



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Integration (P3.5)

International A Level Mathematics · P3: Pure Mathematics 3 (WMA13)

21 questions · 158 marks · Mark schemes included

Past papers January 2020 - June 2025 · 2 sub-topics

 guidemygrades.com

 [@Guidemygrades](https://www.instagram.com/Guidemygrades)

 [@Guidemygrades](https://twitter.com/Guidemygrades)

 [@Guidemygrades](https://www.youtube.com/Guidemygrades)

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
P3.5a INTEGRATING EXPONENTIALS, 1/X & TRIGONOMETRIC FUNCTIONS		21 Qs · 158 marks	2
1. Q1	IAL P3 (WMA13) January 2021	3	3
2. Q9	IAL P3 (WMA13) January 2021	4	5
3. Q5	IAL P3 (WMA13) June 2025	4	7
4. Q8	IAL P3 (WMA13) January 2023	5	9
5. Q5	IAL P3 (WMA13) October 2021	6	11
6. Q3	IAL P3 (WMA13) January 2022	6	13
7. Q3	IAL P3 (WMA13) June 2022	6	15
8. Q3	IAL P3 (WMA13) October 2023	6	17
9. Q10	IAL P3 (WMA13) October 2021	7	19
10. Q3	IAL P3 (WMA13) June 2023	7	22
11. Q2	IAL P3 (WMA13) June 2024	7	24
12. Q5	IAL P3 (WMA13) October 2020	8	28
13. Q3	IAL P3 (WMA13) June 2021	8	30
14. Q4	IAL P3 (WMA13) June 2025	8	34
15. Q1	IAL P3 (WMA13) October 2022	9	38
16. Q6	IAL P3 (WMA13) June 2024	9	42
17. Q4	IAL P3 (WMA13) January 2025	9	46
18. Q8	IAL P3 (WMA13) January 2020	10	50
19. Q9	IAL P3 (WMA13) June 2023	10	54
20. Q7	IAL P3 (WMA13) January 2024	12	58
21. Q9	IAL P3 (WMA13) October 2024	14	62

P3.5b REVERSE CHAIN RULE & AREAS

No questions on this sub-topic in the available past papers.



QUESTIONS

21 questions · 158 marks · 1 sub-topic

SUB-TOPIC P3.5a

INTEGRATING EXPONENTIALS, $1/x$ & TRIGONOMETRIC FUNCTIONS

Sub-topic P3.5a · 21 questions · 158 marks · lowest tariff first



1. Find

$$\int \frac{x^2 - 5}{2x^3} dx \quad x > 0$$

giving your answer in simplest form.

(3)

Leave
blank



Question 1 continued

Leave
blank

(Total 3 marks)

Q1



9. Find

$$(i) \int \frac{3x - 2}{3x^2 - 4x + 5} dx \quad (2)$$

$$(ii) \int \frac{e^{2x}}{(e^{2x} - 1)^3} dx \quad x \neq 0 \quad (2)$$

Leave
blank

Question 9 continued

Leave
blank

(Total 4 marks)

Q9



5. Find

(i) $\int \sin^2 3x \, dx$

(2)

$$(ii) \int x(x^2 + 4)^{\frac{3}{2}} dx$$

(2)



(Total for Question 5 is 4 marks)



8. Find, in simplest form,

$$\int (2 \cos x - \sin x)^2 dx$$

(5)



Question 8 continued

(Total for Question 8 is 5 marks)



Leave
blank

5. (i) Find, by algebraic integration, the exact value of

$$\int_2^4 \frac{8}{(2x-3)^3} dx$$

(4)

- (ii) Find, in simplest form,

$$\int x(x^2 + 3)^7 \, dx$$

(2)

Question 5 continued

Leave
blank

(Total 6 marks)

Q5



3. (i) Find, in simplest form,

$$\int (2x - 5)^7 \, dx$$

(2)

(ii) Show, by algebraic integration, that

$$\int_0^{\frac{\pi}{3}} \frac{4 \sin x}{1 + 2 \cos x} dx = \ln a$$

where a is a rational constant to be found.

(4)

Leave
blank

Question 3 continued

Leave
blank

(Total 6 marks)

Q3



3. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k} \, dx \quad (2)$$

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k} \, dx = \ln 8$$

(b) find the value of k

(4)

Leave
blank

Question 3 continued

Leave
blank

(Total 6 marks)

Q3



3. (a) Using the identity for $\cos(A + B)$, prove that

$$\cos 2A \equiv 2 \cos^2 A - 1 \quad (2)$$

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4 \cos^2 3x) dx \quad (4)$$

Question 3 continued

(Total for Question 3 is 6 marks)



Question 10 continued

Leave
blank



Question 10 continued

Leave
blank

Q10

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

END



3. (i) Find $\frac{d}{dx} \ln(\sin^2 3x)$ writing your answer in simplest form.

(2)

(ii)(a) Find $\frac{d}{dx}(3x^2 - 4)^6$

(2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x(3x^2 - 4)^5 \, dx = R$$

where R is an integer to be found.

(Solutions relying on calculator technology are not acceptable.)

(3)

(Total for Question 3 is 7 marks)





Question 2 continued

(Total for Question 2 is / marks)



5. (a) Show that

$$\sin 3x \equiv 3 \sin x - 4 \sin^3 x \quad (4)$$

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx \quad (4)$$

Leave
blank



Question 5 continued

Leave
blank

Q5

(Total 8 marks)



Question 3 continued

Leave
blank

Question 3 continued

Leave
blank



Question 3 continued

Leave
blank

(Total 8 marks)

Q3



4.

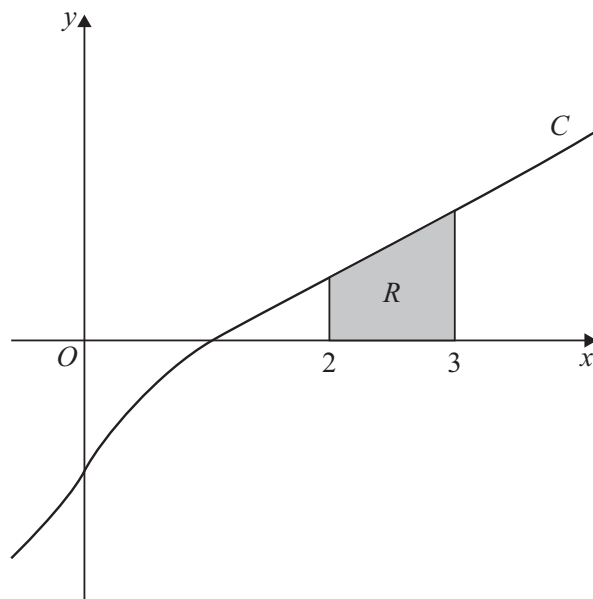


Figure 1

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 1 shows a sketch of part of the curve C with equation $y = g(x)$ where

$$g(x) = \frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1}$$

Given that

$$\frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1} \equiv Ax + B + \frac{Cx + D}{x^2 + 1}$$

(a) (i) find the values of the constants A , B and C

(ii) show that $D = 0$

(4)

The finite region R , shown shaded in Figure 1, is bounded by C , the x -axis and the lines with equations $x = 2$ and $x = 3$

(b) Find the exact area of region R using algebraic integration.

Give the answer in the form $\alpha + \ln \beta$ where α and β are constants to be found.

(4)

Question 4 continued



Question 4 continued

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 8 marks)

Leave
blank

1. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

(a) Show that

$$f(x) \equiv Ax + B + \frac{C(2x + 3)}{x^2 + 3x + 4}$$

where A , B and C are integers to be found.

(4)

(b) Hence, find

$$\int_3^5 f(x) dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)



Question 1 continued

Leave
blank



Question 1 continued

Leave
blank



Question 1 continued

Leave
blank

Q1

(Total 9 marks)



Question 6 continued



Question 6 continued



(Total for Question 6 is 5 marks)



Question 4 continued



Question 4 continued

(Total for Question 4 is 9 marks)



Question 8 continued

Leave
blank

Question 8 continued

Leave
blank



Question 8 continued

Leave
blank

Q8

(Total 10 marks)



Question 9 continued



Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

V

(Total for Question 9 is 10 marks)





Question 7 continued



(Total for Question 7 is 12 marks)



9.

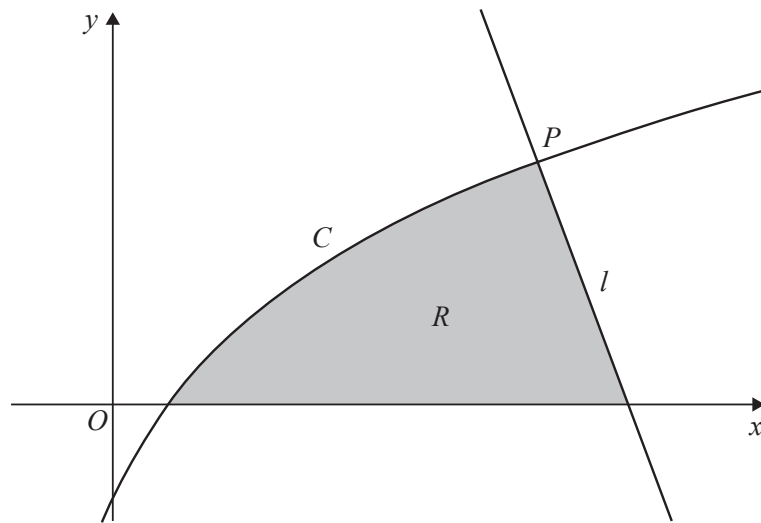


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{6x^2 + 4x - 2}{2x + 1} \quad x > -\frac{1}{2}$$

- (a) Find $f'(x)$, giving the answer in simplest form.

(3)

The line l is the normal to C at the point $P(2, 6)$

- (b) Show that an equation for l is

$$16y + 5x = 106$$

(3)

- (c) Write $f(x)$ in the form $Ax + B + \frac{D}{2x + 1}$ where A , B and D are constants.

(3)

The region R , shown shaded in Figure 5, is bounded by C , l and the x -axis.

- (d) Use algebraic integration to find the exact area of R , giving your answer in the form $P + Q \ln 3$, where P and Q are rational constants.

(Solutions based entirely on calculator technology are not acceptable.)

(5)



Question 9 continued





Question 9 continued

(Total for Question 9 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
9(i)	$\int \frac{3x-2}{3x^2-4x+5} dx = \frac{1}{2} \ln(3x^2-4x+5) (+c)$	M1, A1
		(2)
(ii)	$\int \frac{e^{2x}}{(e^{2x}-1)^3} dx = -\frac{1}{4} (e^{2x}-1)^{-2} (+c)$	M1, A1
		(2)
		Total 4

(i)

M1: Integrates to a form $\alpha \ln(3x^2 - 4x + 5)$ where α is a constant. Condone a missing bracket.

Do not accept $\alpha \ln(3x^2 - 4x + 5) + f(x)$, e.g. $\ln(3x^2 - 4x + 5) + 2x$

If the substitution $u = 3x^2 - 4x + 5$ is attempted, the mark can be awarded for $k \ln u$

It is unlikely but $\alpha \ln \beta(3x^2 - 4x + 5)$ and $\alpha \ln(3x^2 - 4x + 5)^\beta$ are also correct

A1: $\frac{1}{2} \ln(3x^2 - 4x + 5)$ o.e. with or without the $+c$. A bracket or modulus must be present.

ISW after a correct answer.

Do not penalise $\frac{\ln(3x^2 - 4x + 5)}{2}$ or $\ln(3x^2 - 4x + 5)/2$ if the intention is clear

Penalise spurious incorrect notation for the A mark only. So do not allow $\frac{\ln(3x^2 - 4x + 5) dx}{2}$

(ii)

M1: Integrates to a form $\beta(e^{2x} - 1)^{-2}$ where β is a constant.

Do not accept $\beta(e^{2x} - 1)^{-2} + g(x)$, e.g. $\beta(e^{2x} - 1)^{-2} + e^{2x}$

Allow substitutions. So for example,

if the substitution $u = e^{2x} - 1$ is attempted, the mark can be awarded for ku^{-2}

if the substitution $u = e^x$ is attempted, the mark can be awarded for $k(u^2 - 1)^{-2}$

if the substitution $u = e^{2x}$ is attempted, the mark can be awarded for $k(u - 1)^{-2}$

A1: $-\frac{1}{4}(e^{2x} - 1)^{-2}$ or exact equivalent with or without the $+c$

ISW after a correct answer. Need not be simplified

Penalise spurious incorrect notation for the A mark only. So do not allow $\int -\frac{1}{4}(e^{2x} - 1)^{-2}$

Question Number	Scheme	Marks
2(a)	$R = 25$	B1
	$\tan \alpha = \frac{24}{7} \Rightarrow \alpha = \dots$	M1
	$\alpha = 1.287$	A1
		(3)
(b)(i)	$\text{Min} = \frac{5}{90 - 3 \times "25" \times (-1)}$	M1
	$= \frac{1}{33}$	A1
(b)(ii)	$(2x + "1.287") = \pi, \dots \Rightarrow x = \dots$	M1
	$\Rightarrow x = \frac{\pi - "1.287"}{2} = 0.927$	A1
		(4)
		Total 7

Notes:

(a)

B1: Correct value. May be seen embedded in e.g. $(f(x) =) 25 \cos(x + \dots)$.Condone e.g. $R = \sqrt{7^2 + (-24)^2} = 25$ Do not allow ± 25 unless the -25 is rejected.**M1:** Allow for: $\tan \alpha = \pm \frac{24}{7}$, $\tan \alpha = \pm \frac{7}{24}$, $\cos \alpha = \pm \frac{7}{\text{their } R}$, $\sin \alpha = \pm \frac{24}{\text{their } R}$ leading to a value for α .If no method is shown it may be implied by their value for α as awrt 1.3 rads or awrt 74° **A1:** Awrt 1.287 Isw once the mark is awarded. Must be in radians.May be seen embedded in e.g. $(f(x) =) \dots \cos(x + 1.287)$.Apply isw if correct R and α are seen followed by e.g. $(f(x) =) 25 \cos(x - 1.287)$.**Mark (b)(i) and (ii) together.**

(b)(i)

M1: Award for $\frac{5}{90 + 3 \times \text{Their } R}$.

