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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Exponentials & Logarithms (P3.2)

International A Level Mathematics · P3: Pure Mathematics 3 (WMA13)

35 questions · 263 marks · Mark schemes included

Past papers January 2020 - June 2025 · 3 sub-topics

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 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

35 questions · 263 marks · 3 sub-topics

SUB-TOPIC P3.2a

EXPONENTIAL & NATURAL LOG EQUATIONS

Sub-topic P3.2a · 1 question · 12 marks · lowest tariff first



8. The functions f and g are defined by

$$f(x) = \ln(2x - 3) \quad x \in \mathbb{R} \quad x > 2$$

$$g(x) = \frac{3x - 2a}{x - a} \quad x \in \mathbb{R} \quad x \neq a$$

where a is a positive constant.

(a) Find the exact value of $fg(2a)$

(2)

(b) Find f^{-1}

(3)

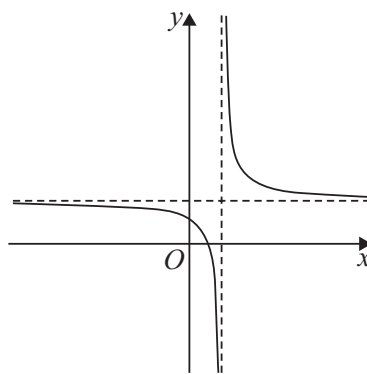


Figure 3

Figure 3 shows a sketch of the curve with equation $y = g(x)$

(c) Sketch a graph of the curve with equation $y = |g(x)|$

Indicate clearly on your sketch the equation of the **horizontal** asymptote.

(3)

The equation $|g(x)| = 10$ has two solutions.

Given that the larger solution is 8

(d) (i) find the value of a ,

(ii) find the value of the smaller solution.

(4)



Question 8 continued

Lined area for writing the answer to Question 8 continued.

Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 12 marks)

SUB-TOPIC P3.2b

EXPONENTIAL MODELLING

Sub-topic P3.2b · 24 questions · 186 marks · lowest tariff first



(Total for Question 2 is 9 marks)



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1. A population of a rare species of toad is being studied.

The number of toads, N , in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \quad t \geq 0, t \in \mathbb{R}$$

According to this model,

- calculate the number of toads in the population at the start of the study, (1)
- find the value of t when there are 420 toads in the population, giving your answer to 2 decimal places. (4)
- Explain why, according to this model, the number of toads in the population can never reach 500 (1)

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Q1

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Q2

Question 3 continued

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(Total 6 marks)

Q3



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- 7 The mass, M kg, of a species of tree can be modelled by the equation

$$\log_{10} M = 1.93 \log_{10} r + 0.684$$

where r cm is the base radius of the tree.

The base radius of a particular tree of this species is 45 cm.

According to the model,

- (a) find the mass of this tree, giving your answer to 2 significant figures.

(2)

- (b) Show that the equation of the model can be written in the form

$$M = pr^q$$

giving the values of the constants p and q to 3 significant figures.

(3)

- (c) With reference to the model, interpret the value of the constant p .

(1)

Question 7 continued

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Lined area for writing the answer to Question 7.

(Total 6 marks)

Q7





(Total for Question 6 is 6 marks)



4. The number of bacteria on a surface is being monitored.

The number of bacteria, N , on the surface, t hours after monitoring began is modelled by the equation

$$\log_{10} N = 0.35t + 2$$

Use the equation of the model to answer parts (a) to (c).

- (a) Find the initial number of bacteria on the surface.

(1)

- (b) Show that the equation of the model can be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of b to 2 decimal places.

(3)

- (c) Hence find the rate of growth of bacteria on the surface exactly 5 hours after monitoring began.

(2)

(Total for Question 4 is 6 marks)



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Q8

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4. The growth of a weed on the surface of a pond is being studied.

The surface area of the pond covered by the weed, $A \text{ m}^2$, is modelled by the equation

$$A = \frac{80pe^{0.15t}}{pe^{0.15t} + 4}$$

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Q4



(Total for Question 4 is / marks)



(Total for Question 3 is 7 marks)

(Total for Question 3 is / marks)



Question 9 continued

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Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 7 marks)

TOTAL FOR PAPER IS 75 MARKS



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Question 8 continued

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(Total 8 marks)

Q8

Answer box for Q8



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9. In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leq \pi$, the equation

$$2 \sec^2 x - 3 \tan x = 2$$

giving the answers, as appropriate, to 3 significant figures.

(4)

(ii) Prove that

$$\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \equiv 2$$

(4)

Question 9 continued

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Q9

Question 7 continued

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Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 8 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 8 marks)



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TOTAL FOR PAPER IS 75 MARKS





TOTAL FOR PAPER IS 75 MARKS



(Total for Question 3 is 7 marks)



8.

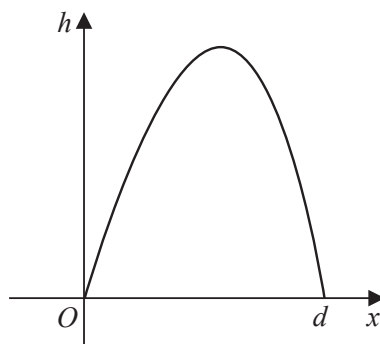


Figure 4

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, h metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball travels a horizontal distance of d metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5xe^{0.02x} \quad 0 \leq x \leq d$$

Use the model to answer parts (a), (b) and (c).

- (a) Find the value of d , giving your answer to 2 decimal places.

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

- (b) Show that the maximum value of h occurs when

$$x = 50 \ln \left(\frac{150}{x + 50} \right)$$

(4)

Using the iteration formula

$$x_{n+1} = 50 \ln \left(\frac{150}{x_n + 50} \right) \quad \text{with } x_1 = 30$$

- (c) (i) find the value of x_2 to 2 decimal places,
 (ii) find, by repeated iteration, the horizontal distance travelled by the golf ball before it reaches its maximum height. Give your answer to 2 decimal places.

(3)



Question 8 continued

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Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

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(Total for Question 8 is 10 marks)



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Q5

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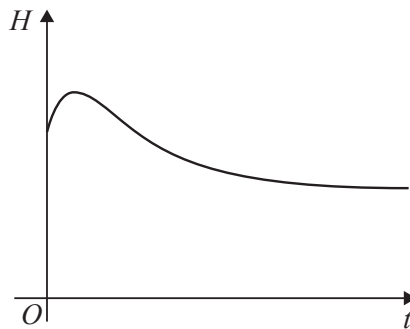


Figure 4

The heart rate of a horse is being monitored.

The heart rate H , measured in beats per minute (bpm), is modelled by the equation

$$H = 32 + 40e^{-0.2t} - 20e^{-0.9t}$$

where t minutes is the time after monitoring began.

Figure 4 is a sketch of H against t .

Use the equation of the model to answer parts (a) to (e).

(a) State the initial heart rate of the horse.

(1)

In the long term, the heart rate of the horse approaches L bpm.

(b) State the value of L .

(1)

The heart rate of the horse reaches its maximum value after T minutes.

(c) Find the value of T , giving your answer to 3 decimal places.

(Solutions based entirely on calculator technology are not acceptable.)

(5)

The heart rate of the horse is 37 bpm after M minutes.

(d) Show that M is a solution of the equation

$$t = 5 \ln \left(\frac{8}{1 + 4e^{-0.9t}} \right)$$

(2)

Using the iteration formula

$$t_{n+1} = 5 \ln \left(\frac{8}{1 + 4e^{-0.9t_n}} \right) \quad \text{with} \quad t_1 = 10$$

(e) (i) find, to 4 decimal places, the value of t_2

(ii) find, to 4 decimal places, the value of M

(3)







(Total for Question 6 is 12 marks)



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Q8

SUB-TOPIC P3.2c

LOGARITHMIC GRAPHS (LOG-LINEAR)

Sub-topic P3.2c · 10 questions · 65 marks · lowest tariff first



Question 3 continued

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(Total 5 marks)

Q3

Mark box for Q3



(Total for Question 3 is 3 marks)



(Total for Question 5 is 5 marks)



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 6 marks)



(Total for Question 5 is 6 marks)



(Total for Question 5 is 6 marks)



Question 5 continued

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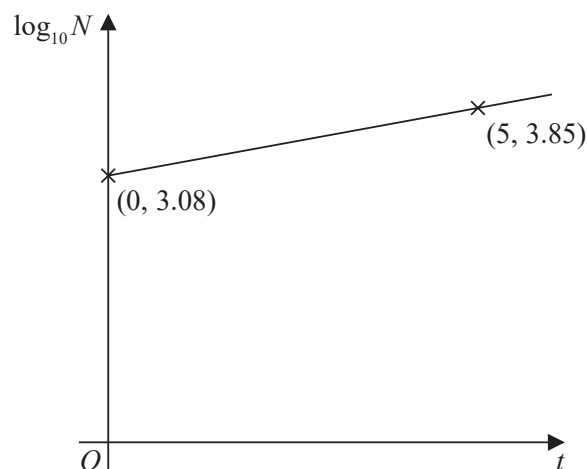


Figure 1

The number of subscribers to an online video streaming service, N , is modelled by the equation

$$N = ab^t$$

where a and b are constants and t is the number of years since monitoring began.

The line in Figure 1 shows the linear relationship between t and $\log_{10} N$

The line passes through the points $(0, 3.08)$ and $(5, 3.85)$

Using this information,

(a) find an equation for this line. (2)

(b) Find the value of a and the value of b , giving your answers to 3 significant figures. (3)

When $t = T$ the number of subscribers is 500 000

According to the model,

(c) find the value of T (2)

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Question 4 continued

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Question 4 continued

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(Total 7 marks)

Q4



9.

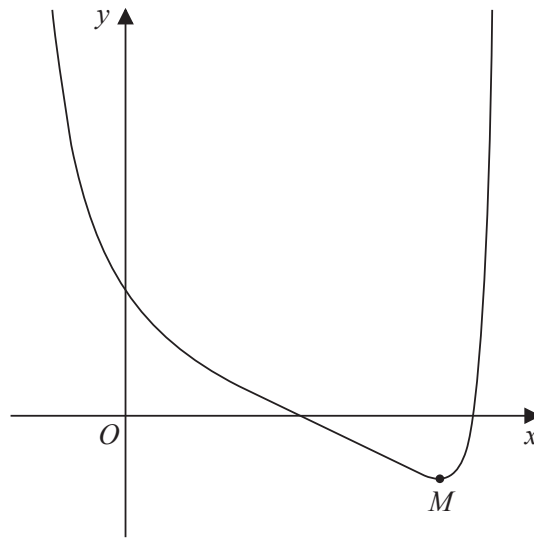
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Figure 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 5 shows the curve with equation

$$y = \frac{1 + 2 \cos x}{1 + \sin x} \quad -\frac{\pi}{2} < x < \frac{3\pi}{2}$$

The point M , shown in Figure 5, is the minimum point on the curve.

(a) Show that the x coordinate of M is a solution of the equation

$$2 \sin x + \cos x = -2 \quad (4)$$

(b) Hence find, to 3 significant figures, the x coordinate of M . (5)

Question 9 continued

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Question 9 continued

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Q9

Mark box for Q9

(Total 9 marks)

END

TOTAL FOR PAPER: 75 MARKS





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(Total for Question 3 is 9 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



And the dM1 for using the double angle formulae $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\cos 2\theta = \pm 1 \pm 2 \sin^2 \theta$ to obtain $\sin 4\theta$ in terms of just $\sin \theta$ and $\cos \theta$

Do not allow candidates just to write out the expansions for $\sin 3\theta$ and $\cos 3\theta$ without their proof

.....
(b)

B1: States or uses $\sec x = \frac{1}{\cos x}$ in the given equation.

It can be implied. So, writing $\sec x \sin 4x$ as $\sin x (4 - 8 \sin^2 x)$ would be B1

M1: Requires

- $\sin 4x$ to be written as $\sin x \cos x ('4' - '8' \sin^2 x)$ using their values for P , Q and n
- a term in $\sin x$ to be cancelled or factorised out
- an attempt at dividing all terms by $\cos^2 x$ to reach an equation involving at least $\sec^2 x$ and $\tan^2 x$ (you don't need to see $\div \cos^2 x$ stated, it can be implied by their terms)

The last two bullets can be done in just one line and implied by all terms $\div (\sin x \cos^2 x)$

A1*: Reaches the given equation showing sufficient and necessary lines of work.

Withhold this mark for mixed variables and/or incorrect notation within the proof.

(c)

M1: Uses $\sec^2 x = \pm 1 \pm \tan^2 x$ to set up a 3TQ in $\tan x$

A1: Correct 3TQ, $4 \tan^2 x + 5 \tan x - 4 = 0$.

The terms do not need to be on one side and the $= 0$ can be implied by further work

dM1: Solves the 3TQ using an appropriate method and proceeds to a value for x via value(s) for $\tan x$

Expect to see " $4 \tan^2 x + 5 \tan x - 4 = 0 \Rightarrow \tan x = \dots \Rightarrow x = \dots$ "

Allow calculator methods of solving the quadratic in $\tan x$ to produce a value for $\tan x$

A1: $x = \text{awrt } 0.506, 2.08$ following M1, A1, dM1.

Ignore any extra solutions outside the range $(0, \pi)$.

Withhold this mark if there are extra solutions in the range.

Note that the demand of the question is that candidates show their working.

So, from the given $4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0$

candidates writing $\Rightarrow 4 \tan^2 x + 5 \tan x - 4 = 0$ followed by $x = \text{awrt } 0.506, 2.08$ scores just the first M1,

A1

There will be alternatives seen in part (c).

Alt (c)

$$4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0$$

$$\times \cos^2 x \Rightarrow 4 - 5 \sin x \cos x - 8 \sin^2 x = 0$$

$$\Rightarrow 4 - 8 \sin^2 x - 5 \sin x \cos x = 0$$

$$\Rightarrow 4 \cos 2x - 2.5 \sin 2x = 0$$

$$\Rightarrow \tan 2x = \frac{8}{5}$$

$$\Rightarrow 2x = 1.0121, 4.1538$$

$$\Rightarrow x = \text{awrt } 0.506, 2.08$$

M1, A1

dM1, A1

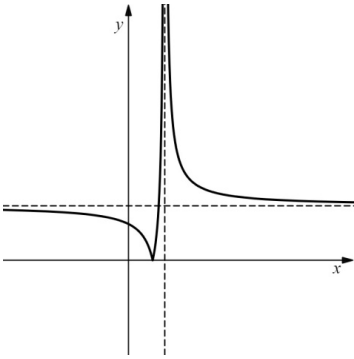
M1: Equation in a single trig ratio

A1: Correct equation

dM1: Full attempt to solve

A1: Correct solutions



Question Number	Scheme	Marks
8. (a)	$g(2a) = \frac{3 \times 2a - 2a}{2a - a} = 4, \quad f(4) = \ln(2 \times 4 - 3) = \ln 5$	M1, A1 (2)
(b)	$y = \ln(2x - 3) \Rightarrow e^y = 2x - 3 \Rightarrow x = \frac{e^y + 3}{2}$ $f^{-1}(x) = \frac{e^x + 3}{2} \quad x > 0$	M1 A1, B1 (3)
(c)	 Shape of curve to right of $x = a$ Shape of curve to left of $x = a$ $y = 3$ marked at correct place on curve	B1 B1 B1 (3)
(d) (i)	$\frac{3 \times 8 - 2a}{8 - a} = 10 \Rightarrow a = \dots$ $\Rightarrow a = 7$	M1 A1
(ii)	$\frac{3x - 2 \times 7}{x - 7} = -10 \Rightarrow x = \dots$ $\Rightarrow x = \frac{84}{13}$	dM1 A1 (4)
		(12 marks)

(a)

 M1: Complete method to find $fg(2a)$. E.g. Finds $g(2a)$ and substitutes into f .

 Some may find $fg(x) = \ln\left(2 \times \frac{3x - 2a}{x - a} - 3\right)$ and substitute $x = 2a$ into this. Condone slips in signs

 A1: $\ln 5$. ISW after a correct answer. Allow this to be written down for both marks.


(b)

M1: Complete method to find $f^{-1}(x)$. Score for the correct order of operations so award this mark forwork proceeding to $x = \frac{e^y \pm 3}{2}$ or $y = \frac{e^x \pm 3}{2}$ o.e if they start with $x = \ln(2y - 3)$ A1: $f^{-1}(x) = \frac{e^x + 3}{2}$ o.e. BUT condone/allow $f^{-1} = \frac{e^x}{2} + \frac{3}{2}$ and $y = \frac{1}{2}(e^x + 3)$ B1: States a domain of $x > 0$. Do not accept $x > \ln 1$ (c) **Do not accept the curve drawn on Figure 3 unless you can clearly see all parts of the curve.****Allow the B1 for the equation of the horizontal asymptote if correct**

B1: Shape of curve to the right of the vertical asymptote (provided that the whole of the original curve is not copied out). Be tolerant of slips of the pen

B1: Shape of curve to the left of the vertical asymptote. It must be a cusp as opposed to a minimum

turning point. Again, be tolerant of slips of the pen and a linear looking section from $x = \frac{2}{3}a$ to $x = a$ B1: For the horizontal asymptote marked as $y = 3$ and not just 3(d)(i) **The following two marks are for a correct method and answer of finding the value of the constant a** **If both equations are used then a correct choice must be made for both marks to be awarded**M1: Sets $\frac{3x-2a}{x-a} = 10$ o.e., substitutes $x = 8$ and solves for a . Note that $3 + \frac{a}{x-a} = 10$ is the same equation

Don't be too worried about the method of solving this equation but the initial equation must be correct

A1: $a = 7$ only(d)(ii) **Marks can only be scored in (ii) only following the award of M1 in (i)**dM1: Sets $\frac{3x-2a}{x-a} = -10$ o.e. and substitutes their a from (d)(i) and solves for x

Don't be too worried about the method of solving this equation

Their a must have been found from solving an equation equivalent to $\frac{3x-2a}{x-a} = 10$ with $x = 8$ A1: $x = \frac{84}{13}$ only (exact answer but ISW after a correct answer).If both solutions are found, $x = 8$ and $x = \frac{84}{13}$, the $\frac{84}{13}$ must be selected and identified as the only solution.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

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$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

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2. Formula

Attempt to use correct formula (with values for a , b and c).

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Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

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Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



- A1 $f^{-1}(x) = \frac{3x}{5-x}$ or $f^{-1}(x) = \frac{-3x}{x-5}$ Must be in terms of x . Condone $f^{-1} = \dots$ (or $f^{-1} = y = \dots$) but **do not allow** just $y = \dots$ or $f^{-1}: y = \dots$
- A1 Correct domain $0 < x < 5$

(c) Mark (i) and (ii) together

- M1 Attempts to use the quotient rule or product rule. Look for an expression of the form $\frac{A(x+3) - 5x}{(x+3)^2}$ or $5x(x+3)^{-1} \rightarrow B(x+3)^{-1} \pm 5x(x+3)^{-2}$ where A and B are non-zero constants.

- A1 $(f'(x) =) \frac{15}{(x+3)^2}$ or $15(x+3)^{-2}$ or $\frac{15}{x^2 + 6x + 9}$ Do not allow $\frac{5}{x+3} - \frac{5x}{(x+3)^2}$ for this mark.

- A1 They achieve $(f'(x) =) \frac{15}{(x+3)^2}$ and states

- (that f is an) **increasing function**
- **since $(x+3)^2$ is positive**

Alt methods in (a)

Taking out common factors

$$M1A1: \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x+4} = 5x \left\{ \frac{1+(x+3)}{(x+3)(x+4)} \right\} \text{ or } = \frac{5x}{(x+4)} \left\{ \frac{1+(x+3)}{(x+3)} \right\}$$

Methods where candidates "split up" fractions.

- M1 Attempts $\frac{5x}{x^2 + 7x + 12}$ OR $\frac{5}{x^2 + 7x + 12}$ by partial fractions

A1 $\frac{5x}{x+3} - \frac{5x}{x+4} + \frac{5x}{x+4}$

- A1* $\frac{5x}{x+3}$ You would expect to see $\frac{5x}{x+3} - \frac{5x}{x+4} + \frac{5x}{x+4}$ before proceeding to the given answer.

Alt method in (b)

- M1 Writes $y = \frac{5x}{x+3}$ as $y = 5 \pm \frac{k}{x+3}$ and then attempts to make x the subject.

A1 $f^{-1}(x) = \frac{15}{5-x} - 3$ or equivalent

A1 $0 < x < 5$

Alt method in (c)

- M1 Writes $y = \frac{5x}{x+3}$ as $y = 5 \pm \frac{k}{x+3}$ and attempts the chain rule to get $\pm \frac{C}{(x+3)^2}$

A1 $(f'(x) =) \frac{15}{(x+3)^2}$ or exact equivalent



A1 States that since $(x+3)^2$ is positive so f is increasing function



Question Number	Scheme	Marks
2 (a)	$\left(\frac{13}{3}, 5\right)$	B1 B1
		(2)
(b)(i)	$f(x) \dots 5$	B1 ft
(ii)	10	B1
		(2)
(c)	Attempts to solve either $16 - 2x \dots 3x - 13 + 5 \Rightarrow$ a value or inequality in x Or $16 - 2x \dots - 3x + 13 + 5 \Rightarrow$ a value or inequality in x	M1
	Both correct critical values $2, \frac{24}{5}$	A1
	Selects inside region for their critical values $2 < x < \frac{24}{5}$	dM1 A1
		(4)
(d)	$a = 4, b = \frac{1}{3}$	B1 ft B1
		(2)
		(10 marks)
Alt(c)	$(11 - 2x)^2 \dots (3x - 13)^2 \Rightarrow 121 - 44x + 4x^2 \dots 9x^2 - 78x + 169$ $5x^2 - 34x + 48 \dots 0 \Rightarrow$ a value or inequality in x $2, \frac{24}{5}$ Selects inside region for their critical values $2 < x < \frac{24}{5}$	M1 A1 dM1 A1

Note – Check the diagram and next to the questions for answers

(a)

B1 One correct coordinate. Allow as $x = \frac{13}{3}, y = 5$ or exact equivalent and condone missing brackets.

B1 Both coordinates correct. Allow as $x = \frac{13}{3}, y = 5$ or exact equivalent and condone missing brackets.

(b)(i)

B1ft $f(x) \dots 5$. Allow equivalent correct answers. E.g. $y \dots 5$, $y \in [5, \infty)$, $f \dots 5$ or using set notation. Follow through on their y coordinate in (a). Do not allow “range $\dots 5$ ”



(b)(ii)

B1 10

(c)

M1 Attempts to solve a correct equation or inequality. Look for the equation or inequality (ignore the direction) after the modulus signs have been removed allowing slips in their rearrangement.

A1 Correct critical values for $x \dots 2, \frac{24}{5}$ or exact equivalent which may be part of an incorrect inequality

dM1 Selects inside region for their critical values or both correct inequalities seen for their values.
Allow this to be written separately and allow the critical values to be included within their region.

A1 $2 < x < \frac{24}{5}$ Allow other exact equivalent forms such as $2 < x < \frac{48}{10}$ or $x \in \left(2, \frac{24}{5}\right)$ or $\frac{24}{5} > x > 2$

The inequalities must be presented together on one line for this mark.

Accept two separate statements such as " $x > 2$ AND $x < \frac{24}{5}$ " but not "

$x > 2$ or $x < \frac{24}{5}$ " or " $x > 2, x < \frac{24}{5}$ "

(d)

B1ft One correct value. Either $a = 4$ or $b = \frac{1}{3}$ but ft on their part (a) so accept

$a = \frac{20}{5}$ or $b = \frac{13}{3} - 4$

B1 $a = 4, b = \frac{1}{3}$ which may be embedded within $y = af(x+b)$

Question Number	Scheme	Marks
3.(a) (i)	$0 = 40 - 30e^{1-0.05t} \Rightarrow 1 - 0.05t = \ln \frac{40}{30}$	M1
	$\Rightarrow (k =) \text{ awrt } 14.2$	A1
	March 1814	A1
(ii)		(3)
(b)	Attempts $G = 40 - 30e^{1-0.05 \times 70} = 37.5$ tonnes	M1 A1
		(2)
(c)	40 (tonnes)	B1
		(1)
		(6 marks)

Mark (a)(i) and (ii) together

(a) (i)

M1 Sets $G = 0$ and proceeds to a linear equation in t or k using a correct method by taking logs of both sides, but condone slips in the rest of their rearrangement.

A1 $(k =) \text{ awrt } 14.2$ or 14.25 but allow the exact value $20 \left(1 - \ln \frac{4}{3} \right)$ or other

equivalent exact expressions such as $\frac{\ln \frac{4}{3} - 1}{-0.05}$ or $20 \left(1 - \ln \frac{4}{3} \right)$ or $20 \ln \left(\frac{3}{4} e \right)$. Isw

after a correct answer.

(a)(ii)

A1 March 1814 but allow April 1814 following "correct" k .
Condone "the third month of 1814" or "the fourth month of 1814" following "correct" k
Withhold this mark if they have two answers of which one is incorrect.

