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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Differentiation: Stationary Points & Optimisation (P2.7)

International A Level Mathematics · P2: Pure Mathematics 2 (WMA12)

14 questions · 118 marks · Mark schemes included

Past papers June 2019 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

14 questions · 118 marks · 2 sub-topics

SUB-TOPIC P2.7a

STATIONARY POINTS & THEIR NATURE

Sub-topic P2.7a · 9 questions · 70 marks · lowest tariff first



(a) Find, using calculus, the x coordinates of the stationary points of C .

(4)

(2)



Question 1 continued

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Lined area for writing the answer to Question 1.

(Total 6 marks)

Q1

Mark box for Q1.



$$y = ax^3 - 3x^2 + 3x + b$$

$$y = ax^3 - 3x^2 + 3x + b$$

Given that

- the point $(2, 5)$ lies on C
- the gradient of the curve at $(2, 5)$ is 7

(4)

(3)

Question 10 continued

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Question 10 continued

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Q10

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

END



2. A curve has equation

$$y = x^3 - x^2 - 16x + 2$$

(a) Using calculus, find the x coordinates of the stationary points of the curve.

(4)

(b) Justify, by further calculus, the nature of all of the stationary points of the curve.

(3)

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Question 2 continued

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(Total 7 marks)

Q2



Solutions relying entirely on calculator technology are not acceptable.

$$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3 \quad x > 0$$


Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 7 marks)



Question 2 continued

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Handwriting practice area with horizontal lines.

(Total 8 marks)

Q2

Answer box for Q2.



Solutions relying entirely on calculator technology are not acceptable.

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \quad x \neq 0$$

(3)

(5)



Question 8 continued

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Handwriting practice area with horizontal lines.

(Total 8 marks)

Q8



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 8 marks)



7. The curve C has equation

$$y = \frac{12x^3(x-7) + 14x(13x-15)}{21\sqrt{x}} \quad x > 0$$

(a) Write the equation of C in the form

$$y = ax^{\frac{7}{2}} + bx^{\frac{5}{2}} + cx^{\frac{3}{2}} + dx^{\frac{1}{2}}$$

where a , b , c and d are fully simplified constants.

(3)

The curve C has three turning points.

Using calculus,

(b) show that the x coordinates of the three turning points satisfy the equation

$$2x^3 - 10x^2 + 13x - 5 = 0$$

(3)

Given that the x coordinate of one of the turning points is 1

(c) find, using algebra, the exact x coordinates of the other two turning points.

(Solutions based entirely on calculator technology are not acceptable.)

(3)

Question 7 continued

Lined area for writing the answer to Question 7 continued.





Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 9 marks)



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$$y = 4x^3 - 9x + \frac{k}{x} \quad x > 0$$

where k is a constant.

The point P with x coordinate $\frac{1}{2}$ lies on C .

Given that P is a stationary point of C ,

- (a) show that $k = -\frac{3}{2}$ (4)

- (b) Determine the nature of the stationary point at P , justifying your answer. (2)

The curve C has a second stationary point.

- (c) Using algebra, find the x coordinate of this second stationary point. (4)

Question 10 continued

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Question 10 continued

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Handwriting area with horizontal lines.

Q10

(Total 10 marks)

END

TOTAL FOR PAPER IS 75 MARKS



SUB-TOPIC P2.7b

OPTIMISATION PROBLEMS

Sub-topic P2.7b · 5 questions · 48 marks · lowest tariff first



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5. A company makes a particular type of watch.

The annual profit made by the company from sales of these watches is modelled by the equation

$$P = 12x - x^{\frac{3}{2}} - 120$$

where P is the annual profit measured in thousands of pounds and $\pounds x$ is the selling price of the watch.

According to this model,

- (a) find, using calculus, the maximum possible annual profit. (6)
- (b) Justify, also using calculus, that the profit you have found is a maximum. (2)

Question 5 continued

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Lined area for writing the answer to Question 5.

(Total 8 marks)

Q5



2.

In this question you must show all stages of your working.

Solutions based entirely on calculator technology are not acceptable.

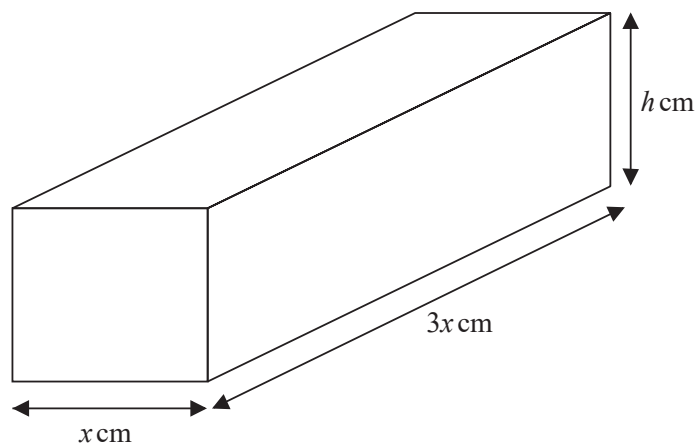


Figure 2

A brick is in the shape of a cuboid with width x cm, length $3x$ cm and height h cm, as shown in Figure 2.

The volume of the brick is 972 cm^3

(a) Show that the surface area of the brick, $S \text{ cm}^2$, is given by

$$S = 6x^2 + \frac{2592}{x} \quad (3)$$

(b) Find $\frac{dS}{dx}$ (1)

(c) Hence find the value of x for which S is stationary. (2)

(d) Find $\frac{d^2S}{dx^2}$ and hence show that the value of x found in part (c) gives the minimum value of S . (2)

(e) Hence find the minimum surface area of the brick. (1)





(Total for Question 2 is 9 marks)



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Question 9 continued

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Lined area for writing the answer to Question 9.

(Total 10 marks)

END

TOTAL FOR PAPER IS 75 MARKS

Q9



3.

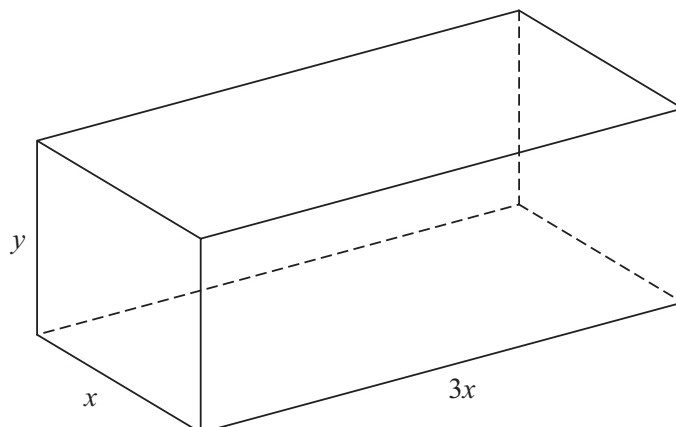


Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is 120m^3

(a) show that the area $A\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

(4)

(c) Find $\frac{d^2A}{dx^2}$ and hence show that the value of x found in part (b) gives the minimum value of A .

(2)





(Total for Question 3 is 10 marks)



9.

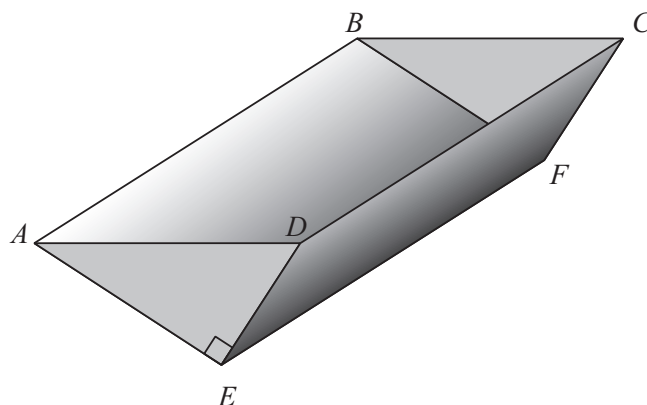


Figure 2

Figure 2 shows a sketch of an open container.

The sides $CDEF$ and $ABFE$ are rectangles.

The ends ADE and BCF are congruent (identical) right-angled triangles.

The container is made from metal of negligible thickness.

Given that

- $AE = BF = 3x$ metres
- $DE = CF = 2x$ metres
- $AB = DC = EF = L$ metres

and the capacity of the container is 12 m^3

(a) show that the area of metal used to make the container, $S\text{ m}^2$, is given by

$$S = Px^2 + \frac{Q}{x}$$

where P and Q are positive integers to be found.

(4)

Given that x can vary,

(b) use algebraic calculus to find the minimum value of S , giving your answer to one decimal place.

(5)

(c) Justify that the value of S found in part (b) is a minimum.

(2)

Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 11 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



n	$x1.02^{(n-1)}$	x5
1	25000	125000
2	25500	127500
3	26010	130050
4	26530	132651
5	27061	135304
6	27602	138010
7	28154	140770
8	28717	143586
9	29291	146457
10	29877	149387
11	30475	152374
12	31084	155422
13	31706	158530
14	32340	161701
	399348	1996742



Question Number	Scheme	Marks
3. (a)	$\left(1 + \frac{x}{4}\right)^{12} = 1 + 12\left(\frac{x}{4}\right)^1 + \frac{12 \times 11}{2}\left(\frac{x}{4}\right)^2 + \frac{12 \times 11 \times 10}{3!}\left(\frac{x}{4}\right)^3 + \dots$ $= 1 + 3x + \frac{33}{8}x^2 + \frac{55}{16}x^3$	M1 B1, A1 (3)
(b)	$\left(\frac{x^2 + 8}{x^5}\right)\left(1 + \frac{x}{4}\right)^{12} =$ Sight of a term independent of $x = \frac{55}{16}$ or $8 \times {}^{12}C_5 \left(\frac{1}{4}\right)^5 (= \frac{99}{16})$ $\frac{55}{16} + 8 \times {}^{12}C_5 \left(\frac{1}{4}\right)^5 = \frac{55}{16} + \frac{99}{16} = \frac{77}{8}$	B1ft, M1 A1 (3) (6 marks)

(a)

M1 For an attempt at the binomial expansion. Score for a correct attempt at term 3 or 4.

Accept sight of ${}^{12}C_2 \left(\frac{x}{4}\right)^2$ or ${}^{12}C_3 \left(\frac{x}{4}\right)^3$ condoning the omission of brackets. Accept any relevant coefficient appearing from Pascal's triangle. FYI ${}^{12}C_2 = 66$, ${}^{12}C_3 = 220$

B1 For $1 + 3x$ Must be simplified. Also allow $1, 3x$ if written as a list.A1 For $+\frac{33}{8}x^2 + \frac{55}{16}x^3$ Accept $+4.125x^2 + 3.4375x^3$

(All four terms do not need to be written as a summation and may be written as a list).

