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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Parametric Equations (P4.3)

International A Level Mathematics · P4: Pure Mathematics 4 (WMA14)

12 questions · 104 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
P4.3a	PARAMETRIC EQUATIONS & CURVES	12 Qs · 104 marks	2
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QUESTIONS

12 questions · 104 marks · 1 sub-topic

SUB-TOPIC P4.3a

PARAMETRIC EQUATIONS & CURVES

Sub-topic P4.3a · 12 questions · 104 marks · lowest tariff first



1. A curve C has parametric equations

$$x = \frac{t}{t-3} \quad y = \frac{1}{t} + 2 \quad t \in \mathbb{R} \quad t > 3$$

Show that all points on C lie on the curve with Cartesian equation

$$y = \frac{ax-1}{bx}$$

where a and b are constants to be found.

(3)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 3 marks)



Question 2 continued

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Lined area for writing the answer to Question 2.

(Total 4 marks)

Q2



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 6 marks)



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4. The curve C is defined by the parametric equations

$$x = \frac{1}{t} + 2 \quad y = \frac{1-2t}{3+t} \quad t > 0$$

- (a) Show that the equation of C can be written in the form $y = g(x)$ where g is the function

$$g(x) = \frac{ax + b}{cx + d} \quad x > k$$

where a, b, c, d and k are integers to be found.

(5)

- (b) Hence, or otherwise, state the range of g .

(2)

Question 4 continued

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Q4

(Total 7 marks)



6.

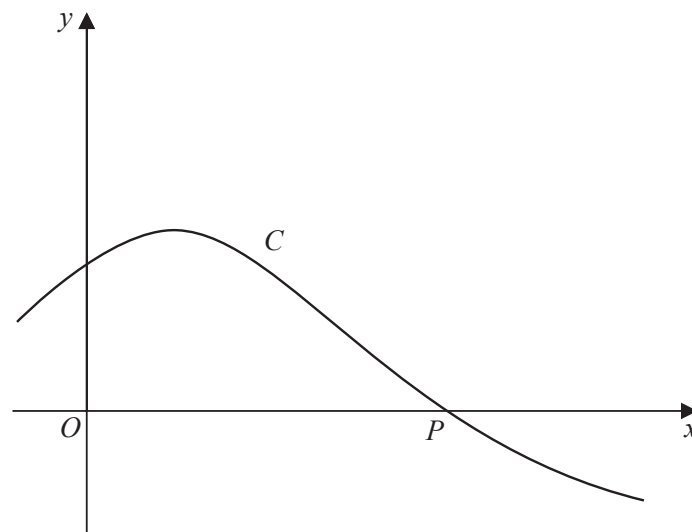


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 1 + 3 \tan t \quad y = 2 \cos 2t \quad -\frac{\pi}{6} \leq t \leq \frac{\pi}{3}$$

The curve crosses the x -axis at point P , as shown in Figure 3.

- (a) Find the equation of the tangent to C at P , writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

The curve C has equation $y = f(x)$, where f is a function with domain $[k, 1 + 3\sqrt{3}]$

- (b) Find the exact value of the constant k .

(1)

- (c) Find the range of f .

(2)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



(Total for Question 6 is 8 marks)



Question 5 continued

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Lined area for writing the answer to Question 5 continued.



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Q5

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8.

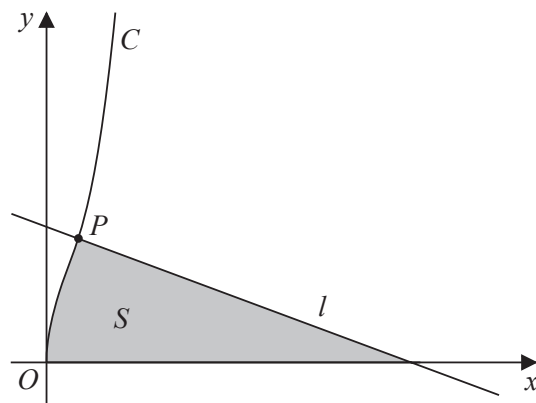


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \quad y = 2 \tan t \quad 0 \leq t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C .

The line l is the normal to C at P , as shown in Figure 3.

(a) Show, using calculus, that an equation for l is

$$8y + 2x = 17 \quad (5)$$

The region S , shown shaded in Figure 3, is bounded by C , l and the x -axis.

(b) Find, using calculus, the exact area of S . (6)

Question 8 continued

Lined area for writing the answer to Question 8 continued.





Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 11 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 11 marks)



4.

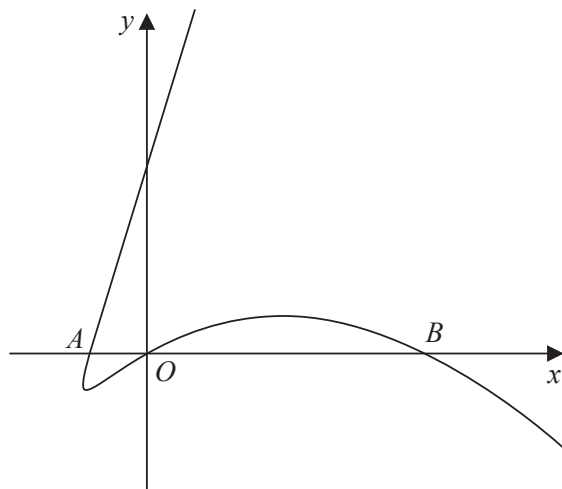


Figure 2

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \quad y = t^3 - 4t, \quad t \in \mathbb{R}$$

The curve cuts the x -axis at the origin and at the points A and B , as shown in Figure 2.

- (a) Find the coordinates of A and show that B has coordinates $(20, 0)$. (3)

- (b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0 \tag{5}$$

The tangent to the curve at B cuts the curve again at the point P .

- (c) Find, using algebra, the x coordinate of P . (4)

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Question 4 continued

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Q4

Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 12 marks)

9.

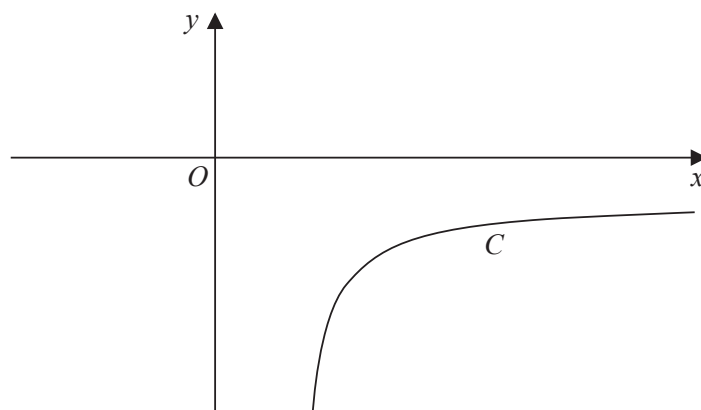


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \sec t \quad y = \sqrt{3} \tan\left(t + \frac{\pi}{3}\right) \quad \frac{\pi}{6} < t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t

(3)

(b) Find an equation for the tangent to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are constants.

(4)

(c) Show that all points on C satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where A and B are constants to be found.

(5)

Question 9 continued

Lined area for writing the answer to Question 9 continued.





Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1 (a)	$\frac{5x+10}{(1-x)(2+3x)} \equiv \frac{A}{1-x} + \frac{B}{2+3x} \Rightarrow$ Value for A or B	M1
	One correct value, either $A=3$ or $B=4$	A1
	Correct PF form $\frac{3}{1-x} + \frac{4}{2+3x}$	A1
		(3)
(b)(i)	$\frac{A}{1-x} = A(1-x)^{-1} = A(1+x+x^2+\dots)$	B1
	$\left\{\frac{B}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1} = \left\{\frac{B}{2}\right\}\left(1+(-1)\frac{3x}{2}+\frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2+\dots\right); = \frac{B}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\dots\right)$	M1; A1
	$f(x) = 3x\left(1+x+x^2+\dots\right) + \frac{4}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\dots\right)$	M1
	$= 5 + \frac{15}{2}x^2 + \dots$	A1
		(5)
(b)(ii)	$ x < \frac{2}{3}$	B1
		(1)
		(9 marks)
(b)(i) Alt 1	$(1-x)^{-1} = 1+x+x^2+\dots$	B1
	$\left\{\frac{1}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1} = \left\{\frac{1}{2}\right\}\left(1+(-1)\frac{3x}{2}+\frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2+\dots\right); = \frac{1}{2}\left(1-\frac{3x}{2}+\frac{9}{4}x^2+\dots\right)$	M1; A1
	$\frac{5x+10}{(1-x)(2+3x)} = (5x+10)\left(1+x+x^2+\dots\right) \times \frac{1}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\dots\right) = 5+\dots x+\dots x^2$	M1
	$= 5 + \frac{15}{2}x^2 + \dots$	A1
		(5)
(b)(i) Alt 2	$\frac{5x+10}{(1-x)(2+3x)} = (5x+10)\left(2+\left(x-3x^2\right)\right)^{-1} = \frac{1}{2}(5x+10)\left(1+\frac{1}{2}\left(x-3x^2\right)\right)^{-1}$	B1
	$(1+p(x))^{-1} = \left(1 \pm p(x) + \frac{(-1)(-2)}{2}(p(x))^2 + \dots\right); \frac{1}{2}\left(1 - \frac{1}{2}\left(x-3x^2\right) + \frac{1}{4}\left(x-3x^2\right)^2 + \dots\right)$	M1; A1
	$(10+5x)\left(\frac{1}{2} - \frac{1}{4}x + \frac{3}{4}x^2 + \frac{1}{8}x^2 + \dots\right) = 5 - \frac{5}{2}x + \frac{35}{4}x^2 + \frac{5}{2}x - \frac{5}{4}x^2 + \dots$	M1

