



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Algebraic Fractions & Functions (P3.1)

International A Level Mathematics · P3: Pure Mathematics 3 (WMA13)

39 questions · 328 marks · Mark schemes included

Past papers January 2020 - June 2025 · 3 sub-topics

 guidemygrades.com

 @Guidemygrades

 @Guidemygrades

 @Guidemygrades

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
P3.1a ALGEBRAIC FRACTIONS		4 Qs · 34 marks	3
1. Q4	IAL P3 (WMA13) June 2025	5	4
2. Q4	IAL P3 (WMA13) January 2023	7	6
3. Q3	IAL P3 (WMA13) January 2021	8	10
4. Q9	IAL P3 (WMA13) October 2020	14	14
P3.1b COMPOSITE & INVERSE FUNCTIONS; DOMAIN & RANGE		17 Qs · 148 marks	19
5. Q1	IAL P3 (WMA13) January 2023	6	20
6. Q4	IAL P3 (WMA13) June 2023	6	22
7. Q3	IAL P3 (WMA13) October 2020	7	24
8. Q2	IAL P3 (WMA13) October 2022	7	28
9. Q2	IAL P3 (WMA13) January 2020	8	30
10. Q2	IAL P3 (WMA13) October 2023	8	34
11. Q6	IAL P3 (WMA13) October 2024	8	38
12. Q6	IAL P3 (WMA13) January 2025	8	42
13. Q1	IAL P3 (WMA13) June 2025	8	46
14. Q1	IAL P3 (WMA13) October 2021	9	50
15. Q2	IAL P3 (WMA13) June 2022	9	54
16. Q4	IAL P3 (WMA13) June 2021	10	58
17. Q6	IAL P3 (WMA13) June 2022	10	62
18. Q7	IAL P3 (WMA13) October 2023	10	66
19. Q5	IAL P3 (WMA13) June 2024	10	70
20. Q6	IAL P3 (WMA13) January 2022	11	74
21. Q4	IAL P3 (WMA13) January 2024	13	78
P3.1c MODULUS FUNCTIONS & TRANSFORMATIONS		18 Qs · 146 marks	82
22. Q1	IAL P3 (WMA13) January 2024	4	83
23. Q1	IAL P3 (WMA13) June 2025	5	85
24. Q2	IAL P3 (WMA13) January 2021	6	87
25. Q1	IAL P3 (WMA13) June 2024	6	89
26. Q3	IAL P3 (WMA13) October 2024	7	91
27. Q4	IAL P3 (WMA13) October 2020	8	95
28. Q6	IAL P3 (WMA13) June 2021	8	99
29. Q5	IAL P3 (WMA13) June 2022	8	103
30. Q6	IAL P3 (WMA13) January 2023	8	107
31. Q7	IAL P3 (WMA13) January 2025	8	111
32. Q6	IAL P3 (WMA13) January 2020	9	113



QUESTION	SOURCE	MARKS	PAGE
33. Q4	IAL P3 (WMA13) January 2021	9	117
34. Q6	IAL P3 (WMA13) June 2023	9	121
35. Q9	IAL P3 (WMA13) October 2023	9	125
36. Q2	IAL P3 (WMA13) October 2021	10	129
37. Q7	IAL P3 (WMA13) January 2022	10	133
38. Q10	IAL P3 (WMA13) June 2025	10	137
39. Q7	IAL P3 (WMA13) October 2022	12	140



QUESTIONS

39 questions · 328 marks · 3 sub-topics

SUB-TOPIC P3.1a

ALGEBRAIC FRACTIONS

Sub-topic P3.1a · 4 questions · 34 marks · lowest tariff first



4. The function f is defined by

$$f(x) = \frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} \quad x > 3$$

(a) Show that

$$f(x) = \frac{7x}{x-3} \quad x > 3 \quad (3)$$

(b) Hence find $f'(x)$ giving your answer in simplest form.

(2)



(Total for Question 4 is 5 marks)



4.

(a) Find the values of the constants A , B , C and D such that

(4)

(b) Hence find,

(3)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(Total for Question 4 is 7 marks)



Leave
blank



Leave
blank



Question 3 continued

Leave
blank

Q3

(Total 8 marks)



Leave
blank

9. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \quad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = g(x)$, where

$$g(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \quad -3 < x < 3.5 \quad x \in \mathbb{R}$$

- (b) Find the equation of the tangent to C at the point where $x = 2$.
Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

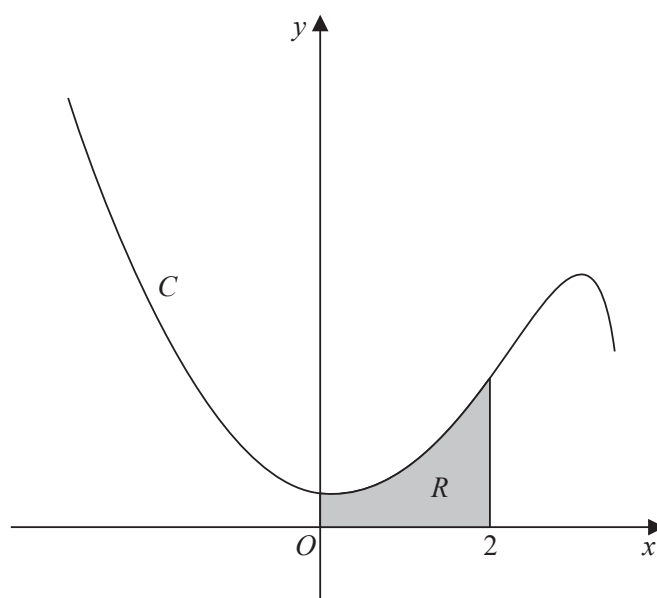


Figure 4

Figure 4 shows a sketch of the curve C .The region R , shown shaded in Figure 4, is bounded by C , the y -axis, the x -axis and the line with equation $x = 2$

- (c) Find the exact area of R , writing your answer in the form $a + b \ln 2$, where a and b are constants to be found.

(5)

Leave
blank

Leave
blank

Leave
blank

Leave
blank

Q9

TOTAL FOR PAPER IS 75 MARKS



SUB-TOPIC P3.1b

COMPOSITE & INVERSE FUNCTIONS; DOMAIN & RANGE

Sub-topic P3.1b · 17 questions · 148 marks · lowest tariff first



1. The functions f and g are defined by

$$f(x) = 9 - x^2 \quad x \in \mathbb{R} \quad x \geq 0$$

$$g(x) = \frac{3}{2x+1} \quad x \in \mathbb{R} \quad x \geq 0$$

(a) Write down the range of f

(1)

(b) Find the value of $fg(1.5)$

(2)

(c) Find g^{-1}

(3)

Question 1 continued

(Total for Question 1 is 6 marks)



Question 4 continued

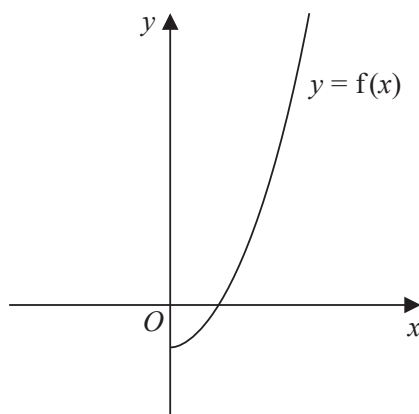


Diagram 1

(Total for Question 4 is 6 marks)



3.

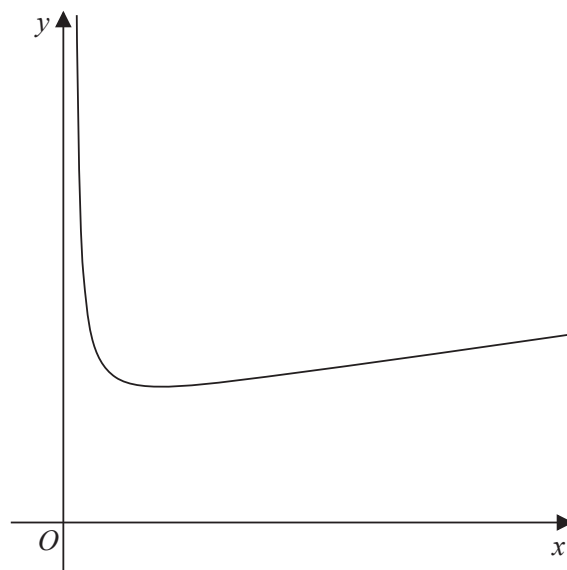


Figure 1

Figure 1 shows a sketch of a curve with equation $y = f(x)$ where

$$f(x) = \frac{2x+3}{\sqrt{4x-1}} \quad x > \frac{1}{4}$$

- (a) Find, in simplest form, $f'(x)$. (4)

- (b) Hence find the range of f . (3)

Leave
blank

Question 3 continued

Leave
blank



Question 3 continued

Leave
blank



Question 3 continued

Leave
blank

(Total 7 marks)

Q3



$$f(x) = 5 - \frac{4}{3x + 2} \quad x \geq 0$$

$$g(x) = \left| 4 \sin \left(\frac{x}{3} + \frac{\pi}{6} \right) \right| \quad x \in \mathbb{R}$$

- (a) Find the range of f (2)
- (b) (i) Find $f^{-1}(x)$
- (ii) Write down the domain of f^{-1} (3)
- (c) Find $fg(-\pi)$ (2)

Question 2 continued

Leave
blank

(Total 7 marks)

Q2



Leave
blank

2. The function f and the function g are defined by

$$f(x) = \frac{12}{x+1} \quad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2} \ln x \quad x > 0, x \in \mathbb{R}$$

(a) Find, in simplest form, the value of $\text{fg}(\text{e}^2)$ (2)

(b) Find f^{-1} (3)

(c) Hence, or otherwise, find all real solutions of the equation

$$f^{-1}(x) = f(x) \quad (3)$$

Question 2 continued

Leave
blank



Question 2 continued

Leave
blank

Question 2 continued

Leave
blank

Q2

(Total 8 marks)



2. The function f is defined by

$$f(x) = \frac{x+3}{x-4} \quad x \in \mathbb{R}, x \neq 4$$

(a) Find $\text{ff}(6)$

(2)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = x^2 + 5 \quad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of a for which

$$\text{gf}(a) = 7$$

(3)



Question 2 continued



Question 2 continued

(Total for Question 2 is 8 marks)



6. The functions f and g are defined by

$$f(x) = 6 - \frac{21}{2x+3} \quad x \geq 0$$

$$g(x) = x^2 + 5 \quad x \in \mathbb{R}$$

(a) Find $gf(2)$

(2)

(b) Find f^{-1}

(3)

(c) Solve the equation

$$\text{gg}(x) = 126$$

(3)







Question 6 continued

(Total for Question 6 is 8 marks)





Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)





Question 1 continued

Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 8 marks)



1. The function f is defined by

$$f(x) = \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} \quad x > 0$$

- (a) Show that $f(x) = \frac{5x}{x+3}$ (3)
- (b) Find f^{-1} (3)
- (c) (i) Find, in simplest form, $f'(x)$.
- (ii) Hence, state whether f is an increasing or a decreasing function, giving a reason for your answer. (3)

Leave
blank

Question 1 continued

Leave
blank



Question 1 continued

Leave
blank



Question 1 continued

Leave
blank

(Total 9 marks)

Q1



$$\begin{aligned} f(x) &= \frac{5-x}{3x+2} & x \in \mathbb{R}, x \neq -\frac{2}{3} \\ g(x) &= 2x-7 & x \in \mathbb{R} \end{aligned}$$

(b) Find f^{-1} (3)

$$f\left(\frac{1}{a}\right) = g(a + 3) \tag{4}$$


Question 2 continued

Leave
blank

Lined area for writing the answer to Question 2.



Leave
blank

Question 2 continued

Leave
blank

Lined area for writing the answer to Question 2.

(Total 9 marks)

Q2

Mark box for Question 2.



Leave
blank

Question 4 continued

Leave
blank



Question 4 continued

Leave
blank

(Total 10 marks)

Q4



6.

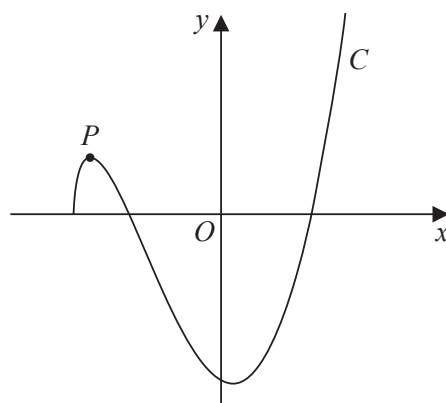


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The function f is defined by

$$f(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \quad x \geq -\frac{9}{4}$$

(a) Show that

$$f'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where k is an integer to be found.

(4)

(b) Hence, find the values of x for which $f'(x) = 0$

(1)

Figure 3 shows a sketch of the curve C with equation $y = f(x)$.

The curve has a local maximum at the point P

(c) Find the exact coordinates of P

(2)

The function g is defined by

$$g(x) = 2f(x) + 4 \quad -\frac{9}{4} \leq x \leq 0$$

(d) Find the range of g

(3)

Leave
blank



Question 6 continued

Leave
blank



Question 6 continued

Leave
blank



Question 6 continued

Leave
blank

Handwriting practice area with 30 horizontal lines.

(Total 10 marks)

Q6





Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)



Question 5 continued



Question 5 continued



Question 5 continued

(Total for Question 5 is 10 marks)



Question 6 continued

Leave
blank



Question 6 continued

Leave
blank



Leave
blank

(Total 11 marks)

Q6



Question 4 continued



Question 4 continued

Question 4 continued

(Total for Question 4 is 13 marks)



SUB-TOPIC P3.1c

MODULUS FUNCTIONS & TRANSFORMATIONS

Sub-topic P3.1c · 18 questions · 146 marks · lowest tariff first



1. The point $P(-4, -3)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

$$(a) \quad y = f(2x) \tag{1}$$

$$(b) \ y = 3f(x - 1) \qquad (2)$$

$$(c) \ y = |f(x)| \tag{1}$$

(Total for Question 1 is 7 marks)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)



2.

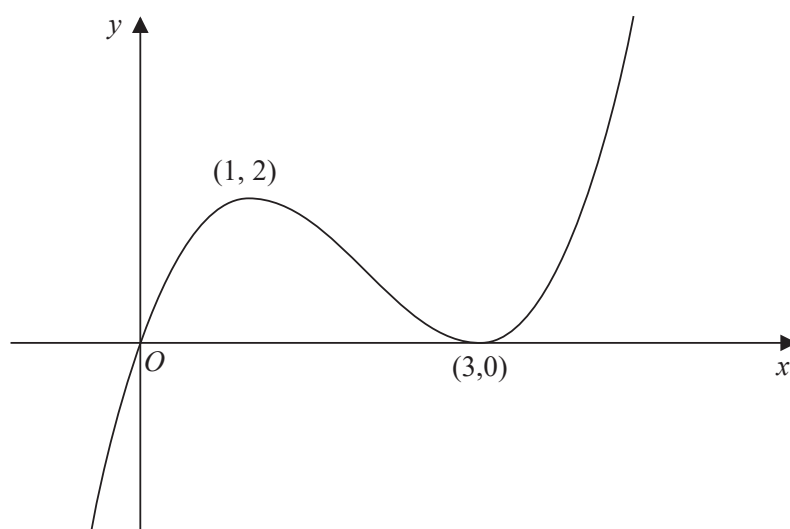
**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where $x \in \mathbb{R}$ and $f(x)$ is a polynomial.

The curve passes through the origin and touches the x -axis at the point $(3, 0)$

There is a maximum turning point at $(1, 2)$ and a minimum turning point at $(3, 0)$

On separate diagrams, sketch the curve with equation

(i) $y = 3f(2x)$ (3)

(ii) $y = f(-x) - 1$ (3)

On each sketch, show clearly the coordinates of

- the point where the curve crosses the y -axis
- any maximum or minimum turning points

Leave
blank

Question 2 continued

Leave
blank

(Total 6 marks)

Q2



1.

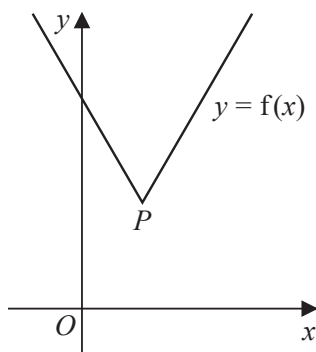


Figure 1

Figure 1 shows a sketch of the graph with equation $y = f(x)$ where

$$f(x) = 2|x - 5| + 10$$

The point P , shown in Figure 1, is the vertex of the graph.

- (a) State the coordinates of P

(2)

- (b) Use algebra to solve

$$2|x - 5| + 10 > 6x$$

(Solutions relying on calculator technology are not acceptable.)

(2)

- (c) Find the point to which P is mapped, when the graph with equation $y = f(x)$ is transformed to the graph with equation $y = 3f(x - 2)$

(2)

Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 6 marks)







(Total for Question 5 is / marks)



Leave
blank

4.

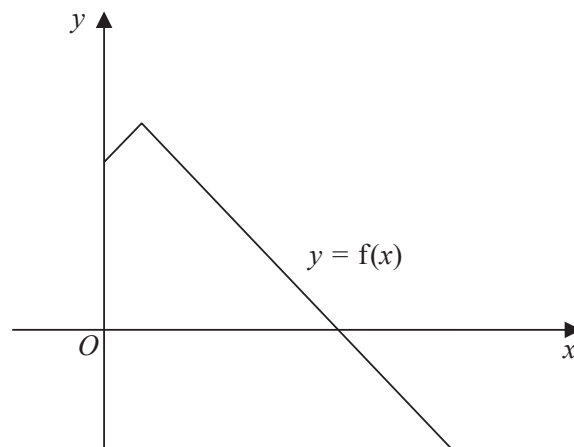
**Figure 2**

Figure 2 shows a sketch of part of the graph with equation $y = f(x)$ where

$$f(x) = 21 - 2|2 - x| \quad x \geq 0$$

(a) Find $ff(6)$ (2)

(b) Solve the equation $f(x) = 5x$ (2)

Given that the equation $f(x) = k$, where k is a constant, has exactly two roots,

(c) state the set of possible values of k . (2)

The graph with equation $y = f(x)$ is transformed onto the graph with equation $y = af(x - b)$

The vertex of the graph with equation $y = af(x - b)$ is $(6, 3)$.

Given that a and b are constants,

(d) find the value of a and the value of b . (2)

Question 4 continued

Leave
blank



Question 4 continued

Leave
blank



Question 4 continued

Leave
blank

Lined area for writing the answer to Question 4.

(Total 8 marks)

Q4

Mark box for Q4.



Leave
blank

6. Given that k is a positive constant,

(a) on separate diagrams, sketch the graph with equation

(i) $y = k - 2|x|$

(ii) $y = \left| 2x - \frac{k}{3} \right|$

Show on each sketch the coordinates, in terms of k , of each point where the graph meets or cuts the axes.

(4)

(b) Hence find, in terms of k , the values of x for which

$$\left| 2x - \frac{k}{3} \right| = k - 2|x|$$

giving your answers in simplest form.

(4)



Question 6 continued

Leave
blank



Leave
blank



Question 6 continued

Leave
blank

(Total 8 marks)

Q6



Leave
blank

5.

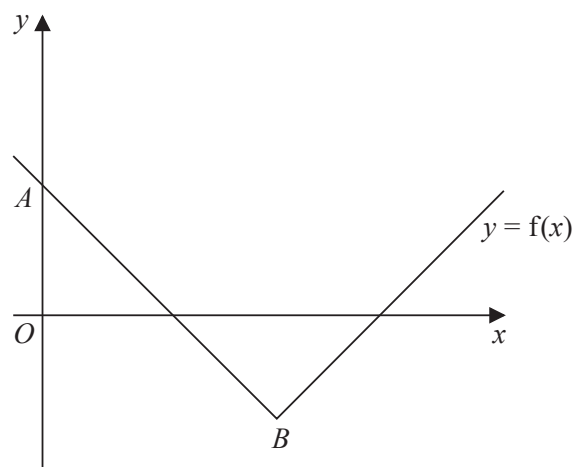
**Figure 2**

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 9| - 2 \quad x \in \mathbb{R}$$

and k is a positive constant.

The graph intersects the y -axis at the point A and has a minimum point at B as shown.

(a) (i) Find the y coordinate of A

(ii) Find, in terms of k , the x coordinate of B

(2)

(b) Find, in terms of k , the range of values of x that satisfy the inequality

$$|kx - 9| - 2 < 0$$

(3)

Given that the line $y = 3 - 2x$ intersects the graph $y = f(x)$ at two distinct points,

(c) find the range of possible values of k

(3)

Leave
blank

Question 5 continued

Leave
blank

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Leave
blank

Lined area for writing the answer to Question 5.

(Total 8 marks)

Q5



6.

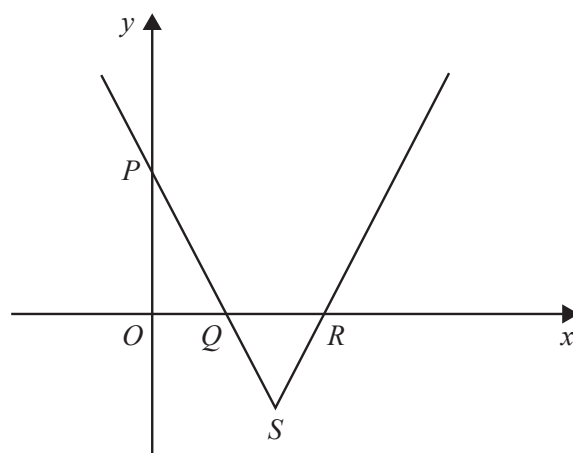


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = |3x - 5a| - 2a$$

where a is a positive constant.

The graph

- cuts the y -axis at the point P
- cuts the x -axis at the points Q and R
- has a minimum point at S

(a) Find, in simplest form in terms of a , the coordinates of

- (i) point P
- (ii) points Q and R
- (iii) point S

(4)

(b) Find, in simplest form in terms of a , the values of x for which

$$|3x - 5a| - 2a = |x - 2a|$$

(4)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(Total for Question 6 is 8 marks)



7. Given that a and b are positive constants with $a > b$,

(a) sketch, on **separate** diagrams, the graph with equation

(i) $y = |3x - a|$

(ii) $y = |3x - a| - b$

Show on each sketch

- the coordinates of the minimum point on the graph
- the coordinates of the point at which the graph crosses the y -axis

(6)

(b) Solve the equation

$$|3x - a| - b = 5x$$

giving any solution for x in terms of a and b .

(2)



(Total for Question 7 is 6 marks)



6.

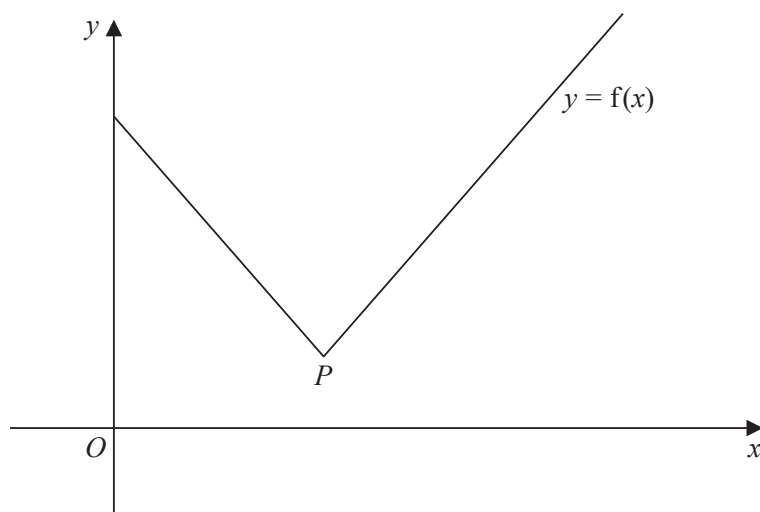
**Figure 2**

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = 2|2x - 5| + 3 \quad x \geq 0$$

The vertex of the graph is at point P as shown.

(a) State the coordinates of P . (2)

(b) Solve the equation $f(x) = 3x - 2$ (4)

Given that the equation

$$f(x) = kx + 2$$

where k is a constant, has exactly two roots,

(c) find the range of values of k . (3)

Leave
blank

Question 6 continued

Leave
blank



Leave
blank



(Total 9 marks)

Leave
blank

4.

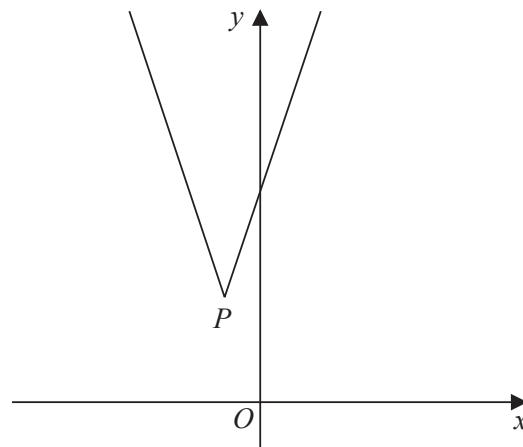
**Figure 2**

Figure 2 shows a sketch of the graph with equation $y = f(x)$, where

$$f(x) = |3x + a| + a$$

and where a is a positive constant.

