\right)^5 = \frac{243}{32}$ or $\left(2 - \frac{2}{5} \times \frac{5}{4}\right)^5 = \left(2 - \frac{1}{2}\right)^5 = \frac{243}{32}$ scores no marks as the $2 - \frac{2}{5} \times \frac{5}{4}$

or $2 - \frac{1}{2}$ must be evaluated for the M mark.



(b)

M1: Attempts the binomial expansion to obtain the correct structure for the x or x^2 term i.e. the correct binomial coefficient with the correct power of 2 and the correct power of $\pm \frac{2}{5}x$.

The binomial coefficients do not have to be evaluated but must be correct if they are.

If awarding this mark for the x^2 term you can condone missing brackets e.g. $\pm^5 C_2 \times 2^3 \times \pm \frac{2}{5}x^2$

A1: For the correct simplified x or x^2 term i.e. $-32x$ or $+\frac{64}{5}x^2$

A1: For $32 - 32x + \frac{64}{5}x^2$ which may be written as a list. Allow equivalents for $\frac{64}{5}$ e.g. 12.8

Condone $32 + (-32x) + \frac{64}{5}x^2$

Ignore any extra terms but do not isw – mark their final answer.

If they don't simplify in (b) do not allow simplified terms in (c) as recovery.

(b) Alternative takes out a power of 2:

$$\left(2 - \frac{2}{5}x\right)^5 = 2^5 \left(1 - \frac{1}{5}x\right)^5 = 2^5 \left(1 - 5 \times \frac{1}{5}x + \frac{5 \times 4}{2} \left(\frac{1}{5}x\right)^2 + \dots\right)$$

Score **M1** for $2^5 \left(\dots \pm 5 \times \pm \frac{1}{5}x + \dots\right)$ or $2^5 \left(\dots \pm \frac{5 \times 4}{2} \left(\pm \frac{1}{5}x\right)^2 + \dots\right)$ condoning $\left(\pm \frac{1}{5}x^2\right)$ as above

Then **A** marks as above.

(c)

M1: Attempts to differentiate their expansion **and** substitutes $x = 0$ which may be implied.

For the differentiation, look for $x^n \rightarrow x^{n-1}$ at least once including $k \rightarrow 0$ or $kx \rightarrow k$

A1ft: -32 following correct differentiation

Or follow through on their q provided

- the expansion in (b) was of the form $p + qx + rx^2$, $p, q, r \neq 0$
- the differentiation is correct for their $p + qx + rx^2$ i.e. $q + 2rx$

In part (c) you can also ignore any extra terms e.g. do not penalise if they have differentiated any of the extra terms incorrectly in an otherwise correct solution.



Question Number	Scheme	Marks
3(a)(i)	$a_1 = \frac{1}{4}$	B1
(ii)	$a_2 = \frac{1}{4}$	B1
(iii)	$a_3 = 1$	B1
		(3)
(b)	$\frac{50}{2}[2+49] (=1275)$ oe	M1A1
	$\sum_{n=1}^{50} \cos^2\left(\frac{n\pi}{3}\right) = 34 \times \frac{1}{4} + 16 \times 1$	M1
	$1275 + \frac{49}{2} = \frac{2599}{2}$	A1
		(4)
		(7 marks)

(a)**B1:** $\frac{1}{4}$ or 0.25**B1:** $\frac{1}{4}$ or 0.25**B1:** 1 (which has clearly not come from a rounded degrees decimal answer)Note that use of degrees gives $a_1 = 0.9996659868\dots$, $a_2 = 0.9986643935\dots$, $a_3 = 0.9969965583\dots$ and scores no marks.**(b)****M1:** Correct attempt to find the sum of $1+2+3+\dots+50$ **A1:** $\frac{50}{2}[2+49]$ or e.g. $\frac{50}{2}[1+50]$ or 1275

Award for any correct numerical expression or for 1275. May be implied or may be seen as part of a complete calculation. A correct answer only of 1275 implies both of the first 2 marks.

M1: Correct attempt to find $\sum_{n=1}^{50} \cos^2\left(\frac{n\pi}{3}\right)$ e.g. by $34 \times \frac{1}{4} + 16 \times 1$ or $17 \times \frac{1}{4} + 17 \times \frac{1}{4} + 16 \times 1$ or $16 \times (\frac{1}{4} + \frac{1}{4} + 1) + \frac{1}{4} + \frac{1}{4}$ or $17 \times (\frac{1}{4} + \frac{1}{4} + 1) - 1$ Must be a **correct** method for the **correct** sequence, e.g. $\frac{1}{4} + \frac{1}{4} + 1 + \frac{1}{4} + \frac{1}{4} + 1 + \frac{1}{4} + \frac{1}{4} + 1 + \dots$ If they **just** write down $49/2$ this scores M0**A1:** $\frac{2599}{2}$ or exact equivalent e.g. 1299.5. Isw once a correct answer is seen.**Note that a method must be seen in part (b) as stated in the question. Correct answer only of $2599/2$ scores no marks.**

Question Number	Scheme	Marks
4(a)	$10 = \log_a 8 - \log_a 4 \Rightarrow \log_a 2 = 10$	M1
	$a^{10} = 2$	M1
	$a = 2^{\frac{1}{10}} = 1.07177... *$	A1*
		(3)
(b)	$w = \log_{1.072}(t+5) - \log_{1.072} 4 \Rightarrow w = \log_{1.072} \left(\frac{t+5}{4} \right)$	M1
	$w = \log_{1.072} \left(\frac{t+5}{4} \right) \Rightarrow \frac{t+5}{4} = 1.072^w \Rightarrow t = ...$	M1
	$t = 4 \times 1.072^w - 5$	A1
		(3)
(b) ALT	$w = \log_{1.072}(t+5) - \log_{1.072} 4 \Rightarrow w + \log_{1.072} 4 = \log_{1.072}(t+5)$	M1
	$\Rightarrow t+5 = 1.072^{w+\log_{1.072} 4}$	M1
	$\Rightarrow t = 1.072^{w+\log_{1.072} 4} - 5$	A1
(c)	$t = 4 \times 1.072^{15} - 5 = ...$	M1
	awrt 6.35	A1
		(2)
		(8 marks)

(a)

M1: Substitutes $t = 3$ and $w = 10$ into the equation and achieves $\log_a 2 = 10$ or e.g. $\log_a \frac{8}{4} = 10$ correctly

M1: Correctly removes the log to obtain $a^{10} = 2$

A1*: Fully correct proof showing $a = 2^{\frac{1}{10}}$ or $a = \sqrt[10]{2}$ or $a = \sqrt[10]{\frac{8}{4}}$ and obtains awrt 1.072

Note that this may be implied by the accuracy of their a e.g. 1.071773463... so allow for 1.0718 (rounded) or 1.0717 (truncated).

May also see logarithm approach e.g. $a^{10} = 2 \Rightarrow \log a^{10} = \log 2 \Rightarrow \log a = \frac{\log 2}{10} \Rightarrow a = 10^{\frac{\log 2}{10}} = ...$

$$\text{or } a^{10} = 2 \Rightarrow \ln a^{10} = \ln 2 \Rightarrow \ln a = \frac{\ln 2}{10} \Rightarrow a = e^{\frac{\ln 2}{10}} = ...$$

False solutions in (a):

$$10 = \log_a 8 - \log_a 4 \Rightarrow \frac{\log_a 8}{\log_a 4} = 10 \Rightarrow \log_a 2 = 10 \Rightarrow a^{10} = 2 \Rightarrow a = \sqrt[10]{2} = 1.072$$

Scores **M0M1A0**

$$10 = \log_a 8 - \log_a 4 \Rightarrow \frac{8 \log a}{4 \log a} = 10 \Rightarrow 2 \log a = 10 \Rightarrow \log a^2 = 10 \Rightarrow a^{10} = 2 \Rightarrow a = \sqrt[10]{2} = 1.072$$

Scores no marks



(b)**M1:** Applies the subtraction law for logs to write the equation as $w = \log_{1.072} \left(\frac{t+5}{4} \right)$ **M1:** Writes the equation as $1.072^w = f(t)$ and proceeds to make t the subject.**A1:** $t = 4 \times 1.072^w - 5$ **Alternative:****M1:** Rearranges to make $\log_{1.072}(t+5)$ the subject correctly**M1:** Removes the logs on lhs to obtain $f(t) = 1.072^{g(w)}$ **A1:** Correct equation. $t = 1.072^{w+\log_{1.072} 4} - 5$ In both cases allow the use of “ a ” or a more accurate value for “ a ” rather than 1.072 for both method marks.**(c)****M1:** Substitutes $w = 15$ into their equation from (b) or possibly the given equation $w = \log_{1.072}(t+5) - \log_{1.072} 4$ and proceeds to find a value for t **A1:** awrt 6.35 (months) (NB If full accuracy used for a answer is 6.3137... and scores A0)

Ignore any units if given.



