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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Proof (P4.1)

International A Level Mathematics · P4: Pure Mathematics 4 (WMA14)

16 questions · 75 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
P4.1a PROOF BY CONTRADICTION		16 Qs · 75 marks	2
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4. Q9	IAL P4 (WMA14) June 2022	4	9
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QUESTIONS

16 questions · 75 marks · 1 sub-topic

SUB-TOPIC P4.1a

PROOF BY CONTRADICTION

Sub-topic P4.1a · 16 questions · 75 marks · lowest tariff first



3. Prove by contradiction that there is no greatest odd integer.

(2)

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Q3

1. Given that n is an integer, use algebra, to prove by contradiction, that if n^3 is even then n is even.

(4)

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Q1

8. Use proof by contradiction to prove that, for all positive real numbers x and y ,

$$\frac{9x}{y} + \frac{y}{x} \geq 6$$

(4)

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Question 8 continued

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Lined area for writing the answer to Question 8.

(Total 4 marks)

Q8

Marking box for Q8.



(4)



Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS



(Total for Question 8 is 4 marks)



7. Use proof by contradiction to prove that $\sqrt{7}$ is irrational.

(You may assume that if k is an integer and k^2 is a multiple of 7 then k is a multiple of 7)



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 4 marks)



(4)

(Total for Question 6 is 4 marks)



(Total for Question 2 is 4 marks)



(4)

(Total for Question 6 is 4 marks)



10.

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Use proof by contradiction to show that for all angles x , where $90^\circ < x < 180^\circ$

$$\left| \frac{\cos 2x}{\cos x - \sin x} \right| < 1$$

(4)



Question 10 continued

Lined area for writing the answer to Question 10 continued.



Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS



6. Three **consecutive** terms in a sequence of real numbers are given by

$$k, 1 + 2k \text{ and } 3 + 3k$$

where k is a constant.

Use proof by contradiction to show that this sequence is not a geometric sequence.

(5)

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Q6

$$k + \frac{9}{k} \geq 6 \tag{4}$$

(1)



(Total for Question 4 is 5 marks)



(5)

Question 10 continued

Lined area for writing the answer to Question 10 continued.



TOTAL FOR PAPER IS 75 MARKS



Question 10 continued

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Lined area for writing the answer to Question 10.



Question 10 continued

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Handwriting practice lines for the answer to Question 10.

Q10

Marking box for Q10.

(Total 6 marks)

TOTAL FOR PAPER: 75 MARKS

END



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9. (i) Relative to a fixed origin O , the points A , B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.

Points A , B and C lie in a straight line, with B lying between A and C .

Given $AB:AC = 1:3$ show that

$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a} \quad (3)$$

- (ii) Given that $n \in \mathbb{N}$, prove by contradiction that if n^2 is a multiple of 3 then n is a multiple of 3
- (5)

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Q9

TOTAL FOR PAPER IS 75 MARKS





TOTAL FOR PAPER IS 75 MARKS



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} = \frac{1}{2}(\dots)$	B1
	$= (1 - 20x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (-20x) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (-20x)^2 + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (-20x)^3 \dots$	M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1 A1
	Special case: If the final answer is left as $\frac{1}{2}(1 - 10x - 50x^2 - 500x^3 + \dots)$ Award SC B1M1A1A1A0	
		(5)
	Alternative by direct expansion	
	$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^{-\frac{1}{2}}(-5x)^1 + \frac{\frac{1}{2} \times -\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$	B1M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1A1
(b)	$\left(\frac{1}{4} - \frac{5}{100}\right)^{\frac{1}{2}} = \left(\frac{1}{5}\right)^{\frac{1}{2}} = \frac{1}{2} - 5 \times \frac{1}{100} - 25\left(\frac{1}{100}\right)^2 - 250\left(\frac{1}{100}\right)^3 + \dots$ $\frac{\sqrt{5}}{5} \approx \frac{1789}{4000} \text{ or } \frac{1}{\sqrt{5}} \approx \frac{1789}{4000}$ $\Rightarrow \sqrt{5} \approx 5 \times \frac{1789}{4000} \text{ or } \sqrt{5} \approx 1 \div \frac{1789}{4000}$	M1
	$\sqrt{5} \approx \frac{1789}{800} \text{ or } \frac{4000}{1789}$	A1
		(2)
		(7 marks)

(a)B1: For taking out a factor of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.M1: Expands $(1 + kx)^{\frac{1}{2}}$, $k \neq \pm 1$ with the correct structure for the third or fourth terme.g. $\pm \frac{\frac{1}{2}(-\frac{1}{2})}{2!} \times (kx)^2$ or $\pm \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!} \times (kx)^3$ with or without the bracket around the kx

A1: For either term three or term four being correct in any form.

E.g. $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times (20x)^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times (-20x)^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times (-20x)^3$ or $-\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times (20x)^3$

The brackets must be present unless they are implied by subsequent work. This mark is independent of the B mark.



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Exact answers

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Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice



Question Number	Scheme	Notes	Marks
5(a)	$4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $-5 + 6\lambda = 4 - 3\mu$	For writing down any 2 of these equations.	M1
	E.g. $4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $\Rightarrow \lambda = \dots$ or $\mu = \dots$	Full method for finding λ or μ	M1
	$\lambda = 2, \mu = -1$	Both correct values	A1
	$-5 + 6\lambda = -5 + 12 = 7$ $4 - 3\mu = 4 + 3 = 7$ So lines intersect	Shows that the parameters satisfy the third equation and makes a conclusion.	B1
	$\lambda = 2 \rightarrow (4 + 4)\mathbf{i} + (4 - 6)\mathbf{j} + (-5 + 12)\mathbf{k}$ or $\mu = -1 \rightarrow (13 - 5)\mathbf{i} + (-1 - 1)\mathbf{j} + (4 + 3)\mathbf{k}$	Uses their λ or μ to find A .	M1
	$8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$	Correct vector or coordinates	A1
			(6)
(b)	$\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} = 10 - 3 - 18 = \sqrt{2^2 + 3^2 + 6^2} \sqrt{5^2 + 1^2 + 3^2} \cos \theta$ Full attempt at the scalar product between the direction vectors		M1
	$\cos \theta = \pm \frac{11}{7\sqrt{35}}$	Correct magnitude for $\cos \theta$ (may be implied by e.g. $\theta = 105.4\dots$ or $74.6\dots$)	A1
	$\theta = 74.6^\circ$	Awrt 74.6	A1
			(3)
(c)	$ 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k} = \sqrt{2^2 + 3^2 + 6^2} = 7$	Finds the magnitude of the direction of l_1	M1
	$35 \div 7 = 5 \Rightarrow \lambda = 5$ $8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k} \pm 5(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	Correct strategy for one of the points	M1
	$(18, -17, 37)$ or $(-2, 13, -23)$	One correct point (ignore labels)	A1
	$P(18, -17, 37)$ and $Q(-2, 13, -23)$	Correct points with correct labels	A1
			(4)
			Total 13

Question Number	Scheme	Notes	Marks
6 Way 1	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \, dx (+c)$ M1: For applying parts to obtain $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		M1A1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left\{ -\frac{1}{3} e^{2x} \cos 3x + \frac{2}{3} \int e^{2x} \cos 3x \, dx \right\} (+c)$ Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \cos 3x \, dx$		M1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} \int e^{2x} \cos 3x \, dx (+c)$ Fully correct application of parts twice		A1
	$\int e^{2x} \cos 3x \, dx + \frac{4}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c)$ $\Rightarrow \frac{13}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		M1
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			(6)
Way 2	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \int e^{2x} \sin 3x \, dx (+c)$ M1: For applying parts to obtain $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		M1A1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \left\{ \frac{1}{2} e^{2x} \sin 3x - \frac{3}{2} \int e^{2x} \cos 3x \, dx \right\} (+c)$ Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \cos 3x \, dx$		M1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x - \frac{9}{4} \int e^{2x} \cos 3x \, dx (+c)$ Fully correct application of parts twice		A1
	$\int e^{2x} \cos 3x \, dx + \frac{9}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c)$ $\Rightarrow \frac{13}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		M1
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			Total 6

Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = \cos t + 6 \cos t \sin t$ $\frac{dy}{dt} = 3 \cos t - 2 \sin t$	B1B1
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3 \cos t - 2 \sin t}{\cos t + 6 \cos t \sin t} = \frac{3 \cos \pi - 2 \sin \pi}{\cos \pi + 6 \cos \pi \sin \pi} = 3$ *	M1A1*
		(4)
(b)	When $t = \pi$, $x = -3$, $y = -2$	B1
	$y - "-2" = 3(x - "-3")$	M1
	$y = 3x + 7$	A1
		(3)
(c)	$y = 3x + 7 \Rightarrow 3 \sin t + 2 \cos t = 3(\sin t - 3 \cos^2 t) + 7$ or $y = 3(x + 3 \cos^2 t) + 2 \cos t \Rightarrow 3x + 7 = 3x + 9 \cos^2 t + 2 \cos t$	M1
	$\Rightarrow 9 \cos^2 t + 2 \cos t - 7 = 0$ *	A1*
		(2)
(d)	$\cos t = \frac{7}{9}$	B1
	$y = 3 \times \frac{\sqrt{32}}{9} + 2 \times \frac{7}{9} = \frac{4\sqrt{2}}{3} + \frac{14}{9}$	M1A1
		(3)
		(12 marks)

Notes

(a)

B1: $\left(\frac{dx}{dt} =\right) \cos t + 6 \cos t \sin t$ or $\cos t + 3 \sin 2t$ B1: $\left(\frac{dy}{dt} =\right) 3 \cos t - 2 \sin t$ M1: Attempts $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ using their $\frac{dx}{dt}$ and their $\frac{dy}{dt}$ and substitutes $t = \pi$. (May substitute π before dividing.)A1*: Achieves $\frac{dy}{dx} = 3$ with full working shown and no errors.

(b)

B1: $x = -3$, $y = -2$ which may be seen within their workingM1: Attempts to find the equation of the tangent with gradient 3. If they use $y = mx + c$ they must proceed as far as $c = \dots$ A1: $y = 3x + 7$

(c)

M1: A full attempt to solve simultaneously the given parametric equations with their equation of the tangent

A1*: Achieves $9 \cos^2 t + 2 \cos t - 7 = 0$ with no errors

(d)

B1: $\cos t = \frac{7}{9}$ seen or implied. Allow if seen in (c).M1: Attempts to find the y coordinate Must attempt to evaluate trig terms. If no substitution/working shown, then score for awrt 3.44 following a correct value for $\cos t$ A1: $\frac{4\sqrt{2}}{3} + \frac{14}{9}$ or exact equivalent. Withhold if additional answers are given.