	$= 5 + \frac{15}{2}x^2 + \dots$	A1
		(5)
Notes:		
a)		
M1: Attempts at correct PF. Correct form identified (may be implicit) and achieves a value for at least one of the constants.		
A1: One correct value or term.		
A1: Correct PF form $\frac{3}{1-x} + \frac{4}{2+3x}$. This may be awarded if seen in (b) but the correct final form (not just values) must be seen somewhere in the question. Accept at $3(1-x)^{-1} + 4(2+3x)^{-1}$		
(b)(i)		
B1: $\frac{A}{1-x} = A(1-x)^{-1} = A(1+x+x^2+\dots)$ which may be unsimplified. Allow with their A or with A = 1.		
M1: Attempts to expand $\frac{1}{2+3x} = (2+3x)^{-1}$ binomially either by taking out the factor 2 first, or directly. Look for $(1+kx)^{-1} = \dots \left(1 \pm kx + \frac{(-1)(-2)}{2}(kx)^2 + \dots \right)$ where $k \neq 1$ following an attempt at taking out a factor 2, or $\frac{1}{2+3x} = (2+3x)^{-1} = \left(2^{-1} \pm 2^{-2}kx + \frac{(-1)(-2)}{2}2^{-3}(kx)^2 + \dots \right)$ by direct expansion. Allow missing brackets on kx^2 in either case.		
A1: $\frac{B}{2+3x} = \frac{B}{2} \left(1 + \frac{3x}{2} \right)^{-1} = \frac{B}{2} \left(1 - \frac{3x}{2} + \frac{9}{4}x^2 + \dots \right)$ oe with their B from (a) or with B = 1		
M1: Uses their coefficients and attempts to add both series.		
A1cao: $5 + \frac{15}{2}x^2 + \dots$ Condone additional higher order terms. Terms may be either order.		
(b)(ii)		
B1: $ x < \frac{2}{3}$ or exact equivalent. This must be clearly identified as the answer. B0 if both ranges are given with no choice of which is correct. (But B1 if formal set notation with $\cap$ used.)		
(b)(i) Alt 1:		
B1: $(1-x)^{-1} = 1+x+x^2+\dots$ which may be unsimplified.		
M1: Same as main scheme.		
A1: Correct expansion (see main scheme, B = 1 allowed).		
M1: Attempts to expand all three brackets, achieving the correct constant term at least.		
A1cso: $5 + \frac{15}{2}x^2 + \dots$ Condone additional higher order terms. Terms may be either order.		
(b)(i) Alt 2		
B1: Writes f(x) as $(5x+10) \left(2 + \left(x-3x^2 \right) \right)^{-1}$ or with the 2 extracted, with the $\left(x-3x^2 \right)$ clear.		

Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} = \frac{1}{2}(\dots)$	B1
	$= (1 - 20x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (-20x) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (-20x)^2 + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (-20x)^3 \dots$	M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1 A1
	Special case: If the final answer is left as $\frac{1}{2}(1 - 10x - 50x^2 - 500x^3 + \dots)$ Award SC B1M1A1A1A0	
		(5)
	Alternative by direct expansion	
	$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^{-\frac{1}{2}}(-5x)^1 + \frac{\frac{1}{2} \times -\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$	B1M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1A1
(b)	$\left(\frac{1}{4} - \frac{5}{100}\right)^{\frac{1}{2}} = \left(\frac{1}{5}\right)^{\frac{1}{2}} = \frac{1}{2} - 5 \times \frac{1}{100} - 25\left(\frac{1}{100}\right)^2 - 250\left(\frac{1}{100}\right)^3 + \dots$ $\frac{\sqrt{5}}{5} \approx \frac{1789}{4000} \quad \text{or} \quad \frac{1}{\sqrt{5}} \approx \frac{1789}{4000}$ $\Rightarrow \sqrt{5} \approx 5 \times \frac{1789}{4000} \quad \text{or} \quad \sqrt{5} \approx 1 \div \frac{1789}{4000}$	M1
	$\sqrt{5} \approx \frac{1789}{800} \quad \text{or} \quad \frac{4000}{1789}$	A1
		(2)
		(7 marks)

(a)B1: For taking out a factor of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.M1: Expands $(1 + kx)^{\frac{1}{2}}$, $k \neq \pm 1$ with the correct structure for the third or fourth terme.g. $\pm \frac{\frac{1}{2}(-\frac{1}{2})}{2!} \times (kx)^2$ or $\pm \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!} \times (kx)^3$ with or without the bracket around the kx

A1: For either term three or term four being correct in any form.

E.g. $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times (20x)^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times (-20x)^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times (-20x)^3$ or $-\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times (20x)^3$

The brackets must be present unless they are implied by subsequent work. This mark is independent of the B mark.



Question Number	Scheme	Marks
6 (a)	For correct parameter $t = \frac{\pi}{4}$ or x coordinate at P $x = 4$ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-4\sin 2t}{3\sec^2 t}$ Equation of tangent $y - 0 = -\frac{2}{3}(x - 4) \Rightarrow y = -\frac{2}{3}x + \frac{8}{3}$	B1 M1 A1 dM1 A1 (5)
(b)	$k = 1 - \sqrt{3}$	B1 (1)
(c)	$-1, f, 2$	M1 A1 (2)
		(8 marks)

(a)

B1: Correct parameter or x coordinate at P .The correct x coordinate $x = 4$ (can even be scored following $x = 45^\circ$)M1: Attempts to find $\frac{dy}{dx}$ using $\frac{dy/dt}{dx/dt}$. Condone poor differentiation.A1: Correct $\frac{dy}{dx} = \frac{-4\sin 2t}{3\sec^2 t}$. You may see other forms for this following use of identities.If you see $\frac{dy}{dx} = \frac{-4\sin 2t}{3\sec^2 x}$ then only allow if they subsequently use this as $\frac{dy}{dx} = \frac{-4\sin 2t}{3\sec^2 t}$ Other possible correct gradients are $\frac{-8\sin t \cos t}{3\sec^2 t}$ and $\frac{-8\sin t \cos^3 t}{3}$

ISW after you see a correct answer. Quite often we see candidates incorrectly adapting a correct answer.

It is also possible for candidates to find $\frac{dy}{dt}$ at $t = \frac{\pi}{4}$, $\frac{dx}{dt}$ at $t = \frac{\pi}{4}$ and divide the two values.dM1: Attempts to find the equation of the tangent at $t = \frac{\pi}{4}$ It is dependent upon the previous M and use of $(4, 0)$ and $t = \frac{\pi}{4}$ Allow substitution into their adapted $\frac{dy}{dx}$ A1: $y = -\frac{2}{3}x + \frac{8}{3}$

(b)

B1: $k = 1 - \sqrt{3}$

(c)

M1: Either end correct. Allow strict inequalities here. E.g $f < 2$ scores M1 but $f > 2$ is M0A1: $-1, f, 2$ o.e such as $[-1, 2]$ and $-1, y, 2$ 

Question Number	Scheme	Marks
1	Attempt at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	Attempt at product rule $3x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$	M1
	Correct differentiation $2 - 8y \frac{dy}{dx} + 6xy + 3x^2 \frac{dy}{dx} = 8x$	A1
	Substitutes $x = 3, y = 2 \Rightarrow \frac{dy}{dx} = -\frac{14}{11}$	M1, A1
	Correct method for normal at $(3, 2) \Rightarrow y - 2 = \frac{11}{14}(x - 3)$ $11x - 14y - 5 = 0$	dM1 A1 (7) (7 marks)

M1: Attempts at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$ where ... is a constant

M1: Attempt at product rule $3x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$ where ... are constants

A1: Correct differentiation $2x - 4y^2 + 3x^2 y = 4x^2 + 8 \rightarrow 2 - 8y \frac{dy}{dx} + 6xy + 3x^2 \frac{dy}{dx} = 8x$ o.e.

You may see $2x - 4y^2 + 3x^2 y = 4x^2 + 8 \rightarrow 2dx - 8ydy + 6xydx + 3x^2 dy = 8xdx$ o.e. which is fine

Note that $\frac{dy}{dx} = 2 - 8y \frac{dy}{dx} + 6xy + 3x^2 \frac{dy}{dx} - 8x$ is A0 unless recovered

M1: Substitutes $x = 3, y = 2$ into a suitable equation and finds a value for $\frac{dy}{dx}$.