(b)

M1 Attempts to find G with $t = 70$ (condone $t = 69$) which may be implied by awrt 37.5
The numbers embedded in the equation is sufficient eg $G = 40 - 30e^{1-0.05 \times 70}$

A1 awrt 37.5 tonnes (requires units)

(c)

B1 40 (tonnes). Condone $G < 40$ or $G = 40$. Do not allow $G > 40$

Question Number	Scheme	Marks
4 (a)	$2 \sin(\theta - 30^\circ) = 5 \cos \theta \Rightarrow 2 \sin \theta \cos 30^\circ - 2 \cos \theta \sin 30^\circ = 5 \cos \theta$	M1
	$\div \cos \theta \Rightarrow 2 \tan \theta \cos 30^\circ - 2 \sin 30^\circ = 5$	dM1
	$\Rightarrow 2 \tan \theta \times \frac{\sqrt{3}}{2} - 2 \times \frac{1}{2} = 5$	A1
	$\Rightarrow \sqrt{3} \tan \theta = 6 \Rightarrow \tan \theta = 2\sqrt{3}^*$	A1*
		(4)
(b)	Attempts $\arctan 2\sqrt{3}$..and then subtracts 20°	M1
	$\Rightarrow x = \text{awrt } 53.9^\circ, 233.9^\circ$	A1, A1
		(3)
		(7 marks)

(a)

M1 Attempts to use $\sin(\theta - 30^\circ) = \sin \theta \cos(\pm 30^\circ) \pm \cos \theta \sin(\pm 30^\circ)$ within the given equation

Condone the omission of a 2 on the second term and a slip on the 5 of $5 \cos \theta$

dM1 Divides by $\cos \theta$ to set up an equation in just $\tan \theta$.

They may collect terms in $\sin \theta$ and $\cos \theta$ before dividing by $\cos \theta$ to set up an equation in just $\tan \theta$

An equation with $\cos 30^\circ$ and $\sin 30^\circ$ still not processed is acceptable.

A1 Fully correct equation in $\tan \theta$ with the $\cos 30^\circ$ and $\sin 30^\circ$ processed

($\sqrt{3} \sin \theta = 6 \cos \theta \Rightarrow \tan \theta = 2\sqrt{3}$ is acceptable for both A marks)

(Note If they proceed directly to the final answer from

$$\tan \theta = \frac{5 + 2 \sin 30^\circ}{2 \cos 30^\circ} \Rightarrow \tan \theta = 2\sqrt{3} \text{ then maximum M1dM1A0A0 unless}$$

$$\tan \theta = \frac{5+1}{\sqrt{3}} \text{ or equivalent is seen before the final given answer.}$$

A1* Correctly proceeds to given answer.

(b) Answers with no working scores 0 marks

M1 Attempts to find a value for x .

Allow $\arctan 2\sqrt{3}$...followed by adding or subtracting 20° . Which may be implied

by

$$\tan(x + 20) = 2\sqrt{3} \Rightarrow x = \arctan(2\sqrt{3}) \pm 20 = \dots$$

Alternatively, attempts to use $\sin(x - 10^\circ) = \sin x \cos 10^\circ \pm \cos x \sin 10^\circ$ within

the given equation, divides by $\cos x$ to set up an equation in just $\tan x$ and proceeds to find an angle for x

$$\tan x = \frac{5 \cos 20 + 2 \sin 10}{2 \cos 10 + 5 \sin 20} \Rightarrow x = \dots$$

A1 One value provided M1 has been scored. Allow either awrt 54° or 234° (or in radians awrt 0.94 or 4.08)

A1 $x = \text{awrt } 53.9^\circ, 233.9^\circ$ and no others inside the range provided M1 has been scored. Ignore any angles outside the range. Must be in degrees



Question Number	Scheme	Marks
3(a)	$(\cos 2A \equiv) \cos A \cos A - \sin A \sin A \equiv \cos^2 A - (1 - \cos^2 A)$	M1
	$\Rightarrow \cos 2A \equiv 2 \cos^2 A - 1$ *	A1*
		(2)
(b)	$\int (3 - 2 \cos 6x) \, dx = 3x - \frac{\sin 6x}{3} \quad (+c)$	M1A1
	$\left[3x - \frac{\sin 6x}{3} \right]_{\frac{\pi}{12}}^{\frac{\pi}{8}} = \left(3 \left(\frac{\pi}{8} \right) - \frac{\sin \left(6 \times \frac{\pi}{8} \right)}{3} \right) - \left(3 \left(\frac{\pi}{12} \right) - \frac{\sin \left(6 \times \frac{\pi}{12} \right)}{3} \right) = \frac{1}{8} \pi + \frac{2 - \sqrt{2}}{6}$	dM1A1
		(4)
		(6 marks)
Notes		
(a)		
M1:	Writes $(\cos 2A \equiv) \cos A \cos A - \sin A \sin A$ and attempts to use (not just stated separately) the identity $\pm \sin^2 A \pm \cos^2 A = \pm 1$ Note going directly to $\cos 2A \equiv \cos^2 A - \sin^2 A$ is allowed M1 as long as a use of the Pythagorean identity is clear. NB Watch out for answers which use double angle identities in the proof (circular reasoning) and score M0 if no Pythagorean identity is used.	
A1*:	Achieves the given answer with no errors seen (and no circular reasoning). Must see $\cos 2A \equiv \cos A \cos A - \sin A \sin A$ and clear attempt at substitution of $\sin^2 A + \cos^2 A = 1$ but allow “LHS” for “ $\cos 2A$ ”. Going directly to $\cos 2A \equiv \cos^2 A - \sin^2 A$ in the first step would score A0. Correct notation must used throughout but condone a missing closing bracket.	
Note :	$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1$ with no further evidence scores M0A0 as they are just writing out known formulae.	
(b)		
M1:	Attempts to substitute in the given result in part (a) and integrates to the form $\dots x \pm \dots \sin 6x$ Must be a full substitution in terms of x , $\cos 2A \rightarrow k \sin 2A$ is M0 unless recovered, e.g by later substitution may be implied by the limits substituted).	
A1:	$3x - \frac{\sin 6x}{3}$	
dM1:	Shows evidence of substituting in both limits into an expression of the form $\dots x \pm \dots \sin 6x$ and subtracts either way round to achieve an exact (not necessarily simplified but trig must be evaluated) answer.	
A1:	Correct answer e.g as scheme or $\frac{1}{8} \pi + \frac{1}{3} - \frac{\sqrt{2}}{6}$ or exact equivalent (and isw after a correct answer).	

Question Number	Scheme	Marks
4(a)	$A = 93$	B1
		(1)
(b)	$100 = 125 - "93" e^{-0.109T} \Rightarrow Ae^{-0.109T} = \dots$	M1
	$Ae^{kT} = B \Rightarrow T = \frac{\ln\left(\frac{B}{A}\right)}{k}$	dM1
	$T = 12.05$	A1
		(3)
(c)	$\frac{dN}{dt} = 0.109 \times "93" e^{-0.109 \times 7}$	M1
	4 730 (total sales per month)	A1
		(2)
(d)	The limit is 125 000 / the model has a limit below 150 000	B1
		(1)
		(7 marks)
Notes		
(a) B1: 93 (b) M1: Sets $100 = 125 - "93" e^{-0.109T}$ and proceeds to $Ae^{-0.109T} = \dots$ May use t instead of T. Condone slips. dM1: Uses correct order of operations and correct log work from an equation of form $Ae^{kT} = B$, $AB > 0$ to proceed to a value for T (must be a solvable equation). Implied by a correct answer for their equation. A1: 12.05 cao Note: Candidates who use 100 000 in the equation will score no marks in this part. Note: Answer only with no working send to review, but 12.05 following a correct equation seen score M1dM1A1. (c) M1: Attempts to differentiate to form $\lambda e^{-0.109 \times t}$, λ constant, and substitutes in $t = 7$. Award for an expression of the form $\dots e^{-0.109 \times 7}$ if no incorrect working is seen. The substitution of $t = 7$ may be implied by a correct value for their derivative of correct form. A1: awrt 4 730 (total sales per month). Accept equivalent forms for the answer, e.g. 4.73 thousand, but just 4.73 is A0. "Sales per month" may be omitted, but score A0 if an incorrect unit is given. Note: Answer only scores no marks. (d) B1: A correct reason given – any factual statements must be correct. E.g The limit is 125 000 / the model has a limit below 150 000 oe. Accept maximum value of N is 125, or N cannot reach 150 as asymptote at 125. Also allow attempts to substitute in 150 and find that they cannot take the log a negative number, but score B0 if a "negative time" is reached and used as reason. Attempts using e.g 150 000 in the equation score B0. There must be a reference to the information given in the answer, just "number is too large" or similar is B0.		

Question Number	Scheme	Marks
5(a)	$\frac{dy}{dx} = \frac{(x^2 + k) \times \frac{2x}{x^2 + k} - 2x \ln(x^2 + k)}{(x^2 + k)^2}$	M1
	$\frac{dy}{dx} = \frac{2x - 2x \ln(x^2 + k)}{(x^2 + k)^2} = \frac{2x(1 - \ln(x^2 + k))}{(x^2 + k)^2}$	M1A1
		(3)
(b)	$x = 0$	B1
	$"1" - \ln(x^2 + k) = 0 \Rightarrow x^2 = e^{-1} \pm k$	M1
	$x = \pm \sqrt{e - k}$	A1ft
		(3)
(c)	Upper limit is e or $k < e$	B1ft
		(1)
		(7 marks)
Notes		
(a) M1: Attempts to apply the quotient rule. Score for an expression of the form $\frac{dy}{dx} = \frac{(x^2 + k) \times \frac{\dots}{x^2 + k} - Qx \ln(x^2 + k)}{(x^2 + k)^2}$ oe (may be simplified) where ... is P or Px and $P, Q > 0$ May also attempt the product rule, in which case look for $-Px(x^2 + k)^{-2} \ln(x^2 + k) + (x^2 + k)^{-1} \times \frac{\dots}{x^2 + k}$ where ... is Q or Qx and $P, Q > 0$ M1: Cancels $x^2 + k$ and correctly takes a factor of $(2)x$ from the numerator to achieve "$B - \ln(x^2 + k)$" (may be done in one step). They must have a factor x in both terms. Note this may be scored following M0 e.g. if a numerator $uv' - vu'$ is used, or if the denominator misses the square. A1: $\left(\frac{dy}{dx} = \frac{2x(1 - \ln(x^2 + k))}{(x^2 + k)^2}\right)$ from fully correct work. Both M's must have been scored. (b) B1: $x = 0$ M1: Sets "$1 - \ln(x^2 + k) = 0$" and proceeds to $x^2 = \dots$ with correct undoing of $\ln$. A1ft: $x = \pm \sqrt{e - k}$ or follow through $x = \pm \sqrt{e^B - k}$ for a numerical B in their $B - \ln(x^2 + k)$ factor (c) B1ft: Upper limit is e or $k < e$ stated. Follow on their (b) of form $x = (\pm) \sqrt{e^B \pm k}$ B0 if $k = e$ or $k \leq e$ is given as the answer without statement about upper limit being e.		

Question Number	Scheme	Marks
6(a)	$\log_{10} S = 4.5 - 0.006 \times 2 \Rightarrow S = 10^{4.5 - 0.006 \times 2} = 30800 \text{ km}^2$	M1A1
		(2)
(b)	$\log_{10} S = 4.5 - 0.006t \Rightarrow S = 10^{4.5 - 0.006t}$ (or $p = 10^{4.5}$ or $q = 10^{-0.006}$)	M1
	$S = 10^{4.5 - 0.006t} = 10^{4.5} \times (10^{-0.006})^t$ (or $p = 10^{4.5}$ and $q = 10^{-0.006}$)	dM1
	$S = 31600 \times (0.986)^t$	A1
		(3)
(c)	E.g. The proportion of area covered by coral reefs retained from year to year.	B1
		(1)
		(6 marks)

Notes

- (a)**
M1: Substitutes in $t = 2$ and proceeds to find **a value** for S via $10^{\dots}$. May be implied by a correct value for their equation. Note awrt 24000 may arise from a slip with $0.006 \times 2 = 0.12$, and is acceptable for M1.
A1: awrt 30800 km² including units, from correct work and isw as long as units are included with a correct value. But e.g. 30760 followed by 307 km² is A0.
- (b)**
M1: Writes $S = 10^{4.5 - 0.006t}$ or award for either $(p =) 10^{4.5}$ or $(q =) 10^{-0.006}$ even if from an incorrect step. Essentially for knowing they need to raise to base 10. May arise from attempts at solving simultaneous equations using the answer to (a) (allowing for rounding errors).
dM1: Writes $S = 10^{4.5} \times (10^{-0.006})^t$ or award for both $p = 10^{4.5}$ and $q = 10^{-0.006}$ (oe forms e.g. accept $\left(\frac{1}{10^{0.006}}\right)$ or $\left(\frac{1}{1.01}\right)$ for q) Must be fully correct method to this point but allow if all that is wrong is rounding errors from other methods such solving simultaneous equations (e.g. 0.990^t).
A1cso: $S = 31600 \times (0.986)^t$ Must be awrt 3 s.f. values. Must be an equation including the S , not just p and q values, but accept if seen in (c).
- (c)**
B1: E.g. accept the following or variations of them
- The proportion of area covered by coral reefs retained from year to year.
 - The seafloor covered by coral reduces by 1.4% each year.
 - The proportional change/decrease in sea floor area covered by coral reefs per year
- Do not accept “rate of change” or “decrease in area of reefs per year” or “ q is the gradient” as the answer as it is an incorrect interpretation. The proportional change should be conveyed. There must be reference to, or equivalents of, the following:
- sea floor area or sea floor or area or seabed
 - coral reefs or coral or reef,
 - each year or year to year or per year
 - (q or its value is) a percentage/proportional decrease/remaining

Some acceptable answers:

- After 1 year the area will reduce to S last year times q . It shows the speed that coral reefs seafloor reduce.
- The percentage of remaining area of sea floor covered by coral reefs to the area of that covered last year.
- q represents the proportion of decreasing of the area of sea floor covered by coral reefs every year.

Some unacceptable answers:

- When there is extra t , the S will increase one more unit of 0.986
- q means the percentage of the part of sea floor covered by coral reefs after t years.
- q is the amount of coral reefs die/shrink in accordance to time.
- The area of the sea floor is decreasing at almost 0.986 times before.
- For every increase in the value of t , the total area of coral reefs will decrease in a rate of 0.986^t



Question Number	Scheme	Marks
2 (a)	$x = 2y^2 + 5y - 6$	
	$\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y + 5 = 0 \Rightarrow y = -\frac{5}{4}$	M1
	Substitutes their $y = -\frac{5}{4}$ into $x = 2y^2 + 5y - 6$ to find x	dM1
	$\left(-\frac{73}{8}, -\frac{5}{4}\right)$	A1
		(3)
		Total 5

Mark as one complete question

Main method via differentiation in (a) and (b)

(a)

M1: Differentiates $2y^2 + 5y - 6$ to a linear term in y AND attempts to take the reciprocal.

Condone for this mark attempts such as $\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y} + \frac{1}{5}$

We may see other methods including implicit differentiation from WMA14.

Look for $x = 2y^2 + 5y - 6 \Rightarrow 1 = ay \frac{dy}{dx} + b \frac{dy}{dx}$ followed by an attempt to make $\frac{dy}{dx}$ the subject

A1: $\frac{dy}{dx} = \frac{1}{4y+5}$ ISW after a correct answer.

(b) **Marks in part (b) can only be awarded after sight of** $\frac{dx}{dy} = \frac{ay+b}{k}$ **or** $\frac{dy}{dx} = \frac{k}{ay+b}$

So full marks can only be awarded after sight of $\frac{dx}{dy} = 4y+5$ **or** $\frac{dy}{dx} = \frac{1}{4y+5}$

M1: Sets ' $ay + b = 0 \Rightarrow y = \dots$ ' following sight of $\frac{dx}{dy} = \frac{ay+b}{k}$ **or** $\frac{dy}{dx} = \frac{k}{ay+b}$

Ignore/condone incorrect statements such as $\frac{dy}{dx} = 0$ here

This can be implied. E.g $\frac{dy}{dx} = \frac{1}{4y+5}$ followed by $y = -\frac{5}{4}$

dM1: Substitutes their solution of their $4y + 5 = 0$ into $x = 2y^2 + 5y - 6$ to find x .

This is dependent upon the previous M

A1: $\left(-\frac{73}{8}, -\frac{5}{4}\right)$ following $\frac{dx}{dy} = 4y+5$ **or** $\frac{dy}{dx} = \frac{1}{4y+5}$

The answer may be given separately, for example, as $x = -9.125$, $y = -1.25$

ISW after a correct answer.



Question Number	Scheme	Marks
8(a)	States $\log a = 0.68$ or $\log b = 0.09$	M1
	$a = 4.79$ or $b = 1.23$	A1
	States $\log a = 0.68$ and $\log b = 0.09$	M1
	$a = 4.79$ and $b = 1.23$ CSO	A1
		(4)
(b)	The percentage of the population with access to the internet at the start of 2005	B1
		(1)
(c)	$P = 4.79 \times 1.23^{10} = \text{awrt } 38$	M1, A1
		(2)
		Total 7

(a)

M1: Either states any of $\log a = 0.68$, $a = 10^{0.68}$, $a = \text{awrt } 4.8$ or any of $\log b = 0.09$, $b = 10^{0.09}$, $b = \text{awrt } 1.2$ A1: Achieves either $a = \text{awrt } 4.79$ **or** $b = \text{awrt } 1.23$ M1: States a correct equation for both a and b . See first M markA1: Achieves $a = 4.79$ **and** $b = 1.23$ **with no incorrect work**.Implied by $P = 4.79 \times 1.23^t$ **with no incorrect work**

These are NOT awrt values

Examples of incorrect work are

- $P = ab^t \Rightarrow \log P = \log a \times t \log b$
- $\log P = 0.68 + 0.09t \Rightarrow P = 10^{0.68} + 10^{0.09t} \Rightarrow P = 4.79 \times 1.23^t$

(b)

B1: A correct interpretation. The emboldened words must be present or stated in a similar way

"The **percentage** of the population with access to the **internet** at the start of **2005**"

A minimal answer is "the percentage with access to the internet in 2005"

Also allow "the initial percentage with internet access".

It is acceptable to state 4.79% of the population had access to the in 2005

(c)

M1: For attempting 4.79×1.23^{10} following through on their 4.79 and 1.23, (Ignore subsequent work)Alternatively attempting $\log P = 0.68 + 10 \times 0.09 \Rightarrow P = \dots$ Condone an attempt at 4.79×1.23^{11}

A1: AWR 38. ISW after sight of awrt 38 and condone misinterpretations such as stating 38 people.

Question Number	Scheme	Marks
2 (a)	$8 \cos \theta = 3 \operatorname{cosec} \theta$ States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ Attempts to use $\sin 2\theta = 2 \sin \theta \cos \theta$ $\sin 2\theta = \frac{3}{4} \quad \text{cso}$	B1 M1 A1 (3)
(b)	Correct order of operations to find θ , Look for $\frac{\arcsin\left(\frac{3}{4}\right)}{2}$ $(\theta =) \text{awrt } 24.3^\circ$	M1 A1 (2) (5 marks)

(a)

B1: States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$. May be implied by the line $8 \sin \theta \cos \theta = 3$.

Also allow with a consistent use of a different variable E.g. $\operatorname{cosec} x = \frac{1}{\sin x}$ but not $\operatorname{cosec} = \frac{1}{\sin}$

unless it has been recovered. Note that $3 \operatorname{cosec} \theta = \frac{1}{3 \sin \theta}$ is B0 unless there is an aside that does state $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$

M1: Attempts to use the identity $\sin 2\theta = 2 \sin \theta \cos \theta$ and proceeds to $\sin 2\theta = k$ where $|k| \leq 1$

Note that an attempt such as $8 \sin \theta \cos \theta = 3 \Rightarrow 8 \sin 2\theta = 3$ would be M0

A1: Achieves $\sin 2\theta = \frac{3}{4}$ o.e. with no errors. Condone the odd notational slip, e.g. $\sin \theta \leftrightarrow \sin$

(b)

M1: Uses the correct order of operations to find any value of θ , in degrees or radians that works for their

$\sin 2\theta = k$ follow through on their k with $|k| \leq 1$.

In general look for a value of θ found from evaluating $\theta = \frac{\arcsin\left(\frac{3}{4}\right)}{2}$.

From a correct equation it is implied by $\theta = \text{awrt } 0.42$ (radians)

A1: $\theta = \text{awrt } 24.3^\circ$ ONLY



Question Number	Scheme	Marks
2(a)	$ff(6) = f\left(\frac{9}{2}\right) = \frac{\frac{9}{2}+3}{\frac{9}{2}-4} = \dots; = 15$	M1; A1
		(2)
(b)	$f^{-1}(x) = \frac{4x+3}{x-1}$	M1A1
	$x \in \mathbb{R}, x \neq 1$	B1
		(3)
(c)	E.g. $\left(\frac{x+3}{x-4}\right)^2 + 5 = 7$ or $\frac{a+3}{a-4} = (\pm)\sqrt{7-5}$	M1
	$\Rightarrow x^2 - 22x + 23 = 0 \Rightarrow x = \dots$ or $(a+3) = (\pm)\sqrt{2}(a-4) \Rightarrow a = \dots$	dM1
	$(a =) 11 + 7\sqrt{2} \text{ oe}$	A1
		(3)
		(8 marks)
Notes		
(a)		
M1:	Substitutes $x = 6$ into f and substitutes the result back into f to find a value for $ff(6)$	
	Alternatively substitutes $x = 6$ into $ff(x) = \frac{\left(\frac{x+3}{x-4}\right)+3}{\left(\frac{x+3}{x-4}\right)-4}$ condoning slips.	
A1:	15	
(b)		
M1:	Changes the subject of $y = \frac{x+3}{x-4}$ or $x = \frac{y+3}{y-4}$ to $y = \frac{\dots x \pm 3}{\dots x \pm 1}$ or $x = \frac{\dots y \pm 3}{\dots y \pm 1}$	
A1:	Achieves $f^{-1}(x) = \frac{4x+3}{x-1}$. Accept $f^{-1} = \dots$ or $y = \dots$ instead of $f^{-1}(x) = \dots$.	
B1:	$x \neq 1$. The omission of $x \in \mathbb{R}$ is condoned. Accept alternative notations.	
(c) Note this is M1A1A1 on open but is being marked as M1dM1A1.		
M1:	Correct attempt to set up an equation in x or a . This will usually be $\left(\frac{x+3}{x-4}\right)^2 + 5 = 7$ but $f(a) = g^{-1}(7)$ may be used first. Score when a suitable equation is set up and allow if a minor slip or miscopy is made if the intention is clear.	
dM1:	Proceeds to solve for x or a , e.g. forms a 3TQ and solves, or take the 5 across, square roots, cross multiplies and makes a the subject, condoning e.g. sign slips when rearranging. Alternatively, uses $(f(x))^2 + 5 = 7 \Rightarrow f(x) = (\pm)\sqrt{2}$ and their part (b) leading to	
	$a = f^{-1}\left(\sqrt{2}\right) = \frac{4\sqrt{2}+3}{\sqrt{2}-1}$. In this method both M's may be scored together.	
A1	$11 + 7\sqrt{2}$ or $\frac{4\sqrt{2}+3}{\sqrt{2}-1}$ Accept $11 + \sqrt{98}$ or $\frac{-4\sqrt{2}-3}{1-\sqrt{2}}$. Common factors should be cancelled. Ignore labelling of x or a . Condone e.g. if $11 - 7\sqrt{2}$ is also given as a solution but A0 if any other extra solutions are included.	

General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$ **Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1(a)	$(-2, -3)$	B1
		(1)
(b)	$(-3, -9)$	B1B1
		(2)
(c)	$(-4, 3)$	B1
		(1)
		(4 marks)

In all parts, if more than 1 coordinate pair is offered then mark the “final” pair.

Accept missing brackets as long as the intention is clear e.g. $-2, -3$ or $(-2, -3$

(a)

B1: $(-2, -3)$. Allow $x = -2, y = -3$.

(b)

B1: One correct coordinate.

May be seen as part of a coordinate pair or written separately as $x = -3$ or $y = -9$.

B1: $(-3, -9)$. Allow $x = -3, y = -9$.

(c)

B1: $(-4, 3)$. Allow $x = -4, y = 3$.



Question Number	Scheme	Marks
2(a)	$(f(2) =) 2^4 - 5(2)^2 + 4(2) - 7 = -3 < 0$ and $(f(3) =) 3^4 - 5(3)^2 + 4(3) - 7 = 41 > 0$	M1
	There is a <u>change of sign</u> and $f(x)$ is <u>continuous</u> so <u>there is a root (in the interval)</u> *	A1*
		(2)
(b)	$x^3 = \frac{5x^2 - 4x + 7}{x} \Rightarrow x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}} \quad *$	B1*
		(1)
(c)(i)	$x_2 = \sqrt[3]{\frac{5(2)^2 - 4(2) + 7}{2}}$	M1
	= awrt 2.1179	A1
(ii)	$\alpha = \text{awrt } 2.1565$	A1
		(3)
		(6 marks)

(a)

M1: Attempts both $f(2)$ and $f(3)$ or a narrower interval that contains the root 2.1565...and considers the signs. Note showing $f(2) \times f(3) < 0$ or “ < 0 ”, “ > 0 ” beside each appropriate root is a consideration of signs and is also sufficient for the “sign change” part of reasoning for the A1.
For the substitution, need to see the values substituted or at least one of $f(2)$ or $f(3)$ correct.

A1*: This mark requires:

- both $f(2)$ and $f(3)$ correct e.g. $f(2) = -3$ and $f(3) = 41$
- a reference to the sign change
- a reference to continuity
- a (minimal) conclusion

For **sign change**, allow equivalent statements e.g. $f(2).f(3) < 0$, $f(2) < 0 < f(3)$ etc.

