



GuideMyGrades

— YOUR GOALS. OUR EXPERTISE. —

EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Integration (P4.5)

International A Level Mathematics · P4: Pure Mathematics 4 (WMA14)

40 questions · 338 marks · Mark schemes included

Past papers October 2020 - June 2025 · 3 sub-topics

 guidemygrades.com

 [@Guidemygrades](https://www.instagram.com/Guidemygrades)

 [@Guidemygrades](https://twitter.com/Guidemygrades)

 [@Guidemygrades](https://www.youtube.com/Guidemygrades)

 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

QUESTION	SOURCE	MARKS	PAGE
P4.5a INTEGRATION BY SUBSTITUTION & BY PARTS		17 Qs · 138 marks	3
1. Q1	IAL P4 (WMA14) June 2024	5	4
2. Q6	IAL P4 (WMA14) October 2024	5	6
3. Q1	IAL P4 (WMA14) June 2025	5	8
4. Q6	IAL P4 (WMA14) (unused paper) January 2022	6	10
5. Q7	IAL P4 (WMA14) January 2025	6	12
6. Q7	IAL P4 (WMA14) June 2025	6	14
7. Q6	IAL P4 (WMA14) October 2021	7	16
8. Q5	IAL P4 (WMA14) January 2021	8	20
9. Q4	IAL P4 (WMA14) June 2021	8	22
10. Q5	IAL P4 (WMA14) June 2022	8	26
11. Q4	IAL P4 (WMA14) June 2025	8	30
12. Q4	IAL P4 (WMA14) January 2023	9	34
13. Q5	IAL P4 (WMA14) June 2023	10	38
14. Q7	IAL P4 (WMA14) June 2025	11	42
15. Q7	IAL P4 (WMA14) October 2020	12	46
16. Q7	IAL P4 (WMA14) October 2022	12	50
17. Q3	IAL P4 (WMA14) October 2023	12	54
P4.5b INTEGRATION USING PARTIAL FRACTIONS & IDENTITIES			
No questions on this sub-topic in the available past papers.			
P4.5c AREAS & VOLUMES OF REVOLUTION		23 Qs · 200 marks	58
18. Q3	IAL P4 (WMA14) January 2023	5	59
19. Q1	IAL P4 (WMA14) January 2025	5	61
20. Q3	IAL P4 (WMA14) October 2020	6	63
21. Q5	IAL P4 (WMA14) October 2022	6	65
22. Q5	IAL P4 (WMA14) October 2020	7	69
23. Q7	IAL P4 (WMA14) January 2021	7	73
24. Q2	IAL P4 (WMA14) June 2021	7	77
25. Q8	IAL P4 (WMA14) October 2021	7	79
26. Q9	IAL P4 (WMA14) June 2025	7	83
27. Q7	IAL P4 (WMA14) January 2022	8	87
28. Q7	IAL P4 (WMA14) January 2024	8	91
29. Q7	IAL P4 (WMA14) October 2024	8	95
30. Q10	IAL P4 (WMA14) October 2024	8	99
31. Q9	IAL P4 (WMA14) June 2024	9	102



QUESTION	SOURCE	MARKS	PAGE
32. Q9	IAL P4 (WMA14) January 2021	10	105
33. Q6	IAL P4 (WMA14) June 2021	10	109
34. Q5	IAL P4 (WMA14) January 2022	10	113
35. Q8	IAL P4 (WMA14) June 2022	10	117
36. Q9	IAL P4 (WMA14) (unused paper) January 2022	11	121
37. Q8	IAL P4 (WMA14) June 2023	12	126
38. Q9	IAL P4 (WMA14) June 2025	12	131
39. Q5	IAL P4 (WMA14) June 2024	13	135
40. Q8	IAL P4 (WMA14) October 2023	14	139



QUESTIONS

40 questions · 338 marks · 2 sub-topics

SUB-TOPIC P4.5a

INTEGRATION BY SUBSTITUTION & BY PARTS

Sub-topic P4.5a · 17 questions · 138 marks · lowest tariff first



1.

In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

Find

$$\int_0^{\frac{\pi}{6}} x \cos 3x \, dx$$

giving your answer in simplest form.

(5)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)

6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3+1}} dx = 2(x^3+1)^k (x^3-A) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)

(Total for Question 6 is 5 marks)



1.

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Find, by algebraic integration,

$$\int_1^3 2x \ln(3x) dx$$

giving the answer in the form $A \ln 3 + B$ where A and B are integers.

(5)



Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)



6. Use integration by parts to show that

$$\int e^{2x} \cos 3x \, dx = pe^{2x} \sin 3x + qe^{2x} \cos 3x + k$$

where p and q are rational numbers to be found and k is an arbitrary constant.

(6)

Leave
blank



7.

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Use the substitution $x = 4 \sin \theta$ to find the exact value of

$$\int_2^{2\sqrt{3}} \frac{1}{(16-x^2)^{\frac{3}{2}}} dx$$

(6)

(Total for Question 7 is 6 marks)



(Total for Question 7 is 6 marks)



Leave
blank

Question 6 continued

Leave
blank

Question 6 continued

Leave
blank

(Total 7 marks)

Q6



Leave
blank

- 5. In this question you should show all stages of your working. Solutions relying on calculator technology are not acceptable.**

Using the substitution $u = 3 + \sqrt{2x - 1}$ find the exact value of

$$\int_1^{13} \frac{4}{3 + \sqrt{2x-1}} \, dx$$

giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(8)

Question 5 continued

Leave
blank

Q5

(Total 8 marks)



Leave
blank

4. Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}} dx$$

Write your answer in the form $4\ln\left(\frac{a}{b}\right)$, where a and b are integers to be found.

(Solutions relying entirely on calculator technology are not acceptable.)

(8)

Question 4 continued

Leave
blank



Question 4 continued

Leave
blank



Leave
blank

(Total 8 marks)

Q4





Question 5 continued

Lined area for writing the answer to Question 5 continued.

Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 8 marks)



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4 continued.



Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 8 marks)







(Total for Question 4 is 9 marks)



5. (i) Find

$$\int x^2 e^x dx$$

(4)

(ii) Use the substitution $u = \sqrt{1 - 3x}$ to show that

$$\int \frac{27x}{\sqrt{1-3x}} dx = -2(1-3x)^{\frac{1}{2}}(Ax+B) + k$$

where A and B are integers to be found and k is an arbitrary constant.

(6)



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 10 marks)



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7 continued.



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 11 marks)



Leave
blank

Leave
blank

Leave
blank

(Total 12 marks)

Q7



Question 7 continued

Lined area for writing the answer to Question 7 continued.





(Total for Question 7 is 12 marks)



Question 3 continued

Lined area for writing the answer to Question 3 continued.





Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 12 marks)



SUB-TOPIC P4.5c

AREAS & VOLUMES OF REVOLUTION

Sub-topic P4.5c · 23 questions · 200 marks · lowest tariff first



(Total for Question 3 is 5 marks)

Question 1 continued

(Total for Question 1 is 5 marks)



Question 3 continued

Leave
blank

Q3

(Total 6 marks)



5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

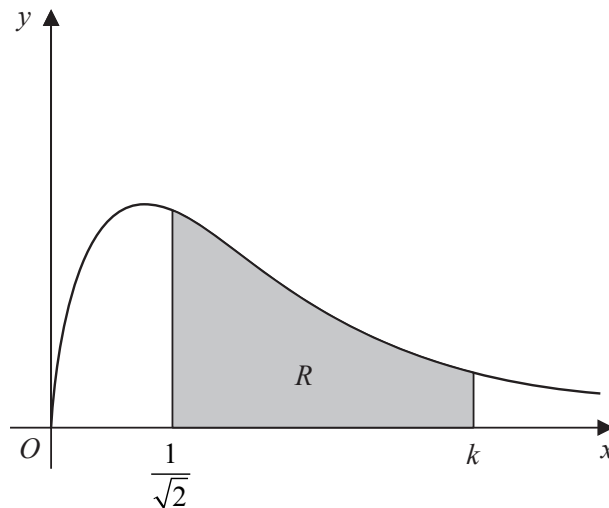


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2 + 3)^{1.5}}$$

The region R , shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{1}{\sqrt{2}}$, the x -axis and the line with equation $x = k$.

This region is rotated through 360° about the x -axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the exact value of the constant k .

(6)







Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 6 marks)



Leave
blank

Question 5 continued

Leave
blank



Q5

Leave
blank



Question 7 continued

Leave
blank

Question 7 continued

Leave
blank

Lined area for writing the answer to Question 7.

(Total 7 marks)

Q7



Leave
blank

Q2

8.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

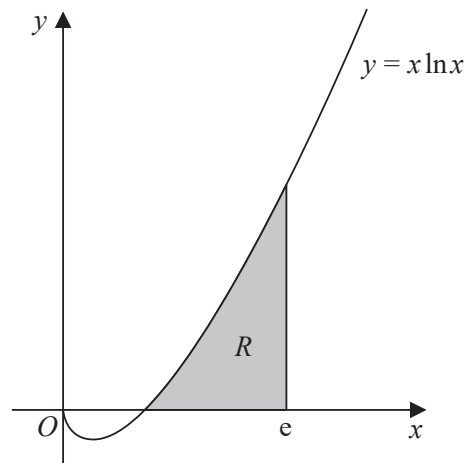


Figure 3

- (a) Find $\int x^2 \ln x \, dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = x \ln x \quad x > 0$$

The region R , shown shaded in Figure 3, lies entirely above the x -axis and is bounded by the curve, the x -axis and the line with equation $x = e$.

This region is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Find the exact volume of the solid formed, giving your answer in simplest form.

(4)

Leave
blank

Leave
blank

Question 8 continued

Leave
blank

Lined area for writing the answer to Question 8 continued.



9. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

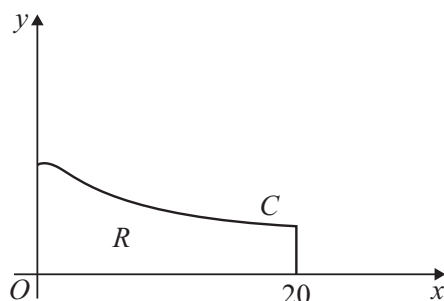


Figure 2

The curve C has equation

$$y = 12 \left(\frac{2x + 3}{x^2 + 3x + 5} \right)^{\frac{1}{2}}$$

The finite region, R , shown in Figure 2, is bounded by the curve C , the line with equation $x = 20$, the x -axis and the y -axis.

The region R is rotated through 2π radians about the x -axis to form a solid, S .

- (a) Use algebraic integration to find the exact volume of S .

Give your answer in the form $k \ln \lambda$ where k and λ are constants.

(4)

Figure 3 shows a hollow vase with identical shape to solid S .

The vase is made of glass, which has negligible thickness.

When the vase is full of water,

$$\text{volume of water in the vase} = \text{volume of } S$$

When the vase contains $360\pi \text{ cm}^3$ of water, the depth of the water is $a \text{ cm}$, as shown in Figure 3.

- (b) Use algebra to find the value of a , giving your answer to one decimal place.

(3)

Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 7 marks)



Leave
blank

7.

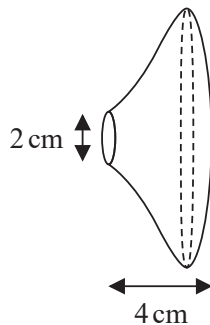


Figure 3

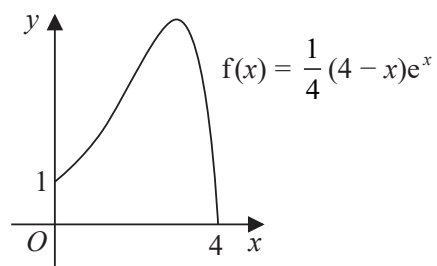


Figure 4

Figure 3 shows the design of a doorknob.

The shape of the doorknob is formed by rotating the curve shown in Figure 4 through 360° about the x -axis, where the units are centimetres.

The equation of the curve is given by

$$f(x) = \frac{1}{4}(4-x)e^x \quad 0 \leq x \leq 4$$

(a) Show that the volume, $V \text{ cm}^3$, of the doorknob is given by

$$V = K \int_0^4 (x^2 - 8x + 16)e^{2x} dx$$

where K is a constant to be found.

(3)

(b) Hence, find the exact value of the volume of the doorknob.

Give your answer in the form $p\pi(e^q + r)\text{cm}^3$ where p , q and r are simplified rational numbers to be found.

(5)

Question 7 continued

Leave
blank

Lined area for writing the answer to Question 7.



Leave
blank

Question 7 continued

Leave
blank

(Total 8 marks)

Q7







(Total for Question 7 is 6 marks)



7.

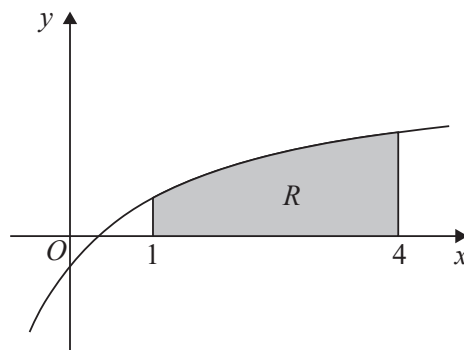


Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = \frac{3x-1}{x+2} \quad x > -2$$

(a) Show that

$$\frac{3x-1}{x+2} \equiv A + \frac{B}{x+2}$$

where A and B are constants to be found.

(2)

The finite region R , shown shaded in Figure 4, is bounded by the curve, the line with equation $x = 4$, the x -axis and the line with equation $x = 1$

This region is rotated through 2π radians about the x -axis to form a solid of revolution.

(b) Use the answer to part (a) and algebraic integration to find the exact volume of the solid generated, giving your answer in the form

$$\pi(p + q \ln 2)$$

where p and q are rational constants.

(6)





(Total for Question 7 is 6 marks)



Question 10 continued



Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS



9.

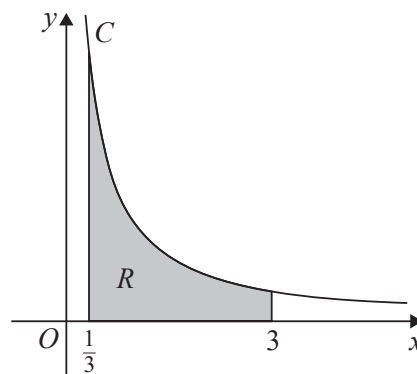


Figure 3

The curve C , shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x} (\arctan \sqrt{x})}$$

The region R , shown shaded in Figure 3, is bounded by C , the line with equation $x = 3$, the x -axis and the line with equation $x = \frac{1}{3}$.

The region R is rotated through 360° about the x -axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

(a) show that the volume V of the solid formed is given by

$$k \int_a^b \frac{1}{u^2} \, du$$

where k , a and b are constants to be found.

(6)

(b) Hence, using algebraic integration, find the value of V in simplest form.

(3)



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS



Leave
blank



Leave
blank



Question 9 continued

Leave
blank

Lined area for writing the answer to Question 9.

(Total 10 marks)

Q9	

6.

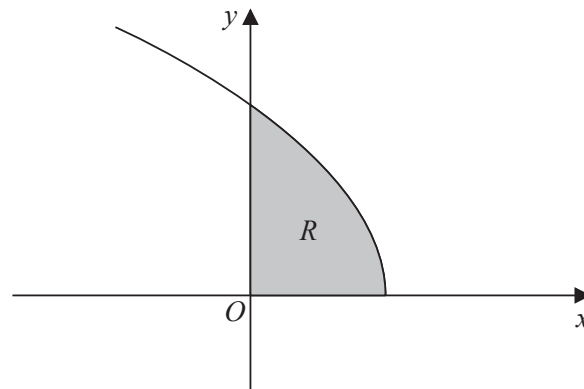


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2 \cos 2t \quad y = 4 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and the y -axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32 \sin^2 t \cos t \, dt$

(ii) Hence find, by algebraic integration, the exact value of the area of R .

(6)

(b) Show that all points on C satisfy $y = \sqrt{ax + b}$, where a and b are constants to be found.

(3)

The curve C has equation $y = f(x)$ where f is the function

$$f(x) = \sqrt{ax + b} \quad -2 \leq x \leq 2$$

and a and b are the constants found in part (b).

(c) State the range of f .

(1)

Leave blank

Leave
blank



Leave
blank

Question 6 continued

Leave
blank

(Total 10 marks)

Q6



Leave
blank

5.

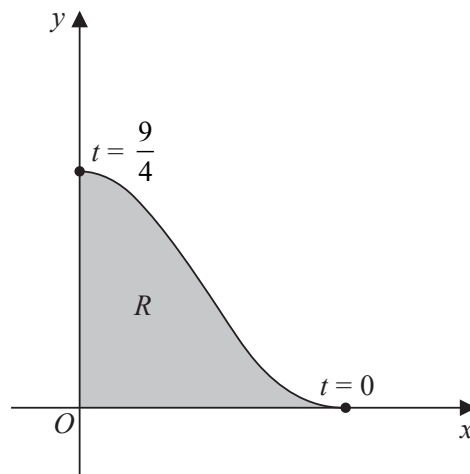


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \quad y = \frac{t^3}{\sqrt{9 + 4t}} \quad 0 \leq t \leq \frac{9}{4}$$

The curve touches the x -axis when $t = 0$ and meets the y -axis when $t = \frac{9}{4}$

The region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the y -axis.

(a) Show that the area of R is given by

$$K \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}} dt$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R .

(Solutions relying on calculator technology are not acceptable.)

(6)



Leave
blank

Question 5 continued

Leave
blank

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Leave
blank

Handwriting practice area with horizontal lines.

(Total 10 marks)

Q5
[] []



8.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

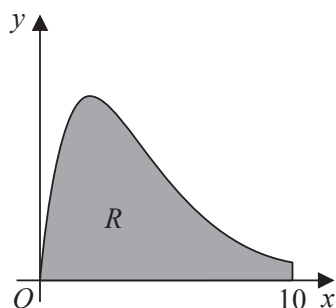


Figure 2

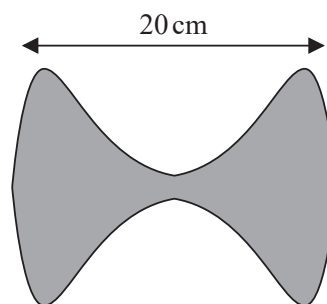


Figure 3

Figure 2 shows the curve with equation

$$y = 10xe^{-\frac{1}{2}x} \quad 0 \leq x \leq 10$$

The finite region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line with equation $x = 10$

The region R is rotated through 2π radians about the x -axis to form a solid of revolution.

(a) Show that the volume, V , of this solid is given by

$$V = k \int_0^{10} x^2 e^{-x} dx$$

where k is a constant to be found.

(2)

(b) Find $\int x^2 e^{-x} dx$

(3)

Figure 3 represents an exercise weight formed by joining two of these solids together.

The exercise weight has mass 5 kg and is 20 cm long.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and using your answers to part (a) and part (b),

(c) find the density of this exercise weight. Give your answer in grams per cm^3 to 3 significant figures.

(5)

Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 10 marks)



Question 9 continued

Leave
blank

Handwriting practice area with 30 horizontal lines.



Question 9 continued

Leave
blank

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Leave
blank

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Leave
blank

Handwriting practice area with horizontal lines.

(Total 11 marks)

END

TOTAL FOR PAPER IS 75 MARKS

Q9



8.

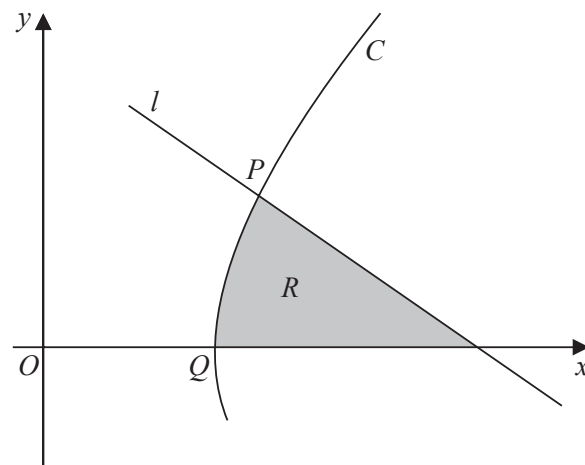


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \quad y = t - \frac{1}{t} \quad t > 0.7$$

The curve C intersects the x -axis at the point Q .

(a) Find the x coordinate of Q .

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

(b) write down the coordinates of P

(1)

(c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

The region, R , shown shaded in Figure 2 is bounded by the curve C , the line l and the x -axis.

(d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x -axis.

(7)

Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS



9. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

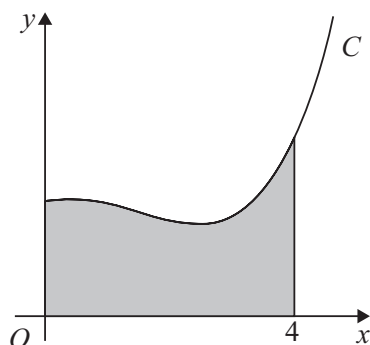


Figure 1

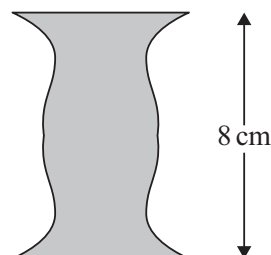


Figure 2

Figure 1 shows a sketch of part of the curve C with equation

$$y = \cos x + \frac{1}{5}e^x$$

The finite region, shown shaded in Figure 1, is bounded by C , the y -axis, the x -axis and the line with equation $x = 4$

The region is rotated through 2π radians about the x -axis to form a solid S .

- (a) Show that the exact volume of S is given by

$$\pi \int_0^4 \left(A + B \cos 2x + Ce^x \cos x + De^{2x} \right) dx$$

where A , B , C and D are constants to be found.

(4)

- (b) Find $\int e^x \cos x \, dx$

(4)

Figure 2 represents a paperweight formed by joining two of these solids together.
The paperweight is 8 cm high.

- (c) Using the answers to parts (a) and (b), find, by algebraic integration, the volume of the paperweight, giving your answer to 2 significant figures.

(4)

Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9 continued.



Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 12 marks)



5.

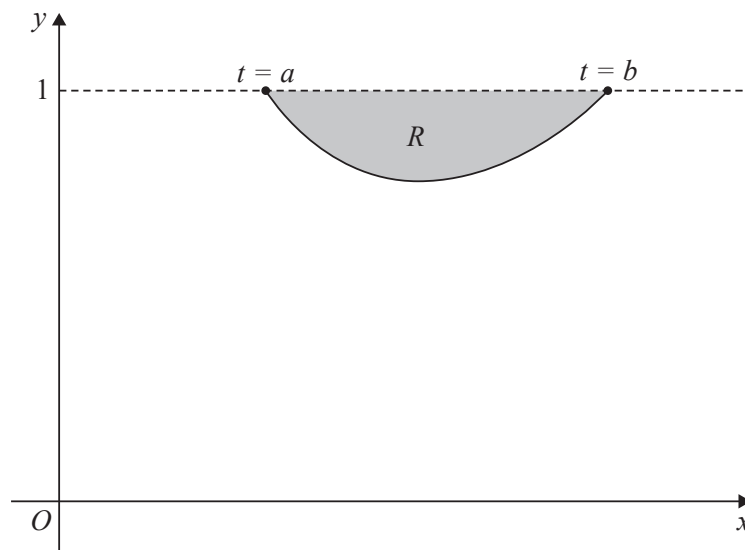


Figure 2

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \quad y = \frac{2}{t(3-t)} \quad a \leq t \leq b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

The region R , shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region R is given by

$$M - k \int_a^b \frac{t+1}{t(3-t)} dt$$

where M and k are constants to be found.

(5)

(c) (i) Write $\frac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of R , giving your answer in simplest form.

(6)



Question 5 continued

Lined area for writing the answer to Question 5 continued.



Question 5 continued

Lined area for writing the answer to Question 5 continued.



(Total for Question 3 is 15 marks)



8.

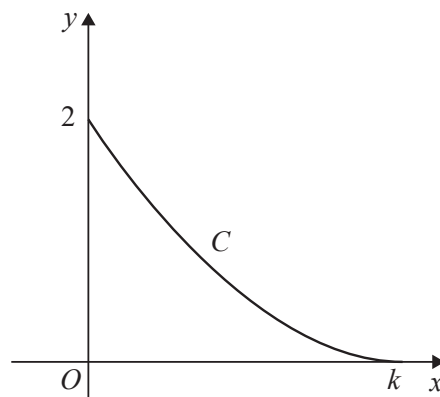


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3 \sin 2t \quad y = 2 \cos t \quad 0 \leq t \leq \frac{\pi}{2}$$

The curve meets the y -axis at 2 and the x -axis at k , where k is a constant.

(a) State the value of k .

(1)

(b) Use parametric differentiation to show that

$$\frac{dy}{dx} = \lambda \operatorname{cosec} t$$

where λ is a constant to be found.

(4)

The point P with parameter $t = \frac{\pi}{4}$ lies on C .

The tangent to C at the point P cuts the y -axis at the point N .

(c) Find the exact y coordinate of N , giving your answer in simplest form.

(3)

The region bounded by the curve, the x -axis and the y -axis is rotated through 2π radians about the x -axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^\alpha \beta (1 - \cos 4t) dt$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.





Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Alt(b) I Solving the differential equation $\frac{dh}{dt} = \frac{75}{6h^2}$ and then using $t = 90$

B1: $\frac{dV}{dh} = \frac{12\pi h^2}{75} \left(= \frac{4\pi h^2}{25} \right)$

M1: Attempts $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} \left(= \frac{75}{6h^2} \right)$ to achieve an expression in h , attempts to solve their

differential equation $\int 6h^2 dh = \int 75 dt \Rightarrow 2h^3 = 75t + c$ o.e. and uses $t = 0, h = 0 \Rightarrow c = 0$

(you may not see the working for c) **to find $h = f(t)$** ($h = (37.5t)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{dh}{dt} \left(= \frac{1}{3} \sqrt[3]{37.5} t^{-\frac{2}{3}} \right)$ and substitutes $t = 90 \Rightarrow \frac{dh}{dt} = \dots$

A1: As above in main scheme

Alt(b) II Equating volumes, rearranging to $h = \dots$ and then using $t = 90$

B1: $V = 2\pi t$

M1: Equates their $2\pi t$ equal to $\frac{4\pi h^3}{75}$ and rearranges to $h = f(t)$ ($h = (37.5t)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{dh}{dt} \left(= \frac{1}{3} \sqrt[3]{37.5} t^{-\frac{2}{3}} \right)$ and substitutes $t = 90 \Rightarrow \frac{dh}{dt} = \dots$

A1: As above in main scheme

Question Number	Scheme	Marks
6	$u = \sqrt{x^3 + 1} \Rightarrow u^2 = x^3 + 1 \Rightarrow 2u \frac{du}{dx} = 3x^2$ or $u = \sqrt{x^3 + 1} \Rightarrow u^2 = x^3 + 1 \Rightarrow 2u = 3x^2 \frac{dx}{du}$ or $u = \sqrt{x^3 + 1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(x^3 + 1)^{-\frac{1}{2}} \times 3x^2$ or $x = (u^2 - 1)^{\frac{1}{3}} \Rightarrow \frac{dx}{du} = \frac{2}{3}u(u^2 - 1)^{-\frac{2}{3}}$	B1
	e.g. $\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = \int \frac{9x^5}{u} \frac{2u}{3x^2} du = \int 6x^3 du = 6 \int (u^2 - 1) du$ or e.g. $\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = \int \frac{9x^5}{\sqrt{x^3 + 1}} \frac{2\sqrt{x^3 + 1}}{3x^2} du = \int 6x^3 du = 6 \int (u^2 - 1) du$	M1A1
	$6 \int (u^2 - 1) du = 2u^3 - 6u (+c) = 2(x^3 + 1)^{\frac{3}{2}} - 6(x^3 + 1)^{\frac{1}{2}} (+c)$	M1
	$= 2(x^3 + 1)^{\frac{1}{2}} [(x^3 + 1) - 3] + c = 2(x^3 + 1)^{\frac{1}{2}} (x^3 - 2) + c$	A1
		(5)
		Total 5

B1: Any correct equation involving $\frac{du}{dx}$ or $\frac{dx}{du}$ or du and dx separately. May be implied by further work.