Question Number	Scheme	Marks
5(a)	$\cos \theta(3 \cos \theta - \tan \theta) = 2 \Rightarrow \cos \theta(3 \cos \theta - \frac{\sin \theta}{\cos \theta}) = 2$ or e.g. $\cos \theta(3 \cos \theta - \tan \theta) = 2 \Rightarrow 3 \cos^2 \theta - \sin \theta = 2$ or e.g. $\cos \theta(3 \cos \theta - \tan \theta) = 2 \Rightarrow 3 \cos^2 \theta - \frac{\sin \theta}{\cos \theta} \cos \theta = 2$	M1
	$3(1 - \sin^2 \theta) - \sin \theta = 2$	M1
	$3 \sin^2 \theta + \sin \theta - 1 = 0$ *	A1*
		(3)
(b)	$(\sin 2x =) \frac{-1 \pm \sqrt{13}}{6}$ (or awrt 0.43 and awrt -0.77 (or truncated -0.76))	M1A1
	$2x = \sin^{-1}(0.434\dots)$ or $2x = \sin^{-1}(-0.77\dots) \Rightarrow x = \dots$	M1
	-1.13, -0.438, 0.225, 1.35	A1A1
		(5)
		(8 marks)

(a)

M1: Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ to write the equation in terms of sine and cosine only.

M1: Uses $\sin^2 \theta + \cos^2 \theta = 1$ to obtain a quadratic equation in sine only.

A1*: Achieves $3 \sin^2 \theta + \sin \theta - 1 = 0$ * with no errors.

Condone one notational slip e.g. $3 \sin \theta^2$ instead of $3 \sin^2 \theta$ or e.g. $3 \sin$ for $3 \sin \theta$ in the working but the printed answer must be correct but allow e.g. $0 = \sin \theta + 3 \sin^2 \theta - 1$

(b)

M1: Attempts to solve the quadratic $3 \sin^2 \theta + \sin \theta - 1 = 0$ which may be in any variable. Usual rules apply for solving a quadratic (via a calculator is also acceptable and may imply this mark). If no working is shown then the roots must be correct.

A1: $\frac{-1 \pm \sqrt{13}}{6}$ or a minimum of $\frac{-1 \pm \sqrt{13}}{2 \times 3}$ (or as decimals awrt 0.43 and awrt -0.77).

Whether subsequently rejected or not. Ignore any labelling just look for these values.

M1: Attempts to find one angle within the range by finding the inverse sine of one of their roots and dividing by 2. May be implied by their values and allow if working in degrees.

A1: Any two of awrt -1.13, -0.438, 0.225, 1.35

A1: All four of awrt -1.13, -0.438, 0.225, 1.35 and no others in the range.

Special case - answers in degrees: -64.9, -25.1, 12.9, 77.1

Score A1 for awrt any 2 of these and then A0 so a maximum of 7/8 for the question.

NB allow answers to be found in degrees which are subsequently converted to radians but then left in terms of π e.g. $\frac{-64.9\pi}{180}$ etc. provided the answers round to the -1.13, -0.438, 0.225, 1.35

Some candidates are obtaining a different quadratic equation in (a) e.g. $3 \sin^2 \theta - \sin \theta - 1 = 0$. In such cases allow both M marks in (b) if they persist with their incorrect quadratic provided there are sign errors only e.g. $\pm 3 \sin^2 \theta \pm \sin \theta \pm 1 = 0$

Question Number	Scheme	Marks
10 (a)	Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ o.e. E.g. $\cos \theta \left(3 \tan \theta + \frac{2}{\tan \theta} \right) \equiv \cos \theta \left(3 \frac{\sin \theta}{\cos \theta} + \frac{2 \cos \theta}{\sin \theta} \right)$	M1
	Uses $\sin^2 \theta + \cos^2 \theta = 1$ E.g. $\equiv 3 \sin \theta + \frac{2 \cos^2 \theta}{\sin \theta} \equiv 3 \sin \theta + \frac{2(1 - \sin^2 \theta)}{\sin \theta}$	dM1, A1
	$\equiv 3 \sin \theta + \frac{2}{\sin \theta} - 2 \sin \theta \equiv \sin \theta + \frac{2}{\sin \theta}$	A1*
(b)	$\sin x + \frac{2}{\sin x} = 4 \sin x - 5 \Rightarrow 3 \sin^2 x - 5 \sin x - 2 = 0$	M1, A1
	$\Rightarrow \sin x = 2, -\frac{1}{3} \Rightarrow x = 3.5$ for example.	dM1
	$\Rightarrow x = 3.48, 5.94$	A1
		(4)
		(8 marks)

(a) Allow use of $=$ for $\equiv$ within this question.

Allow workings on consecutive lines to follow without equals signs or with arrows.

M1: Uses $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ o.e. correctly at least once

dM1: Uses $\cos^2 \theta \equiv \pm 1 \pm \sin^2 \theta$ following the award of the previous M1

You may see $\cos \theta \left(\frac{3 \sin^2 \theta + 2 \cos^2 \theta}{\sin \theta \cos \theta} \right) \equiv \cos \theta \left(\frac{3 \sin^2 \theta + 2(\pm 1 \pm \sin^2 \theta)}{\sin \theta \cos \theta} \right)$ condoning slips in

coefficients

A1: A correct intermediate line in just $\sin \theta$ E.g. $3 \sin \theta + \frac{2(1 - \sin^2 \theta)}{\sin \theta}$

A1*: Proceeds correctly to the given answer with no errors or omissions.

Withold this mark for notational errors. E.g writing $\sin^2 \theta$ as $\sin \theta^2$

Withold this mark if you don't see both sides of the identity (or their equivalents e.g. LHS and RHS) within their solution.

Condone one or two missing θ 's or changes of variable in say, $\sin \theta$, but penalise persistent offences, i.e. > 2 occurrences.

(b)

M1: Uses part (a) and proceeds to a 3TQ in $\sin x$. Condone slips in coefficients.

The multiplication by $\sin x$ must be seen/have occurred on at least three of the four terms

Allow restarts that essentially proceeds to the same equation

A1: Correct quadratic equation. The $= 0$ may be implied by subsequent work

dM1: Solves their quadratic equation in $\sin x$ and proceeds to one solution in the range to 1dp

If their quadratic equation in $\sin x$ is incorrect you may need to use a calculator to check their roots and their value for x . Allow this method to be scored for angles in degrees.

So, for example, $\sin x = -\frac{1}{3} \Rightarrow x = \text{awrt } 199.5 \text{ or } 340.5$ but not -0.339 radians as it is not in the range

A1: AWR T 3.48, 5.94 and no others in the range.

This can only be awarded after the award of all previous marks in (b)

Answers without working scores 0 marks



Question Number	Scheme	Marks
7. (i)	10+8+6.4+...	
(a)	Attempts $S_{\infty} = \frac{a}{1-r}$ with $a=10, r = \frac{8}{10}$ $S_{\infty} = 50$	M1 A1 (2)
(b)	Sets $10 \times 0.8^{k-1} < 0.0005 \Rightarrow 0.8^{k-1} < 0.00005$ e.g. $\Rightarrow k-1 > \frac{\log(0.00005)}{\log(0.8)}$ or e.g. $k-1 > \log_{0.8} 0.00005$ (see notes) $\Rightarrow k-1 > 44.38 \Rightarrow (k =) 46$	M1 M1 A1 (3)
(ii)	Sets $850 + (n-1) \times -7 = 0 \Rightarrow n = 122.4$ States or implies that $n = 122$ $S = \frac{122}{2} \{2 \times 850 + 121 \times -7\} = 52033$	M1 A1 dM1, A1 (4)
		(9 marks)

(i) (a)

M1: Attempts $S_{\infty} = \frac{a}{1-r}$ with $a=10, r = \frac{8}{10}$ but condone slips on an incorrectly evaluated "r" provided the method to finding it is correct and $-1 < r < 1$. The expression with the values substituted in is sufficient.

A1: $(S_{\infty} =) 50$

(b) Note that candidates may misread the number of 0s in 0.0005 but can still score M1M1A0

M1: Sets $10 \times 0.8^{k-1} < 0.0005$ and proceeds to $0.8^{k-1} < 0.00005$ (or they may manipulate correctly to $0.8^k < 0.00005 \times 0.8$). If they take logs before this stage then they will need to proceed to e.g.
 $\log 0.8^{k-1} < \log 0.0005 - \log 10$
 Condone other signs being used e.g. = or >

M1: Uses a correct method to solve an equation or inequality of the type $p^k \dots q$ or $p^{k-1} \dots q$
 It cannot be for just stating the value for k. They only need to proceed as far as $k-1$ on one side of an equation or inequality. Evidence of using logs is required which cannot be e.g. 44.38... otherwise M0A0
 Do not be concerned with the direction of inequalities, if used, in their solution for this mark.