For **continuity**, allow just “continuous” and allow “assuming it is continuous” and “continuous equation” but do **not** allow incorrect statements such as “ x is continuous”, “the interval is continuous”, “there is a change of sign therefore it is continuous”

For the **conclusion**, allow e.g. ✓, #, QED, hence shown, hence root, so it crosses the x -axis, etc., but **not** incorrect statements e.g. so there is a root in the interval $[-3, 41]$

You may have to use your judgement to decide if the A1 conditions are satisfied.

Example: $f(2) = -3$, $f(3) = 41$, there is a change of sign hence $f(x)$ has a continuous root in $[2, 3]$
Scores M1A0 – i.e. it is not the root that is continuous

(b)B1*: Proceeds to given answer with no errors and with $x^3 = \dots$ seen at some stage.

Condone omission of the fraction bar e.g. $x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}}$ as long as there are no algebraic errors. Be tolerant if the radical does not fully encompass the expression but score B0 if the expression is clearly incorrect e.g. $x = \frac{\sqrt[3]{5x^2 - 4x + 7}}{x}$ i.e. where the cube root clearly sits on top of the fraction bar.

Condone working backwards as long as there is a (minimal) conclusion e.g.

$$x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}} \Rightarrow x^3 = \frac{5x^2 - 4x + 7}{x} \Rightarrow x^4 - 5x^2 + 4x - 7 = 0 \quad \checkmark, \#, \text{QED, hence shown, etc.}$$

(c)(i)M1: Attempts to find x_2 using the given iteration formula. Allow for sight of $\sqrt[3]{\frac{5(2)^2 - 4(2) + 7}{2}}$

Must see the correct index and not $\sqrt{\frac{5(2)^2 - 4(2) + 7}{2}}$ unless the “3” is implied by their value(s). May be implied by awrt 2.118 or awrt 2.147 ($= x_3$)

A1: awrt 2.1179 (apply isw if necessary)

(ii)

A1: awrt 2.1565 (provided M1 scored in (c)(i)) (apply isw if necessary)



Question Number	Scheme	Marks
3(a)	$\log_{10} D = 1.04 + 0.38t \Rightarrow D = 10^{1.04+0.38t}$ or $a = 10^{1.04}$ or $b = 10^{0.38}$	M1
	$a = \text{awrt } 10.96$ or $b = \text{awrt } 2.399$	A1
	$D = 10.96 \times 2.399^t$	A1
		(3)
(b)	$45000 = "10.96" \times "2.399"^{12} \Rightarrow T = \dots$ or $\log_{10} 45000 = 1.04 + 0.38T \Rightarrow T = \dots$	M1
	$\text{awrt } 9.51$	A1
		(2)
(c)	$D = "10.96" \times "2.399"^{12} \Rightarrow D = \dots$ or $\log_{10} D = 1.04 + 0.38 \times 12 \Rightarrow D = \dots$ or $350000 = "10.96" \times "2.399"^{12} \Rightarrow t = \dots$ or $\log_{10} 350000 = 1.04 + 0.38 \times t \Rightarrow t = \dots$	M1
	$D = \text{awrt } (\text{£})400\,000 \Rightarrow \text{yes}$ or $t = \text{awrt } 11.9 \Rightarrow \text{yes}$	A1
		(2)
		(7 marks)

(a)

M1: A correct application of log laws to obtain $D = 10^{1.04+0.38t}$ or forms one correct equation for a or b e.g. $a = 10^{1.04}$ or $b = 10^{0.38}$. May be implied by $a = \text{awrt } 11$ or $b = \text{awrt } 2.4$

A1: One of $a = \text{awrt } 10.96$, $b = \text{awrt } 2.399$. May be seen embedded in their equation.

A1: Correct **equation** with correct awrt values e.g. $D = (\text{awrt } 10.96) \times (\text{awrt } 2.399)^t$

Do not allow recovery from incorrect work e.g.

$$\log_{10} D = 1.04 + 0.38t \Rightarrow D = 10^{1.04} + 10^{0.38t} \Rightarrow a = 10.96, b = 2.399 \Rightarrow D = 10.96 \times 2.399^t$$

Scores M1A1A0

Note that a and b may be found in other ways e.g.

$$t = 0 \Rightarrow a = D = 10^{1.04}, t = 1 \Rightarrow b = \frac{D}{a} = \frac{10^{1.42}}{10^{1.04}} = 10^{0.38}$$



(b)

M1: This mark is awarded for proceeding to $T = \dots$ or $t = \dots$ where $\dots$ may be a numerical expression e.g. $\ln(\dots)$ from one of:

- substituting $D = 45000$ into their $D = "10.96" \times "2.399"^{Tt}$ (Must be of this form)
- substituting $D = 45000$ into $\log_{10} D = 1.04 + 0.38T$ (Must be the given equation)

A1: awrt 9.51 (months) (Ignore labelling and just look for the value)

Units are **not** required but if any are given they must be correct.

Allow recovery from incorrect rounding in (a) e.g. 10.97 for a

Correct answer only scores both marks.

Note that part (b) can be done without having attempted part (a)

(c)

M1: This mark is awarded for one of:

- substituting $t = 12$ into their $D = "10.96" \times "2.399"^{12}$ and proceeding to a **value** for D
- substituting $t = 12$ into $\log_{10} D = 1.04 + 0.38t$ proceeding to a value for D
- substituting $D = 350000$ into their $D = "10.96" \times "2.399"^{tt}$ and proceeding to a **value** for t
- substituting $D = 350000$ into $\log_{10} D = 1.04 + 0.38t$ and proceeding to a **value** for t

A1: This mark is awarded for one of:

- obtaining $D =$ awrt (£)400 000 and making a (minimal) conclusion e.g. “yes”
- obtaining $t =$ awrt 11.9 or truncated 11.8 and making a (minimal) conclusion e.g. “yes”

For reference the values of D/t which may be seen are:

$D = 398\,266.278$ (from rounded values of a and b)

$D = 398\,107.1706$ (from full values of a and b)

$t = 11.85236\dots$ (from rounded values of a and b)

$t = 11.85281\dots$ (from full values of a and b)

Allow recovery from incorrect rounding in (a) e.g. 10.97 for a

Note that part (c) can be done without having attempted part (a)

Acceptable alternative for (c):

$\log_{10} 350000 = 5.54\dots$, $1.04 + 0.38 \times 12 = 5.6$ so “yes”

or e.g.

$\log_{10} 350000 = 1.04 + 0.38 \times 12$, $5.544\dots < 5.6$ so “yes”

(Condone e.g. $5.54 \approx 5.6$ so “yes”)

Score M1 for substituting $D = 350\,000$ and $t = 12$ into the given equation and evaluating both sides and then A1 for a correct conclusion with lhs evaluated as awrt 5.54 or truncated 5.5 and rhs evaluated as 5.6

Allow obvious slips in the copying of 45 000 and/or 350 000 used as e.g. 4500 or 35000 for the M marks in (b) and (c)



Question Number	Scheme	Marks
4(a)	$f(x) = \frac{2x^2 - 32}{(3x-5)(x+4)} + \frac{8}{3x-5}$	B1
	$= \frac{2x^2 - 32}{(3x-5)(x+4)} + \frac{8}{3x-5} = \frac{2x^2 - 32 + 8(x+4)}{(3x-5)(x+4)}$ or e.g. $= \frac{2(x-4)(x+4)}{(3x-5)(x+4)} + \frac{8}{3x-5} = \frac{2(x-4)+8}{(3x-5)}$	M1
	$= \frac{2x(x+4)}{(3x-5)(x+4)} = \frac{2x}{3x-5} *$	A1*
		(3)
(b)	$f'(x) = \frac{2(3x-5) - 3 \times 2x}{(3x-5)^2}$	M1A1
	$f'(x) = \frac{-10}{(3x-5)^2}$ As $(3x-5)^2 > 0$ (for $x > 2$) then $f'(x) < 0$ Hence f is a decreasing function *	A1cso*
		(3)
(c)	$g^{-1}(x) = e^{\frac{x-3}{2}}$	M1A1
	$x \geq 3$	B1
		(3)
(d)	$g\left(\frac{2a}{3a-5}\right) = 5 \Rightarrow 3 + 2\ln\left(\frac{2a}{3a-5}\right) = 5$	B1
	$\ln\left(\frac{2a}{3a-5}\right) = 1 \Rightarrow \frac{2a}{3a-5} = e$	M1
	$\frac{2a}{3a-5} = e \Rightarrow a = \dots$	dM1
	$a = \frac{5e}{3e-2}$	A1
		(4)
(d) Way 2	$gf(a) = 5 \Rightarrow \frac{2a}{3a-5} = g^{-1}(5)$	B1
	$\frac{2a}{3a-5} = g^{-1}(5) \Rightarrow \frac{2a}{3a-5} = e$	M1
	$\frac{2a}{3a-5} = e \Rightarrow a = \dots$	dM1
	$a = \frac{5e}{3e-2}$	A1
		(13 marks)

(a)B1: $3x^2 + 7x - 20 = (3x - 5)(x + 4)$ seen or used

M1: Combines fractions with a correct common denominator and the order of terms in the numerator consistent with their common denominator.

A1*: Achieves the given answer with no errors seen but condone e.g. a missing trailing bracket as long as it is “recovered”.

Sufficient working should be shown but allow to go from e.g. $\frac{2(x-4)+8}{3x-5}$ to $\frac{2x}{3x-5}$

Note that candidates may take a longer route:

$$\frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} = \frac{(2x^2 - 32)(3x - 5) + 8(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{M1}$$

$$= \frac{6x^3 + 14x^2 - 40x}{(3x^2 + 7x - 20)(3x - 5)} = \frac{2x(3x^2 + 7x - 20)}{(3x - 5)(x + 4)(3x - 5)} \quad \text{B1}$$

$$= \frac{2x(3x - 5)(x + 4)}{(3x - 5)(x + 4)(3x - 5)} = \frac{2x}{3x - 5} \quad \text{A1*}$$

Or

$$\frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} = \frac{(2x^2 - 32)(3x - 5) + 8(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{M1}$$

$$= \frac{6x^3 + 14x^2 - 40x}{(3x^2 + 7x - 20)(3x - 5)} = \frac{2x(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{B1}$$

In this case the B1 is scored for obtaining a factor of $3x^2 + 7x - 20$ in the numerator and denominator

$$= \frac{2x}{3x - 5} \quad \text{A1*}$$

(b)M1: Attempts to differentiate using the product or quotient rule. Award for an expression of the form $\frac{\alpha(3x-5) - \beta x}{(3x-5)^2}$ or $\alpha(3x-5)^{-1} - \beta x(3x-5)^{-2}$, $\alpha, \beta > 0$ Condone attempts where $(3x-5)^2$ is expanded.Alternatively, attempts to write $\frac{2x}{3x-5}$ as $A + \frac{B}{3x-5}$ before differentiating using thechain rule to obtain $\frac{\pm k}{(3x-5)^2}$

A1: Correct derivative simplified or unsimplified.

A1cso*: Requires a correct simplified derivative $\frac{-10}{(3x-5)^2}$ and then statements that convey:

- $(3x-5)^2 > 0$ or e.g. denominator is positive (condone $(3x-5)^2 \geq 0$)
- so $f'(x) < 0$
- function is decreasing



Question Number	Scheme	Marks
9.(a)	$x = 30e^{-0.2 \times 8} = 6.06 \text{ (mg)}$	M1, A1 (2)
(b)	$x = 30e^{-0.2 \times 10} + 20e^{-0.2 \times 2} = 17.5 \text{ (mg)}^*$	B1* (1)
(c)	$30e^{-0.2 \times (T+8)} + 20e^{-0.2 \times T} = 10$ $(3e^{-1.6} + 2)e^{-0.2T} = 1$ $e^{-0.2T} = \frac{1}{3e^{-1.6} + 2}$ $T = 5 \ln(3e^{-1.6} + 2) = 4.79$	M1 A1 dM1, A1 (4) (7 marks)

(a)

M1: Substitutes $D = 30$ and $t = 8$ into $x = De^{-0.2t}$ and proceeds to a value for x

A1: AWR 6.06 Allow this to be written down for both marks. Units are not important

(b)

B1*: There are many ways to achieve this. Examples of correct expressions are

- $30e^{-0.2 \times 10} + 20e^{-0.2 \times 2}$
- $6.06e^{-0.2 \times 2} + 20e^{-0.2 \times 2}$ (allow with more accurate values than 6.06)
- $26.06e^{-0.2 \times 2}$ (allow with more accurate values than 26.06)

You need to see a correct expression followed by awrt 17.47 or 17.5 (Units not important)

Note that values other than 26.06 in $26.06e^{-0.2 \times 2}$ would still round to 17.5.For instance, expressions such as $26.1 \times e^{-0.2 \times 2}$ and $26.07 \times e^{-0.2 \times 2}$ would round to an appropriate value **but will not score this mark**

(c) Condone $T \leftrightarrow t$ but you must be convinced that they are answering the question.

M1: Sets either $30e^{-0.2 \times (T+8)} + 20e^{-0.2 \times T} = 10$ or $6.06e^{-0.2 \times T} + 20e^{-0.2 \times T} = 10$ o.e

Award the mark for a correct equation in T

A1: For a simplified equation in $e^{-0.2T}$ (that is an equation that has a single $e^{-0.2T}$ term).

Look for $(3e^{-1.6} + 2)e^{-0.2T} = 1$ or awrt $26.06e^{-0.2 \times T} = 10$

Condone for this mark $26.1e^{-0.2 \times T} = 10$, awrt $26.05e^{-0.2 \times T} = 10$ and even awrt $26.07e^{-0.2 \times T} = 10$

dM1: Full method to find T . Condone slips but generally look for

- the candidate making $e^{-0.2T}$ the subject
- then taking lns
- proceeding to a value for T

A1: awrt 4.79 following correct and accurate work. Note exact simplified answer is $5 \ln(3e^{-1.6} + 2)$

If they use their answer to part (a) it must be following an accurate answer to 2dp, e.g.

awrt $26.06e^{-0.2 \times T} = 10$

.....
.....

Alternatives in (c) exist where an equation in T is not found directly

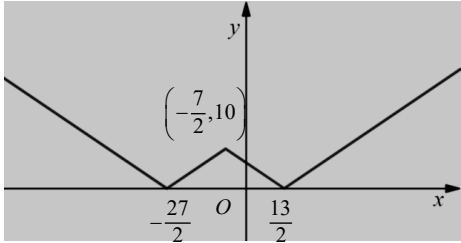
E.g Using the answer to (b)

M1, A1: $17.5e^{-0.2 \times t} = 10$

dM1: $-0.2t = \ln\left(\frac{10}{17.5}\right) \Rightarrow t = \dots$ then add 2 to the result

A1: $T = 4.79$

B1: Fully correct. Accept $5 < ff < 22$, $(5, 22)$. Allow with y or range but $5 < x < 22$ would be B1
B0

Question Number	Scheme	Marks
7 (a)	$\left(-\frac{7}{2}, -10\right)$	B1 B1 (2)
(b)	Attempts to solve either $\frac{1}{2}(2x+7)-10 \dots \frac{1}{3}x+1$ Or $-\frac{1}{2}(2x+7)-10 \dots \frac{1}{3}x+1$ Both correct critical values $x \dots -\frac{87}{8}, \frac{45}{4}$ Selects outside region for their critical values Correct solution $x \leq -\frac{87}{8}, x \geq \frac{45}{4}$	M1 A1 dM1 A1 (4)
(c)		Shape B1 Maximum B1 ft One correct minimum B1 Fully correct B1 (4) (10 marks)

(a) Must be answered in (a). We cannot imply from work in (c)

B1: One correct coordinate. Allow as $x = -\frac{7}{2}, y = -10$. Allow exact equivalents, e.g. $x = -\frac{14}{4}$

B1: Both coordinates correct. Allow as $x = -\frac{7}{2}, y = -10$. Allow exact equivalents, e.g. $x = -\frac{14}{4}$

(b)

M1: Attempts to solve a correct equation or inequality. They must proceed as far as $x \dots$

Condone slips where they attempt to change the subject in an attempt to make $|2x+7|$ the subject.

So either of $\frac{1}{2}|2x+7|-10 \dots \frac{1}{3}x+1 \Rightarrow |2x+7| = ax+b \Rightarrow 2x+7 = ax+b$

or $\frac{1}{2}|2x+7|-10 \dots \frac{1}{3}x+1 \Rightarrow |2x+7| = ax+b \Rightarrow -2x-7 = ax+b$ are condoned for

the M1

A1: Correct critical values $x \dots -\frac{87}{8}, \frac{45}{4}$ which may be part of an incorrect inequality

dM1: Selects outside region for their critical values. Allow these to be written as one inequality. Allow the strict inequalities here. It is dependent upon having attempted to solve **one** correct equation

A1: $x \leq -\frac{87}{8}, x \geq \frac{45}{4}$ These can be given separately but do not isw here. Condone words like "and"

between. Allow other correct forms such as $x \in \left(-\infty, -\frac{87}{8}\right] \cup \left[\frac{45}{4}, \infty\right)$ **Mark their final answer**

Do not allow incorrect answers such as $\frac{45}{4} \leq x \leq -\frac{87}{8}$ or $x \in \left(-\infty, -\frac{87}{8}\right] \cap \left[\frac{45}{4}, \infty\right)$

(c) **There must be a sketch for this. If the text and the graph contradict each other the graph takes precedence**

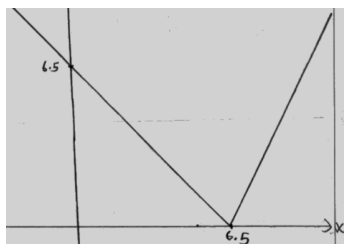
B1: W shape any position. Condone asymmetric arms.

B1ft: Maximum point but follow through on their coordinates from part (a)

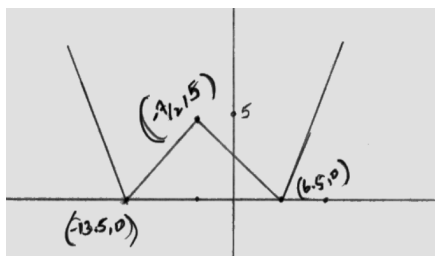
B1: Either minimum value. It must be minimum point and not just the point at which the graph crosses the x -axis. Condone $(0, 6.5)$ for $(6.5, 0)$ as long as it is on the correct axis

B1: Fully correct diagram with **ALL** points correct (allow exact equivalents) Condone asymmetric arms.

Accept a W on the x axis with both $x = \frac{13}{2}$ and $x = -\frac{27}{2}$ marked. Coordinates must be correct



B0 B0 B1 (one local minimum correct) B0

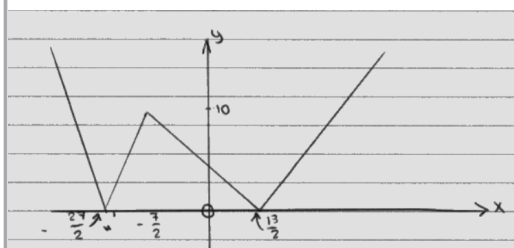


B1: W shape

B1ft: Their part (a) was $\left(-\frac{7}{2}, -5\right)$

B1: One correct local minimum. In fact both are correct

B0: Must be completely correct. Incorrect maximum point



Acceptable for all marks. It is a little asymmetric and the local maximum point is implied

You may see attempts at part (b) via squaring. It is scored in a similar way.

Any attempt must attempt to isolate the modulus term before squaring

$$\frac{1}{2}|2x+7| - 10 \dots \frac{1}{3}x + 1 \Rightarrow \frac{1}{2}|2x+7| \dots \frac{1}{3}x + 11$$

M1: Squares and attempts to solve $\frac{1}{4}(2x+7)^2 \dots \left(\frac{1}{3}x+11\right)^2$ via a quadratic

A1: FYI Quadratic is $32x^2 - 12x - 3915$ which has critical values of $-\frac{87}{8}$ and $\frac{45}{4}$ which may be found via a calculator

dM1: Selects outside region. It is dependent upon a correct $\frac{1}{4}(2x+7)^2 \dots \left(\frac{1}{3}x+11\right)^2$ o.e.

A1: $x \leq -\frac{87}{8}, x \geq \frac{45}{4}$ Allow other forms such as $x \in \left(-\infty, -\frac{87}{8}\right] \cup \left[\frac{45}{4}, \infty\right)$

Question Number	Scheme	Marks
8 (a)	$\log_{10} x = 2.74 - 0.079t$ $\log_{10} x = 2.74 - 0.079t$ $x = 10^{2.74} \times 10^{-0.079t}$ $x = 10^{2.74} \times \left(10^{0.079}\right)^{-t}$ $\log_{10} x = \log_{10} p - t \log_{10} q$ A correct equation for p or q . E.g. either $\log_{10} p = 2.74$ or $\log_{10} q = 0.079$ A correct value for either p or q . Either $p = \text{awrt } 550$ or $q = \text{awrt } 1.2$ A correct equation for both p and q . E.g. both $p = 10^{2.74}$ and $q = 10^{0.079}$ Both $p = \text{awrt } 550$ and $q = \text{awrt } 1.2$ with proof*	M1 A1 dM1 A1* (4)
(b)	"p" is the amount of antibiotic (in mg) in the patient's bloodstream at the start	B1 (1)
(c)	$x = 400 \times 1.4^{-t} \Rightarrow \frac{dx}{dt} = -400 \ln 1.4 \times 1.4^{-t}$ Substitutes $t = 5$ into $\frac{dx}{dt} = \text{awrt } -25$	B1 M1 A1 (3) (8 marks)

(a)

M1: A correct equation in either p or q

For p look for $\log_{10} p = 2.74$ or $p = 10^{2.74}$ For q look for $\log_{10} q = 0.079$ or $q = 10^{0.079}$

This is implied by a correct value for either p or q (See below), $x = 550 \times q^{-t}$, or $x = p \times 1.2^{-t}$

A1: Either $p = \text{awrt } 550$, $q = \text{awrt } 1.2$, $x = 550 \times q^{-t}$, or $x = p \times 1.2^{-t}$

dM1: A correct equation for both p and q . E.g. sight of both $p = 10^{2.74}$ and $q = 10^{0.079}$

This may be implied by $x = 550 \times 1.2^{-t}$

A1*: Both $p = \text{awrt } 550$ and $q = \text{awrt } 1.2$ **with proof*** as it is a "show that" question.

There must be no incorrect working. Correct values of p and q are implied by $x = 550 \times 1.2^{-t}$

This "proof" part can be shown with a minimum of working

Starting with $x = p q^{-t}$ a minimum of $\log_{10} x = \log_{10} p - t \log_{10} q$ followed by correct equations and values is sufficient

Starting with $\log_{10} x = 2.74 - 0.079t$ a minimum of $x = 10^{2.74} \times 10^{-0.079t}$ followed by correct equations and/or values is sufficient



(b)

B1 Award for a statement that refers or alludes to the amount of antibiotic in the bloodstream when $t = 0$

E.g. " p " is the amount of antibiotic (in mg) (in the patient's bloodstream) at the start

Condone that " p " is the dose of antibiotic (in mg) given to the patient

Condone "just / immediately after" but not "before" the dose is given

Don't allow students to give a correct and an incorrect answer. This would be B0

(c)

B1: $x = 400 \times 1.4^{-t} \Rightarrow \frac{dx}{dt} = \pm 400 \ln 1.4 \times 1.4^{-t}$

M1: Substitutes $t = 5$ into a changed function of the form $\left(\frac{dx}{dt}\right) = k \times 1.4^{-t}$ where $k \neq 400$

The left hand side may be incorrect. E.g. $\frac{dy}{dx}$ or similar

A1: $\frac{dx}{dt} = \text{awrt } -25$

Note that candidates who lose the negative sign when differentiating to get $\frac{dx}{dt} = \text{awrt } 25$ score B1 M1

A0

Alt seen;

B1: $x = 400 \times 1.4^{-t} \Rightarrow \ln x = \ln 400 - t \ln 1.4 \Rightarrow \frac{1}{x} \frac{dx}{dt} = \pm \ln 1.4$

M1 A1: As before



Question Number	Scheme	Marks
9 (i)	$2 \sec^2 x - 3 \tan x = 2$ $2(1 + \tan^2 x) - 3 \tan x = 2$ $2 \tan^2 x = 3 \tan x \Rightarrow \tan x = \frac{3}{2}, (\tan x = 0)$ $x = \text{awrt } 0.983, \pi$	M1 dM1 A1, B1 (4)
(ii)	$\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \equiv \frac{\sin 3\theta \cos \theta - \cos 3\theta \sin \theta}{\sin \theta \cos \theta} \equiv \frac{\sin(3\theta - \theta)}{\sin \theta \cos \theta}$ $\equiv \frac{\sin(2\theta)}{\frac{1}{2} \sin 2\theta}$ $\equiv \frac{\sin 2\theta}{\frac{1}{2} \sin 2\theta} \equiv 2 \quad *$	M1 dM1, A1 A1* (4) (8 marks)

(i)

M1: Attempts to use $\sec^2 x = \pm 1 \pm \tan^2 x$ within the given equation, condoning slips

There are alternatives using for example $\sec^2 x = \frac{1}{\cos^2 x}$, $\tan x = \frac{\sin x}{\cos x}$ and $\cos^2 x = 1 - \sin^2 x$

FYI the correct equation using this method would be $2 \sin^2 x - 3 \sin x \cos x = 0$ dM1: Valid method of solving a quadratic equation in $\tan x$. Condone division by $\tan x$

In the alternative the method of solving is similar $\sin x(2 \sin x - 3 \cos x) = 0 \Rightarrow \tan x = \frac{3}{2}$

A1: $x = \text{awrt } 0.983$ as the only solution to $\tan x = \frac{3}{2}$ in the region $(0, \pi]$ B1: $x = \pi$ or awrt 3.14 (as the only solution in the range for $\tan x = 0$ or $\sin x = 0$ in the region $(0, \pi]$).