M1: Attempts to fully change $\int \frac{9x^5}{\sqrt{x^3 + 1}} dx$ into an integral with respect to u . There must be no x 's anywhere. Must include an attempt at replacing dx to get du so M0 if there is no du term present (unless implied by further work) or dx becomes du without an attempt at connecting them first. (i.e. there must have been an attempt at $\frac{du}{dx}$ or $\frac{dx}{du}$). Condone no integral sign for this mark.

A1: Reaches $6 \int (u^2 - 1) du$ o.e. Condone the omission of the integral sign but they must have a du or implied by further work.

M1: Attempts to integrate their expression in terms of u only (usually $\int (Au^2 + B) du$) and attempts to reverse the substitution so that the expression is completely in terms of x (with or without the constant of integration). For the integration look for a correct method to integrate a term i.e. $\dots u^n \rightarrow \dots u^{n+1}$ or $B \rightarrow Bu$ or $\frac{C}{u} \rightarrow \ln u$ at least once. Just seeing $+c$ is not an example of integrating.

A1: Correct expression in the required form including the $+c$



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0 : (x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Notes	Marks
5(a)	$4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $-5 + 6\lambda = 4 - 3\mu$	For writing down any 2 of these equations.	M1
	E.g. $4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $\Rightarrow \lambda = \dots$ or $\mu = \dots$	Full method for finding λ or μ	M1
	$\lambda = 2, \mu = -1$	Both correct values	A1
	$-5 + 6\lambda = -5 + 12 = 7$ $4 - 3\mu = 4 + 3 = 7$ So lines intersect	Shows that the parameters satisfy the third equation and makes a conclusion.	B1
	$\lambda = 2 \rightarrow (4+4)\mathbf{i} + (4-6)\mathbf{j} + (-5+12)\mathbf{k}$ or $\mu = -1 \rightarrow (13-5)\mathbf{i} + (-1-1)\mathbf{j} + (4+3)\mathbf{k}$	Uses their λ or μ to find A .	M1
	$8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$	Correct vector or coordinates	A1
			(6)
(b)	$\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} = 10 - 3 - 18 = \sqrt{2^2 + 3^2 + 6^2} \sqrt{5^2 + 1^2 + 3^2} \cos \theta$ Full attempt at the scalar product between the direction vectors		M1
	$\cos \theta = \pm \frac{11}{7\sqrt{35}}$	Correct magnitude for $\cos \theta$ (may be implied by e.g. $\theta = 105.4\dots$ or $74.6\dots$)	A1
	$\theta = 74.6^\circ$	Awrt 74.6	A1
			(3)
(c)	$ 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k} = \sqrt{2^2 + 3^2 + 6^2} = 7$	Finds the magnitude of the direction of l_1	M1
	$35 \div 7 = 5 \Rightarrow \lambda = 5$ $8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k} \pm 5(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	Correct strategy for one of the points	M1
	$(18, -17, 37)$ or $(-2, 13, -23)$	One correct point (ignore labels)	A1
	$P(18, -17, 37)$ and $Q(-2, 13, -23)$	Correct points with correct labels	A1
			(4)
			Total 13

Question Number	Scheme	Marks
7	$x = 4 \sin \theta \Rightarrow \frac{dx}{d\theta} = 4 \cos \theta$	B1
	$\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx = \int \frac{4 \cos \theta}{(16-16 \sin^2 \theta)^{\frac{3}{2}}} d\theta = \int \frac{4 \cos \theta}{64 \cos^3 \theta} d\theta$	M1 A1
	$= \frac{1}{16} \tan \theta$	dM1
	Uses limits of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or 30° and 60° within their attempted integration and subtracts the right way round e.g. $f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)$ or $f(60^\circ) - f(30^\circ)$	M1
	$= \frac{1}{16} \tan \frac{\pi}{3} - \frac{1}{16} \tan \frac{\pi}{6} = \frac{\sqrt{3}}{24}$	A1
		(6 marks)

Notes:

B1: States or uses $\frac{dx}{d\theta} = 4 \cos \theta$ o.e. e.g. $dx = 4 \cos \theta d\theta$. This may be seen within the integrand.

M1: Simplifies $(16-x^2)^{\frac{3}{2}}$ using the given substitution to $k \cos^3 \theta$ using the Pythagorean identity

This may be implied if the work is correct and they proceed correctly to e.g. $\frac{1}{16} \int \cos^{-2} \theta d\theta$

A1: $\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx = \int \frac{4 \cos \theta}{64 \cos^3 \theta} d\theta$ or exact equivalent e.g. $\frac{1}{16} \int \cos^{-2} \theta d\theta$ or $\int \frac{1}{(4 \cos \theta)^2} d\theta$

or e.g. $\int \frac{4 \cos \theta}{(4 \cos \theta)(4 \cos \theta)(4 \cos \theta)} d\theta$

Condone if the "dθ" is missing but is otherwise correct.

dM1: For $\int k \frac{\cos \theta}{\cos^3 \theta} d\theta = \dots \tan \theta$

M1: Uses limits of $\frac{\pi}{6}(30^\circ)$ and $\frac{\pi}{3}(60^\circ)$ correctly with their attempted integration.

Condone poor integration here.

A1: $\frac{\sqrt{3}}{24}$ or exact equivalent e.g. $\frac{1}{8\sqrt{3}}$



Question Number	Scheme	Marks
7(a)	$u = \tan x \rightarrow \frac{du}{dx} = \sec^2 x$ $u^2 = \tan^2 x = \sec^2 x - 1 \rightarrow \int \frac{u+u^3}{(5+u^2)^3} \times \frac{1}{1+u^2} du$ $\int \frac{u}{(5+u^2)^3} du *$	B1 M1dM1 A1*
		(4)
(b)	$\int \frac{u}{(5+u^2)^3} du = -\frac{1}{4}(5+u^2)^{-2} + c = -\frac{1}{4}(5+\tan^2 x)^{-2} (+c)$	M1A1
		(2)
		(6 marks)

(a)

B1: $\frac{du}{dx} = \sec^2 x$ or exact equivalent seen or used. For example, $\frac{dx}{du} = \frac{1}{1+u^2}$ is correct

M1: Attempts to use $\pm 1 \pm \tan^2 x = \sec^2 x$ in a correct place at least once. Implied by use of $\sec^2 x = \pm 1 \pm u^2$

dM1: Fully substitutes $u = \tan x$ proceeding to an integral in terms of u only.

It is dependent upon the previous M

Look for $\left(\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} dx \right) \rightarrow \int \frac{u + u^3}{(4 \pm 1 \pm u^2)^3} \times f(u) du$ seen on at least one line

Condone a lack of an integral sign. You may never see the integral in terms of x . (It is in the question)

A1*: Proceeds with no errors in the body of their proof to $\int \frac{u}{(5+u^2)^3} du$ including the du .

The dx 's and du 's must be present and correct for their stage of working throughout the proof.

(b)

M1: Attempts to integrate an expression of the form $\int \frac{u}{(k+u^2)^n} du \rightarrow A(k+u^2)^{-n+1}$ where A is a constant and $n \neq 1, n \in \mathbb{N}$

A1: $-\frac{1}{4}(5+\tan^2 x)^{-2} + c$ or equivalent e.g. $-\frac{1}{4}(4+\sec^2 x)^{-2} + c$. (condone omission of $+c$)

This must be in terms of x and you can ISW after sight of a correct answer.

Question Number	Scheme	Marks
3 (a)	Attempts a correct identity $3x^3 + 8x^2 - 3x - 6 \equiv (Ax + B)x(x + 3) + C(x + 3) + Dx$ o.e. Correct method for two constants e.g. $x = 0, x = -3 \Rightarrow C = \dots, D = \dots$ Two correct constants e.g. $C = -2$ and $D = 2$ Correct method to find all four constants (e.g. by comparing coefficients) $A = 3, B = -1, C = -2, D = 2$	M1 dM1 A1 ddM1 A1
(b)	$g(x) = 3x - 1 - \frac{2}{x} + \frac{2}{x+3}$ $\Rightarrow g'(x) = 3 + \frac{2}{x^2} - \frac{2}{(x+3)^2}$	(5) M1 A1ft (2)
(c)	Explains that (if $x > 0$) $\frac{2}{x^2} > \frac{2}{(x+3)^2}$ so $\frac{2}{x^2} - \frac{2}{(x+3)^2} > 0$ and $\Rightarrow g'(x) > 3$	B1 (1)
3 (a) ALT	Via division Attempts to divide $3x^3 + 8x^2 - 3x - 6$ by $x^2 + 3x$ forming a linear quotient Correct method for two constants implied by quotient of $3x + \dots$ Correct quotient $3x - 1$ Correct method to find all four constants (e.g. uses PF on $\frac{rem}{x(x+3)}$) $A = 3, B = -1, C = -2, D = 2$	M1 dM1 A1 ddM1 A1 (5)

(a)

M1: Establishes a method of starting the problem

- either by attempting a correct identity
- or attempting to divide the cubic by $x^2 + 3x$ to obtain a linear quotient

Condone slips in the cubic expression but the intention must be correct

dM1: Attempts a correct method to find two constants. An attempt must be made at a correct equation

For the main method this could be via substituting both $x = 0, x = -3 \Rightarrow C$ and D

They may set up 4 simultaneous equations and solve to find two of the constants.

FYI the equations are $A = 3, 3A + B = 8, 3B + C + D = -3$ and $3C = -6$ For division it will be arriving at a linear quotient of the form $3x \pm \dots$ A1: Achieves **any** two correct constants.Allow $3x - 1$ as the quotient via the alt method. Condone this just written down, i.e. $A = 3, B = -1$

ddM1: Attempts a full method to find all four constants.

In the main method this may be achieved by substituting both $x = 0, x = -3$ followed by comparing coefficients of x^3 and x^2 for instance. Other methods can involve substituting other values of x .In the alternative method it is for dividing and then setting $\frac{\text{remainder}}{x(x+3)} \equiv \frac{C}{x} + \frac{D}{x+3}$ before finding values of C and D A1: Correct values for all four constants $A = 3, B = -1, C = -2, D = 2$

Allow for these to be embedded within the identity

(b)

M1: Attempts to differentiate and achieves $\frac{C}{x} + \frac{D}{x+3} \rightarrow \frac{\dots}{x^2} + \frac{\dots}{(x+3)^2}$ A1ft: Differentiates $Ax + B + \frac{C}{x} + \frac{D}{x+3}$ to $A - \frac{C}{x^2} - \frac{D}{(x+3)^2}$ following through on their constants.

Candidates should simplify their coefficients.

You may award as above with constants A, C and D instead of the numerical values.

(c)

B1: This requires a correct part (a) and a correct (b). Look for a reason and a conclusion

Accept as a minimum $\frac{2}{x^2} - \frac{2}{(x+3)^2} > 0$ so $g'(x) > 3$ Or $\frac{2}{x^2} > \frac{2}{(x+3)^2}$ so $g'(x) > 3$ Also $3 + \frac{2}{x^2} - \frac{2}{(x+3)^2} = 3 + \frac{2(6x+9)}{x^2(x+3)^2}$ and explains that this is $3 + (+ve) > 3$ 

Question Number	Scheme	Marks
4(a)	$\sqrt{1-4x^2} = 1 - \frac{1}{2} \times 4x^2$ $\frac{\frac{1}{2} \times -\frac{1}{2} \times (-4x^2)^2}{2} \quad \text{or} \quad \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} \times (-4x^2)^3}{3!}$ $= 1 - 2x^2 - 2x^4 - 4x^6 + \dots$	B1 M1 A1, A1 (4)
(b)	Substitutes $x = \frac{1}{4}$ into both sides of (a) e.g. $\sqrt{\frac{3}{4}} \approx 1 - 2 \times \left(\frac{1}{4}\right)^2 - 2 \left(\frac{1}{4}\right)^4 - 4 \left(\frac{1}{4}\right)^6$ $\sqrt{3} \approx 1.7324$ cao	M1 A1 (2) (6 marks)

(a)

B1: Correct first two terms which does not need to be simplified, so $1 + \frac{1}{2} \times -4x^2$ is fine.

M1: Correct attempt at the binomial expansion for term 3 **or** term 4.

Look for $\frac{\frac{1}{2} \times -\frac{1}{2} \times (\pm 4x^2)^2}{2}$ or $\frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} \times (\pm 4x^2)^3}{3!}$ but condone a failure to square or cube the 4 (i.e. missing brackets)

Also award this mark for candidates who mistakenly attempt the binomial expansion of

$$\sqrt{1-4x} \text{ to produce either } \frac{\frac{1}{2} \times -\frac{1}{2} \times (\pm 4x)^2}{2} \text{ or } \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} \times (\pm 4x)^3}{3!}$$

$$\text{Or } \sqrt{1-x^2} \text{ to produce either } \frac{\frac{1}{2} \times -\frac{1}{2} \times (\pm x^2)^2}{2} \text{ or } \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} \times (\pm x^2)^3}{3!}$$

A1: Two correct and simplified terms of $-2x^2, -2x^4, -4x^6$

A1: $1 - 2x^2 - 2x^4 - 4x^6$ or exact simplified equivalent such as $1 + (-2)x^2 + (-2)x^4 + (-4)x^6$

Ignore any additional terms. This may be given separately as a list. $1, -2x^2, -2x^4, -4x^6$

(b)

M1: Substitutes $x = \frac{1}{4}$ into both sides of (a) and achieves LHS of $\sqrt{\frac{3}{4}}$ or $\frac{\sqrt{3}}{2}$

$$\text{It would be implied by } (\sqrt{3} =) 2 \times \text{their} \left(1 - 2 \times \left(\frac{1}{4}\right)^2 - 2 \left(\frac{1}{4}\right)^4 - 4 \left(\frac{1}{4}\right)^6 \right) "$$

A1: 1.7324 Correct answer only here. This is not awrt

Note that the calculator answer is 1.7321

Alt (a) It is possible to attempt part (a) as follows but it would be very unusual to get the terms up to x^6

$$\sqrt{1-4x^2} = (1-2x)^{\frac{1}{2}} \times (1+2x)^{\frac{1}{2}} = \left(1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3 \dots \right) \times \left(1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 \dots \right) = 1 \dots$$

Score M1 for such an attempt and award the other marks as in the main scheme. As is the main scheme look for a correct attempt at a term 3 or 4 in either bracket followed by an attempt to multiply out



Question Number	Scheme	Marks
5 (a)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{16 \sec^2 t \tan t}{2 \sec^2 t} = 8 \tan t$ At $x = 3, \tan t = -1 \Rightarrow$ Gradient $= -8$	M1 A1 dM1 A1 (4)
(b)	Attempts to use $1 + \tan^2 t = \sec^2 t \Rightarrow 1 + \frac{(x-5)^2}{4} = \frac{y}{8}$ $y = 2(x-5)^2 + 8$	M1 A1 A1 (3)
(c)	8,, f,, 32	M1 A1 (2) (9 marks)

(a)

M1: Attempts to use the rule $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. Condone incorrect attempts on $\frac{dy}{dt}$ and $\frac{dx}{dt}$

A1: A correct expression for $\frac{dy}{dx} = \frac{16 \sec^2 t \tan t}{2 \sec^2 t}$ or unsimplified equivalent. Note that there may be many different versions of this including ones that have used double angle formulae. Look carefully
dM1: Dependent upon the previous M. It is for

- either substituting $\tan t = -1$ into their $\frac{dy}{dx} = g(t)$ to find the gradient
- or substituting $t = -\frac{\pi}{4}$ into their $\frac{dy}{dx} = g(t)$ to find the gradient. Condone
 $t = \text{awrt } -0.785$

A1: CSO Gradient $= -8$. This cannot be awarded from differentiation of $y = 2(x-5)^2 + 8$



(b)

M1: Attempts to use $\pm 1 \pm \tan^2 t = \pm \sec^2 t$ with $\tan t$ being replaced by an expression in x and $\sec^2 t$ being replaced by an expression in y

A1: A correct unsimplified equation $1 + \frac{(x-5)^2}{4} = \frac{y}{8}$ o.e.

A1: $y = 2(x-5)^2 + 8$. $f(x) = 2(x-5)^2 + 8$ is also fine

NB 1: It is possible to use part (a), find $\frac{dy}{dx} = 4(x-5)$, and then integrate using a point such as (3, 16) to find "c"

NB2: It is possible to use points to generate $f(x)$.

For the M1 you should expect to see $f(x) = ax^2 + bx + c$, or equivalent, and the use 3 points to set up 3 simultaneous equations in a , b and c which must be then solved. At least one of three points used must be correct. Examples of possible points that can be used are $(5, 8)$, $(3, 16)$, $(5 - 2\sqrt{3}, 32)$ and $(7, 16)$

For the A1 you need to see a correct equation, e.g. $y = 2x^2 - 20x + 58$

The final A1 will be $y = 2(x-5)^2 + 8$.

If you see a solution worthy of merit and you cannot see how to award the marks then send to review.

(c)

M1: One correct end found, condoning strict inequalities.

Look for the lower value of the range to be 8 (or their c) or the higher value to be 32

A1: 8,, f,, 32 or equivalent such as $[8, 32]$

Question Number	Scheme	Marks
6	$u = 3 + 4 \sin x \Rightarrow \frac{du}{dx} = 4 \cos x$ $\int \frac{16 \sin 2x}{(3 + 4 \sin x)^2} dx = \int \frac{32 \sin x \cos x}{(3 + 4 \sin x)^2} dx = \int \frac{2(u-3)}{u^2} du$ $= \int \frac{2}{u} - \frac{6}{u^2} du = 2 \ln u + \frac{6}{u}$ Uses limits of 5 and 7 $\Rightarrow 2 \ln 7 + \frac{6}{7} - 2 \ln 5 - \frac{6}{5} = -\frac{12}{35} + \ln \frac{49}{25}$	B1 M1 A1 dM1 A1 M1 A1 (7 marks)

B1: States or uses $\frac{du}{dx} = 4 \cos x$ o.e. This may be seen within the integrand.

M1: Attempts to write all in terms of u .

$$\text{Look for } \int \frac{16 \sin 2x}{(3 + 4 \sin x)^2} dx = \int \frac{\dots \sin x \cos x}{(3 + 4 \sin x)^2} dx = \pm \int \frac{\dots(u \pm 3)}{u^2} du$$

It is common to lose the square on the u^2 which would be M0.

Only the penultimate mark would then be available.

A1: Correct $\int \frac{2(u-3)}{u^2} du$ o.e. Allow this to be unsimplified.

Condone a missing du this can be implied by further work.

dM1: Integrates $\int \frac{\dots(u \pm 3)}{u^2} du$ to $\dots \ln u \pm \frac{\dots}{u}$ o.e. Condone a missing du this can be implied by further work.

Note that $\int \frac{\dots(u \pm 3)}{u^2} du \rightarrow \dots \ln u^2 \pm \frac{\dots}{u}$ is acceptable

You may see attempts using parts.

$$\int \frac{1}{u^2} \times (2u-6) du = -\frac{1}{u} \times (2u-6) - \int -\frac{1}{u} \times 2 du = -\frac{1}{u} \times (2u-6) + 2 \ln u$$

A1: Correct $2 \ln u + \frac{6}{u}$. Note that $\ln u^2 + \frac{6}{u}$ or $-\frac{1}{u} \times (2u-6) + 2 \ln u$ is also completely correct

M1: Uses limits 5 and 7 within their attempted integral and subtracts. Condone poor attempts at the integration. Alternatively converts their answer in u back to x 's using the correct substitution and uses the given limits

A1: $-\frac{12}{35} + \ln \frac{49}{25}$ or equivalent such as $\ln 1.96 - \frac{12}{35}$

ISW after a correct answer.



Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} = \frac{1}{2}(\dots)$	B1
	$= (1 - 20x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (-20x) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (-20x)^2 + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (-20x)^3 \dots$	M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1 A1
	Special case: If the final answer is left as $\frac{1}{2}(1 - 10x - 50x^2 - 500x^3 + \dots)$ Award SC B1M1A1A1A0	
		(5)
	Alternative by direct expansion	
	$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^{-\frac{1}{2}}(-5x)^1 + \frac{\frac{1}{2} \times -\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$	B1M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1A1
(b)	$\left(\frac{1}{4} - \frac{5}{100}\right)^{\frac{1}{2}} = \left(\frac{1}{5}\right)^{\frac{1}{2}} = \frac{1}{2} - 5 \times \frac{1}{100} - 25\left(\frac{1}{100}\right)^2 - 250\left(\frac{1}{100}\right)^3 + \dots$ $\frac{\sqrt{5}}{5} \approx \frac{1789}{4000} \quad \text{or} \quad \frac{1}{\sqrt{5}} \approx \frac{1789}{4000}$ $\Rightarrow \sqrt{5} \approx 5 \times \frac{1789}{4000} \quad \text{or} \quad \sqrt{5} \approx 1 \div \frac{1789}{4000}$	M1
	$\sqrt{5} \approx \frac{1789}{800} \quad \text{or} \quad \frac{4000}{1789}$	A1
		(2)
		(7 marks)

(a)B1: For taking out a factor of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.M1: Expands $(1 + kx)^{\frac{1}{2}}$, $k \neq \pm 1$ with the correct structure for the third or fourth terme.g. $\pm \frac{\frac{1}{2}(-\frac{1}{2})}{2!} \times (kx)^2$ or $\pm \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!} \times (kx)^3$ with or without the bracket around the kx

A1: For either term three or term four being correct in any form.

E.g. $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times (20x)^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times (-20x)^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times (-20x)^3$ or $-\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times (20x)^3$

The brackets must be present unless they are implied by subsequent work. This mark is independent of the B mark.



A1: Two terms correct and simplified of $\frac{1}{2} - 5x - 25x^2 - 250x^3$. Allow if any of the ‘-’ signs are written as “+”.

A1: All four terms correct and simplified of $\frac{1}{2} - 5x - 25x^2 - 250x^3$. Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the ‘-’ signs are written as “+” score A0.

Alternative:

B1: For a first term of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.

M1: For the correct structure for the third or fourth term. E.g. $\frac{\frac{1}{2} \times -\frac{1}{2}}{2} \left(\frac{1}{4}\right)^{-\frac{3}{2}} (kx)^2$ or $\frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!} \left(\frac{1}{4}\right)^{-\frac{5}{2}} (kx)^3$

where $k \neq \pm 1$

A1: For either term three or term four being correct in any form.

$$\text{e.g. } \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times \left(\frac{1}{4}\right)^{-\frac{3}{2}} (-5x)^2 \text{ or } \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times \left(\frac{1}{4}\right)^{-\frac{5}{2}} (-5x)^3$$

The brackets must be present unless they are implied by subsequent work.

A1: Two terms correct and simplified of $\frac{1}{2} - 5x - 25x^2 - 250x^3$. Allow if any of the ‘-’ signs are written as “+”.

A1: All four terms correct and simplified of $\frac{1}{2} - 5x - 25x^2 - 250x^3$. Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the ‘-’ signs are written as “+” score A0.

(b)

M1: Attempts to substitute $x = \frac{1}{100}$ into their part (a) and either multiplies by 5 or finds reciprocal.

A1: $(\sqrt{5}) \frac{1789}{800}$ or $\frac{4000}{1789}$



Question Number	Scheme	Marks
1 (a)	$(1+kx)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (kx) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (kx)^2 + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (kx)^3 \dots$	
(i)	$\frac{1}{2}k = \frac{1}{8} \Rightarrow k = \frac{1}{4}$	M1A1
(ii)	$A = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times "k"{}^2 = -\frac{1}{128} \quad B = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times "k"{}^3 = \frac{1}{1024}$	M1 A1 A1
(b)	Substitutes $x = 0.6 \Rightarrow \sqrt{1.15} = 1 + \frac{1}{8} \times 0.6 - \frac{1}{128} \times 0.6^2 + \frac{1}{1024} \times 0.6^3 = 1.072398$	M1 A1
		(5)
		(2)
		(7 marks)
1(b) alt	$(1+kx)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (kx) - \left(\frac{1}{8}\right) \times (kx)^2 + \left(\frac{1}{16}\right) \times (kx)^3$ Substitutes "kx" = 0.15 $\Rightarrow \sqrt{1.15} = 1 + \frac{1}{2} \times 0.15 - \frac{1}{8} \times 0.15^2 + \frac{1}{16} \times 0.15^3 = 1.072398$	M1A1
		(2)

(a)(i)

 M1: Sets $\frac{1}{2}k = \frac{1}{8}$ or $\frac{1}{2}kx = \frac{1}{8}x$ and proceeds to find k . Implied by a correct value for k

 A1: $k = \frac{1}{4}$ oe such as $\frac{2}{8}$ or 0.25

(a)(ii)

M1: Correct attempt at 3rd or 4th term. Eg. $Ax^2 = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (kx)^2$ or $Bx^3 = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (kx)^3$ with an attempt at substituting in their value for k to find a value for A or a value for B . Condone a missing bracket around the kx terms so allow with k instead of k^2 or k^3 respectively. These may be simplified so award for $A = -\frac{1}{8} \times ("k")^2$ or $B = \frac{1}{16} \times ("k")^3$ again, condoning k instead of k^2 or k^3 respectively.

 A1: $A = -\frac{1}{128}$ o.e. There is no requirement to simplify the fraction

 A1: $B = \frac{1}{1024}$ o.e You may occasionally see $B = \frac{3}{3072}$ which is fine.

(b)

M1: For an attempt to substitute

- either $kx = 0.15$ into an expansion of the form $1 \pm p \times (kx) \pm q \times (kx)^2 \pm r \times (kx)^3$
- or $x = \frac{0.15}{"k"}$ into their $1 + \frac{1}{8}x + "A"x^2 + "B"x^3$

A1: 1.072398 Must be to 6 decimal places.



Question Number	Scheme	Marks
2	Achieves a lower limit of 2 Attempts $\alpha \int y^2 dx = \alpha \int \frac{81}{(2x-3)^{2.5}} dx = \alpha \times \frac{-27}{(2x-3)^{1.5}}$ Correct attempt at volume of solid generated under curve = $= \pi \int_{2''}^6 y^2 dx = \left[\frac{k}{(2x-3)^{1.5}} \right]_{2''}^6 = \left(\left(-\frac{27}{27} \right) - \left(-\frac{27}{1} \right) \right)$ Volume = 26π Correct attempt at volume of solid = $\pi \times 9^2 \times (6 - 2'') - 26\pi$ = 298π	B1 M1, A1 dM1 A1 ddM1 A1 (7 marks)

B1: Finds the lower limit. This may be awarded anywhere. Accept on the diagram or in the limits of an integral

M1: Integrates an expression of the form $\int \frac{\dots}{(2x-3)^{2.5}} dx$ and achieves $\frac{\dots}{(2x-3)^{1.5}}$ oe

A1: Correct integration of $\alpha \int y^2 dx = \alpha \int \frac{81}{(2x-3)^{2.5}} dx$ giving a solution $\alpha \times \frac{-27}{(2x-3)^{1.5}}$ o.e.

following through on their α . Typical values of α will be 1, π or 2π . No need to simplify here

dM1: Attempts to find the volume of the solid generated by rotating the area under the curve.

Look for $\left[\frac{\dots}{(2x-3)^{1.5}} \right]_{2''}^6 = \dots$ It is dependent upon the previous M1.

The top limit must be 6 and the bottom limit must be their solution to $\frac{9}{(2x-3)^{1.25}} = 9$

A1: Correct exact volume for the solid generated by rotating the area under the curve. Volume = 26π

This may be the second part of a larger/complete expression. E.g. $\dots - 26\pi$

ddM1: Correct method for the volume of the solid formed by rotating the area above the curve.

Accept $\pi \times 9^2 \times (6 - 2'') - 26\pi$

It is dependent both previous M's

A1: 298π



Question Number	Scheme	Marks
3 (a)	Attempts to find $\frac{1/3 \times 20^2 \times 24}{160} = 20$ seconds	M1 A1 (2)
(b)	Attempts $\frac{dV}{dh} = h^2 + \frac{8}{3}h$ Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 160 = \left(h^2 + \frac{8}{3}h\right) \times \frac{dh}{dt}$ Substitutes $h = 5 \Rightarrow 160 = \left(5^2 + \frac{40}{3}\right) \times \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{96}{23} \text{ (4.2) cm s}^{-1}$	M1 M1 A1 dM1 A1 (5) (7 marks)

(a)

M1: Attempts to find the volume, using the given formula, at $h = 20$ and dividing by 160. Implied by 20
 Condone slips. For example they may have incorrectly multiplied out/calculated the expression for V

A1: 20 seconds. This requires the correct units as well

(b)

M1: For an attempt to differentiate V

Scored for an attempt to multiply out and then differentiate term by term to achieve $\frac{dV}{dh} = \alpha h^2 + \beta h$

or via the product rule to achieve $V = \frac{1}{3}h^2(h+4) \Rightarrow \frac{dV}{dh} = (h+4) \times ph + qh^2$

M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ (or equivalent) with $\frac{dV}{dt} = 160$ and their $\frac{dV}{dh}$.

