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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Differentiation (P3.4)

International A Level Mathematics · P3: Pure Mathematics 3 (WMA13)

28 questions · 223 marks · Mark schemes included

Past papers January 2020 - June 2025 · 3 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

28 questions · 223 marks · 3 sub-topics

SUB-TOPIC P3.4a

CHAIN, PRODUCT & QUOTIENT RULES

Sub-topic P3.4a · 7 questions · 60 marks · lowest tariff first



6.

$$y = \frac{2 + 3 \sin x}{\cos x + \sin x}$$

Show that

$$\frac{dy}{dx} = \frac{a \tan x + b \sec x + c}{\sec x + 2 \sin x}$$

where a, b and c are integers to be found.

(6)

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Question 6 continued

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Q6

(Total 6 marks)



5. The curve C has equation

$$y = \frac{\ln(x^2 + k)}{x^2 + k} \quad x \in \mathbb{R}$$

where k is a positive constant.

(a) Show that

$$\frac{dy}{dx} = \frac{Ax(B - \ln(x^2 + k))}{(x^2 + k)^2}$$

where A and B are constants to be found.

(3)

Given that C has exactly three turning points,

(b) find the x coordinate of each of these points. Give your answer in terms of k where appropriate.

(3)

(c) find the upper limit to the value for k .

(1)



Question 5 continued

(Total for Question 5 is 7 marks)



6.

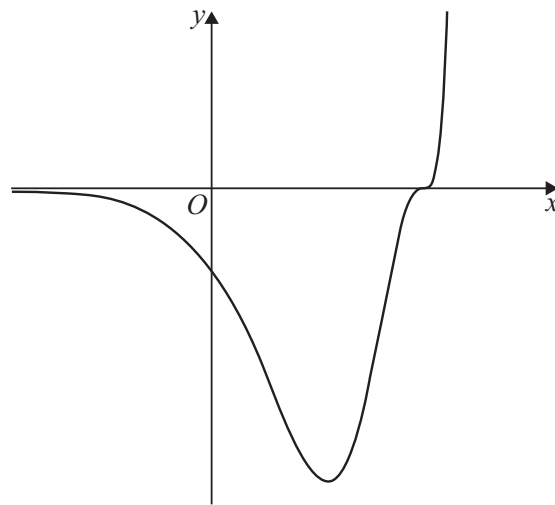
**Figure 2**

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (2x - 3)^3 e^{4x-2}$$

(a) Show that

$$f'(x) = 2(Px + Q)(2x - 3)^n e^{4x-2}$$

where P , Q and n are constants to be found.

(4)

(b) Hence find

- (i) the x coordinates of the stationary points on the curve with equation $y = f(x)$,
- (ii) the range of the function g defined by

$$g(x) = 8(2x - 3)^3 e^{4x-2} \quad x \leq \frac{3}{2}$$

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

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Question 6 continued

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(Total for Question 6 is 8 marks)



Question 8 continued

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Question 8 continued

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Q8

(Total 9 marks)



Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total for Question 8 is 9 marks)



7.

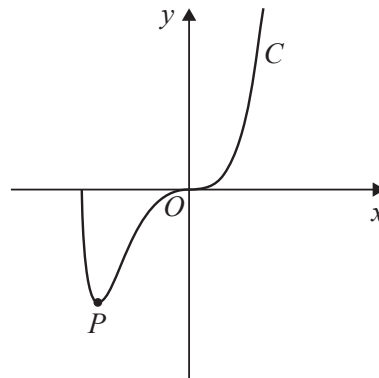


Figure 3

The curve C has equation $y = f(x)$, where

$$f(x) = x^3 \sqrt{4x + 7} \quad x \geq -\frac{7}{4}$$

(a) Show that

$$f'(x) = \frac{kx^2(2x + 3)}{\sqrt{4x + 7}}$$

where k is a constant to be found.

(4)

The point P , shown in Figure 3, is the minimum turning point on C .

(b) Find the coordinates of P .

(2)

(c) Hence find the range of the function g defined by

$$g(x) = -4f(x) \quad x \geq -\frac{7}{4}$$

(2)

The point Q with coordinates $\left(\frac{1}{2}, \frac{3}{8}\right)$ lies on C .

(d) Find the coordinates of the point to which Q is mapped when C is transformed to the curve with equation

$$y = 40f\left(x - \frac{3}{2}\right) - 8$$

(2)



Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)



Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total 11 marks)

Q4

Grading box for Q4.



SUB-TOPIC P3.4b

TANGENTS, NORMALS & STATIONARY POINTS

Sub-topic P3.4b · 11 questions · 85 marks · lowest tariff first



1. Find, using calculus, the x coordinate of the stationary point on the curve with equation

$$y = (2x + 5)e^{3x}$$

(4)

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Question 1 continued

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Q1

(Total 4 marks)



Question 1 continued

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(Total 6 marks)

Q1



Question 1 continued

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Q1

(Total 7 marks)



Question 3 continued

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Q3

(Total 7 marks)



6.

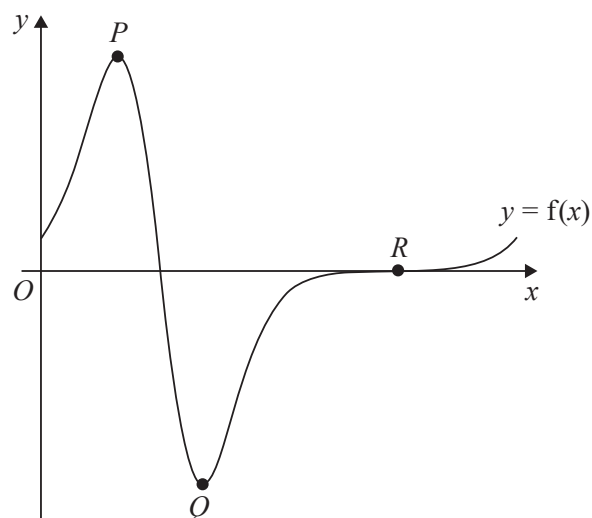


Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = 2e^{3\sin x} \cos x \quad 0 \leq x \leq 2\pi$$

The curve intersects the x -axis at point R , as shown in Figure 1.

(a) State the coordinates of R

(1)

The curve has two turning points, at point P and point Q , also shown in Figure 1.

(b) Show that, at points P and Q ,

$$a \sin^2 x + b \sin x + c = 0$$

where a , b and c are integers to be found.

(4)

(c) Hence find the x coordinate of point Q , giving your answer to 3 decimal places.

(2)



Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)



Question 6 continued

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Lined area for writing the answer to Question 6.

(Total 8 marks)

Q6

Box for marking the question.



Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)



9.

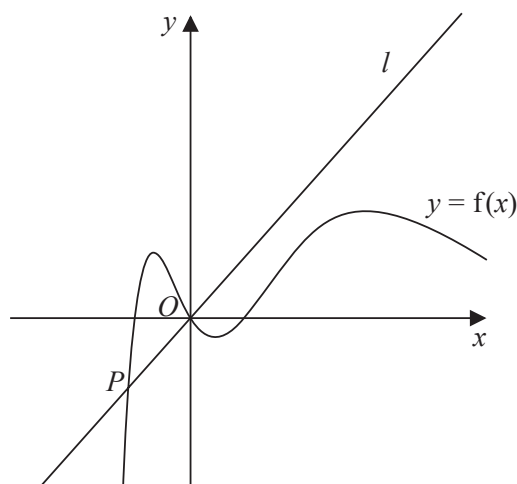


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = x(x^2 - 4)e^{-\frac{1}{2}x}$$

(a) Find $f'(x)$.

(2)

The line l is the normal to the curve at O and meets the curve again at the point P .

The point P lies in the 3rd quadrant, as shown in Figure 3.

(b) Show that the x coordinate of P is a solution of the equation

$$x = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x}}$$

(4)

(c) Using the iterative formula

$$x_{n+1} = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x_n}} \quad \text{with } x_1 = -2$$

find, to 4 decimal places,

(i) the value of x_2

(ii) the x coordinate of P .

(3)

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Question 9 continued

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Question 9 continued

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Question 9 continued

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Handwriting practice area with horizontal lines.

(Total 9 marks)

Q9



7. The curve C has equation

$$x = 3 \tan\left(y - \frac{\pi}{6}\right) \quad x \in \mathbb{R} \quad -\frac{\pi}{3} < y < \frac{2\pi}{3}$$

(a) Show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are integers to be found.

(4)

The point P with y coordinate $\frac{\pi}{3}$ lies on C .

Given that the tangent to C at P crosses the x -axis at the point Q .

(b) find, in simplest form, the exact x coordinate of Q .

(5)

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Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 9 marks)



9.

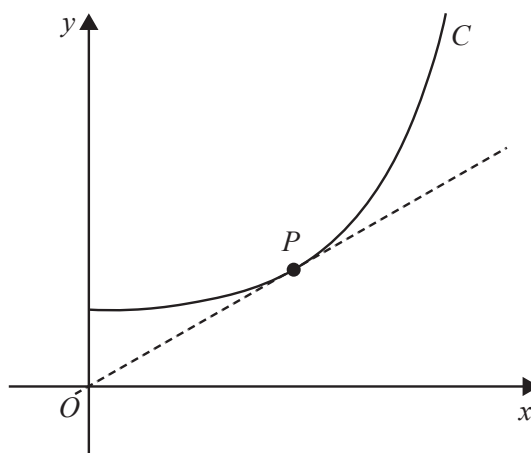


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \sqrt{3 + 4e^{x^2}} \quad x \geq 0$$

- (a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

The point P with x coordinate α lies on C .

Given that the tangent to C at P passes through the origin, as shown in Figure 3,

- (b) show that $x = \alpha$ is a solution of the equation

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

(3)

- (c) Hence show that α lies between 1 and 2

(2)

- (d) Show that the equation in part (b) can be written in the form

$$x = \frac{1}{2}\sqrt{4 + 3e^{-x^2}}$$

(1)

The iteration formula

$$x_{n+1} = \frac{1}{2}\sqrt{4 + 3e^{-x_n^2}}$$

with $x_1 = 1$ is used to find an approximation for α .

- (e) Use the iteration formula to find, to 4 decimal places, the value of

(i) x_3

(ii) α

(3)



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Question 9 continued

(Total for Question 9 is 11 marks)



SUB-TOPIC P3.4c

DIFFERENTIATING $X = F(Y)$ & CONNECTED RATES

Sub-topic P3.4c · 10 questions · 78 marks · lowest tariff first



7. Given that

$$x = 6 \sin^2 2y \quad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{Bx - x^2}}$$

where A and B are integers to be found.

(5)

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Question 7 continued

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Q7

(Total 5 marks)



4.

$$y = \log_{10}(2x + 1)$$

(a) Express x in terms of y .

(2)

(b) Hence, giving your answer in terms of x , find $\frac{dy}{dx}$

(3)

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Question 4 continued

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Q4

(Total 5 marks)



(Total for Question 2 is 5 marks)



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Q10

TOTAL FOR PAPER IS 75 MARKS



Question 10 continued

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Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

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Question 3 continued

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(Total for Question 3 is 8 marks)



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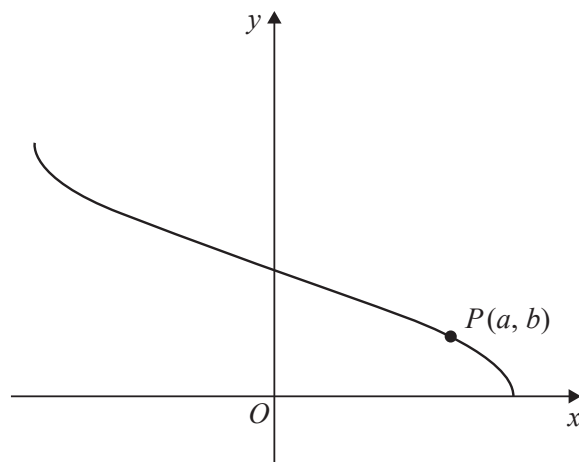


Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3 \cos 2y \quad -3 \leq x \leq 3 \quad 0 \leq y \leq \frac{\pi}{2}$$

(a) Find $\frac{dx}{dy}$ in terms of y .

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\frac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b .

(4)

Question 10 continued

TOTAL FOR PAPER IS 75 MARKS



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Question 10 continued

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Q10

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END



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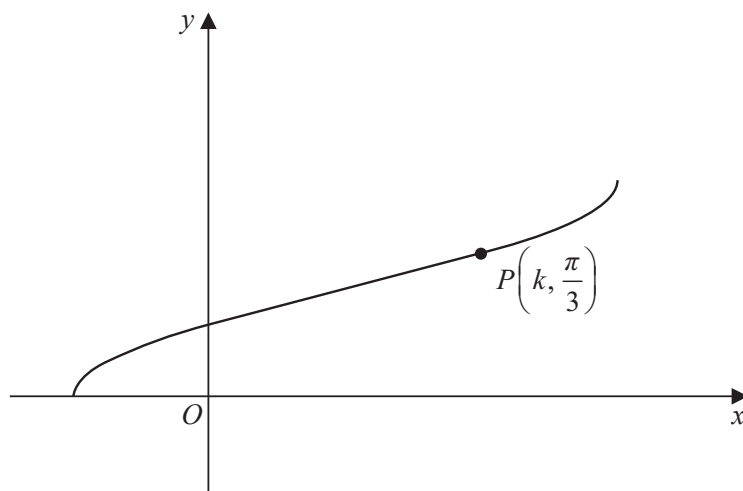


Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \quad 0 \leq y \leq \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

(1)

(b) (i) Find $\frac{dx}{dy}$ in terms of y

(ii) Hence show that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

(6)

The normal to the curve at P cuts the x -axis at the point N .

(c) Find the exact area of triangle OPN , where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants.

(3)



Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS



9.

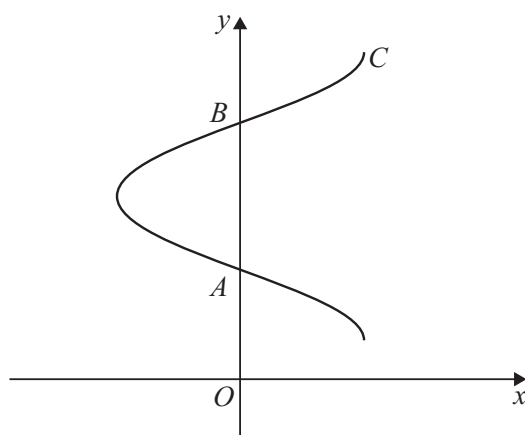


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$x = \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) \quad \frac{\pi}{12} < y < \frac{3\pi}{4}$$

The curve intersects the y -axis at the points A and B as shown.

(a) Find the exact value of the y coordinate of

- point A
- point B

(3)

(b) Show that

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{p - qx^2}$$

where p and q are integers to be found.

(4)

The **normal** to C at A and the **tangent** to C at B intersect at the point D .

Using

- the answer to part (b)
- the sketch of curve C in Figure 2

(c) find, in simplest form, the exact x coordinate of D .

(4)





Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 11 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



Question	Scheme	Marks
1(a)	$2x^3 - 4x - 15 = (Ax + B)(x^2 + 3x + 4) + C(2x + 3) \Rightarrow A = \dots \text{ or}$ $\begin{array}{r} 2x\dots \\ x^2 + 3x + 4 \overline{) 2x^3 + 0x - 4x - 15} \\ 2x^3 + \dots \end{array}$	M1
	$A = 2$	A1
	$B = -6, C = 3$	A1A1
		(4)
(b)	$\int f(x) dx = "A" \frac{x^2}{2} + "B" x + "C" \ln(x^2 + 3x + 4)$	M1 A1ft
	$\int f(x) dx = x^2 - 6x + 3 \ln(x^2 + 3x + 4)$	
	$\int_3^5 f(x) dx = (5^2 - 6 \times 5 + 3 \ln(5^2 + 3 \times 5 + 4)) - (3^2 - 6 \times 3 + 3 \ln(3^2 + 3 \times 3 + 4)) = ..$	dM1
	$= -5 + 3 \ln 44 + 9 - 3 \ln 22 = 4 + 3 \ln \left(\frac{44}{22} \right)$	M1
	$= 4 + \ln 8$	A1
		(5)
		(9 marks)
Notes:		
(a) M1: A correct method leading to at least one of the constants. May attempt long division getting the term A, or multiply through and compare coefficients or substitute values. Note that achieving $A = 2$ implies this mark. A1: $A = 2$ May be seen in the long division or in the expression. A1: Either $B = -6$ or $C = 3$ Stated or as part of the expression A1: Both $B = -6$ and $C = 3$ Stated or as part of the expression		
(b) Note: use of letters A, B and C can score max 3 marks M1A1ftdM0M1A0. M1: Attempts to integrate their answer to (a), look for $x^n \rightarrow x^{n+1}$ at least once and $\frac{2x+3}{x^2+3x+4} \rightarrow k \ln(x^2+3x+4) \text{ Allow with letters used if values have not been found.}$ A1ft: Correct integration following through their A, B and C (or with letters). dM1: Applies the limits to their integral and subtracts. <i>Must have numerical values for this mark..</i> M1: Uses correct log laws to combine the log terms into a single log term. (Need not have used the power law at this stage, unless earlier error leads to mismatch of coefficients.) Allow if letters used, so $3 \ln(44/22)$ can score this mark. A1cao: Correct answer in the form specified.		