Question	Scheme	Marks
8(a)	$V = \pi \int_0^{10} \left(10xe^{\frac{1}{2}x} \right)^2 dx = 100\pi \int_0^{10} x^2 e^{-x} dx$	M1;A1
		(2)
(b)	$\int x^2 e^{-x} dx = -x^2 e^{-x} + 2 \int x e^{-x} dx$	M1
	$= -x^2 e^{-x} + 2 \int x e^{-x} dx = -x^2 e^{-x} + 2 \left\{ -x e^{-x} + \int e^{-x} dx \right\}$	dM1
	$-x^2 e^{-x} - 2x e^{-x} + 2 \int e^{-x} dx = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} (+c)$	A1
		(3)
(c)	Total volume $= 2 \times 100\pi \int_0^{10} x^2 e^{-x} dx$	M1
	$\int_0^{10} x^2 e^{-x} = \left[-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} \right]_0^{10} = \left(-(10)^2 e^{-10} - 2 \times 10 e^{-10} - 2e^{-10} \right) - (-2)$	M1
	$= 2 - 122e^{-10} \quad (1.9944...)$	A1
	Density $= \frac{5000}{200\pi \times 1.9944...}$	dM1
	awrt 3.99 (g / cm ³)	A1
		(5)
		(10 marks)
Notes		
(a) M1: Forms a correct unsimplified expression for the volume. A1: Achieves $100\pi \int_0^{10} x^2 e^{-x} dx$ Condone a missing dx but limits must be present. (b) M1: Attempts integration by parts in the right direction to achieve an expression of the form $\dots x^2 e^{-x} \pm \dots \int x e^{-x} dx$ Condone missing dx dM1: Dependent on the previous method mark. Attempts integration by parts a second time to achieve an expression of the form $\dots x^2 e^{-x} \pm \dots x e^{-x} \pm \dots \int e^{-x} dx$ Condone missing dx A1: $-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} (+c)$ with or without the constant of integration For attempts via the DI (tabular) method, look for first two rows of the table to have correct forms for M1, all rows with correct forms and answer extracted for dM1 and A1 for correct answer. (c) M1: A correct strategy to find the total volume with their values of k. M1: Substitutes the limits of 10 and 0 into their part (b) and subtracts. Alternatively allow M1 for limits 20 and 0 used (as a mistaken attempt to double). A1: $2 - 122e^{-10}$ or awrt 1.99 dM1: Dependent on second M. Attempts to find the density using $\frac{5000}{\text{their Volume}}$. The attempt at the volume need not be correct but an attempt at using (b) must have been made. E.g. if they forget k or forget to double, allow for the attempt with their volume. Must be with 5000 in numerator, or with correct work to reach correct units later. A1: awrt 3.99 (g / cm³) oe. Accept exact simplified answers such as $\frac{5000}{200\pi(2 - 122e^{-10})}$		

Question	Scheme	Marks
9	For question 9 many variations on the proof are possible. Below is a general outline with some examples, which cover many cases. If you see an approach you do not know how to score, consult your team leader. M1: Will be scored for setting up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction and is scored at the point you can see each of these elements. A formal statement of the assumption is not required at this stage. A1: Scored for a correct statement from which it is possible to draw a contradiction. dM1: For making a complete argument that leads to a (full) contradiction of the initial statement, though may be allowed if there are minor gaps or omissions. A1: Correct and complete work with contradiction drawn and conclusion made. There must have been a statement of assumption at the start for which to draw the contradiction, though it may not be technically a correct assumption as long as a relevant assumption has been made. E.g. Accept “Assume $n^2 - 2$ is divisible by 4 for all n”	
9	(Assume that there is an n with $n^2 - 2$ is divisible by 4 so) $n^2 - 2 = 4k$	M1
	then $n^2 = 4k + 2 = 2(2k + 1)$ (so is even)	A1
	Hence n^2 is even so $n (=2m)$ is even hence n^2 is a multiple of 4 As n^2 is a multiple of 4 then $n^2 - 2 = 4m^2 - 2 = 2(2m^2 - 1)$ cannot be a multiple of 4 (as $2m^2 - 1$ is odd) so there is a contradiction.	dM1
	So the original assumption has been shown false. Hence “$n^2 - 2$ is never divisible by 4” is true for all n *	A1*
		(4)
		(4 marks)
Notes		
M1: Sets up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2 - 2 = 4k$ A1: Reaches $n^2 = 2(2k + 1)$ dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Accept explanations such as “as n^2 is even then n is even hence n^2 is a multiple of 4 so $n^2 - 2$ cannot be a multiple of 4 (as 4 does not divide 2)” A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept “Assume $n^2 - 2$ is divisible by 4 for all n”		
9 Alt 1	(Assume that $n^2 - 2$ is divisible by 4 for some n), so $\frac{n^2 - 2}{4}$ is an integer. Then if n is even $n = 2m$ (m integer) so $\frac{n^2 - 2}{4} = \frac{(2m)^2 - 2}{4}$ (oe with odd)	M1
	$= m^2 - \frac{1}{2}$ (which is not an integer)	A1

	Since m^2 is an integer, $m^2 - \frac{1}{2}$ is not, hence n cannot be even, but if n is odd then $\frac{n^2 - 2}{4} = \frac{(2m+1)^2 - 2}{4} = m^2 + m - \frac{1}{4},$ which is again not an integer (since $m^2 + m$ is)	dM1
	Hence there is a contradiction (as n cannot be an integer) Hence “$n^2 - 2$ is never divisible by 4” is true for all n *	A1*
		(4)
		(4 marks)
Notes		
M1: Sets up an algebraic statement in terms of a variable (integer) m or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this Alt, consider case use of $n = 2m$ or $n = 2m + 1$ in $\frac{n^2 - 2}{4}$ is sufficient A1: Reaches $m^2 - \frac{1}{2}$ for n even or $m^2 + m - \frac{1}{4}$ for n odd. dM1: For a complete argument that leads to a contradiction in both cases. See scheme. Allow if minor details are omitted as long as the overall argument is clear. A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept “Assume $n^2 - 2$ is divisible by 4 for all n”		
9 Alt 2	(Assume that $n^2 - 2$ is divisible by 4) $\Rightarrow n^2 - 2 = 4k$	M1
	$\Rightarrow n^2 = 4k + 2 \Rightarrow n = 2\sqrt{k + \frac{1}{2}}$ or $n = \sqrt{2}\sqrt{2k+1}$	A1
	So for some integer m $\sqrt{k + \frac{1}{2}} = \frac{m}{2} \Rightarrow 2k + 1 = \frac{m^2}{2}$ but m^2 is odd if m is odd so $\frac{m^2}{2}$ not an integer, or m^2 is a multiple of 4 if m even, so odd=even or $2k + 1$ is odd, so does not have a factor 2 to combine with the $\sqrt{2}$ outside, hence n must be irrational	dM1
	Hence we have a contradiction. So “$n^2 - 2$ is never divisible by 4” is true for all n *	A1*
		(4)
		(4 marks)
Notes		
M1: Sets up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2 - 2 = 4k$ A1: Reaches $n = 2\sqrt{k + \frac{1}{2}}$ or $n = \sqrt{2}\sqrt{2k+1}$		

dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Must be a valid attempt to show that $2\sqrt{k + \frac{1}{2}} / \sqrt{2}\sqrt{2k+1}$ is not an integer, and this method is a hard route. A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept “Assume $n^2 - 2$ is divisible by 4 for all n”		
9 Alt 3	(Assume that $n^2 - 2$ is divisible by 4) then for n even we have (for some integer m) $n^2 - 2 = 4m^2 - 2$ or for n odd $n^2 - 2 = 4(m^2 + m) - 1$	M1
	$4m^2 - 2$ or $4(m^2 + m) - 1$	A1
	Since 4 divides $n^2 - 2$ and $4m^2$ thus for n even, 4 must divide 2, a contradiction, so n cannot be even, and also 4 divides $4(m^2 + m)$ so for n odd, 4 divides 1, also a contradiction.	dM1
	Hence we have a contradiction for both cases (and as n must be either even or odd). so “$n^2 - 2$ is never divisible by 4” is true for all n *	A1*
		(4)
		(4 marks)
Notes		
M1: Sets up an algebraic statement in terms of a variable (integer) m or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing using n odd or n even to form an expression for $n^2 - 2$ of the form $4 \times \text{integer} \pm \text{non-multiple of 4}$ A1: Reaches $4m^2 - 2$ or $4(m^2 + m) - 1$ dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Both cases must be considered with a reason for the contradiction given (not just stated not divisible by 4). A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept “Assume $n^2 - 2$ is divisible by 4 for all n”		

Question Number	Scheme	Marks
8	$(3x - y) = 25 \text{ and } (x + y) = 1$ Solves one of $\text{or } (3x - y) = 5 \text{ and } (x + y) = 5$ Correct solution of one. Either $\left. \begin{array}{l} 3x - y = 25 \\ x + y = 1 \end{array} \right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, (y = -5.5)$ Or $\left. \begin{array}{l} 3x - y = 5 \\ x + y = 5 \end{array} \right\} \Rightarrow 4x = 10 \Rightarrow x = 2.5, (y = 2.5)$ Solves both equations Both solved correctly with a minimal reason given for the contradiction e.g "not integers" with conclusion "hence there are no integers x and y such that $3x^2 + 2xy - y^2 = 25$"	M1 A1 dM1 A1 (4)

Notes

M1: Attempts to solve one of the two possible cases.

Take as a minimum, one correct pair of equations followed by a value for x or a value for y

A1: Correctly solves one of the two possible cases.

To solve you need only find a value for x or y . Once a correct value is found you can ISW

dM1: Attempts to solve both possible cases

A1: Correctly solves the two possible cases and makes a concluding argument.

To score this mark (i) **all** calculations must be correct.