A1: Correct expression involving $\frac{dh}{dt}$ and h E.g. $160 = \left(h^2 + \frac{8}{3}h\right) \times \frac{dh}{dt}$

dM1: Dependent upon the previous M only. It is for substituting $h = 5$ and proceeding to a value for $\frac{dh}{dt} = \dots$

A1: Either $\frac{96}{23}$ or awrt 4.2 cm s⁻¹ There is no requirement for the units here



Question	Scheme	Marks
2(a)	$\left(\frac{1}{(1+3x)(1-x)} = \frac{A}{1+3x} + \frac{B}{1-x} \Rightarrow 1 = A(1-x) + B(1+3x) \right)$	B1
	when $x=1 \Rightarrow 1=4B \Rightarrow B=...$ or when $x=-\frac{1}{3} \Rightarrow 1=\frac{4}{3}A \Rightarrow A=...$	M1
	$\frac{3}{4(1+3x)} + \frac{1}{4(1-x)}$	A1
		(3)
(b)	$\int \cot y \, dy = \int ... \, dx \Rightarrow " \ln \sin y " = \int ... \, dx$	M1
	$... = \int \left(" \frac{3}{4(1+3x)} " + " \frac{1}{4(1-x)} " \right) dx = ... \ln(1+3x) \pm ... \ln(1-x) \, (+c)$	M1
	$\ln \sin y = \frac{1}{4} \ln(1+3x) - \frac{1}{4} \ln(1-x) \, (+c) \, \text{oe}$	A1ft
	$\ln \sin \left(\frac{\pi}{2} \right) = \frac{1}{4} \ln \left(1+3 \times \frac{1}{2} \right) - \frac{1}{4} \ln \left(1-\frac{1}{2} \right) + c \Rightarrow c = ... \left(= -\frac{1}{4} \ln 5 \right)$	dM1
	$k \ln \sin y = m \ln(...) \Rightarrow \sin^k y = ...^m \, \text{or} \, k \ln \sin y = ... \Rightarrow \sin^k y = \exp(...)$	M1
	$\sin^4 y = \frac{1+3x}{5(1-x)}$	A1
		(6)
		(9 marks)

Notes**(a)**

B1: For a correct suitable identity without fractions, such as $1 = A(1-x) + B(1+3x)$, seen or implied.

M1: Attempts to find one of the constants by either substitution or equating coefficients. May be implied by a correct value for A or B via cover up rule.

A1: $\frac{3}{4(1+3x)} + \frac{1}{4(1-x)}$ oe allow values for A and B to be stated following a correct partial fraction form, or if correct partial fractions see in (b).

(b)

M1: Attempts to separate variables to form $\cot y \frac{dy}{dx} = g(x)$ (oe for $\cot y$) and integrate $\cot y$. Accept any changed function for the attempt but must be attempting to integrate $\cot y$ (oe).

M1: Attempts to integrate their partial fractions from (a) so award for $\frac{...}{(1+3x)} \rightarrow ... \ln(1+3x)$ or $... \ln(4+12x)$

and $\frac{...}{(1-x)} \rightarrow ... \ln(1-x)$ or $... \ln(4-4x)$ oe

A1ft: Correct expression (any equivalent) (both sides). Follow through on their constants for the partial fractions. Condone the absence of the constant of integration.

dM1: Depends on second M, and must have attempted to integrate both sides. Uses the initial conditions in an equation with a constant of integration. May integrate between limits to achieve this. (Accept if a value for c cannot be reached from their equation.)

M1: Attempts to rearrange their equation by correctly using log work to reach the required form $\sin^n y = f(x)$.

Must have had $k \ln \sin y = ...$ (k may be 1). Not dependent - may be gained before finding the constant if $\ln A$ is used, and allow if the constant is missing.

A1: $\sin^4 y = \frac{1+3x}{5(1-x)}$ (oe in correct form)



Question	Scheme	Marks
3(a)	$\frac{dA}{dt} = -0.5$	B1
	$A = \pi x^2 \Rightarrow \frac{dA}{dx} = 2\pi x$	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = \frac{"-0.5"}{"2\pi x"} \quad \left(= \frac{-1}{4\pi x} \right)$	M1
	$\frac{dx}{dt} = -0.011368\dots$	A1cso
		(4)
(b)	$V = \pi x^2 (3x) = 3\pi x^3$	B1
	$\frac{dV}{dx} = 9\pi x^2$	B1ft
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 9\pi x^2 \times \frac{1}{4\pi x} \quad (= -2.25x)$	M1
	$\left(\frac{dV}{dt} = \right) -9 \Rightarrow (\text{Rate of decrease} =) 9 \text{ (mm}^3 \text{ s}^{-1})$	A1
		(4)
		(8 marks)
Notes		
(a)		
B1: $\frac{dA}{dt} = -0.5$ seen or implied from working		
B1: $\frac{dA}{dx} = 2\pi x$ seen or implied from working. Must be in terms of x , but allow recovery if in terms of r and later work uses $r = 7$ to achieve a solution.		
M1: Attempts to use an appropriate chain rule with their $\frac{dA}{dt}$ and $\frac{dA}{dx}$ e.g. $\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = \dots$		
A1: awrt -0.0114 or $-\frac{1}{28\pi}$ cso (must have the negative sign)		
(b)		
B1: $V = \pi x^2 (3x)$ or $V = 3\pi x^3$		
B1ft: $\frac{dV}{dx} = 9\pi x^2$ or ft from their equation for V in one variable		
M1: Their $\frac{dV}{dx} \times$ their $\frac{dx}{dt}$. Note the $\frac{dx}{dt}$ must be in terms of x or with $x = 4$ substituted first, M0 if they use their answer to (a).		
A1: (Rate of decrease =) $9 \text{ (mm}^3 \text{ s}^{-1})$ (with or without the negative sign). May be scored following $\frac{dA}{dt} = 0.5$ in part (a)		

Question	Scheme	Marks
4(a)	$16x^3 - 9kx^2y + 8y^3 = 875$	
	$(8)y^3 \rightarrow (8 \times) 3y^2 \frac{dy}{dx}$	B1
	$-9kx^2y \rightarrow \dots kxy \pm \dots - 9kx^2 \frac{dy}{dx}$	M1
	$48x^2 - 18kxy - 9kx^2 \frac{dy}{dx} + 24y^2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} (24y^2 - 9kx^2) = 18kxy - 48x^2 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2} \quad *$	A1*
		(4)
(b)	$\frac{dy}{dx} = 0, x = \frac{5}{2} \Rightarrow \frac{6k\left(\frac{5}{2}\right)y - 16\left(\frac{5}{2}\right)^2}{8y^2 - 3k\left(\frac{5}{2}\right)^2} = 0 \quad \text{or}$ $x = \frac{5}{2} \Rightarrow 16\left(\frac{5}{2}\right)^3 - 9k\left(\frac{5}{2}\right)^2 y + 8y^3 = 875$	M1
	$15ky - 100 = 0 \quad \text{or} \quad 250 - \frac{225}{4}ky + 8y^3 = 875$	A1
	E.g. $16\left(\frac{5}{2}\right)^3 - 9k\left(\frac{5}{2}\right)^2\left(\frac{20}{3k}\right) + 8\left(\frac{20}{3k}\right)^3 = 875 \Rightarrow k^3 = \dots \left(= \frac{64}{27} \right) \Rightarrow k = \dots$	M1
	$k = \frac{4}{3}$	A1
		(4)
		(8 marks)



Question Number	Scheme	Marks
1(a)	$2y^2 - 3y = 7 + 13 \Rightarrow 2y^2 - 3y - 20 = 0 \Rightarrow y = \dots$	M1
	$y = -\frac{5}{2} \text{ and } y = 4$	A1
		(2)
(b)	$2y^2 \rightarrow 4y \frac{dy}{dx}$	B1
	$-6xy \rightarrow \pm \dots x \frac{dy}{dx} \pm \dots y$	M1
	$4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y = 14e^{2x-1}$	A1
	$4(4) \frac{dy}{dx} - 6\left(\frac{1}{2}\right) \frac{dy}{dx} - 6(4) = 14e^{2\left(\frac{1}{2}\right)-1} \Rightarrow \frac{dy}{dx} = \dots$	M1
	$y - "4" = "\frac{38}{13}" \left(x - \frac{1}{2}\right)$	M1
	$38x - 13y + 33 = 0$	A1
		(6)
		(8 marks)

(a)

M1: Attempts all of the following

- substitutes $x = \frac{1}{2}$ into the equation for C (condoning slips),
- collect terms and forms a 3TQ in y (terms do not need to be on one side of the equation and the '=' may be implied by subsequent working)
- solves their 3TQ in y using usual rules (including via a calculator) to find at least one value for y .
If a calculator is used, the values must be correct for their 3TQ

A1: Achieves both values $-\frac{5}{2}, 4$ following the award of M1

(b)

B1: $2y^2 \rightarrow 4y \frac{dy}{dx}$ o.e. For example, accept $2y^2 \rightarrow 2 \times 2y \frac{dy}{dx}$

M1: Attempts the product rule on $-6xy$. Look for $\pm \alpha x \frac{dy}{dx} \pm \beta y$

Don't be concerned by the signs here unless they state and use an incorrect product rule $vu' - uv'$

A1: Fully correct differentiation $4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y = 14e^{2x-1}$ o.e. Allow un-simplified versions

You may see versions like $4y dy - 6x dy - 6y dx = 14e^{2x-1} dx$ or $4yy' - 6xy' - 6y = 14e^{2x-1}$ which is

acceptable. If they write $\frac{dy}{dx} = 4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y - 14e^{2x-1} = 0$ this will be A0 unless the $\frac{dy}{dx}$ on the left subsequently disappears and is ignored.

M1: Attempts to substitute in $x = \frac{1}{2}$ with their positive y value to find a value for the gradient of the tangent. It

is dependent upon having exactly two terms in $\frac{dy}{dx}$ one each from the differentiation of $2y^2$ and $6xy$. This

can be implied by a correct value for their $\frac{dy}{dx}$. Allow slips if they attempt to make $\frac{dy}{dx}$ the subject before

substituting in $x = \frac{1}{2}$ with their positive y value. FYI $\frac{dy}{dx} = \frac{6y + 14e^{2x-1}}{4y - 6x} \rightarrow \frac{dy}{dx} = \frac{24 + 14}{16 - 3} = \frac{38}{13}$

M1: Uses the value for their $\frac{dy}{dx}$ with the point $\left(\frac{1}{2}, a\right)$ where a is their positive y coordinate to find the

equation of the tangent. If they have two positive y values, use of either one would be OK for this mark. It must follow some attempt at differentiating the equation for C . If the form $y = mx + c$ is used the method must proceed as far as $c = \dots$

A1: $38x - 13y + 33 = 0$ (or any integer multiple of this). The order of the terms is not important



Question Number	Scheme	Marks
2	$\frac{dV}{dt} = \pm k$	B1
	$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dr} = 4\pi r^2$	B1
	$\pm k = \lambda r^2 \times \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{\pm k}{4\pi r^2} \Rightarrow \frac{dr}{dt} \propto \frac{1}{r^2}$ ✓ *	A1*
		(4 marks)

B1: States or uses $\frac{dV}{dt} = k$ or $\frac{dV}{dt} = -k$ (We don't know if k is positive or negative).

Ignore any reference to units even if they are incorrect. E.g. $\frac{dV}{dt} = k \text{ cm}^3 / \text{sec}$

B1: States or uses $\frac{dV}{dr} = 4\pi r^2$ o.e.

M1: Uses a correct chain rule e.g. $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ or equivalent with $\frac{dV}{dt} = \pm k$ and $\frac{dV}{dr} = \lambda r^2$

A1*: Completes the proof by looking for the following three elements

- achieves $\frac{dr}{dt} = \frac{\pm k}{4\pi r^2}$
- Offers a valid reason why $\frac{dr}{dt} \propto \frac{1}{r^2}$
- gives a minimal conclusion

All previous marks must have been awarded

Examples of acceptable reasons and acceptable conclusions are;

E.g. I $\frac{dr}{dt} = -\frac{k}{4\pi r^2} = -\frac{c}{r^2}$, hence proven (Note that c cannot be k)

E.g. II $\frac{dr}{dt} = \frac{k}{4\pi r^2}$, as $\frac{k}{4\pi}$ is a constant $\frac{dr}{dt} \propto \frac{1}{r^2}$

E.g. III $\frac{dr}{dt} = -\frac{k}{4\pi r^2}$ as k and π are constants, it is true

E.g. IV $\frac{dr}{dt} = -\frac{k}{4\pi r^2} = -\frac{k}{4\pi} \times \frac{1}{r^2}$ or $\left(-\frac{k}{4\pi}\right) \times \frac{1}{r^2}$ ✓

E.g. V. $\frac{dr}{dt} : \frac{k}{4\pi} \cdot \frac{1}{r^2}$ Reason

$\frac{dr}{dt} \propto \frac{1}{r^2}$ Conclusion

Question Number	Scheme	Marks
3	$y \cos^2(2x) \frac{dy}{dx} = 3 \sin 2x \rightarrow \int y \, dy = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx$ $\int y \, dy = \int 3 \sec(2x) \tan(2x) \, dx$ $\frac{y^2}{2} = \frac{3}{2} \sec 2x (+c)$ $\frac{16}{2} = \frac{3}{2} \sec\left(2 \frac{\pi}{6}\right) + c \Rightarrow c = \dots (=5)$ $y^2 = 3 \sec 2x + 10$	M1 A1 M1A1 dM1 A1
		(6)

Main method

M1: For an attempt to separate the variables **OR** use of $\frac{\sin 2x}{\cos^2 2x} \equiv \sec 2x \tan 2x$

For the attempt at separation of variables it is not necessary to have the integral signs but the dx and dy must be present and in the correct positions (on at least one line)

E.g. $y \cos^2(2x) \frac{dy}{dx} = 3 \sin 2x \rightarrow \int y \, dy = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx$ or $\int y \frac{dy}{dx} \, dx = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx$

Condone a slip on the position (or omission) of the 3

A1: Correct separation of variables **AND** use of $\frac{\sin 2x}{\cos^2 2x} = \sec 2x \tan 2x$

E.g. $\int y \, dy = \int 3 \sec(2x) \tan(2x) \, dx$ o.e. with or without the integral signs

Note that $\int \frac{1}{3} y \, dy = \int \sec(2x) \tan(2x) \, dx$ is also correct

M1: Correct attempted integration of the $\sec 2x \tan 2x$ to reach $f(y) = \alpha \sec 2x (+c)$ with or without the c

A1: Correct integration.

For example, $\frac{y^2}{2} = \frac{3}{2} \sec 2x (+c)$ Accept with or without the c

Note that that $y^2 = 3 \sec 2x (+c)$ and $\frac{y^2}{3} = \sec 2x (+c)$ are also examples of correct work.

dM1: Substitutes in $y = 4$ and $x = \frac{\pi}{6}$ into an integrated equation of the form $f(y) = \alpha \sec 2x + c$ and proceeds to find c . It is dependent on the previous method mark. You may need to check calculations if the substitution isn't shown. You may see this attempted via $\left[\frac{y^2}{2} \right]_4^y = \left[\frac{3}{2} \sec 2x \right]_{\frac{\pi}{6}}^x$

A1: $y^2 = 3 \sec 2x + 10$ or equivalent such as $y^2 = \frac{3}{\cos 2x} + 10$ but ISW after a correct answer.

The answer is required to be in the form $y^2 =$

A score of 1,0,1,1,1 is possible for candidates who never have a dx and/or a dy when separating the variables, but do everything else correctly.



Question Number	Scheme	Marks
3 ALT	$y \cos^2(2x) \frac{dy}{dx} = 3 \sin 2x \rightarrow \int y \, dy = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx \rightarrow \int y \, dy = 3(\cos 2x)^n$ where $n = -1$ or -3 $\frac{y^2}{2} = \frac{3}{2}(\cos 2x)^{-1} (+c)$ $\frac{16}{2} = \frac{3}{2 \cos\left(\frac{\pi}{3}\right)} + c \Rightarrow c = \dots (=5)$ $y^2 = \frac{3}{\cos 2x} + 10$	M1, A1 M1A1 dM1 A1

Alt versions will integrate $\frac{\sin 2x}{\cos^2 2x}$ using substitution or reverse chain rule

The key here is realising that the integral of this is $(\cos 2x)^n$ or u^n where $u = \cos 2x$

M1: For an attempt to separate the variables **OR** use of $\int \frac{\sin 2x}{\cos^2 2x} dx \equiv (\cos 2x)^n$

For separation of variables look for example for $y \cos^2(2x) \frac{dy}{dx} = 3 \sin 2x \rightarrow \int y \, dy = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx$.
condoning a slip on the position (or omission) of the 3 and omission of the integral signs

A1: Correct separation of variables **AND** use of $\int \frac{\sin 2x}{\cos^2 2x} dx \equiv (\cos 2x)^n$ where $n = -1$ or -3

If the candidate sets $u = \cos 2x$ it is for integrating $\int \frac{\sin 2x}{\cos^2 2x} dx$ to u^n where $n = -1$ or -3

M1: Correct attempted integration of the $\int \frac{\sin 2x}{\cos^2 2x} dx$ to reach $f(y) = \frac{\alpha}{\cos 2x} (+c)$ o.e with or without the c

A1: Correct integration. For example, $\frac{y^2}{2} = \frac{3}{2 \cos 2x} (+c)$ o.e. Accept with or without the c

Note that that $y^2 = \frac{3}{\cos 2x} (+c)$ and $\frac{y^2}{3} = (\cos 2x)^{-1} (+c)$ are also examples of correct work.

dM1: Substitutes in $y = 4$ and $x = \frac{\pi}{6}$ into an integrated equation of the form $f(y) = \frac{\alpha}{\cos 2x} + c$ o.e. and proceeds to find c . It is dependent on the previous method mark.

A1: $y^2 = \frac{3}{\cos 2x} + 10$ or equivalent such as $y^2 = \frac{3}{\cos 2x} + 10$ but ISW after a correct answer.

The answer is required to be in the form $y^2 =$

.....
Also note that you may attempt using integration by parts. FYI $\int 3 \sin 2x \sec^2 2x \, dx = \frac{3}{2} \sin 2x \tan 2x + \frac{3}{2} \cos 2x$

1st A1: Correct separation of variables and $\int 3 \sin 2x \sec^2 2x \, dx = \pm \lambda \sin 2x \tan 2x \pm \kappa \cos 2x \quad \lambda, \kappa > 0$

2nd M1: $\int 3 \sin 2x \sec^2 2x \, dx = \lambda \sin 2x \tan 2x \pm \kappa \cos 2x \quad \lambda, \kappa > 0$

2nd A1: $\int 3 \sin 2x \sec^2 2x \, dx = \frac{3}{2} \sin 2x \tan 2x + \frac{3}{2} \cos 2x$

A1: cao $x = -2$ (ignore any references to the y coordinate). Do not accept $-\frac{10}{5}$
Alt (b)
M1: Solves the parametric equations simultaneously with the line equation to find a value for t
A1: cao Deduces correct value for x .

2 (a) Alt 1	$\frac{dx}{dt} = \frac{(2t+1)-2(t-1)}{(2t+1)^2}$ and $\frac{dy}{dt} = \frac{-12}{(2t+1)^2}$ o.e.	M1 A1
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	dM1
	$\frac{dy}{dx} = \frac{-12}{(2t+1)^2} \div \frac{3}{(2t+1)^2} = -4$ (which is a constant,) hence linear	A1* (4)

Alt 1 (a) via differentiation **Notes:**

M1: Attempts $\frac{dx}{dt}$ and $\frac{dy}{dt}$ using appropriate rule for at least one.

A1: Both correct

dM1: Depends on first M. Attempts $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ leading to constant.

A1: Achieves $\frac{dy}{dx} = -4$ and makes suitable conclusion e.g. “hence linear” *

2 (a) Alt 2	$x = \frac{t-1}{2t+1} \Rightarrow x = A - \frac{B}{2t+1}; x = \frac{1}{2} - \frac{3}{2(2t+1)}$	M1; A1
	$x = \frac{1}{2} - \frac{3}{2(6/y)}$	dM1
	$x = \frac{1}{2} - \frac{1}{4}y$ hence linear *	A1* (4)

Alt 2 (a) via division **Notes:**

M1: Attempts to write x in terms of just $2t+1$. E.g $x = \frac{t-1}{2t+1} \Rightarrow x = A - \frac{B}{2t+1}$.

A1: $x = \frac{1}{2} - \frac{3}{2(2t+1)}$

dM1: Uses $y = \frac{6}{2t+1}$ to form an equation linking x and y

A1: $x = \frac{1}{2} - \frac{1}{4}y$ and states linear*

2 (a) Alt 3	$ax + by = \frac{at - a + 6b}{2t+1} = \frac{k(2t+1)}{2t+1} \Rightarrow a = 2k, 6b - a = k$	M1
	$a = 12b - 2a \Rightarrow a = 4b$	A1
	E.g. $4x + y = \frac{2(2t+1)}{2t+1} = \dots$	dM1
	$4x + y = 2$ (oe) hence linear *	A1* (4)

Alt 3 (a) via elimination Notes:

M1: Writes $ax + by = \dots$ as a single fraction and attempts to compare coefficients of numerator with the denominator.

A1: Correct ratio between a and b deduced.

dM1: Uses their ratio to eliminate t from the equation.

A1: $4x + y = 2$ (oe) and states linear*

Note: It is possible to spot the correct values for a and b directly so the following would gain

full marks: $2x + \frac{1}{2}y = \frac{2t - 2 + 3}{2t + 1} = \frac{2t + 1}{2t + 1} = 1$ hence linear. *

Question Number	Scheme	Marks
3.	States or implies $\text{Volume} = \int_{\sqrt{5}}^5 \pi \left(\sqrt{\frac{3x}{3x^2+5}} \right)^2 dx$	B1
	$\int \left(\sqrt{\frac{3x}{3x^2+5}} \right)^2 dx = \int \frac{3x}{(3x^2+5)} dx = \frac{1}{2} \ln(3x^2+5)$	M1A1
	$\text{Volume} = \left\{ \pi \right\} \left(\frac{1}{2} \ln(3 \times 25 + 5) - \frac{1}{2} \ln(3 \times 5 + 5) \right)$	M1
	$= \pi \ln 2$	A1
		(5 marks)
Notes:		
B1: States or implies $\text{Volume} = \int_{\sqrt{5}}^5 \pi \left(\sqrt{\frac{3x}{3x^2+5}} \right)^2 dx$ o.e. The limits may be implied by subsequent work, and the dx may be missing. This is for knowing the correct formula rather than for notation. The π may be implied by later work. M1: Attempts $\int \left(\sqrt{\frac{3x}{3x^2+5}} \right)^2 dx$ to achieve $k \ln(3x^2+5)$ (oe). May use substitution, either $u = 3x^2$ or $u = 3x^2+5$, in which case they should achieve $k \ln(u+5)$ or $k \ln(u)$ (oe) respectively. Allow if the brackets are missing. A1: Correct result of integration, which may be left unsimplified. May be in terms of u if a substitution has been used. Allow if missing brackets are recovered, but A0 if never recovered. M1: Having achieved an integral of the form $p(x) \ln(3x^2+5)$ (allowing for missing brackets) where $p(x)$ is constant or a polynomial in x (oe in terms of u for substitution), uses the limits within their integral - substitutes correct limits for their variable and subtracts, allowing either way round. The π may be missing for this mark. A1: $\pi \ln 2$ cao. Note $\frac{\pi}{2} \ln 4$ is A0 as form is not as specified.		

Question Number	Scheme	Marks
4 (a)	$a = 3, b = 5$	B1
	E.g $u = \sqrt{2x+1} \Rightarrow \frac{dx}{du} = u$ or $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ o.e.	B1
	$\int \sqrt{8x+4} e^{\sqrt{2x+1}} dx = \int 2u e^u u du$	M1
	$= \int_3^5 2u^2 e^u du$	A1
		(4)
(b)	$\int \sqrt{8x+4} e^{\sqrt{2x+1}} dx = \int 2u^2 e^u du$	
	$= 2u^2 e^u - \int 4u e^u du$	M1
	$= 2u^2 e^u - (4ue^u - 4e^u) = 2u^2 e^u - 4ue^u + 4e^u$	dM1 A1ft
	$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx = \left[2u^2 e^u - 4ue^u + 4e^u \right]_3^5 = (50e^5 - 20e^5 + 4e^5) - (18e^3 - 12e^3 + 4e^3)$	ddM1
	$= 34e^5 - 10e^3$	A1
		(5)
		(9 marks)

Notes:

(a)

B1: For both $a = 3, b = 5$ seen in their solution. Allow if these are recovered in (b).B1: For a correct expression involving $\frac{du}{dx}$ or $\frac{dx}{du}$ or du and dx separately. May be unsimplifiedM1: Attempts to fully change $\int \sqrt{8x+4} e^{\sqrt{2x+1}} dx$ into an integral with respect to u . Must include an attempt at replacing dx to get du so M0 if there are no du terms present or dx becomes du without an attempt at connecting them first (ie there must have been an attempt at $\frac{du}{dx}$ oe).A1: Complete method to show $I = \int_3^5 2u^2 e^u du$. Must include the correct limits and the du .

(b) Note: you may see different ways of presenting the application of parts e.g D/I method.

M1: Use of integration by parts once to obtain $pu^2 e^u - \int qu e^u du$, where $p, q > 0$ (if k is positive, otherwise signs will be opposite) and may be in terms of k (as can the dM mark).dM1: Completely integrates by parts twice to a form $pu^2 e^u - que^u \pm re^u$ where $p, q > 0$ (if $k > 0$ as before). Note they may evaluate in stages, but look for the complete integration overall.A1ft: $\int_a^b ku^2 e^u du = ku^2 e^u - 2kue^u + 2ke^u$ (oe) accepted with k or their value for k . May have the last two terms bracketed but must be seen as a complete answer in their work.ddM1: Substitutes their a and b into a form $pu^2 e^u - que^u \pm re^u$ and subtracts (either way), but must be using a value for k at this stage. May be done in stages.

A1: $34e^5 - 10e^3$ or exact equivalent in a simplified form such as $2e^3(17e^2 - 5)$



(c) **Otherwise:** Applies binomial expansion to $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}}$

B1: Correct simplified expression $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \frac{1}{2}(1-2x)^{\frac{1}{2}}$

M1: Correct structure for 3rd term: $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-2x)^2$ but allow $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm kx)^2$ with or without brackets. Even allow with $k=1$

A1: $\frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$ Ignore extra terms of x^3 and above

Direct expansion in (a)

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{3}{2}} = \left(\frac{1}{4}\right)^{\frac{3}{2}} + \left(-\frac{3}{2}\right)\left(\frac{1}{4}\right)^{\frac{5}{2}}\left(-\frac{1}{2}x\right) + \frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{\frac{7}{2}}\left(-\frac{1}{2}x\right)^2 + \frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)\times\left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{\frac{9}{2}}\left(-\frac{1}{2}x\right)^3$$

B1: Obtains an expansion with a constant term of 8

M1: Attempts the binomial expansion of $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.

Look for $\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{\frac{7}{2}}\left(\pm\frac{1}{2}x\right)^2$ or $\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)\times\left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{\frac{9}{2}}\left(\pm\frac{1}{2}x\right)^3$

with or without the brackets on the $\left(\pm\frac{1}{2}x\right)$

A1: Correct simplified or unsimplified expansion .