Question	Scheme	Marks
2(a)	Either $f(x) < 5$ or $f(x) \dots 3$	M1
	$3,, f(x) < 5$	A1
		(2)
(b)	(i) $y = 5 - \frac{4}{3x+2} \Rightarrow \frac{4}{3x+2} = 5 - y \Rightarrow \frac{4}{5-y} = 3x+2 \Rightarrow x = \dots$	M1
	$(f^{-1}(x)) = \frac{1}{3} \left(\frac{4}{5-x} - 2 \right)$ oe such as $\frac{4}{15-3x} - \frac{2}{3}$ or $\frac{2x-6}{15-3x}$	A1
	(ii) Domain is $3,, x < 5$	B1ft
		(3)
(c)	$fg(-\pi) = f \left(4 \sin \left(-\frac{\pi}{3} + \frac{\pi}{6} \right) \right) = f \left(4 \sin \left(-\frac{\pi}{6} \right) \right) = f(2) = \dots$	M1
	$= 5 - \frac{4}{6+2} = \frac{9}{2}$	A1
		(2)
(7 marks)		

Notes:**(a)**

M1: One correct end of range, though allow with $,,$ or $<$ and $\dots$ or $>$ (in correct direction) at the respective ends, and accept with y instead of $f(x)$, or even with x for the M1.

A1: Correct range, allow with y instead of $f(x)$ or with other correct set notation (but use of x is A0 unless in formal set notation). Accept as two separate inequalities.

(b)(i)

M1: Attempts to make x or a swapped y the subject of the equation. Allow sign slips, but there should be a correct order of operations.

A1: Correct rule, must be in terms of x , accept equivalent and isw after a correct answer is seen. Do not be concerned with the lhs (accept $y = \dots$ and even condone $f(x) = \dots$)

(b)(ii)

B1ft: Follow through on their answer to (a). Accept intervals or set notation answers. Do not accept e.g. $3,, f^{-1}(x) < 5$ or with y

(c)

M1: Attempts to evaluate g at $-\pi$ and substitutes into f . (Allow even if a negative value is found.) Evaluation of $g(-\pi)$ must be attempted, not just substitution into the expression. They may find the expression for $fg(x)$ first and substituted, in which case the trigonometric expression must be evaluated before the mark is awarded.

A1: Correct answer. Accept as decimal. Do not isw if they try to change to degrees.

Question	Scheme	Marks
3(a)	$f'(x) = 2(x-2) \times e^{3x} + (x-2)^2 \times 3e^{3x}$	M1 A1
	$f'(x) = 0 \Rightarrow e^{3x}(x-2)(2+3(x-2)) = 0 \Rightarrow x = \dots$	M1
	$\Rightarrow x = \frac{4}{3}$	A1
	$y = \frac{4}{9}e^4 \quad \left(\text{so } A \text{ is } \left(\frac{4}{3}, \frac{4}{9}e^4 \right) \right)$	A1
		(5)
(b)	$k > 0$ or $k \dots$, their $\frac{4}{9}e^4$ or with ... or $<$ respectively.	M1
	$0 < k \dots$, their $\frac{4}{9}e^4$	A1ft
		(2)
(7 marks)		

Notes:**(a)**

M1: Differentiates using the product rule reaching the form $\alpha(x-2) \times e^{3x} + (x-2)^2 \times \beta e^{3x}$

A1: Correct derivative, need not be simplified.

M1: Sets their derivative equal to zero and attempts to cancel or factorise out the exponential term and the $(x-2)$ term (or multiple thereof) and solve for x . Alternatively, they may expand to a 3 term quadratic $(3x^2 - 10x + 8)$ and attempt to solve this via correct method. Allow recovery on slips in the exponent for this mark (and the A's if a correct quadratic expression is solved).

A1: Correct x value.

A1: Correct simplified y value. Allow if seen in part (b). Isw, after a correct answer, but the follow through in (b) will not apply on an incorrectly simplified answer if this mark has been awarded.

(b)

M1: Identifies one correct boundary for k (the direction of inequality must be correct). Accept in terms of y or $f(x)$ for this mark.

A1ft: Correct range for k following their **positive** y coordinate in (a). Must have correct inequalities. Accept awrt 24.3 for the upper bound. Allow as separate inequalities, or set or interval notation.

Question	Scheme	Marks
4(a)	$y = \log_{10}(2x+1) \Rightarrow 10^y = 2x+1 \Rightarrow x = \dots$	M1
	$\Rightarrow x = \frac{10^y - 1}{2}$	A1
		(2)
(b)	$\left(\frac{dx}{dy}\right) \frac{1}{2} 10^y \ln 10$	M1
	$\frac{dy}{dx} = 1 / \frac{dx}{dy} = \frac{1}{\frac{1}{2} 10^y \ln 10}$	M1
	$\frac{dy}{dx} = \frac{2}{(2x+1) \ln 10}$	A1
		(3)

(5 marks)**Notes:****(a)****M1:** Correctly undoes the logarithm and attempts to rearrange to make x the subject.**A1:** Correct expression for x . Apply isw after a correct answer.**(b)****M1:** Attempts to differentiate 10^y . Accept $\alpha 10^y \rightarrow \beta 10^y \ln 10$ (ignore any extra terms). Do not be concerned about the lhs for this mark.**M1:** Applies reciprocal $\frac{dy}{dx} = 1 / \frac{dx}{dy}$. Variables must be consistent.**A1:** Correct answer. Accept equivalents in terms of x , e.g. with $10^{\log(2x+1)}$ isw after a correct answer.

Alt for (b): Allow use of change of base formula.

M1: $y = \frac{\ln(2x+1)}{\ln 10}$ (effectively a B mark via this method)**M1:** $\frac{dy}{dx} = \left(\frac{1}{\ln 10} \times\right) \frac{1}{2x+1} \times \dots$ Attempts to differentiate using the chain rule.**A1:** Correct answer.Note: Candidates who write $\frac{dy}{dx} = \frac{1}{2x+1} \times \dots \left(\text{or } \frac{dy}{dx} = \frac{1}{10^y} \times \dots \right)$ with no $\ln 10$ term can score M0M1A0 under the Alt.

Alt 2: Implicit Differentiation

M1: As main scheme, attempts to differentiate as part of their work, so $\alpha 10^y \rightarrow \beta 10^y \ln 10 \frac{dy}{dx}$ **M1:** Makes $\frac{dy}{dx}$ the subject. **A1:** Correct answer.

Question	Scheme	Marks
5(a)	$P(1) = \frac{4-1}{10} + \frac{3}{4} \ln\left(\frac{2}{3^2}\right) \quad (= -0.828\dots)$	M1
	$P(1) = -0.828\dots$ (which is negative so a loss of £0.828 million,) so approximately £830 000 loss.	A1*
		(2)
(b)	$P(6) = -0.08799\dots$ and $P(7) = 0.1975\dots$	M1
	There is a sign change and hence as P is continuous on $[6,7]$, so the root for t lies in $[6,7]$.	A1
		(2)
(c)	$P = 0 \Rightarrow \frac{4t-1}{10} = -\frac{3}{4} \ln\left(\frac{t+1}{(2t+1)^2}\right) \Rightarrow t = \dots$	M1
	$\Rightarrow t = \frac{10\left(-\frac{3}{4} \ln\left(\frac{t+1}{(2t+1)^2}\right)\right) + 1}{4} = \frac{1}{4} + \frac{30}{16} \ln\left(\frac{t+1}{(2t+1)^2}\right)^{-1}$ $\Rightarrow t = \frac{1}{4} + \frac{15}{8} \ln\left(\frac{(2t+1)^2}{t+1}\right)^*$	A1*
		(2)
(d)	$t_2 = \frac{1}{4} + \frac{15}{8} \ln\left(\frac{13^2}{7}\right) = \dots (= 6.219978\dots)$	M1
	$t_2 = \text{awrt } 6.220$	A1
	$t_6 = 6.314$	A1
		(3)
(e)	"6.3..." $\times 12 = \dots$ months, or repeated iteration to root 6.31487 gives 75.7785... or allow "0.314..." $\times 12 = "3.768\dots"$	M1
	So it will take 76 months. (Accept 75 or awrt 76 months)	A1
		(2)

(11 marks)**Notes:****(a)**

M1: Attempts to substitute $t = 1$ into the given formula. Allow if there is a slip but an attempt at substitution is seen, or allow for sight of awrt $-0.828\dots$

A1*: Correct value for $P(1)$ seen to at least 2 s.f. if substitution has been shown, or at least 3 s.f. if no substitution was shown, followed by suitable conclusion that it is a loss of approximately £830 000, though accept awrt £830 000. Must mention "loss" and include units (£ or pounds). Negative value given is A0.

(b)

M1: Attempts both $P(6)$ and $P(7)$ with at least one correct to 1 s.f. rounded or truncated. A tighter interval could be used but must contain the root (6.31487)



Question Number	Scheme	Marks
2(a)	$ff(6) = f\left(\frac{9}{2}\right) = \frac{\frac{9}{2}+3}{\frac{9}{2}-4} = \dots = 15$	M1; A1
		(2)
(b)	$f^{-1}(x) = \frac{4x+3}{x-1}$	M1A1
	$x \in \mathbb{R}, x \neq 1$	B1
		(3)
(c)	E.g. $\left(\frac{x+3}{x-4}\right)^2 + 5 = 7$ or $\frac{a+3}{a-4} = (\pm)\sqrt{7-5}$	M1
	$\Rightarrow x^2 - 22x + 23 = 0 \Rightarrow x = \dots \text{ or } (a+3) = (\pm)\sqrt{2}(a-4) \Rightarrow a = \dots$	dM1
	$(a =) 11 + 7\sqrt{2} \text{ oe}$	A1
		(3)
		(8 marks)
Notes		
(a)		
M1:	Substitutes $x = 6$ into f and substitutes the result back into f to find a value for $ff(6)$	
	Alternatively substitutes $x = 6$ into $ff(x) = \frac{\left(\frac{x+3}{x-4}\right)+3}{\left(\frac{x+3}{x-4}\right)-4}$ condoning slips.	
A1:	15	
(b)		
M1:	Changes the subject of $y = \frac{x+3}{x-4}$ or $x = \frac{y+3}{y-4}$ to $y = \frac{\dots x \pm 3}{\dots x \pm 1}$ or $x = \frac{\dots y \pm 3}{\dots y \pm 1}$	
A1:	Achieves $f^{-1}(x) = \frac{4x+3}{x-1}$. Accept $f^{-1} = \dots$ or $y = \dots$ instead of $f^{-1}(x) = \dots$.	
B1:	$x \neq 1$. The omission of $x \in \mathbb{R}$ is condoned. Accept alternative notations.	
(c) Note this is M1A1A1 on open but is being marked as M1dM1A1.		
M1:	Correct attempt to set up an equation in x or a . This will usually be $\left(\frac{x+3}{x-4}\right)^2 + 5 = 7$ but $f(a) = g^{-1}(7)$ may be used first. Score when a suitable equation is set up and allow if a minor slip or miscopy is made if the intention is clear.	
dM1:	Proceeds to solve for x or a , e.g. forms a 3TQ and solves, or take the 5 across, square roots, cross multiplies and makes a the subject, condoning e.g. sign slips when rearranging. Alternatively, uses $(f(x))^2 + 5 = 7 \Rightarrow f(x) = (\pm)\sqrt{2}$ and their part (b) leading to	
	$a = f^{-1}\left(\frac{4\sqrt{2}+3}{\sqrt{2}-1}\right) = \frac{4\sqrt{2}+3}{\sqrt{2}-1}$. In this method both M's may be scored together.	
A1	$11 + 7\sqrt{2}$ or $\frac{4\sqrt{2}+3}{\sqrt{2}-1}$ Accept $11 + \sqrt{98}$ or $\frac{-4\sqrt{2}-3}{1-\sqrt{2}}$. Common factors should be cancelled. Ignore labelling of x or a . Condone e.g. if $11 - 7\sqrt{2}$ is also given as a solution but A0 if any other extra solutions are included.	

Question Number	Scheme	Marks
3(a)	$(\cos 2A \equiv) \cos A \cos A - \sin A \sin A \equiv \cos^2 A - (1 - \cos^2 A)$	M1
	$\Rightarrow \cos 2A \equiv 2 \cos^2 A - 1$ *	A1*
		(2)
(b)	$\int (3 - 2 \cos 6x) \, dx = 3x - \frac{\sin 6x}{3} \quad (+c)$	M1A1
	$\left[3x - \frac{\sin 6x}{3} \right]_{\frac{\pi}{12}}^{\frac{\pi}{8}} = \left(3 \left(\frac{\pi}{8} \right) - \frac{\sin \left(6 \times \frac{\pi}{8} \right)}{3} \right) - \left(3 \left(\frac{\pi}{12} \right) - \frac{\sin \left(6 \times \frac{\pi}{12} \right)}{3} \right) = \frac{1}{8} \pi + \frac{2 - \sqrt{2}}{6}$	dM1A1
		(4)
		(6 marks)
Notes		
(a)		
M1:	Writes $(\cos 2A \equiv) \cos A \cos A - \sin A \sin A$ and attempts to use (not just stated separately) the identity $\pm \sin^2 A \pm \cos^2 A = \pm 1$ Note going directly to $\cos 2A \equiv \cos^2 A - \sin^2 A$ is allowed M1 as long as a use of the Pythagorean identity is clear. NB Watch out for answers which use double angle identities in the proof (circular reasoning) and score M0 if no Pythagorean identity is used.	
A1*:	Achieves the given answer with no errors seen (and no circular reasoning). Must see $\cos 2A \equiv \cos A \cos A - \sin A \sin A$ and clear attempt at substitution of $\sin^2 A + \cos^2 A = 1$ but allow “LHS” for “ $\cos 2A$ ”. Going directly to $\cos 2A \equiv \cos^2 A - \sin^2 A$ in the first step would score A0. Correct notation must used throughout but condone a missing closing bracket.	
Note :	$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1$ with no further evidence scores M0A0 as they are just writing out known formulae.	
(b)		
M1:	Attempts to substitute in the given result in part (a) and integrates to the form $\dots x \pm \dots \sin 6x$ Must be a full substitution in terms of x , $\cos 2A \rightarrow k \sin 2A$ is M0 unless recovered, e.g by later substitution may be implied by the limits substituted).	
A1:	$3x - \frac{\sin 6x}{3}$	
dM1:	Shows evidence of substituting in both limits into an expression of the form $\dots x \pm \dots \sin 6x$ and subtracts either way round to achieve an exact (not necessarily simplified but trig must be evaluated) answer.	
A1:	Correct answer e.g as scheme or $\frac{1}{8} \pi + \frac{1}{3} - \frac{\sqrt{2}}{6}$ or exact equivalent (and isw after a correct answer).	

To score the A1* line it must be fully correct division

(a) **Alternative method via identity**

M1: Sets $3x^3 - 5x^2 + 7x - 5 \equiv (Ax + B)(x^2 + 1) + Cx + D$ and finds values for A , B , and C

B1: States $A = 3$

This is an independent mark and can be awarded without sight of method

A1: $B = -5$ and $C = 4$ following the award of the M mark

A1*: This must be proven/shown and not just stated

For example:

Comparing terms in x^2 , $-5 = B$ I

Comparing constant terms $-5 = B + D$ II

So $D = 0$

Look for a pair of relevant and correct equations which are correctly solved

All previous marks must have been scored

.....
(b)

M1: $\int \frac{Cx}{x^2 + 1} dx = k \ln(x^2 + 1)$ where k is a constant. You may see $k \ln u$ where $u = x^2 + 1$

Allow this to be scored from

$$\int \frac{Cx + D}{x^2 + 1} dx = \int \frac{Cx}{x^2 + 1} dx + \int \frac{D}{x^2 + 1} dx = k \ln(x^2 + 1) + \dots \text{ where } \dots \text{ is not } = m \ln(x^2 + 1)$$

A1ft: $\int 3x - 5 + \frac{4x}{x^2 + 1} dx = \frac{3x^2}{2} - 5x + 2 \ln(x^2 + 1)$ but follow through on their A , B and C (with $D = 0$)

Also accept versions such as $\frac{(3x - 5)^2}{6} + 2 \ln(u)$ where $u = x^2 + 1$

dM1: Substitutes 3 and 2 (o.e) into their integrated function (either way around) and correctly combines the two log terms. The limits must be consistent with their variable, so 3 and 2 for x and 10 and 5 for u . It is dependent upon having integrated to a form $f(x) + \delta \ln(x^2 + 1)$ where $f(x)$ is a quadratic expression

A1: CSO $\frac{5}{2} + \ln 4$ Note that $\frac{5}{2} + 2 \ln 2$ is A0 as the form of the answer has to be $\alpha + \ln \beta$



Question Number	Scheme	Marks
5.(a)	$\frac{dy}{dx} = -10\sin 2x - 24\cos 2x$ $\frac{dy}{dx}\bigg _{x=\frac{\pi}{3}} = -10\sin\left(\frac{2\pi}{3}\right) - 24\cos\left(\frac{2\pi}{3}\right) = -5\sqrt{3} + 12$	M1, A1 A1 (3)
(b)	$R = 13$ $\tan \alpha = \frac{12}{5} \Rightarrow \alpha = \text{awrt } 1.176$	B1 M1A1 (3)
(c)	Sets $-26\sin(2x + 1.176) = 6$ $\sin(2x + "1.176") = -\frac{3}{13}$ $x = \frac{\arcsin\left(-\frac{3}{13}\right) - 1.176}{2} = \text{awrt } 2.44$	M1 A1 ft dM1, A1 (4)
		(10 marks)

(a)

M1: Correct attempt at differentiation. Score for $\frac{dy}{dx} = p\sin 2x + q\cos 2x$

There may be attempts in which the double angle formulae have been used.