(b)

B1ft Sight of $\frac{55}{16}$ or $8 \times {}^{12}C_5 \left(\frac{1}{4}\right)^5 (= \frac{99}{16})$ which must be independent of x

Most candidates do not find the additional term in the expansion of $\left(1 + \frac{x}{4}\right)^{12}$ so typically would only score a maximum of 100 in (b)

M1 For attempting to add their $\frac{55}{16}$ to $8 \times {}^{12}C_5 \left(\frac{1}{4}\right)^5$ or $8 \times 792 \left(\frac{1}{4}\right)^5$ A1 For $\frac{77}{8}$ oe eg 9.625

Question Number	Scheme	Marks
4 (a)	-35	B1 (1)
(b)	Attempts $f\left(\pm\frac{2}{3}\right) = 0 \rightarrow \left(\frac{2}{3}-3\right)\left(3\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right) + a\right) - 35 = 0$ $-\frac{7}{3}(2+a) = 35 \Rightarrow a = -17$ *	M1 A1* (2)
(c)	$f(x) = (x-3)(3x^2 + x - 17) - 35 = 3x^3 - 8x^2 - 20x + 16$ $= (3x-2)(x^2 - 2x - 8)$ $= (3x-2)(x+2)(x-4)$	B1 M1 A1 M1 A1 (5) (8 marks)

(a)

B1 -35 stated

(b)

M1 Attempts to set $f\left(\pm\frac{2}{3}\right) = 0$. Must see $\pm\frac{2}{3}$ substituted into the expression and set equal to zero.

Condone bracket errors for this mark. They may multiply out the expression before substituting so allow errors in the manipulation for this mark. (FYI $3x^3 + x^2 + ax - 9x^2 - 3x - 3a$)
 “=0” may be implied by their working.

A1* Achieves the given answer with no incorrect work (including brackets) and at least one correct intermediate line before proceeding to $a = -17$. “=0” must be seen somewhere for this mark.

Eg. $-\frac{7}{3}(2+a) = 35 \Rightarrow a = -17$

.....
 Alt (b)

M1 Assumes $a = -17$ and attempts $f\left(\pm\frac{2}{3}\right) = 0$ where $f(x) = (x-3)(3x^2 + x - 17) - 35$

A1* Achieves $f\left(\pm\frac{2}{3}\right) = 0$ with no incorrect work, at least one correct intermediate line and states hence proven or some other acknowledgement of the “proof”.



(c)

B1 Multiplies out to achieve $f(x) = 3x^3 - 8x^2 - 20x + 16$. This may be implied by eg $(3x-2)(x^2-2x-8)$ M1 Attempts to divide or factorise out $(3x-2)$ from their cubic.By factorisation look for $3x^3 \pm \dots x^2 \pm \dots x \pm 2\alpha = (3x-2)(x^2 + kx \pm \alpha)$ $k \neq 0$ or they may equate coefficients correctly to find k .

$$\text{By division look for } \begin{array}{r} x^2 \pm \frac{(\beta+2)}{3}x \dots\dots\dots \\ 3x-2 \overline{) 3x^3 \pm \beta x^2 \dots\dots\dots} \\ \underline{3x^3 - 2x^2} \end{array} \quad (\text{Usually } x^2 \pm 6x \dots)$$

There are various other methods but we should be seeing a correct method to find at least two of the coefficients (allow $\pm$) of their quadratic for the method mark. Send to review if unsure how to mark a response.

A1 Correct factors $(3x-2)(x^2-2x-8)$ The two factors do not need to be written together eg this mark can be awarded if the quadratic factor is seen at the top of their algebraic division.M1 Factorises their quadratic factor of the form $Ax^2 + Bx + C$ with $A, B, C \neq 0$. Score for $(Ax^2 + Bx + C) = (dx + e)(fx + g)$ where $|A| = |d \times f|$ or $|C| = |e \times g|$ A1 $(3x-2)(x+2)(x-4)$ on one line following B1M1A1M1

$$\text{isw } (3x-2)(x+2)(x-4) \Rightarrow x = \frac{2}{3}, -2, 4 \quad \text{and allow } 3(x - \frac{2}{3})(x+2)(x-4)$$

Note the question says using algebra and showing each step.

$$f(x) = 3x^3 - 8x^2 - 20x + 16 \Rightarrow (3x-2)(x+2)(x-4) \text{ is 10000.}$$

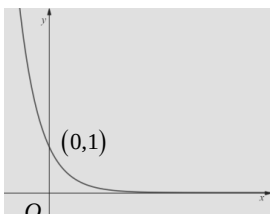
$$\text{Beware } f(x) = 3x^3 - 8x^2 - 20x + 16 = 0 \Rightarrow x = \frac{2}{3}, -2, 4 \Rightarrow (3x-2)(x+2)(x-4) \text{ is 10000}$$

$$f(x) = 3x^3 - 8x^2 - 20x + 16 = 0 \Rightarrow (x - \frac{2}{3})(x+2)(x-4) \text{ is 10000}$$

$$\text{isw } (3x-2)(x+2)(x-4) \Rightarrow x = \frac{2}{3}, -2, 4$$

$$\text{isw } 3(x - \frac{2}{3})(x+2)(x-4) \Rightarrow (x - \frac{2}{3})(x+2)(x-4)$$



Question Number	Scheme	Marks
5.(a)		Shape or intercept at 1
		Fully correct
		$h = 0.5$
(b)	$\text{Area} \approx \frac{0.5}{2} \{4.25 + 16.06 + 2 \times (6.427 + 9.125 + 12.34)\}$ $= \text{awrt } 19.0$	B1
		M1
(c)	$\int_2^4 \left(x(x-3) + \left(\frac{1}{2}\right)^x \right) dx = \int_2^4 \left(x^2 + \left(\frac{1}{2}\right)^x - 3x \right) dx = (b) - \left[\frac{3}{2} x^2 \right]_2^4$ $= \text{awrt } 1.0$	A1
		(7 marks)

(a)

M1 For either the shape in quadrants 1 and 2 only or the y-intercept at 1. Condone slips of the pen but the gradient should be tending to 0 as x increases. Condone (1,0) marked on the correct axis.

A1 Fully correct. Shape in quadrants 1 and 2 only with the y-intercept at 1. Condone slips of the pen towards the x-axis but the graph should not cross the x-axis. Condone (1,0) marked on the correct axis. For a fully correct graph its asymptote should not appear to be $y = 1$. As a rule of thumb look for the asymptote to be at least half way below its y-intercept.

Do not be too concerned about relative position of the curve in the second quadrant as long as the curve gets steeper as $x \rightarrow -\infty$

(b)

B1 For $h = 0.5$ seen or implied by sight of $\frac{0.5}{2}$ in front of the bracket

M1 For the correct bracket structure $\{4.25 + 16.06 + 2 \times (6.427 + 9.125 + 12.34)\}$ condoning slips copying from the table or the omission of the final bracket.

eg $\frac{1}{2} \times 0.5 \times (4.25 + 16.06 + 2 \times (6.427 + 9.125 + 12.34))$

Recovery from missing brackets to achieving the correct answer can still score full marks.

A1 19.0235 but awrt 19.0 will usually score full marks

Note: The calculator answer for the integral is 18.937... which is A0.

Examples of mark traits where full marks are not scored:

$$\frac{1}{2} \times 0.5 \times 4.25 + 16.06 + 2 \times (6.427 + 9.125 + 12.34) \quad (= 72.9065) \text{ is B1M0A0}$$

$$\frac{1}{2} \times 0.5 \times 4.25 + 16.06 + 2 \times 6.427 + 9.125 + 12.34 \quad (= 51.4415) \text{ is B1M0A0}$$

(c)

M1 For realising that $\int_2^4 \left(x(x-3) + \left(\frac{1}{2} \right)^x \right) dx = \int_2^4 \left(x^2 + \left(\frac{1}{2} \right)^x - 3x \right) dx = (b) - \left[\frac{3}{2} x^2 \right]_2^4$

i.e. $(b) - \left[\frac{3}{2} x^2 \right]_2^4$ must be seen or $(b) - \int_2^4 (3x) dx \Rightarrow (b) - 18$ for this mark. The $3x$ term must be isolated from the rest of the integral that is identified in some way to be their part (b). **There must be some integration carried out** so look for $3x \rightarrow \dots x^2$. Condone poor notation.

A1ft 1.0235 but accept awrt 1.0 or follow through on their answer or their unrounded answer from (b) - 18 **provided** M1 has been scored.

Note: The calculator answer for the integral is 0.937 (awrt 0.9) is M0A0



Question Number	Scheme	Marks
6(a)	Attempts line with gradient -2 and point $(4, -1)$ $y + 1 = -2(x - 4)$ $y = -2x + 7$	M1 A1 (2)
(b)	$y = \frac{1}{2}x$ meets $y = -2x + 7$ when $\frac{1}{2}x = -2x + 7 \Rightarrow x = \frac{14}{5}, y = \frac{7}{5}$ oe Attempts $r^2 = \left(4 - \frac{14}{5}\right)^2 + \left(-1 - \frac{7}{5}\right)^2 = \frac{36}{5}$ oe $(x - 4)^2 + (y + 1)^2 = \frac{36}{5}$ oe	M1 A1 dM1 A1 A1 (5) (7 marks)

(a)

M1 Attempts the equation of the line with gradient -2 and point $(4, -1)$
 Condone a slip on one of the signs of the $(4, -1)$. For example $y - 1 = -2(x - 4)$
 If the form $y = mx + c$ is used they must proceed as far as $c = \dots$

A1 $y = -2x + 7$

 Alt (a)

M1 Differentiates implicitly the equation of the circle $(x - 4)^2 + (y + 1)^2 = r^2 \Rightarrow 2(x - 4) + 2(y + 1) \frac{dy}{dx} = 0$
 and substitutes in $\frac{dy}{dx} = \frac{1}{2}$

A1 $y = -2x + 7$

 (b)

M1 Attempts to find where $y = \frac{1}{2}x$ meets their $y = -2x + 7$. Expect to see candidates proceed to $x = \dots$

A1 $\left(\frac{14}{5}, \frac{7}{5}\right)$ or any other form eg $(2.8, 1.4)$ or $x = \frac{14}{5}, y = \frac{7}{5}$

dM1 Attempts to find the distance between their $\left(\frac{14}{5}, \frac{7}{5}\right)$ and $(4, -1)$ using Pythagoras' theorem. Look for an attempt to subtract the coordinates, square and add. It is dependent on the previous method mark. Condone a slip on one of the signs.

A1 For $(r =) \sqrt{\frac{36}{5}}$ or $(r^2 =) \frac{36}{5}$ oe

A1 $(x - 4)^2 + (y + 1)^2 = \frac{36}{5}$ or $(x - 4)^2 + (y + 1)^2 = 7.2$ or $5(x - 4)^2 + 5(y + 1)^2 = 36$ only

.....



Alt (b)

M1 Substitutes $y = \frac{1}{2}x$ into $(x-4)^2 + (y+1)^2 = r^2$, multiplies out and rearranges $\dots = 0$

A1 $5x^2 - 28x + 68 - 4r^2 = 0$ or $5y^2 - 14y + 17 - r^2$

dM1 Uses the discriminant $b^2 - 4ac = 0$ and proceeds to $r = \dots$ or $r^2 = \dots$. It is dependent on the previous method mark.

