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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Integration: Definite Integrals, Areas & the Trapezium Rule (P2.8)

International A Level Mathematics · P2: Pure Mathematics 2 (WMA12)

32 questions · 250 marks · Mark schemes included

Past papers June 2019 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

32 questions · 250 marks · 2 sub-topics

SUB-TOPIC P2.8a

DEFINITE INTEGRALS & AREAS UNDER CURVES

Sub-topic P2.8a · 18 questions · 161 marks · lowest tariff first



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 5 marks)



Question 4 continued

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(Total 6 marks)

Q4



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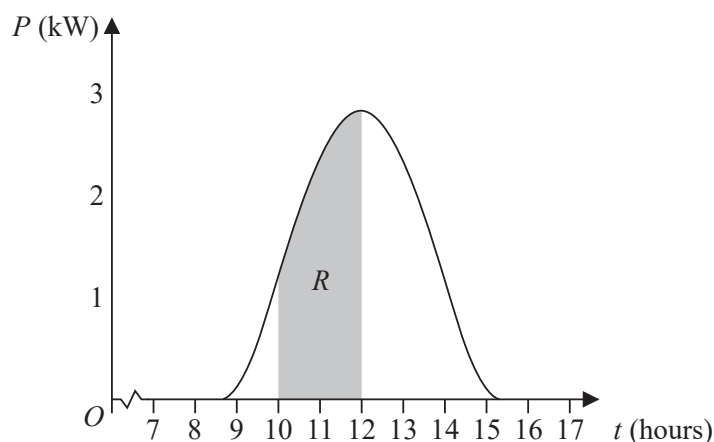


Figure 1

Solar panels are installed on the roof of a building.

The power, P , produced on a particular day, in kW, can be modelled by the equation

$$P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2 \quad 8.5 \leq t \leq 15.2$$

where t is the time in hours after midnight. The graph of P against t is shown in Figure 1.

A table of values of t and P is shown below, with the values of P given to 4 significant figures where appropriate.

Time, t (hours)	10	10.5	11	11.5	12
Power, P (kW)		1.882	2.45		2.95

- (a) Use the given equation to complete the table, giving the values of P to 4 significant figures where appropriate.

(2)

The amount of energy, in kWh, produced between 10:00 and 12:00 can be found by calculating the area of region R , shown shaded in Figure 1.

- (b) Use the trapezium rule, with all the values of P in the completed table, to find an estimate for the amount of energy produced between 10:00 and 12:00. Give your answer to 2 decimal places.

(4)

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Question 7 continued

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(Total 6 marks)

Q7



9.

In this question you must show all stages of your working.

Solutions based entirely on calculator technology are not acceptable.

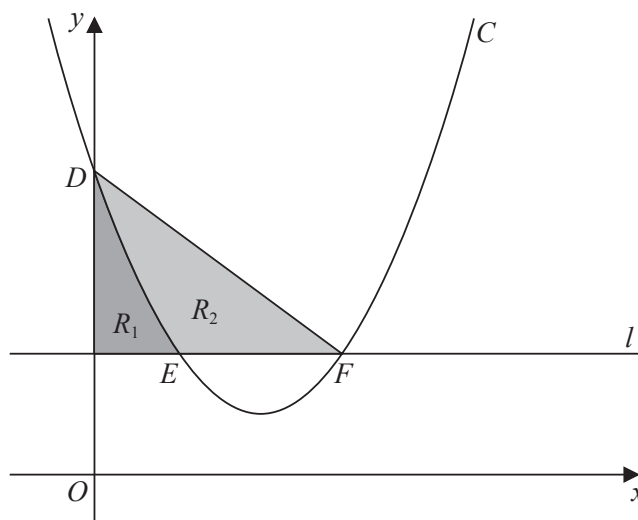


Figure 3

Figure 3 shows

- the curve C with equation $y = x^2 - 4x + 5$
- the line l with equation $y = 2$

The curve C intersects the y -axis at the point D .(a) Write down the coordinates of D .

(1)

The curve C intersects the line l at the points E and F , as shown in Figure 3.(b) Find the x coordinate of E and the x coordinate of F .

(2)

Shown shaded in Figure 3 is

- the region R_1 which is bounded by C , l and the y -axis
- the region R_2 which is bounded by C and the line segments EF and DF

Given that $\frac{\text{area of } R_1}{\text{area of } R_2} = k$, where k is a constant,(c) use algebraic integration to find the exact value of k , giving your answer as a simplified fraction.

(5)







(Total for Question 9 is 8 marks)



9.

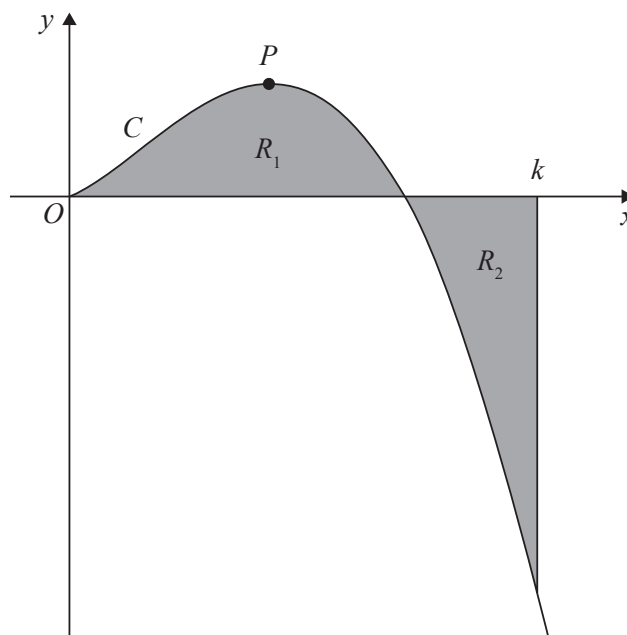


Figure 1

Figure 1 is a sketch of the curve C with equation

$$y = 2x^{\frac{3}{2}}(4 - x) \quad x \geq 0$$

The point P is the stationary point of C .

(a) Find, using calculus, the x coordinate of P .

(4)

The region R_1 , shown shaded in Figure 1, is bounded by C and the x -axis.

The region R_2 , also shown shaded in Figure 1, is bounded by C , the x -axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) find, using calculus, the exact value of k .

(4)

Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 8 marks)



8.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

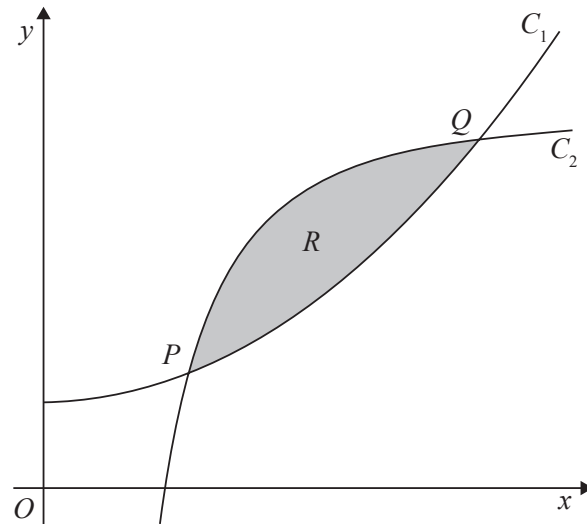


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation

$$y = x^2 + 3 \quad x > 0$$

and part of the curve C_2 with equation

$$y = 13 - \frac{9}{x^2} \quad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q as shown in Figure 1.(a) Use algebra to find the x coordinate of P and the x coordinate of Q .

(4)

The finite region R , shown shaded in Figure 1, is bounded by C_1 and C_2 .(b) Use algebraic integration to find the exact area of R .

(4)



Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)



Question 8 continued

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Question 8 continued

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Question 8 continued

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Q8

(Total 9 marks)



6.

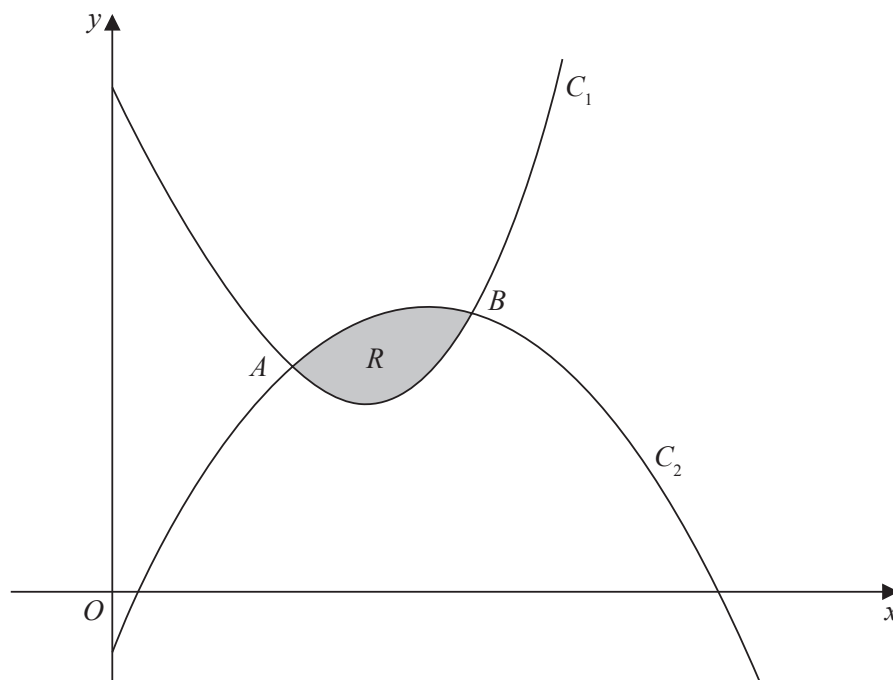


Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2 with equations

$$\begin{aligned} C_1 : y &= x^3 - 6x + 9 & x &\geq 0 \\ C_2 : y &= -2x^2 + 7x - 1 & x &\geq 0 \end{aligned}$$

The curves C_1 and C_2 intersect at the points A and B as shown in Figure 1.

The point A has coordinates $(1, 4)$.

Using algebra and showing all steps of your working,

(a) find the coordinates of the point B .

(4)

The finite region R , shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R .

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total 9 marks)

Q6



Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 10 continued

Lined area for writing the answer to Question 10.



Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS



10.

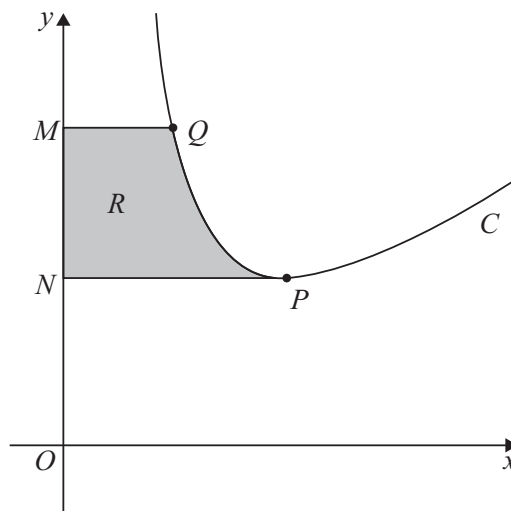


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with equation

$$y = 2x + \frac{64}{x^2} - 3 \quad x > 0$$

The point P , shown in Figure 3, is the stationary point on C .

(a) Show, using calculus, that the x coordinate of P is 4

(4)

The point Q lies on C and has x coordinate 2

The line segments MQ and NP , shown in Figure 3, are parallel to the x -axis.

The region R , shown shaded in Figure 3, is bounded by C , the y -axis and line segments MQ and NP .

(b) Use algebraic integration to find the exact area of R .

You must make your method clear.

(5)

Question 10 continued



Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS



5.

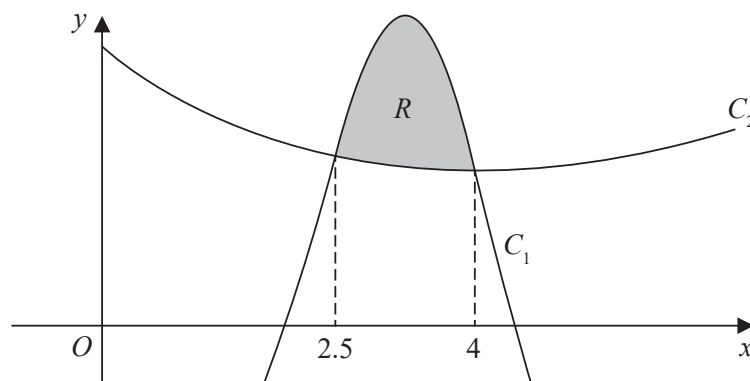


Figure 2

Figure 2 shows a sketch of part of the graph of the curves C_1 and C_2

The curves intersect when $x = 2.5$ and when $x = 4$

A table of values for some points on the curve C_1 is shown below, with y values given to 3 decimal places as appropriate.

x	2.5	2.75	3	3.25	3.5	3.75	4
y	5.453	7.764	9.375	9.964	9.367	7.626	5

Using the trapezium rule with all the values of y in the table,

- (a) find, to 2 decimal places, an estimate for the area bounded by the curve C_1 , the line with equation $x = 2.5$, the x -axis and the line with equation $x = 4$

(4)

The curve C_2 has equation

$$y = x^{\frac{3}{2}} - 3x + 9 \quad x > 0$$

- (b) Find $\int \left(x^{\frac{3}{2}} - 3x + 9 \right) dx$

(3)

The region R , shown shaded in Figure 2, is bounded by the curves C_1 and C_2

- (c) Use the answers to part (a) and part (b) to find, to one decimal place, an estimate for the area of the region R .

(3)

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Question 5 continued

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Question 5 continued

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Question 5 continued

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(Total 10 marks)

Q5



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8. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

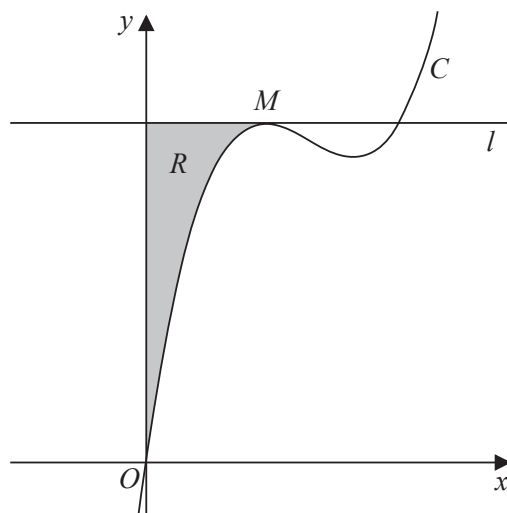


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{4}{3}x^3 - 11x^2 + kx \quad \text{where } k \text{ is a constant}$$

The point M is the maximum turning point of C and is shown in Figure 2.

Given that the x coordinate of M is 2

- (a) show that $k = 28$ (3)
- (b) Determine the range of values of x for which y is increasing. (2)

The line l passes through M and is parallel to the x -axis.

The region R , shown shaded in Figure 2, is bounded by the curve C , the line l and the y -axis.

- (c) Find, by algebraic integration, the exact area of R . (5)

Question 8 continued

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Lined area for writing the answer to Question 8 continued.



Question 8 continued

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Lined area for writing the answer to Question 8 continued.



Question 8 continued

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(Total 10 marks)

Q8



Question 9 continued

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Question 9 continued

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Lined area for writing the answer to Question 9 continued.



Question 9 continued

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(Total 10 marks)

Q9



10. The curve C has equation

$$y = \frac{(x - k)^2}{\sqrt{x}} \quad x > 0$$

where k is a **positive** constant.

(a) Show that

$$\int_1^{16} \frac{(x - k)^2}{\sqrt{x}} \, dx = ak^2 + bk + \frac{2046}{5}$$

where a and b are integers to be found.

(5)

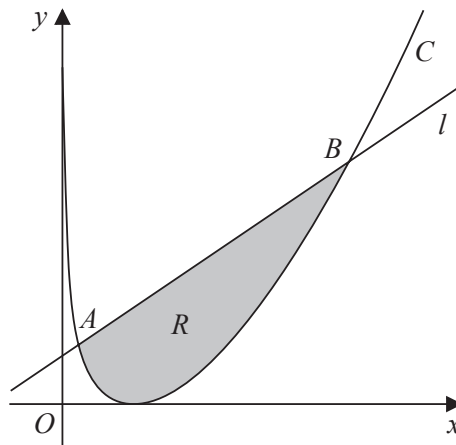


Figure 1

Figure 1 shows a sketch of the curve C and the line l .

Given that l intersects C at the point $A(1, 9)$ and at the point $B(16, q)$ where q is a constant,

(b) show that $k = 4$

(2)

The region R , shown shaded in Figure 1, is bounded by C and l

Using the answers to parts (a) and (b),

(c) find the area of region R

(3)



Question 10 continued

Lined area for writing the answer to Question 10.



Question 10 continued

(Total for Question 10 is 10 marks)



10.

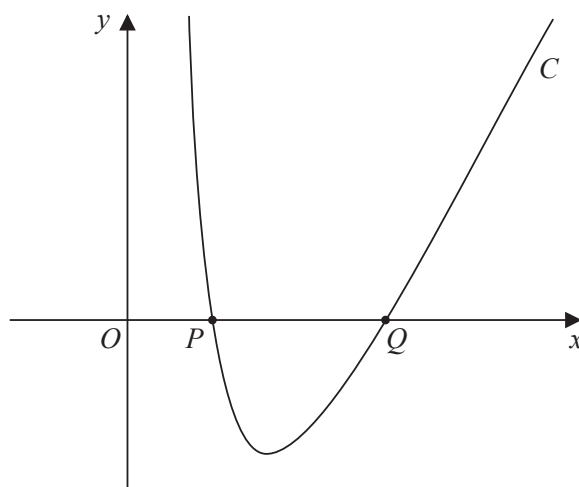
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Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{36}{x^2} + 2x - 13 \quad x > 0$$

Using calculus,

- (a) find the range of values of x for which $f(x)$ is increasing, (4)

- (b) show that $\int_2^9 \left(\frac{36}{x^2} + 2x - 13 \right) dx = 0$ (4)

The point $P(2, 0)$ and the point $Q(6, 0)$ lie on C .

Given $\int_2^6 \left(\frac{36}{x^2} + 2x - 13 \right) dx = -8$

- (c) (i) state the value of $\int_6^9 \left(\frac{36}{x^2} + 2x - 13 \right) dx$
- (ii) find the value of the constant k such that $\int_2^6 \left(\frac{36}{x^2} + 2x + k \right) dx = 0$ (3)

Question 10 continued

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Question 10 continued

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Question 10 continued

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Question 10 continued

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Q10

(Total 11 marks)

END

TOTAL FOR PAPER IS 75 MARKS



9.

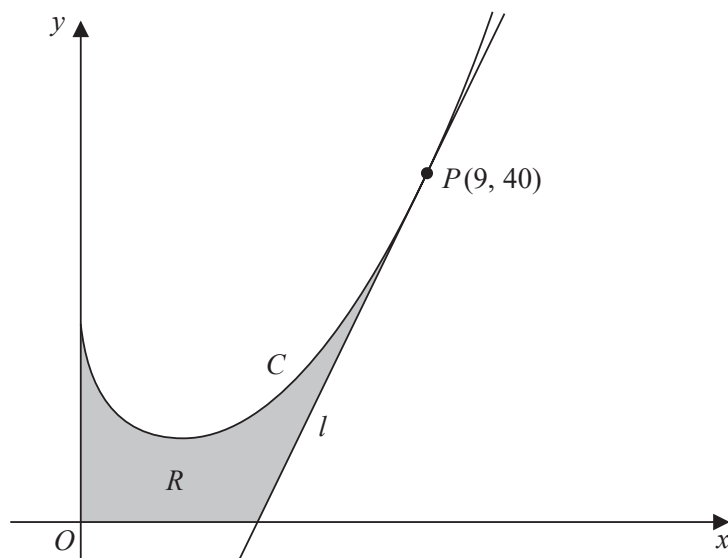


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13 \quad x \geq 0$$

(a) Find, using calculus, the range of values of x for which y is increasing.

(4)

The point P lies on C and has coordinates $(9, 40)$.

The line l is the tangent to C at the point P .

The finite region R , shown shaded in Figure 3, is bounded by the curve C , the line l , the x -axis and the y -axis.

(b) Find, using calculus, the exact area of R .

(8)



Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

(Total for Question 9 is 12 marks)



9.

