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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Differential Equations (P4.6)

International A Level Mathematics · P4: Pure Mathematics 4 (WMA14)

11 questions · 96 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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 info@guidemygrades.com

 Piccadilly Business Centre, Unit C, Aldow Enterprise Park
Blackett Street, Manchester, M12 6AE

HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.
Click any row below to jump straight to that sub-topic or question.
On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.
A gold bar in the left margin marks exactly where the question sits on the original page.
Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.
Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

11 questions · 96 marks · 1 sub-topic

SUB-TOPIC P4.6a

SOLVING & MODELLING WITH DIFFERENTIAL EQUATIONS

Sub-topic P4.6a · 11 questions · 96 marks · lowest tariff first



3. Given that $y = 0$ at $x = 5$ solve the differential equation

$$8y \frac{dy}{dx} = xe^{-y^2} \quad y \geq 0 \quad x \geq 5$$

giving your answer in the form $y^2 = f(x)$

(5)



(Total for Question 5 is 5 marks)



2. Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \quad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$

(6)

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Question 2 continued

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Question 2 continued

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Question 2 continued

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(Total 6 marks)

Q2



3. Given that $y = 4$ at $x = \frac{\pi}{6}$, solve the differential equation

$$y \cos^2(2x) \frac{dy}{dx} = 3 \sin(2x) \quad y > 0 \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

giving your answer in the form $y^2 = g(x)$

(6)

(Total for Question 5 is 6 marks)



Question 9 continued

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Question 9 continued

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Q9

(Total 9 marks)

END

TOTAL FOR PAPER IS 75 MARKS



Question 8 continued

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Question 8 continued

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Question 8 continued

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Q8

(Total 9 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued



Question 6 continued

(Total for Question 6 is 9 marks)



5. (a) Find $\int x^2 \cos 2x \, dx$

(4)

(b) Hence solve the differential equation

$$\frac{dy}{dt} = \left(\frac{t \cos t}{y} \right)^2$$

giving your answer in the form $y^n = f(t)$ where n is an integer.

(5)



Question 5 continued



Question 5 continued



Question 5 continued

(Total for Question 5 is 9 marks)



Question 4 continued



Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)



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7. Water is flowing into a large container and is leaking from a hole at the base of the container.

At time t seconds after the water starts to flow, the volume, $V \text{ cm}^3$, of water in the container is modelled by the differential equation

$$\frac{dV}{dt} = 300 - kV$$

where k is a constant.

- (a) Solve the differential equation to show that, according to the model,

$$V = \frac{300}{k} + Ae^{-kt}$$

where A is a constant.

(5)

Given that the container is initially empty and that when $t = 10$, the volume of water is increasing at a rate of $200 \text{ cm}^3 \text{ s}^{-1}$

- (b) find the exact value of k .

(4)

- (c) Hence find, according to the model, the time taken for the volume of water in the container to reach 6 litres. Give your answer to the nearest second.

(2)



Question 7 continued

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Lined area for writing the answer to Question 7 continued.



Question 7 continued

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Lined area for writing the answer to Question 7.



Question 7 continued

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Lined area for writing the answer to Question 7.

(Total 11 marks)

Q7



Question 9 continued

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Question 9 continued

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Question 9 continued

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Question 9 continued

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(Total 11 marks)

Q9

END

TOTAL FOR PAPER IS 75 MARKS



Question 7 continued

Question 7 continued



Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 11 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0 : (x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1	$\int 2x \ln(3x) dx = x^2 \ln(3x) - \int \frac{1}{x} \times x^2 dx$	M1
	$= \dots + \dots x^2$	dM1
	$x^2 \ln(3x) - \frac{x^2}{2}$	A1
	$9 \ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2} \right) = 17 \ln 3 - 4$	ddM1A1
		(5 marks)

Mark positively in this question and do not penalise poor notation such as a missing “dx” or spurious integral signs, “+ c” etc. as long as the intention is clear.

M1: Attempts integration by parts the right way round achieving $\dots x^2 \ln(3x) \pm \dots \int \frac{1}{x} \times x^2 dx$

(You do not need to be concerned how they arrive at this)

You may see an attempt at the “DI method” e.g.

	D	I
+	$\ln(3x)$	$2x$
–	$\frac{1}{x}$	x^2

but the same condition applies e.g. they must reach $\dots x^2 \ln(3x) \pm \dots \int \frac{1}{x} \times x^2 dx$ which may be

implied by e.g. $\int 2x \ln(3x) dx = \dots x^2 \ln(3x) \pm \dots x^2$

(In such cases, this mark and the following mark will be scored at the same time)

Condone e.g. $\dots \ln 3x \cdot x^2 \pm \dots \int \frac{1}{x} \times x^2 dx$

dM1: For $\dots x^2 \ln(3x) \pm \dots \int \frac{1}{x} \times x^2 dx = \dots \pm \dots x^2$. **Depends on the previous method mark.**

A1: Fully correct integration: $x^2 \ln(3x) - \frac{x^2}{2}$ or unsimplified equivalent e.g. $2 \left\{ \frac{1}{2} x^2 \ln(3x) - \frac{x^2}{4} \right\}$ with

or without a constant of integration.

Any necessary brackets must be present unless they are implied by subsequent work.

Condone e.g. $\ln(3x) \cdot x^2 - \frac{x^2}{2}$ but not e.g. $\ln(3x^3) - \frac{x^2}{2}$ unless $\ln(3x) \cdot x^2 - \frac{x^2}{2}$ is seen first.

ddM1: Substitutes in the limits 3 and 1 into an expression of the form $\alpha x^2 \ln(3x) + \beta x^2$ and subtracts

the correct way round. Condone missing brackets e.g. $9 \ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$

Condone slips in substitution provided the intention is clear.

They may work on each term separately e.g.

$$\left[x^2 \ln(3x) - \frac{x^2}{2} \right]_1^3 = \left[x^2 \ln(3x) \right]_1^3 - \left[\frac{x^2}{2} \right]_1^3 = 9 \ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2} \right)$$

It is dependent on the two previous method marks.

A1: $17 \ln 3 - 4$ Following correct work. Condone $17 \ln 3 + -4$

Note: A common slip is to differentiate $\ln(3x)$ to obtain $\frac{1}{3x}$ leading to :

$$\begin{aligned}\int 2x \ln(3x) dx &= x^2 \ln(3x) - \int \frac{1}{3x} \times x^2 dx \\ &= x^2 \ln(3x) - \frac{x^2}{6} \rightarrow \left[x^2 \ln(3x) - \frac{x^2}{6} \right]_1^3 = 9 \ln 9 - \frac{9}{6} - \left(\ln 3 - \frac{1}{6} \right) = 17 \ln 3 - \frac{4}{3}\end{aligned}$$

and this scores a maximum M1dM1A0ddM1A0

Note: A common incorrect approach is to integrate in the wrong way round with various incorrect attempts to integrate $\ln(3x)$ e.g.

$$\int 2x \ln(3x) dx = \frac{2x}{3x} - \int \frac{2}{3x} dx \text{ etc.}$$

This generally scores no marks.



	Alternative by substitution $u = 3x$	
	$\text{e.g. } u = 3x \Rightarrow \int 2x \ln(3x) dx = \int \frac{2}{3} u \ln u \times \frac{1}{3} du$ $= \frac{1}{9} u^2 \ln u - \frac{2}{9} \int \frac{u^2}{2} \times \frac{1}{u} du$	M1
	$= \dots + \dots u^2$	dM1
	$\frac{1}{9} u^2 \ln u - \frac{2}{9} \times \frac{u^2}{4}$	A1
	$x = 1, 3 \Rightarrow u = 3, 9$ $9 \ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2} \right) = 17 \ln 3 - 4$	ddM1A1

M1: Attempts integration by parts the right way round achieving $\dots u^2 \ln(u) \pm \dots \int \frac{1}{u} \times u^2 du$

Condone e.g. $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \int \frac{u^2}{2} \times \frac{1}{u} du$

dM1: For $\dots u^2 \ln(u) \pm \dots \int \frac{1}{u} \times u^2 du \dots \pm \dots u^2$. **Depends on the previous method mark.**

Condone e.g. $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \times \frac{u^2}{4}$

A1: Fully correct integration for their substitution: $\frac{1}{9} u^2 \ln u - \frac{2}{9} \times \frac{u^2}{4}$ or unsimplified equivalent e.g.

$\frac{2}{9} \left\{ \frac{1}{2} u^2 \ln u - \frac{u^2}{4} \right\}$ with or without a constant of integration.

Condone e.g. $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \times \frac{u^2}{4}$ but not e.g. $\frac{1}{9} \ln u^3 - \frac{2}{9} \times \frac{u^2}{4}$ unless $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \times \frac{u^2}{4}$ is seen first.

Any necessary brackets must be present unless they are implied by subsequent work.

ddM1: Substitutes in the limits 9 **and** 3 into an expression of the form $\alpha u^2 \ln(u) + \beta u^2$ or reverts back to x and substitutes in the limits 3 **and** 1 and subtracts the correct way round.

Condone missing brackets e.g. $9 \ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$

Condone slips in substitution provided the intention is clear.

They may work on each term separately e.g.

$$\left[\frac{1}{9} u^2 \ln u - \frac{u^2}{18} \right]_3^9 = \left[\frac{1}{9} u^2 \ln u \right]_3^9 - \left[\frac{u^2}{18} \right]_3^9 = 9 \ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2} \right)$$

It is dependent on the two previous method marks.