A1 For $r = \sqrt{\frac{36}{5}}$ or $r^2 = \frac{36}{5}$

A1 $(x-4)^2 + (y+1)^2 = \frac{36}{5}$ oe eg $(x-4)^2 + (y+1)^2 = 7.2$ or $5(x-4)^2 + 5(y+1)^2 = 36$

Note: Candidates who substitute $y = -2x + 7$ into $(x-4)^2 + (y+1)^2 = r^2$ and attempt the discriminant approach will score 0 marks in (b)



Question Number	Scheme	Marks
7. (i)	$\log_a \left(\frac{\sqrt{a}}{b} \right) = \frac{1}{2} \log_a a - \log_a b = \frac{1}{2} - k$	M1 A1 (2)
(ii)	$\frac{\log_a a^2 b}{\log_a b^3} = \frac{2 \log_a a + \log_a b}{3 \log_a b} = \frac{2+k}{3k}$	M1 A1 (2)
(iii)	$\sum_{n=1}^{50} (k + \log_a b^n) = 50k + (1k + 2k + 3k + \dots + 50k) \text{ or } (2k + 3k + 4k + \dots + 51k)$ Uses the sum formula an AP with $n = 50, d = k$ $S = 50k + \frac{50}{2}(2k + 49k) \quad S = \frac{50}{2}(2k + 51k)$ $= 1325k$	M1 A1 A1 (3) (7 marks)

(i)

M1 Uses log laws to achieve $\frac{1}{2} \log a - \log b$ or eg $0.5 \log a - \log b$ A1 $\frac{1}{2} - k$ oe

(ii)

M1 Uses correct log laws on the numerator or denominator. Score for $\frac{\dots}{3 \log b}$ or $\frac{\dots}{3k}$ or sight of the numerator $2 \log a + \log b$ which does not have to be part of a fraction (May be implied by $2 + k$)A1 $\frac{2+k}{3k}$ or $\frac{2}{3k} + \frac{1}{3}$ do not isw

(iii)

M1 **Either** uses the sum formula for an AP in terms of k on part or all of the expression with $n = 50, d = k, a = k$ or $n = 50, d = k, a = 2k$. It is sufficient to see the terms substituted in for this mark.

$$\sum_{n=1}^{50} nk = \frac{50}{2}(2k + 49k) \text{ or } \sum_{n=1}^{50} (k + nk) = \frac{50}{2}(2k + 51k).$$

You may see the equivalent sum $\frac{n}{2}(a + L)$ eg $\frac{50}{2}(k + 50k)$ **Or** uses the sum formula for an AP in terms of $\log_a b$ on part of the expression with $n = 50, d = \log b, a = \log b$

$$\sum_{n=1}^{50} \log b^n = \frac{50}{2}(2 \log b + 49 \log b)$$

You may see the equivalent sum $\frac{n}{2}(a + L) = \sum_{n=1}^{50} \log b^n = \frac{50}{2}(\log b + 50 \log b)$ 

A1 A correct unsimplified answer in terms of k .

Eg. $S = 50k + \frac{50}{2}(2k + 49k)$ or $S = \frac{50}{2}(2k + 51k)$

A1 $1325k$



Question Number	Scheme	Marks
8 (i)	$\int \frac{8\sqrt{x}-5}{2x^2} dx = \int 4x^{-\frac{3}{2}} - \frac{5}{2}x^{-2} dx$ $= -8x^{-\frac{1}{2}} + \frac{5}{2}x^{-1} (+C)$ $\int_2^4 \left(4x^{-\frac{3}{2}} - \frac{5}{2}x^{-2} \right) dx = \left(-4 + \frac{5}{8} \right) - \left(-4\sqrt{2} + \frac{5}{4} \right) = 4\sqrt{2} - \frac{37}{8}$	B1 M1 A1 dM1 A1 (5)
8 (ii)	$\int \left(\frac{1}{2}x^2 + k \right) dx = \left[\frac{1}{6}x^3 + kx \right]$ $\int_{-3}^6 \left(\frac{1}{2}x^2 + k \right) dx = 55 \Rightarrow \left[\frac{1}{6}x^3 + kx \right]_{-3}^6 = 55 \Rightarrow (36 + 6k) - \left(-\frac{9}{2} - 3k \right) = 55 \Rightarrow k = \dots$ $k = \frac{29}{18}$	M1 A1 dM1 A1 (4) (9 marks)

(i)

B1 For $\frac{8\sqrt{x}-5}{2x^2} \rightarrow 4x^{-\frac{3}{2}} - \frac{5}{2}x^{-2}$ (seen or implied). Ignore any +C

You may also see a factor of $\frac{1}{2}$ taken out eg $\frac{8\sqrt{x}-5}{2x^2} \rightarrow \frac{1}{2} \left(8x^{-\frac{3}{2}} - 5x^{-2} \right)$

M1 For raising any index by one on one of their terms. Follow through on incorrect indices but do not allow for candidates who attempt to integrate the numerator and denominators separately.

A1 Correct terms (may be unsimplified but the indices must be processed)

dM1 Substitutes 4 and 2 into an integrated function and subtracts either way round. Dependent on the previous method mark. awrt 1.03 following B1M1A1 is sufficient evidence to award this mark.

A1 $4\sqrt{2} - \frac{37}{8}$ or exact equivalent (eg $4\sqrt{2} - 4.625$ or $\frac{32\sqrt{2}-37}{8}$)

Note: Answer only scores 0 marks.

(ii)

M1 Attempts to integrate $\left(\frac{1}{2}x^2 + k \right)$ with one term correct unsimplified so allow eg $\frac{1}{2}x^3$ or kx^1 but the index must be processed.

A1 $\frac{1}{6}x^3 + kx$ (allow unsimplified equivalent expressions as above)

dM1 Substitutes 6 and -3 into $Ax^3 + kx$, subtracts, sets = 55 and proceeds to $k = \dots$. Do not be concerned with the mechanics of their rearrangement.

A1 $k = \frac{29}{18}$ or exact equivalent eg 1.6 $\dot{1}$ but do not allow 1.6 or 1.61 or 1.61....



Question Number	Scheme	Marks
9.(i)	$3\sin(2\theta - 10^\circ) = 1 \Rightarrow (2\theta - 10^\circ) = \arcsin\left(\frac{1}{3}\right)$ $\theta = \frac{19.47 + 10}{2}, \frac{160.53 + 10}{2}$ $\theta = \text{awrt } 14.7^\circ, 85.3^\circ$	M1 dM1 A1, A1 (4)
(ii) (a)	Writes $\frac{1}{\tan \alpha} - \sin \alpha = 2 \sin \alpha - \frac{1}{\tan \alpha}$ oe $\frac{2}{\tan \alpha} = 3 \sin \alpha \Rightarrow \frac{2 \cos \alpha}{\sin \alpha} = 3 \sin \alpha \Rightarrow 2 \cos \alpha = 3 \sin^2 \alpha$ *	M1 dM1 A1* (3)
(b)	$2 \cos \alpha = 3 \sin^2 \alpha \Rightarrow 2 \cos \alpha = 3(1 - \cos^2 \alpha)$ $3 \cos^2 \alpha + 2 \cos \alpha - 3 = 0$ Attempts to solve $3 \cos^2 \alpha + 2 \cos \alpha - 3 = 0 \Rightarrow \cos \alpha = \frac{-2 \pm \sqrt{40}}{6}$ oe $\alpha = 5.517$ radians	M1 A1 dM1 A1 A1 (5) (12 marks)

(i)

M1 For proceeding to $x = \arcsin\left(\frac{1}{3}\right)$ which may be implied by the sight of awrt 19.5° or awrt 160.5°
(Allow awrt 0.340 (radians) for this mark)

dM1 For correct order of operations leading to one answer for θ . May be implied by $14.7^\circ/14.8^\circ$ but cannot be scored by adding 10 to an angle in radians but may be implied by awrt 0.257 / 0.258rad

A1 One of $\theta = \text{awrt } 14.7^\circ, 85.3^\circ$ ignore any others.

A1 Both of $\theta = \text{awrt } 14.7^\circ, 85.3^\circ$ and no others in the range.

Note that solutions based entirely on graphical or numerical methods are not acceptable so answers only will score 0 marks.

(ii)(a)

M1 Uses the terms of an AP to set up a correct equation. Eg $\frac{1}{\tan \alpha} - \sin \alpha = 2 \sin \alpha - \frac{1}{\tan \alpha}$,
 $\frac{\cos \alpha}{\sin \alpha} - \sin \alpha = 2 \sin \alpha - \frac{\cos \alpha}{\sin \alpha}$. They may also do $2\left(\frac{1}{\tan \alpha} - \sin \alpha\right) = 2 \sin \alpha - \sin \alpha$. Condone mixed variables and poor notation for the method marks.



dM1 Uses $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$ within an equation involving $\frac{1}{\tan \alpha}$, $\sin \alpha$ & $2 \sin \alpha$. This mark can be implied but do not award if they just proceed straight to the final answer. It is dependent on the previous method mark. A candidate starting with $\frac{\cos \alpha}{\sin \alpha} - \sin \alpha = 2 \sin \alpha - \frac{\cos \alpha}{\sin \alpha}$ scores M1dM1 straight away.

A1* Proceeds to given answer with no errors or omissions. The equation must start with an equation involving $\tan \alpha$ but see the note below for further guidance. Withhold this mark for incorrect notation eg $\sin \alpha^2$ or if they had mixed variables on the same line. Eg α and θ .

Note: In the example below they can score full marks as $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$ is implied by their second term listed on the first line. Had they not written these first three terms or stated somewhere in their solution $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$ in their working then we would withhold the final mark.

$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$ implied here

(ii)(b)

M1 Attempts to use $\sin^2 \alpha + \cos^2 \alpha = 1$ Eg. $2 \cos \alpha = 3 \sin^2 \alpha \Rightarrow 2 \cos \alpha = 3(\pm 1 \pm \cos^2 \alpha)$. Beware that a candidate may use this identity in part (ii)(a) which would not gain the mark in this part.

A1 $3 \cos^2 \alpha + 2 \cos \alpha - 3 = 0$ The “=0” may be implied by later work but the terms must be collected on one side. Evidence of this may be awarded for correct coefficients substituted into the quadratic formula.

dM1 Attempts to solve their $3 \cos^2 \alpha + 2 \cos \alpha - 3 = 0$ by the formula/completing the square, usual rules for solving a 3TQ apply but do not award for attempted factorisation unless their quadratic factorises.

Award for $(\cos \alpha =) \frac{-1 \pm \sqrt{10}}{3}$ or $(\cos \alpha =)$ awrt 0.72 or awrt -1.4 or equivalent. You may need to check decimal values on your calculator.

A1 $(\cos \alpha =) \frac{-2 + \sqrt{40}}{6}$ oe (typically $(\cos \alpha =) \frac{-1 + \sqrt{10}}{3}$). Implied by $(\cos \alpha =)$ awrt 0.721 or $(\alpha =)$ awrt 5.52 radians.