The graph has a vertex at the point P , as shown in Figure 2.

(a) Find, in terms of a , the coordinates of P .

(2)

(b) Sketch the graph with equation $y = g(x)$, where

$$g(x) = |x + 5a|$$

On your sketch, show the coordinates, in terms of a , of each point where the graph cuts or meets the coordinate axes.

(2)

The graph with equation $y = g(x)$ intersects the graph with equation $y = f(x)$ at two points.

(c) Find, in terms of a , the coordinates of the two points.

(5)

Question 4 continued

Leave
blank



Question 4 continued

Leave
blank



Q4

6.

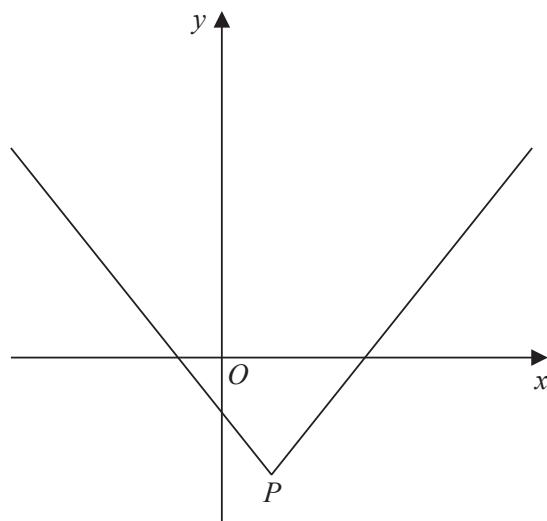
**Figure 2**

Figure 2 shows a sketch of the graph $y = f(x)$, where

$$f(x) = 3|x - 2| - 10$$

The vertex of the graph is at point P , shown in Figure 2.

(a) Find the coordinates of P

(2)

(b) Find $ff(0)$

(2)

(c) Solve the inequality

$$3|x - 2| - 10 < 5x + 10$$

(2)

(d) Solve the equation

$$f(|x|) = 0$$

(3)

Question 6 continued

Lined area for writing the answer to Question 6 continued.





Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 9 marks)



9.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

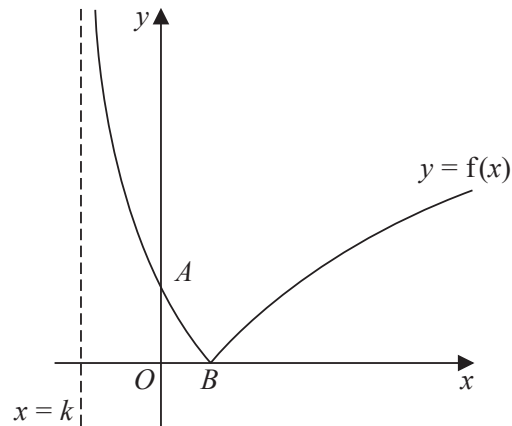


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4 \ln(x + 1)| \quad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the y -axis at point A
- meets the x -axis at point B

as shown in Figure 2,

(a) state the value of k

(1)

(b) (i) find the y coordinate of A (ii) find the exact x coordinate of B

(3)

(c) Using algebra and showing your working, find the set of values of x such that

$$|2 - 4 \ln(x + 1)| > 3$$

(5)





(Total for Question 3 is 11 marks)



Leave
blank

2.

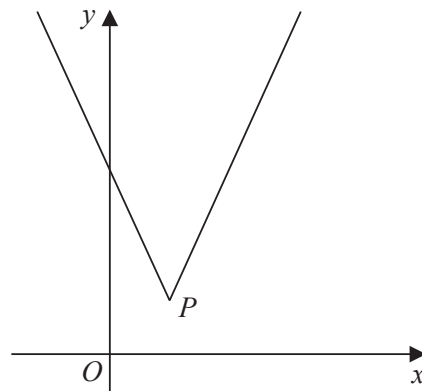


Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |3x - 13| + 5 \quad x \in \mathbb{R}$$

The vertex of the graph is at point P , as shown in Figure 1.

(a) State the coordinates of P . (2)

(b) (i) State the range of f .
(ii) Find the value of $ff(4)$ (2)

(c) Solve, using algebra and showing your working,
 $16 - 2x > |3x - 13| + 5$ (4)

The graph with equation $y = f(x)$ is transformed onto the graph with equation $y = af(x + b)$

The vertex of the graph with equation $y = af(x + b)$ is $(4, 20)$

Given that a and b are constants,

(d) find the value of a and the value of b . (2)

Leave
blank

Leave
blank

Question 2 continued

Leave
blank

Handwriting practice area with horizontal lines.

(Total 10 marks)

Q2



7.

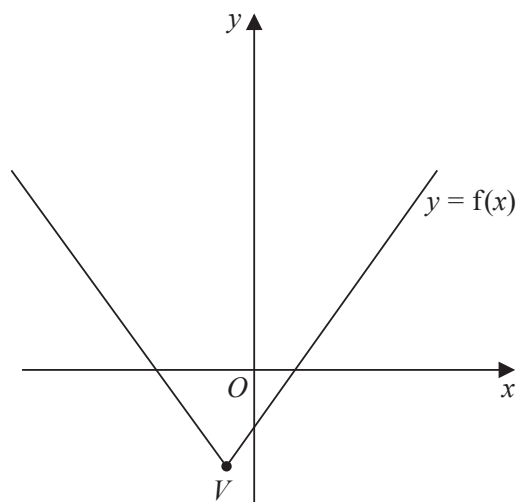


Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = \frac{1}{2} |2x + 7| - 10$$

- (a) State the coordinates of the vertex, V , of the graph.

(2)

- (b) Solve, using algebra,

$$\frac{1}{2} |2x + 7| - 10 \geq \frac{1}{3} x + 1$$

(4)

- (c) Sketch the graph with equation

$$y = |f(x)|$$

stating the coordinates of the local maximum point and each local minimum point.

(4)

Leave
blank

Leave
blank

Question 7 continued

Leave
blank



10.

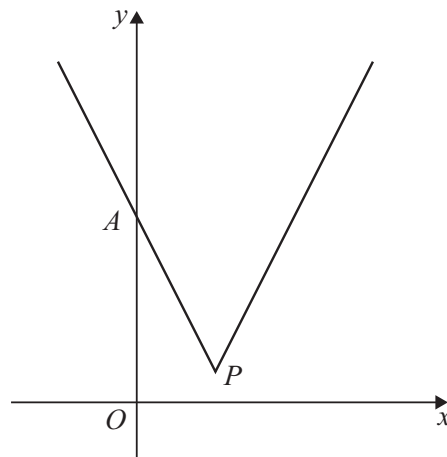
**Figure 3**

Figure 3 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 10| + k \quad x \in \mathbb{R}$$

and k is a positive constant.

The graph

- cuts the y -axis at the point A
- has a vertex at the point P

(a) Find, in simplest form in terms of k ,

- (i) the y coordinate of A
- (ii) the coordinates of P

(3)

(b) Find, in terms of k , the range of values of x which satisfy

$$|kx - 10| + k \geq 2k$$

(3)

Given that the line with equation $y = 3x + 1$ intersects the graph of $y = f(x)$ at 2 distinct points,

(c) find the range of values of k .

(4)

Question 10 continued

Lined area for writing the answer to Question 10 continued.



Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS



7.

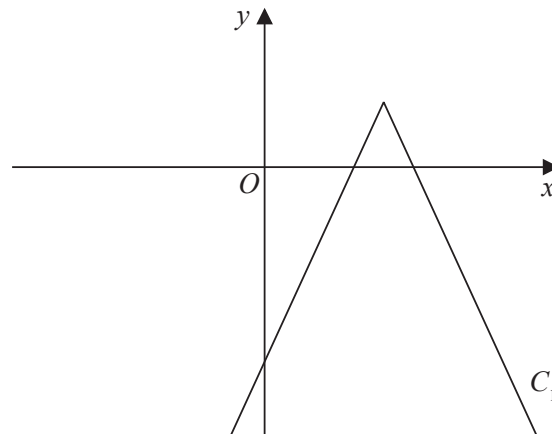
**Figure 3**

Figure 3 shows a sketch of the graph of C_1 with equation

$$y = 5 - |3x - 22|$$

(a) Write down the coordinates of

(i) the vertex of C_1

(ii) the intersection of C_1 with the y -axis.

(2)

(b) Find the x coordinates of the intersections of C_1 with the x -axis.

(2)

Diagram 1, shown on page 21, is a copy of Figure 3.

(c) On Diagram 1, sketch the curve C_2 with equation

$$y = \frac{1}{9}x^2 - 9$$

Identify clearly the coordinates of any points of intersection of C_2 with the coordinate axes.

(3)

(d) Find the coordinates of the points of intersection of C_1 and C_2
(Solutions relying entirely on calculator technology are not acceptable.)

(5)

Leave
blank

Question 7 continued

Leave
blank

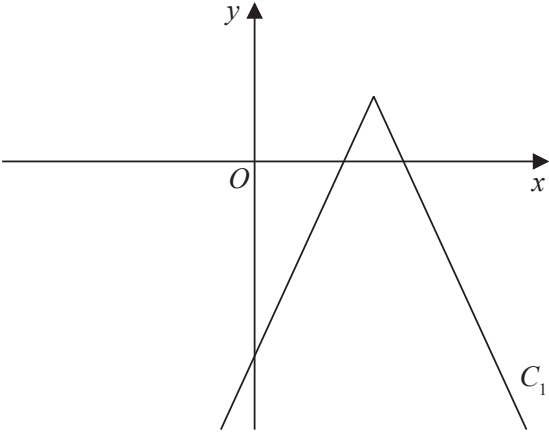


Diagram 1



Question 7 continued

Leave
blank

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Leave
blank

Lined area for writing the answer to Question 7.

(Total 12 marks)

Q7

--	--

MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
1(a)	$gf(1) = g\left(\frac{2(1)}{3 \times 1 + 1}\right) = 4 - \left(\frac{2(1)}{3 \times 1 + 1}\right)^2$	M1
	$= \frac{15}{4}$ oe	A1
		(2)
(b)	$f(x) \dots 0$ or $f(x) < \frac{2}{3}$	B1
	$0 \text{ ,, } f(x) < \frac{2}{3}$	B1
		(2)
Notes: (a) M1: Correct order of operations so requires an attempt to find $f(1)$ and then apply g to their $f(1)$ Allow for an attempt to substitute $x = 1$ into $4 - \left(\frac{2x}{3x+1}\right)^2$ A1: Correct value. Allow equivalents e.g. 3.75, $3\frac{3}{4}$ Correct answer only scores both marks. (b) Allow the use of y or f but not x in place of $f(x)$ for both marks. B1: One correct bound e.g. either $f(x) \dots 0$ or $f(x) < \frac{2}{3}$ which may be seen embedded in another inequality. Allow exact equivalents for $\frac{2}{3}$ including $0.\dot{6}$ B1: Correct range. Allow equivalent notation e.g. $0 \text{ ,, } f < \frac{2}{3}$, $y \in \left[0, \frac{2}{3}\right)$, $\left[0, \frac{2}{3}\right)$, $\frac{2}{3} > y \dots 0$ Allow written as separate inequalities e.g. $f(x) \dots 0$, $f(x) < \frac{2}{3}$, $f(x) \dots 0$ and $f(x) < \frac{2}{3}$ but not e.g. $f(x) \dots 0$ or $f(x) < \frac{2}{3}$ Allow y or f for $f(x)$. Allow exact equivalents for $\frac{2}{3}$ including $0.\dot{6}$		

(c)	$y = \frac{2x}{3x+1} \Rightarrow 3xy + y = 2x \Rightarrow 3xy - 2x = -y \Rightarrow x(3y-2) = -y$ or $x = \frac{2y}{3y+1} \Rightarrow 3xy + x = 2y \Rightarrow 3xy - 2y = -x \Rightarrow y(3x-2) = -x$	M1
	$\Rightarrow f^{-1}(x) = \frac{x}{2-3x}$	A1
		(2)
(d)	$f^{-1}(x) = f(x) \text{ or } f^{-1}(x) = x \text{ or } f(x) = x$ $\text{"}\frac{x}{2-3x}\text{"} = \frac{2x}{3x+1} \text{ or } \text{"}\frac{x}{2-3x}\text{"} = x \text{ or } \frac{2x}{3x+1} = x$ $\Rightarrow x(3x+1) = 2x(2-3x) \text{ or } x = x(2-3x) \text{ or } 2x = x(3x+1)$ $\Rightarrow x = \dots$	M1
	$x = 0, \frac{1}{3}$	A1
		(2)
		Total 8
(c) M1: Attempts to change the subject. If making x the subject, they must proceed as far as getting the two x terms on one side of the equation and the term not in x on the other and factorise out the x. If making y the subject, they must swap x and y first and then proceed as far as getting the two y terms on one side of the equation and the term not in y on the other and factorise out the y. A1: Correct expression $(f^{-1}(x) =) \frac{x}{2-3x}$ oe e.g. $(f^{-1}(x) =) \frac{-x}{3x-2}$ Accept equivalent expressions and isw after a correct answer is seen. Do not be concerned with the lhs and just look for the correct expression in terms of x. You can also ignore any reference to a domain as this is not required. (d) M1: Sets up one of the required equations e.g. $\text{their } f^{-1}(x) = f(x) \text{ or } \text{their } f^{-1}(x) = x \text{ or } f(x) = x$ and proceeds to obtain at least one value for x. If using their $f^{-1}(x)$ it must be a function of x. Condone copying slips as long as the intention is clear. A1: Correct values $x = 0, \frac{1}{3}$. Allow exact equivalents for $\frac{1}{3}$ including $0.\dot{3}$ Do not isw e.g. if they clearly reject $x = 0$ then score A0 Ignore any attempts to find the y values.		

Question Number	Scheme	Marks
2(a)	$R = 25$	B1
	$\tan \alpha = \frac{24}{7} \Rightarrow \alpha = \dots$	M1
	$\alpha = 1.287$	A1
		(3)
(b)(i)	$\text{Min} = \frac{5}{90 - 3 \times "25" \times (-1)}$	M1
	$= \frac{1}{33}$	A1
(b)(ii)	$(2x + "1.287") = \pi, \dots \Rightarrow x = \dots$	M1
	$\Rightarrow x = \frac{\pi - "1.287"}{2} = 0.927$	A1
		(4)
		Total 7

Notes:

(a)

B1: Correct value. May be seen embedded in e.g. $(f(x) =) 25 \cos(x + \dots)$.Condone e.g. $R = \sqrt{7^2 + (-24)^2} = 25$ Do not allow ± 25 unless the -25 is rejected.**M1:** Allow for: $\tan \alpha = \pm \frac{24}{7}$, $\tan \alpha = \pm \frac{7}{24}$, $\cos \alpha = \pm \frac{7}{\text{their } R}$, $\sin \alpha = \pm \frac{24}{\text{their } R}$ leading to a value for α .If no method is shown it may be implied by their value for α as awrt 1.3 rads or awrt 74° **A1:** Awrt 1.287 Isw once the mark is awarded. Must be in radians.May be seen embedded in e.g. $(f(x) =) \dots \cos(x + 1.287)$.Apply isw if correct R and α are seen followed by e.g. $(f(x) =) 25 \cos(x - 1.287)$.**Mark (b)(i) and (ii) together.**

(b)(i)

M1: Award for $\frac{5}{90 + 3 \times \text{Their } R}$.

May be implied by their value (which may be a decimal) and may be seen embedded in e.g. coordinates

 $\left(\frac{\pi - 1.287}{2}, \frac{1}{33} \right)$ which would score (i) M1 A1 and (ii) M1**A1:** For $\frac{1}{33}$ only.Note that both marks are available in (b)(i) even if their α is incorrect.(b)(ii) **Note that (b)(ii) can be marked independently.****M1:** For an attempt to solve $2x \pm "1.287" = \pi$ to obtain a value for x . May be implied by awrt 0.927
Condone a sign slip but the order of operations must be essentially correct e.g.

$$2x \pm "1.287" = \pi \Rightarrow 2x = \pi \pm "1.287" \Rightarrow x = \frac{\pi \pm "1.287"}{2}$$

Condone an attempt to solve e.g. $2x \pm "74^\circ" = 180^\circ$ but not mixed units e.g. $2x \pm "1.287" = 180$ **A1:** For awrt 0.927

Question Number	Scheme	Marks
4(a)	Any correct constant, so for $A = 2$ or $B = 3$ or $C = -1$ or $D = 5$	B1
	$2x^4 + 15x^3 + 35x^2 + 21x - 4 = Ax^2(x+3)^2 + Bx(x+3)^2 + C(x+3)^2 + D$ $\Rightarrow A = \dots, B = \dots, C = \dots, D = \dots$ or $2x^4 + 15x^3 + 35x^2 + 21x - 4 \div (x^2 + 6x + 9) = \dots x^2 + \dots x + \dots + \frac{\dots}{(x+3)^2}$	M1
	2 correct of $A = 2, B = 3, C = -1, D = 5$	A1
	$A = 2, B = 3, C = -1, D = 5$	A1
		(4)
(b)	$\int f(x) dx = \int \left(2x^2 + 3x - 1 + \frac{5}{(x+3)^2} \right) dx = \frac{2x^3}{3} + \frac{3x^2}{2} - x - \frac{5}{x+3} (+c)$	M1A1ftA1
		(3)
		Total 7

(a)

B1: One correct constant or one correct term in $Ax^2 + Bx + C + \frac{D}{(x+3)^2}$

M1: Complete method for finding A, B, C and D

For example substitution/comparing coefficients/long division

Via substitution/comparing coefficients the minimum required is an identity of the correct form (con doning slips) followed by values for A, B, C and D .

See scheme but there are other versions including

$$2x^4 + 15x^3 + 35x^2 + 21x - 4 = (Ax^2 + Bx + C)(x+3)^2 + D \Rightarrow A = \dots, B = \dots, C = \dots, D = \dots$$

Via division look for a divisor of $x^2 + 6x + 9$, a quotient that is a quadratic and a remainder that is either linear or a constant term.

It could be attempted by dividing by $(x+3)$ twice.

FYI, the first division gives $2x^3 + 9x^2 + 8x - 3$ with a remainder of 5

A1: 2 correct constants following the award of M1

For division, when the remainder is a linear term, it would be scored for two correct of $Ax^2 + Bx + C$

A1: All correct following the award of M1

(b)

M1: $\int \frac{D}{(x+3)^2} dx \rightarrow \frac{k}{x+3}$ where k is a constant.

This may be awarded following a term of $\int \frac{\alpha x + D}{(x+3)^2} dx$ following division.

Look for $\int \frac{\alpha x + D}{(x+3)^2} dx$ being correctly split and $\rightarrow \int \frac{\alpha x}{(x+3)^2} dx + \int \frac{D}{(x+3)^2} dx \rightarrow \text{something} + \frac{k}{x+3}$

A1ft: $\int \left(Ax^2 + Bx + C + \frac{D}{(x+3)^2} \right) dx = \frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx - \frac{D}{x+3} (+c).$

Correct integration following through on their non-zero constants.

Allow this to be scored with A , B , C and D as above or with made up values

A1: All correct with or without “+ c ”. Allow $-1x$ for $-x$

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1	$\int \frac{x^2-5}{2x^3} dx = \int Ax^{-1} - Bx^{-3} dx = C \ln x + Dx^{-2} (+c)$	M1 dM1
	$= \frac{1}{2} \ln x + \frac{5}{4} x^{-2} + c$	A1
		(3)
		Total 3

M1: Correct **attempt** to integrate.

Score for an attempt to divide by the x^3 term forming a sum of two terms and then integrating.

Award for achieving one term in the correct form. Either $C \ln x + \dots$ **or** $\dots + Dx^{-2}$

Note that $C \ln ax$ and versions such as $k \ln 2x^3$ are also acceptable for $C \ln x$ so look at responses involving lns carefully. Ignore spurious notation e.g. $\int C \ln x$ for the M marks as long as integration has been attempted

dM1: Achieves both terms in the correct form. Score for $\pm C \ln x \pm Dx^{-2}$ or equivalent

Be aware that $C \ln ax \pm Dx^{-2}$ and other variations are also correct

A1: $\frac{1}{2} \ln x + \frac{5}{4} x^{-2} + c$ or equivalent simplest form **with the** $+ c$. E.g. $\ln \sqrt{x} + \frac{5}{4x^2} + c$

ISW after a correct answer.

Some candidates may incorporate the $+ c$ within the log so $\frac{1}{2} \ln kx + \frac{5}{4} x^{-2}$ where k is an arbitrary constant is ok.

Note that $\frac{1}{2} \ln 2x + \frac{5}{4} x^{-2} + c$ is not the simplest form and is A0. $\int \frac{1}{2} \ln x + \frac{5}{4} x^{-2} + c$ would also be A0

Attempts via integration by parts can be scored in the same way

$$\int \frac{x^2-5}{2x^3} dx = \int (x^2-5) \times \frac{1}{2} x^{-3} dx = (x^2-5) \times -\frac{1}{4} x^{-2} - \int 2x \times -\frac{1}{4} x^{-2} dx = (x^2-5) \times -\frac{1}{4} x^{-2} + \frac{1}{2} \ln x + c$$

M1: For an attempt to integrate by parts the correct way around and achieves $(x^2-5) \times px^{-2} \pm q \ln ax + c$

If the rule is quoted it must be correct.

It is possible to integrate by parts the other way around but unlikely. It can be scored in a similar way.

dM1: Score for

- either then simplifying to an expression of the form $\pm C \ln x \pm Dx^{-2}$ with or without "+ c" which could be numerical
- or integrating to a correct but unsimplified answer $(x^2-5) \times -\frac{1}{4} x^{-2} + \frac{1}{2} \ln ax$ with or without "+ c"

A1: $\frac{1}{2} \ln x + \frac{5}{4} x^{-2} + c$ NOT $\frac{1}{2} \ln x + \frac{5}{4} x^{-2} + \frac{1}{4} + c$ (The answer must be in simplest form and **with the** $+ c$)

Question Number	Scheme	Marks
9 (a)	$\begin{array}{r} x^2 + 2 \\ x^2 - x - 12 \overline{) x^4 - x^3 - 10x^2 + 3x - 9} \\ \underline{x^4 - x^3 - 12x^2} \\ 2x^2 + 3x - 9 \\ \underline{2x^2 - 2x - 24} \\ 5x + 15 \end{array}$ $\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 + x - 12} \equiv x^2 + 2 + \frac{5(x+3)}{(x-4)(x+3)}$ $\equiv x^2 + 2 + \frac{5}{(x-4)} \text{ or stating } P=2, Q=5$	M1A1 M1 A1 (4)
(b)	$g'(x) = 2x - \frac{5}{(x-4)^2}$ Subs $x=2$ into $g'(2) = 2 \times 2 - \frac{5}{(2-4)^2} = \frac{11}{4}$ Uses $m = g'(2) = \left(\frac{11}{4}\right)$ with $(2, g(2)) = \left(2, \frac{7}{2}\right)$ to form equation of tangent $y - \frac{7}{2} = \frac{11}{4}(x-2) \Rightarrow y = \frac{11}{4}x - 2$	M1A1 M1 dM1A1 (5)
(c)	$\int x^2 + 2 + \frac{5}{(x-4)} dx = \frac{1}{3}x^3 + 2x + 5 \ln x-4 $ $\text{Area } R = \left[\frac{1}{3}x^3 + 2x + 5 \ln x-4 \right]_0^2 = \left(\frac{8}{3} + 4 + 5 \ln 2 \right) - (0 + 0 + 5 \ln 4)$ $= \frac{20}{3} + 5 \ln 2 - 5 \ln 4 = \frac{20}{3} - 5 \ln 2$	M1 A1ft dM1 ddM1 A1 (5)
Alt (a)	$x^4 - x^3 - 10x^2 + 3x - 9 \equiv (x^2 + P)(x^2 - x - 12) + Q(x+3)$ Compare terms (OR sub in values) and solve simultaneously to find P and/or Q ie $x^2 \Rightarrow P - 12 = -10, \quad x \Rightarrow -P + Q = 3, \quad \text{const} \Rightarrow -12P + 3Q = -9$ $\Rightarrow P = \dots$ or $Q = \dots$ $P = 2, Q = 5^*$ Award in the order shown here on ePEN.	(14 marks) M1 M1 A1, A1

(a)

M1: Divides to obtain a quadratic quotient and a linear remainder. May divide by $(x-4)$ and then by $(x+3)$ or vice versa to reach these but must be a full process.

FYI: By $x+3$ first gives $x^3 - 4x^2 + 2x - 3$ as quotient, followed by $x^2 + 2 + \frac{5}{x-4}$



By $x-4$ first gives $x^3 + 3x^2 + 2x + 11 + \frac{35}{x-4}$ then $x^2 + 2 + \frac{5}{x+3} + \frac{35}{(x+3)(x-4)} \rightarrow x^2 + 2 + \frac{5}{x-4}$

A1: Obtains a quotient of $x^2 + 2$ and a remainder of $5x + 15$

M1: Writes the given expression in the required form using $x^2 - x - 12 = (x-4)(x+3)$ or divides $x+3$ into their remainder term.