May be implied by their value (which may be a decimal) and may be seen embedded in e.g. coordinates

 $\left(\frac{\pi - 1.287}{2}, \frac{1}{33} \right)$ which would score (i) M1 A1 and (ii) M1**A1:** For $\frac{1}{33}$ only.Note that both marks are available in (b)(i) even if their α is incorrect.(b)(ii) **Note that (b)(ii) can be marked independently.****M1:** For an attempt to solve $2x \pm "1.287" = \pi$ to obtain a value for x . May be implied by awrt 0.927
Condone a sign slip but the order of operations must be essentially correct e.g.

$$2x \pm "1.287" = \pi \Rightarrow 2x = \pi \pm "1.287" \Rightarrow x = \frac{\pi \pm "1.287"}{2}$$

Condone an attempt to solve e.g. $2x \pm "74^\circ" = 180^\circ$ but not mixed units e.g. $2x \pm "1.287" = 180$ **A1:** For awrt 0.927

Question Number	Scheme	Marks
3(a)	$\log_{10} y = \pm \frac{2}{3.5} \log_{10} x - 2$	M1
	$\log_{10} y = -\frac{4}{7} \log_{10} x - 2$	A1
		(2)

Notes:

M1: Obtains an equation of the form $\log_{10} y = \pm \frac{2}{3.5} \log_{10} x - 2$ oe e.g. $\log_{10} y = -\frac{4}{7} \log_{10} x + \log_{10} 0.01$

Note $y = -\frac{4}{7}x - 2$ or $y = \pm \frac{2}{3.5} \log_{10} x - 2$ or $\log_{10} y = -\frac{4}{7}x - 2$ score M0

If this is attempted via simultaneous equations e.g. $0 = -3.5m + c$, $-2 = c$ it is scored for reaching a gradient of $\pm \frac{2}{3.5}$ which may be implied by awrt 0.57 and then $\log_{10} y = \pm \frac{2}{3.5} \log_{10} x - 2$ as above.

Condone log or lg for $\log_{10}$ but not ln or any other incorrect base.

A1: Correct equation $\log_{10} y = -\frac{4}{7} \log_{10} x - 2$ or equivalent.

e.g. $7 \log_{10} y + 4 \log_{10} x + 14 = 0$, $\log_{10} y = -\frac{4}{7} \log_{10} x + \log_{10} 0.01$

Condone $\log_{10} y = -\frac{2}{3.5} \log_{10} x - 2$ and isw once a correct equation is seen.

Condone log or lg for $\log_{10}$ but not ln



(b)	$\log_{10} y = -\frac{4}{7} \log_{10} x - 2 \Rightarrow$ $y = 10^{\left(-\frac{4}{7} \log_{10} x - 2\right)} \text{ or } y = 10^{\left(\log_{10} x^{-\frac{4}{7} - 2}\right)} \text{ or } y = 10^{-\frac{4}{7} \log_{10} x} \times 10^{-2}$ or e.g. $\log_{10} y = \log_{10} x^{-\frac{4}{7}} - 2 \Rightarrow \log_{10} \frac{y}{x^{-\frac{4}{7}}} = -2$ or e.g. $\log_{10} y = -\log_{10} x^{\frac{4}{7}} - 2 \Rightarrow \log_{10} yx^{\frac{4}{7}} = -2$	M1
	$y = 10^{\left(\log_{10} x^{-\frac{4}{7} - 2}\right)} \text{ or } y = 10^{-\frac{4}{7} \log_{10} x} \times 10^{-2} \Rightarrow y = x^{-\frac{4}{7}} \times 10^{-2}$ or e.g. $10^{\log_{10} \frac{y}{x^{-\frac{4}{7}}}} = 10^{-2} \Rightarrow \frac{y}{x^{-\frac{4}{7}}} = 10^{-2} \text{ or } 10^{\log_{10} yx^{\frac{4}{7}}} = 10^{-2} \Rightarrow yx^{\frac{4}{7}} = 10^{-2}$	dM1
	$y = \frac{1}{100} x^{-\frac{4}{7}} \text{ oe}$	A1
		(3)
		Total 5

Do not allow marks for part (a) to be scored in part (b)

Main method: Starting from an equation of the form $\log_{10} y = m \log_{10} x + c$ where m and c are non-zero.

M1: Correctly applies $10^{\text{lhs}} = 10^{\text{rhs}}$ to their $\log_{10} y = m \log_{10} x + c$ including the use of $10^{\log_{10} y} = y$

e.g. $\log_{10} y = m \log_{10} x + c \Rightarrow y = 10^{m \log_{10} x + c}$ or $\log_{10} y = m \log_{10} x + c \Rightarrow y = 10^{m \log_{10} x} \times 10^c$

or combines their $\log_{10} y$ and $m \log_{10} x$ correctly to obtain $\log_{10} yx^m = \pm c$ or equivalent e.g.

$\log_{10} y = m \log_{10} x + c \Rightarrow \log_{10} y = \log_{10} x^m + c \Rightarrow \log_{10} y - \log_{10} x^m = c \Rightarrow \log_{10} \frac{y}{x^m} = \pm c$

May be implied by their work. Note that e.g.

$\log_{10} y = -\frac{4}{7} \log_{10} x - 2 \Rightarrow y = 10^{-\frac{4}{7} \log_{10} x} \times 10^{-2}$ scores M0

dM1: Depends on the previous method mark.

Complete and correct work to obtain an equation in x and y only or for obtaining values for p and q which must not involve logs.

You can condone sign slips with their constant term but the log work must be correct.

A1: Correct equation in the form required $y = \frac{1}{100} x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2} x^{-\frac{4}{7}}$ or e.g.

$y = \frac{x^{-\frac{4}{7}}}{100}$ from correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Note that the correct answer does not imply the method marks and you need to check carefully.



Alternatives for part (b):**Way 2:**

M1: Starts with $y = px^q$ and obtains $\log_{10} y = \log_{10} p + \log_{10} x^q$ oe e.g. $\log_{10} y = \log_{10} p + q \log_{10} x$

dM1: **Depends on the previous method mark.**

Correct attempt at p and q by comparing their equation of the form $\log_{10} y = m \log_{10} x + c$ from part (a) or e.g. identifies the gradient and y -intercept from the sketch to obtain values for p and q not involving logs e.g. $\log_{10} p = c \Rightarrow p = 10^c$ and $\log_{10} x^q = q \log_{10} x \Rightarrow q = m$

A1: Correct **equation** in the form required $y = \frac{1}{100} x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2} x^{-\frac{4}{7}}$ from

correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Note that correct values for p and q with no incorrect work seen scores M1dM1

Note that a correct equation with no incorrect work seen scores M1dM1A1

Way 3:

M1: $7 \log_{10} y + 4 \log_{10} x = -14 \Rightarrow \log_{10} y^7 x^4 = -14 \Rightarrow y^7 x^4 = 10^{-14}$

Multiplies up to obtain integer coefficients, combines the log terms correctly then applies $10^{\text{lhs}} = 10^{\text{rhs}}$

dM1: $y^7 x^4 = 10^{-14} \Rightarrow y = \sqrt[7]{\frac{10^{-14}}{x^4}}$

Complete and correct attempt to obtain y in terms of x .

A1: Correct **equation** in the form required $y = \frac{1}{100} x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2} x^{-\frac{4}{7}}$ or e.g.

$y = \frac{x^{-\frac{4}{7}}}{100}$ from correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Way 4:

M1: $(\log_{10} x, \log_{10} y) = (0, -2) \Rightarrow x = 1, y = 10^{-2} \Rightarrow p = 10^{-2}$

Uses $(0, -2)$ correctly to establish the value of p .

dM1: $(\log_{10} x, \log_{10} y) = (-3.5, 0) \Rightarrow y = 1, x = 10^{-3.5} \Rightarrow 1 = 10^{-2} \times 10^{-3.5q} \Rightarrow -2 - 3.5q = 0 \Rightarrow q = \dots$

Uses $(-3.5, 0)$ correctly with their value of p to obtain a value for q .

A1: Correct **equation** in the form required $y = \frac{1}{100} x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2} x^{-\frac{4}{7}}$ or e.g.

$y = \frac{x^{-\frac{4}{7}}}{100}$ from correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Fully correct answer in (b) with **no** working scores full marks in (b).

Partially correct answer in (b) with no working scores no marks.



Question Number	Scheme	Marks
4(a)	$\frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} = \frac{49x + 7x(x - 3)}{(x + 4)(x - 3)}$ or e.g. $\frac{49x(x + 4)}{(x^2 + x - 12)(x + 4)} + \frac{7x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)} = \frac{49x(x + 4) + 7x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)}$	M1A1
	$\frac{49x + 7x(x - 3)}{(x + 4)(x - 3)} = \frac{49x + 7x^2 - 21x}{(x + 4)(x - 3)} = \frac{7x^2 + 28x}{(x + 4)(x - 3)}$ $= \frac{7x \cancel{(x + 4)}}{\cancel{(x + 4)}(x - 3)} = \frac{7x}{(x - 3)}^*$	A1*
		(3)
Do not allow a mis-read of the numerators of the fractions e.g. $\frac{7x}{x + 4}$ as $\frac{7}{x + 4}$ or $\frac{49x}{x^2 + x - 12}$ as $\frac{49}{x^2 + x - 12}$ M1: Factorises the denominator of the first term to obtain $(x + 4)(x \pm 3)$ and attempts to combine the two fractions using a common denominator to obtain $\frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} = \frac{...x + ...x(x \pm 3)}{(x + 4)(x \pm 3)}$ oe Alternatively multiplies numerator and denominator of both fractions by suitable expressions and attempts to combine the two fractions e.g. $\frac{49x(x + 4)}{(x^2 + x - 12)(x + 4)} + \frac{7x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)} = \frac{...x(x + 4) + ...x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)}$ A1: For a correct unsimplified fraction. Condone missing brackets if they are recovered. A1*: Achieves the printed answer with sufficient working shown. The factorisation required for the cancelling needs to be seen within their fraction so that e.g. $\frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} = \frac{49x + 7x(x - 3)}{(x + 4)(x - 3)} = \frac{7x^2 + 28x}{(x + 4)(x - 3)} = \frac{7x}{(x - 3)}$ scores M1A1A0* Do not allow just $\frac{7x \cancel{(x + 4)}}{\cancel{(x + 4)}(x - 3)}$ without the printed answer written down. For reference if they take the more complicated method, the correct working to reach the printed could be as follows: $= \frac{49x(x + 4) + 7x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)} = \frac{7x(x^2 + 8x + 16)}{(x + 4)(x + 4)(x - 3)} = \frac{7x(x + 4)(x + 4)}{(x + 4)(x + 4)(x - 3)} = \frac{7x}{(x - 3)}^*$		

(b)	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = \frac{(x-3) \times 7 - 7x \times 1}{(x-3)^2}$	M1
	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	A1
		(2)
(b) ALT	$f(x) = \frac{7x}{(x-3)} = 7 + \frac{21}{x-3} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	M1A1
		Total 5

(b)

M1: Attempts to differentiate using the quotient rule or product rule.

 For the quotient rule it requires achieving $f'(x) = \frac{\alpha(x-3) - \beta x}{(x-3)^2} \quad \alpha, \beta > 0$

 If they use a substitution then similar conditions apply e.g. $u = x-3 \Rightarrow f'(u) = \frac{\alpha u - \beta(u+3)}{u^2} \quad \alpha, \beta > 0$

 For the product rule it requires achieving $f'(x) = A(x-3)^{-1} - Bx(x-3)^{-2} \quad A, B > 0$

 If they use a substitution then similar conditions apply e.g. $u = x-3 \Rightarrow f'(u) = Au^{-1} - B(u+3)u^{-2} \quad A, B > 0$
A1: Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$

 Allow $-\frac{21}{x^2 - 6x + 9}$ or e.g. $-21(x^2 - 6x + 9)^{-1}$

Isw once a correct simplified answer is seen.

Alternative:
M1: Expresses $\frac{7x}{(x-3)}$ as $\alpha + \frac{\beta}{(x-3)}$ and differentiates to obtain $\frac{\dots}{(x-3)^2}$ or e.g. $\alpha + \frac{\beta}{u}$ where $u = x-3$

 and differentiates to obtain $\frac{\dots}{u^2}$
A1: Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$

 Allow $-\frac{21}{x^2 - 6x + 9}$ or e.g. $-21(x^2 - 6x + 9)^{-1}$

Isw once a correct simplified answer is seen.