A1: $(k =) 46$ from correct working.

If inequalities are used in their method then the direction must be correct.



(ii) When using either formula condone missing brackets around (-7)

e.g. $850 + (n-1) \times -7$ or e.g. $S_n = \frac{122}{2} [2 \times 850 + (122-1) \times -7]$

M1: Attempts to find the value of n that maximises S_n

May set $850 + (n-1) \times -7 = 0 \Rightarrow -7n + 857 = 0 \Rightarrow n = \dots$ or show trial calculations of $u_{122} = 3$ and $u_{123} = -4$. May be implied by 122.

Alternatively sets $S_n = \frac{n}{2} [2 \times 850 + (n-1) \times -7]$ and either

- differentiates and sets $= 0$: $S_n = \frac{1707}{2}n - \frac{7}{2}n^2 \Rightarrow \frac{1707}{2} - 7n = 0 \Rightarrow n = \dots$
- uses roots: $S_n = \frac{1707}{2}n - \frac{7}{2}n^2 = 0 \Rightarrow n \left(\frac{1707}{2} - \frac{7}{2}n \right) = 0 \Rightarrow n = 0, n = \dots$ and then finds the midpoint of the two roots
- completes the square: $S_n = \frac{1707}{2}n - \frac{7}{2}n^2 = 0 \Rightarrow -\frac{7}{2} \left(n^2 - \frac{1707}{7}n \right) \Rightarrow -\frac{7}{2} \left(n - \frac{1707}{14} \right)^2 \pm \dots \Rightarrow n = \dots$

to find the position of the turning point (approximately 121.9)

In all cases condone the use of n instead of $n-1$

A1: States or implies that $n=122$ (if there are multiple attempts using different values of n then the others must be rejected for this mark to be scored)

dM1: Attempts S_n using a correct formula (they may attempt to find the last term and use $S_n = \frac{n}{2}[a + L]$)

It is dependent upon the previous M.

Condone the incorrect value of n being selected but it must be a positive integer.

May see multiple attempts using different values of n which is acceptable.

A1: 52033 only (i.e. if they have multiple attempts using different values then this answer must be clearly identified or the others all rejected)



Question Number	Scheme	Marks
8 (i)	$3 \tan \left(2x + \frac{\pi}{5} \right) = \sqrt{3} \Rightarrow 2x + \frac{\pi}{5} = \arctan \left(\frac{\sqrt{3}}{3} \right)$ $\Rightarrow 2x + \frac{\pi}{5} = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \Rightarrow x = \dots$ $\Rightarrow x = \frac{29\pi}{60}, \frac{59\pi}{60}$	M1 dM1 A1 (3)
(ii)	$5 \sin \theta \tan \theta = \cos \theta + 4$ $\tan \theta = \frac{\sin \theta}{\cos \theta} \rightarrow 5 \sin \theta \times \frac{\sin \theta}{\cos \theta} = \cos \theta + 4$ $\Rightarrow 5(1 - \cos^2 \theta) = \cos^2 \theta + 4 \cos \theta$ $\Rightarrow 6 \cos^2 \theta + 4 \cos \theta - 5 = 0$ $\Rightarrow \cos \theta = \frac{-4 \pm \sqrt{16 + 120}}{12} (= 0.63849\dots) \Rightarrow \theta =$ $\Rightarrow \theta = 50.3^\circ, 309.7^\circ$	M1 dM1 A1 ddM1 A1 (5) (8 marks)

In both (i) and (ii) condone poor notation, omission of θ or x in their working provided the intention is clear or recovered/implied by further work.

(i)

M1: Correct order of operations $2x + \frac{\pi}{5} = \arctan \left(\frac{\sqrt{3}}{3} \right)$ implied by $2x + \frac{\pi}{5} = \frac{\pi}{6}$ or any correct angle.

Condone decimal values here e.g. $2x + 0.628\dots = 0.52\dots$

Allow to work consistently in degrees. e.g. $2x + 36 = 30$

Alternatively, attempts to use $\tan \theta = \frac{\sin \theta}{\cos \theta}$, squares both sides, attempts to use

$\pm \sin^2 \theta \pm \cos^2 \theta = \pm 1$ and proceeds to a quadratic equation in sine or cosine

e.g.

$$\frac{3 \sin \left(2x + \frac{\pi}{5} \right)}{\cos \left(2x + \frac{\pi}{5} \right)} = \sqrt{3} \Rightarrow 3 \sin \left(2x + \frac{\pi}{5} \right) = \sqrt{3} \cos \left(2x + \frac{\pi}{5} \right) \Rightarrow 9 \sin^2 \left(2x + \frac{\pi}{5} \right) = 3 \left(1 - \sin^2 \left(2x + \frac{\pi}{5} \right) \right)$$

$$\Rightarrow 12 \sin^2 \left(2x + \frac{\pi}{5} \right) = 3 \Rightarrow \sin \left(2x + \frac{\pi}{5} \right) = (\pm) \frac{1}{2} \Rightarrow 2x + \frac{\pi}{5} = \frac{\pi}{6}, \dots$$



dM1: Correct order of operations $2x + \frac{\pi}{5} = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$ to find a value for x which does not need to be in the given range (e.g. condone $-\frac{\pi}{60}$)

Condone decimal answers here (awrt -0.05 , awrt 1.5 , awrt 3.1). It is dependent on the previous method mark. Allow to work consistently in degrees. e.g. $2x + 36 = 30 \Rightarrow x = \frac{30 \pm 36}{2} = \dots$

A1: $(x =) \frac{29\pi}{60}, \frac{59\pi}{60}$ and no other angles in the range. Must be exact and in radians.

Alternative: Using compound angle formula

M1: Forms the equation $\tan 2x = \frac{\frac{\sqrt{3}}{3} - \tan\left(\frac{\pi}{5}\right)}{1 + \frac{\sqrt{3}}{3} \tan\left(\frac{\pi}{5}\right)}$ Allow to work consistently in degrees

dM1: Correct order of operations (arctan followed by dividing by 2)
Allow to work consistently in degrees

A1: $(x =) \frac{29\pi}{60}, \frac{59\pi}{60}$ and no other angles in the range. Must be exact and in radians.

(ii) **Note that there may be other credit worthy methods relying on identities covered in later units. If seen then send to review.**

M1: Attempts to use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ leading to an equation in $\sin \theta$ and $\cos \theta$
e.g. $5\sin \theta \times \frac{\sin \theta}{\cos \theta} = \cos \theta + 4$ Condone slips.

dM1: Multiplies through by $\cos \theta$ (on at least 2 of the 3 terms in the equation) and attempts to use $\pm \sin^2 \theta \pm \cos^2 \theta = \pm 1$ to set up a quadratic equation in $\cos \theta$ (may be in $\sin \theta$ if they make an earlier slip so allow this to score as well)
Condone slips e.g. not multiplying both terms by 5 if $5(1 - \cos^2 \theta)$ is not seen on the left hand side of the equation.
Terms do not need to be collected for this mark.
Condone arithmetical slips in their rearrangement.

A1: $6\cos^2 \theta + 4\cos \theta - 5 = 0$ o.e The $= 0$ may be implied by further work.
Requires 3 terms but allow the constant term to be on the other side of the equation.

ddM1: Solves their **three-term** quadratic equation in $\cos \theta$ (or condone $\sin \theta$) and proceeds to a value for θ

Allow use of a calculator to solve the quadratic otherwise usual rules apply for factorising, using the formula or completing the square.

We must see

- a correct method used to solve the quadratic, or can be implied by a correct root to 1dp for their quadratic equation
- a correct angle for their root by taking arccos of the root (allow to the nearest integer in degrees) You may need to check this.

A correct angle on its own does not imply this mark.

A1: $(\theta =)$ awrt 50.3° , awrt 309.7° and no others in the range. Must be in degrees.