A1 $(\alpha =)$ awrt 5.517 radians and no others in the given range



Question Number	Scheme	Marks
10.(a)	$x = 2, y = 5 \Rightarrow 5 = 8a - 12 + 6 + b$ $\frac{dy}{dx} = 3ax^2 - 6x + 3$ AND $x = 2, \frac{dy}{dx} = 7 \Rightarrow 7 = 12a - 12 + 3$ Solves $11 = 8a + b$ and $7 = 12a - 9 \Rightarrow a = \frac{4}{3}, b = \frac{1}{3}$	M1 M1 A1 A1 (4)
(b)	Sets $\frac{dy}{dx} = 3ax^2 - 6x + 3 = 0$ with their value of a and b $4x^2 - 6x + 3 = 0$ and attempts " $b^2 - 4ac$ " $b^2 - 4ac = -12 < 0$ hence there are no turning points oe	M1 dM1 A1* (3) (7 marks)

(a)

M1 Substitutes $x = 2, y = 5$ into $y = ax^3 - 3x^2 + 3x + b$ to get an equation in a and b (condone slips)M1 Substitutes $x = 2, \frac{dy}{dx} = 7$ into $\frac{dy}{dx} = 3ax^2 - 6x + 3$ to get an equation in a ($\frac{dy}{dx}$ must be correct)A1 $a = \frac{4}{3}$ or exact equivalentA1 $b = \frac{1}{3}$ or exact equivalent

(b)

M1 Sets their $\frac{dy}{dx} = 0$ with their value of a . This may be implied by later working.dM1 Attempts to find $b^2 - 4ac$ or roots via the formula

A1* Achieves $4x^2 - 6x + 3 = 0$, $b^2 - 4ac = -12$ and states $-12 < 0$ and so there are **no turning points** or equivalent. They may attempt to solve the equation and either state that **no real roots so no turning points** or achieve complex roots and state that **no real roots so no turning points**. Note full marks can be scored with an incorrect value for $b = \frac{1}{3}$ but cannot be scored from an incorrect value for $a = \frac{4}{3}$ from part (a)

.....
Alt (b)M1 Attempts to complete the square for $\frac{dy}{dx} = 3ax^2 - 6x + 3$ for their value of a to achieve eg $4(x \pm \dots)^2$ or $(2x \pm \dots)^2$ dM1 $4x^2 - 6x + 3 = 4\left(x \pm \frac{3}{4}\right)^2 \pm \dots$ or $4x^2 - 6x + 3 = \left(2x \pm \frac{3}{2}\right)^2 \pm \dots$

A1* Achieves $4x^2 - 6x + 3 = 4\left(x - \frac{3}{4}\right)^2 + \frac{3}{4}$ or $4x^2 - 6x + 3 = \left(2x - \frac{3}{2}\right)^2 + \frac{3}{4}$ and states there are **no turning points** as $\frac{dy}{dx} > 0$ (for all x) or equivalent.

If you see any other ways that may be credit worthy then send to review



General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1(a)	$h = 1.25$	B1
	$A \approx \frac{1}{2} \times 1.25 \{3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)\}$	M1
	$= 30.9$	A1
		(3)
(b)	$\int_{0.5}^{5.5} (f(x) + 4x) dx = 30.9 + \left[2x^2 \right]_{0.5}^{5.5} = 30.9 + 2 \times 5.5^2 - 2 \times 0.5^2$ or $\int_{0.5}^{5.5} (f(x) + 4x) dx = \int_{0.5}^{5.5} f(x) dx + \frac{1}{2}(4 \times 0.5 + 4 \times 5.5) \times 5 = \dots$	M1
	$= 30.9 + 60 = 90.9$	A1ft
		(2)
		Total 5

(a)

B1: Correct strip width of 1.25. Allow equivalent numerical expressions e.g. $\frac{5.5 - 0.5}{4}$.

May be implied by sight of e.g. $\frac{1.25}{2}$ or $\frac{5}{8}$ or 0.625 in front of the bracket. May also be implied by a correct answer if no incorrect working seen. $h = -1.25$ is B0 unless they recover and subsequently use $h = +1.25$

M1: Correct application of the trapezium rule with their h (which may be 1).

This requires a correct inner bracket structure $3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)$

multiplied by $\frac{1}{2}h$. Condone slips copying values from the table or the omission of the

final bracket on the rhs e.g. $\frac{1}{2} \times "1.25"(3.479 + 5.182 + 2(6.101 + 7.448 + 6.823))$ is M1

but $\frac{1}{2} \times "1.25" \times 3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)$ is M0 unless the missing

brackets are recovered or implied by the correct answer (you may need to check). Also allow for a correct method adding individual trapezia using their h condoning copying errors but the brackets must be correct (or recovered or implied by later work)

e.g. $\frac{1}{2} \times "1.25"(3.479 + 6.101) + \frac{1}{2} \times "1.25"(6.101 + 7.448) + \frac{1}{2} \times "1.25"(7.448 + 6.823) +$

$\frac{1}{2} \times "1.25"(6.823 + 5.182)$

A1: For awrt 30.9. isw once a correct answer is seen. Correct answer with no working scores B1M1A1 but if there is evidence of using $h = -1.25$ then maximum awarded is B0M1A0. The A mark cannot be awarded without both B1M1 being awarded in this question. The exact answer is $\frac{9881}{320}$ and scores A0 unless followed by awrt 30.9

For reference, note that use of $h = 1$ (from using 5 strips) gives 24.7... or $\frac{9881}{400}$

(a), $\int_{0.5}^{5.5} 1.25 (3.479 + 6.101 + 7.448 + 6.823 + 5.182) = 36.3$ Scores B1M0A0 (BOD)



(b)**M1:** Attempts their answer to $(a) + \left[\dots x^2 \right]_{0.5}^{5.5} = \text{their answer to } (a) + \dots 5.5^2 - \dots 0.5^2$ May be implied by e.g. their answer to $(a) + 2 \times 5.5^2 - 2 \times 0.5^2$ if the integration is not seen explicitly.

Or

their answer to $(a) + \frac{1}{2}(4 \times 0.5 + 4 \times 5.5) \times 5$ (trapezium)oe e.g. their answer to $(a) + (4 \times 0.5 \times 5) + \frac{1}{2}(5 \times (5 \times 5.5 - 5 \times 0.5))$ (rectangle + triangle)May be implied by their answer to $(a) + 60$ Condone clear misreads of the 4 in the $4x$ but do not condone a misread of e.g. $4x$ for 4.

Condone clear mis-copy/mis-read of limits as long as they are non-zero E.g. 5 for 5.5.

Condone poor notation e.g. $\int_{0.5}^{5.5} (f(x) + 4x) dx = 30.9 + \int_{0.5}^{5.5} 2x^2 dx = 30.9 + 2 \times 5.5^2 - 2 \times 0.5^2$ **A1ft:** Correct answer of awrt 90.9 or correct ft e.g. $60 +$ their answer to part (a)Allow exact or exact ft answers e.g. $\frac{29081}{320}$ for 90.9Correct answer **only** scores no marks as the questions says “making your method clear”Attempts to use the trapezium rule again by adding $4x$ to the y values score M0 in (b)Note that $30.9 + 4(0.5 + 1.75 + 3 + 4.25 + 5.5) = 90.9$ is fortuitous and scores M0But $30.9 + \frac{1}{2} \times \frac{5}{4}(2 + 22 + 2(7 + 12 + 17)) = 90.9$ scores M1A1

Condone an error with the strip width as in part (a) for this method but the ft is not available in this case.

Examples of minimal acceptable working in (b) for both marks:

$$30.9 + 60 = 90.9$$

$$30.9 + \left[2x^2 \right]_{0.5}^{5.5} = 90.9$$

$$\int_{0.5}^{5.5} f(x) dx + \int_{0.5}^{5.5} 4x dx = 30.9 + 60 = 90.9$$

$$30.9 + 2 \times 5.5^2 - 2 \times 0.5^2 = 90.9$$

All score M1A1



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

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The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice



Question	Scheme	Marks
7	$(x + 2) \text{ a factor} \Rightarrow f(-2) = 0 \Rightarrow -8A + 24 + 8 + B = 0$	M1A1
	$\int f(x)dx = \frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$	M1 A1
	$\int_3^5 f(x)dx = 176 \Rightarrow \left[\frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx \right]_3^5 = 176$ $\Rightarrow \left(\frac{A}{4}5^4 + 2(5^3) - 2(5^2) + 5B \right) - \left(\frac{A}{4}3^4 + 2(3^3) - 2(3^2) + 3B \right) = 176$	dM1
	$\left. \begin{array}{l} 8A - B = 32 \\ 136A + 2B = 12 \end{array} \right\} \Rightarrow A = \dots, B = \dots$	dM1
	$A = \frac{1}{2}, B = -28$	A1
		(7)
(7 marks)		

Notes:

M1: Uses the factor theorem or long division (see below) to find one linear **equation** relating A and B. The “= 0” must be seen or implied by later work.

A1: Correct equation. Award once a correct equation is seen unsimplified or simplified.

So allow e.g. $A(-2)^3 + 6(-2)^2 - 4(-2) + B = 0$ but not e.g. $A \times -2^3 + 6 \times -2^2 - 4 \times -2 + B = 0$

unless the indices are processed correctly subsequently to e.g. $-8A + 24 + 8 + B = 0$

The “= 0” must be seen or implied by later work.

M1: Attempts to integrate the given expression, look for at least one power increased by 1.

This can also be evidenced by $B \rightarrow Bx$

A1: Correct integration. Allow unsimplified e.g. $\frac{6x^{2+1}}{2+1}$ for $2x^3$

Condone poor notation e.g. $\int f(x)dx = \int \frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$ and ignore any “+ c”

dM1: Applies limits 5 and 3 either way round and subtracts and equates to 176 to obtain an equation in A and B. **Depends on previous M.**

Condone poor use of brackets when subtracting e.g. allow

$$\frac{A}{4}5^4 + 2(5^3) - 2(5^2) + 5B - \frac{A}{4}3^4 + 2(3^3) - 2(3^2) + 3B = 176$$

dM1: Solves their two equations in A and B to find values for both constants.

Depends on all previous M marks but allow this mark if the only error was to attempt $f(2) = 0$ at the beginning.

A1: Correct values.

Note that some candidates eliminate A or B by rearranging the first equation and substituting into the integral. This essentially follows the main scheme:



$f(-2) = 0 \Rightarrow -8A + 24 + 8 + B = 0 \Rightarrow B = 8A - 32$	M1A1
$\int f(x)dx = \frac{A}{4}x^4 + 2x^3 - 2x^2 + 8Ax - 32x$	M1A1
$\Rightarrow \left(\frac{A}{4}5^4 + 2(5^3) - 2(5^2) + 40A - 160 \right) - \left(\frac{A}{4}3^4 + 2(3^3) - 2(3^2) + 24A - 96 \right) = 176$	dM1
$\Rightarrow A = \dots, B = \dots$	dM1
$A = \frac{1}{2}, B = -28$	A1
<u>Long Division for reference:</u> $ \begin{array}{r} Ax^2 + (6-2A)x + 4A-16 \\ x+2 \overline{) Ax^3 + 6x^2 - 4x + B} \\ \underline{Ax^3 + 2Ax^2} \\ (6-2A)x^2 - 4x + B \\ \underline{(6-2A)x^2 + (12-4A)x} \\ (4A-16)x + B \\ \underline{(4A-16)x + 8A-32} \\ B - 8A + 32 \end{array} $ Score the first M1 for a complete method to obtain a remainder in terms of A and B that is set = 0 and A1 for a correct equation e.g. $B - 8A + 32 = 0$	