It is dependent upon having exactly two $\frac{dy}{dx}$ terms, one from differentiating each of y^2 and $x^2 y$

A1: $\frac{dy}{dx} = -\frac{14}{11}$ o.e.

dM1: Dependent upon the previous M. It is for a correct method of finding the equation of

the normal at $(3, 2)$. Look for $y - 2 = \frac{11}{14}(x - 3)$ with the negative reciprocal of

their $-\frac{14}{11}$ being used.

If the form $y = mx + c$ is used they must proceed as far as $c = \dots$

A1: $11x - 14y - 5 = 0$ but accept any integer multiple of this. The " $= 0$ " must be seen



Question Number	Scheme	Marks
2	$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \Rightarrow \int \frac{1}{y^2} dy = \int \frac{4}{\sqrt{4x+5}} dx$ $-\frac{1}{y} = 2\sqrt{4x+5} (+c)$ Substitutes $y = \frac{1}{3}, x = -\frac{1}{4} \Rightarrow -3 = 4 + c \Rightarrow c = \dots$ Rearranges $\frac{a}{y} = b\sqrt{4x+5} + c$ to $y = \dots$ $y = \frac{1}{7 - 2\sqrt{4x+5}}$	B1 M1, A1 dM1 ddM1 A1 (6) (6 marks)

B1: Separates the variables. Note that the "4" may be on either side but must be in the correct place
The dy and dx **must** be present **and** in the correct place. Condone missing integral signs

M1: Integrates one side to a correct form. No requirement for $+c$

Look for $\int \frac{1}{y^2} dy \rightarrow \frac{a}{y}$ or $\int \frac{1}{\sqrt{4x+5}} dx \rightarrow k\sqrt{4x+5}$ or equivalent

A1: Correct integration for both sides. Allow unsimplified but there is no requirement for $+c$

Look for $-\frac{1}{y} = 2\sqrt{4x+5}$ or equivalent such as $-\frac{1}{4y} = \frac{\sqrt{4x+5}}{4 \times \frac{1}{2}}$

dM1: Substitutes $y = \frac{1}{3}, x = -\frac{1}{4}$ into their integrated form to find a value for c

It is dependent upon having integrated one side to a correct form.
Condone this being done following poor re-arrangement

ddM1: Rearranges $\frac{a}{y} = b\sqrt{4x+5} + c$ to $y = \dots$ using a correct method. Do not allow each term to be inverted.

It is dependent upon

- integrating BOTH sides to a correct form
- substituting $y = \frac{1}{3}, x = -\frac{1}{4}$ into the correct integrated form to find a value for c
- rearranging $\frac{a}{y} = b\sqrt{4x+5} + c$ to $y = \dots$ using a correct method but condone sign slips

A1: $y = \frac{1}{7 - 2\sqrt{4x+5}}$ or exact equivalent. E.g. $y = \frac{-1}{2(4x+5)^{0.5} - 7}$

Do not isw. So if the candidate then writes $y = \frac{1}{7} - \frac{1}{2\sqrt{4x+5}}$ it is A0

General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**1. Factorisation $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ 2. FormulaAttempt to use correct formula (with values for a , b and c).3. Completing the squareSolving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:**1. DifferentiationPower of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)2. IntegrationPower of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question	Scheme	Marks
1	E.g. $xy^2 \rightarrow (\dots+)2xy \frac{dy}{dx}$ or $x^2y \rightarrow (\dots+)x^2 \frac{dy}{dx}$ but see notes.	M1
	$xy^2 \rightarrow y^2 + kxy \frac{dy}{dx}$ or $x^2y \rightarrow kxy + x^2 \frac{dy}{dx}$	dM1
	$y^2 + 2xy \frac{dy}{dx} = 2xy + x^2 \frac{dy}{dx}$ oe	A1
	$(x^2 - 2xy) \frac{dy}{dx} = y^2 - 2xy \Rightarrow \frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy} \Rightarrow \frac{dy}{dx} \Big _P = \frac{3^2 - 12}{2^2 - 12} = \dots$ or $3^2 + 2 \times 2 \times 3 \frac{dy}{dx} \Big _P = 2 \times 2 \times 3 + 2^2 \frac{dy}{dx} \Big _P \Rightarrow \frac{dy}{dx} \Big _P = \frac{9 - 12}{4 - 12} = \dots$	M1
	Tangent is $y - 3 = \frac{3}{8}(x - 2)$	dM1
	$\Rightarrow 3x - 8y + 18 = 0$	A1
		(6)

(6 marks)

Notes:

M1: For an attempt at implicit differentiation in some form. (Allow y' for $\frac{dy}{dx}$)

For the xy^2 look for a term $kxy \frac{dy}{dx}$ but condone $ky \frac{dy}{dx}$

For the x^2y look for a term $kx^2 \frac{dy}{dx}$ but condone $kx \frac{dy}{dx}$

dM1: For attempting the product rule on either xy^2 or x^2y .

Award for $xy^2 \rightarrow y^2 + kxy \frac{dy}{dx}$ or $x^2y \rightarrow kxy + x^2 \frac{dy}{dx}$

A1: For $y^2 + 2xy \frac{dy}{dx} = 2xy + x^2 \frac{dy}{dx}$ or equivalent.

Ignore any spurious " $\frac{dy}{dx} =$ " in front of their equation.

M1: Attempts to find $\frac{dy}{dx}$ at P . Must substitute **both** x and y values and rearrange (but can be done in

either order). Allow this mark as long as there are at least two terms in $\frac{dy}{dx}$ in their

differentiation not including any spurious " $\frac{dy}{dx} =$ " There must be some correct substitution.

dM1: Uses their **non-zero** $\frac{dy}{dx}$ at $(2, 3)$ to find the equation of the tangent. **Depends on previous M.**

Look for $y - 3 = "m"(x - 2)$ or $y = "m"x + c$ where m is their attempt at the gradient at P , followed by attempt to find c using $(2, 3)$ with the 2 and the 3 **positioned correctly**.

A1: $3x - 8y + 18 = 0$ or any non-zero integer multiple of this equation but with all terms on one side.



Question Number	Scheme	Marks
8 (a)	At $t = \frac{\pi}{4}$ $P = \left(\frac{1}{2}, 2\right)$	B1
	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\sec^2 t}{2\sin t \cos t} = 4$ when $t = \frac{\pi}{4}$	M1 A1
	Equation of l : $y - 2 = -\frac{1}{4}\left(x - \frac{1}{2}\right) \Rightarrow 8y - 16 = -2x + 1 \Rightarrow 8y + 2x = 17^*$	dM1 A1 * cso
		(5)
(b)	$\int y \frac{dx}{dt} dt = \int 2 \tan t \times 2 \sin t \cos t dt$	M1
	$= \int 4 \sin^2 t dt$	A1
	$= \int 2 - 2 \cos 2t dt = 2t - \sin 2t$	dM1 A1
	Total area of $S = \left[2t - \sin 2t\right]_0^{\frac{\pi}{4}} + \frac{1}{2} \times 8 \times 2 = \frac{\pi}{2} - 1 + 8 = \frac{\pi}{2} + 7$	M1 A1
		(6)
		(11 marks)

Notes:

(a)

B1: Correct coordinates for P stated or implied by working.M1: Attempts to find $\frac{dy}{dx}$ using $\frac{dy/dt}{dx/dt}$ at $t = \frac{\pi}{4}$. Condone poor differentiation. Substitution ofthe $\frac{\pi}{4}$ is sufficient for the method. Alternatively, may attempt $\frac{dx}{dy}$ or $-\frac{dx}{dy}$. Accept a valuefollowing finding $\frac{dy}{dx}$ (or its reciprocal etc) as an attempt to evaluate at $t = \frac{\pi}{4}$ if no contrary working is shown **but check carefully** as the correct answer may arise from incorrect working.A1: Correct $\frac{dy}{dx} = 4$ (oe equation) following correct differentiation. May be implied.dM1: Attempts to find the equation of the normal at $t = \frac{\pi}{4}$. It is dependent upon the previousM and use of their P . The value of the gradient used must be correct for their differential.A1*: **cso** Correct proof leading to $8y + 2x = 17$

(b)

M1: Attempts $\int y \frac{dx}{dt} dt = \int 2 \tan t \times "2 \sin t \cos t" dt$ with their $\frac{dx}{dt}$ condoning slips on coefficients.A1: $\int 4 \sin^2 t dt$ 

dM1: Uses $\cos 2t = \pm 1 \pm 2\sin^2 t$ and integrates $\int \pm p \pm q \cos 2t \, dt$ to a form $\pm at \pm b \sin 2t$

See note below.

A1: $\int y \frac{dx}{dt} dt = 2t - \sin 2t$ See note below.