A1: $17 \ln 3 - 4$

General Principles for Core Mathematics Marking*(But note that specific mark schemes may sometimes override these general principles).***Method mark for solving 3 term quadratic:****1. Factorisation** $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$ $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$ **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

3. Completing the squareSolving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$ **Method marks for differentiation and integration:****1. Differentiation**Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)**2. Integration**Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

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Use of a formula

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Normal marking procedure is as follows:

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Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

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Question Number	Scheme	Marks
1(a)	$2y^2 - 3y = 7 + 13 \Rightarrow 2y^2 - 3y - 20 = 0 \Rightarrow y = \dots$	M1
	$y = -\frac{5}{2} \text{ and } y = 4$	A1
		(2)
(b)	$2y^2 \rightarrow 4y \frac{dy}{dx}$	B1
	$-6xy \rightarrow \pm \dots x \frac{dy}{dx} \pm \dots y$	M1
	$4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y = 14e^{2x-1}$	A1
	$4(4) \frac{dy}{dx} - 6\left(\frac{1}{2}\right) \frac{dy}{dx} - 6(4) = 14e^{2\left(\frac{1}{2}\right)-1} \Rightarrow \frac{dy}{dx} = \dots$	M1
	$y - "4" = " \frac{38}{13} " \left(x - \frac{1}{2} \right)$	M1
	$38x - 13y + 33 = 0$	A1
		(6)
		(8 marks)

(a)

M1: Attempts all of the following

- substitutes $x = \frac{1}{2}$ into the equation for C (condoning slips),
- collect terms and forms a 3TQ in y (terms do not need to be on one side of the equation and the '=' may be implied by subsequent working)
- solves their 3TQ in y using usual rules (including via a calculator) to find at least one value for y .
If a calculator is used, the values must be correct for their 3TQ

A1: Achieves both values $-\frac{5}{2}, 4$ following the award of M1

$$\frac{6x^2-16}{(x+1)(2x-3)} = \frac{B}{x+1} + \frac{C}{2x-3} \text{ and via substitution will get "a correct" } \frac{2}{x+1} - \frac{1}{2x-3}$$

$$\int \frac{6x^2-16}{(x+1)(2x-3)} dx = \int \frac{"B"}{x+1} + \frac{"C"}{2x-3} dx = "B" \ln(x+1) + \frac{"C"}{2} \ln(2x-3)$$

Scores SC B0 M0 A0 M1 A1ft A0 for 2 out of 6 if ft integration is correct

Question Number	Scheme	Marks
8 (a)	$\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \Rightarrow \begin{matrix} 4+3\lambda=2+2\mu \\ -3-2\lambda=0-1\mu \\ 2-1\lambda=-9-3\mu \end{matrix} \text{ any two of these}$	M1
	Full method to find either λ or μ eg $(1)+3(3) \Rightarrow \mu = \dots$	M1
	Either $\lambda = -4$ or $\mu = -5$	A1
	Position vector of intersection is $\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - 4 \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = \text{OR} \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} - 5 \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} =$	dM1
	$= \begin{pmatrix} -8 \\ 5 \\ 6 \end{pmatrix}$	A1
(b)	Co-ordinates or position vector of point $Q = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix}$	
	$\overrightarrow{PQ} = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu-8 \\ 7-1\mu \\ -9-3\mu \end{pmatrix}$	M1
	Uses $\overrightarrow{PQ} \cdot \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = 0 \Rightarrow 4\mu - 16 - 7 + \mu + 27 + 9\mu = 0 \Rightarrow \mu = -\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} \Rightarrow Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7} \right)$	ddM1 A1
		(5) (10 marks)



(a)

M1: For writing down **any two equations** that give the coordinates of the point of intersection.Accept two of $4 + 3\lambda = 2 + 2\mu$, $-3 - 2\lambda = 0 - 1\mu$, $2 - 1\lambda = -9 - 3\mu$

There must be an attempt to set the coordinates equal but condone one slip.

Setting the vectors equal to each other is insufficient for this but correct calculations will imply this.

M1: A full method to find **either** λ **or** μ . There are a few answers with limited or no working.In such cases this M mark can be implied only if **either** λ **or** μ are correct for their equations.**A1:** Either value correct $\mu = -5$ or $\lambda = -4$. Correct value(s) following correct equations implies M1 A1**dm1:** Substitutes their value of λ into l_1 to find the coordinates or position vector of the point of intersection.It is dependent upon having scored second method mark. Alternatively substitutes their value of μ into l_2 to find the coordinates or position vector of the point of intersection. It can be implied in caseswhere there is limited working by two correct coordinates for their λ **or** μ .**A1:** Either $\begin{pmatrix} -8 \\ 5 \\ 6 \end{pmatrix}$ or $-8\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ but not the coordinate $(-8, 5, 6)$ However ISW after sight of the correct **vector****

(b)

M1: States or uses a general point on $l_2 = \begin{pmatrix} 2 + 2\mu \\ 0 - 1\mu \\ -9 - 3\mu \end{pmatrix}$ and attempt $\overrightarrow{PQ} = \begin{pmatrix} 2 + 2\mu \\ 0 - 1\mu \\ -9 - 3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu - 8 \\ 7 - 1\mu \\ -9 - 3\mu \end{pmatrix}$ either

way around with their general point. Condone slips

dm1: Uses the scalar product with fact that $\overrightarrow{PQ}$ and $\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$ are perpendicular vectors to find μ

See scheme but again condone slips.

Alternatively uses the scalar product with fact that $\overrightarrow{PQ}$ and $\overrightarrow{XQ}$ are perpendicular vectors to find μ

$$\overrightarrow{PQ} \cdot \overrightarrow{XQ} = 0 \Rightarrow (2\mu - 8)(2\mu + 10) + (7 - \mu)(-\mu - 5) + (-9 - 3\mu)(-15 - 3\mu) = 0 \Rightarrow \mu = \left(-\frac{2}{7}\right)$$

A1: $\mu = -\frac{2}{7}$ **ddM1:** Uses their μ to find the coordinates of Q .If no method is seen imply by two correct coordinates for their μ **A1:** $Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7}\right)$ but not (**) $\frac{10}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{57}{7}\mathbf{k}$ o.e**** Only penalise incorrect notation once, the first time that it occurs. ****

Alt using distances

(b)	Co-ordinates or position vector of point $Q = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu-8 \\ 7-1\mu \\ -9-3\mu \end{pmatrix}$ Uses Pythagoras with $PQ^2 + QX^2 = PX^2 \Rightarrow 28\mu^2 + 148\mu + 544 = 504 \Rightarrow \mu = \cancel{\frac{10}{7}}, -\frac{2}{7}$ Substitutes their μ into $\begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} \Rightarrow Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7}\right)$	M1 dM1 A1 ddM1 A1
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Alt using minimum distance and differentiation

(b)	Co-ordinates or position vector of point $Q = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu-8 \\ 7-1\mu \\ -9-3\mu \end{pmatrix}$ Uses Pythagoras with $PQ^2 = (2\mu-8)^2 + (7-\mu)^2 + (-9-3\mu)^2$ $\frac{d}{d\mu} PQ^2 = 0 \Rightarrow 4(2\mu-8) - 2(7-\mu) - 6(-9-3\mu) = 0 \Rightarrow \mu = -\frac{2}{7}$ Substitutes their μ into $\begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} \Rightarrow Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7}\right)$	M1 dM1 A1 ddM1 A1
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Question Number	Scheme	Marks
9 (a)	$\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{5t^2} \Rightarrow \int \frac{dA}{A^{\frac{3}{2}}} = \int \frac{dt}{5t^2}$ oe	M1
	$-2A^{-\frac{1}{2}} = -\frac{1}{5}t^{-1}(+c)$	M1 M1 A1
	Substitutes $t = 3, A = 2.25 \Rightarrow c = \dots \left(-\frac{19}{15}\right)$	M1
	Uses their $\frac{2}{\sqrt{A}} = \frac{1}{5t} + \frac{19}{15} \Rightarrow A = \left(\frac{30t}{19t+3}\right)^2$	M1, A1
		(7)
(b)	As $t \rightarrow \infty, A \rightarrow \left(\frac{30}{19}\right)^2 = \frac{900}{361}$ or awrt 2.49 cm ²	M1 A1
		(2)
		(9 marks)

(a)

M1: Attempts to separate the variables either $\int \frac{dA}{A^{\frac{3}{2}}} = \int \dots \frac{dt}{t^2}$ oe such as $\int \dots \frac{dA}{A^{\frac{3}{2}}} = \int \frac{dt}{t^2}$

Condone without the integral signs but the $A^{\frac{3}{2}}$, dA , t^2 and dt must be in the correct positions.
Don't be too concerned on the position of the "5"

M1: Integrates one side correctly. Look for $pA^{-\frac{1}{2}}$ or qt^{-1}

dM1: Integrates both sides correctly. Look for $pA^{-\frac{1}{2}}$ and qt^{-1}

A1: Correct intermediate stage (without $+c$).

Either $-2A^{-\frac{1}{2}} = -\frac{1}{5}t^{-1}(+c)$ or $-10A^{-\frac{1}{2}} = -t^{-1}(+c)$ or equivalent such as $-\frac{A^{-0.5}}{0.5} = -\frac{1}{5}t^{-1}(+c)$

M1: Must have a "+c" now. It is for using $t = 3, A = 2.25 \Rightarrow c = \dots$.

