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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Trigonometry (P3.3)

International A Level Mathematics · P3: Pure Mathematics 3 (WMA13)

27 questions · 214 marks · Mark schemes included

Past papers January 2020 - June 2025 · 3 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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P3.3a RECIPROCAL & INVERSE TRIGONOMETRIC FUNCTIONS		10 Qs · 81 marks	2
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2. Q8	IAL P3 (WMA13) October 2021	7	5
3. Q5	IAL P3 (WMA13) June 2023	7	9
4. Q7	IAL P3 (WMA13) June 2022	8	13
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P3.3b COMPOUND & DOUBLE ANGLE FORMULAE		9 Qs · 68 marks	40
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P3.3c R COS($\cdot \pm \cdot$) & SOLVING TRIGONOMETRIC EQUATIONS		8 Qs · 65 marks	74
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QUESTIONS

27 questions · 214 marks · 3 sub-topics

SUB-TOPIC P3.3a

RECIPROCAL & INVERSE TRIGONOMETRIC FUNCTIONS

Sub-topic P3.3a · 10 questions · 81 marks · lowest tariff first



1.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $0 < \theta \leq 360^\circ$, the equation

$$3 \tan^2 \theta + 7 \sec \theta - 3 = 0$$

giving your answers to one decimal place.

(5)



Question 1 continued

(Total for Question 1 is 5 marks)



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8. A curve C has equation $y = f(x)$, where

$$f(x) = \arcsin\left(\frac{1}{2}x\right) \quad -2 \leq x \leq 2 \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

(a) Sketch C .

(1)

(b) Given $x = 2 \sin y$, show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{A - x^2}}$$

where A is a constant to be found.

(3)

The point P lies on C and has y coordinate $\frac{\pi}{4}$

(c) Find the equation of the tangent to C at P . Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)



Question 8 continued

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Question 8 continued

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Question 8 continued

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Q8

(Total 7 marks)



5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$

$$(x - 2)(\sqrt{3} \sec x + 2) = 0 \tag{3}$$

(ii) Solve, for $0 < \theta < 360^\circ$

$$10 \sin \theta = 3 \cos 2\theta \quad (4)$$





Question 5 continued

Lined area for writing the answer to Question 5.



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 7 marks)



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7. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2 \sin \theta (3 \cot^2 2\theta - 7) = 13 \sec \theta$$

can be written as

$$3 \operatorname{cosec}^2 2\theta - 13 \operatorname{cosec} 2\theta - 10 = 0 \quad (4)$$

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$, the equation

$$2 \sin \theta (3 \cot^2 2\theta - 7) = 13 \sec \theta$$

giving your answers to 3 significant figures. (4)

Question 7 continued

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Question 7 continued

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Question 7 continued

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Q7

(Total 8 marks)



8. (a) Prove that

$$2 \operatorname{cosec}^2 2\theta (1 - \cos 2\theta) \equiv 1 + \tan^2 \theta \quad (4)$$

(b) Hence solve for $0 < x < 360^\circ$, where $x \neq (90n)^\circ$, $n \in \mathbb{N}$, the equation

$$2 \operatorname{cosec}^2 2x(1 - \cos 2x) = 4 + 3 \sec x$$

giving your answers to one decimal place.

(Solutions relying entirely on calculator technology are not acceptable.)

Question 8 continued

Question 8 continued



Question 8 continued

(Total for Question 8 is 8 marks)





Question 9 continued

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 8 continued

Lined area for writing the answer to Question 8 continued.





Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 8 marks)



8.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < \pi$

$$3 \operatorname{cosec} \theta = 8 \cos \theta$$

giving your answers, in radians, to 3 significant figures.

(5)

(ii) Solve, for $0 < x < 180^\circ$

$$\frac{\tan 2x - \tan 70^\circ}{1 + \tan 2x \tan 70^\circ} = -\frac{3}{8}$$

giving your answers, in degrees, to one decimal place.

(4)

Question 8 continued



Question 8 continued

(Total for Question 6 is 7 marks)



Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)



8.

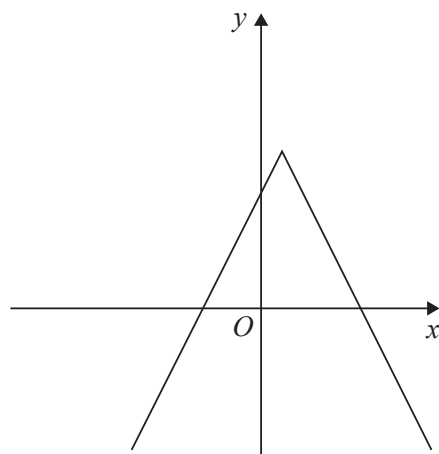


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The graph shown in Figure 2 has equation

$$y = a - |2x - b|$$

where a and b are positive constants, $a > b$

(a) Find, giving your answer in terms of a and b ,

- (i) the coordinates of the maximum point of the graph,
- (ii) the coordinates of the point of intersection of the graph with the y -axis,
- (iii) the coordinates of the points of intersection of the graph with the x -axis.

(5)

On page 24 there is a copy of Figure 2 called Diagram 1.

(b) On Diagram 1, sketch the graph with equation

$$y = |x| - 1$$

(2)

Given that the graphs $y = |x| - 1$ and $y = a - |2x - b|$ intersect at $x = -3$ and $x = 5$

(c) find the value of a and the value of b

(4)



Question 8 continued



Question 8 continued

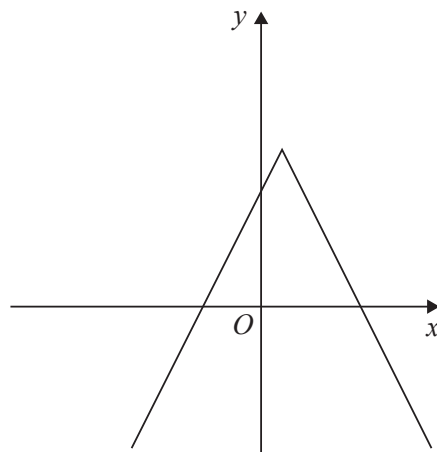


Diagram 1

Question 8 continued

(Total for Question 8 is 11 marks)



SUB-TOPIC P3.3b

COMPOUND & DOUBLE ANGLE FORMULAE

Sub-topic P3.3b · 9 questions · 68 marks · lowest tariff first



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1. Solve, for $0 \leq x < 360^\circ$, the equation

$$2\cos 2x = 7\cos x$$

giving your solutions to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

Question 1 continued

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(Total 5 marks)

Q1



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2. (a) Show that the equation

$$8 \cos \theta = 3 \operatorname{cosec} \theta$$

can be written in the form

$$\sin 2\theta = k$$

where k is a constant to be found.

(3)

- (b) Hence find the smallest positive solution of the equation

$$8 \cos \theta = 3 \operatorname{cosec} \theta$$

giving your answer, in degrees, to one decimal place.

(2)

Question 2 continued

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Handwriting practice area with horizontal lines.

(Total 5 marks)

Q2

Grading box for Q2.



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4. In this question you should show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2 \sin(\theta - 30^\circ) = 5 \cos \theta$$

can be written in the form

$$\tan \theta = 2\sqrt{3} \quad (4)$$

(b) Hence, or otherwise, solve for $0 \leq x \leq 360^\circ$

$$2 \sin(x - 10^\circ) = 5 \cos(x + 20^\circ)$$

giving your answers to one decimal place. (3)



Question 4 continued

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Question 4 continued

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Lined area for writing the answer to Question 4.

(Total 7 marks)

Q4

Mark box for Q4.



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- 5.** (a) Use the substitution $t = \tan x$ to show that the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

can be written in the form

$$5t^4 - 24t^2 - 5 = 0 \quad (4)$$

- (b) Hence solve, for $0 \leq x < 360^\circ$, the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

Show each stage of your working and give your answers to one decimal place.

(4)

Question 5 continued

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Question 5 continued

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Q5

(Total 8 marks)



In this question you must show all stages of your working.

(a) Given that

$$\sqrt{2} \sin(x + 45^\circ) = \cos(x - 60^\circ)$$

show that

$$\tan x = -2 - \sqrt{3} \quad (4)$$

(b) Hence or otherwise, solve, for $0 \leq \theta < 180^\circ$

$$\sqrt{2} \sin(2\theta) = \cos(2\theta - 105^\circ) \quad (4)$$



Question 7 continued

Lined area for writing the answer to Question 7.





Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 8 marks)







(Total for Question 5 is 8 marks)



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Question 7 continued

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Question 7 continued

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(Total 9 marks)

Q7



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Question 2 continued

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Question 2 continued

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(Total 9 marks)

Q2



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Question 9 continued

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Question 9 continued

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Question 9 continued

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Q9

(Total 9 marks)

TOTAL FOR PAPER IS 75 MARKS

END



SUB-TOPIC P3.3c

R COS($\cdot \pm \cdot$) & SOLVING TRIGONOMETRIC EQUATIONS

Sub-topic P3.3c · 8 questions · 65 marks · lowest tariff first



Question 2 continued

(Total for Question 2 is 6 marks)



Question 9 continued

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Question 9 continued

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Q9

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

END



(Total for Question 2 is / marks)



Question 9 continued

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Question 9 continued

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Lined area for writing the answer to Question 9.

(Total 8 marks)

Q9

Box for marking the answer to Question 9.

END

TOTAL FOR PAPER IS 75 MARKS



Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total 9 marks)

Q7



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Question 8 continued

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Question 8 continued

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(Total 9 marks)

Q8







Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 9 marks)



5. In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 5 \cos 2x - 12 \sin 2x \quad -\pi < x < \pi$$

The point A with x coordinate $\frac{\pi}{3}$ lies on C .

- (a) Use algebraic differentiation to find the gradient of the tangent to C at A . Give the answer in simplest form.

(3)

- (b) Express $5 \cos 2x - 12 \sin 2x$ in the form

$$R \cos (2x + \alpha)$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Find the exact value of R and find the value of α to 3 decimal places.

(3)

A point P lies on C .

Given that the gradient of the tangent to C at P is 6

- (c) find the greatest possible value for the x coordinate of P .
Give the answer to 2 decimal places.

(4)





(Total for Question 3 is 16 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
5 (i)	$\int \frac{8}{(2x-3)^3} dx = \frac{-2}{(2x-3)^2} (+c)$	M1 A1
	$\int_2^4 \frac{8}{(2x-3)^3} dx = \left[\frac{-2}{(2x-3)^2} \right]_2^4 = -\frac{2}{25} + 2 = \frac{48}{25}$	dM1 A1
		(4)
(ii)	$\int x(x^2+3)^7 dx = \frac{1}{16}(x^2+3)^8 + c$	M1 A1
		(2)
		(6 marks)
Alt(i)	Let $u = 2x - 3$ $\int \frac{8}{u^3} \times \frac{1}{2} du = -\frac{2}{u^2} (+c)$ $\int_2^4 \frac{8}{(2x-3)^3} dx = \left[-\frac{2}{u^2} \right]_1^5 = -\frac{2}{25} + 2 = \frac{48}{25}$	M1 A1 dM1 A1
Alt(ii)	Let $u = x^2 + 3$ $\int x(x^2+3)^7 dx = \int \frac{u^7}{2} du = \frac{u^8}{16} + c = \frac{1}{16}(x^2+3)^8 + c$	M1A1

(i)

M1 Achieves $\frac{A}{(2x-3)^2}$ or equivalent or in the alternative method $\frac{A}{u^2}$ A1 Achieves $\frac{-2}{(2x-3)^2}$ or in the alternative method $-\frac{2}{u^2}$ (which may be unsimplified but the indices must be processed). There is no requirement for the $+c$ dM1 Substitutes 2 and 4 into $\frac{A}{(2x-3)^2}$ or equivalent or 1 and 5 into $\frac{A}{u^2}$ and subtracts either way round. May be implied but M1 must have been scored.A1 $\frac{48}{25}$ or 1.92 isw after a correct answer

(ii)

M1 Achieves $k(x^2 + 3)^8$ or equivalent or in the alternative method ku^8 . Alternatively multiplies out the expression and integrates achieving an expression of the form

$$\pm \dots x^{16} \pm \dots x^{14} \pm \dots x^{12} \pm \dots x^{10} \pm \dots x^8 \pm \dots x^6 \pm \dots x^4 \pm \dots x^2$$

A1 $\frac{1}{16}(x^2 + 3)^8 + c$ Must be in terms of x and the $+ c$ must be present

Allow $\frac{x^{16}}{16} + \frac{3x^{14}}{2} + \frac{63x^{12}}{4} + \frac{189x^{10}}{2} + \frac{2835x^8}{8} + \frac{1701x^6}{2} + \frac{5103x^4}{4} + \frac{2187x^2}{2} + c$ or simplified equivalent



Question Number	Scheme	Marks
6 (i)	$\left(\frac{dy}{dx} = \right) \frac{6x}{x^2-5} - 8x$	M1 A1
	Stationary point when $\frac{6x}{x^2-5} - 8x = 0 \Rightarrow x^2 = \frac{23}{4} \Rightarrow x = \frac{\sqrt{23}}{2}$ only	dM1 A1
		(4)
(ii)(a) (b)	$\left(\frac{dy}{dx} = \right) 4 - 24 \sin x \cos x$	M1
	$\left(\frac{dy}{dx} = \right) 4 - 12 \sin 2x$	dM1 A1
	Maximum gradient = 16	A1
		(4)
		(8 marks)

(i)

M1 For differentiating $3 \ln(x^2 - 5) \rightarrow \frac{Ax}{x^2 - 5}$ or $\frac{A}{x^2 - 5}$

A1 $\left(\frac{dy}{dx} = \right) \frac{6x}{x^2 - 5} - 8x$ seen or implied by eg $\frac{6x}{x^2 - 5} = 8x$

dM1 For proceeding from $\left(\frac{dy}{dx} = \right) \frac{Ax}{x^2 - 5} - Bx = 0$ to a quadratic or cubic equation. Eg

$$6x - 8x^3 + 40x = 0 \text{ oe}$$

Do not be too concerned with the mechanics of their rearrangement.

A1 For $(x =) \frac{\sqrt{23}}{2}$ only provided dM1 has been scored. Solving the quadratic or cubic with a calculator is acceptable. Condone recovery of slips provided the method is correct.

$x = 0$ and $x = -\frac{\sqrt{23}}{2}$ must be rejected if found or condone some minimal conclusion eg stating $p = 23$

Mark (ii) (a) + (b) together**(ii) Way One**

M1 For differentiating using the chain rule with $\sin^2 x \rightarrow \dots \sin x \cos x$

dM1 and then using $\sin 2x = 2 \sin x \cos x$ to proceed to $\left(\frac{dy}{dx} = \right) A + B \sin 2x$. Condone

the slip writing $A + B \sin x$ provided their coefficient of $\sin x \cos x$ has been halved or writing $2 \sin \cos x$

A1 $\left(\frac{dy}{dx} = \right) 4 - 12 \sin 2x$ or $4 + (-12) \sin 2x$ or $4 - 12 \times \sin 2x$ (or states A and B

following a correct method) Condone recovery of slips provided the method is correct.



A1 Maximum gradient = 16 (the previous A1 must have been scored)

(ii) Way Two

M1 Attempts to use $\cos 2x = \pm 1 \pm 2 \sin^2 x$ in an attempt to write $y = 4x - 12 \sin^2 x$ in terms of $\cos 2x$

dM1 ...and then differentiates $\cos 2x \rightarrow k \sin 2x$ to proceed to $\left(\frac{dy}{dx} = \right) A + B \sin 2x$

A1 $\left(\frac{dy}{dx} = \right) 4 - 12 \sin 2x$ or $4 + (-12) \sin 2x$ or $4 - 12 \times \sin 2x$ (or states A and B following a correct method) Condone recovery of slips provided the method is correct.

A1 Maximum gradient = 16 (the previous A1 must have been scored)

(Note some candidates may find the second derivative and set equal to 0 and then substitute in their angle to find the maximum gradient.)



Question Number	Scheme	Marks
7.(a)	$\log_{10} M = 1.93 \log_{10} 45 + 0.684 \Rightarrow \log_{10} M = 3.8747$	M1
	$(M =) \text{ awrt } 7500 \text{ (kg)}$	A1
		(2)
(b)	$\log_{10} M = 1.93 \log_{10} r + 0.684$	$M = p r^q$ $\log_{10} M = \log_{10} p r^q$ $\log_{10} M = \log_{10} p + \log_{10} r^q$ $\log_{10} M = \log_{10} p + q \log_{10} r$ B1
	$\log_{10} M = \log_{10} r^{1.93} + 0.684$	
	$M = 10^{\log_{10} r^{1.93} + 0.684}$	
	$M = 10^{0.684} \times r^{1.93}$	
	Either $p = 10^{0.684}$ or $q = 1.93$	
	Both $p = \text{awrt } 4.83$ and $q = 1.93$	M1
		A1
		(3)
(c)	"p" is the mass (in kg) of a tree with radius 1 cm	B1
		(1)
		(6 marks)

(a)

M1 Substitutes $r = 45$ in $\log_{10} M = 1.93 \log_{10} r + 0.684$ and proceeds to $\log_{10} M = \dots$
 (which must be a numerical value) (Condone the use of $r = 0.45$).
 Implied by awrt 7500 for $r = 45$ or awrt 1.0 for $r = 0.45$

A1 $(M =) \text{ awrt } 7500 \text{ (kg)}$. Condone 7.5×10^3

Alt (a)

M1 Substitutes $r = 45$ in their $M = p r^q$ and finds M .

A1 $(M =) \text{ awrt } 7500 \text{ (kg)}$



(b) Mark (b) and (c) together

B1 A correct proof showing that $\log_{10} M = 1.93 \log_{10} r + 0.684 \Leftrightarrow M = p r^q$. Condone log or lg for $\log_{10}$ but use of natural logs $\ln$ is B0

Most are starting with $\log_{10} M = 1.93 \log_{10} r + 0.684$. Expect to see the addition (or subtraction) law of logs explicitly used or index law being applied before proceeding to the final answer.

Eg $\log_{10} M = \log_{10} 4.83 + \log_{10} r^{1.93} = \log_{10} (4.83 \times r^{1.93}) \Rightarrow M = 4.83 r^{1.93}$ is B1

$\log_{10} M = \log_{10} r^{1.93} + 0.684 \Rightarrow M = 10^{1.93 \log_{10} r + 0.684} \Rightarrow M = 10^{1.93 \log_{10} r} \times 10^{0.684} = 4.83 r^{1.93}$ is B1

$\log_{10} M = \log_{10} r^{1.93} + 0.684 \Rightarrow M = 10^{1.93 \log_{10} r + 0.684} \Rightarrow M = r^{1.93} \times 10^{0.684} = 4.83 r^{1.93}$ is B1

$\log_{10} M = \log_{10} r^{1.93} + 0.684 \Rightarrow M = 10^{1.93 \log_{10} r} \times 10^{0.684} = 4.83 r^{1.93}$ is B1

$\log_{10} M = \log_{10} 4.83 + \log_{10} r^{1.93} \Rightarrow M = 4.83 r^{1.93}$ is B0 (addition law of logs not seen)

No incorrect working should be seen such as $10^{\log_{10} r^{1.93} + 0.684} = 10^{\log_{10} r^{1.93}} + 10^{0.684} = 10^{\log_{10} r^{1.93}} \times 10^{0.684}$ but condone the omission of the brackets around $\log_{10} 4.83 \times r^{1.93}$ if seen in their working.

