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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Implicit & Parametric Differentiation (P4.4)

International A Level Mathematics · P4: Pure Mathematics 4 (WMA14)

32 questions · 255 marks · Mark schemes included

Past papers October 2020 - June 2025 · 2 sub-topics

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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2. Q5	IAL P4 (WMA14) June 2025	6	5
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QUESTIONS

32 questions · 255 marks · 2 sub-topics

SUB-TOPIC P4.4a

IMPLICIT DIFFERENTIATION

Sub-topic P4.4a · 16 questions · 131 marks · lowest tariff first



1. The curve C has equation

$$xy^2 = x^2y + 6 \quad x \neq 0 \quad y \neq 0$$

Find an equation for the tangent to C at the point $P(2, 3)$, giving your answer in the form $ax + by + c = 0$ where a , b and c are integers.

(6)

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Question 1 continued

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Handwriting practice area with 25 horizontal lines.

(Total 6 marks)

Q1



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 6 marks)

6. A curve C has equation

$$y = x^{\sin x} \quad x > 0 \quad y > 0$$

- (a) Find, by firstly taking natural logarithms, an expression for $\frac{dy}{dx}$ in terms of x and y . **(5)**
- (b) Hence show that the x coordinates of the stationary points of C are solutions of the equation

$$\tan x + x \ln x = 0 \quad (2)$$

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Q6

1. The curve C has equation

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

The point $P(3, 2)$ lies on C .

Find the equation of the normal to C at the point P , writing your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found.

(7)

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Question 1 continued

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Lined area for writing the answer to Question 1.

(Total 7 marks)

Q1







Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 7 marks)



Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 7 marks)



4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$16x^3 - 9kx^2y + 8y^3 = 875$$

where k is a constant.

(a) Show that

$$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$$

(4)

Given that the curve has a turning point at $x = \frac{5}{2}$

(b) find the value of k

(4)

Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4 continued.

(Total for Question 4 is 8 marks)



(Total for Question 2 is 6 marks)



6. A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

- (a) Find $\frac{dy}{dx}$ in terms of x and y .

(5)

The curve crosses the y -axis at the origin and at the point P .

- (b) Find the equation of the normal to the curve at P , writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

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Q6

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Q5

Question 11 continued

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Question 11 continued

Lined area for writing the answer to Question 11.

(Total for Question 11 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS



3.

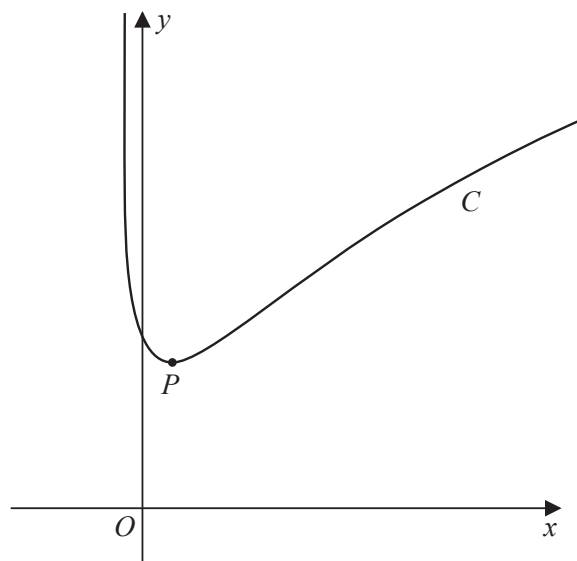


Figure 1

The curve C , shown in Figure 1, has equation

$$y^2x + 3y = 4x^2 + k \quad y > 0$$

where k is a constant.

(a) Find $\frac{dy}{dx}$ in terms of x and y

(5)

The point $P(p, 2)$, where p is a constant, lies on C .

Given that P is the minimum turning point on C ,

(b) find

(i) the value of p

(ii) the value of k

(4)





(Total for Question 5 is 7 marks)







Question 4 continued

Lined area for writing the answer to Question 4 continued.

(Total for Question 4 is 9 marks)



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3. The curve C has equation

$$3y^2 - 11x^2 + 11xy = 20y - 36x + 28$$

(a) Find, in simplest form, $\frac{dy}{dx}$ in terms of x and y .

(5)

The point $P(4, k)$, where k is a constant, lies on C .

Given that $k < 0$

(b) find the value of the gradient of C at P

(5)

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Q3

Question 2 continued

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Question 2 continued

Lined area for writing the answer to Question 2 continued.



(Total for Question 2 is 10 marks)



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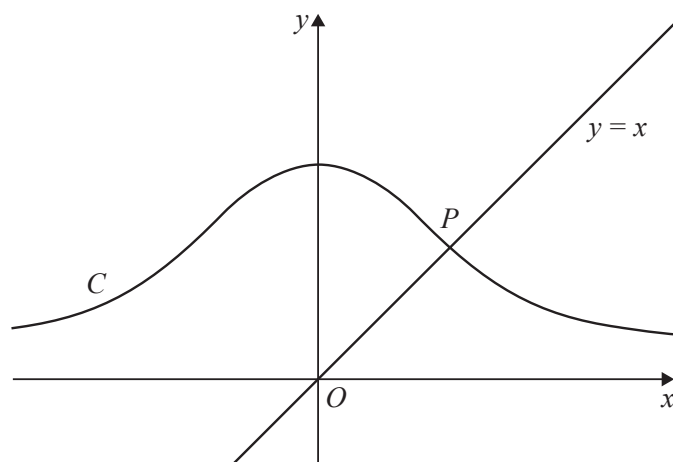


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y^3 - x^2 + 4x^2y = k$$

where k is a positive constant greater than 1

- (a) Find $\frac{dy}{dx}$ in terms of x and y .

(5)

The point P lies on C .

Given that the normal to C at P has equation $y = x$, as shown in Figure 2,

- (b) find the value of k .

(5)

Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5.



(Total for Question 3 is 16 marks)



SUB-TOPIC P4.4b

PARAMETRIC DIFFERENTIATION & RATES OF CHANGE

Sub-topic P4.4b · 16 questions · 124 marks · lowest tariff first



2. A spherical ball of ice with radius r cm is melting.

The volume of the ball of ice, $V \text{ cm}^3$, is decreasing at a constant rate, $k \text{ cm}^3$ per second, where k is a constant.

Given that $V = \frac{4}{3}\pi r^3$, show that the rate of decrease of the radius of the ball of ice with respect to time is inversely proportional to the square of the radius.

(4)



Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 4 marks)



Question 2 continued

(Total for Question 2 is 5 marks)



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 6 marks)



(Total for Question 5 is 6 marks)

Question 3 continued

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Question 3 continued

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(Total 7 marks)

Q3



(Total for Question 2 is 7 marks)

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Question 4 continued

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Question 4 continued

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(Total 8 marks)

Q4



Question 3 continued

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(Total for Question 3 is 8 marks)



Question 10 continued

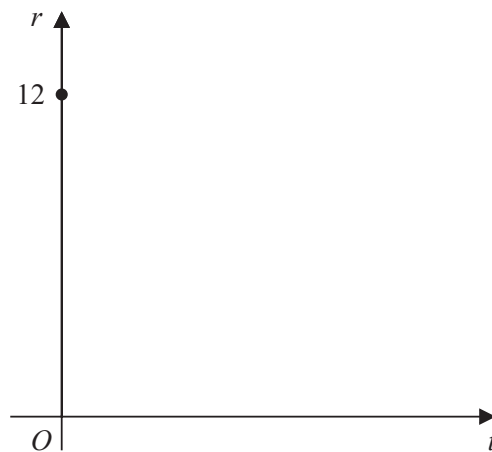


Diagram 1



(Total for Question 10 is 8 marks)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 8 marks)



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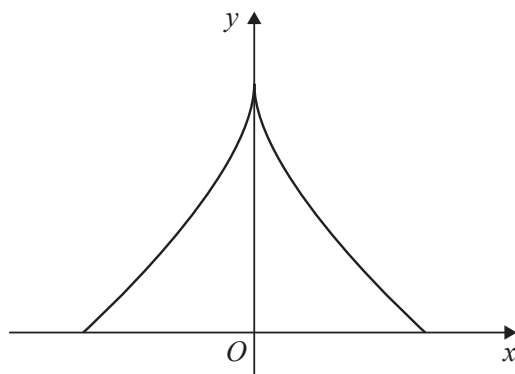


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 3 \sin^3 \theta \quad y = 1 + \cos 2\theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = k \operatorname{cosec} \theta \quad \theta \neq 0$$

where k is a constant to be found.

(3)

The point P lies on C where $\theta = \frac{\pi}{6}$

(b) Find the equation of the tangent to C at P , giving your answer in the form $ax + by + c = 0$ where a , b and c are integers.

(3)

(c) Show that C has Cartesian equation

$$8x^2 = 9(2 - y)^3 \quad -q \leq x \leq q$$

where q is a constant to be found.

(3)

Question 3 continued



(Total for Question 3 is 9 marks)





9.

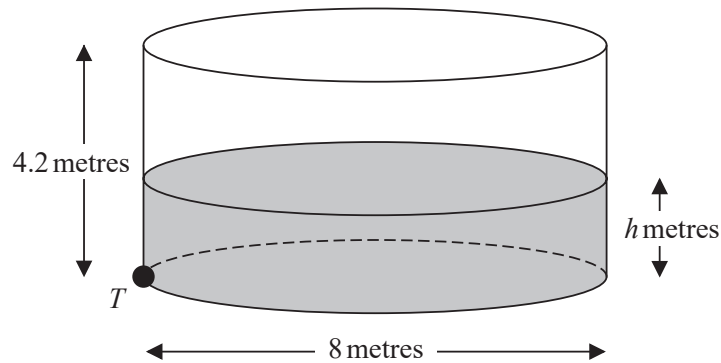
Leave
blank**Figure 4**

Figure 4 shows a cylindrical tank that contains some water.

The tank has an internal diameter of 8 m and an internal height of 4.2 m.

Water is flowing into the tank at a constant rate of $(0.6\pi)\text{ m}^3$ per minute.

There is a tap at point T at the bottom of the tank.

At time t minutes after the tap has been opened,

- the depth of the water is h metres
- the water is leaving the tank at a rate of $(0.15\pi h)\text{ m}^3$ per minute

(a) Show that

$$\frac{dh}{dt} = \frac{12 - 3h}{320} \quad (4)$$

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m. (6)

Question 9 continued

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Question 9 continued

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Question 9 continued

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(Total 10 marks)

Q9







(Total for Question 7 is 12 marks)



9.

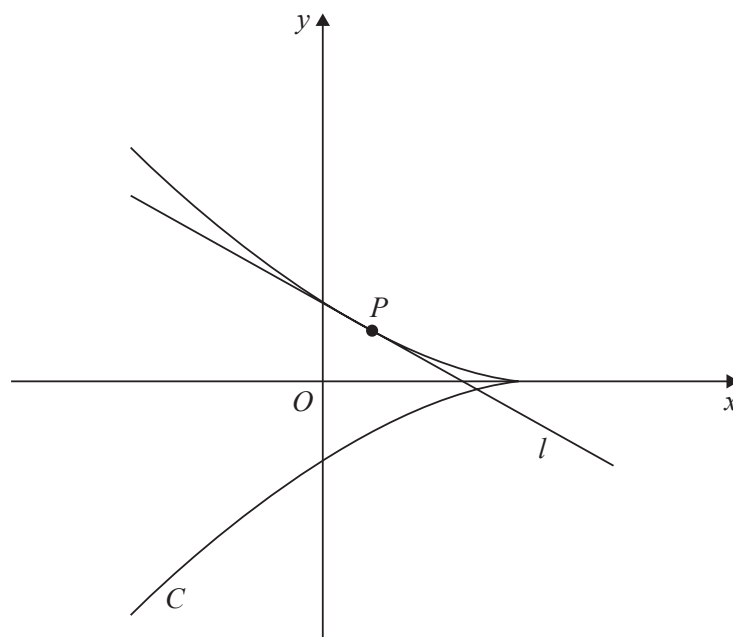


Figure 2

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2 \cos 2t \quad y = \sin^3 t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \frac{\pi}{6}$

The line l , shown in Figure 2, is the tangent to C at P .

(a) Use parametric differentiation to show that

(i) $\frac{dy}{dx} = k \sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

The line l intersects the curve C again at the point Q .

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q .

(6)

Question 9 continued





TOTAL FOR PAPER IS 75 MARKS



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
2(a)	$\frac{dV}{dx} = 3x^2$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3x^2}$	dM1A1
		(3)
Alternative 1		
	$V = x^3 \Rightarrow x = V^{\frac{1}{3}} \Rightarrow \frac{dx}{dV} = \frac{1}{3} V^{-\frac{2}{3}}$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3V^{\frac{2}{3}}} = \frac{5}{3x^2}$	dM1A1
Alternative 2		
	$\frac{dV}{dt} = 5 \Rightarrow V = 5t + c$	M1 (B1 on EPEN)
	$V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 \frac{dx}{dt} = 5$ or $V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 = 5 \frac{dt}{dx}$	dM1
	$\frac{dx}{dt} = \frac{5}{3x^2}$	A1
(b)	$\frac{dx}{dt} = \frac{5}{3 \times 4^2} = \frac{5}{48}$	M1A1
		(2)
		(5 marks)
Note this is now being marked as M1dM1A1 not B1M1A1 (a) Ignore any units associated with any of the expressions/values in this question. M1: Differentiates $V = x^3$ to obtain $\frac{dV}{dx} = \dots x^2$ (seen or implied) dM1: Attempts to use $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ or equivalent e.g. $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$ with their $\frac{dV}{dx}$ and $\frac{dV}{dt}$ where $\frac{dV}{dx} = kx^2$ and $\frac{dV}{dt}$ is a constant to obtain an expression of the form $\frac{\alpha}{x^2}$ oe. May be implied by their working. Depends on the previous mark. A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3} x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.		

Alternative 1:

M1: Differentiates $x = V^{\frac{1}{3}}$ to obtain $\frac{dx}{dV} = \dots V^{-\frac{2}{3}}$ (seen or implied)

dM1: Attempts to use $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ or equivalent e.g. $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$ with their $\frac{dx}{dV}$ and $\frac{dV}{dt}$ where $\frac{dx}{dV} = kV^{-\frac{2}{3}}$ and $\frac{dV}{dt}$ is a constant and changes back to x to obtain an expression of the form $\frac{\alpha}{x^2}$ oe. May be implied by their working.

Depends on the previous mark.

A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3}x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.

Alternative 2:

M1: Uses $\frac{dV}{dt} = 5$ and integrates wrt t to obtain $V = 5t + c$ or $V = 5t$

dM1: Replaces V with x^3 and differentiates wrt t or wrt x to obtain $\alpha x^2 \frac{dx}{dt} = \beta$ or $\alpha x^2 = \beta \frac{dt}{dx}$

Depends on the previous mark.

A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3}x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.

(b)

M1: Substitutes $x = 4$ into their expression for $\frac{dx}{dt}$

A1: $\frac{5}{48}$ or the exact equivalent. Ignore any units, correct or incorrect, and just look for the value.

Do not allow e.g. 0.104... unless the correct exact value is seen previously, then apply isw.

Question Number	Scheme	Marks
3	$8y \frac{dy}{dx} = xe^{-y^2} \rightarrow \int 8ye^{y^2} dy = \int x dx$	B1
	$4e^{y^2} = \frac{x^2}{2} (+c)$	M1A1
	$4 = \frac{5^2}{2} + c \Rightarrow c = \dots$	dM1
	$y^2 = \ln\left(\frac{x^2 - 17}{8}\right)$	A1
		(5 marks)
B1: Separates the variables correctly with some indication of integration. Allow any correct separation e.g. $8 \int \frac{y}{e^{-y^2}} dy = \int x dx$, $\int ye^{y^2} dy = \frac{1}{8} \int x dx$, $\int \frac{y}{e^{-y^2}} dy = \int \frac{x}{8} dx$ Award for any of the above with at least one integral sign. If there are no integral signs then there must be an attempt to integrate at least one side. Condone the omission of dx or dy as long as the intention is clear. M1: For integrating $\dots ye^{y^2}$ correctly to obtain $\dots e^{y^2}$ May be implied by a substitution e.g. $u = y^2 \Rightarrow \int 8ye^{y^2} dy = \int \frac{8y}{2y} e^u du = 4e^u (+c)$ Incorrect attempts to integrate $\dots ye^{y^2}$ e.g. by parts will generally score no more marks. A1: Fully correct equation with or without a constant of integration e.g. $4e^{y^2} = \frac{x^2}{2} (+c)$, $8e^{y^2} = x^2 (+c)$, $e^{y^2} = \frac{x^2}{8} (+c)$ etc. dM1: Depends on the first method mark. Substitutes in $y = 0$ and $x = 5$ and proceeds to find a value for c. A1: Correct equation: $y^2 = \ln\left(\frac{x^2 - 17}{8}\right)$ oe e.g. $y^2 = \ln\left(\frac{x^2}{8} - \frac{17}{8}\right)$, $y^2 = \ln\frac{1}{8}(x^2 - 17)$, $y^2 = \ln(x^2 - 17) - \ln 8$ etc. Brackets must be present if necessary e.g. in $y^2 = \ln\left(\frac{x^2}{8} - \frac{17}{8}\right)$ but not in e.g. $y^2 = \ln \frac{x^2 - 17}{8}$ Apply isw once the correct answer is seen.		

Question Number	Scheme	Marks
4(a)	$4^{\frac{1}{2}} \left(1 + \frac{5}{4}x\right)^{\frac{1}{2}}$	B1
	$\left(1 + \frac{5}{4}x\right)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)\left(\frac{5}{4}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5}{4}x\right)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{5}{4}x\right)^3 + \dots$	M1A1
	$4^{\frac{1}{2}} \left(1 + \frac{5}{4}x\right)^{\frac{1}{2}} = 2 + \frac{5}{4}x - \frac{25}{64}x^2 + \frac{125}{512}x^3$	A1A1
		(5)

(a)

B1: Obtains $4^{\frac{1}{2}}(1+\dots x)^{\frac{1}{2}}$ or e.g. $2(1+\dots x)^{\frac{1}{2}}$ or e.g. $\sqrt{4}(1+\dots x)^{\frac{1}{2}}$ which may be implied by subsequent work.

M1: Attempts the binomial expansion of $(1+kx)^{\frac{1}{2}}$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 which may be unsimplified.

Look for $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}\dots x^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}\dots x^3$ o.e. (you do not need to be concerned with their

$\frac{5}{4}$ which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\left(\frac{1}{2}\right), \left(\frac{1}{2}\right)$ or ${}^{\frac{1}{2}}C_2, {}^{\frac{1}{2}}C_3$ unless interpreted correctly in further work.

May be implied by correct coefficients with x^2 or x^3

A1: Correct third or fourth term unsimplified for $\left(1 + \frac{5}{4}x\right)^{\frac{1}{2}}$ e.g. with the correct sign.

$$\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5}{4}x\right)^2 \quad \text{or} \quad \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{5}{4}x\right)^3$$

Do not condone missing brackets e.g. $\frac{5}{4}x^2$ or $\frac{5}{4}x^3$ unless they are implied by subsequent work.

A1: Two correct simplified terms of $\frac{5}{4}x, -\frac{25}{64}x^2, \frac{125}{512}x^3$ which may be listed.

Condone $\frac{5}{4}x^1$ for this mark.

Condone coefficients given as decimals e.g. $1.25x, -0.390625x^2, 0.24414\dots x^3$

A1: $2 + \frac{5}{4}x - \frac{25}{64}x^2 + \frac{125}{512}x^3$ which may be listed and isw once a correct answer is seen.

Condone coefficients given as decimals e.g. $2 + 1.25x - 0.390625x^2 + 0.244140625x^3$

Do not condone $\frac{5}{4}x^1$ for this mark and do not condone $\dots + -\frac{25}{64}x^2 + \dots$

Note that if their " $4^{\frac{1}{2}}$ " is incorrect they can score a maximum of B0M1A1A0A0



(a) Alternative by direct expansion:

$$(4+5x)^{\frac{1}{2}} = 4^{\frac{1}{2}} + \left(\frac{1}{2}\right)4^{-\frac{1}{2}}(5x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2}4^{-\frac{3}{2}}(5x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}4^{-\frac{5}{2}}(5x)^3 + \dots$$

B1: For $4^{\frac{1}{2}} + \dots$ or $\sqrt{4} + \dots$ or $2 + \dots$

M1: Attempts the binomial expansion of $(4+5x)^{\frac{1}{2}}$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 and the correct power of 4 which may be unsimplified.