Condone the inclusion of 0 as that is outside the range

Condone 180° if both angles have been given in degrees. Ignore any solution outside the range

(ii)

M1: Attempts to form a single fraction and uses

- either the compound angle formula on the numerator. Allow $\sin(3\theta \pm \theta)$
- or the double angle on the denominator. Allow $\sin \theta \cos \theta = k \sin 2\theta$

dM1: Attempts to form a single fraction and uses both

- the compound angle formula on the numerator. Must be $\sin(3\theta - \theta)$
- and the double angle on the numerator or denominator. Allow $\sin \theta \cos \theta = k \sin 2\theta$

A1: For reaching the form $\frac{\sin(3\theta - \theta)}{\frac{1}{2} \sin 2\theta}$ or $\frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta}$

A1*: Fully complete proof.

Alt soln

M1: Attempts compound angle formulae for both $\sin 3\theta$ and $\cos 3\theta$. Condone **only** sign slips

$$\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} = \frac{\sin(2\theta + \theta)}{\sin \theta} - \frac{\cos(2\theta + \theta)}{\cos \theta} = \frac{\sin 2\theta \cos \theta + \cos 2\theta \sin \theta}{\sin \theta} - \frac{\cos 2\theta \cos \theta - \sin 2\theta \sin \theta}{\cos \theta}$$

dM1: This mark is dependent upon the previous one having been awarded. It can be awarded for one of

- attempting to use the double angle formula for $\sin 2\theta$ (condone $\sin 2\theta = k \sin \theta \cos \theta$ if used consistently), then dividing and cancelling out the terms in $\cos 2\theta$ to produce an expression in just $a \cos^2 \theta \pm b \sin^2 \theta$

E.g.
$$\frac{2\sin\theta \cos \theta \cos \theta}{\sin \theta} + \frac{\cos 2\theta \sin \theta}{\sin \theta} - \frac{\cos 2\theta \cos \theta}{\cos \theta} \pm \frac{2\sin \theta \cos \theta \sin \theta}{\cos \theta} = \dots$$

- as above but may use $\cos 2\theta = \pm 1 \pm 2 \sin^2 \theta$ o.e. to produce an expression in just $a \cos^2 \theta + b \sin^2 \theta + c$

E.g.

$$\frac{2\sin\theta \cos \theta \cos \theta}{\sin \theta} + \frac{(\cos^2 \theta - \sin^2 \theta) \sin \theta}{\sin \theta} - \frac{(\cos^2 \theta - \sin^2 \theta) \cos \theta}{\cos \theta} \pm \frac{2\sin \theta \cos \theta \sin \theta}{\cos \theta} = \dots$$

- Uses appropriate expressions for $\sin 2\theta$ and $\cos 2\theta$ then uses a correct common factor to produce a single fraction in just $\sin \theta$ and $\cos \theta$

E.g.

$$\frac{2\sin \theta \cos \theta \cos^2 \theta + (\cos^2 \theta - \sin^2 \theta) \sin \theta \cos \theta}{\sin \theta \cos \theta} - \frac{(\cos^2 \theta - \sin^2 \theta) \sin \theta \cos \theta \pm 2\sin \theta \cos \theta \sin^2 \theta}{\sin \theta \cos \theta} = \dots$$

A1: For a correct intermediate line in single angles that leads to the given answer

In most methods it is for one of

- $\frac{2\sin\theta \cos \theta \cos \theta}{\sin \theta} + \frac{2\sin \theta \cos \theta \sin \theta}{\cos \theta}$ or equivalent
- $2\cos^2 \theta + 1 - 2\sin^2 \theta + 1 - 2\cos^2 \theta + 2\sin^2 \theta$
- $\frac{2\sin \theta \cos^3 \theta + 2\sin^3 \theta \cos \theta}{\sin \theta \cos \theta}$
- $\frac{2\sin\theta \cos \theta \cos \theta + \cos 2\theta \sin \theta}{\sin \theta} - \frac{\cos 2\theta \cos \theta - 2\sin \theta \cos \theta \sin \theta}{\cos \theta}$ or equivalent.

We can see clearly that the $\cos 2\theta$ terms cancel so that it is effectively an expression in single angles

A1*: Fully complete proof with no errors. You may tolerate the odd notational slip.

Withhold this mark if there are obvious and repeated notational errors

.....
Most answers seen will be a combination of these.

Generally the marks are awarded for

M1: Makes a positive step towards achieving the given answer

dM1: Makes all the correct steps towards the proof but allow slips

A1: A correct line that usually only requires the use of the pythagorean identity to reach the given answer

If you see the triple angle identities used (and it is incorrect) please send to review

$$\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \equiv \frac{3\sin \theta - 4\sin^3 \theta}{\sin \theta} - \frac{4\cos^3 \theta - 3\cos \theta}{\cos \theta} \equiv (3 - 4\sin^2 \theta) - (4\cos^2 \theta - 3) \equiv 6 - 4(\sin^2 \theta + \cos^2 \theta) \equiv 2$$

Question Number	Scheme	Marks
6.(a)	$(2, -10)$	B1 B1 (2)
(b)	$ff(0) = f(-4) = \dots$ $= 8$	M1 A1cso (2)
(c)	Attempts to solve $-3(x-2)-10=5x+10 \Rightarrow x = \dots$ $x > -\frac{7}{4}$ only	M1 A1 (2)
(d)	$x(\text{or } x) = \frac{16}{3}$ Attempts $3(x -2)-10=0 \Rightarrow x =k, k > 0$ or $3(-x-2)-10=0 \Rightarrow x=-k$ or $3(x-2)-10=0 \Rightarrow x=k \Rightarrow x=-k$ $x = \left(\frac{16}{3} \text{ and } \right) -\frac{16}{3}$ with no other values	B1 M1 A1 (3) (9 marks)

Notes

- (a)
B1 For one correct coordinate
B1 For $(2, -10)$. Allow $x = \dots, y = \dots$. Do not accept e.g. $6/3$ unless 2 has been seen/identified with this.
- (b)
M1 For a full attempt at $f f(0)$. Can be scored for $f(-4)$. Allow for use of their $f(0)$ even if incorrect as long as the process is clear, e.g. $f(0) = \dots$ stated or calculated first then used. May be scored by first attempting $ff(x)$ before substituting. This mark is for showing the correct process of composites, so may be scored if there are slips or errors with modulus if the intent is clear.
A1cso $ff(0) = 8$ only. A0 if other values given.
- (c)
M1 Attempts to solve $-3(x-2)-10=5x+10 \Rightarrow x = \dots$. Allow with equality or any inequality for the M mark.
Alternatively, rearranges to $|x-2| = ax+b$, squares both sides and solves the quadratic.
A1 $x > -\frac{7}{4}$ (oe) only. If another inequality or value is given and not rejected withhold this mark.
- (d)
Work for (d) must be seen or referred to in (d). Do not accept for work attempted in earlier parts but not used in (d).
B1 For $x = \frac{16}{3}$. Allow when seen even from incorrect working as it could be verified. May be seen on sketch as long as referred to in (d). Allow also for $|x| = \frac{16}{3}$

M1 Correct method to find the root on the negative x -axis. E.g. attempts to solve $3(|x| - 2) - 10 = 0$ to achieve a value for $|x|$, or $3(-x - 2) - 10 = 0$ to achieve a value for x , or for reflecting in the y -axis (making negative) their $\frac{16}{3}$ from an attempt at $3(x - 2) - 10 = 0$. May be part of longer winded attempts. Allow missing brackets for the M.

Note it is possible to arrive at an equation leading to $x = \pm \frac{16}{3}$ from incorrect starting points, and such methods will score M0.

A1 For $x = -\frac{16}{3}$ with no other values (aside their $x = \frac{16}{3}$). Must give the negative value, not just $|x| = \frac{16}{3}$. May be stated on a sketch as long as work seen in (d). Do not isw if they clearly reject this value later or if they try to form an inequality from the values, which is A0 as other values are included.

(b)	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = \frac{(x-3) \times 7 - 7x \times 1}{(x-3)^2}$	M1
	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	A1
		(2)
(b) ALT	$f(x) = \frac{7x}{(x-3)} = 7 + \frac{21}{x-3} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	M1A1
		Total 5

(b)

M1: Attempts to differentiate using the quotient rule or product rule.For the quotient rule it requires achieving $f'(x) = \frac{\alpha(x-3) - \beta x}{(x-3)^2} \quad \alpha, \beta > 0$ If they use a substitution then similar conditions apply e.g. $u = x-3 \Rightarrow f'(u) = \frac{\alpha u - \beta(u+3)}{u^2} \quad \alpha, \beta > 0$ For the product rule it requires achieving $f'(x) = A(x-3)^{-1} - Bx(x-3)^{-2} \quad A, B > 0$ If they use a substitution then similar conditions apply e.g. $u = x-3 \Rightarrow f'(u) = Au^{-1} - B(u+3)u^{-2} \quad A, B > 0$ **A1:** Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$ Allow $-\frac{21}{x^2 - 6x + 9}$ or e.g. $-21(x^2 - 6x + 9)^{-1}$

Isw once a correct simplified answer is seen.

Alternative:**M1:** Expresses $\frac{7x}{(x-3)}$ as $\alpha + \frac{\beta}{(x-3)}$ and differentiates to obtain $\frac{\dots}{(x-3)^2}$ or e.g. $\alpha + \frac{\beta}{u}$ where $u = x-3$ and differentiates to obtain $\frac{\dots}{u^2}$ **A1:** Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$ Allow $-\frac{21}{x^2 - 6x + 9}$ or e.g. $-21(x^2 - 6x + 9)^{-1}$

Isw once a correct simplified answer is seen.

Question Number	Scheme	Marks
10(a)	$(F =) 35$	B1
		(1)
(b)	$200 = \frac{350e^{15k}}{9+e^{15k}} \Rightarrow 1800 + 200e^{15k} = 350e^{15k} \Rightarrow 150e^{15k} = 1800$	M1
	$e^{15k} = \frac{1800}{150} \Rightarrow 15k = \ln 12 \Rightarrow k = \frac{1}{15} \ln 12^*$	dM1A1*
		(3)
(c)	$F = \frac{350e^{kt}}{9+e^{kt}} \Rightarrow \frac{dF}{dt} = \frac{350ke^{kt}(9+e^{kt}) - 350e^{kt}(ke^{kt})}{(9+e^{kt})^2}$	M1A1
	$\frac{3150ke^{kt}}{(9+e^{kt})^2} = 10 \Rightarrow 315ke^{kt} = 81 + 18e^{kt} + e^{2kt} \Rightarrow e^{2kt} + (18-315k)e^{kt} + 81 = 0$	M1
	$e^{2kt} + (18-315k)e^{kt} + 81 = 0 \Rightarrow e^{kt} = \frac{315k - 18 \pm \sqrt{(18-315k)^2 - 4 \times 81}}{2} \Rightarrow kt = \dots$	M1
	$T = \text{awrt } 5.7, 20.8$	A1
		(5)
		Total 9

(a)**B1:** Correct value, 35**(b)****M1:** Uses $F = 200$ and $t = 15$ and reaches $Ae^{15k} = B$ where $A \times B > 0$ **dM1:** Proceeds using correct order of operations to obtain a value for k Look for $Ae^{15k} = B \Rightarrow e^{15k} = \frac{B}{A} \Rightarrow 15k = \ln \frac{B}{A} \Rightarrow k = \dots$ or Alternatively $Ae^{15k} = B \Rightarrow \ln A + 15k = \ln B \Rightarrow 15k = \ln B - \ln A \Rightarrow k = \dots$ **A1*:** Correct proof with all necessary steps shown.Minimum acceptable solution $200 = \frac{350e^{15k}}{9+e^{15k}} \Rightarrow 150e^{15k} = 1800 \Rightarrow e^{15k} = 12 \Rightarrow k = \frac{1}{15} \ln 12$ **(c)** Allow the whole of part (c) to be done with the letter k , the exact value for k or using $k = \text{awrt } 0.166$ **M1:** Correct attempt of the quotient (product or chain) rule.For the Quotient Rule look for $\frac{dF}{dt} = \frac{Pe^{kt}(9+e^{kt}) - Qe^{kt}(e^{kt})}{(9+e^{kt})^2} \quad P, Q > 0$ For the Product Rule look for $\frac{dF}{dt} = Pe^{kt}(9+e^{kt})^{-1} \pm Qe^{kt}e^{kt}(9+e^{kt})^{-2} \quad P, Q > 0$ For Chain Rule look for $F = A \pm \frac{B}{9+e^{kt}} \Rightarrow \frac{dF}{dt} = Qe^{kt}(9+e^{kt})^{-2} \quad Q \text{ is a constant}$ **A1:** Correct differentiation, which may be unsimplified.

Allow for an expression in k , with exact $k = \frac{1}{15} \ln 12$, or using $k = \text{awrt } 0.166$

M1: Sets their derivative = 10 and obtains a 3TQ in e^{kt}
It is dependent upon a reasonable attempt to differentiate.

In almost all cases the M1 will have been awarded but condone an attempt following $\frac{dF}{dt} = \frac{vu' + uv'}{v^2}$

M1: Scored for

- solving a 3TQ in e^{kt} by any method including a calculator (you may need to check with accuracy to 2sf rounded or truncated). Note that the equation may be a quadratic in $12^{\frac{1}{15}t}$ instead of $e^{\left(\frac{1}{15} \ln 12\right)t}$
- then taking $\ln$'s to obtain at least one value for kt

FYI the correct quadratics are;

$$e^{\left(\frac{2}{15} \ln 12\right)t} + (18 - 21 \ln 12)e^{\left(\frac{1}{15} \ln 12\right)t} + 81 = 0,$$

$$\text{or } \left(e^{0.166t}\right)^2 - 34.18e^{0.166t} + 81 = 0 \Rightarrow e^{0.166t} = 2.56, 31.62 \Rightarrow 0.166t = 0.94, 3.45$$

A1: awrt 5.7, 20.8



Question Number	Scheme	Marks
10(a)	$\frac{1}{4} = \sin^2 4y \Rightarrow y = \frac{\pi}{24}$	M1A1
		(2)
(b)	$\frac{dx}{dy} = \underline{\underline{8 \sin 4y \cos 4y}}$	<u>M1A1</u>
		(2)
(c)	$\frac{dx}{dy} = 8 \sin 4y \cos 4y \rightarrow \frac{dy}{dx} = \frac{1}{8 \sin 4y \cos 4y}$	M1
	$\frac{dy}{dx} = \frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{dy}{dx} = \frac{1}{\sqrt{16 - 64\left(x - \frac{1}{2}\right)^2}}$	A1
		(3)
(d)(i)	$x = \frac{1}{2}$	B1ft
(ii)	$\frac{dy}{dx} = \frac{1}{4}$	B1ft
		(2)
		(9 marks)
Notes		
(a) M1: Attempts to substitute in $x = \frac{1}{4}$ and rearranges to find an exact value for y. Look for $y = M \arcsin\left(\pm \frac{1}{2}\right) \rightarrow k\pi$ or $k(30^\circ)$ allowing errors in dividing by the 4, or equivalents criteria via a double angle identity, correct up to sign error. A1: $\frac{\pi}{24}$ Ignore extra solutions outside the domain. (b) M1: Attempts to differentiate achieving a form $\left(\frac{dx}{dy} = \right) A \sin 4y \cos 4y$ or $A \sin 8y$ oe Accept alternative forms e.g. via implicit differentiation $1 = A \sin 4y \cos 4y \frac{dy}{dx}$. The $\frac{dx}{dy}$ may be missing, or labelled $\frac{dy}{dx}$ for this mark. A1: $\frac{dx}{dy} = 8 \sin 4y \cos 4y$ oe eg $\frac{dx}{dy} = 4 \sin 8y$ Coefficients must be simplified. Must include the $\frac{dx}{dy}$ (c) M1: Uses $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$ (allow for the reciprocal of their answer to (b) if left hand side is omitted). Variables must be consistent.		

M1:	Attempts to use $\sin 4y = \pm\sqrt{x}$ and $\cos 4y = \pm\sqrt{1-x}$ to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of x only with no trig terms.	
A1:	$\frac{dy}{dx} = \frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$ and isw after correct answer. Allow with terms under the square root reversed.	
(d) Note to score marks in (d) they must have a derivative which does have a minimum and is of a completed square form (the r may be inside the bracket) or in some other way clearly uses their (c) (e.g. minimising their quadratic).		
(i) B1ft:	$(x =)\frac{1}{2}$ ft their $-s$ provided their $q > 0$, their $r < 0$ and their x is in the range $0 \leq x \leq 1$	
(ii) B1ft:	$\left(\frac{dy}{dx} = \right)\frac{1}{4}$ ft their q provided it is positive and their r was negative.	
(b) Alt	$x = \sin^2 4y \Rightarrow y = \frac{1}{4} \arcsin \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{4} \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{1}{2} x^{-\frac{1}{2}}$	M1
	$\frac{dx}{dy} = 8\sqrt{x}\sqrt{1-x}$	A1
		(2)
(c)	$\frac{dx}{dy} = 8\sin 4y \cos 4y \rightarrow \frac{dy}{dx} = \frac{1}{8\sin 4y \cos 4y}$	M1
	$\frac{dy}{dx} = \frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{dy}{dx} = \frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)
Notes		
Alt by finding y first. Mark (b) and (c) together via such approaches. But note that part (c) says “Hence” and the reciprocal law must have been used at some stage to score full marks.		
(b)		
M1:	Makes y the subject and differentiates to reach form $\frac{dy}{dx} = A \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \dots$ where $\dots$ is a function of x	
A1:	For $\frac{dx}{dy} = 8\sqrt{x}\sqrt{1-x}$ oe with square roots combined. Coefficients must have been gathered.	
(c)		
M1:	For use of the reciprocal rule of derivatives evidenced in the working – allow for it being used in (b) to find $\frac{dx}{dy}$ from $\frac{dy}{dx}$.	
M1:	Award for the correct procedure of having made y the subject and differentiating to reach the correct form for the answer, ie $\frac{dy}{dx} = A \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times Bx^{-\frac{1}{2}}$	
A1:	As main scheme.	

Question Number	Scheme	Marks
3.	$\frac{dy}{dx} = \frac{4(x+3)^2 - 2(4x+1)(x+3)}{(x+3)^4}$ Solves their $4(x+3)^2 - 2(4x+1)(x+3) = 0$ $\Rightarrow (x+3)(10-4x) = 0 \Rightarrow x = \dots$ Critical value of $\frac{5}{2}$; Critical value of -3 C increasing when $\frac{dy}{dx} > 0 \Rightarrow -3 < x < \frac{5}{2}$	M1 A1 M1 A1; B1 A1 (6 marks)

Note this appears as M1A1M1A1M1A1 on open but is being marked as M1A1M1A1B1A1

M1: Attempts the quotient rule and achieves

$$\frac{dy}{dx} = \frac{a(x+3)^2 - b(4x+1)(x+3)}{(x+3)^4} \quad a, b > 0 \quad \text{condoning slips}$$

They may attempt to multiply out the $(x+3)^2$ first which is fine as long as they reach a quadratic

Product rule may be attempted, look for $\frac{dy}{dx} = A(x+3)^{-2} - B(4x+1)(x+3)^{-3}$, $A, B > 0$

Note: Only the numerator is required for this problem, and this mark can be awarded if a student attempts just $u'v - uv' = 0$. If the denominator is considered it must be treated correctly.

A1: For a correct (unsimplified) $\frac{dy}{dx}$ or $u'v - uv'$ (if this is all that is considered). By the Product rule

$$\frac{dy}{dx} = 4(x+3)^{-2} - 2(4x+1)(x+3)^{-3}$$

M1: Solves their numerator/their $\frac{dy}{dx} = 0$ (or with any inequality, and the “=0” may be implied) to

obtain at least one critical value from a non-calculator method. If the numerator is expanded to a quadratic a method of solution must be shown (e.g. factorisation or completing the square or formula seen applied) They may be working with equalities or inequalities for this mark. Note this means students who cancel the $(x+3)$ and lose the second critical value can still score M1.

A1: Critical value of $\frac{5}{2}$ (allow equivalent fractions) found provided the previous M has been scored.

B1: Finds or identifies -3 as the other critical value (no need for method shown).

A1: Depends on first M. For $-3 < x < \frac{5}{2}$ or $-3 < x, \frac{5}{2}$ or equivalent set notation, e.g.

$$x \in \left(-3, \frac{5}{2}\right] \quad \text{Condone } -3, x, \frac{5}{2} \text{ or } -3, x < \frac{5}{2}. \text{ Must be simplified fraction. Allow split}$$

inequalities $-3 < x$ and $x < \frac{5}{2}$, condoning “or” or a comma between but if set notation used it must be intersection not union.

Note: If the denominator of $\frac{dy}{dx}$ is incorrectly treated but the numerator is correct, a correct answer may be achieved. In such cases award SC M0 A0 M1 A1 B1 A1 if the subsequent marks are earned.

Note: answers relying on calculator technology where roots appear from an unsimplified derivative can score maximum M1A1M0A0B1A1



Question Number	Scheme	Marks
4 (a)	$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$	
(i)	Any correct value of $A = 4$, $B = 2$ or $C = -13$ Full method to find values for A , B and C E.g. $4x^3 + 2x^2 + 3x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$ $\text{Or } \begin{array}{r} 4x+2 \\ x^2+0x+4 \overline{) 4x^3+2x^2+3x+8} \\ \underline{-13x} \end{array}$	B1
(ii)	$A = 4, B = 2 \text{ and } C = -13$	M1
(b)	E.g. Using $4x^3 + 2x^2 + 3x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$ For example $x^2 \Rightarrow B = 2$ $x = 0 \Rightarrow 4 \times 2 + D = 8$ $D = 0$ *	A1
	$\int \left(Ax + B + \frac{Cx}{x^2 + 4} \right) dx = \frac{Ax^2}{2} + Bx + \frac{1}{2} C \ln(k(x^2 + 4)) \quad k \text{ any constant}$	B1 ft, M1 A1 ft
	$\int_1^4 \frac{4x^3 + 2x^2 + 3x - 8}{x^2 + 4} dx = \left[2x^2 + 2x - \frac{13}{2} \ln(k(x^2 + 4)) \right]_1^4$	
	$= \left(32 + 8 - \frac{13}{2} \ln 20 \right) - \left(2 + 2 - \frac{13}{2} \ln 5 \right) = 36 + \frac{13}{2} \ln \frac{1}{4} = 36 - 13 \ln 2$	dM1 A1
		(4)
		(5)
		(9 marks)

(a)(i) Note – mark part (a) as a whole.

B1: Any correct value of A , B or C . Must be correctly identified to its letter by label or position in the identity, not just a value.

M1: A full method to find values for A , B and C .

If they attempt $4x^3 + 2x^2 + 3x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$ the expression must be correct.

If they attempt by division then they must proceed to a linear quotient and a linear remainder.

Alternative notations of long division may be seen. Correct values for A , B and C will imply the method mark.

A1: Correct A , B , C or correct expression.

Note: attempts at long division but mixing up the constants to give $-13x + \frac{4x+2}{x^2+4}$ can score the method mark only.

(a)(ii)

B1*: Fully shows that $D = 0$ from clear and correct work.

Using $4x^3 + 2x^2 + 3x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$ they would need to set up (at least) two correct equations and solve, with appropriate substitutions seen, to show that $D = 0$

Using division the working must be fully correct with the remainder shown as just $-13x$ and minimal conclusion or indication $D = 0$

FYI

$$\begin{array}{r}
 4x+2 \\
 x^2+0x+4 \overline{) 4x^3+2x^2+3x+8} \\
 \underline{4x^3+0x^2+16x} \\
 2x^2-13x+8 \\
 \underline{2x^2+0x+8} \\
 -13x
 \end{array}$$

Make sure the remainder is $-13x$ and not just -13

Using verification is also acceptable, e.g. after finding values, expanding both sides with $D = 0$ in place, and checking expressions agree.

(b)

B1ft: Integrates their $Ax + B$ to $\frac{Ax^2}{2} + Bx$ ($A, B \neq 0$)

M1: Integrates their $\frac{Cx}{x^2 + 4}$ to $\dots \ln(k(x^2 + 4))$ where k is any constant (most likely $k = 1$). Condone

invisible brackets, $\ln x^2 + 4$. Condone the answer coming from $\frac{Cx}{2x} \ln(x^2 + 4)$ as long as the spurious x 's are cancelled (which may occur after substitution).

They should be using $D = 0$ to access this mark, $\frac{Cx + D}{x^2 + 4}$ with $D \neq 0$ to $\dots \ln(k(x^2 + 4))$ is M0. If

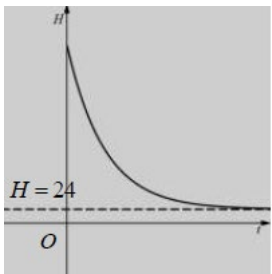
they split the fraction a valid attempt must be made at both parts in order to score the M – send to review in such instances.