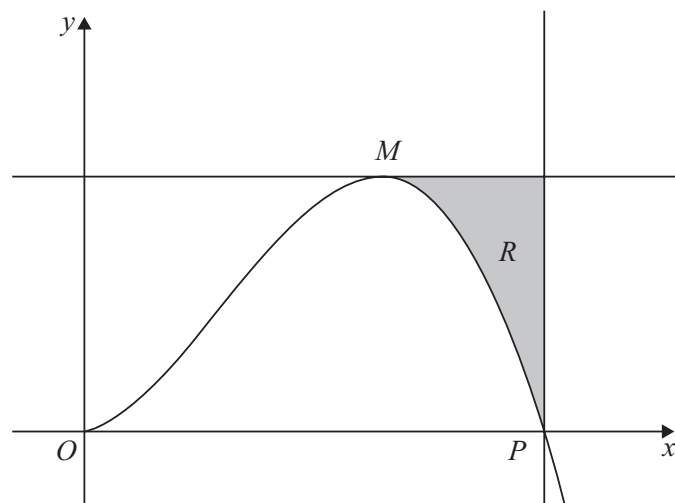


Figure 3

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2(5 - \sqrt{x})}{5} \quad x \geq 0$$

The curve has a turning point at the point M , as shown in Figure 3.

(a) Using calculus, find the coordinates of M .

(5)

The curve crosses the x -axis at the point P , as shown in Figure 3.

(b) Use algebra to find the x coordinate of P .

(2)

The finite region R , shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x -axis and the line through P parallel to the y -axis.

(c) Use algebraic integration to find the area of R , giving your answer to one decimal place.

(5)





Question 9 continued

(Total for Question 9 is 12 marks)



SUB-TOPIC P2.8b

THE TRAPEZIUM RULE

Sub-topic P2.8b · 14 questions · 89 marks · lowest tariff first



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 3 marks)



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Q2

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 5 marks)



(Total for Question 1 is 5 marks)



1.

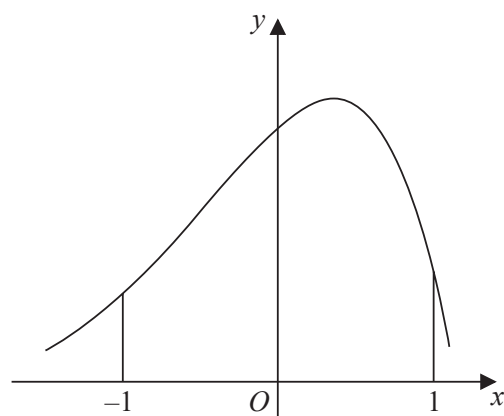


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$

The table below shows some corresponding values of x and y for this curve.

The values of y are given to 3 decimal places.

x	-1	-0.5	0	0.5	1
y	2.287	4.470	6.719	7.291	2.834

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_{-1}^1 f(x) \, dx$$

giving your answer to 2 decimal places.

(3)

(b) Use your answer to part (a) to estimate

(i) $\int_{-1}^1 (f(x) - 2) \, dx$

(ii) $\int_1^3 f(x-2) \, dx$

(3)

(Total for Question 1 is 6 marks)



6.

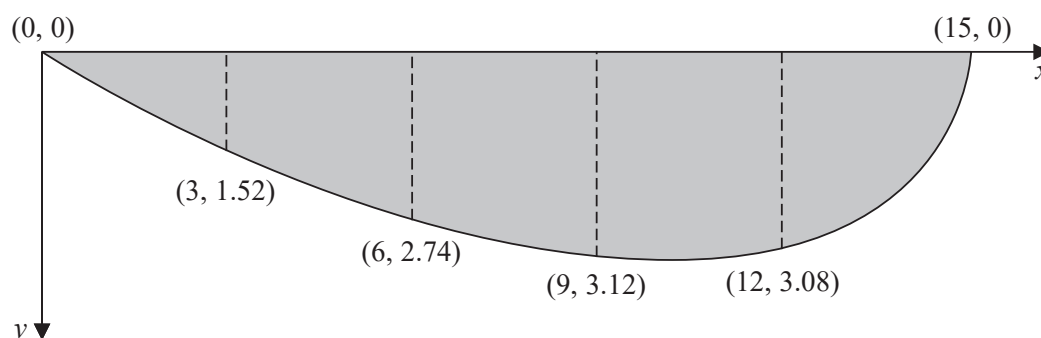


Figure 1

A river is being studied.

At one particular place, the river is 15 m wide.

The depth, y metres, of the river is measured at a point x metres from one side of the river.

Figure 1 shows a plot of the cross-section of the river and the coordinate values (x, y)

- (a) Use the trapezium rule with all the y values given in Figure 1 to estimate the cross-sectional area of the river.

(3)

The water in the river is modelled as flowing at a constant speed of 1.5 m s^{-1} across the whole of the cross-section.

- (b) Use the model and the answer to part (a) to estimate the volume of water flowing through this section of the river each minute, giving your answer in m^3 to 2 significant figures.

(2)

Assuming the model,

- (c) state, giving a reason for your answer, whether your answer for part (b) is an overestimate or an underestimate of the true volume of water flowing through this section of the river each minute.

(1)

(Total for Question 6 is 6 marks)

Question 5 continued

(Total for Question 5 is 6 marks)



Question 1 continued

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Q1

(Total 7 marks)



Question 1 continued

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Lined area for writing the answer to Question 1 continued.



Question 1 continued

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Question 1 continued

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Lined area for writing the answer to Question 1.

(Total 7 marks)

Q1



6. The curve C_1 has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below, with the y values rounded to 4 decimal places where appropriate.

x	0	0.5	1	1.5	2
y	3	2.6833	2.4	2.1466	1.92

(a) Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_0^2 f(x) \, dx$$

giving your answer to 3 decimal places.

(3)

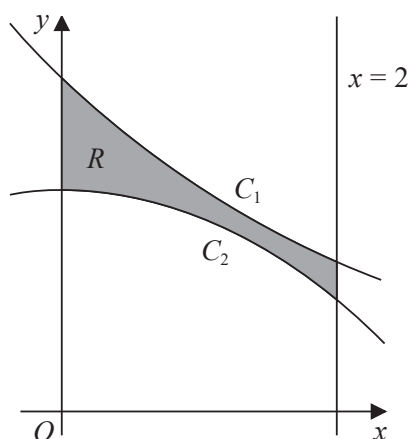


Figure 1

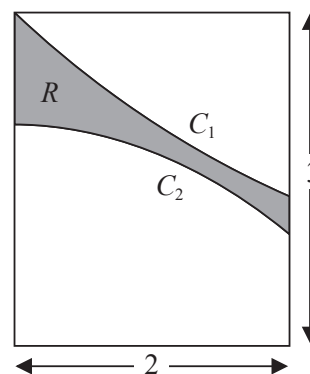


Figure 2

The region R , shown shaded in Figure 1, is bounded by

- the curve C_1
- the curve C_2 with equation $y = 2 - \frac{1}{4}x^2$
- the line with equation $x = 2$
- the y -axis

The region R forms part of the design for a logo shown in Figure 2.

The design consists of the shaded region R inside a rectangle of width 2 and height 3

Using calculus and the answer to part (a),

(b) calculate an estimate for the percentage of the logo which is shaded.

(4)





Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 7 marks)



3.

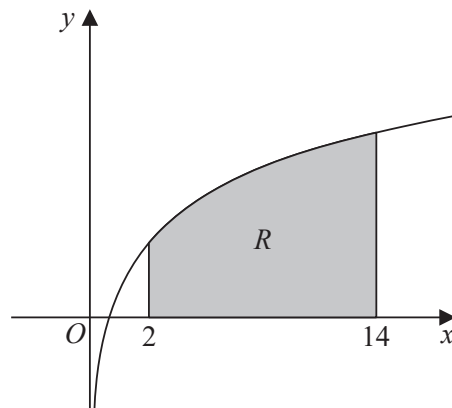


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \log_{10} x$

The region R , shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 2$, the x -axis and the line with equation $x = 14$

Using the trapezium rule with four strips of equal width,

(a) show that the area of R is approximately 10.10 (3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of R . (1)

(c) Using the answer to part (a) and making your method clear, estimate the value of

(i) $\int_2^{14} \log_{10} \sqrt{x} \, dx$

(ii) $\int_2^{14} \log_{10} 100x^3 \, dx$

(4)

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Question 3 continued

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Lined area for writing the answer to Question 3 continued.



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Question 3 continued

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Q3

(Total 8 marks)



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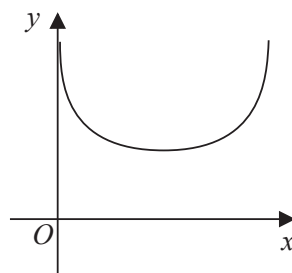


Figure 1

Figure 1 shows the graph of

$$y = 1 - \log_{10}(\sin x) \quad 0 < x < \pi$$

where x is in radians.

The table below shows some values of x and y for this graph, with values of y given to 3 decimal places.

x	0.5	1	1.5	2	2.5	3
y	1.319		1.001		1.223	1.850

(a) Complete the table above, giving values of y to 3 decimal places.

(2)

(b) Use the trapezium rule with all the y values in the completed table to find, to 2 decimal places, an estimate for

$$\int_{0.5}^3 (1 - \log_{10}(\sin x)) dx$$

(3)

(c) Use your answer to part (b) to find an estimate for

$$\int_{0.5}^3 (3 + \log_{10}(\sin x)) dx$$

(3)

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Question 2 continued

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Lined area for writing the answer to Question 2.

(Total 8 marks)

Q2







(Total for Question 7 is 6 marks)



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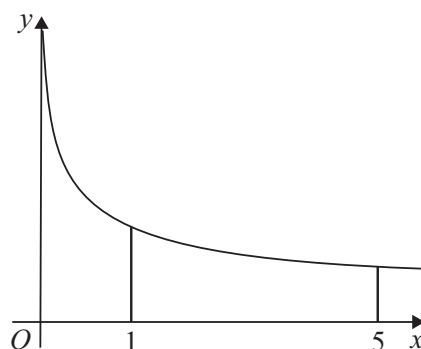


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \log_3(x+1) - \log_3 x$$

The point $P(a, 4)$ lies on the curve.

(a) Find the exact value of the constant a .

(Solutions relying on calculator technology are not acceptable.)

(4)

(b) Use the trapezium rule with 4 strips of equal width to estimate the value of

$$\int_1^5 (\log_3(x+1) - \log_3 x) dx$$

giving the answer in the form $\log_3 k$, where k is a constant to be found.

(4)

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate for

$$\int_1^5 (\log_3(x+1) - \log_3 x) dx$$

(1)

Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4 continued.



(Total for Question 4 is 3 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1 (a)	$h = 3$ $\text{Area} \approx \frac{3}{2} \{2 + 4.81 + 2 \times (3.32 + 4 + 4.46)\}$ $= \text{awrt } 45.6$	B1 M1 A1 (3)
(b)(i)	States or uses $\log_2 4x^2 = 2 \log_2 2x$ Hence $\int_2^{14} \frac{\log_2 4x^2}{5} dx = \frac{2}{5} \times (a) = \text{awrt } 18.2$	M1 A1 ft
(ii)	Attempts to identify $\log_2 2x$ e.g. $\log_2 \frac{2}{x} = \log_2 4 - \log_2 2x = 2 - \log_2 2x$ Hence $\int_2^{14} \log_2 \frac{2}{x} dx = \int_2^{14} (2 - \log_2 2x) dx = 24 - 45.6 = \text{awrt } -21.6$	M1 A1 ft (4) (7 marks)

(a)

B1 For $h = 3$ This is implied by sight of $\frac{3}{2}$ in front of the bracketM1 For a correct bracket condoning slips (e.g. 4.18 instead of 4.81). Look for {first + last + 2(sum of three terms)}, but repeated terms scores M0. Must all be y values. Condone “invisible” outside brackets for the M1.

A1 awrt 45.6 An answer of 45.6 can imply the B and M mark if no contrary working is seen and the M may be recovered from “incorrect bracketing” if the answer implies correct bracketing is used..

(b) (i)

M1 States or uses $\log_2 4x^2 = 2 \log_2 2x$ (oe)A1ft For calculating $\frac{2}{5} \times (a) = \text{awrt } 18.2$ but following through on their answer for (a)

(b)(ii)

M1 Attempts to write equation in terms of $\log_2 2x$ using the subtraction rule work to be able to apply (a).Should be $\log_2 \frac{2}{x} = \log_2 2 - \log_2 x (= 2 - \log_2 2x)$ but allow eg. $\log_2 \frac{2}{x} = \log_2 \frac{2x}{x^2} = \log_2 2x - \log_2 x^2$ for M1.A1ft For calculating $24 -$ their (a) = awrt -21.6 but following through on their answer for (a) and isw if later made positive.Do not allow for answers that evaluate an integral by calculator (so use of $\log_2 2x - \log_2 x^2$ will likely score M1A0).

Note that trapezium rule used again in (b) is M0 in both parts unless the relevant log work has been seen. Watch out for these as they give the same answers as those in the scheme to 1 dp.

Working must be seen in part (b) to gain credit.

$$\text{Alt for (b)(ii)} \quad \int_2^{14} \log_2 \frac{2}{x} dx = \int_2^{14} (\log_2 2 - \log_2 x) dx = 12 - \int_2^{14} \log_2 x dx \quad \text{and} \quad \int_2^{14} \log_2 2x dx = \int_2^{14} (\log_2 2 + \log_2 x) dx = 12 + \int_2^{14} \log_2 x dx$$

solved simultaneously scores the M when the $\log_2 x$ terms are eliminated.

Question Number	Scheme	Marks
5(i)	(As $x \geq 0$ so $\sqrt{3x}$ exists and) $(\sqrt{3x}-1)^2 \geq 0$	M1
	Hence $3x - 2\sqrt{3x} + 1 \geq 0$	M1
	$\Rightarrow 3x + 1 \geq 2\sqrt{3x}$ *	A1*
		(3)
Alt 1	$3x + 1 \geq 2\sqrt{3x} \Leftrightarrow (3x + 1)^2 \geq 12x \Leftrightarrow 9x^2 - 6x + 1 \geq 0$	M1
	$9x^2 - 6x + 1 \geq 0 \Leftrightarrow (3x - 1)^2 \geq 0$	M1
	Square numbers are greater than or equal to zero so $(3x - 1)^2 \geq 0$ is true hence $3x + 1 \geq 2\sqrt{3x}$ *	A1*
		(3)
Alt 2	If $3x + 1 < 2\sqrt{3x}$ then $3x - 2\sqrt{3x} + 1 < 0$	M1
	So $(\dots\sqrt{3x} \pm \dots)^2 < 0$ or $(\sqrt{x} \pm \dots)^2 < 0$	M1
	But $(\sqrt{3x} - 1)^2 \geq 0$ for all $x \geq 0$ so $3x + 1 \geq 2\sqrt{3x}$	A1
(ii)	Shows that it is not true for three consecutive prime numbers Eg $5 + 7 + 11 = 23$ which is not divisible by 5 (so not true)	B1
		(1)
		(4 marks)

(i)

M1 Uses that ($x \geq 0$ and) squares are non-negative to set up a suitable equation.

M1 Squares to achieve 3 terms.

A1 Rearranges correctly to the given result.

Alt 1: (Backwards proof)

M1 Starting with the given statement, attempts to square both sides, expand $(3x+1)^2$ (three terms required) and collect terms on one side of the inequality.

M1 Attempts to complete the square/factorise the expression to achieve a perfect square or (following error) $(..x+..) ^2 + ..$ (inequality not needed here).
Alternatively, uses other valid method (such as discriminant is zero) to show the resulting expression is non-negative. Finding a single solution alone is not sufficient, there must be a reason why the expression is never negative. (E.g. for $9x^2 - 6x + 1 = 0$ discriminant is $(-6)^2 - 4 \times 9 = 0$, so single root hence as positive quadratic, $9x^2 - 6x + 1 \geq 0$)

A1* Achieves $(3x-1)^2 \geq 0$ and a statement such as since this latter equation is true (as squares are never negative) hence $3x+1 \geq 2\sqrt{3x}$. Alternatively, they may achieve an inequality such as $(\sqrt{3x}-1)^2 \geq 0$ and conclude in a similar way.
Note that the proof should really have two way implications at each stage, but allow full credit for proofs that do not show this.

Alt 2: (Contradiction type proof)

M1 **Starts with the negation** of the statement and gathers terms on ones side of the equation.

M1 Attempts to factorise (oe method) the resulting expression to achieve a perfect square.

A1* Achieves $(\sqrt{3x}-1)^2 < 0$ or alternative suitable expression and concludes as this is false the starting assumption was false, hence the original statement is true.

NB They may substitute for $\sqrt{3x} = y$ or similar, which is fine and can score full marks if correctly reasoned.

NB: there are variations between the methods, but in general look for setting up a correct equation/gathering and identifying the underlying quadratic for the first M, attempting to factorise to a perfect square/complete the square to a non-negative expression or expand a perfect square (as appropriate) for the second M, and with all steps correct and suitable conclusion for the final A. Simply rearranging the equation alone is not sufficient for the first M.

(ii)

B1 Provides a correct counter example, such as $5 + 7 + 11 = 23$, or $11 + 13 + 17 = 41$, and gives a conclusion. Must see the sum evaluated correctly. The conclusion may be minimal e.g. “hence shown”, “not divisible by 5” etc.

The conclusion should come from a correct example, so if they conclude from an incorrect example, it will be B0. But if they have multiple examples, at least one of which is a correct one, and a generic conclusion, B1

NB $1+2+3$ is **not** a valid counter example as 1 is not prime.



Question Number	Scheme	Marks
6(a)	$\frac{3 \sin \theta \cos \theta}{2 \sin \theta - 1} = 5 \tan \theta \Rightarrow \frac{3 \sin \theta \cos \theta}{2 \sin \theta - 1} = 5 \frac{\sin \theta}{\cos \theta}$	<u>M1</u>
	$\frac{3 \sin \theta \cos \theta}{2 \sin \theta - 1} = 5 \frac{\sin \theta}{\cos \theta} \Rightarrow 3 \sin \theta \cos^2 \theta = 5 \sin \theta (2 \sin \theta - 1)$	M1
	$3 \sin \theta \cos^2 \theta = 10 \sin^2 \theta - 5 \sin \theta \Rightarrow 3 \sin \theta (1 - \sin^2 \theta) = 10 \sin^2 \theta - 5 \sin \theta *$	M1
	$\Rightarrow 3 \sin^3 \theta + 10 \sin^2 \theta - 8 \sin \theta = 0$	A1*
		(4)
(b)	$3 \sin^3 2x + 10 \sin^2 2x - 8 \sin 2x = 0 \Rightarrow \sin 2x (3 \sin^2 2x + 10 \sin 2x - 8) = 0$ $(3 \sin 2x - 2)(\sin 2x + 4) = 0 \Rightarrow \sin 2x = \dots$	M1
	$\sin 2x = \frac{2}{3}$	A1
	$(x =) 0.365$	A1
	$x = 0$	B1
		(4)
		(8 marks)

(a)

- M1 Attempts to use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ in the given equation. $5 \tan \theta = \frac{5 \sin \theta}{5 \cos \theta}$ is M0 **unless** the correct identity is first stated in which case allow for the attempt to use it.
- M1 Attempts to multiply both sides by both of their denominators. Condone invisible brackets for the method, but penalise in the A mark.
- M1 Uses the identity $\cos^2 \theta = 1 - \sin^2 \theta$ to form an equation in $\sin \theta$ only.
- A1* Achieves the given answer with no errors including brackets, and no **consistent** incorrect notation (allow if just one instance occurs). Examples of incorrect notation are $\sin \theta^2$ or use of just sin.

(b) Note the question says “hence” so there must be an attempt to use the given, or their, answer from part (a) to score the M.

- M1 Attempts to solve the cubic (allow with their cubic if incorrect) by taking out a factor of $\sin 2x$ or cancelling and attempts to solve the quadratic. Usual rules for solving a quadratic. Allow with $2x$ or with θ (or indeed with x or another variable), and allow with $y = \sin 2x$ (oe) used. May be implied by sight of correct value(s) for their cubic/quadratic equation.

- A1 $\sin 2x = \frac{2}{3}$ or allow for $\sin \theta = \frac{2}{3}$, or even $y = \frac{2}{3}$ if y has been identified as $\sin 2x$.

(However, do not accept $x = \frac{2}{3}$ unless recovered.) Ignore any other solutions (whether incorrect or not).

- A1 $(x =)$ awrt 0.365 (isw after a correct value) and no additional solutions in the range other than 0. Must have gained the M mark. (Degrees answer is A0)

- B1 $x = 0$ identified as an answer. Must be x May be scored with no working seen. Condone 0° and 0.000.



Question Number	Scheme	Marks
7(a)	$(t = 10, P =) \quad 1.2$	B1
	$(t = 11.5, P =) \quad 2.821$	B1
		(2)
(b)	$h = 0.5$	B1
	$E \approx \left(\frac{1}{2} \times \frac{1}{2} \right) ["1.2" + 2.95 + 2(1.882 + 2.45 + "2.821")]$	M1 A1ft
	$E \approx 4.61 \quad (\text{kWh})$	A1
		(4)
		(6 marks)

(a)

B1 1.2 or $\frac{6}{5}$. Allow 1.200 May be seen in the table or in their working for part (b).