Question Number	Scheme	Notes
8	$\int (2 \cos x - \sin x)^2 dx = \int (4 \cos^2 x - 4 \sin x \cos x + \sin^2 x) dx$	M1
	$\int 4 \sin x \cos x dx = \int 2 \sin 2x dx = -\cos 2x$ or $\int 4 \sin x \cos x dx = -2 \cos^2 x \quad \text{or} \quad 2 \sin^2 x$	M1
	$\int (4 \cos^2 x + \sin^2 x) dx = \int (1 + 3 \cos^2 x) dx = \int \left(1 + 3 \left(\frac{\cos 2x + 1}{2}\right)\right) dx$ or $\int (4 \cos^2 x + \sin^2 x) dx = \int \left(4 \left(\frac{\cos 2x + 1}{2}\right) + \frac{1 - \cos 2x}{2}\right) dx$	M1
	$\int (2 \cos x - \sin x)^2 dx = \frac{3}{4} \sin 2x + \cos 2x + \frac{5}{2} x (+c)$ or $\int (2 \cos x - \sin x)^2 dx = \frac{3}{4} \sin 2x + 2 \cos^2 x + \frac{5}{2} x (+c)$ or $\int (2 \cos x - \sin x)^2 dx = \frac{3}{4} \sin 2x - 2 \sin^2 x + \frac{5}{2} x (+c)$	A1A1
		(5)
		Total 5

Condone changes in variables throughout this solution as long as the answer is given in terms of x

M1: Expands to the form $p \cos^2 x + q \sin x \cos x + r \sin^2 x$

M1: Correct strategy for **integrating** $q \sin x \cos x$ (i.e. obtains $k \cos 2x$ or $k \sin^2 x$ or $k \cos^2 x$)

M1: Correct strategy for rewriting $p \cos^2 x + r \sin^2 x$ **into a form that can be integrated.**

Score for one of

- writes in terms of just $\sin^2 x$ and then uses $\sin^2 x = \frac{\pm 1 \pm \cos 2x}{2}$
- writes in terms of just $\cos^2 x$ and then uses $\cos^2 x = \frac{\pm 1 \pm \cos 2x}{2}$
- writes both $\sin^2 x$ and $\cos^2 x$ in terms of $\cos 2x$ with **at least one** of these via use of a correct/allowable form. That is $\sin^2 x = \frac{\pm 1 \pm \cos 2x}{2}$ or $\cos^2 x = \frac{\pm 1 \pm \cos 2x}{2}$

A1: Integrates and achieves 2 correct terms (of the 3 required terms)

NB: An unsimplified expression is acceptable for this mark so please check carefully. e.g. $2x + \dots + \frac{1}{2}x$ counts as one correct term.

A1: Correct simplified integration (+ c not required).

An alternative solution is via "R cos" or "R sin" but the last two marks are unlikely to be achieved due to the fact that an exact answer is difficult to arrive at.



Question Number	Scheme	Notes
8	$\int (2 \cos x - \sin x)^2 dx = \int 5 \cos^2 (x + 0.464) dx$	M1
	$= \int \frac{5 \{ \cos (2x + 0.927) + 1 \}}{2} dx$	M1
	$\frac{5 \sin (2x + 0.927)}{4} + \frac{5}{2} x$	M1
	$\frac{3 \sin 2x}{4} + \cos 2x + \frac{5}{2} x + c$	A1A1
		(5)
		Total 5

M1: Writes $2 \cos x - \sin x$ in the form $R \cos(x \pm \alpha)$ o.e. and squares.

Requires a full method so for the form $R \cos(x \pm \alpha)$

- requires $R^2 = 2^2 + 1^2$
- requires $\tan \alpha = \pm \frac{1}{2} \Rightarrow \alpha = \dots$ in radians but condone in degrees

M1: Correct strategy for writing $\cos^2(x \pm \alpha)$ into a form that can be integrated using the double angle formula

$$\cos^2(x \pm \alpha) \rightarrow \frac{\pm 1 \pm \cos(2x \pm 2\alpha)}{2} \text{ but condone } \cos^2(x \pm \alpha) \rightarrow \frac{\pm 1 \pm \cos(2x \pm \alpha)}{2}$$

α should be in radians but condone in degrees

M1: Correct strategy for writing $\cos^2(x \pm \alpha)$ into a form that can be integrated using the double angle formula

$$\cos^2(x \pm \alpha) \rightarrow \frac{\pm 1 \pm \cos(2x \pm 2\alpha)}{2} \text{ but condone } \cos^2(x \pm \alpha) \rightarrow \frac{\pm 1 \pm \cos(2x \pm \alpha)}{2}$$

α should be in radians but condone in degrees

M1: It is for integrating $\cos(2x \pm \delta) \rightarrow \pm \sin(2x \pm \delta)$ following use of an acceptable double angle formula

A1: 2 correct terms of $\frac{3 \sin 2x}{4} + \cos 2x + \frac{5}{2} x + c$

A1: Fully correct and simplified (+ c not required) or $\frac{5 \sin(2x + 2 \arctan 0.5)}{4} + \frac{5}{2} x$



Question Number	Scheme	Notes
8	$\int (2 \cos x - \sin x)^2 dx = \int 5 \sin^2 (x - 1.107) dx$	M1
	$= \int \frac{5\{1 - \cos(2x - 2.21)\}}{2} dx$	M1
	$= \frac{5}{2}x - \frac{5 \sin(2x - 2.21)}{4}$	M1
	$= \frac{5}{2}x + \frac{3 \sin 2x}{4} + \cos 2x + c$	A1A1
		(5)
		Total 5



Question Number	Scheme	Marks
1. (a)	$\frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} = \frac{5x + 5x(x + 3)}{(x + 3)(x + 4)}$ $= \frac{5x^2 + 20x}{(x + 3)(x + 4)} = \frac{5x(x + 4)}{(x + 3)(x + 4)} = \frac{5x}{x + 3} *$	M1 A1
		A1*
		(3)
(b)	$y = \frac{5x}{(x + 3)} \Rightarrow xy + 3y = 5x \Rightarrow 5x - xy = 3y$ $\Rightarrow x = \frac{3y}{5 - y} \quad \text{So } f^{-1}(x) = \frac{3x}{5 - x}$ Domain $0 < x < 5$	M1
		A1
		A1
(c) (i)		(3)
	$f(x) = \frac{5x}{(x + 3)} \Rightarrow (f'(x)) = \frac{5(x + 3) - 5x}{(x + 3)^2} = \frac{15}{(x + 3)^2}$	M1 A1
	(ii) States (f is an) increasing function with a suitable reason E.g. Since $(x + 3)^2$ is positive	A1
		(3)
		(9 marks)

(a)

M1 Attempts to combine the two fractions using a common denominator.
 Allow errors on the numerator but at least one of the terms must have been adapted.
 Usual rules apply for factorising the quadratic denominator.
 Condone invisible brackets and slips on the numerator when combining the two fractions.

A1 Allow the two fractions to be written separately with the same denominator.
 For a correct un-simplified fraction with a quadratic numerator and denominator (which may be implied)

$$\frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} = \frac{5x(x + 4)}{\dots} + \frac{5x(x^2 + 7x + 12)}{\dots} = \frac{5x^3 + 40x^2 + 80x}{(x^2 + 7x + 12)(x + 4)} = \frac{x(5x + 20)(x + 4)}{(x^2 + 7x + 12)(x + 4)} = \frac{x(5x + 20)}{(x^2 + 7x + 12)}$$

A1* Correctly achieves the given answer of $\frac{5x}{x + 3}$ showing intermediate steps (cso). Expect to see the two fractions combined and then both the numerator and denominator factorised before cancelling terms to achieve full marks. In the case of forming a cubic numerator and denominator you must see the terms collected before factorising. Bracket errors at some point in their working is A0*

(b)

M1 Attempts to change the subject on $y = \frac{5x}{x + 3}$ (or $x = \frac{5y}{y + 3}$). Look for cross multiplication with an attempt to collect terms. Do not follow through on their answer to part (a)

- A1 $f^{-1}(x) = \frac{3x}{5-x}$ or $f^{-1}(x) = \frac{-3x}{x-5}$ Must be in terms of x . Condone $f^{-1} = \dots$ (or $f^{-1} = y = \dots$) but **do not allow** just $y = \dots$ or $f^{-1} : y = \dots$
- A1 Correct domain $0 < x < 5$

(c) Mark (i) and (ii) together

- M1 Attempts to use the quotient rule or product rule. Look for an expression of the form $\frac{A(x+3) - 5x}{(x+3)^2}$ or $5x(x+3)^{-1} \rightarrow B(x+3)^{-1} \pm 5x(x+3)^{-2}$ where A and B are non-zero constants.

- A1 $(f'(x) =) \frac{15}{(x+3)^2}$ or $15(x+3)^{-2}$ or $\frac{15}{x^2 + 6x + 9}$ Do not allow $\frac{5}{x+3} - \frac{5x}{(x+3)^2}$ for this mark.

- A1 They achieve $(f'(x) =) \frac{15}{(x+3)^2}$ and states

- (that f is an) **increasing function**
- **since $(x+3)^2$ is positive**

Alt methods in (a)

Taking out common factors

$$M1A1: \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x+4} = 5x \left\{ \frac{1+(x+3)}{(x+3)(x+4)} \right\} \text{ or } = \frac{5x}{(x+4)} \left\{ \frac{1+(x+3)}{(x+3)} \right\}$$

Methods where candidates "split up" fractions.

- M1 Attempts $\frac{5x}{x^2 + 7x + 12}$ OR $\frac{5}{x^2 + 7x + 12}$ by partial fractions

A1 $\frac{5x}{x+3} - \frac{5x}{x+4} + \frac{5x}{x+4}$

- A1* $\frac{5x}{x+3}$ You would expect to see $\frac{5x}{x+3} - \frac{5x}{x+4} + \frac{5x}{x+4}$ before proceeding to the given answer.

Alt method in (b)

- M1 Writes $y = \frac{5x}{x+3}$ as $y = 5 \pm \frac{k}{x+3}$ and then attempts to make x the subject.

A1 $f^{-1}(x) = \frac{15}{5-x} - 3$ or equivalent

A1 $0 < x < 5$

Alt method in (c)

- M1 Writes $y = \frac{5x}{x+3}$ as $y = 5 \pm \frac{k}{x+3}$ and attempts the chain rule to get $\pm \frac{C}{(x+3)^2}$

A1 $(f'(x) =) \frac{15}{(x+3)^2}$ or exact equivalent



General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Marks
1	$y = (2x + 5)e^{3x}$ $\frac{dy}{dx} = 2e^{3x} + (2x + 5)3e^{3x}$ $\frac{dy}{dx} = 0 \Rightarrow 2 + 3(2x + 5) = 0 \Rightarrow x = -\frac{17}{6}$	M1 A1 dM1A1 (4) (4 marks)

M1: Attempts the product rule and achieves a form $\left(\frac{dy}{dx}\right) = Ae^{3x} + B(2x + 5)e^{3x}$ where A and B are positive constants. Condone missing/poor bracketing

Note that this could be attempted from $y = 2xe^{3x} + 5e^{3x} \Rightarrow \frac{dy}{dx} = Ae^{3x} + Bxe^{3x} + Ce^{3x}$

A1: Correct $\frac{dy}{dx} = 2e^{3x} + (2x + 5)3e^{3x}$ o.e. There is no requirement to simplify this

dM1: Dependent upon the previous M, it is for using a correct strategy to find a value for x .

E.g. Sets or implies $\frac{dy}{dx} = 0$, cancels or factorises out the e^{3x} term and solves a linear equation in

x

In most cases where lns are used it will be M0

Example of M0

$$e^{3x}(6x + 17) = 0 \Rightarrow \ln(e^{3x}(6x + 17)) = \ln(0) \Rightarrow 3x(6x + 17) = \dots$$

Example of M1

$$e^{3x}(6x + 17) = 0 \Rightarrow (6x + 17) = 0, \quad (e^{3x} = 0)$$

$$x = \dots$$

A1: $x = -\frac{17}{6}$ o.e. only. If an extra solution is given, e.g. from $e^{3x} = 0$, it is A0

Condone awrt -2.83 following a correct equation. Ignore any attempt to find y



General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$\left(\frac{dy}{dx} = \right) 6 \times 3(3x-2)^5 \quad (=18(3x-2)^5)$	M1A1
		(2)
(b)	$"18"(3 \times \frac{1}{3} - 2)^{5"} = -18$ $-18 \rightarrow \frac{1}{18}$ $y-1 = " \frac{1}{18} " \left(x - \frac{1}{3} \right)$ $3x - 54y + 53 = 0$	M1 M1 dM1 A1
		(4)
		(6 marks)

(a)

M1 Applies the chain rule to achieve a form of $A(3x-2)^5$ which may be unsimplified. The index does not need to be processed for this mark.

Alternatively, multiplies out the brackets to achieve an expression of the form $Ax^6 + Bx^5 + Cx^4 + Dx^3 + Ex^2 + Fx + G$ (where all coefficients do not need to be simplified) and reduces the power by 1 on at least one of the terms. You will need to check this carefully to make sure it is the relevant term where the power has decreased
eg $Bx^5 \rightarrow \dots \times Bx^4$
Condone slips in their expansion.