A1ft: Correct integration for their $\frac{Cx}{x^2 + 4}$ Condone invisible brackets, $\ln x^2 + 4$ (oe)

dM1: Substitutes 1 and 4 into an expression of the form $Px^2 + Qx + R \ln(k(x^2 + 4))$, subtracts either way around and attempts to collect terms (condone slips combining $\ln$ terms) reaching single constant and $\ln$ terms. “Invisible brackets” on a $\ln x^2 + 4$ term must have been recovered to score this mark.

A1: $36 - 13 \ln 2$



Question Number	Scheme	Marks
5 (a)	304(°C)	B1
(b)		B1 (1) M1 Shape or Asymptote A1 Fully correct Shape and Asymptote (2)
(c)	$144 = 280e^{-0.05t} + 24 \Rightarrow 280e^{-0.05t} = 120$ Correct use of lns $\Rightarrow -0.05t = \ln \frac{120}{280} \Rightarrow t = 16.95$	M1 dM1 A1 (3)
(d)	$\left(\frac{dH}{dt}\right) = -0.05 \times 280e^{-0.05t}$ $\frac{dH}{dt} = -0.05 \times (H - 24) \Rightarrow \frac{dH}{dt} = 1.2 - 0.05H$	M1 M1, A1cso (3) (9 marks)

(a)

B1: 304

(b)

M1: Correct shape or correct asymptote. For the shape the curve must be at least in the upper half plane with negative gradient which levels out to 0, but which may be asymptotic to the x-axis. Be tolerant with curves that begin to turn back on themselves for this mark if the intent is clear.

For the asymptote it must be an asymptote for their curve (not just a dashed line with no curve approaching it), and may just be labelled 24 on the y-axis for this mark.

A1: Correct shape and asymptote. For the shape the curve must be in quadrant 1 only, starting on the H axis with negative gradient which levels to 0 at the asymptote which must be above the x-axis. It must not cross the asymptote or clearly turn back on itself, but condone touching the asymptote and slight “pen flicks” at the end.

The asymptote must be an equation, $H=24$ but condone $y=24$. Must be an equation, not just 24 labelled as the intercept of the asymptote.

(c)

M1: Substitutes $H = 144$ into the equation and proceeds at least as far as $Ae^{\pm 0.05t} = B$ condoning slips.

dM1 Uses correct ln work to find t .

$$\text{Method 1: } Ae^{-0.05t} = B \rightarrow e^{\pm 0.05t} = k \rightarrow \pm 0.05t = \ln k \rightarrow t = \dots$$

$$\text{Method 2: } Ae^{-0.05t} = B \rightarrow \ln A - 0.05t = \ln B \rightarrow t = \dots$$

May be implied by a correct f.t. value following reaching an equation $Ae^{\pm 0.05t} = B$

A1 Awrt 16.95. Condone awrt 16.95 minutes.

(d)

M1: Differentiates to a form $ke^{-0.05t}$ where $k \neq 1$ or 280 . May be called $\frac{dy}{dx}$ or unlabelled as long as it is clearly an attempt at the derivative.

M1: Rearranges the equation for H to find $e^{-0.05t}$ or a multiple of $e^{-0.05t}$ which is then substituted into their $ke^{-0.05t}$ (not dependent, so their k may be 280 for this mark). May be implied as long as a derivative has been attempted.

A1: cso $\frac{dH}{dt} = 1.2 - 0.05H$ or with fractions e.g. $\frac{dH}{dt} = \frac{6}{5} - \frac{H}{20}$, which must follow correct work and notation. The $\frac{dH}{dt}$ must have been seen to score this mark.

Alt:

M1: Makes t the subject and differentiates: $H = 280e^{-0.05t} + 24 \Rightarrow t = -20 \ln \frac{H-24}{280} \Rightarrow \left(\frac{dt}{dH} \right) = \frac{-20}{H-24}$

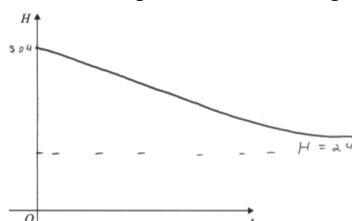
M1: Applies $\frac{dH}{dt} = 1 \div \frac{dt}{dH}$ with their attempt at $\frac{dt}{dH}$

A1: cso per main scheme.

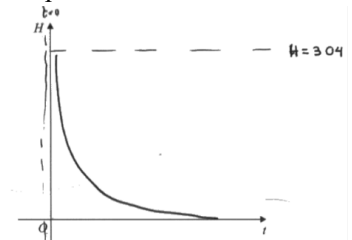
Alt 2: for Second M (first M and A as per scheme, allowing for a and b to be stated as values).

M1: Sets $\frac{dH}{dt}$ equal to $a + bH$, substitutes for H and compares coefficients to find values for a and b .

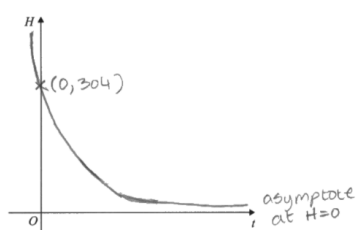
Some examples of scores for graphs:



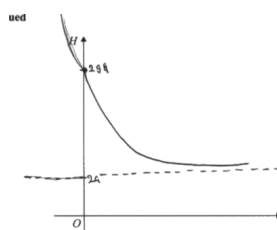
M1A0 Asymptote correct and labelled, shape incorrect



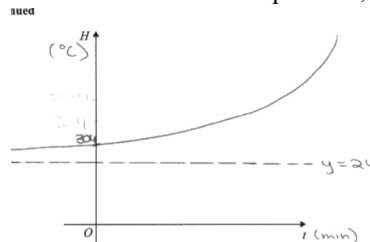
M1A0 Shape acceptable, but asymptote incorrect and does not start on H axis



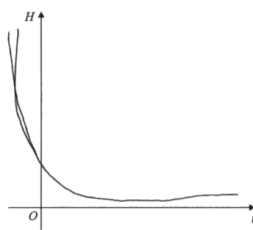
M1A0 Shape acceptable, asymptote incorrect (and curve extends to Q2)



M1A0 Asymptote correct labelled but graph extends into second quadrant, and no equation.



M1A0 Asymptote correct and labelled (and is an asymptote to their graph), shape incorrect



M0A0 Shape and asymptote both incorrect. Gradient increases above zero.



Question Number	Scheme	Marks
6 (a)	$y = (4x - 7)^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx} = \right) 2(4x - 7)^{-\frac{1}{2}}$ (see notes) At $(8, 5)$ gradient of tangent is $2(4 \times 8 - 7)^{-\frac{1}{2}} \left(= \frac{2}{5} \right)$ Equation for l is $y - 5 = -\frac{5}{2}(x - 8)$ $2y - 10 = -5x + 40 \Rightarrow 5x + 2y - 50 = 0$ *	M1 A1 dM1 ddM1 A1* (5)
(b)	$\int (4x - 7)^{\frac{1}{2}} dx = \left[\frac{(4x - 7)^{\frac{3}{2}}}{6} \right]$ (see notes) Complete area = $\int_{\frac{7}{4}}^8 (4x - 7)^{\frac{1}{2}} dx = \left[\frac{(4x - 7)^{\frac{3}{2}}}{6} \right]_{\frac{7}{4}}^8 + 5$ $= \frac{155}{6}$	M1, A1 dM1 A1 (4) (9 marks)

(a)

M1: Attempts to differentiate to achieve $a(4x \pm 7)^{-\frac{1}{2}}$ or equivalent. The index does not need to be processed.

Alternatively, attempts to differentiate implicitly $y^2 = 4x - 7 \Rightarrow Ay \frac{dy}{dx} = B$

A1: Achieves $2(4x - 7)^{-\frac{1}{2}}$ o.e. (which may be unsimplified but the index processed) or using implicit differentiation achieves $2y \frac{dy}{dx} = 4$

dM1: Substitutes $x = 8$ into their $a(4x \pm 7)^{-\frac{1}{2}}$ or substitutes $y = 5$ into their $Ay \frac{dy}{dx} = B$. It does not need to be evaluated. It is dependent on the first method mark but allow this mark to be scored if they have incorrectly manipulated their derivative before substituting in, or made transcription errors including if they lose the $-\frac{1}{2}$ index from the bracket.

ddM1: A full method for the equation of l . Look for

- substitution of $x = 8$ (or $y = 5$ if implicit differentiation) into an attempt at a derivative
- the application of the negative reciprocal rule
- the use of $(8, 5)$ with a correct gradient for their value of m to form an equation for the normal.

$y - 5 = -\frac{5}{2}(x - 8)$ with the coordinates in the correct positions. If $y = -\frac{1}{m}x + c$ is used they must proceed as far as $c = \dots$

It is dependent on both of the previous two method marks.



A1*: Correctly achieves $5x + 2y - 50 = 0$ (in any order on the same side of the equation).

There must be a correct intermediate stage of working following their initial equation for l before achieving the given answer, e.g. $y - 5 = -\frac{5}{2}(x - 8) \Rightarrow 2y - 10 = -5x + 40 \Rightarrow 5x + 2y - 50 = 0$

(b) **Note that if no integration is seen then 0 marks.**

M1: Integrates to achieve a correct form $b(4x \pm 7)^{\frac{3}{2}}$ o.e. (the index does not need to be processed)

May use substitution e.g. $u = 4x \pm 7 \Rightarrow \int \frac{u^{\frac{1}{2}}}{4} du \Rightarrow Qu^{\frac{3}{2}}$ (the integral must be correct but allow a slip on the coefficient for the integrated expression)

A1: Correct integration $\frac{(4x - 7)^{\frac{3}{2}}}{6}$ (which may be unsimplified) with or without a constant of integration. Condone poor notation e.g. if the integral sign and/or dx is still present. Index needs to be processed.

Using the substitution method score for e.g. $\frac{u^{\frac{3}{2}}}{6}$ where $u = 4x - 7$

dM1: Full method to find the shaded area. Note the area of the triangle is $\frac{1}{2} \times (10 - 8) \times 5 = 5$

Look for $\left[b(4x \pm 7)^{\frac{3}{2}} \right]_{\frac{7}{4}}^8 + 5$ o.e. (Provided this is seen then it is sufficient to proceed to the answer)

The limits must be correct for the integration

May see integration used for the triangle

$$\text{e.g. } \int_{\frac{7}{4}}^8 (4x \pm 7)^{\frac{1}{2}} dx + \int_8^{10} \left(-\frac{5}{2}x + 25 \right) dx \Rightarrow \left[b(4x \pm 7)^{\frac{3}{2}} \right]_{\frac{7}{4}}^8 + \left[-\frac{5}{4}x^2 + 25x \right]_8^{10}$$

It is dependent on the previous method mark, but you do not need to see the limits substituted in.

If they do not use the given equation of the straight line l then this mark cannot be scored.

Condone poor notation provided the intention is clear.

If using the substitution you may see e.g. $\left[\frac{u^{\frac{3}{2}}}{6} \right]_0^{25} + 5$ (send to review if unsure)

A1: $\frac{155}{6}$ or exact equivalent e.g. $25\frac{5}{6}$ or $25.8\bar{3}$ provided all the previous marks have been scored.



Question Number	Scheme	Marks
7 (a)	$\sqrt{2} \sin(x + 45^\circ) = \cos(x - 60^\circ)$ $\sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ) = \cos x \cos 60^\circ + \sin x \sin 60^\circ$ $\sin x + \cos x = \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$ $\cos x = (\sqrt{3} - 2) \sin x$ $\tan x \left(= \frac{1}{\sqrt{3} - 2} = \frac{\sqrt{3} + 2}{-1} \right) = -2 - \sqrt{3} \quad *$	M1 A1 M1 A1* (4)
(b)	States or uses $x + 45^\circ = 2\theta$ o.e. Proceeds from e.g. $\tan(2\theta - 45^\circ) = -2 - \sqrt{3} \Rightarrow 2\theta - 45^\circ = 105^\circ, 285^\circ$ Correct order of operations to find one angle $\theta = 75^\circ, 165^\circ$	B1 M1 dM1 A1 (4) (8 marks)

(a) Condone working in mixed variables

M1: Attempts to use the compound angle identities to produce an equation in $\sin x$ and $\cos x$

Look for $\sqrt{2} \times (\sin x \cos 45^\circ \pm \cos x \sin 45^\circ) = \cos x \cos 60^\circ \pm \sin x \sin 60^\circ$ o.e.

May be implied by a first line of e.g. $\sin x \pm \cos x = \frac{1}{2} \cos x \pm \frac{\sqrt{3}}{2} \sin x$ o.e.

Condone missing brackets.

A1: Correct equation in $\sin x$ and $\cos x$.

E.g. $\sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ) = \cos x \cos 60^\circ + \sin x \sin 60^\circ$ o.e. (may be implied by further work). May also see use of equivalent angles e.g. $\cos(-60)^\circ = \cos 60^\circ$

M1: For an attempt to solve the problem. Look for an attempt (condoning slips) at two of the three elements required to complete the proof, namely

- use of correct exact trigonometric values for $\sin 45^\circ, \cos 45^\circ, \cos 60^\circ, \sin 60^\circ$
- collection of like terms in $\sin x$ and $\cos x$ or $\tan x$
- use of $\tan x = \frac{\sin x}{\cos x}$

A1*: Proceeds to the given answer with all previous marks scored.

There should be no errors in the manipulation and no bracket omissions, other than a missing trailing bracket.

e.g. $\sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ) = \cos x \cos 60^\circ + \sin x \sin 60^\circ$

When proceeding from $A \sin x = B \cos x$ (where A or B may be 1) to the given answer it is acceptable

to proceed from $\frac{1}{2} \cos x = \frac{-2 + \sqrt{3}}{2} \sin x$ to the given answer as evidence of the use of $\tan x = \frac{\sin x}{\cos x}$.

Condone poor notation to be recovered provided the final answer line is written correctly. Condone working in mixed variables provided the final given answer is all in terms of x .

Allow surd work to be done via a calculator.



(b)

B1: States or uses $x + 45^\circ = 2\theta$ o.e. e.g. $\theta = \frac{x + 45^\circ}{2}$

This is implied for sight of the equation $\tan(2\theta - 45^\circ) = -2 - \sqrt{3}$

M1: Proceeds from $\tan(2\theta \pm \alpha^\circ) = -2 - \sqrt{3} \Rightarrow 2\theta \pm \alpha^\circ = 105^\circ$ or 285° where $\alpha \neq 0$

The attempt must either achieve an angle of 105° or 285° or equivalent expression (Allow in radians (3sf) which are 1.83, 4.97), or allow a general solution of e.g. $2\theta \pm \alpha^\circ = -75^\circ + (180n)^\circ$

Maybe implied by further work which is not 75° or 165°

dM1: Correct order of operations to solve their $2\theta \pm \alpha^\circ = \dots$

This is dependent on the previous method mark.

Note that $\tan(2\theta - 45^\circ) = -2 - \sqrt{3} \Rightarrow 75^\circ$ does not imply this mark. We must see either e.g.

$2\theta - 45^\circ = 105^\circ \Rightarrow \theta = \dots$ or some intermediate stage before seeing 75°

A1: Both angles $\theta = 75^\circ, 165^\circ$ with no others given within the range

Note that $\tan(2\theta - 45^\circ) = -2 - \sqrt{3} \Rightarrow 2\theta - 45^\circ = 105^\circ \Rightarrow \theta = 75^\circ, 165^\circ$ is acceptable for full marks

Alt (b) via use of $\cos(2\theta - 105^\circ) = \cos 2\theta \cos 105^\circ + \sin 2\theta \sin 105^\circ$

$$\sqrt{2} \sin 2\theta = \cos(2\theta - 105^\circ) \Rightarrow \tan 2\theta = \frac{\cos 105^\circ}{\sqrt{2} - \sin 105^\circ} \Rightarrow \theta = 75^\circ, 165^\circ$$

Note the order of the marks needs to match up to the main scheme so 0110 is possible.

B1: For achieving $\tan 2\theta = -\frac{\sqrt{3}}{3}$ o.e. so allow $\tan 2\theta = \frac{\cos 105^\circ}{\sqrt{2} - \sin 105^\circ} = \text{awrt } -0.58$

Or via double angle identities $\sqrt{3} \tan^2 \theta - 6 \tan \theta - \sqrt{3} = 0$ o.e.

M1: Attempts to use the compound angle identities (allowing sign slips when using them) to reach a form $\tan 2\theta = k$ where k is a constant not $-2 - \sqrt{3}$ (or expression in trigonometric terms such as $\cos 105^\circ$

as seen above). Allow $2\theta = -30^\circ$ o.e. (allow in radians $-\frac{\pi}{6}$) to imply this mark. Do not be concerned

by the mechanics of their rearrangement.

Alternatively, via double angle identities reaches a 3TQ in $\tan \theta$

dM1: Correct order of operations from $\tan 2\theta = k$ proceeding to $\theta = \dots$ e.g. $2\theta = -30^\circ \Rightarrow \theta = -15^\circ$ (**which must be in degrees**) can score this mark. You may need to check this if $\theta = \frac{\tan^{-1} k}{2} = \dots$ is not written

Alternatively, correctly solves their $\sqrt{3} \tan^2 \theta - 6 \tan \theta - \sqrt{3} = 0$ proceeding to $\theta = \dots$

A1: Both angles $\theta = 75^\circ, 165^\circ$ with no others given within the range

Question Number	Scheme	Marks
8	$h = 1.5x - 0.5x e^{0.02x}$	
(a)	$0 = 1.5d - 0.5d e^{0.02d} \Rightarrow e^{0.02d} = 3$ $\Rightarrow 0.02d = \ln 3 \Rightarrow d = 54.93$	M1 dM1 A1 (3)
(b)	$\left(\frac{dh}{dx}\right) = 1.5 - (0.5e^{0.02x} + 0.5x \times 0.02 e^{0.02x})$ Sets $0 = 1.5 - 0.5e^{0.02x} - 0.5x \times 0.02 e^{0.02x} \Rightarrow e^{0.02x} (0.5 + 0.5x \times 0.02) = 1.5$ $\Rightarrow e^{0.02x} = \frac{1.5}{(0.5 + 0.5x \times 0.02)}$ $\Rightarrow e^{0.02x} = \frac{1.5}{(0.5 + 0.5x \times 0.02)} = \frac{150}{50 + x} \Rightarrow x = 50 \ln \left(\frac{150}{50 + x} \right) *$	M1A1 dM1 A1* (4)
(c)	(i) awrt 31.43 (ii) awrt 30.88 metres (including units)	M1 A1 A1 (3) (10 marks)

(a) **Allow to be solved in either x or d**

M1: Sets $h = 0$, cancels by d (or takes out a factor of d) and proceeds to a form $a e^{\pm 0.02d} = b$ where $ab > 0$ and a may be 1. Condone a misread or miscopy proceeding to the form $a e^{\pm 0.2d} = b$

dM1: Solves the equation of the required form using logs. It is dependent on the previous method mark. Look for an expression for d (or x) or a multiple of d (or x) before proceeding to a value for d .

e.g. $e^{0.02d} = 3 \Rightarrow d = \frac{\ln 3}{0.02}$

e.g. $e^{-0.02d} = \frac{1}{3} \Rightarrow -0.02d \ln e = \ln \frac{1}{3} \Rightarrow d =$

Condone the slip on the index to be $\pm 0.2d$

It cannot be implied by a numerical value for d .

A1: 54.93 (m) cao (units not required but if given they must be correct) following correct log work seen and at least one intermediate stage of working. Condone $x = 54.93$. isw if they round after a correct answer is seen.

Minimum acceptable e.g. $e^{0.02d} = 3 \Rightarrow d = \frac{\ln 3}{0.02} = 54.93$ (M1dM1A1)

e.g. $0 = 1.5d - 0.5d e^{0.02d} \Rightarrow 0.02d = \ln 3 \Rightarrow d = 54.93$ (M1dM1A1)

e.g. $0 = 1.5d - 0.5d e^{0.02d} \Rightarrow 3 = e^{0.02d} \Rightarrow d = 54.93$ (M1dM0A0)

(b) **If no attempt is seen for (b) then allow differentiation seen in (a) to score in (b).**

M1: Attempts the product rule on $x e^{0.02x}$ achieving $\pm A x e^{0.02x} \pm B e^{0.02x}$ where A can be 1. It is likely to be part of an expression.

Condone a misread or miscopy proceeding to the form $\pm A x e^{0.2x} \pm B e^{0.2x}$

A1: $\left(\frac{dh}{dx} =\right) 1.5 - \left(0.5 e^{0.02x} + 0.5x \times 0.02 e^{0.02x}\right)$ o.e.

dM1: Sets $0 = 1.5 - "0.5 e^{0.02x} - 0.5x \times 0.02 e^{0.02x}"$, attempts to make $e^{\pm 0.02x}$ (or $C e^{\pm 0.02x}$) the subject and proceeds to the form $C e^{\pm 0.02x} = \frac{D}{E + Fx}$ where C can be 1.

$$\text{May see } 0 = 1.5 - "50 e^{0.02x} - x e^{0.02x}" \Rightarrow e^{0.02x} = \frac{150}{50 + x}$$

Condone slips in their rearrangement but they must take out a factor of $e^{\pm 0.02x}$ (or $C e^{\pm 0.02x}$) and divide by their bracket. If they take logs of both sides first, the rearrangement must be correct. Condone invisible brackets to be recovered or implied by further work.

Condone a misread or miscopy proceeding from $0 = 1.5 - "0.5 e^{0.02x} - 0.5x \times 0.02 e^{0.02x}"$ and attempting to make $e^{\pm 0.2x}$ (or $C e^{\pm 0.2x}$) the subject.

A1*: Achieves the given answer with no errors seen provided all previous marks have been scored. Do not allow this mark to be scored for proceeding directly from

$$e^{0.02x} (0.5 + 0.5x \times 0.02) = 1.5 \text{ to } x = 50 \ln \left(\frac{150}{50 + x} \right).$$

We must see either $e^{0.02x} = \frac{1.5}{(0.5 + 0.5x \times 0.02)}$ o.e. or an unsimplified expression for x

$$\text{e.g. } x = 50 \ln \left(\frac{1.5}{0.5 + 0.01x} \right) \text{ before achieving the given answer.}$$

(c)(i) **Check by the question. If there is a contradiction between answers, the answer in the main body of the work takes precedence.**

M1: Attempts the iteration formula "correctly" seen once.

$$\text{E.g. Award for } x_2 = 50 \ln \left(\frac{150}{80} \right) \text{ or awrt 31.4}$$

A1: awrt 31.43

(c)(ii)

A1: awrt 30.88 m (including units)

Question Number	Scheme	Marks
1	$\int \frac{x^2-5}{2x^3} dx = \int Ax^{-1} - Bx^{-3} dx = C \ln x + Dx^{-2} (+c)$	M1 dM1
	$= \frac{1}{2} \ln x + \frac{5}{4} x^{-2} + c$	A1
		(3)
		Total 3

M1: Correct **attempt** to integrate.

Score for an attempt to divide by the x^3 term forming a sum of two terms and then integrating.

Award for achieving one term in the correct form. Either $C \ln x + \dots$ **or** $\dots + Dx^{-2}$

Note that $C \ln ax$ and versions such as $k \ln 2x^3$ are also acceptable for $C \ln x$ so look at responses involving lns carefully. Ignore spurious notation e.g. $\int C \ln x$ for the M marks as long as integration has been attempted

dM1: Achieves both terms in the correct form. Score for $\pm C \ln x \pm Dx^{-2}$ or equivalent

Be aware that $C \ln ax \pm Dx^{-2}$ and other variations are also correct

A1: $\frac{1}{2} \ln x + \frac{5}{4} x^{-2} + c$ or equivalent simplest form **with the** $+ c$. E.g. $\ln \sqrt{x} + \frac{5}{4x^2} + c$

ISW after a correct answer.

Some candidates may incorporate the $+ c$ within the log so $\frac{1}{2} \ln kx + \frac{5}{4} x^{-2}$ where k is an arbitrary constant is ok.

Note that $\frac{1}{2} \ln 2x + \frac{5}{4} x^{-2} + c$ is not the simplest form and is A0. $\int \frac{1}{2} \ln x + \frac{5}{4} x^{-2} + c$ would also be A0

Attempts via integration by parts can be scored in the same way

$$\int \frac{x^2-5}{2x^3} dx = \int (x^2-5) \times \frac{1}{2} x^{-3} dx = (x^2-5) \times -\frac{1}{4} x^{-2} - \int 2x \times -\frac{1}{4} x^{-2} dx = (x^2-5) \times -\frac{1}{4} x^{-2} + \frac{1}{2} \ln x + c$$

M1: For an attempt to integrate by parts the correct way around and achieves $(x^2-5) \times px^{-2} \pm q \ln ax + c$

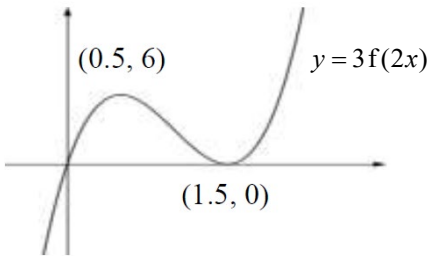
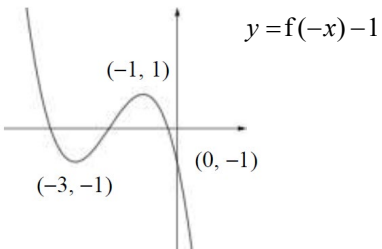
If the rule is quoted it must be correct.

