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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Numerical Methods (P3.6)

International A Level Mathematics · P3: Pure Mathematics 3 (WMA13)

11 questions · 89 marks · Mark schemes included

Past papers January 2020 - June 2025 · 1 sub-topic

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 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
P3.6a LOCATING ROOTS & ITERATION		11 Qs · 89 marks	2
1. Q1	IAL P3 (WMA13) June 2023	5	3
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QUESTIONS

11 questions · 89 marks · 1 sub-topic

SUB-TOPIC P3.6a

LOCATING ROOTS & ITERATION

Sub-topic P3.6a · 11 questions · 89 marks · lowest tariff first



$$g(x) = x^6 + 2x - 1000$$

(2)

$$x_{n+1} = \sqrt[6]{1000 - 2x_n} \quad \text{with } x_1 = 3$$

(3)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)



(Total for Question 1 is 5 marks)



(Total for Question 2 is 6 marks)



(Total for Question 1 is 6 marks)



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



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Question 6 continued

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Q6

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6.

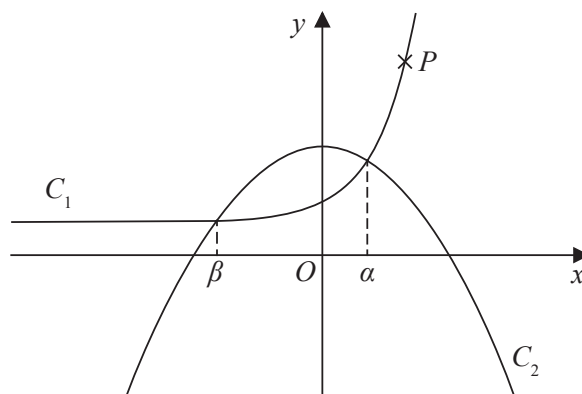
**Figure 3**

Figure 3 shows a sketch of curve C_1 with equation $y = 5e^{x-1} + 3$

and curve C_2 with equation $y = 10 - x^2$

The point P lies on C_1 and has y coordinate 18

- (a) Find the x coordinate of P , writing your answer in the form $\ln k$, where k is a constant to be found.

(3)

The curve C_1 meets the curve C_2 at $x = \alpha$ and at $x = \beta$, as shown in Figure 3.

- (b) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places $\alpha = 1.134$

(3)

The iterative equation

$$x_{n+1} = -\sqrt{7 - 5e^{x_n-1}}$$

is used to find an approximation to β .

Using this iterative formula with $x_1 = -3$

- (c) find the value of x_2 and the value of β , giving each answer to 6 decimal places.

(3)

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Q6

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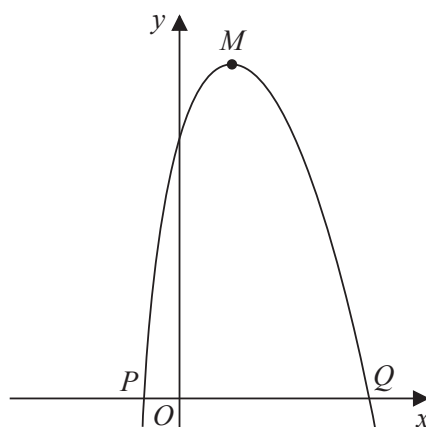


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 6 \ln(2x + 3) - \frac{1}{2}x^2 + 4 \quad x > -\frac{3}{2}$$

The curve cuts the negative x -axis at the point P , as shown in Figure 1.

- (a) Show that the x coordinate of P lies in the interval $[-1.25, -1.2]$

(2)

The curve cuts the positive x -axis at the point Q , also shown in Figure 1.

Using the iterative formula

$$x_{n+1} = \sqrt{12 \ln(2x_n + 3) + 8} \quad \text{with } x_1 = 6$$

- (b) (i) find, to 4 decimal places, the value of x_2
 (ii) find, by continued iteration, the x coordinate of Q . Give your answer to 4 decimal places.

(3)

The curve has a maximum turning point at M , as shown in Figure 1.

- (c) Using calculus and showing each stage of your working, find the x coordinate of M .