B1 awrt 2.821 May be seen in the table or in their working for part (b).

(b)

B1 $h = 0.5$ (seen or implied)

M1 Scored for the correct [.....] bracket structure. It needs to contain their first P value plus the last P value and the inner bracket must be multiplied by 2 and be the summation of the remaining P values in the table with no additional values.

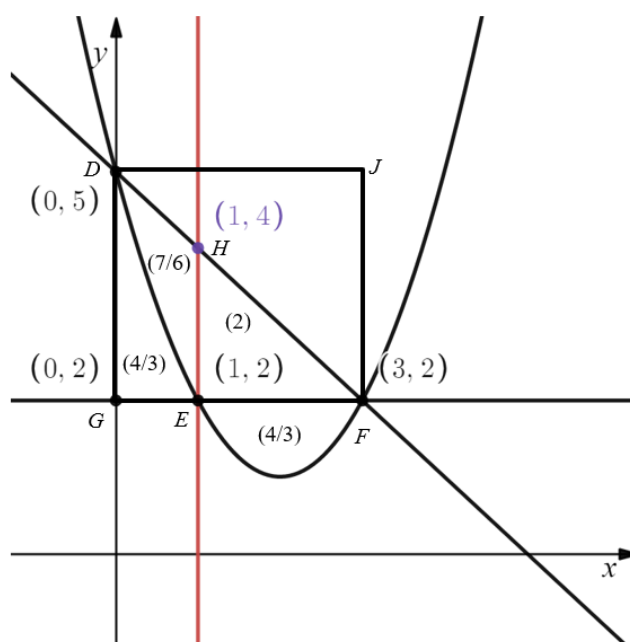
If the only mistake is a copying error or is to omit one value from the inner bracket this may be regarded as a slip and the M mark can be allowed. (An extra repeated term forfeits the M mark however). M0 if values used in brackets are t values instead of P values. Allow recovery of invisible brackets but M0 if never recovered.

A1ft For the correct bracket [.....] following through their values in (a).

A1 cao 4.61 Must be to 2 d.p. (note using integration the answer is 4.643439737)



Question Number	Scheme	Marks
9(a)	$(0, 5)$	B1
		(1)
(b)	$x^2 - 4x + 5 = 2 \Rightarrow x^2 - 4x + 3 = 0$ $\Rightarrow x = \dots$	M1
	$x(E) = 1, x(F) = 3$	A1
		(2)
(c)	Area $R_1 = \int_0^1 (x^2 - 4x + 5 - 2) dx = \left[\frac{x^3}{3} - 2x^2 + 3x \right]_0^1 = \frac{1}{3} - 2 + 3 - 0 = \frac{4}{3}$	M1A1
	Area $R_2 = \frac{1}{2} \times 3 \times 3 - \frac{4}{3} = \frac{19}{6}$ or $\int_0^1 (5 - x - (x^2 - 4x + 5)) dx + \frac{1}{2} \times 2 \times 2 = \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^1 + 2 = \frac{3}{2} - \frac{1}{3} + 2 = \frac{7}{6} + 2 = \frac{19}{6}$ or $\frac{1}{2} \times 3 \times 3 - \int_0^1 (x^2 - 4x + 5 - 2) dx = \frac{9}{2} - \left[\frac{x^3}{3} - 2x^2 + 3x \right]_0^1 = \frac{9}{2} - \left(\left(\frac{1}{3} - 2 + 3 \right) - 0 \right) = \frac{19}{6}$	M1A1
	$\frac{\text{area of } R_1}{\text{area of } R_2} = \frac{\frac{4}{3}}{\frac{19}{6}} = \frac{8}{19}$	A1
		(5)
		Total 8

Diagram for reference:

(a) Mark (a) and (b) together

B1 Correct **coordinates**. Allow to be written as $x = 0$, $y = 5$ (check the diagram as well)
5 on its own is B0

(b)

M1 Sets $C = l$ and solves for x . Expect to see the 3TQ $x^2 - 4x + 3 (= 0)$ oe (terms do not need to be on the same side) before solving the quadratic by factorising, quadratic formula, completing the square or via a calculator to find a value for x .

A1 Correct values following a **correct method**. Condone $x = 1, 3$ and also allow if the correct y coordinates are also given e.g. $(1, 2)$ and $(3, 2)$. Condone if E and F are labelled the wrong way round and check the diagram for the coordinates. If there is a contradiction between Figure 3 and the coordinates stated in the main body of the work then the main body of the work takes precedence. isw after the correct x coordinates are found.

Condone e.g. $E = 1, F = 3$

Answers only scores M0A0 (Minimum seen must be $x^2 - 4x + 5 = 2$ or $x^2 - 4x + 3$ oe)

(c) They are many different strategies for finding the area of R_1 and R_2 so check solutions carefully for alternative credit worthy methods

M1 Fully correct strategy for the area of R_1

Attempting the area under curve C – area under the line $y = 2$ between 0 and “1” Score for either

- attempting to integrate $C - l: \int_0^1 (x^2 - 4x + 5 - 2) dx$ oe
- attempting to integrate $C \int_0^1 (x^2 - 4x + 5) dx$ and subtracting the area of the rectangle

Condone slips in their working but expect to see the power raised by one on at least one of their terms and either substituting in the limits 0 and “1” (or subtracting “2” for the area of the rectangle). Must be the limits based on their part (b) answer.

Their integrated expression must be seen with the limits or an attempt to substitute in and subtract either way round – **the method mark cannot be scored directly from using a calculator**. Condone spurious notation.

A1 $\frac{4}{3}$ or exact equivalent correct area of R_1 provided M1 has been scored.

M1 **Fully correct strategy for the area of R_2 .**

Note that If R_1 is used to find the area of R_2 then R_1 must have been found from a correct strategy.

There are MANY possible strategies including:

- $GDF - "R_1"$
- $EDH + EHF$
- Finding the equation of the line through DF $y = 5 - x$ and attempting to integrate $DF - C$ between 0 and “3”, then finding the area bounded by l and the curve between “1” and “3” and subtracting this from the total
- $DJFG - "R_1" - DJF$

In all cases you need to score for the overall strategy to find R_2 so condone slips in their working and spurious notation but if the method is unclear then the required areas must be correct for their coordinates in (a) and (b). If integration is used on a new area (not R_1) then expect to see the power raised by one on at least one of their

terms and the integrated expression must be seen with the limits or attempting to substitute in and subtracting either way round. Must be the limits based on their part (b) answer.

If there are multiple attempts then score the most complete attempt.

A1 $\frac{19}{6}$ or exact equivalent correct area of R_2 provided M1 has been scored

A1 $\frac{8}{19}$ cao and all previous marks have been scored



Question Number	Scheme	Marks
7 (a)	$x^2 + y^2 + 8x - 10y = 29$	
	Attempts $(x+4)^2 + (y-5)^2 \dots = \dots$	M1
	Correct centre $(-4, 5)$	A1
(b)	Exact radius $\sqrt{70}$	A1
	Attempts distance (or distance ²) from $(-4, 5)$ to $(5, -8)$	M1
	Full method. E.g. compares $\sqrt{9^2 + 13^2}$ to $(\sqrt{70} + \sqrt{52})$	dM1
	States that - distance between centres $= \sqrt{250} = 15.81$ - sum of radii $= \sqrt{70} + \sqrt{52} = 15.58$ Concludes as sum of radii < distance between centres, result proven *	A1*
		(3) (6 marks)

(a)

M1: Attempts to complete the square for **both** variables. Look for $(x \pm 4)^2, (y \pm 5)^2 \dots = \dots$ or may be implied by $(\pm 4, \pm 5)$ by inspection from the equation.

A1: $(-4, 5)$

A1: $\sqrt{70}$ which may be scored following $(x \pm 4)^2 + (y \pm 5)^2 = 70$

(b)

M1: Attempts distance (or distance ²) from $(-4, 5)$ to $(5, -8)$ (condone sign errors)

dM1: Full method. E.g. compares their $\sqrt{9^2 + 13^2}$ to their $(\sqrt{70} + \sqrt{52})$

A1*: Requires correct calculation(s), correct reason and minimal conclusion. Must be evidence to support the inequality – e.g. decimal values – not just the inequality stated. May use the difference between sum of radii and distance between centres.

(b) Alt

M1: Attempts to solve $x^2 + y^2 + 8x - 10y = 29$ and $x^2 + y^2 - 10x + 16y = -37$ forming a linear equation (need not be simplified for this mark)

FYI: Linear equation is $18x - 26y = 66$ o.e. if the calculations are correct

dM1: Attempts to solve their $18x - 26y = 66$ simultaneously with either circle equation reaching a 3 term quadratic for either x or y.

A1*: Requires

- correct equations
- correct "solution" stating no roots and concluding no points of intersection and minimal conclusion

E.g. Substitute $y = \frac{9x-33}{13}$ into $x^2 + y^2 + 8x - 10y = 29$ gives $250x^2 - 412x + 478 = 0$

and $"b^2 - 4ac" = -308256 < 0$ so no solutions

FYI the equation in y (if correct) is $250y^2 + 984y + 1116 = 0$



4(c)(i)	$\left(\frac{d^2y}{dx^2}\right) = -x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}}$ oe e.g. $-x^{-\frac{3}{2}} + 6.75x^{-\frac{5}{2}}$	B1ft
(ii)	$\left(\left(\frac{d^2y}{dx^2}\right)_{x=\frac{9}{4}}\right) = -\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}} \left(= \frac{16}{27}(0.5925\dots)\right)$ $\left(\frac{d^2y}{dx^2}\right) > 0 \text{ so (local) minimum}$	B1
		(2)
(d)	$0 < x < \frac{9}{4}$	B1ft
		(1)
		Total 7

(c)(i)**B1ft:** Correct simplified second derivative.Follow through their first derivative provided there are 2 terms with **different** fractional (non-integer) powers. $\frac{d^2y}{dx^2} =$ is not required.Condone $-1x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}}$ for $-x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}}$

Inclusion of “+ c” scores B0ft as with the first derivative.

(ii)**B1:** Fully correct working, reasoning and conclusion. This requires:

- a correct second derivative
- substitutes $x = \frac{9}{4}$ to obtain $\frac{16}{27}$ or awrt 0.6 or truncated 0.5
- reference to the sign (> 0 or positive). Must see the evaluation as it is not obviously positive but allow if expression is not fully evaluated as long as it is clear it is positive (e.g. $\frac{8}{9} - \frac{8}{27} > 0$).
- reference to minimum

Condone work that doesn't specifically reference $\frac{d^2y}{dx^2}$ e.g.
$$-\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}} = \frac{16}{27} > 0 \text{ so minimum is fine for this mark.}$$
(d)**B1ft:** For $0 < x < \frac{9}{4}$. Allow $0 < x \leq \frac{9}{4}$ and allow equivalent statements e.g.
$$\left(0, \frac{9}{4}\right), \left[0, \frac{9}{4}\right].$$
Follow through their positive x value from part (b). Condone an answer of $x < \frac{9}{4}$ or
$$x \leq \frac{9}{4} \quad \text{Condone } 0 \leq x \leq \frac{9}{4}$$
Do not allow $-\infty < x < \frac{9}{4}$

n	$x1.02^{(n-1)}$	x5
1	25000	125000
2	25500	127500
3	26010	130050
4	26530	132651
5	27061	135304
6	27602	138010
7	28154	140770
8	28717	143586
9	29291	146457
10	29877	149387
11	30475	152374
12	31084	155422
13	31706	158530
14	32340	161701
	399348	1996742



Question Number	Scheme	Notes	Marks
3(a)	$a(-4)^3 - (-4)^2 + b(-4) + 4 = -108$ Attempts to set $f(-4) = -108$ to obtain an equation in a and b. Score when you see “-4” embedded in the equation or 2 correct terms (excluding the “+ 4”) on lhs. May be implied by e.g. $-64a - 16 - 4b + 4 = -108$ Condone minor slips on the lhs e.g. one sign error between terms but must use - 108		M1
	As an alternative for the first mark we will condone an attempt at long division. This requires a complete method to divide $(ax^3 - x^2 + bx + 4)$ by $(x + 4)$ to obtain a remainder in terms of a and b which is then equated to -108 For reference, the quotient is $ax^2 - (1+4a)x + 16a + b + 4$ and the remainder is $-4b - 64a - 12$		
	$-64a - 16 - 4b + 4 = -108$ $\Rightarrow 16a + b = 24 *$	Correct equation obtained with no errors and at least one line of intermediate working if starting with e.g. $a(-4)^3 - (-4)^2 + b(-4) + 4 = -108$	A1*
			(2)
(b)	$a\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 4 = 0$ Attempts to set $f\left(\frac{1}{2}\right) = 0$ to obtain an equation in a and b. Condone slips. Score when you see “$\frac{1}{2}$” embedded in the equation or 2 correct terms (excluding the “+ 4”) on lhs. May be implied by e.g. $\frac{a}{8} - \frac{1}{4} + \frac{b}{2} + 4 = 0$ The “= 0” may be implied when they attempt to solve simultaneously below		M1
	An alternative for the first mark is to attempt long division. This requires a complete method to divide $(ax^3 - x^2 + bx + 4)$ by $(2x - 1)$ to obtain a remainder in a and b which is then equated to 0 For reference, the quotient is $\frac{a}{2}x^2 + \left(\frac{a}{4} - \frac{1}{2}\right)x + \left(\frac{b}{2} - \frac{1}{4} + \frac{a}{8}\right)$ and the remainder is $\frac{15}{4} + \frac{b}{2} + \frac{a}{8}$		
	$16a + b = 24, a + 4b = -30$ $\Rightarrow a = \dots, b = \dots$	Attempts to solve $16a + b = 24$ simultaneously with their equation in a and b. This may be implied if values of a and b are obtained (e.g. calculator)	M1
	$a = 2, b = -8$	Correct values	A1
			(3)
(c)	$f(x) = 2x^3 - x^2 - 8x + 4$ $\Rightarrow f'(x) = 6x^2 - 2x - 8$	Correct derivative (follow through their a and b). Allow unsimplified and apply isw if necessary. Allow with the letters “a” and “b” and a “made up” “a” and “b”.	B1ft
			(1)

(d)	$6x^2 - 2x - 8 = 0$ $\Rightarrow (3x - 4)(x + 1) = 0$ $\Rightarrow x = \dots$	Sets their $f'(x) = 0$ (may be implied) and solves a 3 term quadratic. Apply general guidance if necessary. You may need to check if a calculator has been used.	M1
	$x = \frac{4}{3}, -1 \Rightarrow y = \dots$	Uses at least one of their x values to find a value for y using their $f(x)$ <u>where x is from an attempt to solve $f'(x) = 0$</u> . You may need to check their y values if working is not shown.	M1
	$\left(\frac{4}{3}, -\frac{100}{27}\right) \text{ or } (-1, 9)$ Or e.g. $x = \frac{4}{3}, y = -\frac{100}{27}$ and $x = -1, y = 9$ One correct point. The fractional coordinates must be exact but allow 1.3 with a dot over the 3 and 3.703 with dots over the 7 and 3. Note that it is not necessary for the points to be written as coordinates as long as the pairing is clear. Depends on having scored both previous M marks.		A1
	$\left(\frac{4}{3}, -\frac{100}{27}\right) \text{ and } (-1, 9)$ Or e.g. $x = \frac{4}{3}, y = -\frac{100}{27}$ and $x = -1, y = 9$ Both correct points. The fractional coordinates must be exact but allow 1.3 with a dot over the 3 and 3.703 with dots over the 7 and 3. Note that it is not necessary for the points to be written as coordinates as long as the pairing is clear. Depends on having scored both previous M marks.		A1
	Fully correct answers with no working scores 4/4 following a <u>correct</u> part (c) i.e. $\Rightarrow f'(x) = 6x^2 - 2x - 8$		
			(4)
			Total 10

Question Number	Scheme	Notes	Marks
4(a)(i)	$(-7, 9)$ or e.g. $x = -7, y = 9$	$x = -7$ or $y = 9$	B1
		$x = -7$ and $y = 9$	B1
	Award the marks in (a) once correct answers are seen. Special case: If all you see is $(9, -7)$ award B1B0		
(a)(ii)	<u>Examples:</u> $r = \sqrt{(-3 - ("7"))^2 + (12 - "9")^2}$ or $r = \sqrt{(-11 - ("7"))^2 + (6 - "9")^2}$ or $r = \frac{1}{2}\sqrt{(-3 + 11)^2 + (12 - 6)^2}$	Correct strategy for the radius. Must be a correct method for their centre (if used) but allow 1 sign slip within one of the brackets . A correct answer scores both marks. Must see the $\frac{1}{2}$ if finding the length of the diameter.	M1
	$r = 5$	Correct radius	A1
			(4)
(b)	$(x + 7)^2 + (y - 9)^2 = 5^2$ or e.g. $x^2 + y^2 + 2 \times 7x - 2 \times 9y + 7^2 + 9^2 - 5^2 = 0$ M1: Correct attempt at circle equation using their values. Allow for $(x \pm (\text{their } 7))^2 + (y \pm (\text{their } 9))^2 = (\text{their numerical } r)^2$ or e.g. $x^2 + y^2 \pm 2 \times (\text{their } 7)x \pm 2 \times (\text{their } 9)y + (\text{their } 7)^2 + (\text{their } 9)^2 - (\text{their numerical } r)^2 = 0$ A1: Correct equation in any form		M1A1
			(2)
(c)	$m_{\text{radius}} = \frac{12 - 9}{-3 + 7} \left(= \frac{3}{4} \right) \text{ or }$ $m_{\text{tangent}} = -1 \div \left(\frac{12 - 9}{-3 + 7} \right) \left(= -\frac{4}{3} \right) \text{ or }$ $m_{\text{tangent}} = -\left(\frac{-3 + 7}{12 - 9} \right) \left(= -\frac{4}{3} \right)$	This mark is for an attempt to find the radius gradient or the tangent gradient. If the method is not clear allow one sign error in the numerator or denominator.	M1
	Alternative for the first M: $(x + 7)^2 + (y - 9)^2 = 5^2 \Rightarrow 2(x + 7) + 2(y - 9) \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{x + 7}{9 - y} = \frac{-3 + 7}{9 - 12} \left(= -\frac{4}{3} \right)$ Allow for $(x + 7)^2 + (y - 9)^2 = 5^2 \Rightarrow \alpha(x + 7) + \beta(y - 9) \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \dots$		
	$y - 12 = -\frac{4}{3}(x + 3)$ Uses a correct straight line method for the tangent using the point Q. Must be fully correct work here so must be a clear attempt at the tangent not the radius. So if the radius gradient is found previously, must apply negative reciprocal rule to their radius gradient. If using $y = mx + c$ must reach as far as $c = \dots$		M1
	$4x + 3y - 24 = 0$	Allow any integer multiple	A1
			(3)
			Total 9

(b)

M1: Attempts to use the GP formula in an attempt to find 'r'

Look for $4000 = 300r^{11} \Rightarrow r^{11} = \frac{4000}{300} \Rightarrow r = \sqrt[11]{\frac{4000}{300}}$ condoning slips. Implied by $r = \text{awrt } 1.3$ provided they do not start from an incorrect equation.

M1: A correct attempt to find the **second term** (do not allow a misread) by multiplying 300 by their "1.266..." . It is dependent on an attempt to find r from either $4000 = 300r^{11}$ or $4000 = 300r^{12}$ condoning slips. If no method is seen to find r then it must be correct. e.g. following $4000 = 300r^{12}$ or $\sqrt[12]{\frac{4000}{300}}$.

Condone slips.

A1: awrt 380 (to the nearest 10) (tonnes) isw once a correct answer is seen.

(c) **Condone use of their d and their r even if they have come from incorrect methods**

M1: A correct method to find the sum of either the AP or the GP

For the AP accept an attempt at either $\frac{12}{2}\{300 + 4000\}$ or $\frac{12}{2}\{2 \times 300 + 11 \times "336"\}$

For the GP accept an attempt at either $\frac{300("1.266..."^{12} - 1)}{"1.266..." - 1}$ or $\frac{300(1 - "1.266..."^{12})}{1 - "1.266..."}$

dM1: Both formulae must be attempted "correctly" (see above) and the difference taken
FYI if d and r are correct, the sums are 25 800 tonnes and 17 935

A1: Difference = 7860 or 7870 (tonnes) **This is NOT AWRT.**(Note that use of $r = 1.266$ will lead to a difference of 7810 which is max M1dM1A0)**Alt (c) Listing values**

Accept listing the values

M1: Attempts to add the 12 terms for the arithmetic or geometric series. Must include 300 and 4000, but condone one omission of one of the middle terms. May be implied by calculating the difference between the AP and GP values for each year and adding those together.

dM1: Attempts to add the 12 terms for the arithmetic and geometric series. Must include 300 and 4000 for both sums, but condone one omission of one of the middle terms in each of the summations. May be implied.