Look for $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}4^{-\frac{3}{2}}\dots x^2$ or $\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}4^{-\frac{5}{2}}\dots x^3$ o.e. (you do not need to be concerned with their 5 which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\left(\frac{1}{2}\right)_2$, $\left(\frac{1}{2}\right)_3$ or ${}^{\frac{1}{2}}C_2$, ${}^{\frac{1}{2}}C_3$ unless interpreted correctly in further work.

A1: Correct third or fourth term unsimplified e.g.

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2}4^{-\frac{3}{2}}(5x)^2 \text{ or } \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}4^{-\frac{5}{2}}(5x)^3$$

Do not condone missing brackets e.g. $5x^2$ or $5x^3$ unless they are implied by subsequent work.

A1: Two correct simplified terms of $\frac{5}{4}x$, $-\frac{25}{64}x^2$, $\frac{125}{512}x^3$ which may be listed.

Condone $\frac{5}{4}x^1$ for this mark.

Condone coefficients given as decimals e.g. $1.25x$, $-0.390625x^2$, $0.24414\dots x^3$

A1: $2 + \frac{5}{4}x - \frac{25}{64}x^2 + \frac{125}{512}x^3$ which may be listed and isw once a correct answer is seen.

Condone coefficients given as decimals e.g. $2 + 1.25x - 0.390625x^2 + 0.244140625x^3$

Do not condone $\frac{5}{4}x^1$ for this mark and do not condone $\dots + -\frac{25}{64}x^2 + \dots$

Any attempts to use a Maclaurin series should be sent to review.



(b)	$ x \leq \frac{4}{5}$	B1
		(1)
(c)(i)	$2 - \frac{5}{4}x - \frac{25}{64}x^2 - \frac{125}{512}x^3$	B1ft
		(1)
(c)(ii)	$"2" + "2" + "-\frac{25}{64}"x^2 + "-\frac{25}{64}"x^2 = 4 - \frac{25}{32}x^2$	M1A1
		(2)
		(9 marks)

(b)

B1: $|x| \leq \frac{4}{5}$ or $|x| < \frac{4}{5}$ or accept strict or non-strict inequalities at either end e.g.

$$-\frac{4}{5} < x < \frac{4}{5}, -\frac{4}{5} \leq x \leq \frac{4}{5}, x < \frac{4}{5}, -\frac{4}{5} < x \leq \frac{4}{5}, \frac{4}{5} > x > -\frac{4}{5}, x > -\frac{4}{5} \text{ and } x < \frac{4}{5}$$

or interval notation e.g. $\left(-\frac{4}{5}, \frac{4}{5}\right), \left[-\frac{4}{5}, \frac{4}{5}\right], \left[\frac{4}{5}, \frac{4}{5}\right], \left(-\frac{4}{5}, \frac{4}{5}\right]$

(c)(i)

B1ft: $2 - \frac{5}{4}x - \frac{25}{64}x^2 - \frac{125}{512}x^3$ (follow through their (a))

For their $A + Bx + Cx^2 + Dx^3$ in part (a) they must obtain $A - Bx + Cx^2 - Dx^3$ where A, B, C and D are non-zero for this mark even if they make a re-start.

(c)(ii)

M1: Adds their $A + Bx + Cx^2 + Dx^3$ from part (a) to $A - Bx + Cx^2 - Dx^3$ to give $a + bx^2$ or obtains values for a and b.

May be implied by their answer/values if no incorrect working is seen.

A1: Cao and **cs0** $4 - \frac{25}{32}x^2$ o.e. e.g. $4 - \frac{50}{64}x^2$ Allow $4 + -\frac{25}{32}x^2$

This must follow fully correct work in (a) and (c).

Apply isw once the correct answer is seen.



Question Number	Scheme	Marks
5	$4y^2 \rightarrow 8y \frac{dy}{dx}$	M1
	$e^{xy} \rightarrow \left(y + x \frac{dy}{dx}\right)e^{xy}$ or e.g. $e^{xy} \rightarrow ye^{xy} + x \frac{dy}{dx}e^{xy}$	M1
	$2 + 8y \frac{dy}{dx} = 3 \left(y + x \frac{dy}{dx}\right)e^{xy}$	A1
	$2 = 3 \times \frac{9}{2} \times \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{4}{27}$	M1
	$y = \frac{4}{27} \left(x - \frac{9}{2}\right) \Rightarrow 4x - 27y - 18 = 0$	dM1A1
		(6)
		(6 marks)
M1: Differentiates $4y^2$ to obtain $\dots y \frac{dy}{dx}$ M1: Differentiates $3e^{xy}$ to obtain $\dots \left(y + x \frac{dy}{dx}\right)e^{xy}$ or equivalent. Do not condone missing brackets unless they are implied by subsequent work. A1: Fully correct differentiation $2 + 8y \frac{dy}{dx} = 3 \left(y + x \frac{dy}{dx}\right)e^{xy}$ or equivalent. Condone e.g. $\frac{dy}{dx} = 2 + 8y \frac{dy}{dx} = 3 \left(y + x \frac{dy}{dx}\right)e^{xy}$ where the "$\frac{dy}{dx} =$" is their intention to differentiate. M1: Substitutes $x = \frac{9}{2}$ and $y = 0$ and proceeds to find a value for $\frac{dy}{dx}$. If no substitution is seen then their value must be correct for their $\frac{dy}{dx}$. Must be a constant but may be in terms of e or may be a decimal. Note that they may attempt to make $\frac{dy}{dx}$ the subject first e.g. $\frac{dy}{dx} = \frac{3ye^{xy} - 2}{8y - 3xe^{xy}}$ You do not need to be concerned with the details of the rearrangement. It is dependent on having exactly two $\frac{dy}{dx}$ terms so if they had e.g. $\frac{dy}{dx} = 2 + 8y \frac{dy}{dx} = 3 \left(y + x \frac{dy}{dx}\right)e^{xy}$ and use all 3 $\frac{dy}{dx}$ terms, this scores M0 dM1: Uses their gradient at $\left(\frac{9}{2}, 0\right)$ with $x = \frac{9}{2}$ and $y = 0$ with the values correctly placed, in a correct straight line method for the tangent at P. Their gradient must be non-zero and must exist so do not allow methods leading to e.g. $\frac{\alpha}{0}$ where α is then used for the gradient. If they use $y = mx + c$, they must reach as far as a value for c. It is dependent on the previous method mark. A1: cso $4x - 27y - 18 = 0$ or any integer multiple of this equation. Note that incorrect differentiation can lead to the correct gradient and the correct answer so this mark must follow correct work and all previous marks must be awarded.		

Question Number	Scheme	Marks
4(a)	A and B are where $t^3 - 4t = 0 \Rightarrow t(t^2 - 4) = 0 \Rightarrow t = 2$ or -2 Substitutes $t = 2, x = 2 \times 4 - 6 \times 2 = -4$ Hence $A = (-4, 0)$ When $t = -2, x = 2 \times 4 - 6 \times -2 = 20, (y = 0)$ Hence $B = (20, 0)$ *	M1 A1 B1* (3)
(b)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 4}{4t - 6}$ Sub $t = -2$ into $\frac{dy}{dx} = \frac{3t^2 - 4}{4t - 6} \Rightarrow \text{gradient} = \left(-\frac{4}{7}\right)$ Uses their $\left(-\frac{4}{7}\right)$ and $(20, 0)$ to produce eqn of tangent $\Rightarrow 7y + 4x - 80 = 0$ *	M1A1 M1 M1 A1* (5)
(c)	Substitutes $x = 2t^2 - 6t, y = t^3 - 4t$, into $7y + 4x - 80 = 0$ $\Rightarrow 7(t^3 - 4t) + 4(2t^2 - 6t) - 80 = 0$ $\Rightarrow 7t^3 + 8t^2 - 52t - 80 = 0$ $\Rightarrow (t + 2)^2(7t - 20) = 0$ $t = \frac{20}{7} \Rightarrow x = \dots$ $x = -\frac{40}{49}$	M1 A1 dM1 A1 (4) (12 marks)

(a)

M1: Sets $t^3 - 4t = 0 \Rightarrow t = \dots$ to reach $t = 2$ or -2 A1: Substitutes $t = 2 \Rightarrow x = 2 \times 4 - 6 \times 2 = -4$ and states coordinates of $A = (-4, 0)$ Accept $t = 2 \Rightarrow A = (-4, 0)$ provided the M1 has been awardedB1*: Show that the coordinates of $B = (20, 0)$

Possible ways of this are

1) Solves $t^3 - 4t = 0 \Rightarrow t = -2$ and substitutes into $2t^2 - 6t \Rightarrow x = 20 \Rightarrow B = (20, 0)$.This is a show that and there must be some evidence for the "20", not just sight of $t = -2$ Look for $2 \times (-2)^2 - 6 \times (-2) \Rightarrow x = 20$ or $8 + 12 \Rightarrow x = 20$

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Marks
5 (a)	$y^2 = 2x^2 + 15x + 10y \Rightarrow 2y \frac{dy}{dx} = 4x + 15 + 10 \frac{dy}{dx}$	M1 A1
	$(2y - 10) \frac{dy}{dx} = 4x + 15 \Rightarrow \frac{dy}{dx} = \frac{4x + 15}{2y - 10} \text{ oe}$	M1, A1
		(4)
(b)	Deduces that $2y - 10 = 0 \Rightarrow y = 5$	B1ft
	Substitutes $y = 5$ into $y^2 = 2x^2 + 15x + 10y \Rightarrow 2x^2 + 15x + 25 = 0$ and solves for x	M1
	$(p =) -5, (q =) -\frac{5}{2}$	A1
		(3)
		(7 marks)

Notes:**(a)**

M1: Correct differentiation of one of the y terms, ie $y^2 \rightarrow 2y \frac{dy}{dx}$ or $10y \rightarrow 10 \frac{dy}{dx}$.

A1: Fully correct differentiation $2y \frac{dy}{dx} = 4x + 15 + 10 \frac{dy}{dx}$ o.e.

M1: Rearranges to make $\frac{dy}{dx}$ the subject. The differentiated expression must contain exactly two $\frac{dy}{dx}$ terms - one from each y term, not an extra $\frac{dy}{dx} = \dots$

A1: $\frac{dy}{dx} = \frac{4x + 15}{2y - 10}$ oe

(b)

B1ft: Deduces that $2y - 10 = 0 \Rightarrow y = 5$. Follow through on a denominator of form $ay + b$, $a, b \neq 0$. This deduction may arise from use of the symmetry of a hyperbola.

E.g. $x = 0 \Rightarrow y^2 - 10y = 0 \Rightarrow y = 0, 10$ so p, q when $y = 5$

M1: Substitutes their $y = 5$ into $y^2 = 2x^2 + 15x + 10y (\Rightarrow 2x^2 + 15x + 25 = 0)$ and solves for x (usual rules, if no working shown (by calculator) they must give correct values for their quadratic).

A1: $(p =) -5, (q =) -\frac{5}{2}$ Correct values, do not be concerned about the labels and accept if they give as the end points of the interval (ie accept if they give $\left(-5, -\frac{5}{2}\right)$ as their answer).



(b) Alt method 1

$$y^2 = 2x^2 + 15x + 10y \Rightarrow y^2 - 10y - (2x^2 + 15x) = 0$$

B1: Deduces that roots for x don't exist when $100 + 4 \times (2x^2 + 15x) < 0$

M1: Solves their $2x^2 + 15x + 25 < 0$

A1: $(p =) -5, (q =) -\frac{5}{2}$ Correct values, see note on main scheme.

(b) Alt method 2

B1: $y = 0 \Rightarrow 2x^2 + 15x = 0 \Rightarrow x = 0, -\frac{15}{2}$ Correct values found for x when $y = 0$

M1: Full method to use symmetry to deduce the required values of x . E.g. by symmetry, values required are one third and two thirds way between these $\Rightarrow x = \frac{1}{3} \times -\frac{15}{2}, \frac{2}{3} \times -\frac{15}{2}$

A1: $(p =) -5, (q =) -\frac{5}{2}$ Correct values, see note on main scheme.



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question	Scheme	Marks
1	$\int x \cos 3x \, dx = \frac{1}{3} x \sin 3x - \frac{1}{3} \int \sin 3x \, dx$	M1
	$= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x$	dM1 A1
	$\int_0^{\frac{\pi}{6}} x \cos 3x \, dx = \left[\frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x \right]_0^{\frac{\pi}{6}} = \frac{1}{3} \frac{\pi}{6} \sin \frac{\pi}{2} + \frac{1}{9} \cos \frac{\pi}{2} - \left(0 + \frac{1}{9} \right)$	M1
	$= \frac{\pi}{18} - \frac{1}{9}$	A1
		(5)
(5 marks)		

M1: Applies integration by parts in the correct direction to achieve $\alpha x \sin 3x \pm \beta \int \sin 3x \, dx$

So do not penalise the appearance of a '+' sign unless they state $\int vu' \, dx = uv + \int uv' \, dx$ o.e.

dM1: Completes the process by integrating the $\sin 3x$ term, achieving $\pm \alpha x \sin 3x \pm \beta \cos 3x$

A1: Correct integration which may be left unsimplified. E.g. $\frac{1}{3} x \sin 3x - \frac{1}{3} \left(-\frac{1}{3} \cos 3x \right)$

May have been done in stages (allow if $\frac{1}{3} x \sin 3x$ and $+\frac{1}{9} \cos 3x$ are seen on separate lines.)

Ignore spurious notation as long as the intention is clear.

M1: Applies both the limits to their integral which must be of the form $\pm \alpha x \sin 3x \pm \beta \cos 3x$. The substitution must be seen, or clearly implied by the correct evaluation of their $\pm \alpha x \sin 3x \pm \beta \cos 3x$

If embedded limits are not seen first, then $[\pm \alpha x \sin 3x \pm \beta \cos 3x]_0^{\frac{\pi}{6}} = (\dots) - \underline{0}$ is M0

Condone the subtraction being done either way around.

A1: Correct simplified answer. ISW after a correct answer. Allow $\frac{\pi - 2}{18}$

Alt methods may be seen. E.g. D & I method

Differentiation	Integration
x	$\cos 3x$
1	$\frac{1}{3} \sin 3x$
0	$-\frac{1}{9} \cos 3x$

Score M1 dM1 for sight of $\pm \alpha x \sin 3x \pm \beta \cos 3x$

Question	Scheme	Marks
2	$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$	
(a)	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \dots$	M1
	$= \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix}, 5\mathbf{i} + 6\mathbf{j} - 2\mathbf{k} \text{ or } (5, 6, -2)$	A1
		(2)
(b)	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix} = \dots$	M1
	$\overrightarrow{OC} \cdot \overrightarrow{BC} = 0 \Rightarrow \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} a-5 \\ -1 \\ 1 \end{pmatrix} = 0 \Rightarrow a(a-5) - 5 - 1 = 0$	dM1
	$a^2 - 5a - 6 = 0 \Rightarrow (a-6)(a+1) = 0 \Rightarrow a = \dots$	ddM1
	$a = -1, 6$	A1
		(4)
	(6 marks)	

(a)

M1: Attempts to add $\overrightarrow{OA}$ and $\overrightarrow{AB}$, which may be implied by two correct coordinates out of three if there is no direct statement as in the main scheme.

A1: Correct coordinates, in either vector or Cartesian form. (See main scheme).

Withhold the mark for hybrid versions such as $(5\mathbf{i}, 6\mathbf{j}, -2\mathbf{k})$ but isw after you have seen a correct answer

(b)

M1: Attempts $\overrightarrow{BC}$ by subtracting their $\overrightarrow{OB}$ (look at their answer to (a)) and $\overrightarrow{OC}$, subtracted either way round, which may be implied by two correct coordinates out of three following through on their (a).

dM1: Applies the scalar product with $\overrightarrow{OC}$ and their $\overrightarrow{BC}$, sets equal to 0 and forms a quadratic equation in a . Dependent upon the previous M.

An alternative method is via Pythagoras' theorem $OC^2 + BC^2 = OB^2 \Rightarrow$

$$(a^2 + 5^2 + 1^2) + ((a-5)^2 + 1^2 + 1^2) = 5^2 + 6^2 + (-2)^2 \Rightarrow 2a^2 - 10a - 12 = 0$$

ddM1: Depends on both previous Ms. Solves their 3-term quadratic equation (which must have real roots) by any valid means including a calculator (check roots if done this way).

A1: Correct values for a . If the candidate goes on to reject a value it is A0.



Question	Scheme	Marks
3	$8x^3 - 3y^2 + 2xy = 9$	
	Either $3y^2 \rightarrow ky \frac{dy}{dx}$ or $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$	M1
	Both $3y^2 \rightarrow ky \frac{dy}{dx}$ and $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$	dM1
	$8x^3 - 3y^2 + 2xy = 9 \rightarrow 24x^2 - 6y \frac{dy}{dx} + 2y + 2x \frac{dy}{dx} = 0$	A1
	$(2, 5) : 96 - 30 \frac{dy}{dx} + 10 + 4 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \dots \left(= \frac{53}{13} \right)$	M1
	$\frac{dy}{dx} = \frac{53}{13}$ or $\frac{dx}{dy} = \frac{13}{53}$ o.e.	A1
	Normal: $y - 5 = -\frac{13}{53}(x - 2)$	M1
	$13x + 53y - 291 = 0$	A1
		(7)
(7 marks)		



Question	Scheme	Marks
2(a)	$\left(\frac{1}{(1+3x)(1-x)} = \frac{A}{1+3x} + \frac{B}{1-x} \Rightarrow 1 = A(1-x) + B(1+3x) \right)$	B1
	when $x=1 \Rightarrow 1=4B \Rightarrow B=...$ or when $x=-\frac{1}{3} \Rightarrow 1=\frac{4}{3}A \Rightarrow A=...$	M1
	$\frac{3}{4(1+3x)} + \frac{1}{4(1-x)}$	A1
		(3)
(b)	$\int \cot y \, dy = \int ... \, dx \Rightarrow " \ln \sin y " = \int ... \, dx$	M1
	$... = \int \left(" \frac{3}{4(1+3x)} " + " \frac{1}{4(1-x)} " \right) dx = ... \ln(1+3x) \pm ... \ln(1-x) (+c)$	M1
	$\ln \sin y = \frac{1}{4} \ln(1+3x) - \frac{1}{4} \ln(1-x) (+c) \text{ oe}$	A1ft
	$\ln \sin \left(\frac{\pi}{2} \right) = \frac{1}{4} \ln \left(1+3 \times \frac{1}{2} \right) - \frac{1}{4} \ln \left(1-\frac{1}{2} \right) + c \Rightarrow c = ... \left(= -\frac{1}{4} \ln 5 \right)$	dM1
	$k \ln \sin y = m \ln(...) \Rightarrow \sin^k y = ...^m \text{ or } k \ln \sin y = ... \Rightarrow \sin^k y = \exp(...)$	M1
	$\sin^4 y = \frac{1+3x}{5(1-x)}$	A1
		(6)
		(9 marks)

Notes**(a)**

B1: For a correct suitable identity without fractions, such as $1 = A(1-x) + B(1+3x)$, seen or implied.

M1: Attempts to find one of the constants by either substitution or equating coefficients. May be implied by a correct value for A or B via cover up rule.

A1: $\frac{3}{4(1+3x)} + \frac{1}{4(1-x)}$ oe allow values for A and B to be stated following a correct partial fraction form, or if correct partial fractions see in (b).

(b)

M1: Attempts to separate variables to form $\cot y \frac{dy}{dx} = g(x)$ (oe for $\cot y$) and integrate $\cot y$. Accept any changed function for the attempt but must be attempting to integrate $\cot y$ (oe).

M1: Attempts to integrate their partial fractions from (a) so award for $\frac{...}{(1+3x)} \rightarrow ... \ln(1+3x)$ or $... \ln(4+12x)$

and $\frac{...}{(1-x)} \rightarrow ... \ln(1-x)$ or $... \ln(4-4x)$ oe

A1ft: Correct expression (any equivalent) (both sides). Follow through on their constants for the partial fractions. Condone the absence of the constant of integration.

dM1: Depends on second M, and must have attempted to integrate both sides. Uses the initial conditions in an equation with a constant of integration. May integrate between limits to achieve this. (Accept if a value for c cannot be reached from their equation.)