(4)

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Q5

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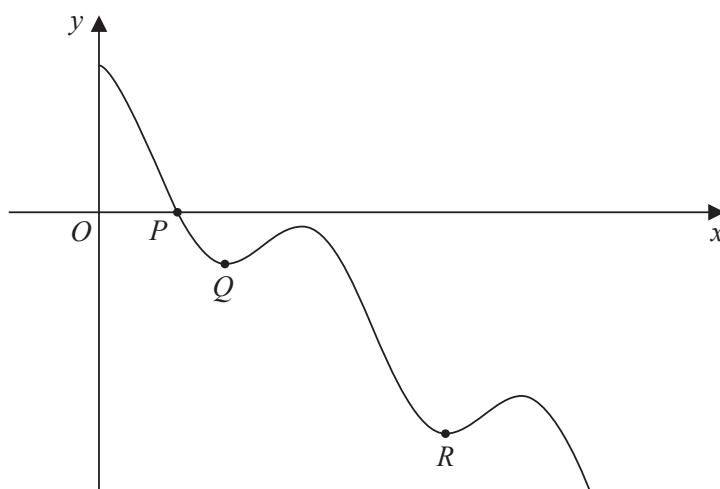


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2 \cos 3x - 3x + 4 \quad x > 0$$

where x is measured in radians.

The curve crosses the x -axis at the point P , as shown in Figure 3.

Given that the x coordinate of P is α ,

- (a) show that α lies between 0.8 and 0.9

(2)

The iteration formula

$$x_{n+1} = \frac{1}{3} \arccos(1.5x_n - 2)$$

can be used to find an approximate value for α .

- (b) Using this iteration formula with $x_1 = 0.8$ find, to 4 decimal places, the value of

(i) x_2

(ii) x_5

(3)

The point Q and the point R are local minimum points on the curve, as shown in Figure 3.

Given that the x coordinates of Q and R are β and λ respectively, and that they are the two smallest values of x at which local minima occur,

- (c) find, using calculus, the exact value of β and the exact value of λ .

(6)

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Q7

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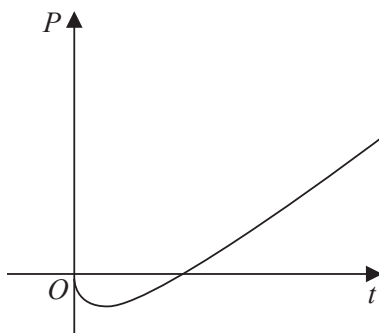


Figure 2

The profit made by a company, £ P million, t years after the company started trading, is modelled by the equation

$$P = \frac{4t-1}{10} + \frac{3}{4} \ln \left[\frac{t+1}{(2t+1)^2} \right]$$

The graph of P against t is shown in Figure 2.

According to the model,

- (a) show that exactly one year after it started trading, the company had made a loss of approximately £830 000 (2)

A manager of the company wants to know the value of t for which $P = 0$

- (b) Show that this value of t occurs in the interval $[6, 7]$ (2)
- (c) Show that the equation $P = 0$ can be expressed in the form

$$t = \frac{1}{4} + \frac{15}{8} \ln \left[\frac{(2t+1)^2}{t+1} \right] \quad (2)$$

- (d) Using the iteration formula

$$t_{n+1} = \frac{1}{4} + \frac{15}{8} \ln \left[\frac{(2t_n+1)^2}{t_n+1} \right] \quad \text{with } t_1 = 6$$

find the value of t_2 and the value of t_6 , giving your answers to 3 decimal places. (3)

- (e) Hence find, according to the model, how many months it takes in total, from when the company started trading, for it to make a profit. (2)

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Q5

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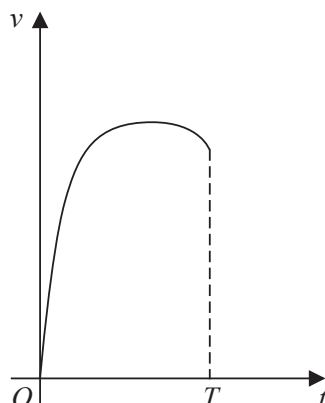


Figure 4

Figure 4 is a graph showing the velocity of a sprinter during a 100 m race.

The sprinter's velocity during the race, $v \text{ m s}^{-1}$, is modelled by the equation

$$v = 12 - e^{t-10} - 12e^{-0.75t} \quad t \geq 0$$

where t seconds is the time after the sprinter begins to run.