A1: Correct answer. ($P=2, Q=5$) May be awarded following an incorrect " $ax^2 + 2$ " quadratic factor.

Alt:

M1: Multiplies though completely by the denominator and cancels the $x-4$ term.

M1: Complete process of comparing coefficients or substituting values to find a value for either P or Q

A1: Either $P=2$ or for **showing** $Q=5^*$ (must have seen a correct equation for Q)

A1: Both $P=2$ and **showing** $Q=5^*$ Note that $Q=5$ is given so it must be shown from correct work, not just stated.

Note M0M1A1A0 is possible if Q is assumed and factorisation of $x^2 - x - 12$ is never seen.

(b)

M1: For $\frac{Q}{x-4} \rightarrow \frac{\dots}{(x-4)^2}$

A1: For $g'(x) = 2x - \frac{5}{(x-4)^2}$. Note that this can be scored from an incorrect P .

M1: Attempts the gradient of C at the point where $x=2$

dM1: Depends on previous M. A complete method of finding the equation of the tangent. If $y = mx + c$ is used, they must proceed as far as finding c .

A1: $y = 2.75x - 2$ or exact equivalent and isw.

Note: This may be attempted from the original function $g(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 + x - 12}$

M1: Scored for an attempt at the quotient rule A1 if correct and so on.

$\rightarrow \frac{(ax^3 + bx^2 + cx + d)(x^2 + x - 12) - (x^4 - x^3 - 10x^2 + 3x - 9)(px + q)}{(x^2 + x - 12)^2}$

(c)

M1: Attempts to integrate with $\int \frac{\dots}{(x-4)} dx \rightarrow \dots \ln|x-4|$ Condone $\ln(x-4)$

A1ft: $\int x^2 + P + \frac{5}{(x-4)} dx = \frac{1}{3}x^3 + Px + 5\ln|x-4|$ following through on their P .

dM1: Dependent on first M mark. Attempt the area of R using the limits 0 and 2 in their integrated function and subtracting the correct way round (or recovered).

ddM1: Depends on both previous M's. Scored for attempting to combine two log terms using correct log work

Allow the method and final accuracy if $\ln(-2) - \ln(-4) \rightarrow \ln\left(\frac{-2}{-4}\right) = -\ln 2$ is used (bod that modulus is meant)

Do not allow if $\ln(-a) \rightarrow -\ln a$ is used.

A1: $\frac{20}{3} - 5 \ln 2$

Note: If a candidate gives correct values of m and c in (b) and of a and b in (c) but has not stated the answer in correct form, then penalise only the first instance.



Question Number	Scheme	Marks
1(a)	$f(x) \text{ „ } 9$	B1
		(1)
(b)	$fg(1.5) = f\left(\frac{3}{2 \times 1.5 + 1}\right) = 9 - \left(\frac{3}{2 \times 1.5 + 1}\right)^2$	M1
	$= \frac{135}{16}$	A1
		(2)
(c)	$g(x) = \frac{3}{2x+1} \Rightarrow g^{-1}(x) = \frac{3-x}{2x}$	M1 A1
	$0 < x \text{ „ } 3$	B1
		(3)
		Total 6

(a)**B1:** Correct range. Allow equivalent notation e.g. $y \text{ „ } 9$, $f \text{ „ } 9$, $y \in (-\infty, 9]$ but not $x \text{ „ } 9$.Condone just " $\text{„ } 9$ " and "less than or equal to 9"**(b)****M1:** Full attempt at method to find $fg(1.5)$ condoning slips. Implied by a correct answer or 8.44 e.g. For a correct order of operations so requires an attempt to apply $g(1.5)$ first and then f to their $g(1.5)$ Also allow for an attempt to substitute $x = 1.5$ into $9 - \left(\frac{3}{2x+1}\right)^2$ condoning slips such as substituting

$$x = 1.5 \text{ into } 9 - \frac{3}{(2x+1)^2}$$

(c)**M1:** Changes the subject of $y = \frac{3}{2x+1}$ and obtains $x = \frac{3 \pm y}{2y}$ or $x = \frac{3}{2y} \pm \frac{1}{2}$ or equivalent.Alternatively changes the subject of $x = \frac{3}{2y+1}$ and obtains $y = \frac{3 \pm x}{2x}$ or $y = \frac{3}{2x} \pm \frac{1}{2}$ or equivalent.**A1:** $g^{-1}(x) = \frac{3-x}{2x}$, $g^{-1}(x) = \frac{1}{2} \left(\frac{3-x}{x} \right)$, $g^{-1}(x) = \frac{3}{2x} - \frac{1}{2}$, $g^{-1}: x \mapsto \frac{3-x}{2x}$ or $g^{-1}: x \mapsto \frac{3}{2x} - \frac{1}{2}$ Condone $y = \frac{3-x}{2x}$ o.e and even $g^{-1} = \frac{3-x}{2x}$ but NOT $f^{-1} = \frac{3-x}{2x}$ o.e. ISW after a correct answer.Don't for this mark allow fractions/fractions that may be misinterpreted such as $g^{-1}(x) = \frac{3-x}{2}$ but answerssuch as $g^{-1}(x) = \frac{\frac{3}{x} - 1}{2}$ and $y = \frac{\left(\frac{3-x}{x}\right)}{2}$ are clear and unambiguous and can score both marks.**B1:** Correct domain. Allow equivalent notation e.g. $x \in (0, 3]$ but not just $(0, 3]$ 

Question Number	Scheme	Marks
2 (a) (i)	$\log_6 T = 4 - 2\log_6 x$	B1
(ii)	E.g. $\log_6 T = 4 - 2\log_6 216 \Rightarrow \log_6 T = 4 - 2 \times 3 = -2 \Rightarrow T = \dots$ $\Rightarrow T = 6^{-2} = \frac{1}{36}$	M1 A1 (3)
(b)	$\log_6 T = 4 - 2\log_6 x \Rightarrow T = 6^{4-2\log_6 x}$ $\Rightarrow T = 6^4 \times 6^{\log_6 x^{-2}}$ $\Rightarrow T = \frac{1296}{x^2}$	M1 dM1 A1 (3)
		(6 marks)

Notes

Mark the question as a whole. Do not be concerned about part labelling.

(a)(i)

B1 Correct linear equation $\log_6 T = 4 - 2\log_6 x$ (oe) The 4 may be written as $\log_6 1296$

(ii)

M1 Substitutes $x = 216$ into an equation linking T and x arising from a linear equation in the logarithms and proceeds to make T the subject. They may have answered (b) first. Do not be concerned about the process for this mark. May be implied by awrt 0.028 following a correct equation.

A1 Correct value $T = \frac{1}{36}$. Do not accept 6^{-2} .

(b)

M1 Makes a first step towards achieving an answer. Use of a correct log rule or law **applied** at some stage in their attempt to eliminate logs from the equation.

As a rule of thumb this can be awarded for e.g.

- application of a power rule $-2\log_6 x = -\log_6 x^{2^*}$ or $4 = \log_6 6^{4^*}$ or $4 \rightarrow 6^4$ (note that e.g. $\log_6 T = -2\log_6 x + 4 \rightarrow x^{-2} + 6^4$ implies this mark)

- an attempt to make T the subject. E.g. $\log_6 T = 4 - 2\log_6 x \Rightarrow T = 6^{4^* - 2^* \log_6 x}$

dM1 Full and complete method in proceeding from an equation of form $\log_6 T = a + b\log_6 x$ ($a, b \neq 0$) to an equation of form $T = k \times x^{\pm n}$ or equivalent. All log work must be correct but allow slips on coefficients.

A1 Achieves $T = \frac{1296}{x^2}$ or equivalent such as $Tx^2 = 1296$ and isw after a correct answer. Allow 6^4 for 1296.

Note: Allow the M marks if a different letter than T is used, e.g. y . But must be correct in terms of T and x for the A mark.



Question Number	Scheme	Marks
3 (i)	$\frac{d}{dx} \ln(\sin^2 3x) = \frac{1}{\sin^2 3x} \times 2 \sin 3x \times 3 \cos 3x = 6 \cot 3x$	M1 A1 (2)
(ii) (a)	$\frac{d}{dx} (3x^2 - 4)^6 = 36x(3x^2 - 4)^5$	M1 A1 (2)
(b)	$\int x(3x^2 - 4)^5 dx = \frac{1}{36} (3x^2 - 4)^6$	B1ft
	$\int_0^{\sqrt{2}} x(3x^2 - 4)^5 dx = \left[\frac{1}{36} (3x^2 - 4)^6 \right]_0^{\sqrt{2}} = \frac{1}{36} (2)^6 - \frac{1}{36} (-4)^6 = -112$	M1 A1cso (3) (7 marks)

Notes

(i)

M1 Attempts to differentiate a $\ln$ function. Award for $\frac{d}{dx} \ln(\sin^2 3x) = \frac{1}{\sin^2 3x} \times \dots$ where $\dots$ could be 1

An alternative could be $\frac{d}{dx} \ln(\sin^2 3x) = \frac{d}{dx} 2 \ln(\sin 3x) = (2 \times) \frac{1}{\sin 3x} \times \dots$ or

$$\frac{d}{dx} \ln\left(\frac{1 - \cos 6x}{2}\right) = \frac{2}{1 - \cos 6x} \times \dots$$

A1 $6 \cot 3x$ o.e. such as $\frac{6 \cos 3x}{\sin 3x}$ or $\frac{6}{\tan 3x}$ or $6(\tan 3x)^{-1}$ but not $6 \tan^{-1} 3x$ Accept also $\frac{6 \sin 6x}{1 - \cos 6x}$ or $\frac{3 \sin 6x}{\sin^2 3x}$ and isw after a suitably simplified answer.

Constant terms must be gathered and no uncanceled common factors in numerator and denominator.

(ii) (a)

M1 Achieves $\frac{d}{dx} (3x^2 - 4)^6 = Ax(3x^2 - 4)^5$ where A is a constant which may be 1.

A1 $\frac{d}{dx} (3x^2 - 4)^6 = 36x(3x^2 - 4)^5$ oe. Need not be simplified. Isw after a correct answer.

(ii) (b)

B1ft $\int x(3x^2 - 4)^5 dx = \frac{1}{36} (3x^2 - 4)^6$ or $\frac{1}{A} (3x^2 - 4)^6$ following through on their (a) provided it is of

the form $\frac{d}{dx} (3x^2 - 4)^6 = Ax(3x^2 - 4)^5$ This may arise from attempts via substitution and can be scored

from a restart if (ii)(a) was incorrect. Need not be simplified and isw if simplified incorrectly. Condone notation errors such as unneeded integral signs - mark the expression that is their attempt at the integration.

M1 Substitutes in both limits and subtracts (either way round) into an expression of the form

$D(3x^2 - 4)^6$ where D is a constant but allow slips such as a missing power if the intention is clear.

Sight of the subtraction is sufficient. Implied by the correct answer for their integral if substitution not seen. If using integration by substitution they must be substituting the **correct** limits for their variable.



A1cso ($R =$) -112 and isw if they make the answer positive after a correct answer seen.

Note: Answer only with no working at all shown scores no marks. Correct integral must be seen.

Note: Attempts at integration by parts are unlikely to succeed, but if done correctly and achieve the correct form of the answer may score the relevant marks.

Note (ii) may be completed by expansion.

(a)

M1 Requires expansion to form $ax^{12} + bx^{10} + cx^8 + dx^6 + ex^4 + fx^2 + g$ followed by an attempt to integrate each term (power decreased by 1)

A1 Requires correct derivative. $8748x^{11} - 58320x^9 + 155520x^7 - 207360x^5 + 138240x^3 - 36864x$

(b)

B1ft Correct answer from a restart, which may be via expansion

$$\frac{81x^{12}}{4} - 162x^{10} + 540x^8 - 960x^6 + 960x^4 - 512x^2$$

M1 Substitutes both limits and subtracts into an expression of the form

$$ax^{12} + bx^{10} + cx^8 + dx^6 + ex^4 + fx^2$$

A1cso As main scheme.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

Question Number	Scheme	Marks
1	$2(2\cos^2 x - 1) = 7\cos x$ $4\cos^2 x - 7\cos x - 2 = 0 \Rightarrow \cos x = -\frac{1}{4}$ $\Rightarrow x = \arccos\left(-\frac{1}{4}\right) = 104.5^\circ, 255.5^\circ$	M1 M1 A1 dM1 A1 (5) (5 marks)

- M1 Attempts to use $\cos 2x = \pm 2\cos^2 x \pm 1$ to form a quadratic equation in $\cos x$
If the other two forms are attempted there must be some attempt to use $\sin^2 x + \cos^2 x = 1$ to form a quadratic equation in $\cos x$
 $2 \times 2\cos^2 x - 1 = 7\cos x$ is M0 unless the correct identity has been previously stated or recovery occurs.
- M1 Attempts to solve a 3TQ in $\cos x$ using an allowable method (the quadratic need not be correct and may have come from incorrect work)
- A1 Reaches $\cos x = -\frac{1}{4}$ or $-\frac{2}{8}$ or -0.25 . (May be implied by a correct value for x) Ignore any reference to $\cos x = 2$ Those who use $y = \cos x$ and stop at a $y = -\frac{1}{4}$ score A0.
- dM1 Depend on the second method mark. Takes arccos of at least one solution (α) of their quadratic where $|\alpha| < 1$ to find at least one solution in range. If substitution not seen then you will need to check.
NB a radian answer of awrt 1.8 or correct 1d.p. answer for their α can imply the method.
- A1 awrt $104.5^\circ, 255.5^\circ$ with no other values in the range. Ignore values outside the range.

General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**1. Factorisation $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ 2. FormulaAttempt to use correct formula (with values for a , b and c).3. Completing the squareSolving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:**1. DifferentiationPower of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)2. IntegrationPower of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1(a)	$(f(1)=)1-5+e=-1.281... < 0$ $(f(2)=)2^2-5 \times 2 + e^2 = 1.389... > 0$	M1
	As there is a <u>change of sign</u> and $f(x)$ is <u>continuous</u> over the interval $[1, 2]$ then <u>there is a root</u> *	A1*
		(2)
(b)(i)	$x_2 = \sqrt{5 \times 1 - e^1} = \text{awrt } 1.5105$	M1A1
(ii)	$\alpha = \text{awrt } 1.7340$	A1
		(3)
		(5 marks)

Notes**(a)**

M1: Attempts $f(1)$ and $f(2)$ with substitution seen or at least one correct to 1 d.p. rounded or truncated **and considers their signs**. Note showing $f(1)f(2) < 0$ is a consideration of signs and is sufficient for the “sign change” part of reasoning for the A1.

A1*: Must have

- both $f(1)$ and $f(2)$ correct (as expressions or correct values imply these, need not be labelled)
- reason, which must mention continuity and state or indicate sign change in some way
- conclusion, “hence root”, or accept e.g. hence “ $f(x)=0$ between $x=1$ and $x=2$ ”

(b)**(i)**

M1: Attempts to find x_2 using the iteration formula. Implied by sight of 1 embedded in the formula or awrt 1.5

A1: awrt 1.5105

(ii)

A1: awrt 1.7340 provided M1 has been scored. Allow 1.734 with trailing zero omitted.

Note 1.7340 only may be from a graphical calculator, so scores M0A0A0. There must be evidence (i.e. the M is scored) of an attempt at least one iteration first.



Question Number	Scheme	Marks
2(a)	$ff(6) = f\left(\frac{9}{2}\right) = \frac{\frac{9}{2}+3}{\frac{9}{2}-4} = \dots; = 15$	M1; A1
		(2)
(b)	$f^{-1}(x) = \frac{4x+3}{x-1}$	M1A1
	$x \in \mathbb{R}, x \neq 1$	B1
		(3)
(c)	E.g. $\left(\frac{x+3}{x-4}\right)^2 + 5 = 7$ or $\frac{a+3}{a-4} = (\pm)\sqrt{7-5}$	M1
	$\Rightarrow x^2 - 22x + 23 = 0 \Rightarrow x = \dots$ or $(a+3) = (\pm)\sqrt{2}(a-4) \Rightarrow a = \dots$	dM1
	$(a =) 11 + 7\sqrt{2}$ oe	A1
		(3)
		(8 marks)

Notes**(a)**M1: Substitutes $x = 6$ into f and substitutes the result back into f to find a value for $ff(6)$

Alternatively substitutes $x = 6$ into $ff(x) = \frac{\left(\frac{x+3}{x-4}\right)+3}{\left(\frac{x+3}{x-4}\right)-4}$ condoning slips.

A1: 15

(b)M1: Changes the subject of $y = \frac{x+3}{x-4}$ or $x = \frac{y+3}{y-4}$ to $y = \frac{\dots x \pm 3}{\dots x \pm 1}$ or $x = \frac{\dots y \pm 3}{\dots y \pm 1}$ A1: Achieves $f^{-1}(x) = \frac{4x+3}{x-1}$. Accept $f^{-1} = \dots$ or $y = \dots$ instead of $f^{-1}(x) = \dots$.B1: $x \neq 1$. The omission of $x \in \mathbb{R}$ is condoned. Accept alternative notations.**(c) Note this is M1A1A1 on open but is being marked as M1dM1A1.**M1: Correct attempt to set up an equation in x or a . This will usually be $\left(\frac{x+3}{x-4}\right)^2 + 5 = 7$ but

$f(a) = g^{-1}(7)$ may be used first. Score when a suitable equation is set up and allow if a minor slip or miscopy is made if the intention is clear.

dM1: Proceeds to solve for x or a , e.g. forms a 3TQ and solves, or take the 5 across, square roots, cross multiplies and makes a the subject, condoning e.g. sign slips when rearranging.Alternatively, uses $(f(x))^2 + 5 = 7 \Rightarrow f(x) = (\pm)\sqrt{2}$ and their part (b) leading to

$a = f^{-1}\left(\frac{9}{2}\right) = \frac{4\sqrt{2}+3}{\sqrt{2}-1}$. In this method both M's may be scored together.

A1 $11 + 7\sqrt{2}$ or $\frac{4\sqrt{2}+3}{\sqrt{2}-1}$ Accept $11 + \sqrt{98}$ or $\frac{-4\sqrt{2}-3}{1-\sqrt{2}}$. Common factors should be cancelled.

Ignore labelling of x or a . Condone e.g. if $11 - 7\sqrt{2}$ is also given as a solution but A0 if any other extra solutions are included.



Question Number	Scheme	Marks
2 (a)	$x = 2y^2 + 5y - 6$	
	$\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y + 5 = 0 \Rightarrow y = -\frac{5}{4}$	M1
	Substitutes their $y = -\frac{5}{4}$ into $x = 2y^2 + 5y - 6$ to find x	dM1
	$\left(-\frac{73}{8}, -\frac{5}{4}\right)$	A1
		(3)
		Total 5

Mark as one complete question

Main method via differentiation in (a) and (b)

(a)

M1: Differentiates $2y^2 + 5y - 6$ to a linear term in y AND attempts to take the reciprocal.

Condone for this mark attempts such as $\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y} + \frac{1}{5}$

We may see other methods including implicit differentiation from WMA14.

Look for $x = 2y^2 + 5y - 6 \Rightarrow 1 = ay \frac{dy}{dx} + b \frac{dy}{dx}$ followed by an attempt to make $\frac{dy}{dx}$ the subject

A1: $\frac{dy}{dx} = \frac{1}{4y+5}$ ISW after a correct answer.

(b) **Marks in part (b) can only be awarded after sight of** $\frac{dx}{dy} = \frac{ay+b}{k}$ **or** $\frac{dy}{dx} = \frac{k}{ay+b}$

So full marks can only be awarded after sight of $\frac{dx}{dy} = 4y+5$ **or** $\frac{dy}{dx} = \frac{1}{4y+5}$

M1: Sets ' $ay + b = 0 \Rightarrow y = \dots$ ' following sight of $\frac{dx}{dy} = \frac{ay+b}{k}$ **or** $\frac{dy}{dx} = \frac{k}{ay+b}$

Ignore/condone incorrect statements such as $\frac{dy}{dx} = 0$ here

This can be implied. E.g $\frac{dy}{dx} = \frac{1}{4y+5}$ followed by $y = -\frac{5}{4}$

dM1: Substitutes their solution of their $4y + 5 = 0$ into $x = 2y^2 + 5y - 6$ to find x .

This is dependent upon the previous M

A1: $\left(-\frac{73}{8}, -\frac{5}{4}\right)$ following $\frac{dx}{dy} = 4y+5$ **or** $\frac{dy}{dx} = \frac{1}{4y+5}$

The answer may be given separately, for example, as $x = -9.125$, $y = -1.25$

ISW after a correct answer.



Further guidance to highlighted sentence.

If (a) is incorrect or incomplete follow the following guidance.

Incorrect (a): If they incorrectly state $\frac{dy}{dx} = 4y + 5$ o.e. and go on to use $4y + 5 = 0 \Rightarrow y = \dots$ and write down a correct set of coordinates for P they will score NO MARKS in (b)

Incomplete (a): If they state $\frac{dx}{dy} = 4y + 5$ in (a) and go on to use $4y + 5 = 0 \Rightarrow y = \dots$ in (b) they will score no marks in (a) but could potentially score full marks in (b)

You can award all 3 marks for $\left(-\frac{73}{8}, -\frac{5}{4}\right)$ following either $\frac{dx}{dy} = 4y + 5$ or $\frac{dy}{dx} = \frac{1}{4y + 5}$

.....
Alternative approach: Part (a) may well be blank or incorrect. Part(b) done via completion of square

It is possible to do part (b) by not doing any differentiating.

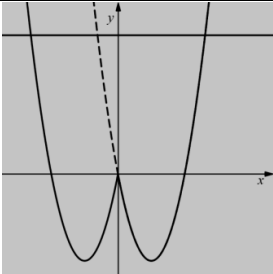
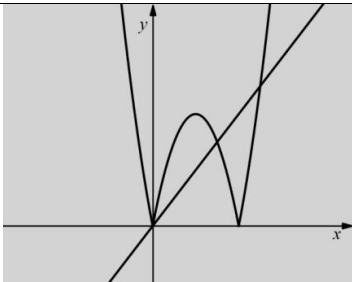
$$x = 2y^2 + 5y - 6 \Rightarrow x = 2\left(y + \frac{5}{4}\right)^2 - \frac{73}{8}$$

M1: An allowable attempt to complete the square followed by a valid attempt at either x or y from their attempt

A1: For a correct x or y

A1: For a correct x and y

.....

Question Number	Scheme	Marks
3 (a)	 Graphical interpretation of $f(x) = 48$	
	One of 8, -8	B1
	Attempts to solve an appropriate equation E.g. $2x^2 - 10x = 48 \Rightarrow x^2 - 5x - 24 = 0 \Rightarrow (x+8)(x-3) = 0 \Rightarrow x = \dots$	M1
	$x = 8, -8$ with no additional values	A1
		(3)
(b)	 Graphical interpretation of $f(x) = \frac{5}{2}x$ $y = \frac{5}{2}x$	
	Attempts to solve $2x^2 - 10x = \frac{5}{2}x \Rightarrow 4x^2 - 25x = 0 \Rightarrow x = \frac{25}{4}$ OR attempts to solve (o.e.) $10x - 2x^2 = \frac{5}{2}x \Rightarrow 4x^2 - 15x = 0 \Rightarrow x = \frac{15}{4}$	M1
	Attempts to solve $2x^2 - 10x = \frac{5}{2}x \Rightarrow 4x^2 - 25x = 0 \Rightarrow x = \frac{25}{4}$ AND attempts to solve (o.e.) $10x - 2x^2 = \frac{5}{2}x \Rightarrow 4x^2 - 15x = 0 \Rightarrow x = \frac{15}{4}$	dM1
	Achieves both critical values $x = \frac{15}{4}, x = \frac{25}{4}$	A1
	Correct set of values $x = \frac{15}{4}$ or $x = \frac{25}{4}$	A1
		(4)
		Total 7

This question demands that some working should be shown

(a) This part is now being marked B1 M1 A1 instead of M1 A1 A1

B1: Correctly obtains 8 or -8 as a solution (for x) . Condone solution appearing as $|x| = 8$

M1: Attempts to solve $f(|x|) = 48$ via an appropriate method including a calculator, factors or formula

So, look for a correct equation followed by an attempt to solve.



The equation must be given in the form of a correct 3TQ, but not necessarily collected on one side of the = sign. Solving is allowed by all methods including via a calculator.

All the following score this M mark

Example 1:

$$2x^2 - 10x = 48 \text{ o.e.} \Rightarrow x = 8, (-3) \dots \text{only the 8 is required}$$

Example 2:

$$2x^2 - 10x = 48 \text{ o.e.} \Rightarrow x^2 - 5x - 24 = 0 \Rightarrow (x - 8)(x + 3) = 0 \Rightarrow x = \dots$$

Example 3:

$$2|x|^2 - 10|x| - 48 = 0 \text{ or } |x|^2 - 5|x| - 24 = 0 \text{ leading to } |x| \text{ or } x = 8, (-3)$$

Example 4:

$$2x^2 + 10x - 48 = 0 \Rightarrow x = \frac{-10 \pm \sqrt{100 - 4(2)(-48)}}{2(2)} = \dots$$

Example 5:

$$2x^2 + 10x = 48 \text{ o.e.} \Rightarrow x = -8, (3) \dots \text{only the -8 is required}$$

A1 $x = 8, -8$ with no additional values unless they have been rejected

(b)

M1: Attempts to solve a correct equation or inequality, leading to a non-zero value for x .