M1: Attempts to rearrange their equation by correctly using log work to reach the required form $\sin^n y = f(x)$.

Must have had $k \ln \sin y = ...$ (k may be 1). Not dependent - may be gained before finding the constant if $\ln A$ is used, and allow if the constant is missing.

A1: $\sin^4 y = \frac{1+3x}{5(1-x)}$ (oe in correct form)



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

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Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



Question Number	Scheme	Marks
2(a)	$\overrightarrow{BA} \cdot \overrightarrow{BC} = -6 \times 2 + 2 \times 5 - 3 \times 8 = (-26)$	M1
	Uses $\overrightarrow{BA} \cdot \overrightarrow{BC} = \overrightarrow{BA} \overrightarrow{BC} \cos \theta \Rightarrow -26 = \sqrt{49} \times \sqrt{93} \cos \theta \Rightarrow \theta = \dots$	dM1
	$\theta = 112.65^\circ$	A1
		(3)
(b)	Attempts to use $ \overrightarrow{BA} \overrightarrow{BC} \sin \theta$ with their θ	M1
	Area = awrt 62.3	A1
		(2)
		(5 marks)

(a)

M1: Attempts the scalar product of $\pm \overrightarrow{AB} \cdot \pm \overrightarrow{BC}$ condone slips as long as the intention is clear**Or** attempts the vector product $\pm \overrightarrow{AB} \times \pm \overrightarrow{BC}$ (see alternative 1)**Or** attempts vector AC (see alternative 2)dM1: Attempts to use $\pm \overrightarrow{AB} \cdot \overrightarrow{BC} = |\overrightarrow{AB}| |\overrightarrow{BC}| \cos \theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus.

For example $\sqrt{2^2 + 5^2 + 8^2} (= \sqrt{93})$ or $\sqrt{6^2 + 2^2 + 3^2} (= 7)$ **Note that we condone poor notation such as:** $\cos \theta = \frac{26}{7\sqrt{93}} = 67.35^\circ$ **Depends on the first mark.****Must be an attempt to find the correct angle.**A1: $\theta = \text{awrt } 112.65^\circ$ Versions finishing with $\theta = \text{awrt } 67.35^\circ$ will normally score M1 dM1 A0Angles given in radians also score A0 (NB $\theta = 1.9661\dots$ or acute $1.1754\dots$)Allow e.g. $\theta = 67.35^\circ \Rightarrow \theta = 180 - 67.35^\circ = 112.65$ and allow $\cos \theta = \frac{26}{7\sqrt{93}} \Rightarrow \theta = 112.65$ **1. Alternative using the vector product:**M1: Attempts the vector product $\pm \overrightarrow{AB} \times \pm \overrightarrow{BC} = \pm \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} \times \pm \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix} = \pm \begin{pmatrix} -31 \\ -42 \\ 34 \end{pmatrix}$ condone slips as long as the intention is

clear

dM1: Attempts to use $\pm \overrightarrow{AB} \times \overrightarrow{BC} = |\overrightarrow{AB}| |\overrightarrow{BC}| \sin \theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus on rhs and attempt at the modulus of the vector product

For example $\sqrt{2^2 + 5^2 + 8^2}$ or $\sqrt{6^2 + 2^2 + 3^2}$ and $\sqrt{31^2 + 42^2 + 34^2} (= \sqrt{3881})$ 

Note that we condone poor notation such as: $\sin \theta = \frac{\sqrt{3881}}{7\sqrt{93}} = 67.35^\circ$ Depends on the first mark.

Must be an attempt to find the correct angle.

A1: $\theta = \text{awrt } 112.65^\circ$ Versions finishing with $\theta = \text{awrt } 67.35^\circ$ will normally score M1 dM1 A0

2. Alternative using cosine rule:

M1: Attempts $\pm \overrightarrow{AC} = \pm (\overrightarrow{AB} + \overrightarrow{BC}) = \pm (8\mathbf{i} + 3\mathbf{j} + 11\mathbf{k})$ condone slips and poor notation as long as the intention is

clear e.g. allow $\begin{pmatrix} 8\mathbf{i} \\ 3\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

dM1: Attempts to use $AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cos \theta$ AND proceeds to a value for θ

Must be an attempt to find the correct angle.

A1: $\theta = \text{awrt } 112.65^\circ$

(b)

M1: Attempts to use $|\overrightarrow{AB}| |\overrightarrow{BC}| \sin \theta$ with their θ . You may see $\frac{1}{2} |\overrightarrow{AB}| |\overrightarrow{BC}| \sin \theta$ found first before it is doubled.

or attempts the magnitude of their vector product e.g. $\sqrt{3881}$

A1: Area = awrt 62.3. If this is achieved from an angle of $\theta = \text{awrt } 67.35^\circ$ full marks can be scored

Note that there are other more convoluted methods for finding the area – score M1 for a complete and correct method using their values and send to review if necessary.

Question Number	Scheme	Marks
3	States the largest odd number and an odd number that is greater E.g. odd number n and $n + 2$	M1
	Fully correct proof including - the assumption: there exists a greatest odd number "n" - a correct statement that their second odd number is greater than their assumed greatest odd number - a minimal conclusion "this is a contradiction, hence proven" You can ignore any spurious information e.g. $n > 0$, $n + 2 > 0$ etc.	A1*
		(2)
		(2 marks)

M1: For starting the proof by **stating** an odd number and a larger odd number.

Examples of an allowable start are

- **odd number** " n " with " $n + 2$ "
- **odd number** " n " with " n^2 "
- " $2k + 1$ " with " $2k + 3$ "
- " $2k + 1$ " with " $(2k + 1)^3$ "
- " $2k + 1$ " with " $2k + 1 + 2k$ "

Note that stating $n = 2k$, even when accompanied by the statement that " n " is odd is M0

A1*: A fully correct proof using contradiction

This must consist of

- 1) An assumption E.g. "(Assume that) there exists a greatest odd number n "
"Let " $2k + 1$ " be the greatest odd number"
- 2) A minimal statement showing their second number is greater than the first,
E.g. If " n " is odd and " $n + 2$ " is greater than n

If " n " is odd and $n^2 > n$

$$2k + 3 > 2k + 1$$

$$2k + 2k + 1 > 2k + 1$$

Any algebra (e.g. expansions) must be correct. So $(2k + 1)^2 = 4k^2 + 2k + 1$ would be A0

- 3) A minimal conclusion which could be

"hence there is no greatest odd number", "hence proven", or simply ✓



Question Number	Scheme	Marks
4(a)	$k = 2$ or $x > 2$	B1
	$t = \frac{1}{x-2} \Rightarrow y = \frac{1 - \frac{2}{x-2}}{3 + \frac{1}{x-2}}$	M1 A1
	$\frac{1 - \frac{2}{x-2}}{3 + \frac{1}{x-2}} = \frac{x-2-2}{\dots}$ or $\frac{\dots}{3(x-2)+1}$	A1 (M1 on EPEN)
	$y = \frac{x-4}{3x-5}$	A1
		(5)
(b)	$-2 < g < \frac{1}{3}$	M1 A1
		(2)
		(7 marks)

(a)

B1: States that $k = 2$ or else states that the domain is $x > 2$. **Must be seen in part (a).**M1: Attempts to find t in terms of x and substitutes into y .Condone poor attempts but you should expect to see $t = f(x)$ found from $x = \frac{1}{t} + 2$ substituted into

$$y = \frac{1-2t}{3+t} \text{ condoning slips.}$$

A1: A correct unsimplified equation involving just x and y

A1(M1 on EPEN): Correct numerator or denominator with fraction removed (allow unsimplified)

A1: $y = \frac{x-4}{3x-5}$ or $g(x) = \frac{x-4}{3x-5}$ (must be $y = \dots$ or $g(x) = \dots$ but allow this mark as long as the $y = \dots$ or $g(x) = \dots$ is present at some point)

Alternative 1 for part (a)M1: Assume $g(x) = \frac{ax+b}{cx+d}$ and substitute in $x = \frac{1}{t} + 2$

$$\text{A1: } g(x) = \frac{a + (b+2a)t}{c + (d+2c)t}$$

A1(M1 on EPEN): Correct numerator or denominator

A1: $y = \frac{x-4}{3x-5}$ or $g(x) = \frac{x-4}{3x-5}$ (must be $y = \dots$ or $g(x) = \dots$ but allow this mark as long as the $y = \dots$ or $g(x) = \dots$ is present at some point)

Alternative 2 for part (a)

M1: Attempts to find t in terms of y and substitutes into x .

Condone poor attempts but you should expect to see $t = f(y)$ found from $y = \frac{1-2t}{3+t}$ substituted into

$$x = \frac{1}{t} + 2 \text{ condoning slips. (NB } t = \frac{1-3y}{y+2} \Rightarrow x = \frac{y+2}{1-3y} + 2)$$

A1: A correct unsimplified equation involving just x and y

A1(M1 on EPEN): Correct numerator or denominator

A1: $y = \frac{x-4}{3x-5}$ or $g(x) = \frac{x-4}{3x-5}$ (must be $y = \dots$ or $g(x) = \dots$ but allow this mark as long as the $y = \dots$ or $g(x) = \dots$ is present at some point)

(b)

M1: For obtaining one of the 2 boundaries (just look for values) e.g. -2 or $\frac{1}{3}$ or for attempting $g(2)$ for their

g or for attempting $\frac{\text{their } a}{\text{their } c}$. Note that for this mark they must be attempting values of y (or $g(x)$).

A1: Correct range: Allow $-2 < g < \frac{1}{3}$, $-2 < g(x) < \frac{1}{3}$, $-2 < y < \frac{1}{3}$, $\left(-2, \frac{1}{3}\right)$, $g > -2$ **and** $g < \frac{1}{3}$

Question Number	Scheme	Marks
5	$u = 3 + \sqrt{2x-1} \Rightarrow x = \frac{(u-3)^2 + 1}{2} \Rightarrow \frac{dx}{du} = u-3$ or $u = 3 + \sqrt{2x-1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2 = \frac{1}{\sqrt{2x-1}} = \frac{1}{u-3}$	M1 A1
	$\int \frac{4}{3+\sqrt{2x-1}} dx = \int \frac{4}{u} \times (u-3) du$	M1
	$\int \frac{4}{u} \times (u-3) du = \int \left(4 - \frac{12}{u}\right) du$	dM1
	$\int \left(4 - \frac{12}{u}\right) du = 4u - 12 \ln u \quad \text{or} \quad k(4u - 12 \ln u)$	ddM1 A1ft
	$\int_1^{13} \frac{4}{3+\sqrt{2x-1}} dx = [4u - 12 \ln u]_4^8 = (4 \times 8 - 12 \ln 8) - (4 \times 4 - 12 \ln 4)$ or $\int_1^{13} \frac{4}{3+\sqrt{2x-1}} dx = \left[4(3+\sqrt{2x-1}) - 12 \ln(3+\sqrt{2x-1})\right]_1^{13} = (4 \times 8 - 12 \ln 8) - (4 \times 4 - 12 \ln 4)$	M1
	$= 16 - 12 \ln 2$	A1
		(8 marks)

M1: Differentiates to get $\frac{du}{dx}$ in terms of x and then obtains $\frac{dx}{du}$ in terms of u

Need to see $\frac{du}{dx} = k(2x-1)^{-\frac{1}{2}} \rightarrow \frac{du}{dx} = \frac{1}{au+b}$ or $\frac{dx}{du} = au+b$

or

Attempts to change the subject of $u = 3 + \sqrt{2x-1}$ and differentiates to get $\frac{dx}{du}$ in terms of u

Need to see $x = \frac{(u \pm 3)^2 \pm 1}{2} \rightarrow \frac{dx}{du} = au+b$

A1: $\frac{dx}{du} = u-3$ or e.g. $\frac{du}{dx} = \frac{1}{u-3}$, $du = \frac{dx}{u-3}$, $dx = (u-3)du$

M1: Attempts to write the integral completely in terms of u .

Need to see $\int \frac{\dots}{u} \times \text{their } \frac{dx}{du} du$ with or without the “ du ” but **not** e.g. $\int \frac{\dots}{u} \times \frac{1}{\frac{dx}{du}} du$

dM1: Divides to reach an integral of the form $\int \left(A + B \times \frac{1}{u}\right) du$. **Depends on both previous M's**

ddM1: Integrates to a form $Au + B \ln u$. **Depends on the previous M.**

An alternative for the previous 2 marks is to use integration by parts:

$$\text{E.g. } \int \frac{4}{u} \times (u-3) \, du = 4(u-3) \ln u - \int 4 \ln u \, du = 4u \ln u - 12 \ln u - 4u \ln u + 4u = 4u - 12 \ln u$$

Score dM1 for $\int \frac{k}{u} \times (Au+B) \, du = k(Au+B) \ln u - \int k \ln u \, du$ and dM1 for integrating to a form $Au + B \ln u$.

A1ft: $4u - 12 \ln u$ or $k(4u - 12 \ln u)$ following through on $\frac{dx}{du} = k(u-3)$ only.

M1: Substitutes 8 and 4 into their $4u - 12 \ln u$ and subtracts **or** substitutes 13 and 1 into their $4u - 12 \ln u$ with $u = 3 + \sqrt{2x-1}$ and subtracts. This mark depends on there having been an attempt to integrate, however poor.

A1: $16 - 12 \ln 2$

Question Number	Scheme	Marks
6(a)	$4y^2 + 3x = 6ye^{-2x}$	
	$4y^2 + 3x \rightarrow 8y \frac{dy}{dx} + 3$	B1
	$6ye^{-2x} \rightarrow -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$	M1 A1
	$8y \frac{dy}{dx} + 3 = -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ oe	M1 A1
		(5)
(b)	Sets $x = 0$ in $4y^2 + 3x = 6ye^{-2x} \Rightarrow y = \frac{3}{2}$ oe	B1
	Substitutes $\left(0, \frac{3}{2}\right)$ in their $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y} = \left(\frac{7}{-2}\right)$	M1
	$m_N = -1 \div \frac{7}{-2} \Rightarrow y = \frac{2}{7}x + \frac{3}{2}$	dM1
	$y = \frac{2}{7}x + \frac{3}{2}$ oe e.g. $y = \frac{6}{21}x + \frac{3}{2}$	A1
		(4)
		(9 marks)

(a)

B1: Differentiates $4y^2 + 3x$ to obtain $8y \frac{dy}{dx} + 3$. Allow unsimplified forms such as $4 \times 2y \frac{dy}{dx} + 3$

M1: Uses the product rule on $6ye^{-2x}$ to obtain an expression of the form $Aye^{-2x} + Be^{-2x} \frac{dy}{dx}$

A1: Differentiates $6ye^{-2x}$ to obtain $-12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$

M1: Collects two terms in $\frac{dy}{dx}$ (one from attempting to differentiate $4y^2$ and one from attempting to differentiate $6ye^{-2x}$) and proceeds to make $\frac{dy}{dx}$ the subject.

A1: $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ or equivalent e.g. $\frac{dy}{dx} = \frac{2e^{-2x} \times 6y + 3}{6e^{-2x} - 8y}$ or $\frac{dy}{dx} = \frac{12y + 3e^{2x}}{6 - 8ye^{2x}}$

You can ignore any spurious " $\frac{dy}{dx} =$ " at the start and allow y' for $\frac{dy}{dx}$.

(b)

B1: Uses $x = 0$ to obtain $y = \frac{3}{2}$ oe e.g. $\frac{6}{4}$ (ignore any reference to $y = 0$)

M1: Substitutes $x = 0$ and their y at $x = 0$ which has come from substituting $x = 0$ into the original equation into their $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ to find a numerical value. Working is normally shown here but you may need to check for evidence. Use of $x = 0$ and $y = 0$ is M0.



Question Number	Scheme	Marks
3 (a)	Attempts to find $\frac{1/3 \times 20^2 \times 24}{160} = 20$ seconds	M1 A1 (2)
(b)	Attempts $\frac{dV}{dh} = h^2 + \frac{8}{3}h$ Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 160 = \left(h^2 + \frac{8}{3}h\right) \times \frac{dh}{dt}$ Substitutes $h = 5 \Rightarrow 160 = \left(5^2 + \frac{40}{3}\right) \times \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{96}{23} \text{ (4.2) cm s}^{-1}$	M1 M1 A1 dM1 A1 (5) (7 marks)

(a)

M1: Attempts to find the volume, using the given formula, at $h = 20$ and dividing by 160. Implied by 20
 Condone slips. For example they may have incorrectly multiplied out/calculated the expression for V

A1: 20 seconds. This requires the correct units as well

(b)

M1: For an attempt to differentiate V

Scored for an attempt to multiply out and then differentiate term by term to achieve $\frac{dV}{dh} = \alpha h^2 + \beta h$

or via the product rule to achieve $V = \frac{1}{3}h^2(h+4) \Rightarrow \frac{dV}{dh} = (h+4) \times ph + qh^2$

M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ (or equivalent) with $\frac{dV}{dt} = 160$ and their $\frac{dV}{dh}$.

A1: Correct expression involving $\frac{dh}{dt}$ and h E.g. $160 = \left(h^2 + \frac{8}{3}h\right) \times \frac{dh}{dt}$

dM1: Dependent upon the previous M only. It is for substituting $h = 5$ and proceeding to a value for $\frac{dh}{dt} = \dots$

A1: Either $\frac{96}{23}$ or awrt 4.2 cm s⁻¹ There is no requirement for the units here



Question	Scheme	Marks
4.	$\int_1^4 \frac{10}{5x+2x\sqrt{x}} dx$ $u = \sqrt{x} \Rightarrow x = u^2 \Rightarrow \frac{dx}{du} = 2u$ $\int \frac{10}{5x+2x\sqrt{x}} dx = \int \frac{10}{5u^2+2u^3} 2u du$ $= \int \frac{20}{5u+2u^2} du = \int \frac{4}{u} - \frac{8}{5+2u} du$ $= 4 \ln u - 4 \ln(5+2u)$ $\text{Limits} = [4 \ln u - 4 \ln(5+2u)]_1^2 = 4 \ln 2 - 4 \ln 9 + 4 \ln 7 = \dots$ $= 4 \ln \left(\frac{14}{9} \right)$	B1 M1 A1 dM1 A1 ddM1 M1 A1 (8 marks)

B1: For $\frac{dx}{du} = 2u$ or $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or equivalent

M1: Attempts to write all terms in the integral in terms of u (inc the dx)

Condone slips for this mark BUT the dx CANNOT just be replaced by du .

Look for either $x \rightarrow u^2$ or $x\sqrt{x} \rightarrow u^3$ with either $dx \rightarrow f(u)du$ or $dx \rightarrow k \times du, k \neq 1$

A1: Correct integrand in terms of just u which may be unsimplified. E.g $\int \frac{10}{5u^2+2u^3} 2u du$

dM1: Attempts to use PF and writes their integrand in terms of its component fractions.

Look for an integral of the form $\int \frac{P}{Qu+Ru^2} du \rightarrow \int \frac{p}{u} + \frac{q}{Q+Ru} du$

A1: Correct PF $\int \frac{4}{u} - \frac{8}{5+2u} (du)$

ddM1: For $\dots \ln u \pm \dots \ln(5+2u)$ but follow through on their PF's which must be of a similar form

M1: Uses the limits 1 and 2 within their attempted integral. The integration may be incorrect.

Alternatively substitutes $u = \sqrt{x}$ and uses the limits 1 and 4 within their attempted integral

A1: $4 \ln \left(\frac{14}{9} \right)$

Question Number	Scheme	Marks
5 (a)	$2y \frac{dy}{dx} = e^{-2x} \frac{dy}{dx} - 2ye^{-2x} - 3$ $(e^{-2x} - 2y) \frac{dy}{dx} = 2ye^{-2x} + 3 \Rightarrow \frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} *$	B1 M1 A1 A1* (4)
(b)	Puts $x = 0$ into the equation of the curve $\Rightarrow y = y^2 \Rightarrow y = 1$ Attempts tangent at $(0,0)$ or $(0,1)$ $y = 3x$ or $y = -5x + 1$ Solves $y = 3x$ with $y = -5x + 1 \Rightarrow R = \left(\frac{1}{8}, \frac{3}{8}\right)$	B1 M1 A1 dM1 A1 (5) (9 marks)

(a) Originally scored M1 B1 A1 A1, now B1 M1 A1 A1

B1: Correct differentiation using the chain rule $y^2 \rightarrow 2y \frac{dy}{dx}$.