According to the model,

- (a) find, using calculus, the sprinter's maximum velocity during the race. (5)

Given that the sprinter runs 100 m in T seconds, such that

$$\int_0^T v \, dt = 100$$

- (b) show that T is a solution of the equation

$$T = \frac{1}{12} (116 - 16e^{-0.75T} + e^{T-10} - e^{-10}) \quad (4)$$

The iteration formula

$$T_{n+1} = \frac{1}{12} (116 - 16e^{-0.75T_n} + e^{T_n-10} - e^{-10})$$

is used to find an approximate value for T

Using this iteration formula with $T_1 = 10$

- (c) find, to 4 decimal places,
- (i) the value of T_2
 - (ii) the time taken by the sprinter to run the race, according to the model. (3)

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Q8

MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme	Marks
1. (a)	$g(3) = -265, \quad g(4) = 3104$ States change of sign, continuous and hence root in $[3, 4]$	M1 A1 (2)
(b)	$x_2 = \sqrt[6]{1000 - 2 \times 3} = 3.1591$ $(\alpha =) 3.1589$	M1 A1 A1 (3)
Notes		
(a)		
M1	Attempts the value of g at 3 and 4 with one correct (accept any value for the other as an attempt). Note narrower ranges are possible but must contain the root and lies in $[3, 4]$.	
A1	Both values correct with reason (Sign change (stated or indicated) and continuous function) and minimal conclusion (root)	
(b)		
M1	Attempts to substitute $x_1 = 3$ into the formula. Implied by sight of expression, awrt 3.159	
A1	awrt 3.1591	
A1	$(\alpha =) 3.1589$ cao - must be to 4 d.p. Do not be concerned about the labelling of the root (x or α etc), mark the final answer of (b)(ii). (Note sight of this value implies the M1 even if x_2 is not seen).	

Question Number	Scheme	Marks
1(a)	$(f(1)=)1-5+e=-1.281... < 0$ $(f(2)=)2^2-5 \times 2 + e^2 = 1.389... > 0$	M1
	As there is a <u>change of sign</u> and $f(x)$ is <u>continuous</u> over the interval $[1, 2]$ then <u>there is a root</u> *	A1*
		(2)
(b)(i)	$x_2 = \sqrt{5 \times 1 - e^1} = \text{awrt } 1.5105$	M1A1
(ii)	$\alpha = \text{awrt } 1.7340$	A1
		(3)
		(5 marks)

Notes**(a)**

M1: Attempts $f(1)$ and $f(2)$ with substitution seen or at least one correct to 1 d.p. rounded or truncated **and considers their signs**. Note showing $f(1)f(2) < 0$ is a consideration of signs and is sufficient for the “sign change” part of reasoning for the A1.

A1*: Must have

- both $f(1)$ and $f(2)$ correct (as expressions or correct values imply these, need not be labelled)
- reason, which must mention continuity and state or indicate sign change in some way
- conclusion, “hence root”, or accept e.g. hence “ $f(x)=0$ between $x=1$ and $x=2$ ”

(b)**(i)**

M1: Attempts to find x_2 using the iteration formula. Implied by sight of 1 embedded in the formula or awrt 1.5

A1: awrt 1.5105

(ii)

A1: awrt 1.7340 provided M1 has been scored. Allow 1.734 with trailing zero omitted.

Note 1.7340 only may be from a graphical calculator, so scores M0A0A0. There must be evidence (i.e. the M is scored) of an attempt at least one iteration first.



General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$ **Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use the correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Marks
6(a)	$(f'(x)) = \cos\left(\frac{x}{3}\right) - \frac{1}{3}x \sin\left(\frac{x}{3}\right)$	M1 A1
		(2)
(b)	$f'(x) = 0 \Rightarrow \cos\left(\frac{x}{3}\right) - \frac{1}{3}x \sin\left(\frac{x}{3}\right) = 0 \Rightarrow 1 - \frac{1}{3}x \tan\left(\frac{x}{3}\right) = 0$	M1
	$\tan\left(\frac{x}{3}\right) = \frac{3}{x} \Rightarrow x = 3 \arctan\left(\frac{3}{x}\right) \quad *$	A1*
		(2)
(c)	$x_2 = 3 \arctan\left(\frac{3}{2.5}\right) = \text{awrt } 2.6$	M1
	$x_2 = \text{awrt } 2.628 \text{ and } x_6 = \text{awrt } 2.586$	A1
		(2)
(d)	$f'(2.5815) = \cos\left(\frac{2.5815}{3}\right) - \frac{1}{3}(2.5815) \sin\left(\frac{2.5815}{3}\right) = -0.000345...$ $f'(2.5805) = \cos\left(\frac{2.5805}{3}\right) - \frac{1}{3}(2.5805) \sin\left(\frac{2.5805}{3}\right) = 0.000346...$ Chooses a suitable interval and attempts both values	M1
	(Both values correct) Sign change and continuous, therefore root	A1
		(2)
		Total 8