There must be some evidence for this award which may be implied by their value for c if no working is seen.

One index must have started out correct

This may be awarded after an incorrect change of subject has occurred.

M1: It is for proceeding to $A = \left(\frac{pt}{qt+r}\right)^2$ from an equation of the form $\frac{p}{\sqrt{A}} = \frac{q}{t} + r$ with correct operations.

Candidates do not need to have found a **value** for r (their constant) to score this mark

The attempt must involve

- using a common factor on the rhs of $\frac{p}{\sqrt{A}} = \frac{q}{t} + r$
- "inverting" both sides not each term
- an attempt to square both sides and not each term to reach $A = \dots$

A1: $A = \left(\frac{30t}{19t+3}\right)^2$ cso. This mark can be awarded independent of the first M1



Be aware that $A = \left(\frac{300t}{190t+30} \right)^2$ is also correct

(b)

M1: $t \rightarrow \infty, A \rightarrow \left(\frac{a}{b} \right)^2$ The form of part (a) must be $A = \left(\frac{at}{bt+c} \right)^2$ where a, b and c are all positive.

The reason for this is that if we had " $bt - c$ ", the area would be infinite at $t = \frac{c}{b}$ and a limit would not exist

A1: $\frac{900}{361}$ or awrt 2.49 cm² following correct work.

Condone for both marks $A < \frac{900}{361}$ following correct work.

A1: $\overrightarrow{OA} = \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$ or equivalent such as $\overrightarrow{OA} = \frac{1}{6} \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$ or $\overrightarrow{OA} = \frac{1}{6}(4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ but **must be in vector form and**

not in coordinate form.

(b)

M1: For an attempt at the co-ordinates or position vector of point $X = (1+4\lambda, -10+4\lambda, -9+2\lambda)$ or $\begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix}$

dM1: For using $\overrightarrow{OX} \cdot \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = 0$ to set up an equation in λ

Alternatively finds $OX^2 = (1+4\lambda)^2 + (-10+4\lambda)^2 + (-9+2\lambda)^2$ and attempts to differentiate and set = 0

May use $OX^2 + OA^2 = AX^2$ to set up an equation in λ

ddM1: Solves for λ This is dependent upon having scored both previous M's

A1: Correct value for $\lambda = 1.5$

A1: Correct coordinates for $X = (7, -4, -6)$. Condone position vector form $7\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$ o.e.

(c)

M1: Finds all elements required to calculate the area.

In the main scheme this would be the distance OX or OX^2 AND distance OA or OA^2 (which is 1)

dM1: Correct method of finding the area of $OXA = \frac{1}{2} \times OX \times 1$

A1: Area = $\frac{\sqrt{101}}{2}$ o.e

.....
There are various alternatives for part (c). Amongst others are;

Alt I via vector product.

$$\frac{1}{2} \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & -6 \\ 2/3 & 2/3 & 1/3 \end{vmatrix} = \frac{1}{2} \times \left| \frac{8}{3}\mathbf{i} - \frac{19}{3}\mathbf{j} + \frac{22}{3}\mathbf{i} \right| = \frac{1}{2} \times \sqrt{\left(\frac{8}{3}\right)^2 + \left(-\frac{19}{3}\right)^2 + \left(\frac{22}{3}\right)^2} = \frac{\sqrt{101}}{2}$$

M1: For an attempt at $\frac{1}{2} \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & -6 \\ 2/3 & 2/3 & 1/3 \end{vmatrix} = \frac{1}{2} \times \left| \frac{8}{3}\mathbf{i} - \frac{19}{3}\mathbf{j} + \frac{22}{3}\mathbf{i} \right|$

dM1: Followed by an attempt at finding the modulus of the resulting vector multiplied by 1/2

Alt II via scalar products

M1: Attempts to find all three components required to find the area triangle OXA

E.g. Angle OXA with length of side OX and XA

Alternatively angle OAX with length of side OA and XA

For this to be scored

- appropriate gradient vectors need to be attempted by subtracting
- a correct attempt at using $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ to find $\cos\theta$ or θ

dM1: Full attempt at area of triangle using $\frac{1}{2}|OX||XA|\sin(OXA)$ or equivalent



Question Number	Scheme	Marks
8 (a)	$\int y^{-\frac{1}{3}} dy = \int 6xe^{-2x} dx$ $\frac{3}{2} y^{\frac{2}{3}} = -3xe^{-2x} + \int 3e^{-2x} dx$ $\frac{3}{2} y^{\frac{2}{3}} = -3xe^{-2x} - \frac{3}{2} e^{-2x} + c$ Substitutes $(0,1) \Rightarrow c = 3$ $y^2 = (-2xe^{-2x} - e^{-2x} + 2)^3$	B1 M1 M1 dM1 A1 M1 A1 (7)
(b)	As $x \rightarrow \infty, e^{-2x} \rightarrow 0$ and $y^2 = (2)^3 \Rightarrow y = 2^{\frac{3}{2}}$	M1 A1 (2) (9 marks)

(a)

B1: Separates the variables either $\int y^{-\frac{1}{3}} dy = \int 6xe^{-2x} dx$ or $\int \frac{1}{6} y^{-\frac{1}{3}} dy = \int xe^{-2x} dx$

Condone with missing integral signs but the dx and dy **must be present** and **in the correct positions**

M1: For integrating the lhs $y^{-\frac{1}{3}} \rightarrow y^{\frac{2}{3}}$

M1: For integrating the rhs by parts the right way around. Look for $\int xe^{-2x} dx \rightarrow \dots xe^{-2x} \pm \int \dots e^{-2x} dx$

dM1: For fully integrating the rhs to obtain $\dots xe^{-2x} \pm \dots e^{-2x}$. Depending upon the previous M

A1: Correct integration with or without '+ c'.

Look for $\frac{3}{2} y^{\frac{2}{3}} = -3xe^{-2x} - \frac{3}{2} e^{-2x} + c$ or equivalent such as $3y^{\frac{2}{3}} = -6xe^{-2x} - 3e^{-2x} + a$

dM1: Must have a "+c" now. Substitutes $(0,1) \Rightarrow c = \dots$

It is dependent upon having a correct attempt to integrate one side so M1 or M2 must have been awarded.

A1: CSO $y^2 = (-2xe^{-2x} - e^{-2x} + 2)^3$

(b)

M1: For $e^{-2x} \rightarrow 0$ $y^2 = (2)^3$

Follow through on their $g(x)$ in $y^2 = g(x)$ but $g(x)$ must be a function of e^{-kx} with $g(0) \neq 0$

Implied by a correct decimal answer for their $y^2 = g(x)$

A1: $y = 2^{\frac{3}{2}}$ o.e such as $y = \sqrt{8}$ cso. ISW after sight of this. Condone $y = \pm 2^{\frac{3}{2}}$ o.e.



Question Number	Scheme	Marks
3(a)	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{A}{2x-1} + \frac{B}{4x-3} \Rightarrow 8x-5 = A(4x-3) + B(2x-1) \Rightarrow A = \dots B = \dots$	M1
	$A=1 \text{ or } B=2$	A1
	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{1}{2x-1} + \frac{2}{4x-3}$	A1
		(3)
(b)	$\int \left(\frac{1}{2x-1} + \frac{2}{4x-3} \right) dx = \frac{1}{2} \ln 2x-1 + \frac{1}{2} \ln 4x-3 (+c)$ Follow through their A and B	M1, A1ft, A1ft
		(3)
(c)	$\left[\frac{1}{2} \ln(2x-1) + \frac{1}{2} \ln(4x-3) \right]_k^{3k} = \frac{1}{2} \ln(6k-1)(12k-3) - \frac{1}{2} \ln(2k-1)(4k-3)$ $= \frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$	M1
	$\frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = \frac{1}{2} \ln 20 \Rightarrow \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = 20$ $\Rightarrow (6k-1)(12k-3) = 20(2k-1)(4k-3) \Rightarrow 88k^2 - 170k + 57 = 0$	dM1A1
	$88k^2 - 170k + 57 = 0 \Rightarrow k = \dots$	ddM1
	$k = \frac{3}{2}$	A1
		(5)
		Total 11

(a)

M1: "Correct" P.F form (condoning slips) with strategy to find at least one constant or partial fraction.

 Look for an attempt at the correct form (condoning slips) $5-8x = A(4x-3) + B(2x-1)$ followed by values for A or B

A1: One correct fraction or constant

A1: Correct partial fractions which may be awarded for sight of correct PF in part (b).

Award if the correct answer is just written down with little or no working.