M1: Attempts to apply the product rule of differentiation on ye^{-2x} to give $e^{-2x} \frac{dy}{dx} \pm \dots ye^{-2x}$

A1: Correct differentiation $2y \frac{dy}{dx} = e^{-2x} \frac{dy}{dx} - 2ye^{-2x} - 3$

Allow $2y \, dy = e^{-2x} \, dy - 2ye^{-2x} \, dx - 3dx$

A1*: Proceeds to the given answer via an intermediate line equivalent to $(e^{-2x} - 2y) \frac{dy}{dx} = 2ye^{-2x} + 3$ with correct bracketing.

(b)

B1: Deduces or implies that $x = 0, y = 1$ at P . May state or use coordinates of $P(0,1)$

M1: Scored for an attempt to find the equation of the tangent at O **or** the equation of the tangent at P

E.g. Substitutes $(0,0)$ into $\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} = \beta$ and states $y = \beta x$

Alternatively substitutes $(0,1)$ into $\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} \rightarrow "k"$ and attempts $y - 1 = "k"x$ or equivalent

A1: Achieves a correct equation for **either** tangent. Look for **either** $y = 3x$ **or** $y = -5x + 1$ o.e.

dM1: Correct attempt **at both** tangents with an attempt to solve simultaneously.

For the attempt to solve accept $y = 3x$, $y = -5x + 1 \Rightarrow x = \dots, y = \dots$

A1: Correct coordinates for $R = \left(\frac{1}{8}, \frac{3}{8}\right)$ oe

Question Number	Scheme	Marks
9	$\begin{pmatrix} 2-\lambda \\ 8+2\lambda \\ 10+3\lambda \end{pmatrix} = \begin{pmatrix} -4+5\mu \\ -1+4\mu \\ 2+8\mu \end{pmatrix}$ Attempts to solve any two of the three equations Either (1) and (2) $\begin{cases} 2-\lambda = -4+5\mu \\ 8+2\lambda = -1+4\mu \end{cases} \Rightarrow \lambda = -\frac{3}{2}, \mu = \frac{3}{2}$ (1) and (3) $\begin{cases} 2-\lambda = -4+5\mu \\ 10+3\lambda = 2+8\mu \end{cases} \Rightarrow \lambda = \frac{8}{23}, \mu = \frac{26}{23}$ (2) and (3) $\begin{cases} 8+2\lambda = -1+4\mu \\ 10+3\lambda = 2+8\mu \end{cases} \Rightarrow \lambda = -10, \mu = -\frac{11}{4}$ Substitutes their values of λ and μ into both sides of the "third" equation E.g. $\lambda = -\frac{3}{2}$ into $10+3\lambda = \frac{11}{2}$ and $\mu = \frac{3}{2}$ into $2+8\mu = 14$ Concludes that lines don't intersect with correct calculations and minimal reason Additionally states that $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ is not parallel to $\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$ with a minimal reason So lines are skew CSO *	M1, A1 dM1 A1 A1* (5) (5 marks)

Notes:

Main method seen

M1: Attempts to solve two of the three equations.

Accept as an attempt, writing down two of the three equations (condoning slips) followed by values for

both λ and μ

A1: Solves two of the three equations to find correct values for both λ and μ , Allow equivalent fractions

dM1: Either: Substitutes their values of λ and μ into both sides of the third equation....or into the equations of both lines to find both coordinates

A1: Having achieved correct values for λ and μ , the values for the third equation are found to enable a comparison to be made. E.g. solving equations (1) and (2) and using equation (3) stating

$10+3 \times -\frac{3}{2} \neq 2+8 \times \frac{3}{2}$ is sufficient. If the values are found they must be correct.

Important: Additionally, to score this mark, a minimal statement must be made that states that the lines do not intersect /cross. Condone statements such as $l_1 \neq l_2$

Stating that the lines are skew at this point is not sufficient to score this mark

In the alternative stating that "as the values are not the same, the lines cannot intersect" is sufficient.

A1*: CSO . Hence all previous marks must have been scored.

In addition to not intersecting there must be a statement, with a minimal reason, that the lines are not parallel and hence skew. Accept statements like, not intersecting, not parallel (with reason), hence proven.



Reasons could be $\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix} \neq k \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ o.e such as $5 = -5 \times -1$ but $4 = 2 \times 2$ so they are not parallel.

Accept an argument based around the scalar product of the direction vectors. If parallel $\cos \theta = 1$

A reason for the lines not being parallel cannot be $\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix} \neq \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$

Note: Other methods are possible and it is important that you look at their complete attempt at proving that they don't intersect.

Alternative 1

For example it is possible to solve equations (1) and (2) to find just λ
 then solve equations (1) and (3) to find just λ
 and then conclude that "as the two values are not the same, the lines don't intersect"

M1 dM1 marks are scored together. Both aspects have to be attempted

Attempts to solve two of the three equations to find λ (or μ)

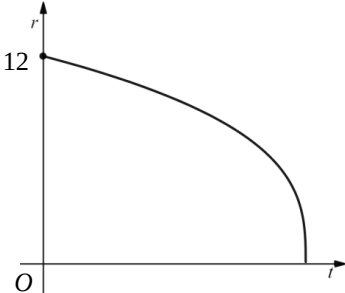
Attempts to solve a different pair of equations to find λ (or μ)

A1: Correct values for λ (or μ).

A1: conclude that "as the two values are not the same, the lines don't intersect"

If you see something that you feel deserves credit AND that you cannot mark, then please send to review



Question Number	Scheme	Marks
10 (a)	States or uses $\frac{dr}{dt} \propto \pm \frac{1}{r^2}$ or $\frac{dr}{dt} = \pm \frac{k}{r^2}$ $\frac{dr}{dt} = \pm \frac{k}{r^2} \Rightarrow \int r^2 dr = \int \pm k dt$ $\frac{1}{3} r^3 = \pm kt + c$ Substitutes $t = 0, r = 12 \Rightarrow c = (576)$ AND $t = 15, r = 6$ and their $c = (576) \Rightarrow k = (\pm 33.6)$ $\frac{1}{3} r^3 = -33.6t + 576 \text{ or } r^3 = -100.8t + 1728 \text{ o.e.}$	B1 M1 A1 M1 A1 (5)
(b)	Sets $r = 0 \Rightarrow -100.8t + 1728 = 0$ $\frac{120}{7} \text{ minutes or awrt } 17.1 \text{ minutes}$	M1 A1 (2)
(c)		B1 (1) (8 marks)

(a)

B1: States or uses $\frac{dr}{dt} \propto \pm \frac{1}{r^2}$, $\frac{dr}{dt} = \pm \frac{k}{r^2}$ or $\frac{dr}{dt} = \pm \frac{1}{kr^2}$ Accept $k \leftrightarrow \lambda$ or other constant. Condone

$$\frac{dr}{dt} = \pm \frac{1}{r^2}$$

M1: Integrates to achieve an expression of the correct form $\dots r^3 = \dots t (+c)$ o.e

There is no requirement for $+c$

A1: Correct integration with two different unknown constants. E.g. look for $\frac{1}{3} r^3 = \pm kt + c$ o.e

So if for example $\frac{dr}{dt} = -\frac{1}{r^2} \Rightarrow \frac{1}{3} r^3 = -t + c$ the score would be B1, M1, A0 then dM0, A0

M1: Uses both boundary conditions to form an equation involving r and t . For this to be awarded there must be two different unknown constants that are initially correctly placed.

It is dependent upon having achieved $\dots r^n = \pm k "t" + "c"$ which may have been achieved from an

incorrect assumption. E.g. $\frac{dr}{dt} = \pm kr^2$



A1: Correct equation. E.g. $\frac{1}{3}r^3 = -33.6t + 576$ or $r^3 = -100.8t + 1728$ o.e. such as

$$t = -\frac{5}{504}r^3 + \frac{120}{7}$$

(b)

M1: Sets $r = 0 \Rightarrow -100.8t + 1728 = 0 \Rightarrow t = \dots$. Follow through on their equation. Condone if this produces a negative value for t . Alt sets $V = 0$ to find $t = \dots$

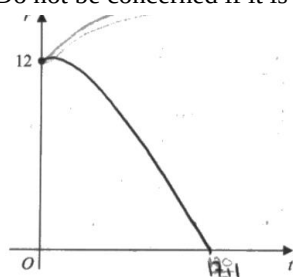
A1: Awrt 17.1 minutes. Must include the units. Also allow 17 minutes 8 seconds or 17 minutes 9 seconds

(c)

B1: For the correct shaped curve only in quadrant 1 starting at (0,12), ignoring value of t .

As $\frac{dr}{dt} = -\frac{k}{r^2}$ the gradient should appear to get increasingly steeper.

If there are two curves given on the axes, both need to be correct for this mark to be awarded
Do not be concerned if it is not infinite at the t axis.



The curve on the left would be at the limit of what we would allow
as the gradient on the left hand side does appear to increase.
We condone its appearance as it approaches the t -axis.
It does not get less steep

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Note that it is possible to use $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$ with $\frac{dr}{dt} = \pm \frac{k}{r^2}$ to set up and solve

$$\frac{dV}{dt} = \pm 4\pi k \text{ or } \frac{dV}{dt} = \pm \beta \quad \text{In this case the equation becomes } V = 2304\pi - \left(\frac{672\pi}{5}\right)t$$

(a)

B1: States or uses either $\frac{dV}{dt} = \pm \beta$ or its exact value which is $\frac{dV}{dt} = -\frac{2016\pi}{15}$ o.e such as

$$\frac{dV}{dt} = -\frac{672\pi}{5}$$

M1: For $\frac{dV}{dt} = \pm \beta \Rightarrow V = \pm \beta t + c$ and attempts to find the values of β and c using the given conditions.

It is dependent upon knowing that $V = \alpha r^3$ and using this to find V at $r = 6$ and 12 with $t = 15$ and $t = 0$

$$A1: V = 2304\pi - \left(\frac{672\pi}{5}\right)t$$

M1: Substitutes $V = \alpha r^3$ to form an equation linking r^3 with t .
It is dependent on the previous M1 in this method

$$A1: \text{Achieves } \frac{4}{3}\pi r^3 = 2304\pi - \left(\frac{672\pi}{5}\right)t$$

(b)

M1: Sets $V = 0$ o.e and finds t



Question Number	Scheme	Marks
11 (a)	$(x+y)^3 + 10y^2 = 108x$ Differentiates $10y^2$ to $20y \frac{dy}{dx}$ Differentiates $(x+y)^3$ to $3(x+y)^2 \left(1 + \frac{dy}{dx}\right)$ $3(x+y)^2 \left(1 + \frac{dy}{dx}\right) + 20y \frac{dy}{dx} = 108$ $3(x+y)^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108 - 3(x+y)^2$ $\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2} \quad *$	B1 M1 A1 dM1 A1* (5)
(b)	Deduces that P and Q are where $108 - 3(x+y)^2 = 0 \Rightarrow (x+y)^2 = 36$ Attempts to substitute $(x+y) = \pm 6$ into $(x+y)^3 + 10y^2 = 108x$ to form an equation in y (or x) Solves $216 + 10y^2 = 108(6-y)$ and finds the negative root Awt 13900 metres	M1 dM1 ddM1 A1 (4) (9 marks)

(a) Allow use of y' for $\frac{dy}{dx}$. This is a proof. Look carefully for evidence of candidates working backwards.

B1: Differentiates $10y^2$ to $20y \frac{dy}{dx}$

M1: Attempts to differentiate $(x+y)^3$ Via the chain rule look for $k(x+y)^2 \left(1 + \frac{dy}{dx}\right)$ See **

Alternatively you will see attempts where $(x+y)^3$ is multiplied out. Terms may not be collected but

look for 3 of $x^3 \rightarrow \dots x^2$, $\dots x^2 y \rightarrow 2xy + x^2 \frac{dy}{dx}$, $\dots xy^2 \rightarrow y^2 + 2xy \frac{dy}{dx}$ and $y^3 \rightarrow 3y^2 \frac{dy}{dx}$

differentiated to the correct form.

A1: Correct differentiation $3(x+y)^2 \left(1 + \frac{dy}{dx}\right) + 20y \frac{dy}{dx} = 108$

or $3x^2 + 3\left(2xy + x^2 \frac{dy}{dx}\right) + 3\left(y^2 + 2xy \frac{dy}{dx}\right) + 3y^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108$

dM1: Collects the terms in $\frac{dy}{dx}$.

Look for either $\frac{dy}{dx}$ being factorised out of the relevant terms

or the terms in $\frac{dy}{dx}$ being set on one side of the equation and the other terms on the other side

It is dependent upon **BOTH** the B and M marks.

A1*: Proceeds to the given answer.

In the approach $3x^2 + 3\left(2xy + x^2 \frac{dy}{dx}\right) + 3\left(y^2 + 2xy \frac{dy}{dx}\right) + 3y^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108$ it is acceptable

to go from $(3x^2 + 6xy + 3y^2 + 20y) \frac{dy}{dx} = 108 - 3x^2 - 6xy - 3y^2$ straight to the given answer

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Watch for candidates who use the given answer and work backwards **.

$$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2} \Rightarrow 3(x+y)^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108 - 3(x+y)^2$$

Therefore candidates who start with

$$3(x+y)^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108 - 3(x+y)^2 \quad \text{or indeed} \quad 3(x+y)^2 \frac{dy}{dx} + 3(x+y)^2 + 20y \frac{dy}{dx} = 108$$

without any other working or evidence can only score B1 M0 A0 M0 A0

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(b)

M1: Deduces that P and Q are where $108 - 3(x+y)^2 = 0 \Rightarrow (x+y)^2 = 36$

This is implied if there is a statement that $(x+y) = \pm 6$

dM1: Attempts to solve $(x+y) = \pm 6$ and $(x+y)^3 + 10y^2 = 108x$ simultaneously to form a quadratic equation in y or x .

ddM1: Solves $216 + 10y^2 = 108(6-y)$ and finds the negative root. Allow if both roots are found

Note that $(x+y) = 6$ must have been used.

If an equation was set up in x , then look for an attempt to find the larger root of

$$216 + 10(6-x)^2 = 108x \quad \text{which should then be used to find a negative } y \text{ value}$$

A1: Awrt 13900 metres or awrt 13.9 km with the correct units. Condone answers like -13.9 km

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
2	Assume that C_1 and C_2 do intersect so that $x^4 + 10x^2 + 8 = 2x^2 - 7$ (and has real solutions).	B1
	$x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 5)(x^2 + 3) = 0 \Rightarrow x^2 = \dots$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow x^2 = \frac{-8 \pm \sqrt{8^2 - 4 \times 15}}{2}$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 4)^2 - 1 = 0 \Rightarrow x^2 = \dots$	M1
	$x^2 = -5, -3$	A1
	e.g. The two values of $x^2 < 0$ (so $x^4 + 8x^2 + 15 = 0$ has no real roots) which is not possible so C_1 and C_2 do not intersect	A1
		(4)
		Total 4

Note that B0M1A1A1 is possible

B1: Sets up the contradiction by

- making a statement assuming that the two curves intersect e.g. let/assume/suppose C_1 and C_2 / the curves intersect/meet/cross
- equating the curves i.e. $x^4 + 10x^2 + 8 = 2x^2 - 7$ o.e. (the equation must be correct)

Condone no reference to the equation having real solutions.

M1: Attempts to solve a three-term quadratic equation in x^2 by any valid (**non-calculator**) method. Usual rules apply. **If the roots are just stated then M0A0A0 follows.**

A1: $x^2 = -5, -3$. Allow e.g. $m^2 = -5, -3$

Do not condone e.g. $y = -5, -3$ without reverting back to $x^2 = \dots$ or making valid deductions about e.g. x^2 being negative

A1: **This mark can only be scored provided the previous M1A1 has been scored.**

Acceptable conclusion which

- refers to x^2 being negative which is not possible or the equation has no real roots or $x^2 = -3, -5$ but $x^2 \dots 0$
- refers to C_1 and C_2 not intersecting
- Correct use of inequalities throughout e.g. $x^2 \dots 0$ (not just $x^2 \geq 0$)

Alternative considering the sign of each term / the expression

B1: As above in main scheme notes

M1: Either one of:

- considers the sign of one side of an equation or inequality which includes either an x^2 or x^4 term. e.g. $x^4 + 8x^2 \dots 0$ e.g. $x^4 + 8x^2 + 15 > 0$ Allow to compare with 0 e.g. $x^2 + 3 \neq 0$
- considers the sign of x^2 or x^4 e.g. $x^2 \dots 0$
- considers separately at least one part of the product of two terms. e.g. $x^2(x^2 + 8) \geq 0$ or $(x^2 + 8) > 0$
- considers the range of values using their completed square form e.g. $(x^2 + 4)^2 \dots 16$ or $x^2 + 4 \dots 4$

Do not be concerned by $>$ or $\dots$

A1: Correct three-term quadratic in x^2 and correct deductions that e.g. $x^4 \dots 0$ and $x^2 \dots 0$

depending on their expression (condone > 0)

This mark cannot be scored if the expression they use to make their deductions is incorrect.

e.g. $x^2(x^2 + 8)$ is positive (or zero) since x^2 and $(x^2 + 8)$ are **both** greater than (or equal to) zero is A1

e.g. $x^4 + 8x^2 \dots 0$ since x^2 and x^4 are both $\dots 0$ is A1

e.g. $x^2 + 4 \dots 4$ since $x^2 \dots 0$ is A1

e.g. $x^4 + 8x^2 \dots 0$ is A0

A1: **This mark can only be scored provided the previous M1A1 has been scored.**

Acceptable conclusion which refers to

- the equation $x^4 + 8x^2 + 15 = 0$ gives no solutions o.e. e.g. $x^4 + 8x^2 + 15$ cannot equal zero, e.g. $x^4 + 8x^2$ cannot equal -15 e.g. $x^2 + 4$ cannot equal -1
- C_1 and C_2 not intersecting
- Correct use of inequalities throughout e.g. $x^2 \dots 0$ (not just $x^2 \geq 0$).

Do not accept e.g. $-3, 0$

Question Number	Scheme	Marks
3(a)	$x = 3\sin^3 \theta \Rightarrow \frac{dx}{d\theta} = 9\sin^2 \theta \cos \theta$ o.e.	B1
	$\frac{dy}{dx} = \frac{-2\sin 2\theta}{9\sin^2 \theta \cos \theta}$	M1
	$\frac{dy}{dx} = \frac{-4\sin \theta \cos \theta}{9\sin^2 \theta \cos \theta} = -\frac{4}{9} \operatorname{cosec} \theta$	A1
		(3)
(b)	$\theta = \frac{\pi}{6} \Rightarrow \frac{dy}{dx} = -\frac{8}{9}$, $x = 3\left(\frac{1}{2}\right)^3 = \dots$, $y = 1 + \frac{1}{2} = \dots$	M1
	$y - \frac{3}{2} = -\frac{8}{9}\left(x - \frac{3}{8}\right)$	dM1
	$16x + 18y - 33 = 0$	A1
		(3)
(c)	e.g. $x = 3\sin^3 \theta \Rightarrow \sin \theta = \sqrt[3]{\frac{x}{3}} \Rightarrow y = 1 + \cos 2\theta = 1 + 1 - 2\sin^2 \theta = 2 - 2\left(\frac{x}{3}\right)^{\frac{2}{3}}$	M1
	$y = 2 - 2\left(\frac{x}{3}\right)^{\frac{2}{3}} \Rightarrow 2\left(\frac{x}{3}\right)^{\frac{2}{3}} = 2 - y \Rightarrow 8x^2 = 9(2 - y)^3$ *	A1*
	$q = 3$	B1
		(3)
Alt(c)	e.g. $8x^2 = 72\sin^6 \theta$, $9(2 - y)^3 = 9(2 - \cos 2\theta - 1)^3 = 9(1 - (1 - 2\sin^2 \theta))^3$	M1
	$72\sin^6 \theta = 9 \times 8\sin^6 \theta \Rightarrow 8x^2 = 9(2 - y)^3$ *	A1*
	$q = 3$	B1
		Total 9

(a)

B1: Correct expression for $\frac{dx}{d\theta}$ which may be part of their expression for $\frac{dy}{dx}$. Condone poor labelling/notation provided the intention is clear. Must be in terms of θ

May use identities before differentiating using the product rule e.g. $\sin \theta \left(\frac{1 - \cos 2\theta}{2} \right)$

M1: Correct method for $\frac{dy}{dx}$. They must be attempting $\frac{dy}{d\theta} \times \frac{d\theta}{dx}$ o.e. condoning copying slips and condoning poor differentiation, labelling /notation provided the intention is clear.