The method **must be shown** with the terms being collected as a minimum response

Look for $2x^2 - 10x = \frac{5}{2}x \Rightarrow ax^2 \pm bx = 0 \Rightarrow x = c, c \neq 0$. Allow with the = as any inequality including <

OR $-2x^2 + 10x = \frac{5}{2}x \Rightarrow ax^2 \pm bx = 0 \Rightarrow x = c, c \neq 0$. Allow with the = as any inequality including <

You may see as an alternative to the 2nd equation/inequality $2x^2 - 10x = -\frac{5}{2}x \Rightarrow ax^2 \pm bx = 0 \Rightarrow x = c, c \neq 0$

It is acceptable to divide through by x , so $2x^2 - 10x = \frac{5}{2}x \Rightarrow 2x - 10 = \frac{5}{2} \Rightarrow 2x = \frac{25}{2} \Rightarrow x = 6.25$

dm1: Attempts to solve BOTH correct equations or inequations leading to two non-zero values for x

Allow even if there are extra equations solved.

A1: Achieves both critical values $x = \frac{15}{4}$ and $x = \frac{25}{4}$ and no other values apart from 0 (and perhaps 5)

following both equations or inequalities. Allow with incorrect inequalities $x \dots \frac{15}{4}$ and $x \dots \frac{25}{4}$, you are looking for the critical values only

A1: Correct range in allowable form. E.g. $x \geq \frac{15}{4}$ or $x \leq \frac{25}{4}$, $\left(-\infty, \frac{15}{4}\right] \cup \left[\frac{25}{4}, \infty\right)$ but condone $x \geq \frac{15}{4}$, $x \leq \frac{25}{4}$

Do NOT incorrect forms such as $x \geq \frac{15}{4}$ and $x \leq \frac{25}{4}$, $\left(-\infty, \frac{15}{4}\right] \cap \left[\frac{25}{4}, \infty\right)$, $\frac{25}{4} \leq x \leq \frac{15}{4}$

The two aspects must be brought together on a single line. Mark their final answer

Alt Method: You may see attempts via squaring. It can be marked using the above guidelines

$$(2x^2 - 10x)^2 = \left(\frac{5}{2}x\right)^2 \Rightarrow x^2(16x^2 - 160x + 375) = 0 \Rightarrow x^2(4x - 15)(4x - 25) = 0$$

Question Number	Scheme	Marks
4 (a)	$\log_{10} N = 2 \Rightarrow N = 100$	B1
		(1)
(b)	$\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2}$	M1
	$\Rightarrow N = 10^{0.35t} \times 10^2$	dM1
	$\Rightarrow N = 100 \times 2.24^t$	A1
		(3)
(c)	Rate of growth at 5 hours is $\frac{dN}{dt} = 100 \times \ln 2.24 \times 2.24^5$	M1
	Allow an answer in the range 4530 to 4550 (bacteria per hour)	A1
		(2)
		Total 6

(a)

B1: For 100 Do not accept just 10^2 and cannot be awarded from 100 in part (b)(b) The demand of the question is ‘show that’, not just find values of a and b Method 1: starting with $\log_{10} N = 0.35t + 2$ M1: Undoes the log. E.g. $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2}$ Give BOD for those who go from $\log_{10} N = 0.35t + 2$ straight to $N = 10^{0.35t} \times 10^2$

dM1: And follows with a correct application of the index law.

Look for $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2} = 10^{0.35t} \times 10^2$ o.e. such as $10^{0.35t} \times 100$ But BOD for $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t} \times 10^2$ o.e. such as $10^{0.35t} \times 100$ And BOD for $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2} \Rightarrow N = 100 \times 2.24^t$ A1: CSO $N = 100 \times 2.24^t$ with no errors following M1, dM1. Allow awrt 2.24Condone $a = 100, b = 2.24$ following the award of M1 dM1

You should expect to see at least one correct line of working and no errors

Method 2: starting with $N = ab^t$ M1: Takes logs of both sides and applies one correct rule. E.g. $N = ab^t \Rightarrow \log N = \log a + \log b^t$ Give BOD for those who write $N = ab^t$ followed by $\log N = \log a + t \log b$ dM1: And follows with $\log N = \log a + t \log b$ before setting $\log a = 2$ and $\log b = 0.35$ and proceeding to values for a and b

A1: Fully shows and produces correct values following M1, dM1

CSO $N = 100 \times 2.24^t$ with no errors. Allow awrt 2.24Condone $a = 100, b = 2.24$ following the award of M1 dM1

You should expect to see at least one correct line of working and no errors

Method 3: A simultaneous approach that finds values but does not show the result

M1: Sets up two simultaneous equations in a and b

E.g. Uses $t = 0, N = 100 \Rightarrow a = 100$ and $t = 1, N = 10^{2.35} \Rightarrow ab^1 = 223.9$

dM1: Solves to find values for a and b .

A0: Has not **shown** the result so cannot be awarded

Common error: Special Case

$$\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t} + 10^2 \text{ so } a = 100 \text{ and } b = 2.24$$

Or even

$$\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t} + 10^2 \Rightarrow N = 100 \times 2.24^t \text{ so } a = 100 \text{ and } b = 2.24$$

Such work resulting in the **correct** values for a and b scores SC 100

(c) Full marks in part (c) can be scored following the SC

M1: Attempts $\frac{dN}{dt} = K \times "2.24"^5$ following through on their 2.24.

This cannot be scored from substituting $t = 5$ into their ab^t

A1: Calculates $\frac{dN}{dt} = 100 \times \ln 2.24 \times 2.24^5$ and achieves an answer in the range 4530 to 4550 (bacteria per hour)

Question Number	Scheme	Marks
5(a)	$\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$	M1
	$= (2 \sin x \cos x) \cos x + (1 - 2 \sin^2 x) \sin x$	dM1
	$= 2 \sin x \cos^2 x + \sin x - 2 \sin^3 x$	A1
	$= 2 \sin x (1 - \sin^2 x) + \sin x - 2 \sin^3 x = 3 \sin x - 4 \sin^3 x$	A1
		(4)
(b)	$2 \sin 3\theta = 5 \sin 2\theta \Rightarrow 2(3 \sin \theta - 4 \sin^3 \theta) = 10 \sin \theta \cos \theta$	M1
	Divides or takes out $\sin \theta$ as a factor and uses $\sin^2 \theta = 1 - \cos^2 \theta$ to set up and solve a 3TQ in $\cos \theta$ E.g. $\Rightarrow 6 \sin \theta - 8 \sin^3 \theta = 10 \sin \theta \cos \theta \Rightarrow 6 - 8(1 - \cos^2 \theta) = 10 \cos \theta$ $\Rightarrow 4 \cos^2 \theta - 5 \cos \theta - 1 = 0$ $\Rightarrow \cos \theta = \frac{5 - \sqrt{41}}{8} = (-0.175...)$	dM1
	Any two of the following four answers $\sin \theta = 0 \Rightarrow \theta = 180^\circ, 360^\circ$ $\cos \theta = \frac{5 - \sqrt{41}}{8} \Rightarrow \theta = \text{awrt } 100^\circ \text{ or awrt } 260^\circ$	A1
	All of $180^\circ, 360^\circ, \text{awrt } 100.1^\circ, \text{awrt } 259.9^\circ$	A1
		(4)
		Total 8

(a)

M1: Attempts $\sin 3x = \sin(2x + x)$ and achieves $\pm \sin 2x \cos x \pm \cos 2x \sin x$. The $= \sin(2x + x)$ may be implied

dM1: Dependent upon the previous mark. It is for using $\sin 2x = 2 \sin x \cos x$ and $\cos 2x = \pm 1 \pm 2 \sin^2 x$, $\pm 1 \pm 2 \cos^2 x$ or $\pm(\cos^2 x - \sin^2 x)$ to form an expression in just $\sin x$ and $\cos x$.

A1: Fully correct AND expanded expression in just $\sin x$ and $\cos x$

Examples of correct expanded expressions are;

$$2 \sin x \cos^2 x + \sin x - 2 \sin^3 x$$

$$2 \sin x \cos^2 x + 2 \sin x \cos^2 x - \sin x$$

$$2 \sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x$$

$$\text{or } 4 \sin x \cos^2 x - \sin x$$

Condone a missing x on a $\sin x$, for example. Condone a slip in notation, e.g. writing $\sin x^3$ instead of $\sin^3 x$ as long as it is subsequently corrected.

A1: Proceeds correctly to $3 \sin x - 4 \sin^3 x$ using $\cos^2 x = 1 - \sin^2 x$

Penalise notational and bracketing errors on this mark only

E.g. Writing $\cos x^2$ for $\cos^2 x$, mixing variables, missing x on a $\sin x$ etc



(b) Allow if solved in terms of another variable, say x

The question states hence or otherwise solve $2\sin 3\theta = 5\sin 2\theta$

In almost all cases they will use their part (a) and proceed by the ‘hence’ route.

‘Otherwise’ routes could start with earlier versions of $2\sin 3\theta$ such as $2(2\sin x \cos^2 x + \sin x - 2\sin^3 x)$

In almost all cases the main mark scheme provides a framework of how the marks are awarded

M1: Uses their part (a) and a double angle for $\sin 2\theta$ of the form $G\sin\theta\cos\theta$ to form an equation in $\sin\theta$ and $\cos\theta$. Look for $R\sin\theta + S\sin^3\theta = T\sin\theta\cos\theta$ o.e. Condone slips such as forgetting to $\times 2$

dM1: The mark is scored for dividing or taking out a factor of $\sin\theta$ and using $\sin^2\theta = \pm 1 \pm \cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$. To solve you should expect to see a valid method used, leading to at least one value for $\cos\theta = k$, $-1 < k < 1$, $k \neq 0$

If a calculator is used to solve the 3TQ in $\cos\theta$ the value(s) must be correct for their equation.

Candidates cannot go straight from an equation e.g. $4\cos^2\theta - 5\cos\theta - 1 = 0$ to a solution e.g. 100.1° without sight of the value for $\cos\theta$ in between.

A1: Finds two angles of 180° , 360° , awrt 100° , awrt 260° following at least one previous M1

The 180° and/or 360° must be found via a correct method with $\sin\theta = 0$ being seen or implied and also produced from a factorised $P\sin\theta + Q\sin^3\theta = R\sin\theta\cos\theta$

Likewise, the 100° and 260° must have been achieved via solving a correct equation.

Allow this mark if the answers are given in radians. So, for two angles of π , 2π , awrt 1.75 , awrt 4.54

A1: All of 180° , 360° , awrt 100.1° , awrt 259.9° with no extra values within the range following M1, dM1

There are no marks for answers appearing without valid method and working

There are other (usually more complicated) ways of attempting part (b) so you must look carefully through each response. If you think a response deserves credit and you are unsure of how to mark, please send to review.

Question Number	Scheme	Marks
6 (a)	$gf(2) = g(3) = 3^2 + 5$	M1
	$= 14$	A1
		(2)
(b)	Correct attempt at inverse $y = 6 - \frac{21}{2x+3} \Rightarrow 2x+3 = \frac{21}{6-y} \Rightarrow x = \left(\frac{21}{6-y} - 3 \right) \div 2$	M1
	$f^{-1}(x) = \frac{3x+3}{2(6-x)}$ or $f^{-1}(x) = \frac{21}{2(6-x)} - \frac{3}{2}$	A1
	$-1, x < 6$	B1
		(3)
(c)	$gg(x) = 126 \Rightarrow (x^2 + 5)^2 + 5 = 126 \Rightarrow x^2 + 5 = \sqrt{121}$	M1
	$\Rightarrow x^2 = "6" \Rightarrow x = (\pm)\sqrt{6}$	dM1
	$\Rightarrow x = \pm\sqrt{6}$	A1
		(3)
		Total 8

(a)

M1: Full attempt at $gf(2)$ applying the rules in the correct order

Look for one of

- An attempt at $6 - \frac{21}{2 \times 2 + 3} \rightarrow k$ followed by an attempt at $k^2 + 5$
- An attempt at substituting $x = 2$ into $\left(6 - \frac{21}{2x+3} \right)^2 + 5$

A1: 14

Allow just 14 with no incorrect working for both marks

(b)

M1: For an attempt at the method of finding the inverse. It is the process you are scoring here.

Look for a process that starts with y as a function of x and proceeds to x as a function of just y .Starting with $y = 6 - \frac{21}{2x+3}$, or possibly a changed function of the form $y = \frac{ax+b}{2x+3}$, which proceeds to aform $x = \text{function of just } y$. Some candidates may swap x and y first. E.g. $x = 6 - \frac{21}{2y+3}$ and proceed tofunction $y = \text{function of just } x$ which is fine.A1: Correct inverse $f^{-1}(x) = \frac{3x+3}{2(6-x)}$ or $f^{-1}(x) = \frac{21}{2(6-x)} - \frac{3}{2}$ o.e. but condone y or f^{-1} for $f^{-1}(x)$ Allow other equivalents such as $y = \frac{-3x-3}{2(x-6)}$, $y = \frac{10.5}{(6-x)} - 1.5$ and $y = \left(\frac{21}{6-x} - 3 \right) \div 2$

ISW following a correct answer

B1: Correct domain written in correct form. E.g. $-1, x < 6$, $[-1, 6)$ 

(c)

M1: For an attempt to "undo" g once in an attempt to find the value of $g(x)$, x^2 or $x^2 + 5$.

This may be scored for any of the following methods (condoning slips)

- $(x^2 + 5)^2 + 5 = 126 \Rightarrow x^2 + 5 = \sqrt{121}$
- $(x^2 + 5)^2 + 5 = 126 \Rightarrow x^4 + 10x^2 - 96 = 0 \Rightarrow (x^2 - 6)(x^2 + 16) = 0 \Rightarrow x^2 = \dots$
- Condone $(x^2 + 5)^2 + 5 = 126 \Rightarrow x^4 + 10x^2 - 96 = 0 \Rightarrow x = \text{awrt } 2.45 \text{ OR } x^2 = 6$
- $g(x) = \sqrt{126 - 5}$

Do not award this mark if the candidate believes that to find x you only need to undo g once.So, answers like $x = 11$ or $x = 6$ will score 0,0,0 unless further work is seen

dM1: For a complete attempt to find the exact solution to the equation. Examples of this would include

- $(x^2 + 5)^2 + 5 = 126 \Rightarrow x^2 + 5 = \sqrt{121} \Rightarrow x = \sqrt{\sqrt{121} \pm 5}$
- $(x^2 + 5)^2 + 5 = 126 \Rightarrow x^4 + 10x^2 - 96 = 0 \Rightarrow (x^2 - 6)(x^2 + 16) = 0 \Rightarrow x^2 = 6 \Rightarrow x = \sqrt{6}$
- $(x^2 + 5)^2 + 5 = 126 \Rightarrow x = \sqrt{6} \text{ or } (x^2 + 5)^2 + 5 = 126 \Rightarrow x = -\sqrt{6}$
- $(x^2 + 5)^2 + 5 = 126 \Rightarrow x^4 + 10x^2 - 96 = 0 \Rightarrow x = \sqrt{6}$
- $x = \sqrt{11 - 5}$

A1: $x = \pm\sqrt{6}$ only. Ignore any reference to $\pm 4i$ if included.Sight of $\pm\sqrt{6}$ without incorrect working scores all 3 marks

Question Number	Scheme	Marks
6 (a)	$y = \frac{4x+3}{x-2} \Rightarrow yx - 2y = 4x + 3$ $\Rightarrow yx - 4x = 2y + 3$ $\Rightarrow x = \frac{2y+3}{y-4} \Rightarrow f^{-1}(x) = \frac{2x+3}{x-4}$ $x \neq 4$	M1 A1 B1 (3)
(b)	$ff(x) = \frac{4 \times \frac{4x+3}{x-2} + 3}{\frac{4x+3}{x-2} - 2} \quad \text{or} \quad ff(x) = 4 + \frac{11}{4 + \frac{11}{x-2} - 2}$ $= \frac{4 \times (4x+3) + 3(x-2)}{4x+3-2(x-2)} = \frac{19x+6}{2x+7}$	M1 dM1 A1 (3)
(c)	Either $x = 1$ or $y = 38$ (1, 38)	M1 A1 (2) (8 marks)

(a)

M1: Attempts to change the subject. May interchange x and y at the start or work towards $x = \dots$ Must proceed as far as getting the two x terms on one side of the equation and the terms not in x on the other or equivalent if x and y are swapped first. Alternatively may divide through first, gather terms and proceed as far as cross multiplying.A1: $f^{-1}(x) = \frac{2x+3}{x-4}$ (oe) but accept $y = \frac{2x+3}{x-4}$ and even $f^{-1} = \frac{2x+3}{x-4}$ or e.g. $f(x)^{-1} = \frac{-2x-3}{4-x}$ Accept equivalent simplified forms.B1: States correct domain $x \neq 4$ ($x \in \mathbb{R}$) Ignore reference to $y \neq 2$ but extra x exclusions is B0.

(b)

M1: Attempts to replace both the x 's in $\frac{4x+3}{x-2}$ by $\frac{4x+3}{x-2}$. Condone a slip such as missing the "+3" as long as the attempt to replace both x 's is seen. Alternatively if dividing through look for the attempt to replace the x by $4 + \frac{11}{x-2}$ condoning a slip missing the extra "-2"dM1: Full attempt to form a fraction of the required form by multiplying all terms in the numerator and denominator by $(x-2)$ If dividing through they must combine to a single fraction.An equally valid alternative is to put both numerator and denominator over common factors of $(x-2)$ A1: $\frac{19x+6}{2x+7}$ but allow equivalent fractions of this form.

(c) Mark only the final attempt here as the candidate may "build up" their answer in stages.

Also watch out for the answer stated next to the question, which can be taken as their final answer.

M1: Either $x = 1$ or $y = 38$

A1: (1, 38) but allow to be written out separately, or with the brackets omitted.



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$\left(\frac{dy}{dx}\right) 2x \cos\left(\frac{1}{2}x\right) - \frac{1}{2}x^2 \sin\left(\frac{1}{2}x\right)$	M1 A1
	Sets $\frac{dy}{dx} = 0 \Rightarrow \tan\left(\frac{1}{2}x\right) = \frac{4}{x} \Rightarrow x = 2 \arctan\left(\frac{4}{x}\right)^*$	dM1 A1*
		(4)
(b)	$x_2 = 2 \arctan\left(\frac{4}{2}\right)$	M1
	$x_2 = \text{awrt } 2.214 \quad x_6 = 2.155 \text{ cao}$	A1, A1
		(3)
		(7 marks)

(a)

M1: Differentiates using the product rule to obtain an expression of the form $Ax \cos\left(\frac{1}{2}x\right) + Bx^2 \sin\left(\frac{1}{2}x\right)$

If the product rule is quoted it must be correct.

A1: For a correct simplified or unsimplified derivative.

dM1: Sets their $\frac{dy}{dx} = 0$ and proceeds to an equation involving $\tan\left(\frac{1}{2}x\right)$. This may be implied by their working.

The M1 must have been scored.

A1*: Correctly achieves the given answer of $x = 2 \arctan\left(\frac{4}{x}\right)$

An intermediate line of $\tan\left(\frac{1}{2}x\right) = \frac{4}{x}$ or $\frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)} = \frac{2x}{\frac{1}{2}x^2}$ must be seen

Note that some may work backwards having differentiated e.g.

$$x = 2 \arctan\left(\frac{4}{x}\right) \Rightarrow \frac{x}{2} = \arctan\left(\frac{4}{x}\right) \Rightarrow \tan\left(\frac{x}{2}\right) = \frac{4}{x} \Rightarrow \frac{x \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = 4 \Rightarrow 4 \cos\left(\frac{x}{2}\right) = x \sin\left(\frac{x}{2}\right)$$

$$\Rightarrow 2x \cos\left(\frac{x}{2}\right) = \frac{1}{2}x^2 \sin\left(\frac{x}{2}\right) \Rightarrow 2x \cos\left(\frac{x}{2}\right) - \frac{1}{2}x^2 \sin\left(\frac{x}{2}\right) = 0$$

Score M1 for a complete method to reach $\frac{dy}{dx} = 0$ and A1 if correct with (minimal) conclusion.

(b)

M1: Attempts to use the given iteration formula. Award for $x_2 = 2 \arctan\left(\frac{4}{2}\right)$ or awrt 2.21

Note that this mark may also be implied by awrt 127 if they are using degrees and no working is shown.

A1: $x_2 = \text{awrt } 2.214$

A1: $x_6 = 2.155$ (Note that this is not awrt)

Question Number	Scheme	Marks
2(a)	States or uses $\sin 2x = 2 \sin x \cos x$	B1
	$\frac{1 - \cos 2x}{2 \sin 2x} = \frac{1 - (1 - 2 \sin^2 x)}{4 \sin x \cos x} = \frac{2 \sin^2 x}{4 \sin x \cos x} = \frac{1}{2} \tan x$	M1 A1
		(3)
(b)	$\frac{9(1 - \cos 2\theta)}{2 \sin 2\theta} = 2 \sec^2 \theta \Rightarrow 9 \times \frac{1}{2} \tan \theta = 2 \sec^2 \theta$	M1
	Attempts $1 + \tan^2 \theta = \sec^2 \theta \Rightarrow 4 \tan^2 \theta - 9 \tan \theta + 4 = 0$	dM1 A1
	$\Rightarrow \tan \theta = \frac{9 \pm \sqrt{17}}{8} (1.64038..., 0.60961...)$	M1
	$\Rightarrow \theta = \text{awrt } 31.4^\circ, 58.6^\circ$	A1 A1
		(6)
(b) Way 2	$\frac{9(1 - \cos 2\theta)}{2 \sin 2\theta} = 2 \sec^2 \theta \Rightarrow 9 \times \frac{1}{2} \tan \theta = 2 \sec^2 \theta$	M1
	$\frac{9}{2} \tan \theta = 2 \sec^2 \theta \Rightarrow \frac{9 \sin \theta}{2 \cos \theta} = \frac{4}{\cos^2 \theta} \Rightarrow 9 \sin \theta \cos \theta = 4 \Rightarrow \sin 2\theta = \frac{8}{9}$	dM1A1
	$\sin 2\theta = \frac{8}{9} \Rightarrow 2\theta = 62.7333..., 117.266... \Rightarrow \theta = ...$	M1
	$\Rightarrow \theta = \text{awrt } 31.4^\circ, 58.6^\circ$	A1 A1
		(9 marks)

(a)

B1: States or uses $\sin 2x = 2 \sin x \cos x$ M1: Attempts to use $1 - \cos 2x = \pm 2 \sin^2 x$ and $2 \sin 2x = 4 \sin x \cos x$ and to proceed to $k \tan x$

A1: Fully correct solution with correct bracketing if seen

(b)

M1: Uses part (a) to form an equation of the form $A \tan \theta = B \sec^2 \theta$ dM1: Replaces $\sec^2 \theta$ by $\pm 1 \pm \tan^2 \theta$ and proceeds to form a 3TQ equation in just $\tan \theta$ A1: Correct 3TQ. E.g. $9 \tan \theta = 4 + 4 \tan^2 \theta$ M1: Correct attempt to find at least one value for $\tan \theta$ for their 3TQ in $\tan \theta$

Allow solutions from calculators and condone decimals as long as accuracy is to at least 2sf.

A1: One of either awrt 31.4° or awrt 58.6° A1: Both awrt 31.4° and awrt 58.6° . Withhold this mark if there are extra solutions in range.