Award if the correct partial fractions are seen in (b)

(b)

 M1: For $\int \frac{\alpha}{2x-1} dx = \beta \ln|2x-1|$ OR $\int \frac{\alpha}{4x-3} dx = \beta \ln|4x-3|$

 Allow for example brackets instead of moduli, e.g. $\ln(2x-1) \leftrightarrow \ln|2x-1|$

 Condone the omission of moduli/brackets, for example $\int \frac{\alpha}{2x-1} dx = \beta \ln 2x-1$ for this mark only

 A1ft: For $\int \frac{A}{2x-1} dx = \frac{A}{2} \ln|2x-1|$ OR $\int \frac{B}{4x-3} dx = \frac{B}{4} \ln|4x-3|$ Follow through their A and B

 Allow for example brackets instead of moduli, e.g. $\ln(2x-1) \leftrightarrow \ln|2x-1|$


A1ft: For $\int \frac{A}{2x-1} dx + \int \frac{B}{4x-3} dx = \frac{A}{2} \ln|2x-1| + \frac{B}{4} \ln|4x-3|$

or $\int \frac{A}{2x-1} dx + \int \frac{B}{4x-3} dx = \frac{A}{2} \ln(2x-1) + \frac{B}{4} \ln(4x-3)$

Follow through their numerical A and B . There is no requirement for the $+c$ ISW after a correct answer. This can be awarded (but not implied) from part (c)

Note that $\frac{1}{2} \ln(2x-1)(4x-3)(+c)$ and $\ln \sqrt{(2x-1)(4x-3)}(+c)$ are also correct

(c)

M1: Attempts the use of the correct limits and uses correct log work to combine terms to obtain a single $\ln$ term.

It is dependent upon having an answer to part (b) involving two $\ln$ terms.

Condone slips/ bracketing errors .

$$\begin{aligned} \text{E.g. } \left\{ \frac{1}{2} \ln(6k-1) + \frac{1}{2} \ln(12k-3) \right\} - \left\{ \frac{1}{2} \ln(2k-1) + \frac{1}{2} \ln(4k-3) \right\} &= \frac{1}{2} \ln(6k-1) + \frac{1}{2} \ln(12k-3) - \frac{1}{2} \ln(2k-1) - \frac{1}{2} \ln(4k-3) \\ &= \frac{1}{2} \ln \frac{(6k-1)(12k-3)(4k-3)}{(2k-1)} \end{aligned}$$

dM1: Eliminates $\ln$'s, multiplies up and collects terms to form a polynomial equation in k .

It is dependent upon the previous M

A1: Correct 3TQ. The $= 0$ may be implied by subsequent work

ddM1: Solves 3TQ. Dependent upon both previous M's.

Can be solved via a graphical calculator (you may need to check the answers)

A1: Correct value and no others. Condone $x = \frac{3}{2}$

Solution with limited working in (c)

Case I

$$\frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = \frac{1}{2} \ln 20 \Rightarrow x \text{ or } k = \frac{3}{2} \quad \text{SC 11000}$$

Case II

$$\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = 20 \Rightarrow x \text{ or } k = \frac{3}{2} \quad \text{SC 11100}$$

Question Number	Scheme	Marks
4(a)	Attempts direction vector by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.	M1
	E.g. $\mathbf{r} = 4\mathbf{i}+8\mathbf{j}+\mathbf{k} + \lambda(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$, $\mathbf{r} = 5\mathbf{i}+6\mathbf{j}+3\mathbf{k} + \mu(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1
		(2)
(b)	E.g. $\overrightarrow{PC} = \begin{pmatrix} 4+\lambda \\ 8-2\lambda \\ 1+2\lambda \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+\lambda \\ 10-2\lambda \\ 2\lambda \end{pmatrix}$	M1
	Uses $\overrightarrow{PC} \cdot (\mathbf{i}-2\mathbf{j}+2\mathbf{k}) = 0$ E.g. $\Rightarrow 1(2+\lambda) - 2(10-2\lambda) + 2 \times 2\lambda = 0 \Rightarrow \lambda = \dots$	dM1
	E.g I $\lambda = 2 \Rightarrow \mathbf{c} = (4+2)\mathbf{i} + (8-4)\mathbf{j} + (1+4)\mathbf{k}$ E.g II $\mu = 1 \Rightarrow \mathbf{c} = (5+1)\mathbf{i} + (6-2)\mathbf{j} + (3+2)\mathbf{k}$	ddM1
	(6, 4, 5)	A1
		(4)
(c)	$\overrightarrow{OP'} = \mathbf{p} + 2\overrightarrow{PC} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} + 2(4\mathbf{i} + 6\mathbf{j} + 4\mathbf{k})$	M1
	(10, 10, 9)	A1
		(2)
(d)	$ \overrightarrow{PP'} = 2 \overrightarrow{PC} = 2\sqrt{4^2 + 6^2 + 4^2} = \dots$	M1
	$4\sqrt{17}$	A1
		(2)
		Total 10

General rule in this question: If no method is shown look for two "correct" components for their vector

(a) **There are many correct versions so please check the candidates answer carefully**

M1: Attempts the direction of l by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.

If no method is shown look for two correct components of $(\pm\mathbf{i} \pm 2\mathbf{j} \pm 2\mathbf{k})$ or a multiple of this

A1: Any correct equation including the lhs of ' $\mathbf{r} =$ '

$$\text{Allow in the form } \mathbf{r} = \begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \text{ but } l = \begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \text{ and is } \mathbf{r} = \begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} \mathbf{i} \\ -2\mathbf{j} \\ 2\mathbf{k} \end{pmatrix} \text{ A0}$$

(b)

M1: Attempts the general vector from P to l forming $\overrightarrow{PC}$ where C is a general point on l .

$$\text{Look for an attempt to subtract their } \begin{pmatrix} 4+\lambda \\ 8-2\lambda \\ 1+2\lambda \end{pmatrix} \text{ and } \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \text{ either way around}$$

dM1: Attempts $\overrightarrow{PC} \cdot (\mathbf{i}-2\mathbf{j}+2\mathbf{k}) = 0$ and solves for λ .

It is dependent upon the previous M and the scalar product of this vector must be attempted with their gradient for the line l to produce and solve a linear equation for λ .

ddM1: Uses their λ to find C . Scored for substituting their correctly found λ into their equation for l

A1: Correct coordinates or vector.

Only penalise an error such as $\begin{pmatrix} 6\mathbf{i} \\ 4\mathbf{j} \\ 5\mathbf{k} \end{pmatrix}$ once, the first time that it is made. Hence award if a mark has already been

withheld for such a mistake.



(c)

M1: Correct strategy for $\overrightarrow{OP'}$ Can be implied by two correct coordinates using their P and C For example look for $\overrightarrow{OP'} = \overrightarrow{OP} + 2 \times \overrightarrow{PC}$ using the coordinates of C found in part (b).

A1: Correct coordinates or vector

(d)

M1: Correct method for $|\overrightarrow{PP'}|$ using their values for P and P' or P and C or P' and C . E.g. $|\overrightarrow{PP'}| = 2 \times |\overrightarrow{PC}|$

There are many ways to do this but it must be a complete method, not a distance squared.

A1: CAO

Alternative part (b) using shortest distanceM1: Attempts the general vector from P to l .

Look for an attempt to subtract their $\begin{pmatrix} 4+\lambda \\ 8-2\lambda \\ 1+2\lambda \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ either way around

dM1: Finds an expression for d or d^2 , differentiates and sets $= 0$ to find λ .

As it is a quadratic expression an equivalent method would be via completing the square.

$$\text{FYI: } D^2 = (2+\lambda)^2 + (10-2\lambda)^2 + (2\lambda)^2 = 9\lambda^2 - 36\lambda + 104 \Rightarrow \frac{dD^2}{d\lambda} = 18\lambda - 36 = 0 \Rightarrow \lambda = 2$$

ddM1: Uses their λ to find C .

A1: Correct coordinates or vector

Alternative part (b) using Pythagoras' TheoremM1: Attempts the general vector from P to l .

Look for an attempt to subtract their $\begin{pmatrix} 4+\lambda \\ 8-2\lambda \\ 1+2\lambda \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ either way around

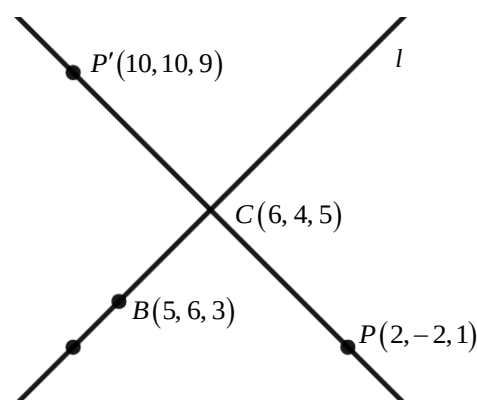
dM1: Uses $PC^2 + CB^2 = PB^2$ or $PC^2 + CA^2 = PA^2$ to set up and solve an equation in λ .E.g. Using $PC^2 + CB^2 = PB^2$ the equations are

$$(2+\lambda)^2 + (10-2\lambda)^2 + (2\lambda)^2 + (\lambda-1)^2 + (2-2\lambda)^2 + (2\lambda-2)^2 = 77$$

$$\Rightarrow 18\lambda^2 - 54\lambda + 36 = 0 \Rightarrow \lambda = 2(1)$$

ddM1: Uses their λ to find C .