A1: Difference = 7860 or 7870 **This is NOT AWRT.**

Year	AP	GP	Difference
1	300	300	0
2	636.3636	379.6544	256.7093
3	972.7273	480.4582	492.2691
4	1309.091	608.0268	701.0641
5	1645.455	769.4668	875.9877
6	1981.818	973.7715	1008.047
7	2318.182	1232.322	1085.86
8	2654.545	1559.521	1095.024
9	2990.909	1973.597	1017.312
10	3327.273	2497.616	829.6567
11	3663.636	3160.77	502.8668
12	4000	4000	0
Sum	25800	17935.2	7864.796



Question Number	Scheme	Marks
8 (i)	Substitutes a value e.g. $n = 6$ into $n^2 + 3n + 1$ where $n^2 + 3n + 1$ is not prime	M1
	Correct calculation for that value e.g. $n^2 + 3n + 1 = 55$ And conclusion "which is not prime"	A1
		(2)
(ii)	Attempts to find $n^2 - 2$ for either odds or evens E.g Attempts $(2p+1)^2 - 2$ or $(2p)^2 - 2$	M1
	Achieves either $(2p+1)^2 - 2 = 4p^2 + 4p - 1$ or $(2p)^2 - 2 = 4p^2 - 2$ and shows or gives a reason why the expression is not a multiple of 4 where required (see notes)	A1
	Attempts to find $n^2 - 2$ for both odds and evens (See above)	dM1
	Achieves both $(2p+1)^2 - 2 = 4p^2 + 4p - 1$ and $(2p)^2 - 2 = 4p^2 - 2$ and shows or gives reasons why these are not multiples of 4 where required (see notes) With a conclusion that they are not multiples of 4. *	A1*
		(4)
		(6 marks)

(i)

M1: Substitutes a value into $n^2 + 3n + 1$ where $n^2 + 3n + 1$ is not prime. (Does not have to be evaluated)Possible values are $n = 6, 11, 13, 16$ etc. Condone slips substituting in or if they have evaluated incorrectly provided the intention was to substitute in a valid value for n . (Note that $n = 0$ is not acceptable)

A1: A correct calculation for the value and a conclusion. There must be some reference to not being prime or they show that the number is divisible by e.g. 5 and state that it is false/not true

e.g. $6^2 + 3 \times 6 + 1 = 55$ which is not prime (so not true) is M1A1e.g. when $n = 11$ then $n^2 + 3n + 1 = 155$ $155 \div 5 = 31$ so false is M1A1e.g. $6^2 + 3 \times 6 + 1 = 55$ so false is M1A0Values of n up to 50 for which $n^2 + 3n + 1$ is not prime:

n	$n^2 + 3n + 1$	n	$n^2 + 3n + 1$
6	55	31	1055
11	155	32	1121
13	209	33	1189
16	305	35	1331
17	341	36	1405
21	505	39	1639
22	551	41	1805
24	649	42	1891
26	755	46	2255
28	869	48	2449
		50	2651

- (ii) **There should be no errors in the algebra but allow e.g. invisible brackets to be “recovered”.
Withhold the final mark if n is used instead of k**

M1: Uses algebra to describe odds e.g. $n = 2p+1$ or evens e.g. $n = 2p$ and attempts $n^2 - 2$ (allow equivalent representation of odd or even e.g. $n = 2k+2$ **or** $2n \pm 5$)

Condone arithmetical slips and condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$

A1: They must

- achieve a correct expanded $n^2 - 2$ for either odds or evens
- show or give an explanation why the expression is not a multiple of 4 (other than the trivial case when $n = 2p$).

e.g. $(2p+1)^2 - 2 = 4p^2 + 4p - 1$ which is 1 less than a multiple of 4 is A1 (explanation)

Do not accept e.g. “you cannot take 4 out as a common factor” as this is insufficient and should be

shown, e.g. $(2p+1)^2 - 2 = 4p^2 + 4p - 1 = 4\left(p^2 + p\right) - 1$ is A1 (shows)

For the case when $n = 2p \Rightarrow (2p)^2 - 2 = 4p^2 - 2$ it does not need to be followed by an explanation as it is clearly not a multiple of 4 but if one is given then it must be correct. e.g. when divided by 4 gives

$$p^2 - \frac{1}{2}$$

They must have evaluated numbers with indices so do not accept $2^2 p^2 - 2$

dM1: Attempts to find $n^2 - 2$ for both odds and evens (See above)

A1: Fully correct proof. They must

- achieve a correct expanded $n^2 - 2$ for both odds and evens
- show or gives a reason as to why each expression is not a multiple of 4 (other than for $n = 2p$)
- make a concluding overall statement. “Hence not a multiple of 4 (for all $n \in \mathbb{N}$)” Accept “hence proven”, “statement proved”, “QED” **if they have stated for each separate case that the expression is not a multiple of 4.**

Attempts using $4k, 4k+1, 4k+2, 4k+3$ should be applied similarly to the main scheme above:

M1: Attempts **at least two of the four cases** from e.g. $4k, 4k+1, 4k+2, 4k+3$ (but could be others e.g. $4k-2, 4k-1, 4k, 4k+1$)

A1: They must

- achieve two correct expanded $n^2 - 2$
- show or give a reason as to why the two expressions are not a multiple of 4 (same examples as in notes above for main scheme).

	$n^2 - 2$
$4k - 3$	$16k^2 - 24k + 7$
$4k - 2$	$16k^2 - 16k + 2$
$4k - 1$	$16k^2 - 8k - 1$
$4k$	$16k^2 - 2$
$4k + 1$	$16k^2 + 8k - 1$
$4k + 2$	$16k^2 + 16k + 2$
$4k + 3$	$16k^2 + 24k + 7$

dM1: Attempts to find $n^2 - 2$ for all four cases (see above)

A1: Fully correct proof (see above)



Question Number	Scheme	Marks
9 (i)	States or uses $\tan x = \frac{\sin x}{\cos x}$	B1
	$\sin x \tan x = 5 \Rightarrow \sin^2 x = 5 \cos x \Rightarrow 1 - \cos^2 x = 5 \cos x$	M1A1
	$\cos^2 x + 5 \cos x - 1 = 0 \Rightarrow (\cos x =) \frac{-5 \pm \sqrt{29}}{2} \Rightarrow x = \text{awrt } 78.9^\circ, 281.1^\circ$	M1dM1A1
		(6)
(ii)	(a) $A = 5$	B1
		(1)
	(b) $2\theta - \frac{3\pi}{8} = \frac{3\pi}{2} \Rightarrow \theta = \dots$	M1
	$\theta = \frac{15\pi}{16}$	A1
	y coordinate $Q = -3$ (or $2 - "A"$)	B1ft
		(3)
	(c) Sets $0 = "5" \sin\left(2\theta - \frac{3\pi}{8}\right) + 2 \Rightarrow \sin\left(2\theta - \frac{3\pi}{8}\right) = \pm \frac{2}{"5"}$	M1
	$\sin\left(2\theta - \frac{3\pi}{8}\right) = \pm \frac{2}{5} \Rightarrow \left(2\theta - \frac{3\pi}{8}\right) = \arcsin\left(\pm \frac{2}{5}\right) = \dots$	dM1
	One of $\theta = 0.38, 2.4, 3.5, 5.5, 6.7, 8.6, 9.8\dots$	A1
	$\theta = \text{awrt } 5.51$	A1
		(4)
		Total 14

(i) **Do not penalise working in another variable or if the variable is omitted at times in their working.**

e.g. $\sin$ instead of $\sin x$ is condoned as a slip or $\cos^2 x$ as $\cos x^2$. They are required to show the stages of working but not to the same level of accuracy in their presentation as a “show” or “prove” question.

B1: States or uses $\tan x = \frac{\sin x}{\cos x}$ (may be seen as $\sin x \tan x \Rightarrow \tan^2 x \cos x$)

M1: Uses $\pm \sin^2 x \pm \cos^2 x = \pm 1$ to set up a quadratic equation in $\cos x$ only. If terms have been collected on one side condone the omission of $= 0$. May form the equation $\tan^2 x \cos x = 5$ and use the identity $\pm 1 \pm \tan^2 x = \pm \sec^2 x$ proceeding to a quadratic in $\cos x$ only

$$\text{e.g. } \Rightarrow \cos x (\sec^2 - 1) = 5 \Rightarrow \frac{1}{\cos x} - \cos x = 5 \Rightarrow 1 - \cos^2 x = 5 \cos x$$

or alternatively, square both sides and proceed to a quartic in $\cos x$ only.

A1: Correct 3TQ (or quartic) in $\cos x$. The three terms do not need to be on the same side of the equation. The $= 0$ may be implied by further work. e.g. attempting to solve the quadratic. Alternatively award for the quartic equation $\cos^4 x - 27 \cos^2 x + 1 = 0$

M1: Attempts to solve their 3TQ (or quartic) in $\cos x$. Usual rules apply for solving the quadratic. Accept via a calculator but at least one root must be correct (you may need to check this)

They cannot proceed from the 3TQ directly to an angle for x ; this is M0 and no further marks can be scored.

Accept either root as a rounded decimal to 2sf e.g. for the correct quadratic it would be awrt 0.19 or awrt -5.2

dM1: Attempts to find one value for x in the given range using their root. You may need to check this. It is dependent on the previous method mark. Accept in radians. It is acceptable to proceed from the roots of their quadratic directly to an angle in the given range to score this mark. May be implied by awrt 79 or awrt 281 (or in radians awrt 1.4 or awrt 4.9)

A1: awrt $x = 78.9^\circ, 281.1^\circ$ (must be in degrees) and no others in the range.



(ii)(a)

B1: $A = 5$ Check by the question. If there are multiple answers, mark the answer in the main body of the work.(ii)(b) **Solutions with no working:** Note if the two coordinates are given with no working then max M0A0B1 $\left(-3, \frac{15\pi}{16}\right)$ (or coordinates given the wrong way round) with no working seen then SC001M1: Attempts to solve $2\theta - \frac{3\pi}{8} = \frac{3\pi}{2}$ or any other minimum value e.g. $2\theta - \frac{3\pi}{8} = -\frac{\pi}{2}$.

They must proceed using the correct order of operations e.g. allow $\theta = \frac{\frac{3\pi}{2} \pm \frac{3\pi}{8}}{2}$, which may be implied by their answer. Answer only with no working though is M0A0.

If they have a mixture of radians and degrees within an equation and they do not “recover” then the method mark cannot be scored.

A1: $\left(\theta = \frac{15\pi}{16}\right)$ must be exact following a correct equation. May be seen as $\left(\frac{15\pi}{16}, -3\right)$ isw if they proceed to

write the coordinates the wrong way round.

B1ft: y coordinate $Q = -3$ or follow through on $2 - "A"$ (may be seen on the diagram or by the question). If there is a contradiction, then the answer in the main body of the work takes precedence. If they proceed to write the coordinates the wrong way round then isw. May be seen as a pair of coordinates.

(ii)(c) **In EPEN this is M1A1dM1A1 but we are marking this M1dM1A1A1****Solutions with no working in (c) scores 0 marks.**M1: Sets $0 = "5" \sin\left(2\theta - \frac{3\pi}{8}\right) + 2$ (which may be implied) and proceeds to $\sin\left(2\theta - \frac{3\pi}{8}\right) = \pm \frac{2}{"5"}$ o.e.May be implied by e.g. $2\theta - \frac{3\pi}{8} = "-0.41..."$ or any other equivalent angle in radians or

$2\theta - 67.5 = "-23.5..."$ or any other equivalent if working in degrees. You may need to check this. Allow use of X for 2θ or $2\theta - \frac{3\pi}{8}$

dM1: Proceeds from $\sin\left(2\theta - \frac{3\pi}{8}\right) = \pm \frac{2}{5}$ to $2\theta - \frac{3\pi}{8} = \arcsin\left(\pm \frac{2}{5}\right) = \dots$ which is one of the values below:

$$2\theta - \frac{3\pi}{8} = \arcsin\left(-\frac{2}{5}\right) = -0.41, 3.6, 5.9, 9.8, 12.2 \quad 2\theta - \frac{3\pi}{8} = \arcsin\left(\frac{2}{5}\right) = 0.41, 2.7, 6.7, 9.0, 13.0, 15.3$$

May be implied by $2\theta = \arcsin\left(\pm \frac{2}{5}\right) + 1.17...$ or allow the expression $= \frac{\arcsin\left(\pm \frac{2}{5}\right) + \frac{3\pi}{8}}{2}$

The sign slip is only condoned before they take $\arcsin(\dots)$

A1: Any one of the values in the table provided M1dM1 has been scored. Do not withhold this mark if other incorrect angles are seen.

	Radians (awrt)	Degrees (awrt)
θ	0.38, 2.4, 3.5, 5.5, 6.7, 8.6, 9.8, 11.8, 12.9	22, 136, 202, 316, 382, 496, 562, 676, 742

A1: $\theta = \text{awrt } 5.51$ only (must be in radians). **(Can only be scored from correct working and all previous marks are scored)**

Question Number	Scheme	Marks
10 (a)	$\left(\frac{dy}{dx} = \right) x - 2187x^{-\frac{5}{2}}$	M1, A1
	Sets $x - 2187x^{-\frac{5}{2}} = 0 \Rightarrow x^{\frac{7}{2}} = 2187$ (or e.g. $x = (\sqrt[7]{2187})^2 \Rightarrow x = 9$ *	dM1A1*
		(4)
(b)	e.g. $\int \left\{ \frac{1}{2}x^2 + 1458x^{-\frac{3}{2}} - 74 \right\} dx = \frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 74x$ or $\int \left\{ \frac{1}{2}x^2 + 1458x^{-\frac{3}{2}} - "94.5" \right\} dx = \frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 94.5x$	M1A1ft
	y value at P is 20.5	B1
	e.g. Area $R = \left[\frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 74x \right]_4^9 - (9-4) \times "20.5"$ $= \left(\frac{1}{6} \times 9^3 - 2916 \times 9^{-\frac{1}{2}} - 74 \times 9 \right) - \left(\frac{1}{6} \times 4^3 - 2916 \times 4^{-\frac{1}{2}} - 74 \times 4 \right) - (9-4) \times "20.5"$	dM1
	$\left(-1516\frac{1}{2} + 1743\frac{1}{3} - 102\frac{1}{2} \right) = 124\frac{1}{3}$	A1
		(5)
		Total 9

(a)

M1: Attempts to differentiate with one index correct e.g. $\dots x^2 \rightarrow \dots x$ or $x^{-\frac{3}{2}} \rightarrow x^{-\frac{3}{2}-1}$ but not for $74 \rightarrow 0$.

A1: Correct differentiation. May be left unsimplified but indices must be processed.

dM1: Either solves their $\frac{dy}{dx} = 0$ of the form $x - \dots x^m = 0$ where m is a fraction via $x^n = A$ or $x = A^{\frac{1}{n}}$. It must

be a solvable equation and they must correctly deal with the indices for their $\frac{dy}{dx}$. Look out for

attempts where $\frac{dy}{dx}$ is manipulated before being set equal to zero which is acceptable.

e.g. $\frac{dy}{dx} = x^{\frac{7}{2}} - 2187 = 0 \Rightarrow x^{\frac{7}{2}} = 2187 \Rightarrow x = \dots$

Alternatively substitutes $x = 9$ into their $\frac{dy}{dx}$ and finds its value.

A1*: Correct calculations and working leading to prove that the x coordinate of P is 9. It is sufficient to achieve $x^n = A$ and proceed directly to $x = 9$ but e.g. $x - 2187x^{-\frac{5}{2}} = 0 \Rightarrow x = 9$ would be dM0A0.

It is also acceptable to proceed from $x - 2187x^{-\frac{5}{2}} = 0$ to an expression for x which is not the given answer before achieving $x = 9$ e.g. $x = (\sqrt[7]{2187})^2$

If using the verification method they should conclude that $x = 9$ (or have a preamble that if $x = 9$ is substituted in then $\frac{dy}{dx} = 0$, then substitute in, achieves 0 followed by e.g. tick, QED etc.)



(b) Note that if no algebraic integration is seen then maximum score is M0A0B1dM0A0

M1: Attempts to integrate with one index correct e.g. $\dots x^2 \rightarrow \dots x^3$ or $x^{-\frac{3}{2}} \rightarrow x^{-\frac{3}{2}+1}$ (indices do not need to be processed for this mark). Also accept $\pm 74 \rightarrow \pm 74x$ or $"-94.5" \rightarrow "-94.5"x$

A1ft: Correct integration. $\frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 74x$ or follow through if curve – line is integrated proceeding to

$\frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - "94.5"x$ (allowing for an error on the coefficient of x when attempting to subtract before integrating)

May be left unsimplified but indices must be processed (may be implied by further working – not just the final answer). Do not be concerned with spurious notation e.g. a dx or integral sign still present.

B1: y value at P is 20.5 or e.g. $\frac{41}{2}$. This may be seen as part of a wider calculation or on the diagram or next to the questions (can also be scored for sight in (a)) This mark can be scored for sight of -94.5 or $-94.5x$ when they are attempting to integrate curve – line.

dM1: Full method to find the area of R . They do not need to proceed as far as finding a value for the area, the values embedded is sufficient. It is dependent on the previous method mark.

The method for finding y at P must be correct (may be implied by e.g. 20.5 or attempting to substitute $x = 9$ into the equation of the curve).

If no integration is seen then this mark cannot be scored.

If they proceed from the integrated expression to the answer without showing some substitution of the limits (or evidence of this which is not the answer) then dM0A0.

e.g. $\left[\frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 74x \right]_4^9 - (9-4) \times 20.5 = 124\frac{1}{3}$ scores dM0A0 (no evidence of limits substituted)

e.g. $-1516\frac{1}{2} + 1743\frac{1}{3} - 102\frac{1}{2} = 124\frac{1}{3}$ scores dM1A1 (evidence of limits substituted)

(Note $\frac{1361}{6} = 226\frac{5}{6}$ is only the area under the curve so this is not a full method to find the required area)

A1: Correct working and calculations seen leading to the answer of $124\frac{1}{3}$ o.e. $\frac{373}{3}$ including $124.\dot{3}$ or

$124.\bar{3}$ $124.33\dots$ but not rounded answers of 124.3

Can only be scored provided all of the previous marks have been scored.



Question Number	Scheme	Marks
10 (a)	$y = \frac{9x - x^2}{2\sqrt{x}} = \frac{9}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{3}{2}} \Rightarrow \frac{dy}{dx} = \frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{4}x^{\frac{1}{2}}$ $\text{Sets } \frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{4}x^{\frac{1}{2}} = 0 \Rightarrow x = \frac{9}{4} \times \frac{4}{3} \Rightarrow x = 3$	M1, A1 dM1, A1
		(4)
(b)	$\int \left\{ \frac{9}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{3}{2}} \right\} dx = 3x^{\frac{3}{2}} - \frac{1}{5}x^{\frac{5}{2}}$ Upper limit is 9 $\text{Area } R = \left[3x^{\frac{3}{2}} - \frac{1}{5}x^{\frac{5}{2}} \right]_1^9 = \left(3 \times 27 - \frac{1}{5} \times 243 \right) - \left(3 \times 1 - \frac{1}{5} \times 1 \right)$ $= \frac{148}{5}$	M1, A1 B1 dM1 A1
		(5)
		Total 9

(a)

M1: Attempts to differentiate and achieves one term with the correct index. For this to be awarded

- y must be written as a sum of two terms with one index correct $y = ax^{\frac{1}{2}} \pm bx^{\frac{3}{2}}$ or $y = ax^{\frac{3}{2}} \pm bx^{\frac{1}{2}}$
- after differentiation a correct index must be achieved. Look for $\alpha x^{-\frac{1}{2}} \pm \beta x^{\frac{1}{2}}$ or $\alpha x^{\frac{1}{2}} \pm \beta x^{\frac{3}{2}}$
- the index cannot be achieved from incorrect working, so calculations such as $y = (9x - x^2) \frac{1}{2} \times \sqrt{x} = \frac{9}{2}x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{5}{2}} \Rightarrow \frac{dy}{dx} = \dots x^{\frac{1}{2}} \pm \dots x^{\frac{3}{2}}$ would score M0

Alternatively, they may use the quotient rule. Look for $\frac{dy}{dx} = \frac{2\sqrt{x}(\alpha - \beta x) - (9x - x^2) \times \delta x^{-\frac{1}{2}}}{\epsilon x}$

A1: Correct differentiation which may be left un-simplified.