A1: Achieves $\frac{dy}{dx} = -\frac{4}{9} \operatorname{cosec} \theta$ with no errors but condone invisible brackets and poor notation

being recovered. Allow equivalent fractions to $-\frac{4}{9}$. Note $\frac{dy}{dx} =$ may be seen in an earlier line of their solution which is acceptable.



Alt (a) Implicit differentiation – note that some candidates may find the cartesian equation and differentiate implicitly or explicitly.

B1: A correctly differentiated cartesian equation. (Do not be concerned with where the cartesian equation has come from for this mark)

e.g. $16x = 9 \times 3 \times (-1) \times (2 - y)^2 \times \frac{dy}{dx}$

M1: Substitutes in for x and y into their differentiated equation and rearranges to achieve an expression for $\frac{dy}{dx}$ in terms of θ . Condone poor differentiation.

A1: Achieves $\frac{dy}{dx} = -\frac{4}{9} \operatorname{cosec} \theta$ with no errors but condone invisible brackets and poor notation

being recovered. Allow equivalent fractions to $-\frac{4}{9}$. They must have shown correctly how they found the cartesian equation, i.e. they cannot just use the given result in (c).

(b)

M1: Attempts to find $\frac{dy}{dx}$, x and y when $\theta = \frac{\pi}{6}$. Must achieve values for all three which may be

implied by further work. Condone slips but if values are only stated then their $\frac{dy}{dx}$ must be correct for their k (look for $2 \times$ their k) and one of x or y must be correct.

Note that they may use their $\frac{dy}{dx}$ in terms of x and y $\left(= -\frac{16x}{27(2-y)^2} \right)$ by either finding the differentiated equation in (a) or by converting in (b).

dM1: Complete method for the tangent using their values. It is dependent on the previous method mark. Condone one sign error when substituting in the coordinates. If they use $y = mx + c$ they must proceed as far as $c = \dots$

A1: $16x + 18y - 33 = 0$ or any integer multiple of this equation. Must have $= 0$.

(c)

M1: A complete method to eliminate θ . Any identities used must be correct when substituted in. Do not allow the parametric equations to be expressed as $\theta =$ and equated. May also be awarded if seen in (a).

A1*: Correct proof with no errors including brackets although condone missing trailing brackets if it does not affect the expression. Must be seen in (c).

B1: Correct value of q . Do not accept $x = 3$ Can be scored for the domain $-3, x, 3$ which must be in terms of x

Alt(c)

M1: A complete method to express lhs and rhs both in terms of $\sin \theta$ only or both in terms $\cos 2\theta$ only. Any identities used must be correct when substituted in. They cannot proceed from $9(2-y)^3 = 72 \sin^6 \theta$ without an intermediate stage but condone $8x^2 = 72 \sin^6 \theta$

A1*: Fully correct work showing lhs = rhs and gives a conclusion. There should be no errors including brackets but condone missing trailing brackets if it does not affect the expression.

B1: As above in the notes for (c)



Question Number	Scheme	Marks
4(a)	$x=1, y=2 \Rightarrow 3x^2 + 2y^2 - 4xy + 8^x - 11 = 3 + 8 - 8 + 8 - 11 = 0^*$	B1*
		(1)
(b)	$8^x \rightarrow 8^x \ln 8$	B1
	$4xy \rightarrow 4x \frac{dy}{dx} + 4y$ or $y^2 \rightarrow 2y \frac{dy}{dx}$	M1
	$6x + 4y \frac{dy}{dx} - 4y - 4x \frac{dy}{dx} + 8^x \ln 8 = 0$ o.e.	A1
	$\frac{dy}{dx}(4y - 4x) = 4y - 6x - 8^x \ln 8 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{4y - 6x - 8^x \ln 8}{4y - 4x}$ o.e. e.g. $\frac{dy}{dx} = \frac{6x + 8^x \ln 8 - 4y}{4x - 4y}$ or e.g. $\frac{dy}{dx} = \frac{4y - 6x - e^{x \ln 8} \ln 8}{4y - 4x}$	A1
		(5)
(c)	$(1, 2) \rightarrow \frac{dy}{dx} = \frac{8 - 6 - 8 \ln 8}{8 - 4} \Rightarrow y - 2 = \frac{4}{8 \ln 8 - 2}(x - 1)$	M1
	$y = 0 \Rightarrow 0 - 2 = \frac{4}{8 \ln 8 - 2}(x - 1) \Rightarrow x = \dots$	M1
	$x = 2 - 12 \ln 2$	A1
		(3)
		Total 9

(a)

B1*: Substitutes $x = 1$ and $y = 2$ and obtains 0 with no errors.Minimum required without evaluating each term e.g. $3 \times 1 + 2 \times 2^2 - 4 \times 1 \times 2 + 8^1 - 11 = 0$. We just need a correct equation which evaluates as 0.Alternatively, substitutes in $x = 1$, forms a quadratic in y i.e. $2y^2 - 4y = 0$ o.e. and solves to find y via any method: $3 + 2y^2 - 4y + 8 - 11 = 0 \Rightarrow 2y^2 - 4y = 0 \Rightarrow y = 2$ (ignore the other solution 0)They may at this stage substitute in $y = 2$ to get 0.

In any of the approaches a conclusion is not required.

(b)

B1: Correct differentiation of 8^x . Award for sight of $\ln 8 \times 8^x$ or $e^{x \ln 8} \times \ln 8$. Do not be concerned with the use of modulus signs around $\ln 8$ M1: For $y^2 \rightarrow ky \frac{dy}{dx}$ or $4xy \rightarrow Ax \frac{dy}{dx} + By$ A1: Fully correct differentiation. Condone the omission of $= 0$ or may be implied by further work. Accept $6x + 4y \frac{dy}{dx} - 4y - 4x \frac{dy}{dx} + e^{x \ln 8} \ln 8 = 0$ Ignore if they start their differentiated line with e.g. $\frac{dy}{dx} =$ 

M1: Attempts to make $\frac{dy}{dx}$ the subject with **2 terms** in $\frac{dy}{dx}$ coming from the correct places. i.e. they must have two terms (one of each) of the form $\alpha x \frac{dy}{dx}$ and $\beta y \frac{dy}{dx}$ where α, β are non-zero constants.

A1: Any correct expression for $\frac{dy}{dx}$. Allow $y' = \dots$ o.e. isw after a correct answer is seen
 Condone poor dividing lines which do not go quite far enough provided an incorrect method was not seen. E.g. $\frac{4y-6x-8^x \ln 8}{4y-4x}$

(c) Note that an incorrect derivative in part (b) can score maximum 110 in part (c)

M1: Attempts to use $x = 1$ and $y = 2$ in their $\frac{dy}{dx}$ to find the gradient at P (condone if this is a decimal) and attempts to form the equation of the normal at P using the negative reciprocal gradient. Condone one sign slip when substituting in the coordinates into the straight line equation.

They may set $y = 0$ at this stage such that $-2 = \frac{4}{8 \ln 8 - 2}(x-1)$ which is acceptable.

If they use $y = mx + c$ they must proceed as far as $c = \dots$

M1: States $y = 0$ or substitutes $y = 0$ into their straight-line equation using a changed gradient from their $\frac{dy}{dx}$ and rearranges to find x . Do not be concerned by the mechanics of their rearrangement.

A1: Correct expression in the required form

General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Notes	Marks
1(a)	$\frac{2}{\sqrt{9-2x}} = \frac{2}{3\sqrt{1-\frac{2}{9}x}}$ or $\frac{2}{\sqrt{9-2x}} = 2(9-2x)^{-\frac{1}{2}} = 2 \times \frac{1}{3} \left(1 - \frac{2}{9}x\right)^{-\frac{1}{2}}$	Obtains $\sqrt{9-2x} = 3\sqrt{1-\dots}$	B1
	$\left(1 - \frac{2}{9}x\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{2}{9}x\right) + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)}{2!}\left(-\frac{2}{9}x\right)^2 + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{2}{9}x\right)^3 + \dots$ M1: Attempts the binomial expansion of $(1+kx)^n$ to get the third and/or fourth term with an acceptable structure. The correct binomial coefficient must be combined with the correct power of x and the correct power of 2. A1: Correct simplified or unsimplified expansion (NB simplified is $= 1 + \frac{1}{9}x + \frac{1}{54}x^2 + \frac{5}{1458}x^3 + \dots$)		M1 A1
	$\frac{2}{\sqrt{9-2x}} = \frac{2}{3} + \frac{2}{27}x + \frac{1}{81}x^2 + \frac{5}{2187}x^3 + \dots$	2 correct simplified terms	A1
		All correct	A1
			(5)
(b)	$x=1 \Rightarrow \frac{2}{\sqrt{9-2}} = \frac{2}{3} + \frac{2}{27} + \frac{1}{81} + \frac{5}{2187} + \dots$ $\Rightarrow \sqrt{7} \approx 2 \div \frac{1652}{2187} \text{ " or } 2 \times \frac{2187}{1652} \text{ "}$ Substitutes $x = 1$ and divides into 2 or equivalent		M1
	$= 2.6477$	Correct approximation	A1
			(2)
Alternative for (b):			
	$x=1 \Rightarrow \frac{2}{\sqrt{9-2}} = \frac{2}{3} + \frac{2}{27} + \frac{1}{81} + \frac{5}{2187} + \dots$ $\frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7} \Rightarrow \sqrt{7} \approx \frac{7}{2} \times \frac{1652}{2187}$ Substitutes $x = 1$ and multiplies by $\frac{7}{2}$		M1
	$= 2.6438$	Correct approximation	A1
			Total 7

Question Number	Scheme	Notes	Marks
2(a)	$\frac{x}{y} = t$	Cao	B1
			(1)
(b)	$y = \frac{\left(\frac{x}{y}\right)^3}{2\left(\frac{x}{y}\right)+1}$ or $x = \frac{\left(\frac{x}{y}\right)^4}{2\left(\frac{x}{y}\right)+1}$	Uses the y coordinate to obtain y in terms of x and y or uses the x coordinate to obtain x in terms of y and x	M1
	$y = \frac{x^3}{2xy^2 + y^3} \Rightarrow y(2xy^2 + y^3) = x^3$ or $x = \frac{x^4}{2xy^3 + y^4} \Rightarrow x(2xy^3 + y^4) = x^4$	Uses correct algebra to eliminate the fractions	M1
	$x^3 - 2xy^3 - y^4 = 0^*$	Cso	A1*
			(3)
			Total 4

Question Number	Scheme	Notes	Marks
3(a)	$3y^2 - 11x^2 + 11xy = 20y - 36x + 28$ $\Rightarrow 6y \frac{dy}{dx} - 22x + 11x \frac{dy}{dx} + 11y = 20 \frac{dy}{dx} - 36$ M1: $y^2 \rightarrow Ay \frac{dy}{dx}$ M1: $11xy \rightarrow px \frac{dy}{dx} + qy$ A1: All correct		M1M1A1
	$(6y + 11x - 20) \frac{dy}{dx} = 22x - 11y - 36 \Rightarrow \frac{dy}{dx} = \dots$ Collects terms in $\frac{dy}{dx}$ (must be 3 and from the appropriate terms) and makes $\frac{dy}{dx}$ the subject		M1
	$\frac{dy}{dx} = \frac{22x - 11y - 36}{6y + 11x - 20}$	Correct expression or correct equivalent	A1
			(5)
(b)	$x = 4 \Rightarrow 3y^2 - 176 + 44y = 20y - 144 + 28$	Substitutes $x = 4$ into C to obtain a 3TQ in y	M1
	$3y^2 + 24y - 60 = 0 \Rightarrow y = \dots$	Solves for y	M1
	$y = -10 \text{ (,2)}$	Correct value	A1
	$(4, -10) \rightarrow \frac{dy}{dx} = \frac{88 + 110 - 36}{-60 + 44 - 20}$	Substitutes $x = 4$ and their negative y into their $\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = -\frac{9}{2}$	Correct value	A1
			(5)
			Total 10

Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} = 8(1-2x)^{-\frac{3}{2}}$	B1
	$(1-2x)^{-\frac{3}{2}} = 1 + \left(-\frac{3}{2}\right)(-2x) + \frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(-2x)^2 + \frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)}{3!}(-2x)^3 + \dots$	M1 A1
	$\left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} = 8 + 24x + 60x^2 + 140x^3 + \dots$	A1, A1
		(5)
(b)	$n = 2$	B1
		(1)
(c)	$\left(\frac{1}{4} - \frac{1}{2}x\right)^2 = \frac{1}{16} - \frac{1}{4}x + \frac{1}{4}x^2$	B1
	$\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \left(\frac{1}{16} - \frac{1}{4}x + \frac{1}{4}x^2\right)(8 + 24x + 60x^2 + 140x^3 + \dots)$ $= 8 \times \frac{1}{16} + 24x \times \frac{1}{16} + 60x^2 \times \frac{1}{16} - 8 \times \frac{1}{4}x - 24x \times \frac{1}{4}x + 8 \times \frac{1}{4}x^2$	M1
	$= \frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$	A1
		(3)
		Total 9

(a)

B1: Takes out the **correct and simplified** common factor to obtain $8(1 \pm \dots)^{-\frac{3}{2}}$ Implied by an expansion $8 + \dots$

M1: Attempts the binomial expansion of $(1 \pm kx)^{-\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.

The correct binomial coefficient must be combined with the correct power of x .

Look for $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(\pm kx)^2$ or $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)}{3!}(\pm kx)^3$ with or without the brackets on the kx

Even allow with $k = 1$

A1: Correct simplified or unsimplified expansion for $(1-2x)^{-\frac{3}{2}}$. (NB simplified is $1 + 3x + \frac{15}{2}x^2 + \frac{35}{2}x^3 + \dots$)

A1: Two correct and simplified terms of $8 + 24x + 60x^2 + 140x^3$

A1: All correct and simplified $8 + 24x + 60x^2 + 140x^3$ Ignore extra terms of x^4 and above

(b)

B1: Correct value, $n = 2$. Do NOT allow incomplete answers such as $\frac{4}{2}$

(c) Hence

B1: Correct expansion of $\left(\frac{1}{4} - \frac{1}{2}x\right)^2$, simplified or unsimplified

M1: Attempts correct strategy to find expansion.

Terms do not need to be collected. There may be other terms as well, for example terms in x^3

Look for an attempt to find 4 or more terms from the following (condoning slips)

$$(A + Bx + Cx^2)(E + Fx + Gx^2 + Hx^3 + \dots) = AE + AFx + AGx^2 + BEx + BFx^2 + CEx^2 + \dots$$

A1: Correct simplified expansion. Ignore extra terms of x^3 and above



(c) **Otherwise:** Applies binomial expansion to $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}}$

B1: Correct simplified expression $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \frac{1}{2}(1 - 2x)^{\frac{1}{2}}$

M1: Correct structure for 3rd term: $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-2x)^2$ but allow $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm kx)^2$ with or without brackets. Even allow with $k=1$

A1: $\frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$ Ignore extra terms of x^3 and above

Direct expansion in (a)

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{3}{2}} = \left(\frac{1}{4}\right)^{\frac{3}{2}} + \left(-\frac{3}{2}\right)\left(\frac{1}{4}\right)^{\frac{5}{2}}\left(-\frac{1}{2}x\right) + \frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{\frac{7}{2}}\left(-\frac{1}{2}x\right)^2 + \frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right) \times \left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{\frac{9}{2}}\left(-\frac{1}{2}x\right)^3$$

B1: Obtains an expansion with a constant term of 8

M1: Attempts the binomial expansion of $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.

Look for $\frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{\frac{7}{2}}\left(\pm\frac{1}{2}x\right)^2$ or $\frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right) \times \left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{\frac{9}{2}}\left(\pm\frac{1}{2}x\right)^3$

with or without the brackets on the $\left(\pm\frac{1}{2}x\right)$

A1: Correct simplified or unsimplified expansion .

Expression at the top of the page is acceptable

A1: Two correct and simplified terms of $8 + 24x + 60x^2 + 140x^3$

A1: All correct and simplified $8 + 24x + 60x^2 + 140x^3$

Direct expansion in (c)

B1: First two terms which may be unsimplified $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} + \frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}\left(-\frac{1}{2}x\right)^1 + \dots$

M1: Correct form for the third term $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}\left(-\frac{1}{2}x\right)^2$

A1: Correct simplified expansion $\frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$

Question Number	Scheme	Marks
1(a)	$(2-5x)^{-2} = \frac{1}{4} \left(1 - \frac{5x}{2}\right)^{-2}$ or e.g. $\frac{8}{4 \left(1 - \frac{5x}{2}\right)^2}$	B1
	$= 8 \times \frac{1}{4} \left(1 + (-2) \times \left(-\frac{5x}{2}\right) + \frac{(-2) \times (-3)}{2!} \times \left(-\frac{5x}{2}\right)^2 + \frac{(-2) \times (-3) \times (-4)}{3!} \times \left(-\frac{5x}{2}\right)^3 \dots \right)$	M1A1
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
		(4)
Alt (a) by direct expansion	$8 \times (2-5x)^{-2}$ $= 8 \times \left(2^{-2} + (-2)2^{-3}(-5x)^1 + \frac{-2 \times -3}{2!} 2^{-4}(-5x)^2 + \frac{-2 \times -3 \times -4}{3!} 2^{-5}(-5x)^3 \right)$	B1, <u>M1A1</u>
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
(b)	$ x < \frac{2}{5}$ o.e.	B1
		(1)
		(5 marks)

(a)

B1: For taking out a factor of 2^{-2} or $\frac{1}{4}$ **from** $(2-5x)^{-2}$ to obtain e.g. $\frac{1}{4}(1 \pm \dots)^{-2}$, $2^{-2}(1 \pm \dots)^{-2}$.

May be implied by a constant term of 2 or by e.g. $2(1 \pm \dots)^{-2}$

The “8” is likely to be present but it is not required for this mark.

M1: For the form of the binomial expansion $(1+ax)^{-2}$

Requires the correct structure for either term three or term four. Allow a slip on the sign.

So allow for either $\frac{(-2) \times (-3)}{2} (\pm ax)^2$ or $\frac{(-2) \times (-3) \times (-4)}{3!} (\pm ax)^3$ where $a \neq 1$, could be -5 but must

be the “a” in their “2” $(1 \pm ax)^{-2}$.

Condone missing brackets around the “ax” for this mark.

A1: Any unsimplified or simplified but correct form of the binomial expansion for $\left(1 - \frac{5x}{2}\right)^{-2}$

Ignore the factor preceding the bracket for this mark and ignore any extra terms if found.

Score for $1 + (-2) \times \left(-\frac{5x}{2}\right) + \frac{(-2) \times (-3)}{2!} \times \left(-\frac{5x}{2}\right)^2 + \frac{(-2) \times (-3) \times (-4)}{3!} \times \left(-\frac{5x}{2}\right)^3$ o.e.

Brackets must be present unless they are implied by later work. Allow $\left(\frac{5x}{2}\right)^2$ for $\left(-\frac{5x}{2}\right)^2$.

The simplified form is $1 + 5x + \frac{75}{4}x^2 + \frac{125}{2}x^3 + \dots$ Allow as a list of terms.

A1: cao $2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$ This must be simplified and allow as a list of terms.

Allow equivalents for $\frac{75}{2}$ e.g. 37.5 and ignore any extra terms.

Do **not** isw and mark the final answer. E.g. Fully correct work leading to $2+10x+\frac{75}{2}x^2+125x^3\ldots$

followed by $=4+20x+75x^2+250x^3\ldots$ loses the final mark.

Alternative to (a) by direct expansion

B1: This is awarded for $(2-5x)^{-2} = 2^{-2} + \ldots$ which may be implied by a final answer of $2 + \ldots$

M1: For the form of the expansion of $(2-5x)^{-2}$

Requires the correct structure for either term three or term four. Allow a slip on the sign.

So allow for either $\frac{(-2) \times (-3)}{2} (2)^{-4} (\pm 5x)^2$ or $\frac{(-2) \times (-3) \times (-4)}{3!} (2)^{-5} (\pm 5x)^3$

Condone missing brackets around the $5x$.

A1: Any unsimplified or simplified but correct form of the binomial expansion for $(2-5x)^{-2}$

Ignore the factor preceding the bracket for this mark and ignore any extra terms if found.