For example, $y = 5\cos 2x - 12\sin 2x = 5(\pm 2\cos^2 x \pm 1) - 24\sin x \cos x$

In such cases look for applications of the chain rule and product rules condoning slips in sign and loss of coefficients.

A1: $\frac{dy}{dx} = -10\sin 2x - 24\cos 2x$ which may be left un-simplified

If the double angle is used you should see $\frac{dy}{dx} = -20\sin x \cos x + 24\sin^2 x - 24\cos^2 x$

A1: Substitutes $x = \frac{\pi}{3}$ into a correct $\frac{dy}{dx}$ and achieves $-5\sqrt{3} + 12$ o.e.

It must follow the award of M1, A1. ISW after a correct answer

(b)

B1: $R = 13$. Allow for example 13.000 but not 12.99 followed by 13

Note that $\sqrt{169}$ or ± 13 is A0 unless followed by 13

M1: $\tan \alpha = \pm \frac{12}{5}$, $\tan \alpha = \pm \frac{5}{12} \Rightarrow \alpha = \dots$ (Condone $\sin \alpha = 12$, $\cos \alpha = 5$ in the working)

If R is used to find α accept $\sin \alpha = \pm \frac{12}{R}$ or $\cos \alpha = \pm \frac{5}{R} \Rightarrow \alpha = \dots$

A1: $\alpha = \text{awrt } 1.176$ ISW after a correct answer so don't penalise if they then write $\sqrt{13} \cos(2x - 1.176)$



(c) Using their answer to part (b) **Working and method must be shown**. Note: If they had $\alpha = 1.176$ but write the equation of C as $y = 13 \cos(2x - 1.176)$ mark (c) as though $\alpha = -1.176$

M1: Differentiates $R \cos(2x + \alpha)$ to $T \sin(2x + \alpha)$ with their α (to at least 2dp) AND sets equal to 6

A1ft: $\sin(2x + '1.176') = \pm \frac{3}{13}$ or decimal equivalent. Condone a sign slip on differentiation and follow

through on their α (to at least 3 dp) so score for $\sin(2x + \text{their } \alpha) = \pm \frac{3}{13}$

dM1: Complete attempt to find any **positive** value for x using a correct order of operations.

We now must be using correct differentiation including the sign but you can fit on their α ONLY

The following values are acceptable following intermediate working

$$\text{E.g. } \sin(2x + \text{awrt } "1.18") = -\frac{3}{13} \Rightarrow 2x + "1.18" = \text{awrt } 3.37 \Rightarrow x = \text{awrt } \frac{3.37 - "1.18"}{2} = \dots$$

$$\text{E.g. } \sin(2x + \text{awrt } 1.18) = -\frac{3}{13} \Rightarrow 2x + "1.18" = \text{awrt } 6.05 \Rightarrow x = \frac{6.05 - "1.18"}{2} = \dots$$

This dependent method mark may be implied by sight of calculations such as

$$\sin(2x + \text{awrt } 1.18) = -\frac{3}{13} \Rightarrow x = \text{awrt } \frac{\arcsin\left(-\frac{3}{13}\right) - 1.18}{2} \text{ o.e., followed by 1.1 or 2.4}$$

A1: awrt 2.44 following the award of M1, A1, dM1. It must be selected and not part of a list of values

Candidates who state $-26 \sin(2x + 1.176) = 6$ o.e and follow this with the correct answer will score 1,0,0,0

Candidates who state $\sin(2x + 1.176) = -\frac{6}{26}$ o.e and follow this with the correct answer will score 1,1,0,0

.....
Candidates can use their answer to part (a) but will not score any marks until they put their derivative into a form in which the equation can be easily solved. **There is no follow through marks via this route.**

Use review if you feel unable to award the marks this way

Example I: $6 = -10 \sin 2x - 24 \cos 2x$

$$\Rightarrow 12 \cos 2x + 5 \sin 2x = -3$$

$$\Rightarrow \cos(2x - 0.395) = -\frac{3}{13} \quad \text{M1, A1}$$

$$\Rightarrow x = \frac{\arcsin\left(-\frac{3}{13}\right) + 0.395}{2}, = 2.44 \quad \text{dM1, A1}$$

Example II: $6 = -10 \sin 2x - 24 \cos 2x$

Uses a more round-about route

$$\Rightarrow 12 \cos 2x + 3 = -5\sqrt{1 - \cos^2 2x}$$

$$\Rightarrow 144 \cos^2 2x + 72 \cos 2x + 9 = 25(1 - \cos^2 2x)$$

$$\Rightarrow 169 \cos^2 2x + 72 \cos 2x - 16 = 0 \quad \text{M1, A1}$$

$$\Rightarrow \cos 2x = 0.1612, -0.5873$$

$$\Rightarrow x = \text{awrt } 1.1 \text{ or } 2.4 \quad \text{dM1}$$

$$\Rightarrow x = \text{awrt } 2.44 \quad \text{A1}$$

Using their answer to part (a) which must be of the form

$$\frac{dy}{dx} = \pm 10 \sin 2x \pm 24 \cos 2x \quad \text{the marks would be}$$

M1: Equation in a single trig ratio which should be

$26 \cos(2x \pm 0.3947..) = \pm 6$ o.e. following use of $R \cos(2x \pm ..)$

or $26 \sin(2x \pm 1.176) = \pm 6$ o.e. following use of $R \sin(2x \pm ..)$

Allow accuracy to 2dp or greater for the angle

A1: Correct equation, which is usually

$$\cos(2x - 0.3947..) = -\frac{6}{26} \text{ or } \sin(2x + 1.176) = -\frac{6}{26}$$

Accuracy must be 3 dp or greater for the angle

$$\text{dM1: Correctly solves } \cos(2x \pm 0.3947..) = -\frac{6}{26} \text{ or}$$

$$\sin(2x \pm 1.176) = -\frac{6}{26} \text{ and proceeds to a positive value for } x.$$

As in the main method. It requires intermediate working

A1: awrt 2.44 only



Question Number	Scheme	Marks
6. (a)	$f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2}$ $= 2(2x-3)^2 e^{4x-2} \{3 + 2(2x-3)\} = 2(4x-3)(2x-3)^2 e^{4x-2}$	M1 A1 dM1 A1 (4)
(b) (i)	$x = \frac{3}{2}, \frac{3}{4}$	B1 ft
(ii)	Attempts $f\left(\frac{3}{4}\right) = \left(-\frac{3}{2}\right)^3 e^1 = -\frac{27}{8}e, \Rightarrow g\left(\frac{3}{4}\right) = -27e$ $-27e, g(x), 0g, g, \zeta$	M1, A1 A1 (4)
		(8 marks)

(a)

M1: Attempts the product rule and achieves $f'(x) = \alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2}$ where $\alpha > 0, \beta > 0$

A1: $f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2}$ which may be left unsimplified

dM1: Correctly takes out a common factor of $(2x-3)^2 e^{4x-2}$ out of $\alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2}$

Look for $f'(x) = \alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2} = (2x-3)^2 e^{4x-2} \{\alpha + \beta(2x-3)\}$

A1: $2(4x-3)(2x-3)^2 e^{4x-2}$ following M1, A1, dM1. The order of the terms is not important.

Allow the dM1 to be implied for candidates who go directly from

$$6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2} \rightarrow 2(4x-3)(2x-3)^2 e^{4x-2}$$

Marks in part (b) can only be scored following a correct method of differentiation in part (a)

So only allow if $f'(x)$ was of the form $\alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2}$

They cannot be scored following made up values for P and Q .

They cannot just appear as this would be relying entirely upon a calculator.

(b)(i)

B1ft: $x = \frac{3}{2}$ and $\frac{3}{4}$ following $f'(x) = 2(4x-3)(2x-3)^2 e^{4x-2}$

Alternatively, candidates can set

$$f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2} = 0 \text{ and solve to reach } x = \frac{3}{2} \text{ and } \frac{3}{4}$$

If part (a) is not correct follow through on their

- factorised $2(Px+Q)(2x-3)^2 e^{4x-2}$ following M1 and some attempt to combine the terms

(not necessarily correctly) in part(a), so award for $x = \frac{3}{2}$ and $-\frac{Q}{P}$

- solution of $\alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2} = 0$ leading to the solutions $x = \frac{3}{2}, \frac{3\beta - \alpha}{2\beta}$



(b)(ii)

M1: Attempts to find $f\left(\frac{3}{4}\right) = \left(-\frac{27}{8}e\right)$ but allow decimal approximation e.g. AWRT -9.17

or attempts to find $g\left(\frac{3}{4}\right) = (-27e)$ but allow decimal approximation e.g. AWRT -73.4 .

The mark is implied by correct or rounded values following the correct method of differentiation

This can only be awarded following a correct method in part (a) and of finding the $f\left(-\frac{Q}{P}\right)$ or

$$g\left(-\frac{Q}{P}\right),$$

A1: Correct exact value for $g\left(\frac{3}{4}\right) = -27e$ which must be exact

A1: Correct range for g . Look for $-27e$, g , 0 or equivalent such as $g \in [-27e, 0]$ or $-27e \leq g \leq 0$

Note that $-27e \leq g < 0$ and $-27e < g \leq 0$ are incorrect



Question Number	Scheme	Marks
8(i)(a)	$g'(x) = 3e^{3x} \sec 2x + 2e^{3x} \sec 2x \tan 2x$	M1 A1 (2)
(b)	$g'(x) = 0 \Rightarrow e^{3x} \sec 2x (3 + 2 \tan 2x) = 0$ $\tan 2x = -1.5 \Rightarrow x = -0.491$	M1 dM1 A1 (3)
(ii)	$x = \ln(\sin y) \Rightarrow \frac{dx}{dy} = \frac{1}{\sin y} \times \cos y$ Attempts $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - e^{2x}}$ or $\frac{dy}{dx} = \frac{\sin y}{\cos y}$ Hence $\frac{dy}{dx} = \frac{\sin y}{\cos y} = \frac{e^x}{\sqrt{1 - e^{2x}}}$	B1 M1 dM1 A1 (4)
		(9 marks)

(i)(a)

M1 Correct attempt at the product rule $g'(x) = Ae^{3x} \sec 2x + Be^{3x} \sec 2x \tan 2x$ For use of the quotient rule look for $\frac{A \cos 2x e^{3x} - B e^{3x} \sin 2x}{\cos^2 2x}$ A1 $g'(x) = 3e^{3x} \sec 2x + 2e^{3x} \sec 2x \tan 2x$ Allow in any form and then iswFor use of the quotient rule look for $\frac{3 \cos 2x e^{3x} + 2e^{3x} \sin 2x}{\cos^2 2x}$

(i)(b)

M1 Sets $g'(x) = 0 \Rightarrow$ and takes out / factorises out $e^{3x} \sec 2x$ to identify a linear factor in $\tan 2x$ For the quotient rule they should be factorising out $\frac{e^{3x}}{\cos^2 2x}$ to leave a linear factor in $\cos 2x$ and $\sin 2x$ dM1 Correct order of operations to $x = \dots$ For quotient rule approach they must use $\tan 2x = \frac{\sin 2x}{\cos 2x}$ (oe correct work)You may need to check their answer if no method is shown. Accept awrt 2.s.f. for their $\tan 2x = \dots$ in either radians or degrees.A1 $x = \text{awrt } -0.491$ only in the range. If extra solutions arise from trying to solve $\sec 2x = 0$ then A0.

(ii)

B1 $\frac{dx}{dy} = \frac{1}{\sin y} \times \cos y$ OR $e^x = \cos y \frac{dy}{dx}$ via $e^x = \sin y$

M1 For one of the two operations needed to complete the proof

- Either an attempt to get $\cos y$ in terms of e^x look for an attempt using $\sin^2 y + \cos^2 y = 1$ with $\sin y$ being replaced by e^x . Alternatively allow use of $\arcsin$
- Or taking the reciprocal and making $\frac{dy}{dx}$ the subject (variables must be consistent)

dM1 Applies both operations to obtain $\frac{dy}{dx}$ in terms of just e^x

A1 $\frac{dy}{dx} = \frac{e^x}{\sqrt{1-e^{2x}}}$ or $\frac{e^x}{\cos(\arcsin e^x)}$ Allow $\frac{1}{\sqrt{1-(e^x)^2}}$ or states $f(x) = \sqrt{1-e^{2x}}$ following a correct expression for $\frac{dy}{dx} = \frac{e^x}{\cos y}$ or similar.

Alt:

B1 $x = \ln(\sin y) \Rightarrow y = \arcsin(e^x)$

M1 $\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-(e^x)^2}} \times \dots$ or $\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-(\dots)^2}} \times e^x$

dM1 both of these

A1 $\frac{e^x}{\sqrt{1-(e^x)^2}}$

Question Number	Scheme	Marks
7.(a)	States or implies that $A = 2\,500$ $10\,000 = 2500e^{k \times 8} \Rightarrow 8k = \ln 4 \Rightarrow k = \dots$ $\Rightarrow k = \frac{1}{8} \ln 4$ or awrt 0.1733	B1 M1 A1 (3)
(b)	$\frac{dN}{dt} = 60\,000 \times -0.6e^{-0.6 \times 5} = -1792$ So decrease is 1790	M1, A1 (2)
(c)	$60\,000e^{-0.6t} = 2500e^{0.1733t}$ $24 = e^{0.1733t + 0.6t} \Rightarrow 0.1733t + 0.6t = \ln 24 \Rightarrow t = \dots$ $T = 4.11$	M1 dM1 A1 (3)
		8 marks

Notes

(a)

B1 States or implies that $A = 2\,500$. E.g award for $N = 2500e^{kt}$

M1 Attempts to use $N = Ae^{kt}$ with $t = 8$, $N = 10\,000$ and their A to set up and solve an equation in k . Correct $\ln$ work must be used to solve their equation. Allow this mark for attempts to find k first by solving simultaneously if they use $t = 1$ for the start of the study: $2500 = Ae^k$, $10\,000 = Ae^{8k} \Rightarrow e^{7k} = 4 \Rightarrow 7k = \ln 4 \Rightarrow k = \dots$ but the index and $\ln$ work must be correct.

A1 $k =$ awrt 0.1733. Accept the exact value $\frac{1}{8} \ln 4$ and isw after seen.

(b)

M1 $\frac{dN}{dt} = Ce^{-0.6 \times 5} = \dots$ where C is a constant. Condone $60\,000e^{-0.6t} \rightarrow 60\,000e^{-0.6t}$ as long as it is clear they think they have found $\frac{dN}{dt}$. Must be correct index (not kt).

A1 Awrt 1790 from a correct derivative. Condone awrt -1790

(c)

M1 Sets $60\,000e^{-0.6t} =$ their $2500e^{0.1733t}$ May use T or another variable instead. Allow a slip on e.g. the 60000 as 6000. Allow with k in place of their "0.1733" as long as they have a value for k in (a).

dM1 Proceeds to rearrange to $e^{mt} = D$ ($D > 0$) and applies $\ln$ to find t . The $\ln$ work must be correct, though there may be slips in the coefficients or index work reaching $e^{mt} = D$ ($D > 0$). May be implied by a correct answer for their $e^{mt} = D$

Alternatively, takes $\ln$ of both sides first and applies correct $\ln$ laws to proceed to make t the

subject: $\ln(60\,000e^{-0.6t}) = \ln(2500e^{0.1733t}) \Rightarrow \ln 60\,000 - 0.6t = \ln 2500 + 0.1733t \Rightarrow t = \dots$

A1 awrt 4.11 Must be a value, not an expression in $\ln$ terms for this mark.

Note Answer only scores no marks, method must be shown and the dM1 must be achieved in order to access the A mark.

Question Number	Scheme	Marks
8. (a)	$f'(x) = 6(2x+1)^2 e^{-4x} - 4(2x+1)^3 e^{-4x}$ $= 2(2x+1)^2 e^{-4x} \{3 - 2(2x+1)\}$ $= 2(2x+1)^2 (1-4x) e^{-4x}$	M1 A1 dM1 A1 (4)
(b)	Sets $f'(x) = 0 \Rightarrow x = -\frac{1}{2}, \frac{1}{4}$ Either $f\left(-\frac{1}{2}\right) = \dots$ or $f\left(\frac{1}{4}\right) = \dots$ Both $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{4}, \frac{27}{8e}\right)$	B1 M1 A1 (3)
(c)	$\left(\frac{9}{4}, \frac{27}{e}\right)$	B1ft B1ft (2) 9 marks

Notes

(a)

M1 Attempts the product rule to achieve $P(2x+1)^2 e^{-4x} \pm Q(2x+1)^3 e^{-4x}$

May also be attempted by the quotient rule - equivalent form after e terms cancel.