Via the quotient rule this would be $\frac{dy}{dx} = \frac{2\sqrt{x}(9 - 2x) - (9x - x^2) \times x^{-\frac{1}{2}}}{4x}$ which may be left un-simplified

dM1: Solves their $\frac{dy}{dx} = 0$ which must have correct indices. Index work when solving must be correct

Look for $\alpha x^{-\frac{1}{2}} \pm \beta x^{\frac{1}{2}} = 0 \Rightarrow \dots x = \dots$ with the index of x being 1

Alternatively squares E.g. $\alpha x^{-\frac{1}{2}} = \pm \beta x^{\frac{1}{2}} \Rightarrow \dots x^{-1} = \dots x^1 \Rightarrow x^2 = \dots \Rightarrow x = \dots$

Via the quotient rule the equation should proceed to a form $px^{\frac{1}{2}}(qx + r) = 0 \Rightarrow x = \dots$

 A1: Correct calculations and working leading to the $x = 3$ at P.

Cannot be scored from an incorrect $\frac{dy}{dx}$ so M1, A0, M1, A1 is NOT possible

Allow $\frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{4}x^{\frac{1}{2}} = 0 \Rightarrow x = 3$ without any intermediate work.



(b)

M1: Attempts to integrate and achieves one term with the correct index. For this to be awarded

- y must be written as a sum of two terms with one index correct $\delta x^{\frac{1}{2}} \pm \epsilon x^{\dots}$ or $\delta x^{\dots} \pm \epsilon x^{\frac{3}{2}}$
- after integration a correct index must be achieved with the indices processed and not left as for example $\left(\frac{1}{2} + 1\right)$. Look for $y = ax^{\frac{3}{2}} \pm bx^{\dots}$ or $y = ax^{\dots} \pm bx^{\frac{5}{2}}$
- an index cannot be achieved from incorrect working. See third bullet point in (a)

A1: Correct integration. Look for $3x^{\frac{3}{2}} - \frac{1}{5}x^{\frac{5}{2}}$ but may be left unsimplified

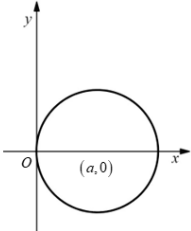
B1: (Upper) limit of the integral is 9. This can be written down without working.

Allow this to be awarded on the Figure for the point at which the curve crosses the x-axis

dM1: Method to find the area of R . It is dependent upon

- having scored the previous M1
- having limits of 1 and 9 or else 1 and a solution of $9x - x^2 = 0$, $\frac{9x - x^2}{2\sqrt{x}} = 0$ or adapted $\frac{9x - x^2}{2\sqrt{x}} = 0$ that isn't 0.

A1: $\frac{148}{5}$ o.e. such as for example 29.6

Question Number	Scheme	Marks
9(a)		B1
(b)	$(x \pm a)^2 + y^2 = \dots$ $(x \pm a)^2 + y^2 = a^2$ Uses (5, 6) in $(x \pm a)^2 + y^2 = a^2$ to form and solve an equation in a E.g. $(5 - a)^2 + 36 = a^2 \Rightarrow 10a = 61 \Rightarrow a = 6.1$ $(x - 6.1)^2 + y^2 = 6.1^2$	(1) M1 A1 dM1 A1 (4) (5 marks)

(a)

B1: A circle lying in quadrants 1 and 4 only which

- passes through (0, 0)
- is symmetrical about the x -axis

Condone slips of the pen. Mark this positively.

Ignore any reference to the point (5, 6)

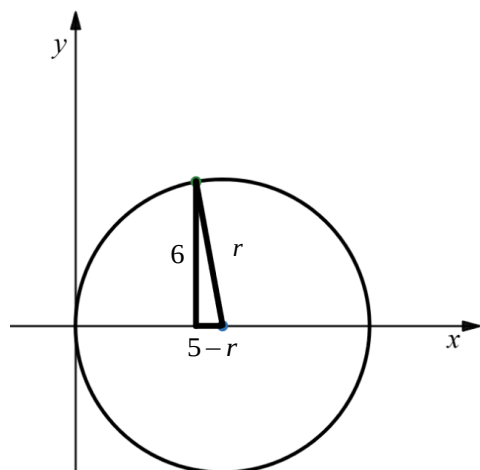
Ignore any reference to the coordinates of the centre

(b) Way One: Candidates start from an appropriate equation involving x^2 and y^2 then use (5, 6)The following equations are mostly written with a constant a . Any suitable constant may be used.The sign of the constant is also not important so accept $(x + a)^2 + y^2 = \dots$ or $(x - a)^2 + y^2 = \dots$ as well as $(a - x)^2 + y^2 = \dots$ M1: Allowable left-hand side for the equation of C For example $(x \pm a)^2 + y^2 = \dots$ or $(a - x)^2 + y^2 = \dots$ This must involve a constant and not be $(x - 5)^2 + y^2 = \dots$ for exampleA1: Correct form for the equation involving a **single** constant.Examples of correct equations are $(x - a)^2 + y^2 = a^2$, $(x + k)^2 + y^2 = k^2$ and $x^2 + y^2 = \pm 2px$ If they write $(x - a)^2 + y^2 = r^2$ and $r^2 = a^2$ this can be implieddM1: Uses (5, 6) in $(x \pm a)^2 + y^2 = a^2$ o.e. to form and solve an equation in a .E.g. $(5 - a)^2 + 36 = a^2 \Rightarrow \pm 10a = 61 \Rightarrow a = \dots$ $5^2 + 6^2 = \pm 10p \Rightarrow p = \dots$ A1: $(x - 6.1)^2 + y^2 = 6.1^2$ o.e. such as $x^2 + y^2 = 12.2x$

.....



Way Two: Candidates use Pythagoras' theorem to set up an equation in single variable



M1: Applies Pythagoras's theorem leading to the equation $(r-5)^2 + 36 = r^2$ or $(5-r)^2 + 36 = r^2$

This may be awarded for any variable

A1: Solves the above equation to find correct value $r = 6.1$

dM1: Uses a correct equation of circle $(x - '6.1')^2 + y^2 = '6.1' ^2$ following through on their 6.1.

This is dependent on the 6.1 being a solution of a correct equation (previous M1)

A1: $(x - 6.1)^2 + y^2 = 6.1^2$ o.e.

Way Three: Bisector of chord method

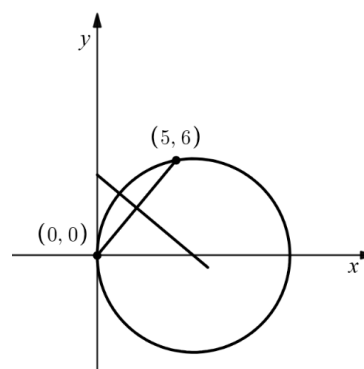
M1: Attempts to find where the perpendicular bisector cuts the x-axis
Look for a good attempt using mid points and perpendicular gradient work.

A1: Achieves equation of perpendicular bisector as $y = -\frac{5}{6}x + \frac{61}{12}$

and point of intersection with the x-axis to be $\frac{61}{10}$ o.e

dM1: Correct form of equation with $(x - '6.1')^2 + y^2 = '6.1' ^2$

A1: $(x - 6.1)^2 + y^2 = 6.1^2$ o.e.



Question Number	Scheme	Marks
10 (a)	$y = 2x + \frac{64}{x^2} - 3$ $\frac{dy}{dx} = 2 - \frac{128}{x^3}$ Attempts to solve $\frac{dy}{dx} \Rightarrow x^3 = 64 \Rightarrow x = 4$ *	M1, A1 dM1, A1* (4)
(b)	$\int 2x + \frac{64}{x^2} - 3 \, dx = x^2 - \frac{64}{x} - 3x \quad (+c)$ Finds the y values at both $x = 2$ and $x = 4$. $M = (0, 17)$ and $N = (0, 9)$ Full attempt at area = $\left[x^2 - \frac{64}{x} - 3x \right]_2^4 - 9 \times 2 + 2 \times (17 - 9) = 22 - 18 + 16 = 20$	M1, A1 B1 dM1, A1 (5) (9 marks)

(a)

M1: Attempts $\frac{dy}{dx}$ with one correct aspect. Allow for $2x \rightarrow 2$ or $x^{-2} \rightarrow x^{-3}$ but not $3 \rightarrow 0$

A1: $\frac{dy}{dx} = 2 - \frac{128}{x^3}$ o.e which may be left un simplified. The lhs is not required

dM1: Sets their $\frac{dy}{dx} = p \pm qx^{-3} = 0$ and proceeds to $\alpha x^{\pm 3} = \beta$

Also allow candidates to substitute 4 into their $\frac{dy}{dx}$ which must be of the form $\frac{dy}{dx} = p \pm qx^{-3}$ and find its value.

A1*: Shows that the x coordinate of P is 4. This is a given answer so there must be sufficient evidence

For a direct proof you should see all of $2 - \frac{128}{x^3} = 0$, $x^3 = 64$ (condone $2x^3 = 128$) and $x = 4$

or $2 - \frac{128}{x^3} = 0$, $x^{-3} = \frac{1}{64}$ (condone $x^{-3} = \frac{2}{128}$) and $x = 4$

For a substitution method look for $\frac{dy}{dx} = 2 - \frac{128}{x^3}$ and at P, $\frac{dy}{dx} = 2 - \frac{128}{4^3} = 0$, hence proven

(o.e.)



(b)

M1: Attempts to integrate $2x + \frac{64}{x^2} - 3$ scored for increasing a correct index by 1 including $3 \rightarrow 3x$

A1: $\int 2x + \frac{64}{x^2} - 3 \, dx = x^2 - \frac{64}{x} - 3x \quad (+c)$ which may be left un simplified

B1: Finds the y values at both $x = 2$ and $x = 4$. $M = (0, 17)$ and $N = (0, 9)$

This mark may be implied by an area calculation $\left[x^2 - \frac{64}{x} - 3x \right]_2^4 - 9 \times 2 + 2 \times 8$

Evidence may be awarded from the diagram or from coordinates for P and Q

dM1: Full attempt at area. It dependent upon having done some correct integration (M1 scored).

Look for the 'correct' areas being combined in the right way, condoning slips in integration and arithmetical slips

Total area = '22' - '18' + '16' = ...

A1: 20

Alt method to b using (curve – line) can be marked in a similar way

M1: Attempts to integrate $\left(2x + \frac{64}{x^2} - 3 \right) - (k)$ scored for increasing a correct index by 1

A1: $\int 2x + \frac{64}{x^2} - 12 \, dx = x^2 - \frac{64}{x} - 12x \quad (+c)$ which may be left un simplified

B1: Finds the y values at both $x = 2$ and $x = 4$. $M = (0, 17)$ and $N = (0, 9)$

This mark may be implied by an area calculation $\left[x^2 - \frac{64}{x} - 12x \right]_2^4 + (17 - 9) \times 2$

dM1: Full attempt at area. It dependent upon having done some correct integration (M1 scored).

$$\text{Shaded area under curve } R_1 = \int_2^4 \left(2x + \frac{64}{x^2} - 3 \right) - (9) \, dx = \left[x^2 - \frac{64}{x} - 12x \right]_2^4 = 4$$

Total area = $4 + 16 = 20$

Look for the 'correct' areas being combined in the right way, condoning slips in integration and arithmetical slips (such as $4 + 16 = 22$)

A1: 20



Question	Scheme	Marks
1(a)	(i) There is a <u>common difference</u> (no common ratio) and so an <u>arithmetic</u> series should be used.	B1
	(ii) $(u_n) = 5 + (n-1)d$ or $(u_n) = 5 + nd$	M1
	$(u_n) = 5 + 0.25(n-1)$ oe	A1
		(3)
(b)	Need $S_n = \frac{n}{2}(2 \times 5 + (n-1) \times 0.25) \geq 350$	M1A1
	$\Rightarrow 0.25n^2 + 9.75n \geq 700 \Rightarrow (n+19.5)^2 - 19.5^2 \geq 2800 \Rightarrow n = \dots$	M1
	So 37 weeks.	A1
		(4)
(7 marks)		

Notes:**(a)(i)**

B1: Identifies that there is a common difference (e.g. goes up in equal amounts) between first and second, and second and third terms, or that the ratio between first and second, and second and third is not the same, and states arithmetic series/sequence to be used.

M1: Attempts general term for A.S. with n or $n-1$ used and their d (which need not be correct).

A1: Correct expression for general term, accept equivalents, eg $4.75 + 0.25n$. May use another label than u_n or no label at all.

(b)

M1: Uses the sum formula for A.S. with their a and d and equates to, or sets inequality with, 350. Accept with any inequality symbol or equality between.

A1: Correct equation/inequality. Accept with any inequality symbol or equality between.

M1: Forms and solves a 3 term quadratic (need not be gathered to one side). Any valid method (including calculator – you may need to check).

A1: 37 weeks selected as answer.

For attempts via listing, send to review.

For use of geometric series:

Case 1: If a student states arithmetic series but uses geometric series formulae, then only the B mark can be scored in (a), but in (b) allow M0A0M1A0 if a correct method is used to solve their equation

$$\frac{5(r^n - 1)}{r - 1} \geq 350 \text{ (or with “=” or “<” etc) to find } n \text{ (so } n = \log_{r,n}(70(r - 1) + 1) \text{ oe)}$$

Case 2: If a student states a geometric series and gives a common ratio, then allow B0M1A0 in (a)

for $(u_n) = 5r^{n-k}$ with $k = n$ or $n-1$ and in (b) allow M1A0 for $\frac{5(r^n - 1)}{r - 1} \geq 350$ and then M1A0 for

a correct method to solve this equation.



Question	Scheme	Marks
2(a)		(i) Correct shape and position for $y = 2^x$ Crosses y axis with same intersection as $y = 4^x$ but with gentler slope. B1
		(ii) Graph of same shape as $y = 4^x$ but translated down B1
		y intercept at -5 B1
		x intercept at $\log_4 6$ B1
		(4)
(b)	$2^x = 4^x - 6$ or $y = (2^x)^2 - 6 = y^2 - 6$	M1
	$\Rightarrow 2^{2x} - 2^x - 6 = 0 \Rightarrow (2^x - 3)(2^x + 2) = 0 \Rightarrow 2^x = \dots$ or $(y - 3)(y + 2) = 0 \Rightarrow y = \dots$	M1
	$y = 2^x = 3$	A1
	$x = \log_2 3$ oe	A1
		(4)
(8 marks)		
Notes:		
(a)(i) Note: Be tolerant of “wobbles” in the graph if it is clear the correct shapes are meant, but the graph must not clearly bend away from the asymptotes. B1: Correct sketch for $y = 2^x$. This should be correct shape, but shallower gradient – look for crossing at same y intercept as $y = 4^x$, above $y = 4^x$ to the left of the y axis and below $y = 4^x$ to the right of the y axis.		
(ii) B1: Graph of $y = 4^x$ translated down by 6 units. Look for the same shape, always below $y = 4^x$ though be tolerant as the graph increases (accept if the graphs don’t “narrow” as shown). B1: Correct y intercepts of -5 B1: Correct x intercept of $\log_4 6$. Accept equivalents such as $\log 6 / \log 4$ or awrt 1.29 Ignore any references to asymptotes for these marks.		
(b) M1: Sets the equations equal to form an equation in x only, or writes 4^x in terms of 2^x and substitutes to achieve an equation in y only. M1: Solves the equation to find a value for 2^x or y A1: Correct y (or 2^x) value of intersection. (May have second solution) A1: Correct x value of intersection. Must have rejected second “solution”. Accept exact equivalents, such as $\log 3 / \log 2$ or $2 \log_4 3$		

Question Number	Scheme	Marks
8 (a)	$y = \frac{4}{3}x^3 - 11x^2 + kx \Rightarrow \frac{dy}{dx} = 4x^2 - 22x + k$ uses $x = 2, \frac{dy}{dx} = 0 \Rightarrow 0 = 16 - 44 + k \Rightarrow k = 28^*$	M1 dM1 A1* (3)
(b)	$\frac{dy}{dx} = 4x^2 - 22x + 28 = 0 \Rightarrow (2x - 4)(2x - 7) = 0 \Rightarrow x = \dots$ $x < 2, x > \frac{7}{2}$	M1 A1 (2)
(c)	$\int \left(\frac{4}{3}x^3 - 11x^2 + 28x \right) dx \Rightarrow \frac{1}{3}x^4 - \frac{11}{3}x^3 + 14x^2$ Correct y coordinate of $M = \frac{68}{3}$ Complete method to find $R = 2 \times \frac{68}{3} - \int_0^2 \left(\frac{4}{3}x^3 - 11x^2 + 28x \right) dx$ $= 2 \times \frac{68}{3} - \left(\frac{1}{3} \times 2^4 - \frac{11}{3} \times 2^3 + 14 \times 2^2 \right)$ $= \frac{40}{3}$	M1 A1 B1 M1 A1 (5) (10 marks)

(a)

M1: Attempts $\frac{dy}{dx}$ with one index correct. Must be seen in part (a).

dM1: Substitutes $x = 2$ into $\frac{dy}{dx}$ and sets $= 0$. The $\frac{dy}{dx}$ must be of the form $ax^2 + bx + k$ and the substitution must be clear. Going directly to $-28 + k = 0$ is M0. The “=0” may be implied by an attempt to solve the equation for this mark.

A1*: Achieves $k = 28$ via a correct intermediate line and no missing “=0”

(b)

M1: Attempts to find the critical values.

A1: $x < 2, x > \frac{7}{2}$ or $x \neq 2, x \neq \frac{7}{2}$ Do not accept $\frac{7}{2} < x < 2$ Accept alternative set notations.



(c)

M1: Attempts to integrate with one index correct

A1: Correct integration $\int \left(\frac{4}{3}x^3 - 11x^2 + 28x \right) dx \Rightarrow \frac{1}{3}x^4 - \frac{11}{3}x^3 + 14x^2$ need not be simplified

B1: Correct y coordinate of $M = \frac{68}{3}$. Accept awrt 22.7 for this mark. May be seen anywhere e.g. on the sketch

M1: Complete method to find $R = 2 \times \left(\frac{68}{3} \right) - \int_0^2 \left(\frac{4}{3}x^3 - 11x^2 + 28x \right) dx$ OR

$\int_0^2 \left(\frac{68}{3} \right) - \left(\frac{4}{3}x^3 - 11x^2 + 28x \right) dx$ The lower limit may be implied. The integral must be a changed function.

A1: $\frac{40}{3}$ or exact equivalent

Question Number	Scheme	Notes	Marks
9(a)	$mx = x - x^2 \Rightarrow m = 1 - x \Rightarrow x = \dots$ Or $y = \frac{y}{m} - \frac{y^2}{m^2} \Rightarrow m^2 = m - y \Rightarrow y = \dots$	Attempts to eliminate either x or y and factors out or cancels x/y to get a linear equation and solve.	M1
	$x = 1 - m$ and $y = m(1 - m)$	Both correct	A1
			(2)
(b)	$\int x - x^2 (-mx) dx = \frac{x^2}{2} - \frac{x^3}{3} \left(-m \frac{x^2}{2} \right)$	$x^n \rightarrow x^{n+1}$ in at least one term	M1
	Area of $R_1 = \int_0^{1-m} \{x - x^2 (-mx)\} dx$ $= \frac{(1-m)^2}{2} - \frac{(1-m)^3}{3} \left(-m \frac{(1-m)^2}{2} \right) - 0$	Uses the limits "1 - m" and 0 in their integrated expression and subtracts (condone the omission of the "- 0")	M1
		Correct expression in m with/without the area under line subtracted.	A1
	Area of $R_1 = \int_0^{1-m} \{x - x^2 - mx\} dx = \frac{(1-m)^2}{2} (1-m) - \frac{(1-m)^3}{3} (-0)$ Correct strategy for the area (may be scored for finding separate areas and subtracting)		dM1
	$= \frac{(1-m)^3}{6} *$	Correct expression	A1*
			(5)
(c)	Area of $(R_1 + R_2) = \int_0^1 (x - x^2) dx = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \dots$ $\left(= \frac{1}{6} \right)$	Correct method for finding the area of $R_1 + R_2$ Alternatively, a correct method for finding the area of R_2	M1
	Alt: Area of $R_2 = \int_{1-m}^1 (x - x^2) dx + \frac{1}{2} ("1 - m") \times m(1 - m)$ $= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_{1-m}^1 + \frac{1}{2} m(1 - m)^2 = \dots \left(= \frac{1}{6} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{2} m(1 - m)^2 \right)$		
	$R_1 = R_2 \Rightarrow \frac{(1-m)^3}{6} = \frac{1}{12} \Rightarrow m = \dots$	Sets up a correct equation using the answer to part (b) and solves for m	dM1
	Alt: $\frac{(1-m)^3}{6} = \frac{1}{6} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{2} m(1 - m)^2 \Rightarrow m = \dots$		
	$m = 1 - \frac{1}{\sqrt[3]{2}}$	Correct exact value in any form	A1
			(3)
			Total 10

Notes**(a)**

M1: Attempts to eliminate either x or y and factors out or cancels x/y to get a linear equation and solves the resulting equation. May be implied by one correct coordinate of the two.



A1: Both coordinates correct. Accept $y = m - m^2$

(b)

M1: Attempts to integrate equation for curve C to find the area, may be part of integrating a difference of curve and line, or may be done separately.