(ii) reason(s) for the contradiction must be written down. Allow "not integers", "x"

and (iii) gives a concluding statement must be given. Allow for example "hence proven"

Ignore any possible cases which would give rise to negative numbers but satisfy

$$(3x - y)(x + y) = 25$$

E.g $(3x - y) = -5, (x + y) = -5$

Withhold the final mark only if they include cases which do not satisfy $(3x - y)(x + y) = 25$

E.g $(3x - y) = 20, (x + y) = 5$



Question Number	Scheme	Marks
7	Assume that $\sqrt{7}$ is rational so that $\sqrt{7} = \frac{a}{b} \Rightarrow 7 = \frac{a^2}{b^2}$ (where a and b have no factors in common)	M1
	$7 = \frac{a^2}{b^2} \Rightarrow 7b^2 = a^2$ So a^2 is a multiple of 7 which means a is a multiple of 7	A1
	$a = 7k \Rightarrow 7b^2 = 49k^2 \Rightarrow b^2 = 7k^2$	M1
	So b^2 is a multiple of 7 which means b is a multiple of 7. As a and b are both multiples of 7, this contradicts the fact that a and b have no factors in common. Hence $\sqrt{7}$ must be irrational.	A1
		(4)
		Total 4

Any variables may be used

M1: Starts the proof by contradicting the given statement and squares both sides.

Condone the omission of the statement about common factors here.

As a minimum allow "assume $\sqrt{7} = \frac{a}{b} \Rightarrow 7 = \frac{a^2}{b^2}$ " or similar "let $\sqrt{7} = \frac{p}{q} \Rightarrow p^2 = 7q^2$ "

A1: Reaches for example $7b^2 = a^2$ and deduces that a^2 is a multiple of 7 which means a is a multiple of 7

Condone these being the "wrong way around" as long as they state both a and a^2 are multiples of 7

Look for

- $7b^2 = a^2$
- states a is a multiple of 7 (which may be implied by setting $a = 7k$ for example)
- states a^2 is a multiple of 7

M1: Sets $a = 7k$ and proceeds to $b^2 = 7k^2$

A1: Fully Correct proof. This must include

- an initial statement that included that a and b had no common factors. Allow equivalent statements such as $\frac{a}{b}$ is a fraction in simplest form or co-prime, relatively prime, mutually prime
- the statements and conclusions the correct way around.
E.g. a^2 is a multiple of 7 which means a is a multiple of 7 (not the other way around)
and b^2 is a multiple of 7 which means b is a multiple of 7 (not the other way around)
- a minimal conclusion

Be tolerant of slight issues with terminology/language used as this may not be the students first language.



A1: $3y \rightarrow 3 \frac{dy}{dx}$ and $4x^2 + k \rightarrow 8x$

dM1: Make $\frac{dy}{dx}$ the subject with correct terms factored (but allow e.g. sign errors rearranging or slips in copying). There must be at least two $\frac{dy}{dx}$ terms in their expression.

A1: Correct expression, accept any equivalent.

(b) Note: allow all marks in (b) from a correct numerator in (a) if the denominator is incorrect.

M1: Sets the numerator of their $\frac{dy}{dx}$ to zero and substitutes $y = 2$ and solves for x . They must have a y term in their numerator to be able to access this mark. If the numerator also contains a k they will need to also use the equation of the curve to find a second equation in p and k and solve simultaneously – look for them reaching a value for p .

A1: Correct value for p or x

M1: Must have been attempting $\frac{dy}{dx} = 0$ to find p . Substitutes back into the curve to find k . May be scored if their numerator was in terms of x only.

A1: Correct value for k

Alt (b)

M1: Substitutes $(p, 2)$ into the equation of the curve and attempts to complete the square or considers the discriminant (or other valid method to be able to deduce the values using a single root)

A1: Correct value for p or k

M1: Substitutes back into the curve to find the remaining unknown or other complete method to find the other value.

A1: Correct value for the other of p and k (so both should be correct).

Question	Scheme	Marks
4(a)	$(l =) \sqrt{25 + r^2}$	B1
		(1)
(b)	$(S = \pi r^2 +) \pi r \sqrt{25 + r^2} \Rightarrow \left(\frac{dS}{dr} = 2\pi r + \right) \pi \sqrt{25 + r^2} + \pi r \cdot \frac{1}{2} (25 + r^2)^{-\frac{1}{2}} \cdot 2r$ Or $(S = \pi r^2 +) \pi \sqrt{25r^2 + r^4} \rightarrow \left(\frac{dS}{dr} = 2\pi r + \right) \pi \cdot \frac{1}{2} (25r^2 + r^4)^{-\frac{1}{2}} \times (50r + 4r^3)$	M1 A1
	$(S = \pi(l^2 - 25)) + \pi l \sqrt{l^2 - 25} \Rightarrow \left(\frac{dS}{dl} = 2\pi l + \right) \pi \sqrt{l^2 - 25} + \pi l \cdot \frac{1}{2} (l^2 - 25)^{-\frac{1}{2}} \cdot 2l$	(M1 A1)
	$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} = \dots \times 3$	M1
	$= 81.5 \text{ (cm}^2 / \text{min)}$	A1
		(4)
(5 marks)		

Notes:**(a)****B1:** Correct expression. Accept $(l =) \sqrt{5^2 + r^2}$ and isw.**(b)****M1:** Differentiates their surface area with use of the product and chain rules on a term of the form
 $Kr\sqrt{\pm L \pm r^2}$ - look the form (oe) $P\sqrt{25 + r^2} + Qr^2(25 + r^2)^{-\frac{1}{2}}$. The base area may be missing for this mark. Alternatively they may work in terms of l and the scheme marks equivalently – see box in main scheme for the derivative in this case.

Alt: Taking r inside the square root look for $K\sqrt{25r^2 + r^4} \rightarrow K \cdot \frac{1}{2} (25r^2 + r^4)^{-\frac{1}{2}} \times (50r + 4r^3)$
A1: Correct differentiation for the curved surface area. Need not be simplified.
Alt: With r inside the square root $\pi\sqrt{25r^2 + r^4} \rightarrow \pi \cdot \frac{1}{2} (25r^2 + r^4)^{-\frac{1}{2}} \times (50r + 4r^3)$
M1: Applies the chain rule correctly with $\frac{dr}{dt} = 3$ and their expression for $\frac{dS}{dr}$ (which may be interms of r and l) or their value for $\frac{dS}{dr}$ at $r = 1.5$ if they find this first. This mark is for the chain rule

being applied, so their previous expressions need not have been correct. If they are working in terms

of l they will need to use $\frac{dS}{dt} = \frac{dS}{dl} \times \frac{dl}{dr} \times \frac{dr}{dt}$.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
3(a)	$B = 6 \times 4^{-\frac{1}{2}} = 3$	B1
	$3 \times \left(-\frac{1}{2}\right) \left(\frac{A}{4}\right) = -\frac{1}{4} \Rightarrow A = \frac{2}{3}$	M1 A1
	$C = 3 \times \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(\frac{A}{4}\right)^2}{2} \Rightarrow C = \frac{1}{32}$	M1 A1
		(5)

Notes:

(a)

B1: For $B = 3$ which may be implied by their expansion e.g. $3 \pm \dots$ **M1:** Attempts to find A condoning the omission of their “3”.Look for $-\frac{1}{2} \times \left(\frac{Ax}{m}\right) = -\frac{1}{4}x$ o.e. where m could be 1 leading to a value for A .Must be consistent with their $\dots \left(1 + \frac{A}{m}x\right)^{-\frac{1}{2}}$ but condone the omission of the “3”.Note the omission of the “3” (if correct) leads to $A = 2$.**A1:** $A = \frac{2}{3}$ or an exact equivalent or e.g. 0.6Allow if seen embedded in $6(4 + Ax)^{-\frac{1}{2}}$ **M1:** Attempt to find C condoning the omission of their “3”.Look for $\frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times \left(\frac{A}{m}\right)^2 = \dots$ or $\frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times \left(\frac{Ax}{m}\right)^2 = \dots$ oe with their numerical value of A leading to a value for C Must be consistent with their $\dots \left(1 + \frac{A}{m}x\right)^{-\frac{1}{2}}$ where m could be 1Note the omission of the “3” (if correct) with a correct A leads to $C = \frac{1}{96}$.**A1:** $C = \frac{1}{32}$ oe e.g. 0.03125

Allow if seen embedded in their expansion.



3(b)	$-6 < x \text{ „ } 6$	B1ft
		(1)
(c)	coefficient of x^3 is $3 \times \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!} \left(\frac{A}{4}\right)^3 = \dots$	M1
	$-\frac{5}{1152}$	A1
		(2)

Notes:

(b)

B1ft: $-6 < x \text{ „ } 6$ but follow through on their A so allow for $-\frac{4}{A} < x \text{ „ } \frac{4}{A}$

Condone strict or non-strict inequalities for either end e.g. condone $|x| < 6$

or e.g. $-6 < x < 6$ but follow through on their A e.g. condone $|x| \text{ „ } \left| \frac{4}{A} \right|$

Accept alternative notation e.g. $x > -6$ and $x \text{ „ } 6$, $(-6, 6)$, $(-6, 6]$ etc.

(c)

M1: Attempt to find the coefficient of x^3 or the term in x^3 to obtain a value condoning the omission of their “3”.

Look for an attempt at $\frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right)}{3!} \times \left(\frac{A}{m}\right)^3 = \dots$ with their numerical value of A .

Must be consistent with their $\dots \left(1 + \frac{A}{m}x\right)^{\frac{1}{2}}$ from part (a) where m could be 1

Note the omission of the “3” with a correct A leads to coefficient of $-\frac{5}{3456}$.

A1: $-\frac{5}{1152}$. Condone $-\frac{5}{1152}x^3$ which may be seen embedded in an expansion.

Correct answer only with correct work in (a) scores both marks.

Expansions for reference

$$6(4 + Ax)^{\frac{1}{2}} = 6 \times 4^{\frac{1}{2}} \left(1 + \frac{A}{4}x\right)^{\frac{1}{2}} = 3 \left(1 - \frac{A}{8}x + \frac{3A^2}{128}x^2 - \frac{5A^3}{1024}x^3 + \dots\right)$$

$$= 3 - \frac{3A}{8}x + \frac{9A^2}{128}x^2 - \frac{15A^3}{1024}x^3 + \dots$$

or e.g. (direct expansion)

$$6(4 + Ax)^{\frac{1}{2}} = 6 \left(4^{\frac{1}{2}} + \left(-\frac{1}{2}\right) 4^{-\frac{1}{2}} (Ax) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2} 4^{-\frac{3}{2}} (Ax)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!} 4^{-\frac{5}{2}} (Ax)^3 + \dots \right)$$

$$= 6 \left(\frac{1}{2} - \frac{1}{16}(Ax) + \frac{3}{256}(Ax)^2 - \frac{5}{2048}(Ax)^3 + \dots \right) = 3 - \frac{3A}{8}x + \frac{9A^2}{128}x^2 - \frac{15A^3}{1024}x^3 + \dots$$



Question Number	Scheme	Marks
4(i)	$\frac{dV}{dt} = 70\pi$	B1
	$\frac{dV}{dr} = 4\pi r^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 70\pi = 4\pi \times 5^2 \times \frac{dr}{dt}$ or e.g. $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} \Rightarrow \frac{dr}{dt} = 70\pi \times \frac{1}{4\pi \times 5^2}$	M1
	$\left(\frac{dr}{dt}\right) = 0.7 \text{ (cm s}^{-1}\text{) oe e.g. } \frac{7}{10}$	A1
		(4)
	Alternative:	
	$V = 70\pi t$	B1
	$\frac{4}{3}\pi r^3 = 70\pi t$	B1
	$\frac{4}{3}\pi r^3 = 70\pi t \Rightarrow 4\pi r^2 \frac{dr}{dt} = 70\pi \Rightarrow 100 \frac{dr}{dt} = 70$ or $\frac{4}{3}\pi r^3 = 70\pi t \Rightarrow r^3 = \frac{105}{2}t \Rightarrow r = \left(\frac{105}{2}t\right)^{\frac{1}{3}}$ $\left(\frac{4}{3}\pi r^3 = 70\pi t \Rightarrow \frac{500}{3}t = 70 \Rightarrow t = \frac{50}{21}\right)$ $\Rightarrow \frac{dr}{dt} = \frac{1}{3}\left(\frac{105}{2}\right)^{\frac{1}{3}} t^{-\frac{2}{3}} = \frac{1}{3}\left(\frac{105}{2}\right)^{\frac{1}{3}} \left(\frac{50}{21}\right)^{-\frac{2}{3}}$	M1
	$\left(\frac{dr}{dt}\right) = 0.7 \text{ (cm s}^{-1}\text{) oe e.g. } \frac{7}{10}$	A1

Notes:

B1: States or uses $\frac{dV}{dt} = 70\pi$ oe e.g. $\frac{dt}{dV} = \frac{1}{70\pi}$

B1: States or uses $\frac{dV}{dr} = 4\pi r^2$ oe e.g. $\frac{dr}{dV} = \frac{1}{4\pi r^2}$ which may be seen as $\frac{dV}{dr} = 100\pi$ or $\frac{dr}{dV} = \frac{1}{100\pi}$ if $r = 5$ has been substituted.

M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ or equivalent with $r = 5$ and their $\frac{dV}{dr}$ and $\frac{dr}{dt}$ correctly placed
e.g. $70\pi = 4\pi \times 5^2 \times \frac{dr}{dt}$ or e.g. $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = \frac{70\pi}{4\pi(5)^2}$

A1: $\left(\frac{dr}{dt}\right) = 0.7$ oe (Units are not required but if any are given they must be correct)



(i) Alternative:**B1:** States or uses $V = 70\pi t$ **B1:** States or uses $\frac{4}{3}\pi r^3 = 70\pi t$ **M1:** For either:

- Differentiating $\frac{4}{3}\pi r^3 = 70\pi t$ to obtain $\dots r^2 \frac{dr}{dt} = \dots$ and substituting $r = 5$ **or**
- Making r the subject from $\frac{4}{3}\pi r^3 = 70\pi t$ to obtain $r = \dots t^{\frac{1}{3}}$ and then differentiating to obtain

$$\frac{dr}{dt} = \dots t^{-\frac{2}{3}} \text{ and then substituting } t \text{ obtained from } \frac{4}{3}\pi(5)^3 = 70\pi t$$

A1: $\frac{dr}{dt} = 0.7$ oe (Units are not required but if any are given they must be correct)

(ii)	$\frac{dh}{dt} = \frac{k}{h^3} \Rightarrow \int h^3 dh = \int k dt \Rightarrow \frac{1}{4}h^4 = kt + c$	M1A1
	$t = 0, h = 4 \Rightarrow \frac{1}{4} \times 4^4 = 0 + c \Rightarrow c = 64$	M1
	$t = 5, h = 6 \Rightarrow \frac{1}{4} \times 6^4 = k \times 5 + 64 \Rightarrow k = (52)$	dM1
	$h = 10 \Rightarrow \frac{1}{4}(10)^4 = 52T + 64 \Rightarrow T = \dots$	dddM1
	46.8 (hours) or exact e.g. $\frac{609}{13}, \frac{9744}{208}$	A1
		(6)
		(10 marks)

Notes:**M1:** Separates variables and integrates both sides to obtain $\dots h^4 = kt(+c)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{1}{4}h^4 = kt(+c)$ or equivalent, with or without a constant of integration.**M1:** Requires a constant of integration and using $t = 0, h = 4$ to find their c .

Condone poor integration and poor attempts to rearrange their equation for this mark.

dM1: Uses $t = 5, h = 6$ to find k .It is dependent upon the **previous** mark and having two correctly placed constants.

Condone poor integration and poor attempts to rearrange their equation for this mark.

dddM1: Dependent upon **all three previous** M's.It is for substituting $h = 10$ into their equation and finding T (or t)**A1:** Awrt 46.8 or exact e.g. $\frac{609}{13}, \frac{9744}{208}$. Allow better accuracy e.g. 46.85.It does not have to be referenced as “ T ” so just look for the correct value.

Units are not required but if any are given they must be correct.

See next pages for alternatives to (ii)

Alternative using definite integration:

$$\frac{dh}{dt} = \frac{k}{h^3} \Rightarrow \int h^3 dh = \int k dt \Rightarrow \left[\frac{1}{4} h^4 \right]_4^6 = [kt]_0^5$$

$$324 - 64 = 5k \Rightarrow k = 52$$

$$\left[\frac{1}{4} h^4 \right]_4^{10} = [52t]_0^T \Rightarrow 2436 = 52T \Rightarrow T = 46.8$$

Score as follows:

M1: Separates variables and integrates both sides to obtain $h^4 = kt (+c)$ or equivalent.
There is no need for a constant of integration for this mark.

A1: $\frac{1}{4}h^4 = kt (+c)$ or equivalent, with or without a constant of integration.

M1: Attempts $\left[\frac{1}{4}h^4 \right]_4^6 = \frac{1}{4} \times 6^4 - \frac{1}{4} \times 4^4 = 260$ Condone poor integration of h^3 .

dM1: Attempts $[kt]_0^5 = 5k$ and solves $\left[\frac{1}{4}h^4 \right]_4^6 = [kt]_0^5$ to find k .

It is dependent upon the **previous** mark.

Condone poor integration and poor attempts to rearrange their equation for this

mark.

dddM1: Dependent upon **all three previous** M's.

It is for attempting $\left[\frac{1}{4}h^4 \right]_4^{10} = [52t]_0^T$ to find T (or t)

A1: Awrt 46.8 or exact e.g. $\frac{609}{13}$, $\frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as “ T ” so just look for the correct value.

Units are not required but if any are given they must be correct.



Alternative when k is placed differently:

$$\frac{dh}{dt} = \frac{k}{h^3} \Rightarrow \int \frac{h^3}{k} dh = \int dt \Rightarrow \frac{h^4}{4k} = t + c$$

$$t = 0, h = 4 \Rightarrow \frac{64}{k} = c \quad \text{or} \quad t = 5, h = 6 \Rightarrow \frac{324}{k} = 5 + c$$

$$\frac{64}{k} = c, \frac{324}{k} = 5 + c \Rightarrow c = \frac{16}{13}, k = 52$$

$$\frac{h^4}{208} = t + \frac{16}{13} \Rightarrow \frac{10^4}{208} = t + \frac{16}{13} \Rightarrow T = \frac{609}{13}$$

Score as follows:

M1: Separates variables and integrates both sides to obtain $\dots \frac{h^4}{k} = t(+c)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{h^4}{4k} = t(+c)$ or equivalent, with or without a constant of integration.

M1: Requires a constant of integration and using either $t = 0, h = 4$ or $t = 5, h = 6$ to obtain an equation connecting c and k .

Condone poor integration for this mark.

dM1: Uses **both** $t = 0, h = 4$ **and** $t = 5, h = 6$ to obtain 2 equations connecting c and k and solves simultaneously to obtain a value for c and a value for k .

You do not need to be concerned how they solve their equations as long as they obtain a value for c and a value for k .

It is dependent upon the **previous** mark and having two correctly placed constants.

Condone poor integration for this mark.

dddM1: Dependent upon **all three previous** M's.

It is for substituting $h = 10$ into their equation and finding T (or t)

A1: Awrt 46.8 or exact e.g. $\frac{609}{13}, \frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as " T " so just look for the correct value.

Units are not required but if any are given they must be correct.



Question Number	Scheme	Marks
5(i)	$\int x^2 e^{4x} dx = \frac{1}{4} x^2 e^{4x} - \int \frac{1}{2} x e^{4x} dx$	M1 A1
	$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \int \frac{1}{8} e^{4x} dx$	dM1
	$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \frac{1}{32} e^{4x} + c$	A1
		(4)

Notes:

(i)

M1: Attempts to integrate by parts once the correct way around. The minus must be present (not +)Look for $ax^2 e^{4x} - \int bx e^{4x} dx$, $a, b > 0$. Condone the omission of the “dx”.**A1:** $\frac{1}{4} x^2 e^{4x} - \int \frac{1}{2} x e^{4x} dx$ oe e.g. $\frac{1}{4} x^2 e^{4x} - \int \frac{2}{4} x e^{4x} dx$ **dM1:** Integrates ... $\int x e^{4x} dx$ by parts the correct way around again. The second sign can be + or –This may be seen in isolation so award for ... $\int x e^{4x} dx = \dots x e^{4x} - \dots \int e^{4x} dx$

Condone the omission of the “dx”.

A1: $\frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \frac{1}{32} e^{4x} (+c)$ with or without a constant of integration.Allow equivalent simplified expressions e.g. $e^{4x} \left(\frac{1}{4} x^2 - \frac{1}{8} x + \frac{1}{32} \right) (+c)$ and apply isw once a correct simplified answer is seen.**Watch for “D and I” method:**

D	I
x^2	e^{4x}
$2x$	$\frac{1}{4} e^{4x}$
2	$\frac{1}{16} e^{4x}$
	$\frac{1}{64} e^{4x}$

In these cases score M1dM1 for obtaining:

$$px^2 e^{4x} \pm qxe^{4x} \pm re^{4x}, \quad p, q, r > 0$$

and then A1 for the **correct** first 2 terms $\frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x}$

and then A1 as in the main scheme.

Note that for this approach M1A1dM0 is not possible.



(ii)	$\frac{2x+11}{(2x+1)(2-x)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2-x)} \Rightarrow A = \dots, B = \dots$	M1
	$= \frac{4}{(2x+1)} + \frac{3}{(2-x)}$	A1
	$\int \frac{2x+11}{(2x+1)(2-x)} dx = 2 \ln 2x+1 - 3 \ln 2-x $	dM1 A1ft
	$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx = (2 \ln 15 - 3 \ln 5) - (2 \ln 9 - 3 \ln 2)$	ddM1
	$= \ln \frac{8}{45}$	A1
		(6)
		(10 marks)

Notes:

(ii)

M1: Attempts partial fractions. Condone slips but should be of the correct form $\frac{A}{(2x+1)} + \frac{B}{(2-x)}$

A1: Correct **fractions** $\frac{4}{(2x+1)} + \frac{3}{(2-x)}$ seen.

It is not for correct “A” and “B” unless the correct fractions are seen or implied by later work.

dM1: Integrates to a correct form e.g. $\dots \ln|2x+1| + \dots \ln|2-x|$ where $\dots$ are non-zero constants.

Moduli are not required, brackets will suffice but condone missing brackets for this mark e.g.

$$\int \frac{2x+11}{(2x+1)(2-x)} dx = 2 \ln 2x+1 - 3 \ln 2-x$$

A1ft: Correct integration but follow through on their A and B.

Moduli are not required, brackets will suffice but must be present now or implied by subsequent work e.g. when they substitute limits. Allow unsimplified e.g. $\frac{4}{2} \ln(2x+1) - 3 \ln(2-x)$

ddM1: Applies the limits 4 and 7 to a function of the correct form e.g. $\dots \ln|2x+1| + \dots \ln|2-x|$ or $\dots \ln(2x+1) + \dots \ln(2-x)$ and subtracts the right way round. Brackets around the $2x+1$ and $2-x$ must be present or implied but condone missing brackets around the expression for the

lower limit e.g. condone $\int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx = 2 \ln 15 - 3 \ln 5 - 2 \ln 9 - 3 \ln 2$

and condone e.g. $\int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx = 2 \ln 15 - 3 \ln(-5) - (2 \ln 9 - 3 \ln(-2))$

May be implied by e.g. $[4 \ln(2x+1)]_4^7 + [3 \ln(2-x)]_4^7 = 4 \ln \frac{15}{9} + 3 \ln \frac{5}{2}$

A1: $\ln \frac{8}{45}$ cao. Do **not** allow $\ln \left| -\frac{8}{45} \right|$.



In (ii) condone slips in copying their PF's if the work is otherwise correct e.g.

$$\begin{aligned}
 \text{ii) } \frac{2x+11}{(2x+1)(2-x)} &= \frac{A}{2x+1} + \frac{B}{2-x} \\
 2x+11 &= A(2-x) + B(2x+1) \\
 x=2 &\Rightarrow 15 = 5B \Rightarrow B=3 \\
 x=-\frac{1}{2} &\Rightarrow 10 = \frac{5}{2}A \Rightarrow A=4 \\
 \frac{2x+11}{(2x+1)(2-x)} &= \frac{4}{2x-1} + \frac{3}{2-x} \\
 \int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx &= \int_4^7 \frac{4}{2x-1} + \frac{3}{2-x} dx \\
 &= [2\ln(2x-1) - 3\ln(2-x)]_4^7 \\
 &= [2\ln(13) - 3\ln(-5)] - [2\ln(7) - 3\ln(-2)] \\
 &= 2\ln 13 - 3\ln(-5) - 2\ln 7 + 3\ln(-2) \\
 &= 2\ln\left(\frac{13}{7}\right) + 3\ln\left(\frac{2}{5}\right) \\
 &= \ln\left(\frac{13}{7}\right)^2 + \ln\left(\frac{2}{5}\right)^3 \\
 &= \ln\left(\frac{1352}{6125}\right) \quad k = \frac{1352}{6125}
 \end{aligned}$$

Would score M1A0M1A0M1A0 in part (ii)

If in doubt use review.



Question Number	Scheme	Marks
6	Assumption: e.g. (Assume) " $n^2 - 4n + 5$ is even and n is even"	B1
	Sets e.g. $n = 2k$ and attempts $(2k)^2 - 4(2k) + 5$	M1
	$4k^2 - 8k + 5$ (oe for their n)	A1
	Completes proof with: (i) Statement "which is odd" (ii) Reason odd E.g. $4(k^2 - 2k) + 5$ or e.g. $4(k^2 - 2k + 1) + 1$ or e.g. $2(2k^2 - 4k + 2) + 1$ or e.g. $(2k - 2)^2 + 1$ Condone $4k^2 - 8k + 5 = \text{even} - \text{even} + \text{odd} = \text{odd}$ (iii) A (minimal) conclusion e.g. "Hence contradiction", "so proven", "the statement is true" etc.	A1*
		(4 marks)

Notes:

Note that B0M1A1A1 is not possible.

**If the same variable is used e.g. $n = 2n$ allow all but the final mark for otherwise correct work.
But allow $n = 2N$ as a different variable.**

B1: Sets up a suitable assumption. Must be in words.

Look for e.g. $n^2 - 4n + 5$ is even and n is even or e.g. if n even then $n^2 - 4n + 5$ is even.

The word "assume" is not required but there must be a statement referring to $n^2 - 4n + 5$ is even and n is even.

Condone e.g. "if $n^2 - 4n + 5$ is even then n is even".

M1: Uses algebra with $n = 2k$ or equivalent e.g. $n = 2k + 2$ or e.g. $n = 4k$ and attempts $n^2 - 4n + 5$

A1: $4k^2 - 8k + 5$ or equivalent for their n such as $4(k^2 - 2k) + 5$ or e.g. $4k^2 + 1$ for $n = 2k + 2$

A1*: Completes proof with

- a correct expression that is clearly odd
- an acceptable statement with reason
- minimal conclusion.

Note that attempts that consider a contradiction such as e.g.

(Assume) " $n^2 - 4n + 5$ is odd and n is odd"

score no marks.

There will be valid approaches to the proof.

See below for some alternatives:



" $n^2 - 4n + 5$ is even and n is even"

$n^2 - 4n + 5 = (n-2)^2 + 1$ is even so $(n-2)^2$ must be odd

If $(n-2)^2$ is odd then $(2k-2)^2$ is odd (as n is even) hence contradiction

So if $n^2 - 4n + 5$ is even then n is odd

" $n^2 - 4n + 5$ is even and n is even"

$n^2 - 4n + 5 = (n-2)^2 + 1 = 2k \Rightarrow (n-2)^2 = 2k-1$ so $(n-2)^2$ must be odd

If $(n-2)^2$ is odd then $(2a-2)^2$ is odd (as n is even) hence contradiction

So if $n^2 - 4n + 5$ is even then n is odd

Mark such attempts as follows:

B1: As main scheme

M1: Completes the square on $n^2 - 4n + 5$ to obtain $(n-2)^2 + \dots$ or uses

$$n^2 - 4n + 5 = n(n-4) + \dots$$

A1: $n^2 - 4n + 5 = (n-2)^2 + 1$ or e.g. $n^2 - 4n + 5 = n(n-4) + 5$

A1: Fully reasoned and convincing argument with a (minimal) conclusion

If you are unsure if a particular response deserves merit then use review.



Question Number	Scheme	Marks
10	Assume there is an angle x, $90^\circ < x < 180^\circ$, $\left \frac{\cos 2x}{\cos x - \sin x} \right \geq 1$ $\frac{\cos 2x}{\cos x - \sin x} \equiv \frac{(\cos x + \sin x)(\cancel{\cos x} - \cancel{\sin x})}{\cancel{\cos x} - \sin x}$ $ \cos x + \sin x \geq 1 \Rightarrow (\cos x + \sin x)^2 \geq 1 \Rightarrow 1 + 2 \sin x \cos x \geq 1 \Rightarrow \sin 2x \geq 0$ This is a contradiction, as $\sin \alpha$ is negative for all angles $180^\circ < \alpha < 360^\circ$, hence true *	B1 M1 dM1 A1*
		(4 marks)

Condone strict inequalities for the first 3 marks in this question

B1: Sets up the argument.

It requires

- words like ‘assume there is an angle x ’ or ‘let there be an angle x ’, condoning ‘for all angles x ’
- the domain ‘ $90^\circ < x < 180^\circ$ ’
- the contradiction $\left| \frac{\cos 2x}{\cos x - \sin x} \right| \geq 1$ but condone $\left| \frac{\cos 2x}{\cos x - \sin x} \right| > 1$

M1: Uses the double angle formula for $\cos 2x$ and simplifies $\frac{\cos 2x}{\cos x - \sin x}$ to $(\cos x + \sin x)$

It may be scored when prematurely removing the modulus before proceeding to $(\cos x + \sin x) \geq 1$

dM1: Squares $|(\cos x + \sin x)| \geq 1$ and uses suitable identities in proceeding to $\sin 2x \geq 0$, condoning $\sin 2x > 0$

It must be in a form in which the contradiction can easily be proven

A1: Deduces contradiction from opening statement and concludes. May consider the signs of $\sin x$ and $\cos x$ in the given range and deduce that the product is negative leading to the contradiction of the original assumption. Withhold this mark if the modulus signs are not correctly dealt with. This mark is withheld for using strict inequalities

There will be many alternative approaches so you will need to look at what they have done to see if there is any credit worthy work. If in doubt please use review.

Alternative using R-alpha method

B1 M1: As above

dM1: Proceeds to the form $|\sqrt{2} \cos(x - 45^\circ)| \geq 1$ with the modulus seen

Alternatively, to the form $|\sqrt{2} \sin(x + 45^\circ)| \geq 1$ with the modulus seen

A1*: Considers $x \in (90^\circ, 180^\circ) \Rightarrow (x - 45^\circ) \in (45^\circ, 135^\circ)$

Deduces that $|\cos(x - 45^\circ)| < \frac{1}{\sqrt{2}}$ which means that $|\sqrt{2} \cos(x - 45^\circ)| < 1$ so meaning that the given statement is true.

Alternatively



Considers $x \in (90^\circ, 180^\circ) \Rightarrow (x + 45^\circ) \in (135^\circ, 225^\circ)$ and deduces that $|\sin(x + 45^\circ)| < \frac{1}{\sqrt{2}}$ which means

that $|\sqrt{2} \sin(x + 45^\circ)| < 1$ so meaning that the given statement is true

Alternative that proceeds via an identity in $\cos 2x$ and $\sin 2x$

M1: Squares $\left| \frac{\cos 2x}{\cos x - \sin x} \right| \geq 1$ and uses suitable identities leading to an inequality in $\sin 2x$ and $\cos 2x$

$$\cos^2 2x \geq (\cos x - \sin x)^2 \Rightarrow \cos^2 2x \geq 1 - \sin 2x$$

dM1: Then uses Pythagoras identity $\Rightarrow 1 - \sin^2 2x \geq 1 - \sin 2x \Rightarrow \sin 2x \geq \sin^2 2x$

A1*: As $90^\circ < x < 180^\circ, \Rightarrow 180^\circ < 2x < 360^\circ$ and $\sin 2x < 0 \Rightarrow \sin 2x \geq 1$ which is impossible and therefore you have a contradiction and the original statement is true

Alternative using logic for the last two marks

dM1 Attempts to explain the contradiction by referencing the domain $90^\circ < x < 180^\circ$ and corresponding range of values of (say) $\sin x, \cos x$. Must involve the modulus in their explanation or have squared previously.

Stating that " $0 < \sin x < 1$ and $-1 < \cos x < 0$ for $90^\circ < x < 180^\circ$ ", followed by

"so $|\sin x + \cos x| < 1$ " would be a minimal but acceptable argument for dM1.

Then the A1* is given for a completely rigorous explanation (plus conclusion)

Question	Scheme	Marks
4(a)	Area of one face = $\frac{1}{2}x \times x \times \sin 60^\circ = \frac{1}{2}x^2 \frac{\sqrt{3}}{2}$ oe	M1
	So surface area of icosahedron is $20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ *	A1*
		(2)
(b)	$\frac{dA}{dx} = 10\sqrt{3}x$ and $\frac{dV}{dx} = \frac{15}{12}(3+\sqrt{5})x^2$	M1
	$\frac{dV}{dA} = \frac{dV}{dx} \times \frac{dx}{dA} = \frac{dV}{dx} \div \frac{dA}{dx} = \dots$ (oe)	M1
	$= \frac{15(3+\sqrt{5})x^2}{12 \times 10\sqrt{3}x} = \frac{(3+\sqrt{5})x}{8\sqrt{3}}$ *	A1*
		(3)
(c)	$\frac{dA}{dt} = 0.025$	B1
	$\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt} = \frac{(3+\sqrt{5})x}{8\sqrt{3}} \times 0.025 = \dots$	M1
	When $x = 2$, $\frac{dV}{dt} = \frac{2(3+\sqrt{5})}{8\sqrt{3}} \times 0.025 = \dots$	
	awrt 0.019 (cm ³ s ⁻¹)	A1
		(3)
(8 marks)		

Notes:**(a)**

M1: Any correct method to establish a surd form for the area of one face. Must evaluate trigonometric terms.

Alternatives include e.g. $\frac{1}{2} \times x \times \sqrt{x^2 - \left(\frac{1}{2}x\right)^2} = \frac{1}{2}x\sqrt{\frac{3}{4}}x^2$

A1*: Correct result achieved after showing side area of one face.

Examples:

$$\text{Area of one face} = \frac{1}{2}x \times x \times \sin 60^\circ = \frac{1}{4}x^2\sqrt{3} \Rightarrow SA = 20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2 \text{ scores M1 A1}$$

$$SA = 20 \times \left(\frac{1}{2} \times x \times x \sin 60 \right) = 5\sqrt{3}x^2 \text{ scores M1 A0 for lack of working}$$

(b)

M1: Attempts derivatives of both A and V wrt x . E.g. $\left(\frac{dA}{dx} = \right)\alpha x$ and $\left(\frac{dV}{dx} = \right)\beta x^2$.

Note that $\frac{5}{12}(3+\sqrt{5})x^3$ is sometimes expanded first as e.g. $\frac{5}{4}x^3 + \frac{5\sqrt{5}}{12}x^3$ and allow this mark

if either term is differentiated to βx^2

M1: Applies the correct chain rule with their derivatives. (Condone poor notation – see below)



A1*: Applies the chain rule to achieve the given result, with suitable intermediate step seen with $\frac{dV}{dA} = \dots$ appearing somewhere in their solution.

Condone poor notation e.g. allow $\frac{dA}{dt} = 10\sqrt{3}x$ and $\frac{dV}{dt} = \frac{15}{12}(3 + \sqrt{5})x^2$, $\frac{dV}{dA} = \frac{dV}{dt} \times \frac{dt}{dA}$ etc. for full marks.

Alt:

M1: Eliminates x from the two equations $V = \frac{5}{12}(3 + \sqrt{5})\left(\frac{A}{5\sqrt{3}}\right)^{\frac{3}{2}}$

M1: Attempts to differentiate $\frac{dV}{dA} = \frac{5}{12}(3 + \sqrt{5}) \times \frac{3}{2}\left(\frac{A}{5\sqrt{3}}\right)^{\frac{1}{2}} \times \frac{1}{5\sqrt{3}}$ – look for correct treatment of the $5\sqrt{3}$ as well as correct power.

A1: Achieves correct result with no errors.

(c)

B1: Interprets the rate of change correctly to state or clearly imply by working that $\frac{dA}{dt} = 0.025$

M1: Applies the chain rule appropriately **to find a value for** $\frac{dV}{dt}$ using $x = 2$

A1: awrt 0.019 condone lack of units.



Question	Scheme	Marks
5(a)	$\frac{dx}{dt} = \frac{1}{2}(9-4t)^{-\frac{1}{2}} \times -4 = \left(\frac{-2}{\sqrt{9-4t}} \right)$	M1
	$A = \int_{(x=0)}^{(x=3)} y \frac{dx}{dt} (dt) = \int_{(t=\frac{9}{4})}^{(t=0)} \frac{t^3}{\sqrt{9+4t}} \times \frac{-2}{\sqrt{9-4t}} (dt)$	M1
	$= -2 \int_{(\frac{9}{4})}^{(0)} \frac{t^3}{\sqrt{81-16t^2}} (dt) \text{ or e.g. } -2 \int_{(\frac{9}{4})}^{(0)} \frac{t^3}{\sqrt{(9-4t)(9+4t)}} (dt)$ Depends on both previous M marks.	ddM1
	$= 2 \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81-16t^2}} dt$	A1
		(4)
(b)	$\frac{du}{dt} = -32t \text{ or } du = -32t dt \text{ oe}$ or $u = 81-16t^2 \Rightarrow t^2 = \frac{1}{16}(81-u) \Rightarrow t = \frac{1}{4}(81-u)^{\frac{1}{2}} \Rightarrow \frac{dt}{du} = -\frac{1}{8}(81-u)^{-\frac{1}{2}}$	B1
	$A = 2 \int_0^{\frac{9}{4}} \frac{t^2}{\sqrt{81-16t^2}} t dt = 2 \int_{(81)}^{(0)} \frac{(81-u)}{16\sqrt{u}} \frac{du}{-32}$ or $A = 2 \int_0^{\frac{9}{4}} \frac{t^2}{\sqrt{81-16t^2}} t dt = 2 \int_{(81)}^{(0)} \frac{\left(\frac{1}{4}(81-u)^{\frac{1}{2}}\right)^3}{\sqrt{u}} \times -\frac{1}{8}(81-u)^{-\frac{1}{2}} du$	M1
	$= k \int_{(81)}^{(0)} 81u^{-\frac{1}{2}} - u^{\frac{1}{2}} du$	A1ft
	$= K \left[\frac{81u^{\frac{1}{2}}}{\frac{1}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_{(81)}^{(0)}$	M1
	$= -\frac{1}{256} \left(0 - \left(162\sqrt{81} - \frac{2}{3}\sqrt{81^3} \right) \right) = \dots$ Depends on all previous M marks.	ddM1
	$= \frac{243}{64}$	A1
		(6)
	(10 marks)	
Notes:		



(a)

M1: Attempts $\frac{dx}{dt}$ achieving the form $k(9-4t)^{-\frac{1}{2}}$ oe

M1: Applies the correct formula for the area, $A = \int y \frac{dx}{dt} dt$, with y and their $\frac{dx}{dt}$

(no need for limits for this mark and no need for “dt”).

ddM1: Combines terms to a single fraction with one square root.

(no need for limits for this mark and no need for “dt”).

Depends on both previous M marks.

A1: Applies difference of squares, or expands, and deals with limits correctly to achieve correct result from correct working.

The “dt” must have made at least one appearance before the final line.

(b)

B1: Correct statement of $\frac{du}{dt}$ or $\frac{dt}{du}$ or any correct equation connecting du with dt

M1: Complete substitution made into the integrand given in part (a) with their K or the letter K or a made up K e.g. $K = 1$ to obtain an integral in terms of u only.

Don’t be concerned with the limits at this stage.

Condone the omission of the “du”.

A1ft: For any multiple of $\int \left(81u^{\frac{1}{2}} - u^{\frac{1}{2}} \right) du$. Limits are not needed.

Condone the omission of the “du”.

M1: Integrates an expression of the form $ku^{\frac{1}{2}} + lu^{\frac{1}{2}} \rightarrow Au^{\frac{3}{2}} + Bu^{\frac{3}{2}}$

ddM1: Applies correct limits to their integral either way round. Correct limits are 0 and 81 if

substituting into their integral in terms of u or 0 and $\frac{9}{4}$ if reverting to t .

Depends on all previous M marks.

A1: Correct answer. Correct work leading to $-\frac{243}{64}$ scores A0 but allow “hence area is $\frac{243}{64}$ ”.

Note that the question says “or otherwise” so some alternatives are:

Uses the substitution $u = \sqrt{81-16t^2}$:

B1: Correct derivative statement $\frac{du}{dt} = \frac{1}{2}(81-16t^2)^{-\frac{1}{2}} \times -32t$ oe such as $2u du = -32t dt$

M1A1ft: Full substitution leading to $2 \int \frac{81-u^2}{16u} \frac{2u}{-32} du = -\frac{1}{128} \int 81-u^2 du$

ddM1: Integrates to $Ku - Lu^3$ **M1:** Applies correct limits 0 and 81 (or returns to t)

Depends on all previous M marks.

A1: Correct.

Integration by parts:

B1: For $\int \frac{t}{\sqrt{81-16t^2}} dt = -\frac{1}{16} \sqrt{81-16t^2}$ seen or implied in working.

M1A1ft: Correct application of parts in the right direction



$$2 \int t^2 \times \frac{t}{\sqrt{81-16t^2}} dt = 2t^2 \times -\frac{1}{16} \sqrt{81-16t^2} - 2 \int 2t \times -\frac{1}{16} \sqrt{81-16t^2} dt$$

M1: Completes the integration process $= -\frac{t^2}{8} \sqrt{81-16t^2} + \frac{1}{4} \times -\frac{\sqrt{81-16t^2}^3}{48}$

ddM1: Applies limits of 0 and $\frac{9}{4}$. **Depends on all previous M marks.**

A1: Correct answer.

Question	Scheme	Marks
6	Assume the sequence is geometric	B1
	So $(r =) \frac{1+2k}{k} = \frac{3+3k}{1+2k}$	M1
	$\Rightarrow (1+2k)^2 = k(3+3k) \Rightarrow k^2 + k + 1 = 0$	A1
	But $k^2 + k + 1 = \left(k + \frac{1}{2}\right)^2 - \frac{1}{4} + 1 \geq \frac{3}{4} > 0$ (since $\left(k + \frac{1}{2}\right)^2 \geq 0$ for all (real) k)	dM1
	This is a contradiction and hence the original assumption is not true. The sequence is not geometric.	A1
		(5)
(5 marks)		

Notes:**(a)****B1:** States an appropriate assumption to set up the contradiction.**M1:** Uses the assumption to set up an equation in k only.

Allow equivalent work e.g. $kr = 1 + 2k$, $kr^2 = 3 + 3k \Rightarrow 3 + 3k = k \left(\frac{1+2k}{k} \right)^2$

Allow use of $\neq$ for $=$ e.g. $\sqrt{\frac{3+3k}{k}} \neq \frac{1+2k}{k}$

This may be implied by e.g. $\frac{1+2k}{k}$ is not the same as $\frac{3+3k}{1+2k}$.

A1: Reaches a correct quadratic equation in k , need not be all on one side, but terms in k and k^2 should be collected. Allow use of $\neq$ for $=$ e.g. $k^2 + k + 1 \neq 0$

dM1: Completes the square, considers the discriminant or other valid means used to reach a point where a contradiction can be deduced. E.g. as scheme, or $b^2 - 4ac = 1^2 - 4(1)(1) = -3 < 0$ may be used. Accept use of calculator to give roots $k = \frac{-1 \pm i\sqrt{3}}{2}$ so k is not real, which contradicts k being a member of the real sequence.

Depends on the previous M.

A1: Correct work leading to a contradiction with deduction of a contradiction made and conclusion given. **This mark is available even if B0 is given at the start. So 01111 is possible.**

If they are using the discriminant or calculator route then there is no need to mention “real” as long as they conclude that e.g. the geometric sequence is not possible. This can score both the dM1 and A1.

Question Number	Scheme	Marks
1(a)	$(2-5x)^{-2} = \frac{1}{4} \left(1 - \frac{5x}{2}\right)^{-2}$ or e.g. $\frac{8}{4 \left(1 - \frac{5x}{2}\right)^2}$	B1
	$= 8 \times \frac{1}{4} \left(1 + (-2) \times \left(-\frac{5x}{2}\right) + \frac{(-2) \times (-3)}{2!} \times \left(-\frac{5x}{2}\right)^2 + \frac{(-2) \times (-3) \times (-4)}{3!} \times \left(-\frac{5x}{2}\right)^3 \dots \right)$	M1A1
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
		(4)
Alt (a) by direct expansion	$8 \times (2-5x)^{-2}$ $= 8 \times \left(2^{-2} + (-2)2^{-3}(-5x)^1 + \frac{-2 \times -3}{2!} 2^{-4}(-5x)^2 + \frac{-2 \times -3 \times -4}{3!} 2^{-5}(-5x)^3 \right)$	B1, <u>M1A1</u>
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
(b)	$ x < \frac{2}{5}$ o.e.	B1
		(1)
		(5 marks)

(a)

B1: For taking out a factor of 2^{-2} or $\frac{1}{4}$ **from** $(2-5x)^{-2}$ to obtain e.g. $\frac{1}{4}(1 \pm \dots)^{-2}$, $2^{-2}(1 \pm \dots)^{-2}$.

May be implied by a constant term of 2 or by e.g. $2(1 \pm \dots)^{-2}$

The “8” is likely to be present but it is not required for this mark.

M1: For the form of the binomial expansion $(1+ax)^{-2}$

Requires the correct structure for either term three or term four. Allow a slip on the sign.

So allow for either $\frac{(-2) \times (-3)}{2} (\pm ax)^2$ or $\frac{(-2) \times (-3) \times (-4)}{3!} (\pm ax)^3$ where $a \neq 1$, could be -5 but must

be the “a” in their “2” $(1 \pm ax)^{-2}$.

Condone missing brackets around the “ax” for this mark.

A1: Any unsimplified or simplified but correct form of the binomial expansion for $\left(1 - \frac{5x}{2}\right)^{-2}$

Ignore the factor preceding the bracket for this mark and ignore any extra terms if found.

Score for $1 + (-2) \times \left(-\frac{5x}{2}\right) + \frac{(-2) \times (-3)}{2!} \times \left(-\frac{5x}{2}\right)^2 + \frac{(-2) \times (-3) \times (-4)}{3!} \times \left(-\frac{5x}{2}\right)^3$ o.e.

Brackets must be present unless they are implied by later work. Allow $\left(\frac{5x}{2}\right)^2$ for $\left(-\frac{5x}{2}\right)^2$.

The simplified form is $1 + 5x + \frac{75}{4}x^2 + \frac{125}{2}x^3 + \dots$ Allow as a list of terms.

A1: cao $2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$ This must be simplified and allow as a list of terms.



Question Number	Scheme	Marks
10 Main	Assume that $p^2 - 4q + 2 = 0$	B1
	$p^2 = 4q - 2$	M1
	$p^2 = 4q - 2 (= 2(2q - 1))$ So p^2 is even so p is even oe e.g. so $p = 2k$	A1
	If $p = 2k$ then e.g. $4k^2 = 4q - 2$ or e.g. $4k^2 = 4q - 2 \Rightarrow q = k^2 + \frac{1}{2}$ or e.g. $q = \frac{2k^2 + 1}{2}$ or e.g. $4k^2 = 4q - 2 \Rightarrow k^2 = q - \frac{1}{2}$ or e.g. $k^2 = \frac{2q - 1}{2}$ or e.g. Let $4k^2 = 4q - 2 \Rightarrow 2q - 2k^2 = 1$	dM1
	$4k^2$ is a multiple of 4 and $4q - 2$ isn't a multiple of 4 or e.g. $q = k^2 + \frac{1}{2}$ which is not an integer or e.g. $k^2 = q - \frac{1}{2}$ which is not an integer or e.g. $2q - 2k^2 = 1$ which is not possible So $p^2 - 4q + 2 \neq 0$	A1
		(5 marks)
Way 1 B1: For setting up the correct contradiction. It must include a word/words such as “assume” or “let” or “if” and $p^2 - 4q + 2 = 0$ As a minimum accept just “assume/let/if” $p^2 - 4q + 2 = 0$ ” M1: Rearranges $p^2 - 4q + 2 = 0$ to obtain $p^2 = \pm 4q \pm 2$ A1: Correct deduction for p following correct work . It is not necessary to factorise $4q - 2$ but if they do it must be correct. It requires e.g. both p^2 is even and so p is even oe e.g. so $p = 2k$ or any other letter but not $p = 2q$ or $p = 2p$ dM1: Depends on the previous method mark. For attempting to use the fact that p is even to obtain a contradiction e.g. uses that as p is even $p = 2k$ and substitutes into the equation to obtain e.g. $4k^2 = 4q - 2$ or $q = k^2 + \frac{1}{2}$ or $k^2 = q - \frac{1}{2}$ or $2q - 2k^2 = 1$ A1: Fully correct work with all previous marks scored that includes - An explanation why there is a contradiction - A (minimal) conclusion e.g. “so $p^2 - 4q + 2 \neq 0$”, QED, “proven”, ✓ Must follow all previous marks and there must be no incorrect statements/algebra in their proof.		

An odds/evens approach:

Alt	Assume that $p^2 - 4q + 2 = 0$	B1
	Evens: Let $p = 2k$, $(2k)^2 - 4q + 2 = 0$ or Odds: Let $p = 2k + 1$, $(2k + 1)^2 - 4q + 2 = 0$	M1
	$4q = 4k^2 + 2 = 4\left(k^2 + \frac{1}{2}\right)$ which is not a multiple of 4 oe e.g. $4k^2 + 2$ is 2 more than a multiple of 4 or $4q = 4k^2 + 4k + 3 = 4\left(k^2 + k + \frac{3}{4}\right)$ which is not a multiple of 4 oe e.g. $4k^2 + 4k + 3$ is 3 more than a multiple of 4	A1
	Evens: Let $p = 2k$, $(2k)^2 - 4q + 2 = 0$ and Odds: Let $p = 2k + 1$, $(2k + 1)^2 - 4q + 2 = 0$	dM1
	$4q = 4k^2 + 2 = 4\left(k^2 + \frac{1}{2}\right)$ which is not a multiple of 4 oe e.g. $4k^2 + 2$ is 2 more than a multiple of 4 and $4q = 4k^2 + 4k + 3 = 4\left(k^2 + k + \frac{3}{4}\right)$ which is not a multiple of 4 oe e.g. $4k^2 + 4k + 3$ is 3 more than a multiple of 4 $p^2 - 4q + 2 \neq 0$	A1

Alt**B1:** For setting up the correct contradiction.It must include a word/words such as “assume” or “let” or “if” and $p^2 - 4q + 2 = 0$ As a minimum accept just “assume/let/if” $p^2 - 4q + 2 = 0$ **M1:** Attempts $p^2 - 4q + 2 = 0$ with p even e.g. $p = 2k$ or equivalent.**or** attempts $p^2 - 4q + 2 = 0$ with p odd e.g. $p = 2k + 1$ or equivalent e.g. $2k - 1$ **A1:** Correct algebra and deduction for odds **or** evens**dM1:** **Depends on the previous method mark.**Attempts $p^2 - 4q + 2 = 0$ with p even e.g. $p = 2k$ or equivalent.**and** attempts $p^2 - 4q + 2 = 0$ with p odd e.g. $p = 2k + 1$ or equivalent e.g. $2k - 1$ **A1:** Fully correct work with all previous marks scored that includes

- Correct arguments for both odds and evens
- A (minimal) conclusion e.g. “so $p^2 - 4q + 2 \neq 0$ ”, QED, “proven”, ✓

Must follow all previous marks and there must be no incorrect statements/algebra in their proof.

Special case:

Assume that $p^2 - 4q + 2 = 0$

$$p^2 = 4q - 2$$

$$p = \sqrt{4q - 2} \Rightarrow p = 2\sqrt{q - \frac{1}{2}}$$

On its own scores B1M1A0dM0A0

The B1M1 score as in the general guidelines.

To score any further marks, they would need a complete proof showing why $2\sqrt{q - \frac{1}{2}}$ cannot be an integer.

See supplementary document for some example responses and suggested marking.

If a response has a mix of approaches, mark the attempt that gives the most credit.

If you are in doubt if a particular approach deserves credit then use Review.



Question Number	Scheme	Marks
10 (a)	Writes $2(4p^3 + 6p^2 + 3p) + 1$ which is odd	B1
(b)	Assumption: E.g. States that there exists integers p and q such that $\sqrt[3]{2} = \frac{p}{q}$ (where $\frac{p}{q}$ is in its simplest form) and then cubes to get $2 = \frac{p^3}{q^3}$ $2 = \frac{p^3}{q^3} \Rightarrow p^3 = 2q^3$ and concludes that p^3 is even so therefore p is even If p is even then it can be written $p = 2m$ so $(2m)^3 = 2q^3$ States that $q^3 = 4m^3$ and concludes that q^3 is even so therefore q is even This contradicts our initial statement, as if they both have a factor of 2 it means that $\frac{p}{q}$ is not in its simplest form, so $\sqrt[3]{2}$ is irrational *	(1) M1 A1 M1 A1 A1* (5) (6 marks)

(a)

B1: See scheme. Requires correct reason/algebra and a statement of the expression being odd.

Allow even + even + even + 1 = odd. Allow $2p(4p^2 + 6p + 3) + 1 = \text{odd}$

(b)

M1: Sets up the contradiction AND cubes. Condone the omission of the fact that $\frac{p}{q}$ is in its simplest form for thismark. Condone as a minimum $\sqrt[3]{2} = \frac{p}{q}$ followed by $2 = \frac{p^3}{q^3}$ o.e.A1: States that $p^3 = 2q^3$ and concludes both that p^3 is even so therefore p is even.

Accept other equivalent statements to even such as "multiple of 2"

Condone poor explanations so long as they state that both p^3 and p are evenM1: Writes $p = 2m$ so $(2m)^3 = 2q^3$ and then attempts to find $q^3 = \dots$ A1: States that $q^3 = 4m^3$ and concludes that both q^3 is even so therefore q is even

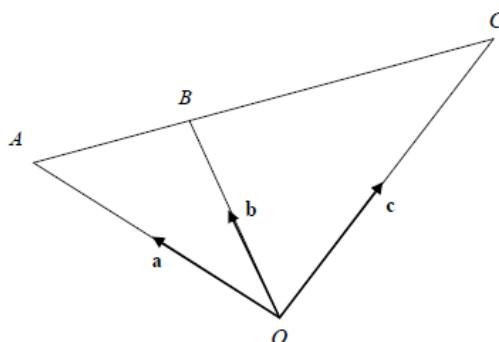
Accept other equivalent statements to even such as "multiple of 2"

Condone poor explanations so long as they state that both q^3 and q are even

A1*: Completely correct proof and conclusion with no missing statements.

To score this final mark the statements **now** need to be the correct way around.E.g. q^3 is even so therefore q is evenIt requires $\frac{p}{q}$ to be in simplest form (or equivalent such as no common factor) in the initial assumption.

Question Number	Scheme	Marks
9 (i)	Attempts two of $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{c} - \mathbf{b}$ either way around Attempts $\mathbf{c} - \mathbf{b} = 2 \times (\mathbf{b} - \mathbf{a})$ or such as $\mathbf{c} - \mathbf{a} = 3 \times (\mathbf{b} - \mathbf{a})$ $\Rightarrow \mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$ *	M1 dM1 A1* (3)
(ii)	Assume that there exists a number n that isn't a multiple of 3 yet n^2 is a multiple of 3 If n is not a multiple of 3 then $m = 3p + 1$ or $m = 3p + 2$ ($p \in \mathbb{N}$) giving $m^2 = (3p + 1)^2 = 9p^2 + 6p + 1$ Or $m^2 = (3p + 2)^2 = 9p^2 + 12p + 4 = 3(3p^2 + 4p + 1) + 1$ $(3p + 1)^2 = 9p^2 + 6p + 1 = 3(3p^2 + 2p) + 1$ is one more than a multiple of 3 $(3p + 2)^2 = 9p^2 + 12p + 4$ is not a multiple of 3 as 3 does not divide into 4 (exactly) Hence if n is a multiple of 3 then n^2 is a multiple of 3	B1 M1 M1 A1 A1 (5) (8 marks)



(i)

M1: Attempts any two of $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

Condone the wrong way around but it must be subtraction.

Allow marked in the correct place on a diagram

dM1: Uses the given information.

Accept $\overrightarrow{AB} = \frac{1}{3}\overrightarrow{AC}$, $\overrightarrow{BC} = 2 \times \overrightarrow{AB}$, $\overrightarrow{BC} = \frac{2}{3} \times \overrightarrow{AC}$ etc condoning slips as in previous M1.A1*: Fully correct work inc bracketing leading to the given answer $\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$ Expect to see the brackets multiplied out. So $\mathbf{c} - \mathbf{b} = 2 \times (\mathbf{b} - \mathbf{a}) \Rightarrow \mathbf{c} - \mathbf{b} = 2\mathbf{b} - 2\mathbf{a} \Rightarrow \mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$ is fine.

(ii)

B1: For setting up the contradiction.

Eg Assume that there exists a number n that isn't a multiple of 3, yet n^2 is a multiple of 3As a minimum accept something like "define a number n such that n is not a multiple of 3 but n^2 is"M1: States that $m = 3p + 1$ or $m = 3p + 2$ and attempts to square.Alternatives exist such as $m = 3p + 1$ or $m = 3p - 1$ Using modulo 3 arithmetic it would be $1 \rightarrow 1$ and $2 \rightarrow 4 = 1$ M1: States that $m = 3p + 1$ AND $m = 3p + 2$ and attempts to square o.e.

A1: Achieves forms that can be argued as to why they are NOT a multiple of 3

E.g. $m^2 = (3p + 1)^2 = 3(3p^2 + 2p) + 1$ or even $9p^2 + 6p + 1$ and $m^2 = (3p + 2)^2 = 3(3p^2 + 4p + 1) + 1$ or even $9p^2 + 12p + 4$

A1: Correct proof which requires

- Correct calculations
- Correct reasons. E.g. $9p^2 + 12p + 4$ is not a multiple of 3 as 4 is not a multiple of 3
There are many ways to argue these. E.g. $m^2 = (3p + 1)^2 = 3(3p^2 + 2p) + 1$ is sufficient as long as followed (or preceded by) "not a multiple of 3"
- Minimal conclusion such as ✓. Note that B0 M1 M1 M1 A1 is possible

Question Number	Scheme	Marks
9 (a)	Let $p = 3k + 2$ then $(3k + 2)^3 = 27k^3 + 54k^2 + 36k + 8$	M1
	$= 3 \times (9k^3 + 18k^2 + 12k + 3) - 1$ not a multiple of 3	A1
	So p cannot be of form $3k + 1$ or $3k + 2$, since p^3 is a multiple of 3. Hence p must be a multiple of 3, a contradiction of our assumption, hence for all integers p , when p^3 is a multiple of 3, then p is a multiple of 3	A1
		(3)
(b)	Assumption: there exist (integers) p and q such that $\sqrt[3]{3} = \frac{p}{q}$ (where p and q have no (non-trivial) common factors.)	B1
	Then $\sqrt[3]{3} = \frac{p}{q} \Rightarrow p^3 = 3q^3$	M1
	So p^3 is a multiple of 3 and (so) p is a multiple of 3	A1
	But $p = 3k \Rightarrow 27k^3 = 3q^3 \Rightarrow q^3 = 9k^3$	dM1
	Hence q^3 is a multiple of 3 so q is a multiple of 3, but as p and q have no (non-trivial) common factors, this is a contradiction. Hence $\sqrt[3]{3}$ is an irrational number.*	A1*
		(5)
		(8 marks)

Notes:

(a)

M1: Attempts to expand $(3k + 2)^3$ or $(3k - 1)^3$.Look for a cubic expression with **4 terms** with at least **two correct** (allowing for incorrect signs).A1: Achieves a correct $3 \times (\dots) + r, |r| < 10$ form for the expansion and states not a multiple of 3.Suitable forms include $(3k + 2)^3 = 3(9k^3 + 18k^2 + 12k + 2) + 2 = 3(9k^3 + 18k^2 + 12k + 3) - 1$ or $3(9k^3 + 18k^2 + 12k) + 8$ or $(3k - 1)^3 = 3(9k^3 - 9k^2 + 3k) - 1$ etc.Alternatively, achieves correct $(3k + 2)^3 = 27k^3 + 54k^2 + 36k + 8$ or $(3k - 1)^3 = 27k^3 - 27k^2 + 9k - 1$ with a **reason** why it is not a multiple of 3 e.g. 3 divides 27, 54 and 36, but not 8 hence not divisible by 3.A1: Completes the proof. Must have scored both previous marks and a reference to **both cases** (in some form) leading to a “**contradiction**” and some indication that **proof is complete**. It is unlikely to be as complete as that shown in the scheme, but all three bold points must be conveyed. E.g. as a minimum after satisfying the first A “both cases give a contradiction hence the original statement is true”.

(b)

May use different letters throughout.



B1: Sets up algebraically the initial statement to be contradicted. Essentially for showing they know what a rational number is algebraically. There is no requirement for this mark to state that p and q are integers with no (non-trivial) common factors (this may be implied for this mark).

M1: Cubes correctly and multiplies through by q^3

A1: Deduces that both p^3 is a multiple of 3 and hence p is a multiple of 3. Jumping directly to p is a multiple of 3 is A0.

dm1: Sets $p = 3k$ and proceeds to find q^3 in terms of k . (May use a different letter.)

A1: Completes the proof. This requires

- Correct algebraic statements
- Correct deductions in correct order. E.g. p^3 is a multiple of 3 so p is a multiple of 3
- initial statement must have included that p and q are integers (or accept natural numbers or a $\frac{p}{q}$ is a fraction) and have no common factors (or are co-prime, or in simplest form)
- correct reason for contradiction and acceptable conclusion
- There must have been no contrary statements during the proof (e.g. that p and q are prime)