(The actual expansion is $64 - 576x + 2160x^2 - 4320x^3 + 4860x^4 - 2916x^5 + 729x^6$)

A1 $\left(\frac{dy}{dx} = \right) 18(3x-2)^5$ or unsimplified equivalent eg $6 \times 3(3x-2)^5$ but the index must be processed. If they have multiplied out then accept $-576 + 4320x - 12960x^2 + 19440x^3 - 14580x^4 + 4374x^5$.
Accept x^1 for x



(b)

M1 Substitutes $x = \frac{1}{3}$ into their $\frac{dy}{dx}$ which must be of the form $A(3x-2)^5$ (or accept a polynomial of degree 5) and finds a value for $\frac{dy}{dx}$

M1 Finds the negative reciprocal of their gradient. May be implied by the equation of their line. It cannot be scored for setting $\frac{dy}{dx} = 0$, solving to find x and then taking the negative reciprocal of this.

dM1 Attempts to find the equation of the normal. Look for $y-1 = \frac{1}{18}\left(x-\frac{1}{3}\right)$ (must be a changed gradient from their tangent gradient). It is dependent on the first method mark. If they use $y = mx + c$ they must proceed as far as $c = \dots \left(= \frac{53}{54} \right)$

A1 $3x - 54y + 53 = 0$ or any integer multiple of this with all terms on one side of the equation eg $-6x + 108y - 106 = 0$ You must see the $= 0$ isw after a correct answer is seen. Eg if they proceed to state values for a , b and c which contradict their equation then take their equation as their final answer.



Question Number	Scheme	Marks
2(a)	$fg(5) = f(3) = \frac{5 - "3"}{3 \times "3" + 2}, = \frac{2}{11}$	M1, A1
		(2)
(b)	$f^{-1}(x) = \frac{5 - 2x}{3x + 1}$	M1A1
	$x \in \mathbb{R}, x \neq -\frac{1}{3}$	B1
		(3)
(c)	$\frac{5 - \frac{1}{a}}{3 \times \frac{1}{a} + 2} = 2(a + 3) - 7$	B1ft
	$\frac{5 - \frac{1}{a}}{3 \times \frac{1}{a} + 2} = 2(a + 3) - 7 \Rightarrow 4a^2 - a - 2 = 0$	M1A1
	$(a =) \frac{1 \pm \sqrt{33}}{8}$	A1
		(4)
		(9 marks)

(a)

M1 Correct order of operations applying g before f on 5

Allow an attempt to substitute $x = 5$ into $fg(x) = \frac{5 - (2x - 7)}{3(2x - 7) + 2}$ condoning slips. They must proceed to find a value.

This may be completed in two steps so allow for an attempt at $g(5) = A \rightarrow f(A) = \dots$

Condoning arithmetical slips and bracket errors/omissions.

A1 $\frac{2}{11}$ or exact equivalent. Accept $0.\dot{1}\dot{8}$ but not eg 0.18 or 0.18....

(b)

M1 Changes the subject of $y = \frac{5 - x}{3x + 2} \rightarrow x = \frac{\dots \pm 2y}{\pm 3y \pm \dots}$ or $x = \frac{5 - y}{3y + 2} \rightarrow y = \frac{\dots \pm 2x}{\pm 3x \pm \dots}$



A1 $f^{-1}(x) = \frac{5-2x}{3x+1}$ or $f^{-1}(x) = \frac{2x-5}{-3x-1}$ or exact equivalent expressions such as $f^{-1}(x) = \frac{5-2x}{3\left(x+\frac{1}{3}\right)}$

.

Accept $f^{-1}: x \rightarrow \frac{5-2x}{3x+1}$ Condone $f^{-1} = \dots$ (or $f^{-1} = y = \dots$) but **do not allow just** $y = \dots$ or $f^{-1}: y = \dots$

B1 $x \neq -\frac{1}{3}$. The omission of $x \in \mathbb{R}$ is condoned.

Note: It is also acceptable to define f^{-1} in any variable eg as $f^{-1}(t) = \frac{5-2t}{3t+1}$ $t \neq -\frac{1}{3}$ as long as the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined eg $f^{-1}(t) = \dots$ not just $f^{-1} = \dots$

(c)

B1ft Sets $\frac{5-\frac{1}{a}}{3 \times \frac{1}{a} + 2} = 2(a+3) - 7$ or their $\frac{1}{a} = f^{-1}(2a-1) \Rightarrow \frac{1}{a} = \frac{"5"- "2"(2a-1)}{"3"(2a-1)+ "1"}$

(may be in terms of x). Follow through on their $f^{-1}(x)$.

May be implied by later work if they have invisible brackets.

M1 They must start with an equation of one of these forms (which may be in x):

- $\frac{A \pm \frac{1}{a}}{B \times \frac{1}{a} \pm C} = Da + E$ or equivalent so typically look for $\frac{5-\frac{1}{a}}{3 \times \frac{1}{a} + 2} = 2(a+3) - 7$

(may be manipulated to $\frac{Aa \pm 1}{B \pm Ca} = Da + E$)

- $\frac{1}{a} = \frac{Aa \pm B}{Ca \pm D}$ or equivalent

Rearranges their equation of an allowable form to a 3TQ. They must be attempting to

solve $f\left(\frac{1}{a}\right) = g(a+3)$ so it cannot be awarded for an attempt to solve $f(a) = g(a)$ or

$$f\left(\frac{1}{a}\right) = g\left(\frac{1}{a}\right)$$

Do not be concerned with the mechanics of their arrangement and condone invisible brackets.

A1 $4a^2 - a - 2 = 0$ or equivalent equation involving 3 terms. Eg $4a^2 - a = 2$ (may be in terms of x). May be implied by awrt ± 0.843 and the $= 0$ may be implied by their attempt to solve their quadratic equation.

A1 $(a =) \frac{1 \pm \sqrt{33}}{8}$ or exact equivalent. Do not accept rounded decimals. Condone $x = \frac{1 \pm \sqrt{33}}{8}$



Question Number	Scheme	Marks
2(a)	$ff(6) = f\left(\frac{9}{2}\right) = \frac{\frac{9}{2}+3}{\frac{9}{2}-4} = \dots; = 15$	M1; A1
		(2)
(b)	$f^{-1}(x) = \frac{4x+3}{x-1}$	M1A1
	$x \in \mathbb{R}, x \neq 1$	B1
		(3)
(c)	E.g. $\left(\frac{x+3}{x-4}\right)^2 + 5 = 7$ or $\frac{a+3}{a-4} = (\pm)\sqrt{7-5}$	M1
	$\Rightarrow x^2 - 22x + 23 = 0 \Rightarrow x = \dots$ or $(a+3) = (\pm)\sqrt{2}(a-4) \Rightarrow a = \dots$	dM1
	$(a =) 11 + 7\sqrt{2}$ oe	A1
		(3)
		(8 marks)

Notes**(a)**M1: Substitutes $x = 6$ into f and substitutes the result back into f to find a value for $ff(6)$

Alternatively substitutes $x = 6$ into $ff(x) = \frac{\left(\frac{x+3}{x-4}\right)+3}{\left(\frac{x+3}{x-4}\right)-4}$ condoning slips.

A1: 15

(b)M1: Changes the subject of $y = \frac{x+3}{x-4}$ or $x = \frac{y+3}{y-4}$ to $y = \frac{\dots x \pm 3}{\dots x \pm 1}$ or $x = \frac{\dots y \pm 3}{\dots y \pm 1}$ A1: Achieves $f^{-1}(x) = \frac{4x+3}{x-1}$. Accept $f^{-1} = \dots$ or $y = \dots$ instead of $f^{-1}(x) = \dots$.B1: $x \neq 1$. The omission of $x \in \mathbb{R}$ is condoned. Accept alternative notations.**(c) Note this is M1A1A1 on open but is being marked as M1dM1A1.**M1: Correct attempt to set up an equation in x or a . This will usually be $\left(\frac{x+3}{x-4}\right)^2 + 5 = 7$ but

$f(a) = g^{-1}(7)$ may be used first. Score when a suitable equation is set up and allow if a minor slip or miscopy is made if the intention is clear.

dM1: Proceeds to solve for x or a , e.g. forms a 3TQ and solves, or take the 5 across, square roots, cross multiplies and makes a the subject, condoning e.g. sign slips when rearranging.Alternatively, uses $(f(x))^2 + 5 = 7 \Rightarrow f(x) = (\pm)\sqrt{2}$ and their part (b) leading to

$a = f^{-1}\left(\sqrt{2}\right) = \frac{4\sqrt{2}+3}{\sqrt{2}-1}$. In this method both M's may be scored together.

A1 $11 + 7\sqrt{2}$ or $\frac{4\sqrt{2}+3}{\sqrt{2}-1}$ Accept $11 + \sqrt{98}$ or $\frac{-4\sqrt{2}-3}{1-\sqrt{2}}$. Common factors should be cancelled.

Ignore labelling of x or a . Condone e.g. if $11 - 7\sqrt{2}$ is also given as a solution but A0 if any other extra solutions are included.



Question Number	Scheme	Marks
10.(a)	$(1 + 2 \cos 2x)^2 = 1 + 4 \cos 2x + 4 \cos^2 2x$ Uses $\cos 4x = 2 \cos^2 2x - 1 \Rightarrow (1 + 2 \cos 2x)^2 = 1 + 4 \cos 2x + 2 \cos 4x + 2$ $= 3 + 4 \cos 2x + 2 \cos 4x$	M1 A1
		(2)
(b)	$a = \frac{2\pi}{3}$ $\int 3 + 4 \cos 2x + 2 \cos 4x \, dx = 3x + 2 \sin 2x + \frac{1}{2} \sin 4x$ $\text{Area} = \left[3x + 2 \sin 2x + \frac{1}{2} \sin 4x \right]_0^{\frac{2\pi}{3}} = 2\pi - \frac{3}{4}\sqrt{3}$	B1 M1 A1 ft dM1 A1
		(5)
		(7 marks)

(a)

M1 Attempts to multiply out $(1 + 2 \cos 2x)^2 = 1 + \dots \cos 2x + \dots \cos^2 2x$ and use $\cos 4x = 2 \cos^2 2x - 1$ to obtain $(1 + 2 \cos 2x)^2$ in the form $p + q \cos 2x + r \cos 4x$.
 Condone slips in the rearrangement of $\cos 4x = 2 \cos^2 2x - 1$ but it must be clear that the identity was correct originally otherwise M0.
 Beware of candidates who write $(1 + 2 \cos 2x)^2 = 1 + 4 \cos 2x + 4 \cos^2 4x$ which is M0A0

A1 $3 + 4 \cos 2x + 2 \cos 4x$

(b)

B1 Deduces that $a = \frac{2\pi}{3}$ (allow 120° for this mark). If more than one angle is found, then look for which one is substituted into their integrated expression.

M1 Integrates $q \cos 2x + r \cos 4x \rightarrow \pm \dots \sin 2x \pm \dots \sin 4x$

A1ft Integrates $p + q \cos 2x + r \cos 4x \rightarrow px + \frac{q}{2} \sin 2x + \frac{r}{4} \sin 4x$ unsimplified where p , q and $r \neq 0$

dM1 Substitutes 0 and $a = \frac{2\pi}{3}$ (or awrt 2.09) (or equivalent) into a valid function (M1 must have been scored) and subtracts either way around. Note $q, r \neq 0$ but they do not need a px term.

Also allow $a = \frac{\pi}{3}$ (or awrt 1.05) or $a = \frac{4\pi}{3}$ (or awrt 4.19) a must be in radians to evaluate correctly.

This mark cannot be scored without a value for a . You do not have to explicitly see 0 substituted in and their answer may imply a correct substitution into their integrated expression.

A1 $2\pi - \frac{3}{4}\sqrt{3}$ or simplified equivalent



Question Number	Scheme	Marks
1. (a)	$g(3) = -265, \quad g(4) = 3104$ States change of sign, continuous and hence root in $[3, 4]$	M1 A1 (2)
(b)	$x_2 = \sqrt[6]{1000 - 2 \times 3} = 3.1591$ $(\alpha =) 3.1589$	M1 A1 A1 (3)
Notes		
(a)		
M1	Attempts the value of g at 3 and 4 with one correct (accept any value for the other as an attempt). Note narrower ranges are possible but must contain the root and lies in $[3, 4]$.	
A1	Both values correct with reason (Sign change (stated or indicated) and continuous function) and minimal conclusion (root)	
(b)		
M1	Attempts to substitute $x_1 = 3$ into the formula. Implied by sight of expression, awrt 3.159	
A1	awrt 3.1591	
A1	$(\alpha =) 3.1589$ cao - must be to 4 d.p. Do not be concerned about the labelling of the root (x or α etc), mark the final answer of (b)(ii). (Note sight of this value implies the M1 even if x_2 is not seen).	



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
4 (a)	$ff(6) = f(13) = -1$	M1 A1 (2)
(b)	Attempts $21 + 2(2 - x) = 5x \Rightarrow x = \dots$ or $21 - 2(x - 2) = 5x \Rightarrow x = \dots$ $x = \frac{25}{7}$ only	M1 A1 (2)
(c)	Either $k < 21$ or $k \geq 17$ $17 \leq k < 21$	M1 A1 (2)
(d)	$a = \frac{1}{7}$ $b = 4$	B1 B1 (2) (8 marks)

(a)

M1 For attempting f "twice". Award for sight of $f(13)$ or $21 - 2|2 - (21 - 2|2 - x|)|$ followed by a value.A1 For -1

(b)

M1 Attempts $21 + 2(2 - x) = 5x \Rightarrow x = \dots$ A1 For $x = \frac{25}{7}$ only. Do not isw if students go on to find additional solutions unless they identify this as the only answer by rejecting others.

(c)

M1 For either $k < 21$ or $k \geq 17$ Condone for this mark $k \leq 21$ or $k > 17$ Alt: Allow for **both** $k = 17$ and $k = 21$ identified as critical values even if no inequalities are given.A1 $17 \leq k < 21$ May be written as separate inequalities. Accept alternative notations for the range such as $[17, 21)$ Do not accept $17 \leq f(x) < 21$

(d)

B1 Either $a = \frac{1}{7}$ **or** $b = 4$ B1 Both $a = \frac{1}{7}$ **and** $b = 4$ Allow both marks if $y = \frac{1}{7}f(x - 4)$ is given.