Score for $2^{-2} + (-2)2^{-3}(-5x)^1 + \frac{-2 \times -3}{2!} 2^{-4} (-5x)^2 + \frac{-2 \times -3 \times -4}{3!} 2^{-5} (-5x)^3$ o.e.

Brackets must be present unless they are implied by later work.

The simplified form is $\frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2 + \frac{125}{8}x^3 + \ldots$ Allow as a list of terms.

A1: cao $2+10x+\frac{75}{2}x^2+125x^3\ldots$ This must be simplified and allow as a list of terms.

Allow equivalents for $\frac{75}{2}$ e.g. 37.5 and ignore any extra terms.

Do **not** isw and mark the final answer. E.g. Fully correct work leading to $2+10x+\frac{75}{2}x^2+125x^3\ldots$

followed by $=4+20x+75x^2+250x^3\ldots$ loses the final mark.

Note: Correct or partially correct answers with no working in (a) should be sent to review.

(b)

B1: $|x| < \frac{2}{5}$ oe e.g. $-\frac{2}{5} < x < \frac{2}{5}$, $x > -\frac{2}{5}$ and $x < \frac{2}{5}$, $\left(-\frac{2}{5}, \frac{2}{5}\right)$

But **not** $-\frac{2}{5} < |x| < \frac{2}{5}$, $x < \frac{2}{5}$, $\left|\frac{5x}{2}\right| < 1$, $\left|-\frac{5x}{2}\right| < 1$, $-1 < \frac{5x}{2} < 1$



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice



Question	Scheme	Marks
1	$(1-4x)^{-3} = 1 \pm 3 \times 4x \pm \frac{-3 \times -4}{2} (\dots x)^2 \pm \frac{-3 \times -4 \times -5}{6} (\dots x)^3 + \dots$	M1
	$(1-4x)^{-3} = 1 + 3 \times 4x + \frac{-3 \times -4}{2} (-4x)^2 + \frac{-3 \times -4 \times -5}{6} (-4x)^3 + \dots$	A1
	$= 1 + 12x + 96x^2 + 640x^3 + \dots$	A1A1
		(4)
(4 marks)		
Notes:		
M1: Attempts the binomial expansion of $(1-4x)^{-3}$ with correct attempts at the (unsimplified, and may be in terms of factorials) binomial coefficients for at least two of the x, x^2, x^3 terms. The “-4” may be missing or have incorrect sign and allow for missing brackets. (May be scored if the x^3 term is omitted.) A1: For a correct unsimplified expansion. (May be in terms of factorials.) A1: Any two terms correct and simplified (of the four, so includes the 1). The M must have been scored. A1: Fully correct and all terms simplified. Ignore any higher order terms. ISW after a correct simplified. Note M1A0A1A0 is possible e.g. if the x and x^3 terms have the wrong signs. Note: listing terms can score maximum of M1A1A1A0.		

Question	Scheme	Marks
2(a)	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$	
	$3x+4 = A(2x+1)^2 + B(x-2)(2x+1) + C(x-2)$ E.g. $x=2 \rightarrow 10 = A \times 25 \Rightarrow A = \dots$ or $x = -\frac{1}{2} \rightarrow \frac{5}{2} = C \times -\frac{5}{2} \Rightarrow C = \dots$ or $x^2: 0 = 4A + 2B, x: 3 = 4A - 3B + C, x^0: 4 = A - 2B - 2C \rightarrow A = \dots$	M1
	One of $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
	$\Rightarrow A = \dots, B = \dots$ and $C = \dots$	M1
	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{2}{5(x-2)} - \frac{4}{5(2x+1)} - \frac{1}{(2x+1)^2}$ or $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
		(4)
(b)	$\int \frac{p}{x-2} dx = \alpha \ln(x-2)$ or $\int \frac{q}{2x+1} dx = \beta \ln(2x+1)$	M1
	Alt: $\int \frac{8x+9}{(2x+1)^2} dx = \int \frac{8x+4}{(2x+1)^2} + \frac{5}{(2x+1)^2} dx = \gamma \ln(2x+1)^2 + \dots$	
	$\int \frac{k}{(2x+1)^2} dx = \frac{K}{2x+1}$	M1
	$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2} dx = \left[\frac{2}{5} \ln(x-2) - \frac{4}{5} \times \frac{1}{2} \ln(2x+1) + \frac{1}{4x+2} \right]_7^{12}$	A1ft
	$= \left(\frac{2}{5} \ln(10) - \frac{2}{5} \ln(25) + \frac{1}{50} \right) - \left(\frac{2}{5} \ln(5) - \frac{2}{5} \ln(15) + \frac{1}{30} \right) = \dots$	DM1
	$= \frac{2}{5} \ln \frac{10}{25} + \frac{1}{50} - \frac{2}{5} \ln \frac{5}{15} - \frac{1}{30} = \frac{2}{5} \ln \frac{10 \times 15}{25 \times 5} + \dots$	M1
	$\frac{2}{5} \ln \frac{6}{5} - \frac{1}{75}$ (oe)	A1
		(6)
(10 marks)		

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$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0 : (x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

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Question Number	Scheme	Marks
5(a)	e.g. $\frac{r}{h} = \frac{12}{30} \Rightarrow r = \frac{12}{30}h \Rightarrow V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{2}{5}h\right)^2 h$	M1
	$V = \frac{4\pi h^3}{75} *$	A1*
		(2)
(b)	$V = \frac{4\pi h^3}{75} \Rightarrow \frac{dV}{dh} = \frac{12\pi h^2}{75} \left(= \frac{4\pi h^2}{25} \right)$	B1
	$V = 1.5 \times 60 \times 2\pi$ $180\pi = \frac{4\pi h^3}{75} \Rightarrow h^3 = 3375 \Rightarrow h = \dots (15)$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt} = \frac{75}{12\pi \times 15^2} \times 2\pi = \dots$	M1
	$\frac{1}{18} \text{ (cm s}^{-1}\text{)}$	A1
		(4)
		Total 6

(a)

M1: Uses the 12 and the 30 to find the ratio between h and r and substitutes into the volume formula to find V in terms of h . Condone poor bracketing / invisible brackets for this mark.

May just state $r = \frac{2}{5}h$

This mark cannot be for substituting in e.g. $r = \frac{2}{5}$ instead of $r = \frac{2}{5}h$

A1*: Correct proof with no errors including brackets. There must be at least one intermediate stage of working between establishing $r = \frac{12}{30}h$ and proceeding to the given answer.

(b) **Note that substituting $h = 1.5$ or $h = 90$ will score M0M0 in (b)**

B1: Correct expression for $\frac{dV}{dh}$ in any form

M1: Attempts their volume (" $2\pi \times t$ ") after 1.5 **minutes** and uses this with the given result from (a) to find h . They may find h by equating their expression for the volume to the given result from (a) before substituting $t = 90$ and rearranging to find h . Do not be concerned by the mechanics of the rearrangement. Condone slips.

M1: Correct chain rule attempt using their h to find $\frac{dh}{dt}$. It is dependent on having used $t = 90$ (or condone $t = 1.5$) when finding h .

A1: Correct value. Allow awrt 0.0556 (cm s⁻¹) Units not required for this mark.



General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question	Scheme	Marks
2(a)	$(1+4x^3)^{\frac{1}{3}} = 1 + \frac{1}{3}(\dots x^{\dots}) + \dots$	M1
	$(1+4x^3)^{\frac{1}{3}} = \dots + \frac{\frac{1}{3}(-\frac{2}{3})}{2}(4x^3)^2 + \dots$	M1
	$= 1 + \frac{4}{3}x^3 + \dots \text{ or } = \dots - \frac{16}{9}x^6$	A1
	$(1+4x^3)^{\frac{1}{3}} = 1 + \frac{4}{3}x^3 - \frac{16}{9}x^6 + \dots$	A1
		(4)
(b)	$1+4x^3 = 1+4 \times \left(\frac{1}{3}\right)^3 = \frac{31}{\dots} \text{ or } 1 + \frac{4}{3}\left(\frac{1}{3}\right)^3 - \frac{16}{9}\left(\frac{1}{3}\right)^6$	M1
	$\sqrt[3]{31} \approx 3 \times \left(1 + \frac{4}{3}\left(\frac{1}{3}\right)^3 - \frac{16}{9}\left(\frac{1}{3}\right)^6\right)$	dM1
	$= \frac{6869}{2187}$	A1
		(3)
(7 marks)		

Notes:**(a)**

M1: For $1 \pm \frac{1}{3}(4x^{\dots})$. Allow if the 4 and/or the power of x is incorrect/missing for this mark.

M1: For the correct structure for the third term. Look for the correct binomial coefficient combined with the correct power of $4x^3$ e.g. $\frac{\frac{1}{3}(\frac{1}{3}-1)}{2}(4x^3)^2$ but condone missing brackets e.g. $\frac{\frac{1}{3}(\frac{1}{3}-1)}{2}4x^{3^2}$

A1: For $1 + \frac{4}{3}x^3 + \dots$ or for $\dots - \frac{16}{9}x^6$.

Allow mixed fractions e.g. $1\frac{1}{3}$ for $\frac{4}{3}$ but do not allow '+-' for '-'.

A1: For $1 + \frac{4}{3}x^3 - \frac{16}{9}x^6 + \dots$. **Ignore any extra terms.**

(b)

M1: Attempts $1 + 4 \times \left(\frac{1}{3}\right)^3$ and reaches $\frac{31}{k}$ (this will usually be seen as e.g. $\sqrt[3]{\frac{31}{27}}$) or substitutes

$x = \frac{1}{3}$ into their expansion from part (a).

dM1: Attempts $3 \times$ (their expansion from part (a) with $x = \frac{1}{3}$ substituted)

Allow if they have included more terms in their expansion for both M's. You need to be convinced that they have attempted this calculation and not just found the cube root of 31 on their calculator e.g. 3.1414...

A1: Cao. Correct fraction.



Question	Scheme	Marks
3(a)	$x = 3 + 2 \sin t \quad y = \frac{6}{7 + \cos 2t} \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$	
	$y = \frac{6}{7 + 1 - 2 \sin^2 t}$	M1
	$\sin t = \frac{x-3}{2} \Rightarrow y = \frac{6}{8 - 2\left(\frac{x-3}{2}\right)^2}$	M1A1
	$\Rightarrow y = \frac{12}{16 - (x-3)^2} = \frac{12}{(4-x+3)(4+x-3)} = \frac{12}{(7-x)(1+x)}^*$	M1A1*
	$\left(t = -\frac{\pi}{2} \Rightarrow x = 1, t = \frac{\pi}{2} \Rightarrow x = 5 \text{ so } \right) p = 1 \text{ and } q = 5$	B1
		(6)
(a) Way 2	$y = \frac{6}{7 + 1 - 2 \sin^2 t}$	M1
	$y = \frac{12}{16 - 4 \sin^2 t} = \frac{12}{(4 + 2 \sin t)(4 - 2 \sin t)}$	M1A1
	$= \frac{12}{(4 + x - 3)(4 - (x - 3))} = \frac{12}{(7 - x)(1 + x)}^*$	M1A1*
	$\left(t = -\frac{\pi}{2} \Rightarrow x = 1, t = \frac{\pi}{2} \Rightarrow x = 5 \text{ so } \right) p = 1 \text{ and } q = 5$	B1
(a) Way 3	$y = \frac{12}{(7-x)(1+x)} = \frac{12}{(4-2 \sin t)(4+2 \sin t)}$	M1
	$= \frac{12}{16 - 4 \sin^2 t}$	M1A1
	$= \frac{12}{16 - 4\left(\frac{1 - \cos 2t}{2}\right)}$	M1
	$= \frac{12}{16 - 2 + 2 \cos 2t} = \frac{12}{14 + 2 \cos 2t} = \frac{6}{7 + \cos 2t} = y^*$	A1*
	$\left(t = -\frac{\pi}{2} \Rightarrow x = 1, t = \frac{\pi}{2} \Rightarrow x = 5 \text{ so } \right) p = 1 \text{ and } q = 5$	B1

(a) Way 4	$y = \frac{6}{7 + \cos 2t} \Rightarrow \cos 2t = \frac{6}{y} - 7 \Rightarrow 1 - 2\sin^2 t = \frac{6}{y} - 7$	M1
	$\sin t = \frac{x-3}{2} \Rightarrow \frac{6}{y} - 7 = 1 - 2\left(\frac{x-3}{2}\right)^2$	M1A1
	$\Rightarrow \frac{12}{y} = -x^2 + 6x + 7 \Rightarrow y = \frac{12}{-x^2 + 6x + 7} = \frac{12}{(7-x)(1+x)}^*$	M1A1*
	$\left(t = -\frac{\pi}{2} \Rightarrow x = 1, t = \frac{\pi}{2} \Rightarrow x = 5 \text{ so}\right) p = 1 \text{ and } q = 5$	B1
(b)	$\frac{12}{(7-x)(1+x)} = \frac{A}{7-x} + \frac{B}{1+x} \Rightarrow 12 = A(x+1) + B(7-x) \Rightarrow A = \dots, B = \dots$	M1
	So $(y =) \frac{3}{2(x+1)} - \frac{3}{2(x-7)}$ oe	A1A1
		(3)

(9 marks)**Notes:****(a)**

M1: Applies $\cos 2t = \pm 1 \pm 2\sin^2 t$ to get y in terms of $\sin t$. This may be implied if they obtain $\cos 2t$ in terms of x with no explicit sight of $\sin t$.

M1: Rearranges equation for x and substitutes into equation for y .

A1: Correct equation in terms of x and y only.

M1: Simplifies and factorises denominator (usual rules). May have the $\frac{1}{2}$ left in the denominator, e.g.

$$y = \frac{6}{8 - \frac{1}{2}(x-3)^2} = \frac{6}{8 - \frac{1}{2}x^2 + 3x - \frac{9}{2}} = \frac{6}{-\frac{1}{2}x^2 + 3x + \frac{7}{2}} = \frac{6}{-\left(\frac{1}{2}x + \frac{1}{2}\right)(x-7)}$$

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality.

Can score anywhere in their response but do not allow $1 \leq t \leq 5$

(a) Way 2

M1: Applies $\cos 2t = \pm 1 \pm 2\sin^2 t$ to get y in terms of $\sin t$

M1: Uses the difference of 2 squares in the denominator

A1: Correct equation with the square completed

M1: Substitutes for $\sin t$ in terms of x

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(a) Way 3

M1: Substitutes for x in terms of t into the given expression for y

M1: Expands denominator.

A1: Correct expression.

M1: Applies $\cos 2t = \pm 1 \pm 2\sin^2 t$ to get $\sin^2 t$ in terms of $\cos 2t$

A1*: Correct result reached with no errors seen and minimal conclusion.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(a) Way 4 (similar to Way 1)

M1: Rearranges to get $\cos 2t$ in terms of y and uses $\cos 2t = \pm 1 \pm 2\sin^2 t$



M1: Rearranges equation for x and substitutes to obtain an equation in x and y .

A1: Correct equation in terms of x and y only.

M1: Simplifies and factorises denominator (usual rules).

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(b)

M1: Correct overall method to apply partial fractions to **the given fraction or equivalent correct fraction** and find at least one constant.

If no method is shown, one correct value can imply the method.

Score M0 for $\frac{12}{(7-x)(1+x)} = \frac{A}{7-x} + \frac{B}{1+x} \Rightarrow 12 = A(7-x) + B(x+1) \Rightarrow A = \dots, B = \dots$

A1: At least one correct fraction in the required form e.g.

$$\frac{3}{2(1+x)}, -\frac{3}{2(x-7)} \text{ or e.g. } \frac{\frac{3}{2}}{1+x}, -\frac{\frac{6}{4}}{x-7} \text{ but not e.g. } \frac{3}{2x+2}, \frac{-3}{2x-14}$$

unless a correct form is seen beforehand.

Or a correct pair of values e.g. $a = \frac{3}{2}, b = 1$ or $c = \frac{3}{2}, d = 1$ or $a = -\frac{3}{2}, b = -7$ or $c = -\frac{3}{2}, d = -7$

A1: Correct expression for y in the given form $\frac{3}{2(x+1)} - \frac{3}{2(x-7)}$ (there is no need for the “ $y =$ ”)

Accept as in scheme or any equivalents in the correct form, e.g. with 1.5 or equivalent as the numerators.

Or all values correct e.g. $\left(a = \frac{3}{2}, b = 1 \text{ and } c = -\frac{3}{2}, d = -7\right)$ or $\left(a = -\frac{3}{2}, b = -7 \text{ and } c = \frac{3}{2}, d = 1\right)$

Note that in (b) some candidates may change the form of y first:

$$\frac{12}{(7-x)(1+x)} = \frac{-12}{(x-7)(1+x)}$$

In such cases the marks can be applied as in the scheme provided the expression is correct.



Question	Scheme	Marks
4(a)	Area of one face $= \frac{1}{2}x \times x \times \sin 60^\circ = \frac{1}{2}x^2 \frac{\sqrt{3}}{2}$ oe	M1
	So surface area of icosahedron is $20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ *	A1*
		(2)
(b)	$\frac{dA}{dx} = 10\sqrt{3}x$ and $\frac{dV}{dx} = \frac{15}{12}(3+\sqrt{5})x^2$	M1
	$\frac{dV}{dA} = \frac{dV}{dx} \times \frac{dx}{dA} = \frac{dV}{dx} \div \frac{dA}{dx} = \dots$ (oe)	M1
	$= \frac{15(3+\sqrt{5})x^2}{12 \times 10\sqrt{3}x} = \frac{(3+\sqrt{5})x}{8\sqrt{3}}$ *	A1*
		(3)
(c)	$\frac{dA}{dt} = 0.025$	B1
	$\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt} = \frac{(3+\sqrt{5})x}{8\sqrt{3}} \times 0.025 = \dots$	M1
	When $x = 2$, $\frac{dV}{dt} = \frac{2(3+\sqrt{5})}{8\sqrt{3}} \times 0.025 = \dots$	
	awrt 0.019 (cm ³ s ⁻¹)	A1
		(3)
(8 marks)		

Notes:**(a)**

M1: Any correct method to establish a surd form for the area of one face. Must evaluate trigonometric terms.

Alternatives include e.g. $\frac{1}{2} \times x \times \sqrt{x^2 - \left(\frac{1}{2}x\right)^2} = \frac{1}{2}x\sqrt{\frac{3}{4}}x^2$

A1*: Correct result achieved after showing side area of one face.

Examples:

$$\text{Area of one face} = \frac{1}{2}x \times x \times \sin 60^\circ = \frac{1}{4}x^2\sqrt{3} \Rightarrow SA = 20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2 \text{ scores M1 A1}$$

$$SA = 20 \times \left(\frac{1}{2} \times x \times x \sin 60^\circ \right) = 5\sqrt{3}x^2 \text{ scores M1 A0 for lack of working}$$

(b)

M1: Attempts derivatives of both A and V wrt x . E.g. $\left(\frac{dA}{dx} = \right)\alpha x$ and $\left(\frac{dV}{dx} = \right)\beta x^2$.

Note that $\frac{5}{12}(3+\sqrt{5})x^3$ is sometimes expanded first as e.g. $\frac{5}{4}x^3 + \frac{5\sqrt{5}}{12}x^3$ and allow this mark

if either term is differentiated to βx^2

M1: Applies the correct chain rule with their derivatives. (Condone poor notation – see below)



General Principles for Pure Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**Attempt to use correct formula (with values for a , b and c).**3. Completing the square**Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.**Answers without working**The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question	Scheme	Marks
1(a)	$A = \frac{1}{9}$	B1
		(1)
(b)	" 3^{-2} " $(1 + (-2)(\frac{kx}{3}) + \frac{(-2)(-3)}{2}(\frac{kx}{3})^2 + \dots)$ or $x: 3^{-2}(-2)(\frac{k}{3})(-\frac{2k}{27})$ and $x^2: 3^{-2}\frac{(-2)(-3)}{2}(\frac{k}{3})^2 = \frac{k^2}{27}$	B1
	$\frac{(-2)(-3)}{2}(\frac{k}{3})^2 = 3 \times (-2)(\frac{k}{3}) \Rightarrow \dots k^2 = \dots k$	M1
	$k^2 + 6k = 0$ *	A1*
		(3)
(c)(i)	$k = -6$	B1
(ii)	$3^{-2} \frac{(-2)(-3)(-4)}{3!} \left(\frac{-6}{3}\right)^3 = \frac{32}{9}$	M1A1
		(3)
		(7 marks)

Notes

Mark parts (a) and (b) as a whole.