A1 $f'(x) = 6(2x+1)^2 e^{-4x} - 4(2x+1)^3 e^{-4x}$ which may be unsimplifieddM1 Correctly takes out a common factor of $(2x+1)^2 e^{-4x}$ from their expression with an intermediate step before the final answer. Allow if there are minor slips in the $(2x+1)^2 e^{-4x}$ as a factor (such as $(2x+)^2 e^{-4x}$) if recovered - look for the correct remaining terms in the bracket $\{ \}$. Allow going from an expanded cubic to a factorised form for this mark:

$$e^{-4x} (2 - 24x^2 - 32x^3) \rightarrow 2(2x+1)^2 (1-4x) e^{-4x}.$$

A1 Achieves $2(2x+1)^2 (1-4x) e^{-4x}$ with no incorrect algebra. Accept with the brackets in either order.

(b)

B1 $x = -\frac{1}{2}, \frac{1}{4}$ o.e. Both required.
M1 Attempts to substitute one of $x = \pm \frac{1}{2}, \pm \frac{1}{4}$ into $f(x)$. If substitution not seen may be implied by either of $\left(-\frac{1}{2}, 0\right)$ or $\left(\frac{1}{4}, \frac{27}{8e}\right)$ o.e (accept awrt 1.24 for this mark) or by either of $\left(\frac{1}{2}, \frac{8}{e^2}\right)$ (awrt 1.08) or $\left(-\frac{1}{4}, \frac{e}{8}\right)$ (awrt 0.340) o.e.

A1 For $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{4}, \frac{27}{8e}\right)$ o.e. must be exact but isw after exact coordinates given.
Allow as $x = \dots, y = \dots$ as long as clearly paired.

Allow the M and A marks if seen in part (c) - mark (b) and (c) together.



Question Number	Scheme	Marks
7 (a)	$f(x) = x^3 \sqrt{4x+7} \Rightarrow f'(x) = 3x^2 \sqrt{4x+7} + 2x^3 (4x+7)^{-\frac{1}{2}}$	M1, A1
	$\Rightarrow f'(x) = 3x^2 \sqrt{4x+7} + \frac{2x^3}{\sqrt{4x+7}} = \frac{3x^2(4x+7) + 2x^3}{\sqrt{4x+7}}$	dM1
	$\Rightarrow f'(x) = \frac{7x^2(2x+3)}{\sqrt{4x+7}}$	A1
		(4)
(b)	Substitutes $x = -\frac{3}{2}$ into $x^3 \sqrt{4x+7} \Rightarrow y = \dots$	M1
	$\left(-\frac{3}{2}, -\frac{27}{8}\right)$	A1
		(2)
(c)	Attempts $-4 \times -\frac{27}{8}$	M1
	$y = \frac{27}{2}$	A1
		(2)
(d)	$(2, 7)$	M1, A1
		(2)
		Total 10

(a)

M1: Attempts the product rule to achieve $f'(x) = Px^2 \sqrt{4x+7} + Qx^3 (4x+7)^{-\frac{1}{2}}$ where P and Q are positive constants.

Other methods are possible so look at each one carefully.

For example: $f(x) = x^3 \sqrt{4x+7} \Rightarrow f(x) = \sqrt{4x^7 + 7x^6} \Rightarrow f'(x) = \frac{1}{2} (4x^7 + 7x^6)^{-\frac{1}{2}} \times (28x^6 + 42x^5)$

The same principles for the main mark scheme can be applied

A1: $f'(x) = 3x^2 \sqrt{4x+7} + 2x^3 (4x+7)^{-\frac{1}{2}}$ o.e. which may be left unsimplified

dM1: Dependent upon previous M1. It is for "correctly" producing a single fraction with a denominator of $\sqrt{4x+7}$.

Look for $f'(x) = Px^2 \sqrt{4x+7} + Qx^3 (4x+7)^{-\frac{1}{2}} \Rightarrow \frac{Px^2(4x+7) + Qx^3}{\sqrt{4x+7}}$ o.e

A1: $f'(x) = \frac{7x^2(2x+3)}{\sqrt{4x+7}}$

.....

It is possible to attempt this by WMA14 methods using $[f(x)]^2 = x^6(4x+7) = 4x^7 + 7x^6$

Score

M1, A1: $2f(x)f'(x) = 28x^6 + 42x^5$

dM1, A1: $f'(x) = \frac{14x^5(2x+5)}{2f(x)} = \frac{14x^5(2x+5)}{2x^3 \sqrt{4x+7}} = \frac{7x^2(2x+5)}{\sqrt{4x+7}}$



Question Number	Scheme	Marks
3.(a)	Implies equation of line is of the form $\log_{10} y = \pm \frac{2}{3} \log_{10} x + 4$	M1
	States $\log_{10} y = -\frac{2}{3} \log_{10} x + 4$ o.e.	A1
		(2)
(b)	Applies one correct log law E.g $\log_{10} y = -\frac{2}{3} \log_{10} x + 4 \rightarrow \log_{10} y = \log_{10} x^{-\frac{2}{3}} + 4$	M1
	Full attempt to undo the logs $\log_{10} y = \log_{10} x^{-\frac{2}{3}} + \log_{10} 10^4 \rightarrow y = x^{-\frac{2}{3}} \times 10^4$	dM1
	$\rightarrow y = 10\,000x^{-\frac{2}{3}}$ o.e.	A1
		(3)
		5 marks

Note that other versions of $\log_{10}$ are acceptable. So allow $\log_{10} \leftrightarrow \log$ and $\log_{10} \leftrightarrow \lg$

(a)

M1 Implies equation of line is of the form $\log_{10} y = \pm \frac{2}{3} \log_{10} x + 4$ oe

(Eg allow for M1 A0 $\log_{10} y = \pm 0.67 \log_{10} x + 4$) Condone "y" = $\pm \frac{2}{3} "x" + 4$ here

A1 $\log_{10} y = -\frac{2}{3} \log_{10} x + 4$ oe such as $3 \log_{10} y + 2 \log_{10} x = 12$

Allow unsimplified equations such as $\log_{10} y = -\frac{4}{6} \log_{10} x + 4$

This may be implied by the constants if the full equation is used in part (b).

So marks in (a) can be awarded from work in (b).

(b) **Main Method: Starting from $\log_{10} y = a \log_{10} x + b$ and working towards $y = px^q$**

M1 Uses one correct log law. Look for either

$$\log_{10} y = a \log_{10} x + b \rightarrow \log_{10} y = \log_{10} x^a + b$$

$$\log_{10} y = a \log_{10} x + b \rightarrow \log_{10} y = a \log_{10} x + \log_{10} 10^b$$

$$\log_{10} y = a \log_{10} x + b \rightarrow y = 10^{a \log_{10} x + b}$$

dM1 A full attempt to get y in terms of x or values of p and q

Cannot be awarded from easier equations, e.g. where $b = 0$

$$\text{Look for } \log_{10} y = a \log_{10} x + b \rightarrow y = x^a \times 10^b$$

A1 $y = 10\,000x^{-\frac{2}{3}}$ but condone $y = 10^4 x^{-\frac{2}{3}}$

This requires the equation and not just the values of p and q

It is acceptable to just write down the answer, but it must follow a correct part (a)

.....

Method Two: Starting from $y = px^q$ and working towards $\log_{10} y = a \log_{10} x + b$.

M1 Takes $\log_{10}$ of both sides and uses one correct log law

E.g. proceeds to $\log_{10} y = \log_{10} p + \log_{10} x^q$

dM1 Proceeds to $\log_{10} y = \log_{10} p + q \log_{10} x$ and finds p and q using $\log_{10} p = '4'$ and $q = '-\frac{2}{3}'$

A1 $y = 10\,000x^{-\frac{2}{3}}$ or $y = 10^4 x^{-\frac{2}{3}}$

Condone $\log_{10} \leftrightarrow \log$ throughout

This requires the equation and not just the values of p and q

.....

Alt (b) using the alternative equation in (a)

Method Three: Starting from $a \log_{10} y + b \log_{10} x = c$ and working towards $y = px^q$

M1 Uses correct log laws to combine the two terms on the lhs

Eg: $3 \log_{10} y + 2 \log_{10} x = 12 \Rightarrow \log_{10} y^3 x^2 = 12$

dM1 Undoes the logs and makes y the subject $\Rightarrow y^3 x^2 = 10^{12} \Rightarrow y^3 x^2 = 10^{12} \Rightarrow y = \sqrt[3]{\frac{10^{12}}{x^2}}$

A1 $y = 10\,000x^{-\frac{2}{3}}$ or $y = 10^4 x^{-\frac{2}{3}}$

.....

Method 4: Using the coordinates

M1: Finds either p or q using one of the coordinates.

Look for either

using $(\log_{10} x, \log_{10} y) = (0, 4) \Rightarrow x = 1, y = 10000$ and then substituting into $y = px^q \Rightarrow p = 10^4$

Condone slips here for the method mark

Or using $(\log_{10} x, \log_{10} y) = (6, 0) \Rightarrow x = 10^6, y = 1$ and the substituting into

$y = 10^4 \times x^q \Rightarrow 1 = 10^4 \times 10^{6q} \Rightarrow q = \dots$ Condone slips here for the method mark

dM1 Finds both p and q using both coordinates. (See above)

A1 $y = 10\,000x^{-\frac{2}{3}}$

.....

Partially correct answers without any work

If you see any partially correct answers, e.g. $y = 4x^{-\frac{2}{3}}$, without working, award SC 100.

General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

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Attempt to use the correct formula (with values for a, b and c).

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Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



(a)B1: $3x^2 + 7x - 20 = (3x - 5)(x + 4)$ seen or used

M1: Combines fractions with a correct common denominator and the order of terms in the numerator consistent with their common denominator.

A1*: Achieves the given answer with no errors seen but condone e.g. a missing trailing bracket as long as it is “recovered”.

Sufficient working should be shown but allow to go from e.g. $\frac{2(x-4)+8}{3x-5}$ to $\frac{2x}{3x-5}$

Note that candidates may take a longer route:

$$\frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} = \frac{(2x^2 - 32)(3x - 5) + 8(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{M1}$$

$$= \frac{6x^3 + 14x^2 - 40x}{(3x^2 + 7x - 20)(3x - 5)} = \frac{2x(3x^2 + 7x - 20)}{(3x - 5)(x + 4)(3x - 5)} \quad \text{B1}$$

$$= \frac{2x(3x - 5)(x + 4)}{(3x - 5)(x + 4)(3x - 5)} = \frac{2x}{3x - 5} \quad \text{A1*}$$

Or

$$\frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} = \frac{(2x^2 - 32)(3x - 5) + 8(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{M1}$$

$$= \frac{6x^3 + 14x^2 - 40x}{(3x^2 + 7x - 20)(3x - 5)} = \frac{2x(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{B1}$$

In this case the B1 is scored for obtaining a factor of $3x^2 + 7x - 20$ in the numerator and denominator

$$= \frac{2x}{3x - 5} \quad \text{A1*}$$

(b)M1: Attempts to differentiate using the product or quotient rule. Award for an expression of the form $\frac{\alpha(3x-5) - \beta x}{(3x-5)^2}$ or $\alpha(3x-5)^{-1} - \beta x(3x-5)^{-2}$, $\alpha, \beta > 0$ Condone attempts where $(3x-5)^2$ is expanded.Alternatively, attempts to write $\frac{2x}{3x-5}$ as $A + \frac{B}{3x-5}$ before differentiating using thechain rule to obtain $\frac{\pm k}{(3x-5)^2}$

A1: Correct derivative simplified or unsimplified.

A1cso*: Requires a correct simplified derivative $\frac{-10}{(3x-5)^2}$ and then statements that convey:

- $(3x-5)^2 > 0$ or e.g. denominator is positive (condone $(3x-5)^2 \geq 0$)
- so $f'(x) < 0$
- function is decreasing

- A1 $f^{-1}(x) = \frac{3x}{5-x}$ or $f^{-1}(x) = \frac{-3x}{x-5}$ Must be in terms of x . Condone $f^{-1} = \dots$ (or $f^{-1} = y = \dots$) but **do not allow** just $y = \dots$ or $f^{-1} : y = \dots$
- A1 Correct domain $0 < x < 5$

(c) Mark (i) and (ii) together

- M1 Attempts to use the quotient rule or product rule. Look for an expression of the form $\frac{A(x+3) - 5x}{(x+3)^2}$ or $5x(x+3)^{-1} \rightarrow B(x+3)^{-1} \pm 5x(x+3)^{-2}$ where A and B are non-zero constants.

- A1 $(f'(x) =) \frac{15}{(x+3)^2}$ or $15(x+3)^{-2}$ or $\frac{15}{x^2 + 6x + 9}$ Do not allow $\frac{5}{x+3} - \frac{5x}{(x+3)^2}$ for this mark.

- A1 They achieve $(f'(x) =) \frac{15}{(x+3)^2}$ and states

- (that f is an) **increasing function**
- **since $(x+3)^2$ is positive**

Alt methods in (a)

Taking out common factors

$$M1A1: \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x+4} = 5x \left\{ \frac{1+(x+3)}{(x+3)(x+4)} \right\} \text{ or } = \frac{5x}{(x+4)} \left\{ \frac{1+(x+3)}{(x+3)} \right\}$$

Methods where candidates "split up" fractions.

- M1 Attempts $\frac{5x}{x^2 + 7x + 12}$ OR $\frac{5}{x^2 + 7x + 12}$ by partial fractions

A1 $\frac{5x}{x+3} - \frac{5x}{x+4} + \frac{5x}{x+4}$

- A1* $\frac{5x}{x+3}$ You would expect to see $\frac{5x}{x+3} - \frac{5x}{x+4} + \frac{5x}{x+4}$ before proceeding to the given answer.

Alt method in (b)

- M1 Writes $y = \frac{5x}{x+3}$ as $y = 5 \pm \frac{k}{x+3}$ and then attempts to make x the subject.

A1 $f^{-1}(x) = \frac{15}{5-x} - 3$ or equivalent

A1 $0 < x < 5$

Alt method in (c)

- M1 Writes $y = \frac{5x}{x+3}$ as $y = 5 \pm \frac{k}{x+3}$ and attempts the chain rule to get $\pm \frac{C}{(x+3)^2}$

A1 $(f'(x) =) \frac{15}{(x+3)^2}$ or exact equivalent



(ii) Alt method 2

M1: Applies double angle formula and multiplies through to achieve a quadratic in $\tan x$ (condoning slips and not necessarily gathered)

$$\frac{\frac{2 \tan x}{1 - \tan^2 x} - \tan 70^\circ}{1 + \frac{2 \tan x}{1 - \tan^2 x} \tan 70^\circ} = -\frac{3}{8} \Rightarrow 16 \tan x - 8 \tan 70^\circ (1 - \tan^2 x) = -3(1 - \tan^2 x + 2 \tan x \tan 70^\circ)$$

$$(\Rightarrow t^2 (8 \tan 70^\circ - 3) + t (16 + 6 \tan 70^\circ) + 3 - 8 \tan 70^\circ = 0 \quad (18.98t^2 + 32.48t - 18.98 = 0))$$

A1: A correct value for $\tan x$, $\tan x = \text{awrt } 0.4604$ or $\text{awrt } -2.172$ dM1: Applies arctan to achieve at least one value for x .

A1: As main scheme.

Alt (i)	States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ (or equivalent identity)	B1
	$3 \operatorname{cosec} \theta = 8 \cos \theta \Rightarrow 9 \operatorname{cosec}^2 \theta = 64 \cos^2 \theta$ and uses both $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	M1
	and $\sin^2 \theta + \cos^2 \theta = 1$ to get equation in one trig term	
	$\Rightarrow 9 = 64 \sin^2 \theta (1 - \sin^2 \theta) \Rightarrow \sin^2 \theta = \frac{4 \pm \sqrt{7}}{8}$	A1
	$\Rightarrow \theta = \arcsin \left(\sqrt{\frac{4 \pm \sqrt{7}}{8}} \right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)

B1: States or uses $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ or in variations a similar correct reciprocal identity (e.g. $\cot \theta = \frac{\cos \theta}{\sin \theta}$) during the proof.

M1: Squares both sides and uses both reciprocal identity and Pythagorean identity to set up an equation in one trig term only.

A1: $\sin^2 \theta = \frac{4 \pm \sqrt{7}}{8}$ note it is the same roots for $\cos^2 \theta$ M1: Correct method of solving an equation of the form $\sin^2 \theta = k$ or $\cos^2 \theta = k$, $0 < k < 1$ to find at least one value for θ . May be solving in degrees for this mark ($24.30^\circ, 65.70^\circ$)A1: $\theta = \text{awrt } 0.424, \text{awrt } 1.15$ and no other values in the range. Under this method the extra solutions formed via squaring will need to be rejected to score this mark.