Question	Scheme	Marks
8(a)	$\left(\frac{dy}{dx} = \right) Ax^3 + B + \frac{C}{x^3}$	M1
	$\left(\frac{dy}{dx} = \right) 4 \times 256x^3 - 304 - 2 \times 27x^{-3}$ oe e.g. $1024x^3 - 304 - \frac{54}{x^3}$	A1A1
		(3)
(b)	$\frac{dy}{dx} = 0$ at SP $\Rightarrow 1024x^3 - 304 - \frac{54}{x^3} = 0 \Rightarrow 1024x^6 - 304x^3 - 54 = 0$	M1
	$\Rightarrow 512(x^3)^2 - 152x^3 - 27 = 0 \Rightarrow (8x^3 + 1)(64x^3 - 27) = 0 \Rightarrow x^3 = \dots$	dM1
	$x^3 = -\frac{1}{8}$ or $x^3 = \frac{27}{64}$ or $x = -\frac{1}{2}$ or $x = \frac{3}{4}$	A1
	$y = 256\left(-\frac{1}{2}\right)^4 - 304\left(-\frac{1}{2}\right) - 35 + \frac{27}{\left(-\frac{1}{2}\right)^2} = \dots$ or with $x = \frac{3}{4}$	dM1
	Coordinates are $\left(-\frac{1}{2}, 241\right)$ and $\left(\frac{3}{4}, -134\right)$	A1
		(5)
(8 marks)		
Notes:		
(a) M1: Attempts the derivative, with at least two terms of the correct form (ie $x^n \rightarrow \dots x^{n-1}$ at least twice). A1: At least two terms correct, need not be simplified. A1: Fully correct derivative, need not be simplified. Isw after a correct (unsimplified) answer. Note that there is no need to see the "$\frac{dy}{dx} =$" in (a) just look for the differentiation. Ignore any spurious "$= 0$" (b) M1: Sets their derivative equal to zero and multiplies through by "x^3" to achieve a polynomial equation. If in doubt at least 2 terms must be multiplied. Allow equivalent work e.g. $1024x^3 - 304 - \frac{54}{x^3} = 0 \Rightarrow \frac{1}{x^3}(1024x^6 - 304x^3 - 54) = 0 \Rightarrow 1024x^6 - 304x^3 - 54 = 0$ To score this mark, the derivative must have a negative power of x so allow for e.g. $\left(\frac{dy}{dx} = \right) 1024x^3 - 304 - 54x^{-1} = 0 \Rightarrow 1024x^4 - 304x - 54 = 0$ dM1: Solves a 3 term quadratic in x^3 by any valid means (including calculator). Must come from an attempt at $\frac{dy}{dx} = 0$ not $\frac{d^2y}{dx^2} = 0$ or $\int y dx = 0$ Condone use of inequalities rather than "$=$" as long as they solve an equation.		



Note that here we accept e.g. $512(x^3)^2 - 152x^3 - 27 = 0 \Rightarrow (8y+1)(64y-27) = 0 \Rightarrow y = \dots$

Or even $512(x^3)^2 - 152x^3 - 27 = 0 \Rightarrow (8x+1)(64x-27) = 0 \Rightarrow x = \dots$

A1: Achieves at least one correct value for x or x^3 . It must be clear that they are values for x^3 here so $y = \dots$ is acceptable if $y = x^3$ is seen or implied. If they have $x = \dots$ then they must have cube rooted. So e.g. $(8x+1)(64x-27) = 0 \Rightarrow x = -\frac{1}{8}$ unless they recover and recognise they have x^3

then this is A0

dm1: Proceeds to find the y coordinate for at least one value of x . Must **cube root** and not e.g. square root the solution from the quadratic first. The x must have come from a 2 or 3 term quadratic in x^3 .

If no method shown, accept any value for y having found an x value but score M0 if there is no evidence of cube rooting or clear evidence that they have substituted into something other than the curve equation but condone if they clearly just mis-copy the equation.

Depends on having scored at least 1 of the previous M marks in (b)

A1: Both correct pairs of coordinates. Accept if given as e.g. $x = -\frac{1}{2}, y = 241$ and $x = \frac{3}{4}, y = -134$

And no other values.

Allow equivalent fractions or values for the $-\frac{1}{2}$ and $\frac{3}{4}$



Question Number	Scheme	Marks
6a	$\frac{k}{2k-15} = \frac{k+4}{k} \Rightarrow k^2 = 2k^2 + 8k - 15k - 60 \Rightarrow k^2 - 7k - 60 = 0$ *	M1A1*
		(2)
b	$k = 12$ $\Rightarrow \frac{"12"}{2 \times "12" - 15} \left(= \frac{4}{3} \right)$ e.g. $("12" + 4) \times \frac{4}{3} = \dots$ or $(2 \times "12" - 15) \times \left(\frac{4}{3} \right)^3 = \dots$ $= 21300$	B1 M1 M1 A1
		(4)
c	$\text{e.g. } \frac{9 \left(1 - \left(\frac{4}{3} \right)^N \right)}{1 - \frac{4}{3}} > 3000 \Rightarrow \left(\frac{4}{3} \right)^N > \frac{1009}{9}$ $\log \left(\frac{4}{3} \right)^N > \log \left(\frac{1009}{9} \right) \Rightarrow N > \frac{\log \left(\frac{1009}{9} \right)}{\log \left(\frac{4}{3} \right)}$ $N > 16.405 \dots \Rightarrow N = 17$	M1 dM1 A1
		(3)
		(9 marks)

(a)

M1 Forms a correct equation linking the three terms and attempts to rearrange to the given quadratic (it cannot be awarded for just proceeding in one step from the starting equation to the given answer and usually will involve attempting to multiply out brackets or dealing with any fractions). Must proceed as far as a quadratic with terms collected. Condone slips in their rearrangement and invisible brackets. Possible starting equations will involve using the n th term formula or the sum of a geometric series. The ones below are not exhaustive.

$$\frac{k}{2k-15} = \frac{k+4}{k} \quad \text{or} \quad k^2 = (k+4)(2k-15) \quad \left((u_2)^2 = u_1 \times u_3 \right) \quad \text{or} \quad (2k-15) \left(\frac{k+4}{k} \right) = k \quad \text{or}$$

$$k \left(\frac{k}{2k-15} \right) = k+4 \quad \text{or} \quad 2k-15 + k + k+4 = \frac{(2k-15) \left(1 - \left(\frac{k}{2k-15} \right)^3 \right)}{1 - \frac{k}{2k-15}} \quad (\text{sum of 3 terms})$$

A1* Rearranges to achieve $k^2 - 7k - 60 = 0$ with no errors seen including brackets. There must be at least some intermediate working (e.g. brackets multiplied out or multiplying to remove any fractions) between their starting equation and the given answer.

(b) **Note a candidate who uses k as r can only score maximum B1 in (b) and no marks in (c)**

B1 12 (ignore -5) seen or used in (b) (may be implied by the value of the second term)

M1 Substitutes their 12 (must be positive) into the expressions for two of the terms and attempts to find a value for r . Condone arithmetical slips but they the division the correct way round.



- M1 Attempts to find the 4th term using their positive k and their positive r eg $(12+4) \times \frac{4}{3} = \dots$
 (may be implied by awrt 21.3). Alternatively, they may find the first term a and use this with their r to find the 4th term. Condone slips in finding a
 Condone r to be rounded from their earlier work and condone missing brackets but not incorrectly placed brackets.
- A1 21300 or 21330 or 21333 only or their equivalences e.g. 21.3 thousand

(c) Typically, candidates will work in base 10, but other bases such as their r is acceptable

- M1 Uses their $a = 9$ or $a = 9000$ and their positive $r = \frac{4}{3}$ in the sum of a geometric formula $\Rightarrow$

$$\frac{9 \left(1 - \left(\frac{4}{3} \right)^N \right)}{1 - \frac{4}{3}} \dots 3 \dots \text{(may be = or any inequality in between)} \text{ and proceeds to } A \left(\frac{4}{3} \right)^N \dots B$$

(condone $A \times B$ to be negative), where A could be 1 and B could be unsimplified. Do not be concerned with the number of 0s after the 3.

Do not be concerned by the mechanics of their rearrangement and may be implied by further work if logarithms are used correctly. Condone r to be rounded from their earlier work and condone missing brackets but not incorrectly placed brackets.

- dM1 Attempts to find N using logarithms correctly. Do not be concerned with the use of equals or the direction of any inequality used for this mark. May be implied by a correct expression (not a value) for N . It is dependent on the previous method mark.

This mark can only be scored if $A \times B > 0$ for their $A \left(\frac{4}{3} \right)^N \dots B$

- A1 17 cso (if an incorrect inequality sign is used at some point in their working then withhold this final mark) This mark can only be scored provided there are **no arithmetical errors** and **correct log work** is seen as part of their solution. Do not withhold the mark for invisible brackets around their $\left(\frac{4}{3} \right)^N$. Condone if r is rounded from $\frac{4}{3}$. If a value for N is found before deducing $N = 17$ then it must be 16.4 or better. 17 with no working seen is 0 marks.

Note e.g. $N > \log_{\frac{4}{3}} \left(\frac{991}{9} \right) \Rightarrow N = 17$ is A0 (arithmetical slip combining $1 + \frac{1000}{9}$)

$$\frac{9 \left(1 - \left(\frac{4}{3} \right)^N \right)}{1 - \frac{4}{3}} > 3000 \Rightarrow \left(\frac{4}{3} \right)^N > \frac{1009}{9} \Rightarrow N = 17 \text{ scores M1dM0A0 (insufficient working)}$$

$$\frac{9 \left(1 - \left(\frac{4}{3} \right)^N \right)}{1 - \frac{4}{3}} < 3000 \Rightarrow N < \frac{\log \left(\frac{1009}{9} \right)}{\log \left(\frac{4}{3} \right)} \Rightarrow N = 17 \text{ scores M1dM1A0 (inequality sign errors)}$$

$$\frac{9000 \left(1 - \left(\frac{4}{3} \right)^N \right)}{1 - \frac{4}{3}} > 3000000 \Rightarrow N > \log_{\frac{4}{3}} \left(\frac{1009}{9} \right) \Rightarrow N = 17 \text{ scores M1dM1A1 (implied M marks)}$$

Note: Methods relying on trial and improvement, or the equation solver score maximum M1dM0A0



Question Number	Scheme	Marks
7(a)	$\frac{12x^3(x-7)+14x(13x-15)}{21\sqrt{x}} = \frac{12x^4-84x^3+182x^2-210x}{21\sqrt{x}}$	M1
	$\frac{4}{7}x^{\frac{7}{2}}, -4x^{\frac{5}{2}}, +\frac{26}{3}x^{\frac{3}{2}}, -10x^{\frac{1}{2}}$	A1
	$(y =) \frac{4}{7}x^{\frac{7}{2}} - 4x^{\frac{5}{2}} + \frac{26}{3}x^{\frac{3}{2}} - 10x^{\frac{1}{2}}$	A1
		(3)
(b)	$y = \frac{4}{7}x^{\frac{7}{2}} - 4x^{\frac{5}{2}} + \frac{26}{3}x^{\frac{3}{2}} - 10x^{\frac{1}{2}}$ $\left(\frac{dy}{dx}\right) = 2x^{\frac{5}{2}} - 10x^{\frac{3}{2}} + 13x^{\frac{1}{2}} - 5x^{-\frac{1}{2}}$	M1A1ft
	$2x^3 - 10x^2 + 13x - 5 = 0 *$	A1*
		(3)
(c)	$(2x^3 - 10x^2 + 13x - 5) \div (x-1) = (2x^2 \pm \dots x \pm \dots)$	M1
	$2x^2 - 8x + 5$	A1
	$x = \frac{4 \pm \sqrt{6}}{2}$	A1
		(3)
		(9 marks)

(a)**M1:** Attempts to multiply out numerator (at least 2 correct terms obtained).

May be done by e.g. 2 separate fractions.