(a)

M1: Attempts the product rule and obtains a derivative of the form $\alpha \cos\left(\frac{x}{3}\right) \pm \beta x \sin\left(\frac{x}{3}\right)$

If the rule is stated or implied to be $'vu' - uv'$ it is M0

A1: $\cos\left(\frac{x}{3}\right) - \frac{1}{3}x \sin\left(\frac{x}{3}\right)$ which may be unsimplified

(b)

M1: Sets their $f'(x) = \alpha \cos\left(\frac{x}{3}\right) \pm \beta x \sin\left(\frac{x}{3}\right) = 0$ and proceeds to an equation involving $\tan\left(\frac{x}{3}\right)$

A1*: CSO Proceeds to $x = 3 \arctan\left(\frac{3}{x}\right)$ following an intermediate line of $\tan\left(\frac{x}{3}\right) = \frac{3}{x}$ or $\tan\left(\frac{x}{3}\right) = \frac{1}{\left(\frac{x}{3}\right)}$

Do not condone $\tan^{-1}$ notation unless correct notation is also given.

(c)

M1: Uses a formula of the type $x = \alpha \arctan\left(\beta \times \frac{1}{x}\right)$ with the 2.5 to find the value of x_2 correct to one dp.

So when $x = 3 \arctan\left(\frac{3}{x}\right)$ M1 is scored for $x_2 = \text{awrt } 2.6$

and when $x = 3 \arctan\left(\frac{1}{x}\right)$ M1 is scored for $x_2 = \text{awrt } 1.1$

A1: $x_2 = \text{awrt } 2.628$ and $x_6 = \text{awrt } 2.586$



(d)

M1: Chooses a suitable function for **their** $f'(x) = 0$ and attempts its value at both 2.5815 and 2.5805.

For the attempt we need to see embedded values as in scheme or one value correct to 1sf for their $f'(x)$

Allowable functions are $f'(x) = \cos\left(\frac{x}{3}\right) - \frac{1}{3}x \sin\left(\frac{x}{3}\right)$ and multiples of this. See scheme.

Follow through on their $\cos\left(\frac{x}{3}\right) - \frac{1}{3}x \sin\left(\frac{x}{3}\right)$

Allow the function to be stated as just $f'(x)$ (or mistakenly written as $f(x)$) as long as one of $f'(2.5805) = \dots$ or $f'(2.5815) = \dots$ is correct to 1sf

Other functions are possible, for example $h(x) = x - 3 \arctan\left(\frac{3}{x}\right)$ and multiples of this.

Follow through on their $x - k \arctan\left(\frac{k}{x}\right)$ Unlikely, but it is also acceptable to pick a tighter interval

A1: Requires correct differentiation and

- both values correct (rounded or truncated to 1sf)
Note that $h(2.5815) = 0.000786\dots$, $h(2.5805) = -0.000788\dots$
- a valid reason that includes both a reference to the sign change and continuity. Condone a mention of continuity of $f(x)$ instead of $f'(x)$
- a minimal conclusion which could be ✓, QED, root.



Question Number	Scheme	Marks
6 (a)	$5e^{x-1} + 3 = 18 \Rightarrow e^{x-1} = 3$ $\Rightarrow x = \ln 3 + 1$ or $e^x = 3e$ $\Rightarrow x = \ln 3e$	M1 A1 A1 (3)
(b)	Sets $5e^{x-1} + 3 = 10 - x^2$ and proceeds to find and use a suitable function. Eg $(f(x) =) 7 - x^2 - 5e^{x-1}$ Attempts $f(1.1335) = 0.001$ and $f(1.1345) = -0.007$ Correct values with reason(change of sign and continuous) and conclusion, hence α is 1.134 to 3dp	B1 M1 A1 (3)
(c)	$x_2 = -\sqrt{7 - 5e^{-3-1}} = -2.628388$ $\beta = -2.620330$	M1 A1 A1 (3)
		(9 marks)

(a)

M1 For attempting to proceed from $5e^{x-1} + 3 = 18$ to $e^{x-1} = \dots$ or $e^x = \dots$ A1 $x = \ln 3 + 1$ or for $e^x = 3e$ A1 $x = \ln 3e$ Accept $x = \ln 3e^1$

(b)

B1 Sets the equations equal to each other and finds a suitable function which is then used.