M1: Full method to find area of region S. Finds the sum of their values for $\int_0^{\frac{\pi}{4}} y \frac{dx}{dt} dt$ and

$\frac{1}{2} \left(\frac{17}{2} - P_x \right) \times P_y$. Condone poor integration for this mark as long as they are attempting to apply

the correct limits to their result. They may attempt the area under the line by integration:

$\int_{P_x}^{\frac{17}{2}} -\frac{1}{4}x + \frac{17}{8} dx$ In such a method condone minor slips, but must be attempting correct limits.

A1: $\frac{\pi}{2} + 7$

Note: If the t 's becomes x 's during the integration, then allow the M's and the A's if recovered but if $2x - \sin 2x$ or with mixed variables is found and x values substituted then it is M1A0 for the integral and M0 for the method for area.

8 (a) Alt	At $t = \frac{\pi}{4}$ $P = \left(\frac{1}{2}, 2 \right)$	B1
	$y = \frac{2 \sin t}{\cos t} = \frac{2\sqrt{x}}{\sqrt{1-x}} \quad y^2 = \frac{4x}{1-x}$ $\frac{dy}{dx} = \frac{x^{-\frac{1}{2}}(1-x)^{\frac{1}{2}} - 2\sqrt{x} \times -\frac{1}{2}(1-x)^{-\frac{1}{2}}}{1-x} \Rightarrow \frac{dy}{dx} \Big _{x=\frac{1}{2}} = 4 \text{ or}$ $2y \frac{dy}{dx} = \frac{4(1-x) - 4x \times -1}{(1-x)^2} \Rightarrow \frac{dy}{dx} \Big _{x=\frac{1}{2}, y=2} = 4 \text{ oe}$	M1 A1
	Equation of l : $y - 2 = -\frac{1}{4} \left(x - \frac{1}{2} \right) \Rightarrow 8y - 16 = -2x + 1 \Rightarrow 8y + 2x = 17^*$	dM1 A1 * cso
		(5)
(a) B1: Correct coordinate for $P \left(\frac{1}{2}, 2 \right)$ stated or implied by working. M1: Attempts to find Cartesian equation for C, any form, and attempts $\frac{dy}{dx}$ (or an equivalent as main scheme) with appropriate differentiation methods for their Cartesian form, allowing for slips and finds x and/or y using $t = \frac{\pi}{4}$ and evaluate the derivative with these values.		

A1: Correct $\frac{dy}{dx} = 4$ (oe equation) following correct differentiation and from correct work.

dM1: Attempts to find the equation of the normal at their x and y values.
It is dependent upon the previous M and use of their P .

A1*: **cs0** Correct proof leading to $8y + 2x = 17$



Question Number	Scheme	Marks
6(a)	$\frac{dy}{dx} = 6 \cos t \times \frac{1}{-8 \sin t} = k \frac{\cos t}{\sin t}$	M1
	$\frac{dy}{dx} = -\frac{3}{4} \cot\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{4}$	dM1A1
		(3)
Mark (a) and (b) together.		
(a)		
M1: Attempts to differentiate both parametric equations to obtain $\frac{dx}{dt} = \dots \sin t$ and $\frac{dy}{dt} = \dots \cos t$ and attempts $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ or equivalent e.g. $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$		
Note that this may be done numerically e.g.		
$\frac{dx}{dt} = -8 \sin t = -4\sqrt{3}, \frac{dy}{dt} = 6 \cos t = 3 \Rightarrow \frac{dy}{dx} = 3 \times \frac{1}{-4\sqrt{3}}$		
dM1: Substitutes in $t = \frac{\pi}{3}$ and proceeds to find a value for $\frac{dy}{dx}$ which may be implied.		
Depends on the previous mark.		
A1: $-\frac{\sqrt{3}}{4}$ from fully correct work or any exact equivalent e.g. $-\frac{3}{4\sqrt{3}}$.		
Isw once a correct answer is seen.		

(b)	$(6, 3\sqrt{3}-3)$	B1B1
	$-\frac{\sqrt{3}}{4} \rightarrow \frac{4}{\sqrt{3}}$	B1ft
	$y - 3\sqrt{3} + 3 = \frac{4}{\sqrt{3}}(x - 6)$ or $y = \frac{4}{\sqrt{3}}x + c \Rightarrow -3\sqrt{3} + 3 = \frac{4}{\sqrt{3}} \times 6 + c \Rightarrow c = (-3 - 5\sqrt{3})$	M1
	When $y = 0$ $-3\sqrt{3} + 3 = \frac{4}{\sqrt{3}}(x - 6) \Rightarrow x = \dots$	dM1
	$\left(\frac{15 + 3\sqrt{3}}{4}, 0 \right)$	A1
		(6)

(b)

B1: One of $x = 6$, $y = 3\sqrt{3} - 3$ **B1:** For both coordinates correct for P : $x = 6$, $y = 3\sqrt{3} - 3$ or e.g. $(6, 3\sqrt{3} - 3)$

Note that the above B marks may be implied and may be seen embedded in their attempt at the equation of normal and can be scored if seen anywhere in their solution.

B1ft: Gradient of the normal to C at P is $\frac{4}{\sqrt{3}}$ or e.g. $\frac{4\sqrt{3}}{3}$ (follow through from (a)) and can be scored anywhere in their solution.

M1: Attempts the equation of the normal with values correctly placed using

- their coordinates for P which have come from substituting $t = \frac{\pi}{3}$ into the parametric equations
- a changed gradient from that found in (a)

If using $y = mx + c$ they must proceed to find a value for c .

If candidates use P in terms of t e.g. $x = 8\cos t + 2$ and $y = 6\sin t - 3$ in their normal equation

then this mark will only score at the point they substitute $t = \frac{\pi}{3}$

The first 2 B marks may also be implied if the x and y coordinates can clearly be implied or seen as $x = 6$ and/or $y = 3\sqrt{3} - 3$ you may need to check.

dM1: Substitutes $y = 0$ into their equation of the normal and proceeds to find a value for x .

It is dependent on the previous method mark.

Note that attempts to solve $6\sin t - 3 = 0$ score M0

A1: Correct coordinates or correct value of $x \left(\frac{15 + 3\sqrt{3}}{4}, 0 \right)$ or $x = \frac{15 + 3\sqrt{3}}{4}$ o.e. e.g.

$$x = \frac{9 + 15\sqrt{3}}{4\sqrt{3}}, \frac{9\sqrt{3} + 45}{12}, \frac{3}{4}(5 + \sqrt{3}) \text{ and isw once a correct answer is seen.}$$

Allow unsimplified e.g. $x = (5\sqrt{3} + 3) \div \frac{4\sqrt{3}}{3}$



(c)	$x = 8\cos t + 2 \Rightarrow \cos t = \frac{x-2}{8} \quad y = 6\sin t - 3 \Rightarrow \sin t = \frac{y+3}{6}$	M1
	$\frac{(x-2)^2}{64} + \frac{(y+3)^2}{36} = 1$	A1
		(2)
		(11 marks)
(c) M1: Attempts to rearrange either of the parametric equations to obtain $\cos t$ or $\cos^2 t$ in terms of x or $\sin t$ or $\sin^2 t$ in terms of y which may be implied by their working. You can condone slips as long as the intention is clear e.g. $x = 8\cos t + 2 \Rightarrow x^2 = 8\cos^2 t + 4 \Rightarrow \cos^2 t = \frac{x^2 - 4}{8}$ Also condone an attempt to obtain $\cos t$ or $\cos^2 t$ in terms of x or $\sin t$ or $\sin^2 t$ in terms of y if they have errors in trig identities but the intention is clear e.g. $\sin^2 t = 1 - \frac{x-2}{8}$ A1: $\frac{(x-2)^2}{64} + \frac{(y+3)^2}{36} = 1$ o.e. e.g. $y = 6\sqrt{1 - \left(\frac{x-2}{8}\right)^2} - 3$ or e.g. $y = \pm 6\sqrt{1 - \left(\frac{x-2}{8}\right)^2} - 3$ There will be many different alternatives. Once a correct equation is seen, award the A mark and apply isw.		

Question Number	Scheme	Marks
1	Assume that there exists a number m such that when m^3 is even, m is odd If m is odd then $m = 2p + 1$ (where p is an integer) and $m^3 = (2p + 1)^3 = \dots$ $= 8p^3 + 12p^2 + 6p + 1$ $2 \times (4p^3 + 6p^2 + 3p) + 1$ is odd and hence we have a contradiction so if n^3 is even, then n is even.	B1 M1 A1 A1 (4) (4 marks)

B1: For setting up the contradiction.

Eg Assume that there exists a number m such that when m^3 is even, m is odd

Condone a contra-positive statement here

"Assume that there exists a number m such that when m^3 is even, m is not even"

As a minimum accept "assume if m^3 is even then m is odd."

Condone the other way around "assume if n is odd then n^3 is even"

M1: Attempts to cube an odd number. Accept an attempt at $(2p + 1)^3$, $(2p - 1)^3$

Look for $(2p + 1)^3 = \dots p^3 \dots$

A1: $(2p + 1)^3 = 8p^3 + 12p^2 + 6p + 1$ or simplified equivalent such as $2 \times (4p^3 + 6p^2 + 3p) + 1$.