It is possible to integrate by parts the other way around but unlikely. It can be scored in a similar way.

dM1: Score for

- either then simplifying to an expression of the form $\pm C \ln x \pm Dx^{-2}$ with or without "+ c" which could be numerical
- or integrating to a correct but unsimplified answer $(x^2-5) \times -\frac{1}{4} x^{-2} + \frac{1}{2} \ln ax$ with or without "+ c"

A1: $\frac{1}{2} \ln x + \frac{5}{4} x^{-2} + c$ NOT $\frac{1}{2} \ln x + \frac{5}{4} x^{-2} + \frac{1}{4} + c$ (The answer must be in simplest form and **with the** $+ c$)

Question Number	Scheme	Marks
2(i)		Shape (two way stretch) B1
	Maximum at (0.5, 6)	B1
	Minimum at (1.5, 0)	B1
		(3)
(ii)		Shape and position B1
	Minimum at (-3, -1) and maximum at (-1, 1)	B1
	Crosses y-axis at (0, -1)	B1
		(3)
		Total 6

(i)

B1: Same shape **passing through the origin** with evidence of a two way stretch.Minimum must be on the x -axis and the graph must be in quadrants 1 and 3Evidence is $(3, 0) \rightarrow (a, 0)$ where $a \neq 3$ and $(1, 2) \rightarrow (b, c)$ where $b \neq 1$ and $c \neq 2$ Condone slips of the pen and **mark positively** but the curve should **neither bend significantly** at either end **nor** consist of three straight linesB1: Maximum at (0.5, 6). Condone a " $\wedge$ " shape to the curve here.

There must be a sketch for this to be awarded.

The maximum point may be implied by the sight of 0.5 and 6 being marked on the correct axes in the correct position.

B1: Minimum at (1.5, 0). Condone a " $\vee$ " shape to the curve here.

There must be a sketch for this to be awarded.

Allow this with 1.5 marked on the x -axis (at the minimum point) and condone marked (0, 1.5) on the x -axis

(ii)

B1: Reflection in the y -axis followed by a vertical translation. Look for a $-x^3$ shaped **crossing the y -axis but not at the origin** with turning points to the left of the y -axis. Don't be concerned about the coordinates or relative "heights" of the turning points or the y intercept for this mark.

See conditions for shape in (i).

B1: A minimum at (-3, -1) and a maximum at (-1, 1) and at only these points. These may be implied. See part i second B mark. They must be in the correct quadrants and be turning points, not just points on the curve

B1: Award for a curve crossing the y -axis at (0, -1). May be awarded for a curve stopping at the y -axis at (0, -1) There must be a sketch for this to be awarded.Allow this with -1 marked on the y -axis and condone marked (-1, 0) on the y -axis

Question Number	Scheme	Marks
3(a)	$2x^2 - 3x - 5 = (2x - 5)(x + 1)$	B1
	$3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2-3x-5} = \frac{3(x+1)(2x-5) - (x-2)(2x-5) + 5x+26}{(x+1)(2x-5)}$	M1 A1
	$= \frac{(4x+1)(x+1)}{(x+1)(2x-5)} = \frac{4x+1}{2x-5}$	A1
		(4)
(b)	Correct attempt at inverse $y = \frac{4x+1}{2x-5} \Rightarrow x = \dots$	M1
	$f^{-1}(x) = \frac{5x+1}{2x-4}$	A1
		(2)
(c)	$2 < x < \frac{17}{3}$	M1 A1
		(2)
		Total 8

(a)

B1: Correct factorisation, can be scored anywhere. Sight of $2x^2 - 3x - 5 = (2x - 5)(x + 1)$ Condone $2(x - 2.5)(x + 1)$ M1: Attempts to combine **all three terms** using a common denominator. Allow the terms to be separate.There must be an **attempt** to adapt the numerators of the first two terms, one of them must be adapted correctly.

So allow for example $3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2-3x-5} = \frac{3(x+1)(2x-5) - (x-2)(2x-5) + 5x+26}{(x+1)(2x-5)}$

This may be done in stages but is only scored when all three terms are combined.

Condone a fraction where the denominator $(x+1)(2x^2 - 3x - 5)$ is used. (In this case there must be an attempt to adapt the numerators of the all terms and two of the three numerators must be adapted correctly)

$$3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2-3x-5} = \frac{3(x+1)(2x^2-3x-5)}{(x+1)(2x^2-3x-5)} - \frac{(x-2)(2x^2-3x-5)}{(x+1)(2x^2-3x-5)} + \frac{(5x+26)(x+1)}{(x+1)(2x^2-3x-5)} **$$

A1: Correct fraction with denominator $(x+1)(2x-5)$ or equivalent such as $2x^2 - 3x - 5$

Allow this to be given separately

$$3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2-3x-5} = \frac{3(x+1)(2x-5)}{(x+1)(2x-5)} - \frac{(x-2)(2x-5)}{(x+1)(2x-5)} + \frac{5x+26}{(x+1)(2x-5)}$$

If ** was given then they must proceed to $\frac{(x+1)(4x^2+5x+1)}{(x+1)(2x^2-3x-5)}$

A1: Correct fraction (or correct values). Proceeds to $\frac{4x+1}{2x-5}$ via $\frac{(4x+1)(x+1)}{(x+1)(2x-5)}$ oe.

(b)

M1: Attempts at the method for finding the inverse.

Score for an attempt to change the subject for their $y = \frac{ax+b}{cx+d}$ or possibly $y = \frac{a}{c} \pm \frac{e}{cx+d}$ Look for a minimum of cross multiplying by $cx+d$ and proceeding to a form $x = g(y)$ Some candidates may swap x and y first e.g. $x = \frac{ay+b}{cy+d}$ and proceed to $y = ..$ which is fine (same conditions)Allow this to be scored if one (but not more) of a , b or $d = 0$ Allow this to be scored for candidates who don't finish (a) and attempt to change the subject for $y = \frac{ax+b}{cx+d}$ A1: Correct inverse $f^{-1}(x) = \frac{5x+1}{2x-4}$ but condone $y = \frac{5x+1}{2x-4}$ and $f^{-1} = \frac{5x+1}{2x-4}$ Allow other equivalents such as $y = \frac{-5x-1}{4-2x}$, $y = \frac{-5x/2 - 1/2}{2-x}$ or $y = \frac{5}{2} + \frac{11}{2x-4}$

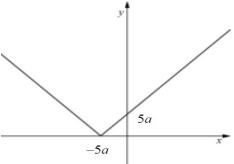
(c)

M1: For finding one "end" of the domain. Ignore any inequalities.

This must be numerical.....and you are just looking for the number, not the variable so $y = ...$ is OKSight of either $\frac{17}{3}$ or their $f(4)$ which may need to be checkedor 2 or $f(x)$ as $x \rightarrow \infty$ that is their $\frac{a}{c}$ (Can be scored for $x \neq 2$)

A1: Correct domain with allowable notation.

Allowable equivalent forms are e.g. $\left(2, \frac{17}{3}\right)$, $x > 2$ and $x < \frac{17}{3}$. Condone "or"

Question Number	Scheme	Marks
4(a)	Either $x = -\frac{a}{3}$ or $y = a$	B1
	Correct coordinates $\left(-\frac{a}{3}, a\right)$	B1
		(2)
(b)	 V shape in Quad 1 and 2 with the vertex on the negative x-axis Intersects/meets axes at $(0, 5a)$ and $(-5a, 0)$	B1
		B1
		(2)
(c)	Attempts to solve a correct equation E.g. either $x + 5a = -3x - a + a \Rightarrow x = \dots$ or $x + 5a = 3x + a + a \Rightarrow x = \dots$	M1
	$x = -\frac{5}{4}a$ or $x = \frac{3}{2}a$	A1
	$x = -\frac{5}{4}a$ and $x = \frac{3}{2}a$	A1
	$x = \dots \Rightarrow y = \dots$ using either equation	dM1
	$\left(-\frac{5}{4}a, \frac{15}{4}a\right)$ and $\left(\frac{3}{2}a, \frac{13}{2}a\right)$	A1
		(5)
		Total 9

(a)

B1: One of $x = -\frac{a}{3}$ **or** $y = a$ B1: Both coordinates correct. Allow this to be written separately as $x = -\frac{a}{3}$ **and** $y = a$

(b)

B1: Correct shape and position in both quadrant 1 and quadrant 2. Condone an asymmetric graph but the vertex must be on the negative x - axis. Condone a free hand sketch as long as the intention was to have two straight lines. Ignore any "dotted" linesB1: Correct intercepts given as coordinates or as marked in the scheme. Must be on the correct axes in the correct positions. Condone for example $(5a, 0)$ for $(0, 5a)$ if marked on the correct axis. Additional intersections is B0
If the graph is **only** in quadrant 2 this can be scored for meeting the axes at $(-5a, 0)$ and $(0, 5a)$

(c)

M1: Attempts to solve either of the correct equations. (Ones that don't involve a modulus)

Allow $x + 5a = -3x$ for $x + 5a = -3x - a + a$ and $x + 5a = 3x + 2a$ for $x + 5a = 3x + a + a$ Do not condone attempts where a candidate incorrectly combines the modulus terms even if it leads to a correct value for x . E.g. $|3x + a| - |x + 5a| = -a \Rightarrow |2x| = 3a \Rightarrow$ scores M0A1: One correct value for x . Either $x = -\frac{5}{4}a$ or $x = \frac{3}{2}a$ (following a correct non modulus equation)

Allow this mark even if the candidate subsequently rejects the solution

A1: Both values correct $x = -\frac{5}{4}a$ and $x = \frac{3}{2}a$ with no additional values given

This cannot be scored if the candidate rejects either of these solutions

dM1: Correct method to obtain at least one y value using either equation in y .Evidence would be embedded values leading to $y =$ which may be unsimplified or a correct calculation for their x A1: Both sets of coordinates $\left(-\frac{5}{4}a, \frac{15}{4}a\right)$ and $\left(\frac{3}{2}a, \frac{13}{2}a\right)$ which may be given $x = \dots, y = \dots$ with no extras.

Special case: B1 For either of the two correct coordinates which may be scored with no working or following an attempt that incorrectly combines the modulus terms. Scored 1 0 0 0 0



Question Number	Scheme	Marks
5(a)	$18 = A - 180 \times 1 \Rightarrow A = \dots$	M1
	$A = 198$	A1
		(2)
(b)	$90 = 198 - 180e^{-5k} \Rightarrow 180e^{-5k} = 108$	M1, A1
	$e^{-5k} = \frac{108}{180} \Rightarrow -5k = \ln 0.6 \Rightarrow k = \dots$	dM1
	$k = -\frac{1}{5} \ln \frac{3}{5} \text{ or } k = \frac{1}{5} \ln \frac{5}{3}$	A1
		(4)
(c)	$\theta = 198 - 180e^{-9 \times \frac{1}{5} \ln \frac{5}{3}} \Rightarrow \theta = \dots$	M1
	$\theta = 126^\circ\text{C}$ (awrt)	A1
		(2)
(d)	$\left\{ \frac{d\theta}{dt} \right\} = -180 \left(-\frac{1}{5} \ln \frac{5}{3} \right) e^{-\frac{t}{5} \ln \frac{5}{3}} = \dots$	B1ft
	$\left\{ \frac{d\theta}{dt} \right\} = -180 \left(-\frac{1}{5} \ln \frac{5}{3} \right) e^{\frac{9}{5} \ln \frac{5}{3}} = \dots$	M1
	$= 7.33 \text{ }^\circ\text{C min}^{-1}$ (awrt)	A 1
		(3)
		Total 11



(a)

M1: Substitutes $\theta = 18$ and $t = 0$ uses $e^0 = 1$ and proceeds to find a value for A . Look for $18 = A - 180 \times 1 \Rightarrow A = \dots$ A1: $A = 198$. Condone $A = 198^\circ\text{C}$. Sight of 198 is sufficient to award both marks.

(b)

M1: Substitutes $\theta = 90$ and $t = 5$ with their value for A and proceeds to an equation of the form $Pe^{\pm 5k} = Q$ A1: Correct equation $180e^{-5k} = 108$ o.e.dM1: Correct order of operations using ln's to make k the subject. Do not award if taking $\log_{10}$'s. ($\log \frac{3}{5} = -0.22\dots$)

$$Pe^{\pm 5k} = Q \Rightarrow \pm 5k = \ln\left(\frac{P}{Q}\right) \Rightarrow k = \dots \text{ with } \frac{P}{Q} > 0 \quad \text{OR} \quad Pe^{\pm 5k} = Q \Rightarrow \ln P \pm 5k = \ln Q \Rightarrow k = \dots \text{ with } P, Q > 0$$

It would be implied by a decimal equivalent to 3sf. So for correct values accept $k = \text{awrt } 0.102$ A1: Cao. Allow equivalent correct exact answers in the required form. E.g. $k = -0.2 \ln 0.6$ and $k = \frac{1}{5} \ln \frac{180}{108}$

(c)

M1: Substitutes their A and their k with $t = 9$ to find a value for θ .

Sight of embedded values is sufficient evidence and condone sign slips which may be common.

A1: Awrt 126°C but condone a lack of units. Sight of awrt 126 following $k = \text{awrt } 0.102$ is sufficient to award both marks.

(d)

B1ft: Correct differentiation. Follow through their k (even as a decimal or as " k "). Allow for $180ke^{-kt}$ M1: Substitutes $t = 9$ into a function of the form $\left(\frac{d\theta}{dt} = \right) \beta e^{-kt}$ following through on their k . Condone $\frac{d\theta}{dt} = \frac{dy}{dx}$

A1: Awrt 7.33 Ignore units.

Award B0 M1 A1 for candidates who achieve $\pm \text{awrt } 7.33$ following a sign error in their $\frac{d\theta}{dt}$

Answers without working in (d).

The rubric states that candidates are required to show sufficient working to make their method clear.

They must have a value of k to do this part.

SC: B1ft scored for

- either stating $\frac{d\theta}{dt}$ at $t = 9$ is You will need check. Allow accuracy to 3sf and ft on their k
- or following correct k just writing down awrt 7.33



Question Number	Scheme	Marks
7 (a)	$f(x) = x^3 \sqrt{4x+7} \Rightarrow f'(x) = 3x^2 \sqrt{4x+7} + 2x^3 (4x+7)^{-\frac{1}{2}}$	M1, A1
	$\Rightarrow f'(x) = 3x^2 \sqrt{4x+7} + \frac{2x^3}{\sqrt{4x+7}} = \frac{3x^2(4x+7) + 2x^3}{\sqrt{4x+7}}$	dM1
	$\Rightarrow f'(x) = \frac{7x^2(2x+3)}{\sqrt{4x+7}}$	A1
		(4)
(b)	Substitutes $x = -\frac{3}{2}$ into $x^3 \sqrt{4x+7} \Rightarrow y = \dots$	M1
	$\left(-\frac{3}{2}, -\frac{27}{8}\right)$	A1
		(2)
(c)	Attempts $-4 \times -\frac{27}{8}$	M1
	$y = \frac{27}{2}$	A1
		(2)
(d)	$(2, 7)$	M1, A1
		(2)
		Total 10

(a)

M1: Attempts the product rule to achieve $f'(x) = Px^2 \sqrt{4x+7} + Qx^3 (4x+7)^{-\frac{1}{2}}$ where P and Q are positive constants.

Other methods are possible so look at each one carefully.

For example: $f(x) = x^3 \sqrt{4x+7} \Rightarrow f(x) = \sqrt{4x^7 + 7x^6} \Rightarrow f'(x) = \frac{1}{2} (4x^7 + 7x^6)^{-\frac{1}{2}} \times (28x^6 + 42x^5)$

The same principles for the main mark scheme can be applied

A1: $f'(x) = 3x^2 \sqrt{4x+7} + 2x^3 (4x+7)^{-\frac{1}{2}}$ o.e. which may be left unsimplified

dM1: Dependent upon previous M1. It is for "correctly" producing a single fraction with a denominator of $\sqrt{4x+7}$.

Look for $f'(x) = Px^2 \sqrt{4x+7} + Qx^3 (4x+7)^{-\frac{1}{2}} \Rightarrow \frac{Px^2(4x+7) + Qx^3}{\sqrt{4x+7}}$ o.e

A1: $f'(x) = \frac{7x^2(2x+3)}{\sqrt{4x+7}}$

.....
It is possible to attempt this by WMA14 methods using $[f(x)]^2 = x^6(4x+7) = 4x^7 + 7x^6$

Score

M1, A1: $2f(x)f'(x) = 28x^6 + 42x^5$

dM1, A1: $f'(x) = \frac{14x^5(2x+5)}{2f(x)} = \frac{14x^5(2x+5)}{2x^3 \sqrt{4x+7}} = \frac{7x^2(2x+5)}{\sqrt{4x+7}}$



(b)

M1: Attempts to substitute $x = -\frac{3}{2}$ into $x^3\sqrt{4x+7}$ to find the y value of the minimum point.

This may be implied by the correct y value.

It can be attempted even if part (a) is missing or incorrect

If they ignore the given form of $f'(x)$ and use their own version, they must

- Set their $f'(x)$ which must be of the form $Px^2\sqrt{4x+7} + Qx^3(4x+7)^{-\frac{1}{2}} = 0$
- Solve $Px^2\sqrt{4x+7} + Qx^3(4x+7)^{-\frac{1}{2}} = 0$ by multiplying by $(4x+7)^{\frac{1}{2}}$ o.e. to achieve a non-zero value for x
- Substitute this non zero value for x in into $x^3\sqrt{4x+7}$ to find y

A1: $\left(-\frac{3}{2}, -\frac{27}{8}\right)$ which may be awarded separately as, for example, $x = -1.5, y = -3.375$

Ignore (0, 0) if also given.

(c)

M1: Attempts $\pm 4 \times -\frac{27}{8}$ which may be seen within an inequality.

A1: $y, \frac{27}{2}$ which must be correctly simplified.

Allow it to be written in other correct forms such as $g, \frac{27}{2}, g(x), \frac{27}{2}$ or $(-\infty, 13.5]$

The original function is f so $f(x), \frac{27}{2}$ would be M1 A0

(d)

M1: For one correct coordinate. Look for $(2, \dots)$ or $(\dots, 7)$. Allow for either $x = 2$ or $y = 7$

If the coordinates have been built up mark the final answer.

E.g. $\left(\frac{1}{2}, \frac{3}{8}\right) \rightarrow \left(2, \frac{3}{8}\right) \rightarrow (2, 15) \rightarrow (-6, 15)$ would score M0 A0

A1: $(2, 7)$. Allow $x = 2$ and $y = 7$ ISW after a correct answer

Question Number	Scheme	Marks
8 (a)	52 b.p.m.	B1
		(1)
(b)	32 b.p.m.	B1
		(1)
(c)	$\frac{dH}{dt} = -8e^{-0.2t} + 18e^{-0.9t}$	M1, A1
	Sets $-8e^{-0.2t} + 18e^{-0.9t} = 0 \Rightarrow 4e^{0.7t} = 9$	dM1
	$\Rightarrow 0.7t = \ln \frac{9}{4} \Rightarrow t = \dots$	M1
	$T = 1.158 \text{ (minutes)}$	A1
		(5)
(d)	$37 = 32 + 40e^{-0.2t} - 20e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1 + 4e^{-0.9t}}{8}$	M1
	$\Rightarrow e^{0.2t} = \frac{8}{1 + 4e^{-0.9t}} \Rightarrow t = 5 \ln \left(\frac{8}{1 + 4e^{-0.9t}} \right)$	A1*
		(2)
(e)	$t_2 = 5 \ln \left(\frac{8}{1 + 4e^{-0.9 \times 10}} \right) = \dots$	M1
	$(t_2) = \text{awrt } 10.3947$	A1
	$(M) = 10.3955$	A 1
		(3)
		Total 12

(a)

B1: 52 b.p.m. Units are not required. Check for answer given in the body of the question

(b)

B1: 32 b.p.m. . Units are not required. Check for answer given in the body of the question

(c)

M1: Achieves $\frac{dH}{dt} = P e^{-0.2t} + Q e^{-0.9t}$ where P and Q are constantsA1: $\frac{dH}{dt} = -8e^{-0.2t} + 18e^{-0.9t}$ but allow un-simplified versionsdM1: Sets $P e^{-0.2t} + Q e^{-0.9t} = 0 \Rightarrow m e^{0.7t} = n$ or $m e^{-0.7t} = n$ where $m \times n > 0$ M1: Solves an equation of the form $m e^{\pm kt} = n$ where $m \times n > 0$ using correct $\ln$ work.
It must proceed from a correct form of the derivative (so the first M1).Look for $m e^{\pm kt} = n \Rightarrow e^{\pm kt} = \frac{n}{m} \Rightarrow \pm kt = \ln \left(\frac{n}{m} \right) \Rightarrow t = \dots$ A1: $T = \text{awrt } 1.158 \text{ minutes}$ following the award of all previous marks. You can ignore units.....
An alternative for the last three marks in part (c) are;dM1: $P e^{-0.9t} = Q e^{-0.2t} \Rightarrow \ln P - 0.9t = \ln Q - 0.2t$ M1: Collects terms in t and proceeds to $t = \dots$ Under this method it is dependent upon both previous M'sA1: $T = \text{awrt } 1.158 \text{ minutes}$ following the award of all previous marks. Units are not required

Question Number	Scheme	Marks
8(a)	200	B1
		(1)
(b)	600	B1
		(1)
(c)	$500 = \frac{600e^{0.3t}}{2 + e^{0.3t}} \Rightarrow 100e^{0.3t} = 1000, \Rightarrow e^{0.3t} = 10$	M1, A1
	$\Rightarrow t = \frac{\ln 10}{0.3} = 7 \text{ years } 8 \text{ months}$	dM1 A1
		(4)
(d)	$N = \frac{600e^{0.3t}}{2 + e^{0.3t}} \Rightarrow \frac{dN}{dt} = \frac{(2 + e^{0.3t}) \times 180e^{0.3t} - 180e^{0.3t} \times e^{0.3t}}{(2 + e^{0.3t})^2}$	M1 A1
	$\Rightarrow \frac{dN}{dt} = \frac{360e^{0.3t}}{(2 + e^{0.3t})^2}$	A1
(e)		(3)
	$8 = \frac{360e^{0.3t}}{(2 + e^{0.3t})^2} \Rightarrow 8(e^{0.3t})^2 - 328(e^{0.3t}) + 32 = 0$	M1
	$e^{0.3t} = \frac{41 + \sqrt{1665}}{2}$	dM1
	$(t =) \frac{\ln\left(\frac{41 + \sqrt{1665}}{2}\right)}{0.3}$	ddM1
	$(T =) \text{awrt } 12.4$	A1
		(4)
		(13 marks)

(a)

B1: 200

(b)

B1: 600

(c)

M1: Sets $N = 500$ and proceeds to $Ce^{0.3t} = D$ or equivalentA1: $e^{0.3t} = 10$. If ln's are taken earlier it would be for $\ln 100 + 0.3t = \ln 1000$ dM1: Full method to find a value for t using correct log work.A1: 7 years 8 months. Accept 7 years 9 months following a correct value of t .

(d)

M1: Uses the quotient rule to obtain an expression of the form $\frac{(2 + e^{0.3t}) \times ae^{0.3t} - be^{0.3t} \times e^{0.3t}}{(2 + e^{0.3t})^2}$

In general condone missing brackets for the M mark.

A correct rule may be implied by their u, v, u', v' followed by applying $\frac{vu' - uv'}{v^2}$ etc.

E.g. if they quote $u = 600e^{0.3t}$ and $v = 2 + e^{0.3t}$ and do not make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: Correct derivative in any form e.g. $\frac{dN}{dt} = \frac{(2 + e^{0.3t}) \times 180e^{0.3t} - 180e^{0.3t} \times e^{0.3t}}{(2 + e^{0.3t})^2}$

A1: Correctly obtains $\frac{dN}{dt} = \frac{360e^{0.3t}}{(2 + e^{0.3t})^2}$

Withhold this mark if you see $e^{0.3t} \times e^{0.3t}$ written as $e^{0.3t^2}$ or $e^{0.09t}$

Alt (d)

M1 A1: $N = \frac{600e^{0.3t}}{2 + e^{0.3t}} \Rightarrow N = 600 - \frac{1200}{2 + e^{0.3t}} \Rightarrow \frac{dN}{dt} = \frac{1200 \times 0.3e^{0.3t}}{(2 + e^{0.3t})^2}$

Score M1 for splitting $\frac{600e^{0.3t}}{2 + e^{0.3t}}$ into $A \pm \frac{B}{2 + e^{0.3t}}$ leading to $\frac{dN}{dt} = \frac{ke^{0.3t}}{(2 + e^{0.3t})^2}$

and A1 for correct derivative in any form.

A1: $\frac{dN}{dt} = \frac{360e^{0.3t}}{(2 + e^{0.3t})^2}$ (following fully correct work)

May also see product rule in (d):

$$\begin{aligned} N &= \frac{600e^{0.3t}}{2 + e^{0.3t}} = 600e^{0.3t} (2 + e^{0.3t})^{-1} \Rightarrow \frac{dN}{dt} = 180e^{0.3t} (2 + e^{0.3t})^{-1} - 180e^{0.3t} \times e^{0.3t} (2 + e^{0.3t})^{-2} \\ &= \frac{180e^{0.3t}}{2 + e^{0.3t}} - \frac{180e^{0.6t}}{(2 + e^{0.3t})^2} = \frac{360e^{0.3t} + 180e^{0.6t} - 180e^{0.6t}}{(2 + e^{0.3t})^2} = \frac{360e^{0.3t}}{(2 + e^{0.3t})^2} \end{aligned}$$

Score M1 for an expression of the form $ae^{0.3t} (2 + e^{0.3t})^{-1} - be^{0.3t} \times e^{0.3t} (2 + e^{0.3t})^{-2}$ and A1 A1 as above.