M1: Applies the limits of 0 (may be implied) and their x from (a) (which must be in terms of m) to the integral (which must be a changed function), and subtracts the correct way round. Again, may or may not be including the line at this point.

A1: Correct expression in m for the area under C between 0 and $1 - m$. If the line equation has been subtracted and combined already, then it is for an unsimplified correct overall expression, but if it is separate then the A1 can be scored just for the area under C . If

$$\text{combined it is } \text{Area} = \frac{(1-m)^2}{2}(1-m) - \frac{(1-m)^3}{3}(-0)$$

dM1: Depends on previous M. Correct overall strategy for the area. Can be scored if they have attempted curve – line, or may be scored for finding separate areas and subtracting the triangle area from area under curve. For the area under line, if not as part of integral, look for $\frac{1}{2} \times \text{their } x \times \text{their } y$ from part (a) subtracted from the curve.

A1: Fully correct work leading to the given answer with sufficient evidence of combination of terms before the given answer. Look for reaching an expression of the form

$$\frac{(1-m)^2}{2}(1-m) - \frac{(1-m)^3}{3} \text{ with } (1-m)^3 \text{ terms before the final answer. Going from}$$

$$= \frac{(1-m)^2}{2} - \frac{(1-m)^3}{3} - m \frac{(1-m)^2}{2} \text{ to the given answer with no intermediate step is A0.}$$

Alternatively, they may fully expand their expression and factorise, or fully expand their expression and the given answer and compare and conclude it is the same – check carefully that all working is correct.

(c)

M1: Correct method for finding the area of $R_1 + R_2$. Look for the integral of C with limits 0 to 1 applied, the subtraction of 0 may be implied. Allow for $\frac{1}{6}$ from a correctly set up integral if the integration is not shown.

Alternatively score for a correct method for finding the area of R_2 . Look for the integral of C from $1 - m$ to 1 with the area of the triangle added.

dM1: Sets up a correct equation using the answer to part (b) and reaches a value for m .

A1: $m = 1 - \frac{1}{\sqrt[3]{2}}$ Correct exact value in any form



Question Number	Scheme	Marks
10a	$\frac{(x-k)^2}{\sqrt{x}} = \frac{x^2 - 2kx + k^2}{x^{\frac{1}{2}}} = \frac{x^2}{x^{\frac{1}{2}}} - \frac{2kx}{x^{\frac{1}{2}}} + \frac{k^2}{x^{\frac{1}{2}}} = x^{\frac{3}{2}} - 2kx^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$	M1
	$\int_1^{16} x^{\frac{3}{2}} - 2kx^{\frac{1}{2}} + k^2x^{-\frac{1}{2}} dx = \left[\frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}kx^{\frac{3}{2}} + 2k^2x^{\frac{1}{2}} \right]_1^{16}$	M1A1
	$\frac{2}{5}(16)^{\frac{5}{2}} - \frac{4}{3}k(16)^{\frac{3}{2}} + 2k^2(16)^{\frac{1}{2}} - \frac{2}{5} + \frac{4}{3}k - 2k^2 = 6k^2 - 84k + \frac{2046}{5}$	dM1A1
		(5)
b	$9 = \frac{(1-k)^2}{\sqrt{1}} \Rightarrow 9 = (1-k)^2$	M1
	$k = 4 \text{ only} \quad *$	A1*
		(2)
c	$q = 36$	B1
	$\frac{1}{2}(9 + "36") \times 15 - \left("6"(4)^2 - "84"(4) + \frac{2046}{5} \right) = \frac{1683}{10}$	M1A1
		(3)
		(10 marks)

(a)

- M1 Attempts to multiply out the numerator and split the fraction into separate terms. Score for one term with a correct index (which must be processed). Any attempts via substitution e.g. setting $u = \sqrt{x}$ send to review. Condone poor attempts at multiplying out the brackets such that $(x-k)^2 \Rightarrow x^2 \pm k^2$
- M1 Attempts to raise the power (which must be a fraction or decimal) by one on one of their terms. (The index does not need to be processed).
- A1 $\frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}kx^{\frac{3}{2}} + 2k^2x^{\frac{1}{2}}$ or unsimplified equivalent. Indices must be processed. Ignore any spurious notation around the expression. May be with or without $+c$
- dM1 Substitutes in 16 and 1 for x and subtracts either way round. The values embedded are sufficient and condone invisible brackets. It is dependent on the previous method mark.
- A1 $6k^2 - 84k + \frac{2046}{5}$ o.e. e.g. $6k^2 - 84k + 409.2$ isw after a correct answer is seen, but withhold this mark if there is spurious notation such as $\int$ or dx at either the start or end of the answer.

(b)

- M1 Substitutes the coordinates of A into the equation for C or, alternatively, substitutes in the coordinates of A and $k = 4$ into the equation for C.
- A1* Solves either:
- $9 = (1-k)^2 \Rightarrow k = \dots$
 - $k^2 - 2k - 8 = 0$ with terms collected to a 3TQ via any allowable method including via a calculator.



With either method they must proceed to $k = 4$ only with no errors seen, including invisible brackets. If -2 is found it must be rejected or 4 clearly selected. If they use $k = 4$ in their solution, then there must be a minimal conclusion.

(c)

B1 $q = 36$ seen (may be in (b) or on the diagram) or implied for example by a correct area of a triangle or trapezium.

M1 Attempts to find the area of R using their q and **their answer to part (a)**. Condone arithmetical slips and do not be concerned with poor notation for this mark. Must proceed to find a value for the area but condone subtracting areas the wrong way round which may result in a negative area.

Possible methods include:

e.g. Line – curve: $\int_1^{16} \frac{9}{5}x + \frac{36}{5} dx$ – part (a) with $k = 4$

e.g. Area of trapezium ($= 337.5$) – part (a) with $k = 4$

Note: the correct values if the trapezium is split into a rectangle and a triangle are 135 and 202.5, respectively.

For evidence of use of part (a), look for $k = 4$ to be substituted into their part (a) answer, or may be implied by a correct value for their $6k^2 - 84k + \frac{2046}{5}$ with $k = 4$

Condone $\int_1^{16} \frac{(x-k)^2}{\sqrt{x}} dx = 169.2$ provided their part (a) is correct

A1 $\frac{1683}{10}$ oe (ignore units) Condone spurious notation.

Withhold this mark if their method results in a negative area but they just omit the negative sign. They would need to indicate area is positive for e.g. $A = \dots - \dots = -\frac{1683}{10}$ so $A = \frac{1683}{10}$

Question Number	Scheme	Marks
10 (a)	$(f'(x)) = -\frac{72}{x^3} + 2$ Attempts to solve $f'(x) = 0 \Rightarrow x = \dots$ via $x^{\pm n} = k$, $k > 0$ $x > \sqrt[3]{36}$ oe	M1 A1 dM1 A1 (4)
(b)	$\int \frac{36}{x^2} + 2x - 13 \, dx = -\frac{36}{x} + x^2 - 13x (+c)$ Uses limits 9 and 2 $= \left(-\frac{36}{9} + 9^2 - 13 \times 9\right) - \left(-\frac{36}{2} + 2^2 - 13 \times 2\right) = 0 *$	M1 A1 dM1 A1* (4)
(c)(i)	8	B1
(ii)	$\int_2^6 \left(\frac{36}{x^2} + 2x + k\right) dx = 0 \Rightarrow \left[-\frac{36}{x} + x^2 + kx\right]_2^6 = 0 \Rightarrow (30 + 6k) - (-14 + 2k) = 0$ $44 + 4k = 0 \Rightarrow k = -11$	M1 A1 (3) (11 marks)

Notes**(a)**M1 Attempts $f'(x)$ with one index correct. Allow for $x^{-2} \rightarrow x^{-3}$ or $2x \rightarrow 2$ A1 $f'(x) = -\frac{72}{x^3} + 2$ correct but may be unsimplified $f'(x) = 36 \times -2x^{-3} + 2$ dM1 Attempts to find where $f'(x) = 0$. Score for $x^n = k$ where $k > 0$ and $n \neq \pm 1$ leading to $x = \dots$
Do not allow this to be scored from an equation that is adapted incorrectly to get a positive k .
Allow this to be scored from an attempt at solving $f'(x) \dots 0$ where $\dots$ can be any inequalityA1 Achieves $x > \sqrt[3]{36}$ or $x > 6^{\frac{2}{3}}$ Allow $x \geq \sqrt[3]{36}$ or $x \geq 6^{\frac{2}{3}}$ but not $x > \left(\frac{1}{36}\right)^{-\frac{1}{3}}$

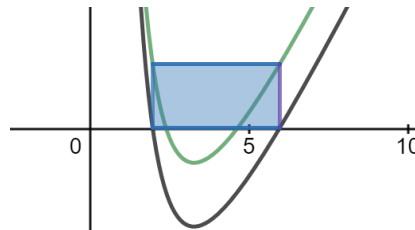
We require an exact value but remember to isw. An answer of 3.302 usually implies the first 3 marks.

(b)M1 For $x^n \rightarrow x^{n+1}$ seen on either $\frac{36}{x^2}$ or $2x$. Indices must be processed. eg $x^{1+1} \rightarrow x^2$ A1 $\int \frac{36}{x^2} + 2x - 13 \, dx = -\frac{36}{x} + x^2 - 13x$ which may be unsimplified. Eg $x^2 \leftrightarrow \frac{2x^2}{2}$ Allow with $+c$ dM1 Substitutes 9 and 2 into their integral and subtracts either way around. Condone missing brackets
Dependent upon the previous MA1* Completely correct integration with either embedded values seen or calculated values
 $(-40) - (-40)$

Note that this is a given answer and so the bracketing must be correct.



Question Number	Scheme	Marks
(c)(i) B1	For sight of 8. Allow this to be scored from a restart, from a calculator or ... = 8	
(c)(ii) M1	This may be awarded in a variety of ways - A restart (See scheme). For this to be awarded all terms must be integrated with $k \rightarrow kx$, the limits 6 and 2 applied, the linear expression in k must be set equal to 0 and a solution attempted. - An attempt at solving $\int_2^6 k + 13 \, dx = 8$ or equivalent. Look for the linear equation $-8 + 4(13 + k) = 0$ or $4(13 + k) = 8$ and a solution attempted. - Recognising that the curve needs to be moved up 2 units. - Sight of $\frac{8}{6-2}$ or $-13 + 2$	
A1	$k = -11$. This alone can be awarded both marks as long as no incorrect working is seen.	



Question Number	Scheme	Marks
9 (a)	$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13$ $\frac{dy}{dx} = \frac{4}{3}x - \frac{9}{2\sqrt{x}}$ Attempts to solve $\frac{dy}{dx} > 0 \Rightarrow x^{\frac{3}{2}} > \frac{27}{8} \Rightarrow x > \frac{9}{4}$	M1, A1 dM1, A1 (4)
(b)	$\int \frac{2}{3}x^2 - 9\sqrt{x} + 13 \, dx = \frac{2}{9}x^3 - 6x^{\frac{3}{2}} + 13x \quad (+c)$ $\text{Area} = \left[\frac{2}{9}x^3 - 6x^{\frac{3}{2}} + 13x \right]_0^9 = 117$ Substitutes $x=9$ into $\frac{dy}{dx} = \frac{4}{3}x - \frac{9}{2\sqrt{x}} \Rightarrow \frac{dy}{dx} = \left(\frac{21}{2}\right)$ Finds where tangent meets x-axis $0 - 40 = \frac{21}{2}(x-9) \Rightarrow x = \frac{109}{21}$ $\text{Area of } R = \left[\frac{2}{9}x^3 - 6x^{\frac{3}{2}} + 13x \right]_0^9 - \frac{1}{2} \times 40 \times \left(9 - \frac{109}{21}\right)$ $= 117 - \frac{1600}{21} = \frac{857}{21}$	M1 dM1, A1 M1 dM1 A1 ddM1 A1 (8) (12 marks)

There are no marks for integrating in part (a) or differentiating in (b). This is a problem-solving question and candidates should be awarded these marks only for work in the correct part.

(a)

M1: Attempts $\frac{dy}{dx}$ with one index correct. Allow for $x^2 \rightarrow x^1$ or $\sqrt{x} \rightarrow x^{\frac{1}{2}}$

A1: $\frac{dy}{dx} = \frac{4}{3}x - \frac{9}{2\sqrt{x}}$ o.e which may be left un simplified. The LHS is not required

dM1: Attempts to find where $\frac{dy}{dx} > 0$ and proceeds to $x^{\frac{3}{2}} > k$ or $x^3 > k$ following correct squaring

Also allow $\frac{dy}{dx} \geq 0$ and proceeds to $x^{\frac{3}{2}} \geq k$ but also condone $\frac{dy}{dx} = 0$ leading to $x^{\frac{3}{2}} = k$

If a candidate has a correct derivative, then condone $\frac{4}{3}x - \frac{9}{2\sqrt{x}} \dots 0 \Rightarrow x \dots \frac{9}{4}$ where ... is =, > or $\geq$

A1: States $x > \frac{9}{4}$ or $x \geq \frac{9}{4}$ o.e. following the award of both M's. Do not accept other ranges such as $x < 0$



(b)

M1: Attempts to integrate $\frac{2}{3}x^2 - 9\sqrt{x} + 13$ scored for increasing a correct index by 1 which could include $13 \rightarrow 13x$

dM1: Uses limits of 9 (and 0) either way around and finds area under curve between 0 and 9.
It is dependent upon the previous M mark. Award when values are embedded

A1: Achieves 117 as the area under the curve between 0 and 9.

Be careful when awarding this mark as sight of 117 does not necessarily merit this award

You must see M1 and dM1 and have correct integration. Allow for $\left[\frac{2}{9}x^3 - 6x^{\frac{3}{2}} + 13x \right]_0^9 = 117$

The use of a calculator to integrate is not allowed

M1: Attempts the gradient of the tangent.

Substitutes $x = 9$ into their $\frac{dy}{dx}$ (from part a or restarted) which must have one index correct.

A correct value (or follow through value) can imply this method only if there is a $\frac{dy}{dx}$. It cannot be

awarded if there is not a calculation for $\frac{dy}{dx}$ in the question

dM1: Correct method of finding where tangent meets the x -axis. Dependent upon previous M
For this to be awarded you must see all elements or their equivalent. E.g.

- Substitution of $x = 9$ into their $\frac{dy}{dx}$ (from part a or restarted) which must have one index correct
- use of (9, 40) with their $\frac{dy}{dx}$ to form the equation of the tangent at P
- substitution of $y = 0$ into their correctly found tangent to find where tangent crosses x -axis

A1: Correct value $x = \frac{109}{21}$

ddM1: Complete method of finding the area of R . This is dependent upon all previous M's

It is usually scored for the area under the curve between 0 and 9 – the area of the triangle.

The area of the triangle may be attempted by integration. Look for a correct attempt at the method for the equation of a tangent, integrated to a form $\alpha x^2 + \beta x$ with limits of the x intercept of their tangent and 9

A1: Area $R = \frac{857}{21}$

.....

Note that there are other ways in which the area could be attempted. Look carefully at their attempt please.

ALT I: (Area under curve between 0 and x intercept) + (Area between curve and line between x intercept and 9)

$$\text{E.g. } \int_0^{\frac{109}{21}} \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13 \right) dx + \int_{\frac{109}{21}}^9 \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13 \right) - \left(\frac{21}{2}x - \frac{109}{2} \right) dx$$

M1: As the main method. Attempts to integrate $\frac{2}{3}x^2 - 9\sqrt{x} + 13$, scored for increasing a correct index by 1

dM1: As main method with same dependency. In this case it would be for applying the limits 0 to k and k to 9 to the integrated $\frac{2}{3}x^2 - 9\sqrt{x} + 13$

A1: Implied by correct final answer

M1, dM1, A1: As main method

$$\text{ddM1: } \int_0^{\frac{109}{21}} \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13 \right) dx + \int_{\frac{109}{21}}^9 \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13 \right) - \left(\frac{21}{2}x - \frac{109}{2} \right) dx$$

Complete method of finding the area of R. This is dependent upon all previous M's. See main scheme for general principles for awarding this mark.

$$\text{A1: Area } R = \frac{857}{21}$$

ALT II: (Area between curve and line between 0 and 9) - (Area of triangle under axis)

$$\text{E.g. } \int_0^9 \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13 \right) - \left(\frac{21}{2}x - \frac{109}{2} \right) dx - \frac{1}{2} \times \frac{109}{2} \times \frac{109}{21}$$

M1: As the main method. Attempts to integrate $\frac{2}{3}x^2 - 9\sqrt{x} + 13$, scored for increasing a correct index by 1

dM1: As main method with same dependency. Look for applying the limits 0 and 9 to the integrated $\frac{2}{3}x^2 - 9\sqrt{x} + 13$

A1: Area between curve and line, between 0 and 9, is $\frac{729}{4}$

M1, dM1, A1: As main method

$$\text{ddM1: } \int_0^9 \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13 \right) - \left(\frac{21}{2}x - \frac{109}{2} \right) dx - \frac{1}{2} \times \frac{109}{2} \times \frac{109}{21}$$

Complete method of finding the area of R. This is dependent upon all previous M's. See main scheme for general principles for awarding this mark.

$$\text{A1: Area } R = \frac{857}{21}$$

Question Number	Scheme	Marks
9(a)	$y = \frac{9x^2(5-\sqrt{x})}{5}$ $y = 9x^2 - \frac{9}{5}x^{\frac{5}{2}} \Rightarrow \frac{dy}{dx} = 18x - \frac{9}{2}x^{\frac{3}{2}}$ $18x - \frac{9}{2}x^{\frac{3}{2}} = 0 \Rightarrow x^{\frac{1}{2}} = (4)$ $(16, 460.8)$	M1, A1 dM1 A1, A1 (5)
(b)	$5 - \sqrt{x} = 0 \Rightarrow x = 25$	M1, A1 (2)
(c)	$\int 9x^2 - \frac{9}{5}x^{\frac{5}{2}} dx = 3x^3 - \frac{18}{35}x^{\frac{7}{2}}$ $\text{Shaded area} = \frac{(25-16) \times 460.8}{1} - \left[3x^3 - \frac{18}{35}x^{\frac{7}{2}} \right]_{16}^{25}$ $= 1312.7$	M1A1 <u>M1</u> , dM1 A1 (5) (12 marks)

(a)

M1: Attempts to differentiate. In most cases this will involve multiplying out and differentiating each term. Look for a sum of two terms with one correct index.

So, score for either $y = ax^2 \pm \dots \Rightarrow \frac{dy}{dx} = px \pm \dots$ or $y = \dots \pm bx^{\frac{5}{2}} \Rightarrow \frac{dy}{dx} = \dots \pm qx^{\frac{3}{2}}$

It is possible to differentiate using the product rule.

In this case look for the correct form. E.g $y = \frac{9x^2(5-\sqrt{x})}{5} \Rightarrow \frac{dy}{dx} = (5-\sqrt{x}) \times \dots x + \dots x^2 \times \dots x^{-\frac{1}{2}}$

You may see attempts using the product or quotient rules so look carefully at what is written

A1: $\frac{dy}{dx} = 18x - \frac{9}{2}x^{\frac{3}{2}}$ o.e but allow this un-simplified.

FYI, using the product rule, the correct unsimplified answer is $(5-\sqrt{x}) \times \frac{18x}{5} + \frac{9x^2}{5} \times -\frac{1}{2}x^{-\frac{1}{2}}$

dM1: Sets their $px \pm qx^{\frac{3}{2}} = 0$ or $px \pm qx^{\frac{1}{2}} = 0$ (from a slip on the fractional index) and proceeds using correct index work to $\alpha x^{\pm \frac{1}{2}} = \beta$

Other methods exist, such as squaring, but look for correct index work with the coefficients squared as well. The demand of candidates is to show their working and it will be applied in this question on this mark.

If candidates go from $18x - \frac{9}{2}x^{\frac{3}{2}} = 0$ to $x = 16$ without intermediate working they will lose this mark.

They can be awarded the next two marks however.

A1: x coordinate of 16 **following correct differentiation and** $\frac{dy}{dx} = 0$.



Note that errors stemming from a misunderstanding of the $\div 5$ can lead to $x = 16$ but should be scored A0

A1: (16, 460.8) following award of previous A1. Allow this to be scored from correct coordinate in part (c)

Also allow this to be given separately as $x = 16$, $y = \frac{2304}{5}$ o.e.

(b)

M1: Deduces that $5 - \sqrt{x} = 0$ or $\sqrt{x} = 5$. Squaring methods are also possible (with coefficients squared) usually leading to $25 - x = 0$ o.e.

A1: $x = 25$ following award of the M1

25 on its own without any working scores M0 A0

(c) In open this is marked M, A, M, A, A but is now being scored M, A, M, dM, A
Method integrating just curve

M1: Attempts to integrate. In most cases this will involve multiplying out and integrating each term. Look for a sum of two terms with one correct index. Ignore any spurious integral signs if it is obvious they have integrated.