For $(2p - 1)^3 = 8p^3 - 12p^2 + 6p - 1$ or equivalent such as $2 \times (4p^3 - 6p^2 + 3p - 1) + 1$

A1: For a fully correct proof. Requires correct calculations with reason and conclusion

E.g. 1 Correct calculations $(2p + 1)^3 = 8p^3 + 12p^2 + 6p + 1 =$

Reason (even + 1) = odd

Conclusion "hence we have a contradiction, so if n^3 is even, then n is even."

E.g. 2 Correct calculations $(2p + 1)^3 = 8p^3 + 12p^2 + 6p + 1$

Reason $= 2 \times (4p^3 + 6p^2 + 3p) + 1 =$ odd

Conclusion "this is contradiction, so proven."

E.g. 3 Correct calculations $(2p - 1)^3 = 8p^3 - 12p^2 + 6p - 1$

Reason $= 8p^3 - 12p^2 + 6p$ is even so $8p^3 - 12p^2 + 6p - 1$ is odd



Conclusion: So if n^3 is even then n must be even

Note that B0 M1 A1 A1 is possible

Question Number	Scheme	Marks
2(a)	$4^{-\frac{1}{2}} \text{ or } \frac{1}{4^{\frac{1}{2}}} \text{ or } \frac{1}{2}$ $(4-5x)^{-\frac{1}{2}} = \frac{1}{2} \left(1 - \frac{5x}{4}\right)^{-\frac{1}{2}}$ $= \dots \left(1 + \left(-\frac{1}{2}\right) \times \left(-\frac{5x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2!} \times \left(-\frac{5x}{4}\right)^2 + \dots \right)$ $= \frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots$ $\frac{2+kx}{(2-3x)^3} = (2+kx) \left(\frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots \right)$	B1 M1A1 A1 (4)
(b)	Compares x terms leading to $k = \dots$ E.g. $\frac{10}{16} + \frac{k}{2} = \frac{3}{10} \Rightarrow k = -\frac{13}{20}$	M1 A1 (2)
(c)	Compares x^2 terms leading to $m = \dots$ E.g. $m = \frac{75}{128} + \frac{5}{16} \times -\frac{13}{20} \Rightarrow m = \frac{49}{128}$	M1 A1 (2)
		(8 marks)
Alt (a)	$4^{-\frac{1}{2}} \text{ or } \frac{1}{4^{\frac{1}{2}}}$ $(4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) 4^{-\frac{3}{2}} (-5x) + \frac{-\frac{1}{2} \times -\frac{3}{2}}{2} 4^{-\frac{5}{2}} (-5x)^2$ $= \frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots$	B1 M1A1 A1 (4)



Question	Scheme	Marks
5(a)	$1 = 2 \sin u \Rightarrow p = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$	B1
	$x = 2 \sin u \Rightarrow \frac{dx}{du} = 2 \cos u$ oe	M1
	$\int \frac{3x+2}{(4-x^2)^{\frac{3}{2}}} dx = \int \frac{6 \sin u + 2}{(4-4 \sin^2 u)^{\frac{3}{2}}} 2 \cos u du = \int \frac{6 \sin u + 2}{(4 \cos^2 u)^{\frac{3}{2}}} 2 \cos u du$	M1
	$= \int \frac{12 \sin u}{8 \cos^2 u} + \frac{2}{4 \cos^2 u} du = \int_0^{\frac{\pi}{6}} \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du$ *	A1*
		(4)
(b)	$\int \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du = \frac{3}{2} \sec u + \frac{1}{2} \tan u$	M1A1
	$\left[\frac{3}{2} \sec u + \frac{1}{2} \tan u \right]_0^{\frac{\pi}{6}} = \left(\frac{3}{2} \sec \left(\frac{\pi}{6} \right) + \frac{1}{2} \tan \left(\frac{\pi}{6} \right) \right) - \left(\frac{3}{2} \sec 0 + \frac{1}{2} \tan 0 \right) = \dots$	M1
	$= \sqrt{3} + \frac{\sqrt{3}}{6} - \frac{3}{2} = \frac{7\sqrt{3}}{6} - \frac{3}{2} \left(= \frac{7\sqrt{3}-9}{6} \right)$	A1
		(4)
		(8 marks)
Notes		
(a)		
B1:	$p = \frac{\pi}{6}$ Allow if seen anywhere, even in (b). $p = 30$ is B0.	
M1:	$x = 2 \sin u \Rightarrow \frac{dx}{du} = \pm \dots \cos u$ or any rearrangement of this equation.	
M1:	Full substitution from an integral in terms of x to an integral in terms of u and uses the identity $\sin^2 u + \cos^2 u = 1$ in the denominator. Do not be concerned with the limits for this mark.	
A1*:	Achieves given answer include du (with their p) with no errors and at least one intermediate step with the fractional power simplified. Condone missing du in intermediate lines.	
(b)		
M1:	$\int \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du = \dots \sec u + \dots \tan u$	
A1:	$\frac{3}{2} \sec u + \frac{1}{2} \tan u$ ignore any constant c	
M1:	Depends on having one term of the correct form, attempts to substitute in their p ($\neq 1$) and 0, subtracting either way round. The substitution must be seen or clearly implied, e.g. by correct values for each term in an intermediate step before the answer (allowing missing 0's).	
A1:	$\frac{7\sqrt{3}}{6} - \frac{3}{2}$ or exact equivalent eg $\frac{7\sqrt{3}-9}{6}$ Allow if $p = 30^\circ$ was used.	



Question	Scheme	Marks
6(a)	$\overrightarrow{AB} = \begin{pmatrix} 5-1 \\ 3-4 \\ -2-3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix} = 4\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$	M1
	e.g. $\mathbf{r} = \mathbf{i} - 4\mathbf{j} + 3\mathbf{k} + \lambda(4\mathbf{i} + 7\mathbf{j} - 5\mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} + 7\mathbf{j} - 5\mathbf{k})$	M1A1
		(3)
(b)	$\overrightarrow{AC} = \begin{pmatrix} 3-1 \\ p-4 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 2 \\ p+4 \\ -4 \end{pmatrix} = 2\mathbf{i} + (p+4)\mathbf{j} - 4\mathbf{k}$	M1
	$\begin{pmatrix} 2 \\ p+4 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix} = 8 + 7p + 28 + 20 = 0 \Rightarrow p = -8$	M1A1
		(3)
(c)	$ \overrightarrow{AB} = \sqrt{4^2 + 7^2 + (-5)^2} = \sqrt{90} \text{ or } \overrightarrow{AC} = \sqrt{2^2 + (-4)^2 + (-4)^2} = 6$	M1
	Area $\frac{1}{2} \times \sqrt{90} \times 6 = 9\sqrt{10}$	dM1A1
		(3)
		(9 marks)

Notes

Accept either vector form throughout but extra $\mathbf{i}, \mathbf{j}, \mathbf{k}$ in column vectors will lose A mark in (a).

(a) This is now being marked MMA

M1: Attempts to find $\overrightarrow{AB}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates.

M1: Attempts equation for the line, score for $\overrightarrow{OA} + \lambda \times \text{their } \overrightarrow{AB}$ or $\overrightarrow{OB} + \lambda \times \text{their } \overrightarrow{AB}$. No need for $\mathbf{r} =$ for this mark.

A1: Any correct equation. Must be $\mathbf{r} = \dots$ ($l = \dots$ is A0)

(b)

M1: Attempts to find $\overrightarrow{AC}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates.

M1: Takes scalar product of their $\overrightarrow{AB}$ and their $\overrightarrow{AC}$ to form and solve a linear equation in p

A1: $p = -8$

(c)

M1: Attempts to find the magnitude of either their $\overrightarrow{AB}$ or their $\overrightarrow{AC}$ using their p

dM1: Attempts to find the exact area of the triangle ABC . It is dependent on the previous method mark. There

most common method will be $\frac{1}{2} |\overrightarrow{AB}| |\overrightarrow{AC}|$ as in scheme but other methods are possible. E.g.

$$\cos \angle ABC = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}| |\overrightarrow{BC}|} \Rightarrow A = \frac{1}{2} |\overrightarrow{BA}| |\overrightarrow{BC}| \sin \angle ABC. \text{ Such a method must be complete, including use of}$$

Pythagorean identity to find $\sin \angle ABC$. Other more advanced methods (such as cross products) are also possible. If you see something you feel is worthy of some credit but does not fit the scheme, send to Review.