Question Number	Scheme	Marks
8(i)	$5 \sin(3x + 0.1) + 2 = 0$ $\Rightarrow 5 \sin(3x + 0.1) = -2$ $\Rightarrow \sin(3x + 0.1) = -\frac{2}{5}$	M1
	$\sin(3x + 0.1) = -\frac{2}{5}$ $\Rightarrow 3x + 0.1 = \sin^{-1}\left(-\frac{2}{5}\right)$ $\Rightarrow x = \frac{\sin^{-1}\left(-\frac{2}{5}\right) - 0.1}{3}$	dM1
	$x = -0.94, -0.17, 1.15, 1.92$	A1A1
		(4)
(ii)	$2 \tan \theta \sin \theta = \cos \theta + 5$ $\Rightarrow 2 \sin^2 \theta = \cos^2 \theta + 5 \cos \theta$	M1
	$\Rightarrow 2(1 - \cos^2 \theta) = \cos^2 \theta + 5 \cos \theta$	M1
	$\Rightarrow 3 \cos^2 \theta + 5 \cos \theta - 2 = 0$	A1
	$\cos \theta = \frac{1}{3}, -2$ $\Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right) = \dots$	M1
	$(\theta =) 70.5^\circ, 289.5^\circ$	A1
		(5)
		Total 9

Answers with no working score 0 marks**(i)**

M1 For ± 2 and then dividing by 5 to reach $\sin(3x + 0.1) = \pm \frac{2}{5}$ oe

May be implied by $3x + 0.1 = \pm 0.411\dots$ or allow a different variable to be used such that

$$y = 3x + 0.1 \Rightarrow \sin(y) = \pm \frac{2}{5}.$$

Condone $3x + 0.1 = \pm 23.57\dots$ if they have incorrectly worked in degrees for this mark only.



dM1 Correct strategy for finding x . Allow $x = \frac{\pm 2\pi n + \sin^{-1}\left(\pm \frac{2}{5}\right) \pm 0.1}{3}$ or

e.g. $x = \frac{\pi - \sin^{-1}\left(\pm \frac{2}{5}\right) \pm 0.1}{3}$ May be implied by a correct angle, but they must have proceeded as far

as $\sin(3x + 0.1) = -\frac{2}{5}$ before achieving an angle.

Must be working in radians OR entirely in degrees (if the 0.1 radians is converted first)

It is dependent on the first method mark.

A1 Two of awrt $-0.94, -0.17, 1.15, 1.92$. Must be in radians.

A1 All of awrt $-0.94, -0.17, 1.15, 1.92$ and no extras in range

Beware of $5\sin(3x + 0.1) = -2 \Rightarrow 15\sin x + 0.5 = -2 \Rightarrow x = -0.17$ **which scores 0 marks**

Note: There are other credit worthy methods such as squaring $5\sin(3x + 0.1) = -2$ and using

$\pm \sin^2 \theta = \pm 1 \pm \cos^2 \theta$, then solving a quadratic in $\cos \theta$. For the first M1 they would have to proceed as far $3x + 0.1 = \dots$ but allow the mark if slips in rearranging are made.

(ii)

M1 For using $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and attempting to multiply by $\cos \theta$ (may even be $\cos^2 \theta$ or higher which is

acceptable). Look for the denominator being removed from $\frac{\sin^2 \theta}{\cos \theta}$ and multiplying at least one other term by $\cos \theta$ (or $\cos^2 \theta$ etc).

M1 Attempts to use $\pm \sin^2 \theta = \pm 1 \pm \cos^2 \theta$ and proceeds to an equation in $\cos \theta$ only or $\sin \theta$ only

A1 $3\cos^2 \theta + 5\cos \theta - 2 (= 0)$. Not all need to be on the same side of the equation and condone the

omission of $= 0$ if all on one side. Condone poor notation for $\cos^2 \theta$ eg $\cos \theta^2$

M1 Solves their 3TQ in $\cos \theta$ and takes inverse cos of one of their roots to obtain at least one value for θ . As a minimum, expect to see the root(s) to their 3TQ before proceeding to an angle which may need to be checked. Condone their angle to be in radians eg typically awrt 1.23 or awrt 5.05

A1 awrt 70.5, awrt 289.5 and no others in the range. Must be in degrees not radians.

Question	Scheme	Marks
7(a)	$2 = \log_3 3^2$ oe seen or implied by working	B1
	$3 \log_3 (2x-1) = \log_3 (2x-1)^3$ or $\log_3 "3^2" + \log_3 (14x-25) = \log_3 ("3^2"(14x-25))$	M1
	$\Rightarrow (2x-1)^3 = 9(14x-25)$	dM1
	$\Rightarrow 8x^3 - 12x^2 + 6x - 1 = 126x - 225$ $\Rightarrow 8x^3 - 12x^2 - 120x + 224 = 0 \Rightarrow 2x^3 - 3x^2 - 30x + 56 = 0^*$	A1*
		(4)
(b)	$2(\pm 4)^3 - 3(\pm 4)^2 - 30(\pm 4) + 56 = \dots$	M1
	$2(-4)^3 - 3(-4)^2 - 30(-4) + 56 = -128 - 48 + 120 + 56 = 0$ Hence -4 is a root of the equation.	A1
		(2)
Alt (b)	$2x^3 - 3x^2 - 30x + 56 = (x \pm 4)(2x^2 + \dots \pm 14)$	M1
	$2x^3 - 3x^2 - 30x + 56 = (x+4)(2x^2 - 11x + 14)$ so -4 is a root of the equation.	A1
		(2)
(c)	$2x^3 - 3x^2 - 30x + 56 = 0 \Rightarrow (x+4)(2x^2 + \dots \pm 14) = 0$	M1
	$(x+4)(2x^2 - 11x + 14) = 0$	A1
	$(x+4)(2x-7)(x-2) = 0 \Rightarrow x = \dots$	dM1
	(Equation not defined for $x = -4$ so) solutions are 2 and $\frac{7}{2}$	A1
		(4)
(10 marks)		

Notes:**(a)****B1:** States or implies $2 = \log_3 3^2$ or equivalent. May be gained via $\log_3 f(x) = 2 \Rightarrow f(x) = 9$ **M1:** Uses a correct log law, either power law on $3 \log_3 (2x-1) = \log_3 (2x-1)^3$ or sum law on $\log_3 "3^2" + \log_3 (14x-25) = \log_3 ("3^2"(14x-25))$ or equivalent work (e.g. difference law if combining the two log terms on one side first).**dM1:** Uses correct log work to remove the logs. Allow slips, but all log work must have been correct.**A1*:** Expands and simplifies to the given cubic. Must see an intermediate step between the factorised sides and final answer.

(b)

M1: Substitutes ± 4 into the cubic and attempts to evaluate it.

A1: Evaluates to 0 with intermediate working shown, and gives conclusion that -4 is a root.

Alt:

M1: Attempts to take a factor of $(x \pm 4)$ out of the equation or uses long division. Look for first term $2x^2$ and last term ± 14 in the quadratic factor.

A1: Correct factorisation with conclusion. If via long division, all work must be correct with zero remainder found.

(c)

M1: Attempts to take a factor of $(x + 4)$ out of the cubic, or attempts long division by $x + 4$

A1: Correct quadratic factor (either by factorisation or by long division)

Note: the first two marks may be awarded for work seen in part (b).

dM1: Depends on first M. Solves the resulting quadratic (any means).

A1: Correct two solutions **only** identified. Withhold if -4 is listed as a solution.

Since the question says “hence, using algebra and showing each step of your working”, solutions derived from a calculator with no working shown score no marks. The intermediate quadratic must be attempted to gain marks in part (c)

Question	Scheme	Marks
8(i)	$3 \sin(\theta + 30^\circ) = 7 \cos(\theta + 30^\circ) \Rightarrow \tan(\theta + 30^\circ) = \frac{7}{3}$	B1
	$\theta + 30^\circ = \arctan\left(\frac{7}{3}\right) (= 66.8^\circ)$	M1
	$\theta = 36.8^\circ$ or 216.8° (awrt)	A1A1
		(4)
(ii)	(a) $3 \sin^3 x = 5 \sin x - 7 \sin x \cos x \Rightarrow 3 \sin x (1 - \cos^2 x) = 5 \sin x - 7 \sin x \cos x$	M1
	$\Rightarrow \sin x (5 - 7 \cos x + 3 \cos^2 x - 3) = 0$	
	$\Rightarrow \sin x (3 \cos^2 x - 7 \cos x + 2) = 0$	A1
	(b) $\sin x = 0 \Rightarrow x = 0$	B1
	$\Rightarrow \sin x (3 \cos x - 1)(\cos x - 2) = 0 \Rightarrow \cos x = \dots$	M1
	$3 \cos x = 1 \Rightarrow x = \arccos\left(\frac{1}{3}\right) = (\pm)1.23$	M1
	Both of $x =$ awrt $-1.23, 1.23$	A1
		(6)

(10 marks)**Notes:****(i)****Note: answers only scores no marks.****B1:** Divides through by $\cos(\theta + 30^\circ)$ to get correct equation in tan.**M1:** Applies arctan to find a value for $\theta + 30^\circ$. If using other approaches it is for correct work to achieve one value for $\theta + 30^\circ$ or just θ as appropriate. (Allow if “radians” are used.)**A1:** One of awrt 36.8° or awrt 216.8° **A1:** Both of awrt 36.8° and awrt 216.8° with no other solutions in the range.**(ii)(a)****Part (ii) may be marked as a whole.****M1:** Uses $\sin^2 x = 1 - \cos^2 x$ in the equation and gathers terms on one side and factors out the $\sin x$ (Allow the M if the $\sin x$ is cancelled) May be seen in (b)**A1:** Achieves suitable form. Accept non-zero multiples of the quadratic in $\cos x$. (Allow if $\sin x$ is cancelled and put back in later). Result must be clearly stated but may be seen in (b).**(b)****B1:** For $x = 0$ as one solution.**M1:** Solves the quadratic in $\cos x$, usual rules (may be implied by correct answers).**M1:** Takes arccos of at least one value that has modulus less than 1 which is a root of their quadratic. Allow if in degrees, but must reach a value.**A1:** Both solutions from cosine equation correct and no others in the range.