A1: Correct coordinates or vector



Question Number	Scheme	Marks
5(i)	$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx (+c)$	M1A1
	$\int x e^x dx = x e^x - \int e^x dx (+c)$	M1
	$\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x (+c)$ Also allow $\int x^2 e^x dx = x^2 e^x - (2x e^x - 2e^x)(+c)$	A1
		(4)
(ii)	$u = (1-3x)^{\frac{1}{2}} \Rightarrow u^2 = 1-3x \Rightarrow 2u \frac{du}{dx} = -3$ or $u = (1-3x)^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = -\frac{3}{2}(1-3x)^{-\frac{1}{2}}$	B1
	$\int \frac{27x}{\sqrt{1-3x}} dx = 27 \int \frac{\frac{1-u^2}{3}}{u} \left(-\frac{2u}{3}\right) du$ or $\int \frac{27x}{\sqrt{1-3x}} dx = 27 \int \frac{\frac{1-u^2}{3}}{\sqrt{1-3x}} \left(-\frac{2}{3}\sqrt{1-3x}\right) du$	M1
	$6 \int (u^2 - 1) du$ or $-6 \int (1 - u^2) du$	A1
	$6 \left(\frac{u^3}{3} - u \right) (+k)$	A1ft
	$6 \left(\frac{(1-3x)^{\frac{3}{2}}}{3} - (1-3x)^{\frac{1}{2}} \right) (+k) = 2(1-3x)^{\frac{1}{2}} (1-3x-3)(+k)$	M1
	$= -2(1-3x)^{\frac{1}{2}} (2+3x)(+k)$ or $= -2(1-3x)^{\frac{1}{2}} (3x+2)(+k)$	A1
		(6)
	Total 10	

(i) Condone missing dx's here

M1: For applying parts to obtain $x^2 e^x - k \int x e^x dx (+c)$ where $k > 0$

A1: Correct expression. No requirement for + c

M1: Applies parts again to $\int x e^x dx$ and obtains $\int x e^x dx = x e^x - \alpha \int e^x dx (+c)$ where $\alpha > 0$

Note that if the whole expression is written out then this mark is implied by

$$\int x^2 e^x dx = x^2 e^x - \alpha x e^x \pm \int \beta e^x dx \quad \text{or} \quad \int x^2 e^x dx = x^2 e^x - \alpha x e^x \pm \beta e^x \quad \text{where } \alpha > 0$$



A1: Correct answer (+ c not required)

Watch for $\int x^2 e^x dx = x^2 e^x - (2x e^x - 2e^x)(+c)$ which scores all 4 marks. ISW after sight of this

Watch for D & I method which may be seen. You will just see the answer here.

D	I
x^2	e^x
$2x$	e^x
2	e^x
0	e^x

$$= x^2 e^x - 2x e^x + 2e^x$$

Score M2 for $x^2 e^x \pm 2x e^x \pm 2e^x$
and then A2 for $x^2 e^x - 2x e^x + 2e^x (+c)$

(ii)

B1: Any correct expression involving $\frac{du}{dx}$ which may be "split" E.g. $du = -\frac{3}{2}(1-3x)^{-\frac{1}{2}} dx$

M1: Substitutes $x = f(u^2)$ to fully change the integrand from $\frac{27x}{\sqrt{1-3x}} dx$ to $k \frac{f(u^2)}{u} u du$ o.e.

Note that the u 's may have been cancelled.

Condone the du not being present but it cannot appear as dx

A1: Correct simplified integral. Look for the u 's being cancelled and some simplification/collection of the constant terms

Look for, for example. integrals such as $6 \int (u^2 - 1) du$, $\int 6u^2 - 6 du$, $-6 \int (1 - u^2) du$ or even $-\frac{2}{3} \int (9 - 9u^2) du$ The brackets may be implied by subsequent work

Condone the du not being present as long as it is implied by subsequent integration

A1ft: Correct follow through integration on their simplified integral.

It is dependent upon having achieved an integral of the form above

E.g. $\alpha \int (u^2 - 1) du$ or $\beta \int (1 - u^2) du$ $\alpha, \beta > 0$. No requirement for $+k$

M1: Achieves an integral of $Pu^3 + Qu$ and

- "correctly" takes out a common factor of u or $(1-3x)^{\frac{1}{2}}$ for their $Pu^3 + Qu$ or $\alpha(1-3x)^{\frac{3}{2}} + \beta(1-3x)^{\frac{1}{2}}$ AND proceeds to the form $(1-3x)^{\frac{1}{2}}(\delta(1-3x) + \varepsilon)$ or better. Condone slips on coefficients
- substitutes $u = (1-3x)^{\frac{1}{2}}$

A1: CAO. No requirement for $+k$



Question Number	Scheme	Marks
6(a)	$\frac{d\theta}{dt} = -k(\theta - 15)^2 \Rightarrow \int \frac{d\theta}{(\theta - 15)^2} = \int -k dt$	B1
	$\int \frac{d\theta}{(\theta - 15)^2} = -(\theta - 15)^{-1}$	M1
	$-\frac{1}{\theta - 15} = -kt + c$	A1
	$t = 0, \theta = 85 \Rightarrow -\frac{1}{70} = c$	M1
	$t = 10, \theta = 40 \Rightarrow \frac{1}{25} = 10k + \frac{1}{70} \Rightarrow k = \dots \left(\frac{9}{3500} \right)$	M1
	$\frac{1}{\theta - 15} = \frac{9t}{3500} + \frac{1}{70} \Rightarrow \theta = \dots$	M1
	$\theta = \frac{135t + 4250}{9t + 50}$	A1
		(7)
(b)	$20 = \frac{135t + 4250}{9t + 50} \Rightarrow t = \dots$	M1
	$t = \text{awrt } 72$	A1
		(2)
		Total 9

(a) **Note that candidates cannot work backwards from the answer using differentiation**

B1: Correct separation of variables.

The integral signs do not have to be present, but the $d\theta$ and dt do, and be in the correct positions

M1: Integrates $\int \frac{1}{(\theta - 15)^2} d\theta$ to $\frac{\alpha}{\theta - 15}$

A1: Correct equation including a k and another **different** constant.

M1: Uses $t = 0, \theta = 85$ to find “ c ”. May be awarded after incorrect integration. May be also awarded if the constant k had been assigned a value using given values of θ and t .

Their initial equation may have been (incorrectly) adapted but it must be solvable with $t = 0, \theta = 85$ used leading to $c = \dots$

M1: Uses $t = 10$ and $\theta = 40$ with their value of “ c ” to find k .

May be awarded after incorrect integration but there must have been two constants. Their initial equation may have been adapted but it must be solvable with $t = 10, \theta = 40$ AND their $c = \dots$

M1: Rearranges using correct algebra to obtain θ in terms of t .

For this to be awarded the integral must be in the correct form and both constants found using a correct method. Condone slips in working but the overall process should be sound.

Look for $\frac{1}{\theta - 15} = \alpha t + \beta \Rightarrow \theta = \frac{ct + d}{et + f}$

A1: Correct expression. Allow integer multiples of the given answer E.g. $\theta = \frac{297500 + 9450t}{3500 + 630t}$



(b)

 M1: Uses their answer to part (a) or equivalent (or possibly an earlier equation) to find t when $\theta = 20$.

 For this to be awarded, substitute $\theta = 20$ into a "correct" form such as $\frac{1}{\theta - 15} = \alpha t + \beta \Rightarrow t = \dots$

 Or substitute $\theta = 20$ into a "correct" form such as $\theta = \frac{ct + d}{et + f} \Rightarrow t = \dots$ where $c, d, e, f \neq 0$

 They cannot just make values up for the c, d, e and f for example. It must follow their work in (a)

 A1: Correct value which must have come from a correct equation. Exact value is $\frac{650}{9}$ but allow awrt 72

Of course candidates can move the "k" or "-k" term over to the LHS or use limits.

6(a)	$\frac{d\theta}{dt} = -k(\theta - 15)^2 \Rightarrow \int \frac{d\theta}{-k(\theta - 15)^2} = \int dt$	B1
	$\int \frac{d\theta}{(\theta - 15)^2} = \alpha(\theta - 15)^{-1}$	M1
	$\frac{1}{k(\theta - 15)} = t + \beta$	A1
	$t = 0, \theta = 85 \Rightarrow \frac{1}{70k} = \beta$	M1
	$t = 10, \theta = 40 \Rightarrow \frac{1}{25k} = \beta + 10 \Rightarrow k = \dots \left(\frac{9}{3500} \right) \quad \beta = \dots \left(\frac{50}{9} \right)$	M1
	$\frac{3500}{9(\theta - 15)} = t + \frac{50}{9} \Rightarrow \theta = \dots$	M1
	$\theta = \frac{135t + 4250}{9t + 50}$	A1

6(a)	$\frac{d\theta}{dt} = -k(\theta - 15)^2 \Rightarrow \int \frac{d\theta}{(\theta - 15)^2} = \int -k dt$	B1
	$\int \frac{d\theta}{(\theta - 15)^2} = -(\theta - 15)^{-1}$	M1
	Correct equation using limits $\left[-\frac{1}{\theta - 15} \right]_{85}^{40} = [-kt]_0^{10}$ Or $\left[\frac{1}{\theta - 15} \right]_{85}^{40} = [kt]_0^{10}$	A1
	Correct attempt to find k $-\frac{1}{25} + \frac{1}{70} = -10k \Rightarrow k = \dots \left(\frac{9}{3500} \right)$	M1
	Correct attempt to find equation linking θ and t E.g. $\left[-\frac{1}{\theta - 15} \right]_{85}^{\theta} = [-kt]_0^t$	M1
	$-\frac{1}{\theta - 15} + \frac{1}{70} = -\frac{9t}{3500} \Rightarrow \theta = \dots$	M1
	$\theta = \frac{135t + 4250}{9t + 50}$	A1