Suitable functions are $f(x) = \pm(7 - x^2 - 5e^{x-1})$, $g(x) = \pm(x - \sqrt{7 - 5e^{x-1}})$ or $h(x) = \pm\left(x - 1 - \ln\left(\frac{7 - x^2}{5}\right)\right)$

M1 Substitutes $x = 1.1335$ and $x = 1.1345$, or suitable values for a tighter interval (either side of 1.133634..), into a suitable function and obtains one correct value to 1sf rounded or truncated.

FYI for the + options above

$$f(1.1335) = 0.001 \text{ and } f(1.1345) = -0.007$$

$$g(1.1335) = -0.0005 \text{ and } g(1.1345) = 0.003$$

$$h(1.1335) = -0.0002 \text{ and } h(1.1345) = 0.001$$

A1 Requires both values to be correct (1sf rounded or truncated), a reason (change of sign and continuous) and a minimal conclusion

(c)

M1 For an attempt at substituting -3 into the iterative equation.This is implied by the sight of the expression or awrt -2.63 A1 awrt -2.628388 A1 $\beta =$ awrt -2.620330 condone -2.62033

It is possible in part (b) to score B0 M1 A1 by comparing y values for $y = 5e^{x-1} + 3$ and $y = 10 - x^2$ at $x = 1.1335$ and $x = 1.1345$ For the A1 apply the similar criteria as for the main scheme with values to 3d.p..

$$\text{At } x = 1.1335, y|_{1.1335} = 5e^{x-1} + 3 = 8.714 \text{ and } y|_{1.1335} = 10 - x^2 = 8.715$$

$$\text{At } x = 1.1345, y|_{1.1345} = 5e^{x-1} + 3 = 8.720 \text{ and } y|_{1.1345} = 10 - x^2 = 8.713$$



Question Number	Scheme	Marks
3 (i)	$\int (2x-5)^7 dx = \frac{(2x-5)^8}{8} + c$	M1 A1 (2)
(ii)	$\int \frac{4 \sin x}{1+2 \cos x} dx = -2 \ln(1+2 \cos x) + c$ $\int_0^{\frac{\pi}{3}} \frac{4 \sin x}{1+2 \cos x} dx = [-2 \ln(1+2 \cos x)]_0^{\frac{\pi}{3}} = -2 \ln 2 + 2 \ln 3 = \ln \frac{9}{4}$	M1 A1 dM1 A1 (4) (6 marks)

(i)

M1: Achieves $a(2x-5)^8$ or equivalent where a is a constant. Alternatively achieves au^8 with

$$u = (2x-5)$$

Allow this mark from a miscopy such as $\int (2x-3)^7 dx = k(2x-3)^8$

A1: Achieves $\frac{(2x-5)^8}{8} + c$ or exact simplified equivalent such as $\frac{1}{8}(2x-5)^8 + c$. **The +c (o.e) must be present.**

Any attempts that start by multiplying out $(2x-5)^7$ are likely to end in failure. They are unlikely to get an expression of the form $a(2x-5)^8$

FYI

$$(2x-5)^7 = 128x^7 - 2240x^6 + 16800x^5 - 70000x^4 + 175000x^3 - 262500x^2 + 218750x - 78125$$

Score B1 SC for at least 5 out of 8 **correct** terms of

$$\frac{128}{8}x^8 - \frac{2240}{7}x^7 + \frac{16800}{6}x^6 - \frac{70000}{5}x^5 + \frac{175000}{4}x^4 - \frac{262500}{3}x^3 + \frac{218750}{2}x^2 - 78125x$$

(ii)

M1: Achieves $b \ln(1+2 \cos x)$ or $b \ln|1+2 \cos x|$ where b is a constant. Condone a missing bracketAlternatively achieves $b \ln u$ with $u = 1+2 \cos x$ (You may see $b \ln ku$ which is also correct)