	Alternatives to (i)	
8(i) Alt 1	$\sin(\theta + 30^\circ) = \frac{7}{\sqrt{58}}$ or $\cos(\theta + 30^\circ) = \frac{3}{\sqrt{58}}$	B1
	$(3 \sin(\theta + 30^\circ))^2 = 7(\cos(\theta + 30^\circ))^2 \Rightarrow \dots \sin^2(\theta + 30^\circ) = \dots (1 - \sin^2(\theta + 30^\circ))$ $\Rightarrow \sin(\theta + 30^\circ) = \dots \Rightarrow \theta + 30^\circ = \dots$ Or $(3 \sin(\theta + 30^\circ))^2 = 7(\cos(\theta + 30^\circ))^2 \Rightarrow \dots (1 - \cos^2(\theta + 30^\circ)) = \dots \cos^2(\theta + 30^\circ)$ $\Rightarrow \cos(\theta + 30^\circ) = \dots \Rightarrow \theta + 30^\circ = \dots$	M1
	$\theta = 36.8^\circ$ or 216.8° (awrt)	A1A1
		(4)
Alt: B1: For reaching $\sin(\theta + 30^\circ) = \frac{7}{\sqrt{58}}$ or $\cos(\theta + 30^\circ) = \frac{3}{\sqrt{58}}$ M1: For a complete method of squaring both sides, applying $\sin^2(\dots) + \cos^2(\dots) = 1$ to achieve an equation in just sin or cos and then applying the inverse function to their solution to achieve one value for $\theta + 30^\circ$ or just θ as appropriate. . (Allow if “radians” are used.) A1A1: As above, must have rejected any extra solutions from squaring.		
8(i) Alt 2	$3 \sin(\theta + 30^\circ) = 7 \cos(\theta + 30^\circ) = \frac{3\sqrt{3}}{2} \sin \theta + \frac{3}{2} \cos \theta = \frac{7\sqrt{3}}{2} \cos \theta - \frac{7}{2} \sin \theta$ oe	B1
	$3 \sin(\theta + 30^\circ) = 7 \cos(\theta + 30^\circ) \Rightarrow$ $3(\sin \theta \cos 30^\circ \pm \cos \theta \sin 30^\circ) = 7(\cos \theta \sin 30^\circ \pm \sin \theta \cos 30^\circ)$ $\Rightarrow \frac{\sin \theta}{\cos \theta} = \tan \theta = \dots \Rightarrow \theta = \arctan(\dots) = \dots$	M1
	$\theta = 36.8^\circ$ or 216.8° (awrt)	A1A1
		(4)
Alt 2: B1: Correct application of compound angle formula and trig ratios to reach $\frac{3\sqrt{3}}{2} \sin \theta + \frac{3}{2} \cos \theta = \frac{7\sqrt{3}}{2} \cos \theta - \frac{7}{2} \sin \theta$ oe M1: Attempts compound angle formula (allow sign slips), rearranges to make $\tan \theta$ the subject and applies arctan to find at least one value for θ . (Allow if “radians” are used.) A1A1: As main scheme.		

Question Number	Scheme	Marks
10 (i)	$\tan^2\left(2x + \frac{\pi}{4}\right) = 3 \Rightarrow 2x + \frac{\pi}{4} = \pm \frac{\pi}{3}, \pm \frac{2\pi}{3}$ $\Rightarrow x = \left(\dots - \frac{\pi}{4}\right) \div 2$ $\Rightarrow x = \frac{\pi}{24}, -\frac{11\pi}{24}, \frac{5\pi}{24}, -\frac{7\pi}{24}$	M1 dM1 A1; A1, A1 (5)
(ii)	$(2\sin\theta - \cos\theta)^2 = 1 \Rightarrow 4\sin^2\theta - 4\sin\theta\cos\theta + \cos^2\theta = 1$ Attempts to use $\sin^2\theta + \cos^2\theta = 1 \Rightarrow 4\sin^2\theta - 4\sin\theta\cos\theta = \sin^2\theta$ $\Rightarrow \sin\theta(3\sin\theta - 4\cos\theta) = 0$ $\tan\theta = \frac{4}{3}$ $\theta = 53.1^\circ, 233.1^\circ, \underline{180^\circ}$	M1 dM1 A1 A1, <u>B1</u> (5) (10 marks)

(i)

M1: Correct order of operations. Takes square root followed by arctan.

Implied by $2x + \frac{\pi}{4} = \frac{\pi}{3}$, $2x + \frac{\pi}{4} = 1.047$. Condone for this mark only $2x + \frac{\pi}{4} = 60$.Allow if they use θ for $2x + \frac{\pi}{4}$

A longer method is to do $\frac{\sin^2\left(2x + \frac{\pi}{4}\right)}{\cos^2\left(2x + \frac{\pi}{4}\right)} = 3 \Rightarrow \sin^2\left(2x + \frac{\pi}{4}\right) = 3\cos^2\left(2x + \frac{\pi}{4}\right)$ and

use $\sin^2\theta + \cos^2\theta = 1$ to produce an equation in either sin or cos, before taking the inverse. They must have a correct method up to slips in rearranging, and reach the stage of taking arcsin or arccos in order to score the M.

dM1: Complete attempt to find one value for x .This would involve an attempt to move the $\frac{\pi}{4}$ before dividing by 2. Condone

$$x = \left(1.047 \pm \frac{\pi}{4}\right) \div 2$$

A1: One value of $\frac{\pi}{24}, -\frac{11\pi}{24}$. Condone decimals here awrt 0.13, -1.44A1: One value of $\frac{5\pi}{24}, -\frac{7\pi}{24}$. Condone decimals here awrt 0.65, -0.92A1: $\frac{\pi}{24}, \frac{5\pi}{24}, -\frac{7\pi}{24}, -\frac{11\pi}{24}$ and no other values in the range

(ii)

 M1: Attempts to multiply out to at least three terms, and use $\sin^2 \theta + \cos^2 \theta = 1$ somewhere in the equation.

 dM1: Cancels or factorises out the $\sin \theta$ term to produce a factor $a \sin \theta \pm b \cos \theta$ or an equation of the form $a \sin \theta \pm b \cos \theta = 0$ or (the “=0” may be implied)

A1: $\tan \theta = \frac{4}{3}$

 A1: $\theta = 53.1^\circ, 233.1^\circ$ and no others in the range

 (Note: 0 and 360° are outside the range so ignore if given as solutions.)

 B1: $\theta = 180^\circ$ Award when seen and allow however it arises.

Question Number	Scheme	Marks
10(ii) Alt	$(2 \sin \theta - \cos \theta)^2 = 1 \Rightarrow 2 \sin \theta - \cos \theta = \pm 1$ $\Rightarrow 2 \sin \theta = \cos \theta \pm 1$ $\Rightarrow 4 \sin^2 \theta = \cos^2 \theta \pm 2 \cos \theta + 1$ then attempts to use $\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow 4 - 4 \cos^2 \theta = \cos^2 \theta \pm 2 \cos \theta + 1$ $\Rightarrow 5 \cos^2 \theta \pm 2 \cos \theta - 3 = 0 \Rightarrow \cos \theta = \dots$ $\cos \theta = \frac{3}{5}, -1$ or $\cos \theta = -\frac{3}{5}, 1$ $\theta = 53.1^\circ, 233.1^\circ, 180^\circ$	M1 dM1 A1 A1, B1 (5) (10 marks)

 M1: Attempts at least one of $2 \sin \theta - \cos \theta = \pm 1$, rearranges and squares then attempts to use $\sin^2 \theta + \cos^2 \theta = 1$ to produce an equation in just $\cos \theta$ or just $\sin \theta$. FYI in $\sin \theta$ it is $4 \sin^2 \theta \pm 4 \sin \theta + 1 = 1 - \sin^2 \theta$

 dM1: Solves their quadratic in $\cos \theta$ or $\sin \theta$ to obtain at least one value for $\cos \theta$ or $\sin \theta$

 A1: One correct pair of solutions. FYI in $\sin \theta$ it is $\sin \theta = 0, \frac{4}{5}$ or $\sin \theta = 0, -\frac{4}{5}$

 A1: $\theta = 53.1^\circ, 233.1^\circ$ and no others in the range. (Note via this method some extra solutions in the range are likely)

 B1: $\theta = 180^\circ$ Award when seen and allow however it arises.