Question Number	Scheme	Marks
9. (a)	$\frac{1}{4}$	B1 (1)
(b)	$f'(x) = \frac{3}{\sqrt{x}} \ln(4x) + 6\sqrt{x} \times \frac{1}{x}$ Sets $\frac{3}{\sqrt{x}} \ln(4x) + 6\sqrt{x} \times \frac{1}{x} = 0 \Rightarrow \ln(4x) = -2$ $Q\left(\frac{1}{4e^2}, -\frac{6}{e}\right)$	M1 A1 dM1 A1 A1 (5)
(c)	Attempts $-2 \times y$ co-ordinate of Q Range $g(x)$ „ $\frac{12}{e}$	M1 A1ft (2)
		(8 marks)

(a)

B1: $\frac{1}{4}$ or exact equivalent. May be seen as the x coordinate in a coordinate pair.

(b)

M1: Attempts to differentiate using the product rule and achieves $f'(x) = \frac{a}{\sqrt{x}} \ln(4x) + b\sqrt{x} \times \frac{1}{x}$ A1: $f'(x) = \frac{3}{\sqrt{x}} \ln(4x) + 6\sqrt{x} \times \frac{1}{x}$. Note the $\frac{1}{x}$ may be seen as $\frac{4}{4x}$ dM1: Sets $f'(x) = 0$ and proceeds to $\ln(4x) = k$. The “=0” may be implied by the attempt to solve.Condoned attempts where multiplying through by $\sqrt{x}$ occurs before setting equal to 0.A1: $x = \frac{1}{4e^2}$ oe but allow awrt 0.034 following correct equation.A1: $Q\left(\frac{1}{4e^2}, -\frac{6}{e}\right)$ Both coordinates correct and simplified, but may be listed separately, $x = \dots, y = \dots$ and allow if y is found in (c). Accept negative powers of e , but must be simplified coefficients and simplified powers.

(c)

M1: Attempts $-2 \times$ their y co-ordinate of Q . Also allow following decimal answerA1ft: Range $g(x)$ „ $\frac{12}{e}$ following through on their negative y value for Q . Accept with y instead of $g(x)$ and with negative powers or expression in e following their (b), e.g. y „ $12e^{-1}$ Accept decimals following a decimal answer to (b). Do not accept $f(x)$.

Question Number	Scheme	Marks
9 (a)	$f(x) = (x^3 - 4x)e^{-\frac{1}{2}x} \Rightarrow f'(x) = (3x^2 - 4)e^{-\frac{1}{2}x} - \frac{1}{2}(x^3 - 4x)e^{-\frac{1}{2}x}$	M1 A1
		(2)
(b)	$f'(0) = -4$ so equation of normal is $y = -\frac{1}{4}x$ $y = \frac{1}{4}x$ Sets $\frac{1}{4}x = x(x^2 - 4)e^{-\frac{1}{2}x} \Rightarrow x^2 - 4 = \frac{1}{4}e^{\frac{1}{2}x}$ $\Rightarrow x^2 = \frac{16 + e^{\frac{1}{2}x}}{4} \Rightarrow x = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x}}$ *	M1 B1 M1 A1*
		(4)
(c)	(i) $x_2 = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x-2}} = -2.0229$ (ii) $(x =) -2.0226$	M1 A1 A1
		(3)
		(9 marks)

(a)

M1 Uses a valid method to differentiate. This could be:

(i) using the product rule on $f(x) = (x^3 - 4x)e^{-\frac{1}{2}x}$ so score for an expression of the

form $(f'(x) =) \pm A(x^3 - 4x)e^{-\frac{1}{2}x} \pm (Bx^2 \pm C)e^{-\frac{1}{2}x}$. ($A, B, C \neq 0$) Condone the squared missing on the Bx^2 term

(ii) using the quotient rule on $f(x) = \frac{x^3 - 4x}{e^{\frac{1}{2}x}}$ so score for an expression of the form
$$(f'(x) =) \frac{\pm e^{\frac{1}{2}x}(Bx^2 \pm C) - Ae^{\frac{1}{2}x}(x^3 - 4x)}{\left(e^{\frac{1}{2}x}\right)^2} \quad (A, B, C \neq 0) \text{ Condone the squared}$$

missing on the Bx^2 term or an attempt at (iii) $f'(x) = uvw' + uv'w + u'vw$ which could look like:

$$(x^2 - 4)e^{-\frac{1}{2}x} + x \frac{d\left((x^2 - 4)e^{-\frac{1}{2}x}\right)}{dx} = \pm \dots (x^2 - 4)e^{-\frac{1}{2}x} \pm x(\dots)e^{-\frac{1}{2}x} \pm \dots x(x^2 - 4)e^{-\frac{1}{2}x}$$

A1 Correct $f'(x)$ but may be unsimplified. Isw after a correct unsimplified expression

$$(3x^2 - 4)e^{-\frac{1}{2}x} - \frac{1}{2}(x^3 - 4x)e^{-\frac{1}{2}x} \text{ or } (x^2 - 4)e^{-\frac{1}{2}x} + x \frac{d\left((x^2 - 4)e^{-\frac{1}{2}x}\right)}{dx} = (x^2 - 4)e^{-\frac{1}{2}x} + x(2x)e^{-\frac{1}{2}x} - \frac{1}{2}x(x^2 - 4)e^{-\frac{1}{2}x}$$

(b) Note on EPEN it is M1A1M1A1 but we are marking this M1B1M1A1

M1 Full method to find the equation of the normal through O .

Look for an attempt at $f'(0)$ followed by the equation $y = -\frac{1}{f'(0)}x$

B1 Equation of normal is $y = \frac{1}{4}x$ (seen or implied) (which may follow an incorrect

$f'(x)$ from part (a))

M1 Equates their $y = \frac{1}{4}x$ (which must be a straight line through the origin) with

$f(x) = x(x^2 - 4)e^{-\frac{1}{2}x}$, divides through or factorises out the x term and attempts to make x^2 (or allow $4x^2$) the subject

A1* Full proof showing all steps. There is no requirement to justify the $-$ sign. Note that

A1* cannot be scored if A0 in part (a), unless they restart in (b).

(c)(i)

M1 Substitutes $x = -2$ into the iteration formula and finds x_2 . May also be implied by

-2.0228 or -2.0229

A1 awrt -2.0229

(ii)

A1 $(x =) -2.0226$ correct to 4 dp

Question Number	Scheme	Marks
7(a)	$x = 3 \tan \left(y - \frac{\pi}{6} \right) \Rightarrow \frac{dx}{dy} = 3 \sec^2 \left(y - \frac{\pi}{6} \right)$	B1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{3 \sec^2 \left(y - \frac{\pi}{6} \right)}$	M1
	$\frac{1}{3 \left(1 + \tan^2 \left(y - \frac{\pi}{6} \right) \right)} = \frac{1}{3 \left(1 + \left(\frac{x}{3} \right)^2 \right)}$	dM1
	$= \frac{3}{x^2 + 9}$	A1
		(4)
(b)	$y = \frac{\pi}{3} \Rightarrow x = 3 \tan \frac{\pi}{6} = \sqrt{3}$	B1
	$x = \sqrt{3} \Rightarrow \frac{dy}{dx} = \frac{3}{(\sqrt{3})^2 + 9}$ or $y = \frac{\pi}{3} \Rightarrow \frac{dy}{dx} = \frac{1}{3 \sec^2 \left(\frac{\pi}{3} - \frac{\pi}{6} \right)} = \dots$	M1
	$y - \frac{\pi}{3} = \frac{1}{4} (x - \sqrt{3})$	dM1
	$y = 0 \Rightarrow 0 - \frac{\pi}{3} = \frac{1}{4} (x - \sqrt{3}) \Rightarrow x = \dots$	ddM1
	$x = \sqrt{3} - \frac{4\pi}{3}$	A1
		(5)
		Total 9

(a)

B1: Correct derivative, including correct lhs. Condone $\frac{dx}{dy} = k \sec^2 \left(y - \frac{\pi}{6} \right)$ where k is a constant.

M1: Either (1) attempts to apply the reciprocal rule $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

Don't be too concerned by the position of the constant, it is the function and variable that is important.

Or (2) attempts to apply $\sec^2 \left(y - \frac{\pi}{6} \right) = 1 + \tan^2 \left(y - \frac{\pi}{6} \right)$ with $\tan \left(y - \frac{\pi}{6} \right)$ being replaced by $\frac{x}{3}$ to get $\frac{dx}{dy}$ in terms of x

dM1: Attempts both (1) and (2) to obtain $\frac{dy}{dx}$ in terms of x

A1: Correct answer

(b)

B1: Correct value for x . Condone awrt 1.73. This can be awarded from the sight of this value in an equation

M1: Uses a correct method to find the value of $\frac{dy}{dx}$ which may be a decimal.

Note that this can be scored either from answer to part (a) e.g. $x = \sqrt{3} \Rightarrow \frac{dy}{dx} = \frac{3}{(\sqrt{3})^2 + 9}$

Or via their $\frac{dx}{dy} = 3\sec^2\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$ followed by an attempt at the reciprocal

dM1: Correct straight line method for the tangent at $\left(\sqrt{3}, \frac{\pi}{3}\right)$ using their correctly found m .

It is dependent upon having found the gradient and x value using a correct method.

If they use the form $y = mx + c$ they must proceed as far as $c = \dots$

ddM1: Uses $y = 0$ to find x . It is dependent upon having scored the previous mark

A1: Correct value or exact equivalent for example $\frac{3\sqrt{3} - 4\pi}{3}$

Alt (a) via arctan

B1: $x = 3 \tan\left(y - \frac{\pi}{6}\right) \Rightarrow y = \frac{\pi}{6} + \arctan\left(\frac{x}{3}\right) \Rightarrow \frac{dy}{dx} = 0 + \frac{1}{1 + \left(\frac{x}{3}\right)^2} \times \dots$ where ... could be any values even 1

M1: $\frac{dy}{dx} = 0 + \frac{1}{1 + \left(\frac{x}{3}\right)^2} \times \dots$ where ... could be any value even 1

dM1: $\frac{dy}{dx} = 0 + \frac{1}{1 + \left(\frac{x}{3}\right)^2} \times \frac{1}{3}$

A1*: $\frac{dy}{dx} = \frac{3}{x^2 + 9}$

Alt (a) via compound angle identity they could pick up the first two marks

$$x = \frac{3\left(\tan y - \tan \frac{\pi}{6}\right)}{1 + \tan \frac{\pi}{6} \tan y} \Rightarrow \frac{dx}{dy} = \frac{\left(1 + \tan \frac{\pi}{6} \tan y\right) \times 3\sec^2 y - 3\left(\tan y - \tan \frac{\pi}{6}\right) \times \tan \frac{\pi}{6} \sec^2 y}{\left(1 + \tan \frac{\pi}{6} \tan y\right)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(1 + \tan \frac{\pi}{6} \tan y\right)^2}{\left(1 + \tan \frac{\pi}{6} \tan y\right) \times 3\sec^2 y - 3\left(\tan y - \tan \frac{\pi}{6}\right) \times \tan \frac{\pi}{6} \sec^2 y}$$

(b)	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = \frac{(x-3) \times 7 - 7x \times 1}{(x-3)^2}$	M1
	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	A1
		(2)
(b) ALT	$f(x) = \frac{7x}{(x-3)} = 7 + \frac{21}{x-3} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	M1A1
		Total 5

(b)

M1: Attempts to differentiate using the quotient rule or product rule.For the quotient rule it requires achieving $f'(x) = \frac{\alpha(x-3) - \beta x}{(x-3)^2}$ $\alpha, \beta > 0$ If they use a substitution then similar conditions apply e.g. $u = x-3 \Rightarrow f'(u) = \frac{\alpha u - \beta(u+3)}{u^2}$ $\alpha, \beta > 0$ For the product rule it requires achieving $f'(x) = A(x-3)^{-1} - Bx(x-3)^{-2}$ $A, B > 0$ If they use a substitution then similar conditions apply e.g. $u = x-3 \Rightarrow f'(u) = Au^{-1} - B(u+3)u^{-2}$ $A, B > 0$ **A1:** Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$ Allow $-\frac{21}{x^2 - 6x + 9}$ or e.g. $-21(x^2 - 6x + 9)^{-1}$

Isw once a correct simplified answer is seen.

Alternative:**M1:** Expresses $\frac{7x}{(x-3)}$ as $\alpha + \frac{\beta}{(x-3)}$ and differentiates to obtain $\frac{\dots}{(x-3)^2}$ or e.g. $\alpha + \frac{\beta}{u}$ where $u = x-3$ and differentiates to obtain $\frac{\dots}{u^2}$ **A1:** Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$ Allow $-\frac{21}{x^2 - 6x + 9}$ or e.g. $-21(x^2 - 6x + 9)^{-1}$

Isw once a correct simplified answer is seen.

Question Number	Scheme	Marks
5(i)	$\cos 6x = 1 - 2\sin^2 3x \Rightarrow \sin^2 3x = \frac{1}{2} - \frac{1}{2}\cos 6x$ $\int \sin^2 3x \, dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6x \right) dx$	M1
	$= \frac{1}{2}x - \frac{1}{12}\sin 6x (+c)$	A1
		(2)
(ii)	$\int x(x^2 + 4)^{\frac{3}{2}} \, dx = \frac{1}{5}(x^2 + 4)^{\frac{5}{2}} (+c)$	M1A1
		(2)
		Total 4

Notes:

(i)

M1: Uses $\cos 6x = \pm 1 \pm 2\sin^2 3x$ in an attempt to obtain an integrable form.

 An alternative would be to change $\int \sin^2 3x \, dx$ to $\int (1 - \cos^2 3x) \, dx$ and then use $\cos 6x = \pm 2\cos^2 3x \pm 1$ in an attempt to obtain an integrable form.

 May be implied by an answer of $\alpha x + \beta \sin 6x (+c)$
A1: Correct integration. Condone the omission of “+ c”

 Condone spurious notation e.g. $\int \sin^2 3x \, dx = \frac{1}{2}x - \frac{1}{12}\sin 6x \, dx$

(ii)

M1: Obtains $\int x(x^2 + 4)^{\frac{3}{2}} \, dx = k(x^2 + 4)^{\frac{5}{2}} (+c)$

 May be implied by e.g. $\int x(x^2 + 4)^{\frac{3}{2}} \, dx = \frac{2}{5}x(x^2 + 4)^{\frac{5}{2}} \times \frac{1}{2x} (+c)$

Note this may also be implied by an attempt at substitution e.g.

$$u = x^2 + 4 \Rightarrow \int x(x^2 + 4)^{\frac{3}{2}} \, dx = \int x(x^2 + 4)^{\frac{3}{2}} \frac{dx}{du} du = \int xu^{\frac{3}{2}} \frac{1}{2x} du = \frac{1}{5}u^{\frac{5}{2}} + c$$

 Score M1 for a complete method leading to $ku^{\frac{5}{2}} (+c)$
A1: Correct integration. Condone the omission of “+ c”

 Allow equivalent correct expressions but do not allow $\frac{x(x^2 + 4)^{\frac{5}{2}}}{5x}$ without the x's cancelled.

 Condone spurious notation e.g. $\int x(x^2 + 4)^{\frac{3}{2}} \, dx = \frac{1}{5}(x^2 + 4)^{\frac{5}{2}} \, dx$


Question Number	Scheme	Marks
6(a)	$\theta = 75, t = 0 \Rightarrow 75 = 21 + A \Rightarrow A = \dots$	M1
	$A = 54$	A1
		(2)
(b)	$\theta = 21 + 54e^{-kt} \Rightarrow 25 = 21 + 54e^{-5k}$	M1
	$54e^{-5k} = 4 \Rightarrow e^{-5k} = \frac{2}{27} \Rightarrow -5k = \ln \frac{2}{27} \Rightarrow k = \dots$	M1
	$k = -\frac{1}{5} \ln \frac{2}{27} = 0.521$	A1
		(3)

Notes:

(a)

M1: Substitutes $\theta = 75$ and $t = 0$ into the given equation, uses $e^0 = 1$ and proceeds to find a value for A. Look for $75 = 21 + A \times 1 \Rightarrow A = \dots$

A1: For $A = 54$

Sight of 54 with no incorrect working scores both marks.

(b)

M1: Uses $\theta = 25$ and $t = 5$ with their A in $\theta = 21 + Ae^{\pm kt}$ so e.g. $25 = 21 + 54e^{\pm 5k}$ is all that is required.

M1: Proceeds from an equation of the form $e^{\pm 5k} = \alpha$ where $\alpha > 0$ to $\pm 5k = \ln \alpha \Rightarrow k = \dots$ to obtain a value for k. Condone slips but the log work must be correct. Must be using $\ln$ and not e.g. $\log_{10}$ which for reference gives $k = 0.226$

May be implied by the correct exact value for k following through their A.

A1: For awrt 0.521 from correct and sufficient work.

Allow the exact value e.g. $-\frac{1}{5} \ln \frac{2}{27}$ and apply isw if subsequently rounded incorrectly.