Question Number	Scheme	Marks
5 (a)	$\sin 3x \equiv \sin(2x + x) \equiv \sin 2x \cos x + \cos 2x \sin x$ $\equiv 2 \sin x \cos x \cos x + (1 - 2 \sin^2 x) \sin x$ $\equiv 2 \sin x (1 - \sin^2 x) + (1 - 2 \sin^2 x) \sin x$ $\equiv 3 \sin x - 4 \sin^3 x \quad *$	M1 M1 ddM1 A1* (4)
(b)	$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \int_0^{\frac{\pi}{3}} \frac{3}{4} \sin x - \frac{1}{4} \sin 3x \, dx$ $= \left[-\frac{3}{4} \cos x + \frac{1}{12} \cos 3x \right]_0^{\frac{\pi}{3}}$ $= \frac{5}{24}$	M1 dM1 A1 A1 (4) (8 marks)

(a)

M1 Uses $\sin 3x = \sin(2x + x) = \pm \sin 2x \cos x \pm \cos 2x \sin x$ M1 Uses the correct identity for $\sin 2x = 2 \sin x \cos x$ and any correct identity for $\cos 2x$ ddM1 Dependent upon both previous M's. It is for using $\cos^2 x = 1 - \sin^2 x$ to get an equation in only $\sin x$ A1* Fully correct solution with correct notation within their proof. Examples of incorrect notation include use of $\sin x^2$ instead of $\sin^2 x$ or use of $\sin$ instead of $\sin x$ and so on. Penalise in the A mark only for such.

Note: The ddM mark and final A mark may be score by substituting $\sin^2 x = 1 - \cos^2 x$ into the right hand side of the equation to reach an identical expression to an expanded left hand side ("working from both sides"), with the A mark then awarded for correct work leading to identical expressions **and** an minimal conclusion given (e.g. //)

Note You may see use of De Moivre's Theorem from an FP3 candidate. This can score full credit if carried out correctly. If there are errors or you are unsure then send to review.

If attempted in reverse:M1 $3 \sin x - 4 \sin^3 x = 3 \sin x - 2 \sin^2 x \sin x - 2 \sin x (1 - \cos^2 x) = \sin x - 2 \sin x \frac{1}{2} (1 - \cos 2x) + \sin 2x \cos x$

Uses $\sin^3 x = \sin x \times \sin^2 x$ with either $\sin^2 x = \frac{1}{2} (1 - \cos 2x)$ OR $\sin^2 x = 1 - \cos^2 x$ and $2 \sin x \cos x = \sin 2x$

M1 Uses both steps above to get to an equation with $\sin 2x$ and $\cos 2x$ ddM1 Gathers terms to reach $= \cos 2x \sin x + \sin 2x \cos x$ A1 Completes the proof uses $\cos 2x \sin x + \sin 2x \cos x = \sin 3x$ with no errors seen.

(b)

M1 Attempts to use part (a) to simplify. Accept $\int \sin^3 x \, dx = \int A \sin x + B \sin 3x \, dx$ dM1 $\int A \sin x + B \sin 3x \, dx \rightarrow a \cos x + b \cos 3x$ A1 $-\frac{3}{4} \cos x + \frac{1}{12} \cos 3x$ oe (not a multiple of this (unless recovered))A1 CSO = $\frac{5}{24}$

Note an answer of A1 $\frac{5}{24}$ with no supporting working scores no marks as algebraic integration is specified. But alternative methods of integration are permissible. Two alternatives are:

Question Number	Scheme	Marks
5 (b) Alt 1	$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \int_0^{\frac{\pi}{3}} \sin x (1 - \cos^2 x) \, dx = \int_0^{\frac{\pi}{3}} \sin x - \sin x \cos^2 x \, dx$ $= \left[-\cos x - \left(-\frac{\cos^3 x}{3} \right) \right]_0^{\frac{\pi}{3}}$ $= \frac{5}{24}$	M1 dM1 A1 A1 (4)
(b) Alt 2	$u = \cos x \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow \int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \int_1^{\frac{1}{2}} u^2 - 1 \, du$ $= \left[\frac{u^3}{3} - u \right]_1^{\frac{1}{2}}$ $= \frac{5}{24}$	M1 dM1 A1 A1 (4) (8 marks)

Notes: First three marks as follows (final A is as main scheme).

Alt 1

M1: Splits as $\sin^3 x = \sin x \sin^2 x$ and applies $\sin^2 x = 1 - \cos^2 x$ to get the integrand into and integrable form.

dM1 for $\sin x \rightarrow \pm \cos x$ and $\sin x \cos^2 x \rightarrow K \cos^3 x$

A1 $-\cos x - \left(-\frac{\cos^3 x}{3} \right)$ oe (not a multiple of this (unless recovered))

Alt 2

M1 Sets $u = \cos x$, finds $\frac{du}{dx} = \pm \sin x$ and makes a full substitution using both of these to get an integral in terms of u only. (Limits not needed for this mark).

dM1 for $au^2 - b \rightarrow Au^3 - bu$

A1 For reaching $\left[\frac{u^3}{3} - u \right]_1^{\frac{1}{2}}$ including correct limits or for undoing the substitution and reaching $\frac{\cos^3 x}{3} - \cos x$



General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Marks
1(a)	$\left(\frac{dy}{dx}\right) 2x \cos\left(\frac{1}{2}x\right) - \frac{1}{2}x^2 \sin\left(\frac{1}{2}x\right)$	M1 A1
	Sets $\frac{dy}{dx} = 0 \Rightarrow \tan\left(\frac{1}{2}x\right) = \frac{4}{x} \Rightarrow x = 2 \arctan\left(\frac{4}{x}\right)^*$	dM1 A1*
		(4)
(b)	$x_2 = 2 \arctan\left(\frac{4}{2}\right)$	M1
	$x_2 = \text{awrt } 2.214$ $x_6 = 2.155 \text{ cao}$	A1, A1
		(3)
		(7 marks)

(a)

M1: Differentiates using the product rule to obtain an expression of the form $Ax \cos\left(\frac{1}{2}x\right) + Bx^2 \sin\left(\frac{1}{2}x\right)$

If the product rule is quoted it must be correct.

A1: For a correct simplified or unsimplified derivative.

dM1: Sets their $\frac{dy}{dx} = 0$ and proceeds to an equation involving $\tan\left(\frac{1}{2}x\right)$. This may be implied by their working.

The M1 must have been scored.

A1*: Correctly achieves the given answer of $x = 2 \arctan\left(\frac{4}{x}\right)$

An intermediate line of $\tan\left(\frac{1}{2}x\right) = \frac{4}{x}$ or $\frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)} = \frac{2x}{\frac{1}{2}x^2}$ must be seen

Note that some may work backwards having differentiated e.g.

$$x = 2 \arctan\left(\frac{4}{x}\right) \Rightarrow \frac{x}{2} = \arctan\left(\frac{4}{x}\right) \Rightarrow \tan\left(\frac{x}{2}\right) = \frac{4}{x} \Rightarrow \frac{x \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = 4 \Rightarrow 4 \cos\left(\frac{x}{2}\right) = x \sin\left(\frac{x}{2}\right)$$

$$\Rightarrow 2x \cos\left(\frac{x}{2}\right) = \frac{1}{2}x^2 \sin\left(\frac{x}{2}\right) \Rightarrow 2x \cos\left(\frac{x}{2}\right) - \frac{1}{2}x^2 \sin\left(\frac{x}{2}\right) = 0$$

Score M1 for a complete method to reach $\frac{dy}{dx} = 0$ and A1 if correct with (minimal) conclusion.

(b)

M1: Attempts to use the given iteration formula. Award for $x_2 = 2 \arctan\left(\frac{4}{2}\right)$ or awrt 2.21

Note that this mark may also be implied by awrt 127 if they are using degrees and no working is shown.

A1: $x_2 = \text{awrt } 2.214$

A1: $x_6 = 2.155$ (Note that this is not awrt)



Question Number	Scheme	Marks
1 (a)	$(2, -4)$	B1, B1 (2)
(b)	$(8, 6)$	B1, B1 (2)
(c)	$(-2, 6)$	B1 (1)
		(5 marks)

Notes:

1. Candidates may build up their answer in stages.

E.g. $(6, -2) \rightarrow (8, -2) \rightarrow (8, 6)$ In such cases you mark the final coordinates only

2. Condone the omission of brackets. So, allow 8, 6 for $(8, 6)$

3. Allow for example, $x = 2$ for $(2, \dots)$ and $x = 2, y = -4$ for $(2, -4)$

4. There are no marks for transposed coordinates i.e $(6, 8)$ for $(8, 6)$

(a)

B1: For either $(2, \dots)$ or $(\dots, -4)$

B1: $(2, -4)$

(b)

B1: For either $(8, \dots)$ or $(\dots, 6)$

B1: $(8, 6)$

(c)

B1: $(-2, 6)$



Question Number	Scheme	Marks
2. (a)	$f(1) = 4 \times 1^3 + 2 \times 1^2 - 12 = -6$ AND $f(2) = 4 \times 2^3 + 2 \times 2^2 - 12 = 28$ States change of sign, continuous and hence root	M1 A1 (2)
(b)	$4x^3 + 2x^2 - 12 = 0 \Rightarrow x^2(4x + 2) = 12$ $\Rightarrow x^2 = \frac{12}{4x + 2} = \frac{6}{2x + 1} \Rightarrow x = \sqrt{\frac{6}{2x + 1}}$	M1 A1 (2)
(c) (i)	$\left(x_2 = \sqrt{\frac{6}{2+1}} = 1.4... \right), \quad x_3 = 1.2519$	M1, A1
(ii)	$\alpha = 1.2934$	B1 (3)
		(7 marks)

(a)

M1: Attempts the value of $f(x)$ at 1 and 2 with at least one correct.Candidates who state $f(1) < 0$, $f(2) > 0$ without finding the values will score M0, A0

A smaller interval could be chosen but unlikely. The root must lie within the interval

A1: Both values correct with reason and minimal conclusion (root)

For the reason accept, for example,

- ‘Sign change and continuous function’
- ‘ $f(1) = -6 < 0$, $f(2) = 28 > 0$ and cts’

For the minimal conclusion accept, for example

- Root
- ✓
- QED
- Hence α in the range 1 to 2

(b)

M1: Either writes $4x^3 + 2x^2 - 12 = 0$ OR states $f(x) = 0$ with $4x^3 + 2x^2 = 12$ o.e.

and ‘correctly’ reaches one of the following factorised forms

- $x^2(4x + 2) = \pm 12$
- $x^2(2x + 1) = \pm 6$
- $2x^2(2x + 1) = \pm 12$

Alternatively, they reach $4x^3 + 2x^2 = 12$ with evidence as stated above in line 1.

- and then divide each term by $2x^2$ giving $2x + 1 = \frac{12}{2x^2}$ o.e

The key step in the above is an attempt that isolates the $2x + 1$ or $4x + 2$ termA1: Shows **each** of the following steps in correctly showing $x = \sqrt{\frac{6}{2x + 1}}$ 

- States or sets $f(x) = 0$
- $x^2(4x+2) = 12$ o.e. for instance $2x+1 = \frac{12}{2x^2}$
- $x^2 = \frac{6}{2x+1}$ or $x^2 = \frac{12}{4x+2}$ o.e
- $x = \sqrt{\frac{6}{2x+1}}$

.....
 ...

Alt (b)

It is possible to do this backwards

M1: States $x = \sqrt{\frac{k}{2x+1}}$ squares and cross multiplies to $x^2(2x+1) = k$

A1: Compares $4x^3 + 2x^2 - 12 = 0$ with $2x^3 + x^2 - k = 0$ o.e and states that $f(x) = 0$ when $k = 6$

.....
 (c)

M1: Attempts to use the iteration formula at least once with $x_1 = 1$. Follow through on their k

It cannot be scored from a made-up value for k .

Look for $x_2 = \sqrt{\frac{6}{2 \times 1 + 1}}$ but it is implied by $x_2 = \sqrt{2}$ or awrt 1.4... for correct k

Also implied by $x_3 = \text{awrt } 1.25$ for correct k

A1: $x_3 = \text{awrt } 1.2519$

B1: $\alpha = 1.2934$ CAO following evidence of some iteration (and must follow $k = 6$)

Accept as sight of iteration any decimal answer in the range $(1, 2)$ for an intermediate value

x_n

Question Number	Scheme	Marks
3. (a)	$\frac{dx}{dy} = \frac{8y(2y+1) - 2(4y^2 - 3)}{(2y+1)^2}$ $\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2 + 8y + 6}$	M1 A1 dM1, A1 (4)
(b)	Sets $\frac{(2y+1)^2}{8y^2 + 8y + 6} = \frac{1}{3} \Rightarrow 4y^2 + 4y - 3 = 0$ $\Rightarrow (2y-1)(2y+3) = 0 \Rightarrow y = \frac{1}{2}, x = \dots$ $P = \left(-1, \frac{1}{2}\right)$ only	M1, A1 dM1 A1 (4) (8 marks)

(a)

M1: Attempts the quotient rule to achieve $\frac{Ay(2y+1) - B(4y^2 - 3)}{(2y+1)^2} \quad A > 0, B > 0$.