A1: Two of $\frac{4}{7}x^{\frac{7}{2}}, -4x^{\frac{5}{2}}, +\frac{26}{3}x^{\frac{3}{2}}, -10x^{\frac{1}{2}}$ where the coefficient may be unsimplified but the indexmust be processed so allow for any 2 of e.g. $\frac{12}{21}x^{\frac{7}{2}}, -\frac{84}{21}x^{\frac{5}{2}}, +\frac{182}{21}x^{\frac{3}{2}}, -\frac{210}{21}x^{\frac{1}{2}}$ **A1:** $y = \frac{4}{7}x^{\frac{7}{2}} - 4x^{\frac{5}{2}} + \frac{26}{3}x^{\frac{3}{2}} - 10x^{\frac{1}{2}}$ or exact simplified equivalent. Allow as a list.**(b)****M1:** Differentiates to achieve an expression of the form $\left(\frac{dy}{dx}\right) = \dots x^{\frac{5}{2}} \pm \dots x^{\frac{3}{2}} \pm \dots x^{\frac{1}{2}} \pm \dots x^{-\frac{1}{2}}$ **A1ft:** Correct differentiation $2x^{\frac{5}{2}} - 10x^{\frac{3}{2}} + 13x^{\frac{1}{2}} - 5x^{-\frac{1}{2}}$ Follow through their *a, b, c* and *d* with simplified coefficients. " $\frac{dy}{dx} =$ " is not required.**A1*:** $2x^3 - 10x^2 + 13x - 5 = 0$

Reaches this printed answer from a correct derivative where the "=" has appeared at least

once before the final answer. Must be as shown and not just e.g. $x^{-\frac{1}{2}}(2x^3 - 10x^2 + 13x - 5) = 0$ but allow e.g. $x^{-\frac{1}{2}}(2x^3 - 10x^2 + 13x - 5) = 0$ followed by a minimal conclusion e.g. hence proven.

Question Number	Scheme	Marks
10.(a)	$y = 4x^3 - 9x + \frac{k}{x} \Rightarrow \frac{dy}{dx} = 12x^2 - 9 - \frac{k}{x^2}$ $x = \frac{1}{2}, \frac{dy}{dx} = 0 \Rightarrow 0 = 12 \times \left(\frac{1}{2}\right)^2 - 9 - 4k \Rightarrow 4k = -6 \Rightarrow k = -\frac{3}{2} *$	M1 A1 dM1 A1* (4)
(b)	Attempts $\frac{d^2y}{dx^2} = 24x - \frac{3}{x^3}$ at $x = \frac{1}{2}$ $\Rightarrow \frac{d^2y}{dx^2} = -12 < 0$ (Local) Maximum	M1 A1 (2)
(c)	$12x^2 - 9 + \frac{3}{2x^2} = 0 \Rightarrow 24x^4 - 18x^2 + 3 = 0$ $\Rightarrow 3(4x^2 - 1)(2x^2 - 1) = 0$ $\Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \frac{\sqrt{2}}{2} \text{ oe}$	M1 A1 dM1 A1 (4) (10 marks)

(a)

M1 Attempts to differentiate reducing any power of x by oneA1 For $12x^2 - 9 - \frac{k}{x^2}$ oe need not see $\frac{dy}{dx}$ dM1 Substitutes $x = \frac{1}{2}, \frac{dy}{dx} = 0$ into their $\frac{dy}{dx}$ to set up a linear equation in k . (Allow the M if “=0” implied)A1* Proceeds via $ak + b = 0$ or $ak = b$ to achieve $k = -\frac{3}{2}$ following correct work. The “=0” must have been seen for this mark.

(b) Attempts at first derivative test – send to review if they seem worthy of merit.

M1 Attempts to find the value of $\frac{d^2y}{dx^2}$ at $x = \frac{1}{2}$. Follow through on their $\frac{dy}{dx}$ with one term correct.A1 Fully correct $\frac{d^2y}{dx^2} = -12 < 0 \Rightarrow$ (Local) Maximum. Allow with $12 - 24 < 0$

(c)

M1 Sets their $\frac{dy}{dx} = 0$ and proceeds to a three term quartic equation in x or factorise to give
$$k\left(ax + \frac{b}{x}\right)\left(cx + \frac{d}{x}\right) = 0 \text{ where } kac = 24 \text{ and } kbd = 3 \text{ (though } k = 3 \text{ may be cancelled).}$$
A1 $3(4x^2 - 1)(2x^2 - 1) = 0$ oe. This is implied by $x^2 = \frac{1}{2}$ following use of quadratic formula or having identified a quadratic in x^2 e.g. by $y = x^2$ and $24y^2 - 18y + 3 \rightarrow y = \dots$ dM1 Solves their $x^2 = \frac{1}{2} \Rightarrow x = \pm \frac{\sqrt{2}}{2}$ by taking square root of their positive rootA1 $x = \frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$ or $\sqrt{\frac{1}{2}}$ but must be positive solution only. Ignore the $x = \frac{1}{2}$ but A0 if others given.

NB Answers directly from the quartic with no further working score M1A0dM0A0



Question Number	Scheme	Marks
2.(a)	Attempts $(x \pm 2)^2 + (y \pm 5)^2 \dots = 0$ (i) Centre $(-2, 5)$ (ii) Radius $\sqrt{50}$ or $5\sqrt{2}$	M1 A1 B1 (3)
(b)	Gradient of radius $= \frac{(5)-4}{(-2)-5} = -\frac{1}{7}$ which needs to be in simplest form Uses $m_2 = -\frac{1}{m_1}$ to find gradient of tangent Equation of tangent $y - 4 = -\frac{1}{7}(x - 5) \Rightarrow y = -\frac{1}{7}x + \frac{29}{7}$	B1ft M1 M1 A1 (4) (7 marks)
Notes		
(a)	Note that the open set up here is M1 M1 B1	
M1	Attempts to complete the square on both terms or states the centre as $(\pm 2, \pm 5)$ For completing the square look for $(x \pm 2)^2 + (y \pm 5)^2 \dots = \dots$	
A1	Centre $(-2, 5)$ Allow $x = -2, y = 5$ This alone can score both marks even following incorrect lines eg $(x + 2)^2 + (y - 5)^2 = \dots$ where $\dots$ could be $\dots$, for example a minus sign or blank	
A1	Radius $\sqrt{50}$ or $5\sqrt{2}$ You may award after a correct answer.	
	If a candidate attempts to use $x^2 + y^2 + 2fx + 2gy + c = 0$ then M1 may be awarded for a centre of $(\pm 2, \pm 5)$	
(b)	Note that the open set up here is M1 M1 M1 A1	
B1ft	Correct answer for the gradient of the line joining $P(5, 4)$ to their centre. You may award on their centre but the value must be fully simplified.	
M1	Awarded for using $m_2 = -\frac{1}{m_1}$ to find gradient of tangent. Do be aware that some good candidates may do the first two marks at once so you may need to look at what value they are using for the gradient of the tangent.	
M1	For an attempt to find the equation of the tangent using $P(5, 4)$ and a changed gradient. Condone bracketing slips only. If the candidate uses the form $y = mx + c$ they must use x and y the correct way around and proceed as far as $c = \dots$	
A1	$y = 7x - 31$ stated. It must be written in this form. (It cannot be awarded from $y = mx + c$ by just stating $c = -31$)	
	 Attempts at (b) using differentiation.	
B1	$x^2 + y^2 + 4x - 10y - 21 = 0 \rightarrow 2x + 2y \frac{dy}{dx} + 4 - 10 \frac{dy}{dx} = 0$.	
M1	Substitutes $P(5, 4)$ into an expression of the form $ax + by \frac{dy}{dx} + c + d \frac{dy}{dx} = 0$ AND finds the value of $\frac{dy}{dx} = (7)$. The values of a, b, c and d must be non-zero.	

M1 Uses $m = \left. \frac{dy}{dx} \right|_{x=5}$ with $P(5, 4)$ to find equation of tangent

A1 $y = 7x - 31$



Question Number	Scheme	Marks
3. (i)	$(x-4)^2 \geq 2x-9 \Rightarrow x^2-10x+25 \dots 0$ $\Rightarrow (x-5)^2 \dots 0$ Explains that "square numbers are greater than or equal to zero" hence (as $x \in \mathbb{R}$), $\Rightarrow (x-4)^2 \geq 2x-9$ *	M1 A1 A1* (3)
(ii)	Shows that it is not true for a value of n Eg. When $n=3$, $2^n+1=8+1=9 \times$ Not prime	B1 (1) (4 marks)
Notes		
(i) A proof starting with the given statement M1 Attempts to expand $(x-4)^2$ and work from form $(x-4)^2 \dots 2x-9$ to form a 3TQ on one side of an equation or an inequality A1 Achieves both $x^2-10x+25$ and $(x-5)^2$. Allow $(x-5)^2$ written as $(x-5)(x-5)$ A1* For a correct proof. Eg "square numbers are greater than or equal to zero", hence (as $x \in \mathbb{R}$), $(x-5)^2 \geq 0$ $\Rightarrow (x-4)^2 \geq 2x-9$ This requires (1) Correct algebra throughout, (2) a correct explanation concerning square numbers and (3) a reference back to the original statement Answers via b^2-4ac are unlikely to be correct. Whilst it is true that there is only one root and therefore it touches the x-axis, it does not show that it is always positive. The explanation could involve a sketch of $y=(x-5)^2$ but it must be accurate with a minimum on the +ve x axis with some statement alluding to why this shows $(x-5)^2 \geq 0$ Approaches via odd and even numbers will usually not score anything. They would need to proceed using the main scheme via $(2m-4)^2 \geq 4m-9$ and $(2m-1-4)^2 \geq 2(2m-1)-9$ Alt to (i) via contradiction Proof by contradiction is acceptable and marks in a similar way M1 For setting up the contradiction 'Assume that there is an x such that $(x-4)^2 < 2x-9 \Rightarrow x^2-10x+25 \dots 0$ A1 $\Rightarrow (x-5)^2 \dots 0$ or $(x-5)(x-5) \dots 0$ A1* This is not true as square numbers are always greater than or equal to 0, hence $(x-4)^2 \geq 2x-9$ Alt to part (i) States $(x-5)^2 \geq 0$ $\Rightarrow x^2-10x+25 \geq 0$ $\Rightarrow x^2-8x-16 \geq 2x-9$ $\Rightarrow (x-4)^2 \geq 2x-9$		

Question Number	Scheme	Marks
M1	States $(x-5)^2 \geq 0$ and attempts to expand. There is no explanation required here	
A1	Rearranges to reach $x^2 - 8x - 16 \geq 2x - 9$	
A1*	Reaches the given answer $(x-4)^2 \geq 2x - 9$ with no errors	
.....		
(ii)		
B1	Shows that it is not true for a value of n This requires a calculation (and value found) with a minimal statement that it is not true Eg. ‘ $2^6 + 1 = 65$ which is not prime’ or ‘ $2^5 + 1 = 33 \times$ ’ Condone sloppily expressed proofs. Eg. ‘ $2^7 + 1 = \frac{129}{3} = 43$ which is not prime’ Condone implied proofs where candidates write $2^5 + 1 = 33$ which has a factor of 11 If there are lots of calculations mark positively. Only one value is required to be found (with the relevant statement) to score the B1 The calculation cannot be incorrect. Eg. $2^3 + 1 = 10$ which is not prime	

Question Number	Scheme	Marks
4.(a)	$\left(2 - \frac{1}{4}x\right)^6 = 2^6 + {}^6C_1 2^5 \left(-\frac{1}{4}x\right)^1 + {}^6C_2 2^4 \left(-\frac{1}{4}x\right)^2 + {}^6C_3 2^3 \left(-\frac{1}{4}x\right)^3 + \dots$ $= 64 - 48x + 15x^2 - 2.5x^3$	B1, M1 A1 A1 (4)
(b)	$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 = (64 - 48x + 15x^2 - 2.5x^3) + (64 + 48x + 15x^2 + 2.5x^3)$ $\approx 128 + 30x^2$	M1 B1ft A1 (3) (7 marks)

Notes

(a)

B1 For either 2^6 or 64. Award for an unsimplified ${}^6C_0 2^6 \left(-\frac{1}{4}x\right)^0$

M1 For an attempt at the binomial expansion. Score for a correct attempt at term 2, 3 or 4.