A1: Achieves $-2 \ln(1+2 \cos x)$, $-2 \ln|1+2 \cos x|$ or $-2 \ln u$ o.e. with $u = 1+2 \cos x$ oe. There is no need for $+c$. This may be left unsimplified. Only condone a missing bracket if subsequent work implies one

dM1: Substitutes **both** 0 and $\frac{\pi}{3}$ into an expression of the form $b \ln(1+2 \cos x)$ or $b \ln|1+2 \cos x|$

and subtracts either way around. There must have been some attempt to evaluate the trig terms

Alternatively substitutes both 3 and 2 into an expression of the form $k \ln u$ and subtracts

$$A1: \ln \frac{9}{4}$$

Note that algebraic integration must be seen here. Candidates using their calculators to just write down

$$\int_0^{\frac{\pi}{3}} \frac{4 \sin x}{1+2 \cos x} dx = 0.81093... = \ln \frac{9}{4} \text{ should be awarded 0 marks}$$



Question Number	Scheme	Marks
6.(a)	(2.5, 3) oe	B1 B1 (2)
(b)	Attempts one solution usually $4x - 10 + 3 = 3x - 2 \Rightarrow x = 5$ Attempts both solutions $-4x + 10 + 3 = 3x - 2 \Rightarrow x = \frac{15}{7}$	M1 A1 dM1 A1 (4)
(c)	Attempts to solve $y = kx + 2$ with $x = 2.5, y = 3$ or states that $k < 4$ $k \dots \frac{2}{5}$ States $\frac{2}{5} < k < 4$	M1 A1 A1 (3) 9 marks

(a)

B1 For one correct coordinate, either $x = 2.5$ or $y = 3$ B1 For (2.5, 3). Allow $x = \dots, y = \dots$ Allow exact equivalents

If a candidate reverses these and just writes down (3, 2.5) score SC 10

(b)

M1 For a correct method of finding one point of intersection. The signs of the terms must be correct

Accept $2(2x - 5) + 3 = 3x - 2 \Rightarrow x = \dots$ oe or $-2(2x - 5) + 3 = 3x - 2 \Rightarrow x = \dots$ oeNote that equations such as $2(2x - 5) + 3 = -3x + 2 \Rightarrow x = \dots$ are incorrect and score M0A1 For 5 or $\frac{15}{7}$ Ignore any reference to the y coordinate

dM1 For a correct method of finding both intersections

A1 For both values 5 and $\frac{15}{7}$ (with no extra solutions)Ignore any reference to the y coordinates or $x = \frac{2}{3}$ within the body of their solution.

(c)

M1 Attempts to solve $y = kx + 2$ with THEIR $x = 2.5, y = 3$ to find k or deduces that $k < 4$ An equivalent is attempting to set k equal to the gradient between (0, 2) and (2.5, 3)A1 Finds that $k = \frac{2}{5}$ is a critical valueA1 $\frac{2}{5} < k < 4$ 

.....
 Alt to (b) via squaring. Cannot be scored via squaring each term.

In reality expect the squaring to start from the point where $2|2x-5|=3x-2\pm 3$ is simplified

M1A1 $4(2x-5)^2 = (3x-5)^2 \Rightarrow 7x^2 - 50x + 75 = 0$

dM1A1 Solve 5 and $\frac{15}{7}$

.....

Alternatives to (c)

Alt I

M1A1 Deduces that it must hit the lhs once so $kx+2 = -4x+13 \Rightarrow x = \frac{11}{k+4} < \frac{5}{2} \Rightarrow k > \frac{2}{5}$

Variations on this are

$$kx+2 = 4x-7 \Rightarrow x = \frac{9}{4-k} > \frac{5}{2} \Rightarrow k > \frac{2}{5} \quad \text{for M1 A1}$$

$$\frac{11}{4+k} < \frac{9}{4-k} \Rightarrow k > \frac{2}{5} \text{ for M1 A1}$$

Alt II via squaring

M1 Sets $2|2x-5| = kx+2\pm 3$, collects terms, squares and writes in the form $Ax^2 + Bx + C = 0$

FYI they should get $(16-k^2)x^2 + (2k-80)x + 99 = 0$

Then attempts to use $b^2 - 4ac = 0$ with at least a, b in terms of k reaching a value or values for k

A1 $k = \frac{2}{5}$

.....