A correct rule may be implied by their u, v, u', v' followed by applying $vu' + uv'$ etc.

(e)

M1: Sets $\frac{dN}{dt} = 8$ and proceeds to a quadratic in $e^{0.3t}$

dm1: Correct attempt to solve the quadratic in $e^{0.3t}$. Condone both roots to be found

ddM1: Correct attempt to find the value of t for $e^{0.3t} = k$ where $k > 0$ using correct log work

A1: Achieves ($T =$) awrt 12.4



Question Number	Scheme	Marks
1 (a)	$P_0 = 300$	B1 (1)
(b)	$420 = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \Rightarrow 60e^{0.12t} = 420$ Correct use of lns $\Rightarrow t = \frac{\ln 7}{0.12} = 16.22$	M1 A1 dM1 A1 (4)
(c)	States that maximum number (upper limit) is 450 so cannot reach 500	B1 (1)
		6 marks

(a)

B1 300

(b)

M1 Substitutes $N = 420$ and proceeds to $Ae^{0.12t} = B$ condoning slipsA1 $60e^{0.12t} = 420$ oedM1 Uses correct ln work to find t . This must be from a solvable equation.Method 1: $Ae^{0.12t} = B \rightarrow e^{0.12t} = k \rightarrow 0.12t = \ln k \rightarrow t = \dots$ ($k > 0$)Method 2: $Ae^{0.12t} = B \rightarrow \ln A + 0.12t = \ln B \rightarrow t = \dots$ ($A, B > 0$)

A1 Awrt 16.22 (years)

Note: **Answers without working** (even to accuracy of 1 dp) can score SC 1100.

Eg. $420 = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \Rightarrow t = 16.2$



(c)

B1 May be tackled in a variety of ways. **Requires a reason and a conclusion**

E.g.

- States that the **upper** limit (or maximum value) is 450 so cannot reach 500. Note that it is acceptable to state that the number of toads cannot exceed 449. (Allow here $N < 450$ for upper limit is 450)
- The conclusion can be implied by a statement such as "as the maximum value is 450"
- Alternatively substitutes $500 = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \Rightarrow 100e^{0.12t} = -500$ or similar and states this cannot be found as ln's cannot be taken of negative numbers, hence cannot ever be 500. (Allow the candidate to state maths error, exponentials cannot be negative to form part of their reason)

The calculations here must be correct and the proof complete.

The following 3 examples score B0 as they are incomplete and/or incorrect

1) $500 = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \Rightarrow e^{0.12t} = -5$ which cannot be found. (Requires a conclusion)

2) The limit is 450 so they cannot reach 500. (Requires upper limit)

3) $500 = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \Rightarrow e^{0.12t} = -0.01$ which cannot be found and so they cannot reach 500.

(Requires correct calculation)



Question Number	Scheme	Marks
2.(a)	$fg(e^2) = f\left(\frac{5}{2}\ln e^2\right) = \frac{12}{\frac{5}{2}\ln e^2 + 1}, = 2$	M1, A1 (2)
(b)	$f(x) = \frac{12}{x+1}$ $f^{-1}(x) = \frac{12}{x} - 1$ $0 < x < 12$	M1 A1 B1 (3)
(c)	$\frac{12}{x+1} = \frac{12}{x} - 1 \Rightarrow 12x = 12(x+1) - x(x+1)$ $\Rightarrow x^2 + x - 12 = 0 \Rightarrow x = \dots$ $x = 3$ only	Must be 3TQ dM1 A1 (3) 8 marks
(c) Alts	Solves $f^{-1}(x) = x \Rightarrow \frac{12}{x} - 1 = x$ leading to quadratic equation, or solves $f(x) = x \Rightarrow \frac{12}{x+1} = x$ leading to quadratic equation $\Rightarrow x^2 + x - 12 = 0 \Rightarrow x = \dots$ $x = 3$ only	M1 dM1 A1 (3)

(a)

M1 Correct order of operations applying g before f on e^2 .

Allow an attempt to substitute $x = e^2$ into $fg(x) = \frac{12}{\frac{5}{2}\ln x + 1}$

This must be a complete attempt using all aspects of both functions but allow for slips on the coefficients

A1 cso 2.



(b)

M1 Changes the subject of $y = \frac{12}{x+1}$ or $x = \frac{12}{y+1}$ to one of the appropriate forms. See below.

Allow for $x = \frac{12}{y} \pm 1$ or $x = \frac{12 \pm y}{y}$ (Only error allowed is a slip in sign)

Or $y = \frac{12}{x} \pm 1$ or $y = \frac{12 \pm x}{x}$ when x and y are swapped

A1 $f^{-1}(x) = \frac{12}{x} - 1$ or $f^{-1}(x) = \frac{12-x}{x}$ **with correct notation**. Condone $f^{-1}(x) = y = \frac{12}{x} - 1$

$f^{-1} : x \mapsto \frac{12}{x} - 1$ is fine as is any other variable used consistently, e.g. $f^{-1}(y) = \frac{12}{y} - 1$

but $f^{-1} = \frac{12}{x} - 1$ is A0 (as it is incomplete) and $y = \frac{12}{x} - 1$ is also A0 (not set in the appropriate form)

B1 Gives a correct domain for the function $0 < x < 12$ or equivalent "Domain $\in (0, 12)$ "

(c)

M1 Attempts to set $f^{-1}(x) = f(x)$, $f^{-1}(x) = x$ or $f(x) = x$ and proceeds to a quadratic equation

The quadratic equation does not need to be simplified. Allow this mark for using **their** $f^{-1}(x)$

dM1 Solves a 3TQ leading to at least one value for x . Apply the usual rules.

This is dependent upon the previous M mark

A1 $x = 3$ only

Question Number	Scheme	Marks
3.(a)	Implies equation of line is of the form $\log_{10} y = \pm \frac{2}{3} \log_{10} x + 4$	M1
	States $\log_{10} y = -\frac{2}{3} \log_{10} x + 4$ o.e.	A1
(b)	Applies one correct log law E.g $\log_{10} y = -\frac{2}{3} \log_{10} x + 4 \rightarrow \log_{10} y = \log_{10} x^{-\frac{2}{3}} + 4$	M1
	Full attempt to undo the logs $\log_{10} y = \log_{10} x^{-\frac{2}{3}} + \log_{10} 10^4 \rightarrow y = x^{-\frac{2}{3}} \times 10^4$	dM1
	$\rightarrow y = 10\,000x^{-\frac{2}{3}}$ o.e.	A1
		(3) 5 marks

Note that other versions of $\log_{10}$ are acceptable. So allow $\log_{10} \leftrightarrow \log$ and $\log_{10} \leftrightarrow \lg$

(a)

M1 Implies equation of line is of the form $\log_{10} y = \pm \frac{2}{3} \log_{10} x + 4$ oe

(Eg allow for M1 A0 $\log_{10} y = \pm 0.67 \log_{10} x + 4$) Condone " $y = \pm \frac{2}{3} x + 4$ " here

A1 $\log_{10} y = -\frac{2}{3} \log_{10} x + 4$ oe such as $3 \log_{10} y + 2 \log_{10} x = 12$

Allow unsimplified equations such as $\log_{10} y = -\frac{4}{6} \log_{10} x + 4$

This may be implied by the constants if the full equation is used in part (b).

So marks in (a) can be awarded from work in (b).

(b) **Main Method: Starting from $\log_{10} y = a \log_{10} x + b$ and working towards $y = px^q$**

M1 Uses one correct log law. Look for either

$$\log_{10} y = a \log_{10} x + b \rightarrow \log_{10} y = \log_{10} x^a + b$$

$$\log_{10} y = a \log_{10} x + b \rightarrow \log_{10} y = a \log_{10} x + \log_{10} 10^b$$

$$\log_{10} y = a \log_{10} x + b \rightarrow y = 10^{a \log_{10} x + b}$$

dM1 A full attempt to get y in terms of x or values of p and q

Cannot be awarded from easier equations, e.g. where $b = 0$

$$\text{Look for } \log_{10} y = a \log_{10} x + b \rightarrow y = x^a \times 10^b$$

A1 $y = 10\,000x^{-\frac{2}{3}}$ but condone $y = 10^4 x^{-\frac{2}{3}}$

This requires the equation and not just the values of p and q

It is acceptable to just write down the answer, but it must follow a correct part (a)

Question Number	Scheme	Marks
3(a)	$\log_{10} y = \frac{5}{16}x + 1.5$	M1A1
		(2)
(b)	$\log_{10} y = \frac{5}{16}x + 1.5 \Rightarrow y = 10^{\frac{5}{16}x + 1.5}$	M1
	$\Rightarrow y = 10^{\frac{5}{16}x} \times 10^{1.5}$	M1
	$y = 31.6 \times 2.05^x$	A1
		(3)
		Total 5

(a)**M1:** Scored for a complete attempt to get the equation of the line condoning $\log_{10} y \leftrightarrow y \leftrightarrow l$ and an incorrectsign on the gradient. So, allow for $(\log_{10} y) = \pm \frac{1.5}{4.8}x + 1.5$ o.e. and for $\frac{y-0}{x+4.8} = \pm \frac{1.5}{4.8}$ If this is attempted via simultaneous equations the mark is scored when the candidate reaches $m = \pm 0.3125$
 $c = 1.5$ **A1:** Correct equation. e.g. $\log_{10} y = \frac{5}{16}x + 1.5$ or equivalent such as $16\log_{10} y = 5x + 24$ The $\log_{10}$ must not appear as "ln" but allow as "log" or "lg"**(b)**Main Method: Starting with their $\log_{10} y = mx + c$ **M1:** "Removes" the logs in their equation. e.g. $\log_{10} y = "m"x + "c" \Rightarrow y = 10^{m"x + "c"}$ **M1:** "Correct" strategy to obtain values of k and b or else proceeding correctly to a form $y = 10^{\frac{5}{16}x} \times 10^{1.5}$ Allow for $k = 10^{1.5}$ ($= 31.6$) and $b = 10^{\frac{5}{16}} = (2.05)$. Note that you may see $10^{1.5}$ as $10\sqrt{10}$ **A1:** Correct equation produced, $y = 31.6 \times 2.05^x$, and no errors seen.Condone correct working followed by $k = 31.6, b = 2.05$ with the equation being impliedcso the values must be 31.6 and 2.05, not values rounding to these numbers or exact values like $10\sqrt{10}$.**Note:** A solution may be fudged from working similar to the following

$$\log_{10} y = mx + c \Rightarrow y = 10^{mx} + 10^c \Rightarrow y = 31.6 + 2.05^x \Rightarrow y = 31.6 \times 2.05^x$$

This will score Special case: M0 M1 A0Alternative Method: Starting with $y = kb^x$ **M1:** Takes logs of both sides and applies at least one correct log lawe.g. $\log_{(10)} kb^x = \log_{(10)} k + \log_{(10)} b^x$, $\log_{(10)} b^x = x \log_{(10)} b$ **M1:** "Correct" strategy to obtain values for k and b from their $y = mx + c$ So $\log_{10} k = "c" \Rightarrow k = 10^c$ and $\log_{10} b = "m" \Rightarrow b = 10^m$

A1: Correct equation produced $y = 31.6 \times 2.05^x$ and no errors seen.

Condone correct working followed by $k = 31.6, b = 2.05$ with the equation being implied

cso the values must be 31.6 and 2.05, not values rounding to these numbers.

Correct answers for k and b without any working scores M0 M1 A0. Instructions on the paper state that they should show sufficient working to make their method clear.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1. (a)	$g(3) = -265, \quad g(4) = 3104$ States change of sign, continuous and hence root in $[3, 4]$	M1 A1 (2)
(b)	$x_2 = \sqrt[6]{1000 - 2 \times 3} = 3.1591$ $(\alpha =) 3.1589$	M1 A1 A1 (3)
(5 marks)		
Notes		
(a)		
M1	Attempts the value of g at 3 and 4 with one correct (accept any value for the other as an attempt). Note narrower ranges are possible but must contain the root and lies in [3,4].	
A1	Both values correct with reason (Sign change (stated or indicated) and continuous function) and minimal conclusion (root)	
(b)		
M1	Attempts to substitute $x_1 = 3$ into the formula. Implied by sight of expression, awrt 3.159	
A1	awrt 3.1591	
A1	$(\alpha =) 3.1589$ cao - must be to 4 d.p. Do not be concerned about the labelling of the root (x or α etc), mark the final answer of (b)(ii). (Note sight of this value implies the M1 even if x_2 is not seen).	

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1. Factorisation

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$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

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Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

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Question Number	Scheme	Marks
1 (a)	(5,10)	B1 B1 (2)
(b)	Attempts to solve e.g. $-2(x-5)+10 \dots 6x \Rightarrow x \dots \frac{5}{2}$ $x < \frac{5}{2}$ o.e.	M1 A1 (2)
(c)	(7, 30)	B1ft, B1ft (2) (6 marks)

Condone missing brackets around pairs of coordinates provided the meaning is clear

(a) If there is a contradiction between coordinates on the graph, by the question and in the main body of the work then the work in the main body of the work takes precedence.

B1: One correct coordinate. Allow as $x = 5$ or $y = 10$ Do not allow them to be the wrong way round

B1: Both coordinates correct. Allow as $x = 5, y = 10$

(b) Answers with no working scores 0 marks

M1: Attempts to solve a correct equation or inequality with at least one intermediate stage of working. Allow this mark even if there are additional equations/inequalities. The modulus signs must have been removed.

Alternatively, they may attempt to square both sides and then attempt to solve the quadratic (usual rules apply). If via a calculator the root(s) must be correct (you may need to check these)

Condone slips including when squaring and multiplying out

$$|x-5| \dots 3x-5 \Rightarrow x^2 - 10x + 25 \dots 9x^2 - 30x + 25 \Rightarrow 0 \dots 8x^2 - 20x \Rightarrow x \dots$$

In either method condone slips in their rearrangement and do not be concerned by the direction of the inequality sign for this mark.

A1: $x < \frac{5}{2}$ o.e. **only**. isw if they attempt to simplify their fraction incorrectly. Must be identified as their only inequality e.g. circling, underlining.

Allow other forms such as $x \in \left(-\infty, \frac{5}{2}\right)$ Do not accept $x \leq \frac{5}{2}$

Do not withhold this mark for incorrect use of inequality signs in earlier work provided the final answer is correct.

(c)

B1ft: One correct coordinate. Allow as $x = 7$ or $y = 30$

Follow through on their $((a)+2, (a) \times 3)$ Allow as $x = (a)+2$ or $y = (a) \times 3$ **(Does not have to be evaluated)**

B1ft: Correct coordinates (7, 30) Allow as $x = 7, y = 30$

or correct ft $((a)+2, (a) \times 3)$. Allow as $x = (a)+2, y = (a) \times 3$ **(Must be evaluated)**



Question Number	Scheme	Marks
2	$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$	
(a)	Sets $2x^2 - 5x + 8 = (Ax + B)(x - 2) + C$ with an attempt at one constant Attempts all 3 constants. E.g. Sets $x = 2 \Rightarrow C = \dots, -2B + C = 8 \Rightarrow B = \dots$ and compares x^2 terms gives $A = \dots$ $2x - 1 + \frac{6}{x - 2}$	M1 dM1 A1 (3)
(b)	$\int 2x - 1 + \frac{6}{x - 2} dx = x^2 - x + 6 \ln(x - 2)$ $\int_4^8 2x - 1 + \frac{6}{x - 2} dx = \left[x^2 - x + 6 \ln(x - 2) \right]_4^8 = 56 + 6 \ln 6 - 12 - 6 \ln 2$ $= 44 + 6 \ln 3$	M1 A1ft dM1 A1 (4) (7 marks)

ANSWERS IN (a) of $A = 2, B = -5, C = 2$, and/or $28 + 2 \ln 3$ in (b) please send to review

(a) **Note that correct values for A, B, and C imply the first two marks**

M1: Attempts to find one of the constants via

- an identity e.g. $2x^2 - 5x + 8 = (Ax + B)(x - 2) + C$

Note that if an identity is used it must be correct. Condone arithmetical slips when evaluating.

The identity may be implied by correct equations

e.g. $x^2 : A = 2$, $x : -2A + B = -5$, constant : $-2B + C = 8$

- division is used look for a quotient as far as $2x$

$$\begin{array}{r} 2x-1 \\ x-2 \overline{) 2x^2-5x+8} \\ \underline{2x^2-4x+8} \\ 6 \end{array}$$

Note that the division may appear in many different formats or may be by inspection so $2x$ is sufficient to score this mark.

$$\text{e.g. } \frac{2x^2 - 5x + 8}{x - 2} = \frac{2x(x - 2) + 4x - 5x + 8}{x - 2} = 2x + \frac{-(x - 2) - 2 + 8}{x - 2} = 2x - 1 + \frac{6}{x - 2}$$

e.g.

$$\begin{array}{r|rrr} & 2x & -1 & \\ \hline x & 2x^2 & -x & 6 \\ -2 & -4x & 2 & \end{array}$$



Question Number	Scheme	Marks
1. (a)	$f(x) = 2\sec x + 6x - 3$ $f(0.1) = -0.39 \quad f(0.2) = 0.24$ States change of sign, continuous and hence root	M1 A1 (2)
(b)	Sets $f(x) = 0$, uses $\sec x = \frac{1}{\cos x}$ and makes x of $6x$ the subject $\Rightarrow 6x = 3 - \frac{2}{\cos x} \Rightarrow x = \frac{1}{2} - \frac{1}{3\cos x}^*$	B1* (1)
(c)	(i) $x_2 = \frac{1}{2} - \frac{1}{3\cos 0.15} = 0.16..$ $x_2 = \text{awrt } 0.1629$ (ii) $\alpha = 0.1622$	M1 A1 A1 (3) (6 marks)

Note: If the student uses degrees mode they will get values -0.39999 and 0.200001 in (a) and $0.1666...$ in (c). An answer in (c) of $0.16666...$ would be an indicator of the mode used for part (a) if rounded values are given see notes below.

(a)

M1: Attempts the value of $f(x)$ at 0.1 and 0.2 with one **correct** to at least 1 sf rounded or truncated.

Note that degrees mode gives -0.39999 and 0.200001 and these can be accepted for the M mark.

A1: Both values correct to at least 1sf (but see warning), rounded or truncated with reason (Sign change **and** continuous function) and minimal conclusion (root). For “sign change” accept $f(0.1) < 0$, $f(0.2) > 0$ shown, or $f(0.1)f(0.2) < 0$

Warning: Degrees mode answers should score A0, so if the values -0.39999 and 0.200001 are used score A0. However, if just $f(0.1) = -0.4$ and $f(0.2) = 0.2$ are used with no contrary evidence – see note above – the A1 will be scored if reason and conclusion are given.

Note: Use of a narrower interval is possible. You may need to check values, but score as per the main scheme with their end points as long as their interval contains the root.

(b)

B1*: Shows all necessary steps to show given result. Allow if α is used in place of x .

Sets $f(x) = 0$, uses $\sec x = \frac{1}{\cos x}$ and makes the x of the $6x$ the subject. The “=0” must be seen at some stage.

(c) (i)

M1: Uses iterative formula and $x_1 = 0.15$ to find $x_2 = 0.16..$ There must be some evidence of use of the iterative formula, either substitution seen or awrt 0.16 **other than** just $0.1622(315...)$, which may be from a calculator solve. If all that is seen is $0.1622(315...)$ with no other evidence then M0A0A0 will be scored.

A1: $(x_2 =)$ awrt 0.1629 Allow if the x_2 is omitted, so just awrt 0.1629 can score M1A1

(c)(ii) A1: $(\alpha =)$ 0.1622 following the award of the M mark (see note).

An answer of $0.1666...$ may score the first M in (c) as long as there is evidence of the iterative formula used but the value only will be M0 (calculator solve gives this value).

Question Number	Scheme	Marks
2 (a)	10 m^2	B1 (1)
(b)	$\log_{10} 25 = 1 + 0.03T \Rightarrow T = \dots$ $T = 13.26$	M1 A1cso (2) (3 marks)

(a)

B1: 10 m^2 . Requires the units

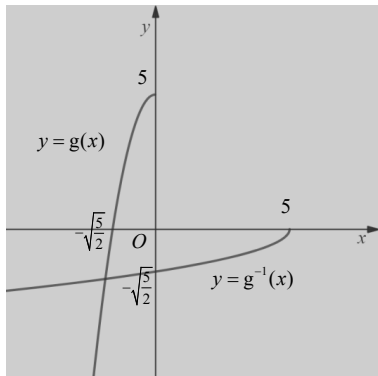
(b)

M1: Substitutes $A = 25$ and finds value or exact expression for T or t . Condone poor algebra in the log work for this mark. Look for the process of substituting A and reaching a value/expression for T or t .

You may see attempts at making T the subject first. Look for substituting $A=25$ into their expression, condoning any poor algebra in the rearrangement and proceeding to a value/expression for T or t .

A1cso: Awrt 13.26 but condone awrt 13.26 weeks and isw rounding errors after a correct answer. Must have come from fully correct log work.

Do not accept expressions, they must evaluate to give a decimal correct to at least 2d.p.

Question Number	Scheme	Marks	
4(a)	$fg(x) = \frac{4(5 - 2x^2) + 6}{5 - 2x^2 - 5}$	M1	
	$fg(x) = 3 \Rightarrow \frac{26 - 8x^2}{-2x^2} = 3 \Rightarrow x = -\sqrt{13}$	dM1 A1 A1	
		(4)	
(b)	$y = \frac{4x + 6}{x - 5} \Rightarrow yx - 5y = 4x + 6 \Rightarrow yx - 4x = 5y + 6$	M1	
	$\Rightarrow x = \frac{5y + 6}{y - 4}$	A1	
	$\Rightarrow f^{-1}(x) = \frac{5x + 6}{x - 4} \quad x \in \mathbb{R}, x \neq 4$	A1	
		(3)	
(c)		Shape and position for $y = g(x)$. A parabola starting on the y -axis that extends down in quadrant 2 to at least the x -axis. Condone slips of the pen e.g. if the curve heads back towards the y -axis in quadrant 3 slightly.	B1
		Shape and position for $y = g(x)$ as above that crosses the x -axis and $y = g^{-1}(x)$ a parabola starting on the x -axis with graphs crossing as shown in quadrant 3. Condone slips of the pen e.g. if $g^{-1}(x)$ heads back towards the x -axis in quadrant 3 slightly.	B1
		Correct intercepts. This mark is for the 4 correct intercepts as shown. Allow as values or coordinates and allow coordinates the wrong way round (e.g. (0, 5) for (5, 0)) as long as they are in the correct places. The negative intercepts must be exact as shown or equivalent. The graphs must cross the axes at these points and you can ignore any other intercepts.	B1
		Note that labels for $g(x)$ and $g^{-1}(x)$ are not required	
			(3)
		(10 marks)	

(a)

M1: For attempting $fg(x)$ in the correct order.dM1: For proceeding to $x^2 = \dots$ **Depends on the first mark.**A1: For $x^2 = 13$ which may be implied by $x = \sqrt{13}$ A1: For $x = -\sqrt{13}$ only

Note that in (a) some candidates may take an alternative approach for the first mark:

$$fg(x) = 3 \Rightarrow g(x) = f^{-1}(3) = -21 \Rightarrow 5 - 2x^2 = -21$$

Score M1 for finding $f^{-1}(3)$ and setting $g(x) = f^{-1}(3)$ then as main scheme

or

$$fg(x) = 3 \Rightarrow g(x) = f^{-1}(3) = -21 \Rightarrow x = g^{-1}(-21)$$

Score M1 for finding $f^{-1}(3)$ and setting $g^{-1}f^{-1}(3) = x$ then as main scheme

(b)

M1: For attempting to change the subject. In the attempt shown in the mark scheme score when both terms in x have been isolated and set on one side of the equation

A1: For $x = \frac{5y+6}{y-4}$ or equivalent such as $x = 5 + \frac{26}{y-4}$

If the x and y have been swapped award for $y = \frac{5x+6}{x-4}$ or equivalent.

A1: Requires both the expression in x **and** the domain.

$$f^{-1}(x) = \frac{5x+6}{x-4} \quad x \neq 4 \quad \text{or} \quad f^{-1}(x) = \frac{5x+6}{x-4} \quad x \in \mathbb{R}, x \neq 4 \quad (\text{Condone } f^{-1} = \dots)$$

Do **not** allow $y = \dots$

(c) See scheme

A1 $f^{-1}(x) = \frac{5-2x}{3x+1}$ or $f^{-1}(x) = \frac{2x-5}{-3x-1}$ or exact equivalent expressions such as $f^{-1}(x) = \frac{5-2x}{3\left(x+\frac{1}{3}\right)}$

.

Accept $f^{-1}: x \rightarrow \frac{5-2x}{3x+1}$ Condone $f^{-1} = \dots$ (or $f^{-1} = y = \dots$) but **do not allow just** $y = \dots$ or $f^{-1}: y = \dots$

B1 $x \neq -\frac{1}{3}$. The omission of $x \in \mathbb{R}$ is condoned.