(a)

B1: $A = \frac{1}{9}$

(b)

B1: Correct unsimplified coefficients for x and x^2 either in an expansion or separate for $(3 + kx)^{-2}$ or for

$\left(1 + \frac{k}{3}\right)^{-2}$ (accept the 3^{-2} missing or incorrect). May be implied. Accept $B = -\frac{2k}{3}$ and $C = \frac{k^2}{3}$ if they forget

the multiple outside. B0 if brackets on $\left(\frac{k}{3}\right)^2$ missing unless implied by recovery.

M1: Sets their coefficient of x^2 equal to 3 times their coefficient of x to produce a two term quadratic equation in terms of k .

A1*: Achieves given answer from a correct equation, but condone if B and C both missed the 3^{-2} . May be scored if A was incorrect.

(c)(i)

B1: $k = -6$ only. The $k = 0$ solution must be rejected.

(ii)

M1: Substitutes their non-zero value for k into a correct expression for the coefficient of x^3 . Must include the 3^{-2}

A1: $\frac{32}{9}$ oe



General Principles for Pure Mathematics Marking*(NB specific mark schemes may sometimes override these general principles)***Method mark for solving 3 term quadratic:**

1. Factorisation

 $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$ **Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1(a)	$(8-3x)^{-\frac{1}{3}} = \frac{1}{2} \left(1 - \frac{3}{8}x\right)^{-\frac{1}{3}}$	B1
	$\left(1 - \frac{3}{8}x\right)^{-\frac{1}{3}} = 1 + \left(-\frac{1}{3}\right)\left(-\frac{3}{8}x\right) + \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}\left(-\frac{3}{8}x\right)^2 + \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}\left(-\frac{3}{8}x\right)^3 + \dots$	M1
	$(8-3x)^{-\frac{1}{3}} = \frac{1}{2} + \frac{1}{16}x + \frac{1}{64}x^2 + \frac{7}{1536}x^3 + \dots$	A1 A1
		(4)
(b)	$\frac{1}{2} + \frac{1}{16}\left(\frac{2}{3}\right) + \frac{1}{64}\left(\frac{2}{3}\right)^2 + \frac{7}{1536}\left(\frac{2}{3}\right)^3 + \dots = \frac{2851}{5184} \Rightarrow \sqrt[3]{6} = \left(\frac{2851}{5184}\right)^{-1} = \dots$	M1
	$= \frac{5184}{2851} \text{ or } 1\frac{2333}{2851}$	A1
		(2)
		Total 6

Note a misread of $(8-3x)^{\frac{1}{3}}$ can score a maximum of B1M1A0A0 M0A0

(a)

B1: Obtains $\frac{1}{2}(1 \pm \dots x)^{-\frac{1}{3}}$. $8^{-\frac{1}{3}}$ must be evaluated. May be implied by further work e.g. $\frac{1}{2} \pm \dots$ provided this has not come from an incorrect method.

e.g. $(8-3x)^{-\frac{1}{3}} = \frac{1}{(8-3x)^{\frac{1}{3}}} = \frac{1}{2} + \frac{1}{\frac{1}{2}\sqrt[3]{3}x} + \frac{1}{\frac{1}{2}(\sqrt[3]{3}x)^2} + \dots$ is B0.

M1: Attempts the binomial expansion of $(1+kx)^n$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 which may be unsimplified.

Look for $\frac{-\frac{1}{3}(-\frac{1}{3}-1)}{2!} \dots x^2$ or $\frac{-\frac{1}{3}(-\frac{1}{3}-1)(-\frac{1}{3}-2)}{3!} \dots x^3$ o.e. (you do not need to be concerned

with their $-\frac{3}{8}$ which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix}$, $\begin{pmatrix} -\frac{1}{3} \\ 2 \end{pmatrix}$ unless interpreted correctly in further work.

May be implied by correct coefficients with x^2 or x^3

A1: For 2 correct **simplified** terms of $\frac{1}{16}x$, $\frac{1}{64}x^2$, $\frac{7}{1536}x^3$ (allow x^1 for x for this mark) which may be listed.

Condone if coefficients are given as decimals i.e. $0.0625x$, $0.015625x^2$, $0.004557 \dots x^3$

Also allow for $\left(1 - \frac{3}{8}x\right)^{-\frac{1}{3}} = 1 + \frac{1}{8}x + \frac{1}{32}x^2 + \frac{7}{768}x^3 + \dots$ (Note B0M1A1A0 is possible)

A1: $\frac{1}{2} + \frac{1}{16}x + \frac{1}{64}x^2 + \frac{7}{1536}x^3$. Isw following a correct answer.

Condone the position of x^n provided it is not clearly on the denominator. e.g. condone $\frac{1}{16}x$

but not $\frac{1}{16x}$. Ignore terms with higher powers of x . Do not accept $\frac{1}{16}x^1$

(b)

M1: Attempts to substitute $x = \frac{2}{3}$ into their expansion from part (a) to achieve a fraction and

attempts to find the reciprocal of that fraction. May be implied by their fraction. You may need to check this on your calculator. They may attempt to find an additional term which is fine. Do not be concerned as to whether the fraction is correct for their binomial expansion if they have shown the substitution. Condone slips including losing one of their terms.

$$\left(\frac{1}{2} + \frac{1}{16} \left(\frac{2}{3} \right) + \frac{1}{64} \left(\frac{2}{3} \right)^2 + \frac{7}{1536} \left(\frac{2}{3} \right)^3 \right) = \frac{A}{B} \Rightarrow \frac{B}{A} \text{ scores M1}$$

$$1 \div \frac{2851}{5184} \text{ scores M1}$$

$$\frac{1}{\left(\frac{1}{2} + \frac{1}{16} \left(\frac{2}{3} \right) + \frac{1}{64} \left(\frac{2}{3} \right)^2 + \frac{7}{1536} \left(\frac{2}{3} \right)^3 \right)} \text{ on its own scores M0}$$

A1: $\frac{5184}{2851}$ or $1\frac{2333}{2851}$. Isw once a correct answer is seen. Correct answer scores M1A1.



Question Number	Scheme	Marks
4(a)	e.g. $5+17x-10x^2 = A(2x+1)(1-x) + Bx(2x+1) + Cx(1-x)$ $\Rightarrow A=..., B=..., C=...$	M1
	$A=5, B=4, C=8 \Rightarrow f(x) = \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)}$	A1A1
		(3)
(b)	$\int \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)} dx = 5 \ln x - 4 \ln 1-x + 4 \ln 2x+1 (+c)$	M1A1ftA1
	$\left[5 \ln x - 4 \ln 1-x + 4 \ln 2x+1 \right]_2^4$ $= (5 \ln 4 - 4 \ln 3 + 4 \ln 9) - (5 \ln 2 - 4 \ln 1 + 4 \ln 5) = 5 \ln 2 + 4 \ln \left(\frac{3}{5} \right)$	dM1A1
		(5)
		(8 marks)

(a)

M1: Sets $5+17x-10x^2 = A(2x+1)(1-x) + Bx(2x+1) + Cx(1-x)$ condoning slips on the coefficients of $5+17x-10x^2$ only and finds values for A, B and C . The right-hand side must be correct. Do not be concerned how the values are formed, e.g. via substitution or simultaneous equations. Just look for a suitable identity followed by values for A, B and C .

A1: Two (out of three) correct values of $A=5, B=4, C=8$

A1: Accept either $A=5, B=4, C=8$ or $\frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)}$ but condone $\frac{5}{x} - \frac{4}{(x-1)} + \frac{8}{(2x+1)}$ which is also correct (but not in the form requested).

(b)

M1: Attempts to integrate their $\frac{"5"}{x} + \frac{"4"}{(1-x)} + \frac{"8"}{(2x+1)}$.

Score for any one $\frac{"5"}{x} \rightarrow \alpha \ln|x|$ $\frac{"4"}{(1-x)} \rightarrow \beta \ln|1-x|$ $\frac{"8"}{(2x+1)} \rightarrow \delta \ln|2x+1|$

(Condone invisible modulus signs or invisible brackets if subsequent work implies them)

Condone $\beta \ln|1-x| \rightarrow \beta \ln|x-1|$ **throughout (b)**

Be aware that if they incorrectly find C , e.g. $C = \frac{7}{2}$, then $\frac{7}{(4x+2)} \rightarrow \frac{7}{4} \ln(4x+2)$ is a correct follow through

A1ft: Two of $5 \ln|x| - 4 \ln|1-x| + 4 \ln|2x+1|$ follow through their A, B and C .

Allow use of brackets instead of modulus signs or allow further work to imply them.

A1: $5 \ln|x| - 4 \ln|1-x| + 4 \ln|2x+1|$ with or without the $+c$

Allow use of brackets instead of modulus signs or allow further work to imply them.



dM1: Substitutes in the limits, subtracts either way round and applies laws of logs correctly to achieve either an $p \ln 2$ term or a $q \ln \frac{3}{5}$ term.

It is dependent on the previous M and having integrated to a form $\alpha \ln|x| + \beta \ln|1-x| + \delta \ln|2x+1|$
or $\alpha \ln|x| + \beta \ln|x-1| + \delta \ln|2x+1|$

This is most likely going to be scored via the $\ln 2$ term.

The term in $\ln 2$ must be found from $\alpha \ln 4 - \alpha \ln 2 = \alpha \ln 2$ so the value of p and α must be the same

The term in $\ln\left(\frac{3}{5}\right)$ must be found from $\beta \ln 3 + 2\delta \ln 3 - \delta \ln 5$

Condone calculations such as $\ln(-3) - \ln(-5) = \ln \frac{3}{5}$

A1: CSO: $5 \ln 2 + 4 \ln\left(\frac{3}{5}\right)$

It cannot be scored following incorrect integration. ISW after sight of this

Question Number	Scheme	Marks
5(a)	$\frac{dx}{dt} = \frac{2(1-t) - (-1)(3+2t)}{(1-t)^2} = \frac{5}{(1-t)^2}$ and $\frac{dy}{dt} = -2t$	
	At $t = 2 \Rightarrow \frac{dy}{dx} = \frac{(-1)^2}{5} \times (-2(2)) = -\frac{4}{5}$	M1A1
	$(-7, -3)$	B1
	$y+3 = \frac{5}{4}(x+7)$	M1
	$5x-4y+23=0$	A1
		(5)
(b)	$x = \frac{3+2t}{1-t} \rightarrow t = \frac{x-3}{x+2}$	M1
	$y = 1 - \left(\frac{x-3}{x+2}\right)^2 = \frac{(x+2)^2 - (x-3)^2}{(x+2)^2} = \frac{10x-5}{(x+2)^2}$	dM1A1
	$x \neq -2$	B1
		(4)
		(9 marks)

FYI $x = \frac{3+2t}{1-t} = \frac{5}{1-t} - 2$

(a) Note: A cartesian approach in (a) can score a maximum of 1 mark (the B1) as it does not satisfy the demand of the question.

M1: Attempts to find $\frac{dy}{dx}$ at $t = 2$ by finding $\frac{dy}{dt} \div \frac{dx}{dt}$.

Condone slips when differentiating $\frac{dx}{dt}$ and $\frac{dy}{dt}$ but look for a minimum of

- $\frac{dx}{dt} = \frac{\alpha(1-t) \pm \beta(3+2t)}{(1-t)^2}$ with $\alpha > 0$ or equivalent using the product rule or division and chain rule
- $\frac{dy}{dt} = \pm 2t$
- Substitution of $t = 2$ into $\frac{dy}{dt} \div \frac{dx}{dt}$ which may have been simplified incorrectly. Note that a simplified $\frac{dy}{dx} = \frac{-2t(1-t)^2}{5}$

Note that using the quotient rule the wrong way around $\frac{uv' - vu'}{v^2}$ would be M0

A1: $-\frac{4}{5}$ following M1. It may be implied by a normal gradient of $\frac{5}{4}$

B1: $(-7, -3)$ which may be implied from terms within an attempt at the equation of the normal



- M1: Attempts to find the equation of the normal at P . They must be using the negative reciprocal of their $\frac{dy}{dx}$ at $t = 2$ with their point P . It is dependent on having found $\frac{dy}{dx}$ from $\left(\text{their } \frac{dy}{dt}\right) \div \left(\text{their } \frac{dx}{dt}\right)$ Condone poor attempts at $\frac{dx}{dt}$ and $\frac{dy}{dt}$ or what they believe are $\frac{dx}{dt}$ and $\frac{dy}{dt}$ so 0, 0, 1, 1, 0 is possible. Condone one sign slip when using their coordinate of P . e.g $(-7, -3) \rightarrow (7, -3)$
If the form $y = mx + c$ is used the method must proceed as far as $c = \dots$
- A1: $5x - 4y + 23 = 0$ or any integer multiple with terms on one side of an equation. The order of the terms is not important. ISW after sight of a correct answer in a correct form
-

Other more complicated methods do exist so look carefully at what is attempted. For example, candidates could find $\frac{dx}{dy}$ at $t = 2$ by finding $\frac{dx}{dt} \div \frac{dy}{dt}$ and use the value of $-\frac{dx}{dy}$ as the gradient of the normal.

.....

(b)

- M1: Attempts to rearrange the parametric equation for x to make t the subject. Expect to see cross multiplying (or equivalent) and collection of terms with $t = \frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are linear functions in x .

dM1: Substitutes their expression for t in terms for x into the parametric equation for y and attempts to manipulate the right-hand side into a single fraction. It is dependent on the previous method mark.

$$\text{For example, look for } y = 1 - \left(\frac{f(x)}{g(x)}\right)^2 = \frac{(g(x))^2 - (f(x))^2}{(g(x))^2}$$

A1: $y = \frac{10x - 5}{(x + 2)^2}$

- B1: $x \neq -2$ or $k = -2$ but must follow correct work. E.g. Sight of $(x + 2)$ o.e. on the denominator of either the expression for t or the expression for y .
-

Other more complicated methods do exist but are unlikely to lead to any success. If you see a solution that you feel deserves credit please send to review and alert your TL.

For example, if they start with $t = \sqrt{1 - y}$ and then write down $x = \frac{3 + 2\sqrt{1 - y}}{1 - \sqrt{1 - y}}$ the first M mark will not be scored until they make $\sqrt{1 - y}$ the subject (in terms of just x) which reverts to the original method

Question Number	Scheme	Marks
8 (a)	$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \times \frac{1}{x} \, dx$ $= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 (+c)$	M1 dM1 A1 (3)
(b)	$\int x^2 \ln^2 x \, dx = \frac{1}{3}x^3 \ln^2 x - \int \frac{1}{3}x^3 \times 2 \ln x \times \frac{1}{x} \, dx$ $= \frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \right)$ $\text{Volume} = \int_1^e \pi x^2 \ln^2 x \, dx = \pi \times \left[\frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \right) \right]_1^e$ $= \frac{5}{27}\pi e^3 - \frac{2}{27}\pi$	M1 A1ft dM1 A1 (4) (7 marks)

(a) Note that no marks are available for integrating incorrect functions

M1: Attempts to integrate by parts the correct way around. Look for $\int x^2 \ln x \, dx = \dots x^3 \ln x - \int \dots x^3 \times \frac{1}{x} \, dx$ oe

dM1: And then integrates again to a form $px^3 \ln x - qx^3$ where p and q are positive constants.

A1: $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$ Condone a missing $+c$.

Candidates who go on to write $\ln x \times \frac{x^3}{3} - \frac{1}{9}x^3 = \ln \frac{x^4}{3} - \frac{1}{9}x^3$ o.e. withhold this final A1

(b)

M1: Integrates $\int x^2 \ln^2 x \, dx$ by parts, the correct way around to reach a form $\dots x^3 \ln^2 x - \dots \int x^3 \times \ln x \times \frac{1}{x} \, dx$

You may see a π or 2π in front of this expression which is fine.

A1ft: Achieves $\int x^2 \ln^2 x \, dx = \frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \times \text{their answer to (a)}$.

Accept $\int \pi x^2 \ln^2 x \, dx = \frac{1}{3}\pi x^3 \ln^2 x - \frac{2}{3} \times \pi \times \text{their answer to (a)}$ or other incorrect factors such as

$\int 2\pi x^2 \ln^2 x \, dx = \frac{2}{3}\pi x^3 \ln^2 x - \frac{4}{3} \times \pi \times \text{their answer to (a)}$

FYI The correct answer is $\frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \right)$

dM1: Applies the limits 1 and e to a function of the form $\pi \times \left[\alpha x^3 \ln^2 x - \beta x^3 \ln x \pm \chi x^3 \right]_1^e$

and subtracts where α, β, χ are positive constants. The π must now be seen.

A1: $\frac{5}{27}\pi e^3 - \frac{2}{27}\pi$ or exact simplified equivalent such as $\frac{\pi}{27}(5e^3 - 2)$

Alt (b)

(b)	$\int x^2 \ln^2 x \, dx = \int \ln x \times x^2 \ln x \, dx$ $= \left(\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \right) \ln x - \int \left(\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \right) \times \frac{1}{x} \, dx$ $= \left(\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \right) \ln x - \int \left(\frac{1}{3} x^2 \ln x - \frac{1}{9} x^2 \right) \, dx$ $= \left(\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \right) \ln x - \frac{1}{3} \left(\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \right) + \frac{1}{27} x^3$ Volume = $\int_1^e \pi x^2 \ln^2 x \, dx = \pi \times \left[\left(\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \right) \ln x - \frac{1}{3} \left(\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \right) + \frac{1}{27} x^3 \right]_1^e$ $= \frac{5}{27} \pi e^3 - \frac{2}{27} \pi$	M1 A1ft dM1 A1 (4)
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Question Number	Scheme	Marks
9 (a)	States or uses $V = \pi \times 4^2 \times h \Rightarrow \frac{dV}{dh} = 16\pi$	B1
	States or uses $\frac{dV}{dt} = 0.6\pi - 0.15\pi h$	B1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 0.6\pi - 0.15\pi h = 16\pi \times \frac{dh}{dt}$	M1
	$\frac{dh}{dt} = \frac{12-3h}{320}$ *	A1*
		(4)
(b)	$\frac{dh}{dt} = \frac{12-3h}{320} \Rightarrow \int \frac{dh}{12-3h} = \int \frac{1}{320} dt$	
	$-\frac{1}{3} \ln(12-3h) = \frac{1}{320} t + c$	M1 A1
	Uses $t = 0, h = 0.5$ $-\frac{1}{3} \ln\left(\frac{21}{2}\right) = c \Rightarrow -\frac{1}{3} \ln(12-3h) = \frac{1}{320} t - \frac{1}{3} \ln\left(\frac{21}{2}\right)$	M1 A1
	Substitutes $h = 3.5$ $-\frac{1}{3} \ln\left(\frac{3}{2}\right) = \frac{1}{320} t - \frac{1}{3} \ln\left(\frac{21}{2}\right) \Rightarrow t = \dots$	dM1
	208 minutes cso	A1
		(6)
		(10 marks)

(a)

 B1: States or uses $V = \pi \times 4^2 \times h \Rightarrow \frac{dV}{dh} = 16\pi$ o.e.

 B1: States or uses $\frac{dV}{dt} = 0.6\pi - 0.15\pi h$

 M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ or equivalent with their $\frac{dV}{dh}$ and $\frac{dV}{dt}$

 A1*: Proceeds to $\frac{dh}{dt} = \frac{12-3h}{320}$ from a correct statement with no errors seen. The main scheme is acceptable.

Note that the answer is given and there may be many "fudged" attempts.

 If you see $\frac{dh}{dt} = \frac{1}{16\pi} \times (0.6\pi - 0.15\pi h) = \frac{12-3h}{320}$ or similar without any explanation or sight of $\frac{dV}{dt}$ etc you can award B1 (implied), B1 (implied), M1 (implied), A0* (not fully shown)


(b)

M1: Separates variables and integrates both sides. Look for $\pm a \ln(12-3h) = \pm bt + c$ or equivalent.Note that $\pm a \ln(4-h) = \pm bt + k$ is also correct. There is no need for a constant at this stage

$$\text{Alternatively } \frac{dt}{dh} = \frac{320}{12-3h} \Rightarrow t = \dots \ln(12-3h) + c$$

Watch for variations on this such as $t = \ln(4-h) + k$

$$\text{A1: } -\frac{1}{3} \ln(12-3h) = \frac{1}{320}t + c \text{ with or without the } + c$$

M1: Uses $t=0$, $h=0.5$ to find c .