Question	Scheme	Marks
2(a)	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$	
	$3x+4 = A(2x+1)^2 + B(x-2)(2x+1) + C(x-2)$ E.g. $x=2 \rightarrow 10 = A \times 25 \Rightarrow A = \dots$ or $x = -\frac{1}{2} \rightarrow \frac{5}{2} = C \times -\frac{5}{2} \Rightarrow C = \dots$ or $x^2: 0 = 4A + 2B, x: 3 = 4A - 3B + C, x^0: 4 = A - 2B - 2C \rightarrow A = \dots$	M1
	One of $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
	$\Rightarrow A = \dots, B = \dots$ and $C = \dots$	M1
	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{2}{5(x-2)} - \frac{4}{5(2x+1)} - \frac{1}{(2x+1)^2}$ or $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
		(4)
(b)	$\int \frac{p}{x-2} dx = \alpha \ln(x-2)$ or $\int \frac{q}{2x+1} dx = \beta \ln(2x+1)$	M1
	Alt: $\int \frac{8x+9}{(2x+1)^2} dx = \int \frac{8x+4}{(2x+1)^2} + \frac{5}{(2x+1)^2} dx = \gamma \ln(2x+1)^2 + \dots$	
	$\int \frac{k}{(2x+1)^2} dx = \frac{K}{2x+1}$	M1
	$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2} dx = \left[\frac{2}{5} \ln(x-2) - \frac{4}{5} \times \frac{1}{2} \ln(2x+1) + \frac{1}{4x+2} \right]_7^{12}$	A1ft
	$= \left(\frac{2}{5} \ln(10) - \frac{2}{5} \ln(25) + \frac{1}{50} \right) - \left(\frac{2}{5} \ln(5) - \frac{2}{5} \ln(15) + \frac{1}{30} \right) = \dots$	DM1
	$= \frac{2}{5} \ln \frac{10}{25} + \frac{1}{50} - \frac{2}{5} \ln \frac{5}{15} - \frac{1}{30} = \frac{2}{5} \ln \frac{10 \times 15}{25 \times 5} + \dots$	M1
	$\frac{2}{5} \ln \frac{6}{5} - \frac{1}{75}$ (oe)	A1
		(6)
(10 marks)		



Question Number	Scheme	Marks
2(a)	$2^x \rightarrow 2^x \ln 2$	B1
	$y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	$4x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$	M1
	$3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$	A1
	$(10y + 4x^2) \frac{dy}{dx} = 10 \times 2^x \ln 2 - 3 - 8xy \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$	A1
		(6)

B1: Correct differentiation $2^x \rightarrow 2^x \ln 2$

M1: For $y^2 \rightarrow \dots y \frac{dy}{dx}$ where ... is a non-zero constant.

M1: For $4x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$ where ... are non-zero constants.

A1: Correct differentiation e.g.

$$3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2 \text{ or e.g. } 3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy - 10 \times 2^x \ln 2 = 0$$

Condone a spurious " $\frac{dy}{dx} =$ " e.g. $\frac{dy}{dx} = 3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$

M1: A valid attempt to make $\frac{dy}{dx}$ the subject by factorising $\frac{dy}{dx}$ from exactly two **different** terms in

$\frac{dy}{dx}$ which have come from differentiating $5y^2$ and $4x^2 y$ and then dividing by the terms in

the bracket. Note that here, 2 **different** terms means terms such as $y \frac{dy}{dx}$ and $x^2 \frac{dy}{dx}$ and not

e.g. $y \frac{dy}{dx}$ and $3y \frac{dy}{dx}$. Look for $(\dots \pm \dots) \frac{dy}{dx} = \dots \Rightarrow \frac{dy}{dx} = \dots$ which may be implied.

$\frac{dy}{dx}$ must be non-zero and in terms of x **and** y .

Condone slips provided the intention is clear and the above conditions are satisfied.

For those candidates who had a spurious $\frac{dy}{dx} = \dots$ at the start, they may incorporate this in their

rearrangement in which case they will have 3 terms in $\frac{dy}{dx}$ and so score M0.

A1: $\frac{dy}{dx} = \frac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$ or any equivalent correct expression e.g. $\frac{2^x \ln 2 - 0.3 - 0.8xy}{y + 0.4x^2}$

Note that some candidates may divide through by 10 initially e.g.

$$3x + 5y^2 + 4x^2 y = 10(2^x) + 35 \Rightarrow 0.3x + 0.5y^2 + 0.4x^2 y = 2^x + 3.5 \text{ and full marks are available for}$$

equivalent work in (a) and (b). Note that 2^x may appear correctly as $e^{x \ln 2}$.



Question Number	Scheme	Notes	Marks
5(a)	$4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $-5 + 6\lambda = 4 - 3\mu$	For writing down any 2 of these equations.	M1
	E.g. $4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $\Rightarrow \lambda = \dots$ or $\mu = \dots$	Full method for finding λ or μ	M1
	$\lambda = 2, \mu = -1$	Both correct values	A1
	$-5 + 6\lambda = -5 + 12 = 7$ $4 - 3\mu = 4 + 3 = 7$ So lines intersect	Shows that the parameters satisfy the third equation and makes a conclusion.	B1
	$\lambda = 2 \rightarrow (4+4)\mathbf{i} + (4-6)\mathbf{j} + (-5+12)\mathbf{k}$ or $\mu = -1 \rightarrow (13-5)\mathbf{i} + (-1-1)\mathbf{j} + (4+3)\mathbf{k}$	Uses their λ or μ to find A .	M1
	$8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$	Correct vector or coordinates	A1
			(6)
(b)	$\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} = 10 - 3 - 18 = \sqrt{2^2 + 3^2 + 6^2} \sqrt{5^2 + 1^2 + 3^2} \cos \theta$ Full attempt at the scalar product between the direction vectors		M1
	$\cos \theta = \pm \frac{11}{7\sqrt{35}}$	Correct magnitude for $\cos \theta$ (may be implied by e.g. $\theta = 105.4\dots$ or $74.6\dots$)	A1
	$\theta = 74.6^\circ$	Awrt 74.6	A1
			(3)
(c)	$ 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k} = \sqrt{2^2 + 3^2 + 6^2} = 7$	Finds the magnitude of the direction of l_1	M1
	$35 \div 7 = 5 \Rightarrow \lambda = 5$ $8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k} \pm 5(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	Correct strategy for one of the points	M1
	$(18, -17, 37)$ or $(-2, 13, -23)$	One correct point (ignore labels)	A1
	$P(18, -17, 37)$ and $Q(-2, 13, -23)$	Correct points with correct labels	A1
			(4)
			Total 13

(b) Way 3	$\int_0^4 (x^2 - 8x + 16)e^{2x} dx = \int_0^4 x^2 e^{2x} dx - \int_0^4 8xe^{2x} dx + \int_0^4 16e^{2x} dx$	
	$= \left(\frac{\pi}{16} \right) \left(x^2 \frac{e^{2x}}{2} - \int 2x \times \frac{e^{2x}}{2} dx + \dots \right)$	M1A1
	$\int_0^4 x^2 e^{2x} dx - \int_0^4 8xe^{2x} dx + \int_0^4 16e^{2x} dx$ $= \left(\frac{\pi}{16} \right) \left(x^2 \frac{e^{2x}}{2} - \left\{ x \times \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx \right\} - 8 \left\{ x \times \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx \right\} + 8e^{2x} \right)$	dM1
	Depends on the first method mark. $\left(\frac{\pi}{16} \right) \left[\frac{x^2 e^{2x}}{2} - \frac{9xe^{2x}}{2} + \frac{41e^{2x}}{4} \right]_0^4 = \frac{\pi}{16} \left(8e^8 - 18e^8 + \frac{41}{4}e^8 - \frac{41}{4} \right)$	ddM1
	Depends on both previous method marks. $= \frac{\pi}{64} (e^8 - 41) \text{ (cm}^3\text{)}$	A1

(8 marks)**Notes:****(a)**

M1: Sets up the volume using the volume formula, but allow limits and π and the “dx” to be missing. No need to square $f(x)$ for this mark as long as the intent to do so is clear.

There should be an integral

M1: Attempts to square $f(x)$ (need not be part of an integral). The power of e should be correct and the bracket expanded, but allow if the $\frac{1}{4}$ is not squared.

A1: Correct expression for the integral, π and limits must be included, and no errors seen in working so the “dx” must be present throughout.

(b)

M1: Attempts integration by parts the correct way at least once on the given integral to obtain

$$\alpha(x^2 - 8x + 16)e^{2x} \pm \beta \int (ax - b)e^{2x} dx$$

A1: Correct first application of parts: $(x^2 - 8x + 16) \frac{e^{2x}}{2} - \int (2x - 8) \times \frac{e^{2x}}{2} dx$ or equivalent.

dM1: Attempts integration by parts again in the same direction on $\int (2x - 8) \times \frac{e^{2x}}{2} dx$ and proceeds

to complete the integration. Look for $K \left(\dots + \alpha(ax - b)e^{2x} \pm \beta \int e^{2x} dx \right)$.

ddM1: Applies the limits to their integral and subtracts, (but this need not have the K or π for this mark.) May be done in stages, so look for reaching a value using correct limits on each part of their integral.

A1: Correct answer achieved from correct work and the π must have been introduced before the final step, it should have been part of their answer to (a). Condone lack of units.