Product rule attempts may be seen

Condone missing brackets and ignore the LHS, so $\frac{dx}{dy}$ may be $\frac{dy}{dx}$

A1: $\frac{dx}{dy} = \frac{8y(2y+1) - 2(4y^2 - 3)}{(2y+1)^2}$ with both left-hand and right-hand sides correct (unsimplified)

Via the product rule you should see $\frac{dx}{dy} = 8y(2y+1)^{-1} - 2(4y^2 - 3)(2y+1)^{-2}$

dM1: Uses $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$. It is dependent upon the previous M1

A1: $\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2 + 8y + 6}$ o.e such as $\frac{4y^2 + 4y + 1}{8y^2 + 8y + 6}$ ISW after a correct answer.

Allow partially factorised forms such as $\frac{dy}{dx} = \frac{(2y+1)^2}{2(2y+1)^2 + 4}$ but $\frac{dy}{dx} = \frac{-4y^2 - 4y - 1}{-8y^2 - 8y - 6}$ is A0

.....
.....

Alt via division

M1: Divides $(4y^2 - 3)$ by $(2y+1)$ to achieve $2y + \alpha + \frac{\beta}{2y+1}$ and then differentiates to

$$2 + \frac{\delta}{(2y+1)^2}$$



A1: Divides $(4y^2 - 3)$ by $(2y + 1)$ to achieve $2y - 1 - \frac{2}{2y+1}$ and then differentiates to

$$\frac{dx}{dy} = 2 + \frac{4}{(2y+1)^2}$$

dM1: Requires $\frac{dx}{dy} = 2 + \frac{\delta}{(2y+1)^2}$ with attempt to find the reciprocal to reach $\frac{dy}{dx} = \frac{(2y+1)^2}{2(2y+1)^2 + \delta}$

A1: Simplifies to $\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2 + 8y + 6}$ or $\frac{4y^2 + 4y + 1}{8y^2 + 8y + 6}$

.....

(b)

M1: Sets their $\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2 + 8y + 6} = \frac{1}{3}$ and proceeds to a simplified quadratic equation in y .

Alternatively sets their $\frac{dx}{dy} = \frac{8y^2 + 8y + 6}{(2y+1)^2} = 3$ and proceeds to a simplified quadratic equation

in y .

A1: Correct 3TQ E.g. $4y^2 + 4y - 3 = 0$ but allow variations such as $4y^2 = 3 - 4y$

dM1: Solves quadratic via a correct method, finds a positive y value and the associated x coordinate.

Allow the quadratic to be solved via a calculator but it must be the simplified 3TQ

It is dependent upon the previous M mark.

This can be awarded if they find both values and cross out the correct answer.

A1: $P = \left(-1, \frac{1}{2}\right)$ only. Allow $x = -1, y = \frac{1}{2}$ ISW after a correct answer

If they find both values $\left(-1, \frac{1}{2}\right)$ and $\left(-3, -\frac{3}{2}\right)$ AND don't reject $\left(-3, -\frac{3}{2}\right)$ they will lose the last mark only

The demand is that solutions relying on calculator technology are not acceptable.

If a candidate uses their calculator to solve a correct $\frac{(2y+1)^2}{8y^2 + 8y + 6} = \frac{1}{3}$ o.e they can be awarded M1

for

$y = \frac{1}{2}$ and A1 for $x = -1, y = \frac{1}{2}$ for a total of 1100 in (b)

.....

For candidates who have studied WMA14 you may see versions of implicit differentiation

E.g. $2xy + x = 4y^2 - 3 \Rightarrow 2y + 2x \frac{dy}{dx} + 1 = 8y \frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{2y+1}{8y-2x}$$

Then substitute $x = \text{into}$ $\frac{dy}{dx} = \frac{2y+1}{8y-2x} \Rightarrow \frac{2y+1}{8y-2 \times \frac{4y^2-3}{2y+1}} = \frac{(2y+1)^2}{8y^2 + 8y + 6}$

Score M1 for an attempt at both product and chain rule on xy and y^2 terms

A1 for a correct $\frac{dy}{dx}$ in terms of both x and y

dM1 for the substitution and attempt at simplification

A1 Fully correct and simplified

Note that part (b) can be attempted from $\frac{2y+1}{8y-2x} = \frac{1}{3} \Rightarrow x = y - \frac{3}{2}$ o.e which can then be substituted into the equation of the curve.



General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**1. Factorisation $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ 2. FormulaAttempt to use correct formula (with values for a , b and c).3. Completing the squareSolving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:**1. DifferentiationPower of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)2. IntegrationPower of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
5 (a)	$f^{-1}(22) \Rightarrow 2 + 5 \ln x = 22 \Rightarrow \ln x = 4 \Rightarrow (x =) e^4$	M1 A1 (2)
(b)	$g(x) = \frac{6x-2}{2x+1} \Rightarrow g'(x) = \frac{6(2x+1) - 2(6x-2)}{(2x+1)^2}$ States $(g'(x) =) \frac{10}{(2x+1)^2} > 0$ hence increasing *	M1 A1 A1* (3)
(c)	$y = \frac{6x-2}{2x+1} \Rightarrow 2xy + y = 6x - 2 \Rightarrow 2xy - 6x = -y - 2$ $\Rightarrow x = \frac{-y-2}{2y-6}$ So $g^{-1}(x) = \frac{-x-2}{2x-6}$ o.e. Domain $0 < x < 3$	M1 A1 B1 (3)
(d)	Range fg is $fg < 2 + 5 \ln 3$	M1, A1 (2) (10 marks)

(a)

M1: Sets $2 + 5 \ln x = 22$ and rearranges to $\ln x = \dots$ (may be implied by further work)Alternatively, may attempt to find $f^{-1}(x)$ and substitute 22 into the expression. i.e. $f^{-1}(x) = e^{\frac{x+2}{5}}$.

Maybe implied by awrt 55.

Condone sign slips only in their rearrangement.

A1: e^4 (must be simplified and exact) isw once a correct answer is seen.

(b)

M1: Either

- attempts to use the quotient rule proceeding to the form $\frac{A(2x+1) - B(6x-2)}{(2x+1)^2}$ o.e. where A and

B are both positive. Do not award this mark if the quotient rule is applied the wrong way round.

- attempts to use the product rule proceeding to the form

$$(6x-2)(2x+1)^{-1} \Rightarrow \pm C(6x-2)(2x+1)^{-2} \pm D(2x+1)^{-1}$$

- writes $g(x)$ as $3 - \frac{5}{2x+1} = 3 - 5(2x+1)^{-1}$ and attempts to use the chain rule proceeding to the form $E(2x+1)^{-2}$

In all methods invisible brackets can be recovered or implied by further work.A1: Correct unsimplified e.g. $\frac{6(2x+1) - 2(6x-2)}{(2x+1)^2}$ or $-2(6x-2)(2x+1)^{-2} + 6(2x+1)^{-1}$ or $10(2x+1)^{-2}$

A1*: Requires

- a reason as to why the expression is always positive e.g. by finding $(g'(x) = \frac{10}{(2x+1)^2} > 0$
- a minimal conclusion

e.g. writes $g'(x)$ as $\frac{10}{(2x+1)^2}$ **and deduces** > 0 with minimal conclusion e.g. “it is an increasing

function” or “hence increasing” **Condone use of** $\geq$

They may consider just the denominator separately, but they would still need to simplify the numerator to 10 and state e.g. $(2x+1)^2 > 0$ o.e. with a minimal conclusion

Do not accept the conclusion $g'(x)$ is an increasing function if stated instead of $g(x)$.

Note that they cannot just state e.g. “ $-2(6x-2)(2x+1)^{-2} + 6(2x+1)^{-1} > 0$ so increasing” as it is not clear from the expression that this will be the case.

Do not withhold this mark if invisible brackets are recovered but A0* if there are errors in the method.

(c)

M1: Attempts to change the subject. Look for cross multiplication with an attempt to collect terms achieving the form $\pm 2xy \pm 6x = \pm y \pm 2$ or $\pm 2xy \pm 6y = \pm x \pm 2$. May be implied by further work.

If they start from their $y = \alpha \pm \frac{\beta}{2x+1}$ look for an alternative rearrangement to $\frac{\beta}{y \pm \alpha} = 2x+1$ (or

$$\frac{\beta}{x \pm \alpha} = 2y+1)$$

A1: $g^{-1}(x) = \frac{-x-2}{2x-6}$ or equivalent such as $g^{-1}(x) = \frac{x+2}{6-2x}$ or may be seen as $g^{-1}(x) = \frac{\frac{5}{3-x}-1}{2}$ or $g^{-1}(x) = \frac{5}{6-2x} - 0.5$

Accept $g^{-1}: x \rightarrow \frac{x+2}{6-2x}$ Condone $g^{-1} = \dots$ or $g^{-1} = y = \dots$ but do not allow just $y = \frac{x+2}{6-2x}$

B1: Correct domain $0 < x < 3$ o.e. e.g. $0 < x \cap x < 3$ or $]0, 3[$ but not $0 < y < 3$ or $0 < g^{-1}(x) < 3$

Note: It is also acceptable to define g^{-1} in any variable e.g. as $g^{-1}(t) = \frac{t+2}{6-2t}$, $0 < t < 3$ as long as

the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined e.g. $g^{-1}(t) = \dots$ not just $g^{-1} = \dots$

(d)

M1: $2 + 5\ln 3$ found. Allow awrt 7.5 instead of $2 + 5\ln 3$ for this mark only. Do not be concerned by the direction of the inequality if seen.

A1: Fully correct. $(-\infty <) fg < 2 + 5\ln 3$ o.e. $5\ln 3$ must be exact. Do not be too concerned by alternative notation to fg as long as it is not x . There are many alternatives including $y < 2 + 5\ln 3$ or $(-\infty, 2 + 5\ln 3)$ but not $[-\infty, 2 + 5\ln 3]$ as requires a strict inequality at the upper bound. Do not allow $f < 2 + 5\ln 3$ or $g < 2 + 5\ln 3$

Question Number	Scheme	Marks
3.	$\frac{dy}{dx} = \frac{4(x+3)^2 - 2(4x+1)(x+3)}{(x+3)^4}$ Solves their $4(x+3)^2 - 2(4x+1)(x+3) = 0$ $\Rightarrow (x+3)(10-4x) = 0 \Rightarrow x = \dots$ Critical value of $\frac{5}{2}$; Critical value of -3 C increasing when $\frac{dy}{dx} > 0 \Rightarrow -3 < x < \frac{5}{2}$	M1 A1 M1 A1; B1 A1 (6 marks)

Note this appears as M1A1M1A1M1A1 on open but is being marked as M1A1M1A1B1A1

M1: Attempts the quotient rule and achieves

$$\frac{dy}{dx} = \frac{a(x+3)^2 - b(4x+1)(x+3)}{(x+3)^4} \quad a, b > 0 \quad \text{condoning slips}$$

They may attempt to multiply out the $(x+3)^2$ first which is fine as long as they reach a quadratic

Product rule may be attempted, look for $\frac{dy}{dx} = A(x+3)^{-2} - B(4x+1)(x+3)^{-3}$, $A, B > 0$

Note: Only the numerator is required for this problem, and this mark can be awarded if a student attempts just $u'v - uv' = 0$. If the denominator is considered it must be treated correctly.

A1: For a correct (unsimplified) $\frac{dy}{dx}$ or $u'v - uv'$ (if this is all that is considered). By the Product rule

$$\frac{dy}{dx} = 4(x+3)^{-2} - 2(4x+1)(x+3)^{-3}$$

M1: Solves their numerator/their $\frac{dy}{dx} = 0$ (or with any inequality, and the “=0” may be implied) to

obtain at least one critical value from a non-calculator method. If the numerator is expanded to a quadratic a method of solution must be shown (e.g. factorisation or completing the square or formula seen applied) They may be working with equalities or inequalities for this mark. Note this means students who cancel the $(x+3)$ and lose the second critical value can still score M1.

A1: Critical value of $\frac{5}{2}$ (allow equivalent fractions) found provided the previous M has been scored.

B1: Finds or identifies -3 as the other critical value (no need for method shown).

A1: Depends on first M. For $-3 < x < \frac{5}{2}$ or $-3 < x, \frac{5}{2}$ or equivalent set notation, e.g.

$$x \in \left(-3, \frac{5}{2}\right] \quad \text{Condone } -3, x, \frac{5}{2} \text{ or } -3, x < \frac{5}{2}. \text{ Must be simplified fraction. Allow split}$$

inequalities $-3 < x$ and $x < \frac{5}{2}$, condoning “or” or a comma between but if set notation used it must be intersection not union.

Note: If the denominator of $\frac{dy}{dx}$ is incorrectly treated but the numerator is correct, a correct answer may be achieved. In such cases award SC M0 A0 M1 A1 B1 A1 if the subsequent marks are earned.

Note: answers relying on calculator technology where roots appear from an unsimplified derivative can score maximum M1A1M0A0B1A1



Question Number	Scheme	Marks
8.(i)	$\int \frac{2}{3x-1} dx = \frac{2}{3} \ln(3x-1)$ $\int_3^{42} \frac{2}{3x-1} dx = \frac{2}{3} \ln(125) - \frac{2}{3} \ln(8) = \ln \frac{25}{4}$	M1 A1 dM1 A1 (4)
(ii)	$\frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} = 2x + B + \frac{C}{(x-1)^2}$ Full method to find values of A , B and C $\frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} = 2x - 3 + \frac{4}{(x-1)^2}$ $\int h(x) dx = \int Ax + B + \frac{C}{(x-1)^2} dx$ $= \frac{1}{2} Ax^2 + Bx - \frac{C}{(x-1)}$ $= x^2 - 3x - \frac{4}{(x-1)} \quad (+c)$	B1 M1 A1 M1 A1 ft A1 (6)
		10 marks
(ii) Alt I first 3 marks	$2x^3 - 7x^2 + 8x + 1 = Ax(x-1)^2 + B(x-1)^2 + C$ Any of $A = 2, B = -3$, or $C = 4$ Either substitution or equating coefficients to get two values All values correct $A = 2, B = -3, C = 4$	B1 M1 A1 (3)
(ii) Alt II first 3 marks	$\begin{array}{r} 2x-3 \\ x^2-2x+1 \overline{) 2x^3-7x^2+8x+1} \\ \underline{2x^3-4x^2+2x-3} \\ 3x^2-5x+4 \\ \underline{3x^2-6x+3} \\ 9x+1 \\ \underline{9x-18} \\ 19 \end{array}$ Likely to be $2x$ For attempt at division Correct quotient and remainder	B1 M1 A1 (3)

(i)



M1 Integrates to $k \ln(3x-1)$ condoning slips. Condone with a missing bracket.