Expression at the top of the page is acceptable

A1: Two correct and simplified terms of $8 + 24x + 60x^2 + 140x^3$

A1: All correct and simplified $8 + 24x + 60x^2 + 140x^3$

Direct expansion in (c)

B1: First two terms which may be unsimplified $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} + \frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}\left(-\frac{1}{2}x\right)^1 + \dots$

M1: Correct form for the third term $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}\left(-\frac{1}{2}x\right)^2$

A1: Correct simplified expansion $\frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$

Question Number	Scheme	Marks
2(a)	$x = 2 \Rightarrow 4 - 8y + y^2 = 13 \Rightarrow (y^2 - 8y - 9 = 0 \Rightarrow) y = \dots$	M1
	$y = 9$	A1
		(2)
(b)	$2^x \rightarrow 2^x \ln 2$	B1
	$-4xy \rightarrow -4x \frac{dy}{dx} - 4y$ OR $y^2 \rightarrow 2y \frac{dy}{dx}$	M1
	$2^x \ln 2 - 4x \frac{dy}{dx} - 4y + 2y \frac{dy}{dx} = 0$	A1
	$\frac{dy}{dx}(2y - 4x) = 4y - 2^x \ln 2 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{4y - 2^x \ln 2}{2y - 4x}$ or $\frac{dy}{dx} = \frac{2^x \ln 2 - 4y}{4x - 2y}$ or $\frac{dy}{dx} = \frac{\ln 2 e^{x \ln 2} - 4y}{4x - 2y}$	A1
		(5)
(c)	$(2, 9) \rightarrow \frac{dy}{dx} = \frac{4(9) - 2^2 \ln 2}{2(9) - 4(2)}$ $\Rightarrow y - "9" = \frac{36 - 4 \ln 2}{10} (x - 2)$	M1
	$y = 0 \Rightarrow 0 - "9" = \frac{36 - 4 \ln 2}{10} (x - 2) \Rightarrow x = \dots$	dM1
	$x = \frac{4 \ln 2 + 9}{2 \ln 2 - 18}$ o.e. e.g. $x = \frac{-8 \ln 2 - 18}{-4 \ln 2 + 36}$	A1
	.	(3)
		Total 10

(a)

M1: Substitutes $x = 2$ into the equation for C, forms a quadratic equation in y and then solves to obtain at least one value for y A1: $y = 9$ only(b) It is acceptable to use $y' \leftrightarrow \frac{dy}{dx}$ in this questionB1: Correct differentiation of 2^x . Allow also $2^x = e^{x \ln 2} \rightarrow e^{x \ln 2} \ln 2$ M1: Differentiates $-4xy \rightarrow \pm Ax \frac{dy}{dx} \pm By$ **OR** differentiates $y^2 \rightarrow k y \frac{dy}{dx}$ A1: Fully correct differentiation. Allow versions such as $2^x \ln 2 dx - 4x dy - 4y dx + 2y dy = 0$ M1: Attempts to make $\frac{dy}{dx}$ the subject with the 2 terms in $\frac{dy}{dx}$ coming from differentiating y^2 and $-4xy$ A1: Any correct expression for $\frac{dy}{dx}$. Note that you can isw after a correct answer. $\frac{2y - 2^{x-1} \ln 2}{y - 2x}$ **is** correct

(c)

M1: Uses $x = 2$ and their y value from part (a) to find the gradient at P and attempts to form the equation of the tangent at P . Condone poor attempts at differentiation but $\frac{dy}{dx}$ must have both x and y terms.

If the form $y = mx + c$ is used they must proceed as far as $c = \dots$

dM1: Substitutes $y = 0$ into their tangent equation and rearranges to find x . Dependent upon previous M. It is possible for a method to combine this with the previous M.

Look for methods like the following, which score M1, dM1

$$-9 = m(x - 2) \Rightarrow x = \dots \text{ where 'm' is the value of their } \frac{dy}{dx} \text{ at } x = 2$$

$$\text{or } \frac{9}{2 - x} = m \Rightarrow x = \dots \text{ where 'm' is the value of their } \frac{dy}{dx} \text{ at } x = 2$$

A1: Correct expression in the required form. Must come from a correct $\frac{dy}{dx}$

Do not be concerned about the order of the terms on the numerator and denominator

Question Number	Scheme	Marks
3(a)	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{A}{2x-1} + \frac{B}{4x-3} \Rightarrow 8x-5 = A(4x-3) + B(2x-1) \Rightarrow A = \dots B = \dots$	M1
	$A=1 \text{ or } B=2$	A1
	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{1}{2x-1} + \frac{2}{4x-3}$	A1
		(3)
(b)	$\int \left(\frac{1}{2x-1} + \frac{2}{4x-3} \right) dx = \frac{1}{2} \ln 2x-1 + \frac{1}{2} \ln 4x-3 (+c)$ Follow through their A and B	M1, A1ft, A1ft
		(3)
(c)	$\left[\frac{1}{2} \ln(2x-1) + \frac{1}{2} \ln(4x-3) \right]_k^{3k} = \frac{1}{2} \ln(6k-1)(12k-3) - \frac{1}{2} \ln(2k-1)(4k-3)$ $= \frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$	M1
	$\frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = \frac{1}{2} \ln 20 \Rightarrow \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = 20$ $\Rightarrow (6k-1)(12k-3) = 20(2k-1)(4k-3) \Rightarrow 88k^2 - 170k + 57 = 0$	dM1A1
	$88k^2 - 170k + 57 = 0 \Rightarrow k = \dots$	ddM1
	$k = \frac{3}{2}$	A1
		(5)
		Total 11

(a)

M1: "Correct" P.F form (condoning slips) with strategy to find at least one constant or partial fraction.

Look for an attempt at the correct form (condoning slips) $5-8x = A(4x-3) + B(2x-1)$ followed by values for A or B

A1: One correct fraction or constant

A1: Correct partial fractions which may be awarded for sight of correct PF in part (b).

Award if the correct answer is just written down with little or no working.

Award if the correct partial fractions are seen in (b)

(b)

M1: For $\int \frac{\alpha}{2x-1} dx = \beta \ln|2x-1|$ OR $\int \frac{\alpha}{4x-3} dx = \beta \ln|4x-3|$ Allow for example brackets instead of moduli, e.g. $\ln(2x-1) \leftrightarrow \ln|2x-1|$ Condone the omission of moduli/brackets, for example $\int \frac{\alpha}{2x-1} dx = \beta \ln 2x-1$ for this mark onlyA1ft: For $\int \frac{A}{2x-1} dx = \frac{A}{2} \ln|2x-1|$ OR $\int \frac{B}{4x-3} dx = \frac{B}{4} \ln|4x-3|$ Follow through their A and BAllow for example brackets instead of moduli, e.g. $\ln(2x-1) \leftrightarrow \ln|2x-1|$

Question Number	Scheme	Marks
7(a)	$\frac{du}{dx} = e^x$	B1
	$\int \frac{24e^x - 2}{(2e^x - 1)^2} dx = \int \frac{24\left(u + \frac{1}{2}\right) - 2}{(2u)^2} \times \frac{du}{\left(u + \frac{1}{2}\right)}$	M1
	$= \int \frac{24u + 12 - 2}{(2u)^2} \times \frac{2}{2u + 1} du = \int \frac{48u + 20}{4u^2(2u + 1)} du = \int \frac{12u + 5}{u^2(2u + 1)} du$	A1
	$x = \ln 3 \rightarrow a = \frac{5}{2}, \quad x = \ln 8 \rightarrow b = \frac{15}{2}$	B1
		(4)
(a) Note that apart from the limits, this is a given answer so check the work carefully. B1: Any correct equation connecting dx and du e.g. $\frac{du}{dx} = e^x, \quad \frac{dx}{du} = \frac{1}{e^x}, \quad du = e^x dx, \quad 1 = e^x \frac{dx}{du}, \quad dx = \frac{du}{e^x}, \quad dx = \frac{du}{u + \frac{1}{2}} \text{ etc.}$ M1: A full attempt to substitute $u = e^x - \frac{1}{2}$ proceeding to an expression in terms of u only (which is not the given answer). Condone slips such as a poor rearrangement of $\frac{du}{dx} = e^x$ but it must include an attempt to replace dx with a function of u. A1: Achieves the given integral with sufficient and no incorrect working seen including invisible brackets. You can ignore any limits for this mark. There must be at least one intermediate line of working following the stage at which they obtain their integral totally in terms of u. This may be seen in “side-working”. You can condone e.g. a missing “du” in the working as long as it is present in the final answer. B1: Correct limits seen at any stage and they do not need to be seen attached to the integral but if there is any contradiction, the limits attached to the integral take precedence. Note that an <u>example</u> of minimum acceptable work for the first 3 marks in (a) could be: $\frac{du}{dx} = e^x, \quad \int \frac{24e^x - 2}{(2e^x - 1)^2} dx = \int \frac{24\left(u + \frac{1}{2}\right) - 2}{(2u)^2} \times \frac{du}{\left(u + \frac{1}{2}\right)} = \int \frac{24u + 10}{(2u)^2} \times \frac{du}{u + \frac{1}{2}} = \int \frac{12u + 5}{u^2(2u + 1)} du$		

(b)	$12u + 5 \equiv A(2u + 1) + Bu(2u + 1) + Cu^2 \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$ or e.g. $12u + 5 \equiv (2B + C)u^2 + (2A + B)u + A \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$	M1
	2 of: $A = 5, B = 2, C = -4$	A1
	$\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)}$	A1
		(3)
(b) M1: Attempts to find at least one of the values for A, B and C by multiplying up and substituting values or by multiplying up and comparing coefficients. When multiplying up must have at least one of the terms on the rhs correct for their lhs. A1: Two of $A = 5, B = 2, C = -4$ A1: Correct fractions : $\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)}$ oe. Allow $\frac{5}{u^2} + \frac{2}{u} + -\frac{4}{(2u+1)}$ If the correct fractions are not seen in part (b), allow this mark to score if the correct fractions are seen in part (c).		
(c)	$\int \frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)} du = -5u^{-1} + 2\ln u - 2\ln(2u+1) (+c)$	M1A1ft
	$\left[-5u^{-1} + 2\ln u - 2\ln(2u+1) \right]_{2.5}^{7.5}$ $= (-5 \times 7.5^{-1} + 2\ln 7.5 - 2\ln(16)) - (-5 \times 2.5^{-1} + 2\ln 2.5 - 2\ln(6))$ or $\left[-5\left(e^x - \frac{1}{2}\right)^{-1} + 2\ln\left(e^x - \frac{1}{2}\right) - 2\ln\left(2\left(e^x - \frac{1}{2}\right) + 1\right) \right]_{\ln 3}^{\ln 8}$ $= (-5 \times 7.5^{-1} + 2\ln 7.5 - 2\ln(16)) - (-5 \times 2.5^{-1} + 2\ln 2.5 - 2\ln(6))$	dM1
	$\frac{4}{3} + \ln\left(\frac{81}{64}\right)$	A1
		(4)
		(11 marks)
(c) M1: Integrates either $\frac{"2"}{u} \rightarrow \dots \ln ku$ or $\frac{"4"}{(2u+1)} \rightarrow \dots \ln k(2u+1)$ A1ft: $-5u^{-1} + 2\ln u - 2\ln(2u+1)$ oe following through on their non-zero A, B and C from part (b). Allow unsimplified. dM1: Substitutes in their changed limits from part (a) or transforms back in terms of x i.e. $\ln 3$ and $\ln 8$ and subtracts either way round. Depends on the previous method mark. A1: $\frac{4}{3} + \ln\left(\frac{81}{64}\right)$ or equivalent in the required form i.e. $p + \ln q$		

Question Number	Scheme	Marks
4(a)	A and B are where $t^3 - 4t = 0 \Rightarrow t(t^2 - 4) = 0 \Rightarrow t = 2$ or -2 Substitutes $t = 2, x = 2 \times 4 - 6 \times 2 = -4$ Hence $A = (-4, 0)$ When $t = -2, x = 2 \times 4 - 6 \times -2 = 20, (y = 0)$ Hence $B = (20, 0)$ *	M1 A1 B1* (3)
(b)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 4}{4t - 6}$ Sub $t = -2$ into $\frac{dy}{dx} = \frac{3t^2 - 4}{4t - 6} \Rightarrow \text{gradient} = \left(-\frac{4}{7}\right)$ Uses their $\left(-\frac{4}{7}\right)$ and $(20, 0)$ to produce eqn of tangent $\Rightarrow 7y + 4x - 80 = 0$ *	M1A1 M1 M1 A1* (5)
(c)	Substitutes $x = 2t^2 - 6t, y = t^3 - 4t$, into $7y + 4x - 80 = 0$ $\Rightarrow 7(t^3 - 4t) + 4(2t^2 - 6t) - 80 = 0$ $\Rightarrow 7t^3 + 8t^2 - 52t - 80 = 0$ $\Rightarrow (t + 2)^2(7t - 20) = 0$ $t = -\frac{20}{7} \Rightarrow x = \dots$ $x = -\frac{40}{49}$	M1 A1 dM1 A1 (4) (12 marks)

(a)

M1: Sets $t^3 - 4t = 0 \Rightarrow t = \dots$ to reach $t = 2$ or -2

A1: Substitutes $t = 2 \Rightarrow x = 2 \times 4 - 6 \times 2 = -4$ and states coordinates of $A = (-4, 0)$

Accept $t = 2 \Rightarrow A = (-4, 0)$ provided the M1 has been awarded

B1*: Show that the coordinates of $B = (20, 0)$

Possible ways of this are

1) Solves $t^3 - 4t = 0 \Rightarrow t = -2$ and substitutes into $2t^2 - 6t \Rightarrow x = 20 \Rightarrow B = (20, 0)$.

This is a show that and there must be some evidence for the "20", not just sight of $t = -2$

Look for $2 \times (-2)^2 - 6 \times (-2) \Rightarrow x = 20$ or $8 + 12 \Rightarrow x = 20$

2) Alternatively sets $2t^2 - 6t = 20 \Rightarrow t = -2$ and substitutes into $y = t^3 - 4t \rightarrow y = 0 \Rightarrow B = (20, 0)$

This is a show that and there must be some evidence for the "0", not just sight of $t = -2$

(b)

M1: Attempts to differentiate $x(t)$ and $y(t)$ and calculates $\frac{dy}{dx}$ by using $\frac{dy/dt}{dx/dt}$ in part (b)

A1: $\frac{dy}{dx} = \frac{3t^2 - 4}{4t - 6}$

M1: Sub 'their' $t = -2$ into their $\frac{dy}{dx}$ to find the gradient of the tangent at B .

M1: Uses $(20, 0)$ and their $\frac{dy}{dx}$ to find an equation of the tangent.

If they use $y = mx + c$ they need to proceed as far as $c = \dots$

A1*: Achieves given answer. $cs0 \Rightarrow 7y + 4x - 80 = 0$ The $= 0$ must be seen

ISW after sight of this

(c) **The demand here is to use algebra which is satisfied by the setting up of the cubic equation.**

M1: Attempts to substitute $x = 2t^2 - 6t$, $y = t^3 - 4t$ into $7y + 4x - 80 = 0$

A1: Correct simplified equation in t : $7t^3 + 8t^2 - 52t - 80 = 0$ The $= 0$ may be implied by further work

dM1: Uses a correct method to find x

- Using a calculator this can be scored for finding a value of $t \neq -2$ and substituting this in $x(t)$. You may have to check this on a calculator if their equation is incorrect. An accuracy of 2 sf is required for non exact solutions. Note that the equation may be in different forms under this method so $7t^3 + 8t^2 - 52t = 80 \Rightarrow t = \frac{20}{7}$ is fine

Note: $7(t^3 - 4t) + 4(2t^2 - 6t) - 80 = 0 \Rightarrow t = \frac{20}{7} \Rightarrow x = -\frac{40}{49}$ would score M1 A0 dM1 A0

- Via factorisation look for factors where the ends work,
E.g. " $7t^3 + 8t^2 - 52t - 80$ " = $(Pt + a)(Qt + b)(Rt + c)$ with $PQR = "7"$ and $abc = "\pm 80"$
followed by substitution of a value of $t \neq -2$ in $x(t)$

A1: CSO $x = -\frac{40}{49}$ Do not accept decimals here but remember to isw following a correct answer.

Ignore any references to the y coordinate. Penalise extra solutions, incorrect factorisation etc on this mark.

Question Number	Scheme	Marks
7 (i)	$u = e^x - 3 \Rightarrow \frac{du}{dx} = e^x$ $\int \frac{4e^{3x}}{e^x - 3} dx = \int \frac{4(u+3)^3}{u(u+3)} du = \int \frac{4(u+3)^2}{u} du$ $= \int \frac{4u^2 + 24u + 36}{u} du$ $= \int 4u + 24 + \frac{36}{u} du = 2u^2 + 24u + 36 \ln u$ $= \left[2u^2 + 24u + 36 \ln u \right]_{u=2}^{u=4}$ $= 72 + 36 \ln 2$	B1 M1 A1 dM1 A1 M1 A1 (7)
(ii)	$\int 3e^x \cos 2x dx = \frac{3}{2} e^x \sin 2x - \int \frac{3}{2} e^x \sin 2x dx$ $= \frac{3}{2} e^x \sin 2x - \left(-\frac{3}{4} e^x \cos 2x + \int \frac{3}{4} e^x \cos 2x dx \right)$ Collect terms $\frac{15}{4} \int e^x \cos 2x dx = \frac{3}{2} e^x \sin 2x + \frac{3}{4} e^x \cos 2x$ $\int 3e^x \cos 2x dx = \frac{6}{5} e^x \sin 2x + \frac{3}{5} e^x \cos 2x (+c)$	M1 A1 dM1 ddM1 A1 (5)

(12 marks)

(i) The 4 must be present to score the A marks. No misreads

B1: States or uses $\frac{du}{dx} = e^x$ o.e. such as $\frac{dx}{du} = \frac{1}{u+3}$. This may be seen within the integrand.M1: A "correct" attempt to write the integrand in terms of u .Look for $\int \frac{4e^{3x}}{e^x - 3} dx = \int \frac{P(u \pm 3)^3}{u(u \pm 3)} du$ which does not need to be simplified.Condone a missing du but dx is M0A1: Correct $\int \frac{4(u+3)^2}{u} du$. Condone a missing du (cannot be dx).

dM1: Integrates by attempting to multiply out and integrating each term.

Look for two "correct" terms, one of which must be $\int \frac{P}{u} du = P \ln u$.It is dependent upon having achieved $\int \frac{P(u \pm 3)^2}{u} du$ A1: Correct $2u^2 + 24u + 36 \ln u$ o.e.

M1: Uses limits 2 and 4 within their attempted integral and subtracts. Condone poor attempts at the integration

Alternatively converts their answer in u back to x 's using the correct substitution and uses the given limits.There must have been some attempt to use the substitution $u = e^x - 3$ at some point.

A1: $72 + 36 \ln 2$ or $36 \ln 2 + 72$

(ii) Condone missing dx's here

M1: Attempts to integrate by parts $\int 3e^x \cos 2x \, dx = Ae^x \sin 2x \pm \int Be^x \sin 2x \, dx$

A1: $\int 3e^x \cos 2x \, dx = \frac{3}{2}e^x \sin 2x - \int \frac{3}{2}e^x \sin 2x \, dx$ Condone missing dx's

The 3 may have been "removed"/factorised out and replaced at the final answer.

If so you can this would be awarded at the sight of a correct answer.

dM1: Attempts to integrate by parts again. See below of what to look for

Look for $\int 3e^x \cos 2x \, dx = Ae^x \sin 2x \pm Be^x \cos 2x \pm \int C e^x \cos 2x \, dx$

ddM1: Dependent upon both previous M's

It is awarded for attempting to collect both terms in $\int e^x \cos 2x \, dx$

A1: $\int 3e^x \cos 2x \, dx = \frac{6}{5}e^x \sin 2x + \frac{3}{5}e^x \cos 2x (+c)$ condone the omission of the + c.

Alternative method for (ii) . This can be scored in a similar way to the above.

(ii)	$\int 3e^x \cos 2x \, dx = 3e^x \cos 2x + \int 6e^x \sin 2x \, dx$	M1 A1
	$= 3e^x \cos 2x + 6e^x \sin 2x - \int 12e^x \cos 2x \, dx$	dM1
	$15 \int e^x \cos 2x \, dx = 6e^x \sin 2x + 3e^x \cos 2x$	ddM1
	$\int 3e^x \cos 2x \, dx = \frac{6}{5}e^x \sin 2x + \frac{3}{5}e^x \cos 2x (+c)$	A1
		(5)

1stM1: $\int 3e^x \cos 2x \, dx = Pe^x \cos 2x \pm Q \int e^x \sin 2x \, dx$

2ndM1: $\int 3e^x \cos 2x \, dx = Pe^x \cos 2x \pm Qe^x \sin 2x \pm R \int e^x \cos 2x \, dx$



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



	$= 5 + \frac{15}{2}x^2 + \dots$	A1
		(5)
Notes:		
a)		
M1: Attempts at correct PF. Correct form identified (may be implicit) and achieves a value for at least one of the constants.		
A1: One correct value or term.		
A1: Correct PF form $\frac{3}{1-x} + \frac{4}{2+3x}$. This may be awarded if seen in (b) but the correct final form (not just values) must be seen somewhere in the question. Accept at $3(1-x)^{-1} + 4(2+3x)^{-1}$		
(b)(i)		
B1: $\frac{A}{1-x} = A(1-x)^{-1} = A(1+x+x^2+\dots)$ which may be unsimplified. Allow with their A or with A = 1.		
M1: Attempts to expand $\frac{1}{2+3x} = (2+3x)^{-1}$ binomially either by taking out the factor 2 first, or directly. Look for $(1+kx)^{-1} = \dots \left(1 \pm kx + \frac{(-1)(-2)}{2}(kx)^2 + \dots \right)$ where $k \neq 1$ following an attempt at taking out a factor 2, or $\frac{1}{2+3x} = (2+3x)^{-1} = \left(2^{-1} \pm 2^{-2}kx + \frac{(-1)(-2)}{2}2^{-3}(kx)^2 + \dots \right)$ by direct expansion. Allow missing brackets on kx^2 in either case.		
A1: $\frac{B}{2+3x} = \frac{B}{2} \left(1 + \frac{3x}{2} \right)^{-1} = \frac{B}{2} \left(1 - \frac{3x}{2} + \frac{9}{4}x^2 + \dots \right)$ oe with their B from (a) or with B = 1		
M1: Uses their coefficients and attempts to add both series.		
A1cao: $5 + \frac{15}{2}x^2 + \dots$ Condone additional higher order terms. Terms may be either order.		
(b)(ii)		
B1: $ x < \frac{2}{3}$ or exact equivalent. This must be clearly identified as the answer. B0 if both ranges are given with no choice of which is correct. (But B1 if formal set notation with $\cap$ used.)		
(b)(i) Alt 1:		
B1: $(1-x)^{-1} = 1+x+x^2+\dots$ which may be unsimplified.		
M1: Same as main scheme.		
A1: Correct expansion (see main scheme, B = 1 allowed).		
M1: Attempts to expand all three brackets, achieving the correct constant term at least.		
A1cso: $5 + \frac{15}{2}x^2 + \dots$ Condone additional higher order terms. Terms may be either order.		
(b)(i) Alt 2		
B1: Writes f(x) as $(5x+10) \left(2 + \left(x-3x^2 \right) \right)^{-1}$ or with the 2 extracted, with the $\left(x-3x^2 \right)$ clear.		

M1: Attempts the binomial expansion on $(1 + p(x))^{-1}$ or $(2 + p(x))^{-1}$ for $p(x)$ of form $ax + bx^2$.

Same conditions as for main scheme.

A1: Correct expansion. For direct expansion $\left(\frac{1}{2} - \frac{1}{4}(x - 3x^2) + \frac{1}{8}(x - 3x^2)^2 + \dots \right)$

M1: Expands the brackets achieving at least the correct constant term.

A1cao: $5 + \frac{15}{2}x^2 + \dots$ Condone additional higher order terms. Terms may be either order.



Question Number	Scheme	Marks
2 (a)	E.g. $x = \frac{t-1}{2t+1} \Rightarrow t = \frac{x+1}{1-2x}$ or $y = \frac{6}{2t+1} \Rightarrow t = \frac{6-y}{2y}$	M1
	E.g. $y = \frac{6}{2t+1} \Rightarrow y = \frac{6}{2 \times \left(\frac{x+1}{1-2x} \right) + 1}$ or $t = \frac{6-y}{2y} \Rightarrow x = \frac{\frac{6-y}{2y} - 1}{2 \times \frac{6-y}{2y} + 1}$	A1
	E.g. $y = \frac{6}{2 \times \left(\frac{x+1}{1-2x} \right) + 1} \Rightarrow y = \frac{6(1-2x)}{2 \times (x+1) + 1(1-2x)} = ax + b$	dM1
	E.g. $y = \frac{6(1-2x)}{3}, y = 2(1-2x)$ oe so linear *	A1*
		(4)
(b)	$y = 2(1-2x)$ and $y = x+12 \Rightarrow 2(1-2x) = x+12 \Rightarrow x = \dots$	M1
	$x = -2$	A1cao
		(2)
Alt (b)	$\frac{6}{2t+1} = \frac{t-1}{2t+1} + 12 \Rightarrow t = \left(-\frac{1}{5} \right)$	M1
	$x = \frac{-\frac{1}{5} - 1}{2 \times -\frac{1}{5} + 1} = -2$	A1
		(2)
		(6 marks)

Notes:

(a) Do not recover marks for part (a) from part (b) if there is an attempt at part (a). If there is no labelling mark as a whole.

M1: For an attempt to get t in terms of x or y **or** x and y $\frac{x}{y} = \frac{t-1}{6} \Rightarrow t = \frac{6x+y}{y}$ or full

method to eliminate t from the equations.

A1: Forms a correct equation linking x and y only. Other forms are possible using t in terms of x and y in either equation for x or y etc.

dM1: Depends on first M. Attempts to simplify the fraction reaching a linear form in x and y . Allow if there are slips but an unsimplified equation of form $ax + by = c$ must be achieved.

A1: Achieves $y = 2(1-2x)$ o.e. (and isw after a correct linear equation) and states linear or hence on line etc. There must be a reference to linearity in some form (similarly for the Alts).

(b)

M1: Solves their " $y = 2(1-2x)$ " (may not be linear) with $y = x+12$, E.g. $2(1-2x) = x+12 \Rightarrow x = \dots$

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:**1. Factorisation**

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \dots$$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0, \text{ leading to } x = \dots$$

Method marks for differentiation and integration:**1. Differentiation**

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice



Question Number	Scheme	Marks
1	Assume that there exists a number m such that when m^3 is even, m is odd If m is odd then $m = 2p + 1$ (where p is an integer) and $m^3 = (2p + 1)^3 = \dots$ $= 8p^3 + 12p^2 + 6p + 1$ $2 \times (4p^3 + 6p^2 + 3p) + 1$ is odd and hence we have a contradiction so if n^3 is even, then n is even.	B1 M1 A1 A1 (4) (4 marks)

B1: For setting up the contradiction.

Eg Assume that there exists a number m such that when m^3 is even, m is odd

Condone a contra-positive statement here

"Assume that there exists a number m such that when m^3 is even, m is not even"

As a minimum accept "assume if m^3 is even then m is odd."

Condone the other way around "assume if n is odd then n^3 is even"

M1: Attempts to cube an odd number. Accept an attempt at $(2p + 1)^3$, $(2p - 1)^3$

Look for $(2p + 1)^3 = \dots p^3 \dots$

A1: $(2p + 1)^3 = 8p^3 + 12p^2 + 6p + 1$ or simplified equivalent such as $2 \times (4p^3 + 6p^2 + 3p) + 1$.

For $(2p - 1)^3 = 8p^3 - 12p^2 + 6p - 1$ or equivalent such as $2 \times (4p^3 - 6p^2 + 3p - 1) + 1$

A1: For a fully correct proof. Requires correct calculations with reason and conclusion

E.g. 1 Correct calculations $(2p + 1)^3 = 8p^3 + 12p^2 + 6p + 1 =$

Reason (even + 1) = odd

Conclusion "hence we have a contradiction, so if n^3 is even, then n is even."

E.g. 2 Correct calculations $(2p + 1)^3 = 8p^3 + 12p^2 + 6p + 1$

Reason $= 2 \times (4p^3 + 6p^2 + 3p) + 1 =$ odd

Conclusion "this is contradiction, so proven."