May be seen embedded e.g. $\theta = 21 + 54e^{-0.521t}$

Minimum for full marks in (b) is: $25 = 21 + 54e^{-5k} \Rightarrow k = -\frac{1}{5} \ln \frac{4}{54}$

$25 = 21 + 54e^{-5k} \Rightarrow e^{-5k} = \frac{4}{54} \Rightarrow k = 0.521$ scores M1dM0A0

$25 = 21 + 54e^{-5k} \Rightarrow k = 0.521$ scores M1dM0A0

Note that some candidates are using $\theta = 50$ and $t = 5$ leading to $k = 0.124$ and this scores a maximum of **M0M1A0** in (b)



(c)	$\theta = 21 + 54e^{-0.521t} \Rightarrow \frac{d\theta}{dt} = -28.1...e^{-0.521t}$	M1
	$-28.1...e^{-0.521T} = -9 \Rightarrow e^{-0.521T} = \frac{9}{28.1...} \Rightarrow -0.521T = \ln\left(\frac{9}{28.1...}\right)$ $\Rightarrow T = \ln\left(\frac{9}{28.1...}\right) \div -0.521$	dM1
	$= 2.19$	A1
		(3)

(c)

M1: Differentiates to obtain $\frac{d\theta}{dt} = \alpha e^{-kt}$ where $\alpha \neq A$

dM1: Depends on the previous mark.

Sets their $\frac{d\theta}{dt} = -9$ or sets $-\left(\text{their } \frac{d\theta}{dt}\right) = 9$ and proceeds to obtain a value for t (or T).

You do not need to be concerned about the processing as long as they reach a value.

For this mark they must have obtained $\frac{d\theta}{dt} = \alpha e^{-kt}$ with $\alpha < 0$ and $k > 0$

Do **not** condone work where candidates ignore/cross out signs to make their equation solvable.

A1: For awrt 2.19

Just look for this value so ignore any units, correct or incorrect, and apply isw once awrt 2.19 is seen.

Must follow M1dM1 previously.

Note candidates who used $\theta = 50$ and $t = 5$ leading to $k = 0.124$ in (b) who proceed correctly in (c) will obtain $T = -2.38$ and can score a maximum of **M1dM1A0** in (c)

Note correct answer with no working scores no marks in (c).



Question Number	Scheme	Marks
7(a)	$y = e^{-x^2} \sin 3x \Rightarrow \frac{dy}{dx} = 3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x$	M1A1
	$3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x = 0 \Rightarrow 3 \cos 3x - 2x \sin 3x = 0$ $\Rightarrow 3 \cos 3x = 2x \sin 3x \Rightarrow \tan 3x = \frac{3}{2x}$	dM1
	$x = \frac{1}{3} \arctan\left(\frac{3}{2x}\right) *$	A1*
		(4)
(b)(i)	$x_1 = 0.4 \Rightarrow x_2 = \frac{1}{3} \arctan\left(\frac{3}{2 \times 0.4}\right)$	M1
	$(x_2 =) 0.4367$	A1
(ii)	$(x_4 =) 0.4307$	A1
		(3)
Notes: (a) M1: Differentiates using the product rule to obtain $\alpha e^{-x^2} \cos 3x + \beta x e^{-x^2} \sin 3x$, $\alpha, \beta \neq 0$ A1: Correct derivative in any form dM1: Depends on the first method mark. Sets their $\frac{dy}{dx} = 0$ (which may be implied), cancels e^{-x^2} and rearranges to $\pm \tan 3x = \dots$ A1*: Correct proof with no errors seen. Allow e.g. $\tan^{-1}$ or e.g. arctan for arctan and allow e.g. $x = \frac{1}{3} \arctan \frac{3}{2x}$ Ignore any reference or attempts to solve $e^{-x^2} = 0$ (b) M1: Attempts to substitute $x = 0.4$ into the formula. Allow if seen embedded in the formula or may be implied by $x_2 = \text{awrt } 0.44$ A1: Awrt 0.4367 A1: 0.4307 cao not awrt 0.4307 Both values correct as defined with no working scores M1A1A1		



(c)	e.g. $f(x) = x - \frac{1}{3} \arctan\left(\frac{3}{2x}\right)$ $f(0.4305) = 0.4305 - \frac{1}{3} \arctan\left(\frac{3}{2 \times 0.4305}\right) (= 6.38 \times 10^{-5})$ $f(0.4295) = 0.4295 - \frac{1}{3} \arctan\left(\frac{3}{2 \times 0.4295}\right) (= -1.141... \times 10^{-3})$ or e.g. $f(x) = 3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x$ $f(0.4305) = 3e^{-(0.4305)^2} \cos 3(0.4305) - 2(0.4305)e^{-(0.4305)^2} \sin 3(0.4305) (= -4.968... \times 10^{-4})$ $f(0.4295) = 3e^{-(0.4295)^2} \cos 3(0.4295) - 2(0.4295)e^{-(0.4295)^2} \sin 3(0.4295) (= 8.885... \times 10^{-3})$	M1
	Sign change therefore x is 0.430 to 3dp	A1
		(2)
		Total 9

(c)

M1: Chooses a suitable function and attempts its value at both 0.4305 and 0.4295

For the attempt we need to see embedded values as in scheme or one value correct to 1sf for their suitable function.

The function they are using must be seen or implied by embedded values.

Allowable functions are:

- $x - \frac{1}{3} \arctan\left(\frac{3}{2x}\right)$ or multiples of this
- $3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x$ or multiples of this following through on their $\frac{dy}{dx}$
- $3 \cos 3x - 2x \sin 3x$ or multiples of this following through on their $\frac{dy}{dx}$

Unlikely, but it is also acceptable to pick a tighter interval e.g. 0.4305 and 0.4301

A1: Requires correct differentiation (if $\frac{dy}{dx}$ is used)

- both values correct (rounded or truncated to 1sf)
- a valid reason that includes a reference to the sign change e.g. $< 0, > 0$
- a minimal conclusion which could be $\checkmark$, QED, root

Note we condone the omission of a reference to continuity.

Attempts to use repeated iteration score no marks.

Question Number	Scheme	Marks
9(a)	$y = \sqrt{3+4e^{x^2}} = (3+4e^{x^2})^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}(3+4e^{x^2})^{-\frac{1}{2}} \times 8xe^{x^2}$	M1
	$= 4xe^{x^2} (3+4e^{x^2})^{-\frac{1}{2}}$	A1
		(2)
(b)	$\frac{(3+4e^{x^2})^{\frac{1}{2}}}{x} = 4xe^{x^2} (3+4e^{x^2})^{-\frac{1}{2}}$	M1
	$\frac{(3+4e^{x^2})}{x} = 4xe^{x^2}$	dM1
	$4x^2e^{x^2} - 4e^{x^2} - 3 = 0^*$	A1*
		(3)
(c)	$f(x) = 4x^2e^{x^2} - 4e^{x^2} - 3 \Rightarrow f(1) = -3 \text{ AND } f(2) = 652 \dots$	M1
	Change of sign and $f(x)$ is continuous hence root in (1, 2)	A1
		(2)
(d)	$4x^2e^{x^2} - 4e^{x^2} - 3 = 0 \Rightarrow x^2 = \frac{4e^{x^2} + 3}{4e^{x^2}} = \frac{4+3e^{-x^2}}{4} \Rightarrow x = \frac{1}{2}\sqrt{4+3e^{-x^2}}^*$	B1*
		(1)
(e)(i)	$x_1 = 1 \Rightarrow x_2 = \frac{1}{2}\sqrt{4+3e^{-1}}$	M1
	$x_3 = 1.0997$	A1
(ii)	$\alpha = 1.1051$	A1
		(3)
		Total 11

(a)

M1: Differentiates using the chain rule to obtain $kxe^{x^2} (3+4e^{x^2})^{-\frac{1}{2}}$ OR $ke^{x^2} (3+4e^{x^2})^{-\frac{1}{2}}$

If they are using implicit differentiation accept $\frac{kxe^{x^2}}{y}$ or $\frac{ke^{x^2}}{y}$

A1: Correct derivative in simplest form. Also award for $\frac{dy}{dx} = \frac{4xe^{x^2}}{y}$ o.e. simplified answer

(b)

M1: Sets their $\frac{dy}{dx} = \frac{(3+4e^{x^2})^{\frac{1}{2}}}{x}$ or equivalent such as use of $y = mx$ with $m =$ their $\frac{dy}{dx}$ and $y = (3+4e^{x^2})^{\frac{1}{2}}$.

Allow with another variable $x \rightarrow \alpha$

dM1: Multiplies up to eliminate the square root. Allow a consistent use of another variable $x \rightarrow \alpha$

Dependent upon the previous M1 and a suitable $\frac{dy}{dx}$, that is one of the form that scored an M1 in part (a)



A1*: Correct proof

An alternative method may be seen using the equation of the line with gradient $\frac{dy}{dx}$ and point $\left(\alpha, \sqrt{3+4e^{\alpha^2}}\right)$

M1: For $y - \sqrt{3+4e^{\alpha^2}} = \left.\frac{dy}{dx}\right|_{x=\alpha} (x - \alpha)$ and sets $(x, y) = (0, 0)$

dM1: Proceeds as the main method with the same condition on the gradient

A1*: Correct proof. Condone with variable α

(c)

M1: Attempts both $f(1)$ and $f(2)$ (or tighter) and obtains at least one value correct to 1sf rounded or truncated.

Condone $f(2) = 12e^4 - 3$ If f or y is not stated assume that $f(x) = 4x^2e^{x^2} - 4e^{x^2} - 3$

A1: Requires

- both values either correct or rounded/ truncated with accuracy to at least 1 sf
- a reference to a sign change e.g. $f(1) = -3 < 0$, $f(2) = 652 > 0$ or $f(1) \times f(2) < 0$
- a mention of continuity
- a minimal conclusion, e.g. hence root

(d)

B1*: Correct proof.

Look for the following evidence

- the line $4x^2e^{x^2} - 4e^{x^2} - 3 = 0$ but may be implied by $4x^2e^{x^2} = 4e^{x^2} + 3$ o.e.

- the line $x^2 = \frac{4e^{x^2} + 3}{4e^{x^2}}$ o.e. $x^2 = 1 + \frac{3}{4e^{x^2}}$ or $4x^2 = \frac{4e^{x^2} + 3}{e^{x^2}}$ o.e. $4x^2 = 4 + \frac{3}{e^{x^2}}$
- correct square root work leading to the given answer

(e)

M1: Attempts to use the iteration formula.

This may be implied by awrt 1.13 for x_2 or awrt 1.10 for x_3 or sight of a correctly embedded value.

It cannot be awarded for just the value of α

A1: awrt 1.0997

A1: 1.1051 cao following the award M1

Question Number	Scheme	Marks
7	$x = 6 \sin^2 2y \Rightarrow \frac{dx}{dy} = 24 \sin 2y \cos 2y$	M1 A1
	Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$	M1
	Attempts to use both $\sin 2y = \sqrt{\frac{x}{6}}$ and $\cos 2y = \sqrt{1 - \sin^2 2y} = \sqrt{1 - \frac{x}{6}}$	M1
	$\frac{dy}{dx} = \frac{1}{24 \sin 2y \cos 2y} = \frac{1}{24 \times \sqrt{\frac{x}{6}} \times \sqrt{1 - \frac{x}{6}}} = \frac{1}{4\sqrt{6x - x^2}}$	A1
		(5)
Way 2	$x = 6 \sin^2 2y = 3 - 3 \cos 4y \Rightarrow \frac{dx}{dy} = 12 \sin 4y$	M1A1
	Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$	M1
	Attempts to use $\sin 4y = \frac{\sqrt{6x - x^2}}{3}$	M1
	$\frac{dy}{dx} = \frac{1}{12 \sin 4y} = \frac{1}{12 \times \frac{\sqrt{6x - x^2}}{3}} = \frac{1}{4\sqrt{6x - x^2}}$	A1
Way 3	$x = 6 \sin^2 2y = 24 \sin^2 y \cos^2 y \Rightarrow \frac{dx}{dy} = 48 \sin y \cos^3 y - 48 \sin^3 y \cos y$	M1A1
	Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$	M1
	$\frac{dy}{dx} = \frac{1}{48 \sin y \cos^3 y - 48 \sin^3 y \cos y} = \frac{1}{48 \sin y \cos y (\cos^2 y - \sin^2 y)}$ $\frac{dy}{dx} = \frac{1}{24 \sin 2y \cos 2y} = \frac{1}{24 \times \sqrt{\frac{x}{6}} \times \sqrt{1 - \frac{x}{6}}}$	M1
	$= \frac{1}{4\sqrt{6x - x^2}}$	A1
		(5 marks)



In general, apply the following marking guidance for this question:

M1: Attempts to differentiate to obtain $\frac{dx}{dy}$ in a correct form

A1: Correct derivative

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

M1: Change fully to a function of x .

A1: All correct

E.g.

M1: Attempts to use the chain rule on the rhs to achieve $k \sin 2y \cos 2y$

A1: Fully correct derivative $\frac{dx}{dy} = 24 \sin 2y \cos 2y$

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

Condone a slip on the coefficient. E.g. $\frac{dx}{dy} = 24 \sin 2y \cos 2y \Rightarrow \frac{dy}{dx} = \frac{24}{\sin 2y \cos 2y}$

Do not condone slips/errors on the variable. E.g. $\frac{dx}{dy} = 24 \sin 2y \cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{24 \sin 2x \cos 2x}$

M1: Attempts to change both $\sin 2y$ and $\cos 2y$ to functions in x .

Expect to see $\sin 2y = p\sqrt{x}$ and $\cos 2y = \sqrt{1 - qx}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x - x^2}}$

Way 2

M1: Differentiates to obtain $k \sin 4y$

A1: Fully correct derivative $\frac{dx}{dy} = 12 \sin 4y$

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

Condone a slip on the coefficient. E.g. $\frac{dx}{dy} = 12 \sin 4y \Rightarrow \frac{dy}{dx} = \frac{12}{\sin 4y}$

M1: Attempts to change both $\sin 4y$ to a function in x .

Expect to see $\sin 4y = p\sqrt{6x - x^2}$ or equivalent e.g. $\sin 4y = p\sqrt{9 - (9 - 6x + x^2)}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x - x^2}}$



Way 3

M1: Differentiates to obtain $A \sin y \cos^3 y + B \sin^3 y \cos y$

A1: Fully correct derivative $\frac{dx}{dy} = 48 \sin y \cos^3 y - 48 \sin^3 y \cos y$

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

M1: Attempts to change both $\sin 2y$ and $\cos 2y$ to functions in x .

Expect to see $\sin 2y = p\sqrt{x}$ and $\cos 2y = \sqrt{1-qx}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x-x^2}}$

A possible alternative:

$$x = 6 \sin^2 2y \Rightarrow \sin 2y = \sqrt{\frac{x}{6}} \Rightarrow 2y = \sin^{-1} \sqrt{\frac{x}{6}} \Rightarrow y = \frac{1}{2} \sin^{-1} \sqrt{\frac{x}{6}}$$

M1: For a correct attempt to make y or $2y$ the subject

A1: Correct rearrangement

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{1-\frac{x}{6}}} \times \frac{1}{12} \left(\frac{x}{6} \right)^{-\frac{1}{2}}$$

M1: For $\frac{A}{\sqrt{1-\frac{x}{6}}} \times \dots$ M1: For $\dots \times B \left(\frac{x}{6} \right)^{-\frac{1}{2}}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x-x^2}}$



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
10 (a)	$x = ye^{2y} \Rightarrow \frac{dx}{dy} = e^{2y} + 2ye^{2y}$ $\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 2x$ $\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{x}{y} + 2x}$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x + 2xy} = \frac{y}{x(1 + 2y)}^*$	M1, A1 dM1 A1* (4)
(b)	Deduces $y = -\frac{1}{2}$ Substitutes $y = -\frac{1}{2} \Rightarrow x = -\frac{1}{2e}$ Range for k $-\frac{1}{2e} < k < 0$	B1 M1, A1 (3) (7 marks)

(a)

 M1: For attempting to differentiate with respect to y .

 Uses the product rule on $ye^{2y} \Rightarrow e^{2y} + \dots ye^{2y}$. The left hand side may be missing/incorrect

 A1: Correct differentiation E.g. $\frac{dx}{dy} = e^{2y} + 2ye^{2y}$

 dM1: Full method to get $\frac{dy}{dx}$ in terms of just x and y .

This requires, in any order

- a correct attempt to invert, e.g. not inverting each term in a sum of terms
- e^{2y} being fully replaced. You should see e^{2y} being replaced by $\frac{x}{y}$ or equivalent and ye^{2y} being replaced by x

A1*: Correct proof. All relevant steps should be shown and there should be no errors.

If you feel that it hasn't been fully shown then please award M1 A1 dM1 A0

 (b) **Now being marked B1 M1 A1. On open it is set up M1 A1 A1**

 B1: For deducing left hand end occurs when $y = -\frac{1}{2}$.

 M1: For attempting to find x when $y = -\frac{1}{2} \Rightarrow x = \dots$. This may be implied by $k = \text{awrt } -0.183$ or -0.184

 A1: $-\frac{1}{2e} < k < 0$ or **exact** equivalent such as e.g. $-\frac{1}{2}e^{-1} < k < 0$, $k > -\frac{1}{2}e^{-1}$ **and** $k < 0$,

$$\left\{k : k > -\frac{1}{2e}\right\} \cap \left\{k : k < 0\right\}$$

This must be correct so the candidate cannot have two separate inequalities with an "or" between

 This must be the range for k not x

.....