(b) Way 2

M1: Uses part (a) to form an equation of the form equation $A \tan \theta = B \sec^2 \theta$ dM1: Uses $\sec^2 \theta = \frac{1}{\cos^2 \theta}$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin 2\theta = 2 \sin \theta \cos \theta$ to producean equation of the form $\sin 2\theta = C$ A1: $\sin 2\theta = \frac{8}{9}$ M1: Full method to find at least one value of θ from $\sin 2\theta = C$ where $-1 < C < 1$.E.g. $2\theta = \sin^{-1} C$, $\theta = \frac{1}{2} \sin^{-1} C$. (NB radians gives $0.547...$, $1.02...$ and implies the M mark)

A1, A1: As above

Question Number	Scheme	Marks
3(i)	$\int \frac{12}{(2x-1)^2} dx = \frac{-6}{(2x-1)} (+c)$	M1 A1
		(2)
(ii)(a)	(a) $\frac{4x+3}{x+2} = 4 - \frac{5}{x+2}$	M1, A1
		(2)
(ii)(b)	(b) $\int \frac{4x+3}{x+2} dx = \int 4 - \frac{5}{x+2} dx = 4x - 5 \ln x+2 (+c)$	M1 A1ft
	Hence $\int_{-8}^{-5} \frac{4x+3}{x+2} dx = (-20 - 5 \ln 3) - (-32 - 5 \ln 6)$	dM1
	$= 12 + 5 \ln 2$	A1
		(4)
		(8 marks)

(i)

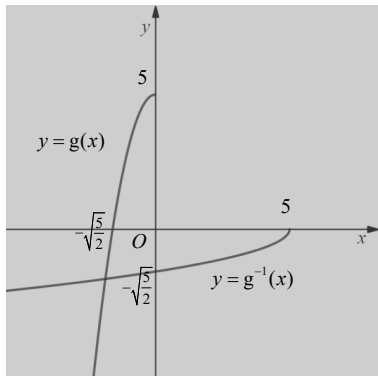
M1: Achieves $\frac{A}{(2x-1)}$ or equivalent e.g. $A(2x-1)^{-1}$ A1: Achieves $\frac{-6}{(2x-1)}$ or $-6(2x-1)^{-1}$. There is no requirement for the $+c$

(ii)(a)

M1: For using division or any other suitable method to find either A or B Using division look for $x+2 \overline{)4x+3}$ Using an identity look for $4x+3 = A(x+2) + B$ followed by a value for either A or B It may be implied by either $A = 4$ or $B = -5$ A1: Correct value for A and B or a correct expression $4 - \frac{5}{x+2}$ or $4 + \frac{-5}{x+2}$

(ii)(b)

M1: Integrates $\frac{B}{x+2} \rightarrow \dots \ln|x+2|$. Condone $\frac{B}{x+2} \rightarrow \dots \ln(x+2)$ and condone $\frac{B}{x+2} \rightarrow \dots \ln x + 2$ A1ft: Integrates $A + \frac{B}{x+2} \rightarrow Ax + B \ln|x+2|$. Condone $A + \frac{B}{x+2} \rightarrow Ax + B \ln(x+2)$ But do **not** condone missing brackets unless they are implied by subsequent work.dM1: Substitutes -5 and -8 into their integrated function containing $\ln|x+2|$ or $\ln(x+2)$ A1: $12 + 5 \ln 2$ or equivalent exact answer such as $12 + \ln 32$ SC: For candidates who proceed to $12 + 5 \ln 2$ or equivalent exact answervia $(-20 - 5 \ln(-3)) - (-32 - 5 \ln(-6))$ without sight of modulus signs withhold just the final A1

Question Number	Scheme	Marks	
4(a)	$fg(x) = \frac{4(5 - 2x^2) + 6}{5 - 2x^2 - 5}$	M1	
	$fg(x) = 3 \Rightarrow \frac{26 - 8x^2}{-2x^2} = 3 \Rightarrow x = -\sqrt{13}$	dM1 A1 A1	
		(4)	
(b)	$y = \frac{4x + 6}{x - 5} \Rightarrow yx - 5y = 4x + 6 \Rightarrow yx - 4x = 5y + 6$	M1	
	$\Rightarrow x = \frac{5y + 6}{y - 4}$	A1	
	$\Rightarrow f^{-1}(x) = \frac{5x + 6}{x - 4} \quad x \in \mathbb{R}, x \neq 4$	A1	
		(3)	
(c)		Shape and position for $y = g(x)$. A parabola starting on the y -axis that extends down in quadrant 2 to at least the x -axis. Condone slips of the pen e.g. if the curve heads back towards the y -axis in quadrant 3 slightly.	B1
		Shape and position for $y = g(x)$ as above that crosses the x -axis and $y = g^{-1}(x)$ a parabola starting on the x -axis with graphs crossing as shown in quadrant 3. Condone slips of the pen e.g. if $g^{-1}(x)$ heads back towards the x -axis in quadrant 3 slightly.	B1
		Correct intercepts. This mark is for the 4 correct intercepts as shown. Allow as values or coordinates and allow coordinates the wrong way round (e.g. (0, 5) for (5, 0)) as long as they are in the correct places. The negative intercepts must be exact as shown or equivalent. The graphs must cross the axes at these points and you can ignore any other intercepts.	B1
		Note that labels for $g(x)$ and $g^{-1}(x)$ are not required	
			(3)
		(10 marks)	

(a)

M1: For attempting $fg(x)$ in the correct order.dM1: For proceeding to $x^2 = \dots$ **Depends on the first mark.**A1: For $x^2 = 13$ which may be implied by $x = \sqrt{13}$ A1: For $x = -\sqrt{13}$ only

$$b^2 - 4ac = 0 \Rightarrow (18k - 20)^2 - 4(4 - k^2) \times (-56) = 0 \Rightarrow 100k^2 - 720k + 1296 = 0 \Rightarrow k = \dots$$

(allow two values for k from a quadratic)

In each method do not be too concerned by their use of $=, \geq, \leq, >, <$ throughout their working.

Condone arithmetical slips.

A1 ($k =$) 3.6 oe

Question Number	Scheme	Marks
6(a)	$(f'(x) =) 5(x^2 - 2) \times \frac{1}{2} \times 4 \times (4x + 9)^{-\frac{1}{2}} + 10x(4x + 9)^{\frac{1}{2}}$	M1A1
	$(f'(x) =) \frac{10(x^2 - 2) + 10x(4x + 9)}{(4x + 9)^{\frac{1}{2}}} = \frac{50x^2 + 90x - 20}{(4x + 9)^{\frac{1}{2}}}$	dM1
	$(f'(x) =) \frac{10(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$	A1
		(4)
(b)	Sets $f'(x) = 0$ to give $x = -2, \frac{1}{5}$	B1
		(1)
(c)	$y = 5(("-2")^2 - 2)(4 \times (" - 2") + 9)^{\frac{1}{2}} = 10$ $(-2, 10)$	M1 A1cao
		(2)
(d)	$f(0) = -30$ Upper bound $= 2 \times "10" + 4 = \dots$ or Lower bound $= 2 \times " - 30" + 4 = \dots$ $-56 \leq g(x) \leq 24$	B1 M1 A1
		(3)
		(10 marks)

A1 $k > 3.6$ or eg $k > \frac{18}{5}$ but not $k \geq \frac{18}{5}$ If $k < -2$ is found then it must be rejected

(a)

M1 Attempts the product rule. Award for $\pm A(5x^2 - 10)(4x + 9)^{-\frac{1}{2}} \pm Bx(4x + 9)^{\frac{1}{2}}$ or equivalent where $A, B \neq 0$ The indices do not need to be processed for this mark.

If they quote the product rule eg $uv' + u'v$ and state the expressions for u, u', v and v' then allow a miscopy when substituted. They must have

eg $u = 5x^2 - 10 \rightarrow u' = Ax$ and $v = (4x + 9)^{\frac{1}{2}} \rightarrow v' = B(4x + 9)^{-\frac{1}{2}}$

May also attempt the product rule on $5x^2(4x + 9)^{\frac{1}{2}} - 10(4x + 9)^{\frac{1}{2}}$ so award for
 $\pm Ax(4x + 9)^{\frac{1}{2}} \pm Bx^2(4x + 9)^{-\frac{1}{2}} \pm C(4x + 9)^{-\frac{1}{2}}$ $A, B, C \neq 0$

A1 Correct unsimplified expression for $f'(x)$ eg $2(5x^2 - 10)(4x + 9)^{-\frac{1}{2}} + 10x(4x + 9)^{\frac{1}{2}}$ oe

dM1 Proceeds to a single fraction, multiplies out the brackets on the numerator and proceeds to the form $\frac{3TQ}{(4x + 9)^{\frac{1}{2}}}$ or $3TQ(4x + 9)^{-\frac{1}{2}}$. They cannot proceed straight to the given 3TQ or a multiple of it without some working seen to score this mark.

Alternatively, you may only see manipulation of the numerator and then $\frac{3TQ}{(4x + 9)^{\frac{1}{2}}}$ which is

acceptable: $10(x^2 - 2) + 10x(4x + 9) = 10(x^2 - 2 + 4x^2 + 9x) = 10(5x^2 + 9x - 2) \Rightarrow \frac{10(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$

Condone arithmetical slips in their working but their method to proceeding to a single fraction must be correct.

It is dependent on the previous method mark.

A1 $(f'(x) =) \frac{10(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$ with no errors seen in the main body of their solution.

Accept $\frac{10(5x^2 + 9x - 2)}{\sqrt{4x + 9}}$

Condone invisible brackets as long as they are recovered, or their subsequent working makes their intention clear. They do not need to explicitly state $k = 10$ but if there is a contradiction between their expression and their stated value then the expression takes precedence. Do not be concerned with what they have before the =

(b) Mark (b) and (c) together

B1 Sets $f'(x) = 0$ to give $-2, \frac{1}{5}$ Withhold if one is rejected. Condone $(-2, 0) \left(\frac{1}{5}, 0 \right)$. May be seen in (c)

(c)

M1 Substitutes their smaller root from solving $5x^2 + 9x - 2 = 0$ into $y = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}}$ and proceeds to find a value for y . Condone bracket omissions, miscopying or arithmetical slips as long as it is clear that they are attempting to substitute their smaller root into $f(x)$

A1cao $(-2, 10)$ only (Allow $x = -2, y = 10$)



(d)

B1 $(f(0) =) -30$ seen (which may be implied by sight of -56)

M1 A correct attempt at the lower or upper bound.

- Either attempts $2 \times 10 + 4$ and identifies this as the upper bound. Follow through on their part (c) y coordinate. < 24 or ≤ 24 is acceptable
- Or attempts $2 \times -30 + 4$ and identifies this as their lower bound. > -56 or ≥ -56 is acceptable for this mark. It cannot be scored for using $2 \times f\left(\frac{1}{5}\right) + 4$ as their lower bound.

Note that $g > -56$ or $g \geq -56$ will score B1M1.

Allow use of max or min for upper and lower bound for this mark.

A1 $-56 \leq g(x) \leq 24$ or eg “ $\{g(x) \in \mathbb{R} : -56 \leq g(x) \cap g(x) \leq 24\}$ ” or “ $-56 \leq g(x)$ AND $g(x) \leq 24$ ” or $g \in [-56, 24]$ or other variations similar to these. Condone g for $g(x)$ and $-56 \leq y \leq 24$ or $-56 \leq 2f(x) + 4 \leq 24$ but not $-56 \leq f \leq 24$

DO NOT ACCEPT “ $-56 \leq g(x) \cup g(x) \leq 24$ ” or “ $-56 \leq g(x)$ OR $g(x) \leq 24$ ”

Question Number	Scheme	Marks
7(a)	$f(0) = (0-3)^2 = 9$	M1
	$0 \leq f(x) \leq 9$	A1
		(2)
(b)	$f'(x) = -2xe^{-x^2}(2x^2-3)^2 + e^{-x^2} \times 8x(2x^2-3)$	M1A1
	$= 2x(2x^2-3)e^{-x^2}(-(2x^2-3)+4) = 2xe^{-x^2}(2x^2-3)(7-2x^2)$	dM1A1
		(4)
(c)	$x^2 = \frac{3}{2}, \frac{7}{2} \Rightarrow f\left(\sqrt{\frac{7}{2}}\right) = e^{-\frac{7}{2}}\left(2 \times \frac{7}{2} - 3\right)^2 = 16e^{-\frac{7}{2}}$	M1A1
	$16e^{-\frac{7}{2}} < k < 9$	dM1A1
		(4)
		(10 marks)
Notes		
(a) Work for part (a) must be seen in part (a) not recovered in part (c). M1: Substitutes $x = 0$ and proceeds to find a value for y. Implied by sight of 9. A1: $0 \leq f(x) \leq 9$ Accept with y or just f, but not with x. Accept interval, $[0, 9]$, or set notation. (b) M1: Attempts the product rule and chain rule achieving $\pm Pxe^{-x^2}(2x^2 \pm 3)^2 + e^{-x^2} \times Qx(2x^2 \pm 3)$ with $P, Q > 0$. Alternatively, attempts the quotient rule on $\frac{(2x^2-3)^2}{e^{x^2}}$ achieving $\frac{Px(2x^2 \pm 3)e^{x^2} - Qxe^{x^2}(2x^2 \pm 3)^2}{(e^{x^2})^2}$ $P, Q > 0$ A1: $-2xe^{-x^2}(2x^2-3)^2 + e^{-x^2} \times 8x(2x^2-3)$ oe (need not be simplified) dM1: Must have scored previous M. Attempts to take out a factor of $2xe^{-x^2}(2x^2-3)$ to obtain a factor $(\pm C \pm Dx^2)$ which may be unsimplified. Allow if e.g. the x or e^{-x^2} is dropped when taking out the factor. A1: Achieves $2xe^{-x^2}(2x^2-3)(7-2x^2)$ with no errors seen. (c) Note : allow all the marks in (c) from answers which were a constant multiple out in (b) or missing the factor x, which lead to the correct answers here. M1: Attempts to find a y value (which may be zero) for at least one of the valid non-zero roots for $f'(x)=0$. Allow for an attempt at either $f\left(\pm\sqrt{\frac{3}{2}}\right)$ or $f\left(\pm\sqrt{\frac{A}{B}}\right)$ leading to a value, where A and B are their values from (b) with $AB > 0$. Note e.g. $f\left(\frac{A}{B}\right)$ attempted is M0. A1: For obtaining $16e^{-\frac{7}{2}}$ Accept awrt 0.483 for this mark following the award of M. dM1: Attempts the inside region, allowing $\leq$, using their y intercept from (a) and their positive y value, less than their intercept, from an attempt at the y value of the maxima. Allow a positive y value from an attempt at any non-zero root of $f'(x)$ as such an attempt. It is dependent on the previous method mark. A1: $16e^{-\frac{7}{2}} < k < 9$ or in any equivalent form e.g. interval notation $k \in \left(\frac{16}{e^{\frac{7}{2}}}, 9\right)$ Allow y instead of k Ignore references to "$k = 0$"		

Question Number	Scheme	Marks
4(a)	$f(x) = 8\sin x \cos x + 4\cos^2 x - 3$	M1
	States or uses $\sin 2x = 2\sin x \cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$	dM1
	Uses $\sin 2x = 2\sin x \cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ in $f(x)$ $(f(x) =) 8\sin x \cos x + 4\cos^2 x - 3 = 4\sin 2x + 2\cos 2x - 1$	A1
		(3)
(b)	$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20} \text{ or } 2\sqrt{5}$	B1ft
	$\tan \alpha = \frac{b}{a} \Rightarrow \alpha = \dots$ ("awrt 0.464")	M1
	$(f(x) =) 2\sqrt{5} \sin(2x + 0.464) - 1$	A1
		(3)
(c)	(i) Maximum value = " $2\sqrt{5} - 1$ "	B1 ft
	(ii) Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = \dots$	M1
	$(x =) \text{awrt } 3.69 \text{ (or } (x =) \text{awrt } 3.70)$	A1
		(3)
		(9 marks)

(a) For the method marks condone working in mixed variables provided the intention is clear

M1: **Either** states one of the correct identities $\sin 2x = 2\sin x \cos x$ or $\cos 2x = 2\cos^2 x - 1$

or uses $\sin 2x = 2\sin x \cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$ (may be implied by use of e.g. $\cos^2 x = \pm 1 \pm \sin^2 x$ and $\cos 2x = \pm 1 \pm 2\sin^2 x$ or $\cos 2x = \pm \cos^2 x \pm \sin^2 x$)

Can be implied by **either of**

- $a = 4$
- $b = \pm 2$ **and** $c = -5$ or -1

dM1: **Uses** $\sin 2x = 2\sin x \cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ (which may be implied as above for the first method mark) in $f(x)$ to produce an expression of the required form $a\sin 2x + b\cos 2x + c$
 Condone slips when substituting in.

Can be implied by $a = 4$ **and** $b = \pm 2$ **and** $c = -5$ or -1

A1: $4\sin 2x + 2\cos 2x - 1$ Correct answer scores all three marks. Must be in terms of x .

(b) **Full marks can only be scored provided correct values for a , b and c are found in (a).**

B1ft: Finds the exact value of R using $R^2 = a^2 + b^2$ (or may use trigonometry using their value for α)

Can be implied by a correct exact value.

Follow through on their a or b but the correct values should give $R = \sqrt{20}$ or $2\sqrt{5}$



M1: Uses a "correct" method to find α using their a and b . Accept for example $\tan \alpha = \pm \frac{b}{a} \Rightarrow \alpha = \dots$ or $\tan \alpha = \pm \frac{a}{b} \Rightarrow \alpha = \dots$ (You may need to check this on your calculator if only the angle is seen)

May also be seen using e.g. $\sin \alpha = \pm \frac{b}{"R"} \Rightarrow \alpha = \dots$ or $\cos \alpha = \pm \frac{a}{"R"} \Rightarrow \alpha = \dots$. Allow the angle to be found in degrees for this mark. $\alpha = \text{"awrt } 26.6^\circ \text{"}$ (1dp)

A1cso: $(f(x) =) 2\sqrt{5} \sin(2x + \text{awrt } 0.464) - 1$ o.e. e.g. $(f(x) =) \sqrt{20} \sin(2x + 0.464) - 1$ (may be awarded if seen in (c)) Can only be scored provided correct values for a , b and c are found in (a).

(c) (i)

B1ft: $"2\sqrt{5} - 1"$ o.e. e.g. $"\sqrt{20} - 1"$ but follow through on their $R + c$ as long as **R has been correctly found** (ie awarded in (b)). Ft on their c which may be different in (a) and (b) but **cannot be 0**

Allow to be a decimal if R was given as a decimal in (b) at any point. isw following a correct answer.

(c)(ii)

M1: Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = \dots$ using their α found in (b) (note that this is still the equation which will need to be solved if they differentiate $f(x)$ first and set equal to 0). You may need to check this on your calculator.

Allow this mark to be scored even if there are additional equations formed (and possibly solved).

They must be consistent in their use of degrees or radians in an equation that they are solving.

$\frac{5\pi}{2} = 7.85\dots$ may be used in their working.

A1: $(x =)$ awrt 3.69 but also allow $(x =)$ awrt 3.70. If several angles are found then they must indicate which angle is their final answer by e.g. underlining, circling

Question Number	Scheme	Marks
3 (i)	$\int (2x-5)^7 dx = \frac{(2x-5)^8}{8} + c$	M1 A1 (2)
(ii)	$\int \frac{4 \sin x}{1+2 \cos x} dx = -2 \ln(1+2 \cos x) + c$ $\int_0^{\frac{\pi}{3}} \frac{4 \sin x}{1+2 \cos x} dx = [-2 \ln(1+2 \cos x)]_0^{\frac{\pi}{3}} = -2 \ln 2 + 2 \ln 3 = \ln \frac{9}{4}$	M1 A1 dM1 A1 (4) (6 marks)

(i)

M1: Achieves $a(2x-5)^8$ or equivalent where a is a constant. Alternatively achieves au^8 with

$$u = (2x-5)$$

Allow this mark from a miscopy such as $\int (2x-3)^7 dx = k(2x-3)^8$

A1: Achieves $\frac{(2x-5)^8}{8} + c$ or exact simplified equivalent such as $\frac{1}{8}(2x-5)^8 + c$. **The +c (o.e) must be present.**

Any attempts that start by multiplying out $(2x-5)^7$ are likely to end in failure. They are unlikely to get an expression of the form $a(2x-5)^8$

FYI

$$(2x-5)^7 = 128x^7 - 2240x^6 + 16800x^5 - 70000x^4 + 175000x^3 - 262500x^2 + 218750x - 78125$$

Score B1 SC for at least 5 out of 8 **correct** terms of

$$\frac{128}{8}x^8 - \frac{2240}{7}x^7 + \frac{16800}{6}x^6 - \frac{70000}{5}x^5 + \frac{175000}{4}x^4 - \frac{262500}{3}x^3 + \frac{218750}{2}x^2 - 78125x$$

(ii)

M1: Achieves $b \ln(1+2 \cos x)$ or $b \ln|1+2 \cos x|$ where b is a constant. Condone a missing bracketAlternatively achieves $b \ln u$ with $u = 1+2 \cos x$ (You may see $b \ln ku$ which is also correct)

A1: Achieves $-2 \ln(1+2 \cos x)$, $-2 \ln|1+2 \cos x|$ or $-2 \ln u$ o.e. with $u = 1+2 \cos x$ oe. There is no need for $+c$. This may be left unsimplified. Only condone a missing bracket if subsequent work implies one

dM1: Substitutes **both** 0 and $\frac{\pi}{3}$ into an expression of the form $b \ln(1+2 \cos x)$ or $b \ln|1+2 \cos x|$

and subtracts either way around. There must have been some attempt to evaluate the trig terms

Alternatively substitutes both 3 and 2 into an expression of the form $k \ln u$ and subtracts

A1: $\ln \frac{9}{4}$

Note that algebraic integration must be seen here. Candidates using their calculators to just write down

$$\int_0^{\frac{\pi}{3}} \frac{4 \sin x}{1+2 \cos x} dx = 0.81093... = \ln \frac{9}{4} \text{ should be awarded 0 marks}$$

Question Number	Scheme	Marks
4	$A = \frac{80pe^{0.15t}}{pe^{0.15t} + 4}$	
(a)	$30 = \frac{80p}{p+4}$ $30p + 120 = 80p \Rightarrow p = \frac{120}{50} = 2.4 \quad *$	M1 A1* (2)
(b)	$50 = \frac{80 \times 2.4e^{0.15T}}{2.4e^{0.15T} + 4} \Rightarrow 72e^{0.15T} = 200$ $\Rightarrow 0.15T = \ln\left(\frac{200}{72}\right) \Rightarrow T = 6.8$	M1 A1 dM1 A1 (4)
(c)	80 m ²	B1 (1) (7 marks)

(a)

 M1: Sets $A = 30$ and $e^{0.15 \times 0} = 1$ to set up an equation in p .

 A1*: Achieves $p = 2.4$ with no significant (*) errors and with one correct **linear (non fractional)**

 equation in p . * Condone minor slips as long as they are recovered before reaching the given answer.

An example of this would be

$$p + 4(30) = 80p \Rightarrow 30p + 120 = 80p \Rightarrow 50p = 120 \Rightarrow p = 2.4$$

Alt method

 M1: Sets $p = 2.4$, $e^{0.15 \times 0} = 1$ and attempts the value of $(A) = \frac{80 \times 2.4 \times 1}{2.4 \times 1 + 4}$

 A1*: Achieves $A = 30$ with no errors and concludes that $p = 2.4$. Condone $30 = \frac{192}{6.4}$ ✓

 (b) Allow $t \leftrightarrow T$ here

 M1: Sets $A = 50$, $p = 2.4$ and proceeds to an equation of the form $ce^{0.15t} = d$ $c \times d > 0$

 Condone slips, e.g. on the 0.15. You may see $de^{-0.15t} = c$ $c \times d > 0$

 A1: Achieves $72e^{0.15t} = 200$ o.e.

 dM1: Correct order of operations to find T/t

$$\text{For example } 0.15T = \ln\left(\frac{200}{72}\right) \Rightarrow T = \dots \text{ or}$$

$$\ln 72 + 0.15T = \ln 200 \Rightarrow 0.15T = \dots \Rightarrow T = \dots$$

A1: AWR 6.8

(c)

 B1: Requires units as well. 80 m²

Students lacking work in part (b)

 Example 1: $50 = \frac{80 \times 2.4e^{0.15T}}{2.4e^{0.15T} + 4} \Rightarrow T = \text{awrt } 6.8$ can be awarded SC 1000

Example 2: $50 = \frac{80 \times 2.4e^{0.15T}}{2.4e^{0.15T} + 4} \Rightarrow 72e^{0.15T} = 200 \Rightarrow T = \text{awrt } 6.8$ score M1 A1 via scheme and then

SC 10

Question Number	Scheme	Marks
5 (a)	Attempts to find y at -1.25 and -1.2 with one correct to 1sf Achieves $y(-1.25) = -0.9$ and $y(-1.2) = 0.2$ With reason (change of sign and continuous) and Conclusion	M1 A1 (2)
(b)	(i) Attempts $\sqrt{12 \ln(15) + 8}$ $x_2 = \text{awrt } 6.3637$ (ii) $x = 6.4142$	M1 A1 B1 (3)
(c)	$\frac{dy}{dx} = \frac{12}{2x+3} - x$ Stationary point when $\frac{12}{2x+3} = x \Rightarrow 2x^2 + 3x - 12 = 0 \Rightarrow x =$ $\Rightarrow x = \frac{-3 + \sqrt{105}}{4}$ or awrt 1.81 ONLY	M1 A1 dM1 A1 (4) (9 marks)

(a)

M1: Attempts to find y at -1.25 and -1.2 with one correct to at least 1sf.

FYI $y(-1.25) = -0.9$ and $y(-1.2) = 0.2$

A1: Achieves $y(-1.25) = \text{awrt } -0.9$ and $y(-1.2) = \text{awrt } 0.2$ with a reason and minimal conclusion.

Acceptable reasons are; "sign change and continuous", " $y(-1.25) \times y(-1.2) < 0$ and continuous" Minimal conclusions are; "hence proven", "hence root", " $\checkmark$ ", " $\square$ "

Note: A smaller interval could be chosen but it must span -1.20998 and the final conclusion must refer to the given interval to score both marks.