A1: $9\sqrt{10}$



A1: For awrt 81.5, do not need the units. Accept $\left(9 + \frac{177\sqrt{109}}{109}\right)\pi$ oe (FYI it is 81.535404...)		
(b) Alt 1	$\frac{dr}{dt} = 3 \Rightarrow r = 3t + c \Rightarrow S = 9\pi(t+c)^2 + 3\pi(t+c)\sqrt{25+9(t+c)^2}$ $\Rightarrow \frac{dS}{dt} = 18\pi(t) + 3\pi\sqrt{25+9(t)^2} + 3\pi(t) \cdot \frac{1}{2}(25+9(t)^2)^{-\frac{1}{2}} \cdot 18(t)$ Or $\frac{dS}{dt} = 18\pi(t) + 3\pi \cdot \frac{1}{2}(25(t)^2 + 9(t)^4)^{-\frac{1}{2}} \times (50(t) + 36(t)^3)$ where (t) may be $(t+c')$ throughout	M1 A1
	$r = 1.5 \Rightarrow t+c' = 0.5 \Rightarrow \frac{dS}{dt} = \dots$	M1
	$= 81.5 \text{ (cm}^2 \text{ / min)}$	A1
		(4)
Notes:		
Alt 1 (b) M1: Uses $\frac{dr}{dt} = 3$ and integrates to $r = 3t + c$, where c may be omitted (without loss of generality the time frame can be set to $t = 0$ when $r = 0$), and substitutes into the surface area formula, with their l to form an expression of the correct form and proceeds to differentiate and find $\frac{dS}{dt}$ achieving a suitable form for the curved surface area part (as per main scheme, the only difference is the constant multiples). A1: Correct differentiation for the curved surface area. Need not be simplified. Accept equivalents and may be working to t or $t + c$ throughout. M1: Finds $t + c$ when $r = 1.5$ and substitutes into their $\frac{dS}{dt}$. A1: For awrt 81.5, or as per main scheme.		
(b) Alt 2	$\frac{dl}{dt} = \frac{dl}{dr} \frac{dr}{dt} = 3 \times \frac{1}{2}(r^2 + 25)^{-\frac{1}{2}} \times 2r$	M1 A1
	$S = \pi r^2 + \pi r l \Rightarrow \frac{dS}{dt} = 2\pi r \frac{dr}{dt} + \pi \frac{dr}{dt} l + \pi r \frac{dl}{dt}$	M1
	$= 81.5 \text{ (cm}^2 \text{ / min)}$	A1
		(4)
Notes:		
Alt 2 (b) M1: Uses the chain rule and their expression from (a) to find $\frac{dl}{dt}$ and uses $\frac{dr}{dt} = 3$.		



A1: Correct derivative.

M1: Uses implicit differentiation to find $\frac{dS}{dt}$ in terms of $\frac{dr}{dt}$ and $\frac{dl}{dt}$.

A1: As main scheme.

Question	Scheme	Marks
5(a)	$\int x^2 \cos 2x \, dx = x^2 \cdot \frac{1}{2} \sin 2x - \int 2x \cdot \frac{1}{2} \sin 2x \, dx$	M1 A1
	$= x^2 \cdot \frac{1}{2} \sin 2x - \left(-\frac{1}{2} x \cos 2x - \int -\frac{1}{2} \cos 2x \, dx \right)$	M1
	$= \frac{1}{2} x^2 \sin 2x + \frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x (+c)$	A1
		(4)
(b)	$\frac{dy}{dt} = \frac{t^2 \cos^2 t}{y^2} \Rightarrow \text{e.g. } \int y^2 \frac{dy}{dt} dt \text{ or } \int y^2 dy = \int t^2 \cos^2 t \, dt$	B1
	$\Rightarrow \frac{y^3}{3} = \dots ; \dots = \frac{1}{2} \int t^2 + t^2 \cos 2t \, dt$	M1 ; M1
	$\dots = \frac{1}{6} t^3 + \frac{1}{2} \left(\frac{1}{2} t^2 \sin 2t + \frac{1}{2} t \cos 2t - \frac{1}{4} \sin 2t \right) (+c)$	M1
	$\Rightarrow y^3 = \frac{1}{2} t^3 + \frac{3}{4} t^2 \sin 2t + \frac{3}{4} t \cos 2t - \frac{3}{8} \sin 2t + c$	A1ft
		(5)
(9 marks)		

Notes:

(a)

M1: Attempts parts in the correct direction – look for $Ax^2 \sin 2x \pm \int Bx \sin 2x \, dx$ unless they explicitly state an incorrect formula (in which case it is M0).

A1: Correct first application of parts.

M1: Attempts parts again in the same direction. $\int x \sin 2x \, dx \rightarrow Kx \cos 2x \pm L \int \cos 2x \, dx$

A1: Correct answer with or without constant of integration. ISW after a correct answer.

(b)

Note: accept use of x in place of t (or even mixed x 's and t 's) for the method marks if the intention is clear, but the final A mark needs to be correct in terms of t .

B1: Correct separation of variables with indication to integrate (e.g. statement as shown in scheme, or an attempt to integrate one side).

M1: For $y^n \rightarrow \dots y^{n+1}$ on the left hand side.



M1: Applies double angle formula to right hand side. Accept $\cos^2 t \rightarrow \frac{1}{2}(\pm 1 \pm \cos 2t)$ for this mark.

M1: Applies their result from (a) (allow with t or x here) or redoes integration by parts and reaches the correct form or same form as their (a), and integrates the t^2 term (no need for $+c$)

A1ft: Achieves the correct result, accepting equivalent forms, but must include a constant of integration ($+c$ or other labelling is fine). Follow through on their answer to (a) *providing* isw was not applied. So for $y^3 = \frac{1}{2}t^3 + \frac{3}{2} \times$ their answer to (a) $+c$

5(a) By DI method	S	D	I	M1 A1
	+	x^2	$\cos 2x$	
	-	$2x$	$\frac{1}{2} \sin 2x$	
	+	2	$-\frac{1}{4} \cos 2x$	
	-	0	$-\frac{1}{8} \sin 2x$	
	$\int x^2 \cos 2x \, dx = x^2 \times \frac{1}{2} \sin 2x - 2x \times -\frac{1}{4} \cos 2x + 2 \times -\frac{1}{8} \sin 2x$			M1
	$= \frac{1}{2} x^2 \sin 2x + \frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x \, (+c)$			A1

Notes

M1: Sets up a correct table or equivalent method with x^2 and $\cos x$ as the leads.

A1: Correct tables of derivatives and integrals.

M1: Extracts the answer from the table achieving the correct form.

A1: Correct answer.

Note an answer of the form $Ax^2 \sin 2x + Bx \cos 2x + C \sin 2x$ with no incorrect working seen can score M1A0M1A0 if the coefficients are incorrect.



Question	Scheme	Marks
6(a)	$(3\mathbf{i} + p\mathbf{j} + 7\mathbf{k}) + \lambda(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}) = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ $3 + 2\lambda = 8 + 4\mu \quad (1)$ $\Rightarrow p - 5\lambda = -2 + \mu \quad (2)$ $7 + 4\lambda = 5 + 2\mu \quad (3)$	M1
	e.g. $(3) - 2 \times (1): 1 = -11 - 6\mu \Rightarrow \mu = -2 \left(\text{or } \lambda = -\frac{3}{2} \right)$	M1
	$\mu = -2, \lambda = -\frac{3}{2} \Rightarrow p = -2 - 2 - \frac{15}{2} = \dots$	M1
	$p = -\frac{23}{2}$	A1
		(4)
(b)	Intersect at $(8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + -2(4\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = \dots$	M1
	$= -4\mathbf{j} + \mathbf{k}$	A1
		(2)
(c)	$(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}) \cdot (4\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 2 \times 4 - 5 \times 1 + 4 \times 2 = \dots$	M1
	$\cos \theta = \frac{\text{"11"}}{\sqrt{2^2 + 5^2 + 4^2} \sqrt{4^2 + 1^2 + 2^2}} = \dots \left(= \frac{11}{3\sqrt{105}} = 0.3578\dots \right)$	M1
	$\theta = \cos^{-1} \frac{11}{3\sqrt{105}} = 69.0^\circ$	A1
		(3)
(d)	$\lambda = 2 \Rightarrow \overrightarrow{OA} = \left(7\mathbf{i} - \frac{43}{2}\mathbf{j} + 15\mathbf{k} \right)$	B1ft
	$\overrightarrow{AB} = \pm \left((8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k}) - \left(7\mathbf{i} - \frac{43}{2}\mathbf{j} + 15\mathbf{k} \right) \right)$ $= \pm \left((1 + 4\mu)\mathbf{i} + \left(\frac{39}{2} + \mu \right)\mathbf{j} + (-10 + 2\mu)\mathbf{k} \right)$	M1
	$\left((1 + 4\mu)\mathbf{i} + \left(\frac{39}{2} + \mu \right)\mathbf{j} + (-10 + 2\mu)\mathbf{k} \right) \cdot (4\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 0 \Rightarrow \mu = \dots \left(= -\frac{1}{6} \right)$	M1
	$\overrightarrow{OB} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + -\frac{1}{6}(4\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = \dots$	dM1
	$= \frac{22}{3}\mathbf{i} - \frac{13}{6}\mathbf{j} + \frac{14}{3}\mathbf{k} \text{ or } \left(\frac{22}{3}, -\frac{13}{6}, \frac{14}{3} \right)$	A1
		(5)
(14 marks)		



Notes:

Accept alternative notations, such as column or row vector notation, throughout. Condone if **i**, **j** and **k** are left in columns etc. except for in the final answers to (b) and (d).