Condone poor integration and poor attempts to re arrange the equation for this mark.

A1: Correct equation linking h and t .

$$\text{This may be implied by an equation with } h \text{ replaced by } 0.5. \text{ E.g. } \ln(12-3 \times 3.5) = -\frac{3}{320}t + \ln 10.5$$

$$\text{Condone } c \text{ being a decimal (3sf) for this mark. So } \ln(12-3h) = -\frac{3}{320}t + 2.35 \text{ is fine}$$

dM1: Substitutes $h=3.5$ to find t . Dependent upon previous M

$$\text{Note that } \left[-\frac{1}{3} \ln(12-3h) \right]_{0.5}^{3.5} = \left[\frac{1}{320}t \right]_0^T \text{ can score 3 marks at once provided they reach a value for } T$$

A1: States 207- 208 minutes with correct units following correct work.

$$\text{Accept any value between and including these. Condone } \frac{320}{3} \ln 7 \text{ minutes}$$

$$\begin{aligned} \int \frac{1}{12-3h} dh &= \int \frac{1}{320} dt \\ -\frac{1}{3} \ln(12-3h) &= \frac{1}{320}t + c \\ \text{or} \\ t &= -\frac{320}{3} \ln(12-3h) - 320c \\ \text{when } h &= 3.5 \\ t_1 &= -\frac{320}{3} \ln \frac{3}{2} - 320c \\ \text{when } h &= 0.5 \\ t_2 &= -\frac{320}{3} \ln \frac{11}{2} - 320c \\ t_1 - t_2 &= 207.56 \text{ mins.} \end{aligned}$$

This method is similar to

$$\left[-\frac{1}{3} \ln(12-3h) \right]_{0.5}^{3.5} = \left[\frac{1}{320}t \right]_0^T$$

and can score 3 marks on the final line when they subtract

Question Number	Scheme	Marks
7 (a)	$\frac{dV}{dr} = 4\pi r^2$	B1 (1)
(b) (i)	$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \Rightarrow V = -\frac{450}{2t+3} + c$ (oe)	M1 A1
	$t = 0, V = 0 \Rightarrow 0 = -\frac{450}{3} + c \Rightarrow c = \dots$	M1
	$V = 150 - \frac{450}{2t+3} = \frac{300t + 450 - 450}{2t+3} = \frac{300t}{2t+3}$ *	A1 *
(ii)	150 cm^3	B1
		(5)
(c)	$t = 3 \Rightarrow V = \frac{300 \times 3}{2 \times 3 + 3} = (100)$	M1
	$100 = \frac{4}{3}\pi r^3 \Rightarrow r = 2.88 \text{ cm}$	dM1 A1cao (3)
(d)	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{900}{(2t+3)^2} = 4\pi r^2 \times \frac{dr}{dt}$	M1
	$t = 3, r = "2.88" \Rightarrow \frac{900}{81} = 4\pi \times 2.88^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \dots$	dM1
	$\Rightarrow \frac{dr}{dt} = \text{awrt } 0.11 \text{ cm s}^{-1}$	A1 (3)
		(12 marks)

Notes: Mark the question as a whole. Penalise only once for missing/incorrect units in the question.

(a)

B1: cao See scheme.

(b)(i)

M1: Integrates to a form $V = \frac{k}{2t+3}$ (oe) with or without + c. Condone a sign error in $2t - 3$.

A1: $V = -\frac{450}{2t+3} (+c)$ (oe). There is no need for + c

M1: Substitutes $V = 0, t = 0$ and proceeds to find a value for c. There must have been an attempt at integrating to achieve a function in V and t with a constant of integration.

A1*: Correct integration and value for c with at least one intermediate step with c substituted back in the equation before proceeding to the given answer.

(b)(ii)

B1: 150 cm^3 . Must include units.

(c)

M1: Attempts to substitute $t = 3$ into the equation for V. Allow if there is a slip in substitution.

dM1: Uses their V in $V = \frac{4}{3}\pi r^3$ to find a value for r

A1: cao $r = 2.88 \text{ cm}$. Must be to 3 s.f.. Must include units unless already penalised in (b)(ii).

(d)

M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ (oe) with the given formula for $\frac{dV}{dt}$ and an attempt at

substituting their $\frac{dV}{dr}$ (allow if this substitution is not in the correct place if a correct chain rule has been stated.)

dM1: Substitutes both $t = 3$ and their value for r and proceeds to find a value for $\frac{dr}{dt}$. If no substitution shown, the answer must be correct for their r to imply the method (may need to check).
A1: awrt 0.11 cm s^{-1} . Must include units unless already penalised in (b)(ii) or (c).



$$\text{Starting from } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} : \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \frac{5}{3}\overline{AB} \quad \text{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \frac{7}{3}\overline{AB}$$

$$\text{Starting from } A \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} : \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} + 2\overline{AB} \quad \text{or} \quad \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} - 2\overline{AB}$$

$$\text{Starting from } B \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} : \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} + \overline{AB} \quad \text{or} \quad \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - 3\overline{AB}$$

A1: One correct point $(22, 9, -2)$ or $(-26, -15, 10)$

Condone if given as a vector e.g. $22\mathbf{i} + 9\mathbf{j} - 2\mathbf{k}$ or $-26\mathbf{i} - 15\mathbf{j} + 10\mathbf{k}$

dM1: Attempts a complete and correct method for finding both possible positions for D .

(May be implied by at least 2 correct or correct ft components)

Examples:

$$\text{Starting from } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} : \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\text{Starting from } A \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} : \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} + 6 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} - 6 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\text{Starting from } B \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} : \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - 9 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

Note that some candidates may use their $\overline{AB}$ rather than the direction vector e.g.

$$\text{Starting from } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} : \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \frac{5}{3}\overline{AB} \quad \text{and} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \frac{7}{3}\overline{AB}$$

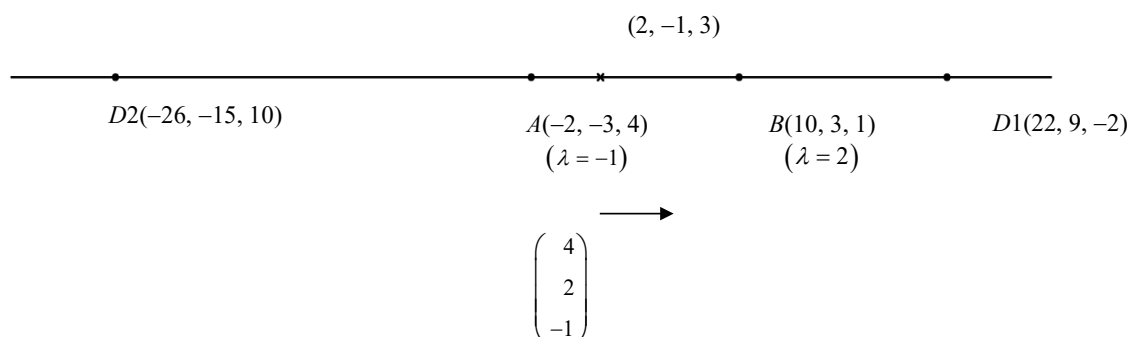
$$\text{Starting from } A \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} : \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} + 2\overline{AB} \quad \text{and} \quad \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} - 2\overline{AB}$$

$$\text{Starting from } B \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} : \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} + \overline{AB} \quad \text{and} \quad \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - 3\overline{AB}$$

A1: Gives both possible coordinates $(-26, -15, 10)$ and $(22, 9, -2)$

Condone if given as vectors e.g. $22\mathbf{i} + 9\mathbf{j} - 2\mathbf{k}$ and $-26\mathbf{i} - 15\mathbf{j} + 10\mathbf{k}$



Configuration in (d):Note that there may be more convoluted methods in (d) e.g.

$$\text{Area } CAD = 2 \times \text{Area } CAB \Rightarrow AD = 2\sqrt{12^2 + 6^2 + 3^2} \Rightarrow AD^2 = 4 \times 189$$

$$\overrightarrow{AD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4\lambda + 4 \\ 2\lambda + 2 \\ -\lambda - 1 \end{pmatrix}$$

$$(4\lambda + 4)^2 + (2\lambda + 2)^2 + (-\lambda - 1)^2 = 756$$

$$\Rightarrow 21\lambda^2 + 42\lambda - 735 = 0 \Rightarrow \lambda = 5, -7$$

$$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

In such cases, marks can be awarded as above for:

- **M1:** A complete and correct method to find one position for D
(May be implied by at least 2 correct or correct ft components)
- **A1:** One correct position for D
- **dM1:** A complete and correct method to find both positions for D
(May be implied by at least 2 correct or correct ft components)
- **A1:** As main scheme

If you are in any doubt whether a method is sound or not use review.



Can also be done via the area e.g.

$$\text{Area } CAB = \frac{1}{2} AB \times AC \sin \theta = \frac{1}{2} \sqrt{189} \sqrt{172} \sin \theta$$

$$\cos \theta = \frac{25}{\sqrt{43} \sqrt{21}} \Rightarrow \sin \theta = \sqrt{\frac{278}{903}}$$

$$\text{Area } CAD = \frac{1}{2} AD \times AC \sin \theta = \sqrt{189} \sqrt{172} \sqrt{\frac{278}{903}}$$

$$\overrightarrow{AD} = \begin{pmatrix} 4\lambda + 4 \\ 2\lambda + 2 \\ -\lambda - 1 \end{pmatrix} \therefore \frac{1}{2} \sqrt{(4\lambda + 4)^2 + (2\lambda + 2)^2 + (-\lambda - 1)^2} \sqrt{172} \sqrt{\frac{278}{903}} = \sqrt{189} \sqrt{172} \sqrt{\frac{278}{903}}$$

$$\Rightarrow (4\lambda + 4)^2 + (2\lambda + 2)^2 + (-\lambda - 1)^2 = 756$$

$$\Rightarrow 21\lambda^2 + 42\lambda - 735 = 0 \Rightarrow \lambda = 5, -7$$

$$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

The same marking principles apply

- **M1:** A complete and correct method to find one position for D
(May be implied by at least 2 correct or correct ft components)
- **A1:** One correct position for D
- **dM1:** A complete and correct method to find both positions for D
(May be implied by at least 2 correct or correct ft components)
- **A1:** As main scheme

For this method it is unlikely that candidates will work in exact terms and will revert to decimals.

This is acceptable but is unlikely to result in correct exact coordinates but the method marks are available as long as the method is complete and correct.

If candidates do work in decimals and then round their inexact values for the coordinates to the correct ones the A marks should be withheld.



Question Number	Scheme	Marks
9(a)(i)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\sin^2 t \cos t}{-4\sin 2t}$	M1A1
	$\frac{3\sin^2 t \cos t}{-4\sin 2t} = \frac{3\sin^2 t \cos t}{-8\sin t \cos t} = -\frac{3}{8}\sin t$ or e.g. $\frac{3\sin^2 t \cos t}{-4\sin 2t} = \frac{\frac{3}{2}\sin 2t \sin t}{-4\sin 2t} = -\frac{3}{8}\sin t$	A1
(ii)	$t = \frac{\pi}{6} \Rightarrow x = 1, y = \frac{1}{8}$	B1
	$m = -\frac{3}{8}\sin\left(\frac{\pi}{6}\right) = -\frac{3}{16} \Rightarrow y - \frac{1}{8} = -\frac{3}{16}(x - 1)$	M1
	$3x + 16y - 5 = 0$ *	A1*
		(6)
(b)	$3(2\cos 2t) + 16(\sin^3 t) - 5 = 0$	M1
	$6(1 - 2\sin^2 t) + 16\sin^3 t - 5 = 0$	dM1
	$16\sin^3 t - 12\sin^2 t + 1 = 0$	A1
	$(2\sin t - 1)^2(4\sin t + 1) = 0 \Rightarrow \sin t = -\frac{1}{4}$	ddM1
	$\sin t = -\frac{1}{4} \Rightarrow x = 2\cos 2t = \dots$ and $y = \sin^3 t = \dots$	dddM1
	$Q\left(\frac{7}{4}, -\frac{1}{64}\right)$	A1
		(6)
		(12 marks)

Notes:(a)(i) **Note that (a)(i) is now being marked as M1A1A1 not M1dM1A1****M1:** Attempts to use the rule $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. Condone incorrect attempts on $\frac{dy}{dt}$ and $\frac{dx}{dt}$ Note that x and/or y may have been adapted before differentiation.**A1:** Correct $\frac{dy}{dx}$ in any form **from correct work**.**A1:** Achieves $\frac{dy}{dx} = -\frac{3}{8}\sin t$ **from fully correct work so must follow M1A1 so M1A0A1 is not possible.**You may need to check the working carefully as the value of k can be deduced from the given tangent equation.

e.g. $3x + 16y - 5 = 0 \Rightarrow m = -\frac{3}{16}, m = k \sin \frac{\pi}{6} \Rightarrow k = -\frac{3}{8}$

(a)(ii)

B1: Correct coordinates for P $x=1, y=\frac{1}{8}$ **M1:** Correct method for tangent. It is awarded for:

- substituting $t = \frac{\pi}{6}$ into their $k \sin t$ or their $\frac{dy}{dx}$ to find m
- using $y - y_1 = m(x - x_1)$ where x_1 and y_1 are their values of x and y when $t = \frac{\pi}{6}$ or using $y = mx + c$ with their values of x and y when $t = \frac{\pi}{6}$ and proceeds to $c = \dots$

A1*: cso. Proceeds with a correct method and correct work to the given answer with sufficient working. Note that this cso so that if $-\frac{3}{8}\sin t$ has been obtained fortuitously or from incorrect work in part (i), this mark should be withheld.

(b)

M1: Substitutes $x = 2\cos 2t$ and $y = \sin^3 t$ into $3x + 16y - 5 = 0$

Condone slips as long as the intention is clear.

dm1: Uses $\cos 2t = \pm 1 \pm 2\sin^2 t$ or equivalent work to obtain a cubic equation in $\sin t$.The equivalent work could be e.g. $6\cos 2t = 6(\cos^2 t - \sin^2 t) = 6(1 - \sin^2 t - \sin^2 t)$ etc.**A1:** Correct cubic equation with terms collected and all on one side e.g. $16\sin^3 t - 12\sin^2 t + 1 = 0$

The “= 0” may be implied by later work e.g. by their attempt to solve.

ddM1: Solves cubic equation via any appropriate means including via a calculator to obtain $\sin t = \alpha$ where $|\alpha| < 1$ and $\alpha \neq \frac{1}{2}$ **dddM1:** Substitutes their $\sin t = -\frac{1}{4}$ into both $x = 2\cos 2t$ and $y = \sin^3 t$ to obtain a value for x and y This may be using appropriate identities for finding x or via a calculator e.g. $x = 2\cos\left(2\sin^{-1}\left(-\frac{1}{4}\right)\right)$ **A1:** $Q\left(\frac{7}{4}, -\frac{1}{64}\right)$ Allow any equivalent exact values and allow as $x = \dots, y = \dots$

Note that it is possible to answer (b) by eliminating t :

1. Via x :

$$\begin{aligned}x &= 2 \cos 2t = 2(1 - 2 \sin^2 t) \Rightarrow \sin^2 t = \frac{2-x}{4} \Rightarrow y = \sin^3 t = \left(\frac{2-x}{4}\right)^{\frac{3}{2}} \\3x + 16\left(\frac{2-x}{4}\right)^{\frac{3}{2}} - 5 &= 0 \Rightarrow 16\left(\frac{2-x}{4}\right)^{\frac{3}{2}} = 5 - 3x \Rightarrow 4(2-x)^3 = (5-3x)^2 \\&\Rightarrow 4x^3 - 15x^2 + 18x - 7 = 0 \\&\Rightarrow x = \frac{7}{4} \Rightarrow y = -\frac{1}{64}\end{aligned}$$

Score as:

M1: Uses the x coordinate and $\cos 2t = \pm 1 \pm 2 \sin^2 t$ or equivalent work to find an equation connecting

$\sin t$ and x e.g. $\sin^2 t = \frac{2-x}{4}$ and uses this to find y in terms of x .

dm1: Substitutes their y in terms of x into $3x + 16y - 5 = 0$ to obtain an equation in terms of x only.

A1: Correct cubic equation with terms collected and all on one side e.g.

$$4x^3 - 15x^2 + 18x - 7 = 0$$

The “= 0” may be implied by later work e.g. by their attempt to solve.

ddm1: Solves cubic equation via any appropriate means including via a calculator to obtain a value for x where $x \neq 1$

dddM1: Uses their value of x in $3x + 16y - 5 = 0$ to find a value of y .

A1: $\mathcal{O}\left(\frac{7}{4}, -\frac{1}{64}\right)$ Allow any equivalent exact values and allow as $x = \dots, y = \dots$

2. Via y :

$$\begin{aligned}y &= \sin^3 t \Rightarrow \sin t = y^{\frac{1}{3}}, x = 2(1 - 2 \sin^2 t) = 2 - 4y^{\frac{2}{3}} \\3\left(2 - 4y^{\frac{2}{3}}\right) + 16y - 5 &= 0 \Rightarrow 12y^{\frac{2}{3}} = 16y + 1 \Rightarrow 1728y^2 = (16y + 1)^3 \\&\Rightarrow 4096y^3 - 960y^2 + 48y + 1 = 0 \\&\Rightarrow y = -\frac{1}{64} \Rightarrow x = \frac{7}{4}\end{aligned}$$

Score as:

M1: Uses the y coordinate to find an equation connecting $\sin t$ and y e.g. $\sin t = y^{\frac{1}{3}}$ and uses this to find x in terms of y .

dm1: Substitutes their x in terms of y into $3x + 16y - 5 = 0$ to obtain an equation in terms of y only.

A1: Correct cubic equation with terms collected and all on one side e.g.

$$4096y^3 - 960y^2 + 48y + 1 = 0$$

The “= 0” may be implied by later work e.g. by their attempt to solve.



ddM1: Solves cubic equation via any appropriate means including via a calculator to obtain a value for y where $y \neq \frac{1}{8}$

dddM1: Uses their value of y in $3x + 16y - 5 = 0$ to find a value of x .

A1: $\mathcal{Q}\left(\frac{7}{4}, -\frac{1}{64}\right)$ Allow any equivalent exact values and allow as $x = \dots, y = \dots$

Note that some candidates are bypassing the cubic and clearly using the “solve” function on their calculator. This is not satisfying the rubric of the question or the rubric on the front of the question paper. These will be marked as special cases.

Examples:

$$x = 2 \cos 2t = 2(1 - 2 \sin^2 t) \Rightarrow \sin^2 t = \frac{2-x}{4} \Rightarrow y = \sin^3 t = \left(\frac{2-x}{4}\right)^{\frac{3}{2}}$$

$$3x + 16\left(\frac{2-x}{4}\right)^{\frac{3}{2}} - 5 = 0 \Rightarrow x = \frac{7}{4}$$

$$3\left(\frac{7}{4}\right) + 16y - 5 = 0 \Rightarrow y = -\frac{1}{64}$$

Scores SC M1M1A0M0M1A1

$$x = 2 \cos 2t \Rightarrow t = \frac{1}{2} \arccos \frac{x}{2} \Rightarrow y = \sin^3 \left(\frac{1}{2} \arccos \frac{x}{2}\right)$$

$$3x + 16 \sin^3 \left(\frac{1}{2} \arccos \frac{x}{2}\right) - 5 = 0 \Rightarrow x = \frac{7}{4}$$

$$3\left(\frac{7}{4}\right) + 16y - 5 = 0 \Rightarrow y = -\frac{1}{64}$$

Scores SC M1M0A0M0M1A1

In such cases award marks as follows:

M1: Uses the x coordinate and $\cos 2t = \pm 1 \pm 2 \sin^2 t$ or equivalent work to find an equation connecting

$\sin t$ and x e.g. $\sin^2 t = \frac{2-x}{4}$ and uses this to find y in terms of x .

dM1: Substitutes their y in terms of x into $3x + 16y - 5 = 0$ to obtain an equation in terms of x only that is algebraic i.e. does not involve inverse trig functions.

A0: Not available

M0: Not available

M1: Uses their value of x in $3x + 16y - 5 = 0$ to find a value of y .

A1: $\mathcal{Q}\left(\frac{7}{4}, -\frac{1}{64}\right)$ Allow any equivalent exact values and allow as $x = \dots, y = \dots$