(b)(i)

M1: Attempts to apply the iteration formula once. Accept $\sqrt{12 \ln(15) + 8} = \dots$ or awrt 6.4

A1: $x_2 = \text{awrt } 6.3637$

(b)(ii)

B1: CAO $x = 6.4142$ There must be some evidence of the M or continued iteration for this to be awarded. A minimum would be an attempt at x_2 (the M) or an attempt at any intermediate term.

(c)

M1: Attempts to differentiate with $\ln(2x+3) \rightarrow \frac{\dots}{2x+3}$

A1: $\frac{dy}{dx} = \frac{12}{2x+3} - x$ which may be left unsimplified. No requirement to see lhs

dM1: Sets their $\frac{\dots}{2x+3} \pm \dots x = 0$ and proceeds to a value for x via a correct method of solving a 3TQ

There must be some evidence of working but allow candidates to use a calculator to write down the solution of 3TQ. If they do, it must be correct for their 3TQ



A1: cso $x = \frac{-3 + \sqrt{105}}{4}$ or awrt 1.81 ONLY. If $x = \frac{-3 - \sqrt{105}}{4}$ or awrt -3.31 is also written down it must be rejected. ISW after a correct answer. There must be evidence of dM1 to award this mark

Question Number	Scheme	Marks
6. (a)	$f(x) = \frac{5x-3}{x-4} \Rightarrow f'(x) = \frac{5(x-4) - (5x-3)}{(x-4)^2} = \frac{k}{(x-4)^2}$ States that $f'(x) = \frac{-17}{(x-4)^2} \Rightarrow f'(x) < 0$ hence decreasing * cso	M1 dM1 A1* (3)
(b)	$y = \frac{5x-3}{x-4} \Rightarrow xy - 4y = 5x - 3 \Rightarrow xy - 5x = 4y - 3$ $\Rightarrow x = \frac{4y-3}{y-5} \quad \text{So } f^{-1}(x) = \frac{4x-3}{x-5}$ Domain $x > 5$	M1 A1 B1 (3)
(c) (i)	$ff(x) = \frac{5 \times \frac{5x-3}{x-4} - 3}{\frac{5x-3}{x-4} - 4}$ $ff(x) = \frac{5 \times (5x-3) - 3(x-4)}{5x-3-4(x-4)} = \frac{22x-3}{x+13}$	M1 dM1 A1
(ii)	$5 < ff(x) < 22$	B1, B1 (5) (11 marks)

(a)

M1: Attempts to use the quotient rule to achieve a form $\frac{p(x-4) - q(5x-3)}{(x-4)^2}$ with $p, q > 0$ OR

attempts the product rule to achieve a form $p(x-4)^{-1} \pm (5x-3)(x-4)^{-2}$

Condone attempts in which terms such as $(x-4)^2$ may have been multiplied out incorrectly

dM1: Proceeds to $f'(x) = \frac{k}{(x-4)^2}$ Allow the lhs to be $\frac{dy}{dx}$

A1*: Requires a correct $f'(x)$, a correct statement such as " $f'(x) < 0$ " o.e (such as $f'(x)$ is negative) and a minimal conclusion which could be $\checkmark$ or QED or $\square$

It cannot be awarded from substituting in single values of x . It cannot be based on incorrect mathematics.

"There is a minus sign" or " $-17 < 0$ " is insufficient without the further statement $f'(x) < 0$ (o.e) followed by a minimal conclusion.

(b)

M1 Attempts to change the subject. Look for cross multiplication with an attempt to collect terms with x 's (or replaced y 's) on one side of the equation and non x terms on the other side

A1 $f^{-1}(x) = \frac{4x-3}{x-5}$ o.e Condone the lhs as f^{-1} and even y . The notation $f^{-1}: x \mapsto \frac{4x-3}{x-5}$ is fine



B1 Correct domain $x > 5$

(c) (i)

M1 Attempts to substitute $\frac{5x-3}{x-4}$ into f . Condone minor slips but the form of the expression must be correct

dM1 Multiplies all terms on numerator and denominator by $x-4$. Condone missing brackets
Alternatively writes both the numerator and denominator as single fractions over $(x-4)$

A1 $(ff(x)) = \frac{22x-3}{x+13}$ Condone a missing left hand side or being set as $y =$

(c)(ii)

B1: Achieves the value of one "end". It is not just for the number so $f < 5$ is B0

Condone non strict inequalities here so $5 \leq ff < 21$ would be fine for B1 B0

B1: Fully correct. Accept $5 < ff < 22$, $(5, 22)$. Allow with y or range but $5 < x < 22$ would be B1

B0

Methods where candidates "split up" fraction $\frac{5x-3}{x-4} = 5 + \frac{17}{x-4}$

(a)

M1: Attempts to split into form $5 + \frac{17}{x-4}$ AND attempts to use the chain rule.

Look for $\frac{5x-3}{x-4} \rightarrow A + \frac{B}{x-4}$ with a differential of $\frac{k}{(x-4)^2}$

dM1: Proceeds to $f'(x) = \frac{k}{(x-4)^2}$. Allow the lhs to be $\frac{dy}{dx}$

A1*: $f'(x) = \frac{-17}{(x-4)^2}$ and states $f'(x) < 0$ (for all x) so f is decreasing function

(b)

M1: Attempts to split $y = \frac{5x}{x+3}$ into form $y = 5 + \frac{17}{x-4}$ and then attempts to make x the subject.

Look for $y = \frac{5x-3}{x-4} = A + \frac{B}{x-4}$ with progress to a form $x-4 = \frac{B}{\pm y \pm A}$

A1: $f^{-1}(x) = \frac{17}{x-5} + 4$ or equivalent

B1: $x > 5$

(c)

M1: Attempts to split $y = \frac{5x}{x+3}$ into form $y = 5 + \frac{17}{x-4}$ and then attempts $ff(x) = 5 + \frac{17}{5 + \frac{17}{x-4} - 4}$

dM1: Uses a correct method to combine into a single fraction of the required form

A1: $ff(x) = \frac{22x-3}{x+13}$

(c)(ii)

B1: Achieves the value of one "end". It is not just for the number so $ff < 5$ is B0

Condone non strict inequalities here so $5 \leq ff < 21$ would be fine for B1 B0

Question Number	Scheme	Marks
2(a)	$(f(2) =) 2^4 - 5(2)^2 + 4(2) - 7 = -3 < 0$ and $(f(3) =) 3^4 - 5(3)^2 + 4(3) - 7 = 41 > 0$	M1
	There is a <u>change of sign</u> and $f(x)$ is <u>continuous</u> so <u>there is a root (in the interval)</u> *	A1*
		(2)
(b)	$x^3 = \frac{5x^2 - 4x + 7}{x} \Rightarrow x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}} \quad *$	B1*
		(1)
(c)(i)	$x_2 = \sqrt[3]{\frac{5(2)^2 - 4(2) + 7}{2}}$	M1
	= awrt 2.1179	A1
(ii)	$\alpha = \text{awrt } 2.1565$	A1
		(3)
		(6 marks)

(a)

M1: Attempts both $f(2)$ and $f(3)$ or a narrower interval that contains the root 2.1565...and considers the signs. Note showing $f(2) \times f(3) < 0$ or “ < 0 ”, “ > 0 ” beside each appropriate root is a consideration of signs and is also sufficient for the “sign change” part of reasoning for the A1.
For the substitution, need to see the values substituted or at least one of $f(2)$ or $f(3)$ correct.

A1*: This mark requires:

- both $f(2)$ and $f(3)$ correct e.g. $f(2) = -3$ and $f(3) = 41$
- a reference to the sign change
- a reference to continuity
- a (minimal) conclusion

For **sign change**, allow equivalent statements e.g. $f(2).f(3) < 0$, $f(2) < 0 < f(3)$ etc.

For **continuity**, allow just “continuous” and allow “assuming it is continuous” and “continuous equation” but do **not** allow incorrect statements such as “ x is continuous”, “the interval is continuous”, “there is a change of sign therefore it is continuous”

For the **conclusion**, allow e.g. ✓, #, QED, hence shown, hence root, so it crosses the x -axis, etc., but **not** incorrect statements e.g. so there is a root in the interval $[-3, 41]$

You may have to use your judgement to decide if the A1 conditions are satisfied.

Example: $f(2) = -3$, $f(3) = 41$, there is a change of sign hence $f(x)$ has a continuous root in $[2, 3]$
Scores M1A0 – i.e. it is not the root that is continuous

General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$ **Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1	$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3(\sec^2\theta - 1) + 7\sec\theta - 3 = 0$	M1
	$3\sec^2\theta + 7\sec\theta - 6 = 0$	A1
	$(3\sec\theta - 2)(\sec\theta + 3) = 0 \Rightarrow \sec\theta = \dots \Rightarrow \cos\theta = \dots$	dM1
	$\theta = 109.5^\circ, 250.5^\circ$	A1, A1
		(5)
		Total 5

Main method: Also allow if another variable is used e.g. $x \leftrightarrow \theta$

M1: Attempts to set up a quadratic equation in $\sec\theta$ using $\tan^2\theta = \pm 1 \pm \sec^2\theta$

Condone bracketing slips, for example, where $3\tan^2\theta$ is replaced by $\pm 3\sec^2\theta \pm 1$

A1: Collects terms to form a correct 3TQ, $3\sec^2\theta + 7\sec\theta - 6 = 0$ o.e

Terms do not need to be all on one side of the equation and the $= 0$ can be implied by subsequent work.

dM1: Dependent upon the previous M1, it is scored for the method of producing a solvable non zero solution for $\cos\theta$

Look for;

- A valid way of solving their quadratic equation in $\sec\theta$ using any allowable method including a calculator leading to $\sec\theta = \dots$
- Use of $\sec\theta = \frac{1}{\cos\theta}$ to produce a solution $\cos\theta = k$ where $-1 < k < 1, k \neq 0$

E.g. $3\sec^2\theta + 7\sec\theta - 6 = 0 \Rightarrow \sec\theta = -3 \Rightarrow \cos\theta = -\frac{1}{3}$. You can ignore the 2nd solution

If the error had been $\tan^2\theta = \sec^2\theta + 1$ look for $3\sec^2\theta + 7\sec\theta = 0 \Rightarrow \sec\theta = -\frac{7}{3} \Rightarrow \cos\theta = -\frac{3}{7}$

A1: One of $\theta = \text{awrt } 109.5^\circ, \text{awrt } 250.5^\circ$ following M1, A1, dM1

Allow either answer in radians here so score for awrt 1.91 or awrt 4.37

A1: CSO. Both $\theta = \text{awrt } 109.5^\circ$ and $\text{awrt } 250.5^\circ$ and no extras in the range

.....

Other methods do exist but the same marking principles will apply

Alt method:

M1: For an attempt to set up a quadratic equation in a single trigonometric term, e.g. $\cos\theta$

Replaces $\tan^2\theta$ by $\frac{\sin^2\theta}{\cos^2\theta}$, $\sec\theta$ by $\frac{1}{\cos\theta}$ and then uses $\sin^2\theta = \pm 1 \pm \cos^2\theta \Rightarrow$ quadratic equation in

$\cos\theta$ E.g. $3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3\frac{\sin^2\theta}{\cos^2\theta} + 7 \times \frac{1}{\cos\theta} - 3 = 0 \Rightarrow 3\sin^2\theta + 7\cos\theta - 3\cos^2\theta = 0$

and then uses $\sin^2\theta = \pm 1 \pm \cos^2\theta \Rightarrow 3(1 - \cos^2\theta) + 7\cos\theta - 3\cos^2\theta = 0$.

Condone slips

A1: For a correct 3TQ in $\cos\theta$. FYI $6\cos^2\theta - 7\cos\theta - 3 = 0$. Terms do not have to be on one side

dM1: Solves the quadratic equation in $\cos\theta$ using an appropriate method including a calculator.

It must lead to a solution $\cos\theta = k$ where $-1 < k < 1, k \neq 0$



If a calculator is used the solution must be correct for their equation.

E.g. $6\cos^2\theta - 7\cos\theta - 3 = 0 \Rightarrow \cos\theta = -\frac{1}{3}$

A1: One of $\theta = \text{awrt } 109.5^\circ, \text{awrt } 250.5^\circ$ following M1, A1, dM1

Allow either answer in radians here so score for awrt 1.91 or awrt 4.37

A1: CSO. Both $\theta = \text{awrt } 109.5^\circ$ and $\text{awrt } 250.5^\circ$ and no extras in the range.

If both solutions to the quadratic were given then both must be correct

Methods using calculator technology and not showing all stages of working as required by question:

Example 1:

$$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow \theta = 109.5^\circ, 250.5^\circ$$

No marks

Example 2:

$$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3(\sec^2\theta - 1) + 7\sec\theta - 3 = 0$$

$$\Rightarrow 3\sec^2\theta + 7\sec\theta - 6 = 0 \Rightarrow \sec\theta = -3 \Rightarrow \theta = 109.5^\circ, 250.5^\circ$$

Scores M1, A1, dM0, A0, A0

Example 3:

$$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 6\cos^2\theta - 7\cos\theta - 3 = 0 \text{ via identities followed by } \Rightarrow \theta = 109.5^\circ, 250.5^\circ$$

Scores M1, A1, dM0, A0, A0

Methods using calculator technology and showing sufficient stages of working:

Example 4: $3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3\sec^2\theta + 7\sec\theta - 6 = 0$ via identity followed by

$$\sec\theta = -3 \Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \theta = \pm 109.5^\circ$$

Scores M1, A1, dM1, A1, A0 (only one of the two solutions given)

Example 5: $3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 6\cos^2\theta - 7\cos\theta - 3 = 0$ via identities followed by

$$\cos\theta = -\frac{1}{3} \Rightarrow \theta = 109.5^\circ, 250.5^\circ \text{ M1, A1, dM1, A1, A1}$$

Question Number	Scheme	Marks
2.(a)	Sight of $10^{1.478}$ or $10^{0.0646}$ or $10^{0.0646t+1.478}$ $(a =)$ awrt 30 or $(b =)$ awrt 1.16 $\log_{10} N = 0.0646t + 1.478 \rightarrow N = 10^{0.0646t+1.478} = 10^{0.0646t} 10^{1.478}$ $= "30" \times "1.16"$ $N = 30 \times 1.16^t$	M1 A1 dM1 A1 (4)
(b)	Attempts $N = 30 \times 1.16^{30} =$ awrt 2600	M1 A1 (2) (6 marks)

(a) **NB This shows as MMAA on ePEN but is being marked as MAMA.**

M1 Sight of $10^{1.478}$ or $10^{0.0646}$ or $10^{0.0646t+1.478}$ (allowing slips copying the values) anywhere in their solution.

This mark is implied by seeing awrt 30 or awrt 1.16

A1 Sight of either awrt 30 or awrt 1.16

dM1 Applies correct index laws and proceeds to find values for a and b . $N = \left(10^{0.0646}\right)^t \times 10^{1.478} = "1.16" \times "30"$

For this mark there must be evidence of correct index work so expect to see at least $10^{0.0646t+1.478}$ before a final answer and no incorrect index work.

A1 $a =$ awrt 30 and $b =$ awrt 1.16 as long as there is no contrary work, or states that $N = 30 \times 1.16^t$ (awrt values)

(b)

M1 Attempts $N = 30 \times 1.16^{30}$ with their values of a and b .

Alternatively $\log_{10} N = 0.0646 \times 30 + 1.478 \Rightarrow N = 10^{3.416}$

A1 awrt 2600, isw after a correct answer.

.....
 Alt (a) using $N = ab^t$ as a starting point.

M1 As main scheme.

A1 $a =$ awrt 30 or $b =$ awrt 1.16 (must be correctly assigned)

dM1 Takes $\log_{10}$ of both sides and proceeds to at least $\log_{10} N = \log_{10} a + t \log_{10} b$ and attempts to find a value for both of the constants with no incorrect log work.

A1 $a =$ awrt 30 and $b =$ awrt 1.16 as long as there is no contrary work, or states that $N = 30 \times 1.16^t$ (awrt values)

Question Number	Scheme	Marks
3. (a)	$\frac{dy}{dx} = \frac{(4x-1)^{\frac{1}{2}} \times 2 - (2x+3) \times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)}$ $\frac{(4x-1)^{\frac{1}{2}} \times 2 - (2x+3) \times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)} \times \frac{(4x-1)^{\frac{1}{2}}}{(4x-1)^{\frac{1}{2}}} = \frac{4x-8}{(4x-1)^{\frac{3}{2}}}$	M1 A1 dM1 A1 (4)
(b)	Turning point where $\frac{dy}{dx} = 0 \Rightarrow x = 2$ Find value of f at $x = 2 \Rightarrow f(x) = \sqrt{7}$ Hence range is $f \geq \sqrt{7}$	M1 dM1 A1 (3) (7 marks)

(a)

M1 Attempts the quotient rule and **achieves** $\frac{dy}{dx} = \frac{p(4x-1)^{\frac{1}{2}} - q(2x+3)(4x-1)^{-\frac{1}{2}}}{(4x-1)}$ condoning slips in coefficients

Allow the denominator to appear as $(4x-1)^{\frac{1}{2} - 2}$ or even $(4x-1)^{\frac{1}{4}}$

Alternatively attempts the product rule with $(2x+3)$ and $(4x-1)^{-\frac{1}{2}}$ and **achieves**

$$\frac{dy}{dx} = A(4x-1)^{-\frac{1}{2}} \pm B(2x+3)(4x-1)^{-\frac{3}{2}}$$

A1 For a correct un-simplified $\frac{dy}{dx}$. Look for $\frac{dy}{dx} = \frac{(4x-1)^{\frac{1}{2}} \times 2 - (2x+3) \times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)}$ oe via the quotient rule

Look for $\frac{dy}{dx} = 2(4x-1)^{-\frac{1}{2}} - 2(2x+3)(4x-1)^{-\frac{3}{2}}$ oe via the product rule

dM1 Multiplies both numerator and denominator by $(4x-1)^{\frac{1}{2}}$ and simplifies to a linear numerator, or correctly takes a common factor of $(4x-1)^{\frac{1}{2}}$ out of the numerator to leave a linear factor.

If the product rule is used look for a correct attempt to factor out $(4x-1)^{\frac{3}{2}}$

A1 $\frac{4x-8}{(4x-1)^{\frac{3}{2}}}$ or exact simplified equivalent Must have a single term in $(4x-1)$

NB Allow marks for (a) if simplification of $f'(x)$ is seen in (b) (but not having set $f'(x) = 0$ and rearranging).

(b)

M1 Attempts $\frac{dy}{dx} = 0 \Rightarrow x = \dots$

dM1 Finds the value of $f(\text{their } 2)$, where their $2 > \frac{1}{4}$

A1 $f \geq \sqrt{7}$ Allow e.g. $y \geq \sqrt{7}$, $f(x) \geq \sqrt{7}$ or range is $[\sqrt{7}, \infty)$ Allow with $\frac{7}{\sqrt{7}}$ oe or awrt 2.65

Do not allow $x \geq \sqrt{7}$ or just $\geq \sqrt{7}$

Special Case: If candidates square the function first in (a) they will get $(f(x)^2)' = 1 - \frac{49}{(4x-1)^2} = \frac{16x^2 - 8x - 48}{(4x-1)^2}$.

They will score no marks in (a) but can achieve full marks in (b) as the stationary point occurs at the same x coordinate.

Special Case: A correct range given following no attempt, or an incorrect answer to (a), can score SC M1M0A0.

Note that in (a) some candidates may take an alternative approach for the first mark:

$$fg(x) = 3 \Rightarrow g(x) = f^{-1}(3) = -21 \Rightarrow 5 - 2x^2 = -21$$

Score M1 for finding $f^{-1}(3)$ and setting $g(x) = f^{-1}(3)$ then as main scheme

or

$$fg(x) = 3 \Rightarrow g(x) = f^{-1}(3) = -21 \Rightarrow x = g^{-1}(-21)$$

Score M1 for finding $f^{-1}(3)$ and setting $g^{-1}f^{-1}(3) = x$ then as main scheme

(b)

M1: For attempting to change the subject. In the attempt shown in the mark scheme score when both terms in x have been isolated and set on one side of the equation

A1: For $x = \frac{5y+6}{y-4}$ or equivalent such as $x = 5 + \frac{26}{y-4}$

If the x and y have been swapped award for $y = \frac{5x+6}{x-4}$ or equivalent.

A1: Requires both the expression in x **and** the domain.

$$f^{-1}(x) = \frac{5x+6}{x-4} \quad x \neq 4 \quad \text{or} \quad f^{-1}(x) = \frac{5x+6}{x-4} \quad x \in \mathbb{R}, x \neq 4 \quad (\text{Condone } f^{-1} = \dots)$$

Do **not** allow $y = \dots$

(c) See scheme

Question Number	Scheme	Marks
5(a)	States or implies that $\log_{10} p = 0.32$ or $\log_{10} q = \frac{0.56 - 0.32}{8}$	M1
	$p = \text{awrt } 2.089$ or $q = \text{awrt } 1.072$	A1
	States or implies that $\log_{10} p = 0.32$ and $\log_{10} q = \frac{0.56 - 0.32}{8}$	M1
	$p = \text{awrt } 2.089$ and $q = \text{awrt } 1.072$	A1
		(4)
(b)	States or implies that $\frac{dA}{dt} = p \ln q \times q'$ with their values for p and q	M1 A1
	Rate of increase in pond weed after 6 days is 0.22 (m ² /day)	A1
		(3)
		(7 marks)

(a)

M1: States or implies that $\log_{10} p = 0.32$ **or** $\log_{10} q = \frac{0.56 - 0.32}{8}$ or equivalent equationsA1: $p = \text{awrt } 2.089$ **or** $q = \text{awrt } 1.072$ M1: States or implies that $\log_{10} p = 0.32$ **and** $\log_{10} q = \frac{0.56 - 0.32}{8}$ or equivalent equationsA1: $p = \text{awrt } 2.089$ **and** $q = \text{awrt } 1.072$

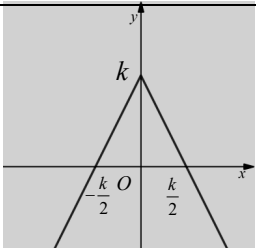
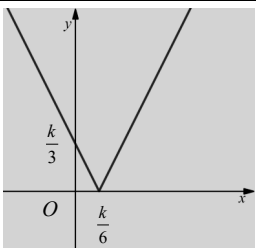
(b)

M1: Uses $\frac{d}{dt} q' \rightarrow kq'$ $k \neq 1$ A1: States or implies that $\frac{dA}{dt} = p \ln q \times q'$ with their values for p and q

A1: awrt 0.22 Units are not required

.....
Alt (b) using $\log_{10} A = 0.03t + 0.32$ as a starting pointM1: Attempts to differentiate and reaches $\frac{1}{A} \frac{dA}{dt} = k$ or equivalentA1: $\frac{1}{A \ln 10} \frac{dA}{dt} = 0.03$

A1: awrt 0.22 Units are not required

Question Number	Scheme	Marks
6(a)(i)		M1
	Correct intercepts.	A1
(ii)		M1
	Correct intercepts.	A1
		(4)
(b)	Attempts to solve either $k + 2x = -2x + \frac{k}{3}$ or $k - 2x = 2x - \frac{k}{3}$	M1
	Achieves either $x = -\frac{1}{6}k$ or $x = \frac{1}{3}k$ oe	A1
	Attempts to solve both $k + 2x = -2x + \frac{k}{3}$ and $k - 2x = 2x - \frac{k}{3}$	M1
	Achieves either $x = -\frac{1}{6}k$ and $x = \frac{1}{3}k$	A1
		(4)
		(8 marks)

(a)

M1: For the shape of $y = k - 2|x|$. Score for an upside down V with maximum on the y-axis with the branches at least reaching the x-axis.

A1: Intercepts at k on the y-axis and $\pm \frac{k}{2}$ on the x-axis. Allow as values or coordinates and allow coordinates the wrong way round (e.g. $(k, 0)$ for $(0, k)$) as long as they are in the correct places. The graph must cross the axes at these points.

M1: For the shape of $y = \left| 2x - \frac{k}{3} \right|$. Score for a V shape in quadrants 1 and 2 with a minimum on the x-axis and at least reaches the y-axis.

A1: Intercepts at $\frac{k}{3}$ on the y-axis and $\frac{k}{6}$ on the x-axis. Allow as values or coordinates and allow coordinates the wrong way round (e.g. $(k/3, 0)$ for $(0, k/3)$) as long as they are in the correct places. The graph must cross the y-axis and touch the x-axis.

(b)

M1: Attempts to solve one correct equation not involving moduli (or equivalent equations)

A1: One correct solution. Allow unsimplified forms such as $-\frac{2k}{12}$

M1: Attempts to solve two correct equations not involving moduli (or equivalent equations)

A1: Two correct solutions (simplified) and no other solutions given.

Some may square in (b) e.g.

$$\left(2x - \frac{k}{3}\right)^2 = (k - 2x)^2 \Rightarrow 4x^2 - \frac{4kx}{3} + \frac{k^2}{9} = k^2 - 4kx + 4x^2 \Rightarrow \frac{8kx}{3} = \frac{8k^2}{9} \Rightarrow x = \frac{k}{3}$$

Score M1 for squaring to obtain at least 3 terms on both sides and solving for x and A1 for $x = \frac{k}{3}$

Candidates are unlikely to find the other root using this approach.