E.g. 3 Correct calculations $(2p - 1)^3 = 8p^3 - 12p^2 + 6p - 1$

Reason $= 8p^3 - 12p^2 + 6p$ is even so $8p^3 - 12p^2 + 6p - 1$ is odd



Question Number	Scheme	Marks
4(a)	$\frac{1}{\sqrt{4-x^2}} = (4-x^2)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}(1-\dots)$ $\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{1}{4}x^2\right)^2 + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$ $\frac{1}{\sqrt{4-x^2}} = \frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$	B1 M1, A1 A1, A1 (5)
(b)	$ x < 2$	B1
(c)	Substitutes an appropriate value of x in both sides with LHS in terms of $\sqrt{3}$ E.g. with $x = 1$ $\sqrt{3} = \frac{2048}{1181}$ or $\frac{3543}{2048}$	M1 A1 (2)
		(8 marks)

(a) Note that the first mark in (a) is M1 on open. We are now marking it as B1

B1: Correct constant term/ factor.

Accept as $4^{-\frac{1}{2}}$ or $\frac{1}{2}$ as the constant term or as the factor with the bracket starting (1....

M1: Correct attempt at the third or the fourth term of binomial expansion of form $(1+ax^2)^{-\frac{1}{2}}$

Look for a correct binomial coefficient with a correct power of x

E.g. $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(\pm ax^2\right)^2$ or $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(\pm ax^2\right)^3$ where a could be

1

A1: Correct unsimplified

For example

$$\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(-\frac{1}{4}x^2\right)^2 + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$$

A1: Two correct and simplified terms of $\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$

A1: $\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$. Accept as a list, ignore any terms with indices greater than these.

(b)

B1: States a correct range. E.g. $|x| < 2$ or $-2 < x < 2$ o.e.

(c) For example: See **

M1: Substitutes an exact value of x into both sides of their expansion that enables $\sqrt{3}$ to be found

.



$$\text{E.g. } x=1 \quad \frac{1}{\sqrt{4-x^2}} = \frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$$

Note that $x = -1$ and $x^2 = 1$ all work with the same results

Alternatively correctly solves $\frac{1}{\sqrt{4-x^2}} = \sqrt{3}$ o.e. and substitutes this into their expansion

A1: With $x=1$ $\sqrt{3} = \frac{2048}{1181}$ or $\frac{3543}{2048}$. This can only be scored from a correct expansion

******There are lots of values of x that work, all more difficult than the one above.

$$\text{E.g. } x^2 = \frac{8}{3} \Rightarrow \frac{1}{\sqrt{4-x^2}} = \frac{\sqrt{3}}{2} \quad \text{and} \quad x^2 = \frac{11}{3} \Rightarrow \frac{1}{\sqrt{4-x^2}} = \sqrt{3}$$

In both these cases the fractions become much more difficult to find.

$$\text{For } x^2 = \frac{8}{3} \Rightarrow \sqrt{3} = \frac{43}{27} \quad \text{and for } x^2 = \frac{11}{3} \Rightarrow \sqrt{3} = \frac{55687}{55296}$$

Alt I (a) By direct expansion

$$\frac{1}{\sqrt{4-x^2}} = \left(4-x^2\right)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) \times 4^{-\frac{3}{2}} \left(-x^2\right)^1 + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times 4^{-\frac{5}{2}} \left(-x^2\right)^2 + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right)}{3!} \times 4^{-\frac{7}{2}} \left(-x^2\right)^3$$

B1: For $4^{-\frac{1}{2}} +$

M1: A correct attempt at the third or fourth terms condoning sign slips on the $-x^2$

A1: Correct and unsimplified expansion. See above

A1: Two correct and simplified terms of $\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$

A1: $\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$. Accept as a list

Alt II (a) By difference of two squares

$$\frac{1}{\sqrt{4-x^2}} = \left(2-x\right)^{-\frac{1}{2}} \times \left(2+x\right)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \dots$$

B1: For $4^{-\frac{1}{2}} +$

M1: For correctly writing $\frac{1}{\sqrt{4-x^2}} = \left(2-x\right)^{-\frac{1}{2}} \times \left(2+x\right)^{-\frac{1}{2}} = \dots \left(1-\frac{x}{2}\right)^{-\frac{1}{2}} \times \left(1+\frac{x}{2}\right)^{-\frac{1}{2}} =$

with a correct attempt at the third or fourth term in either expansion, followed by an attempt to combine both the expansions.

A1: One correct term in x^2 , x^4 or x^6

A1: Two correct and simplified terms of $\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$ with no terms in x , x^3 or x^5

A1: Fully correct

Question Number	Scheme	Marks
5.	States or implies Volume = $\int_{\frac{1}{\sqrt{2}}}^k \pi \left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}} \right)^2 dx$ Attempts $\int \left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}} \right)^2 dx = \int \frac{144x}{(2x^2+3)^3} dx = -18(2x^2+3)^{-2}$ Sets $-\frac{18\pi}{(2k^2+3)^2} + \frac{18\pi}{(2 \times \frac{1}{2} + 3)^2} = \frac{713}{648}\pi$ $\Rightarrow -\frac{18}{(2k^2+3)^2} + \frac{9}{8} = \frac{713}{648} \Rightarrow (2k^2+3)^2 = 729 \Rightarrow k^2 = \dots$ $k = 2\sqrt{3}$	B1 M1A1 M1 ddM1 A1 (6 marks)

B1: States or implies Volume = $\int_{\frac{1}{\sqrt{2}}}^k \pi \left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}} \right)^2 dx$ o.e. including limits and π .

You can condone a missing dx and the limits the wrong way around.

You may not see the limits until the integration has been attempted which is fine.

M1: Attempts $\int \left(\frac{\sqrt{x}}{(2x^2+3)^{1.5}} \right)^2 dx$ to achieve $A(2x^2+3)^{-2}$ o.e.

With substitution of $u = 2x^2 + 3$ look for Au^{-2}

A1: Correct integration of $\int \left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}} \right)^2 dx$ to $-18(2x^2+3)^{-2}$ or $-18u^{-2}$ which may be left unsimplified.

Note that this can be scored with or without the π and may even have an incorrect coefficient of 2π

M1: Uses the limits (subtracts either way around) within their attempted integration of y^2 NOT y

and sets equal to $\frac{713}{648}\pi$. Condone poor attempts at integrating the y^2 here.

Note that if they change limits to u where $u = 2x^2 + 3$ look for limits of $2k^2 + 3$ and 4

ddM1: Dependent upon both previous M's. It is for proceeding to

either $(2k^2+3)^{\pm 2} = B$ where $B > 0$ leading to a value for k^2 or k

or setting up a quadratic equation in k^2 leading to a value for k^2 or k

FYI

$$k^4 + 3k^2 - 180 = 0 \Rightarrow (k^2 - 12)(k^2 + 15) = 0 \Rightarrow k^2 = 12$$

A1: $k = 2\sqrt{3}$ or exact equivalent such as $\sqrt{12}$ Condone if $k = \pm 2\sqrt{3}$ o.e.

Conclusion: So if n^3 is even then n must be even

Note that B0 M1 A1 A1 is possible

Question Number	Scheme	Marks
2(a)	$4^{-\frac{1}{2}} \text{ or } \frac{1}{4^{\frac{1}{2}}} \text{ or } \frac{1}{2}$ $(4-5x)^{-\frac{1}{2}} = \frac{1}{2} \left(1 - \frac{5x}{4}\right)^{-\frac{1}{2}}$ $= \dots \left(1 + \left(-\frac{1}{2}\right) \times \left(-\frac{5x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2!} \times \left(-\frac{5x}{4}\right)^2 + \dots \right)$ $= \frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots$ $\frac{2+kx}{(2-3x)^3} = (2+kx) \left(\frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots \right)$	B1 M1A1 A1 (4)
(b)	Compares x terms leading to $k = \dots$ E.g. $\frac{10}{16} + \frac{k}{2} = \frac{3}{10} \Rightarrow k = -\frac{13}{20}$	M1 A1 (2)
(c)	Compares x^2 terms leading to $m = \dots$ E.g. $m = \frac{75}{128} + \frac{5}{16} \times -\frac{13}{20} \Rightarrow m = \frac{49}{128}$	M1 A1 (2)
		(8 marks)
Alt (a)	$4^{-\frac{1}{2}} \text{ or } \frac{1}{4^{\frac{1}{2}}}$ $(4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) 4^{-\frac{3}{2}} (-5x) + \frac{-\frac{1}{2} \times -\frac{3}{2}}{2} 4^{-\frac{5}{2}} (-5x)^2$ $= \frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots$	B1 M1A1 A1 (4)

(a)

B1: For taking out a factor of $4^{-\frac{1}{2}}$ or $\frac{1}{2}$

For a direct expansion look for $4^{-\frac{1}{2}} + \dots$ or equivalent.

M1: For the form of the binomial expansion $(1+ax)^{\frac{1}{2}}$ where $a \neq 1$ or -5

To score M1 it is sufficient to see either term two or term three. Allow a slip on the sign.

So allow for either $\left(-\frac{1}{2}\right)(\pm ax)$ or $\frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2}(\pm ax)^2$

In the alternative version look for $\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x)$ or $\frac{-\frac{1}{2} \times -\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$ condoning sign slips

A1: Any (unsimplified) but correct form of the binomial expansion for $\left(1-\frac{5x}{4}\right)^{-\frac{1}{2}}$

Ignore the factor preceding the bracket for this mark

Score for $1 + \left(-\frac{1}{2}\right) \times \left(-\frac{5x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2!} \times \left(-\frac{5x}{4}\right)^2$ o.e.

In the alternative version look for $(4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x) + \frac{-\frac{1}{2} \times -\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$

A1: cao $\frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots$ This must be simplified

$$(2+kx)(P+Qx+Rx^2+\dots) = 1 + \frac{3}{10}x + mx^2$$

(b)

M1: For a correct equation in k formed by comparing the x terms. It must lead to a value for k

Follow through on their expansion. So look for $Pk+2Q = \frac{3}{10} \Rightarrow k = \dots$ Condone slips, i.e copying errors.

Condone $Pk+2Qx = \frac{3}{10}x$ as long as it leads to a value for k

A1: $k = -\frac{13}{20}$

(c)

M1: Correctly compares the x^2 terms, following through on their expansion and their value for k leading to a value for m . Condone slips, i.e copying errors.

Look for $Qk+2R = m$ Condone $Qkx^2+2Rx^2 = mx^2$ as long as it leads to a value for m



A1: $m = \frac{49}{128}$ oe Condone sight of $\frac{49}{128}x^2$ as evidence for a correct value for m .

Question Number	Scheme	Marks
3	Calculates the upper limit as $2 \ln 2$ or $\ln 4$ $\int (e^{0.5x} - 2)^2 dx = \int (e^x - 4e^{0.5x} + 4) dx = e^x - 8e^{0.5x} + 4x$ Volume = $\pi \left[e^x - 8e^{0.5x} + 4x \right]_0^{2 \ln 2} = \dots$ Volume = $8\pi \ln 2 - 5\pi$	B1 M1 A1 A1 dM1 A1 (6 marks)

B1: Calculates the upper limit as $2 \ln 2$ or $\ln 4$ only. Do not accept 1.386, $\frac{\ln 2}{0.5}$ or anything else.

Recovery from $\frac{\ln 2}{0.5}$ is allowed so if the candidate progresses to $8\pi \ln 2 - 5\pi$ do not withhold this mark.

M1: Attempts to square $(e^{0.5x} - 2)$ **with** $\int e^{kx} dx \rightarrow e^{kx}$ seen at least once. Ignore any factor of π

This may be awarded when candidates fails to square $e^{0.5x}$ correctly

So for example $\int (e^{0.5x})^2 - 4e^{0.5x} + 4 dx \rightarrow \dots \pm \dots e^{0.5x}$ is sufficient

A1: Two terms correct $e^x - 8e^{0.5x} + 4x$ Ignore any factor of π

A1: All three terms correct $e^x - 8e^{0.5x} + 4x$ Ignore any factor of π

dM1: A full and complete method of finding the volume of the solid generated.

Look for an attempt to find $\pi \int_0^{a''} (e^{0.5x} - 2)^2 dx$ with a being $2 \ln 2$, $\ln 4$, 1.38.. or $\frac{\ln 2}{2}$

It is dependent upon the previous M so some correct integration must be seen as well as use of both limits. There must be some evidence for the 0. Those terms cannot just "disappear" or be set to 0.

A1: cao $8\pi \ln 2 - 5\pi$ or $-5\pi + 8\pi \ln 2$ Condone $\pi(8 \ln 2 - 5)$

(Note that a requirement of the question is to write in the form $a \ln 2 + b$)

Question Number	Scheme	Marks
6(a)	$4y^2 + 3x = 6ye^{-2x}$	
	$4y^2 + 3x \rightarrow 8y \frac{dy}{dx} + 3$	B1
	$6ye^{-2x} \rightarrow -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$	M1 A1
	$8y \frac{dy}{dx} + 3 = -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ oe	M1 A1
		(5)
(b)	Sets $x = 0$ in $4y^2 + 3x = 6ye^{-2x} \Rightarrow y = \frac{3}{2}$ oe	B1
	Substitutes $\left(0, \frac{3}{2}\right)$ in their $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y} = \left(\frac{7}{-2}\right)$	M1
	$m_N = -1 \div \frac{7}{-2} \Rightarrow y = \frac{2}{7}x + \frac{3}{2}$	dM1
	$y = \frac{2}{7}x + \frac{3}{2}$ oe e.g. $y = \frac{6}{21}x + \frac{3}{2}$	A1
		(4)
		(9 marks)

(a)

B1: Differentiates $4y^2 + 3x$ to obtain $8y \frac{dy}{dx} + 3$. Allow unsimplified forms such as $4 \times 2y \frac{dy}{dx} + 3$

M1: Uses the product rule on $6ye^{-2x}$ to obtain an expression of the form $Aye^{-2x} + Be^{-2x} \frac{dy}{dx}$

A1: Differentiates $6ye^{-2x}$ to obtain $-12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$

M1: Collects two terms in $\frac{dy}{dx}$ (one from attempting to differentiate $4y^2$ and one from attempting to differentiate $6ye^{-2x}$) and proceeds to make $\frac{dy}{dx}$ the subject.

A1: $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ or equivalent e.g. $\frac{dy}{dx} = \frac{2e^{-2x} \times 6y + 3}{6e^{-2x} - 8y}$ or $\frac{dy}{dx} = \frac{12y + 3e^{2x}}{6 - 8ye^{2x}}$

You can ignore any spurious " $\frac{dy}{dx} =$ " at the start and allow y' for $\frac{dy}{dx}$.

(b)

B1: Uses $x = 0$ to obtain $y = \frac{3}{2}$ oe e.g. $\frac{6}{4}$ (ignore any reference to $y = 0$)

M1: Substitutes $x = 0$ and their y at $x = 0$ which has come from substituting $x = 0$ into the original equation into their $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ to find a numerical value. Working is normally shown here but you may need to check for evidence. Use of $x = 0$ and $y = 0$ is M0.



General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
7 (a)	Co-ordinates or position vector of a point on $l = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$ $\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$ Uses $\overrightarrow{AX} \cdot \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0 \Rightarrow 20+16\lambda-15+9\lambda-25+25\lambda=0 \Rightarrow \lambda = \frac{2}{5}$ (i) Substitutes their λ into $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} \Rightarrow X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$ (ii) $\overrightarrow{AX} = -\frac{33}{5}\mathbf{i} + \frac{19}{5}\mathbf{j} - 3\mathbf{k}$ $\text{Shortest distance} = \sqrt{\left(-\frac{33}{5}\right)^2 + \left(\frac{19}{5}\right)^2 + (-3)^2} = \sqrt{67}$	M1 dM1 A1 dM1 A1 M1 A1 (7)
(b)	Uses $\overrightarrow{AX} = \overrightarrow{XB}$ or similar correct method point B has position vector $-\frac{21}{5}\mathbf{i} + \frac{23}{5}\mathbf{j} - 4\mathbf{k}$	M1 A1 (2) (9 marks)
7 (a) ALT way	Co-ordinates or position vector of a point on $l = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$ $\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$ Uses $d^2 = (-5-4\lambda)^2 + (5-3\lambda)^2 + (-5+5\lambda)^2$ which is smallest when its gradient $= 0 \Rightarrow -8(-5-4\lambda) - 6(5-3\lambda) + 10(-5+5\lambda) = 0 \Rightarrow \lambda = \frac{2}{5}$ (Note that this could be done by completing the square) (i) Substitutes their λ into $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} \Rightarrow X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$ (ii) Shortest distance $= \left(-5-4 \times \frac{2}{5}\right)^2 + \left(5-3 \times \frac{2}{5}\right)^2 + \left(-5+5 \times \frac{2}{5}\right)^2 \Rightarrow d = \sqrt{67}$	M1 dM1 A1 dM1 A1 M1 A1 (7)

(a)

M1: States or uses a general point on $l = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$ and attempt $\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$

either way around with their general point. Condone slips

dM1: Uses a correct method to find a value for λ

This could be

- via scalar products $\overrightarrow{AX} \cdot \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0$
- via differentiation using minimum distance of $(-5-4\lambda)^2 + (5-3\lambda)^2 + (-5+5\lambda)^2$ in this method condone slips on the differentiation

A1: $\lambda = \frac{2}{5}$

(i)

dM1: Uses their λ to find the coordinates (or position vector) of X This is dependent upon the previous M

A1: Finds coordinates for $X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$ Condone use of position vector

(ii)

M1: Uses a correct method to find distance AX or distance AX^2 using A and their X .

Award if a correct method is seen for two of the three coordinates

Usually look for an attempt at $(-5-4\lambda)^2 + (5-3\lambda)^2 + (-5+5\lambda)^2$ with their λ .

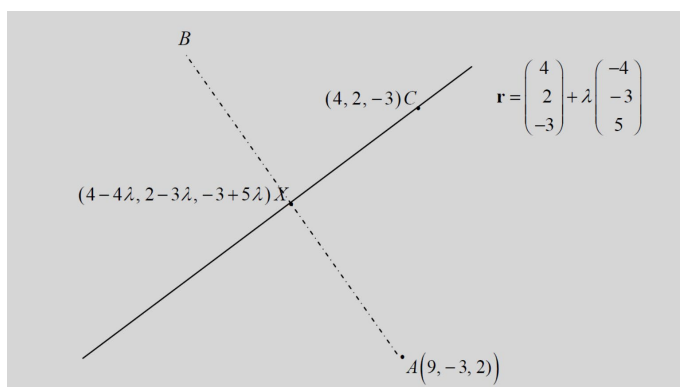
A1: $\sqrt{67}$ Note that $d = \sqrt{67}$ so $d = 67$ is quite common. This is fine as d is the constant given in the question

(b)

M1: Uses a correct method to find B .

The method can be implied by two correct coordinates for their coordinates for X and A .

A1: Correct position vector for B . $-\frac{21}{5}\mathbf{i} + \frac{23}{5}\mathbf{j} - 4\mathbf{k}$. Condone use of coordinates



Note that it is possible (though unlikely) to attempt this questions via scalar product using $\overrightarrow{CA}$ and $\overrightarrow{CX}$.

$$\text{FYI } |\overrightarrow{CX}| = |\overrightarrow{CA}| \cos \theta = \frac{\overrightarrow{CA} \cdot \overrightarrow{CX}}{|\overrightarrow{CX}|} = 2\sqrt{2}$$

$$\text{Hence } \lambda = \frac{2\sqrt{2}}{\sqrt{50}} = \frac{2}{5}$$

Question Number	Scheme	Marks
8 (a)	$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \times \frac{1}{x} \, dx$ $= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 (+c)$	M1 dM1 A1 (3)
(b)	$\int x^2 \ln^2 x \, dx = \frac{1}{3}x^3 \ln^2 x - \int \frac{1}{3}x^3 \times 2 \ln x \times \frac{1}{x} \, dx$ $= \frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \right)$ $\text{Volume} = \int_1^e \pi x^2 \ln^2 x \, dx = \pi \times \left[\frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \right) \right]_1^e$ $= \frac{5}{27}\pi e^3 - \frac{2}{27}\pi$	M1 A1ft dM1 A1 (4) (7 marks)

(a) Note that no marks are available for integrating incorrect functions

M1: Attempts to integrate by parts the correct way around. Look for $\int x^2 \ln x \, dx = \dots x^3 \ln x - \int \dots x^3 \times \frac{1}{x} \, dx$ oe

dM1: And then integrates again to a form $px^3 \ln x - qx^3$ where p and q are positive constants.

A1: $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$ Condone a missing $+c$.

Candidates who go on to write $\ln x \times \frac{x^3}{3} - \frac{1}{9}x^3 = \ln \frac{x^4}{3} - \frac{1}{9}x^3$ o.e. withhold this final A1

(b)

M1: Integrates $\int x^2 \ln^2 x \, dx$ by parts, the correct way around to reach a form $\dots x^3 \ln^2 x - \dots \int x^3 \times \ln x \times \frac{1}{x} \, dx$

You may see a π or 2π in front of this expression which is fine.

A1ft: Achieves $\int x^2 \ln^2 x \, dx = \frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \times \text{their answer to (a)}$.

Accept $\int \pi x^2 \ln^2 x \, dx = \frac{1}{3}\pi x^3 \ln^2 x - \frac{2}{3} \times \pi \times \text{their answer to (a)}$ or other incorrect factors such as

$\int 2\pi x^2 \ln^2 x \, dx = \frac{2}{3}\pi x^3 \ln^2 x - \frac{4}{3} \times \pi \times \text{their answer to (a)}$

FYI The correct answer is $\frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \right)$

dM1: Applies the limits 1 and e to a function of the form $\pi \times \left[\alpha x^3 \ln^2 x - \beta x^3 \ln x \pm \chi x^3 \right]_1^e$

and subtracts where α, β, χ are positive constants. The π must now be seen.



Question Number	Scheme	Marks
9(a)	Volume = $144\pi \int_0^{20} \frac{2x+3}{x^2+3x+5} dx$ or e.g. $\pi \int_0^{20} \left(12 \left(\frac{2x+3}{x^2+3x+5} \right)^{\frac{1}{2}} \right)^2 dx$	B1
	$144\pi \left[\ln(x^2+3x+5) \right]_0^{20}$	M1
	$= 144\pi (\ln 465 - \ln 5) = 144\pi \ln 93 \text{ (units}^3\text{)}$	dM1A1
		(4)
(a) B1: Correct integral for the volume of R which may be simplified or unsimplified. The 144 (or 12^2) and the π and the limits 0 and 20 must be present or implied by subsequent work. Condone the omission of “dx”. $2\pi \times 144 \int_0^{20} \frac{2x+3}{x^2+3x+5} dx$ scores B0 M1: Integrates $\frac{2x+3}{x^2+3x+5}$ to obtain $\dots \ln(x^2+3x+5)$ Condone missing brackets e.g. $\dots \ln x^2+3x+5$ Condone poor notation e.g. $144\pi \int \frac{2x+3}{x^2+3x+5} dx = \int 144\pi \left[\ln(x^2+3x+5) \right]_0^{20} dx$ This may be implied by substitution e.g. $u = x^2+3x \Rightarrow \int \frac{2x+3}{x^2+3x+5} dx = \dots \ln(u+5)$ or $u = x^2+3x+5 \Rightarrow \int \frac{2x+3}{x^2+3x+5} dx = \dots \ln u$ dM1: Substitutes 20 and 0 into their expression of the form $\dots \ln(x^2+3x+5)$, subtracts either way round and proceeds to find an exact expression for the volume e.g. $144\pi (\ln 465 - \ln 5)$ If they use substitution, the limits must be correct for their substitution e.g. $u = x^2+3x \Rightarrow u = 0, 460$ or e.g. $u = x^2+3x+5 \Rightarrow u = 5, 465$ Do not allow e.g. $144\pi (\ln 465 - 0)$ i.e. no evidence of the use of the lower limit. Depends on the previous mark. A1: $144\pi \ln 93$ (condone lack of units) Note if π is missing throughout in (a), a maximum of B0M1dM1A0 is possible.		

Example working if a substitution is used e.g.

$$u = x^2 + 3x + 5 \Rightarrow \frac{du}{dx} = 2x + 3 \Rightarrow 144\pi \int \frac{2x+3}{u} \frac{dx}{2x+3} = 144\pi [\ln u]_5^{465}$$

$$144\pi (\ln 465 - \ln 5) = 144\pi \ln 93$$



(b)	$144\pi \left[\ln(x^2 + 3x + 5) \right]_p^{20} = 360\pi$ $\Rightarrow 144\pi (\ln 465 - \ln(p^2 + 3p + 5)) = 360\pi$ or e.g. Let $p = 20 - a$: $144\pi \left[\ln(x^2 + 3x + 5) \right]_{20-a}^{20} = 360\pi$ $\Rightarrow 144\pi (\ln 465 - \ln((20-a)^2 + 3(20-a) + 5)) = 360\pi$ or e.g. $144\pi \left[\ln(x^2 + 3x + 5) \right]_0^{20-a} = 144\pi \ln 93 - 360\pi$ $\Rightarrow 144\pi (\ln((20-a)^2 + 3(20-a) + 5) - \ln 5) = 144\pi \ln 93 - 360\pi$ or e.g. $144\pi \left[\ln(x^2 + 3x + 5) \right]_0^p = 144\pi \ln 93 - 360\pi$ $\Rightarrow 144\pi (\ln(p^2 + 3p + 5) - \ln 5) = 144\pi \ln 93 - 360\pi$	M1
	$\frac{5}{2} = \ln\left(\frac{465}{p^2 + 3p + 5}\right) \Rightarrow e^{\frac{5}{2}} = \frac{465}{p^2 + 3p + 5}$ $\Rightarrow p^2 + 3p + 5 - 465e^{-\frac{5}{2}} = 0 \Rightarrow p = \dots$ or e.g. $\ln\left(\frac{a^2 - 43a + 465}{5}\right) = \frac{144\pi \ln 93 - 360\pi}{144\pi}$ $\Rightarrow \frac{a^2 - 43a + 465}{5} = e^{\left(\frac{144\ln 93 - 360}{144}\right)} \Rightarrow a^2 - 43a + 465 - 5e^{\left(\frac{144\ln 93 - 360}{144}\right)} = 0 \Rightarrow a = \dots$ or e.g. $\ln\left(\frac{p^2 + 3p + 5}{5}\right) = \frac{144\pi \ln 93 - 360\pi}{144\pi}$ $\Rightarrow \frac{p^2 + 3p + 5}{5} = e^{\left(\frac{144\ln 93 - 360}{144}\right)} \Rightarrow p^2 + 3p + 5 - 5e^{\left(\frac{144\ln 93 - 360}{144}\right)} = 0 \Rightarrow p = \dots$	dM1
	$a = \text{awrt } 15.5$	A1
		(3)
		(7 marks)

(b)

M1: Sets an expression of the form $\dots \left[\ln(x^2 + 3x + 5) \right]_p^{20}$ equal to 360π and substitutes in the limits 20 and p (or any other letter – condone a) and subtracts either way round.

Alternatively sets an expression of the form $\dots \left[\ln(x^2 + 3x + 5) \right]_0^{20-a}$ or e.g. $\dots \left[\ln(x^2 + 3x + 5) \right]_0^p$ (or any other letter – condone a) equal to 360π – their answer to part (a) and substitutes in the limits $20 - a$ and 0 or e.g. 0 and p and subtracts either way round.

For this mark, their method, if executed correctly would either lead directly to the value of a or would enable a to be found by e.g. $a = 20$ – their value.

dM1: Depends on the previous mark.

Removes lns, rearranges to form a three term quadratic and attempts to solve, proceeding to a value for p . In the alternative method this mark is scored for proceeding to find a value for

A1: a . The usual rules apply for solving their 3TQ although this is most likely to be done on a calculator so you may need to check.
 cao awrt 15.5 with or without units
 Note that if they find “ p ” first then then need to find $20 - “p”$ to obtain the correct value of a .
 For reference, “ p ” = 4.45143.....