(a) Note: work for (a) must be shown in part (a) not recovered from later parts.

M1: Equates equations of the lines and extracts at least two simultaneous equations (usually (1) and (3)) by equating coefficients.

M1: Solves the equations to find at least one of λ or μ

M1: Uses their values or other suitable method to find p allowing for slips.

A1: Correct value.

(b)

M1: Uses either parameter to find the point of intersection. May be implied by two correct entries if no method shown.

A1: Correct point. Accept as coordinates or in vector form.

(c)

M1: Attempts the scalar product with the two direction vectors (or any multiples of them). Allow if there are miscopies etc if the intent of the two correct vectors is clear. Note that $-3\mathbf{i} + \frac{15}{2}\mathbf{j} - 6\mathbf{k}$ and $-8\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ are commonly seen correct direction vectors.

M1: Applies the scalar product formula to find the value for $\cos \theta$. Allow if wrong “direction” vectors are used, or if a slip is made but the intent is clear. This mark is for demonstrating a correct use of the formula with any vectors.

A1: Correct angle, awrt 69.0° , and isw if they give on to give the obtuse angle. But A0 if another incorrect angle is found.

(d)

B1ft: Correct coordinates for A, follow through their p

M1: Attempts to find $\overline{AB}$ in terms of μ with their $\overline{OA}$ and a general point on l_2 . Allow subtraction either way round. May be implied by two correct entries if no method shown. Note, if

$\overline{AB} = (x-7)\mathbf{i} + \left(y + \frac{43}{2}\right)\mathbf{j} + (z-15)\mathbf{k}$ is used this mark is not scored until the point where they identify x , y and z in terms of μ .

M1: Takes scalar product of their $\overline{AB}$ with direction of l_2 , sets equal to zero and solves to find μ

dM1: Depends on previous M mark. Uses their parameter in the correct line to find the coordinates. Implied by 2 correct coordinates if no method is shown.

A1: Correct answer. Accept as coordinates or in vector form.



Alt (d)	$\lambda = 2 \Rightarrow \overrightarrow{OA} = \left(7\mathbf{i} - \frac{43}{2}\mathbf{j} + 15\mathbf{k} \right)$	B1ft
E.g.	$\overrightarrow{XA} = \pm \left(\left(7\mathbf{i} - \frac{43}{2}\mathbf{j} + 15\mathbf{k} \right) - (-4\mathbf{j} + \mathbf{k}) \right)$ $\overrightarrow{XB} = \pm \left((8+4\mu)\mathbf{i} + (-2+\mu)\mathbf{j} + (5+2\mu)\mathbf{k} - (-4\mathbf{j} + \mathbf{k}) \right)$	M1
	$AX \cos \theta = BX \Rightarrow \left(7^2 + \left(-\frac{35}{2} \right)^2 + 14^2 \right) \left(\frac{121}{945} \right) = (8+4\mu)^2 + (2+\mu)^2 + (4+2\mu)^2$ $\Rightarrow 36\mu^2 + 144\mu + 23 = 0 \Rightarrow \mu = \dots \left(-\frac{1}{6}, -\frac{23}{6} \right)$	M1
	$\overrightarrow{OB} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \left(-\frac{1}{6} \right) (4\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = \dots$	dM1
	$= \frac{22}{3}\mathbf{i} - \frac{13}{6}\mathbf{j} + \frac{14}{3}\mathbf{k}$ or $\left(\frac{22}{3}, -\frac{13}{6}, \frac{14}{3} \right)$	A1
		(5)

Notes
Alt (d) – by right triangle

B1ft: Correct coordinates for A, follow through their p

M1: Attempts to find at least two sides of the triangle AXB where X is the intersection point. (Note if no further progress in this method is made the main scheme can apply to allow M1 for just $\overrightarrow{AB}$) in terms of μ where relevant, using their $\overrightarrow{OA}$ and a general point on l_2 . Allow subtraction either way round. May be implied by two correct entries if no method shown.

M1: For a full, correct method leading to a value for μ . There are variation to that shown in the scheme, such as use of Pythagoras on all three sides, or via $AX \sin \theta = AB$. The correct side must be used for the hypotenuse, but allow use of decimals in the trig work but must be working in the correct mode to 3s.f..

dM1: Depends on previous M mark. Uses one of their parameters in the correct line to find the coordinates. Implied by 2 correct coordinates if no method is shown. They may use either solution to their quadratic for this mark.

A1: Correct answer only. Must have selected the root which corresponds to A and reject the other. Accept as coordinates or in vector form. Allow awrt 3 s.f. for the coordinates.



Question	Scheme	Marks
7(a) Alt by inspection for first 4 marks	$u = 4x + 2 \sin 2x \Rightarrow \frac{du}{dx} = 4 + 4 \cos 2x$ oe	M1 A1
	$\frac{du}{dx} = 4 + 4 \cos 2x = 8 \cos^2 x \Rightarrow \int_{(0)}^{\left(\frac{\pi}{2}\right)} e^{4x+2\sin 2x} \cos^2 x \, dx = \int_{(0)}^{(2\pi)} \frac{e^u}{8} du$	M1
	$= \left[\frac{1}{8} e^u \right]_0^{2\pi}$ or $\left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	A1ft
	$\int e^{4x+2\sin 2x} \cos^2 x \, dx = \int e^{4x+2\sin 2x} \times \frac{1}{2} (1 \pm \cos 2x) \, dx$	(M1 A1)
	$= k e^{4x+2\sin 2x}; = \left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	(M1; A1ft)
	$= \left[\frac{1}{8} e^u \right]_0^{2\pi}$ or $\left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}} = \frac{1}{8} (e^{2\pi} - 1) *$	A1*cs0
		(5)
(b)	$V \left[= \pi \int_0^{\frac{\pi}{2}} y^2 \, dx = \pi \int_0^{\frac{\pi}{2}} \left(6 e^{2x+2\sin 2x} \cos x \right)^2 \, dx \right] = 36\pi \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx$	B1
	$K \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx = \frac{K}{8} (e^{2\pi} - 1)$	M1
	$= \frac{9\pi}{2} (e^{2\pi} - 1)$	A1
		(3)
(8 marks)		

Notes:**(a)**

M1: Attempts the derivative of u . Look for $\sin 2x \rightarrow \dots \cos 2x$ May apply double angle formula first, look for $u = 4x + k \sin x \cos x \rightarrow \frac{du}{dx} = (4 +) k (\cos^2 x - \sin^2 x)$ oe. Form should be correct but there may be sign errors.

A1: Correct derivative (any form).

M1: Applies a double angle formula – must be seen – and carries out the full substitution to get an integral in terms of u . Must include an attempt to replace dx . Do not be concerned with the limits for this mark.

A1ft: Integrates to achieve the correct form with their constant with correct limits assigned (either in u or reverted in x)– must have scored the M1.



A1*cs0: Correct answer from fully correct working. Must have identified **correct** limits in terms of u or have reverted back to x with the original limits before giving the final answer.

Alt for first 4 marks, by inspection:

M1: Applies double angle formula for $\cos^2 x$ (allowing for sign error) in the integrand in the form shown – must be this form to be able to apply recognition.

A1: Correct expression for the integrand in this form.

M1: Integrates, achieving the correct form.

A1ft: Achieves the correct integral from correct work with correct limits assigned – the double angle formula must have been correct (follow through if e.g they lose the $\frac{1}{2}$ from the double angle formula before integrating).

(b)

B1: Applies correct volume formula with π seen somewhere in the answer and squares correctly to achieve the correct y^2 . The limits may be missing for this mark. May be implied by the correct answer.

M1: Makes the connection with part (a) and multiplies the answer to (a) by their constant formed from their attempt at y^2 . The use of (a) may be via reuse of the substitution. May be implied by an expression

$M(e^{2\pi} - 1)$, $M \neq \frac{1}{8}$ if no incorrect working is shown for this (but a clearly incorrect method seen is

M0).

A1: cao. As shown or $\frac{9\pi}{2}e^{2\pi} - \frac{9\pi}{2}$



Question	Scheme	Marks
8	$y = 2x + x^3 + \cos x$	
	Assume (there is a stationary point so) $\frac{dy}{dx} = 0$ for some value of x	B1
	So for this x $\frac{dy}{dx} = 2 + 3x^2 - \sin x$	M1
	$\frac{dy}{dx} = 0 \Rightarrow \sin x = 3x^2 + 2 \dots 2$	dM1
	This is a <u>contradiction</u> as $ \sin x \leq 1$ for all x , hence the assumption is false and so the function has <u>no stationary points</u> .	A1cso
		(4)
(4 marks)		
Notes:		
B1: For a clear, suitable assumption with reference to $\frac{dy}{dx} = 0$ or implication that it means $\frac{dy}{dx} = 0$ at some point in their proof. Need not mention “for some x”, but stating “for all x” is B0. Accept just “Assume $\frac{dy}{dx} = 0$” M1: Attempts the derivative achieving $2 + 3x^2 \pm \sin x$ dM1: Makes progress with a correct method leading to a contradiction. E.g. set their derivative equal to 0, makes $\sin x$ the subject and deduces greater than or equal to 2. Other methods are possible, e.g. states $x^2 \geq 0$ and $\sin x \leq 1$ and deduces $2 + 3x^2 - \sin x \geq 1$ (hence cannot be zero). It must be clear what the contradiction will be but allow if there are some inaccuracies in details e.g. condone if a strict inequality is used for this mark. There must be reference to both $(3)x^2 \dots 0$ and the range of $\sin x$ in some way. Do not accept “this equation has no solutions” without proof as to why there can be no solutions. Note: A sketch is not a proof. An algebraic approach is required, but accept verbal arguments (e.g. stating “the minimum of $2 + 3x^2$ is 2” and “$\sin x$ cannot exceed one” etc), which may accompany a sketch. A1cso: Fully correct proof, with derivative correct and contradiction made clear and conclusion drawn referring back to no stationary points. All necessary details must have been given. Condone cases where strict inequalities have been given instead of unstrict ones.		