Question Number	Scheme	Marks
4(a)	$\text{Gradient} = \frac{3.85 - 3.08}{5 - 0} \quad (=0.154 \text{ or } \frac{77}{500})$ $\log_{10} N = 3.08 + 0.154t \quad \text{oe}$	M1 A1
		(2)
(b)	$a = 10^{3.08} = \text{awrt } 1200$ $\log_{10} b = "0.154" \text{ or } b = 10^{".154"}$ $b = \text{awrt } 1.43$	B1 M1 A1
		(3)
(c)	$\log_{10} 500\,000 = 3.08 + 0.154T \Rightarrow T = \dots \text{ or } T = \log_{".143"} \left(\frac{500\,000}{".1200"} \right) = \dots$ $(T =) \text{awrt } 17$	M1 A1
		(2)
		(7 marks)

Mark (a) and (b) together
(a)

M1 A correct attempt to find the gradient between the given coordinates. This may be implied by $10^{0.154}$ or awrt 1.43 or $\frac{77}{500}$. The values embedded in the expression $\frac{3.85 - 3.08}{5 - 0}$ or $\frac{3.08 - 3.85}{-5}$ is sufficient. Check Figure 1 and may be seen in (b)

A1 $\log_{10} N = 3.08 + 0.154t$ Must be in terms of $\log_{10} N$ and t (may be seen in (b))
 Condone $\log N$ or $\lg N$ for $\log_{10} N$ but do not accept $\ln N$
 Accept equivalent equations such as $\log_{10} N = \frac{77}{25} + \frac{77}{500}t$ or $500 \log_{10} N = 1540 + 77t$ or

$$t = \frac{500}{77} \log_{10} N - \frac{1540}{77}$$

 Do not accept $N = 1200 \times 1.43^t$



(b)

B1 awrt 1200

M1 A correct equation to find b using their gradient in (a). Can be awarded for $\log_{10} b = "0.154"$ or $b = 10^{0.154}$ (may be implied by awrt 1.43). Allow log or lg but not ln.

A1 awrt 1.43

(c)

M1 Substitutes in $N = 500\,000$ into their $N = ab^t$ using their values for a and b and uses a valid method to find a value or expression for T (can be left as a logarithm)

This can be either by:

- using correct inverse laws
- using a calculator and proceeding directly to a value for t

$$\text{Typically, } "1200" \times "1.43"^T = 500\,000 \Rightarrow "1.43"^T = \frac{500\,000}{"1200"} \Rightarrow T = \frac{\log_{10} \frac{500\,000}{1200}}{\log_{10} "1.43"} = \dots$$

$$\Rightarrow "1.43"^T = \frac{500\,000}{1200} \Rightarrow T = \log_{1.43} \left(\frac{500\,000}{1200} \right) = \dots$$

Alternatively uses their $\log_{10} N = "3.08" + "0.154"t$ and rearranges to find a value for T allowing sign slips only:

$$\log_{10} 500\,000 = "3.08" + "0.154"T \Rightarrow T = \frac{\log_{10} 500\,000 - "3.08"}{"0.154"} = \dots$$

May be implied by a correct answer.

A1 ($T =$) awrt 17 isw after a correct answer is seen (cannot be left as a logarithm)

This mark can only be scored provided EITHER their equation in (a) is correct OR their values for a and b are correct in (b).



Question Number	Scheme	Marks
5(a)		
(i)	7	B1
(ii)	$\frac{9}{k}$	B1
		(2)
(b)	$kx - 9 - 2 = 0$ and $-kx + 9 - 2 = 0 \Rightarrow \frac{\dots}{k}, \frac{\dots}{k}$ CVs $\frac{7}{k}, \frac{11}{k}$ $\frac{7}{k} < x < \frac{11}{k}$	M1 A1 A1
		(3)
(c)	$"-2" = -2 \times \frac{9}{k} + 3 \Rightarrow k = \dots$ $(k =) 3.6$ $k > 3.6$	M1 A1 A1
		(3)
		(8 marks)
Alt(b)	$-2 < kx - 9 < 2$ $\frac{\dots}{k} < x < \frac{\dots}{k}$ $\frac{7}{k} < x < \frac{11}{k}$	M1 A1A1

(a) Check Figure 2 and by the questions**(i)**

B1 7 Condone (0, 7) but not (7, 0)



(ii)

 B1 $\frac{9}{k}$ oe Condone $\left(\frac{9}{k}, -2\right)$ but not $\left(-2, \frac{9}{k}\right)$

(b)

M1 A correct attempt to find the two critical values. Eg

- Forms two correct equations (or linear inequalities) with the modulus signs removed and solves to find the values of x where the graph crosses the x -axis (see main scheme) or the values of x such that $|kx - 9| - 2 < 0$
- Solves the given inequality to isolate x in the middle (Alt(b)) by:
 $|kx - 9| - 2 < 0 \Rightarrow |kx - 9| < 2 \Rightarrow -2 < kx - 9 < 2 \Rightarrow \dots < x < \dots$
- Solves the given inequality by squaring both sides and solving:

$$(kx - 9)^2 < 4 \Rightarrow k^2 x^2 - 18kx + 77 < 0 \Rightarrow x^2 - \frac{18}{k}x + \frac{77}{k^2} < 0 \Rightarrow \left(x - \frac{9}{k}\right)^2 < \frac{4}{k^2} \Rightarrow \dots < x < \dots$$

 Do not be too concerned by their use of $=, \geq, \leq, >, <$ throughout their working.

 A1 Correct critical values $\frac{7}{k}, \frac{11}{k}$ cao which may be seen in their final answer.

 A1 $\frac{7}{k} < x < \frac{11}{k}$ or simplified equivalent eg " $x > \frac{7}{k} \cap x < \frac{11}{k}$ " or " $\left\{x \in \mathbb{R} : \frac{7}{k} < x < \frac{11}{k}\right\}$ " or

 $x \in \left(\frac{7}{k}, \frac{11}{k}\right)$ Must be in terms of x

 There must be some attempt to bring the two inequalities together if separate. The mark cannot be scored for $x > \frac{7}{k}$ $x < \frac{11}{k}$ DO NOT ACCEPT eg " $x > \frac{7}{k}$ or $x < \frac{11}{k}$ " $x \in \left[\frac{7}{k}, \frac{11}{k}\right]$

(c)

 M1 Uses a correct method to find a value for k . For example:

- Substitutes in $\left(\frac{9}{k}, "-2"\right)$ into $y = 3 - 2x$ and attempts to find a value for k . They may solve
 $"-2" = 3 - 2x \Rightarrow x = "2.5" \Rightarrow \frac{9}{k} < "2.5" \Rightarrow k > \dots$
- Finds when $y = 3 - 2x$ intersects with AB ($y = -kx + 7$) and then when $y = 3 - 2x$ intersects with B and forms the inequality $\frac{4}{k-2} < \frac{9}{k}$ or similar.
- Finds the value of x when $y = 3 - 2x$ intersects with AB , $y = -kx + 7$, $\left(= \frac{4}{k-2}\right)$ and then when
 $y = 3 - 2x$ intersects $y = kx - 11$ $\left(= \frac{14}{k+2}\right)$ and sets equal to each other to find the value for k (or solves simultaneously to find $\Rightarrow x = "2.5" \Rightarrow \frac{9}{k} < "2.5" \Rightarrow k > \dots$)
- Finds the gradient between $(0, 3)$ and B and deduces that $-2 > \frac{"-2"-3}{\frac{9}{k}-0}$ oe leading $k = \dots$
- Sets $f(x) = 3 - 2x$ and attempts the discriminant leading to a value for k :
 $3 - 2x = |kx - 9| - 2 \Rightarrow (5 - 2x)^2 = (kx - 9)^2 \Rightarrow (4 - k^2)x^2 + (18k - 20)x - 56 = 0$

Question Number	Scheme	Marks
6(a)	(i) $P(0, 3a)$	B1
	(ii) $Q(a, 0)$ $R\left(\frac{7}{3}a, 0\right)$	B1 B1
	(iii) $S\left(\frac{5}{3}a, -2a\right)$	B1
		(4)
(b)	$3x - 5a - 2a = x - 2a \Rightarrow x = \dots$ or $-(3x - 5a) - 2a = -(x - 2a) \Rightarrow x = \dots$	M1
	$x = \frac{5}{2}a$ or $x = \frac{1}{2}a$	A1
	$3x - 5a - 2a = x - 2a \Rightarrow x = \dots$ and $-(3x - 5a) - 2a = -(x - 2a) \Rightarrow x = \dots$	dM1
	$x = \frac{5}{2}a$ and $x = \frac{1}{2}a$	A1
		(4)
		Total 8

(a) Question states simplest form so don't accept e.g. $\frac{10}{6}a$ for $\frac{5}{3}a$

(i)

B1: For $(0, 3a)$. Condone just $y = 3a$ as long as there isn't a value for x coordinate (apart from 0).
Note that either of $P = 3a$ or $(3a, 0)$ is B0 unless $y = 3a$ is previously seen

(ii)

B1: For either coordinate $(a, 0)$ **or** $\left(\frac{7}{3}a, 0\right)$ which may or may not be linked correctly to Q and R

Condone just $x = a$ ($x = 1a$) or $x = \frac{7}{3}a$ as long as there isn't a y coordinate (that isn't 0).

Note that an answer such as $Q = a$ or $(0, a)$ is B0 unless $x = a$ is previously seen.

Likewise an answer such as $R = \frac{7}{3}a$ or $(0, 7a/3)$ is B0 unless $x = 7a/3$ is previously seen

B1: For $(a, 0)$ **and** $\left(\frac{7}{3}a, 0\right)$. Allow the coordinates to be given separately. They do not need to be linked to the correct letter. Condone just $x = a$ ($x = 1a$) and $x = \frac{7}{3}a$ as long as there isn't a y coordinate (that isn't 0).

SC: $Q = a$ **AND** $R = \frac{7}{3}a$ or vice versa is B1 B0

(iii)

B1: $\left(\frac{5}{3}a, -2a\right)$. Allow this to be given as $x = \dots, y = \dots$

(b)

M1: Attempts to solve one **correct** equation. It must be an equation without moduli and be found correctly
For example, look for

- Either taking both positive aspects of the modulus equation $|3x - 5a| - 2a = |x - 2a|$

So $3x - 5a - 2a = x - 2a$ or exact equivalent such as $3x - 7a = x - 2a \Rightarrow x = \dots$

Condone for this mark $\Rightarrow a = \dots$

- Or taking both negative aspects of the modulus equation $|3x - 5a| - 2a = |x - 2a|$

So $-3x + 5a - 2a = -x + 2a$ or exact equivalent such as $-3x + 3a = -x + 2a \Rightarrow x = \dots$

Condone for this mark $\Rightarrow a = \dots$

The $2a$ may be moved over first so it is possible to solve $|3x - 5a| = |x - 2a| + 2a$

So both positive will give an equation $3x - 5a = x - 2a + 2a$ or $3x - 5a = x$

or both negative would give $-3x + 5a = -x + 2a + 2a$ or $-3x + 5a = -x + 4a$

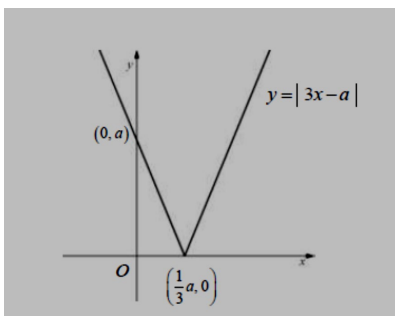
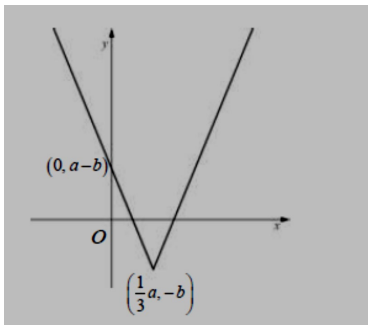
A1: One correct value of x which must be found from a correctly produced equation

dM1: Attempts to solve both of these equations.

There may be other equations which should be ignored for this mark

A1: Both correct values of x (coming from correct equations) with no other values given.

Also allow if 4 answers are found, for example, followed by the 2 correct answers being chosen and the other 2 incorrect answers discarded.

Question Number	Scheme	Marks
7 (a)(i)		Shape B1 Vertex $\left(\frac{1}{3}a, 0\right)$ B1 y intercept $(0, a)$ B1
(a)(ii)		Shape B1ft Vertex $\left(\frac{1}{3}a, -b\right)$ B1ft y intercept $(0, a - b)$ B1ft
(b)	Attempts to solve $-3x + a - b = 5x \Rightarrow x = \dots$ $\Rightarrow x = \frac{a-b}{8}$ ONLY	M1 A1 (2)
		(6)
		(8 marks)

Note: accept both diagrams on the same sketch as long as the information for each is clear.

If two graphs are given unlabelled, treat their first as part (a)(i) and their second as part (a)(ii).

(a)(i)

B1: A V shaped curve sitting on the positive x -axis and appearing in at least quadrant 1. Don't be too concerned about the graph being non symmetrical unless it appears wildly incorrect

B1: The vertex is on the positive x -axis and is marked $\left(\frac{1}{3}a, 0\right)$ but condone $(x =) \frac{1}{3}a$ which may be seen as the intercept on the graph. May be scored from an incorrect shape for the graph, but must have a single vertex on the x -axis.

B1: The graph passes through the positive y -axis at $(0, a)$ but condone $(y =) a$ which may be seen as the intercept on the graph.

SC If a “Y” shape is given, if part is dashed ignore the dashed line, if all are solid withhold the first B but allow B0B1B1 if vertex and positive intercept are correct.

(a)(ii)

B1ft: A V shaped curve with vertex in quadrant 4 or follow through a V shape that is a translation down from their answer in (a)(i)

B1ft: The vertex is in quadrant 4 and is marked $\left(\frac{1}{3}a, -b\right)$, which may be indicated by appropriate labelling on the graph. Follow through on their x intercept of the vertex in (i) (which must have been on positive the x -axis)

B1ft: The graph passes through the positive y -axis at $(0, a-b)$, following through on their y intercept from (i), but condone $(y =) a - b$ which may be seen as the intercept on the graph.

SC If a “Y” shape is given, if part is dashed ignore the dashed line, if all are solid withhold the first B but all B0B1B1 if vertex and positive intercept are correct.

(b)

M1: Attempts to solve $-3x + a - b = 5x \Rightarrow x = \dots$ There may be other equations attempted

A1: $x = \frac{a-b}{8}$ or exact equivalent only. If multiple values are found the incorrect ones must be rejected or this correct answer clearly indicated as their final answer (e.g. by underlining). Ignore any reasoning given, mark the value.

Question Number	Scheme	Marks
6.(a)	(2.5, 3) oe	B1 B1 (2)
(b)	Attempts one solution usually $4x - 10 + 3 = 3x - 2 \Rightarrow x = 5$ Attempts both solutions $-4x + 10 + 3 = 3x - 2 \Rightarrow x = \frac{15}{7}$	M1 A1 dM1 A1 (4)
(c)	Attempts to solve $y = kx + 2$ with $x = 2.5, y = 3$ or states that $k < 4$ $k \dots \frac{2}{5}$ States $\frac{2}{5} < k < 4$	M1 A1 A1 (3) 9 marks

(a)

B1 For one correct coordinate, either $x = 2.5$ or $y = 3$ B1 For (2.5, 3). Allow $x = \dots, y = \dots$ Allow exact equivalents

If a candidate reverses these and just writes down (3, 2.5) score SC 10

(b)

M1 For a correct method of finding one point of intersection. The signs of the terms must be correct

Accept $2(2x - 5) + 3 = 3x - 2 \Rightarrow x = \dots$ oe or $-2(2x - 5) + 3 = 3x - 2 \Rightarrow x = \dots$ oeNote that equations such as $2(2x - 5) + 3 = -3x + 2 \Rightarrow x = \dots$ are incorrect and score M0A1 For 5 or $\frac{15}{7}$ Ignore any reference to the y coordinate

dM1 For a correct method of finding both intersections

A1 For both values 5 and $\frac{15}{7}$ (with no extra solutions)Ignore any reference to the y coordinates or $x = \frac{2}{3}$ within the body of their solution.

(c)

M1 Attempts to solve $y = kx + 2$ with THEIR $x = 2.5, y = 3$ to find k or deduces that $k < 4$ An equivalent is attempting to set k equal to the gradient between (0, 2) and (2.5, 3)A1 Finds that $k = \frac{2}{5}$ is a critical valueA1 $\frac{2}{5} < k < 4$ 

Question Number	Scheme	Marks
5 (i)	States $x = 2$ $\sqrt{3} \sec x + 2 = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \dots$ $x = \frac{5\pi}{6}$	B1 M1 A1 (3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2 \theta$ $6\sin^2 \theta + 10\sin \theta - 3 = 0$ $\sin \theta = \frac{-5 \pm \sqrt{43}}{6} (= -1.926\dots, 0.2595\dots) \Rightarrow \theta = \arcsin(\dots)$ $\theta = 15.0^\circ, 165^\circ$	M1 A1 M1 A1 (4) (7 marks)
Notes		
(i)		
B1	States $x = 2$. May be seen anywhere in (i) and don't be concerned where it comes from.	
M1	For a correct process to solve $\sqrt{3} \sec x + 2 = 0$ E.g. $\sec x = \frac{1}{\cos x} \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \dots$ Allow slips in rearranging but must attempt to solve $\cos x = k, k < 1$ or $\sec x = k, k > 1$ Degree value (150°) following a correct equation implies the M mark. Note some may use $\sec^2 x = 1 + \tan^2 x$ and form a quadratic in $\tan x$. These will need a correct identity, correct method to solve a quadratic (which may be by calculator) and attempt to solve $\tan x = k, k \neq 0$	
A1	$x = \frac{5\pi}{6}$ and no other extra solutions in the range. Accept awrt 2.62 (and isw).	
Note that $\sqrt{3} \sec x + 2 = 0 \rightarrow x = \frac{5\pi}{6}$ can score M1A1 as no incorrect work is seen, method implied.		
Question required working to be shown $x = \frac{5\pi}{6}$ without seeing at least $\sqrt{3} \sec x + 2 = 0$ extracted first is M0A0.		
(ii)		
M1	Attempts to use $\cos 2\theta = \pm 1 \pm 2\sin^2 \theta$ to form a quadratic equation in $\sin \theta$. If using alternative forms for the identity, must also use $\cos^2 \theta = 1 - \sin^2 \theta$ before gaining this mark.	
A1	Correct 3 term quadratic equation $6\sin^2 \theta + 10\sin \theta - 3 = 0$ or a multiple of this. Alternatively may be scored for $6\sin^2 \theta + 10\sin \theta = 3$ if followed by completing the square on LHS to solve.	
M1	Full attempt to find one value for θ from a quadratic in $\sin \theta$. Must involved - correct method to solve the quadratic in $\sin \theta$ (usual rules, may use calculator) to produce a value for $\sin \theta$ - use of $\arcsin(\dots)$ to reach the value for θ (you may need to check the values if $\arcsin(\dots)$ is not shown). Radian answers can imply the mark (awrt 0.263, 2.88 if correct). May be scored from an incorrect identity as long as a quadratic is achieved. Accept arcsin expression for the M	
A1	$\theta = \text{awrt } 15.0^\circ, 165^\circ$ and no other solutions in the range. Accept just 15° for 15.0° (but not awrt 15° if it does not round to 15.0°)	
Condone a different variable used than θ throughout.		

Question Number	Scheme	Marks
6.(a)	$(2, -10)$	B1 B1 (2)
(b)	$ff(0) = f(-4) = \dots$ $= 8$	M1 A1cso (2)
(c)	Attempts to solve $-3(x-2)-10=5x+10 \Rightarrow x = \dots$ $x > -\frac{7}{4}$ only	M1 A1 (2)
(d)	$x(\text{or } x) = \frac{16}{3}$ Attempts $3(x -2)-10=0 \Rightarrow x =k, k > 0$ or $3(-x-2)-10=0 \Rightarrow x=-k$ or $3(x-2)-10=0 \Rightarrow x=k \Rightarrow x=-k$ $x = \left(\frac{16}{3} \text{ and } \right) -\frac{16}{3}$ with no other values	B1 M1 A1 (3) (9 marks)

Notes

- (a)
B1 For one correct coordinate
B1 For $(2, -10)$. Allow $x = \dots, y = \dots$. Do not accept e.g. $6/3$ unless 2 has been seen/identified with this.
- (b)
M1 For a full attempt at $f f(0)$. Can be scored for $f(-4)$. Allow for use of their $f(0)$ even if incorrect as long as the process is clear, e.g. $f(0) = \dots$ stated or calculated first then used. May be scored by first attempting $ff(x)$ before substituting. This mark is for showing the correct process of composites, so may be scored if there are slips or errors with modulus if the intent is clear.
A1cso $ff(0) = 8$ only. A0 if other values given.
- (c)
M1 Attempts to solve $-3(x-2)-10=5x+10 \Rightarrow x = \dots$. Allow with equality or any inequality for the M mark.
Alternatively, rearranges to $|x-2| = ax+b$, squares both sides and solves the quadratic.
A1 $x > -\frac{7}{4}$ (oe) only. If another inequality or value is given and not rejected withhold this mark.
- (d)
Work for (d) must be seen or referred to in (d). Do not accept for work attempted in earlier parts but not used in (d).
B1 For $x = \frac{16}{3}$. Allow when seen even from incorrect working as it could be verified. May be seen on sketch as long as referred to in (d). Allow also for $|x| = \frac{16}{3}$

Question Number	Scheme	Marks
8(a)	Starting with the LHS: $2\operatorname{cosec}^2 2\theta(1 - \cos 2\theta) = \frac{2 - 2\cos 2\theta}{\sin^2 2\theta}$	M1
	$= \frac{2 - 2(1 - 2\sin^2 \theta)}{4\sin^2 \theta \cos^2 \theta}$	M1dM1
	$= \sec^2 \theta = 1 + \tan^2 \theta \equiv \text{RHS} \quad *$	A1*
		(4)
(b)	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = \dots$	M1
	$\cos x = \frac{1}{4} \quad (\text{ignore } -1)$	A1
	$\cos x = \frac{1}{4} \Rightarrow x = \dots$	dM1
	$x = 75.5^\circ, 284.5^\circ$	A1
		(4)
		(8 marks)
Notes		
(a) The most common method and where to award marks is as follows:		
M1:	Uses $\operatorname{cosec} 2\theta = \frac{1}{\sin 2\theta}$ (oe) at some stage in the proof to convert the cosec into sine.	
M1	Attempts to use one of $\cos 2\theta = \pm 1 \pm 2\sin^2 \theta$ or $\sin 2\theta = 2\sin \theta \cos \theta$ but condone $\sin^2 2\theta = 2\sin^2 \theta \cos^2 \theta$ as an attempt at squaring.	
dM1:	Depends on previous M. Attempts to use both $\cos 2\theta = 1 - 2\sin^2 \theta$ oe and $\sin 2\theta = 2\sin \theta \cos \theta$	
A1*:	Achieves the RHS with no mathematical errors seen and all stages of working shown. Must see the $\sec^2 \theta$ before the final answer. Condone minor notational errors (e.g. a missing θ) but not consistent poor notation throughout. There will be other solutions but if they work from both sides then they need a conclusion at the end.	
For other methods or variations apply scheme as follows:		
M1:	Uses a correct identity for either $\operatorname{cosec} 2\theta$ or $\operatorname{cosec}^2 2\theta$ or $\cot^2 2\theta$ to get the equation in terms of sin and cosine only or to introduce $\operatorname{cosec} 2\theta$ if working in reverse. Condone the 2 becoming $\frac{1}{2}$.	
M1:	Attempts to apply a double angle identity correct up to sign errors, e.g. $\cos 2\theta = \pm 1 \pm 2\sin^2 \theta$	
dM1:	Depends on previous M. Use of at least two correct double angle identities and/or Pythagorean identities to <i>progress sufficiently towards the goal</i> (e.g. reduces all terms to single argument form in cosine or sine only, or identify an appropriate factor to cancel to such, or to get all double arguments if working in reverse with the $\sin 2\theta$ identified). There may be algebraic slips, but all trig identities used up to this point must be correct (e.g. see stag reach in main scheme).	
A1*:	Fully correct proof with sufficient stages shown including use of $\sec^2 \theta = 1 + \tan^2 \theta$ or equivalent identity (e.g. other Pythagorean relation), and if necessary, a conclusion given.	
For example $1 + \tan^2 \theta = \sec^2 \theta = \frac{1}{\cos^2 \theta} = \frac{2}{1 + \cos 2\theta}$ 2 nd M1, use of a suitable double angle identity		
$= \frac{2(1 - \cos 2\theta)}{(1 + \cos 2\theta)(1 - \cos 2\theta)} = \frac{2(1 - \cos 2\theta)}{1 - \cos^2 2\theta} = \frac{2(1 - \cos 2\theta)}{\sin^2 2\theta}$ 3 rd dM1, correct initial double angles		
identity and Pythagorean identity used to identify the sin in the denominator, all identities correct		
$= 2\operatorname{cosec}^2 2\theta(1 - \cos 2\theta)$ 1 st M1, use of correct identity for $\operatorname{cosec}^2 2\theta$. (A1 if all correct)		

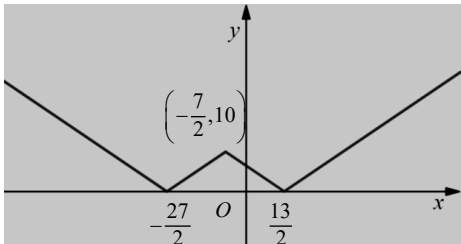
(b) Note: allow with x or θ and allow recovery of mixed variables in this part.		
M1: Uses part (a) to forms a 3 term quadratic in $\sec x$ or $\cos x$, solves their quadratic and finds $\cos x = \dots$ or $\sec x = \dots$ for at least one solution. For reference the quadratic in $\cos$ is $4\cos^2 x + 3\cos x - 1 = 0$		
A1: $\cos x = \frac{1}{4}$ (ignore any reference to $\cos x = -1$) Must have a correct cosine or implied by a correct answer after reaching a secant if no incorrect working seen. May be implied if not seen explicitly.		
dM1: Correct method to find a value for x from $\cos x = k$ where $ k < 1$ as a solution for their equation. May be implied by one correct angle from $\sec x = k$ or $\cos x = k$. Note radian answer awrt 1.3 implies dM1.		
A1: awrt 75.5° and awrt 284.5° Allow with or without 180° included, but there must be no other angles in given interval. All previous marks must have been scored. Ignore answers outside the domain.		
(b)Alt	$1 + \tan^2 x = 4 + 3\sec x \Rightarrow 9\sec^2 x = \tan^4 x - 6\tan^2 x + 9 \Rightarrow \tan^4 x - 15\tan^2 x = 0$ $\Rightarrow \tan^2 x = \dots (\neq 0)$	M1
	$\tan x = (\pm)\sqrt{15}$	A1
	$\tan x = k \neq 0 \Rightarrow x = \dots$	dM1
	$x = 75.5^\circ, 284.5^\circ$	A1
		(4)
Notes		
(b) Alt		
M1: Makes $\sec x$ the subject, squares and solves the resulting quadratic in $\tan^2 x$ leading to a value for $\tan^2 x$. Note there may be variations on this approach, such as $1 + \tan^2 x = 4 + 3\sqrt{1 + \tan^2 x}$ being used before squaring, or solving for “ $1 + \tan^2 x$ ”. Score for a correct approach leading to a value for $\tan^2 x$.		
A1: Correct value for $\tan x$. Need not give both values.		
dM1: Solves from their $\tan x = k, k \neq 0$ to find a value for x		
A1: Both correct values obtained and no invalid solutions in the range (ignore 180° as main scheme). Must reject extra solutions.		
Note: other methods or variations may be seen, which can be marked according to the same principles, first M for correct approach to find a value for a trig ratio, A1 correct non-trivial value, dM1 solves for x .		