Way 2

M1: Attempts integration by parts the correct way at least once on the given integral to obtain

$$\alpha(4-x)^2 e^{2x} \pm \beta \int (4-x) e^{2x} dx.$$

A1: Correct first application of parts: $(4-x)^2 \frac{e^{2x}}{2} + \int (4-x) e^{2x} dx$ or equivalent.

dM1: Attempts integration by parts again in the same direction on $\int (4-x) e^{2x} dx$ and proceeds

to complete the integration. Look for $K \left(\dots + \alpha(4-x) e^{2x} \pm \beta \int e^{2x} dx \right)$ for the parts.

ddM1: Applies the limits to their integral and subtracts, (but this need not have the K or π for this mark.) May be done in stages, so look for reaching a value using correct limits on each part of their integral.

A1: Correct answer achieved from correct work and the π must have been introduced before the final step, it should have been part of their answer to (a). Condone lack of units.

Way 3

M1: Splits the integral and attempts integration by parts the correct way at least once on $\int x^2 e^{2x} dx$.

Look for $\alpha x^2 e^{2x} \pm \beta \int x e^{2x} dx$.

A1: Correct first application of parts: $x^2 \frac{e^{2x}}{2} - \int 2x \frac{e^{2x}}{2} dx$ or equivalent.

dM1: Applies parts a second time on both occurrences of $\int x e^{2x} dx$ and completes the integration.

ddM1: Applies the limits to their integral and subtracts, (but this need not have the K or π for this mark.) May be done in stages, so look for reaching a value using correct limits on each part of their integral.

A1: Correct answer achieved from correct work and the π must have been introduced before the final step, it should have been part of their answer to (a). Condone lack of units.

Question	Scheme	Marks
8(a)	Attempts $\pm \left(\begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix} - \begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} \right) = \pm \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix}$	M1
	$\mathbf{r} = \begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 6 \\ -5 \end{pmatrix}$ oe	A1
		(2)
(b)	Meet if $\begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$ so $\begin{cases} 6 = 3 + \mu \\ 6 - 6\lambda = 1 + 5\mu \\ 2 + 5\lambda = 4 + 9\mu \end{cases}$	M1
	From first equation $\mu = 3$ Alt: solves equations 2 and 3 to give either $\mu = \frac{13}{79}$ or $\lambda = \frac{55}{79}$	A1
	Then need $\lambda = \frac{5 - 5 \times 3}{6} = -\frac{5}{3}$ from 2 nd equation, $\lambda = \frac{2 + 9 \times 3}{5} = \frac{29}{5}$ from 3 rd equation Alt: checks their μ in the first equation	M1
	Values of λ do not agree and hence lines do not meet.	A1
		(4)
(c)	$\overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -5 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 2 \\ -4 \\ -5 \end{pmatrix} - \begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -10 \\ -7 \end{pmatrix}$ Accept $\pm$. Depends on first mark.	dM1
	$\frac{\pm \overrightarrow{AC} \cdot (\mathbf{i} + 5\mathbf{j} + 9\mathbf{k})}{ \overrightarrow{AC} \mathbf{i} + 5\mathbf{j} + 9\mathbf{k} } = \pm \frac{-4 \times 1 + -10 \times 5 + -7 \times 9}{\sqrt{16 + 100 + 49} \sqrt{1 + 25 + 81}}$ Depends on both previous marks.	ddM1A 1
	$\Rightarrow \theta = \arccos \left \frac{-117}{\sqrt{165} \sqrt{107}} \right = 28.3^\circ$ or e.g. $\Rightarrow \theta = 90^\circ - \arcsin \left \frac{-117}{\sqrt{165} \sqrt{107}} \right = 28.3^\circ$	A1
		(5)
(11 marks)		



Notes:**(a)****M1:** Attempts the difference between the given two vectors. Implied by 2 out of three correct coordinates if no method is shown. Can be either way round.**A1:** Any correct equation for the line in the form given and **must have** “ $\mathbf{r} = \dots$ ”. Two examples are shown in the scheme, but they may use any point on the line as the starting point, and any multiple of the direction vector.

Allow in $\mathbf{i}, \mathbf{j}, \mathbf{k}$ form but not e.g. $\mathbf{r} = \begin{pmatrix} 6\mathbf{i} \\ 6\mathbf{j} \\ 2\mathbf{k} \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -6\mathbf{j} \\ 5\mathbf{k} \end{pmatrix}$ but condone missing brackets on the position vector so

condone e.g. $\mathbf{r} = 6 + \lambda \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix}$ but not on the direction so $\mathbf{r} = \begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix}$ is A0

(b)**M1:** Equates their line and the given line and extracts at least one equation in λ and μ **A1:** “Correct” value for one parameter. This will depend on which equation(s) they solve – see main scheme **but must follow M1**.**M1:** Substitutes into both other equations to compare results.**A1:** Correct values for λ found with conclusion that the lines do not meet.

Alt: If equation 1 is not first used to find μ then the 1st A mark can be awarded for any correct value for λ or μ from the equations they solve, and the M mark for checking their solutions for consistency in the remaining equation.

For the final A, all work must have been correct with a suitable conclusion made.

When checking, allow e.g. $2 - \frac{25}{3} \neq 4 + 27$ so they do not meet. I.e. the calculations may not

be fully carried out as long as the result is obvious. However if either side is subsequently evaluated incorrectly, withhold the final mark.

FYI: equations (1) and (2) give $\lambda = -\frac{5}{3}$ or $\mu = 3$; equations (1) and (3) give $\lambda = \frac{29}{5}$ or $\mu = 3$ and

equations (2) and (3) give $\lambda = \frac{55}{79}$ or $\mu = \frac{13}{79}$ (use of the negative direction vectors reverse the signs of λ s)

If $\mathbf{r} = \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 6 \\ -5 \end{pmatrix}$ is used equations (1) and (2) give $\lambda = \frac{8}{3}$ or $\mu = 3$; equations (1) and (3) give $\lambda = -\frac{24}{5}$

or $\mu = 3$ and equations (2) and (3) give $\lambda = \frac{24}{79}$ or $\mu = \frac{13}{79}$ (use of the negative direction vectors reverse the signs of λ s)

Note that the maximum marks in (b) if their line in (a) is incorrect is 1010



There will be many different approaches to part (b).

The general structure for marking is as follows:

M1: Equates lines and obtains at least one equation in λ and μ

A1: Correct value for one of the parameters

M1: Attempts the checking process to show the lines do not meet

A1: All correct with a conclusion

(c)

M1: Substitutes $\mu = -1$ into l_2 to find the coordinates for C or the vector $\overrightarrow{OC}$

dM1: Uses their $\overrightarrow{OC}$ to find $\pm \overrightarrow{AC}$. Look for an attempt at the difference of vectors with at least two correct coordinates.

ddM1: Attempts $\frac{\pm \overrightarrow{AC} \cdot (\mathbf{i} + 5\mathbf{j} + 9\mathbf{k})}{|\overrightarrow{AC}| |\mathbf{i} + 5\mathbf{j} + 9\mathbf{k}|}$ with their $\overrightarrow{AC}$

This requires an attempt at the scalar product of their $\overrightarrow{AC}$ with $\mathbf{i} + 5\mathbf{j} + 9\mathbf{k}$ in the numerator with at least 2 “components” correct if no method is shown, and correct attempts at the product of the magnitudes of $\overrightarrow{AC}$ and the direction of $\mathbf{i} + 5\mathbf{j} + 9\mathbf{k}$ in the denominator.

A1: A correct expression simplified or unsimplified e.g. $\pm \left(\frac{-4 \times 1 + -10 \times 5 + -7 \times 9}{\sqrt{16 + 100 + 49} \sqrt{1 + 25 + 81}} \right)$, $\pm \frac{117}{\sqrt{17655}}$

A1: Correct answer. Accept awrt 28.3

Question	Scheme	Marks
9(a)	$\frac{d}{dy}(1+2\ln y)^{-2} = \alpha(1+2\ln y)^{-3} \times \frac{\beta}{y} = \frac{K}{y(1+2\ln y)^3}$	M1
	$= \frac{-4}{y(1+2\ln y)^3}$ oe	A1
		(2)
(b)	$3\operatorname{cosec}(2x)\frac{dy}{dx} = y(1+2\ln y)^3$ $\Rightarrow \int \dots dy = \int k \sin(2x) dx$ or $\int \frac{k}{y(1+2\ln y)^3} dy = \int \dots dx$ and $\Rightarrow \int \dots dy = \int k \sin(2x) dx \Rightarrow \dots = A \cos 2x$ oe or $\Rightarrow \int \frac{k}{y(1+2\ln y)^3} dy = \int \dots dx \Rightarrow \frac{A}{(1+2\ln y)^2} = \dots$	M1
	$\Rightarrow \int \frac{A}{y(1+2\ln y)^3} dy = \int B \sin(2x) dx \Rightarrow \frac{C}{(1+2\ln y)^2} = D \cos 2x$ oe	M1
	One side integrated correctly $\int \frac{3}{y(1+2\ln y)^3} dy = -\frac{3}{4(1+2\ln y)^2}, \int \frac{1}{y(1+2\ln y)^3} dy = -\frac{1}{4(1+2\ln y)^2}$ or $\int \frac{1}{3} \sin(2x) dx = -\frac{1}{6} \cos(2x)$ oe, $\int \sin(2x) dx = -\frac{1}{2} \cos(2x)$ oe	A1
	$-\frac{3}{4(1+2\ln y)^2} = -\frac{1}{2} \cos(2x)(+c)$ or $-\frac{1}{4(1+2\ln y)^2} = -\frac{1}{6} \cos(2x)(+c)$	A1
		(4)
	$y=1, x=\frac{\pi}{6} \Rightarrow -\frac{3}{4(1+2\ln 1)} = -\frac{1}{2} \cos\left(\frac{\pi}{3}\right) + c \Rightarrow c = \dots$	M1
(c)	$-\frac{3}{4(1+2\ln y)^2} = -\frac{1}{2} \cos(2x) - \frac{1}{2}$ or $-\frac{1}{4(1+2\ln y)^2} = -\frac{1}{6} \cos(2x) - \frac{1}{6}$	A1
	$-\frac{3}{4(1+2\ln y)^2} = -\frac{1}{2} \cos(2x) - \frac{1}{2} \Rightarrow \frac{3}{4(1+2\ln y)^2} = \frac{1}{2} + \frac{1}{2} \cos(2x) = \cos^2 x$	M1
	$\Rightarrow (1+2\ln y)^2 = \frac{3}{4} \sec^2 x \Rightarrow \ln y = \frac{1}{2} \left(\frac{\sqrt{3}}{2} \sec x - 1 \right)$ Depends on both previous method marks.	ddM1
	$\Rightarrow y = e^{\frac{\sqrt{3}}{4} \sec x - \frac{1}{2}}$	A1
		(5)