Accept sight of ${}^6C_1 2^5 \left(\pm \frac{1}{4}x\right)^1 {}^6C_2 2^4 \left(\pm \frac{1}{4}x\right)^2 {}^6C_3 2^3 \left(\pm \frac{1}{4}x\right)^3$ condoning omission of brackets.

Accept any coefficient appearing from Pascal's triangle. FYI 6, 15, 20

A1 For any two simplified terms of $-48x + 15x^2 - 2.5x^3$

A1 For $64 - 48x + 15x^2 - 2.5x^3$ ignoring terms with greater powers. This may be awarded in (b) if it is not fully simplified in (a). Allow the terms to be listed $64, -48x, 15x^2, -2.5x^3$. Isw after sight of correct values. The expression written out without any method can be awarded all 4 marks.

(b) **Note that this is now marked M1 B1 A1**

M1 For adding two sequences that must be of the correct form with the correct signs.

Look for $(A - Bx + Cx^2 - Dx^3) + (A + Bx + Cx^2 + Dx^3)$ but condone

$$(A - Bx + Cx^2) + (A + Bx + Cx^2)$$

For this to be scored there must be some negative terms in (a)

B1ft For one correct term (follow through). Usually $a = 128$ but accept either $a = 2 \times \text{'their' +ve } 64$ or $b = 2 \times \text{'their' +ve } 15$

A1 For $128 + 30x^2$. CSO so must be from $(64 - 48x + 15x^2 - 2.5x^3) + (64 + 48x + 15x^2 + 2.5x^3)$

Allow $a = 128, b = 30$ following correct work.

This is a show that question so M1 must be awarded. It must be their final answer so do not isw here.

Alternative method in (a):

$$\left(2 - \frac{1}{4}x\right)^6 = 2^6 \left(1 - \frac{1}{8}x\right)^6 = 2^6 \left(1 + 6\left(-\frac{1}{8}x\right) + \frac{6 \times 5}{2} \left(-\frac{1}{8}x\right)^2 + \frac{6 \times 5 \times 4}{3!} \left(-\frac{1}{8}x\right)^3 + \dots\right)$$

B1 For sight of factor of either 2^6 or 64

M1 For an attempt at the binomial expansion seen in at least one term within the brackets. Score for a correct attempt at term 2, 3 or 4.



Question Number	Scheme	Marks
1(a)	$h = 0.5$	B1
	$A \approx \frac{1}{2} \times \frac{1}{2} \{2.287 + 2.834 + 2(4.470 + 6.719 + 7.291)\}$	M1
	= awrt 10.52	A1
		(3)
(b)(i)	$A \approx "10.52" - 4$	M1
(b)(ii)	= 6.52	A1ft
	$A \approx "10.52"$	B1ft
		(3)
		Total 6

(a)

B1 $h = 0.5$ oe seen or implied by sight of $\frac{0.5}{2}$ in front of the bracket. May also be implied by a correct answer if no incorrect working seen. $h = -0.5$ is B0

M1 Correct application of the trapezium rule with their h . Look for a correct bracket structure, condoning slips copying values from the table or the omission of the final brackets on the rhs

e.g. $\frac{1}{2} \times \frac{1}{2} (2.287 + 2.834 + 2(4.470 + 6.719 + 7.291))$ is M1

but $\frac{1}{2} \times \frac{1}{2} \times 2.287 + 2.834 + 2(4.470 + 6.719 + 7.291)$ is M0 unless the brackets are recovered or implied by their answer (you may need to check this).

Also allow for a correct method adding individual trapezia using their h condoning copying errors but the brackets must be correct (or recovered or implied by later work)

e.g. $\frac{1}{2} \times \frac{1}{2} (2.287 + 4.470) + \frac{1}{2} \times \frac{1}{2} (4.470 + 6.719) + \frac{1}{2} \times \frac{1}{2} (6.719 + 7.291) + \frac{1}{2} \times \frac{1}{2} (7.291 + 2.834)$

A1 awrt 10.52 isw once a correct answer is seen. Correct answer with no working scores B1M1A1 but if there is evidence of using $h = -0.5$ then maximum awarded is B0M1A0

(b)(i) Note that they must be using their answer to part (a) to score marks in (b)

M1 Attempts to subtract 4 from their answer to part (a) or allow for the expression $"10.52" - [2x]_{-1}^1$ or e.g. $"10.52" - (2 + 2)$. Do not condone omission of brackets or arithmetical errors here. This mark cannot be scored from a new attempt at the trapezium rule.
 $"10.52" - (2 - 2)$ is M0

A1ft awrt 6.52 correct answer or correct ft (allowing if they proceed to round a more accurate answer in (a) to 2dp). In both cases they must be using their answer to part (a) function which could even come from integration.

(ii)

B1ft awrt 10.52 Writes down the same answer as part (a) (follow through their (a)). Condone "same as **part (a)**" (again allow if they proceed to round a more accurate answer in (a) to 2dp)

Note (a) 10.5 (b)(i) 6.5 (b)(ii) 10.5 could score B1M1A0M1A1ftB1ft



Question Number	Scheme	Marks
2(a)	$(S =) 6x^2 + 6xh + 2xh$	B1
	eg $V = 3x^2h = 972 \Rightarrow h = \frac{972}{3x^2}$ or eg $hx = \frac{324}{x}$ $\Rightarrow (S =) 6x^2 + 8x\left(\frac{972}{3x^2}\right)$ or $\Rightarrow (S =) 6x^2 + 8\left(\frac{324}{x}\right)$	M1
	$S = 6x^2 + \frac{2592}{x} *$	A1*
		(3)
(b)	$\left(\frac{dS}{dx} =\right) 12x - \frac{2592}{x^2}$	B1
		(1)
(c)	$12x - \frac{2592}{x^2} = 0 \Rightarrow 12x^3 = 2592$ $\Rightarrow x = \sqrt[3]{\frac{2592}{12}}$	M1
	$x = 6$	A1
		(2)
(d)	$\left(\frac{d^2S}{dx^2} =\right) 12 + \frac{5184}{x^3}$	B1ft
	$\frac{d^2S}{dx^2} > 0$ when $x = 6$ so minimum	B1
		(2)
(e)	$S = 6(6)^2 + \frac{2592}{6} = 648 \text{ (cm}^2\text{)}$	B1
		(1)
		Total 9

(a)

B1 Correct expression for the surface area in any form.

It may be implied by a correct expression for the surface area in terms of x which is not the given answer. Once a correct expression is seen then award B1.

M1 Uses the given volume in a dimensionally correct formula to obtain h or hx in terms of x and substitutes into their expression for S of the form $\dots xh + \dots x^2$ or equivalent e.g. $\dots xh + \dots xh + \dots x^2$

It may be implied by a correct expression for the surface area in terms of x which is not the given answer.

A1* Achieves the given answer with no errors including bracketing omissions.

As a minimum you must see:

- the separate equation for the volume
- a substitution before seeing the $\frac{2592}{x}$ term
- $S =$ in their solution at some point (or e.g. SA = or surface area = or Area=)



Examples:

e.g. $6x^2 + 6x\left(\frac{324}{x^2}\right) + 2x\left(\frac{324}{x^2}\right)$ scores B1M1A0

e.g. $6x^2 + 8x\left(\frac{972}{3x^2}\right) \Rightarrow S = 6x^2 + \frac{2592}{x}$ scores B1M1A0 (does not write a separate equation for the volume)

e.g. $3x^2h = 972 \Rightarrow S = 6x^2 + 6x\left(\frac{324}{x^2}\right) + 2x\left(\frac{324}{x^2}\right)$
 $= 6x^2 + \frac{2592}{x}$

scores B1M1A1 (separate equation for the volume, a substitution before seeing the $\frac{2592}{x}$ term and $S =$ seen in their solution at some point.

(b) Mark (b), (c), (d) and (e) altogether

B1 $\left(\frac{dS}{dx} = \right) 12x - \frac{2592}{x^2}$ in any form eg $12x - 2592x^{-2}$

(c)

M1 Starts from a derivative of the form $Ax - Bx^{-2}$ oe, sets equal to 0 (seen or implied) where $A \times B > 0$, and solves via a correct method to find an expression or value for $x = \sqrt[3]{\frac{B}{A}}$ (you may need to check this)

$\sqrt[3]{\frac{2592}{12}}$ scores M1.

Stating 6 without a **correct derivative from (b)** is M0.

M1 6 cao with no incorrect working seen in their method. Withhold this mark if ± 6 seen

Minimum acceptable for full marks is $12x - \frac{2592}{x^2} = 0 \Rightarrow x = 6$

Note this final mark is only possible from a correct derivative

(Beware of incorrect derivatives leading to the correct value of x e.g. $6x - \frac{1296}{x^2} = 0 \Rightarrow x = 6$)

(d)

B1ft Correct second derivative. Follow through their first derivative of the form $Ax - Bx^{-2}$ so award for $A + \frac{2B}{x^3}$ or equivalent.

B1 Fully correct justification with a conclusion (or preamble with eg tick/QED etc) following fully correct work to find $\frac{d^2S}{dx^2}$. They must either:

- substitute $x = 6$ into $12 + \frac{5184}{x^3}$ (if evaluated achieving 36) and state e.g. $(36) > 0$ oe **hence minimum**



Question	Scheme	Marks
9(a)	$V = hl^2 \Rightarrow 250000 = hl^2$ or $l = \frac{500}{\sqrt{h}}$ oe (may use e.g. $hl = \sqrt{250000h}$)	B1
	$S = l^2 + 4hl = \frac{250000}{h} + 4h \times \frac{500}{\sqrt{h}}$	M1
	$S = \frac{250000}{h} + 2000\sqrt{h}$ *	A1*
		(3)
(b)	$\frac{dS}{dh} = -\frac{250000}{h^2} + 2000 \times \frac{1}{2} h^{-\frac{1}{2}}$ oe	M1A1
	$\frac{dS}{dh} = 0 \Rightarrow -\frac{250000}{h^2} + 2000 \times \frac{1}{2} h^{-\frac{1}{2}} = 0 \Rightarrow h^k = \dots$	dM1
	$\Rightarrow h^{\frac{3}{2}} = 250 \Rightarrow h = \dots$	ddM1
	$h = 250^{\frac{2}{3}}$	A1
		(5)
(c)	$\frac{d^2S}{dh^2} = \frac{500000}{h^3} - 500h^{-\frac{3}{2}}$	M1
	$\left. \frac{d^2S}{dh^2} \right _{h=39.7} = 6 > 0$ hence gives the minimum value.	A1
		(2)
(10 marks)		

Notes:**(a)****B1:** Correct equation linking h and l seen or implied by substitution.**M1:** Attempts the surface area and substitutes to eliminate l . Allow $l^2 + 2(hl + hl)$ or $2(l^2 + hl + hl)$ (or equivalents) for the attempt at surface area.**A1*:** Achieves the given result with no errors.**(b)**Allow if $\frac{dS}{dh}$ is missing or called $\frac{dy}{dx}$ throughout.**M1:** Attempts derivative of the area function – power decreased by one in at least one term.**A1:** Correct derivative. Need not be simplified.**dM1:** Sets derivative equal to zero and attempts to solve as far as a power of h equal to something. Depends on first M**ddM1:** Depends on both M marks, attempts to solve an equation with a fractional index to find h **A1:** $h = 250^{\frac{2}{3}}$ or states $k = \frac{2}{3}$ 

(c)

M1: Attempts the second derivative, form $\frac{A}{h^3} - Bh^{-\frac{3}{2}}$ reached ($A, B > 0$). Note: the question specifies by further differentiation, so first derivative test attempts score M0.