Question Number	Scheme	Marks
9(a)	$k = -1$	B1
		(1)
(b)(i)	$f(0) = 2 - 4\ln(0+1) = 2 - 0 = 2$	B1
(ii)	$0 = 2 - 4\ln(x+1) \Rightarrow \ln(x+1) = \frac{1}{2} \Rightarrow x = e^p + q$	M1
	$x = e^{\frac{1}{2}} - 1$	A1
		(3)
(c)	$2 - 4\ln(x+1) = 3 \Rightarrow \ln(x+1) = \dots$ or $-2 + 4\ln(x+1) = 3 \Rightarrow \ln(x+1) = \dots$	M1
	$2 - 4\ln(x+1) = 3 \Rightarrow x = \dots$ and $-2 + 4\ln(x+1) = 3 \Rightarrow x = \dots$	dM1
	CVs $e^{-\frac{1}{4}} - 1, e^{\frac{5}{4}} - 1$	A1
	"-1" $< x < e^{-\frac{1}{4}} - 1$ or $x > e^{\frac{5}{4}} - 1$	ddM1A1ft
		(5)
		(9 marks)

Notes**(a)**B1: For $(k =) -1$. Accept $x = -1$ or even $x > -1$ **(b) (i)**B1: For 2 seen as the y coordinate. May be identified as y or $f(0)$ or $(0, 2)$.**(ii)**M1: Sets $0 = 2 - 4\ln(x+1)$, rearranges and proceeds to $x = \dots$ of the correct form $e^p + q$, $q \neq 0$. May be implied by awrt 0.65.A1: $x = e^{\frac{1}{2}} - 1$ or accept $\sqrt{e} - 1$ or even $e^{\frac{2}{4}} - 1$ May be seen as part of a coordinate pair.**(c) Note this is M1A1A1M1A1 on open but is being marked as M1M1A1M1A1.**M1: Forms one valid equation with the modulus signs removed and attempts to solve at least as far as $\ln(x+1)$. Accept with $>$ or $<$ etc instead of $=$ for the M.dM1: Both equations attempted, allowing for " $=$ " or any inequality used, to achieve values (may be decimals) for each via $\ln(x+1) = \dots$ and suitable attempt to undo $\ln$ (raises to a base power, allow an error with the base). For reference the decimals are -0.221 and 2.49 to 3 s.f.A1: Both $e^{-\frac{1}{4}} - 1, e^{\frac{5}{4}} - 1$ Must be exact.ddM1: Chooses the outside region for their values. Allow with strict or non-strict inequalities. It is dependent on both previous method marks and requires two distinct critical values. The left hand bound -1 may be missing for this mark (i.e. allow for $x < e^{-\frac{1}{4}} - 1$ or $x > e^{\frac{5}{4}} - 1$) Allow with decimal values for this mark. ddM0 if the middle section is also included but isw if they choose the correct outside region but later reject the left hand portion due to confusion with $x > -1$ Watch out for $x > \dots$ for both inequalities, which is ddM0.A1ft: "-1" $< x < e^{-\frac{1}{4}} - 1$ or $x > e^{\frac{5}{4}} - 1$ or any equivalent form following through on their **negative** k from (a). Must be exact (not decimals). Allow "and" instead of "or" – ie both regions are identified as the solution. But use of $\cap$ in set notation is A0 as it is incorrect.

B1: Fully correct. Accept $5 < x < 22$, $(5, 22)$. Allow with y or range but $5 < x < 22$ would be B1

B0

Question Number	Scheme	Marks
7 (a)	$\left(-\frac{7}{2}, -10\right)$	B1 B1 (2)
(b)	Attempts to solve either $\frac{1}{2}(2x+7)-10 \dots \frac{1}{3}x+1$ Or $-\frac{1}{2}(2x+7)-10 \dots \frac{1}{3}x+1$ Both correct critical values $x \dots -\frac{87}{8}, \frac{45}{4}$ Selects outside region for their critical values Correct solution $x \leq -\frac{87}{8}, x \geq \frac{45}{4}$	M1 A1 dM1 A1 (4)
(c)		Shape B1 Maximum B1 ft One correct minimum B1 Fully correct B1 (4) (10 marks)

(a) Must be answered in (a). We cannot imply from work in (c)

B1: One correct coordinate. Allow as $x = -\frac{7}{2}, y = -10$. Allow exact equivalents, e.g. $x = -\frac{14}{4}$

B1: Both coordinates correct. Allow as $x = -\frac{7}{2}, y = -10$. Allow exact equivalents, e.g. $x = -\frac{14}{4}$

(b)

M1: Attempts to solve a correct equation or inequality. They must proceed as far as $x \dots$

Condone slips where they attempt to change the subject in an attempt to make $|2x+7|$ the subject.

So either of $\frac{1}{2}|2x+7|-10 \dots \frac{1}{3}x+1 \Rightarrow |2x+7| = ax+b \Rightarrow 2x+7 = ax+b$

or $\frac{1}{2}|2x+7|-10 \dots \frac{1}{3}x+1 \Rightarrow |2x+7| = ax+b \Rightarrow -2x-7 = ax+b$ are condoned for

the M1

A1: Correct critical values $x \dots -\frac{87}{8}, \frac{45}{4}$ which may be part of an incorrect inequality

dM1: Selects outside region for their critical values. Allow these to be written as one inequality. Allow the strict inequalities here. It is dependent upon having attempted to solve **one** correct equation

Question Number	Scheme	Marks
10(a)(i)	$(y=)10+k$	B1
(ii)	$x = \frac{10}{k}$ or $y = k$	B1
	$x = \frac{10}{k}$ and $y = k$	B1
		(3)
Remember to look for answers written within the question and on Figure 3. If there is any contradiction, answers in the body of the script take precedence. (a)(i) B1: Correct expression. The “y =” is not required and condone $x = 10 + k$ or $A = 10 + k$ Allow e.g. $(0, 10+k)$ and condone $(10+k, 0)$ Do not allow $(y=) -10 +k$ (a)(ii) B1: One correct coordinate. May be seen as $x = \dots$ or $y = \dots$ or in coordinate form e.g. $\left(\frac{10}{k}, \dots\right)$ or $(\dots, k)$ Condone missing brackets e.g. $\frac{10}{k}, \dots$ or $\dots, k$ Do not condone e.g. $P = k$ unless k is clearly identified as the y coordinate. Do not condone coordinates the wrong way round unless they are correctly assigned first and then apply isw. B1: Both correct. May be seen as $x = \dots$ and $y = \dots$ or as e.g. $\left(\frac{10}{k}, k\right)$ and no other points. Condone missing brackets e.g. $\frac{10}{k}, k$ Do not condone coordinates the wrong way round unless they are correctly assigned first and then apply isw		

(b)	$kx - 10 + k = 2k \Rightarrow x = \dots$ or $-kx + 10 + k = 2k \Rightarrow x = \dots$	M1
	$kx - 10 + k = 2k \Rightarrow x = \dots$ and $-kx + 10 + k = 2k \Rightarrow x = \dots$	dM1
	$x \geq \frac{10-k}{k}$ or $x \leq \frac{10+k}{k}$ oe	A1
		(3)
	(b) Alternative by squaring.	
	$ kx - 10 + k = 2k \Rightarrow kx - 10 = k \Rightarrow k^2x^2 - 20kx + 100 = k^2$ $\Rightarrow k^2x^2 - 20kx + 100 - k^2 = 0$	M1
	$k^2x^2 - 20kx + 100 - k^2 = 0 \Rightarrow x = \frac{20k \pm \sqrt{400k^2 - 4k^2(100 - k^2)}}{2k^2}$ $= \frac{10 \pm k}{k}$	dM1
	$x \geq \frac{10-k}{k}$ or $x \leq \frac{10+k}{k}$ oe	A1

(b)

M1: Attempts to solve one of these equations to find one of the limits in terms of k .Condone use of any inequality for either equation e.g. $kx - 10 + k \dots 2k$ where $\dots$ is any inequality or “=”

May be implied by one correct limit.

dM1: Attempts to solve both of these equations to find both limits.Condone use of any inequality for either equation e.g. $kx - 10 + k \dots 2k$ where $\dots$ is any inequality or “=”.

May be implied by both correct limits.

A1: Correct range. Allow equivalent notation e.g. the inequalities written separately or e.g. $x \geq \frac{10-k}{k}, x \leq \frac{10+k}{k}$ or $\left(-\infty, \frac{10-k}{k}\right] \cup \left[\frac{10+k}{k}, \infty\right)$ Condone $x \geq \frac{10-k}{k}$ and $x \leq \frac{10+k}{k}$ but **not** $x \geq \frac{10-k}{k} \cap x \leq \frac{10+k}{k}$ Do not allow e.g. $\frac{10+k}{k} \geq x \geq \frac{10-k}{k}$ unless e.g. $x \geq \frac{10-k}{k}, x \leq \frac{10+k}{k}$ is seen first then apply isw.

Allow equivalent correct expressions for the limits

e.g. $\frac{10}{k} - 1, -\frac{k-10}{k}, \frac{k-10}{-k}$ for $\frac{10-k}{k}$ and $\frac{10}{k} + 1$ for $\frac{10+k}{k}$ etc.**Alternative by squaring:****M1:** Isolates the $|kx - 10|$, squares both sides, obtaining at least 3 terms on the lhs and collects terms to obtain a 3TQ = 0**dM1:** Attempts to solve their 3TQ using a correct method to find both limits.**A1:** Correct range as above.

(c)	$k > 3$	B1
	Maximum value of k occurs when $y = 3x + 1$ passes through vertex e.g. when $"k" = 3 \times \frac{10}{k} + 1 \Rightarrow k = \dots$ See below for alternative approaches.	M1
	$k^2 - k - 30 = 0 \Rightarrow (k + 5)(k - 6) = 0 \Rightarrow k = 6$ So maximum k is 6	A1
	$3 < k < 6$	A1
		(4)
		Total 10
B1: For the correct minimum value for k. Condone $k \dots 3$ or "minimum value is 3". This may be seen embedded in an inequality but it must be identified as the minimum value. M1: Correct strategy for the maximum value of k. E.g. sets $f(x) = 3x + 1$, substitutes their $x = \frac{10}{k}$ and $y = k$ or their y from (a)(ii) and attempts to solve for k. An equivalent approach would be to substitute their $x = \frac{10}{k}$ and $y = k$ or their y from (a)(ii) into $y = 3x + 1$ and attempt to solve for k. For this mark the value obtained may not be the maximum value for their range. See below for alternative methods for finding the maximum value of k. A1: For the correct maximum value for k Condone $k \dots 6$ This may be seen embedded in an inequality but it must be the maximum value. A1: Correct range in terms of k. Allow equivalent notation e.g. $k > 3$ and $k < 6$ or $(3, 6)$ but not $k > 3$ or $k < 6$ or $k > 3, k < 6$		

Alternative methods for finding the maximum value of k :

$$kx - 10 + k = 3x + 1, \quad -kx + 10 + k = 3x + 1$$

$$\Rightarrow x = \frac{11-k}{k-3}, \quad x = \frac{9+k}{3+k} \Rightarrow \frac{11-k}{k-3} = \frac{9+k}{3+k}$$

$$\Rightarrow 2k^2 - 2k - 60 = 0 \Rightarrow k = -5, 6$$

or

$$kx - 10 + k = 3x + 1, \quad -kx + 10 + k = 3x + 1$$

$$\Rightarrow k = \frac{3x+11}{x+1}, \quad k = \frac{9-3x}{x-1} \Rightarrow \frac{3x+11}{x+1} = \frac{9-3x}{x-1}$$

$$\Rightarrow 3x^2 + x - 10 = 0 \Rightarrow x = \frac{5}{3}, -2$$

$$x = \frac{5}{3} \Rightarrow k = \frac{3\left(\frac{5}{3}\right) + 11}{\frac{5}{3} + 1} = 6 \quad \text{or} \quad \frac{9 - 3\left(\frac{5}{3}\right)}{\frac{5}{3} - 1} = 6 \quad \text{or} \quad 3\left(\frac{5}{3}\right) + 1 = 6$$

or

$$kx - 10 + k = 3x + 1 \Rightarrow x = \frac{11-k}{k-3}$$

$$\frac{10}{k} = \frac{11-k}{k-3} \Rightarrow k^2 - k - 30 = 0 \Rightarrow k = -5, 6$$

or

$$-kx + 10 + k = 3x + 1 \Rightarrow k = \frac{9-3x}{x-1}$$

$$k = \frac{9 - 3\left(\frac{10}{k}\right)}{\left(\frac{10}{k}\right) - 1} \Rightarrow k^2 - k - 30 = 0 \Rightarrow k = -5, 6$$

**These are some examples – there will be other correct methods.
If you are unsure if a particular approach deserves credit, use review.**

In general allow M1 for a complete overall method condoning slips.
A1 can then be scored when $k = 6$ is seen as the **maximum** value.



Question	Scheme	Marks
5(a)	$P(1) = \frac{4-1}{10} + \frac{3}{4} \ln\left(\frac{2}{3^2}\right) \quad (= -0.828\dots)$	M1
	$P(1) = -0.828\dots$ (which is negative so a loss of £0.828 million,) so approximately £830 000 loss.	A1*
		(2)
(b)	$P(6) = -0.08799\dots$ and $P(7) = 0.1975\dots$	M1
	There is a sign change and hence as P is continuous on $[6,7]$, so the root for t lies in $[6,7]$.	A1
		(2)
(c)	$P = 0 \Rightarrow \frac{4t-1}{10} = -\frac{3}{4} \ln\left(\frac{t+1}{(2t+1)^2}\right) \Rightarrow t = \dots$	M1
	$\Rightarrow t = \frac{10\left(-\frac{3}{4} \ln\left(\frac{t+1}{(2t+1)^2}\right)\right) + 1}{4} = \frac{1}{4} + \frac{30}{16} \ln\left(\frac{t+1}{(2t+1)^2}\right)^{-1}$ $\Rightarrow t = \frac{1}{4} + \frac{15}{8} \ln\left(\frac{(2t+1)^2}{t+1}\right)^*$	A1*
		(2)
(d)	$t_2 = \frac{1}{4} + \frac{15}{8} \ln\left(\frac{13^2}{7}\right) = \dots (= 6.219978\dots)$	M1
	$t_2 = \text{awrt } 6.220$	A1
	$t_6 = 6.314$	A1
		(3)
(e)	"6.3..." $\times 12 = \dots$ months, or repeated iteration to root 6.31487 gives 75.7785... or allow "0.314..." $\times 12 = "3.768\dots"$	M1
	So it will take 76 months. (Accept 75 or awrt 76 months)	A1
		(2)

(11 marks)**Notes:****(a)**

M1: Attempts to substitute $t = 1$ into the given formula. Allow if there is a slip but an attempt at substitution is seen, or allow for sight of awrt $-0.828\dots$

A1*: Correct value for $P(1)$ seen to at least 2 s.f. if substitution has been shown, or at least 3 s.f. if no substitution was shown, followed by suitable conclusion that it is a loss of approximately £830 000, though accept awrt £830 000. Must mention "loss" and include units (£ or pounds). Negative value given is A0.

(b)

M1: Attempts both $P(6)$ and $P(7)$ with at least one correct to 1 s.f. rounded or truncated. A tighter interval could be used but must contain the root (6.31487)



A1: Both correct to 1 s.f. rounded or truncated with suitable conclusion made. Must mention sign change and continuity as well as conclusion about root in the interval.

(c)

M1: Attempts to isolate at from the $\frac{4t-1}{10}$ after setting equal to zero.

A1*: Correct work to reach the given answer with no incorrect work seen and at least one intermediate step with either $at = \dots$ reached or with the power law applied on the $\ln$ term (need not see the power explicitly used, but must have been applied correctly).

(d)

M1: Attempts to use the formula with $t_1 = 6$. Accept with 6 embedded in formula followed by a value, or awrt 6.2 (or even 6.3) as implying the attempt.

A1: awrt 6.220 Accept 6.22.

A1: Correct and given to 3 d.p.

(e)

M1: Multiplies their final root from (d) by 12, or uses repeated iteration to narrow further then multiplies by 12. Allow the method if they multiply the fractional part only by 12 and get, e.g., 6 years 3 months.

A1: Accept 75 or 76 months, or anything that rounds to 76 months.

Question	Scheme	Marks
6	$\frac{d}{dx}(\cos x + \sin x) = -\sin x + \cos x$	B1
	$\frac{dy}{dx} = \frac{(\cos x + \sin x)(3\cos x) - (-\sin x + \cos x)(2 + 3\sin x)}{(\cos x + \sin x)^2}$	M1 A1
	$= \frac{3\cos^2 x + \cancel{3\sin x \cos x} + 2\sin x + 3\sin^2 x - 2\cos x - \cancel{3\sin x \cos x}}{\cos^2 x + 2\sin x \cos x + \sin^2 x}$ $= \frac{3 + 2\sin x - 2\cos x}{1 + 2\sin x \cos x}$	M1
	$= \frac{3 + 2\sin x - 2\cos x}{1 + 2\sin x \cos x} \times \frac{\sec x}{\sec x} = \frac{3\sec x + 2\sec x \sin x - 2}{\sec x + 2\sin x}$	M1
	$= \frac{2\tan x + 3\sec x - 2}{\sec x + 2\sin x}$	A1
		(6)

(6 marks)**Notes:**

B1: Correct differentiation of $\cos x + \sin x \rightarrow -\sin x + \cos x$ seen somewhere in the proof. This can be scored if seen in workings for quotient rule, or even in the denominator of an incorrect attempt at u'/v' .

M1: Differentiates using the quotient rule or product rule. For quotient rule look for

$$\frac{(\cos x + \sin x)(\pm a \cos x) - (\pm \sin x \pm \cos x)(2 + 3\sin x)}{(\cos x + \sin x)^2}$$

For product rule look for $(\cos x + \sin x)^{-1}(\pm a \cos x) \pm (\cos x + \sin x)^{-2}(\pm \cos x \pm \sin x)(2 + 3\sin x)$

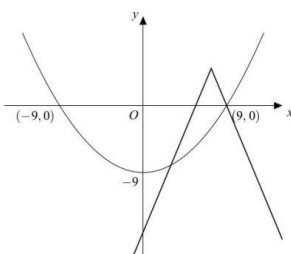
A1: Fully correct derivative.

M1: Expands numerator or denominator and applies $\sin^2 x + \cos^2 x = 1$ or other appropriate correct Pythagorean identity at least once in the proof.

M1: Attempts to multiply through by $\sec x$ in numerator and denominator (to achieve the $2\sin x$ term). Not dependent and may be scored before the previous M.

A1: Correct answer. Terms may be in different order. Allow minor slips in notation (e.g. a single missing x) and recovery from missing brackets if the intent is clear, but A0 for persistent incorrect notation throughout (e.g. no x 's in the trig terms).



Question	Scheme		Marks
7(a)	(i) $\left(\frac{22}{3}, 5\right)$		B1
	(ii) $(0, -17)$		B1
			(2)
(b)	$5 - (3x - 22) = 0 \Rightarrow x = \dots$ and $5 + (3x - 22) = 0 \Rightarrow x = \dots$		M1
	$x = 9$ and $x = \frac{17}{3}$		A1
			(2)
(c)		Correct U shape symmetric about y-axis with vertex on negative y-axis.	B1
		Graphs meet $(9, 0)$ with $(-9, 0)$ also shown.	B1
		Intercept at $(0, -9)$ stated or labelled.	B1
			(3)
(d)	Intersect at $(9, 0)$		B1
	Or when $5 + 3x - 22 = \frac{1}{9}x^2 - 9$		M1
	$\Rightarrow (x - 3)(x - 24) = 0 \Rightarrow x = \dots$		dM1
	Need smaller root $x = 3 \Rightarrow y = \dots$		dM1
	$(3, -8)$		A1
			(5)
			(12 marks)

Notes:**(a)****Mark (a) and (b) as a whole****(i) B1:** Correct coordinates. May be listed as separate coordinates, $x = \dots, y = \dots$ **(ii) B1:** Correct y intercept (and no other). Accept as coordinates, or stated as $y = \dots$ (allow without the $x = 0$) or seen on graph as the y intercept. However just -17 on its own is B0.**(b)****M1:** Attempts to solve **both** equations, though allow sign slips when expanding the brackets.**A1:** Both values correct.

(c)

B1: Correct U shape symmetric about y-axis with vertex on negative y-axis. Allow a little tolerance with shape, but the curve must not clearly bend back on itself, and the minimum should be clearly intended as on the y-axis.

B1: Graphs meet at (9,0) both x intercepts labelled or clearly stated (and no others). Shape need not be correct for this mark e.g. inverted parabola may be shown). Must pass through the same point as the modulus graph at (9,0).

B1: States or labels the y intercept at (0, -9) and intercept must be on the negative y-axis.

(d)

B1: Deduces one point of intersection is (9,0). Must be seen in (d), or clearly stated as the answer if parts are not labelled, do not accept just this marked on the diagram.

M1: Sets up equation for the intersection of the quadratic with the positive gradient line segment, accept $5 + 3x \pm 22 = \frac{1}{9}x^2 - 9$

dM1: Solves their equation, any valid means. After the equation is seen you may see just the correct root, or 24, which implies the M. Alternatively, expanding and solving the quadratic by usual rules.

dM1: Depends on first M mark. Selects the correct (smaller) root of their quadratic and attempts to find the y value (accept any y value appearing after choosing the correct root as an attempt). The larger root must be rejected. Following a correct equation, if no working is shown for finding the root (see above) they must have achieved $x = 3$ and go on to find y for this mark. From an incorrect equation/incorrect method to solve, if only one root is given it must correspond to the smaller root of their equation/method.

A1: (3, -8) only

Alternative by squaring.

B1: As main scheme.

M1: Rearranges to $|3x - 22| = a + bx^2 \left(= 14 - \frac{1}{9}x^2 \right)$ and squares both sides to reach a quartic equation. If

correct it is $\frac{1}{81}x^4 - \frac{109}{9}x^2 + 132x - 288 = 0$

dM1: Solves the resulting quartic (any means). If no method is shown, a correct value from the correct equation will imply the mark.

dM1: Depends on first M mark Selects correct root (second of the four in ascending order) and finds the y coordinate. If no method shown, but the quartic was correct, they must be using $x = 3$.

A1: As scheme.