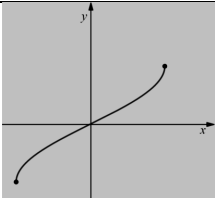
M1 Either $p = 10^{0.684}$ or $q = 1.93$ This may be implied by $M = p r^q$ with a "correct" p or q

A1 Both $p = \text{awrt } 4.83$ and $q = 1.93$ (B0M1A1 can be scored) This may be implied by $M = 4.83 r^{1.93}$

(c)

B1 "p" is the mass of a tree with radius 1 cm



Question Number	Scheme	Marks
8 (a)		Shape and position B1 (1)
(b)	$x = 2 \sin y \Rightarrow \left(\frac{dx}{dy} = \right) 2 \cos y \text{ and attempts to use both } \frac{dy}{dx} = 1 \div \frac{dx}{dy} \text{ and}$ $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - \dots x^2}$ $\left(\frac{dy}{dx} = \right) \frac{1}{2\sqrt{1 - \frac{x^2}{4}}}$ $\left(\frac{dy}{dx} = \right) \frac{1}{\sqrt{4 - x^2}}$	M1 A1 A1cso (3)
(c)	Substitutes $x = \sqrt{2}$ into their $\frac{dy}{dx} = \frac{1}{\sqrt{4 - x^2}} \quad \left(= \frac{\sqrt{2}}{2} \right)$ Finds the equation of the tangent at P $y - \frac{\pi}{4} = \frac{\sqrt{2}}{2} (x - \sqrt{2})$ $y = \frac{\sqrt{2}}{2} x - 1 + \frac{\pi}{4}$	M1 dM1 A1 (3)
		(7 marks)
Alt(b)	$y = \arcsin\left(\frac{x}{2}\right) \Rightarrow \left(\frac{dy}{dx} = \right) \frac{1}{2} \times \frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}}$ $\left(\frac{dy}{dx} = \right) \frac{1}{\sqrt{4 - x^2}}$	M1A1 A1

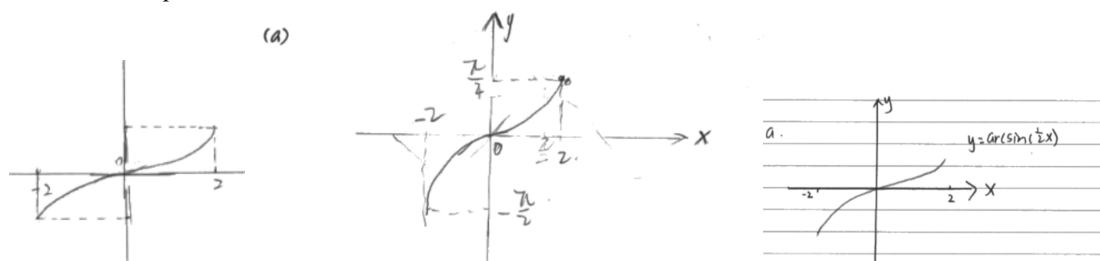
(a)

B1

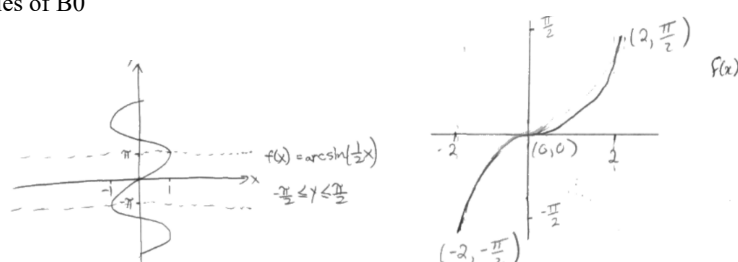
Correct shape and position: Look for a curve in quadrants 1 and 3 with non zero gradient at the origin and gradient $\rightarrow \infty$ at both ends. Ignore values labelled on the axes.

If there is more than one attempt, mark the one which appears in the main body of the work as their fullest attempt. If there is more than one sketch on the same graph, then it must be clearly labelled which is the one to be marked.

Examples of B1



Examples of B0

**(b) Note on EPEN it is M1M1A1 which is now marked as M1A1A1**

M1 $x = 2 \sin y \Rightarrow \frac{dx}{dy} = \pm k \cos y$ where k is non zero and attempts to use both

$\frac{dy}{dx} = 1 \div \frac{dx}{dy}$ and $\sin^2 y + \cos^2 y = 1$ in order to obtain $\frac{dy}{dx}$ in terms of x . Allow slips with dealing with their “2”

Allow working leading to $\frac{1}{\sqrt{1-x^2}}$ to score M1.

A1 A correct expression for $\frac{dy}{dx}$ as a function of x which may be unsimplified.

eg $\left(\frac{dy}{dx}\right) = \frac{1}{2\sqrt{1-\frac{x^2}{4}}}$ or $\frac{1}{\sqrt{4-\frac{x^2}{4}}}$ or $\frac{1}{2\sqrt{4-x^2}}$

A1cso $\left(\frac{dy}{dx}\right) = \frac{1}{\sqrt{4-x^2}}$

Alt (b)

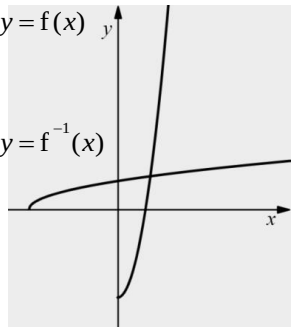
M1 $y = \arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = k \times \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$ (where k can equal 1) then A1A1 as above

SC $y = \arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{4-x^2}}$ with no correct intermediate working score as 100.

(c)

- M1 Attempts to find the value of $\frac{dy}{dx}$ or $\frac{dx}{dy}$ at P . See scheme but allow sub of $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = \pm k \cos y$
- dM1 Full method to find the equation of the tangent at P . Their $\frac{dy}{dx}$ may not be exact. If they use $y = mx + c$ they must proceed as far as $c = \dots$
- A1 Correct equation and in the correct form. The coefficient of x and the constant must be exact. Isw

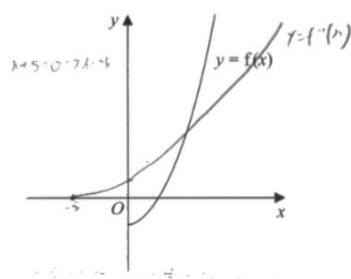


Question Number	Scheme	Marks
4.(a)	$f \geq -5$	B1
(b)	 Curve starting on negative x-axis and passing through positive y-axis, in quadrants 1 and 2 only. Shape and position correct.	M1 A1
(c)	$2x^2 - 5 = x$ or $2x^2 - 5 = \sqrt{\frac{x+5}{2}}$ or $x = \sqrt{\frac{x+5}{2}}$ or $2(2x^2 - 5)^2 - 5 = x$ Full attempt to solve $2x^2 - x - 5 = 0 \Rightarrow x = \dots$ exact $x = \frac{1 + \sqrt{41}}{4}$	B1 M1 A1
		(3) 6 marks

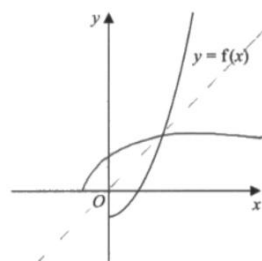
Notes

- (a) Mark the question as a whole - if (c) answered as (b) allow the marks.
- B1 Correct range. Accept $y \geq -5$, $f(x) \geq -5$, $f \in [-5, \infty)$ or correct formal set notation but not just $x \geq -5$.
- (b) Note: if a sketch is redrawn score for the sketch of the inverse only.
- M1 For a curve starting on the negative x -axis and passing through the positive y -axis, in quadrants 1 and 2 only.
- A1 Correct shape (curvature) and position. Must be increasing (not bending back on itself) with decreasing gradient, though be tolerant with pen slips at the end. Do not penalise incorrect intercepts.
- (c)
- B1 Sets up a correct equation for the solution, as shown in scheme or equivalents. Should be an equation but allow “=0” implied if there is an attempt to solve. Just $2x^2 - x - 5$ is B0 with no further working.
- M1 Full attempt to solve a correct equation leading to exact answers. Attempts via $f(x) = f^{-1}(x)$ (oe) will lead a quartic ($8x^4 - 40x^2 - x + 45 = 0$ if correct) but will likely not lead to exact answers. Note exact answers following a quadratic is fine, but method should be shown for a quartic. Decimal answer only is M0.
- A1 $x = \frac{1 + \sqrt{41}}{4}$ ONLY.

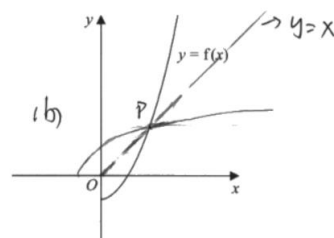
Some examples of curves for question 4(b).



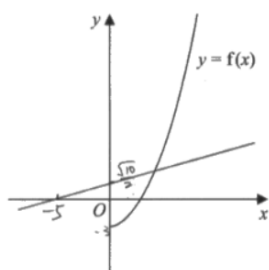
M1A0: Meets conditions for M1 but gradient not decreasing.



M1A1bod: Curve slips downward at the end due to slip of pen. This is a borderline case.



M1A1bod: Gradient is decreasing on the whole.



M0A0: Does not start on negative x-axis. Curvature would be unacceptable.

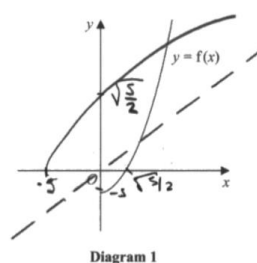
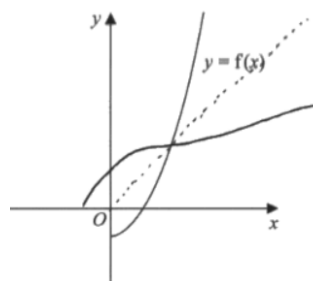


Diagram 1

M1A1bod: Position and curvature correct – ignore possible $y = x$ line, mark the curve.



M1A0: Meets condition for M1 but curvature out of tolerance, gradient increases after intersection.

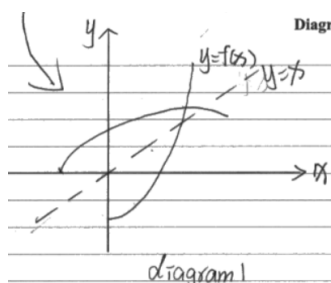


Diagram 1

M1A0: Curve is clearly going downward on the right-hand side.

Question Number	Scheme	Marks
5 (i)	States $x = 2$ $\sqrt{3} \sec x + 2 = 0 \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \dots$ $x = \frac{5\pi}{6}$	B1 M1 A1 (3)
(ii)	Attempts to use $\cos 2\theta = 1 - 2\sin^2 \theta$ $6\sin^2 \theta + 10\sin \theta - 3 = 0$ $\sin \theta = \frac{-5 \pm \sqrt{43}}{6} (= -1.926\dots, 0.2595\dots) \Rightarrow \theta = \arcsin(\dots)$ $\theta = 15.0^\circ, 165^\circ$	M1 A1 M1 A1 (4) (7 marks)
Notes		
(i)		
B1	States $x = 2$. May be seen anywhere in (i) and don't be concerned where it comes from.	
M1	For a correct process to solve $\sqrt{3} \sec x + 2 = 0$ E.g. $\sec x = \frac{1}{\cos x} \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \dots$ Allow slips in rearranging but must attempt to solve $\cos x = k, k < 1$ or $\sec x = k, k > 1$ Degree value (150°) following a correct equation implies the M mark. Note some may use $\sec^2 x = 1 + \tan^2 x$ and form a quadratic in $\tan x$. These will need a correct identity, correct method to solve a quadratic (which may be by calculator) and attempt to solve $\tan x = k, k \neq 0$	
A1	$x = \frac{5\pi}{6}$ and no other extra solutions in the range. Accept awrt 2.62 (and isw).	
Note that $\sqrt{3} \sec x + 2 = 0 \rightarrow x = \frac{5\pi}{6}$ can score M1A1 as no incorrect work is seen, method implied.		
Question required working to be shown $x = \frac{5\pi}{6}$ without seeing at least $\sqrt{3} \sec x + 2 = 0$ extracted first is M0A0.		
(ii)		
M1	Attempts to use $\cos 2\theta = \pm 1 \pm 2\sin^2 \theta$ to form a quadratic equation in $\sin \theta$. If using alternative forms for the identity, must also use $\cos^2 \theta = 1 - \sin^2 \theta$ before gaining this mark.	
A1	Correct 3 term quadratic equation $6\sin^2 \theta + 10\sin \theta - 3 = 0$ or a multiple of this. Alternatively may be scored for $6\sin^2 \theta + 10\sin \theta = 3$ if followed by completing the square on LHS to solve.	
M1	Full attempt to find one value for θ from a quadratic in $\sin \theta$. Must involved - correct method to solve the quadratic in $\sin \theta$ (usual rules, may use calculator) to produce a value for $\sin \theta$ - use of $\arcsin(\dots)$ to reach the value for θ (you may need to check the values if $\arcsin(\dots)$ is not shown). Radian answers can imply the mark (awrt 0.263, 2.88 if correct). May be scored from an incorrect identity as long as a quadratic is achieved. Accept arcsin expression for the M	
A1	$\theta = \text{awrt } 15.0^\circ, 165^\circ$ and no other solutions in the range. Accept just 15° for 15.0° (but not awrt 15° if it does not round to 15.0°)	
Condone a different variable used than θ throughout.		

Question Number	Scheme	Marks
7(a)	$3\cot^2 2\theta - 7 = \frac{13}{2\sin\theta\cos\theta}$ or $2\sin\theta(3(\operatorname{cosec}^2 2\theta - 1) - 7) = 13\sec\theta$ oe	M1
	$3(\operatorname{cosec}^2 2\theta - 1) - 7 = \frac{13}{2\sin\theta\cos\theta}$	dM1
	$\Rightarrow 3\operatorname{cosec}^2 2\theta - 10 = 13\operatorname{cosec} 2\theta \Rightarrow 3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0^*$	ddM1A1*
		(4)
(b)	$3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0$	
	$(3\operatorname{cosec} 2\theta + 2)(\operatorname{cosec} 2\theta - 5) = 0 \Rightarrow (\operatorname{cosec} 2\theta) = 5 \Rightarrow (\sin 2\theta) = \frac{1}{5}$	M1A1
	$\theta = \frac{\sin^{-1}\left(\frac{1}{5}\right)}{2} = \dots$	dM1
	$(\theta =) \text{ awrt } 0.101, 1.47$	A1
		(4)
		(8 marks)
Alt(a)	$2\sin\theta\left(\frac{3(1-\sin^2 2\theta)}{\sin^2 2\theta} - 7\right) = 13\sec\theta$ $\frac{3(1-\sin^2 2\theta)}{\sin^2 2\theta} - 7 = \frac{13}{2\sin\theta\cos\theta}$ $\frac{3}{\sin^2 2\theta} - 10 = \frac{13}{\sin 2\theta}$ $3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0^*$	M1 dM1 ddM1 A1*

(a)M1 Attempts **EITHER**

- to divide both sides by $(2)\sin\theta$ and use $\sec\theta = \frac{1}{\cos\theta}$ (or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$) to achieve a term of

the form $\frac{A}{B\sin\theta\cos\theta}$ **oe** (implied by $\frac{\dots}{\dots\sin 2\theta}$ or $\dots\operatorname{cosec}\theta\sec\theta$).

Note $\sin 2\theta(3\cot^2 2\theta - 7) = 13$ does not score this mark. They have not divided by $(2)\sin\theta$

- to use $\pm 1 \pm \cot^2 2\theta = \pm \operatorname{cosec}^2 2\theta$ to replace the $\cot^2 2\theta$ term in the equation or in the alternative method it is for using $\cot^2 2\theta = \frac{\cos^2 2\theta}{\sin^2 2\theta}$ and using $\pm 1 \pm \sin^2 2\theta = \pm \cos^2 2\theta$ to replace the $\cos^2 2\theta$

Condone arithmetical slips and invisible brackets.



dM1 Attempts **BOTH**

- to divide both sides by $(2)\sin\theta$ and use $\sec\theta = \frac{1}{\cos\theta}$ (or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$) to achieve a term of the form $\frac{A}{B\sin\theta\cos\theta}$ or (implied by $\frac{\dots}{\dots\sin 2\theta}$ or $\dots \operatorname{cosec}\theta\sec\theta$). May be implied
- to use $\pm 1 \pm \cot^2 2\theta = \pm \operatorname{cosec}^2 2\theta$ to replace the $\cot^2 2\theta$ term in the equation or in the alternative method it is for using $\cot^2 2\theta = \frac{\cos^2 2\theta}{\sin^2 2\theta}$ and using $\pm 1 \pm \sin^2 2\theta = \pm \cos^2 2\theta$ to replace the $\cos^2 2\theta$

Condone arithmetical slips and invisible brackets. It is dependent on the first mark.

ddM1 A full attempt to achieve a 3TQ in $\operatorname{cosec} 2\theta$ only. It is dependent on both of the previous method marks. They must

- use the identity $\sin 2\theta = 2\sin\theta\cos\theta$ (or $\operatorname{cosec} 2\theta = \frac{1}{2}\operatorname{cosec}\theta\sec\theta$) and proceed to a $\operatorname{cosec} 2\theta$ term
- multiply out the brackets and collect terms on one side to form a 3TQ in $\operatorname{cosec} 2\theta$ only.

Do not be concerned with the mechanics of their arrangement and they may proceed to the printed answer at this stage. Condone arithmetical slips and invisible brackets.

A1* Achieves $3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0$ with no arithmetical errors in the main body of their solution. You must see either $\sin\theta\cos\theta$ or $\sin 2\theta$ within their working. Allow recovery of invisible brackets in their solution.

Withhold the final mark for poor notation such as $\operatorname{cosec}^2 2\theta$ being written as $\operatorname{cosec} 2\theta^2$ or if the 2 is missing from 2θ

They may work in a different variable, but the final answer must be in terms of 2θ

(b) ***Be aware that answers with no intermediate working seen is not acceptable***

M1 Attempts to solve the quadratic and finds the reciprocal of one of their roots (usual rules apply for solving a quadratic). Do not be concerned by their labelling.

Look for eg $\dots = 5 \rightarrow \frac{1}{5}$ or eg $\dots = -\frac{3}{2} \rightarrow -\frac{2}{3}$

A1 $(\sin 2\theta =) \frac{1}{5}$ (Ignore any reference to $-\frac{3}{2}$) $\frac{1}{5}$ on its own is sufficient. Condone eg

$\sin^2 2\theta = \frac{1}{5}$ or $\sin X = \frac{1}{5}$ or $\sin\theta = \frac{1}{5}$

dM1 Correct method to find a value for θ by finding $\arcsin(\text{their } \frac{1}{5})$ and dividing by 2.

May be implied by awrt 0.10 or awrt 1.5 (or in degrees awrt 5.8 or awrt 84).

It is dependent on seeing either the quadratic solved or their root from the quadratic (either ≥ 1 or ≤ -1) or the reciprocal of the root being stated first.

It cannot be scored for solving eg $\sin^2 2\theta = \dots$

A1 $(\theta =)$ awrt 0.101, awrt 1.47 and no other angles within the given interval.

Some intermediate working must be shown. Examples of minimum working shown in (b):



$$\sin 2\theta = \frac{1}{5} \Rightarrow (\theta =) 0.101, 1.47$$

$$\operatorname{cosec} 2\theta = 5 \Rightarrow (\theta =) 0.101, 1.47$$

Question Number	Scheme	Marks
8(a)	$\frac{dv}{dt} = -e^{t-10} + 9e^{-0.75t} = 0 \Rightarrow 9 = e^{1.75t-10} \text{ or } 9e^{-0.75t} = e^{t-10}$	M1A1
	$9 = e^{1.75t-10} \Rightarrow t = \frac{10 + \ln 9}{1.75} \text{ or } \ln 9 - 0.75t = t - 10 \Rightarrow t = \frac{10 + \ln 9}{1.75}$	M1
	$t = \text{awrt } 6.97$	A1
	$\text{awrt } 11.9 \text{ (ms}^{-1}\text{)}$	A1
		(5)
(b)	$\int 12 - 12e^{-0.75t} - e^{(t-10)} dt = 12t - e^{(t-10)} + 16e^{-0.75t} (+C)$	M1A1
	$\left[12t - e^{(t-10)} + 16e^{-0.75t} \right]_0^T = 100 \Rightarrow 12T = \dots$	M1
	$T = \frac{1}{12} (116 - 16e^{-0.75T} + e^{T-10} - e^{-10}) *$	A1*
		(4)
(c) (i) (ii)	$T_2 = \frac{1}{12} (116 - 16e^{-0.75 \times 10} + e^{10-10} - e^{-10})$	M1
	$T_2 = \text{awrt } 9.7493$	A1
	9.7293 (seconds)	A1
		(3)
		(12 marks)

(a) * Be aware this can be solved directly on a calculator. Calculus must be seen*

M1 Differentiates to $\pm Ae^{t-10} \pm Be^{-0.75t}$ and sets equal to 0 Must be seen in (a) Do not accept differentiating to eg $\dots te^{t-10}$. Condone an expression of $12 \pm Ae^{t-10} \pm Be^{-0.75t}$ only

A1 $9 = e^{1.75t-10}$ or equivalent eg $9e^{-0.75t} - e^{t-10} = 0$

M1 Proceeds from their equation of the form $Ae^{Ct+D} - Be^{Et+F} = 0$ or equivalent leading to a value or an expression (which may be unsimplified) for t . If log work is seen it must be correct.

Eg $\frac{40 + 4 \ln 9}{7}$ or $\frac{10 + \ln 9}{1.75}$ will score this mark

$9e^{-0.75t} - e^{t-10} = 0 \Rightarrow \text{awrt } 6.97$ can also score this mark

This cannot be scored from an unsolvable equation.