Examples in (b) that score no marks:

$$144\pi \left[\ln(x^2 + 3x + 5) \right]_0^p = 360\pi$$

$$144\pi [\ln u]_5^{p^2+3p+5} = 360\pi$$

(Having substituted $u = x^2 + 3x + 5$ in (a))

Example of correct working if a substitution is used e.g. $u = x^2 + 3x + 5$

$$\text{Let } p = 20 - a: 144\pi [\ln u]_{p^2+3p+5}^{465} = 360\pi$$

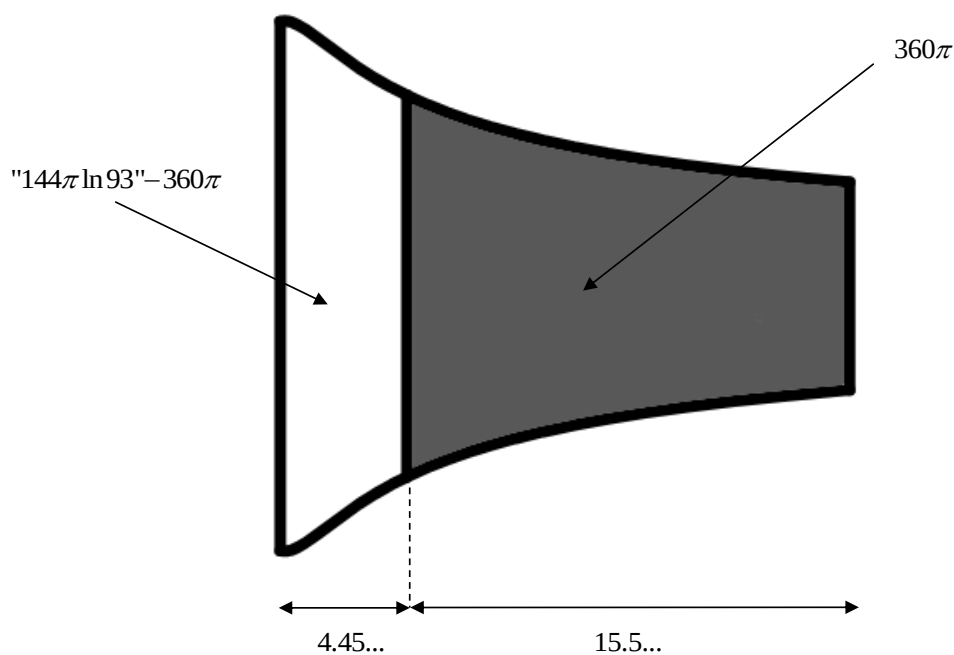
$$\Rightarrow \frac{5}{2} = \ln \left(\frac{465}{p^2 + 3p + 5} \right) \Rightarrow e^{\frac{5}{2}} = \frac{465}{p^2 + 3p + 5} \text{ etc.}$$

or e.g.

$$144\pi [\ln u]_5^{(20-a)^2+3(20-a)+5} = 144\pi \ln 93 - 360\pi$$

$$\Rightarrow 144\pi \left(\ln \left((20-a)^2 + 3(20-a) + 5 \right) - \ln 5 \right) = 144\pi \ln 93 - 360\pi \text{ etc.}$$

For reference:



Question 10

Note that there are many different ways to answer this question.

General Guidance:

B1: For setting up the correct contradiction.

It must include a word/words such as “assume” or “let” or “if” and $p^2 - 4q + 2 = 0$

As a minimum accept just “assume/let/if $p^2 - 4q + 2 = 0$ ”

M1: Attempts to make p^2 the subject.

A1: Makes a correct deduction following correct work.

dM1: Continues the argument/algebra that will lead to a contradiction.

A1: For a fully correct proof.

Requires

- all previous marks scored
- no incorrect assumptions/statements
- correct calculations/algebra
- a reason: e.g. $2q - 2k^2 = 1$ is not true
- E.g. hence $p^2 - 4q + 2 \neq 0$, hence proven, QED, etc. but not just “contradiction”

Question	Scheme	Marks
7(a)	$V = (\pi) \int_{(0)}^{(4)} \left(\frac{1}{4}(4-x)e^x \right)^2 dx$	M1
	$\int (f(x))^2 dx = \frac{1}{16} \int (4-x)^2 e^{2x} dx = \dots$	M1
	$V = \frac{\pi}{16} \int_0^4 (x^2 - 8x + 16) e^{2x} dx$	A1
		(3)
(b)	$= \left(\frac{\pi}{16} \right) \left((x^2 - 8x + 16) \frac{e^{2x}}{2} - \int (2x - 8) \times \frac{e^{2x}}{2} dx \right)$	M1 A1
	$= \left(\frac{\pi}{16} \right) \left((x^2 - 8x + 16) \frac{e^{2x}}{2} - \left[(2x - 8) \frac{e^{2x}}{4} - \int 2 \times \frac{e^{2x}}{4} dx \right] \right)$ $= \left(\frac{\pi}{16} \right) \left((x^2 - 8x + 16) \frac{e^{2x}}{2} - (2x - 8) \frac{e^{2x}}{4} + \frac{e^{2x}}{4} \right)$ Depends on the first method mark.	dM1
	$V = \frac{\pi}{16} \left[(x^2 - 8x + 16) \frac{e^{2x}}{2} - (2x - 8) \frac{e^{2x}}{4} + \frac{e^{2x}}{4} \right]_0^4 = \frac{\pi}{16} \left(0 + 0 + \frac{e^8}{4} - \frac{16}{2} - \frac{8}{4} - \frac{1}{4} \right)$ Depends on both previous method marks.	ddM1
	$= \frac{\pi}{64} (e^8 - 41) \text{ (cm}^3\text{)}$	A1
		(5)
(b) Way 2	$= \left(\frac{\pi}{16} \right) \left((4-x)^2 \frac{e^{2x}}{2} + \int (4-x) e^{2x} dx \right)$	M1 A1
	$= \left(\frac{\pi}{16} \right) \left((4-x)^2 \frac{e^{2x}}{2} + \frac{1}{2} (4-x) e^{2x} + \int \frac{1}{2} e^{2x} dx \right)$ $= \left(\frac{\pi}{16} \right) \left((4-x)^2 \frac{e^{2x}}{2} + \frac{1}{2} (4-x) e^{2x} + \frac{1}{4} e^{2x} \right)$ Depends on the first method mark.	dM1
	$= \left(\frac{\pi}{16} \right) \left[(4-x)^2 \frac{e^{2x}}{2} + \frac{1}{2} (4-x) e^{2x} + \frac{1}{4} e^{2x} \right]_0^4 = \left(\frac{\pi}{16} \right) \left(\frac{1}{4} e^8 - 8 - 2 - \frac{1}{4} \right)$ Depends on both previous method marks.	ddM1
	$= \frac{\pi}{64} (e^8 - 41) \text{ (cm}^3\text{)}$	A1



Question	Scheme	Marks
3(a)	$y^2x + 3y = 4x^2 + k$	
	$y^2x \rightarrow \dots \times y \frac{dy}{dx} (+\dots)$ or $3y \rightarrow 3 \frac{dy}{dx}$	M1
	$y^2x \rightarrow 2xy \frac{dy}{dx} + y^2$	A1
	$\dots + 3y = 4x^2 + k \rightarrow \dots + 3 \frac{dy}{dx} = 8x$	A1
	$2xy \frac{dy}{dx} + y^2 + 3 \frac{dy}{dx} = 8x \Rightarrow \frac{dy}{dx} = \dots$	dM1
	$\frac{dy}{dx} = \frac{8x - y^2}{2xy + 3}$ oe	A1
		(5)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 8x - 4 = 0 \Rightarrow x = \dots$	M1
	$p = \frac{1}{2}$	A1
	$k = 2^2 \times \frac{1}{2} + 3 \times 2 - 4 \times \frac{1}{4} = \dots$	M1
	$k = 7$	A1
		(4)
Alt (b)	$(p, 2) \text{ on } C \Rightarrow 4p + 6 = 4p^2 + k \Rightarrow 4p^2 - 4p + k - 6 = 0$ $\Rightarrow 4\left(p - \frac{1}{2}\right)^2 - 1 + k - 6 = 0$ or $b^2 - 4ac = 16 - 16(k - 6)$	M1
	(Min, single solution) $\Rightarrow p = \frac{1}{2}$ or $16 - 16(k - 6) = 0 \Rightarrow k = 7$	A1
	$\Rightarrow k = 7 - 4\left(\frac{1}{2} - \frac{1}{2}\right)^2$ or $4p^2 - 4p + 7 - 6 = 0 \Rightarrow p = \dots$	M1
	The other of $p = \frac{1}{2}$ and $k = 7$	A1
		(4)
(9 marks)		
Notes:		
(a) M1: Some evidence of an attempt at implicit differentiation. Look for one of the terms being differentiated with either the $3 \frac{dy}{dx}$ or a term with $kxy \frac{dy}{dx}$ or $y^2 \rightarrow ky \frac{dy}{dx}$ as part of a product rule seen appropriately. A1: y^2x differentiated correctly		

Question Number	Scheme	Marks
7(a)	$\frac{3x-1}{x+2} = 3 + \frac{\dots}{x+2}$ or $\dots - \frac{7}{x+2}$	M1
	$\frac{3x-1}{x+2} = 3 - \frac{7}{x+2}$	A1
		(2)
(b)	$V = (\pi) \int \left(3 - \frac{7}{x+2}\right)^2 dx = \dots$	M1
	$= (\pi) \int \left(9 - \frac{42}{x+2} + \frac{49}{(x+2)^2}\right) dx = \dots - 42 \ln(x+2) + \dots$ or $= (\pi) \int \left(9 - \frac{42}{x+2} + \frac{49}{(x+2)^2}\right) dx = \dots + \dots - \frac{49}{x+2}$	M1
	$= (\pi) \int \left(9 - \frac{42}{x+2} + \frac{49}{(x+2)^2}\right) dx = \dots - 42 \ln(x+2) + \dots$ and $= (\pi) \int \left(9 - \frac{42}{x+2} + \frac{49}{(x+2)^2}\right) dx = \dots + \dots - \frac{49}{x+2}$	dM1
	$= (\pi) \int \left(9 - \frac{42}{x+2} + \frac{49}{(x+2)^2}\right) dx = 9x - 42 \ln(x+2) - \frac{49}{x+2}$	A1
	$V = (\pi) \left[9x - 42 \ln(x+2) - \frac{49}{x+2}\right]_1^4 = (\pi) \left\{36 - 42 \ln 6 - \frac{49}{6} - \left(9 - 42 \ln 3 - \frac{49}{3}\right)\right\}$	M1
	$= \pi \left(\frac{211}{6} - 42 \ln 2\right)$	A1
		(6)
		Total 8

(a)

M1: Obtains either $A = 3$ or $B = -7$ A1: $3 - \frac{7}{x+2}$ Just stating the values of A and B is insufficient but you may award this mark if itis seen in (b). Allow $3 + \frac{-7}{x+2}$ and $3 + -\frac{7}{x+2}$ (b) **Condone transcription errors for the method marks** e.g. $\frac{\dots}{x+2} \rightarrow \dots \ln(x-2)$ **provided****no incorrect method is seen. Condone invisible brackets for the M marks**M1: Uses their result from part (a) and applies the volume formula (with or without the π and condone 2π) and attempts to square the bracket. Condone if they just square the two terms in the brackets or if their squaring is poor.

M1: Integrates to obtain the correct form for one of the fractional terms i.e. $\frac{\dots}{x+2} \rightarrow \dots \ln(x+2)$

or $\frac{\dots}{(x+2)^2} \rightarrow \frac{\dots}{x+2}$. This can be scored even if they have not squared any terms originally

if they have one of these terms of the required form. Condone invisible brackets for $\ln x+2$

dM1: Integrates to obtain the correct form for both fractional terms i.e. $\frac{\dots}{x+2} \rightarrow \dots \ln(x+2)$ and

$\frac{\dots}{(x+2)^2} \rightarrow \frac{\dots}{x+2}$. It is dependent on the previous method mark. Condone invisible brackets for $\ln x+2$

A1: $9x - 42 \ln(x+2) - \frac{49}{x+2}$ o.e. May be implied by further work.

M1: Applies the limits 1 and 4 to an expression of the form $Ax \pm B \ln(x+2) \pm C(x+2)^n$ where $A, B, C, n \neq 0$ and subtracts the correct way round. The expression with values embedded is sufficient. Condone slips and invisible brackets.

A1: $\pi \left(\frac{211}{6} - 42 \ln 2 \right)$

Correct expression in the **required form**. The fraction does not need to be in simplest form.

Alternative methods (including not using part (a)) requiring use of a substitution e.g. $u = x+2$

or e.g. $u = \frac{7}{x+2}$. **Watch out for these longer methods which can still score full marks.**

M1: If using $u = x+2$ then $(\pi) \int \left(\frac{3x-1}{x+2} \right)^2 dx \Rightarrow (\pi) \int 9 - \frac{42}{u} + \frac{49}{u^2} du$ (They will need to divide their numerator by the denominator to achieve $\alpha + \frac{\beta x + \gamma}{(x+2)^2}$). Condone slips.

If using $u = \frac{7}{x+2}$ then $(\pi) \int \frac{42}{u} - \frac{63}{u^2} - 7 du$ Condone slips.

M1: As above in the main scheme notes for one of the fractional terms i.e. $\frac{\dots}{u} \rightarrow \dots \ln u$ or

$\frac{\dots}{u^2} \rightarrow \frac{\dots}{u}$.

dM1: Integrates to obtain the correct form for both fractional terms

A1: $9u - 42 \ln u - \frac{49}{u}$ if using $u = x+2$ **or** $42 \ln u + \frac{63}{u} - 7u$ if using $u = \frac{7}{x+2}$

M1: Applies the limits 1 and 4 to an expression of the required form (see above in main scheme notes) if the expression is converted back to being in terms of x

or applies the limits 3 and 6 if in terms of u if using $u = x+2$

or e.g. $\frac{7}{3}, \frac{7}{6}$ if using $u = \frac{7}{x+2}$

A1: As above in main scheme notes.

Question Number	Scheme	Marks
9(a)	$\frac{1}{x(2x-1)} \equiv \frac{A}{x} + \frac{B}{2x-1} \Rightarrow 1 \equiv A(2x-1) + Bx \Rightarrow A = \dots \text{or } B = \dots$	M1
	$\frac{1}{x(2x-1)} \equiv \frac{2}{2x-1} - \frac{1}{x}$	A1
		(2)
(b)	$\frac{dh}{dt} = \frac{1}{50} h(2h-1) \cos\left(\frac{t}{10}\right) \Rightarrow \int \frac{1}{h(2h-1)} dh = \int \frac{1}{50} \cos\left(\frac{t}{10}\right) dt$	M1
	$\int \frac{1}{h(2h-1)} dh = \int \left(\frac{2}{2h-1} - \frac{1}{h} \right) dh = \ln(2h-1) - \ln h$	M1
	$\int \left(\frac{2}{2h-1} - \frac{1}{h} \right) dh = \int \frac{1}{50} \cos\left(\frac{t}{10}\right) dt \Rightarrow \ln(2h-1) - \ln h = \frac{1}{5} \sin\left(\frac{t}{10}\right) (+c)$	A1ft
	e.g. $t = 0, h = 2.5 \Rightarrow \ln(2 \times 2.5 - 1) - \ln 2.5 = c$ or e.g. $\ln \frac{2h-1}{h} = \frac{1}{5} \sin\left(\frac{t}{10}\right) + c \Rightarrow \frac{2h-1}{h} = X e^{\frac{1}{5} \sin\left(\frac{t}{10}\right)} \quad t = 0, h = 2.5 \Rightarrow X = 1.6$	M1
	$\ln(2h-1) - \ln h = \frac{1}{5} \sin\left(\frac{t}{10}\right) + \ln 1.6 \Rightarrow \frac{2h-1}{h} = 1.6 e^{\frac{1}{5} \sin\left(\frac{t}{10}\right)}$ $\Rightarrow 2h - 1.6 h e^{\frac{1}{5} \sin\left(\frac{t}{10}\right)} = 1 \Rightarrow h = \dots$	M1
	$h = \frac{5}{10 - 8e^{\frac{1}{5} \sin\left(\frac{t}{10}\right)}}$	A1
		(6)
(c)	$\sin\left(\frac{t}{10}\right) = 1 \Rightarrow \frac{t}{10} = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots \Rightarrow t = \dots$	M1
	$t = 45\pi = 141 \text{ (s)}$	A1
		(2)
		Total 10

(a)

M1: Correct attempt at partial fractions. Sets $\frac{1}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}$ and uses a correct method to

identify at least one constant. Incorrect work e.g. $\frac{A}{x} + \frac{B}{2x-1} \Rightarrow 1 = Ax + B(2x-1)$ etc. is M0.

A1: Correct partial fractions. Note that this mark is not just for the correct constants, it is for the correct written fractions but allow this mark to be scored if it is seen in part (b) (in terms of h or x).

Condone $\frac{2}{2x-1} + \frac{-1}{x}$ or written as e.g. $2(2x-1)^{-1} - x^{-1}$

(b) **Note that they may work in x throughout, but the final answer must be $h = \dots$**

M1: Attempts to separate the variables. Do not be concerned by slips with the constants so look for as a minimum $\int \frac{1}{h(2h-1)} = \int \cos(\dots t)$ o.e. May omit an integral sign on one of the sides or may be implied by their integrated expression.

M1: For $\int \left(\frac{A}{h} + \frac{B}{2h-1} \right) dh = \alpha \ln h + \beta \ln(2h-1)$

or e.g. $\lambda \int \left(\frac{"A"}{h} + \frac{"B"}{2h-1} \right) dh = \alpha \ln h + \beta \ln(2h-1)$ (if they attempt some rearranging first)

Condone invisible brackets.

A1ft: Fully correct equation following through their A and B . The “+ c ” is not required here.

$$\int \left(\frac{A}{h} + \frac{B}{2h-1} \right) dh = \int \frac{1}{50} \cos\left(\frac{t}{10}\right) dt \Rightarrow A \ln h + \frac{B}{2} \ln(2h-1) = \frac{1}{5} \sin\left(\frac{t}{10}\right) (+c) \text{ o.e.}$$

Condone invisible brackets.

M1: States or uses $t = 0, h = 2.5$ followed by $c = \dots$ is sufficient for M1. You do not need to check their substitution/rearrangement but if there is just a value for the constant then you may need to check they used $t = 0, h = 2.5$. May be implied by e.g. $\ln 8 - \ln 5, \ln 1.6$, awrt 0.47 following correct integration. Note that they may have removed lns first.

M1: To score this mark they must have integrated and achieved an expression of the form

$$\pm p \ln h \pm q \ln(2h-1) = \pm r \sin\left(\frac{t}{10}\right) + c \text{ o.e.}$$

Rearranges using correct algebra to make h the subject including when combining logs and when dealing with the constant. May occur before finding their constant but there must have been a constant.

Expect to see terms in h collected on one side as part of their manipulation rather than proceeding to the given answer once they have established the value of k . Alternatively, they may cancel e.g. $\frac{2h-1}{h} \rightarrow 2 - \frac{1}{h}$ and rearrange to make h the subject using correct algebra.

Do not condone sign slips for this mark.

A1: Correct expression (including $h = \dots$) in the required form. Requires

- Correct integration
- Correct algebra

Do not penalise brackets recovered in their work or working in x rather than h until the final answer.

(c) **Note that an answer with no working seen scores M0A0**

M1: Solves $\sin\left(\frac{t}{10}\right) = 1$ to find a value for t . Do not allow this mark to be implied by

e.g. $5\pi, 25\pi, 45\pi$ but allow for e.g. $\frac{t}{10} = \frac{\pi}{2} \Rightarrow t = \dots$ Must be working in radians.

Note that attempts that do not use part (b) and just use the differential equation proceeding to $\cos\left(\frac{t}{10}\right) = 0$ score M0A0. (**Note if their k is negative condone if they solve $\sin\left(\frac{t}{10}\right) = -1$**)

A1: awrt 141 (but do not allow just 45π) Units not required.



Question Number	Scheme	Marks
10(a)	$\frac{dx}{dt} = 6t$ o.e.	B1
	$\text{Area} = \left(\int \sin t \sin 2t \times 6t \, dt \right) \int \sin t \times 2 \sin t \cos t \times 6t \, dt$	M1
	$= 12 \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, dt$	A1
		(3)
(b)	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - (12) \int_0^{\frac{\pi}{2}} \frac{1}{3} \sin^3 t \, dt$	M1
	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - \left(\frac{12}{3} \right) \int_0^{\frac{\pi}{2}} \sin t (1 - \cos^2 t) \, dt$	M1
	$= \left[4t \sin^3 t + 4 \cos t - \frac{4}{3} \cos^3 t \right]_0^{\frac{\pi}{2}}$	A1
	$= 2\pi - \left(4 - \frac{4}{3} \right)$	M1
	$= 2\pi - \frac{8}{3}$	A1
		(5)
		Total 8

(a) On EPEN this is M1M1A1 but we are marking this as B1M1A1

It is acceptable to work in a different variable such as θ instead of t provided the final answer is in terms of t but withhold the final mark if there is a mixture of variables in the same line of working.

B1: $\frac{dx}{dt} = 6t$ seen or implied.

$6t \sin t \sin 2t$ implies B1 but $6t \sin^2 t \cos t$ does not imply B1.

M1: Uses $\sin 2t = 2 \sin t \cos t$ within the expression for y i.e. $y = \sin t \sin 2t = 2 \sin^2 t \cos t$ or may be seen in their integral. May be implied by e.g. $\int pt \sin t \sin 2t \, (dt) \rightarrow \int 2pt \sin^2 t \cos t \, (dt)$

Condone the omission of t . i.e. $y = 2 \sin^2 \cos$

A1: Correct integral including dt with $k = 12$ **with at least one intermediate stage of working** (which may only be e.g. $\frac{dx}{dt} = 6t$) The integral sign, dt , limits only need to be present in the

final answer. Ignore any attempts to verify the limits of 0 and $\frac{\pi}{2}$.

Do not condone poor notation e.g. $\sin t^2$ or the absence of variables in the main body of the solution.

$12t \sin^2 t \cos t$ with or without integration notation and with no other working implies B1M1A0

(b) Condone working in mixed variables provided the intention is clear or they recover in further work. Condone the absence of variables in trigonometric functions e.g. $\sin$ for $\sin t$ or $\cos$ for $\cos t$

M1: Attempts integration by parts in the correct direction to achieve an expression of the form $\dots t \sin^3 t \pm \dots \int \sin^3 t \, dt$ Condone the omission of their 12

M1: Uses $\sin^3 t = \sin t(1 - \cos^2 t)$ to reach the integrable form $\dots t \sin^3 t \pm \dots \int \sin t(1 - \cos^2 t) \, dt$ o.e.

May be seen as $\dots t \sin^3 t \pm \dots \int \sin t - \sin t \cos^2 t \, dt$.

Alternatively uses compound angle formulae to substitute $\sin^3 t = \dots \sin t \pm \dots \sin 3t$ to reach an integrable form. $\dots t \sin^3 t \pm \dots \int (\dots \sin t \pm \dots \sin 3t) \, dt$ $(4t \sin^3 t - \int (3 \sin t - \sin 3t) \, dt)$

In either approach it may be implied by their integrated expression. Condone the omission of their 12

A1: Fully correct integration of $\int t \sin^2 t \cos t \, dt$

or alternatively $4t \sin^3 t - \int (3 \sin t - \sin 3t) \, dt = 4t \sin^3 t + 3 \cos t - \frac{1}{3} \cos 3t$.

Condone any multiple and condone missing variables e.g. $4t \sin^3 + 4 \cos - \frac{4}{3} \cos^3$

M1: Applies the limits 0 and $\frac{\pi}{2}$ to an expression of the form $At \sin^3 t + B \cos t + C \cos^3 t$ where

$A, B, C \neq 0$ or in the alternative $\alpha \cos t + \beta \cos 3t$ where $\alpha, \beta \neq 0$

A1: Correct answer in the required form. $\frac{8}{3}$ does not need to be in simplest form. isw once a correct answer is seen.

Alternative substitution method using $u = \sin t$

M1: Attempts to use the substitution $u = \sin t$ to proceed to $(12) \int u^2 \sin^{-1} u \, du$ and attempts integration by parts in the correct direction to achieve an expression of the form

$(12) \left(\frac{u^3}{3} \sin^{-1} u - \frac{1}{3} \int \frac{u^3}{\sqrt{1-u^2}} \, du \right)$ but condone coefficient slips.

Condone the omission of their 12

M1: Proceeds to an integrable form by using the substitution $v = \sqrt{1-u^2}$

e.g. $(12) \left(\frac{1}{3} u^3 \sin^{-1} u + \frac{1}{3} \int (1-v^2) \, dv \right)$ Condone the omission of their 12

A1: Fully correct integration $\frac{1}{3} u^3 \sin^{-1} u + \frac{1}{3} \sqrt{1-u^2} - \frac{1}{9} (\sqrt{1-u^2})^3$ (may not have reverted back to being in terms of u). Condone any multiple and condone missing variables

M1: Applies the limits 0 and 1 to an expression of the form $Du^3 \sin^{-1} u + E \sqrt{1-u^2} + F (\sqrt{1-u^2})^3$ where D, E and $F \neq 0$

A1: Correct answer in the required form. $\frac{8}{3}$ does not need to be in simplest form. isw once a correct answer is seen.

Alt (b) – Integration by parts twice

M1: Attempts integration by parts in the correct direction to achieve an expression of the form $\dots t \sin^3 t \pm \dots \int \sin^3 t \, dt$ Condone the omission of their 12

M1: Proceeds to an integrable form by attempting integration by parts again achieving the form $\dots t \sin^3 t \pm \dots \cos t \sin^2 t \pm \dots \int \sin t \cos^2 t \, dt$ Condone the omission of their 12

A1: $(12) \left(\frac{t}{3} \sin^3 t + \frac{1}{3} \cos t \sin^2 t + \frac{2}{9} \cos^3 t \right)$ o.e. Condone any multiple and condone missing variables

M1: Applies the limits 0 and $\frac{\pi}{2}$ to an expression of the form $pt \sin^3 t + q \sin^2 t \cos t + r \cos^3 t$ where p, q and $r \neq 0$

A1: As above in main scheme notes

Question	Scheme	Marks
9(a)	Volume, $V = \pi \int_{\frac{1}{3}}^3 y^2 dx = \pi \int_{\frac{1}{3}}^3 \frac{x^{-\frac{1}{2}}}{(1+x)\left(\arctan(\sqrt{x})\right)^2} dx$	B1
	$\tan u = \sqrt{x} \Rightarrow \sec^2 u \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$ oe	M1 A1
	$\Rightarrow V = (\pi) \int \frac{2 \times \frac{1}{2} x^{-\frac{1}{2}}}{\left(\arctan(\sqrt{x})\right)^2 (1+x)} dx = (\pi) \int \frac{2}{u^2 (1+\tan^2 u)} \sec^2 u du$	dM1
	$= \pi \int \frac{2}{u^2 \sec^2 u} \sec^2 u du = 2\pi \int \frac{1}{u^2} du$	A1
	$\left. \begin{array}{l} x=3 \Rightarrow u = \arctan \sqrt{3} = \frac{\pi}{3} \\ x=\frac{1}{3} \Rightarrow u = \arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6} \end{array} \right\} \Rightarrow V = 2\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{u^2} du$	B1
		(6)
(b)	$V = (2\pi) \left[-\frac{1}{u} \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1
	$= (2\pi) \left[-\frac{1}{\pi/3} - \left(-\frac{1}{\pi/6} \right) \right]$	dM1
	Or $= (2\pi) \left[-\frac{1}{\arctan \sqrt{x}} \right]_{\frac{1}{3}}^3 = (2\pi) \left(-\frac{1}{\arctan \sqrt{3}} - \left(-\frac{1}{\arctan \sqrt{1/3}} \right) \right)$	
	$= 2\pi \left(-\frac{3}{\pi} + \frac{6}{\pi} \right) = 6$	A1
		(3)
(9 marks)		

B1: For a fully correct integral in x or u with π , dx or du , correct limits and bracketing.

May be implied by a fully correct (simplified or un-simplified) integral in u

May be seen at any stage in the working following correct work.

The limits must be correct for the variable they are working in. So for an integral in u the limits must be in radians

$$\text{Look for } V = \pi \int_{\frac{1}{3}}^3 \frac{x^{-\frac{1}{2}}}{(1+x)(\arctan(\sqrt{x}))^2} dx \quad \text{OR} \quad V = \pi \int_{\frac{1}{3}}^3 \left(\frac{x^{-\frac{1}{4}}}{\sqrt{1+x} \arctan(\sqrt{x})} \right)^2 dx$$

$$\text{If the correct integral in } x \text{ is not seen score for } V = \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2}{u^2(1+\tan^2 u)} \sec^2 u \, du \quad \text{o.e.}$$

M1: Attempts the differentiation of the given substitution achieving the correct form with suitable placement for du and dx .