It is possible someone could have studied WMA13.

 M1: Attempts to write $2 \sin \theta - \cos \theta$ in the form $R \sin(\theta - \alpha)$ o.e. FYI
 $2 \sin \theta - \cos \theta = \sqrt{5} \sin(\theta - 26.6^\circ)$

 dM1: Proceeds from $R^2 \sin^2(\theta - \alpha) = 1$ to $\sin(\theta - \alpha) = \pm \frac{1}{R}$

$$A1: \sin(\theta - 26.6^\circ) = \pm \frac{1}{\sqrt{5}}$$

$$A1: \theta = 53.1^\circ, 233.1^\circ \text{ and no others in the range}$$

(Note: 0 and 360° are outside the range so ignore if given as solutions.)

$$B1: \theta = 180^\circ \text{ Award when seen and allow however it arises.}$$



n	$x1.02^{(n-1)}$	x5
1	25000	125000
2	25500	127500
3	26010	130050
4	26530	132651
5	27061	135304
6	27602	138010
7	28154	140770
8	28717	143586
9	29291	146457
10	29877	149387
11	30475	152374
12	31084	155422
13	31706	158530
14	32340	161701
	399348	1996742



Question Number	Scheme	Marks
7 (a)	$x^2 + y^2 + 8x - 10y = 29$	
	Attempts $(x+4)^2 + (y-5)^2 \dots = \dots$	M1
	Correct centre $(-4, 5)$	A1
(b)	Exact radius $\sqrt{70}$	A1
	Attempts distance (or distance ²) from $(-4, 5)$ to $(5, -8)$	M1
	Full method. E.g. compares $\sqrt{9^2 + 13^2}$ to $(\sqrt{70} + \sqrt{52})$	dM1
	States that - distance between centres $= \sqrt{250} = 15.81$ - sum of radii $= \sqrt{70} + \sqrt{52} = 15.58$ Concludes as sum of radii < distance between centres, result proven *	A1*
		(3) (6 marks)

(a)

M1: Attempts to complete the square for **both** variables. Look for $(x \pm 4)^2, (y \pm 5)^2 \dots = \dots$ or may be implied by $(\pm 4, \pm 5)$ by inspection from the equation.

A1: $(-4, 5)$

A1: $\sqrt{70}$ which may be scored following $(x \pm 4)^2 + (y \pm 5)^2 = 70$

(b)

M1: Attempts distance (or distance ²) from $(-4, 5)$ to $(5, -8)$ (condone sign errors)

dM1: Full method. E.g. compares their $\sqrt{9^2 + 13^2}$ to their $(\sqrt{70} + \sqrt{52})$

A1*: Requires correct calculation(s), correct reason and minimal conclusion. Must be evidence to support the inequality – e.g. decimal values – not just the inequality stated. May use the difference between sum of radii and distance between centres.

(b) Alt

M1: Attempts to solve $x^2 + y^2 + 8x - 10y = 29$ and $x^2 + y^2 - 10x + 16y = -37$ forming a linear equation (need not be simplified for this mark)

FYI: Linear equation is $18x - 26y = 66$ o.e. if the calculations are correct

dM1: Attempts to solve their $18x - 26y = 66$ simultaneously with either circle equation reaching a 3 term quadratic for either x or y.

A1*: Requires

- correct equations
- correct "solution" stating no roots and concluding no points of intersection and minimal conclusion

E.g. Substitute $y = \frac{9x-33}{13}$ into $x^2 + y^2 + 8x - 10y = 29$ gives $250x^2 - 412x + 478 = 0$

and $"b^2 - 4ac" = -308256 < 0$ so no solutions

FYI the equation in y (if correct) is $250y^2 + 984y + 1116 = 0$



Question Number	Scheme	Marks
3 (a)	$f(x) = (3x^2 - 4x - 5)(x - k) - 5$ States -5	B1 (1)
(b)	Sets $f(-2) = 25 \rightarrow (3 \times 4 - 4 \times -2 - 5)(-2 - k) - 5 = 25$ $15(-2 - k) = 30 \Rightarrow -2 - k = 2 \Rightarrow k = -4$ *	M1 A1* (2)
(c)	$(3x^2 - 4x - 5)(x + 4) - 5 = 3x^3 + 8x^2 - 21x - 25$ Attempts $3x - 1 \overline{) 3x^3 + 8x^2 - 21x - 25}$ to achieve quotient of $\dots x^2 + \dots x + \dots$ and a remainder that is a constant Quotient = $x^2 + 3x - 6$ OR Remainder = -31 Quotient = $x^2 + 3x - 6$ AND Remainder = -31	B1 M1 A1ft A1 (4) (7 marks)

(a)

 B1: States -5

 You may see $f(k)$ or a division sum attempted, which is fine. You are just awarding the sight of -5

 Don't be too concerned about what -5 is called. You may see $f(x)$, $f(k)$ or even x

(b)

 M1: Sets $f(-2) = 25$ to form an equation in k Condone sign slips within the calculations but setting

 $f(-2) = -25$ or $f(2) = 25$ is M0. Some may attempt to substitute -2 into the expanded form of

 $f(x) = 3x^3 - (4 + 3k)x^2 + (4k - 5)x + 5k - 5$ which is fine. Condone slips but expect to see linear coefficients for k in each of x^2 , x and the constant terms.

 Don't be concerned about poor attempts to solve the equation in k for this mark

 A1*: Shows sufficient working with no errors to prove that $k = -4$.

The answer must follow a simplified and (easily) solvable equation.

 (See scheme for possible route but allow simplified linear equations like $-30 - 15k = 30$)

 M1: Alternatively substitutes $k = -4$ into $f(x)$ to get $(3x^2 - 4x - 5)(x + 4) - 5$ and attempt the value of $f(-2)$

 A1: Shows that $f(-2) = (12 + 8 - 5)(-2 + 4) - 5 = 15 \times 2 - 5 = 25$ and makes a minimal conclusion e.g. ✓

(c)

B1: Multiplies out $(3x^2 - 4x - 5)(x + 4) - 5$ to achieve a simplified $3x^3 + 8x^2 - 21x - 25$

M1: Scored for a full method of finding **both** the quotient and the remainder.

The two main ways of scoring this mark are;

Either attempting to divide their expanded $f(x)$ by $(3x - 1)$ to form a quotient of the form

$Ax^2 + B.x + C$ and a remainder that is a constant.

Alternatively, setting up an identity of the form $(Ax^2 + B.x + C)(3x - 1) + R$ followed by an attempt to find the values of A , B , C and R .

Watch for candidates who find $f\left(\frac{1}{3}\right) = -31$ and then adapt $f(x)$ to $3x^3 + 8x^2 - 21x + 6$. The M mark

would be scored for dividing their $3x^3 + 8x^2 - 21x + 6$ by $(3x - 1)$ to form a quotient of the form

$Ax^2 + B.x + C$ and a remainder that is a constant (which should be 0 in a correct division).