Note: It is also acceptable to define f^{-1} in any variable eg as $f^{-1}(t) = \frac{5-2t}{3t+1}$ $t \neq -\frac{1}{3}$ as long as the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined eg $f^{-1}(t) = \dots$ not just $f^{-1} = \dots$

(c)

B1ft Sets $\frac{5-\frac{1}{a}}{3 \times \frac{1}{a} + 2} = 2(a+3) - 7$ or their $\frac{1}{a} = f^{-1}(2a-1) \Rightarrow \frac{1}{a} = \frac{"5"- "2"(2a-1)}{"3"(2a-1)+ "1"}$

(may be in terms of x). Follow through on their $f^{-1}(x)$.

May be implied by later work if they have invisible brackets.

M1 They must start with an equation of one of these forms (which may be in x):

- $\frac{A \pm \frac{1}{a}}{B \times \frac{1}{a} \pm C} = Da + E$ or equivalent so typically look for $\frac{5-\frac{1}{a}}{3 \times \frac{1}{a} + 2} = 2(a+3) - 7$

(may be manipulated to $\frac{Aa \pm 1}{B \pm Ca} = Da + E$)

- $\frac{1}{a} = \frac{Aa \pm B}{Ca \pm D}$ or equivalent

Rearranges their equation of an allowable form to a 3TQ. They must be attempting to

solve $f\left(\frac{1}{a}\right) = g(a+3)$ so it cannot be awarded for an attempt to solve $f(a) = g(a)$ or

$$f\left(\frac{1}{a}\right) = g\left(\frac{1}{a}\right)$$

Do not be concerned with the mechanics of their arrangement and condone invisible brackets.

A1 $4a^2 - a - 2 = 0$ or equivalent equation involving 3 terms. Eg $4a^2 - a = 2$ (may be in terms of x). May be implied by awrt ± 0.843 and the $= 0$ may be implied by their attempt to solve their quadratic equation.

A1 $(a =) \frac{1 \pm \sqrt{33}}{8}$ or exact equivalent. Do not accept rounded decimals. Condone $x = \frac{1 \pm \sqrt{33}}{8}$

Question Number	Scheme	Marks
3(a)	$\int \frac{9x}{3x^2 + k} dx = \frac{3}{2} \ln(3x^2 + k) + C$	M1A1
		(2)
(b)	$\frac{3}{2} \ln(75 + k) - \frac{3}{2} \ln(12 + k) = \ln 8$	M1
	$\frac{3}{2} \ln\left(\frac{75 + k}{12 + k}\right) = \ln 8 \quad \text{oe}$	dM1
	$\frac{75 + k}{12 + k} = 4 \Rightarrow k = \dots$	ddM1
	$(k =) 9$	A1
		(4)
		(6 marks)

(a)

M1 Integrates to $A \ln(3x^2 + k)$ with or without the + C. Allow $A = 1$ for this mark. Condone invisible brackets.

A1 $\frac{3}{2} \ln(3x^2 + k) + C$ or $\frac{3}{2} \ln B(3x^2 + k)$ oe Must include the constant of integration that is different from k . Must have brackets around $3x^2 + k$
 Also allow eg $\ln \left[B(3x^2 + k) \right]^{\frac{3}{2}}$



(b) ***Be aware they can solve equations on the calculator which is not acceptable***

M1 Attempts to substitute in 5 and 2 into their changed expression which must include x and k , subtracting either way round and sets equal to $\ln 8$. The values embedded in an equation is sufficient to score this mark. Condone slips if they attempted to evaluate when substituting in. Ignore the $+C$.

dM1 Attempts to apply the subtraction (or addition) law of logarithms once to an equation of the form $\pm A \ln(B+k) \pm A \ln(C+k) = \ln D$ or equivalent (allow $A = 1$).

Typically look for $A \ln(B+k) - A \ln(C+k) = \ln D \Rightarrow A \ln\left(\frac{B+k}{C+k}\right) = \ln D$ or equivalent

Do not be concerned with arithmetical errors and condone slips dealing with their A

Condone invisible brackets.

ddM1 Solves their equation of the form $\ln\left(\frac{B+k}{C+k}\right) = \ln D^{\frac{1}{A}}$ or $\ln\left(\frac{B+k}{C+k}\right)^A = \ln D$

where $A \neq 1$, and the A is correctly dealt with by

- removing lns correctly
- rearranging to form a linear equation in k
- proceeding to find a value for k .

eg $\ln\left(\frac{75+k}{12+k}\right) = \ln 8^{\frac{2}{3}} \Rightarrow 75+k = 48+4k \Rightarrow k = \dots$

It is dependent on both of the previous method marks.

Condone arithmetical slips in their rearrangement and condone invisible brackets as long as the intention is clear. Also condone this mark to be scored if k is negative.

A1 9 cao (with intermediate working seen)

They cannot proceed from $\frac{3}{2} \ln(75+k) - \frac{3}{2} \ln(12+k) = \ln 8$ to $(k =) 9$ without some intermediate working seen (eg an equation with the lns removed)

Note an answer of $\ln(3x^2 + k) (+C)$ in (a) will score a maximum of M1dM1ddM0A0 in (b)

$$\sin 2\theta = \frac{1}{5} \Rightarrow (\theta =) 0.101, 1.47$$

$$\operatorname{cosec} 2\theta = 5 \Rightarrow (\theta =) 0.101, 1.47$$

Question Number	Scheme	Marks
8(a)	$\frac{dv}{dt} = -e^{t-10} + 9e^{-0.75t} = 0 \Rightarrow 9 = e^{1.75t-10} \text{ or } 9e^{-0.75t} = e^{t-10}$	M1A1
	$9 = e^{1.75t-10} \Rightarrow t = \frac{10 + \ln 9}{1.75} \text{ or } \ln 9 - 0.75t = t - 10 \Rightarrow t = \frac{10 + \ln 9}{1.75}$	M1
	$t = \text{awrt } 6.97$	A1
	$\text{awrt } 11.9 \text{ (ms}^{-1}\text{)}$	A1
		(5)
(b)	$\int 12 - 12e^{-0.75t} - e^{(t-10)} dt = 12t - e^{(t-10)} + 16e^{-0.75t} (+C)$	M1A1
	$\left[12t - e^{(t-10)} + 16e^{-0.75t} \right]_0^T = 100 \Rightarrow 12T = \dots$	M1
	$T = \frac{1}{12} (116 - 16e^{-0.75T} + e^{T-10} - e^{-10}) *$	A1*
		(4)
(c) (i)	$T_2 = \frac{1}{12} (116 - 16e^{-0.75 \times 10} + e^{10-10} - e^{-10})$	M1
	$T_2 = \text{awrt } 9.7493$	A1
	9.7293 (seconds)	A1
(ii)		(3)
		(12 marks)

(a) * Be aware this can be solved directly on a calculator. Calculus must be seen*

M1 Differentiates to $\pm Ae^{t-10} \pm Be^{-0.75t}$ and sets equal to 0 Must be seen in (a) Do not accept differentiating to eg $\dots te^{t-10}$. Condone an expression of $12 \pm Ae^{t-10} \pm Be^{-0.75t}$ only

A1 $9 = e^{1.75t-10}$ or equivalent eg $9e^{-0.75t} - e^{t-10} = 0$

M1 Proceeds from their equation of the form $Ae^{Ct+D} - Be^{Et+F} = 0$ or equivalent leading to a value or an expression (which may be unsimplified) for t . If log work is seen it must be correct.

Eg $\frac{40 + 4 \ln 9}{7}$ or $\frac{10 + \ln 9}{1.75}$ will score this mark

$9e^{-0.75t} - e^{t-10} = 0 \Rightarrow \text{awrt } 6.97$ can also score this mark

This cannot be scored from an unsolvable equation.



A1 awrt 6.97 or awrt 6.96 including exact unsimplified expressions such as $\frac{40 + 4\ln 9}{7}$ or $\frac{10 + \ln 9}{1.75}$.

A1 awrt 11.9 (ms⁻¹) with all previous marks scored. Condone lack of units

Note: awrt 6.97 and awrt 11.9 with no calculus scores 0 marks

(b)

M1 Attempts to integrate the expression for v . Award for $12t \pm Ae^{(t-10)} \pm Be^{-0.75t}$.

Do not accept eg $12t \pm Ae^{(t-10)+1} \pm Be^{-0.75t+1}$

Allow if their attempt to integrate appears in part (a) as long as it is a clear attempt at integration.
Limits may have been substituted in.

A1 $12t - e^{(t-10)} + 16e^{-0.75t}$ (with or without +C) and allow unsimplified eg $12t - e^{(t-10)} - \frac{12}{-0.75}e^{-0.75t}$

Do not withhold this mark if they write $12x$ but subsequently replace it with T later on.

M1 Substitutes T and 0 into their changed expression with three different terms in t , sets equal to 100 and attempts to make $12T$ or T the subject.

Eg $(12T - e^{T-10} + 16e^{-0.75T}) - (-e^{-10} + 16) = 100 \Rightarrow 12T = \dots$

Condone arithmetical slips in their rearrangement.

A1* $T = \frac{1}{12}(116 - 16e^{-0.75T} + e^{T-10} - e^{-10})$ with no errors including bracket errors/omissions.

Must have $T = \dots$

(c)

(i)

M1 Substitutes 10 into the iterative formula. The expression with 10 embedded is sufficient to score this mark.

May be implied by awrt 9.7492 or awrt 9.7493. May also be implied by 9.7306

A1 awrt 9.7493

(ii) *Be aware that this can be found directly on a calculator which is not acceptable*

A1 9.7293 (seconds) (cannot be awarded without the method mark)

SC If they over round in (i) and (ii) and achieve 9.749 and 9.729 then award M1A0A1

Question Number	Scheme	Marks
9(a)	$\frac{dy}{dx} = \frac{(1 + \sin x)(-2 \sin x) - (1 + 2 \cos x) \cos x}{(1 + \sin x)^2}$	M1A1
	$(1 + \sin x)(-2 \sin x) - (1 + 2 \cos x) \cos x = 0$ $\Rightarrow -2 \sin x - \cos x - 2 = 0$ $2 \sin x + \cos x = -2 \quad *$	dM1 A1*
		(4)
(b)		
	Possible solutions from $R \sin(x + \alpha)$ or $R \cos(x - \alpha)$: $2 \sin x + \cos x = -2 \Rightarrow \sqrt{5} \sin(x + 0.464) = -2$ or $\sqrt{5} \cos(x - 1.107) = -2$ or eg solutions from squaring $(2 \sin x + \cos x)^2 = 4 \Rightarrow \cos x(3 \cos x - 4 \sin x) = 0$ ----- $\sqrt{5} \sin(x + 0.464) = -2$ or $\sqrt{5} \cos(x - 1.107) = -2 \Rightarrow x = \dots$ or $\tan x = \frac{3}{4} \Rightarrow x = \dots$ $x = \text{awrt } 3.79$	M1M1A1 dM1 A1
		(5)
		(9 marks)

(a)

M1 Attempts to differentiate by the product rule or quotient rule and proceeds to:

Quotient rule: $\frac{\pm A(1 + \sin x)(\sin x) \pm (1 + 2 \cos x) \cos x}{\dots}$ (Scored for the numerator only)Product rule: $\pm \cos x(1 + 2 \cos x)(1 + \sin x)^{-2} \pm A \sin x(1 + \sin x)^{-1}$ -----
Alternatively, they express $\frac{1 + 2 \cos x}{1 + \sin x} = \frac{1}{1 + \sin x} + \frac{2 \cos x}{1 + \sin x}$ and proceed to eitherQuotient rule: $\pm \frac{\cos x}{(1 + \sin x)^2} \pm \frac{A \sin x(1 + \sin x) \pm \cos x(2 \cos x)}{(1 + \sin x)^2}$ Product rule: $\pm \cos x(1 + \sin x)^{-2} \pm A \cos^2 x(1 + \sin x)^{-2} \pm B \sin x(1 + \sin x)^{-1}$

Condone arithmetical slips and invisible brackets.

A1 $\left(\frac{dy}{dx} = \right) \frac{(1 + \sin x)(-2 \sin x) - (1 + 2 \cos x) \cos x}{(1 + \sin x)^2}$ or equivalent (which may be unsimplified)

If it is via the product rule then it may appear as

$\left(\frac{dy}{dx}\right) = -\cos x(1+2\cos x)(1+\sin x)^{-2} - 2\sin x(1+\sin x)^{-1}$ or equivalent (which may be unsimplified).

- dM1 Sets the numerator of their quotient equal to zero, multiplies out the brackets and attempts to use the identity $\pm \sin^2 x \pm \cos^2 x = \pm 1$ (which may be implied by their working) leading to an equation in $\sin x$ and $\cos x$ only. There must be no squared terms.
If it is via the product rule they will need to either combine their terms into a single fraction and then set the numerator equal to zero (or multiply both sides by $(1+\sin x)^2$), and then attempt to use the identity $\pm \sin^2 x \pm \cos^2 x = \pm 1$ leading to an equation in $\sin x$ and $\cos x$ only. There must be no squared terms.
Condone slips and invisible brackets. It is dependent on the first method mark.

- A1* Achieves $2\sin x + \cos x = -2$ with no errors or missing brackets around $1+2\cos x$ or $1+\sin x$ when needed in the main body of their solution.
(Do not penalise the fraction line not being quite long enough for the numerator.)

It is acceptable to proceed to a correct $\frac{dy}{dx}$, multiply out the brackets and proceed directly from

$-2\sin x - 2\sin^2 x - \cos x - 2\cos^2 x = 0 \Rightarrow 2\sin x + \cos x = -2$ to score full marks. You do not need to explicitly see $\sin^2 x + \cos^2 x = 1$ stated or substituted in.

Note if they only ever use the numerator of the quotient formula: $vu' - uv'$ from the beginning
Maximum score is M1A0dM1A0*

(b) *Be aware this can be solved entirely using a calculator which is not acceptable*

Solution by expressing $2\sin x + \cos x$ in the form $R\sin(x \pm \alpha)$ or $R\cos(x \pm \alpha)$

M1 $R = \sqrt{2^2 + 1^2} = \sqrt{5}$

M1 $(\alpha =) \tan^{-1}\left(\pm \frac{1}{2}\right)$ or $(\alpha =) \tan^{-1}(\pm 2)$ (or $\cos^{-1}\left(\pm \frac{2}{\sqrt{5}}\right)$ $\sin^{-1}\left(\pm \frac{1}{\sqrt{5}}\right)$)

A1 $"\sqrt{5}"\sin(x + "0.464") = -2$ or $"\sqrt{5}"\cos(x - "1.107") = -2$ oe (or in another variable)

- dM1 Attempts to solve their equation and proceeds to find a value for x .

eg $x = \left(\pi - \sin^{-1}\left(\frac{-2}{\sqrt{5}}\right)\right) \pm "0.464"$ or $x = \sin^{-1}\left(\frac{-2}{\sqrt{5}}\right) \pm "0.464"$

You may need to check their angle on your calculator. Condone slips in their rearrangement.
It is dependent on both of the previous method marks.

- A1 awrt 3.79 only

Solutions squaring both sides and using the identity $\sin^2 x + \cos^2 x \equiv 1$

- M1 Attempts to square both sides and multiplies out the brackets. Condone sign slips only on the brackets they are multiplying out. Do not accept just squaring all the individual terms.

- M1 Attempts to use $\pm \sin^2 x \pm \cos^2 x = \pm 1$ and proceeds to a quadratic equation in either $\sin x$ or $\cos x$ or an equation of the form $\pm A\cos^2 x \pm B\cos x \sin x = 0$.

The use of the identity may be done before squaring both sides. Eg look for

$2\sin x + 2 = \cos x \Rightarrow 2\sin x + 2 = \sqrt{1 - \sin^2 x} \Rightarrow \dots$

Condone sign slips only.

A1 $3\cos^2 x - 4\sin x \cos x = 0$ oe or $5\sin^2 x + 8\sin x + 3 = 0$ oe or $5\cos^2 x + 4\cos x = 0$ oe
The $= 0$ may be implied by later work and terms do not need to be collected on one side.

dM1 Attempts to solve their quadratic equation in either $\sin x$ or $\cos x$ or an equation of the form $\pm A\cos^2 x \pm B\cos x \sin x = 0$ and proceeds to find a value for x . Do not allow for solving $\cos x = 0$
Usual rules apply for solving a quadratic. They may even just state the roots from their calculator.
It is dependent on both of the previous method marks.

A1 awrt 3.79 only
The table below may help with following through a particular approach:

	$(2\sin x + \cos x)^2 = 4$	$(2\sin x + 2)^2 = \cos^2 x$	$(\cos x + 2)^2 = 4\sin^2 x$
M1	$4\sin^2 x + 4\sin x \cos x + \cos^2 x = 4$	$4\sin^2 x + 8\sin x + 4 = \cos^2 x$	$\cos^2 x + 4\cos x + 4 = 4\sin^2 x$
M1	Attempts to use $\pm \sin^2 x \pm \cos^2 x = \pm 1$		
A1	$3\cos^2 x - 4\sin x \cos x = 0$ oe	$5\sin^2 x + 8\sin x + 3 = 0$ oe	$5\cos^2 x + 4\cos x = 0$ oe
dM1	Eg $\cos x(3\cos x - 4\sin x) = 0$ $\Rightarrow x = \dots$	Eg $(5\sin x + 3)(\sin x + 1) = 0$ $\Rightarrow x = \dots$	Eg $\cos x(5\cos x + 4) = 0$ $\Rightarrow x = \dots$
A1	awrt 3.79		

It is possible that they may divide the equation by either $\sin x$ or $\cos x$ and use $1 + \tan^2 x = \sec^2 x$ or $1 + \cot^2 x = \operatorname{cosec}^2 x$ resulting in the following equations (or equivalent simplified rearrangements of these) to solve:

- $3\cot^2 x = 4\cot x$
- $\cot x(5\cot x + 4\cot x \operatorname{cosec} x) = 0$
- $3\operatorname{cosec}^2 x + 8\operatorname{cosec} x + 5 = 0$
- $4\tan x = 3$
- $4\sec x + 5 = 0$

The mark scheme should be applied in the same way but looking for use of $\pm 1 \pm \tan^2 x = \pm \sec^2 x$ or $\pm 1 \pm \cot^2 x = \pm \operatorname{cosec}^2 x$ and proceeding to equations involving $\cot x$, $\operatorname{cosec} x$, $\tan x$ or $\sec x$
Send to review if unsure.

Solutions using the t method substitution

Using $t = \tan\left(\frac{x}{2}\right) \Rightarrow \sin x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$

M1 Attempts to express $2\sin x + \cos x = -2$ in terms of t . Attempts to use $\sin x = \frac{\pm 2t}{\pm 1 \pm t^2}$ and $\cos x = \frac{\pm 1 \pm t^2}{\pm 1 \pm t^2}$.

M1 Proceeds from an equation of the form $2\left(\frac{2t}{1+t^2}\right) + \frac{1-t^2}{1+t^2} = -2$ to a 3TQ
 $\frac{4t}{1+t^2} + \frac{1-t^2}{1+t^2} = -2 \Rightarrow 4t + 1 - t^2 = -2(1+t^2) \Rightarrow \dots$



A1 $t^2 + 4t + 3 = 0$ or equivalent. The $= 0$ may be implied by later work and the three terms do not need to be on the same side of the equation.

dM1 Correctly rearranges their equation to find a value for x
 $t^2 + 4t + 3 = 0 \Rightarrow t = -3 \Rightarrow x = 2(\arctan(-3) + \pi)$ Ignore any reference to $t = -1$
It is dependent on both of the previous method marks.

A1 awrt 3.79

There may be other variants of these methods, but the mark scheme should still be able to be applied. If in doubt send to review.

Answer only scores 0 marks

Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x - \tan^2 x \equiv \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} \equiv \frac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$	M1
	$\equiv \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x \cos^2 x} \equiv \frac{\cos 2x}{\dots} \text{ or } \frac{\dots}{\left(\frac{1}{2} \sin 2x\right)^2}$	dM1
	$\equiv \frac{\cos 2x}{\left(\frac{1}{2} \sin 2x\right)^2}$	A1
	$\equiv 4 \frac{\cos 2x}{\sin 2x \sin 2x} \equiv 4 \cot 2x \operatorname{cosec} 2x^*$	A1*
	(4)	
(b)	$4 \cot 2\theta \operatorname{cosec} 2\theta = 2 \tan^2 \theta \Rightarrow \cot^2 \theta - \tan^2 \theta = 2 \tan^2 \theta \Rightarrow \cot^2 \theta - 3 \tan^2 \theta = 0$	M1
	$\cot^2 \theta - 3 \tan^2 \theta = 0 \Rightarrow \frac{1}{\tan^2 \theta} - 3 \tan^2 \theta = 0 \Rightarrow \tan^4 \theta = \frac{1}{3}$	A1
	$\tan^4 \theta = \frac{1}{3} \Rightarrow \tan \theta = \pm \sqrt[4]{\frac{1}{3}} = \pm 0.7598.. \Rightarrow \theta = \dots$	M1
	$\theta = \text{awrt } 0.65, -0.65$	A1A1
	(5)	
		Total 9

(a) Way One LHS to RHS

M1: Changes the LHS to $\sin x$ and $\cos x$ and attempts to make a single fraction using a correct common denominator. Condone errors/slips on the numerator

dM1: Attempts/ applies

- Either $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos^2 x - \sin^2 x = \cos 2x$ on the numerator
- Or $\sin 2x = 2 \sin x \cos x$ to the denominator condoning bracketing slip

A1: Applies both of the above correctly to achieve a correct expression in terms of $\cos 2x$ and $\sin 2x$

A1*: Reaches the right hand side with sufficient working shown. Expect to see $\sin^2 2x$ split into $\sin 2x \sin 2x$. Penalise consistent (not once or twice) use of poor notation on this mark only.

Examples $\cos^2 x \leftrightarrow \cos x^2$, $\sin^2 \leftrightarrow \sin^2 x$, constantly switching between $x \leftrightarrow \theta$

(b) Allow use of $x \leftrightarrow \theta$

M1: Uses part (a) and attempt to collect terms. (See Appendix III for ways not using part (a))

See below for equations in $\sin \theta$ or $\cos \theta$ where this mark is awarded for an equation in just $\sin \theta$ or $\cos \theta$

A1: Reaches a correct equation in a single term, usually $\tan^4 \theta$. Look for $\tan^4 \theta = \frac{1}{3}$ o.e. such as $3 \tan^4 \theta = 1$

Other correct intermediate forms are $2 \sin^4 \theta + 2 \sin^2 \theta - 1 = 0$ and $2 \cos^4 \theta - 6 \cos^2 \theta + 3 = 0$

M1: Takes the 4th root of their $\frac{1}{3}$ (o.e) and uses $\tan^{-1}$ (you may need to check) to obtain at least one value for θ

For the other intermediate forms look for working such as $\sin^2 \theta = \frac{\sqrt{3}-1}{2} \Rightarrow \sin \theta = \sqrt{\frac{\sqrt{3}-1}{2}} \Rightarrow \theta = \dots$



A1: Either awrt 0.65 or awrt -0.65. Allow either answer in degrees, so awrt $\pm 37.2^\circ$

A1: Both answers in radians, awrt ± 0.65 , and no extras in range

There are many different ways to do part (a). Generally, this is how they will be marked.

Most cases can be aligned to one of the three cases.

RHS to LHS

(a) Way 2	$4 \cot 2x \operatorname{cosec} 2x \equiv 4 \frac{\cos 2x}{\sin 2x} \times \frac{1}{\sin 2x} \equiv \frac{4(\cos^2 x - \sin^2 x)}{\dots} \text{ or } \frac{\dots}{4 \sin^2 x \cos^2 x}$	M1
	$\frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} \equiv \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \equiv \operatorname{cosec}^2 x - \sec^2 x$	dM1A1
	$\equiv 1 + \cot^2 x - 1 - \tan^2 x \equiv \cot^2 x - \tan^2 x^*$	A1*

M1: Changes to $\sin 2x$ and $\cos 2x$ or $\tan 2x$ and $\sin 2x$ and attempts single angles in $\sin x$ and $\cos x$

dM1: Changes to single angles throughout and splits into 2 separate fractions (which don't need to be simplified)

A1: Correct expression in terms of the single angles $\operatorname{cosec} x$ and $\sec x$

A1*: Reaches the left hand side with sufficient working shown

Working on both sides: One possible way

(a) Way 3	$\cot^2 x - \tan^2 x \equiv 4 \cot 2x \operatorname{cosec} 2x$ $\frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} \equiv 4 \times \frac{\cos 2x}{\sin 2x} \times \frac{1}{\sin 2x}$ $\cos^4 x - \sin^4 x \equiv 4 \sin^2 x \cos^2 x \frac{\cos 2x}{\sin 2x} \times \frac{1}{\sin 2x}$	M1
	$\cos^4 x - \sin^4 x \equiv 4 \times \sin^2 x \cos^2 x \frac{(\cos^2 x - \sin^2 x)}{(2 \sin x \cos x)^2}$	dM1A1
	$(\cos^4 x - \sin^4 x) \equiv (\cos^2 x - \sin^2 x)$ $(\cos^2 x - \sin^2 x) \frac{(\cos^2 x + \sin^2 x)}{1} \equiv (\cos^2 x - \sin^2 x) \text{ Hence true}$	A1*

M1: Changes to $\sin x$, $\cos x$, $\sin 2x$ and $\cos 2x$ and attempts to cross multiply

dM1: Applies

- either $\cos^2 x - \sin^2 x = \cos 2x$ to the numerator
- or $\sin 2x = 2 \sin x \cos x$ to the denominator

A1: Correct identity in terms of $\cos x$ and $\sin x$

A1*: Reaches a point where both sides are equal and makes a minimal comment

Example of how you mark a "different" approach.