Question Number	Scheme	Marks
7(a)	$f(0) = (0-3)^2 = 9$	M1
	$0 \leq f(x) \leq 9$	A1
		(2)
(b)	$f'(x) = -2xe^{-x^2}(2x^2-3)^2 + e^{-x^2} \times 8x(2x^2-3)$	M1A1
	$= 2x(2x^2-3)e^{-x^2}(-(2x^2-3)+4) = 2xe^{-x^2}(2x^2-3)(7-2x^2)$	dM1A1
		(4)
(c)	$x^2 = \frac{3}{2}, \frac{7}{2} \Rightarrow f\left(\sqrt{\frac{7}{2}}\right) = e^{-\frac{7}{2}}\left(2 \times \frac{7}{2} - 3\right)^2 = 16e^{-\frac{7}{2}}$	M1A1
	$16e^{-\frac{7}{2}} < k < 9$	dM1A1
		(4)
		(10 marks)
Notes		
(a) Work for part (a) must be seen in part (a) not recovered in part (c). M1: Substitutes $x = 0$ and proceeds to find a value for y. Implied by sight of 9. A1: $0 \leq f(x) \leq 9$ Accept with y or just f, but not with x. Accept interval, $[0, 9]$, or set notation. (b) M1: Attempts the product rule and chain rule achieving $\pm Pxe^{-x^2}(2x^2 \pm 3)^2 + e^{-x^2} \times Qx(2x^2 \pm 3)$ with $P, Q > 0$. Alternatively, attempts the quotient rule on $\frac{(2x^2-3)^2}{e^{x^2}}$ achieving $\frac{Px(2x^2 \pm 3)e^{x^2} - Qxe^{x^2}(2x^2 \pm 3)^2}{(e^{x^2})^2}$ $P, Q > 0$ A1: $-2xe^{-x^2}(2x^2-3)^2 + e^{-x^2} \times 8x(2x^2-3)$ oe (need not be simplified) dM1: Must have scored previous M. Attempts to take out a factor of $2xe^{-x^2}(2x^2-3)$ to obtain a factor $(\pm C \pm Dx^2)$ which may be unsimplified. Allow if e.g. the x or e^{-x^2} is dropped when taking out the factor. A1: Achieves $2xe^{-x^2}(2x^2-3)(7-2x^2)$ with no errors seen. (c) Note : allow all the marks in (c) from answers which were a constant multiple out in (b) or missing the factor x, which lead to the correct answers here. M1: Attempts to find a y value (which may be zero) for at least one of the valid non-zero roots for $f'(x)=0$. Allow for an attempt at either $f\left(\pm\sqrt{\frac{3}{2}}\right)$ or $f\left(\pm\sqrt{\frac{A}{B}}\right)$ leading to a value, where A and B are their values from (b) with $AB > 0$. Note e.g. $f\left(\frac{A}{B}\right)$ attempted is M0. A1: For obtaining $16e^{-\frac{7}{2}}$ Accept awrt 0.483 for this mark following the award of M. dM1: Attempts the inside region, allowing $\leq$, using their y intercept from (a) and their positive y value, less than their intercept, from an attempt at the y value of the maxima. Allow a positive y value from an attempt at any non-zero root of $f'(x)$ as such an attempt. It is dependent on the previous method mark. A1: $16e^{-\frac{7}{2}} < k < 9$ or in any equivalent form e.g. interval notation $k \in \left(\frac{16}{e^{\frac{7}{2}}}, 9\right)$ Allow y instead of k Ignore references to "$k = 0$"		

(d) Way 2

B1: Sets up a valid equation in a e.g. $\frac{2a}{3a-5} = g^{-1}(5)$ seen or implied.

M1: Attempts $g^{-1}(5)$ and reaches $\frac{2a}{3a-5} = e^k$ oe

dM1: Rearranges an equation of the form $\frac{2a}{3a-5} = e^k$ to obtain $a = \frac{...e^k}{...e^k \pm ...}$ or equivalent.

Depends on the previous method mark.

A1: $(a =) \frac{5e}{3e-2}$ oe e.g. $(a =) \frac{-5e}{2-3e}$ and apply isw once the correct exact answer is seen.
Accept e^1 for e .

Alternative for (d) which has been seen:

$$g\left(\frac{2a}{3a-5}\right) = 5 \Rightarrow 3 + 2\ln\left(\frac{2a}{3a-5}\right) = 5: \text{ B1 as above}$$

$$\Rightarrow \ln\left(\frac{2a}{3a-5}\right)^2 = 2 \Rightarrow \left(\frac{2a}{3a-5}\right)^2 = e^2: \text{ M1 rearranges to reach } \left(\frac{2a}{3a-5}\right)^2 = e^2$$

$$\Rightarrow (9e^2 - 4)a^2 - 30e^2a + 25e^2 = 0$$

$$\Rightarrow a = \frac{30e^2 \pm \sqrt{900e^4 - 100e^2(9e^2 - 4)}}{2(9e^2 - 4)}: \text{ dM1 forms and solves 3TQ in } a \text{ (usual rules)}$$

$$= \frac{5e}{3e-2} \text{ oe: A1 as above}$$

Question Number	Scheme	Marks
5(a)	(A=)8	B1 (1)
(b)	$16 = 10 + "8" e^{-Bt} \Rightarrow e^{-Bt} = \dots$	M1
	$e^{-45B} = "3/4" \Rightarrow -45B = \ln "3/4" \Rightarrow B = \dots$	dM1
	$B = \text{awrt } 0.00639$	A1
		(3)
(c)	$\frac{dT}{dt} = -"8" \times \frac{1}{45} \ln \left(\frac{4}{3} \right) \times e^{-\frac{1}{45} \ln \left(\frac{4}{3} \right) \times 2} = \dots$	M1
	$= -0.0505$	A1
		(2)
(d)	The temperature has a (lower) limit of 10°C or $5 = 10 + "8" e^{-Bt} \Rightarrow e^{-Bt} = -\frac{5}{"8"}$ e.g. which is not possible or cannot be solved or you cannot find the log of a negative number	B1 (1)
		(7 marks)

(a)

B1: (A=)8 or award for (T=)10+8e^{-Bt}. Note that "A=" is not required, just look for 8.

(b)

M1: Sets $16 = 10 + "8" e^{-Bt}$ and rearranges to the form $e^{-Bt} = \dots$ with or without $t = 45$

dM1: Uses $t = 45$, takes lns of both sides and proceeds to find a numerical expression for B

Depends on the previous method mark.

A1: Achieves awrt 0.00639 or 6.39×10^{-3} or e.g. $\frac{1}{45} \ln \left(\frac{4}{3} \right)$ or $-\frac{1}{45} \ln \left(\frac{3}{4} \right)$ or $\frac{-\ln(0.75)}{45}$ etc.

and apply isw once the correct value is seen. **Correct answer only scores no marks.**

(c)

M1: Differentiates using the chain rule to obtain $\dots e^{-Bt}$ (not $\dots e^{Bt}$) where $\dots$ is a constant and substitutes in $t = 2$ to obtain a value for $\frac{dT}{dt}$.

Condone $\frac{dT}{dt} = "8" e^{-Bt}$ as long as it is clear they think they have found $\frac{dT}{dt}$.

If they lose the minus sign in $\dots e^{-Bt}$ they obtain $\pm 0.0518\dots$ and this scores M0

Note that if there is evidence that $\frac{dT}{dt} = \dots t e^{-Bt}$ is being used, score M0

A1: -0.0505 cao. Accept 0.0505 **decreasing** or equivalent **following correct work**.

Ignore any units, correct or incorrect)

Correct answer only scores no marks.

(d)

B1: Either states that the (lower) limit of the model is 10°C or there is an asymptote at $T = 10$ or as t tends to ∞ T tends to 10. But do not allow incorrect statements e.g. the maximum temperature is 10. Also allow “the maximum the temperature can drop to is 10”

or

shows that the calculation cannot be carried out and makes a statement to that effect. The working must be correct for their A and/or B where $A > 0$ and $B > 0$ and they must reach at least $e^{-Bt} = \dots$

Allow comments such as: “which is not possible”, “cannot be done”, “you cannot find the log of a negative number” etc. but do not allow ambiguous/incorrect statements e.g. “logs cannot be negative”, “you cannot have a negative time”

Also accept equivalent arguments e.g. “ Ae^{-Bt} would need to be < 0 and this is not possible” (provided their $A > 0$)

If you are unsure if the reasoning is acceptable or not, use review.



Question Number	Scheme	Marks
6(a)	$\left(\frac{3\pi}{2}, 0\right)$	B1
		(1)
(b)	$f'(x) = (6 \cos^2 x) e^{3 \sin x} - (2 \sin x) e^{3 \sin x}$	M1A1
	$f'(x) = (6 \cos^2 x) e^{3 \sin x} - (2 \sin x) e^{3 \sin x}$ $\Rightarrow (6(1 - \sin^2 x) - 2 \sin x)(=0)$	dM1
	$\Rightarrow 3 \sin^2 x + \sin x - 3 = 0$ oe	A1
		(4)
(c)	$\sin x = \frac{-1 + \sqrt{37}}{6} \Rightarrow x = \dots$	M1
	$x = 2.131$	A1
		(2)
		(7 marks)

(a)

B1: $\left(\frac{3\pi}{2}, 0\right)$ or accept the identification that $x = \frac{3\pi}{2}$ at R e.g. $\cos x = 0$ so $x = \frac{3\pi}{2}$

Accept $\frac{3\pi}{2}$ written on the diagram at the point R or just $\frac{3\pi}{2}$ seen as the answer as long as there is no evidence that y is anything other than zero. Must be exact.

(b)

M1: Attempts the product rule. Award for the form $f'(x) = \pm \alpha \cos^2 x e^{3 \sin x} \pm \beta \sin x e^{3 \sin x}$

A1: Correct derivative in any form

dM1: Attempts to use the identity $\pm \sin^2 x \pm \cos^2 x = \pm 1$ to produce a 3 term quadratic expression in $\sin x$ which may be seen embedded but with the $e^{3 \sin x}$ factored.

$$\text{e.g. } e^{3 \sin x} (6(1 - \sin^2 x) - 2 \sin x)$$

Depends on the previous method mark.

A1: $3 \sin^2 x + \sin x - 3 = 0$ or any integer multiple of this equation e.g. $6 - 6 \sin^2 x - 2 \sin x = 0$ and apply isw once a correct equation is seen.

Must be “extracted” but allow if seen, used or implied in part (c) and allow recovery from missing brackets if the quadratic expression is extracted correctly.

Beware:

$$f'(x) = (-6 \cos^2 x) e^{3 \sin x} + (2 \sin x) e^{3 \sin x} = 0 \text{ leading to } 3 \sin^2 x + \sin x - 3 = 0 \text{ scores}$$

M1A0dM1A0

But allow full recovery in (c)



(c)

M1: Solves their 3-term quadratic in $\sin x$ (allow any method including calculator) and proceeds to find a value of x using $\sin^{-1}(\text{their } \sin x)$ where $|\sin x| < 1$. You may need to check their value.

The attempt to solve their quadratic may be implied by at least 1 correct value for $\sin x$ (allow 2dp if inexact).

A1: awrt 2.131 only and no other values offered.

Correct answer only scores no marks.



Question Number	Scheme	Marks
7(a)	$\left(\frac{dy}{dx}\right) = -\frac{16}{3}(3x-k)^{-2}$	M1A1
		(2)
(b)	$-\frac{16}{3}(3x-k)^{-2} = -12 \Rightarrow (3x-k)^2 = \dots$	M1
	$3-k = \pm \frac{2}{3} \Rightarrow k = \dots$	dM1
	$k = \frac{7}{3}, \frac{11}{3}$	A1
		(3)
(c)	$y = \frac{16}{9\left(3(1)-\frac{7}{3}\right)}$	M1 (B1 on ePEN)
	$y - \frac{8}{3} = \frac{1}{12}(x-1)$	dM1
	$12y - x - 31 = 0$	A1
		(3)
(d)	$\int \frac{16}{9(3x-k)} dx = \frac{16}{27} [\ln(3x-k)]$	M1
	$= \frac{16}{27} \left[\ln\left(3x - \frac{7}{3}\right) \right]$	A1ft
	$\frac{16}{27} \left[\ln\left(3x - \frac{7}{3}\right) \right]_1^3 = \frac{16}{27} \left(\ln\left(3(3) - \frac{7}{3}\right) - \ln\left(3(1) - \frac{7}{3}\right) \right)$	dM1
	$= \frac{16}{27} \ln(10)$	A1
		(4)
		(12 marks)

(a)M1: Attempts to differentiate to the form $A(3x-k)^{-2}$ oe e.g. $A(27x-9k)^{-2}$ A1: $\left(\frac{dy}{dx}\right) = -\frac{16}{3}(3x-k)^{-2}$ oee.g. $-\frac{16}{3(3x-k)^2}$ or $-\frac{16}{3(9x^2-6kx+k^2)}$ or $-\frac{16}{27x^2-18kx+3k^2}$ but **not** $-432(27x-9k)^{-2}$ or $-\frac{432}{(27x-9k)^2}$

as there is a common factor in the numerator and denominator.



(b)

- M1: Sets their derivative of the form $A(3x-k)^{-2}$ (or equivalent) equal to -12 (not 12) and rearranges to $\dots(3x-k)^2 = \dots$ or equivalent e.g. $\dots(27x-9k)^2 = \dots$ or $\dots(9x^2 - 6kx + k^2) = \dots$
 Condone poor squaring e.g. allow $\dots(9x^2 + k^2) = \dots$
 May have already substituted $x = 1$

dM1: Depends on having obtained $A < 0$ (otherwise the equation has no real solutions):

Substitutes $x = 1$ and solves to find 2 values for k .

If the $(3x-k)^2$ is expanded then the usual rules apply for solving a 3TQ and allow using a calculator. FYI correct 3TQ is $9k^2 - 54k + 77 = 0$

Depends on the previous method mark.

- A1: Achieves $k = \frac{7}{3}$ and $k = \frac{11}{3}$ from a correct method. Accept equivalent exact fractions or recurring decimals $2.\dot{3}$, $3.\dot{6}$ but not rounded decimals e.g. 2.33, 3.67

(c) Note we are scoring the first mark as an M mark not a B mark.

M1(B1 on ePEN):

Uses a value of k from part (b) (or a 'made up' k) and $x = 1$ to find the value of y at P .

- dM1:** Attempts to find the equation of the normal using their y -coordinate with the gradient $\frac{1}{12}$.

If they use $y = mx + c$ they must proceed as far as $c = \dots$

Depends on the previous method mark.

- A1: $12y - x - 31 = 0$ or any equivalent integer multiple of this equation.

Note if $k = \frac{11}{3}$ is used they should get $x - 12y - 33 = 0$ and would generally score 110

(d)

- M1: Integrates to the form $B \ln(3x-k)$ or e.g. $B \ln \alpha(3x-k)$

- A1ft: $\frac{16}{27} \left[\ln \left(3x - \frac{7}{3} \right) \right]$ which may be unsimplified and isw once correct integration is seen.

Follow through their k and allow the letter k and allow if their k is not less than 3.

Ignore any reference to $+c$.

You may need to check their integration carefully.

$$\text{E.g. } \frac{16}{27} \left[\ln(27x - 21) \right] \text{ is also correct (for } k = \frac{7}{3} \text{)}$$

Ignore any spurious integral signs after a correct integral is seen.

dM1: Substitutes in the limits 3 and 1 and subtracts either way round. Must have a numeric k now.

It is dependent on the first method mark.

A1: $\frac{16}{27} \ln(10)$ or exact equivalent e.g. $\frac{32}{54} \ln(10)$

Use of an incorrect k in (d) scores a maximum of M1A1ft dM1A0

Note that in part (d), some candidates may use substitution e.g.

$$u = 3x - k \Rightarrow \frac{du}{dx} = 3 \Rightarrow \int \frac{16}{9(3x-k)} dx = \frac{16}{9} \int \frac{1}{u} \frac{1}{3} du = \frac{16}{27} \ln u$$

Score M1 for integrating to the correct form e.g. $k \ln u$

and A1 for $\frac{16}{27} \ln u$ following through their k or the letter k as above

then dM1 for

$$\left[\frac{16}{27} \ln u \right]_{\frac{2}{3}}^{\frac{20}{3}} = \frac{16}{27} \ln \frac{20}{3} - \frac{16}{27} \ln \frac{2}{3} \text{ or } \left[\frac{16}{27} \ln \left(3x - \frac{7}{3} \right) \right]_1^3 = \frac{16}{27} \ln \frac{20}{3} - \frac{16}{27} \ln \frac{2}{3}$$

i.e. applies the correct changed limits or reverts to x and uses 3 and 1

$$= \frac{16}{27} \ln(10) \quad \text{A1}$$

Note if $k = \frac{11}{3}$ is used they should get $= \frac{16}{27} \ln(8)$ and would generally score 1110

Note:

If you see any responses where the denominator in part (a) is expanded incorrectly

$$\text{e.g. } \frac{16}{9(3x-k)} = \frac{16}{27x-k}$$

and candidates persist with this incorrect expansion then send to review.

A typical response with an expanded denominator:

Question Number	Scheme	Marks
7(a)	$y = \frac{16}{9(3x-k)} = \frac{16}{27x-9k} \Rightarrow \frac{dy}{dx} = -\frac{432}{(27x-9k)^2}$	M1A0
(b)	$-\frac{432}{(27x-9k)^2} = -12 \Rightarrow 12(27x-9k)^2 = 432$	M1
	$12(27x-9k)^2 = 432 \Rightarrow (27x-9k)^2 = 36 \Rightarrow 27-9k = \pm 6 \Rightarrow k = \dots$	dM1
	$k = \frac{7}{3}, \frac{11}{3}$	A1
(c)	$y = \frac{16}{27-21}$	M1 (B1 on ePEN)
	$y - \frac{8}{3} = \frac{1}{12}(x-1)$	dM1
	$12y - x - 31 = 0$	A1
(d)	$\int \frac{16}{27x-9k} dx = \frac{16}{27} [\ln(27x-9k)]$	M1
	$= \frac{16}{27} [\ln(27x-21)]$	A1ft
	$\frac{16}{27} [\ln(27x-21)]_1^3 = \frac{16}{27} (\ln(27(3)-21) - \ln(27-21))$	dM1
	$= \frac{16}{27} \ln(10)$	A1

Question Number	Scheme	Marks
8(a)(i)	$\left(\frac{b}{2}, a\right)$	B1B1
(ii)	$(0, a-b)$	B1
(iii)	$\left(\frac{b-a}{2}, 0\right)$ and $\left(\frac{a+b}{2}, 0\right)$	B1B1
		(5)

(a) (i)

B1: One correct coordinate $x = \frac{1}{2}b$ or $y = a$

B1: $\left(\frac{b}{2}, a\right)$ or $x = \frac{1}{2}b$ and $y = a$

May be seen on the sketch.

(ii)

B1: $(0, a-b)$ **or** $x = 0$ and $y = a-b$ **or** just $y = a-b$ without the $x = 0$

May be seen on the sketch and if so, allow just $a-b$ marked in the correct place.

If it is on the sketch, condone $(a-b, 0)$ marked in the **correct place**.

If more than 1 point is offered and no clear decision is made then score B0

(iii)

B1: $\left(\frac{b-a}{2}, 0\right)$ **or** $\left(\frac{a+b}{2}, 0\right)$ Allow just $x = \frac{b-a}{2}$ or $x = \frac{a+b}{2}$ without the $y = 0$

May be seen on the sketch and allow just $\frac{b-a}{2}$ or $\frac{a+b}{2}$ marked in the **correct place**.

If they are on the sketch, condone $\left(0, \frac{b-a}{2}\right)$ or $\left(0, \frac{a+b}{2}\right)$ marked in the **correct place**.

Allow equivalent expressions e.g. $\frac{-a-b}{-2}$ for $\frac{a+b}{2}$ or e.g. $\frac{a-b}{-2}$ for $\frac{b-a}{2}$

B1: $\left(\frac{b-a}{2}, 0\right)$ **and** $\left(\frac{a+b}{2}, 0\right)$ Allow just $x = \frac{b-a}{2}$ or $x = \frac{a+b}{2}$ without the $y = 0$

May be seen on the sketch and allow just $\frac{b-a}{2}$ and $\frac{a+b}{2}$ marked in the **correct place**.

If they are on the sketch, condone $\left(0, \frac{b-a}{2}\right)$ and $\left(0, \frac{a+b}{2}\right)$ marked in the **correct place**.