Question Number	Scheme	Marks
7. (a)	$y _{0.8} = 2 \cos 2.4 - 2.4 + 4 = 0.13$ AND $y _{0.9} = 2 \cos 2.7 - 2.7 + 4 = -0.51$ States change of sign, continuous and hence root	M1 A1 (2)
(b) (i)	$(x_2) = \frac{1}{3} \arccos(1.5 \times 0.8 - 2) = 0.8327$	M1 A1
(ii)	$x_5 = 0.8110$	A1
(c)	$\frac{dy}{dx} = -6 \sin 3x - 3$ Attempts $\frac{dy}{dx} = 0 \Rightarrow \sin 3x = -\frac{1}{2} \Rightarrow x = \dots$ via correct order of operations Achieves either $\frac{7\pi}{18}$ or $\frac{19\pi}{18}$ Correct attempt to find the third positive solution e.g. $x = \frac{\frac{\pi}{6} + 3\pi}{3}$ $\beta = \frac{7\pi}{18}$ and $\lambda = \frac{19\pi}{18}$	M1 A1 dM1 A1 ddM1 A1 (6)
		11 marks

(a)

M1 Attempts the value of y at 0.8 **AND** 0.9 with at least one correct to 1 sf rounded or truncated.

Note that it is possible to choose a tighter interval containing the root but to score the A1 the conclusion must refer to the given interval.

A1 Both values correct to 1sf rounded or truncated, with reason (Sign change **and** continuous function) and minimal conclusion (root)

If the candidate chooses 0.8 and 0.9 the minimal conclusion does not need to mention the interval.

So e.g. $y|_{0.8} = 0.1 > 0$, $y|_{0.9} = -0.5 < 0$ and function is continuous, so ✓ would be acceptable

(b) (i)

M1 Attempts to substitute $x_1 = 0.8$ into the formula. Implied by sight of embedded values in expression or awrt 0.83

A1 AWRT 0.8327

(b)(ii)

A1 $x_5 = 0.8110$ CAO. This is not awrt and 0.811 is A0 unless preceded by 0.8110

If it is clearly marked (b)(ii) then you don't need the x_5

(c)

M1 For $\left(\frac{dy}{dx}\right) = A \sin 3x + B$

A1 $\left(\frac{dy}{dx}\right) = -6 \sin 3x - 3$

dM1 Attempts $\frac{dy}{dx} = 0 \Rightarrow \sin 3x = a$, $|a| < 1 \Rightarrow x = \dots$ It is dependent upon the previous M

Look for correct order of operations, invsina then $\div 3$ leading to a value for x .

When $\sin 3x = -\frac{1}{2}$ it is implied, for example, by $-\frac{\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}$ (2nd soln), $\frac{19\pi}{18}$ amongst others or

if answers are given as decimals, for example, by awrt -0.17 , awrt 1.2 , awrt 1.92 or awrt 3.32

For $\sin 3x = +\frac{1}{2}$ it would be implied, for example, by values such as $\frac{\pi}{18}, \frac{5\pi}{18}$ awrt 0.17 or awrt 0.87

The calculations must be using radians. If degrees are used initially they must be converted to radians

A1 For recognising that either $\frac{7\pi}{18}$ or $\frac{19\pi}{18}$ is a solution to $\sin 3x = -\frac{1}{2}$

ddM1 Attempts to find the solution for λ in the correct quadrant.

Look for the **3rd positive solution** for their $\sin 3x = k$

So for $k > 0$ it would be for $x = \frac{2\pi + \arcsin|k|}{3}$

And for $k < 0$ it would be for $x = \frac{3\pi + \arcsin|k|}{3}$

A1 States that $\beta = \frac{7\pi}{18}$ and $\lambda = \frac{19\pi}{18}$ Labels must be correct