Please note that $k \ln(3ax-a)$ where a is a positive constant is also correct

If a substitution is made, i.e. $u = 3x-1$, then they must proceed to $k \ln u$ where k is a constant

A1 $\frac{2}{3} \ln(3x-1)$

Also accept $\frac{2}{3} \ln(3ax-a)$ where a is a positive constant is also correct or $\frac{2}{3} \ln u$ where $u = 3x-1$,

Do not allow with the missing bracket unless subsequent work implies that it is present

dM1 Substitutes in both limits and applies one $\ln$ law correctly. May be subtraction law or power law.

If a substitution has been made then the correct limits must be used. With u they are 125 and 8

A1 $\ln \frac{25}{4}$ or simplified equivalent such as $\ln 25 - \ln 4$, $2 \ln 5 - 2 \ln 2$ or $2 \ln \frac{5}{2}$

ISW if followed by decimals

(ii)

B1 Any correct value of A , B or C seen or implied.

M1 A full method to find values of A , B and C .

If they attempt $2x^3 - 7x^2 + 8x + 1 = Ax(x-1)^2 + B(x-1)^2 + C$ via this route, this expression must be correct.

If they attempt by division then they must proceed to a linear quotient but may get a linear remainder.

A1 Correct A , B , C or correct expression. This may be implied by a correct quotient and remainder.

M1 $\int \frac{P}{(x-1)^2} dx \rightarrow \frac{Q}{(x-1)^1}$ o.e. where P and Q could be the same

Award for $\int \frac{P}{(x-1)^2} dx \rightarrow \frac{Q}{u}$ where they have previously set $u = x-1$

A1ft $\int Ax + B + \frac{C}{(x-1)^2} dx = \frac{1}{2} Ax^2 + Bx - \frac{C}{(x-1)}$ or unsimplified equivalent.

So allow $\frac{1}{2} Ax^2 + Bx + \frac{C}{-1}(x-1)^{-1}$ with the indices processed

Also allow with non-numerical values.

Also score for $\int Ax + B + \frac{C}{(x-1)^2} dx = \frac{1}{2} Ax^2 + Bx - \frac{C}{u}$ where they have previously set $u = x-1$

A1 $x^2 - 3x - \frac{4}{(x-1)}$ (+c) or exact simplified equivalent with or without the +c

So allow $x^2 - 3x - 4(x-1)^{-1}$ (+c)

Question Number	Scheme	Marks
8. (a)	$f'(x) = 6(2x+1)^2 e^{-4x} - 4(2x+1)^3 e^{-4x}$ $= 2(2x+1)^2 e^{-4x} \{3 - 2(2x+1)\}$ $= 2(2x+1)^2 (1-4x) e^{-4x}$	M1 A1 dM1 A1 (4)
(b)	Sets $f'(x) = 0 \Rightarrow x = -\frac{1}{2}, \frac{1}{4}$ Either $f\left(-\frac{1}{2}\right) = \dots$ or $f\left(\frac{1}{4}\right) = \dots$ Both $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{4}, \frac{27}{8e}\right)$	B1 M1 A1 (3)
(c)	$\left(\frac{9}{4}, \frac{27}{e}\right)$	B1ft B1ft (2) 9 marks

Notes

(a)

M1 Attempts the product rule to achieve $P(2x+1)^2 e^{-4x} \pm Q(2x+1)^3 e^{-4x}$

May also be attempted by the quotient rule - equivalent form after e terms cancel.

A1 $f'(x) = 6(2x+1)^2 e^{-4x} - 4(2x+1)^3 e^{-4x}$ which may be unsimplifieddM1 Correctly takes out a common factor of $(2x+1)^2 e^{-4x}$ from their expression with an intermediate step before the final answer. Allow if there are minor slips in the $(2x+1)^2 e^{-4x}$ as a factor (such as $(2x+)^2 e^{-4x}$) if recovered - look for the correct remaining terms in the bracket $\{ \}$. Allow going from an expanded cubic to a factorised form for this mark:

$$e^{-4x} (2 - 24x^2 - 32x^3) \rightarrow 2(2x+1)^2 (1-4x) e^{-4x}.$$

A1 Achieves $2(2x+1)^2 (1-4x) e^{-4x}$ with no incorrect algebra. Accept with the brackets in either order.

(b)

B1 $x = -\frac{1}{2}, \frac{1}{4}$ o.e. Both required.
M1 Attempts to substitute one of $x = \pm\frac{1}{2}, \pm\frac{1}{4}$ into $f(x)$. If substitution not seen may be implied by either of $\left(-\frac{1}{2}, 0\right)$ or $\left(\frac{1}{4}, \frac{27}{8e}\right)$ o.e (accept awrt 1.24 for this mark) or by either of $\left(\frac{1}{2}, \frac{8}{e^2}\right)$ (awrt 1.08) or $\left(-\frac{1}{4}, \frac{e}{8}\right)$ (awrt 0.340) o.e.

A1 For $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{4}, \frac{27}{8e}\right)$ o.e. must be exact but isw after exact coordinates given.
Allow as $x = \dots, y = \dots$ as long as clearly paired.

Allow the M and A marks if seen in part (c) - mark (b) and (c) together.



(c)

B1ft One correct aspect applied correctly to one of their points. So for either 2 added to one of their x coordinates, or a non-zero y coordinate multiplied by 8. E.g. either $\left(\frac{9}{4}, \dots\right)$ or $\left(\dots, \frac{27}{e}\right)$ or follow through on $\left(\frac{1}{4} + 2, \dots\right)$ or $\left(\dots, 8 \times \frac{27}{8e}\right)$ etc.

B1ft $\left(\frac{9}{4}, \frac{27}{e}\right)$ only or follow through on the y coordinate only so $\left(\frac{9}{4}, 8 \times \frac{27}{8e}\right)$ (oe) **only**. B0 if another point is given. Accept awrt 9.93 for second ordinate but note 9.92 is a correct follow through on 1.24. Allow as $x = \frac{9}{4}, y = \dots$.

SC allow B1B0 if coordinates given wrong way round.



Question Number	Scheme	Marks
9(a)	$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \frac{\cos 2x}{\sin x} + \frac{2 \sin x \cos x}{\cos x} \quad (\text{One Correct identity})$ $= \frac{1 - 2 \sin^2 x}{\sin x} + \frac{2 \sin x \cancel{\cos x}}{\cancel{\cos x}}$ $= \frac{1}{\sin x} - \frac{2 \sin^2 x}{\sin x} + 2 \sin x = \frac{1}{\sin x} = \operatorname{cosec} x \quad *$	B1 M1 A1* (3)
(b)	E.g. Equation is $\operatorname{cosec}^2 \theta = 6 \cot \theta - 4 \Rightarrow 1 + \cot^2 \theta = 6 \cot \theta - 4$ E.g. $\cot^2 \theta - 6 \cot \theta + 5 = 0$ E.g. $\tan \theta = \frac{1}{5}, 1$ $\theta = 0.197, \frac{\pi}{4}$	M1 A1 dM1 A1, A1 (5)
(c)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \cot x \, dx = \left[-\operatorname{cosec} x \right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ $= 2 - \sqrt{2}$	M1 A1 (2)
		10 marks

Notes(a) **There are lots of ways of proving this statement. In general score as follows**

B1 For applying at least one CORRECT double or compound angle identity during the proof or for forming a CORRECT single fraction initially.

M1 For a correct overall strategy, e.g. applying double angle identities to reduce terms to single angle arguments and cancelling down terms to eliminate $\cos x$ terms (score at the stage $\cos x$ terms could be eliminated), or attempting a single fraction and applying relevant identities to achieve single angle argument with common factor $\cos x$ in the numerator. Allow slips in signs, such as $\cos 2x = 1 \pm 2 \sin^2 x$ for the M but otherwise identities used must be correct.A1* Fully correct proof showing all necessary steps, though the left hand side may be implied (and may follow initial lines of aside working). Must see the $\frac{1}{\sin x} \rightarrow \operatorname{cosec} x$ during the proof. Do not penalise minor notational slips such as missing an x in one term.(b) M1 Correctly applies the result of (a) and attempts to use relevant identities, allowing sign errors e.g. $\pm 1 \pm \cot^2 \theta = \operatorname{cosec}^2 \theta$ to produce an equation in $\cot \theta$ or other single trig term only. An alternative is

$$\frac{1}{\sin^2 \theta} = 6 \frac{\cos \theta}{\sin \theta} - 4 \Rightarrow 1 = 6 \sin \theta \cos \theta - 4 \sin^2 \theta \Rightarrow (1 + 4 \sin^2 \theta)^2 = 36 \sin^2 \theta (1 - \sin^2 \theta)$$

A1 Correct quadratic $\cot^2 \theta - 6 \cot \theta + 5 = 0$ or $5 \tan^2 \theta - 6 \tan \theta + 1 = 0$. In the alternative, a correct quadratic in $\sin^2 \theta$ or $\cos^2 \theta$ e.g. $52 \sin^4 \theta - 28 \sin^2 \theta + 1 = 0$. The “=0” may be implied by an attempt to solve. May be implied by correct solutions following an unsimplified quadratic.

dM1 Attempts to solve quadratic to find at least one value for their trig term used. Usual rules, may use calculator.

A1 One correct value for θ following from a correct value for the trig term they are working in — must have solved a correct quadratic in the dM. Accept awrt 0.197 or 0.785. Degrees answer are A0A0.

Question Number	Scheme	Marks
A1	Both values correct and no other values in the range. Accept awrt 0.197 and $\frac{\pi}{4}$ only (must be exact but isw after correct value seen).	
Note:	Allow if a different variable used (such as x). For mixed variables allow the M's but only allow the first A (and final A's) if recovered.	
Note	Answers without working score no marks.	
(c)		
M1	For using part (a) and achieving $\pm k \operatorname{cosec} x$ oe for the integral (limits not required for this mark. May arise from longer methods, but must achieve the correct form.	
A1	$2 - \sqrt{2}$ Must have scored the M - answer only with no integration shown is M0A0.	
(a) ALT I	$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \frac{\cos 2x \cos x + \sin 2x \sin x}{\sin x \cos x}$ Correct single fraction $= \frac{\cos x (1 - 2 \sin^2 x) + 2 \sin x \cos x \sin x}{\sin x \cos x}$ Single fraction with single arguments and common factor $\cos x$ in numerator $= \frac{\cancel{\cos x}}{\sin x \cancel{\cos x}} = \frac{1}{\sin x} = \operatorname{cosec} x^*$	B1 M1 A1* (3)
(a) ALT II	$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \frac{\cos 2x \cos x + \sin 2x \sin x}{\sin x \cos x}$ Correct single fraction $= \frac{\cos(2x - x)}{\sin x \cos x}$ Applies identity to reach single fraction with single arguments and common factor $\cos x$ in numerator $= \frac{\cancel{\cos x}}{\sin x \cancel{\cos x}} = \frac{1}{\sin x} = \operatorname{cosec} x^*$ Note $\cos(x - 2x)$ is equally correct for the M1.	B1 M1 A1* (3)
(a) ALT III	$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x} + \frac{2 \sin x \cos x}{\cos x}$ Correct identity $= \frac{\cos^3 x - \sin^2 x \cos x + 2 \sin^2 x \cos x}{\sin x \cos x}$ Single fraction with single arguments and common factor $\cos x$ in numerator $= \frac{(\cos^2 x + \sin^2 x) \cancel{\cos x}}{\sin x \cancel{\cos x}} = \frac{1}{\sin x} = \operatorname{cosec} x^*$	B1 M1 A1* (3)

(a)B1: $3x^2 + 7x - 20 = (3x - 5)(x + 4)$ seen or used

M1: Combines fractions with a correct common denominator and the order of terms in the numerator consistent with their common denominator.

A1*: Achieves the given answer with no errors seen but condone e.g. a missing trailing bracket as long as it is “recovered”.