Allow equivalent expressions e.g. $\frac{-a-b}{-2}$ for $\frac{a+b}{2}$ or e.g. $\frac{a-b}{-2}$ for $\frac{b-a}{2}$



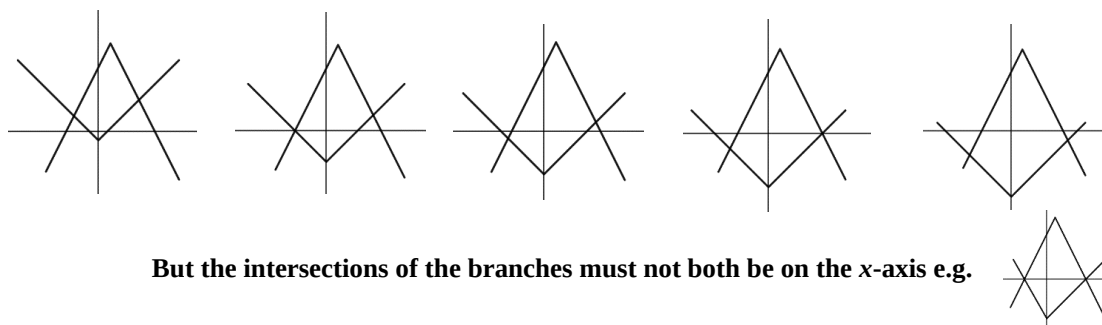
(b)		B1B1
		(2)
(c)	$-x-1=2x+a-b, x=-3 \Rightarrow 2=-6+a-b$ or $x-1=a+b-2x, x=5 \Rightarrow 5-1=a+b-10$	M1
	$-x-1=2x+a-b, x=-3 \Rightarrow 2=-6+a-b$ and $x-1=a+b-2x, x=5 \Rightarrow 5-1=a+b-10$	dM1 (A1 on ePEN)
	$a-b=8 \Rightarrow a=\dots$ or $b=\dots$ $a+b=14$	ddM1
	$a=11, b=3$	A1
		(4)
		(11 marks)

(b)

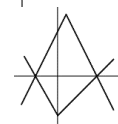
B1: A V shaped graph anywhere on the set of axes

B1: Correct shape with a minimum point on the y-axis below the x-axis and with an intention for the graph to be symmetrical about the y-axis. The left branch of $y = |x| - 1$ must intersect the left branch of $y = a - |2x - b|$ and the right branch of $y = |x| - 1$ must intersect the right branch of $y = a - |2x - b|$

These are all acceptable for both marks:



But the intersections of the branches must not both be on the x-axis e.g.



Some candidates may use Figure 2 to do their sketch which is acceptable but if both Diagram 1 and Figure 2 are used and neither is “rejected” then Diagram 1 takes precedence.

If they draw a sketch of $y = |x| - 1$ on their own axes, only the first B mark is available unless they draw the given graph as well in which case the 2nd B is available.

(c) Note that $x = -3 \Rightarrow y = 2$ and $x = 5 \Rightarrow y = 4$ and may be used to find the equations.

M1: Forms one correct equation in a and b only with modulus signs removed, simplified or unsimplified. There may be several equations so you will need to check if any of them are correct.

dm1(A1 on ePEN):

Forms 2 correct equations in a and b only with modulus signs removed, simplified or unsimplified. There may be several equations so you will need to check if 2 of them are correct.

Note that for their second equation, some candidates may substitute in for a or b from their 1st equation but may make a slip when rearranging. Condone this if the 2nd equation is otherwise correct.

ddM1: Solves the correct equations in a and b e.g. $8 = a - b$ and $14 = a + b$ or equivalent to find a value for a or a value for b .

A1: $a = 11$, $b = 3$ (Correct answers with no working scores no marks)



Question Number	Scheme	Marks
9(a)	$\frac{3\sin\theta\cos\theta}{\cos\theta+\sin\theta} = (2+\sec 2\theta)(\cos\theta-\sin\theta)$ $\Rightarrow \frac{3}{2}\sin 2\theta = (2+\sec 2\theta)(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)$ or $\Rightarrow 3\sin\theta\cos\theta = (2+\sec 2\theta)\cos 2\theta$	M1
	$\Rightarrow \frac{3}{2}\sin 2\theta = (2+\sec 2\theta)\cos 2\theta$	dM1
	$\Rightarrow \frac{3}{2}\sin 2\theta = (2+\sec 2\theta)\cos 2\theta$ $\Rightarrow \frac{3}{2}\sin 2\theta = 2\cos 2\theta + 1 \Rightarrow 3\sin 2\theta - 4\cos 2\theta = 2^*$	A1*
		(3)

(a)M1: Attempts to use $\sin 2\theta = 2\sin\theta\cos\theta$ **or** $\cos 2\theta = \pm\cos^2\theta \pm \sin^2\theta$ Accept going from $(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$ to $\cos 2\theta$ dM1: Attempts to use $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\cos 2\theta = \pm\cos^2\theta \pm \sin^2\theta$ Accept going from $(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$ to $\cos 2\theta$ **Depends on the previous method mark.**A1*: Uses the identity $\sec 2\theta = \frac{1}{\cos 2\theta}$ to achieve an equation in just $\sin 2\theta$ and $\cos 2\theta$ and

proceeds to the given answer with all previous stages of working seen.

Note we would accept going from e.g. $6\sin\theta\cos\theta = 4\cos 2\theta + 2$ to the given answer.**You can condone e.g. a missing θ or a missing bracket but withhold the A mark if there are clear errors such as a missing “2” or a missing pair of brackets that are not recovered.**

There are various alternatives in part (b) and in each case:

- the first 3 marks are for obtaining a suitable equation
- the final 2 marks are for solving their equation

Condone the use of a variable other than x in (b) e.g. θ

(b)	Way 1 using $R\sin(2x-\alpha)$ or $R\cos(2x+\alpha)$	
	$\Rightarrow R = \sqrt{3^2 + 4^2} = \dots(5)$ or $(\alpha =) \tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right) = \dots$	M1
	$\Rightarrow R = \sqrt{3^2 + 4^2} = \dots(5)$ and $(\alpha =) \tan^{-1}\left(\frac{4}{3} \text{ or } \frac{3}{4}\right) = \dots$	dM1
	$3\sin 2x - 4\cos 2x = 2 \Rightarrow 5\sin(2x - 0.927) = 2$ or $3\sin 2x - 4\cos 2x = 2 \Rightarrow 5\cos(2x + 0.644) = -2$	A1
	$5\sin(2x - 0.927) = 2 \Rightarrow x = \frac{\sin^{-1}\frac{2}{5} + 0.927}{2}$ or $5\cos(2x + 0.644) = -2 \Rightarrow x = \frac{\cos^{-1}\frac{-2}{5} - 0.644}{2}$	ddM1
	$x = \text{awrt } 3.81$	A1
		(5)
		(8 marks)

Note that it is not necessary to state the form $R\sin(2x-\alpha)$ or $R\cos(2x+\alpha)$ for the first 2 marks

M1: $R = \sqrt{3^2 + 4^2} (=5)$ Condone obtaining ± 5

or $(\alpha =) \tan^{-1}\left(\pm\frac{4}{3}\right), \tan^{-1}\left(\pm\frac{3}{4}\right), \cos^{-1}\left(\pm\frac{3}{5}\right), \sin^{-1}\left(\pm\frac{4}{5}\right), \cos^{-1}\left(\pm\frac{4}{5}\right), \sin^{-1}\left(\pm\frac{3}{5}\right)$

dM1: $R = \sqrt{3^2 + 4^2} (=5)$ Condone obtaining ± 5

and $(\alpha =) \tan^{-1}\left(\pm\frac{4}{3}\right), \tan^{-1}\left(\pm\frac{3}{4}\right), \cos^{-1}\left(\pm\frac{3}{5}\right), \sin^{-1}\left(\pm\frac{4}{5}\right), \cos^{-1}\left(\pm\frac{4}{5}\right), \sin^{-1}\left(\pm\frac{3}{5}\right)$

Depends on the previous method mark.

A1: Correct equation: $5\sin(2x - 0.927) = 2$ **or** $5\cos(2x + 0.644) = -2$ which may be implied.

Accept awrt 0.93 or 0.64 for " α ".

Do not allow e.g. $\pm 5\sin(2x - 0.927) = 2$ or $\pm 5\cos(2x + 0.644) = -2$ unless they recover by solving the correct equation.

ddM1: Attempts to solve their equation of the correct form e.g. with $(2x \pm \dots)$ as the argument and proceeds to find a value for x .

The order of operations should be correct for their equation but condone $\pm "0.927"$ or $\pm "0.644"$

e.g. $x = \frac{\left(\sin^{-1}\left(\frac{2}{5}\right)\right) \pm "0.927"}{2}$ or $x = \frac{\cos^{-1}\frac{-2}{5} \pm 0.644}{2}$



You may need to check their angle on your calculator.

It is dependent on both of the previous method marks.

(b)	Way 2 Squaring	
	$3\sin 2x - 4\cos 2x = 2 \Rightarrow 3\sin 2x = 2 + 4\cos 2x$ $\Rightarrow 9\sin^2 2x = 4 + 16\cos 2x + 16\cos^2 2x$ or $3\sin 2x - 4\cos 2x = 2 \Rightarrow 3\sin 2x - 2 = 4\cos 2x$ $\Rightarrow 9\sin^2 2x - 12\sin 2x + 4 = 16\cos^2 2x$	M1
	$9(1 - \cos^2 2x) = 4 + 16\cos 2x + 16\cos^2 2x$ $\Rightarrow 25\cos^2 2x + 16\cos 2x - 5 = 0$ or $9\sin^2 2x - 12\sin 2x + 4 = 16(1 - \sin^2 2x)$ $\Rightarrow 25\sin^2 2x - 12\sin 2x - 12 = 0$	ddM1A1
	$25\cos^2 2x + 16\cos 2x - 5 = 0 \Rightarrow \cos 2x = \frac{-8 \pm 3\sqrt{21}}{25} (0.23, -0.87)$ or $25\sin^2 2x - 12\sin 2x - 12 = 0 \Rightarrow \sin 2x = \frac{6 \pm 4\sqrt{21}}{25} (0.97, -0.49)$ $\Rightarrow x = \dots$	ddM1
	$x = \text{awrt } 3.81$	A1

A1: $x = \text{awrt } 3.81$ and no other values in range. ($x = \pi + 0.669$ scores A0)

M1: Isolates 1 trig term on one side and then attempts to square both sides and multiply out the brackets. Do **not** condone poor squaring for this mark. e.g. do not condone

$$3\sin 2x = 2 + 4\cos 2x \Rightarrow 9\sin^2 2x = 4 + 16\cos^2 2x$$

ddM1: Attempts to use $\pm \sin^2 2x \pm \cos^2 2x = \pm 1$ and proceeds to a 3-term quadratic equation in either $\sin 2x$ or $\cos 2x$

Depends on the previous method mark.

A1: $25\cos^2 2x + 16\cos 2x - 5 = 0$ or $25\sin^2 2x - 12\sin 2x - 12 = 0$ oe

ddM1: Attempts to solve their 3TQ in either $\sin 2x$ or $\cos 2x$ and proceeds to find value for x by taking $\sin^{-1}$ or $\cos^{-1}$ and dividing by 2.

Usual rules apply for solving a quadratic. They may even just state the roots from their calculator. It is dependent on both of the previous method marks.

You may need to check their angle on your calculator.

It is dependent on both of the previous method marks.

A1: $\text{awrt } 3.81$ and no other values in range. ($x = \pi + 0.669$ scores A0)



(b)	Way 3 Using double angle formulae	
	$3\sin 2x - 4\cos 2x = 2 \Rightarrow 6\sin x \cos x - 4\cos^2 x + 4\sin^2 x = 2$	M1
	$6\sin x \cos x - 4\cos^2 x + 4\sin^2 x = 2 \Rightarrow 6\tan x - 4 + 4\tan^2 x = 2\sec^2 x$ $\Rightarrow 6\tan x - 4 + 4\tan^2 x = 2(1 + \tan^2 x)$ $\Rightarrow \tan^2 x + 3\tan x - 3 = 0$	dM1A1
	$\Rightarrow \tan^2 x + 3\tan x - 3 = 0 \Rightarrow \tan x = \frac{-3 \pm \sqrt{21}}{2} (0.79, -3.79)$ $\Rightarrow x = \dots$	ddM1
	$x = \text{awrt } 3.81$	A1

M1: Attempts to use $\sin 2x = 2\sin x \cos x$ and $\cos 2x = \pm \sin^2 x \pm \cos^2 x$ oe
e.g. $\cos 2x = \pm 1 \pm 2\cos^2 x$

dM1: Divides by $\cos^2 x$ and attempts to use $\sec^2 x = \pm 1 \pm \tan^2 x$ and proceeds to a 3-term quadratic equation in $\tan x$. Alternatively may divide by $\sin^2 x$ and proceed to a 3-term quadratic equation in $\cot x$.

It is dependent on the previous method mark.

A1: $\tan^2 x + 3\tan x - 3 = 0$ oe $3\cot^2 x - 3\cot x - 1 = 0$

ddM1: Attempts to solve their 3TQ in $\tan x$ (or $\cot x$) and proceeds to find a value for x .
Usual rules apply for solving a quadratic. They may even just state the roots from their calculator. It is dependent on both of the previous method marks.
You may need to check their angle on your calculator.

It is dependent on both of the previous method marks

A1: awrt 3.81 and no other values in range. ($x = \pi + 0.669$ scores A0)



(b)	Way 4 Divides by $\cos 2x$ and squares	
	$3\sin 2x - 4\cos 2x = 2 \Rightarrow 3\tan 2x - 4 = 2\sec 2x$ $\Rightarrow 9\tan^2 2x - 24\tan 2x + 16 = 4\sec^2 2x$	M1
	$\Rightarrow 9\tan^2 2x - 24\tan 2x + 16 = 4(1 + \tan^2 2x)$ $\Rightarrow 5\tan^2 2x - 24\tan 2x + 12 = 0$	dm1A1
	$5\tan^2 2x - 24\tan 2x + 12 = 0 \Rightarrow \tan 2x = \frac{12 \pm 2\sqrt{21}}{5} (4.23, 0.57)$ $\Rightarrow x = \dots$	ddM1
	$x = \text{awrt } 3.81$	A1

M1: Divides by $\cos 2x$ and then attempts to square both sides and multiply out the brackets.
Do not condone poor squaring for this mark e.g. $(3\tan 2x - 4)^2 = 9\tan^2 2x + 16$

dm1: Attempts to use $\sec^2 2x = \pm 1 \pm \tan^2 2x$ and proceeds to a 3-term quadratic equation in $\tan 2x$
It is dependent on the previous method mark.

A1: $5\tan^2 2x - 24\tan 2x + 12 = 0$ oe

ddM1: Attempts to solve their 3TQ in $\tan 2x$ and proceeds to find a value for x by taking $\tan^{-1}$ and dividing by 2.
Usual rules apply for solving a quadratic. They may even just state the roots from their calculator. It is dependent on both of the previous method marks.
You may need to check their angle on your calculator.
It is dependent on both of the previous method marks.

A1: awrt 3.81 and no other values in range. ($x = \pi + 0.669$ scores A0)

Other methods may be seen.
Use review if you are not sure if an attempt deserves credit.

Question Number	Scheme	Marks
8(a)	$\tan 3x \equiv \tan(2x + x) \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ or e.g. $\tan 3x \equiv \frac{\sin 3x}{\cos 3x} \equiv \frac{\sin(2x + x)}{\cos(2x + x)} \equiv \frac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x} \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$	M1
	$\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} \equiv \frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x}$	dM1
	$\frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x} \equiv \frac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2 \tan^2 x}$ $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} *$	A1*
		(3)
Condone use of a different variable e.g. θ for x. Condone the use of mixed variables for the M marks as long as the intention is clear. Condone clear use of “tan” written differently e.g. “tg” M1: Writes $\tan 3x$ as $\tan(2x + x)$ which may be implied and applies the addition formula for $\tan(A + B)$ to obtain an expression in terms of $\tan x$ and $\tan 2x$ only. Condone one sign slip only. May also reach this point by writing $\tan 3x$ as $\frac{\sin 3x}{\cos 3x}$ and then uses the addition formulae for $\sin(A + B)$ and $\cos(A + B)$ to obtain an expression in terms of $\tan x$ and $\tan 2x$ only. Condone one sign slip only. dM1: Replaces $\tan 2x$ with $\frac{2 \tan x}{1 - \tan^2 x}$ to express $\tan 3x$ in terms of $\tan x$ only. A1*: Reaches the right hand side with sufficient working shown. For this mark condone minor notational errors (e.g. a missing x) but not consistent poor notation throughout such as mixed variables or writing $\tan^2 x$ as $\tan x^2$ Attempts to use higher multiples of x e.g. $\tan 3x \equiv \tan(4x - x)$ should be sent to review.		

Way 2	$\tan 3x \equiv \frac{\sin 3x}{\cos 3x} \equiv \frac{\sin(2x+x)}{\cos(2x+x)} \equiv \frac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x}$ $\equiv \frac{2 \sin x \cos^2 x + (\cos^2 x - \sin^2 x) \sin x}{(\cos^2 x - \sin^2 x) \cos x - 2 \sin^2 x \cos x}$	M1
	$\equiv \frac{2 \sin x \cos^2 x + \cos^2 x \sin x - \sin^3 x}{\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x} \equiv \frac{3 \sin x \cos^2 x - \sin^3 x}{\cos^3 x - 3 \sin^2 x \cos x}$	dM1
	$\equiv \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} *$	A1*

Condone use of a different variable e.g. θ for x .

Condone the use of mixed variables for the M marks as long as the intention is clear.

- M1:** Writes $\tan 3x$ as $\frac{\sin 3x}{\cos 3x}$ and then uses the addition formulae for $\sin(A+B)$ and $\cos(A+B)$ condoning one sign slip and then applies $\sin 2x = 2 \sin x \cos x$ and $\cos 2x \equiv \pm \cos^2 x \pm \sin^2 x$ or equivalent e.g. $\cos 2x \equiv \pm 1 \pm 2 \sin^2 x$ or $\cos 2x \equiv \pm 2 \cos^2 x \pm 1$ to obtain an expression in terms of $\sin x$ and $\cos x$ only.
- dM1:** Expands and collects terms to obtain 2 terms in the numerator and 2 terms in the denominator. There must have been no errors in their identities for $\sin 2x$ or $\cos 2x$
- A1*:** Reaches the right hand side with sufficient working shown.
For this mark condone minor notational errors (e.g. a missing x) but not consistent poor notation throughout such as mixed variables.



(b)	$\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = 2 \sec^2 3\theta - 8 \Rightarrow \tan 3\theta = 2 \sec^2 3\theta - 8$ $\Rightarrow \tan 3\theta = 2(1 + \tan^2 3\theta) - 8$	M1
	$\Rightarrow 2 \tan^2 3\theta - \tan 3\theta - 6 = 0 \quad \text{or e.g.} \Rightarrow 2 \tan^2 3\theta - \tan 3\theta = 6$	A1
	$(2 \tan 3\theta + 3)(\tan 3\theta - 2) = 0 \Rightarrow \tan 3\theta = -\frac{3}{2}, 2$ $\tan 3\theta = -\frac{3}{2} \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots \quad \text{or} \quad \tan 3\theta = 2 \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots$	dM1
	$\theta = 0.37, 0.72, 1.42$	A1A1
		(5)
		Total 8
Allow if a different variable used (such as x). For mixed variables allow the M's but only allow the first A (and final A's) if recovered or interpreted correctly as the same variable. M1: Writes the lhs as $k \tan 3\theta$ and applies $\sec^2 3\theta = \pm 1 \pm \tan^2 3\theta$ on the rhs. A1: Reaches the correct 3TQ in $\tan 3\theta$ with terms collected but not necessarily all on one side. dM1: Solves their 3TQ in $\tan 3\theta$ by any method (usual rules) including a calculator (you may need to check) to obtain at least one value for $\tan 3\theta$ and uses correct processing to obtain at least one value for θ e.g. finds inverse tan and divides by 3. A1: Awrt at least one of these answers. Must be in radians and must come from the correct 3TQ. A1: Awrt these answers and no extras in range. Must be in radians.		