Question	Scheme	Marks
1(a)	$2x^3 - 4x - 15 = (Ax + B)(x^2 + 3x + 4) + C(2x + 3) \Rightarrow A = \dots$ or $\begin{array}{r} 2x^3 - 4x - 15 \\ x^2 + 3x + 4 \overline{) 2x^3 + 0x^2 - 4x - 15} \\ \underline{2x^3 + 6x^2 + 8x + 12} \\ -6x^2 - 12x - 27 \\ \underline{-6x^2 - 18x - 24} \\ 6x + 3 \end{array}$	M1
	$A = 2$	A1
	$B = -6, C = 3$	A1A1
		(4)
(b)	$\int f(x) dx = "A" \frac{x^2}{2} + "B" x + "C" \ln(x^2 + 3x + 4)$	M1 A1ft
	$\int f(x) dx = x^2 - 6x + 3 \ln(x^2 + 3x + 4)$	
	$\int_3^5 f(x) dx = (5^2 - 6 \times 5 + 3 \ln(5^2 + 3 \times 5 + 4)) - (3^2 - 6 \times 3 + 3 \ln(3^2 + 3 \times 3 + 4)) = ..$	dM1
	$= -5 + 3 \ln 44 + 9 - 3 \ln 22 = 4 + 3 \ln \left(\frac{44}{22} \right)$	M1
	$= 4 + \ln 8$	A1
		(5)
		(9 marks)
Notes:		
(a) M1: A correct method leading to at least one of the constants. May attempt long division getting the term A, or multiply through and compare coefficients or substitute values. Note that achieving $A = 2$ implies this mark. A1: $A = 2$ May be seen in the long division or in the expression. A1: Either $B = -6$ or $C = 3$ Stated or as part of the expression A1: Both $B = -6$ and $C = 3$ Stated or as part of the expression		
(b) Note: use of letters A, B and C can score max 3 marks M1A1ftdM0M1A0. M1: Attempts to integrate their answer to (a), look for $x^n \rightarrow x^{n+1}$ at least once and $\frac{2x+3}{x^2+3x+4} \rightarrow k \ln(x^2+3x+4) \text{ Allow with letters used if values have not been found.}$ A1ft: Correct integration following through their A, B and C (or with letters). dM1: Applies the limits to their integral and subtracts. <i>Must have numerical values for this mark..</i> M1: Uses correct log laws to combine the log terms into a single log term. (Need not have used the power law at this stage, unless earlier error leads to mismatch of coefficients.) Allow if letters used, so $3 \ln(44/22)$ can score this mark. A1cao: Correct answer in the form specified.		

$$\sin 2\theta = \frac{1}{5} \Rightarrow (\theta =) 0.101, 1.47$$

$$\operatorname{cosec} 2\theta = 5 \Rightarrow (\theta =) 0.101, 1.47$$

Question Number	Scheme	Marks
8(a)	$\frac{dv}{dt} = -e^{t-10} + 9e^{-0.75t} = 0 \Rightarrow 9 = e^{1.75t-10} \text{ or } 9e^{-0.75t} = e^{t-10}$	M1A1
	$9 = e^{1.75t-10} \Rightarrow t = \frac{10 + \ln 9}{1.75} \text{ or } \ln 9 - 0.75t = t - 10 \Rightarrow t = \frac{10 + \ln 9}{1.75}$	M1
	$t = \text{awrt } 6.97$	A1
	$\text{awrt } 11.9 \text{ (ms}^{-1}\text{)}$	A1
		(5)
(b)	$\int 12 - 12e^{-0.75t} - e^{(t-10)} dt = 12t - e^{(t-10)} + 16e^{-0.75t} (+C)$	M1A1
	$\left[12t - e^{(t-10)} + 16e^{-0.75t} \right]_0^T = 100 \Rightarrow 12T = \dots$	M1
	$T = \frac{1}{12} (116 - 16e^{-0.75T} + e^{T-10} - e^{-10}) *$	A1*
		(4)
(c) (i) (ii)	$T_2 = \frac{1}{12} (116 - 16e^{-0.75 \times 10} + e^{10-10} - e^{-10})$	M1
	$T_2 = \text{awrt } 9.7493$	A1
	9.7293 (seconds)	A1
		(3)
		(12 marks)

(a) * Be aware this can be solved directly on a calculator. Calculus must be seen*

M1 Differentiates to $\pm Ae^{t-10} \pm Be^{-0.75t}$ and sets equal to 0 Must be seen in (a) Do not accept differentiating to eg $\dots te^{t-10}$. Condone an expression of $12 \pm Ae^{t-10} \pm Be^{-0.75t}$ only

A1 $9 = e^{1.75t-10}$ or equivalent eg $9e^{-0.75t} - e^{t-10} = 0$

M1 Proceeds from their equation of the form $Ae^{Ct \pm D} - Be^{Et \pm F} = 0$ or equivalent leading to a value or an expression (which may be unsimplified) for t . If log work is seen it must be correct.

Eg $\frac{40 + 4 \ln 9}{7}$ or $\frac{10 + \ln 9}{1.75}$ will score this mark

$9e^{-0.75t} - e^{t-10} = 0 \Rightarrow \text{awrt } 6.97$ can also score this mark

This cannot be scored from an unsolvable equation.