A1ft: For either the quotient = $x^2 + 3x - 6$. Allow this to be scored within the division sum

Or the remainder = -31 (may be scored within the division sum)

It must follow M1 so attempting $f\left(\frac{1}{3}\right)$ and reaching -31 will not score this mark without further

work

Follow through on finding a correct quotient or remainder for their $3x^3 + 8x^2 - 21x - 25$ (4 term cubic expression). Note that any $3x^3 + 8x^2 - 21x + k$ will have a quotient of $x^2 + 3x - 6$

A1: States that the Quotient = $x^2 + 3x - 6$ AND the Remainder = -31 . These cannot be scored within a division sum and must be stated separately to this. All previous marks must have been scored in (c) so cannot be awarded following an incorrect $f(x)$

Question Number	Scheme	Marks
4. (a)	$\log_3(a+1) - \log_3 a = 4 \Rightarrow \log_3\left(\frac{a+1}{a}\right) = 4$ $\Rightarrow \left(\frac{a+1}{a}\right) = 3^4$ $\Rightarrow a+1 = 81a \Rightarrow a = \frac{1}{80}$	M1 A1 dM1, A1 (4)
(b)	$\text{Area} \approx \frac{1}{2} \left\{ \log_3\left(\frac{2}{1}\right) + \log_3\left(\frac{6}{5}\right) + 2 \left(\log_3\left(\frac{3}{2}\right) + \log_3\left(\frac{4}{3}\right) + \log_3\left(\frac{5}{4}\right) \right) \right\}$ $= \frac{1}{2} \log_3\left(\frac{2}{1} \times \frac{6}{5} \times \frac{3^2}{2^2} \times \frac{4^2}{3^2} \times \frac{5^2}{4^2}\right) = \frac{1}{2} \log_3 15 = \log_3 \sqrt{15}$	M1, A1 dM1, A1 (4)
(c)	States 'increase the number of strips'	B1 (1) (9 marks)

(a) Allow with $a \leftrightarrow x$ as seen in the notes

M1: Sets $\log_3(x+1) - \log_3 x = 4$ and then applies the subtraction law. Condone the omission of the 3 in $\log_3$

Alternatively writes the equation $\log_3(x+1) = 4 + \log_3 x$ as $\log_3(x+1) = \log_3 k + \log_3 x$ ($k \neq 4$)

and then uses the addition law $\log_3(x+1) = \log_3(kx)$

A1: Correct equation not involving logs. For example, $\left(\frac{x+1}{x}\right) = 3^4$

dM1: Correct attempt to solve an equation of the form $\frac{x+1}{x} = k$ o.e. where k is an integer $k \neq 3$ or 4.

Look for $\frac{x+1}{x} = k \Rightarrow x = \frac{1}{k-1}$ where $k \neq 3$.

A1: a or $x = \frac{1}{80}$ or 0.0125 following the award of M1, A1, dM1.

Working must be seen as required by the question.

Special Case:

You may see attempts that proceed towards the correct answer using incorrect laws.

$$\text{E.g. } \log_3(x+1) - \log_3 x = 4 \Rightarrow \frac{\log_3(x+1)}{\log_3 x} = 4 \Rightarrow \left(\frac{x+1}{x}\right) = 3^4 \Rightarrow x = \frac{1}{80}$$

Candidates who achieve the correct answer using the above incorrect line can score SC 0, 0, 1, 1

(b)

Condone the omission of the 3 in $\log_3$ except for the final A mark where they must arrive at $\log_3 \sqrt{15}$.

Any use of calculators to produce decimal equivalents to these values will result in a maximum score of 1,0,0,0 marks as an exact value could never be achieved.

M1: Score for a reasonable attempt at applying the trapezium rule.

Look for **an attempt** at the correct form $\frac{h}{2} \{ \text{first term} + \text{last term} + 2(\text{sum of the other terms}) \}$ with at least the following

- (1) Correct first **or** last term
- (2) At least two correct 'other terms'
- (3) $h=1$

The bracketing must be correct but condone a missing trailing bracket.

So, look for a minimum of $\frac{1}{2} \{ a + e + 2 \times (b + c + d) \}$ with a or e correct, two out of three of b , c and d correct with the missing trailing bracket

FYI the decimal solution is $\frac{1}{2} \{ 0.631 + 0.166 + 2(0.369 + 0.262 + 0.203) \}$ allow awrt 2 dp

A1: A fully correct application of the trapezium rule in un-simplified form.

Amongst various correct alternatives are

$$\text{Area} \approx \frac{1}{2} \left\{ \log_3 \left(\frac{2}{1} \right) + \log_3 \left(\frac{6}{5} \right) + 2 \left(\log_3 \left(\frac{3}{2} \right) + \log_3 \left(\frac{4}{3} \right) + \log_3 \left(\frac{5}{4} \right) \right) \right\}$$

$$\text{Area} \approx \frac{1}{2} \{ \log_3 2 + \log_3 6 - \log_3 5 + 2(\log_3 3 - \log_3 2 + \log_3 4 - \log_3 3 + \log_3 5 - \log_3 4) \}$$

$$\text{Area} \approx \frac{1}{2} \{ \log_3 2 + \log_3 6 - \log_3 5 + 2(-\log_3 2 + \log_3 5) \}$$

$$\text{Area} \approx \frac{1}{2} \left\{ \log_3 2 + \log_3 \frac{3}{2} \right\} + \frac{1}{2} \left\{ \log_3 \frac{3}{2} + \log_3 \frac{4}{3} \right\} + \frac{1}{2} \left\{ \log_3 \frac{4}{3} + \log_3 \frac{5}{4} \right\} + \frac{1}{2} \left\{ \log_3 \frac{5}{4} + \log_3 \frac{6}{5} \right\}$$

FYI: A fully simplified expression for the approximate area is $\frac{1}{2} \{ \log_3 6 + \log_3 5 - \log_3 2 \}$

Condone the omission of the trailing bracket as long as subsequent work implies its presence.

dM1: Correct use of logs in an attempt to fully combine their terms following the award of the previous M.

This must include sight of at least one correct use of two of the three log laws.

So, use of just addition and subtraction laws is fine as would addition and power laws

This mark can be implied if they go directly from a correct exact expression to the correct answer.

Candidates must proceed as far as $\frac{1}{2} \log p$ or $\log q$

A1: $\log_3 \sqrt{15}$ following M1, A1 dM1

(c)

B1: States

- 'increase the number of strips'
- 'decrease the width of the strips'
- 'make more strips'

without any incorrect statements such as increase the strips, so try $\int_1^{20} \log_3 (x+1) - \log_3 x \, dx$

Do not allow vague statements such as 'add more numbers of x '

Question Number	Scheme	Marks
5. (a)	$u_2 = 1 - \frac{1}{4} = \frac{3}{4}, \quad u_3 = 1 - \frac{4}{3} = -\frac{1}{3},$ $u_4 = 1 - \left(-\frac{3}{1}\right) = 4 \text{ with statement saying the sequence repeats every 3 terms o.e}$	M1 A1 A1* (3)
(b)	$\sum_{n=1}^{180} (5n+3) = \frac{180}{2} (8+903) = (81990)$ $\sum_{n=1}^{180} (u_n) = 60 \times \left(4 + \frac{3}{4} + -\frac{1}{3}\right) = (265)$ $\sum_{n=1}^{180} (5n+3+u_n) = 81990 + 265 = 82255$	M1 M1 A1, A1 (4) (7 marks)

(a)

M1: Attempts to use the iterative sequence correctly at least once.

Most often this is for $u_2 = 1 - \frac{1}{4}$ or $u_2 = \frac{3}{4}$ o.e.

Condone omission of the lhs as long as it is obvious. E.g. 4, 0.75

A1: Achieves a correct value for both u_2 and u_3 . Condone $u_3 = -0.\dot{3}$ Do not allow $u_3 = -0.33$ unless you see the exact value beforehand.Allow (4), $\frac{3}{4}$, $-\frac{1}{3}$ if it is obviously values for both u_2 and u_3 .

A1*: This is a show that question and requires

- correct values for u_2 and u_3
- a correct calculation and value seen for u_4 E.g. $u_4 = 1 - \left(-\frac{3}{1}\right) = 4$
- a minimal reason/statement 'the sequence repeats every 3 terms' o.e such as 'as $u_4 = u_1$,
- a minimal conclusion such as sequence is periodic of order 3' or 'hence true'

(b) **All stages of working must be shown so don't accept just answers as evidence of method**

M1: Establishes a viable method to find $\sum_{n=1}^{180} (5n+3)$ using the fact that it is an AP.

Look for use of $\frac{n}{2}\{a+l\}$ or $\frac{n}{2}\{2a+(n-1)d\}$ o.e. with $n=180, a=3, 5$ or 8 and $d=5$

Alternatively attempts $\sum_{n=1}^{180} (5n+3) = \sum_{n=1}^{180} 5n + \sum_{n=1}^{180} 3 = 5 \times \frac{180}{2} \{180+1\} + 180 \times 3$ o.e.

Do not just accept $\sum_{n=1}^{180} (5n+3) = 81990$ without sight of an intermediate calculation

This may be awarded from terms within a more complicated expression to complete $\sum_{n=1}^{180} (5n+3+u_n)$

M1: Establishes a viable method to find $\sum_{n=1}^{180} (u_n)$.