So score for either $\int (ax^2 \pm \dots) dx \Rightarrow rx^3 \pm \dots$ or $\int (\dots \pm bx^{\frac{5}{2}}) dx \Rightarrow \dots \pm sx^{\frac{7}{2}}$

This can be scored in a similar way if they have attempted (line – curve)

A1: $3x^3 - \frac{18}{35}x^{\frac{7}{2}}$ but allow this unsimplified

If (line – curve) has been attempted look for $\dots - \left(3x^3 - \frac{18}{35}x^{\frac{7}{2}}\right)$ OR $\dots - 3x^3 + \frac{18}{35}x^{\frac{7}{2}}$

M1: Correct method of finding the **area of rectangle** or **area of right-angled triangle** (with MP as the hypotenuse).

E.g. For the rectangle look for ('25' – '16') $\times$ '460.8' or $\int_{16}^{25} ('460.8' - \dots) dx = ['460.8'x - \dots]_{16}^{25} = \dots$

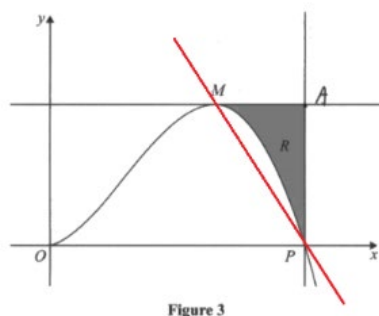
dM1: Complete attempt to find area of R involving the correct combination of areas, usually

(area of rectangle – area under curve) or $(2 \times \text{area of triangle} - \text{area under curve})$.

It is dependent upon both previous M's. For the area under the curve look for a minimum of one term being correctly integrated and limits of their 16 from (a) and their 25 from (b)

A1: AWR 1312.7

Other more complicated methods are possible; If they integrate (line – curve) we score as follows



The first M1, A1 can be awarded as in the main method.
The following M1 is a correct method for triangle AMP
The dM1 is for the full method to find area of R as before.

Question Number	Scheme	Marks
1	$h = 0.2$ $\frac{1}{2} \times 0.2 \times [9.2 + 8.6 + 2(8.4556 + 3.8512 + 5.0342 + 7.8297)]$ 6.814	B1 M1 A1
		(3 marks)

B1 $h = 0.2$ o.e. (including $\frac{5-4}{5}$) seen or implied by sight of e.g. $\frac{0.2}{2}$ or $\frac{1}{10}$ in front of the bracket.
 May also be implied by a correct answer if no incorrect working seen. $h = -0.2$ is B0

M1 Correct application of the trapezium rule with their h (which may be 1 or h).
 Look for a correct inner bracket structure $9.2 + 8.6 + 2(8.4556 + 3.8512 + 5.0342 + 7.8297)$
 condoning slips copying values from the table or the omission of the final brackets on the rhs

e.g. $\frac{1}{2} \times 0.2 \times (9.2 + 8.6 + 2(8.4556 + 3.8512 + 5.0342 + 7.8297))$ is M1

but $\frac{1}{2} \times 0.2 \times 9.2 + 8.6 + 2(8.4556 + 3.8512 + 5.0342 + 7.8297)$ is M0 unless the brackets are recovered or implied by the correct answer (you may need to check this)

Also allow for a correct method adding individual trapezia using their h condoning copying errors but the brackets must be correct (or recovered or implied by later work)

e.g. $\frac{1}{2} \times 0.2 \times (9.2 + 8.4556) + \frac{1}{2} \times 0.2 \times (8.4556 + 3.8512) + \frac{1}{2} \times 0.2 \times (3.8512 + 5.0342) +$

$\frac{1}{2} \times 0.2 \times (5.0342 + 7.8297) + \frac{1}{2} \times 0.2 \times (7.8297 + 8.6)$

A1 awrt 6.814 isw once a correct answer is seen. Correct answer with no working scores B1M1A1 but if there is evidence of using $h = -0.2$ then maximum awarded is B0M1A0
 The A mark cannot be awarded without both B1M1 being awarded on this question.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$ **Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

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General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1(a)	$h = 0.5$	B1
	$A \approx \frac{1}{2} \times \frac{1}{2} \{2.287 + 2.834 + 2(4.470 + 6.719 + 7.291)\}$	M1
	= awrt 10.52	A1
		(3)
(b)(i)	$A \approx "10.52" - 4$	M1
(b)(ii)	= 6.52	A1ft
	$A \approx "10.52"$	B1ft
		(3)
		Total 6

(a)

B1 $h = 0.5$ oe seen or implied by sight of $\frac{0.5}{2}$ in front of the bracket. May also be implied by a correct answer if no incorrect working seen. $h = -0.5$ is B0

M1 Correct application of the trapezium rule with their h . Look for a correct bracket structure, condoning slips copying values from the table or the omission of the final brackets on the rhs

e.g. $\frac{1}{2} \times \frac{1}{2} (2.287 + 2.834 + 2(4.470 + 6.719 + 7.291))$ is M1

but $\frac{1}{2} \times \frac{1}{2} \times 2.287 + 2.834 + 2(4.470 + 6.719 + 7.291)$ is M0 unless the brackets are recovered or implied by their answer (you may need to check this).

Also allow for a correct method adding individual trapezia using their h condoning copying errors but the brackets must be correct (or recovered or implied by later work)

e.g. $\frac{1}{2} \times \frac{1}{2} (2.287 + 4.470) + \frac{1}{2} \times \frac{1}{2} (4.470 + 6.719) + \frac{1}{2} \times \frac{1}{2} (6.719 + 7.291) + \frac{1}{2} \times \frac{1}{2} (7.291 + 2.834)$

A1 awrt 10.52 isw once a correct answer is seen. Correct answer with no working scores B1M1A1 but if there is evidence of using $h = -0.5$ then maximum awarded is B0M1A0

(b)(i) Note that they must be using their answer to part (a) to score marks in (b)

M1 Attempts to subtract 4 from their answer to part (a) or allow for the expression $"10.52" - [2x]_{-1}^1$ or e.g. $"10.52" - (2 + 2)$. Do not condone omission of brackets or arithmetical errors here. This mark cannot be scored from a new attempt at the trapezium rule.
 $"10.52" - (2 - 2)$ is M0

A1ft awrt 6.52 correct answer or correct ft (allowing if they proceed to round a more accurate answer in (a) to 2dp). In both cases they must be using their answer to part (a) function which could even come from integration.

(ii)

B1ft awrt 10.52 Writes down the same answer as part (a) (follow through their (a)). Condone "same as **part (a)**" (again allow if they proceed to round a more accurate answer in (a) to 2dp)

Note (a) 10.5 (b)(i) 6.5 (b)(ii) 10.5 could score B1M1A0M1A1ftB1ft

Question Number	Scheme	Marks
6. (a)	Identifies $h = 3$	B1
	Area = $\frac{3}{2}\{0 + 0 + 2(1.52 + 2.74 + 3.12 + 3.08)\}$ = 31.38 m ²	M1 A1
		(3)
(b)	Calculates "31.38" $\times 1.5 \times 60$ = awrt 2800 m ³	M1 A1
		(2)
(c)	Underestimate with valid reason, e.g. area of trapezia are smaller than cross sectional area	B1
		(1)
		(6 marks)

(a)

B1: Identifies $h = 3$. This may be stated or implied by the sight of $\frac{3}{2}\{\dots + 2(\dots + \dots + \dots)\}$

M1: Correct method of finding area using the trapezium rule. The form of the rule is given so look for the form

$\frac{3}{2}\{0 + 0 + 2(1.52 + 2.74 + 3.12 + 3.08)\}$ but you can condone slips on the digits of the heights, e.g. 3.21 for 3.12. You may not see the zeros or the trailing } which is fine. The bracketing may be hard to decipher

but the correct answer implies the method. If they state another value of h e.g. $\frac{15}{6} = 2.5$ then look for

$\frac{2.5}{2}\{0 + 0 + 2(1.52 + 2.74 + 3.12 + 3.08)\}$ condoning slips. If h is not stated, then this mark can be implied by

$\frac{\dots}{2}\{0 + 0 + 2(1.52 + 2.74 + 3.12 + 3.08)\}$ condoning slips.

Condone an attempt using two triangles and three trapezia. Use the same criteria for this award. The area of any triangle and the area of any trapezium must be found from a correct formula.

A1: Accept 31.38 or awrt 31.4 (m²) following correct expression for area.

Units are not necessary. The exact fractional answer is $\frac{1569}{50}$ is also acceptable

As the area is below the axis, a minority of candidates are taking h or the y values to be negative. This is fine for the first two marks and all three marks provided the modulus is taken.

(b)

M1: Attempts their (a) $\times 1.5 \times 60$ or equivalent such as their (a) $\times 90$

A1: AWRT 2800 m³ Units are not necessary

(c)

B1: States **underestimate** AND gives a **valid reason**.

The reason, in almost all cases, should explicitly compare the area of the trapezia with the shaded area.

All reasons must make some reference to 'trapezia' or 'area of a trapezium'

The Figure could be used to explain their reasoning so look carefully at what the candidate states

Condone 'the trapezia are smaller than the cross section'

'the bottom lines of the trapezia are above the curve'

'the curve is above the lines of the trapezia' (remember the y axis is pointing down)

Do not accept any reference to convex or concave without further clarification.

Merely stating "the area we get from the trapezium rule is less than the shaded area" is too vague. There needs to be some explanation of why it is an underestimate, not just the fact that it is.



Question Number	Scheme	Marks
3	$\int_1^4 3x + \frac{16}{x^2} - 8 \, dx = \frac{3}{2}x^2 - \frac{16}{x} - 8x \quad (+c)$ e.g. Area = $\frac{3}{2}(5+11) - \left[\frac{3}{2}x^2 - \frac{16}{x} - 8x \right]_1^4 = 24 - \left(-12 + \frac{45}{2} \right) = \frac{27}{2}$	M1, A1, A1 dM1, A1 (5) (5 marks)

M1: Attempts to integrate the curve and achieves at least one term with the correct index i.e. $x \rightarrow x^2$, $x^{-2} \rightarrow x^{-1}$ or $-8 \rightarrow \pm 8x$ (indices do not need to be processed for this mark e.g. $x \rightarrow x^{1+1}$)

A1: Two correct terms from $\frac{3}{2}x^2 - \frac{16}{x} - 8x$, may be unsimplified but indices must be processed.

Allow equivalent forms.

A1: $\frac{3}{2}x^2 - \frac{16}{x} - 8x$, may be unsimplified. Allow the presence of the constant of integration.

Do not be concerned with spurious notation or any limits.

dM1: Full method including correct area (value or expression) of trapezium. The expression with correct limits substituted in or partially evaluated as evidence is sufficient for this mark and condone subtracting either way round. They cannot just proceed to the final answer without additional evidence of using the limits. Note area of trapezium = 24, area under curve = 10.5.

Condone slips in evaluating some of the terms provided the intention is clear.

A1: 13.5 o.e following all previous marks scored. Note if they use the limits the wrong way round resulting in a negative value for the integral then we must see that calculated correctly before they state the area is 13.5

Alternative: line – curve

$$\int_1^4 (13 - 2x) - \left(3x + \frac{16}{x^2} - 8 \right) dx = \int_1^4 \left(-5x - \frac{16}{x^2} + 21 \right) dx = \left[-\frac{5x^2}{2} + \frac{16}{x} + 21x \right]_1^4 = 48 - \frac{69}{2} = \frac{27}{2}$$

Note: They may not fully simplify/collect terms in this method so score according to the main scheme for the first 3 marks.

M1: Attempts to integrate and achieves at least one term with the correct index i.e. $x \rightarrow x^2$, $x^{-2} \rightarrow x^{-1}$ or "21" $\rightarrow$ " ± 21 " x (allow slips in their collecting terms). Indices do not need to be processed for this mark.

A1: Two correct terms from $-\frac{5x^2}{2} + \frac{16}{x} + 21x$, which may be unsimplified but indices must be processed.

Allow equivalent forms including terms which have not been collected.

A1: Correct integration, may be unsimplified. Allow the presence of the constant of integration.

Allow equivalent forms including terms which have not been collected.

Do not be concerned with spurious notation or any limits.



dM1: Full method (condoning an incorrect equation for the line). Scored for

e.g. $\int_1^4 (13-2x) - \left(3x + \frac{16}{x^2} - 8\right) dx = \left[\left(13x - x^2\right) - \left(\frac{3}{2}x^2 - \frac{16}{x} - 8x\right) \right]_1^4 = \dots$

The expression with correct limits substituted in or partially evaluated is sufficient for this mark and condone subtracting either way round.

Condone slips in finding the straight line, evaluating some terms or collecting terms incorrectly provided the intention is clear.

A1: 13.5 o.e following all previous marks scored. Note if they use the limits the wrong way round resulting in a negative value then we must see that calculated correctly before they state the area is 13.5

There will be variations of these two methods but should mark similarly. Send to review if you are unsure.

Note for those who attempt curve-line

$$\int_1^4 \left(3x + \frac{16}{x^2} - 8\right) - (13 - 2x) - dx = \int_1^4 \left(5x + \frac{16}{x^2} - 21\right) dx = \left[\frac{5x^2}{2} - \frac{16}{x} - 21x \right]_1^4 = -48 + \frac{69}{2} = -\frac{27}{2}$$

Allow to score M1A1A1dM1A0 but accept the final A1 to be scored if they subsequently state the area is $\frac{27}{2}$

(i.e. they cannot just write $\frac{27}{2}$ as the answer to this integral)

Question Number	Scheme	Marks
4(a)	$f(x) = ax^3 + bx^2 + 5x - 3$ Sets $f(-3) = 0 \rightarrow a \times (-3)^3 + b \times (-3)^2 + 5 \times (-3) - 3 = 0$ $\Rightarrow -27a + 9b = 18 \Rightarrow b = 3a + 2^*$	M1 A1* (2)
(b)	Sets $f\left(\frac{1}{2}\right) = \frac{7}{4} \rightarrow a \times \left(\frac{1}{2}\right)^3 + b \times \left(\frac{1}{2}\right)^2 + 5 \times \left(\frac{1}{2}\right) - 3 = \frac{7}{4}$ $a + 2b = 18$ Solves $b = 3a + 2$ with their $a + 2b = 18 \Rightarrow a = \dots, b = \dots$ $a = 2, b = 8$	M1 A1 M1 A1 (4)
(c)	e.g. $\frac{2x^3 + 8x^2 + 5x - 3}{x - 2} = (\dots x^2 + \dots x + \dots) + \text{Remainder}$ (Quotient =) $2x^2 + 12x + 29$ Remainder = 55	M1 A1 B1 (3) (9 marks)

(a)

M1: Sets $f(-3) = 0$ and proceeds to an equation in a and b Condone sign slips.Score when you see embedded values within the equation or two correct simplified terms on the lhs of the equation. The $= 0$ may be implied by further work which is not the given answer.A1*: Completes proof with no errors including brackets and at least one intermediate stage of working such as $-27a + 9b = 18$ and $= 0$ **must be seen** somewhere in their solution.**Alt I**M1: Attempts to divide by $x + 3$ and sets their remainder in a and $b = 0$. May be seen in a variety of forms such as the grid or box method. Condone slips provided they are attempting to divide by $x + 3$ and set their remainder in a and b equal to 0. The $= 0$ may be implied by further work which is not the given answer.

$$\begin{array}{r}
 \overline{ax^2 + (b-3a)x + 5 - 3(b-3a)} \\
 x+3 \overline{) ax^3 + 5x } \\
 \underline{ax^3 } \\
 + 3ax^2 \\
 \underline{(b-3a)x^2 } \\
 + 5x \\
 \underline{(b-3a)x^2 + 3(b-3a)x} \\
 (5-3(b-3a))x \\
 \underline{(5-3(b-3a))x + 15 - 9(b-3a)} \\
 - 3
 \end{array}
 \Rightarrow -3 - (15 - 9(b-3a)) = 0$$

A1*: Completes proof with no errors seen and an intermediate stage of working such as $-18 + 9(b-3a) = 0 \Rightarrow b = 3a + 2^*$ ($= 0$ **must be seen** somewhere in their solution)

Alt II

M1: Substitutes in $b = 3a + 2$, sets $f(-3) = 0 \rightarrow$ equation in a or b only Condone sign slips.

Score when you see embedded values within the equation or two correct terms on the lhs of the equation.

A1*: Completes proof and achieves $0 = 0$ concluding that $b = 3a + 2$

(b) Note they may have substituted in $b = 3a + 2$ early so the equation will be in a only or b only

M1: Sets $f\left(\frac{1}{2}\right) = \frac{7}{4}$ o.e. and proceeds to an equation in a and b .

Condone slips provided they set their remainder in a and b equal to $\frac{7}{4}$ o.e. (e.g. $f\left(\frac{1}{2}\right) - \frac{7}{4} = 0$)

May use algebraic division or e.g. grid method.

$$\begin{array}{r}
 \frac{a}{2}x^2 + \frac{1}{2}\left(b + \frac{a}{2}\right)x + \frac{5}{2} + \frac{1}{4}\left(b + \frac{a}{2}\right) \\
 2x-1 \overline{) ax^3 \quad + bx^2 \quad + 5x \quad - 3} \\
 \underline{ax^3 \quad - \frac{a}{2}x^2} \\
 \left(b + \frac{a}{2}\right)x^2 + 5x \Rightarrow -3 - \left(-\frac{5}{2} - \frac{1}{4}\left(b + \frac{a}{2}\right)\right) = \frac{7}{4} \\
 \underline{\left(b + \frac{a}{2}\right)x^2 \quad - \frac{1}{2}\left(b + \frac{a}{2}\right)x} \\
 \left(5 + \frac{1}{2}\left(b + \frac{a}{2}\right)\right)x - 3 \\
 \underline{\left(5 + \frac{1}{2}\left(b + \frac{a}{2}\right)\right)x - \frac{5}{2} - \frac{1}{4}\left(b + \frac{a}{2}\right)} \\

 \end{array}$$

A1: $a + 2b = 18$ or other simplified equivalent including $\frac{1}{8}a + \frac{1}{4}b = \frac{9}{4}$ which may only be in a or only in b .

e.g. $7a = 14$

M1: Solves $b = 3a + 2$ with their $a + 2b = 18 \Rightarrow a = \dots, b = \dots$

You do not need to be concerned with the mechanics of the rearrangement. Award as long as values appear for both a and b (both non zero) (which may be found using a calculator)

A1: $a = 2, b = 8$

(c) In EPEN this is M1A1A1 but we are marking this M1A1B1

M1: Attempts to divide the cubic with their values for a and b by $(x - 2)$ and proceeds to a quadratic

quotient $ax^2 + \dots x + \dots$ where a is the value found in (b). It is not a requirement to proceed to a value for R for this mark, but their method must allow for it to be found. If R is stated as 0 then M0.

Condone slips.

If they attempt $2x^3 + 8x^2 + 5x - 3 = (x - 2)(ax^2 + bx + c) + R \Rightarrow a = \dots, b = \dots, c = \dots, (R = \dots)$

A1: Quotient $= 2x^2 + 12x + 29$ which can be extracted from their working e.g. on the top line of their algebraic division or $(x - 2)(2x^2 + 12x + 29) + R$ isw

B1: Remainder $= 55$ which can be extracted from their working. isw

May also be written in the expression $(x - 2)(2x^2 + 12x + 29) + 55$



Question Number	Scheme	Marks
5(a)	Identifies $h = 1.5$ $\text{Area} = \frac{1.5}{2} \{12 + 0.023 + 2(4.243 + 1.5 + 0.530 + 0.188 + 0.066)\}$ $= 18.8$	B1 M1 A1 (3)
(b) (i)	$\int_{-2}^7 3\left(\frac{1}{2}\right)^{x+2} dx = \frac{1}{4} \int_{-2}^7 3\left(\frac{1}{2}\right)^x dx = \frac{1}{4} \times "18.8" = 4.7$	B1ft
(ii)	$\int_{-2}^7 (2^{-x} + 2x) dx = \frac{1}{3} \times "18.8" + \left[x^2\right]_{-2}^7 = 51.3$	M1, A1ft (3) (6 marks)

(a) Note that awrt 18.8 with no working is 0 marks

B1: Identifies $h = 1.5$ or may be implied by e.g. 0.75 oe outside of the bracket in the expression for the area

M1: Correct method of finding the area. Requires $\frac{1}{2} \times \text{their } h \times \text{correct bracket structure}$ but condone the omission of a trailing bracket. Allow $h = 1$ if stated but if only an expression is seen then it must be correct. Allow equivalent expressions e.g. they may add together individual trapezia. Condone invisible brackets to be implied by further work or awrt 18.8 but do not allow just awrt 18.8 with no working seen to score this mark. Condone slips on the digits of the heights, e.g. 4.234 for 4.243

A1: 18.8 or better (18.80775) following M1. i.e. if they have carried out the calculation and their 18.8 does not come from 18.0775 (allow truncating or rounding) then A0. Isw
Note that the calculator answer is 17.2785...