Question	Scheme	Marks
9(a)	$\frac{dx}{dt} = \sec t \tan t$ and $\frac{dy}{dt} = \sqrt{3} \sec^2 \left(t + \frac{\pi}{3} \right)$	B1
	$\left(\frac{dy}{dx} = \right) \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\sqrt{3} \sec^2 \left(t + \frac{\pi}{3} \right)}{\sec t \tan t}$ oe	M1 A1
		(3)
(a) Alt	$x = \sec t \quad y = \sqrt{3} \frac{\tan t + \sqrt{3}}{1 - \sqrt{3} \tan t}$	
	$\frac{dx}{dt} = \sec t \tan t$ and $\frac{dy}{dt} = \sqrt{3} \frac{(1 - \sqrt{3} \tan t) \sec^2 t - (\tan t + \sqrt{3})(-\sqrt{3} \sec^2 t)}{(1 - \sqrt{3} \tan t)^2}$ (oe)	B1
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \sqrt{3} \frac{(1 - \sqrt{3} \tan t) \sec^2 t - (\tan t + \sqrt{3})(-\sqrt{3} \sec^2 t)}{\sec t \tan t (1 - \sqrt{3} \tan t)^2}$ oe	M1 A1
(b)	$t = \frac{\pi}{3} \Rightarrow x = 2, y = -3$	B1
	$t = \frac{\pi}{3} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{3} \sec^2 \frac{2\pi}{3}}{\sec \frac{\pi}{3} \tan \frac{\pi}{3}} = \dots (= 2)$	M1
	Tangent is $y + 3 = 2(x - 2) \Rightarrow y = 2x - 7$ cso	M1A1
		(4)
(c)	$y = \sqrt{3} \frac{\tan t \pm \tan \frac{\pi}{3}}{1 \pm \tan t \tan \frac{\pi}{3}}$	M1
	$x^2 = \sec^2 t = 1 + \tan^2 t \Rightarrow \tan t = \sqrt{x^2 - 1} \Rightarrow y = \sqrt{3} \frac{\sqrt{x^2 - 1} + \sqrt{3}}{1 - \sqrt{3} \sqrt{x^2 - 1}}$	M1 A1
	$= \sqrt{3} \frac{\sqrt{x^2 - 1} + \sqrt{3}}{1 - \sqrt{3} \sqrt{x^2 - 1}} \times \frac{1 + \sqrt{3} \sqrt{x^2 - 1}}{1 + \sqrt{3} \sqrt{x^2 - 1}} = ..$	
	$= \sqrt{3} \frac{\sqrt{x^2 - 1} + \sqrt{3}(x^2 - 1) + \sqrt{3} + 3\sqrt{x^2 - 1}}{1 - (3x^2 - 3)} = \frac{.. \sqrt{x^2 - 1} + .. x^2}{4 - 3x^2}$	M1
	$= \frac{3x^2 + 4\sqrt{3x^2 - 3}}{4 - 3x^2}$	A1
		(5)

(12 marks)
Notes:
(a) B1: Both derivatives given correctly. See Alt for an alternative form for y M1: Divides their $\frac{dy}{dt}$ by their $\frac{dx}{dt}$ A1: Correct expression and isw after a correct answer, accept any equivalent – e.g. see the Alt form in scheme.
(b) B1: Correct values for x and y stated or implied. M1: Some evidence of substituting in $t = \frac{\pi}{3}$ to their gradient expression to find the gradient of the tangent. Must be evaluated. You may need to check their value matches their $\frac{dy}{dx}$ if no substitution or other evidence is shown. M1: Correct method for the equation of the tangent, using their evaluated gradient. Must use their $\frac{dy}{dx}$ for the gradient but allow a slip in sign on one side for the coordinates in the formula. If they use $y = mx + c$ they need to proceed as far as finding c. A1cso: Correct equation in the form stated. Must have come from a correct $\frac{dy}{dx}$.
(c) M1: Attempts the compound angle formula on y, accept with sign errors. Allow if the $\sqrt{3}$ is missing. Must be seen or used in part (c). M1: Uses $\sec^2 t = 1 + \tan^2 t$ to find $\tan t$ or $\tan^2 t$ in terms of x and substitutes to produce an equation linking y and x. Allow if there are errors rearranging after a correct formula. A1: Any correct Cartesian formula, with trig terms evaluated. M1: Full method to reach an answer in the required form, so multiplies through numerator and denominator by appropriate factor for their denominator and expands the numerator. The denominator work may be implied. A1: Achieves the correct answer.

Alt (c)	$y = \sqrt{3} \frac{\tan t \pm \tan \frac{\pi}{3}}{1 \pm \tan t \tan \frac{\pi}{3}}$	M1
	$\left(\Rightarrow y(1 \pm \sqrt{3} \tan t) = \sqrt{3} \tan t \pm 3 \Rightarrow \tan t = \frac{\pm y \pm 3}{\pm y \sqrt{3} \pm \sqrt{3}} \right)$ $x^2 = \sec^2 t = 1 + \tan^2 t = 1 + \left(\frac{y-3}{\sqrt{3}(y+1)} \right)^2$	M1 A1
	$\Rightarrow 3x^2 - 3 = \left(\frac{y-3}{y+1} \right)^2 \Rightarrow \frac{y-3}{y+1} = \sqrt{3x^2 - 3} \left(\text{as } y < -1 \Rightarrow \frac{y-3}{y+1} > 0 \right)$	
	$\Rightarrow y = \frac{3 + \sqrt{3x^2 - 3}}{1 - \sqrt{3x^2 - 3}} = \frac{(3 + \sqrt{3x^2 - 3})(1 + \sqrt{3x^2 - 3})}{1 - 3x^2 + 3} = \frac{\dots x^2 + \dots \sqrt{3x^2 - 3}}{4 - 3x^2}$	M1
	$= \frac{3x^2 + 4\sqrt{3x^2 - 3}}{4 - 3x^2}$	A1
		(5)

Alt (c)

M1: Attempts the compound angle formula on y , accept with sign errors. Allow if the $\sqrt{3}$ is missing. Must be seen or used in part (c).

M1: Rearranges to find $\tan t$ and uses $\sec^2 t = 1 + \tan^2 t$ to find x or x^2 in terms of $\tan t$ with their attempt at y to achieve an equation linking y and x . Allow if there are errors rearranging after a correct formula and the $\sqrt{3}$ may have been missing.

A1: Any correct Cartesian formula with all trig terms evaluated.

M1: Full method to reach an answer in the required form, takes the 1 across, square roots (no need for justification of which sign, and allow if wrong sign is chosen), makes y the subject, multiplies numerator and denominator by appropriate factor for their denominator and expands numerator. The denominator work may be implied.

A1: Achieves the correct answer.

Alt (c)	$y = \sqrt{3} \frac{\sin\left(t + \frac{\pi}{3}\right)}{\cos\left(t + \frac{\pi}{3}\right)} = \sqrt{3} \frac{\sin t \cos \frac{\pi}{3} + \cos t \sin \frac{\pi}{3}}{\cos t \cos \frac{\pi}{3} - \sin t \sin \frac{\pi}{3}}$	M1
	$\left(\Rightarrow y(\cos t - \sqrt{3} \sin t) = \sqrt{3} \sin t + 3 \cos t \right) \Rightarrow y \left(\frac{1}{x} - \sqrt{3} \sqrt{1 - \frac{1}{x^2}} \right) = \sqrt{3} \sqrt{1 - \frac{1}{x^2}} + \frac{3}{x}$	M1 A1

Alt (c) By sine and cosine, or variants, apply as follows.

M1: Writes $\tan$ in terms of $\sin$ and $\cos$ and attempts the compound angle formula on y , accept with sign errors. Allow if the $\sqrt{3}$ is missing. Must be seen or used in part (c).

M1: Applies $\sin^2 t = 1 - \cos^2 t$ throughout to get an equation linking y and x . The $\sqrt{3}$ may have been missing.

A1: Any correct Cartesian formula with all trig terms evaluated.

M1A1: Per main scheme.