Note that there may be more convoluted methods which include writing the lhs as $k \frac{\sin 3\theta}{\cos 3\theta}$ e.g.

$$\begin{aligned} \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = 2 \sec^2 3\theta - 8 &\Rightarrow \frac{\sin 3\theta}{\cos 3\theta} = 2 \sec^2 3\theta - 8 = \frac{2}{\cos^2 3\theta} - 8 \\ &\Rightarrow \sin 3\theta \cos 3\theta = 2 - 8 \cos^2 3\theta \Rightarrow \frac{1}{2} \sin 6\theta = 2 - 8 \left(\frac{\cos 6\theta + 1}{2} \right) \\ &\Rightarrow \sin 6\theta + 8 \cos 6\theta = -4 \\ &\Rightarrow \sqrt{65} \sin(6\theta + 1.4464) = -4 \Rightarrow \sin(6\theta + 1.4464) = -\frac{4}{\sqrt{65}} \Rightarrow \theta = \dots \\ &\theta = 0.37, 0.72, 1.42 \end{aligned}$$

In such cases, the first M mark will only be scored when they reach an equation they can solve directly e.g. in the above example when they reach $\sqrt{65} \sin(6\theta + 1.4464) = -4$

If you are in doubt whether an approach deserves any credit use Review.



(a)(ii)

B1ft: A V shaped curve with vertex in quadrant 4 or follow through a V shape that is a translation down from their answer in (a)(i)

B1ft: The vertex is in quadrant 4 and is marked $\left(\frac{1}{3}a, -b\right)$, which may be indicated by appropriate labelling on the graph. Follow through on their x intercept of the vertex in (i) (which must have been on positive the x -axis)

B1ft: The graph passes through the positive y -axis at $(0, a-b)$, following through on their y intercept from (i), but condone $(y =) a - b$ which may be seen as the intercept on the graph.

SC If a “Y” shape is given, if part is dashed ignore the dashed line, if all are solid withhold the first B but all B0B1B1 if vertex and positive intercept are correct.

(b)

M1: Attempts to solve $-3x + a - b = 5x \Rightarrow x = \dots$ There may be other equations attempted

A1: $x = \frac{a-b}{8}$ or exact equivalent only. If multiple values are found the incorrect ones must be rejected or this correct answer clearly indicated as their final answer (e.g. by underlining). Ignore any reasoning given, mark the value.



Question Number	Scheme	Marks
8 (i)	States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	B1
	Uses both $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\sin 2\theta = 2 \sin \theta \cos \theta$	M1
	$3 \operatorname{cosec} \theta = 8 \cos \theta \Rightarrow \sin 2\theta = \frac{3}{4}$	A1
	$\Rightarrow \theta = \frac{1}{2} \arcsin\left(\frac{3}{4}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)
(ii)	$\frac{\tan 2x - \tan 70^\circ}{1 + \tan 2x \tan 70^\circ} = -\frac{3}{8} \Rightarrow \tan(2x - 70^\circ) = -\frac{3}{8}$	M1 A1
	Correct order of operations $x = \frac{\arctan\left(-\frac{3}{8}\right) + 70^\circ}{2}$	dM1
	awrt 24.7° , awrt 114.7°	A1
		(4)
		(9 marks)

(i)

B1: States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ M1: Uses both $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\sin 2\theta = 2 \sin \theta \cos \theta$ to set up an equation in $\sin 2\theta$ A1: $\sin 2\theta = \frac{3}{4}$ M1: Correct method of solving an equation of the form $\sin 2\theta = k$, $|k| < 1$ to find at least one value for θ . May be solving in degrees for this mark ($24.30^\circ, 65.70^\circ$)A1: $\theta = \text{awrt } 0.424, \text{awrt } 1.15$ and no other values in the range (isw for incorrect rounding).

(ii) Main scheme

M1: Uses the compound angle identities to reach $\tan(2x \pm 70^\circ) = \pm \frac{3}{8}$ A1: $\tan(2x - 70^\circ) = -\frac{3}{8}$ dM1: Correct order of operations to find a value for x . Condone if radians mode used, e.g.

$$2x - 70 = -0.3587... \Rightarrow x = \frac{70 - 0.3587...}{2} = 34.82... \text{ May be scored from}$$

$$2x + 70 = ... \rightarrow x = \frac{-70 - ...}{2} \text{ Condone errors in finding the second solution such at } 180 - PV.$$

A1: Both awrt 24.7° , awrt 114.7° and no other values in the range (isw for incorrect rounding)

(ii) Alt method 1

M1: Cross multiplies and makes $\tan 2x$ the subject. Condone $\tan 70^\circ$ being replaced by 2.75

$$\text{A1: } \tan 2x = \frac{8 \tan 70^\circ - 3}{8 + 3 \tan 70^\circ} = \text{awrt } 1.17$$

dM1: Correct order of operations to find one value for x . See note on main scheme.Implied by a correct value for their $\tan 2x = \text{awrt } 1.17$ provided the previous M has been scoredA1: Both awrt 24.7° , awrt 114.7° and no other values in the range (isw for incorrect rounding)

(ii) Alt method 2

M1: Applies double angle formula and multiplies through to achieve a quadratic in $\tan x$ (condoning slips and not necessarily gathered)

$$\frac{\frac{2 \tan x}{1 - \tan^2 x} - \tan 70^\circ}{1 + \frac{2 \tan x}{1 - \tan^2 x} \tan 70^\circ} = -\frac{3}{8} \Rightarrow 16 \tan x - 8 \tan 70^\circ (1 - \tan^2 x) = -3(1 - \tan^2 x + 2 \tan x \tan 70^\circ)$$

$$(\Rightarrow t^2 (8 \tan 70^\circ - 3) + t (16 + 6 \tan 70^\circ) + 3 - 8 \tan 70^\circ = 0 \quad (18.98t^2 + 32.48t - 18.98 = 0))$$

A1: A correct value for $\tan x$, $\tan x = \text{awrt } 0.4604$ or $\text{awrt } -2.172$ dM1: Applies arctan to achieve at least one value for x .

A1: As main scheme.

Alt (i)	States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ (or equivalent identity)	B1
	$3 \operatorname{cosec} \theta = 8 \cos \theta \Rightarrow 9 \operatorname{cosec}^2 \theta = 64 \cos^2 \theta$ and uses both $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	M1
	and $\sin^2 \theta + \cos^2 \theta = 1$ to get equation in one trig term	
	$\Rightarrow 9 = 64 \sin^2 \theta (1 - \sin^2 \theta) \Rightarrow \sin^2 \theta = \frac{4 \pm \sqrt{7}}{8}$	A1
	$\Rightarrow \theta = \arcsin \left(\sqrt{\frac{4 \pm \sqrt{7}}{8}} \right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)

B1: States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ or in variations a similar correct reciprocal identity (e.g. $\cot \theta = \frac{\cos \theta}{\sin \theta}$) during the proof.

M1: Squares both sides and uses both reciprocal identity and Pythagorean identity to set up an equation in one trig term only.

A1: $\sin^2 \theta = \frac{4 \pm \sqrt{7}}{8}$ note it is the same roots for $\cos^2 \theta$ M1: Correct method of solving an equation of the form $\sin^2 \theta = k$ or $\cos^2 \theta = k$, $0 < k < 1$ to find at least one value for θ . May be solving in degrees for this mark ($24.30^\circ, 65.70^\circ$)A1: $\theta = \text{awrt } 0.424, \text{awrt } 1.15$ and no other values in the range. Under this method the extra solutions formed via squaring will need to be rejected to score this mark.

Question Number	Scheme	Marks
7 (a)	$\sin 4\theta \equiv 2 \sin 2\theta \cos 2\theta$ $\equiv 2(2 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta)$ $\equiv 4 \sin \theta \cos \theta - 8 \sin^3 \theta \cos \theta$ $\equiv \sin \theta \cos \theta (4 - 8 \sin^2 \theta)$	M1 dM1 A1 (3)
(b)	$\sec x \sin 4x = 5 \sin^3 x \cot x$ $\frac{1}{\cos x} \times \cos x \sin x (4 - 8 \sin^2 x) = 5 \sin^3 x \cot x$ $\div \sin x \Rightarrow 4 - 8 \sin^2 x = 5 \sin^2 x \cot x$ $\div \cos^2 x \Rightarrow 4 \sec^2 x - 8 \tan^2 x = 5 \tan^2 x \cot x$ $\Rightarrow 4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0 \quad *$	B1 M1 A1* (3)
(c)	Uses $\sec^2 x = 1 + \tan^2 x$ $\Rightarrow 4 \tan^2 x + 5 \tan x - 4 = 0$ $\Rightarrow \tan x = \frac{-5 \pm \sqrt{89}}{8} \Rightarrow x = \dots$ $\Rightarrow x = \text{awrt } 0.506, 2.08$	M1 A1 dM1 A1 (4) (10 marks)

(a)

M1: Uses the double angle formula for $\sin 4\theta$ to get $2 \sin 2\theta \cos 2\theta$ o.e. such as $\sin 2\theta \cos 2\theta + \cos 2\theta \sin 2\theta$

dM1: Uses the double angle formulae $\sin 2\theta = K \sin \theta \cos \theta$ and $\cos 2\theta = \pm 1 \pm 2 \sin^2 \theta$ to obtain $\sin 4\theta$ in terms of just $\sin \theta$ and $\cos \theta$. If they use one of the other identities for $\cos 2\theta$, either

$\pm 2 \cos^2 \theta \pm 1$ or $\cos^2 \theta - \sin^2 \theta$, they must subsequently change the $\cos^2 \theta$ terms to $1 - \sin^2 \theta$

A1: Reaches $\sin 4\theta \equiv \sin \theta \cos \theta (4 - 8 \sin^2 \theta)$

Withold this mark if there are mixed variables e.g. $\sin 4\theta \equiv 2 \sin 2x \cos 2x$ and incorrect notation e.g. $\cos 2\theta \equiv 1 - 2 \sin \theta^2$ used within the body of the proof.

.....

.....

You may well see more complicated versions of this proof.

For example,

$$\sin 4\theta \equiv \sin(3\theta + \theta) = \sin 3\theta \cos \theta + \cos 3\theta \sin \theta$$

followed by
$$= \sin(2\theta + \theta) \cos \theta + \cos(2\theta + \theta) \sin \theta$$

$$= \sin 2\theta \cos^2 \theta + \cos 2\theta \sin \theta \cos \theta + \cos 2\theta \cos \theta \sin \theta - \sin 2\theta \sin \theta \sin \theta$$

In this method the M1 is scored for the line above but you can condone sign slips



And the dM1 for using the double angle formulae $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\cos 2\theta = \pm 1 \pm 2 \sin^2 \theta$ to obtain $\sin 4\theta$ in terms of just $\sin \theta$ and $\cos \theta$

Do not allow candidates just to write out the expansions for $\sin 3\theta$ and $\cos 3\theta$ without their proof

.....
(b)

B1: States or uses $\sec x = \frac{1}{\cos x}$ in the given equation.

It can be implied. So, writing $\sec x \sin 4x$ as $\sin x (4 - 8 \sin^2 x)$ would be B1

M1: Requires

- $\sin 4x$ to be written as $\sin x \cos x ('4' - '8' \sin^2 x)$ using their values for P , Q and n
- a term in $\sin x$ to be cancelled or factorised out
- an attempt at dividing all terms by $\cos^2 x$ to reach an equation involving at least $\sec^2 x$ and $\tan^2 x$ (you don't need to see $\div \cos^2 x$ stated, it can be implied by their terms)

The last two bullets can be done in just one line and implied by all terms $\div (\sin x \cos^2 x)$

A1*: Reaches the given equation showing sufficient and necessary lines of work.

Withhold this mark for mixed variables and/or incorrect notation within the proof.

(c)

M1: Uses $\sec^2 x = \pm 1 \pm \tan^2 x$ to set up a 3TQ in $\tan x$

A1: Correct 3TQ, $4 \tan^2 x + 5 \tan x - 4 = 0$.

The terms do not need to be on one side and the $= 0$ can be implied by further work

dM1: Solves the 3TQ using an appropriate method and proceeds to a value for x via value(s) for $\tan x$

Expect to see " $4 \tan^2 x + 5 \tan x - 4 = 0 \Rightarrow \tan x = \dots \Rightarrow x = \dots$ "

Allow calculator methods of solving the quadratic in $\tan x$ to produce a value for $\tan x$

A1: $x = \text{awrt } 0.506, 2.08$ following M1, A1, dM1.

Ignore any extra solutions outside the range $(0, \pi)$.

Withhold this mark if there are extra solutions in the range.

Note that the demand of the question is that candidates show their working.

So, from the given $4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0$

candidates writing $\Rightarrow 4 \tan^2 x + 5 \tan x - 4 = 0$ followed by $x = \text{awrt } 0.506, 2.08$ scores just the first M1,

A1

There will be alternatives seen in part (c).

Alt (c)

$$4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0$$

$$\times \cos^2 x \Rightarrow 4 - 5 \sin x \cos x - 8 \sin^2 x = 0$$

$$\Rightarrow 4 - 8 \sin^2 x - 5 \sin x \cos x = 0$$

$$\Rightarrow 4 \cos 2x - 2.5 \sin 2x = 0$$

$$\Rightarrow \tan 2x = \frac{8}{5}$$

$$\Rightarrow 2x = 1.0121, 4.1538$$

$$\Rightarrow x = \text{awrt } 0.506, 2.08$$

M1, A1

dM1, A1

M1: Equation in a single trig ratio

A1: Correct equation

dM1: Full attempt to solve

A1: Correct solutions



(a)B1: $3x^2 + 7x - 20 = (3x - 5)(x + 4)$ seen or used

M1: Combines fractions with a correct common denominator and the order of terms in the numerator consistent with their common denominator.

A1*: Achieves the given answer with no errors seen but condone e.g. a missing trailing bracket as long as it is “recovered”.

Sufficient working should be shown but allow to go from e.g. $\frac{2(x-4)+8}{3x-5}$ to $\frac{2x}{3x-5}$

Note that candidates may take a longer route:

$$\frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} = \frac{(2x^2 - 32)(3x - 5) + 8(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{M1}$$

$$= \frac{6x^3 + 14x^2 - 40x}{(3x^2 + 7x - 20)(3x - 5)} = \frac{2x(3x^2 + 7x - 20)}{(3x - 5)(x + 4)(3x - 5)} \quad \text{B1}$$

$$= \frac{2x(3x - 5)(x + 4)}{(3x - 5)(x + 4)(3x - 5)} = \frac{2x}{3x - 5} \quad \text{A1*}$$

Or

$$\frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} = \frac{(2x^2 - 32)(3x - 5) + 8(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{M1}$$

$$= \frac{6x^3 + 14x^2 - 40x}{(3x^2 + 7x - 20)(3x - 5)} = \frac{2x(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{B1}$$

In this case the B1 is scored for obtaining a factor of $3x^2 + 7x - 20$ in the numerator and denominator

$$= \frac{2x}{3x - 5} \quad \text{A1*}$$

(b)M1: Attempts to differentiate using the product or quotient rule. Award for an expression of the form $\frac{\alpha(3x-5) - \beta x}{(3x-5)^2}$ or $\alpha(3x-5)^{-1} - \beta x(3x-5)^{-2}$, $\alpha, \beta > 0$ Condone attempts where $(3x-5)^2$ is expanded.Alternatively, attempts to write $\frac{2x}{3x-5}$ as $A + \frac{B}{3x-5}$ before differentiating using thechain rule to obtain $\frac{\pm k}{(3x-5)^2}$

A1: Correct derivative simplified or unsimplified.

A1cso*: Requires a correct simplified derivative $\frac{-10}{(3x-5)^2}$ and then statements that convey:

- $(3x-5)^2 > 0$ or e.g. denominator is positive (condone $(3x-5)^2 \geq 0$)
- so $f'(x) < 0$
- function is decreasing



Some candidates may attempt to differentiate the original $f(x)$ e.g.:

$$f'(x) = \frac{4x(3x^2 + 7x - 20) - (2x^2 - 32)(6x + 7)}{(3x^2 + 7x - 20)^2} - \frac{24}{(3x - 5)^2}$$

Score M1 for:

$$f'(x) = \frac{Ax(3x^2 + 7x - 20) - (2x^2 - 32)(Cx + D)}{(3x^2 + 7x - 20)^2} - \frac{E}{(3x - 5)^2}$$

and (first) A1 if they reach $\frac{-10}{(3x - 5)^2}$

(c)

M1: Rearranges $y = 3 + 2 \ln x$ to $x = e^{f(y)}$ or $x = 3 + 2 \ln y$ to $y = e^{f(x)}$

A1: Obtains $g^{-1}(x) = e^{\frac{x-3}{2}}$ or equivalent e.g. $g^{-1}(x) = \sqrt{e^{x-3}}$, $g^{-1}(x) = e^{\frac{3-x}{-2}}$ and isw.

Accept $g^{-1}(x) = \dots$, $g^{-1} = \dots$, $y = \dots$ but not e.g. $f^{-1}(x) = \dots$

B1: Correct domain: $x \geq 3$ or using correct notation e.g. $[3, \infty)$

(d) Condone the miscopy of $f(x)$ for the M marks as long as it has the correct form e.g. $\frac{\dots a}{\dots a \pm \dots}$

and allow use of x instead of a for all marks in (d)

B1: Sets up a valid equation in a e.g. $3 + 2 \ln \left(\frac{2a}{3a-5} \right) = 5$

M1: Rearranges to obtain $\ln \left(\frac{2a}{3a-5} \right) = k$ and uses the inverse law of logarithms to remove

the $\ln$ correctly to reach $\frac{2a}{3a-5} = e^k$ or

dM1: Rearranges an equation of the form $\frac{2a}{3a-5} = e^k$ to obtain $a = \frac{\dots e^k}{\dots e^k \pm \dots}$ or equivalent.

Depends on the previous method mark.

A1: $(a =) \frac{5e}{3e-2}$ or e.g. $(a =) \frac{-5e}{2-3e}$ and apply isw once the correct exact answer is seen.

Accept e^1 for e .



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1. (a)	$\frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} = \frac{5x + 5x(x + 3)}{(x + 3)(x + 4)}$ $= \frac{5x^2 + 20x}{(x + 3)(x + 4)} = \frac{5x(x + 4)}{(x + 3)(x + 4)} = \frac{5x}{x + 3} *$	M1 A1
		A1*
		(3)
(b)	$y = \frac{5x}{(x + 3)} \Rightarrow xy + 3y = 5x \Rightarrow 5x - xy = 3y$ $\Rightarrow x = \frac{3y}{5 - y} \quad \text{So } f^{-1}(x) = \frac{3x}{5 - x}$ Domain $0 < x < 5$	M1
		A1
		A1
(c) (i)		(3)
	$f(x) = \frac{5x}{(x + 3)} \Rightarrow (f'(x)) = \frac{5(x + 3) - 5x}{(x + 3)^2} = \frac{15}{(x + 3)^2}$	M1 A1
	(ii) States (f is an) increasing function with a suitable reason E.g. Since $(x + 3)^2$ is positive	A1
		(3)
		(9 marks)

(a)

M1 Attempts to combine the two fractions using a common denominator.
 Allow errors on the numerator but at least one of the terms must have been adapted.
 Usual rules apply for factorising the quadratic denominator.
 Condone invisible brackets and slips on the numerator when combining the two fractions.

A1 Allow the two fractions to be written separately with the same denominator.
 For a correct un-simplified fraction with a quadratic numerator and denominator (which may be implied)

$$\frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} = \frac{5x(x + 4)}{\dots} + \frac{5x(x^2 + 7x + 12)}{\dots} = \frac{5x^3 + 40x^2 + 80x}{(x^2 + 7x + 12)(x + 4)} = \frac{x(5x + 20)(x + 4)}{(x^2 + 7x + 12)(x + 4)} = \frac{x(5x + 20)}{(x^2 + 7x + 12)}$$

A1* Correctly achieves the given answer of $\frac{5x}{x + 3}$ showing intermediate steps (cso). Expect to see the two fractions combined and then both the numerator and denominator factorised before cancelling terms to achieve full marks. In the case of forming a cubic numerator and denominator you must see the terms collected before factorising. Bracket errors at some point in their working is A0*

(b)

M1 Attempts to change the subject on $y = \frac{5x}{x + 3}$ (or $x = \frac{5y}{y + 3}$). Look for cross multiplication with an attempt to collect terms. Do not follow through on their answer to part (a)



Question Number	Scheme	Marks
4. (i) (a)	$f'(x) = \frac{4(x-3)(2x+5) - (2x+5)^2}{(x-3)^2}$ or $\frac{(x-3)(8x+20) - (4x^2 + 20x + 25)}{(x-3)^2}$	M1 A1
	$= \frac{(2x+5)(2x-17)}{(x-3)^2}$	M1 A1
	(b) Attempts both critical values or finds one "correct" end $x < -2.5, x > 8.5$ (accept $x \leq -2.5, x \geq 8.5$)	M1 A1
	(ii) Attempts the chain rule on $(\sin 4x)^{\frac{1}{2}} \rightarrow A(\sin 4x)^{-\frac{1}{2}} \times \cos 4x$	M1
	$g(x) = x(\sin 4x)^{\frac{1}{2}} \Rightarrow g'(x) = (\sin 4x)^{\frac{1}{2}} + x \times \frac{1}{2}(\sin 4x)^{-\frac{1}{2}} 4 \cos 4x$	M1 A1
	Sets $g'(x) = 0 \rightarrow (\sin 4x)^{\frac{1}{2}} + x \times \frac{2 \cos 4x}{(\sin 4x)^{\frac{1}{2}}} = 0$ and $\times \frac{(\sin 4x)^{\frac{1}{2}}}{\cos 4x}$ oe $\rightarrow \tan 4x + 2x = 0$	M1 A1
		(6) (5) 11 marks

(i)(a)

M1 Attempts the quotient rule and achieves

$$f'(x) = \frac{A(x-3)(2x+5) - B(2x+5)^2}{(x-3)^2} \quad A, B > 0 \text{ condoning slips}$$

Alternatively uses the product rule and achieves

$$\frac{d}{dx} \{ (x-3)^{-1} (2x+5)^2 \} = \pm A(2x+5)^2 (x-3)^{-2} + B(x-3)^{-1} (2x+5) \quad A, B > 0$$

They may attempt to multiply out the $(2x+5)^2$ first which is fine as long as they reach a 3TQ.A1 Score for correct unsimplified $f'(x)$ M1 Attempts to take out a factor of $(2x+5)$ or multiplies out and attempts to factorise the numerator.

The method must be seen $\frac{(x-3)4(2x+5) \pm (2x+5)^2}{\dots} = \frac{(2x+5)\{(x-3)4 \pm (2x+5)\}}{\dots}$ condoning slips.

If the method is not seen it may be implied by a correct result for their fraction

This can be achieved from an incorrect quotient or product rule. E.g. $\frac{vu' + uv'}{v^2}$ or $\frac{vu' - uv'}{v}$

It can be scored by candidates who multiply out their numerators and then factorise by taking out a factor of $(2x+5)$

If the product rule is used it would be for writing as a single fraction and taking out, from the numerator, a common factor of $(2x+5)$.

A1 $\frac{(2x+5)(2x-17)}{(x-3)^2}$ but accept expressions such as $\frac{4(x+2.5)(x-8.5)}{(x-3)^2}$ or $\frac{(2x+5)(2x-17)}{(x-3)(x-3)}$

Note the final two marks in (i)(a) may be scored in (i)(b), ONLY IF the correct work is done on the complete fraction, not just the numerator

(i)(b)



- M1 Achieves the two critical values from the quadratic numerator of their $f'(x)$
 Alternatively finds one correct end for their $(2x+5)(2x-17) > 0$ or $(2x+5)(2x-17) \geq 0$
 So award for either $x < -2.5$ or $x > 8.5$ which may be scored from an intermediate line.
- A1 $x < -2.5, x > 8.5$ (accept $x \leq -2.5, x \geq 8.5$).
 Ignore any references to "and" or "or" so condone $x < -2.5$ and $x > 8.5$
 Mark the final response. This is not isw
 It may follow working such as $(2x+5)(2x-17) > 0 \Rightarrow x > -\frac{5}{2}, x > \frac{17}{2}$. So $x < -2.5, x > 8.5$
 Accept alternative forms such as $(-\infty, -2.5] \cup [8.5, \infty)$

(ii)

- M1 Attempts the chain rule on $(\sin 4x)^{\frac{1}{2}} \rightarrow A(\sin 4x)^{-\frac{1}{2}} \times \cos 4x$
- M1 For an attempt at the product rule.
 If they state $u = x, v = (\sin 4x)^{\frac{1}{2}}, u' = 1, v' = \dots$ award for $(\sin 4x)^{\frac{1}{2}} + x \times \text{their } v'$
 If this is not stated or implied by their $uv' + vu'$ then award for $(\sin 4x)^{\frac{1}{2}} + x \times (\sin 4x)^{-\frac{1}{2}} \dots$
- A1 $(g'(x)) = (\sin 4x)^{\frac{1}{2}} + 2x(\sin 4x)^{-\frac{1}{2}} \cos 4x$ which may be unsimplified.
 You may not see the lhs which is fine. Condone $\sin 4x^{\frac{1}{2}}$ for $(\sin 4x)^{\frac{1}{2}}$ if subsequent work is correct
- M1 Sets their $g'(x)$ which must be of the form $(\sin 4x)^{\frac{1}{2}} + kx(\sin 4x)^{-\frac{1}{2}} \cos 4x$ equal to 0
 and proceeds with correct work to an equation of the correct form. Allow $\tan 4x = kx$ here
- A1 cso $\tan 4x + 2x = 0$

.....
 Alt to (a) via division which may not be very common

- M1 Score for $\frac{(2x+5)^2}{x-3} \rightarrow Ax + B + \frac{C}{x-3}$ differentiating to $A \pm \frac{C}{(x-3)^2}$
- A1 $4 - \frac{121}{(x-3)^2}$
- M1 Forms a single fraction and attempts to factorise out $(2x+5)$ from the numerator (which must be a 3TQ)
- A1 $\frac{(2x+5)(2x-17)}{(x-3)^2}$

Alt to (b) via squaring

$$[g(x)]^2 = x^2 \sin 4x \Rightarrow 2g(x)g'(x) = 2x \sin 4x + 4x^2 \cos 4x$$

- M1 Correct form for the rhs. Apply the same rules as the main method. Condone slips on coefficients
- dM1 Correct form for the left hand side as well as the right hand side. Condone a slip on the coefficient
- A1 $2g(x)g'(x) = 2x \sin 4x + 4x^2 \cos 4x$
- ddM1 Sets $g'(x) = 0$ and proceeds with correct work to an equation of the correct form.
- A1 cso $\tan 4x + 2x = 0$

Question Number	Scheme	Marks
5 (a)	$12 \tan 2x + 5 \cot x \sec^2 x = 0$ $12 \times \frac{2t}{1-t^2} + 5 \times \frac{1}{t} (1+t^2) = 0$ $12 \times 2t^2 + 5(1+t^2)(1-t^2) = 0 \rightarrow 5t^4 - 24t^2 - 5 = 0 *$	B1 M1 A1 A1* (4)
(b)	$5t^4 - 24t^2 - 5 = 0$ $(5t^2 + 1)(t^2 - 5) = 0$ Correct order of operations $t = (\pm)\sqrt{5} \Rightarrow x = ..$ Two of awrt $x = 66^\circ, 114^\circ, 246^\circ, 294^\circ$ All four of awrt $x = 65.9^\circ, 114.1^\circ, 245.9^\circ, 294.1^\circ$	M1 dM1 A1 A1 (4) 8 marks

(a)

B1 Any **correct** identity used within the given equation either in terms of $\tan x$ or in terms of t

Eg: Attempts to replace either $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$, $\cot x = \frac{1}{\tan x}$ or $\sec^2 x = 1 + \tan^2 x$

M1 Uses $\tan 2x = \frac{2 \tan x}{1 \pm \tan^2 x}$, $\cot x = \frac{1}{\tan x}$, and $\sec^2 x = \pm 1 \pm \tan^2 x$ or with $t = \tan x$ to produce an equation in terms of t or $\tan x$

A1 Correct intermediate equation in t or $\tan x$ The $= 0$ may be implied by later workA1* For proceeding to the correct answer with a correct intermediate line. Must have $= 0$.

There cannot be any notational or bracketing errors within the body of the solution if this mark is to be awarded. A notational error is $\tan x^2 \leftrightarrow \tan^2 x$

The intermediate line should be of a form in which the given answer could immediately follow. See main scheme for such an example; the fractional terms have been dealt with in this case.

Condone partially completed lines if the candidate is only working on one side of the equation

(b)

M1 Correct attempt to solve.

Allow an attempt to factorise $5t^4 - 24t^2 - 5 = 0 \Rightarrow (at^2 + b)(ct^2 + d) = 0$ with $ac = \pm 5, bd = \pm 5$

Alt lets $u = t^2$ and attempts to factorise $5u^2 - 24u - 5 = 0 \Rightarrow$ with usual rules.

Allow use of calculator giving $t^2 = 5$ or $\tan^2 x = 5$ (You may ignore the negative root).

It is also implied by $t = \sqrt{5}$ or $t = -\sqrt{5}$ Watch out for $\tan x = 5$ which is M0

dM1 For using the correct order of operations and finding one value of x for their $t^2 = k$ where k is a positive constant. Allow accuracy to either the nearest degree or correct to 1dp in radians.

It is dependent upon them having scored the previous M1.

A1 Any two of awrt $x = 66^\circ, 114^\circ, 246^\circ, 294^\circ$ May be implied by awrt two of 1.15, 1.99, 4.29, 5.13A1 All four of awrt $x = 65.9^\circ, 114.1^\circ, 245.9^\circ, 294.1^\circ$ AND no extras within the range.

Question Number	Scheme	Marks
6 (a)	$y = (4x - 7)^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx} = \right) 2(4x - 7)^{-\frac{1}{2}}$ (see notes) At $(8, 5)$ gradient of tangent is $2(4 \times 8 - 7)^{-\frac{1}{2}} \left(= \frac{2}{5} \right)$ Equation for l is $y - 5 = -\frac{5}{2}(x - 8)$ $2y - 10 = -5x + 40 \Rightarrow 5x + 2y - 50 = 0$ *	M1 A1 dM1 ddM1 A1* (5)
(b)	$\int (4x - 7)^{\frac{1}{2}} dx = \left[\frac{(4x - 7)^{\frac{3}{2}}}{6} \right]$ (see notes) Complete area = $\int_{\frac{7}{4}}^8 (4x - 7)^{\frac{1}{2}} dx = \left[\frac{(4x - 7)^{\frac{3}{2}}}{6} \right]_{\frac{7}{4}}^8 + 5$ $= \frac{155}{6}$	M1, A1 dM1 A1 (4) (9 marks)

(a)

M1: Attempts to differentiate to achieve $a(4x \pm 7)^{-\frac{1}{2}}$ or equivalent. The index does not need to be processed.

Alternatively, attempts to differentiate implicitly $y^2 = 4x - 7 \Rightarrow Ay \frac{dy}{dx} = B$

A1: Achieves $2(4x - 7)^{-\frac{1}{2}}$ o.e. (which may be unsimplified but the index processed) or using implicit differentiation achieves $2y \frac{dy}{dx} = 4$

dM1: Substitutes $x = 8$ into their $a(4x \pm 7)^{-\frac{1}{2}}$ or substitutes $y = 5$ into their $Ay \frac{dy}{dx} = B$. It does not need to be evaluated. It is dependent on the first method mark but allow this mark to be scored if they have incorrectly manipulated their derivative before substituting in, or made transcription errors including if they lose the $-\frac{1}{2}$ index from the bracket.

ddM1: A full method for the equation of l . Look for

- substitution of $x = 8$ (or $y = 5$ if implicit differentiation) into an attempt at a derivative
- the application of the negative reciprocal rule
- the use of $(8, 5)$ with a correct gradient for their value of m to form an equation for the normal.

$y - 5 = -\frac{5}{2}(x - 8)$ with the coordinates in the correct positions. If $y = -\frac{1}{m}x + c$ is used they must proceed as far as $c = \dots$

It is dependent on both of the previous two method marks.

Question Number	Scheme	Marks
2 (a)	$x = 2y^2 + 5y - 6$	
	$\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y + 5 = 0 \Rightarrow y = -\frac{5}{4}$	M1
	Substitutes their $y = -\frac{5}{4}$ into $x = 2y^2 + 5y - 6$ to find x	dM1
	$\left(-\frac{73}{8}, -\frac{5}{4}\right)$	A1
		(3)
		Total 5

Mark as one complete question

Main method via differentiation in (a) and (b)

(a)

M1: Differentiates $2y^2 + 5y - 6$ to a linear term in y AND attempts to take the reciprocal.

Condone for this mark attempts such as $\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y} + \frac{1}{5}$

We may see other methods including implicit differentiation from WMA14.

Look for $x = 2y^2 + 5y - 6 \Rightarrow 1 = ay \frac{dy}{dx} + b \frac{dy}{dx}$ followed by an attempt to make $\frac{dy}{dx}$ the subject

A1: $\frac{dy}{dx} = \frac{1}{4y+5}$ ISW after a correct answer.

(b) **Marks in part (b) can only be awarded after sight of** $\frac{dx}{dy} = \frac{ay+b}{k}$ **or** $\frac{dy}{dx} = \frac{k}{ay+b}$

So full marks can only be awarded after sight of $\frac{dx}{dy} = 4y+5$ **or** $\frac{dy}{dx} = \frac{1}{4y+5}$

M1: Sets ' $ay + b = 0 \Rightarrow y = \dots$ ' following sight of $\frac{dx}{dy} = \frac{ay+b}{k}$ **or** $\frac{dy}{dx} = \frac{k}{ay+b}$

Ignore/condone incorrect statements such as $\frac{dy}{dx} = 0$ here

This can be implied. E.g $\frac{dy}{dx} = \frac{1}{4y+5}$ followed by $y = -\frac{5}{4}$

dM1: Substitutes their solution of their $4y + 5 = 0$ into $x = 2y^2 + 5y - 6$ to find x .

This is dependent upon the previous M

A1: $\left(-\frac{73}{8}, -\frac{5}{4}\right)$ following $\frac{dx}{dy} = 4y+5$ **or** $\frac{dy}{dx} = \frac{1}{4y+5}$

The answer may be given separately, for example, as $x = -9.125$, $y = -1.25$

ISW after a correct answer.



Question Number	Scheme	Marks
7(a)	Uses $\sin 2x = 2 \sin x \cos x$ AND $\cos 2x = 1 - 2 \sin^2 x$ o.e. in $\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x}$	M1
	$\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} = \frac{2 \sin x \cos x}{\cos x} + \frac{1 - 2 \sin^2 x}{\sin x}$ $= \frac{2 \sin x \cancel{\cos x}}{\cancel{\cos x}} + \frac{1}{\sin x} - \frac{2 \sin^2 x}{\cancel{\sin x}} = \frac{1}{\sin x} = \operatorname{cosec} x^*$	dM1 A1*
		(3)
(b)	Uses part (a) $\Rightarrow 7 + \operatorname{cosec} 2\theta = 3 \cot^2 2\theta$	B1
	Either Uses $\pm 1 \pm \cot^2 2\theta = \pm \operatorname{cosec}^2 2\theta \rightarrow 3\text{TQ in } \operatorname{cosec} 2\theta$ Or alternatively replaces $\operatorname{cosec} 2\theta$ with $1/\sin 2\theta$, $\cot^2 2\theta$ with $\cos^2 2\theta/\sin^2 2\theta$, multiplies by $\sin^2 2\theta$ and uses $\pm \cos^2 2\theta = \pm 1 \pm \sin^2 2\theta \rightarrow 3\text{TQ in } \sin 2\theta$	M1
	$3 \operatorname{cosec}^2 2\theta - \operatorname{cosec} 2\theta - 10 = 0$ or $10 \sin^2 2\theta + \sin 2\theta - 3 = 0$	A1
	$(3 \operatorname{cosec} 2\theta + 5)(\operatorname{cosec} 2\theta - 2) = 0$ or $(5 \sin 2\theta + 3)(2 \sin 2\theta - 1) = 0$ $\Rightarrow \operatorname{cosec} 2\theta = -\frac{5}{3}, 2$ or $\Rightarrow \sin 2\theta = -\frac{3}{5}, \frac{1}{2}$ $\Rightarrow \sin 2\theta = -\frac{3}{5}, \frac{1}{2} \Rightarrow \theta = \dots$	dM1
	$\theta = \frac{\pi}{12} (0.262), \frac{5\pi}{12} (1.31), -0.322, -1.25$ (awrt these values)	A1, A1
		(6)
		Total 9

(a)

M1: Uses

- $\sin 2x = 2 \sin x \cos x$
- AND $\cos 2x = 1 - 2 \sin^2 x$ or equivalent. Condone sign slips on the versions of $\cos 2x$

in an attempt to write $\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x}$ as an expression in $\sin x$ and $\cos x$

dM1: Adopts a valid approach that can be followed and completes the proof.

All necessary steps may not be shown and condone errors such as writing $\cos$ for $\cos x$ or $\sin x^2$ for $\sin^2 x$

A1*: Correct proof showing all necessary intermediate steps with no errors (seen within the body of the solution) or omissions of any of the steps shown. The LHS starting point does not need to be seen

See main mark scheme and below for examples showing all steps and scoring full marks

$$\begin{aligned} \frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} &= \frac{\sin x \sin 2x + \cos x \cos 2x}{\sin x \cos x} \\ &= \frac{2 \sin^2 x \cos x + \cos x (1 - 2 \sin^2 x)}{\sin x \cos x} \\ &= \frac{\cancel{\cos x}}{\sin x \cancel{\cos x}} \\ &= \operatorname{cosec} x \end{aligned}$$

$$\begin{aligned} \frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} &= \frac{2 \sin x \cancel{\cos x}}{\cancel{\cos x}} + \frac{\cos 2x}{\sin x} \\ &= \frac{2 \sin^2 x + (\cos^2 x - \sin^2 x)}{\sin x} \\ &= \frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} \\ &= \operatorname{cosec} x \end{aligned}$$



Alt part (a)

M1: For using compound angle formula $\sin x \sin 2x + \cos x \cos 2x = \cos(2x - x)$

dM1: As in the main scheme, it is for adopting a valid approach that can be followed and completing the proof

A1: Correct proof showing all necessary steps (See below) with no errors or omissions

$$\begin{aligned}\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} &= \frac{\sin x \sin 2x + \cos x \cos 2x}{\sin x \cos x} \\ &= \frac{\cos(2x - x)}{\sin x \cos x} \\ &= \frac{\cancel{\cos x}}{\sin x \cancel{\cos x}} \\ &= \operatorname{cosec} x\end{aligned}$$

.....
(b)

B1: States $7 + \operatorname{cosec} 2\theta = 3 \cot^2 2\theta$ or exact equivalent which may be implied by subsequent work

OR $7 + \operatorname{cosec} x = 3 \cot^2 x$ with $x = 2\theta$

Watch for and do not allow $7 + \operatorname{cosec} \theta = 3 \cot^2 \theta$

M1: Attempts to use part (a) and uses $\pm 1 \pm \cot^2 2\theta = \pm \operatorname{cosec}^2 2\theta$ to form a 3TQ in $\operatorname{cosec} 2\theta$

Condone $3 \cot^2 2\theta$ being replaced by $3 \times \pm \operatorname{cosec}^2 2\theta \pm 1$ with or without the bracket.

Condone when the "7" is missing but these attempts will score a maximum of 2 marks. This mark and dM1
The terms don't need to be collected for this mark.

Alternatively replaces $\operatorname{cosec} 2\theta$ with $1/\sin 2\theta$, $\cot^2 2\theta$ with $\cos^2 2\theta / \sin^2 2\theta$ within an equation of the form
 $a + b \operatorname{cosec} 2\theta = c \cot^2 2\theta$ multiplies by $\sin^2 2\theta$ and uses $\pm \cos^2 2\theta = \pm 1 \pm \sin^2 2\theta \rightarrow 3\text{TQ in } \sin 2\theta$

A1: Correct equation $3 \operatorname{cosec}^2 2\theta - \operatorname{cosec} 2\theta - 10 = 0$ or $10 \sin^2 2\theta + \sin 2\theta - 3 = 0$

The $= 0$ may be implied by further work, e.g. solution of the equation

Allow this mark even for the correct equation in a different forms. E.g. $3 \operatorname{cosec}^2 2\theta - \operatorname{cosec} 2\theta = 10$

dM1: For a correct attempt to solve their 3TQ $\sin 2\theta$ or $\operatorname{cosec} 2\theta$ leading to a value for θ

If they state that $\sin \theta = -\frac{3}{5}, \frac{1}{2}$ and do not proceed to take arcsin and $\div 2$ it is M0

A1: For two of awrt $\theta = \frac{\pi}{12}(0.262), \frac{5\pi}{12}(1.31), -0.322, -1.25$

A1: For awrt $\theta = \frac{\pi}{12}(0.262), \frac{5\pi}{12}(1.31), -0.322, -1.25$ with no additional values within the range.

.....
If you see other worthwhile solutions and the scheme cannot be applied, e.g. t formula, please send to review

.....
How to mark when other variables are used, e.g. $x = 2\theta$

B1: $7 + \operatorname{cosec} x = 3 \cot^2 x$

M1: Uses $\pm 1 \pm \cot^2 x = \pm \operatorname{cosec}^2 x$ to form 3TQ in $\operatorname{cosec} x$ or the equivalent in $\sin x$

A1: Correct equation $3 \operatorname{cosec}^2 x - \operatorname{cosec} x - 10 = 0$ or $10 \sin^2 x + \sin x - 3 = 0$

ddM1: For this to be scored there must be an attempt to halve the values, otherwise M0.

Allow full marks to be scored for a candidate who uses a different variable correctly and reaches 4 correct answers

General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question	Scheme	Marks
9(a)	$\frac{\cos^2 \theta}{\cos 2\theta - \sin 3\theta} = \frac{\cos^2 \theta}{\cos 2\theta - (\sin 2\theta \cos \theta + \cos 2\theta \sin \theta)}$	M1
	$= \frac{\cos^2 \theta}{1 - 2\sin^2 \theta - 2\sin \theta \cos^2 \theta - \sin \theta (1 - 2\sin^2 \theta)}$	M1
	$= \frac{1 - \sin^2 \theta}{1 - 3\sin \theta - 2\sin^2 \theta + 4\sin^3 \theta} = \frac{(1 - \sin \theta)(1 + \sin \theta)}{(1 - \sin \theta)(1 - 2\sin \theta - 4\sin^2 \theta)}$	M1
	$= \frac{1 + \sin \theta}{1 - 2\sin \theta - 4\sin^2 \theta} *$	A1*
		(4)
(b)	$\frac{1 + \sin \theta}{1 - 2\sin \theta - 4\sin^2 \theta} = 2\operatorname{cosec} \theta \Rightarrow (1 + \sin \theta) \sin \theta = 2(1 - 2\sin \theta - 4\sin^2 \theta)$	M1
	$\Rightarrow 9\sin^2 \theta + 5\sin \theta - 2 = 0$	A1
	$\Rightarrow \sin \theta = \frac{-5 \pm \sqrt{25 - 4 \times 9 \times -2}}{18} = \dots \Rightarrow \theta = \sin^{-1} \dots$	M1
	Two of $\theta = 15.6^\circ, 164.4^\circ, 235.6^\circ, 304.4^\circ$	A1
	All of $\theta = 15.6^\circ, 164.4^\circ, 235.6^\circ, 304.4^\circ$	A1
		(5)
(9 marks)		
Notes:		
(a) M1: Applies the compound angle formula to $\sin 3\theta$. Accept $\pm \sin \theta \cos 2\theta \pm \sin 2\theta \cos \theta$. Allow for students who apply $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$ directly. May be seen anywhere (e.g. in separate working). M1: Uses correct double angle formula for $\cos 2\theta$ and $\sin 2\theta$ (or assumed used correctly if correct $\sin 3\theta$ formula was used) to achieve all terms in single angle. (May be scored after the third M.) M1: Uses Pythagorean identity to identify the factor of $1 - \sin \theta$ in numerator. They need not have achieved single angle arguments for all terms for this mark. (May be scored before the second M.) There must be an intermediate step before the given answer with $1 - \sin \theta$ cancelled if the factor is not explicitly seen in the numerator. A1: Cancels the $1 - \sin \theta$ from numerator and denominator and simplifies to the given result with no incorrect steps or consistently incorrect notation. (b) M1: Substitutes the result from (a) and cross multiplies to get an equation in $\sin \theta$ only. A1: Correct simplified quadratic in $\sin \theta$ M1: Solves the resulting quadratic by formula, completing square or calculator (allow by factorisation if their quadratic factorises) and applies inverse sine to at least one root. May be implied by any correct arcsin of one of their solutions, such as -55.6°. May be implied by a radians answer (e.g. 0.272, 2.869...) A1: Any two correct solutions in the range, accept awrt. Answers in radians scores A0. A1: Awrt all four solutions correct and no others in the range.		
9(a)	$\frac{\cos^2 \theta}{\cos 2\theta - \sin 3\theta} = \frac{\cos^2 \theta}{\cos 2\theta - (\sin 2\theta \cos \theta + \cos 2\theta \sin \theta)}$	M1

Example Alt	$= \frac{1 - \sin^2 \theta}{\cos 2\theta(1 - \sin \theta) - 2 \sin \theta(1 - \sin^2 \theta)}$	3rd M1
	$= \frac{(1 - \sin \theta)(1 + \sin \theta)}{(1 - \sin \theta) \cos 2\theta - 2 \sin \theta(1 + \sin \theta)(1 - \sin \theta)}$	
	$= \frac{(1 - \sin \theta)(1 + \sin \theta)}{(1 - \sin \theta)(1 - 2 \sin^2 \theta - 2 \sin \theta - 2 \sin^2 \theta)}$	2nd M1
	$= \frac{1 + \sin \theta}{1 - 2 \sin \theta - 4 \sin^2 \theta} *$	A1*
		(4)
Notes The above approach can be marked via the main scheme but scores the 3 rd M before the 2 nd M. Alternative approaches will hopefully fit the scheme similarly. Other approaches can be similarly applied as each of the three method marks will generally be required. E.g. see below (allowing first M for students who apply $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$ directly) Note in Alt 2 below it is more likely the 1 st M will be gained by aside work to expand $\sin 3\theta$ with substitution later seen into the equation.		
9(a) Alt2 In reverse	$\frac{1 + \sin \theta}{1 - 2 \sin \theta - 4 \sin^2 \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} = \frac{1 - \sin^2 \theta}{1 - 3 \sin \theta - 2 \sin^2 \theta + 4 \sin^3 \theta}$	3rd M1
	$= \frac{\cos^2 \theta}{1 - 3 \sin \theta - 2 \sin^2 \theta + 4 \sin^3 \theta}$	
	$= \frac{\cos^2 \theta}{1 - 2 \sin^2 \theta - 3 \sin \theta + 4 \sin^3 \theta} = \frac{\cos^2 \theta}{\cos 2\theta - (3 \sin \theta - 4 \sin^3 \theta)}$	2nd M1
	$= \frac{\cos^2 \theta}{\cos 2\theta - \sin \theta(3 - 4 \sin^2 \theta)} = \frac{\cos^2 \theta}{\cos 2\theta - \sin \theta(1 + 2(1 - \sin^2 \theta) - 2 \sin^2 \theta)}$	1st M1
	$= \frac{\cos^2 \theta}{\cos 2\theta - \sin \theta(1 + 2 \cos^2 \theta - (1 - \cos 2\theta))}$	
	$= \frac{\cos^2 \theta}{\cos 2\theta - (2 \sin \theta \cos^2 \theta + \sin \theta \cos 2\theta)}$	
	$= \frac{\cos^2 \theta}{\cos 2\theta - (\sin 2\theta \cos \theta + \cos 2\theta \sin \theta)}$	
	$= \frac{\cos^2 \theta}{\cos 2\theta - \sin 3\theta} *$	A1*
		(4)



9(a) Alt3 By cross multiplying	$\frac{\cos^2 \theta}{\cos 2\theta - \sin 3\theta} = \frac{1 + \sin \theta}{1 - 2\sin \theta - 4\sin^2 \theta} \Leftrightarrow$ $\cos^2 \theta (1 - 2\sin \theta - 4\sin^2 \theta) = (1 + \sin \theta)(\cos 2\theta - \sin 3\theta) \Leftrightarrow$ $\cos^2 \theta (1 - 2\sin \theta - 4\sin^2 \theta) = (1 + \sin \theta)(\cos 2\theta - \sin 2\theta \cos \theta - \cos 2\theta \sin \theta)$	M1
	$\Leftrightarrow \cos^2 \theta (1 - 2\sin \theta - 4\sin^2 \theta)$ $= (1 + \sin \theta)(1 - 2\sin^2 \theta - 2\sin \theta \cos^2 \theta - (1 - 2\sin^2 \theta)\sin \theta)$	M1
	$\Leftrightarrow \cos^2 \theta - \cancel{2\sin \theta \cos^2 \theta} - 4\sin^2 \theta \cos^2 \theta = 1 - 2\sin^2 \theta - \cancel{2\sin \theta \cos^2 \theta}$ $- \cancel{\sin \theta} + \cancel{2\sin^3 \theta} + \cancel{\sin \theta} - \cancel{2\sin^3 \theta} - 2\sin^2 \theta \cos^2 \theta - \sin^2 \theta + 2\sin^4 \theta$ $\Leftrightarrow \cos^2 \theta - 2\sin^2 \theta \cos^2 \theta = 1 - 3\sin^2 \theta + 2\sin^2 \theta (1 - \cos^2 \theta)$	M1
	$\Leftrightarrow \cos^2 \theta - \cancel{2\sin^2 \theta \cos^2 \theta} = 1 - 3\sin^2 \theta + 2\sin^2 \theta - \cancel{2\sin^2 \theta \cos^2 \theta}$ $\Leftrightarrow \cos^2 \theta = 1 - \sin^2 \theta$ which is true, hence given result is true.*	A1*
		(4)

Notes:**(a)**

M1: Applies the compound angle formula to $\sin 3\theta$ at some stage in the working. Accept $\pm \sin \theta \cos 2\theta \pm \sin 2\theta \cos \theta$. Allow for students who apply $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$ directly. May be seen anywhere (e.g. in separate working).

M1: Uses correct double angle formula for $\cos 2\theta$ **and** $\sin 2\theta$ (or $\sin 3\theta$) to achieve all terms in single angle. (May be before or after cross multiplying.)

M1: Cross multiplies and uses Pythagorean identity somewhere in the working in an attempt to achieve a true statement by reaching the same expression on both sides, or reducing to a known true trigonometric identity (as shown in scheme). Award at the stage the identity is applied to an appropriate term in an expression where the angles have been reduced to just θ .

A1: Reaches a correct true statement with no errors seen **and** makes conclusion that the original result is true.



Question Number	Scheme	Marks
2(a)	$R = \sqrt{5}$	B1
	$\tan \alpha = \frac{2}{1} \Rightarrow \alpha = \dots$	M1
	$\alpha = 1.107$	A1
		(3)
(b)(i)	$\text{Max} = 3 + 7\sqrt{5}$	B1ft
(b)(ii)	$(2x - "1.107") = \pi \Rightarrow x = \dots$	M1
	$\Rightarrow x = \frac{\pi + "1.107"}{2} = 2.12$	A1
		(3)
		Total 6

(a)**B1:** Correct exact value (Condone $R = \pm\sqrt{5}$).isw after a correct answer. e.g. $R = \sqrt{5} = 2.24$ **M1:** Allow for: $\tan \alpha = \pm \frac{2}{1}$, $\tan \alpha = \pm \frac{1}{2}$, $\cos \alpha = \pm \frac{1}{"R"}$, $\sin \alpha = \pm \frac{2}{"R"}$ leading to a value for α If no method is shown imply by the sight of awrt 1.1 rads or awrt 63° **A1:** awrt 1.107**(b)(i)****B1ft:** Award for $3 + 7 \times \text{their } R$ where $R > 0$.

Also follow through on decimal answers from (a) e.g.

Condone solutions such as $3 - 7 \times -\sqrt{5} = 18.65\dots$ **(b)(ii)****M1:** For an attempt to solve $(2x \pm "1.107") = \pi$ or $(2x \pm "1.107") = -\pi$. May be implied by awrt 2.12Condone a bracketing slip $\cos(2(x \pm "1.107")) = -1 \Rightarrow 2(x \pm "1.107") = \pi \Rightarrow x = \dots$ (condone $-\pi$ as above)Also condone an attempt to solve $(2x \pm "63^\circ") = 180^\circ$ but not mixed units, e.g. $(2x \pm "1.107") = 180^\circ$ **A1:** For awrt 2.12. This cannot be given in a list, the 2.12 must be selected.

Question Number	Scheme	Marks
9.(a)	$R = \sqrt{41}$ $\tan \alpha = \frac{4}{5} \Rightarrow \alpha = \text{awrt } 0.675$	B1 M1A1 (3)
(b)	(i) Describes stretch: stretch in the y direction by " $\sqrt{41}$ " (ii) Describes translation: E.g. translate by $\begin{pmatrix} -\arctan \frac{4}{5} \\ 0 \end{pmatrix}$	B1 ft B1 ft (2)
(c)	Attempts either $g(\theta) = \frac{90}{4 + (\sqrt{41})^2}$ OR $g(\theta) = \frac{90}{4}$ Range $2 \leq g(\theta) \leq 22.5$	M1 A1 (2)
		7 marks

(a)

 B1 $R = \sqrt{41}$

 Condone $R = \pm\sqrt{41}$ (Do not allow decimals for this mark Eg 6.40 but remember to isw after $\sqrt{41}$)

 M1 $\tan \alpha = \pm \frac{4}{5}, \tan \alpha = \pm \frac{5}{4} \Rightarrow \alpha = \dots$ Condone $\sin \alpha = 4, \cos \alpha = 5 \Rightarrow \tan \alpha = \frac{4}{5}$

 If R is used to find α accept $\sin \alpha = \pm \frac{4}{R}$ or $\cos \alpha = \pm \frac{5}{R} \Rightarrow \alpha = \dots$

 A1 $\alpha = \text{awrt } 0.675$

 Note that the degree equivalent $\alpha = \text{awrt } 38.7^\circ$ is A0

(b)(i)

 B1ft Fully describes the stretch. Follow through on their R . **Requires the size and the direction**
Allow responses such as

- stretch in the y direction by " $\sqrt{41}$ "
- multiplies all the y coordinates/values by " $\sqrt{41}$ "
- stretch in $\uparrow$ direction by " $\sqrt{41}$ "
- vertical stretch by " $\sqrt{41}$ "
- Scale Factor " $\sqrt{41}$ " in just the y direction

 Do **not** award for y is translated/transformed by " $\sqrt{41}$ "

(b)(ii)

B1 ft Fully describes the translation. **Requires the size and the direction**Follow through on their 0.675 or $\alpha = \arctan 38.7^\circ$ or $\arctan \frac{4}{5}$ **Allow** responses such as

- translates left by 0.675
- horizontal by -0.675
- condone "transforms" left by 0.675. (question asks for the translation)
- moves $\leftarrow$ by 38.7°
- x values move back by 0.675
- shifts in the negative x direction by $\arctan \frac{4}{5}$
- $\begin{pmatrix} -0.675 \\ 0 \end{pmatrix}$

Do **not** award for translates left by -0.675 (double negative...wrong direction)
horizontal shift of 0.675 (no direction)

If there are no labels score in the order given but do allow these to be written in any order as long as the candidate clearly states which one they are answering. For example it is fine to write

translation is.....

stretch is

If the candidate does not label correctly, or states which one they are doing, but otherwise gets both completely correct then award SC B1 B0

(c)

M1 Score for either end achieved by a correct method

Look for $\frac{90}{4}$ (implied by 22.5), $\frac{90}{4 + \text{their}(\sqrt{41})^2}$, $g \dots 22.5$ or $g \dots 2$ etc

A1 See scheme but allow 22.5 to be written as $\frac{90}{4}$

Accept equivalent ways of writing the interval such as $[2, 22.5]$

Condone $2 \leq g(x) \leq 22.5$ or $2 \leq y \leq 22.5$



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
9(a)	$R = 13$	B1
	$\tan \alpha = \frac{5}{12} \Rightarrow \alpha = \text{awrt } 0.395$	M1A1
		(3)
(b)	$g(\theta) = 10 + 13 \sin\left(2\theta - \frac{\pi}{6} - 0.395\right)$	
(i)	(i) Minimum value is -3	B1 ft
(ii)	$2\theta - \frac{\pi}{6} - 0.395 = \frac{3\pi}{2} \Rightarrow \theta = \text{awrt } 2.82$	M1 A1
		(3)
(c)	$h(\beta) = 10 - 169 \sin^2(\beta - 0.395)$	
	$-159 \leq h \leq 10$	M1 A1
		(2)
		(8 marks)

(a)

B1: $R = 13$ ($R = \pm 13$ is B0)M1: $\tan \alpha = \pm \frac{5}{12}$, $\tan \alpha = \pm \frac{12}{5} \Rightarrow \alpha = \dots$ If R is used to find α accept $\sin \alpha = \pm \frac{5}{R}$ or $\cos \alpha = \pm \frac{12}{R} \Rightarrow \alpha = \dots$ A1: $\alpha = \text{awrt } 0.395$ Note that the degree equivalent $\alpha = \text{awrt } 22.6^\circ$ is A0

(b)(i)

B1ft: States the value of $10 - R$ following through their R .

(b)(ii)

M1: Attempts to solve $2\theta - \frac{\pi}{6} \pm "0.395" = \frac{3\pi}{2} \Rightarrow \theta = \dots$ A1: $\theta = \text{awrt } 2.82$. No other values should be given

(c)

M1: Achieves one of the end values, either -159 (or $10 - (\text{their } R)^2$ evaluated) or 10 A1: Fully correct range $-159 \leq h \leq 10$, $-159 \leq h(\beta) \leq 10$, $-159 \leq \text{range} \leq 10$, $-159 \leq h(x) \leq 10$, $[-159, 10]$ or equivalent correct ranges.

Question Number	Scheme	Marks
6 (a)	$5e^{x-1} + 3 = 18 \Rightarrow e^{x-1} = 3$ $\Rightarrow x = \ln 3 + 1$ or $e^x = 3e$ $\Rightarrow x = \ln 3e$	M1 A1 A1 (3)
(b)	Sets $5e^{x-1} + 3 = 10 - x^2$ and proceeds to find and use a suitable function. Eg $(f(x) =) 7 - x^2 - 5e^{x-1}$ Attempts $f(1.1335) = 0.001$ and $f(1.1345) = -0.007$ Correct values with reason(change of sign and continuous) and conclusion, hence α is 1.134 to 3dp	B1 M1 A1 (3)
(c)	$x_2 = -\sqrt{7 - 5e^{-3-1}} = -2.628388$ $\beta = -2.620330$	M1 A1 A1 (3)
		(9 marks)

(a)

 M1 For attempting to proceed from $5e^{x-1} + 3 = 18$ to $e^{x-1} = \dots$ or $e^x = \dots$

 A1 $x = \ln 3 + 1$ or for $e^x = 3e$

 A1 $x = \ln 3e$ Accept $x = \ln 3e^1$

(b)

B1 Sets the equations equal to each other and finds a suitable function which is then used.

 Suitable functions are $f(x) = \pm(7 - x^2 - 5e^{x-1})$, $g(x) = \pm(x - \sqrt{7 - 5e^{x-1}})$ or $h(x) = \pm\left(x - 1 - \ln\left(\frac{7 - x^2}{5}\right)\right)$

 M1 Substitutes $x = 1.1335$ and $x = 1.1345$, or suitable values for a tighter interval (either side of 1.133634..), into a suitable function and obtains one correct value to 1sf rounded or truncated.

FYI for the + options above

$$f(1.1335) = 0.001 \text{ and } f(1.1345) = -0.007$$

$$g(1.1335) = -0.0005 \text{ and } g(1.1345) = 0.003$$

$$h(1.1335) = -0.0002 \text{ and } h(1.1345) = 0.001$$

A1 Requires both values to be correct (1sf rounded or truncated), a reason (change of sign and continuous) and a minimal conclusion

(c)

 M1 For an attempt at substituting -3 into the iterative equation.

 This is implied by the sight of the expression or awrt -2.63

 A1 awrt -2.628388

 A1 $\beta =$ awrt -2.620330 condone -2.62033

.....

 It is possible in part (b) to score B0 M1 A1 by comparing y values for $y = 5e^{x-1} + 3$ and $y = 10 - x^2$ at $x = 1.1335$ and $x = 1.1345$ For the A1 apply the similar criteria as for the main scheme with values to 3d.p..

$$\text{At } x = 1.1335, y|_{1.1335} = 5e^{x-1} + 3 = 8.714 \text{ and } y|_{1.1335} = 10 - x^2 = 8.715$$

$$\text{At } x = 1.1345, y|_{1.1345} = 5e^{x-1} + 3 = 8.720 \text{ and } y|_{1.1345} = 10 - x^2 = 8.713$$

Question Number	Scheme	Marks
7.(a)	$R = \sqrt{17}$ $\tan \alpha = 4 \Rightarrow \alpha = \text{awrt } 1.326$	B1 M1A1 (3)
(b)	Minimum height = $\frac{24}{3 + "R"} = 3.37$ (metres)	M1 A1 (2)
(c)	Uses part (a) $10 = \frac{24}{3 + \sqrt{17} \cos\left(\frac{1}{2}t - 1.326\right)} \Rightarrow \cos\left(\frac{1}{2}t - 1.326\right) = \frac{-0.6}{\sqrt{17}}$ $t = \text{awrt } 6.09$	M1 A1 M1 A1 (4) (9 marks)

(a)

B1 $R = \sqrt{17}$ Condone $R = \pm\sqrt{17}$ (Do not allow decimals for this mark Eg 4.12 but remember to isw after $\sqrt{17}$)M1 $\tan \alpha = \pm 4, \tan \alpha = \pm \frac{1}{4} \Rightarrow \alpha = \dots$ If R is used to find α accept $\sin \alpha = \pm \frac{4}{R}$ or $\cos \alpha = \pm \frac{1}{R} \Rightarrow \alpha = \dots$ A1 $\alpha = \text{awrt } 1.326$ Note that the degree equivalent $\alpha = \text{awrt } 75.96^\circ$ is A0

(b)

M1 Attempts minimum height by stating or finding $\frac{24}{3 + "R"}$

Attempts via differentiation must be complete methods with correct work up to slips in coefficients. They are unlikely to succeed.

$$\text{FYI } \frac{dH}{dt} = -\frac{24\left(2\cos\left(\frac{t}{2}\right) - \frac{1}{2}\sin\left(\frac{t}{2}\right)\right)}{\left(4\sin\left(\frac{t}{2}\right) + \cos\left(\frac{t}{2}\right) + 3\right)^2} = 0 \Rightarrow \tan\left(\frac{t}{2}\right) = 4 \Rightarrow H = \frac{24}{3 + \cos(1.326) + 4\sin(1.326)}$$

A1 3.37 (metres) Assume metres unless otherwise stated, but 3.37 cm is A0. Accept 337 cm as long as the units are stated, but do not accept $\frac{24}{3 + \sqrt{17}}$ and do not isw if incorrect units are given following 3.37.

(c)

M1 Attempts to use their answer to part (a) (including their R and their α) AND proceeds to

$$\cos(\beta t \pm "1.326") = k, \quad -1 < k < 1 \text{ and } \beta = 1 \text{ or } \frac{1}{2}$$

A1 $\cos(\beta t \pm "1.326") = \frac{-0.6}{\sqrt{17}}$ or awrt -0.146 where $\beta = 1$ or $\frac{1}{2}$ M1 Full method to make t the subject from an equation of the form $\cos\left(\frac{1}{2}t \pm "1.326"\right) = k, \quad -1 < k < 1$ Look for $2 \times (\text{their } \arccos(k) \pm \text{their } \alpha)$ A1 awrt $t = 6.09$ (Ignore any extra solutions outside the domain, but A0 if extras inside are given.)

Question	Scheme	Marks
8(a)	$R = 17$	B1
	$\tan \alpha = \frac{15}{8}$	M1
	$\alpha = 1.081$	A1
		(3)
(b)	(i) $\text{Min } f(x) = \frac{15}{41 + 2 \times "17"}$	M1
	$= \frac{1}{5}$	A1
	(ii) Occurs when $\sin(x - 1.081) = 1 \Rightarrow x - 1.081 = \frac{\pi}{2} \Rightarrow x = \dots$	M1
	$x = \text{awrt } 2.65$	A1
		(4)
(c)	$-\frac{23}{5}$ (or -4.6)	B1ft
		(1)
(d)	Awrt 1.33	B1ft
		(1)
(9 marks)		

Notes:**(a)****B1:** For 17 only.

M1: Attempts an equation in α . Accept $\tan \alpha = \pm \frac{15}{8}$ or $\tan \alpha = \pm \frac{8}{15}$. If using R accept $\cos \alpha = \pm \frac{8}{"17"}$ or $\sin \alpha = \pm \frac{15}{"17"}$. Implied by a correct value for α

A1: Awrt 1.081. Must be in radians.**(b)(i)**

M1: Attempts to apply the result from (a) to find the minimum. Allow for $\frac{15}{41 \pm 2 \times \text{"their } R"}$

A1: cao.**(ii)**

M1: Attempts to solve $x \pm \alpha = \frac{\pi}{2}$

A1: cao**(c)**

B1ft: Correct answer or follow through $2 \times \left(\text{their } \frac{1}{5} \right) - 5$ and no other solutions.

(d)

B1ft: For awrt 1.33 or follow through $0.5 \times \text{their } 2.65$ and no other solutions.



Question Number	Scheme	Marks
2	$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$	
(a)	Sets $2x^2 - 5x + 8 = (Ax + B)(x - 2) + C$ with an attempt at one constant Attempts all 3 constants. E.g. Sets $x = 2 \Rightarrow C = \dots, -2B + C = 8 \Rightarrow B = \dots$ and compares x^2 terms gives $A = \dots$ $2x - 1 + \frac{6}{x - 2}$	M1 dM1 A1 (3)
(b)	$\int 2x - 1 + \frac{6}{x - 2} dx = x^2 - x + 6 \ln(x - 2)$ $\int_4^8 2x - 1 + \frac{6}{x - 2} dx = \left[x^2 - x + 6 \ln(x - 2) \right]_4^8 = 56 + 6 \ln 6 - 12 - 6 \ln 2$ $= 44 + 6 \ln 3$	M1 A1ft dM1 A1 (4) (7 marks)

ANSWERS IN (a) of $A = 2, B = -5, C = 2$, and/or $28 + 2 \ln 3$ in (b) please send to review

(a) **Note that correct values for A, B, and C imply the first two marks**

M1: Attempts to find one of the constants via

- an identity e.g. $2x^2 - 5x + 8 = (Ax + B)(x - 2) + C$

Note that if an identity is used it must be correct. Condone arithmetical slips when evaluating.

The identity may be implied by correct equations

e.g. $x^2 : A = 2$, $x : -2A + B = -5$, constant : $-2B + C = 8$

- division is used look for a quotient as far as $2x$

$$\begin{array}{r} 2x-1 \\ x-2 \overline{) 2x^2-5x+8} \\ \underline{2x^2-4x-8} \\ 6 \end{array}$$

Note that the division may appear in many different formats or may be by inspection so $2x$ is sufficient to score this mark.

$$\text{e.g. } \frac{2x^2 - 5x + 8}{x - 2} = \frac{2x(x - 2) + 4x - 5x + 8}{x - 2} = 2x + \frac{-(x - 2) - 2 + 8}{x - 2} = 2x - 1 + \frac{6}{x - 2}$$

e.g.

$$\begin{array}{r|rrr} & 2x & -1 & \\ \hline x & 2x^2 & -x & 6 \\ -2 & -4x & 2 & \end{array}$$



dM1: Attempts to find all 3 constants which are all non-zero. It is dependent upon the previous mark. Via an identity: If three simultaneous equations are formed allow this mark to be scored for proceeding to values for A , B and C . You do not need to check this. Allow sign slips only in forming the equations. Via division: Look for a quotient of $2x \pm c$ and a constant remainder

A1: $2x - 1 + \frac{6}{x-2}$ The expression must be written; it cannot be awarded for just stating the correct values of A , B and C , unless it is later written correctly as an expression in (b) (may be seen within an integral)

(b)

M1: Attempts to integrate the term $\frac{C}{x-2}$ proceeding to $D \ln|x-2|$ C does not have to be equal D (do not be concerned with brackets or modulus sign)

A1ft: Correct integration of $g(x)$ for their A , B and C in part (a).

They must be integrating an expression of the form $Ax + B + \frac{C}{x-2} \Rightarrow \frac{Ax^2}{2} + Bx + C \ln|x-2|$ o.e.

(where $A, B, C \neq 0$) which may be unsimplified with or without a constant of integration.

Do not be concerned by the absence of the bracket or modulus on the $x-2$ (as both values substituted in will give positive lns)

dM1: Attempts to proceed to the form $p + q \ln r$ by

- using the limits 8 and 4 in an expression of the form $Ex^2 + Fx + D \ln|x-2|$ o.e. (where $D, E, F \neq 0$) and subtracting either way round

- combining their $\ln$ terms **correctly**. e.g. this mark cannot be scored for

$$\ln 6 - \ln 2 = \frac{\ln 6}{\ln 2} = \ln 3$$

but treat as a sign slip in front of the log rather than incorrect log work e.g.

$$6 \ln 6 - 6 \ln 2 = -6 \ln 3$$

Condone slips when evaluating or multiplying out brackets only (condone a spurious $+c$ present for this mark). The substitution and combining of their $\ln$ terms can be implied by their $p + q \ln r$ provided no incorrect log work is seen. Note that $\ln r$ does not have to be $\ln 3$ e.g. it may be $\ln 729$.

A1: $44 + 6 \ln 3$ Withhold the final mark if there is an integral and dx around the answer or $+c$

Alt (b) Attempts via substitution

Note there may be attempts using substitution e.g. $u = x - 2$ on **some or all** of their expression.

$$\text{e.g. } \int_4^8 \frac{6}{x-2} dx \rightarrow \int_2^6 \frac{6}{u} du = [6 \ln u]_2^6$$

M1: Correct attempt to integrate $\frac{C}{u}$ to $D \ln|u|$ C does not have to be equal D

A1ft: $A(u+2) + B + \frac{C}{u} \Rightarrow \frac{Au^2}{2} + (2A+B)x + C \ln|u|$ or condone $\frac{Ax^2}{2} + Bx + C \ln|u|$

dM1: Attempts to either substitute in the limits 6 and 2, subtracts either way round and combines their $\ln$ terms correctly, or substitutes back to an expression in terms of x and proceeds as in the main scheme and notes. If they are working in a mixture of variables, $x^2 - x + 6 \ln u$ then the correct limits must be substituted into the appropriate terms.

A1: $44 + 6 \ln 3$ Withhold the final mark if there is an integral and dx around the answer or $+c$



Question Number	Scheme	Marks
3 (i)	$y = \frac{10^6}{x^3} \Rightarrow \log_{10} y = \log_{10} 10^6 - \log_{10} x^3 \Rightarrow \log_{10} y = 6 - 3\log_{10} x$	B1 B1 B1 (3)
(ii)	States $\log_3 N = 2t + 4$ or $\log_3 N = \log_3 a + t \log_3 b$ $N = 3^{2t+4} = 3^{2t} \times 3^4$ or $\log_3 a = 4 \Rightarrow a = \dots$ or $\log_3 b = 2 \Rightarrow b = \dots$ $N = 81 \times 9^t$	B1 M1 A1cso (3) (6 marks)

(i) **Condone invisible brackets around coordinates provided the meaning is clear.**

Coordinates may be stated away from the graph which is fine provided a graph is drawn.

Note that they may draw a curve so B0B1B1 is possible as the marks are earned independently of each other.

Note that if they draw on Figure 2 the axis labels are incorrect so maximum score is B0B1B1

- B1: For a straight line with negative gradient anywhere on a set of axes. It must intersect both axes and not stop at these points. Do not penalise if the line is not perfectly straight and do not be concerned with the steepness provided there is no intention to draw a curve or a horizontal line. Axes do not need to be labelled but if they are then the y-axis needs to be $\log y$ and the horizontal axis must be $\log x$ (with or without base 10 labelled)
- B1: For an intercept of 6 on the $\log_{10} y$ axis for their graph. Can be labelled as just 6 instead of (0, 6). The line must pass through (or start/stop at) this point. Condone y intercept labelled as (6, 0) provided the point is on the positive y-axis. Do not be concerned by the axis labelling for this mark.
- B1: For an intercept of 2 on the $\log_{10} x$ axis for their graph. Can be labelled as just 2 instead of (2, 0). The line must pass through (or start/stop at) this point. Condone x intercept being labelled as (0, 2) provided the point is on the positive x-axis. Do not be concerned by the axis labelling for this mark.



(ii) **Note that $N = 81 \times 9^t$ with no working is 0 marks**

May work in terms of x instead of t which is fine, but the final answer must be in terms of t

A misread writing a different base to 3 (including 10) can score a maximum B0M1A0.

B1: For a correct equation $\log_3 N = 2t + 4$ which may be left unsimplified or states

$$\log_3 N = \log_3 a + t \log_3 b$$

Condone lack of base 3 written but base 10 is B0. May be implied by further work which is not the final answer.

If they write down an incorrect equation e.g. $\log_3 N = -2t + 4$ as well as $\log_3 N = \log_3 a + t \log_3 b$ then allow this mark to be scored.

M1: For using the laws of logs correctly and moving from $\log_3 N = at + b$ to $N = 3^{at} \times 3^b$

If they write $3^{2t} + 3^4$ then M0A0 but if it is ambiguous between a + and $\times$ give the benefit of the doubt.

This may involve using their $\log_3 N = at + b$ for two different values for t to find N using logs correctly. They then form two simultaneous equations using $N = ab^t$ and proceed to find a value for a or b .

$$\text{e.g. } t=1 \Rightarrow \log_3 N = 6 \Rightarrow N = 729, \quad t=2 \Rightarrow \log_3 N = 8 \Rightarrow N = 6561$$

$$\Rightarrow ab = 729, \quad ab^2 = 6561 \Rightarrow b = \frac{6561}{729} \text{ or } a = 81$$

$$\text{e.g. } (0, 81), (-2, 1) \Rightarrow 81 = ab^0, \quad 1 = ab^{-2} \Rightarrow a = 81 \text{ or } b = 9$$

Alternatively, forms an equation in a or b and proceeds to find a value for a or b using logs correctly.

(may be implied by their values for a or b **provided no incorrect log work is seen**)

If they misread and write a different base this mark can be scored for working correctly in that base.

$$\text{e.g. } \log_{10} a = 4 \Rightarrow a = 10^4 \text{ o.e. or e.g. } N = 10^4 \times 10^{2t}$$

A1: $N = 81 \times 9^t$ cso **Just stating the values of a and b does not score this mark. Must be in terms of t**
We must see

- $\log_3 N = 2t + 4$ or $N = 3^{2t+4}$ or $N = 3^{2t} \times 3^4$ before proceeding to the answer with no incorrect log work seen
- $\log_3 N = \log_3 a + t \log_3 b$ **or** both $\log_3 a = 4 \Rightarrow a = 3^4$ o.e. **and** $\log_3 b = 2 \Rightarrow b = 3^2$ o.e. before proceeding to $N = 81 \times 9^t$

Note that e.g. $N = 3^{2t+4} = 3^{2t} + 3^4 \Rightarrow N = 81 \times 9^t$ is B1M0A0 (incorrect log work – the values of 81 and 9 cannot imply the method mark as we see incorrect log work)



Question Number	Scheme	Marks
4(a)	$f(x) = 8\sin x \cos x + 4\cos^2 x - 3$	M1
	States or uses $\sin 2x = 2\sin x \cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$	dM1
	Uses $\sin 2x = 2\sin x \cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ in $f(x)$	A1
	$(f(x) =) 8\sin x \cos x + 4\cos^2 x - 3 = 4\sin 2x + 2\cos 2x - 1$	(3)
(b)	$R^2 = a^2 + b^2 \Rightarrow R = \sqrt{20} \text{ or } 2\sqrt{5}$	B1ft
	$\tan \alpha = \frac{b}{a} \Rightarrow \alpha = \dots$ ("awrt 0.464")	M1
	$(f(x) =) 2\sqrt{5} \sin(2x + 0.464) - 1$	A1
		(3)
(c)	(i) Maximum value = " $2\sqrt{5} - 1$ "	B1 ft
	(ii) Solves $2x + \alpha = \frac{5\pi}{2} \Rightarrow x = \dots$	M1
	$(x =) \text{awrt } 3.69 \text{ (or } (x =) \text{awrt } 3.70)$	A1
		(3)
		(9 marks)

(a) For the method marks condone working in mixed variables provided the intention is clear

M1: **Either** states one of the correct identities $\sin 2x = 2\sin x \cos x$ or $\cos 2x = 2\cos^2 x - 1$

or uses $\sin 2x = 2\sin x \cos x$ or $\cos 2x = \pm 2\cos^2 x \pm 1$ (may be implied by use of e.g. $\cos^2 x = \pm 1 \pm \sin^2 x$ and $\cos 2x = \pm 1 \pm 2\sin^2 x$ or $\cos 2x = \pm \cos^2 x \pm \sin^2 x$)

Can be implied by **either of**

- $a = 4$
- $b = \pm 2$ **and** $c = -5$ or -1

dM1: **Uses** $\sin 2x = 2\sin x \cos x$ and $\cos 2x = \pm 2\cos^2 x \pm 1$ (which may be implied as above for the first method mark) in $f(x)$ to produce an expression of the required form $a \sin 2x + b \cos 2x + c$
Condone slips when substituting in.

Can be implied by $a = 4$ **and** $b = \pm 2$ **and** $c = -5$ or -1

A1: $4\sin 2x + 2\cos 2x - 1$ Correct answer scores all three marks. Must be in terms of x .

(b) **Full marks can only be scored provided correct values for a , b and c are found in (a).**

B1ft: Finds the exact value of R using $R^2 = a^2 + b^2$ (or may use trigonometry using their value for α)

Can be implied by a correct exact value.

Follow through on their a or b but the correct values should give $R = \sqrt{20}$ or $2\sqrt{5}$

Question Number	Scheme	Marks
4.(a)	$ \begin{array}{r} 3x-5 \\ x^2+0x+1 \overline{) 3x^3-5x^2+7x-5} \\ \underline{3x^3+0x^2+3x} \\ -5x^2+4x \\ \underline{-5x^2+0x-5} \\ 4x \end{array} $ $A=3, \quad B=-5, C=4$ $D=0$ *	M1 <u>B1</u>, A1 A1* (4)
(b)	$ \int 3x-5 + \frac{4x}{x^2+1} dx = \frac{3x^2}{2} - 5x + 2 \ln(x^2+1) + c $ $ \begin{aligned} \text{Area} &= \left(\frac{3 \times 3^2}{2} - 5 \times 3 + 2 \ln(3^2+1) \right) - \left(\frac{3 \times 2^2}{2} - 5 \times 2 + 2 \ln(2^2+1) \right) \\ &= \frac{5}{2} + \ln 4 \end{aligned} $	M1, A1ft dM1, A1 (4) (8 marks)

(a) **Method via division**

M1: Full attempt to divide producing a linear quotient ($\dots x + \dots$) and linear remainder ($\dots x + \dots$)

B1: States $A = 3$ which may be awarded from the quotient line in the division sum $3x...$

This is an independent mark and can be awarded without sight of method

A1: $B = -5$ and $C = 4$ following the award of the M mark. May be awarded from within the division sum.

A1*: Requires

- fully correct division (see main scheme) with no constant within the remainder which proves that $D = 0$
- all previous marks to have been scored
- all constants stated or correct expression seen in (a) or (b)

If the candidates don't put in the '0's you would need to see something like the following work

$$\begin{array}{r} 3x-5 \\ x^2+1 \overline{) 3x^3-5x^2+7x-5} \\ \underline{3x^3 } \\ -5x^2+4x-5 \\ \underline{-5x^2 } -5 \\ 4x \end{array}$$

The remainder should be $4x$ and not $4x \pm 5$

To score the A1* line it must be fully correct division

(a) **Alternative method via identity**

M1: Sets $3x^3 - 5x^2 + 7x - 5 \equiv (Ax + B)(x^2 + 1) + Cx + D$ and finds values for A , B , and C

B1: States $A = 3$

This is an independent mark and can be awarded without sight of method

A1: $B = -5$ and $C = 4$ following the award of the M mark

A1*: This must be proven/shown and not just stated

For example:

Comparing terms in x^2 , $-5 = B$ I

Comparing constant terms $-5 = B + D$ II

So $D = 0$

Look for a pair of relevant and correct equations which are correctly solved

All previous marks must have been scored

.....
(b)

M1: $\int \frac{Cx}{x^2 + 1} dx = k \ln(x^2 + 1)$ where k is a constant. You may see $k \ln u$ where $u = x^2 + 1$

Allow this to be scored from

$$\int \frac{Cx + D}{x^2 + 1} dx = \int \frac{Cx}{x^2 + 1} dx + \int \frac{D}{x^2 + 1} dx = k \ln(x^2 + 1) + \dots \text{ where } \dots \text{ is not } = m \ln(x^2 + 1)$$

A1ft: $\int 3x - 5 + \frac{4x}{x^2 + 1} dx = \frac{3x^2}{2} - 5x + 2 \ln(x^2 + 1)$ but follow through on their A , B and C (with $D = 0$)

Also accept versions such as $\frac{(3x - 5)^2}{6} + 2 \ln(u)$ where $u = x^2 + 1$

dM1: Substitutes 3 and 2 (o.e) into their integrated function (either way around) and correctly combines the two log terms. The limits must be consistent with their variable, so 3 and 2 for x and 10 and 5 for u . It is dependent upon having integrated to a form $f(x) + \delta \ln(x^2 + 1)$ where $f(x)$ is a quadratic expression

A1: CSO $\frac{5}{2} + \ln 4$ Note that $\frac{5}{2} + 2 \ln 2$ is A0 as the form of the answer has to be $\alpha + \ln \beta$