(b) Note in part (b) they may have rounded their value in part (a) as their final answer (e.g. 18.8) but then used an earlier unrounded value in part (b). They must give an answer in part (b) that is at least 1dp for the B1ft in (i) and/or A1ft (ii) to be awarded. You will need to check their follow through answers if (a) is incorrect.

Do not accept solutions in (b) which involve re-doing the table of values and attempting the trapezium rule again.

(b)(i) Note if they have awrt 17.3 in (a) then this mark cannot be scored unless we see $\frac{1}{4} \times \text{awrt } 17.3$

B1ft: Calculates $\frac{1}{4} \times "18.8" = "4.7"$

Note that the calculator answer is 4.3196...

(b)(ii) Note if they have awrt 17.3 in (a) then A1ft can be scored following M1.

M1: Recognises that $\int_{-2}^7 (2^{-x}) dx = \frac{1}{3} \times \int_{-2}^7 3\left(\frac{1}{2}\right)^x dx$ AND integrates $2x$ to x^2 . We must see the x^2 or allow to be implied by limits substituted in and subtracting e.g. $7^2 - (-2)^2$ (or $7^2 - 2^2$ or $49 - 4$) but not just 45

A1ft: 51.3 or ft on their answer to part (a) by adding 45 to $\frac{1}{3} \times "18.8"$. Allow fractions e.g. $\frac{769}{15}$

Note that the calculator answer is 50.7595...



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \dots$$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0, \quad \text{leading to } x = \dots$$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

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Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

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Question Number	Scheme	Marks
5(a)	$\cos \theta(3 \cos \theta - \tan \theta) = 2 \Rightarrow \cos \theta(3 \cos \theta - \frac{\sin \theta}{\cos \theta}) = 2$ or e.g. $\cos \theta(3 \cos \theta - \tan \theta) = 2 \Rightarrow 3 \cos^2 \theta - \sin \theta = 2$ or e.g. $\cos \theta(3 \cos \theta - \tan \theta) = 2 \Rightarrow 3 \cos^2 \theta - \frac{\sin \theta}{\cos \theta} \cos \theta = 2$	M1
	$3(1 - \sin^2 \theta) - \sin \theta = 2$	M1
	$3 \sin^2 \theta + \sin \theta - 1 = 0$ *	A1*
		(3)
(b)	$(\sin 2x =) \frac{-1 \pm \sqrt{13}}{6}$ (or awrt 0.43 and awrt -0.77 (or truncated -0.76))	M1A1
	$2x = \sin^{-1}(0.434\dots)$ or $2x = \sin^{-1}(-0.77\dots) \Rightarrow x = \dots$	M1
	-1.13, -0.438, 0.225, 1.35	A1A1
		(5)
		(8 marks)

(a)

M1: Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ to write the equation in terms of sine and cosine only.

M1: Uses $\sin^2 \theta + \cos^2 \theta = 1$ to obtain a quadratic equation in sine only.

A1*: Achieves $3 \sin^2 \theta + \sin \theta - 1 = 0$ * with no errors.

Condone one notational slip e.g. $3 \sin \theta^2$ instead of $3 \sin^2 \theta$ or e.g. $3 \sin$ for $3 \sin \theta$ in the working but the printed answer must be correct but allow e.g. $0 = \sin \theta + 3 \sin^2 \theta - 1$

(b)

M1: Attempts to solve the quadratic $3 \sin^2 \theta + \sin \theta - 1 = 0$ which may be in any variable. Usual rules apply for solving a quadratic (via a calculator is also acceptable and may imply this mark). If no working is shown then the roots must be correct.

A1: $\frac{-1 \pm \sqrt{13}}{6}$ or a minimum of $\frac{-1 \pm \sqrt{13}}{2 \times 3}$ (or as decimals awrt 0.43 and awrt -0.77).

Whether subsequently rejected or not. Ignore any labelling just look for these values.

M1: Attempts to find one angle within the range by finding the inverse sine of one of their roots and dividing by 2. May be implied by their values and allow if working in degrees.

A1: Any two of awrt -1.13, -0.438, 0.225, 1.35

A1: All four of awrt -1.13, -0.438, 0.225, 1.35 and no others in the range.

Special case - answers in degrees: -64.9, -25.1, 12.9, 77.1

Score A1 for awrt any 2 of these and then A0 so a maximum of 7/8 for the question.

NB allow answers to be found in degrees which are subsequently converted to radians but then left in terms of π e.g. $\frac{-64.9\pi}{180}$ etc. provided the answers round to the -1.13, -0.438, 0.225, 1.35

Some candidates are obtaining a different quadratic equation in (a) e.g. $3 \sin^2 \theta - \sin \theta - 1 = 0$. In such cases allow both M marks in (b) if they persist with their incorrect quadratic provided there are sign errors only e.g. $\pm 3 \sin^2 \theta \pm \sin \theta \pm 1 = 0$



Question Number	Scheme	Marks
6(a)	$h = 0.5$	B1
	$\frac{1}{2} \times 0.5 \times [3 + 1.92 + 2(2.6833 + 2.4 + 2.1466)]$	M1
	4.845	A1
		(3)
(b)	$\int_0^2 2 - \frac{1}{4}x^2 \, dx = \left[2x - \frac{x^3}{12} \right]_0^2 = \frac{10}{3}$	M1A1
	% of logo shaded = $\frac{4.845 - \frac{10}{3}}{6}$	dM1
	= 25.2(%)	A1
		(4)
		(7 marks)

(a)**B1:** $h = 0.5$ seen or implied.**M1:** A full attempt at the trapezium rule.Look for $\frac{\text{their } h}{2} \{3 + 1.92 + 2(2.6833 + 2.4 + 2.1466)\}$ but condone copying slips.Note that $\frac{\text{their } h}{2} 3 + 1.92 + 2(2.6833 + 2.4 + 2.1466)$ scores M0 unless the missing brackets are recovered or implied by their answer. (You may need to check)

Allow this mark if they add the areas of individual trapezia e.g.

$$\frac{\text{their } h}{2} \{3 + 2.6833\} + \frac{\text{their } h}{2} \{2.6833 + 2.4\} + \frac{\text{their } h}{2} \{2.4 + 2.1466\} + \frac{\text{their } h}{2} \{2.1466 + 1.92\}$$

Condone copying slips but must be a complete method using all the trapezia.

A1: awrt 4.845. Apply isw once awrt 4.845 is seen.**(b)****M1:** Attempts to integrate $2 - \frac{1}{4}x^2$. Award for either $2x$ or $\dots x^3$.**A1:** $\frac{10}{3}$ seen or implied.**dM1:** Attempts to find the difference (either way round) between their answer to part (a) and their attempt at the area under C_2 which must be positive and divides by 6
(Must follow an attempt to integrate C_2 and not an attempt to use the trapezium rule again)**A1:** awrt 25.2(%) (the % symbol is not required). Do not allow -25.2% or e.g. 0.252Allow $\frac{907}{36}$ (%)Some candidates are not doing the integration and are using their calculators for the area under C_2 . If they then go on and use the $10/3$ to get awrt 25.2% then we will allow a Special Case of M0A0dM0A1

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**1. Factorisation $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ 2. FormulaAttempt to use correct formula (with values for a , b and c).3. Completing the squareSolving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:**1. DifferentiationPower of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)2. IntegrationPower of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
6(a)	States or uses $M = (5, 4)$ or $\text{grad } AB = \frac{1}{3}$ o.e.	B1
	States or uses $M = (5, 4)$ and $\text{grad } AB = \frac{1}{3}$ o.e.	B1
	Equation of l is $y - 4 = -3(x - 5)$	M1
	$y = -3x + 19$	A1
(b)	$k = -2$	B1 ft
(c)	Attempts radius or radius ² E.g. $(11 - 7)^2 + (6 - -2)^2$	M1
	States $(x - 7)^2 + (y + 2)^2 = (11 - 7)^2 + (6 - -2)^2$	dM1
	$(x - 7)^2 + (y + 2)^2 = 80$	A1
		(3)
		(8 marks)

(a)

B1: States or uses $M = (5, 4)$ **or** $\text{grad } AB = \frac{1}{3}$ o.e. such as $\frac{4}{12}$ Note that gradient AB may be implied by use of a gradient for l of -3 o.e.B1: States or uses $M = (5, 4)$ **and** $\text{grad } AB = \frac{1}{3}$ o.e. such as $\frac{4}{12}$ Note that gradient AB may be implied by use of a gradient for l of -3 o.e.M1: Attempts equation of l using their M and their correctly adapted gradient.The gradient used must be the negative reciprocal of their gradient of AB . If it just appears without any working it must be correct.If the form $y = mx + c$ is used this mark is only scored when they proceed as far as $c = \dots$ A1: CAO $y = -3x + 19$

(b)

B1ft: $k = -2$ or follow through on the value of their $'-3 \times 7 + 19'$ Allow this to be scored if the coordinates of P are given.For example, as $(7, -2)$ or even $y = -2$ following use of 7

(c)

M1: Attempts either the radius² or the radius using their $(7, k)$ and one of the given points

$$\text{E.g. } (11 - 7)^2 + (6 - -2)^2 \text{ or } \sqrt{(7 - (-1))^2 + (-2 - 2)^2}$$

It does not need to be correctly identified as r or r^2 dM1: Attempts equation of circle using their radius² and their $(7, k)$ Note that $(x - 7)^2 + (y - k)^2 = (11 - 7)^2 + (6 - k)^2$ would score M1 dM1A1: $(x - 7)^2 + (y + 2)^2 = 80$ The rhs of the equation must be simplified and ISW after a correct answer.

Alt (c)

M1: Writes equation of circle as $x^2 + y^2 - 14x - 2k'y = \lambda$ following through on their k dM1: Substitutes $(-1, 2)$ or $(11, 6)$ into the equation of the circle and finds a value for λ 

$$A1: x^2 + y^2 - 14x + 4y = 27$$

Question Number	Scheme	Marks
7 (i)	$\frac{1}{2}\{\log 4 + \log 9 + 2(\log 5 + \log 6 + \log 7 + \log 8)\}$ $= \frac{1}{2}\log(4 \times 9) + \log(5 \times 6 \times 7 \times 8) = \log 10080$	B1 M1 A1 (3)
(ii)	$2\log_5(5-a) - \log_5(a+25) = 1$ $2\log_5(5-a) = \log_5(5-a)^2 \text{ OR } 1 = \log_5 5$ $\log_5 \frac{(5-a)^2}{(a+25)} = 1$ $a^2 - 15a - 100 = 0$ $(a-20)(a+5) = 0$ $a = -5$	B1 M1 A1 dM1 A1 (5) (8 marks)

(i) Do not be concerned if the 10's completely disappear or disappear and reappear from the $\log_{10}$

B1: For writing down a correct expression for the approximate area using the trapezium rule.

The bracketing must be correct (condoning a missing trailing bracket) or implied to be correct by subsequent work. It is not implied by answers rounding to 4.0

Allow the calculation with separate trapezia. E.g.

$$\frac{1}{2}(\log 4 + \log 5) + \frac{1}{2}(\log 5 + \log 6) + \frac{1}{2}(\log 6 + \log 7) + \frac{1}{2}(\log 7 + \log 8) + \frac{1}{2}(\log 8 + \log 9)$$

M1: Attempts some correct log work seen at least once within an attempted trapezium rule.

This will usually be for a correct use of the addition law at some point but could be for $2\log 5 = \log 25$.

It cannot be scored for $\log k = 4.00 \Rightarrow k = 10^{4.00}$

A1: $\log 10080$ following the award of the M mark.

This cannot be scored by candidates who approximate the result to be 4.003...for instance and then use their calculators to get the $\log 10080$

(ii) Do not be concerned if a different variable is used or the base 5 is omitted

B1: For $2\log_5(5-a) = \log_5(5-a)^2$ OR replacing 1 by $\log_5 5$

M1: Correctly combines any two terms which will usually be

$$2\log_5(5-a) - \log_5(a+25) = 1 \Rightarrow \log_5 \frac{(5-a)^2}{(a+25)} = \dots$$

Alternatives exist. E.g. $2\log_5(5-a) = \log_5(a+25) + 1 \Rightarrow \dots = \log_5(5a+125)$

A1: Correct 3TQ $a^2 - 15a - 100 = 0$ o.e.

dM1: Correct attempt to find at least one value of a . It is dependent upon previous M and having reached a quadratic equation. Apply usual rules for solving a quadratic.

Allow calculator solutions but you will need to check that they are/it is correct

A1: $a = -5$ ONLY

If $a = 20$ is written down it must be rejected



Note that a solution where the log terms are incorrectly combined can score SC 10001 as long as they reach $a = -5$ only

$$\frac{\log_5 (5-a)^2}{\log_5 (a+25)} = 1 \Rightarrow a^2 - 15a - 100 = 0 \Rightarrow a = -5$$



Question Number	Scheme					Marks
1.	<i>a</i>	<i>b</i>	<i>c</i>	<i>abc</i>	One correct set <i>a</i> , <i>b</i> & <i>c</i>	B1
	2	15	3	90		
	3	9	8	216	Two correct rows	M1
	4	1	15	60		
	All values of <i>abc</i> are multiples of 6				Fully correct + statement	A1
						(3 marks)

Main method: Numerical answer finding all known solutions

B1: Any correct set of values for *a*, *b* and *c*. You can ignore any attempt at the product *abc*

M1: Any two fully correct rows including the correct calculations for the products.
Condone any extra or incorrect rows

A1: Fully correct. The solution must have

- the three correct rows with correct calculations for *abc*
- any extra (or incorrect) rows that may be present must be deleted or somehow excluded
- a minimal statement showing that they are aware that 90, 216 and 60 are multiples of 6.

E.g they are all divisible by 6, use of a tick ✓, proven, QED, calculations such as $\frac{90}{6} = 15$. They can be divided by 6 (on its own) is insufficient without further clarification such as "to give whole numbers"

- no incorrect calculations. E.g if their attempt to prove that 90 being a multiple of 6 includes $\frac{90}{6} = 10$ then it is A0

Algebraic solution or solutions that rely on explanations: These will be rare and unlikely to lead to completely correct solutions.

Example of an attempt that has some merit based upon odds and evens

If *a* is even

Then $a = 2n$, leading to $c = 4n^2 - 1 = (2n+1)(2n-1)$ and $b = 21 - 4n^2 - 2n$

giving $abc = 2n(21 - 4n^2 - 2n)(2n+1)(2n-1)$

If *a* is odd.

Then $a = 2n+1$, leading to $c = 4n^2 + 4n = 4n(n+1)$ and $b = 19 - 4n^2 - 6n$

giving $abc = (2n+1)(19 - 4n^2 - 6n)4n(n+1)$

B1: Attempts both odd and even and achieves correct algebraic expressions for *a*, *b* and *c* in both cases.

M1: Correct products with attempts when $n = 1, 2$ and achieves two fully correct rows

A1: Fully correct with no additional rows

Question Number	Scheme	Marks
2.(a)	E.g. $64 \times \left(-\frac{1}{2}\right)^3 = -8 \checkmark *$	M1, A1* (2)
(b)	Finds the value of a . E.g. $64 \div \left(-\frac{1}{2}\right)^2 = 256$ Uses $S_{\infty} = \frac{a}{1-r} = \frac{256}{1-\left(-\frac{1}{2}\right)} = \frac{512}{3}$	M1, A1 M1, A1 (4) (6 marks)

(a)

M1: Any acceptable attempt to prove the statement.

This usually involves getting to an equation in r ...

E.g. I $ar^2 = 64, ar^5 = -8 \Rightarrow 64r^3 = -8$ o.e. such as $ar^2 = 64, ar^5 = -8 \Rightarrow r^{-3} = -8$

E.g. II $ar^2 = 64, ar^5 = -8 \Rightarrow \frac{64}{r^2} = \frac{-8}{r^5}$

(the numbers used in the attempt, that is 64 and -8 must be correct)

but could involve using the given answer

E.g. III $64 \times \left(-\frac{1}{2}\right)^3, 64 \times -\frac{1}{2} \times -\frac{1}{2} \times -\frac{1}{2}$ or $-8 \div \left(-\frac{1}{2}\right)^3$

Condone the following types of slips/errors

E.g. I or II with incorrect indices on terms as long as they differ by 3;

$$ar^3 = 64, ar^6 = -8 \Rightarrow r^3 = -\frac{8}{64}$$

E.g. III with missing brackets $64 \times -\frac{1}{2}^3$

A1*: Fully correct proof. A minimal proof could be $r^3 = -\frac{8}{64} \Rightarrow r = -\frac{1}{2}$ o.eAs the result is given a direct proof must have a penultimate line equivalent of $r^3 = -\frac{8}{64}$ or $r^{-3} = -8$ If they state $64r^3 = -8$ followed by $r = -\frac{1}{2}$ without any intermediate work then it is M1 A0If they assume the answer then expect to see equivalent to $64 \times \left(-\frac{1}{2}\right)^3 = -8$ which must be followed by a concluding statement (tick sufficient) is required.

Equivalent work would be $64 \xrightarrow{\times -\frac{1}{2}} -32 \xrightarrow{\times -\frac{1}{2}} 16 \xrightarrow{\times -\frac{1}{2}} -8 \checkmark$

This mark cannot be awarded from incorrect indices on r or missing brackets (that are necessary)

(b) **The marks for the following can only be awarded from work in part (b)**

M1: Uses a suitable method to find the value of a allowing $r = \pm \frac{1}{2}$

E.g. Attempts $a \times \left(\pm \frac{1}{2}\right)^2 = 64 \Rightarrow a = \dots$

Condone a missing bracket e.g. $a \times -\frac{1}{2}^2 = 64 \Rightarrow a = \dots$

A1: States or implies that $a = 256$ via the correct $r = -\frac{1}{2}$.

Stating the value scores the two marks. Note that $a \times \frac{1}{4} = 64 \Rightarrow a = 256$ is M1, A1 (nothing incorrect)

This can be awarded following a missing bracket. (It isn't a proof)

M1: Uses $S_{\infty} = \frac{a}{1-r}$ with a value of $a \neq 64, -8$ and $r = \pm \frac{1}{2}$

So award $S_{\infty} = \frac{'a'}{1 - \frac{1}{2}} = \dots$ or $S_{\infty} = \frac{'a'}{1 - \left(-\frac{1}{2}\right)} = \dots$ but if an incorrect formula is stated score it M0

A1: $\frac{512}{3}$ o.e. ISW after a correct answer.

Condone $170.\dot{6}$ but do not allow awrt 170.7 unless the exact correct answer is seen first.

Question Number	Scheme	Marks
3 (a)	$f(x) = (3x^2 - 4x - 5)(x - k) - 5$ States -5	B1 (1)
(b)	Sets $f(-2) = 25 \rightarrow (3 \times 4 - 4 \times -2 - 5)(-2 - k) - 5 = 25$ $15(-2 - k) = 30 \Rightarrow -2 - k = 2 \Rightarrow k = -4$ *	M1 A1* (2)
(c)	$(3x^2 - 4x - 5)(x + 4) - 5 = 3x^3 + 8x^2 - 21x - 25$ Attempts $3x-1 \overline{)3x^3 + 8x^2 - 21x - 25}$ to achieve quotient of $\dots x^2 + \dots x + \dots$ and a remainder that is a constant Quotient = $x^2 + 3x - 6$ OR Remainder = -31 Quotient = $x^2 + 3x - 6$ AND Remainder = -31	B1 M1 A1ft A1 (4) (7 marks)

(a)

B1: States -5 You may see $f(k)$ or a division sum attempted, which is fine. You are just awarding the sight of -5 Don't be too concerned about what -5 is called. You may see $f(x)$, $f(k)$ or even x

(b)

M1: Sets $f(-2) = 25$ to form an equation in k Condone sign slips within the calculations but setting $f(-2) = -25$ or $f(2) = 25$ is M0. Some may attempt to substitute -2 into the expanded form of $f(x) = 3x^3 - (4 + 3k)x^2 + (4k - 5)x + 5k - 5$ which is fine. Condone slips but expect to see linear coefficients for k in each of x^2 , x and the constant terms.Don't be concerned about poor attempts to solve the equation in k for this markA1*: Shows sufficient working with no errors to prove that $k = -4$.

The answer must follow a simplified and (easily) solvable equation.

(See scheme for possible route but allow simplified linear equations like $-30 - 15k = 30$).....
M1: Alternatively substitutes $k = -4$ into $f(x)$ to get $(3x^2 - 4x - 5)(x + 4) - 5$ and attempt the value of $f(-2)$ A1: Shows that $f(-2) = (12 + 8 - 5)(-2 + 4) - 5 = 15 \times 2 - 5 = 25$ and makes a minimal conclusion e.g. ✓

.....