Examples;

- $\tan u = \sqrt{x} \Rightarrow \sec^2 u \, du = \alpha x^{-\frac{1}{2}} dx$
- $x = \tan^2 u \Rightarrow \frac{dx}{du} = \dots \tan u \sec^2 u$
- $u = \arctan \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{1+x} \times \dots x^{-\frac{1}{2}}$

A1: Correct differentiation, any form. Score when first seen, don't penalise an incorrect rearrangement.

Examples

- $\tan u = \sqrt{x} \Rightarrow \sec^2 u \, du = \frac{1}{2} x^{-\frac{1}{2}} dx$
- $x = \tan^2 u \Rightarrow \frac{dx}{du} = 2 \tan u \sec^2 u$
- $u = \arctan \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{1+x} \times \frac{1}{2} x^{-\frac{1}{2}}$

dM1: Makes a complete substitution for x into $\int y^2 dx$.

No need for the π or limits for this mark. It is dependent upon the previous M

Look for the form $\int \frac{c}{u^k(1+\tan^2 u)} \sec^2 u \, (du)$ (which may be un-simplified) where $k=1$ or 2

Condone a missing du . It can be implied if the substitution for dx has been attempted.

Condone missing brackets if the intention is clear.

A1: Correct simplified integral in terms of u including the π from correct working.

Allow recovery from missing brackets. No need for limits for this mark.

Condone a missing du . It can be implied if the substitution for dx has been attempted

B1: Correct limits for the integral found. Allow where seen, even in part (b) and even if not attached to the integral. Must be in radians

(b)

M1: Carries out the integration.

The π etc may be missing or incorrect for this mark. Accept $\frac{1}{u^2} \rightarrow \pm \frac{a}{u}$

dM1: Substitutes appropriate limits into their integral and subtracts either way round.

If the integral is in terms of u the limits must be in radians, not degrees.

If in terms of x they should be using $\arctan \sqrt{3}$ and $\arctan \frac{1}{\sqrt{3}}$

A1: cso. Cannot be scored if k is guessed or found incorrectly

Question Number	Scheme	Marks
9(a)	$\tan \theta = \sqrt{3} \Rightarrow k = \frac{\pi}{3} \text{ (or } 60^\circ \text{)} \text{ (Allow } x = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} \text{ (or } 60^\circ \text{))}$	B1
	$V = (\pi) \int y^2 dx = (\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta \text{ oe}$	M1A1
	$4(\pi) \int \sin^2 2\theta \sec^2 \theta d\theta = 4(\pi) \int 4 \sin^2 \theta \cancel{\cos^2 \theta} \times \frac{1}{\cancel{\cos^2 \theta}} d\theta$	dM1
	$= 16(\pi) \int \sin^2 \theta d\theta \text{ oe e.g. } 16(\pi) \int (1 - \cos^2 \theta) d\theta$	A1
	$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \Rightarrow 16(\pi) \int \sin^2 \theta d\theta = 16(\pi) \int \frac{1 - \cos 2\theta}{2} d\theta$	dM1
	$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi (1 - \cos 2\theta) d\theta$	A1 Cso
		(7)
(b)	$\int (1 - \cos 2\theta) d\theta \rightarrow \theta - \frac{\sin 2\theta}{2}$	B1
	$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi (1 - \cos 2\theta) d\theta = [8\pi\theta - 4\pi \sin 2\theta]_0^{\frac{\pi}{3}} = \frac{8}{3}\pi^2 - 2\sqrt{3}\pi$	M1 A1
		(3)
		(10 marks)

(a)

B1: States or uses $\tan \theta = \sqrt{3} \Rightarrow k = \frac{\pi}{3}$ (Allow 60° here). May be implied by their integral.

Allow if seen anywhere in the question either stated or used as their upper limit.

M1: Attempts volume = $(A\pi) \int y^2 dx = (A\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta$ with or without π or “ $d\theta$ ”.

Condone bracketing errors

A1: For a volume of $(A\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta$ with or without π or “ $d\theta$ ”. The brackets must be

present but may be implied by subsequent work.

dM1: Uses $\sin 2\theta = 2 \sin \theta \cos \theta$ and proceeds to $\text{Volume} = B \int \sin^2 \theta d\theta$ with or without “ $d\theta$ ”. (No requirement for limits yet). Note that if $(2 \sin 2\theta)^2$ becomes $2 \sin^2 \theta \cos^2 \theta$ with no evidence of a correct identity then score dM0

Depends on the first M.

A1: Volume = $(A\pi) \int 16 \sin^2 \theta d\theta$ oe e.g. $(A\pi) \int 16(1 - \cos^2 \theta) d\theta$ with or without π or “ $d\theta$ ”. (No requirement for limits yet)

dM1: Attempts to use $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ or $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$ and obtains $\text{Volume} = \int (P \pm Q \cos 2\theta) d\theta$



Depends on the first M.

A1: CSO $\int_0^{\frac{\pi}{3}} 8\pi(1 - \cos 2\theta) d\theta$. Fully correct integral with both limits and the “dθ” but the 8 and/or the π can be either side of the integral sign.

Note this alternative solution for part (a):

$$V = (A\pi) \int y^2 dx = (A\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta = (A\pi) \int \frac{4 \sin^2 2\theta}{\cos^2 \theta} d\theta \quad \mathbf{M1 \ A1 \ as \ above}$$

$$= (A\pi) \int \frac{4 \sin^2 2\theta}{\frac{1}{2}(1 + \cos 2\theta)} d\theta$$

dM1: uses $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$ in the denominator. **A1:** Correct integral

$$= 8(A\pi) \int \frac{1 - \cos^2 2\theta}{1 + \cos 2\theta} d\theta = 8(A\pi) \int \frac{(1 + \cos 2\theta)(1 - \cos 2\theta)}{1 + \cos 2\theta} d\theta$$

dM1: Uses $\sin^2 2\theta = 1 - \cos^2 2\theta$ and the difference of 2 squares in the numerator and cancels

$$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi(1 - \cos 2\theta) d\theta \quad \mathbf{A1 \ CSO}$$

Note that a Cartesian approach in part (a) essentially follows the main scheme e.g.

$$V = (\pi) \int y^2 dx = (A\pi) \int (4x \cos^2 \theta)^2 \sec^2 \theta d\theta = 4(A\pi) \int 4 \sin^2 \theta \cancel{\cos^2 \theta} \times \frac{1}{\cancel{\cos^2 \theta}} d\theta \text{ etc.}$$

If in doubt whether such attempts deserve credit send to review.

(b)

B1: States or uses $\int (1 - \cos 2\theta) d\theta \rightarrow \theta - \frac{\sin 2\theta}{2}$

M1: Volume = $\int_0^{\frac{\pi}{3}} p(1 - \cos 2\theta) d\theta = [p\theta \pm kp \sin 2\theta]_0^{\frac{\pi}{3}}$ and uses the limit $\frac{\pi}{3}$ (not 60°).

(The limit of 0 may not be seen)

A1: $\frac{8}{3}\pi^2 - 2\sqrt{3}\pi$ or e.g. $8\pi\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)\frac{8}{3}$, $\pi^2 - 2\sqrt{3}\pi$, $\frac{2\pi}{3}(4\pi - 3\sqrt{3})$, $\frac{8\pi^2 - 6\sqrt{3}\pi}{3}$



Question Number	Scheme	Marks
6 (a)(i)	$\text{Area } R = \int y \frac{dx}{dt} dt = \int 4 \sin t \times -4 \sin 2t dt$ $= - \int 32 \sin^2 t \cos t dt$ $x = 0 \Rightarrow t = \frac{\pi}{4} \text{ and } y = 0 \Rightarrow t = 0 \Rightarrow \text{Area} = - \int_{\frac{\pi}{4}}^0 32 \sin^2 t \cos t dt = \int_0^{\frac{\pi}{4}} 32 \sin^2 t \cos t dt \quad *$	M1 A1 dM1 A1* (4)
(ii)	$\text{Area} = \left[\frac{32}{3} \sin^3 t \right]_0^{\frac{\pi}{4}} = \frac{32}{3} \times \frac{2\sqrt{2}}{8} = \frac{8\sqrt{2}}{3}$	M1 A1 (2)
(b)	Attempts to use $\cos 2t = 1 - 2 \sin^2 t \Rightarrow \frac{x}{2} = 1 - 2 \left(\frac{y}{4} \right)^2$ $y = \sqrt{8 - 4x}$	M1A1 A1 (3)
(c)	Range is $0 \leq f \leq 4$	B1 (1)
		(10 marks)

(a)(i)

 M1: Attempts to multiply y by $\frac{dx}{dt}$ to achieve an integrand of the form $\pm k \sin t \sin 2t$

 Look for area $R = \int y \frac{dx}{dt} (dt) = \int (4) \sin t \times \pm k \sin 2t (dt)$ but condone a missing 4 or dt

 A1: Correct integrand. Achieves area $R = \int y \frac{dx}{dt} (dt) = \int 4 \sin t \times -4 \sin 2t (dt)$

 Condone a missing dt and do not be concerned by the limits. Allow unsimplified.

 dM1: Substitutes $\sin 2t = 2 \sin t \cos t \rightarrow$ seen or implied $= \pm \int A \sin^2 t \cos t dt$

 Dependent upon previous M. Condone a missing dt and do not be concerned by the limits.

 A1*: Achieves area $\left(= \int_0^2 y dx \right) = - \int_{\frac{\pi}{4}}^0 32 \sin^2 t \cos t dt \rightarrow \int_0^{\frac{\pi}{4}} 32 \sin^2 t \cos t dt$

This is a given answer so you must see

- evidence of the dt in at least one line other than the given answer
- correct application of the limits as seen above with as a minimum $-\int_{\frac{\pi}{4}}^0 \dots dt \rightarrow \int_0^{\frac{\pi}{4}} \dots dt$

(a)(ii)

 M1: Integrates to a form $\left[A \sin^3 t \right]$ with some attempt to apply the limit(s)

 You may see a substitution $u = \sin t$ which is fine. Just look for Au^3 with some attempt to apply the adapted limits

 A1: Correct answer $\frac{8\sqrt{2}}{3}$ o.e. following correct **algebraic** integration.

Cannot be scored without the M mark.

(b)

 M1: Attempts to use a double angle formula of the form $\cos 2t = \pm 1 \pm 2 \sin^2 t$ with the parametric equations to get a Cartesian equation.

 If the parametric equations are substituted into the given form of the answer $y = \sqrt{ax+b}$, marks are only scored when the double angle formula is used

 A1: Any correct un-simplified equation $\frac{x}{2} = 1 - 2 \left(\frac{y}{4} \right)^2$

On the alternative method, this A mark is scored when the candidate writes down

$$a + 2b = 0, -4b = 16 \Rightarrow a = \dots, b = \dots$$

 A1: $y = \sqrt{8-4x}$ or $y = \sqrt{-4x+8}$ ONLY

(c)

 B1: Range is $0 \leq f \leq 4$

 Allow other acceptable forms such as $0 \leq y \leq 4$, $0 \leq f(x) \leq 4$ and $[0,4]$

 Examples of unacceptable forms are $0 \leq x \leq 4$, $[0,4)$

Question	Scheme	Marks
4(a)	Area of one face = $\frac{1}{2}x \times x \times \sin 60^\circ = \frac{1}{2}x^2 \frac{\sqrt{3}}{2}$ oe	M1
	So surface area of icosahedron is $20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ *	A1*
		(2)
(b)	$\frac{dA}{dx} = 10\sqrt{3}x$ and $\frac{dV}{dx} = \frac{15}{12}(3+\sqrt{5})x^2$	M1
	$\frac{dV}{dA} = \frac{dV}{dx} \times \frac{dx}{dA} = \frac{dV}{dx} \div \frac{dA}{dx} = \dots$ (oe)	M1
	$= \frac{15(3+\sqrt{5})x^2}{12 \times 10\sqrt{3}x} = \frac{(3+\sqrt{5})x}{8\sqrt{3}}$ *	A1*
		(3)
(c)	$\frac{dA}{dt} = 0.025$	B1
	$\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt} = \frac{(3+\sqrt{5})x}{8\sqrt{3}} \times 0.025 = \dots$	M1
	When $x = 2$, $\frac{dV}{dt} = \frac{2(3+\sqrt{5})}{8\sqrt{3}} \times 0.025 = \dots$	
	awrt 0.019 (cm ³ s ⁻¹)	A1
		(3)
(8 marks)		

Notes:**(a)**

M1: Any correct method to establish a surd form for the area of one face. Must evaluate trigonometric terms.

Alternatives include e.g. $\frac{1}{2} \times x \times \sqrt{x^2 - \left(\frac{1}{2}x\right)^2} = \frac{1}{2}x\sqrt{\frac{3}{4}}x^2$

A1*: Correct result achieved after showing side area of one face.

Examples:

$$\text{Area of one face} = \frac{1}{2}x \times x \times \sin 60^\circ = \frac{1}{4}x^2\sqrt{3} \Rightarrow SA = 20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2 \text{ scores M1 A1}$$

$$SA = 20 \times \left(\frac{1}{2} \times x \times x \sin 60^\circ \right) = 5\sqrt{3}x^2 \text{ scores M1 A0 for lack of working}$$

(b)

M1: Attempts derivatives of both A and V wrt x . E.g. $\left(\frac{dA}{dx} = \right)\alpha x$ and $\left(\frac{dV}{dx} = \right)\beta x^2$.

Note that $\frac{5}{12}(3+\sqrt{5})x^3$ is sometimes expanded first as e.g. $\frac{5}{4}x^3 + \frac{5\sqrt{5}}{12}x^3$ and allow this mark

if either term is differentiated to βx^2

M1: Applies the correct chain rule with their derivatives. (Condone poor notation – see below)

Question	Scheme	Marks
6(a)	$\overrightarrow{AB} = \begin{pmatrix} 5-1 \\ 3-4 \\ -2-3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix} = 4\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$	M1
	e.g. $\mathbf{r} = \mathbf{i} - 4\mathbf{j} + 3\mathbf{k} + \lambda(4\mathbf{i} + 7\mathbf{j} - 5\mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} + 7\mathbf{j} - 5\mathbf{k})$	M1A1
		(3)
(b)	$\overrightarrow{AC} = \begin{pmatrix} 3-1 \\ p-4 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 2 \\ p+4 \\ -4 \end{pmatrix} = 2\mathbf{i} + (p+4)\mathbf{j} - 4\mathbf{k}$	M1
	$\begin{pmatrix} 2 \\ p+4 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix} = 8 + 7p + 28 + 20 = 0 \Rightarrow p = -8$	M1A1
		(3)
(c)	$ \overrightarrow{AB} = \sqrt{4^2 + 7^2 + (-5)^2} = \sqrt{90} \text{ or } \overrightarrow{AC} = \sqrt{2^2 + (-4)^2 + (-4)^2} = 6$	M1
	Area $\frac{1}{2} \times \sqrt{90} \times 6 = 9\sqrt{10}$	dM1A1
		(3)
		(9 marks)

Notes

Accept either vector form throughout but extra $\mathbf{i}, \mathbf{j}, \mathbf{k}$ in column vectors will lose A mark in (a).

(a) This is now being marked MMA

M1: Attempts to find $\overrightarrow{AB}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates.

M1: Attempts equation for the line, score for $\overrightarrow{OA} + \lambda \times \text{their } \overrightarrow{AB}$ or $\overrightarrow{OB} + \lambda \times \text{their } \overrightarrow{AB}$. No need for $\mathbf{r}$ for this mark.

A1: Any correct equation. Must be $\mathbf{r} = \dots$ ($l = \dots$ is A0)

(b)

M1: Attempts to find $\overrightarrow{AC}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates.

M1: Takes scalar product of their $\overrightarrow{AB}$ and their $\overrightarrow{AC}$ to form and solve a linear equation in p

A1: $p = -8$

(c)

M1: Attempts to find the magnitude of either their $\overrightarrow{AB}$ or their $\overrightarrow{AC}$ using their p

dM1: Attempts to find the exact area of the triangle ABC . It is dependent on the previous method mark. There

most common method will be $\frac{1}{2} |\overrightarrow{AB}| |\overrightarrow{AC}|$ as in scheme but other methods are possible. E.g.

$$\cos \angle ABC = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}| |\overrightarrow{BC}|} \Rightarrow A = \frac{1}{2} |\overrightarrow{BA}| |\overrightarrow{BC}| \sin \angle ABC. \text{ Such a method must be complete, including use of}$$

Pythagorean identity to find $\sin \angle ABC$. Other more advanced methods (such as cross products) are also possible. If you see something you feel is worthy of some credit but does not fit the scheme, send to Review.

A1: $9\sqrt{10}$



Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = \cos t + 6 \cos t \sin t$ $\frac{dy}{dt} = 3 \cos t - 2 \sin t$	B1B1
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3 \cos t - 2 \sin t}{\cos t + 6 \cos t \sin t} = \frac{3 \cos \pi - 2 \sin \pi}{\cos \pi + 6 \cos \pi \sin \pi} = 3$ *	M1A1*
		(4)
(b)	When $t = \pi$, $x = -3$, $y = -2$	B1
	$y - "-2" = 3(x - "-3")$	M1
	$y = 3x + 7$	A1
		(3)
(c)	$y = 3x + 7 \Rightarrow 3 \sin t + 2 \cos t = 3(\sin t - 3 \cos^2 t) + 7$ or $y = 3(x + 3 \cos^2 t) + 2 \cos t \Rightarrow 3x + 7 = 3x + 9 \cos^2 t + 2 \cos t$	M1
	$\Rightarrow 9 \cos^2 t + 2 \cos t - 7 = 0$ *	A1*
		(2)
(d)	$\cos t = \frac{7}{9}$	B1
	$y = 3 \times \frac{\sqrt{32}}{9} + 2 \times \frac{7}{9} = \frac{4\sqrt{2}}{3} + \frac{14}{9}$	M1A1
		(3)
		(12 marks)

Notes

(a)

B1: $\left(\frac{dx}{dt} =\right) \cos t + 6 \cos t \sin t$ or $\cos t + 3 \sin 2t$ B1: $\left(\frac{dy}{dt} =\right) 3 \cos t - 2 \sin t$ M1: Attempts $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ using their $\frac{dx}{dt}$ and their $\frac{dy}{dt}$ and substitutes $t = \pi$. (May substitute π before dividing.)A1*: Achieves $\frac{dy}{dx} = 3$ with full working shown and no errors.

(b)

B1: $x = -3$, $y = -2$ which may be seen within their workingM1: Attempts to find the equation of the tangent with gradient 3. If they use $y = mx + c$ they must proceed as far as $c = \dots$ A1: $y = 3x + 7$

(c)

M1: A full attempt to solve simultaneously the given parametric equations with their equation of the tangent

A1*: Achieves $9 \cos^2 t + 2 \cos t - 7 = 0$ with no errors

(d)

B1: $\cos t = \frac{7}{9}$ seen or implied. Allow if seen in (c).M1: Attempts to find the y coordinate Must attempt to evaluate trig terms. If no substitution/working shown, then score for awrt 3.44 following a correct value for $\cos t$ A1: $\frac{4\sqrt{2}}{3} + \frac{14}{9}$ or exact equivalent. Withhold if additional answers are given.

Question Number	Scheme	Notes	Marks
6 Way 1	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \, dx (+c)$ M1: For applying parts to obtain $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		M1A1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left\{ -\frac{1}{3} e^{2x} \cos 3x + \frac{2}{3} \int e^{2x} \cos 3x \, dx \right\} (+c)$ Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \cos 3x \, dx$		M1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} \int e^{2x} \cos 3x \, dx (+c)$ Fully correct application of parts twice		A1
	$\int e^{2x} \cos 3x \, dx + \frac{4}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c)$ $\Rightarrow \frac{13}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		M1
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			(6)
Way 2	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \int e^{2x} \sin 3x \, dx (+c)$ M1: For applying parts to obtain $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		M1A1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \left\{ \frac{1}{2} e^{2x} \sin 3x - \frac{3}{2} \int e^{2x} \cos 3x \, dx \right\} (+c)$ Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \cos 3x \, dx$		M1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x - \frac{9}{4} \int e^{2x} \cos 3x \, dx (+c)$ Fully correct application of parts twice		A1
	$\int e^{2x} \cos 3x \, dx + \frac{9}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c)$ $\Rightarrow \frac{13}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		M1
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			Total 6

Question Number	Scheme	Notes	Marks
7(a)	$\frac{dV}{dt} = 300 - kV \Rightarrow \int \frac{dV}{300 - kV} = \int dt$	Correct separation of variables	B1
	$\int \frac{dV}{300 - kV} = -\frac{1}{k} \ln(300 - kV)$	$\int \frac{dV}{300 - kV} = \alpha \ln(300 - kV)$	M1
	$-\frac{1}{k} \ln(300 - kV) = t + c$	Correct equation including a constant of integration	A1
	$-\frac{1}{k} \ln(300 - kV) = t + c \Rightarrow \ln(300 - kV) = -kt + d$ $\Rightarrow 300 - kV = e^{-kt+d}$ Correct processing to remove the “ln”		M1
	$kV = 300 - e^{-kt+d} \Rightarrow V = \frac{300}{k} - Be^{-kt}$ $V = \frac{300}{k} + Ae^{-kt} *$	Correct proof	A1*
			(5)
(b)	$V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$	Uses $V = 0$ when $t = 0$ to find A in terms of k	M1
	$V = \frac{300}{k} - \frac{300}{k} e^{-kt} \Rightarrow \frac{dV}{dt} = 300e^{-kt}$	$\frac{dV}{dt} = \alpha e^{-kt}$	M1
	$300e^{-10k} = 200 \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$	Uses $\frac{dV}{dt} = 200$ when $t = 10$ and correct processing to find k	M1
	$k = -\frac{1}{10} \ln \frac{2}{3}$	Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$	A1
			(4)
(b) Way 2	$V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$	Uses $V = 0$ when $t = 0$ to find A in terms of k	M1
	$\frac{dV}{dt} = 200, t = 10 \Rightarrow 200 = 300 - kV$ $\Rightarrow kV = 100$	Uses $\frac{dV}{dt} = 200$ when $t = 10$ to find a value for kV	M1
	$V = \frac{300}{k} + Ae^{-kt} \Rightarrow kV = 300 - 300e^{-10k}$ $\Rightarrow 100 = 300 - 300e^{-10k} \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$	Substitutes for kV , kA and $t = 10$ and uses correct processing to find k	M1
	$k = -\frac{1}{10} \ln \frac{2}{3}$	Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$	A1
(c)	$6000 = \frac{3000}{\ln 1.5} - \frac{3000}{\ln 1.5} e^{-\frac{t}{10} \ln 1.5}$ $\Rightarrow e^{-\frac{t}{10} \ln 1.5} = 1 - 2 \ln 1.5$ $\Rightarrow -\frac{t}{10} \ln 1.5 = \ln(1 - 2 \ln 1.5)$	Correct strategy using $V = 6000$ to reach $at = \dots$	M1
	$t = 41$	Correct value	A1
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
8	Assume that there exist positive real numbers x and y such $\frac{9x}{y} + \frac{y}{x} < 6$	Starts the proof by contradicting the given statement	B1
	$\frac{9x}{y} + \frac{y}{x} < 6 \Rightarrow 9x^2 + y^2 < 6xy$ as x and y are both positive	Multiplies through by xy	M1
	$\Rightarrow 9x^2 + y^2 - 6xy < 0$ $\Rightarrow (3x - y)^2 < 0$	Reaches a correct contradictory statement	A1
	As x and y are positive real numbers, this is a contradiction and so $\frac{9x}{y} + \frac{y}{x} < 6$ must be incorrect and so $\frac{9x}{y} + \frac{y}{x} \dots 6^*$	Makes a suitable conclusion	A1*
			(4)
			Total 4

Question Number	Scheme	Notes	Marks
9(a)	$V = \pi \int y^2 dx = \pi \int y^2 \frac{dx}{d\theta} d\theta$ $= \pi \int (3 \sin \theta - \sin 2\theta)^2 (-5 \sin \theta) d\theta$	Applies $V = \pi \int y^2 \frac{dx}{d\theta} d\theta$ with or without the π	M1
	$= \pi \int (3 \sin \theta - 2 \sin \theta \cos \theta)^2 (-5 \sin \theta) d\theta$	Applies $\sin 2\theta = 2 \sin \theta \cos \theta$	M1
		Fully correct integral in terms of $\sin \theta$ and $\cos \theta$ only (π not needed)	A1
	$= \pi \int \sin^2 \theta (3 - 2 \cos \theta)^2 (-5 \sin \theta) d\theta$ $V = -5\pi \int \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = -5\pi \int_{\pi}^0 \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = 5\pi \int_0^{\pi} \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta^*$	Completes correctly with correct limits and no incorrect statements previously. The factor of π must be present throughout.	A1*
			(4)
(b)	$u = \cos \theta \Rightarrow V = 5\pi \int \sin^3 \theta (3 - 2u)^2 \frac{du}{-\sin \theta}$	Applies the substitution correctly	M1
	$\theta = 0 \Rightarrow u = 1, \theta = \pi \Rightarrow u = -1$	Attempts to change θ limits to u limits	M1
	$V = -5\pi \int \sin^2 \theta (3 - 2u)^2 du = -5\pi \int (1 - u^2)(3 - 2u)^2 du$ Correct integral in terms of u only		A1
	$(1 - u^2)(3 - 2u)^2 = (1 - u^2)(9 - 12u + 4u^2)$ $= 9 - 12u - 5u^2 + 12u^3 - 4u^4$	Attempt to expand	M1
		Correct expansion	A1
	$V = 5\pi \int_{-1}^1 (9 - 12u - 5u^2 + 12u^3 - 4u^4) du$ $= 5\pi \left[9u - 6u^2 - \frac{5u^3}{3} + 3u^4 - \frac{4u^5}{5} \right]_{-1}^1 = \dots$ Integrates and applies their u limits		M1
	$= \frac{196}{3} \pi$	Cao	A1
			(7)
			Total 11



Question Number	Scheme	Marks
8(a)	$x = 2$	B1
		(1)
(b)	$(2.5, 1.5)$	B1
		(1)
(c)	$\frac{dy}{dx} = \frac{1 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$	M1
	$t = 2 \Rightarrow \frac{dy}{dx} = \frac{5}{3} \Rightarrow y - 1.5 = -\frac{3}{5}(x - 2.5)$	dM1
	$3x + 5y = 15^*$	A1*
		(3)
(d)	l crosses x -axis at $x = 5$	B1
	$\frac{1}{3} \pi \times 1.5^2 (5 - 2.5) \left(= \frac{15}{8} \pi \right)$	M1
	$V = \pi \int y^2 dx = \pi \int y^2 \frac{dx}{dt} dt = \pi \int \left(t - \frac{1}{t} \right)^2 \left(1 - \frac{1}{t^2} \right) dt$	M1
	$\pi \int \left(t^2 - 2 + \frac{1}{t^2} \right) \left(1 - \frac{1}{t^2} \right) dt = \pi \int \left(t^2 + \frac{3}{t^2} - \frac{1}{t^4} - 3 \right) dt = \dots$	M1
	$\pi \left[\frac{t^3}{3} - \frac{3}{t} + \frac{1}{3t^3} - 3t \right]$	A1
	$\pi \left[\frac{t^3}{3} - \frac{3}{t} + \frac{1}{3t^3} - 3t \right]_1^2 = \pi \left(\left(\frac{8}{3} - \frac{3}{2} + \frac{1}{24} - 6 \right) - \left(\frac{1}{3} - 3 + \frac{1}{3} - 3 \right) \right)$	M1
	$V = \frac{13}{24} \pi + \frac{15}{8} \pi = \frac{29}{12} \pi$ Cao	A1
		(7)
		Total 12

(a)

 B1: Correct value. Just look for 2 and condone $Q = 2$

(b)

 B1: Correct coordinates. Allow these to be given separately $x = 2.5$, $y = 1.5$

(c)

 M1: Attempts to differentiate both parameters and use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ to achieve $\frac{1 \pm \frac{1}{t^2}}{1 \pm \frac{1}{t^2}}$ or equivalent via

quotient rule

 dM1: Uses $t = 2$ in their $\frac{dy}{dx}$ and uses the negative reciprocal gradient with their point P to form equation

of normal. It is dependent upon the previous M

A1*: Obtains the printed answer with no errors and sufficient working shown.