A1: Second derivative correct and substitutes h to get awrt 6, and makes correct conclusion referring to sign (>0 or positive). Must see the substitution as it is not obviously positive but allow if expression is not fully simplified as long as it is clear it is positive (e.g. $8 - 2 > 0$).



Question Number	Scheme		Marks
3.(a)	OPEN TOPPED $120 = 3x^2y \Rightarrow y = \frac{40}{x^2}$ $A = 3x^2 + 8xy$ $= 3x^2 + 8x \times \frac{40}{x^2} = 3x^2 + \frac{320}{x}$	CLOSED CONTAINER $120 = 3x^2y \Rightarrow y = \frac{40}{x^2}$ $A = 6x^2 + 8xy$ $= 6x^2 + 8x \times \frac{40}{x^2} = 6x^2 + \frac{320}{x}$	M1, A1
			dM1, A1
(b)	$\frac{dA}{dx} = 6x - \frac{320}{x^2}$ $\frac{dA}{dx} = 6x - \frac{320}{x^2} = 0$ $\Rightarrow x^3 = \frac{160}{3} \Rightarrow x = \text{awrt } 3.76$	$\frac{dA}{dx} = 12x - \frac{320}{x^2}$ $\frac{dA}{dx} = 12x - \frac{320}{x^2} = 0$ $\Rightarrow x^3 = \frac{80}{3} \Rightarrow x = \text{awrt } 2.99$	M1, A1ft
			dM1, A1
(c)	Attempts $\left. \frac{d^2A}{dx^2} \right _{x=3.76} = 6 + \frac{640}{x^3}$ $= 6 + \frac{640}{3.76^3} = ..$ $\left. \frac{d^2A}{dx^2} \right _{x=3.76} = \text{awrt } 18 > 0 \text{ MINIMUM*}$	Attempts $\left. \frac{d^2A}{dx^2} \right _{x=2.99} = 12 + \frac{640}{x^3}$ $= 12 + \frac{640}{2.99^3} = ..$ $\left. \frac{d^2A}{dx^2} \right _{x=2.99} = \text{awrt } 36 > 0 \text{ MINIMUM*}$	M1
			A1*
			(10 marks)

Note that candidates may be confused regarding open topped or closed container so both cases can potentially score full marks if the case is consistently applied.

(a)

M1: Sets $120 = kx^2y$ o.e such as $120 = kx \times x \times y$

A1: $y = \frac{40}{x^2}$ or $xy = \frac{40}{x}$ or equivalent such as $y = \frac{120}{3x^2}$

It can be scored from an embedded expression within an attempt at A .

It can be implied following a correct $120 = 3x^2y$ o.e. and a correct expression for A in terms of x

dM1: Achieves an expression of the form $(A =) px^2 + qxy$, $p, q > 0$ **AND** substitutes in their y or xy of

the form $y = \frac{K}{x^2}$ or $xy = \frac{K}{x}$ which may be implied by a correct expression for A in terms of x

A1: Achieves the answer of Area / $A = 3x^2 + \frac{320}{x}$ **or** Area/ $A = 6x^2 + \frac{320}{x}$

Open Container

Closed Container

The LHS $A =$ or an equivalent such as 'Area =' must appear at some point.

Condone A appearing, for example, as SA or S or Area (of sheet metal) or other but a LHS must be seen.



Mark parts (b) and (c) together.

(b) Condone $\frac{dA}{dx}$ appearing as $\frac{dy}{dx}$, y' or A' or the absence of a LHS **throughout part (b)**

M1: $\frac{dA}{dx} = \alpha x \pm \frac{\beta}{x^2}$.

Allow this mark to be scored with the constants P and Q or for made up values of P and Q

A1ft: $\frac{dA}{dx} = 6x - \frac{320}{x^2}$ o.e. or $\frac{dA}{dx} = 12x - \frac{320}{x^2}$ following through on their numerical P and Q

Open Container

Closed Container

It can be scored for made up values of P and Q

dM1: Sets $\frac{dA}{dx} = \alpha x - \frac{\beta}{x^2} = 0$ and proceeds to a numerical $x^3 = k$ $k > 0$ but allow $\alpha x^3 = \beta$ $\alpha\beta > 0$

Condone $\frac{dA}{dx} = \alpha x - \frac{\beta}{x^2} = 0$ leading to a correct value for x for their $\frac{dA}{dx} = 0$

A1: $x = \text{awrt } 3.76$ or $x = \text{awrt } 2.99$

Open Container

Closed Container

(c) Condone $\frac{d^2 A}{dx^2}$ being written as A'' throughout part (c)

M1: Achieves $\frac{d^2 A}{dx^2} = \delta \pm \frac{\epsilon}{x^3}$ and

Open Container

Closed Container

- Either attempts to find its value at their 3.76 or 2.99
- Or states it is positive (even following an incorrect value)

Condone $\frac{d^2 A}{dx^2}$ as $\frac{d^2 y}{dx^2}$ for this mark only

Allow this mark to be scored with the constants P and Q or for made up values of P and Q

A1*: Scored for

- A correct $\frac{d^2 A}{dx^2}$ including the LHS. So look for $\frac{d^2 A}{dx^2} = 6 + \frac{640}{x^3}$ or $\frac{d^2 A}{dx^2} = 12 + \frac{640}{x^3}$

Open Container

Closed Container

- A correct statement and conclusion

Open Container

Closed Container

Either $\left. \frac{d^2 A}{dx^2} \right|_{x=3.76} = \text{awrt } 18 > 0$ and MINIMUM $\left. \frac{d^2 A}{dx^2} \right|_{x=2.99} = \text{awrt } 36 > 0$ and MINIMUM

Or $\frac{d^2 A}{dx^2} = 6 + \frac{640}{x^3} > 0$ as $x > 0$ MINIMUM $\frac{d^2 A}{dx^2} = 12 + \frac{640}{x^3} > 0$ as $x > 0$ MINIMUM

Or $\frac{d^2 A}{dx^2} = 6 + \frac{640}{x^3} > 0$ when $x = 3.76$, MINIMUM $\frac{d^2 A}{dx^2} = 12 + \frac{640}{x^3} > 0$ when $x = 2.99$



MINIMUM

Open Container Closed Container

The mark can only be awarded following $x = \text{awrt } 3.76$ or $x = \text{awrt } 2.99$

Condone statements like ' $x = 3.76$ is the minimum value of A as $\frac{d^2 A}{dx^2} > 0$ '.

Look for the key aspects ' $\frac{d^2 A}{dx^2} > 0$ ' and 'minimum'. If in doubt use review



Question Number	Scheme	Marks
9 (a)	Attempts $12 = 3x^2 \times L \Rightarrow L = \frac{4}{x^2}$ or $Lx = \frac{4}{x}$	M1, A1
	$S = 6x^2 + 5xL = 6x^2 + \frac{20}{x}$	dM1, A1
		(4)
(b)	$\frac{dS}{dx} = 12x - \frac{20}{x^2}$	M1, A1ft
	$\frac{dS}{dx} = 0 \Rightarrow x^3 = \frac{5}{3} \Rightarrow x = 1.18563\dots$	dM1
	$S _{x=1.18563} = 6 \times "1.18563"^2 + \frac{20}{"1.18563"} = 25.3(\text{m}^2)$	ddM1, A1
		(5)
(c)	$\left(\frac{d^2S}{dx^2}\right) = 12 + \frac{40}{x^3}$	M1
	Justifies $\frac{d^2S}{dx^2} > 0$ (at $x = 1.18563\dots$), hence minimum *	A1*
		(2)
		(11 marks)

(a)

M1: Attempts to use a volume formula to make L or Lx the subject. May be seen substituted into a surface area expression which is not in the required form of the final answer.

Allow $12 = \alpha x^2 L \Rightarrow L = \frac{\beta}{x^2}$ or $Lx = \frac{\beta}{x}$

A1: $L = \frac{4}{x^2}$ OR $Lx = \frac{4}{x}$ o.e. Maybe seen substituted into the equation for the surface area.

dM1: Substitutes $L = \frac{"4"}{x^2}$ OR $Lx = \frac{"4"}{x}$ into $S = Rx^2 + TxL$ which may be unsimplified to produce a formula of the correct form. It is dependent on the previous method mark.

A1: $S = 6x^2 + \frac{20}{x}$

Must see " $S =$ " or e.g. "(surface) area =" connected at some point with $6x^2 + \frac{20}{x}$



They may work algebraically using P and Q . They can still score M1A1ft in (b) but they would have to substitute in their values to score any further marks in (b) or any marks in (c)

- (b) **Condone other letters used instead of S and x for $\frac{dS}{dx}$ including y for S .**

Accept use of e.g. S'

M1: Correct form for the derivative $\left(\frac{dS}{dx} = \right) ax + \frac{b}{x^2}$

A1ft: Follow through on the derivative of their $S = Px^2 + \frac{Q}{x} \Rightarrow 2Px - \frac{Q}{x^2}$

dM1: Solves $\frac{dS}{dx} = 0 \Rightarrow x = \dots$ Do not be concerned by the mechanics of the rearrangement as long as a value for x is achieved. $=0$ may be implied by further work or if just a correct value for x then you may need to check this. It is dependent on the previous method mark.

ddM1: Substitutes their value of x in S to find S . Condone slips. It is dependent on the previous two method marks.

If only a value is stated you may need to check this.

A1: awrt 25.3 (m^2) following correct differentiation. Units not required.

- (c) **Note the final A1* can only be scored using a correct expression for $\frac{d^2S}{dx^2}$ or $\frac{dS}{dx}$ (depending on the approach taken). Condone other letters used instead of S and x for $\frac{d^2S}{dx^2}$ including y for S . Accept use of e.g. S''**

If second differentiation is seen in (b) then it must be seen or used in (c) to score.

M1: Differentiates and finds $\frac{d^2S}{dx^2}$ of the form $M + \frac{N}{x^3}$ (do not be concerned about signs for this mark).

Alternatively, substitutes in values of x above and below their 1.1856.. and proceeds to find values for $\frac{dS}{dx}$ at these points.

A1*: **Either** justifies $\frac{d^2S}{dx^2} > 0$, which must be a correct expression AND gives a minimal conclusion

Requires one of:

- a correct value for $\frac{d^2S}{dx^2}$ (awrt 36 if using 1.18(56..) but allow awrt 35.1 if using 1.2) from a correct expression, a correct comparison with 0 and a conclusion e.g. min
- deduces that as 12 and 40 are both positive and the correct value of x (awrt 1.2) > 0 then $\frac{d^2S}{dx^2} > 0$ with a conclusion e.g. proven

Or justifies there is a change of sign for $\frac{dS}{dx}$ AND gives a minimal conclusion

Requires both calculations to be correct using a correct expression for $\frac{dS}{dx}$ (you will need to check this)