Alt via ln's

$$x = y e^{2y} \Rightarrow \ln x = \ln y + 2y \quad \text{o.e.}$$

M1: Attempts to differentiate wrt x . Consider just rhs $\ln y + 2y \rightarrow \frac{1}{y} \frac{dy}{dx} + \dots \frac{dy}{dx}$

Alternatively attempts to differentiate wrt y . Consider both sides $\dots \frac{dx}{dy} = \frac{1}{y} + 2$

A1: Correct differentiation $\frac{1}{x} = \frac{1}{y} \frac{dy}{dx} + 2 \frac{dy}{dx}$ or $\frac{1}{x} \frac{dx}{dy} = \frac{1}{y} + 2$

.....

Alt via differentiating wrt x

$$x = y e^{2y} \Rightarrow 1 = e^{2y} \frac{dy}{dx} + y \times 2e^{2y} \frac{dy}{dx}$$

$$1 = \frac{x}{y} \frac{dy}{dx} + 2x \frac{dy}{dx}$$

$$1 = \left(\frac{x + 2xy}{y} \right) \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x(1 + 2y)} \quad *$$

M1: For attempting to differentiate wrt x .

Uses the product rule on $y e^{2y} \Rightarrow e^{2y} \frac{dy}{dx} + \dots y \frac{dy}{dx} e^{2y}$. The left hand side may be missing/incorrect

Uses the quotient rule on $y = \frac{x}{e^{2y}} \Rightarrow \frac{dy}{dx} = \frac{e^{2y} - 2xe^{2y} \times \frac{dy}{dx}}{e^{4y}}$ condoning slips on the coefficient

A1: Correct differentiation including the lhs

dM1: Full method to get $\frac{dy}{dx}$ in terms of x and y .

Question Number	Scheme	Marks
10 (a)	$x = \frac{2y^2 + 6}{3y - 3} \Rightarrow \left(\frac{dx}{dy} = \right) \frac{4y(3y - 3) - 3(2y^2 + 6)}{(3y - 3)^2}$ $\frac{dx}{dy} = \frac{6y^2 - 12y - 18}{9(y - 1)^2} = \frac{2y^2 - 4y - 6}{3(y - 1)^2} \text{ o.e.}$	M1 A1 dM1, A1 (4)
(b)	P and Q are where $\frac{dx}{dy} = 0$ or where $2y^2 - 4y - 6 = 0$ Solves $2y^2 - 4y - 6 = 0 \Rightarrow 2(y - 3)(y + 1) = 0 \Rightarrow y = 3, -1$ Subs $y = -1$ and 3 in $x = \frac{2y^2 + 6}{3y - 3} \Rightarrow x = ..$ Achieves $x = -\frac{4}{3}$ and $x = 4$	B1 M1 dM1 A1cso (4) 8 marks
Notes		
(a)		
M1	Attempts the quotient rule. Condone slips on the coefficients - look for $\frac{Ay(3y - 3) - B(2y^2 + 6)}{(3y - 3)^2}$ $A, B > 0$. Allow a product rule attempt: $x = (2y^2 + 6)(3y - 3)^{-1} \Rightarrow \left(\frac{dx}{dy} = \right) Ay(3y - 3)^{-1} + (2y^2 + 6) \times -B(3y - 3)^{-2}$	
A1	Correct differentiation which may be unsimplified. Allow if the $\frac{dx}{dy}$ is missing or called $\frac{dy}{dx}$ for this mark. By product rule $4y(3y - 3)^{-1} + (2y^2 + 6) \times -3(3y - 3)^{-2}$ Condone missing brackets if recovered.	
dM1	Requires an attempt to get a single fraction with some attempt to simplify. For the quotient rule look for a simplification of the numerator with like terms collected giving a 3TQ. Attempts via the product rule will require a correct method to put as a single fraction.	
A1	$\left(\frac{dx}{dy} = \right) \frac{2y^2 - 4y - 6}{3y^2 - 6y + 3}$ or exact simplified equivalent such as $\frac{2(y - 3)(y + 1)}{3(y - 1)^2}$ isw after a correct simplified answer. Common factor 3 must have been cancelled. Must be seen in part (a). A0 if called $\frac{dy}{dx}$ but allow A1 if LHS is not stated.	
Attempts at $\frac{dy}{dx}$ can score the first 3 marks if correct. Allow use of x in place of y for the Ms.		
(b)		
B1	Indicates P and Q are where $\frac{dx}{dy} = 0$ or where their $2y^2 - 4y - 6 = 0$ (which may be the denominator of $\frac{dy}{dx}$ if they found this instead).	
M1	Solves their 3TQ from an attempt at $\frac{dx}{dy} = 0$ (or denominator of their $\frac{dy}{dx} = 0$), usual rules.	

dM1 Substitutes both their solutions to $2y^2 - 4y - 6 = 0$ into $x = \frac{2y^2 + 6}{3y - 3}$. Condone slips if the attempt is clear. At least one should be correct if no method is shown. A1cso Achieves $x = -\frac{4}{3}$ and $x = 4$ only. Must be equations not just values but isw after correct equations seen as long as no contrary work is shown (such as giving horizontal lines). Accept equivalents. Must have come from a correct derivative - though allow from an isw form if a numerical factor was lost in the numerator. Must be exact. Answers from no working score 0/4 as the question instructs use of part (a), so must see the attempt at setting $\frac{dx}{dy} = 0$		
Alt (a)	$x = \frac{2y^2 + 6}{3y - 3} \Rightarrow 3xy - 3x = 2y^2 + 6 \Rightarrow 3x + 3y \frac{dx}{dy} - 3 \frac{dx}{dy} = 4y$ $\frac{dx}{dy} = \frac{4y - 3x}{3(y - 1)}$	M1 A1 dM1, A1 (4)
(b) First 2 marks.	States that P and Q are where $\frac{dx}{dy} = 0$ or where $4y - 3x = 0$ $\Rightarrow \frac{4}{3}y = \frac{2y^2 + 6}{3y - 3} \Rightarrow 4y^2 - 4y = 2y^2 + 6 \Rightarrow \text{as main scheme}$	B1 M1
Alt II (a)	$x = \frac{2y^2 + 6}{3y - 3} = \frac{2y}{3} + \frac{2}{3} + \frac{8}{3(y - 1)} \Rightarrow \frac{dx}{dy} = \frac{2}{3} - \frac{8}{3(y - 1)^2}$ $\frac{dx}{dy} = \frac{2(y - 1)^2 - 8}{3(y - 1)^2} = \frac{2y^2 - 4y - 6}{3(y - 1)^2} \text{ oe}$	M1 A1 dM1, A1 (4)
Notes		
(a)		
M1	Attempts long division or other method to achieve $Ay + B + \frac{C}{3y - 3}$ oe and differentiates.	
A1	Correct differentiation.	
dM1	Attempts to get a single fraction and simplifies numerator to 3TQ or uses difference of squares to factorise.	
A1	Correct answer.	

General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1 (a)	$(2, -4)$	B1, B1 (2)
(b)	$(8, 6)$	B1, B1 (2)
(c)	$(-2, 6)$	B1 (1)
		(5 marks)

Notes:

1. Candidates may build up their answer in stages.

E.g. $(6, -2) \rightarrow (8, -2) \rightarrow (8, 6)$ In such cases you mark the final coordinates only

2. Condone the omission of brackets. So, allow 8, 6 for $(8, 6)$

3. Allow for example, $x = 2$ for $(2, \dots)$ and $x = 2, y = -4$ for $(2, -4)$

4. There are no marks for transposed coordinates i.e. $(6, 8)$ for $(8, 6)$

(a)

B1: For either $(2, \dots)$ or $(\dots, -4)$

B1: $(2, -4)$

(b)

B1: For either $(8, \dots)$ or $(\dots, 6)$

B1: $(8, 6)$

(c)

B1: $(-2, 6)$

Question Number	Scheme	Marks
10.(a)	$x = 3 \cos 2y \Rightarrow \left(\frac{dx}{dy} = \right) -6 \sin 2y$	M1 A1 (2)
(b)	E.g. $\frac{dx}{dy} = -6\sqrt{1-kx^2}$ or $\frac{dy}{dx} = \frac{1}{\text{their } \frac{dx}{dy}}$ $\frac{dy}{dx} = -\frac{1}{6\sqrt{1-\cos^2 2y}} = -\frac{1}{6\sqrt{1-\left(\frac{x}{3}\right)^2}} = -\frac{1}{2\sqrt{9-x^2}}$	M1 dM1 A1 (3)
(c)	Sets $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}} = -\frac{1}{4}$ $x^2 = 5 \Rightarrow a = \sqrt{5}$ $\sqrt{5} = 3 \cos 2y \Rightarrow b = \frac{1}{2} \arccos\left(\frac{\sqrt{5}}{3}\right)$	M1 A1 dM1 A1
Alt (c)	Sets $\frac{dy}{dx} = -\frac{1}{6 \sin 2y} = -\frac{1}{4}$ $\Rightarrow \sin 2y = \frac{2}{3} \Rightarrow y = \frac{1}{2} \arcsin\left(\frac{2}{3}\right)$ $x = 3 \cos 2y = 3\sqrt{1-\sin^2\left(\arcsin\frac{2}{3}\right)} = \sqrt{5}$	M1A1 dM1 A1 (4) (9 marks)

(a)

 M1: $\left(\frac{dx}{dy} = \right) k \sin 2y$ Alt: $x = 3(2 \cos^2 y - 1) \Rightarrow \left(\frac{dx}{dy} = \right) k \cos y \sin y$ (oe with sine identity etc)

 A1: $\left(\frac{dx}{dy} = \right) -6 \sin 2y$ Alt: $\left(\frac{dx}{dy} = \right) -12 \cos y \sin y$

(b)

M1: Either

- Uses $\sin^2 2y + \cos^2 2y = 1$ with $x = 3 \cos 2y$ to find $\sin 2y$ in terms of x (condone slips with the 3 when rearranging)
- Or uses $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$

 dM1: Uses both of the above to find $\frac{dy}{dx}$ in terms of x .

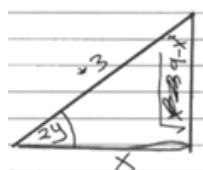
A1: $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}}$ including the $\frac{dy}{dx}$ seen at some stage associated with the answer, may have the $-1/2$

in the numerator seen and isw attempts to take the 2 into the numerator.

Alt for first bullet point: Longer ways may be possible, but look for correct formulae used, allowing algebraic slips simplifying, and must reach an expression for $\sin 2y$ in terms of x . E.g.

$$\sin 2y = 2 \sin y \cos y = 2 \sqrt{\frac{1}{2} \left(1 - \frac{x}{3}\right)} \sqrt{\frac{1}{2} \left(1 + \frac{x}{3}\right)}$$

Another alternative for finding $\sin 2y$ in terms of x is use of appropriate triangles, e.g.



To score the M they must proceed as far as identifying $\sin 2y$ in terms of x , here $\sin 2y = \frac{\sqrt{9-x^2}}{3}$. The trig ratios and application of Pythagoras theorem must be correct.

Notes: The question says “hence” and so answer via arcos formula are not being credited any marks.

You may allow recovery of marks for part (b) if the work is done in part (c) instead.

(c)

M1: Sets $\frac{dy}{dx} = -\frac{1}{4}$ and proceeds to $x^2 = p, p > 0$. Condone slips in rearranging.

A1: $a = \sqrt{5}$ from a correct answer to (b) Accept $x = \dots$

dM1: Substitutes the value of x in $x = 3 \cos 2y$ and uses arccos to find y (allowing decimal values – you may need to check).

A1: $b = \frac{1}{2} \arccos\left(\frac{\sqrt{5}}{3}\right)$ following a correct answer to (b). Accept $y = \dots$ Accept alternative notation such

as $\frac{1}{2} \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$ or even $\arccos$.

Alt method

M1: Sets $\frac{dy}{dx} = -\frac{1}{4}$ and proceeds to $\sin 2y = p, |p| < 1$. Condone slips in rearranging.

A1: $b = \frac{1}{2} \arcsin \frac{2}{3}$ Accept $y = \dots$ Accept alternative notation such as $\frac{1}{2} \sin^{-1}\left(\frac{2}{3}\right)$ or even $\arcsin$

dM1: Substitutes the value of y in $x = 3 \cos 2y$ and uses Pythagorean identity to find exact value of x .

A1: $a = \sqrt{5}$ Accept $x = \dots$

Question Number	Scheme	Marks
10(a)	$\frac{dx}{dy} = 12 \sec^2 2y \tan 2y$	M1, A1
		(2)
(b)	$\frac{dx}{dy} = 12 \left(\frac{x}{3} \right) \sqrt{\sec^2 2y - 1} \Rightarrow \frac{dx}{dy} = 12 \left(\frac{x}{3} \right) \sqrt{\frac{x}{3} - 1}$	M1, A1ft
	$\frac{dy}{dx} = \frac{\sqrt{3}}{4x\sqrt{x-3}}$	A1
		(3)
(c)	$y = \frac{\pi}{12} \Rightarrow x = 4$	B1
	$\frac{dx}{dy} = 12 \times \frac{4}{3} \times \frac{1}{\sqrt{3}}$ or $\frac{dy}{dx} = \frac{1}{12 \left(\frac{4}{3} \right) \sqrt{\frac{4}{3} - 1}}$	M1
	Correct $m_N = -\frac{16}{\sqrt{3}}$ o.e	A1
	$y - \frac{\pi}{12} = -\frac{16}{\sqrt{3}}(x - 4)$	dM1
	$y = -\frac{16\sqrt{3}}{3}x + \frac{64\sqrt{3}}{3} + \frac{\pi}{12}$	A1
		(5)
		Total 10

(a)

M1: Differentiates to a form on the rhs of $\alpha \sec^2 2y \tan 2y$ which may be written $\dots \sec 2y \times \dots \sec 2y \tan 2y$ Note that the same scheme can also be applied to students who adapt $x = 3 \sec^2 2y$ to $x = \pm 3 \tan^2 2y \pm 3$ A1: $\frac{dx}{dy} = 12 \sec^2 2y \tan 2y$. If the lhs is included it must be correct. So $\frac{dy}{dx} = 12 \sec^2 2y \tan 2y$ is M1 A0Condone this to be unsimplified $\frac{dx}{dy} = 6 \sec 2y \times 2 \sec 2y \tan 2y$ ISW after sight of correct answer

(b)

M1: For an attempt to

- replace $\sec^2 2y$ with αx
- use the identity $\pm 1 \pm \tan^2 2y = \pm \sec^2 2y$ and replaces $\tan 2y = b\sqrt{\pm 1 \pm d x}$

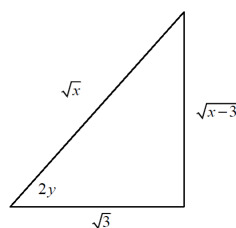
to obtain an expression for $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of x only.The expression in part (a) must have had $\frac{dx}{dy}$ as a function of both $\sec 2y$ and $\tan 2y$ o.e.A1ft: Requires a substitution of both $\sec^2 2y$ with $\frac{x}{3}$ and $\tan 2y = \sqrt{\frac{x}{3} - 1}$ to obtain a correct expression for $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of x . Follow through on their $\frac{dx}{dy} = \alpha \sec^2 2y \tan 2y$ 

For a correct $\frac{dx}{dy}$ it is awarded for $\frac{dy}{dx} = \frac{1}{4x\sqrt{\frac{1}{3}x-1}}$

A1: Correct answer in the required form.

Allow equivalents e.g. $\frac{3\sqrt{3}}{12x\sqrt{x-3}}$. Form required is $\frac{p}{qx\sqrt{x-3}}$ where p is irrational and q is an integer

Alt method for (a) and (b) which can be marked in a similar way



(a) M1 A1: $x = 3(\cos 2y)^{-2} \Rightarrow \frac{dx}{dy} = 12(\cos 2y)^{-3} \sin 2y$

(b) If $x = 3(\cos 2y)^{-2} \Rightarrow \cos 2y = \frac{\sqrt{3}}{\sqrt{x}}$

Score in a similar way to the main scheme

M1 A1: $\frac{dx}{dy} = \frac{12 \sin 2y}{(\cos 2y)^3} = \frac{12 \sqrt{\frac{x-3}{x}}}{\left(\frac{\sqrt{3}}{\sqrt{x}}\right)^3}$

Alt (b) via arccos

$$y = \frac{1}{2} \arccos \sqrt{\frac{3}{x}} \Rightarrow \frac{dy}{dx} = \frac{1}{2} \frac{1}{\sqrt{1 - \left(\sqrt{\frac{3}{x}}\right)^2}} \times \frac{\sqrt{3}}{2} x^{-\frac{3}{2}}$$

M1: For $\frac{dy}{dx} = \lambda \frac{1}{\sqrt{1 - \left(\sqrt{\frac{\alpha}{x}}\right)^2}} \times -x^{-\frac{3}{2}}$ A1: Correct and unsimplified A1: Correct and in the required form

(c)

B1: Correct value for x

M1: Attempts to find the value of $\frac{dx}{dy}$ using their part (a) with $y = \frac{\pi}{12}$

the value of $\frac{dy}{dx}$ from an inverted $\frac{dx}{dy}$ using their part (a) with $y = \frac{\pi}{12}$

or the value of $\frac{dy}{dx}$ using their part (b) with their value of x found using $y = \frac{\pi}{12}$.