What is shown in the MS is sufficient

(d)

B1: Correct x intercept for l . This may appear on the x -axis on Figure 2.

It may be implied by the value appearing in the volume of the cone but not the volume under the curve

M1: Fully correct method for the cone volume $\frac{1}{3}\pi \times 1.5^2 (5 - 2.5) =$

$$\text{May be awarded for } \int_{2.5}^5 \pi(3-0.6x)^2 dx = \left[\frac{\pi(3-0.6x)^3}{-1.8} \right]_{2.5}^5 = \frac{\pi(3-0.6 \times 2.5)^3}{1.8}$$

Condone slips. If they expand $(3-0.6x)^2$ look for a quadratic expression integrating to a cubic of the correct form followed by use of the limits 5 and their 2.5

M1: Correct strategy for the volume generated by rotating the area under the curve

$$\text{Look for } V = \pi \int y^2 dx = \pi \int y^2 \frac{dx}{dt} dt \text{ leading to } \pi \int \left(t - \frac{1}{t}\right)^2 \left(1 \pm \frac{k}{t^2}\right) dt, \text{ where } k \text{ is a constant}$$

Condone a sign slip on $\frac{dx}{dt}$ so allow $\left(1 \pm \frac{1}{t^2}\right)$. The π may be added later in the solution.

The dt may be omittedM1: Attempt to expand $\left(t - \frac{1}{t}\right)^2 \left(1 \pm \frac{k}{t^2}\right)$ and integrate term by term.

Look for $\left(t - \frac{1}{t}\right)^2 \left(1 \pm \frac{k}{t^2}\right) \rightarrow$ polynomial in t containing positive and negative indices followed by some correct integration for both positive and negative indices

A1: Correct integration of $\left(t - \frac{1}{t}\right)^2 \left(1 - \frac{1}{t^2}\right) \rightarrow \frac{t^3}{3} - \frac{3}{t} + \frac{1}{3t^3} - 3t$ which does not need to be simplifiedM1: Applies the t limits “1” and 2 to an attempt at the integral of $(\pi) \int \left(t - \frac{1}{t}\right)^2 \left(1 \pm \frac{k}{t^2}\right) dt$ The π does not need to be included.A1: $\frac{29}{12}\pi$ or exact equivalent......
Part (c) using a calculator

Whilst it is possible to do the integration for the area under the curve using a calculator, it will not score all of the marks.

$$\text{E.g. } \pi \int_1^2 \left(t - \frac{1}{t}\right)^2 \left(1 - \frac{1}{t^2}\right) dt = 0.5416\pi$$

The marks in (c) available to such a candidate are:

B1(for $x = 5$),

M1 (for volume of cone),

M1 (Sight of correct integral),

M0 (no expansion),

A0 ,

M1(sight of limits),

A0
.....

See below for Cartesian approach for (c) and (d)

(c)	$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t} \Rightarrow x^2 - y^2 = 4 \Rightarrow 2x - 2y \frac{dy}{dx} = 0$ or $y^2 = x^2 - 4 \Rightarrow y = (x^2 - 4)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = x(x^2 - 4)^{-\frac{1}{2}}$ Attempts to differentiate a Cartesian equation of the form $x^2 - y^2 = k$ to obtain either $\alpha x + \alpha y \frac{dy}{dx} = 0$ or $\frac{dy}{dx} = kx(x^2 - 4)^{-\frac{1}{2}}$	M1
	$\frac{dy}{dx} = \frac{x}{y} = \frac{t + \frac{1}{t}}{t - \frac{1}{t}} = \frac{2.5}{1.5} \quad \text{or} \quad \frac{dy}{dx} = 2.5(2.5^2 - 4)^{-\frac{1}{2}}$ $\Rightarrow y - 1.5 = -\frac{3}{5}(x - 2.5)$ Uses $t = 2$ in their $\frac{dy}{dx}$ and uses the negative reciprocal gradient with their point P. If using explicit differentiation, must attempt x using $t = 2$ first	M1
	$3x + 5y = 15^*$ Obtains the printed answer with no errors and sufficient working shown.	A1*
		(3)
(d)	l crosses x -axis at $x = 5$	B1
	$\frac{1}{3} \pi \times 1.5^2 (5 - 2.5) \left(= \frac{15}{8} \pi \right)$ Fully correct method for the cone volume	M1
	$V = \pi \int y^2 dx = \pi \int (x^2 - 4) dx$ Correct strategy for the other volume Condone for this $V = \pi \int y^2 dx = \pi \int (x^2 - k) dx$	M1
	$\pi \int (x^2 - 4) dx = \pi \left[\frac{x^3}{3} - 4x \right]$ M1: Attempt to integrate $\int (x^2 - k) dx$ A1: Correct integration of $\int (x^2 - 4) dx$	M1A1
	$\pi \left[\frac{x^3}{3} - 4x \right]_2^{2.5}$ Applies the limits “2” and their x from $t = 2$ to an attempted integral of $(\pi) \int (x^2 - k) dx$	M1
	$V = \frac{13}{24} \pi + \frac{15}{8} \pi = \frac{29}{12} \pi$ Cao	A1
		(7)

Question Number	Scheme		Marks
9(a)	$(V) = \pi \int_0^4 \left(\cos x + \frac{1}{5} e^x \right)^2 dx$		B1
	$\left(\cos x + \frac{1}{5} e^x \right)^2 = \cos^2 x + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x}$ $= \frac{\cos 2x + 1}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x}$ $= \pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$		M1 dM1 A1
			(4)
(b)	Way I $\int e^x \cos x \, dx = \pm e^x \sin x \pm \int e^x \sin x \, dx$ $= e^x \sin x - \int e^x \sin x \, dx$	Way Two $\int e^x \cos x \, dx = e^x \cos x \pm \int e^x \sin x \, dx$ $= e^x \cos x + \int e^x \sin x \, dx$	M1 A1
	$\int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x - \int e^x \cos x \, dx$ $2 \int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x$		dM1
	$\int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x) + (c)$		A1
			(4)
(c)	$\pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$ $= (\pi) \left[\frac{1}{2} x + \frac{\sin 2x}{4} + \frac{2}{5} \left(\frac{1}{2} e^x (\sin x + \cos x) \right) + \frac{1}{50} e^{2x} \right]_0^4$ $2\pi \left(2 + \frac{\sin 8}{4} + \frac{2}{5} \left(\frac{1}{2} e^4 (\sin(4) + \cos(4)) \right) + \frac{1}{50} e^8 - \frac{11}{50} \right)$ $= \text{awrt } 290 \text{ (cm}^3\text{)}$		M1A1ft dM1 A1
			(4)
			(12 marks)

(a)

B1: A correct expression for the volume including limits and dx E.g. $\pi \int_0^4 \left(\cos x + \frac{1}{5} e^x \right)^2 dx$

The correct limits may be implied by subsequent work. It can be awarded from a correct final answer (as it is not a given answer). Don't be concerned by the non- appearance of a V . Condone it being termed 'Area'

M1: Attempts to multiply out $\left(\cos x + \frac{1}{5} e^x \right)^2$ leading to $\cos^2 x \pm \dots e^x \cos x \pm \dots e^{2x}$

dM1: Uses the identity $\cos 2x = \pm 2 \cos^2 x \pm 1$. Dependent upon previous M

A1: Fully correct $\pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$



For this final mark

- condone either a missing dx or limits (but not both) if the candidate has already lost the B mark for the same reason
- condone missing limits if they have already appeared within an expression for the volume anywhere in part (a)

(b) Condone lack of dx 's throughout part (b).

If you see other methods that you feel deserve credit, and you cannot apply the scheme, then please send to review.

M1: Attempts integration by parts. Be generous unless they explicitly state the rule is $\int v du = uv + \int u dv$

Score for Way One $\int e^x \cos x \, dx = \pm e^x \sin x \pm \int e^x \sin x \, dx$

Or Way Two $\int e^x \cos x \, dx = e^x \cos x \pm \int e^x \sin x \, dx$

A1: Correct intermediate form

Score for Way One $\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$

Or Way Two $\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$

dM1: Attempts integration by parts a second time. It is dependent upon having scored the previous M.

Score for $\int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x - \int e^x \cos x \, dx$

AND rearranges to $2 \int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x$

A1: CSO $\int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x) + c$ o.e. condone lack of $+c$ and the dx

Withold this mark if there is incorrect or unexplained working within the proof

(c)

M1: Attempts to integrate their (a) condoning transcription errors on coefficients achieving

$\alpha x + \beta \sin 2x + \chi \left(\frac{1}{2} e^x (\cos x + \sin x) \right) + \delta e^{2x}$ where α, β, χ and δ are constants following through on their

attempted integration of $e^x \cos x$

They must have an attempted integration for part (b), which may be incorrect, which is then used in (c).

A1ft: $\left(\pi \right) \left(\frac{1}{2} x + \frac{\sin 2x}{4} + \frac{2}{5} \left(\frac{1}{2} e^x (\cos x + \sin x) \right) + \frac{1}{50} e^{2x} \right)$

Condone the absence of π and follow through their coefficients A, B, C and D from (a). The integration from part (c) must be correct.

So, look for $\int_0^4 \left(A + B \cos 2x + C e^x \cos x + D e^{2x} \right) dx \rightarrow \left(Ax + \frac{B}{2} \sin 2x + C \left(\frac{1}{2} e^x (\cos x + \sin x) \right) + \frac{D}{2} e^{2x} \right)$

dM1: This requires

- the limits 4 and 0 must be substituted into an expression of the form $ax + b \sin 2x + ce^x (\cos x \pm \sin x) + de^{2x}$ either way around and subtracted. This may be implied by a decimal answer which you may need to check.
- the answer to the above multiplied by 2π (An appreciation that the shape consists of two solids)

A1: awrt 290 cm^3 (condone lack of units).

If they have correct integration, correct limits followed by awrt 290, the last two marks can be awarded.

Note that an answer of 460 is achieved when candidates are working in degrees and scores the method mark but not the accuracy mark.



Question	Scheme	Marks
3	$8x^3 - 3y^2 + 2xy = 9$	
	Either $3y^2 \rightarrow ky \frac{dy}{dx}$ or $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$	M1
	Both $3y^2 \rightarrow ky \frac{dy}{dx}$ and $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$	dM1
	$8x^3 - 3y^2 + 2xy = 9 \rightarrow 24x^2 - 6y \frac{dy}{dx} + 2y + 2x \frac{dy}{dx} = 0$	A1
	$(2, 5) : 96 - 30 \frac{dy}{dx} + 10 + 4 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \dots \left(= \frac{53}{13} \right)$	M1
	$\frac{dy}{dx} = \frac{53}{13}$ or $\frac{dx}{dy} = \frac{13}{53}$ o.e.	A1
	Normal: $y - 5 = -\frac{13}{53}(x - 2)$	M1
	$13x + 53y - 291 = 0$	A1
		(7)
(7 marks)		



M1: Differentiates one term in y to a correct form. Award for either $-3y^2 \rightarrow \pm ky \frac{dy}{dx}$ or

$2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$ Accept equivalents if they differentiate with respect to y instead.

dM1: Differentiates both y terms to a correct form

Award for **Both** $-3y^2 \rightarrow \pm ky \frac{dy}{dx}$ and $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$

A1: Fully correct differentiation. (o.e. if with respect to y). Allow y' for $\frac{dy}{dx}$

You may see $24x^2 dx - 6y dy + 2y dx + 2x dy = 0$ which is fine.

Note that $\frac{dy}{dx} = 24x^2 - 6y \frac{dy}{dx} + 2y + 2x \frac{dy}{dx} = 0$ is A0 unless the extra $\frac{dy}{dx}$ is subsequently omitted

M1: For

- either rearranging to find $\frac{dy}{dx}$ (or $\frac{dx}{dy}$) in terms of **both** x and y and then substituting **both** (2, 5)
- or substituting **both** (2, 5) then rearranging to find $\frac{dy}{dx}$ (or $\frac{dx}{dy}$)

FYI $\frac{dy}{dx} = \frac{24x^2 + 2y}{6y - 2x}$ o.e.

It is dependent upon having exactly two terms in $\frac{dy}{dx}$, one each coming from differentiating the $3y^2$ and the $2xy$.

A1: Correct value for either $\frac{dy}{dx} = \frac{53}{13}$ (accept awrt 4.08) or $\frac{dx}{dy} = \frac{13}{53}$ (awrt 0.245)

M1: Correct method for the equation of the normal at (2, 5). E.g. $y - 5 = -\frac{13}{53}(x - 2)$

It is dependent upon them having attempted differentiation (may be only one term in $\frac{dy}{dx}$) and using

$x = 2, y = 5$ to find the value of $-\frac{dx}{dy}$ which must be the negative reciprocal of their $\frac{dy}{dx}$

If the form $y = mx + c$ is used then the method must proceed as far as $c = \dots$

A1: Correct answer or any integer multiple of this. The $= 0$ must be seen

(a)

B1: Correct coordinates or position vector e.g. as shown or $x = 2, y = 3, z = -7$ or $2\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ or $\begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix}$

Condone $(2\mathbf{i}, 3\mathbf{j}, -7\mathbf{k})$

(b)

M1: Attempts $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ condoning slips. Note that any non-zero multiples of these vectors can be used.

If the method is not explicit then this mark may be implied by 2 correct components.

dM1: Full attempt at $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ where $\mathbf{a}$ and $\mathbf{b}$ are the direction vectors or multiples of them.

Depends on the first method mark.

Note that $\cos \theta = \frac{4 - 2 + 16}{\sqrt{1^2 + 2^2 + 2^2} \sqrt{4^2 + 1^2 + 8^2}}$ would imply both method marks.

A1: $\cos \theta = \frac{2}{3}$

(b) Alternative using the cosine rule:

M1: Attempts $\pm \left(\alpha \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} - \beta \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right)$

dM1: Full attempt at the cosine rule using the appropriate lengths.

Depends on the first method mark.

A1: $\cos \theta = \frac{2}{3}$

(c)

M1: Uses $\lambda = 6$ to find the length of PQ . E.g. uses Pythagoras to calculate $|6\mathbf{i} + 12\mathbf{j} + 12\mathbf{k}|$ or $6|\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}|$

If the method is not explicit then the attempt at $\overline{PQ}$ may be implied by 2 correct components.

M1: Full attempt at $\frac{1}{2}ab \sin C$ where $a = b = |\overline{PQ}|$ and C is their θ which may be attempted in decimals.

Their θ or $\sin \theta$ must follow an attempt to use their $\cos \theta$.

A1: $54\sqrt{5}$ or exact equivalent e.g. $\frac{162\sqrt{5}}{3}$

(d)

M1: Attempts a **correct** method of finding at least one value for μ .

Pythagoras must be used correctly and both sides of their equation must be consistent e.g.

$$(4\mu)^2 - \mu^2 + (8\mu)^2 = 18^2 \text{ and } (4\mu)^2 + \mu^2 + (8\mu)^2 = 18^2 \text{ both score M0}$$

Must be correct work here so must be using the length of PQ not OQ .

dM1: Attempts one correct position $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} \pm 2 \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ **Depends on the first method mark.**

A1: Gives both possible coordinates $(10, 1, 9)$ and $(-6, 5, -23)$ Allow as coordinates or vectors or as $x = \dots, y = \dots, z = \dots$ 

Question Number	Scheme	Marks
7(a)	Substitutes $x = 3$ and $t = 0$ into $x = \frac{k(9t+5)}{4t+3}$	M1
	$\Rightarrow 3 = \frac{5k}{3} \Rightarrow k = 1.8$ *	A1*
		(2)
(b)	4050	B1
		(1)
(c)	$\frac{3}{x(9-2x)} \equiv \frac{A}{x} + \frac{B}{9-2x}$	M1
	Either $A = \frac{1}{3}$ or $B = \frac{2}{3}$	A1
	$\frac{3}{9x-2x^2} \equiv \frac{1}{3x} + \frac{2}{3(9-2x)}$	A1
		(3)
(d)	$3 \frac{dx}{dt} = x(9-2x) \Rightarrow \int \frac{3}{x(9-2x)} dx = \int dt$ Or $3 \frac{dx}{dt} = x(9-2x) \Rightarrow \int \frac{1}{x(9-2x)} dx = \int \frac{1}{3} dt$ $\Rightarrow \frac{1}{3} \ln x - \frac{1}{3} \ln(9-2x) = t(+c)$ or $\frac{1}{9} \ln x - \frac{1}{9} \ln(9-2x) = \frac{1}{3} t(+c)$	M1 A1ft
	Substitutes $t = 0, x = 3 \Rightarrow c = 0$	M1
	$\frac{1}{3} \ln x - \frac{1}{3} \ln(9-2x) = t \Rightarrow \left(\frac{x}{9-2x} \right) = e^{3t}$	ddM1
	$\left(\frac{9-2x}{x} \right) = e^{-3t} \Rightarrow \frac{9}{x} - 2 = e^{-3t} \Rightarrow x = \frac{9}{2+e^{-3t}}$ *	A1*
		(5)
(e)	4500	B1
		(1)
		(12 marks)

(a)

M1: Substitutes $x = 3$ and $t = 0$ into the given equationA1*: Shows that $k = 1.8$ (oe e.g. $\frac{9}{5}$) with no errors and with at least one correct line of the form $ak = b$

$$\text{e.g. } 3 = \frac{k(9 \times 0 + 5)}{4 \times 0 + 3} \Rightarrow k = 1.8, \quad 3 = \frac{k(0 + 5)}{0 + 3} \Rightarrow k = 1.8, \quad 3 = \frac{5k}{3} \Rightarrow k = 1.8 \quad \text{all score M1A1}$$

Alternative by verification:

M1: Substitutes $k = 1.8$ and $t = 0$ into the given equationA1*: Shows that $x = 3$ with at least one correct intermediate line and a concluding statement that this is 3000

Special cases:

1. $x = \frac{k(9 \times 0 + 5)}{4 \times 0 + 3} \Rightarrow k = 1.8$ scores M1A0

2. Substitutes $x = 3000$ and $t = 0$ into the given equation to obtain $k = 1800$ (hence $k = 1.8$) scores M1A0

(b)

B1: 4050 cao. Allow 4.05 thousand.

(c)

M1: Sets $\frac{3}{x(9-2x)} \equiv \frac{A}{x} + \frac{B}{9-2x}$ or equivalent e.g. $3 \equiv A(9-2x) + Bx$

A1: One correct value for 'A' or 'B' or one correct fraction.

A1: Correct **fractions** e.g. $\frac{3}{x(9-2x)} \equiv \frac{1}{3x} + \frac{2}{3(9-2x)}$ oe e.g. $\frac{1/3}{x} + \frac{2/3}{(9-2x)}$, $\frac{1}{3x} + \frac{2}{27-6x}$

This mark is for the correct partial fractions not for the values of the constants.

Award once the correct fractions are seen and allow if seen in (d) if not seen here.

(d)

M1: Attempts to separate the variables and integrate to obtain a kt term and one of $\alpha \ln \beta x$ or $\gamma \ln \delta(9-2x)$.

Condone missing brackets e.g. $\ln 9-2x$

The $\ln$ terms must come from a partial fraction of the form $\frac{A}{x}$ or $\frac{B}{9-2x}$.

A1ft: $\frac{1}{3} \ln x - \frac{1}{3} \ln(9-2x) = t(+c)$ but ft on their A and B so award for $A \ln x - \frac{B}{2} \ln(9-2x) = t(+c)$ oe

There is no requirement for $+c$

Brackets must be present unless they are implied by later work.

Note that there are various correct alternatives here e.g. $\frac{1}{3} \ln 3x - \frac{1}{3} \ln(27-6x) = t(+c)$

M1: Sets $t = 0$ and $x = 3$ in an attempt to find "c" having made some attempt to integrate at least one of their partial fractions to obtain a $\ln$ term.

If this step is not attempted only the first two marks are available in this part.

They can "state" e.g. $c = 0$ or e.g. $K = 1$ provided it follows correct work and there was a $+c$.

ddM1: Dependent upon both previous Method marks.

It is for using correct work to remove the $\ln$'s having found a constant of integration.

A1*: Correct work to reach the given answer showing all necessary steps.

The scheme shows one such way with acceptable minimal working.

Note that some candidates may rearrange first before finding the constant of integration e.g.

$$\begin{aligned}
 3 \frac{dx}{dt} &= x(9-2x) \Rightarrow \int \frac{3}{x(9-2x)} dx = \int dt \\
 &\Rightarrow \frac{1}{3} \ln x - \frac{1}{3} \ln(9-2x) = t + c \\
 \Rightarrow \frac{1}{3} \ln \frac{x}{9-2x} &= t + c \Rightarrow \ln \frac{x}{9-2x} = 3t + d \Rightarrow \frac{x}{9-2x} = Ke^{3t} \\
 t = 0 \text{ and } x = 3 &\Rightarrow K = 1 \\
 \frac{x}{9-2x} &= e^{3t} \Rightarrow 9e^{3t} - 2xe^{3t} = x \Rightarrow x(2e^{3t} + 1) = 9e^{3t} \\
 \Rightarrow x &= \frac{9e^{3t}}{(2e^{3t} + 1)} = \frac{9}{2 + e^{-3t}} *
 \end{aligned}$$

In these cases, the first 2 marks are as already defined and then award M1 at the point $t = 0$ and $x = 3$ are used in an attempt to find “c” and then **ddM1** for using correct work to remove the ln’s and **A1*** for correct work to reach the given answer showing all necessary steps.

As in the main scheme, if there is no “+ c” then only the first 2 marks are available in (d).

Note that in (d) it is possible to start again e.g.

$$\begin{aligned}
 3 \frac{dx}{dt} &= x(9-2x) \Rightarrow \int \frac{1}{x(9-2x)} dx = \int \frac{1}{3} dt \\
 \frac{1}{x(9-2x)} &\equiv \frac{A}{x} + \frac{B}{9-2x} \equiv \frac{1}{9x} + \frac{2}{9(9-2x)} \\
 &\text{etc.}
 \end{aligned}$$

(e)

B1: 4500 cao or 4.5 thousand

Question Number	Scheme	Marks
8(a)	3π	B1
		(1)
(b)	$\frac{dy}{dx} = \frac{-2\sin t}{6-6\cos 2t}$	M1 A1
	$= \frac{-2\sin t}{6-6(1-2\sin^2 t)} = \frac{-2\sin t}{12\sin^2 t} = -\frac{1}{6}\operatorname{cosec} t$	M1 A1*
		(4)
(c)	$P = \left(\frac{3\pi}{2} - 3, \sqrt{2} \right)$	B1
	E.g. $y - \sqrt{2} = -\frac{\sqrt{2}}{6} \left(0 - \left(\frac{3\pi}{2} - 3 \right) \right) \Rightarrow y = \dots$ or $\sqrt{2} = -\frac{\sqrt{2}}{6} \left(\frac{3\pi}{2} - 3 \right) + c \Rightarrow c = \dots$	M1
	$\frac{\pi\sqrt{2}}{4} + \frac{\sqrt{2}}{2}$ o.e	A1
		(3)
(d)(i)	Attempts $y^2 \frac{dx}{dt} = 4\cos^2 t (6-6\cos 2t)$	M1 A1
	$= (2\cos 2t + 2)(6-6\cos 2t) = 6 \left(2 - 2\cos^2 2t \right) = 6 \left(2 - (1 + \cos 4t) \right)$	dM1
	Volume $= \int_0^{\frac{\pi}{2}} \pi y^2 \frac{dx}{dt} dt = \int_0^{\frac{\pi}{2}} 6\pi (1 - \cos 4t) dt$	A1
(ii)	$\left[6\pi t - \frac{6\pi \sin 4t}{4} \right]_0^{\frac{\pi}{2}}, = 3\pi^2$	M1, A1
		(6)
		(14 marks)

(a)

B1: For 3π . Condone $(3\pi, 0)$ and allow $x = 3\pi$. Remember to check the diagram as the 3π may appear there.

Do not allow decimals here.

(b)

M1: Attempts to differentiate $x(t)$ and $y(t)$ and calculates $\frac{dy}{dx}$ by using $\frac{dy}{dt} \div \frac{dx}{dt}$.

Condone poor differentiation but one of the terms must be of the correct form so it requires

 $x \rightarrow \alpha + \beta \cos 2t$ or $y \rightarrow A \sin t$ A1: $\frac{dy}{dx} = \frac{-2\sin t}{6-6\cos 2t}$ or equivalent e.g. $\frac{dy}{dx} = \frac{-2\sin t}{6-6(\cos^2 t - \sin^2 t)}$ 

M1: Uses appropriate trigonometry to reach $\frac{dy}{dx} = \alpha \operatorname{cosec} t$.

Condone sign errors only with any identities that are used.

A1: $\frac{dy}{dx} = -\frac{1}{6} \operatorname{cosec} t$

You can condone poor notation along the way e.g. $\frac{dy}{dx} = \frac{-2 \sin t}{6 - 6(\cos^2 - \sin^2)}$ as long as the intention is clear.

An alternative for the final M1A1: $\frac{-2 \sin t}{6 - 6 \cos 2t} = \lambda \operatorname{cosec} t \Rightarrow \lambda = \frac{-2 \sin^2 t}{6 - 6 \cos 2t} = \frac{-2 \sin^2 t}{12 \sin^2 t} = -\frac{1}{6}$

(c)

B1: At P , $x = \frac{3\pi}{2} - 3$, $y = \sqrt{2}$ or exact equivalents

May be implied by being seen embedded in their tangent equation.

M1: Full method of finding the y coordinate of N

Requires:

- substitution of $t = \frac{\pi}{4}$ into their $\frac{dy}{dx} = \alpha \operatorname{cosec} t$ to find gradient of tangent
- use of their $\left(\frac{3\pi}{2} - 3, \sqrt{2}\right)$ (which may be inexact) in the formation of the equation of the tangent or the substitution of these values into $y = mx + c$ with the coordinates correctly placed.
- the setting of $x = 0$ as well as finding the y value or rearranging to find “ c ”

A1: $\frac{\pi\sqrt{2}}{4} + \frac{\sqrt{2}}{2}$ or exact simplified equivalent. E.g. $\frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}$.

Does not need to be identified as a value for y so just look for the correct exact expression (may be $c = \dots$).

(d) (i) Note that the first 3 marks only involve the consideration of $y^2 \frac{dx}{dt}$ i.e. not an integral.

Condone poor notation e.g. $\cos^2 t$ for $\cos^2 t$ as long as it is recovered and the intention is clear.

M1: Attempts $y^2 \frac{dx}{dt}$ with their $\frac{dx}{dt}$ condoning slips

A1: $y^2 \frac{dx}{dt} = 4 \cos^2 t (6 - 6 \cos 2t)$ oe. Allow e.g. $(2 \cos t)^2 (6 - 6 \cos 2t)$ with both sets of brackets present unless they are implied by subsequent work.

dM1: A full method to form an expression in terms of $\cos 4t$.

Condone sign errors only with any identities that are used. **Depends on the first method mark.**

There are many ways of achieving the result so you will need to check their work carefully.

E.g.: $4 \cos^2 t (6 - 6 \cos 2t) = 4 \cos^2 t (12 \sin^2 t) = 12 \sin^2 2t = 6 - 6 \cos 4t$

or

$$4 \cos^2 t (6 - 6 \cos 2t) = 4 \cos^2 t (12 \sin^2 t) = 48 \cos^2 t - 48 \cos^4 t$$

$$\cos 4t = \cos^2 2t - \sin^2 2t = \left(2 \cos^2 t - 1\right)^2 - 4 \sin^2 t \cos^2 t = 8 \cos^4 t - 8 \cos^2 t + 1$$

$$\Rightarrow 48 \cos^2 t - 48 \cos^4 t = 6(1 - \cos 4t)$$



A1: Volume = $\int_0^{\frac{\pi}{2}} 6\pi(1 - \cos 4t) dt$. All correct with correct limits and including the “dt”.

(d)(ii)

M1: For $\int \beta(1 - \cos 4t) dt \rightarrow \left[\beta \left(t - \frac{\sin 4t}{4} \right) \right]$. Allow with their β , a made up β or the letter β .

A1: $3\pi^2$ following **full marks in (d)(i)** and **correct integration** and **correct limits**.