Sufficient working should be shown but allow to go from e.g. $\frac{2(x-4)+8}{3x-5}$ to $\frac{2x}{3x-5}$

Note that candidates may take a longer route:

$$\frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} = \frac{(2x^2 - 32)(3x - 5) + 8(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{M1}$$

$$= \frac{6x^3 + 14x^2 - 40x}{(3x^2 + 7x - 20)(3x - 5)} = \frac{2x(3x^2 + 7x - 20)}{(3x - 5)(x + 4)(3x - 5)} \quad \text{B1}$$

$$= \frac{2x(3x - 5)(x + 4)}{(3x - 5)(x + 4)(3x - 5)} = \frac{2x}{3x - 5} \quad \text{A1*}$$

Or

$$\frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} = \frac{(2x^2 - 32)(3x - 5) + 8(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{M1}$$

$$= \frac{6x^3 + 14x^2 - 40x}{(3x^2 + 7x - 20)(3x - 5)} = \frac{2x(3x^2 + 7x - 20)}{(3x^2 + 7x - 20)(3x - 5)} \quad \text{B1}$$

In this case the B1 is scored for obtaining a factor of $3x^2 + 7x - 20$ in the numerator and denominator

$$= \frac{2x}{3x - 5} \quad \text{A1*}$$

(b)M1: Attempts to differentiate using the product or quotient rule. Award for an expression of the form $\frac{\alpha(3x-5) - \beta x}{(3x-5)^2}$ or $\alpha(3x-5)^{-1} - \beta x(3x-5)^{-2}$, $\alpha, \beta > 0$ Condone attempts where $(3x-5)^2$ is expanded.Alternatively, attempts to write $\frac{2x}{3x-5}$ as $A + \frac{B}{3x-5}$ before differentiating using thechain rule to obtain $\frac{\pm k}{(3x-5)^2}$

A1: Correct derivative simplified or unsimplified.

A1cso*: Requires a correct simplified derivative $\frac{-10}{(3x-5)^2}$ and then statements that convey:

- $(3x-5)^2 > 0$ or e.g. denominator is positive (condone $(3x-5)^2 \geq 0$)
- so $f'(x) < 0$
- function is decreasing



Question Number	Scheme	Marks
8 (a)	52 b.p.m.	B1
		(1)
(b)	32 b.p.m.	B1
		(1)
(c)	$\frac{dH}{dt} = -8e^{-0.2t} + 18e^{-0.9t}$	M1, A1
	Sets $-8e^{-0.2t} + 18e^{-0.9t} = 0 \Rightarrow 4e^{0.7t} = 9$	dM1
	$\Rightarrow 0.7t = \ln \frac{9}{4} \Rightarrow t = \dots$	M1
	$T = 1.158 \text{ (minutes)}$	A1
		(5)
(d)	$37 = 32 + 40e^{-0.2t} - 20e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1 + 4e^{-0.9t}}{8}$	M1
	$\Rightarrow e^{0.2t} = \frac{8}{1 + 4e^{-0.9t}} \Rightarrow t = 5 \ln \left(\frac{8}{1 + 4e^{-0.9t}} \right)$	A1*
		(2)
(e)	$t_2 = 5 \ln \left(\frac{8}{1 + 4e^{-0.9 \times 10}} \right) = \dots$	M1
	$(t_2) = \text{awrt } 10.3947$	A1
	$(M) = 10.3955$	A 1
		(3)
		Total 12

(a)

B1: 52 b.p.m. Units are not required. Check for answer given in the body of the question

(b)

B1: 32 b.p.m. . Units are not required. Check for answer given in the body of the question

(c)

M1: Achieves $\frac{dH}{dt} = P e^{-0.2t} + Q e^{-0.9t}$ where P and Q are constantsA1: $\frac{dH}{dt} = -8e^{-0.2t} + 18e^{-0.9t}$ but allow un-simplified versionsdM1: Sets $P e^{-0.2t} + Q e^{-0.9t} = 0 \Rightarrow m e^{0.7t} = n$ or $m e^{-0.7t} = n$ where $m \times n > 0$ M1: Solves an equation of the form $m e^{\pm kt} = n$ where $m \times n > 0$ using correct $\ln$ work.
It must proceed from a correct form of the derivative (so the first M1).Look for $m e^{\pm kt} = n \Rightarrow e^{\pm kt} = \frac{n}{m} \Rightarrow \pm kt = \ln \left(\frac{n}{m} \right) \Rightarrow t = \dots$ A1: $T = \text{awrt } 1.158$ minutes following the award of all previous marks. You can ignore units.....
An alternative for the last three marks in part (c) are;dM1: $P e^{-0.9t} = Q e^{-0.2t} \Rightarrow \ln P - 0.9t = \ln Q - 0.2t$ M1: Collects terms in t and proceeds to $t = \dots$ Under this method it is dependent upon both previous M'sA1: $T = \text{awrt } 1.158$ minutes following the award of all previous marks. Units are not required

.....
 There may be answers produced from a calculator. In cases such as this we are going to apply the rule 'answers without full working may not gain credit.'

$$\frac{dH}{dt} = -8e^{-0.2t} + 18e^{-0.9t} = 0 \Rightarrow t = 1.158 \text{ scores } 1,1,0,0,0$$

.....
 (d)

M1: Sets $H = 37$ and proceeds to make $e^{\pm 0.2t}$ the subject

A1*: Proceeds to the given answer showing all necessary steps.

Note that this is a given answer so the steps must be logical without huge jumps

Acceptable proofs:

Example 1

$$\begin{aligned} 37 &= 32 + 40e^{-0.2t} - 20e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1 + 4e^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1 + 4e^{-0.9t}}{8}\right) \\ &\Rightarrow t = -5\ln\left(\frac{1 + 4e^{-0.9t}}{8}\right) \\ &\Rightarrow t = 5\ln\left(\frac{8}{1 + 4e^{-0.9t}}\right) \end{aligned}$$

Example 2

$$37 = 32 + 40e^{-0.2t} - 20e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1 + 4e^{-0.9t}}{8} \Rightarrow e^{0.2t} = \frac{8}{1 + 4e^{-0.9t}} \Rightarrow 0.2t = \ln\left(\frac{8}{1 + 4e^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1 + 4e^{-0.9t}}\right)$$

Unacceptable proof:

$$\begin{aligned} 37 &= 32 + 40e^{-0.2t} - 20e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1 + 4e^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1 + 4e^{-0.9t}}{8}\right) \\ &\Rightarrow t = 5\ln\left(\frac{8}{1 + 4e^{-0.9t}}\right) \end{aligned}$$

They need to deal with 0.2 and $-$ sign in separate steps

.....
 Alt (d) lns can be taken earlier. The M1 is scored for making $0.2t$ the subject. See below

$$\begin{aligned} 37 &= 32 + 40e^{-0.2t} - 20e^{-0.9t} \\ \Rightarrow 40e^{-0.2t} &= 5 + 20e^{-0.9t} \Rightarrow \ln 40 - 0.2t = \ln(5 + 20e^{-0.9t}) \Rightarrow 0.2t = \ln 40 - \ln(5 + 20e^{-0.9t}) \end{aligned}$$

We would need to see three further steps. E.g.

$$0.2t = \ln\left(\frac{40}{5 + 20e^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{40}{5 + 20e^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1 + 4e^{-0.9t}}\right)$$

.....
 (e)

M1: Uses the iteration formula once. Allow for the embedded 10 leading to a value or sight of $t_2 = \text{awrt } 10.4$

A1: $(t_2) = \text{awrt } 10.3947$

A1: $(M) = 10.3955$ following some evidence of previous iterations, e.g. sight of

$$t_2 = 5 \ln \left(\frac{8}{1 + 4e^{-0.9 \times 10}} \right) = \text{awrt } 10.4 \dots \quad \text{This is NOT awrt so just } 10.3955 \text{ here}$$

Question Number	Scheme	Marks
9 (a)	$f'(x) = \frac{(2x+1) \times (12x+4) - 2(6x^2+4x-2)}{(2x+1)^2}$	M1 A1
	$= \frac{12x^2 + 12x + 8}{(2x+1)^2} \text{ o.e}$	A1
		(3)
(b)	At $x=2 \Rightarrow f'(x) = \frac{12 \times 2^2 + 12 \times 2 + 8}{(2 \times 2 + 1)^2} = \left(\frac{16}{5} \right)$	M1
	Full method of normal $y - 6 = -\frac{5}{16}(x - 2)$	dM1
	$16y - 96 = -5x + 10 \Rightarrow 16y + 5x = 106 \quad *$	A1*
		(3)
(c)	Any correct value $A = 3, B = \frac{1}{2} \text{ or } D = -\frac{5}{2}$	B1
	$6x^2 + 4x - 2 = (Ax + B)(2x + 1) + D \Rightarrow \text{Values of } A, B \text{ and } D$	M1
	$3x + \frac{1}{2} + \frac{-5/2}{2x+1} \text{ o.e}$	A1
		(3)
(d)	$\int 3x + \frac{1}{2} + \frac{-5/2}{2x+1} dx = \frac{3}{2}x^2 + \frac{1}{2}x - \frac{5}{4} \ln(2x+1)$	M1, A1 ft
	Area under curve = $\left[\frac{3}{2}x^2 + \frac{1}{2}x - \frac{5}{4} \ln(2x+1) \right]_1^2 = \left(\frac{3}{2} \times 2^2 + \frac{1}{2} \times 2 - \frac{5}{4} \ln(5) \right) - \left(\frac{3}{2} \times \left(\frac{1}{3} \right)^2 + \frac{1}{2} \times \left(\frac{1}{3} \right) - \frac{5}{4} \ln \left(\frac{5}{3} \right) \right)$	dM1
	Area of $R = \int_{\frac{1}{3}}^2 3x + \frac{1}{2} + \frac{-5/2}{2x+1} dx + \frac{1}{2} \times 6 \times \left(\frac{106}{5} - 2 \right)$	M1
	Area of $R = \frac{964}{15} - \frac{5}{4} \ln 3$	A1
		(5)
		Total 14

(a)

M1: Condoning slips, this is for an attempt using

- the quotient rule to obtain $f'(x) = \frac{(2x+1) \times (Ax+B) - C(6x^2+4x-2)}{(2x+1)^2}$ Must be '-' on the numerator
- the chain rule on $3x + A + \frac{B}{2x+1} \Rightarrow f'(x) = 3 \pm D(2x+1)^{-2}$
- the product rule on $(6x^2+4x-2)(2x+1)^{-1} \Rightarrow f'(x) = (2x+1)^{-1} \times (Ax+B) - C(6x^2+4x-2)(2x+1)^{-2}$ where A, B and C are > 0

A1: Correct (unsimplified) expression for $f'(x)$. Look for

- $f'(x) = \frac{(2x+1) \times (12x+4) - 2(6x^2 + 4x - 2)}{(2x+1)^2}$ via the quotient rule
- $f'(x) = (2x+1)^{-1} \times (12x+4) + (6x^2 + 4x - 2) \times -2(2x+1)^{-2}$ via the product rule
- $3 + 5(2x+1)^{-2}$ following division. Condone an incorrect value for A.

A1: Simplified answers such as $\frac{4(3x^2 + 3x + 2)}{(2x+1)^2}$, $\frac{12x^2 + 12x + 8}{4x^2 + 4x + 1}$ or $3 + 5(2x+1)^{-2}$ following correct A.

ISW following a correct answer

(b)

M1: Substitutes $x = 2$ into their attempt at $f'(x)$.

If $f'(x)$ is incorrect then look for correctly embedded 2's or a correct value.

dM1: Full method for finding the equation of the normal at $(2, 6)$. Look for $y - 6 = -\frac{1}{f'(2)}(x - 2)$

A1*: $y - 6 = -\frac{5}{16}(x - 2)$ o.e with at least one correct intermediate line leading to $16y + 5x = 106$

If $y = mx + c$ is used look for $6 = -\frac{5}{16} \times 2 + c \Rightarrow c = \frac{106}{16}$ You can allow $y = -\frac{5}{16}x + \frac{106}{16}$ going straight to the given answer. ISW following sight of correct answer. It must follow a correct derivative in (a)

(c)

B1: Any correct value for $A = 3$, $B = \frac{1}{2}$ or $D = -\frac{5}{2}$.

They may be embedded within an expression e.g. $3x$ in the quotient of a division sum

M1: Correct method for finding all three constants.

Via comparison/inspection look for $6x^2 + 4x - 2 = (Ax + B)(2x + 1) + D \Rightarrow$ Values of A, B and D

Via division look for a linear quotient and a constant remainder

A1: $3x + \frac{1}{2} + \frac{-5/2}{2x+1}$ o.e such as $3x + \frac{1}{2} - \frac{5}{2(2x+1)}$ which may be seen within an integral in (d).

It is not just for values of A, B and D.

ISW following a correct answer (e.g. they may go on to $\times 2$ but then the A1 ft in (d) would be unavailable)



(d)

M1: Attempts to integrate their $Ax + B + \frac{D}{2x+1}$. Look for two terms in the correct form, one of which must

be the $\ln$ term. Award for $\dots x^2 + \dots + k \ln(2x+1)$ or $\dots + \dots x + k \ln(2x+1)$

Also note that $k \ln(4x+2)$ and any multiple of $(2x+1)$ within the $\ln$ is also correct

A1ft: $\frac{3}{2}x^2 + \frac{1}{2}x - \frac{5}{4}\ln(2x+1)$ o.e. but follow through on their values of A , B and D

dM1: A correct method of finding the area under the curve using limits of $\frac{1}{3}$ and 2 either way around.

Dependent upon the previous M mark. Award if the intention is clear.

M1: For a correct method of finding the area of R . It is scored for adding the area under the curve to the correct calculation for the area of the triangle. The integration of $f(x)$ need not be correct but the limits must be applied the correct way around. The area of the triangle may be done by integration. Look for

$$\int_{\frac{1}{3}}^2 f(x) \, dx + \frac{1}{2} \times 6 \times \left(\frac{106}{5} - 2 \right) = \left[I(x) \right]_{\frac{1}{3}}^2 + \frac{1}{2} \times 6 \times \left(\frac{106}{5} - 2 \right) = I(2) - I\left(\frac{1}{3}\right) + \frac{1}{2} \times 6 \times \left(\frac{106}{5} - 2 \right) = \dots$$

where $I(x)$ is their attempt at $\int f(x) \, dx$

A1: $\frac{964}{15} - \frac{5}{4}\ln 3$