These may be called m or f' and not identified as $\frac{dx}{dy}$ or $\frac{dy}{dx}$

A1: Correct normal gradient

dM1: Attempt at the equation of normal at $y = \frac{\pi}{12}$.

The gradient should be either an attempt at the value of $-\frac{dx}{dy}$ at $y = \frac{\pi}{12}$ for their $\frac{dx}{dy}$

or an attempt at the negative reciprocal of their $\frac{dy}{dx}$ at their "4" which must have been found from $y = \frac{\pi}{12}$

A1: Fully correct equation in the required form. ISW after a correct answer



Allow equivalent exact forms e.g. $y = -\frac{16}{\sqrt{3}}x + \frac{64}{\sqrt{3}} + \frac{\pi}{12}$, $y = -\frac{16}{\sqrt{3}}x + \frac{256\sqrt{3} + \pi}{12}$



Question Number	Scheme	Marks
9 (a)	$(k =) 4\sin^2\left(\frac{\pi}{3}\right) - 1 = 2$ *	B1*
		(1)
(b)	(i) $x = 4\sin^2 y - 1 \Rightarrow \frac{dx}{dy} = 8\sin y \cos y$ o.e.	M1 A1
	(ii) Attempts either $\sin^2 y = \frac{\pm x + 1}{4}$ or $\cos^2 y = \pm 1 \pm \frac{x+1}{4}$ (both for dM1)	M1 dM1
	$\frac{dy}{dx} = \frac{1}{8\sin y \cos y} = \frac{1}{8 \times \sqrt{\frac{x+1}{4}} \times \sqrt{\pm 1 \pm \frac{x+1}{4}}} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ *	ddM1 A1*
		(6)
(c)	At $x = 2$, gradient of curve $= \frac{1}{2\sqrt{3}} \Rightarrow$ gradient of normal is $-2\sqrt{3}$	M1
	Point N (base length of triangle) is solution of	
	$x - \frac{\pi}{3} = -2\sqrt{3}(x - 2) \Rightarrow x = 2 + \frac{\pi}{6\sqrt{3}}$ (= 2.3...)	dM1
	Area $= \frac{1}{2} \times \left(2 + \frac{\pi}{6\sqrt{3}}\right) \times \frac{\pi}{3} = \frac{\pi}{3} + \frac{\pi^2}{36\sqrt{3}}$	A1
		(3)
		(10 marks)

(a)

B1*: Verifies that $k = 2$ with no errors seen. (Condone $x = 2$) Do not allow this mark if rounded numbers are seen in their solution for e.g. $\frac{\pi}{3}$. They may work in degrees and substitute in 60° which is

acceptable. Look for as a minimum e.g. $(k =) 4\sin^2 \frac{\pi}{3} - 1 = 2$ Do not allow $k = 4\sin\left(\frac{\pi}{3}\right)^2 - 1 = 2$

If they rearrange the equation to e.g. $y = \sin^{-1}\left(\sqrt{\frac{x+1}{4}}\right)$ and substitute in $x = 2$ proceeding to e.g. $\frac{\pi}{3}$ then they must conclude that $k = 2$ (or a preamble followed by a minimal conclusion e.g. QED, tick)

(b) (i)

M1: Differentiates to a form $p \sin y \cos y$ or equivalent. Indices do not need to be processed for this mark. e.g. $\dots \sin^{2-1} y \cos y$

May use the identity $\cos 2y = \pm 1 \pm 2\sin^2 y \Rightarrow x = 4\sin^2 y - 1 = 1 - 2\cos 2y \Rightarrow \frac{dx}{dy} = q \sin 2y$

A1: Correct lhs and rhs. For $\frac{dx}{dy} = 8\sin y \cos y$ or $4\sin 2y$. Note that $\frac{dy}{dx} = \dots$ is A0. isw for this mark

after a correct answer is seen. You may see $\frac{dx}{dy}$ (or allow x') on an earlier line which is fine for this mark. May be seen at the start of (b)(ii). Indices must be processed for this mark.



(b)(ii) **On EPEN this is M1A1dM1A1* we are marking this as M1dM1ddM1A1***

Note this is a given answer so full working must be seen.

M1: Attempts to find either $\sin y$ (or $\sin^2 y$) **or** $\cos y$ (or $\cos^2 y$) in terms of x (or possibly multiples of

these) Typically look for $\sin^2 y = \frac{\pm x \pm 1}{4}$ or $\sin y = \sqrt{\frac{\pm x \pm 1}{4}}$ Also allow e.g. $4\sin^2 y = \pm x \pm 1$

May attempt to use $\pm \sin^2 y \pm \cos^2 y = \pm 1 \Rightarrow \cos^2 y = \pm 1 \pm \sin^2 y \Rightarrow \cos^2 y = \pm 1 \pm \frac{x \pm 1}{4}$ or

$\cos y = \sqrt{\pm 1 \pm \frac{x \pm 1}{4}}$ Ignore $\pm \sqrt{\dots}$ for this mark. (may be seen within their expression for $\frac{dy}{dx}$ or $\frac{dx}{dy}$)

dM1: Attempts to find both $\sin y$ (or $\sin^2 y$) **and** $\cos y$ (or $\cos^2 y$) or possibly multiples of these in terms of x . (See previous method mark for guidance) It is dependent on the previous method mark.

ddM1: Full method to get $\frac{dy}{dx}$ of the form $\frac{1}{p \sin y \cos y}$ o.e. in terms of x e.g. $\frac{dy}{dx} = \frac{1}{\text{"8"} \sqrt{\frac{\pm x \pm 1}{4}} \sqrt{\pm 1 \pm \frac{\pm x \pm 1}{4}}}$

Condone square root notation which does not fully go over any fraction provided it is implied by further work which is not the given answer. Condone missing arguments for $\sin$ and $\cos$ provided the intention is clear or implied. Condone missing brackets to be recovered **before** achieving the given answer. If they have rearranged or simplified incorrectly for $\cos y$ before substituting, ddM1 can still be scored.

A1*: Achieves the given answer **provided all previous method marks have been scored in (b) and no errors**. Look for

- a correct expression for $\cos^2 y$ or $\cos y$ in x which has not already been simplified to $\dots \sqrt{3-x}$
- a correct expression for $\sin^2 y$ or $\sin y$ in x which has not already been simplified to $\dots \sqrt{x+1}$

$\frac{dy}{dx}$ (or y') may appear on an earlier line which is acceptable. Note that if they work backwards by

substituting $x = 4\sin^2 y$ into the given answer there must be a conclusion that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ isw

(c)

M1: Full method for the gradient of the normal using the x or y value. Condone arithmetical slips but the gradient must be the negative reciprocal of the gradient of the tangent to the curve.

Note that they may use an earlier incorrect line in part (b) for $\frac{dy}{dx}$ (or even $\frac{dx}{dy}$).

Allow use of an earlier line provided it is an attempt at a correct method substituting in x or y and

proceeding from e.g. $\frac{dx}{dy} = \alpha \Rightarrow$ gradient of the normal $= -\alpha$

May be implied by awrt -3.5 If no method is shown where the gradient of the tangent or normal has come from, then it must be correct.

dM1: Attempts to find the x coordinate of N . It is dependent on the previous method mark. Uses their gradient of the normal with $\left(2, \frac{\pi}{3}\right)$, sets $y = 0$ in $y - y_1 = m(x - x_1)$ and proceeds to find x . May also use $y = mx + c$ to find the equation for the straight line and then sets $y = 0$ and rearranges to find x . Do not be concerned by the mechanics of the rearrangement for this mark.

A1: $\frac{\pi}{3} + \frac{\pi^2}{36\sqrt{3}}$ or exact equivalent such as $\frac{\pi}{3} + \frac{\pi^2\sqrt{3}}{108}$ which is in the form $a\pi + b\pi^2$



Question Number	Scheme	Marks
9(a)	$x = 0 \Rightarrow \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) = 0 \Rightarrow 3y + \frac{\pi}{4} = \pi, 2\pi, \dots \Rightarrow y = \dots$	M1
	$y = \frac{\pi}{4} \text{ or } y = \frac{7\pi}{12}$	A1
	$y = \frac{\pi}{4} \text{ and } y = \frac{7\pi}{12}$	A1
		(3)

Condone any confusion about which point is which and just look for the correct values.

Look for answers given in the body of the question or on Figure 2

M1: Correct strategy to find y at A or B.

Requires an attempt to solve either $3y + \frac{\pi}{4} = \pi$ or $3y + \frac{\pi}{4} = 2\pi$

May be implied by at least one correct value and may be seen embedded e.g. $\left(0, \frac{\pi}{4}\right), \left(0, \frac{7\pi}{12}\right)$

Condone e.g. $\left(\frac{\pi}{4}, 0\right), \left(\frac{7\pi}{12}, 0\right)$

Allow equivalent work e.g. $3y + \frac{\pi}{4} = 0 \Rightarrow y = -\frac{\pi}{12} \Rightarrow y = -\frac{\pi}{12} + \frac{\pi}{3}, -\frac{\pi}{12} + \frac{2\pi}{3}$

May be implied by decimal answers e.g. awrt 0.79 or awrt 1.8

A1: One correct value. Allow simplified or unsimplified and may be seen embedded e.g. $\left(0, \frac{\pi}{4}\right), \left(0, \frac{7\pi}{12}\right)$

Condone unsimplified for this mark e.g. $y = \frac{1}{3}\left(\pi - \frac{\pi}{4}\right)$ for $\frac{\pi}{4}$ or e.g. $y = \frac{2\pi}{3} - \frac{\pi}{12}$ for $\frac{7\pi}{12}$

A1: Both correct and simplified and may be seen embedded e.g. $\left(0, \frac{\pi}{4}\right), \left(0, \frac{7\pi}{12}\right)$

Condone e.g. $\left(\frac{\pi}{4}, 0\right), \left(\frac{7\pi}{12}, 0\right)$ and condone unsimplified as long as they are single fractions e.g. $\frac{2\pi}{8}$

Alternative:

$$\frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) = 0 \Rightarrow \frac{2}{3} \sin 3y \cos \frac{\pi}{4} + \frac{2}{3} \cos 3y \sin \frac{\pi}{4} = 0 \Rightarrow \tan 3y = -1 \Rightarrow 3y = \frac{3\pi}{4}, \frac{7\pi}{4}$$

Score M1 for an attempt to solve either $3y = \frac{3\pi}{4}$ or $3y = \frac{7\pi}{4}$ and then A1A1 as above.



(b)	$x = \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) \Rightarrow \frac{dx}{dy} = \frac{2}{3} \times 3 \cos\left(3y + \frac{\pi}{4}\right) \left(= 2 \cos\left(3y + \frac{\pi}{4}\right)\right)$	B1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2 \cos\left(3y + \frac{\pi}{4}\right)} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4 \cos^2\left(3y + \frac{\pi}{4}\right)} = \frac{1}{4 \left(1 - \sin^2\left(3y + \frac{\pi}{4}\right)\right)}$	M1
	$= \frac{1}{4 \left(1 - \left(\frac{3x}{2}\right)^2\right)}$	dM1
	$= \frac{1}{4 - 9x^2}$	A1
		(4)
B1: Correct $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form. Do not condone missing brackets e.g. $2 \cos 3y + \frac{\pi}{4}$ unless recovered or implied by subsequent work. M1: For any 2 of the following applied to their $\frac{dx}{dy}$: • finds reciprocal (may have already happened if they find $\frac{dy}{dx}$) • squares • applies $\cos^2\left(3y + \frac{\pi}{4}\right) = 1 - \sin^2\left(3y + \frac{\pi}{4}\right)$ Note that this mark (and the next one) may be implied by equivalent work e.g. $x = \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) \Rightarrow \sin\left(3y + \frac{\pi}{4}\right) = \frac{3x}{2} \Rightarrow \cos\left(3y + \frac{\pi}{4}\right) = \frac{\sqrt{4 - 9x^2}}{2}$ $\frac{dx}{dy} = 2 \cos\left(3y + \frac{\pi}{4}\right) = \sqrt{4 - 9x^2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4 - 9x^2}$ dM1: Uses all 3 of the above correctly to obtain $\left(\frac{dy}{dx}\right)^2$ in terms of x. A1: Correct answer. You can condone e.g. missing trailing brackets within their working e.g. $\frac{1}{4(1 - \sin^2(3y + \frac{\pi}{4}))}$ Special case: allow this mark if the only error in their solution is to obtain $\frac{dx}{dy} = -\frac{2}{3} \times 3 \cos\left(3y + \frac{\pi}{4}\right)$		

(b) Way 2	$x = \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) \Rightarrow 3y + \frac{\pi}{4} = \sin^{-1} \frac{3x}{2}$	B1
	$3y + \frac{\pi}{4} = \sin^{-1} \frac{3x}{2} \Rightarrow 3 \frac{dy}{dx} = \frac{1}{\sqrt{1 - \frac{9x^2}{4}}} \times \frac{3}{2}$	M1
	$3 \frac{dy}{dx} = \frac{1}{\sqrt{1 - \frac{9x^2}{4}}} \times \frac{3}{2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \left(\frac{1}{2\sqrt{1 - \frac{9x^2}{4}}}\right)^2$	dM1
	$= \frac{1}{4 - 9x^2}$	A1

Way 2:**B1:** Correct equation in any form connecting y and $\sin^{-1} \frac{3x}{2}$.**M1:** Differentiates to obtain $\frac{dy}{dx} = \frac{k}{\sqrt{1 - \left(\frac{3x}{2}\right)^2}}$ or equivalent. Condone missing brackets around $\frac{3x}{2}$.**dM1:** Squares and makes $\left(\frac{dy}{dx}\right)^2$ the subject.**A1:** Correct answer.

(c)	Identifies that one of the gradients is $\sqrt{4}$ or that one of the gradients is $\frac{1}{\sqrt{4}}$ or Identifies that one of the gradients is 2 or that one of the gradients is $\frac{1}{2}$	B1ft
	At A: $y - \frac{\pi}{4} = 2x$ and At B: $y - \frac{7\pi}{12} = \frac{1}{2}x$	M1
	$2x + \frac{\pi}{4} = \frac{1}{2}x + \frac{7\pi}{12} \Rightarrow x = \dots$	dM1
	$x = \frac{2\pi}{9}$	A1
		(4)
		Total 11
Note that in this part the question says to use the answer to part (b) i.e. their $\frac{1}{p - qx^2}$ If they use their $\frac{1}{p - qx^2}$ then the ft is available as shown. If candidates “start again” or e.g. use their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of y with their y values from part (a) then the value(s) must be <u>correct</u>. B1ft: Using the answer to part (b): Award this mark for one gradient identified as $\sqrt{p}$ or $\frac{1}{\sqrt{p}}$ where p is their value from part (b). or not using the answer to part (b): One gradient identified as 2 or $\frac{1}{2}$ M1: Correct straight line method for the equation of the normal at A and the tangent at B. They must have 2 values for y from part (a) (ignore how these were labelled) - both gradients positive For the normal at A this requires a gradient of $\sqrt{p}$ or 2 with $x = 0$ and their smaller value of y from part (a) If they use $y = mx + c$ they must proceed as far as $c = \dots$ For the tangent at B this requires a gradient of $\frac{1}{\sqrt{p}}$ or $\frac{1}{2}$ with $x = 0$ and their larger value of y from part (a) If they use $y = mx + c$ they must proceed as far as $c = \dots$ dM1: Correct method for both lines as defined above and solves simultaneously (may be via a calculator) to find the x coordinate of D. Depends on the previous mark. A1: Cso. Correct value $x = \frac{2\pi}{9}$. Ignore any attempts to find the y coordinate. If they use part (b) then full marks can be awarded following an incorrect value for q provided all the other work is correct.		

Beware: It is possible to obtain the correct answer in (c) from incorrect values in (a) e.g.

$$\begin{aligned} \text{c) } \frac{dy}{dx} &= \pm \sqrt{\frac{1}{4-9x^2}} = \frac{1}{\sqrt{4-9x^2}} \quad \text{or} \quad -\frac{1}{\sqrt{4-9x^2}} \\ \text{When } x=0 \quad \frac{dy}{dx} &= \frac{1}{\sqrt{4-9(0)^2}} = \frac{1}{2} \\ \frac{dy}{dx} &= -\frac{1}{\sqrt{4-9(0)^2}} = -\frac{1}{2} \\ \text{A \& normal: } y - \frac{\pi}{12} &= 2(x-0) \rightarrow y = 2x + \frac{\pi}{12} \\ \text{B tangent: } y - \frac{\pi}{12} &= \frac{1}{2}(x-0) \rightarrow y = \frac{1}{2}x + \frac{\pi}{12} \\ 2x + \frac{\pi}{12} &= \frac{1}{2}x + \frac{\pi}{12} \\ 1.5x &= \frac{\pi}{3} \\ x &= \frac{2}{9}\pi \end{aligned}$$

This scores B1M1dM1A0

Diagram for reference:

