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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Partial Fractions & the Binomial Expansion (P4.2)

International A Level Mathematics · P4: Pure Mathematics 4 (WMA14)

27 questions · 225 marks · Mark schemes included

Past papers October 2020 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
P4.2a PARTIAL FRACTIONS		10 Qs · 100 marks	2
1. Q2	IAL P4 (WMA14) October 2022	7	3
2. Q3	IAL P4 (WMA14) October 2021	8	5
3. Q4	IAL P4 (WMA14) (unused paper) January 2022	9	9
4. Q2	IAL P4 (WMA14) June 2022	9	13
5. Q2	IAL P4 (WMA14) January 2024	10	17
6. Q9	IAL P4 (WMA14) October 2024	10	21
7. Q5	IAL P4 (WMA14) January 2025	10	25
8. Q3	IAL P4 (WMA14) June 2023	11	29
9. Q7	IAL P4 (WMA14) October 2023	12	33
10. Q10	IAL P4 (WMA14) January 2021	14	37
P4.2b BINOMIAL EXPANSION FOR ANY RATIONAL POWER		17 Qs · 125 marks	42
11. Q1	IAL P4 (WMA14) January 2024	4	43
12. Q1	IAL P4 (WMA14) October 2023	5	45
13. Q4	IAL P4 (WMA14) October 2021	6	47
14. Q1	IAL P4 (WMA14) October 2024	6	49
15. Q1	IAL P4 (WMA14) January 2021	7	51
16. Q1	IAL P4 (WMA14) June 2021	7	53
17. Q1	IAL P4 (WMA14) (unused paper) January 2022	7	57
18. Q2	IAL P4 (WMA14) January 2022	7	59
19. Q1	IAL P4 (WMA14) June 2022	7	61
20. Q2	IAL P4 (WMA14) October 2020	8	63
21. Q4	IAL P4 (WMA14) October 2022	8	67
22. Q8	IAL P4 (WMA14) June 2024	8	69
23. Q3	IAL P4 (WMA14) January 2025	8	73
24. Q1	IAL P4 (WMA14) January 2023	9	75
25. Q1	IAL P4 (WMA14) June 2023	9	79
26. Q4	IAL P4 (WMA14) June 2025	9	83
27. Q8	IAL P4 (WMA14) June 2025	10	87



QUESTIONS

27 questions · 225 marks · 2 sub-topics

SUB-TOPIC P4.2a

PARTIAL FRACTIONS

Sub-topic P4.2a · 10 questions · 100 marks · lowest tariff first



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



Question 3 continued

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Q3

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Q4

Question 2 continued

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Question 2 continued

Lined area for writing the answer to Question 2 continued.



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 9 marks)







(Total for Question 2 is 10 marks)



Question 9 continued

Lined area for writing the answer to Question 9 continued.





(Total for Question 2 is 10 marks)



Question 5 continued

Lined area for writing the answer to Question 5.





(Total for Question 3 is 16 marks)



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3 continued.



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 11 marks)



7. The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x , in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where k is a constant and t is the number of years after monitoring began.

- (a) Show that $k = 1.8$ (2)
- (b) Hence calculate the long-term population of goats predicted by this model. (1)

- (b) Hence calculate the long-term population of goats predicted by this model. (1)

In a **second** model, the number of goats, x , in thousands, is modelled by the differential equation

$$3 \frac{dx}{dt} = x(9 - 2x)$$

- (c) Write $\frac{3}{x(9-2x)}$ in partial fraction form. (3)
- (d) Solve the differential equation with the initial condition to show that

- (d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}} \quad (5)$$

- (e) Find the long-term population of goats predicted by this **second** model. (1)



Question 7 continued

Lined area for writing the answer to Question 7 continued.





Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 12 marks)



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Q10

END



SUB-TOPIC P4.2b

BINOMIAL EXPANSION FOR ANY RATIONAL POWER

Sub-topic P4.2b · 17 questions · 125 marks · lowest tariff first



1. Find, in ascending powers of x up to and including the term in x^3 , the binomial expansion of

$$(1 - 4x)^{-3} \quad |x| < \frac{1}{4}$$

fully simplifying each term.

(4)



Question 1 continued

(Total for Question 1 is 4 marks)



1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\frac{8}{(2-5x)^2}$$

writing each term in simplest form.

(4)

- (b) Find the range of values of x for which this expansion is valid.

(1)



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4. $f(x) = \sqrt{1 - 4x^2}$ $|x| < \frac{1}{2}$

- (a) Find, in ascending powers of x , the first four non-zero terms of the binomial expansion of $f(x)$. Give each coefficient in simplest form.

(4)

- (b) By substituting $x = \frac{1}{4}$ into the binomial expansion of $f(x)$, obtain an approximation for $\sqrt{3}$

Give your answer to 4 decimal places.

(2)

Question 4 continued

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Q4

(Total 6 marks)



1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$(8 - 3x)^{-\frac{1}{3}} \quad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

- (b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)



Question 1 continued

(Total for Question 1 is 6 marks)



- $$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} \quad |x| < \frac{1}{20}$$

(5)

(b) find an approximation for $\sqrt{5}$

(2)



Question 1 continued

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Q1

(Total 7 marks)



Question 1 continued

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Question 1 continued

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Question 1 continued

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(Total 7 marks)

Q1



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1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$\frac{2}{\sqrt{9-2x}} \quad |x| < \frac{9}{2}$$

giving each coefficient as a simplified fraction.

(5)

By substituting $x = 1$ into the answer to part (a),

- (b) find an approximation for $\sqrt{7}$, giving your answer to 4 decimal places.

(2)



Question 1 continued

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Q1

(Total 7 marks)



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2. (a) Find, in ascending powers of x , the first three non-zero terms of the binomial series expansion of

$$\sqrt[3]{1+4x^3} \qquad |x| < \frac{1}{\sqrt[3]{4}}$$

giving each coefficient as a simplified fraction.

(4)

- (b) Use the expansion from part (a) with $x = \frac{1}{3}$ to find a rational approximation to $\sqrt[3]{31}$

(3)

Question 2 continued

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(Total 7 marks)

Q2



1. The binomial expansion of

$$(3 + kx)^{-2} \quad |kx| < 3$$

where k is a non-zero constant, may be written in the form

$$A + Bx + Cx^2 + Dx^3 + \dots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that $C = 3B$

(b) show that

$$k^2 + 6k = 0$$

(3)

(c) Hence (i) find the value of k

(ii) find the value of D

(3)

Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 7 marks)



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Question 2 continued

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Question 2 continued

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(Total 8 marks)

Q2



4.

$$g(x) = \frac{1}{\sqrt{4-x^2}}$$

- (a) Find, in ascending powers of x , the first four non-zero terms of the binomial expansion of $g(x)$. Give each coefficient in simplest form.

(5)

- (b) State the range of values of x for which this expansion is valid.

(1)

- (c) Use the expansion from part (a) to find a fully simplified rational approximation for $\sqrt{3}$

Show your working and make your method clear.

(2)

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 8 marks)



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 8 marks)



3. Given that the binomial expansion, in ascending powers of x , of

$$6(4 + Ax)^{-\frac{1}{2}}$$

is $B - \frac{1}{4}x + Cx^2 + \dots$

- (a) find the values of the constants A , B and C . (5)
- (b) State the range of values of x for which this expansion is valid. (1)
- For the expansion,
- (c) find the coefficient of the term in x^3 (2)



Question 3 continued

(Total for Question 3 is 8 marks)



$$f(x) = \frac{5x + 10}{(1 - x)(2 + 3x)}$$

(3)

(5)

(1)







Question 1 continued

(Total for Question 1 is 9 marks)



Question 1 continued

Lined area for writing the answer to Question 1 continued.



Question 1 continued

Lined area for writing the answer to Question 1 continued.



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 9 marks)



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 9 marks)



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 10 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

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Question Number	Scheme	Marks
1	Attempt at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	Attempt at product rule $3x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$	M1
	Correct differentiation $2 - 8y \frac{dy}{dx} + 6xy + 3x^2 \frac{dy}{dx} = 8x$	A1
	Substitutes $x = 3, y = 2 \Rightarrow \frac{dy}{dx} = -\frac{14}{11}$	M1, A1
	Correct method for normal at $(3, 2) \Rightarrow y - 2 = \frac{11}{14}(x - 3)$ $11x - 14y - 5 = 0$	dM1 A1 (7) (7 marks)

M1: Attempts at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$ where ... is a constant

M1: Attempt at product rule $3x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$ where ... are constants

A1: Correct differentiation $2x - 4y^2 + 3x^2 y = 4x^2 + 8 \rightarrow 2 - 8y \frac{dy}{dx} + 6xy + 3x^2 \frac{dy}{dx} = 8x$ o.e.

You may see $2x - 4y^2 + 3x^2 y = 4x^2 + 8 \rightarrow 2dx - 8ydy + 6xydx + 3x^2 dy = 8xdx$ o.e. which is fine

Note that $\frac{dy}{dx} = 2 - 8y \frac{dy}{dx} + 6xy + 3x^2 \frac{dy}{dx} - 8x$ is A0 unless recovered

M1: Substitutes $x = 3, y = 2$ into a suitable equation and finds a value for $\frac{dy}{dx}$.

It is dependent upon having exactly two $\frac{dy}{dx}$ terms, one from differentiating each of y^2 and $x^2 y$

A1: $\frac{dy}{dx} = -\frac{14}{11}$ o.e.

dM1: Dependent upon the previous M. It is for a correct method of finding the equation of

the normal at $(3, 2)$. Look for $y - 2 = \frac{11}{14}(x - 3)$ with the negative reciprocal of

their $-\frac{14}{11}$ being used.

If the form $y = mx + c$ is used they must proceed as far as $c = \dots$

A1: $11x - 14y - 5 = 0$ but accept any integer multiple of this. The "= 0" must be seen

Question Number	Scheme	Notes	Marks
3(a)	$3y^2 - 11x^2 + 11xy = 20y - 36x + 28$ $\Rightarrow 6y \frac{dy}{dx} - 22x + 11x \frac{dy}{dx} + 11y = 20 \frac{dy}{dx} - 36$ M1: $y^2 \rightarrow Ay \frac{dy}{dx}$ M1: $11xy \rightarrow px \frac{dy}{dx} + qy$ A1: All correct		M1M1A1
	$(6y + 11x - 20) \frac{dy}{dx} = 22x - 11y - 36 \Rightarrow \frac{dy}{dx} = \dots$ Collects terms in $\frac{dy}{dx}$ (must be 3 and from the appropriate terms) and makes $\frac{dy}{dx}$ the subject		M1
	$\frac{dy}{dx} = \frac{22x - 11y - 36}{6y + 11x - 20}$	Correct expression or correct equivalent	A1
			(5)
(b)	$x = 4 \Rightarrow 3y^2 - 176 + 44y = 20y - 144 + 28$	Substitutes $x = 4$ into C to obtain a 3TQ in y	M1
	$3y^2 + 24y - 60 = 0 \Rightarrow y = \dots$	Solves for y	M1
	$y = -10 \text{ (,2)}$	Correct value	A1
	$(4, -10) \rightarrow \frac{dy}{dx} = \frac{88 + 110 - 36}{-60 + 44 - 20}$	Substitutes $x = 4$ and their negative y into their $\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = -\frac{9}{2}$	Correct value	A1
			(5)
			Total 10



Question Number	Scheme	Notes	Marks
4(a)	$\frac{4-4x}{x(x-2)^2} \equiv \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$	Correct form for the partial fractions	B1
	$4-4x = A(x-2)^2 + Bx(x-2) + Cx$ $\Rightarrow A = \dots$ or $B = \dots$ or $C = \dots$	Uses a correct strategy to find at least one of their constants	M1
	$\frac{4-4x}{x(x-2)^2} \equiv \frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2}$	2 correct constants	A1
		All correct	A1
			(4)
(b)	$\int \left(\frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2} \right) dx = \ln x - \ln(x-2) + \frac{2}{x-2} (+c)$ M1 for $\int \frac{\alpha}{x} dx = \beta \ln x$ or $\int \frac{\alpha}{x-2} dx = \beta \ln(x-2)$ M1 for $\int \frac{\alpha}{(x-2)^2} dx = \frac{\beta}{x-2}$ A1: All correct		M1
			M1
			A1
			(3)
(c)	$\left[\ln x - \ln(x-2) + \frac{2}{x-2} \right]_3^5 = \left(\ln 5 - \ln 3 + \frac{2}{3} \right) - (\ln 3 - \ln 1 + 2)$ $= \ln \frac{5}{9} - \frac{4}{3}$ M1: Correct use of limits and reaches the required form using log rules A1: Correct answer		M1
			A1
			(2)
			Total 9



General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

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$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

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Question Number	Scheme	Marks
9(a)	$\frac{1}{x(2x-1)} \equiv \frac{A}{x} + \frac{B}{2x-1} \Rightarrow 1 \equiv A(2x-1) + Bx \Rightarrow A = \dots \text{ or } B = \dots$	M1
	$\frac{1}{x(2x-1)} \equiv \frac{2}{2x-1} - \frac{1}{x}$	A1
		(2)
(b)	$\frac{dh}{dt} = \frac{1}{50} h(2h-1) \cos\left(\frac{t}{10}\right) \Rightarrow \int \frac{1}{h(2h-1)} dh = \int \frac{1}{50} \cos\left(\frac{t}{10}\right) dt$	M1
	$\int \frac{1}{h(2h-1)} dh = \int \left(\frac{2}{2h-1} - \frac{1}{h} \right) dh = \ln(2h-1) - \ln h$	M1
	$\int \left(\frac{2}{2h-1} - \frac{1}{h} \right) dh = \int \frac{1}{50} \cos\left(\frac{t}{10}\right) dt \Rightarrow \ln(2h-1) - \ln h = \frac{1}{5} \sin\left(\frac{t}{10}\right) (+c)$	A1ft
	e.g. $t = 0, h = 2.5 \Rightarrow \ln(2 \times 2.5 - 1) - \ln 2.5 = c$ or e.g. $\ln \frac{2h-1}{h} = \frac{1}{5} \sin\left(\frac{t}{10}\right) + c \Rightarrow \frac{2h-1}{h} = X e^{\frac{1}{5} \sin\left(\frac{t}{10}\right)} \quad t = 0, h = 2.5 \Rightarrow X = 1.6$	M1
	$\ln(2h-1) - \ln h = \frac{1}{5} \sin\left(\frac{t}{10}\right) + \ln 1.6 \Rightarrow \frac{2h-1}{h} = 1.6 e^{\frac{1}{5} \sin\left(\frac{t}{10}\right)}$ $\Rightarrow 2h - 1.6 h e^{\frac{1}{5} \sin\left(\frac{t}{10}\right)} = 1 \Rightarrow h = \dots$	M1
	$h = \frac{5}{10 - 8e^{\frac{1}{5} \sin\left(\frac{t}{10}\right)}}$	A1
		(6)
(c)	$\sin\left(\frac{t}{10}\right) = 1 \Rightarrow \frac{t}{10} = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots \Rightarrow t = \dots$	M1
	$t = 45\pi = 141 \text{ (s)}$	A1
		(2)
		Total 10

(a)

M1: Correct attempt at partial fractions. Sets $\frac{1}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}$ and uses a correct method to

identify at least one constant. Incorrect work e.g. $\frac{A}{x} + \frac{B}{2x-1} \Rightarrow 1 = Ax + B(2x-1)$ etc. is M0.

A1: Correct partial fractions. Note that this mark is not just for the correct constants, it is for the correct written fractions but allow this mark to be scored if it is seen in part (b) (in terms of h or x).

Condone $\frac{2}{2x-1} + \frac{-1}{x}$ or written as e.g. $2(2x-1)^{-1} - x^{-1}$



Question Number	Scheme	Marks
2(a)	$2^x \rightarrow 2^x \ln 2$	B1
	$y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	$4x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$	M1
	$3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$	A1
	$(10y + 4x^2) \frac{dy}{dx} = 10 \times 2^x \ln 2 - 3 - 8xy \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$	A1
		(6)

B1: Correct differentiation $2^x \rightarrow 2^x \ln 2$

M1: For $y^2 \rightarrow \dots y \frac{dy}{dx}$ where ... is a non-zero constant.

M1: For $4x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$ where ... are non-zero constants.

A1: Correct differentiation e.g.

$$3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2 \text{ or e.g. } 3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy - 10 \times 2^x \ln 2 = 0$$

Condone a spurious " $\frac{dy}{dx} =$ " e.g. $\frac{dy}{dx} = 3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$

M1: A valid attempt to make $\frac{dy}{dx}$ the subject by factorising $\frac{dy}{dx}$ from exactly two **different** terms in

$\frac{dy}{dx}$ which have come from differentiating $5y^2$ and $4x^2 y$ and then dividing by the terms in

the bracket. Note that here, 2 **different** terms means terms such as $y \frac{dy}{dx}$ and $x^2 \frac{dy}{dx}$ and not

e.g. $y \frac{dy}{dx}$ and $3y \frac{dy}{dx}$. Look for $(\dots \pm \dots) \frac{dy}{dx} = \dots \Rightarrow \frac{dy}{dx} = \dots$ which may be implied.

$\frac{dy}{dx}$ must be non-zero and in terms of x **and** y .

Condone slips provided the intention is clear and the above conditions are satisfied.

For those candidates who had a spurious $\frac{dy}{dx} = \dots$ at the start, they may incorporate this in their

rearrangement in which case they will have 3 terms in $\frac{dy}{dx}$ and so score M0.

A1: $\frac{dy}{dx} = \frac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$ or any equivalent correct expression e.g. $\frac{2^x \ln 2 - 0.3 - 0.8xy}{y + 0.4x^2}$

Note that some candidates may divide through by 10 initially e.g.

$$3x + 5y^2 + 4x^2 y = 10(2^x) + 35 \Rightarrow 0.3x + 0.5y^2 + 0.4x^2 y = 2^x + 3.5 \text{ and full marks are available for}$$

equivalent work in (a) and (b). Note that 2^x may appear correctly as $e^{x \ln 2}$.

2(b)	At $x = 0$ $5y^2 = 45 \Rightarrow y = 3$ $x = 0, y = 3 \Rightarrow \frac{dy}{dx} = \frac{10 \times 2^0 \ln 2 - 3 - 8 \times 0 \times 3}{10 \times 3 + 4 \times 0^2} = \frac{10 \ln 2 - 3}{30}$	M1 A1
		(2)
Notes: M1: Full method of finding the gradient. Requires: - substituting $x = 0$ into the equation for C and finding a non-zero value for y. Do not be concerned about the processing as long as they are using $x = 0$ in the equation for C and finding a non-zero value for y - substituting $x = 0$ and their non-zero y at $x = 0$ into their $\frac{dy}{dx}$ to find a numerical value This may be implied by their value. Condone slips in copying their $\frac{dy}{dx}$ from part (a) if the intention is clear. If they have a “$\ln x$” term on the rhs then condone “$\ln 0$” appearing as part of their substitution but if no substitution is shown in such cases, score M0 A1: $\frac{10 \ln 2 - 3}{30}$ oe e.g. $\frac{1}{3} \ln 2 - \frac{1}{10}$, $\frac{\ln 2^{10} - 3}{30}$, $\frac{\ln 1024 - 3}{30}$ <u>from correct work.</u> Apply isw once a correct answer is seen. There must be no other answers e.g. some candidates use $y = \pm 3$ to give 2 gradients. There are many different incorrect expressions for $\frac{dy}{dx}$ that will give the correct answer here so this must follow a correct $\frac{dy}{dx}$ in part (a).		
		(8 marks)

(c) **Otherwise:** Applies binomial expansion to $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}}$

B1: Correct simplified expression $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \frac{1}{2}(1 - 2x)^{\frac{1}{2}}$

M1: Correct structure for 3rd term: $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-2x)^2$ but allow $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm kx)^2$ with or without brackets. Even allow with $k=1$

A1: $\frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$ Ignore extra terms of x^3 and above

Direct expansion in (a)

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{3}{2}} = \left(\frac{1}{4}\right)^{\frac{3}{2}} + \left(-\frac{3}{2}\right)\left(\frac{1}{4}\right)^{\frac{5}{2}}\left(-\frac{1}{2}x\right) + \frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{\frac{7}{2}}\left(-\frac{1}{2}x\right)^2 + \frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right) \times \left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{\frac{9}{2}}\left(-\frac{1}{2}x\right)^3$$

B1: Obtains an expansion with a constant term of 8

M1: Attempts the binomial expansion of $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.

Look for $\frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{\frac{7}{2}}\left(\pm\frac{1}{2}x\right)^2$ or $\frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right) \times \left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{\frac{9}{2}}\left(\pm\frac{1}{2}x\right)^3$

with or without the brackets on the $\left(\pm\frac{1}{2}x\right)$

A1: Correct simplified or unsimplified expansion .

Expression at the top of the page is acceptable

A1: Two correct and simplified terms of $8 + 24x + 60x^2 + 140x^3$

A1: All correct and simplified $8 + 24x + 60x^2 + 140x^3$

Direct expansion in (c)

B1: First two terms which may be unsimplified $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} + \frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}\left(-\frac{1}{2}x\right)^1 + \dots$

M1: Correct form for the third term $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}\left(-\frac{1}{2}x\right)^2$

A1: Correct simplified expansion $\frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$

(a)

B1: Correct coordinates or position vector e.g. as shown or $x = 2, y = 3, z = -7$ or $2\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ or $\begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix}$

Condone $(2\mathbf{i}, 3\mathbf{j}, -7\mathbf{k})$

(b)

M1: Attempts $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ condoning slips. Note that any non-zero multiples of these vectors can be used.

If the method is not explicit then this mark may be implied by 2 correct components.

dM1: Full attempt at $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where $\mathbf{a}$ and $\mathbf{b}$ are the direction vectors or multiples of them.

Depends on the first method mark.

Note that $\cos\theta = \frac{4 - 2 + 16}{\sqrt{1^2 + 2^2 + 2^2} \sqrt{4^2 + 1^2 + 8^2}}$ would imply both method marks.

A1: $\cos\theta = \frac{2}{3}$

(b) Alternative using the cosine rule:

M1: Attempts $\pm \left(\alpha \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} - \beta \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right)$

dM1: Full attempt at the cosine rule using the appropriate lengths.

Depends on the first method mark.

A1: $\cos\theta = \frac{2}{3}$

(c)

M1: Uses $\lambda = 6$ to find the length of PQ . E.g. uses Pythagoras to calculate $|6\mathbf{i} + 12\mathbf{j} + 12\mathbf{k}|$ or $6|\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}|$

If the method is not explicit then the attempt at $\overline{PQ}$ may be implied by 2 correct components.

M1: Full attempt at $\frac{1}{2}ab\sin C$ where $a = b = |\overline{PQ}|$ and C is their θ which may be attempted in decimals.

Their θ or $\sin\theta$ must follow an attempt to use their $\cos\theta$.

A1: $54\sqrt{5}$ or exact equivalent e.g. $\frac{162\sqrt{5}}{3}$



Question Number	Scheme	Marks
10(a)	$\frac{1}{(H-5)(H+3)} = \frac{A}{H-5} + \frac{B}{H+3} \Rightarrow A = \dots \text{ or } B = \dots$	M1
	$A = \frac{1}{8} \text{ or } B = -\frac{1}{8}$	A1
	$\frac{1}{(H-5)(H+3)} = \frac{1}{8(H-5)} - \frac{1}{8(H+3)} \text{ or } \frac{\frac{1}{8}}{(H-5)} - \frac{\frac{1}{8}}{(H+3)} \text{ or } \frac{\frac{1}{8}}{(H-5)} + \frac{-\frac{1}{8}}{(H+3)}$ or $\frac{1}{8H-40} - \frac{1}{8H+24}$	A1
		(3)
(b)	$\frac{dH}{dt} = -\frac{(H-5)(H+3)}{40}$ $\int \frac{40}{(H-5)(H+3)} dH = \int -1 dt \text{ or e.g. } \int \frac{1}{(H-5)(H+3)} dH = \int -\frac{1}{40} dt$ $\int \left(\frac{5}{(H-5)} - \frac{5}{(H+3)} \right) dH = \int -1 dt \text{ or e.g. } \frac{1}{8} \int \left(\frac{1}{(H-5)} - \frac{1}{(H+3)} \right) dH = \int -\frac{1}{40} dt$	M1
	$5 \ln H-5 - 5 \ln H+3 = -t(+c) \text{ oe e.g. } \frac{1}{8} \ln H-5 - \frac{1}{8} \ln H+3 = -\frac{1}{40} t(+c)$ Or e.g. $5 \ln(8H-40) - 5 \ln(8H+24) = -t(+c) \text{ etc.}$	M1 A1ft
	Substitutes $t=0, H=13 \Rightarrow c = \dots$ Note that this may happen at a later stage e.g. may attempt to remove logs and then substitute to find the constant	M1
	$5 \ln H-5 - 5 \ln H+3 = -t + 5 \ln\left(\frac{1}{2}\right) \text{ oe e.g.}$ $\frac{1}{8} \ln H-5 - \frac{1}{8} \ln H+3 = -\frac{1}{40} t + \frac{1}{8} \ln\left(\frac{1}{2}\right)$	A1
	$5 \ln\left(2 \left \frac{H-5}{H+3} \right \right) = -t \Rightarrow \frac{H-5}{H+3} = \frac{1}{2} e^{-0.2t} \Rightarrow H = \dots$	dddM1
	$H = \frac{10 + 3e^{-0.2t}}{2 - e^{-0.2t}} *$	A1*
		(7)
(c)	Sets $\frac{10 + 3e^{-0.2t}}{2 - e^{-0.2t}} = 8 \Rightarrow e^{-0.2t} = \left(\frac{6}{11}\right)$	M1
	$\Rightarrow t = -5 \ln\left(\frac{6}{11}\right) = \text{awrt } 3.03 \text{ days}$	dM1 A1
		(3)
(d)	$k = 5$	B1
		(1)
		(14 marks)

(a)

M1: Attempts any correct method to find either constant. It is implied by one correct constant

A1: One correct constant



A1: Correct partial fractions: $\frac{1}{8(H-5)} - \frac{1}{8(H+3)}$. Note that this mark is not just for the correct constants, it is for the correctly stated fractions either in part (a) or used in part (b). Allow 0.125 for 1/8.

(b)

M1: Separates the variables and uses part (a) to reach: $\int \frac{P}{(H-5)} + \frac{Q}{(H+3)} dH = \int \pm k dt$ with or without the integral signs

M1: Attempts to integrate both sides to reach: $\alpha \ln|H-5| + \beta \ln|H+3| = kt$

or e.g. $\alpha \ln|8H-40| + \beta \ln|8H+24| = kt$ Condone $| \leftrightarrow ()$ and condone the omission of brackets e.g. allow $\alpha \ln H - 5 + \beta \ln H + 3 = kt$ or e.g. $\alpha \ln 8H - 40 + \beta \ln 8H + 24 = kt$

A1ft: Correct integration of both sides following through on their PF in (a). Condone $| \leftrightarrow ()$ and condone the omission of $+c$ but brackets must be present unless they are implied by subsequent work.

Also follow through on a MR of $\frac{dH}{dt} = \frac{(H-5)(H+3)}{40}$ for $\frac{dH}{dt} = -\frac{(H-5)(H+3)}{40}$

E.g. obtains $\frac{1}{8} \ln|H-5| - \frac{1}{8} \ln|H+3| = \frac{1}{40}t (+c)$

M1: Substitutes $t=0, H=13 \Rightarrow c=...$ For this to be scored there must have been a $+c$ and depends on some attempt at integration of both sides however poor.

Alternatively attempts $\int_{13}^H \frac{1}{(H-5)} - \frac{1}{(H+3)} dH = \int_0^t -\frac{1}{5} dt \Rightarrow \left[\ln \frac{H-5}{H+3} \right]_{13}^H = \left[-\frac{1}{5}t \right]_0^t \Rightarrow \ln \frac{H-5}{H+3} - \ln \frac{1}{2} = -\frac{1}{5}t$

A1: For a correct equation in H and t . Condone $| \leftrightarrow ()$ but brackets must be present unless they are implied by subsequent work.

dddM1: A correct attempt to make H the subject of the formula. **All previous M's in (b) must have been scored.**

A1*: $H = \frac{10+3e^{-0.2t}}{2-e^{-0.2t}}$ cso with sufficient working shown and no errors.

Note that marks in (b) may need to be awarded retrospectively:

E.g. First 3 marks gained to reach $\ln|H-5| - \ln|H+3| = -\frac{1}{5}t + c$ and then:

$$\ln \frac{H-5}{H+3} = -\frac{1}{5}t + c \Rightarrow \frac{H-5}{H+3} = Ae^{-0.2t} \Rightarrow H = \frac{5+3Ae^{-0.2t}}{1-Ae^{-0.2t}}$$

$$H=13, t=0 \Rightarrow 13 = \frac{5+3A}{1-A} \Rightarrow A = \frac{1}{2} \Rightarrow H = \frac{5+\frac{3}{2}e^{-0.2t}}{1-\frac{1}{2}e^{-0.2t}} = \frac{10+3e^{-0.2t}}{2-e^{-0.2t}} *$$

The M3 can be awarded when they attempt to find "A", the dddM4 can be awarded for a correct attempt to make H the subject and then A2 and A3 can be awarded together at the end.

(c)

M1: Sets $\frac{10+3e^{-0.2t}}{2-e^{-0.2t}} = 8$ or possibly an earlier version of H or possibly their t in terms of H and reaches

$$Ae^{\pm 0.2t} = p, \quad p > 0$$

dM1: Correct processing of an equation of the form $Ae^{\pm 0.2t} = p$ with correct log work leading to $t=...$

Depends on the first M.

A1: $t = -5 \ln\left(\frac{6}{11}\right)$ or $t = 5 \ln\left(\frac{11}{6}\right)$ or awrt 3.03 (days)

(d)

B1: $k = 5$ (Allow $H = 5$ or just “5”)



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

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Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



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Attempt to use the correct formula (with values for a, b and c).

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Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice



Question Number	Scheme	Marks
3 (a)	Attempts $(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) - (8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ $\overrightarrow{RQ} = -6\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	M1 A1 (2)
(b)	Attempts $\overrightarrow{PQ} \cdot \overrightarrow{RQ} = 2 \times -6 + -3 \times 2 + 4 \times 1$ Full attempt to find $\cos PQR$ E.g. $2 \times -6 + -3 \times 2 + 4 \times 1 = \sqrt{29} \sqrt{41} \cos PQR$ Angle $PQR = 114^\circ$	M1 dM1 A1 (3) (5 marks)

(a)

M1: Attempts to subtract vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ either way around. Look for $\overrightarrow{PQ} - \overrightarrow{PR}$ or $\overrightarrow{PR} - \overrightarrow{PQ}$. If a method is not shown it can be implied by two correct components of $\pm 6\mathbf{i} \pm 2\mathbf{j} \pm \mathbf{k}$

Note that an attempt such as $\overrightarrow{PR} - \overrightarrow{QP}$ is M0

A1: $\overrightarrow{RQ} = -6\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ o.e. such as $\begin{pmatrix} -6 \\ 2 \\ 1 \end{pmatrix}$ Do not accept coordinates or indeed $\begin{pmatrix} -6\mathbf{i} \\ 2\mathbf{j} \\ \mathbf{k} \end{pmatrix}$

(b)

M1: Attempts scalar product of $\overrightarrow{PQ}$ and their $\overrightarrow{RQ}$. Look for an attempt at multiplying together the components and adding. There will be some confusion over direction so allow for sight of

$$(\pm 2 \times \pm 6) + (\pm 3 \times \pm 2) + (\pm 4 \times \pm 1)$$

This cannot be scored if they attempt a scalar product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ for instance.

dM1: Full attempt to find $\cos PQR$ using $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ using vectors $\pm \overrightarrow{PQ}$ and their $\pm \overrightarrow{RQ}$.

There must be an attempt at both moduli with at least one correct (which may be unsimplified)

but you should fit on their $\overrightarrow{RQ}$. Don't be concerned whether the angle is acute or obtuse.

A1: Angle $PQR = \text{awrt } 114^\circ$ ISW after sight of this, e.g followed by 66°

Alt (b)

M1: Attempts all three lengths or all three lengths² using Pythagoras' Theorem. Look for an attempt to square and add with at least one modulus or modulus² correct.

dM1: Attempts to use the cosine rule with the lengths in the correct positions in order to find angle PQR

$$\text{Look for } \cos PQR = \frac{PQ^2 + QR^2 - PR^2}{2 \times PQ \times QR}$$

There are more round about methods including finding other angles first and then using the sine rule

but this method mark can only be awarded when an attempt is made at angle PQR

A1: Angle $PQR = \text{awrt } 114^\circ$

It must be found using "correct" vectors, e.g. $\pm(-6\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\pm(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$

If, for example $\overrightarrow{RQ}$ is incorrect, e.g. $6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, A0 will be awarded even if 114° is stated

Question Number	Scheme	Marks
4(a)	$\frac{1}{\sqrt{4-x^2}} = (4-x^2)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}(1-\dots)$ $\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{1}{4}x^2\right)^2 + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$ $\frac{1}{\sqrt{4-x^2}} = \frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$	B1 M1, A1 A1, A1 (5)
(b)	$ x < 2$	B1
(c)	Substitutes an appropriate value of x in both sides with LHS in terms of $\sqrt{3}$ E.g. with $x = 1$ $\sqrt{3} = \frac{2048}{1181}$ or $\frac{3543}{2048}$	M1 A1 (2)
		(8 marks)

(a) Note that the first mark in (a) is M1 on open. We are now marking it as B1

B1: Correct constant term/ factor.

Accept as $4^{-\frac{1}{2}}$ or $\frac{1}{2}$ as the constant term or as the factor with the bracket starting (1....

M1: Correct attempt at the third or the fourth term of binomial expansion of form $\left(1+ax^2\right)^{-\frac{1}{2}}$

Look for a correct binomial coefficient with a correct power of x

E.g. $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(\pm ax^2\right)^2$ or $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(\pm ax^2\right)^3$ where a could be

1

A1: Correct unsimplified

For example

$$\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(-\frac{1}{4}x^2\right)^2 + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$$

A1: Two correct and simplified terms of $\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$

A1: $\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$. Accept as a list, ignore any terms with indices greater than these.

(b)

B1: States a correct range. E.g. $|x| < 2$ or $-2 < x < 2$ o.e.

(c) For example: See **

M1: Substitutes an exact value of x into both sides of their expansion that enables $\sqrt{3}$ to be found

.



Question	Scheme	Marks
8(a)	$(8-3x)^{\frac{4}{3}} = 16\left(1-\frac{3}{8}x\right)^{\frac{4}{3}}$ but condone $16(1-kx)^{\frac{4}{3}}, k \neq 3$	M1
	"Correct" term 3 or term 4 in $(1-kx)^{\frac{4}{3}} = 1 \pm \frac{4}{3}kx + \frac{\frac{4}{3} \times \frac{1}{3}}{2}(\pm kx)^2 + \frac{\frac{4}{3} \times \frac{1}{3} \times \left(-\frac{2}{3}\right)}{6}(\pm kx)^3 + \dots$	M1
	Two terms correct of $16-8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$ following M1, M1 and $k = \pm \frac{3}{8}$	A1
	$= 16-8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$	A1
		(4)
(b)	Assume the curves meet so $8x - \frac{15x^2}{2} + 8 = "16-8x + \frac{x^2}{2} + \frac{x^3}{24}"$ (for some x)	M1
	$\Rightarrow 0 = 8-16x+8x^2 + \frac{x^3}{24} + \dots \Rightarrow 0 = 8(x^2-2x)+8+\frac{x^3}{24} + \dots \Rightarrow 8(x-1)^2 \pm \dots$	M1
	$\Rightarrow 0 = 8(x-1)^2 + \frac{x^3}{24}$	A1
	Requires - Argument that $8(x-1)^2 + \frac{x^3}{24} > 0$. E.g. $8(x-1)^2 \geq 0$ and $\frac{x^3}{24} > 0$ (for $0 < x < \frac{8}{3}$) so $8(x-1)^2 + \frac{x^3}{24} > 0$ - Minimal statement (contradiction) and conclusion	A1
		(4)
(8 marks)		

(a)

M1: Attempts to take out a factor of 16 or $8^{\frac{4}{3}}$. Look for $(8-3x)^{\frac{4}{3}} = 16(1-kx)^{\frac{4}{3}}, k \neq 3$

or $(8-3x)^{\frac{4}{3}} = 8^{\frac{4}{3}}(1-kx)^{\frac{4}{3}}, k \neq 3$ Condone incorrect values of k other than 3

M1: Attempts the binomial expansion on $(1-kx)^{\frac{4}{3}}$ achieves at least a "correct" term 3 or term 4 (which may be unsimplified) for their k . Even allow with $k=1$

Look for either $\frac{\frac{4}{3} \times \frac{1}{3}}{2}(\pm kx)^2$ or $\frac{\frac{4}{3} \times \frac{1}{3} \times \left(-\frac{2}{3}\right)}{6}(\pm kx)^3$ with the correct binomial coefficient and the correct power of x . Condone missing brackets on the (kx) terms.

If they expand $\left(1-\frac{3}{8}x\right)^{\frac{4}{3}}$ without showing the unsimplified terms it is implied by $\pm \frac{x^3}{384}$

A1: For two correct and simplified terms of $16-8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$ following M1, M1 and $k = \pm \frac{3}{8}$

A1: Correct expansion from correct work. Terms may be as a list for both accuracy marks



Alt (a) by direct expansion. $(8 \pm 3x)^{\frac{4}{3}} = 8^{\frac{4}{3}} + \frac{4}{3} \left(8^{\frac{1}{3}} \right) (\pm 3x) + \frac{\frac{4}{3} \times \frac{1}{3}}{2} \left(8^{\frac{2}{3}} \right) (\pm 3x)^2 + \frac{\frac{4}{3} \times \frac{1}{3} \times \left(-\frac{2}{3} \right)}{6} \left(8^{\frac{5}{3}} \right) (\pm 3x)^3 + \dots$

M1: For $8^{\frac{4}{3}}$ or 16 or seen as the constant term E.g. $= 16 \pm \dots x + \dots$

M1: For the correct term 3 or term 4 of the expansion. Condone missing brackets on the $(\pm 3x)^2$, $(\pm 3x)^3$ terms

A1: For two correct and simplified terms of $16 - 8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$ Allow as a list

A1: Correct expansion from correct work. Allow as a list

(b)

M1: Sets up the contradiction hypothesis using the result of their (a) and setting equal to the quadratic equation. There must be some words so look for a minimum of

Assume they meet (or simply let) $8x - \frac{15x^2}{2} + 8 = "16" - 8x + \frac{x^2}{2} + \frac{x^3}{24}$

M1: Rearranges terms and attempts to complete the square (or factorises as $k(x-1)(x-1)$) on the quadratic terms or other means to show the quadratic part is always positive (e.g. considers discriminant). The work on the quadratic terms of the expression, which must be of the form $8x^2 - 16x + (A-8)$ should take the form of one of

- $8x^2 - 16x + (A-8) = 8(x-1)^2 + \dots$
- $8x^2 - 16x + (A-8)$ and attempts $b^2 - 4ac = 16^2 - 4 \times 8(A-8) = \dots$

A1: Correctly completes the square to form equation $0 = 8(x-1)^2 + \frac{x^3}{24}$.

Via the discriminant you would need $b^2 - 4ac = 16^2 - 4 \times 8 \times 8 = 0$,

A1: Completes the argument by drawing together the contradiction.

States that $8(x-1)^2 \geq 0$ and $\frac{x^3}{24} > 0$ (in the interval), hence $0 \neq 8(x-1)^2 + \frac{x^3}{24}$ so proven

Stating that $8(x-1)^2 + \frac{x^3}{24} > 0$ is insufficient without some explanation. (See above)

Stating that both terms are positive (as $x > 0$) is not enough as $8(x-1)^2 = 0$ at $x = 1$

For discriminant must mention $b^2 - 4ac = 0$, hence single root (at 1) and positive quadratic or similar, $\frac{x^3}{24} > 0$ hence $0 \neq 8(x-1)^2 + \frac{x^3}{24}$ so proven.

.....
There will be alternatives for the middle two marks in (b), so consider carefully what is attempted.

For example, candidates could work on the cubic $8 - 16x + 8x^2 + \frac{x^3}{24}$. Condone use of calculus which would involve some calculator work that proves that the function has a minimum value at $x = \text{awrt } 0.99$ $y = \text{awrt } 0.04$ and hence $8 - 16x + 8x^2 + \frac{x^3}{24} \neq 0$ (for $0 < x < \frac{8}{3}$) proving that there is a contradiction. **M1:**

Differentiates, sets $= 0$ and solves to find x and y in the interval. **A1:** Correct with proof of minimum

Note that merely solving $8 - 16x + 8x^2 + \frac{x^3}{24} = 0$ on a calculator to get $x = -194$ and stating this is not in the range is **M0**.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \dots$$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0, \text{ leading to } x = \dots$$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Marks
1	Volume = $\int_0^8 \frac{16\pi}{(x+2)^2} (dx)$ oe e.g. $\pi \int_0^8 \left(\frac{4}{x+2}\right)^2 (dx)$	B1
	$\int \frac{16}{(x+2)^2} dx = \frac{-16}{(x+2)}$ or e.g. $u = x+2 \Rightarrow \int \frac{16}{(x+2)^2} dx = \int \frac{16}{u^2} du = \frac{-16}{u}$	M1 A1
	$\left[\frac{-16\pi}{(x+2)} \right]_0^8 = \frac{-16\pi}{10} - \left(\frac{-16\pi}{2} \right)$ or e.g. $\left[\frac{-16\pi}{u} \right]_2^{10} = \frac{-16\pi}{10} - \left(\frac{-16\pi}{2} \right)$	M1
	$= \frac{32}{5}\pi$	A1
		(5)
		(5 marks)
B1: Correct expression for the volume in terms of x including the limits (which may be implied by later work). Condone omission of the “dx” but the correct integral with the correct limits and the 16π included must be seen or implied. M1: For $\int \frac{\dots}{(x+2)^2} (dx) \rightarrow \frac{\dots}{x+2}$ oe e.g. $\dots(x+2)^{-1}$ where “...” are non-zero constants. Or e.g. $\int \frac{\dots}{(x+2)^2} dx = \int \frac{\dots}{u^2} (du) \rightarrow \frac{\dots}{u}$ oe e.g. $\dots u^{-1}$ where “...” are non-zero constants. A1: $\int \frac{16}{(x+2)^2} (dx) = \frac{-16}{(x+2)}$ or $\frac{-16}{u}$ where $u = x+2$. Ignore any reference to π which could be 2π or e.g. 180°. The limits are not required for this mark so you can ignore any that are given. M1: Substitutes the limits 0 and 8 into their attempted integration of $\frac{\dots}{(x+2)^2}$ or e.g. 2 and 10 into their attempted integration of $\frac{\dots}{u^2}$ and <u>subtracts</u> either way round. Allow this mark following poor attempts at integration, e.g. $\int \frac{\dots}{(x+2)^2} (dx) \rightarrow \ln(x+2)^2$ but it must be a “changed” function e.g. not $\int \frac{\dots}{(x+2)^2} (dx) \rightarrow \frac{\dots}{(x+2)^2}$ and it must be an expression that can be evaluated e.g. not expressions containing $\frac{1}{x}$ or e.g. $\ln x$ that cannot be evaluated at $x = 0$ If the integration is incorrect and the substitution of limits is not seen explicitly then you may need to check their answer. A1: $\frac{32}{5}\pi$ or exact equivalent e.g. $\frac{64\pi}{10}$, 6.4π following fully correct work. $\frac{32}{5}\pi + c$ is A0. Note $\int_0^8 \frac{16\pi}{(x+2)^2} dx = \frac{32}{5}\pi$ on its own scores B1M0A0M0A0		

Question Number	Scheme	Marks
1 (a)	$\frac{5x+10}{(1-x)(2+3x)} \equiv \frac{A}{1-x} + \frac{B}{2+3x} \Rightarrow$ Value for A or B	M1
	One correct value, either $A=3$ or $B=4$	A1
	Correct PF form $\frac{3}{1-x} + \frac{4}{2+3x}$	A1
		(3)
(b)(i)	$\frac{A}{1-x} = A(1-x)^{-1} = A\left(1+x+x^2+\dots\right)$	B1
	$\left\{\frac{B}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1} = \left\{\frac{B}{2}\right\}\left(1+(-1)\frac{3x}{2}+\frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2+\dots\right); = \frac{B}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\dots\right)$	M1; A1
	$f(x) = 3x\left(1+x+x^2+\dots\right) + \frac{4}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\dots\right)$	M1
	$= 5 + \frac{15}{2}x^2 + \dots$	A1
		(5)
(b)(ii)	$ x < \frac{2}{3}$	B1
		(1)
		(9 marks)
(b)(i) Alt 1	$(1-x)^{-1} = 1+x+x^2+\dots$	B1
	$\left\{\frac{1}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1} = \left\{\frac{1}{2}\right\}\left(1+(-1)\frac{3x}{2}+\frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2+\dots\right); = \frac{1}{2}\left(1-\frac{3x}{2}+\frac{9}{4}x^2+\dots\right)$	M1; A1
	$\frac{5x+10}{(1-x)(2+3x)} = (5x+10)\left(1+x+x^2+\dots\right) \times \frac{1}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\dots\right) = 5+\dots x+\dots x^2$	M1
	$= 5 + \frac{15}{2}x^2 + \dots$	A1
		(5)
(b)(i) Alt 2	$\frac{5x+10}{(1-x)(2+3x)} = (5x+10)\left(2+\left(x-3x^2\right)\right)^{-1} = \frac{1}{2}(5x+10)\left(1+\frac{1}{2}\left(x-3x^2\right)\right)^{-1}$	B1
	$(1+p(x))^{-1} = \left(1 \pm p(x) + \frac{(-1)(-2)}{2}(p(x))^2 + \dots\right); \frac{1}{2}\left(1 - \frac{1}{2}\left(x-3x^2\right) + \frac{1}{4}\left(x-3x^2\right)^2 + \dots\right)$	M1; A1
	$(10+5x)\left(\frac{1}{2} - \frac{1}{4}x + \frac{3}{4}x^2 + \frac{1}{8}x^2 + \dots\right) = 5 - \frac{5}{2}x + \frac{35}{4}x^2 + \frac{5}{2}x - \frac{5}{4}x^2 + \dots$	M1

Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} = 8(1-2x)^{-\frac{3}{2}}$	B1
	$(1-2x)^{-\frac{3}{2}} = 1 + \left(-\frac{3}{2}\right)(-2x) + \frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(-2x)^2 + \frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)}{3!}(-2x)^3 + \dots$	M1 A1
	$\left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} = 8 + 24x + 60x^2 + 140x^3 + \dots$	A1, A1
		(5)
(b)	$n = 2$	B1
		(1)
(c)	$\left(\frac{1}{4} - \frac{1}{2}x\right)^2 = \frac{1}{16} - \frac{1}{4}x + \frac{1}{4}x^2$	B1
	$\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \left(\frac{1}{16} - \frac{1}{4}x + \frac{1}{4}x^2\right)(8 + 24x + 60x^2 + 140x^3 + \dots)$ $= 8 \times \frac{1}{16} + 24x \times \frac{1}{16} + 60x^2 \times \frac{1}{16} - 8 \times \frac{1}{4}x - 24x \times \frac{1}{4}x + 8 \times \frac{1}{4}x^2$	M1
	$= \frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$	A1
		(3)
		Total 9

(a)

B1: Takes out the **correct and simplified** common factor to obtain $8(1 \pm \dots)^{-\frac{3}{2}}$ Implied by an expansion $8 + \dots$

M1: Attempts the binomial expansion of $(1 \pm kx)^{-\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.

The correct binomial coefficient must be combined with the correct power of x .

Look for $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(\pm kx)^2$ or $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)}{3!}(\pm kx)^3$ with or without the brackets on the kx

Even allow with $k = 1$

A1: Correct simplified or unsimplified expansion for $(1-2x)^{-\frac{3}{2}}$. (NB simplified is $1 + 3x + \frac{15}{2}x^2 + \frac{35}{2}x^3 + \dots$)

A1: Two correct and simplified terms of $8 + 24x + 60x^2 + 140x^3$

A1: All correct and simplified $8 + 24x + 60x^2 + 140x^3$ Ignore extra terms of x^4 and above

(b)

B1: Correct value, $n = 2$. Do NOT allow incomplete answers such as $\frac{4}{2}$

(c) Hence

B1: Correct expansion of $\left(\frac{1}{4} - \frac{1}{2}x\right)^2$, simplified or unsimplified

M1: Attempts correct strategy to find expansion.

Terms do not need to be collected. There may be other terms as well, for example terms in x^3

Look for an attempt to find 4 or more terms from the following (condoning slips)

$$(A + Bx + Cx^2)(E + Fx + Gx^2 + Hx^3 + \dots) = AE + AFx + AGx^2 + BEx + BFx^2 + CEx^2 + \dots$$

A1: Correct simplified expansion. Ignore extra terms of x^3 and above



	Alternative by substitution $u = 3x$	
	$\text{e.g. } u = 3x \Rightarrow \int 2x \ln(3x) dx = \int \frac{2}{3} u \ln u \times \frac{1}{3} du$ $= \frac{1}{9} u^2 \ln u - \frac{2}{9} \int \frac{u^2}{2} \times \frac{1}{u} du$	M1
	$= \dots + \dots u^2$	dM1
	$\frac{1}{9} u^2 \ln u - \frac{2}{9} \times \frac{u^2}{4}$	A1
	$x = 1, 3 \Rightarrow u = 3, 9$ $9 \ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2} \right) = 17 \ln 3 - 4$	ddM1A1

M1: Attempts integration by parts the right way round achieving $\dots u^2 \ln(u) \pm \dots \int \frac{1}{u} \times u^2 du$

Condone e.g. $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \int \frac{u^2}{2} \times \frac{1}{u} du$

dM1: For $\dots u^2 \ln(u) \pm \dots \int \frac{1}{u} \times u^2 du \dots \pm \dots u^2$. **Depends on the previous method mark.**

Condone e.g. $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \times \frac{u^2}{4}$

A1: Fully correct integration for their substitution: $\frac{1}{9} u^2 \ln u - \frac{2}{9} \times \frac{u^2}{4}$ or unsimplified equivalent e.g.

$\frac{2}{9} \left\{ \frac{1}{2} u^2 \ln u - \frac{u^2}{4} \right\}$ with or without a constant of integration.

Condone e.g. $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \times \frac{u^2}{4}$ but not e.g. $\frac{1}{9} \ln u^3 - \frac{2}{9} \times \frac{u^2}{4}$ unless $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \times \frac{u^2}{4}$ is seen first.

Any necessary brackets must be present unless they are implied by subsequent work.

ddM1: Substitutes in the limits 9 **and** 3 into an expression of the form $\alpha u^2 \ln(u) + \beta u^2$ or reverts back to x and substitutes in the limits 3 **and** 1 and subtracts the correct way round.

Condone missing brackets e.g. $9 \ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$

Condone slips in substitution provided the intention is clear.

They may work on each term separately e.g.

$$\left[\frac{1}{9} u^2 \ln u - \frac{u^2}{18} \right]_3^9 = \left[\frac{1}{9} u^2 \ln u \right]_3^9 - \left[\frac{u^2}{18} \right]_3^9 = 9 \ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2} \right)$$

It is dependent on the two previous method marks.

A1: $17 \ln 3 - 4$

	Alternative by substitution $u = \ln(3x)$.	
	$\text{e.g. } u = \ln(3x) \Rightarrow \int 2x \ln(3x) dx = \int \frac{2}{3} e^u u \times \frac{1}{3} e^u du = \frac{2}{9} \int u e^{2u} du$ $= \frac{1}{9} u e^{2u} - \frac{1}{9} \int e^{2u} du$	M1
	$= \dots + \dots e^{2u}$	dM1
	$\frac{1}{9} u e^{2u} - \frac{1}{18} e^{2u}$	A1
	$x = 1, 3 \Rightarrow u = \ln 3, \ln 9$ $9 \ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2} \right) = 17 \ln 3 - 4$	ddM1A1
M1: Attempts integration by parts the right way round achieving $\dots u e^{2u} \pm \dots \int e^{2u} du$ dM1: For $\dots u e^{2u} \pm \dots \int e^{2u} du = \dots \pm \dots e^{2u}$. Depends on the previous method mark. A1: Fully correct integration for their substitution: $\frac{1}{9} u e^{2u} - \frac{1}{18} e^{2u}$ or unsimplified equivalent e.g. $\frac{2}{9} \left(\frac{1}{2} u e^{2u} - \frac{1}{4} e^{2u} \right)$ with or without a constant of integration. Any necessary brackets must be present unless they are implied by subsequent work. ddM1: Substitutes in the limits $\ln 9$ and $\ln 3$ into an expression of the form $\alpha u e^{2u} + \beta e^{2u}$ or reverts back to x and substitutes in the limits 3 and 1 and subtracts the correct way round. Condone missing brackets e.g. $9 \ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$ Condone slips in substitution provided the intention is clear. They may work on each term separately e.g. $\left[\frac{1}{9} u e^{2u} - \frac{1}{18} e^{2u} \right]_{\ln 3}^{\ln 9} = \left[\frac{1}{9} u e^{2u} \right]_{\ln 3}^{\ln 9} - \left[\frac{1}{18} e^{2u} \right]_{\ln 3}^{\ln 9} = 9 \ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2} \right)$ It is dependent on the two previous method marks. A1: $17 \ln 3 - 4$		

Question Number	Scheme	Marks
2(a)	$\frac{dV}{dx} = 3x^2$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3x^2}$	dM1A1
		(3)
	Alternative 1	
	$V = x^3 \Rightarrow x = V^{\frac{1}{3}} \Rightarrow \frac{dx}{dV} = \frac{1}{3} V^{-\frac{2}{3}}$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3V^{\frac{2}{3}}} = \frac{5}{3x^2}$	dM1A1
	Alternative 2	
	$\frac{dV}{dt} = 5 \Rightarrow V = 5t + c$	M1 (B1 on EPEN)
	$V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 \frac{dx}{dt} = 5$ or $V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 = 5 \frac{dt}{dx}$	dM1
	$\frac{dx}{dt} = \frac{5}{3x^2}$	A1
(b)	$\frac{dx}{dt} = \frac{5}{3 \times 4^2} = \frac{5}{48}$	M1A1
		(2)
		(5 marks)
Note this is now being marked as M1dM1A1 not B1M1A1 (a) Ignore any units associated with any of the expressions/values in this question. M1: Differentiates $V = x^3$ to obtain $\frac{dV}{dx} = \dots x^2$ (seen or implied) dM1: Attempts to use $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ or equivalent e.g. $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$ with their $\frac{dV}{dx}$ and $\frac{dV}{dt}$ where $\frac{dV}{dx} = kx^2$ and $\frac{dV}{dt}$ is a constant to obtain an expression of the form $\frac{\alpha}{x^2}$ oe. May be implied by their working. Depends on the previous mark. A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3} x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.		

Question Number	Scheme	Marks
8(a)	$\frac{1}{\sqrt{4+x}} = 4^{-\frac{1}{2}} \left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$	M1
	$\left(1 + \frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{x}{4}\right)^2 + \dots$	M1
	$\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$	A1A1
		(4)
(b)	$\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2 + \dots$	B1ft
		(1)
(c)	$\left(\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2\right)\left(\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2\right) = \frac{1}{4} + \frac{1}{128}x^2$	M1A1
		(2)
(d)	$\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128} = \frac{33}{128}$	M1
	$\frac{128}{33} \times 3 = \frac{128}{11} \text{ (see alt (d) for alternative answer)}$	dM1A1
		(3)
Alt(d)	$\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128} = \frac{33}{128}$	M1
	$\frac{1}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15} \approx \frac{33}{128} \Rightarrow \sqrt{135} = 3\sqrt{15} \approx 3 \times 15 \times \frac{33}{128} = \frac{1485}{128}$	dM1A1
		(10 marks)

(a) It is scored B, M, A, A on open. We are scoring it M, M, A, A

M1: Takes out a factor of 4 proceeding to $4^{-\frac{1}{2}} \left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$ o.e such as $\frac{1}{2} \times \left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$

M1: Attempts the binomial expansion.

Look for the correct binomial coefficient for the **third term with the correct power of x**. So, award for

$$\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(\dots x)^2 \text{ or } \frac{3}{8} \times \dots x^2 \text{ where } \dots \text{ could even be 1. If there is no intermediate working it is}$$

implied by the correct coefficient of $\frac{3}{128}x^2$ in the expansion of $\left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$ or $\frac{3}{256}x^2$ in the expansion of $(4+x)^{-\frac{1}{2}}$

A1: Two correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$ So 1,0,1, 0 is possible

A1: All three correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$



Accept as a list. Condone any extra terms even if incorrect

Direct expansion: $(4+x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) \times 4^{-\frac{3}{2}}x + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times 4^{-\frac{5}{2}}x^2 + \dots = \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$

M1: For a constant term of $4^{-\frac{1}{2}}$ or $\frac{1}{2}$

M1: For a correct third term which may be un-simplified $\frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times 4^{-\frac{5}{2}}x^2$

It is implied if they get $\frac{3}{256}x^2$ for the term in x^2

A1: Two correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$

A1: All three correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$

Accept as a list. Condone any extra terms even if incorrect

(b)

B1ft: For a correct answer $\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2 + \dots$ which can be scored following a restart.

Allow as a list. Condone any extra terms even if incorrect.

Or follow through on their (a) $\alpha - \beta x + \delta x^2 + \dots$ where α, β, δ are positive to (b) $\alpha + \beta x + \delta x^2 + \dots$

Answers must be seen in (b).

(c)

M1: Attempts to multiply out the correct $\left(\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2\right)\left(\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2\right)$ or follow through using their answers to parts (a) and (b) and attempts to collect terms. **It must be an attempt** on $(\alpha - \beta x + \delta x^2)(\alpha + \beta x + \delta x^2)$ where α, β, δ are positive leading to $\alpha^2 + \kappa x^2$. Condone extra terms where the index is greater than 2, such as a term in x^4

Alternatively, attempts the expansion $(16 - x^2)^{-\frac{1}{2}} = 16^{-\frac{1}{2}} \left(1 - \frac{1}{16}x^2\right)^{-\frac{1}{2}}$ leading to $\frac{1}{4} \pm \kappa x^2$

A1: $\frac{1}{4} + \frac{1}{128}x^2$ ONLY Any extra terms would be A0. Condone = for $\approx$

(d) **This cannot be scored from made up values of a and b.**

M1: Substitutes $x=1$ into both sides of their part (c) to achieve $\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128}$

dM1: Uses the fact that $\sqrt{135} = 3 \times \sqrt{15}$ and finds a rational approximation for $\sqrt{135}$

Method 1: $\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128} \Rightarrow \frac{\sqrt{15}}{15} = \frac{1}{4} + \frac{1}{128} \Rightarrow \sqrt{135} = \left(\frac{1}{4} + \frac{1}{128}\right) \times 45$

Method 2: $\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128} \Rightarrow \frac{1}{\sqrt{15}} = \frac{1}{4} + \frac{1}{128} \Rightarrow \sqrt{135} = \frac{3}{\left(\frac{1}{4} + \frac{1}{128}\right)}$

The method marks M1, dM1 may be implied, e.g. by the calculation $\left(\frac{1}{4} + \frac{1}{128}\right) \times 45$ without sight of $\sqrt{135}$

A1: Either $\frac{128}{11}$ (or $11\frac{7}{11}$) or $\frac{1485}{128}$ (or $11\frac{77}{128}$) (or accept both if found)