For example, as it repeats every 3 terms look for $60 \times (4+u_2+u_3)$ o.e such as
 $60 \times 4 + 60 \times u_2 + 60 \times u_3$

Do not accept $\sum_{n=1}^{180} (u_n) = 265$ without sight of an intermediate calculation

This may be awarded from terms within a more complicated expression to complete $\sum_{n=1}^{180} (5n+3+u_n)$

A1: Scored for either $\sum_{n=1}^{180} (5n+3) = 81990$ or $\sum_{n=1}^{180} (u_n) = 265$ following the award of the equivalent M mark

It can be implied, so for instance 82255 following the award of both M's but it is the value, not the calculation that must be seen or implied.

A less obvious award would be for $\sum_{n=1}^{180} (u_n+3) = 60 \times \left(4 + \frac{3}{4} + -\frac{1}{3} + 9\right) = 805$ which implies the 265

A1: 82255 which must follow the award of M1, M1 and A1



Question Number	Scheme	Marks
6 (i) (a)	Uses $\sin^2 \theta + \cos^2 \theta = 1$ with $\cos \theta = \frac{1}{\sqrt{5}} \Rightarrow \sin^2 \theta = \frac{4}{5}$ $\Rightarrow \sin \theta = -\frac{2}{\sqrt{5}}$	M1 A1
(b)	Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ with $\cos \theta = \frac{1}{\sqrt{5}}$ and their $\sin \theta = -\frac{2}{\sqrt{5}}$ $\Rightarrow \tan \theta = -2$	M1 A1
(ii) (a)	1 (m)	B1 (4) (1)
(b)	$30t - 40 = 180 \Rightarrow t = 7.33$ Hence 7:20 am	M1, A1, A1 (3)
(c)	$4 + 3 \cos(30T - 40)^\circ = 3.5 \Rightarrow \cos(30T - 40)^\circ = -\frac{1}{6}$ $\Rightarrow 30T - 40 = 99.6, 260.4, \underline{459.6}$ $\Rightarrow T = \frac{"459.6" + 40}{30} = 16.65$	M1 A1 dM1, A1 (4) (12 marks)

(i)

Clear use of trigonometric identities must be shown. Answers without method don't score marks

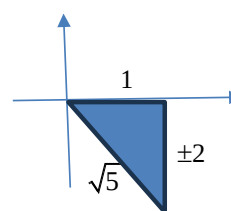
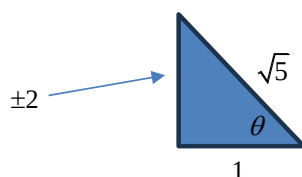
Look for work within the body of the script which may be common

(a)

M1: Uses $\sin^2 \theta + \cos^2 \theta = 1$ with $\cos \theta = \frac{1}{\sqrt{5}} \Rightarrow \sin^2 \theta + \frac{1}{5} = 1$ o.e.

Alternatively shows a 'triangle' method and uses Pythagoras' theorem to reach $\sin \theta = \pm \frac{2}{\sqrt{5}}$

E.g.



A1: $\sin \theta = -\frac{2}{\sqrt{5}}$ or simplified equivalent such as $-\frac{2\sqrt{5}}{5}$ following the award of the M1. ISW after a

correct answer. Note that $\sin \theta = -\sqrt{\frac{4}{5}}$ is A0 as it is not simplified

(b)

M1: Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ with $\cos \theta = \frac{1}{\sqrt{5}}$ and either $\sin \theta = \frac{2}{\sqrt{5}}$ or $\sin \theta = -\frac{2}{\sqrt{5}}$ from (a) which must have

been awarded from a correct method.

Alternatively, from a triangle method, shows the opposite side = 2 in a right-angled triangle and takes the correct ratio

A1: $\tan \theta = -2$ following the award of the M1 in both (i) and (ii)

(ii)(a)

B1: 1 (m) Units are not necessary here but incorrect units e.g. 1 cm would be B0

(ii)(b)

M1: Sets $30t - 40$ equal to any value in degrees where the cosine is -1

So $30t - 40 = 180(2n+1)$, for any $n \in \mathbb{Z}$ is acceptable. Condone $30t - 40 = (2n+1)\pi$,

Condone the result obtained from differentiation with $\sin(30t - 40) = 0 \Rightarrow 30t - 40 = 180(2n+1)$

A1: $30t - 40 = 180 \Rightarrow t = 7\frac{1}{3}$ or 7.33 but condone 7.3. It must follow the award of the M mark

A1: Achieves 7:20 am or 07:20 or any other acceptable way of establishing the exact **time of day** time in hours and minutes. So 7 hours 20 minutes after midnight is fine. **Condone 7:20 but 7:20pm is A0 and 7 hours 20 minutes without reference to after midnight is A0. It must follow correct working and cannot be rounded from a different time value such as 7:19.5**

(ii)(c) Radian attempts can only usually score the first M1.

M1: Sets $4 + 3\cos(30T - 40)^\circ$ equal to 3.5 and proceeds to $\cos(30T - 40)^\circ = k$, $|k| < 1$

A1: Achieves $30T - 40$ equal to any of 99.6, 260.4, 459.6

dM1: Proceeds from $30T - 40$ equal to any of awrt 100, 260, 460 AND uses correct order of operations and working leading to a value for T .

For example, allow for $30T - 40 = \text{awrt } 100 \Rightarrow T = \frac{140}{30}$ or awrt 4.7

$30T - 40 = \text{awrt } 260 \Rightarrow T = \text{awrt } 10$

$30T - 40 = \text{awrt } 460 \Rightarrow T = \frac{500}{30}$ or awrt 16.7

A1: awrt 16.65 following the award of M1, A1 dM1

ISW after a correct answer for T . Ignore any attempts to convert the T value to a time.

A clear method must be shown here, so solutions produced entirely from a calculator will not score.

E.g. $4 + 3\cos(30T - 40)^\circ = 3.5 \Rightarrow T = 16.65$ is 0 marks

A minimal acceptable response for 4 marks would be

$$4 + 3\cos(30T - 40)^\circ = 3.5 \Rightarrow \cos(30T - 40)^\circ = -\frac{1}{6}$$

$$\Rightarrow 30T - 40 = 459.6$$

$$\Rightarrow T = 16.65$$



(b)

M1: Attempts to use the GP formula in an attempt to find 'r'

Look for $4000 = 300r^{11} \Rightarrow r^{11} = \frac{4000}{300} \Rightarrow r = \sqrt[11]{\frac{4000}{300}}$ condoning slips. Implied by $r = \text{awrt } 1.3$ provided they do not start from an incorrect equation.

M1: A correct attempt to find the **second term** (do not allow a misread) by multiplying 300 by their "1.266..." . It is dependent on an attempt to find r from either $4000 = 300r^{11}$ or $4000 = 300r^{12}$ condoning slips. If no method is seen to find r then it must be correct. e.g. following $4000 = 300r^{12}$ or $\sqrt[12]{\frac{4000}{300}}$.

Condone slips.

A1: awrt 380 (to the nearest 10) (tonnes) isw once a correct answer is seen.

(c) **Condone use of their d and their r even if they have come from incorrect methods**

M1: A correct method to find the sum of either the AP or the GP

For the AP accept an attempt at either $\frac{12}{2}\{300 + 4000\}$ or $\frac{12}{2}\{2 \times 300 + 11 \times "336"\}$

For the GP accept an attempt at either $\frac{300("1.266..."^{12} - 1)}{"1.266..." - 1}$ or $\frac{300(1 - "1.266..."^{12})}{1 - "1.266..."}$

dM1: Both formulae must be attempted "correctly" (see above) and the difference taken

FYI if d and r are correct, the sums are 25 800 tonnes and 17 935A1: Difference = 7860 or 7870 (tonnes) **This is NOT AWRT.**(Note that use of $r = 1.266$ will lead to a difference of 7810 which is max M1dM1A0)**Alt (c) Listing values**

Accept listing the values

M1: Attempts to add the 12 terms for the arithmetic or geometric series. Must include 300 and 4000, but condone one omission of one of the middle terms. May be implied by calculating the difference between the AP and GP values for each year and adding those together.

dM1: Attempts to add the 12 terms for the arithmetic and geometric series. Must include 300 and 4000 for both sums, but condone one omission of one of the middle terms in each of the summations. May be implied.

A1: Difference = 7860 or 7870 **This is NOT AWRT.**

Year	AP	GP	Difference
1	300	300	0
2	636.3636	379.6544	256.7093
3	972.7273	480.4582	492.2691
4	1309.091	608.0268	701.0641
5	1645.455	769.4668	875.9877
6	1981.818	973.7715	1008.047
7	2318.182	1232.322	1085.86
8	2654.545	1559.521	1095.024
9	2990.909	1973.597	1017.312
10	3327.273	2497.616	829.6567
11	3663.636	3160.77	502.8668
12	4000	4000	0
Sum	25800	17935.2	7864.796