(11 marks)

Notes:**(a)**

M1: Differentiates to achieve the form $\frac{K}{y(1+2\ln y)^3}$.

You can ignore what they call it so just look for the expression.

A1: Correct derivative, accept any equivalent form, need not be simplified.

You can ignore what they call it so just look for the expression.

(b)

M1: Attempts to separate the variables with one side correct ignoring coefficients and with at least

one side integrated to the correct form. The separation may have $\frac{k}{\operatorname{cosec} 2x}$ rather than $k \sin 2x$.

Do not be concerned about the presence or absence of the “dx” and “dy” or even if they are wrongly position – just look for the expressions appearing on either side.

M1: Attempt to separate the variables with both sides correct ignoring coefficients and with both

sides integrated to the correct form. The separation may have $\frac{k}{\operatorname{cosec} 2x}$ rather than $k \sin 2x$.

Do not be concerned about the presence or absence of the “dx” and “dy” or even if they are wrongly position – just look for the expressions appearing on either side.

A1: One side integrated **correctly**.

For the LHS this will be either $-\frac{1}{4(1+2\ln y)^2}$ or $-\frac{3}{4(1+2\ln y)^2}$

For the RHS this will be either $-\frac{1}{2}\cos(2x)$ or $-\frac{1}{6}\cos(2x)$ but note that if $\sin 2x$ is changed to

$2\sin x \cos x$ first, the integration may appear as e.g. $\frac{1}{3}\sin^2 x$ or $-\frac{1}{3}\cos^2 x$ or $\sin^2 x$ or $-\cos^2 x$

A1: A **fully correct** general solution, need not be simplified and a constant of integration is not required.

Note that M1M0A1A0 is possible as a mark profile.

(c)

Note that if a candidate rearranges to an explicit form in part (b) the work can be credited in part (c) but there is no credit for work for part (c) in part (b) unless part (c) is attempted.

M1: Substitutes the boundary values into their equation to find their constant which does not to be evaluated for this mark.

This may be done later in the solution, but they must have a constant in the equation.

A1: Correct equation in any form with their constant evaluated. **Must follow A1 in (b).**

M1: Applies $\cos 2x = \pm 2\cos^2 x \pm 1$ at some point in their solution.

May see attempts to rearrange to $y = \dots$ first **and note that if candidates integrate the $\sin 2x$ in part (b) to $\dots\cos^2 x$ then they can score this mark by implication if part (c) is attempted.**

ddM1: Rearranges the equation, **including square rooting**, to reach $\ln y = \dots$

Depends on both previous method marks.

A1: Correct answer achieved.

You may see candidates rearrange first in (b) to achieve e.g.



$$C - \frac{3}{4(1+2\ln y)^2} = -\frac{1}{2}\cos(2x) \Rightarrow C + \frac{1}{2}\cos(2x) = \frac{3}{4(1+2\ln y)^2} \Rightarrow (1+2\ln y)^2 = \frac{3}{4C+2\cos(2x)}$$

$$\Rightarrow \ln y = \frac{1}{2}\sqrt{\frac{3}{4C+2\cos(2x)}} - \frac{1}{2} \Rightarrow y = \exp\left(\frac{1}{2}\sqrt{\frac{3}{4C+2\cos(2x)}} - \frac{1}{2}\right)$$

The first M in (c) will be scored when they substitute the given conditions into their rearranged equation that contains a constant and the first A mark for a correct equation. They will also score the second M if they use the double angle formula. The dM mark will be as described e.g. Rearranges the equation, **including square rooting**, to reach $\ln y = \dots$ and **depends on both previous method marks**.

There are no marks in (c) for substituting the initial conditions into the given answer to find the value of A. If you see any attempts to use this answer to show it satisfies the differential equation in (b) let your TL know.

Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = k - 3x \Rightarrow \int \frac{1}{k-3x} dx = \int dt$ oe	M1
	$\Rightarrow -\frac{1}{3} \ln(k-3x) = t (+c)$	M1 A1
	$t = 0, x = 0 \Rightarrow -\frac{1}{3} \ln k = c$	M1
	$-\ln(k-3x) = 3t - \ln k \Rightarrow \ln \frac{k-3x}{k} = -3t \Rightarrow 1 - \frac{3}{k} x = e^{-3t} \Rightarrow x = \dots$	M1
	$x = \frac{k}{3} (1 - e^{-3t})$	A1
		(6)
(b)	$t \rightarrow \infty \Rightarrow e^{-3t} \Rightarrow 0 \Rightarrow x \rightarrow \frac{k}{3} \Rightarrow \frac{k}{3} = 7 \Rightarrow k = \dots$	M1
	$k = 21$	A1
		(2)
(c)	$x = 5 \Rightarrow 7(1 - e^{-3t}) = 5 \Rightarrow e^{-3t} = \dots$	M1
	$\Rightarrow t = -\frac{1}{3} \ln \frac{2}{7}$	dM1
	Awrt 0.42 seconds	A1
		(3)
(11 marks)		

Alt (a): Use the same criteria for the first 3 marks. See notes

7(a)	$\frac{dx}{dt} = k - 3x \Rightarrow \int_0^x \frac{1}{k-3x} dx = \int_0^t dt$ oe	M1
	$\Rightarrow \left[-\frac{1}{3} \ln(k-3x) \right]_0^x = \left[t \right]_0^t$	M1 A1
	$-\frac{1}{3} \ln(k-3x) + \frac{1}{3} \ln k = t$	M1
	$\ln k - \ln(k-3x) = 3t \Rightarrow \ln \frac{k}{k-3x} = 3t \Rightarrow \frac{k}{k-3x} = e^{3t} \Rightarrow k = ke^{3t} - 3xe^{3t} \Rightarrow x = \dots$	M1
	$x = \frac{k}{3} (1 - e^{-3t})$	A1

Mark as one complete question. For example, it is possible to get (b) without doing (a)

(a)

M1: Separates the variables correctly, with or without the integration signs shown. The dx and dt must be shown in the correct positions

M1: Attempts the integration of the LHS with $\frac{1}{k-3x} \rightarrow \pm a \ln(k-3x)$ A missing bracket should be penalised unless recovered later in the solution

A1: Fully correct integration, with or without the + c for this mark.

A missing bracket should be penalised unless recovered later in the solution

M1: Attempts to find the constant of integration following **any** attempt at integration to an equation involving a constant. This involves $t = 0, x = 0 \Rightarrow c = \dots$ where c is non zero. For a definite integral we would require the $x = 0$ to be clearly substituted to obtain a constant term.

M1: Correct log work to make x the subject. May be scored before the previous M if the constant is found later. Allow if the constant is zero, or is missing. Condone slips on signs.

You may see versions of the following showing correct exponential work:

$$-\frac{1}{3} \ln(k - 3x) = t + c \Rightarrow \ln(k - 3x) = -3t + d \Rightarrow k - 3x = e^{-3t+d} \Rightarrow k - 3x = Ae^{-3t} \Rightarrow x = \frac{k - Ae^{-3t}}{3}$$

A1: Correct answer or exact equivalent. E.g. $x = \frac{k(e^{3t} - 1)}{3e^{3t}}$.

(b)

M1: Correctly identifies $e^{-3t} \rightarrow 0$ (which may be implied) and deduces the long-term limit for their expression and sets their long-term limit to 7 and solves for k. The equation must be of the form

$x = p - qe^{-3t}$ with p in terms of k and $q > 0$. Alternatively, if they have an equation of the form

$$x = \frac{k(e^{3t} - \beta)}{\delta e^{3t}} \text{ they would need to solve } 7 = \frac{k}{\delta} \text{ to find the value of } k.$$

Condone work on equations of the same form where they have lost the 3 and used another positive value,

usually 1. So, score for equations $x = p - qe^{-\omega t}$ or $x = \frac{k(e^{\omega t} - \beta)}{\delta e^{\omega t}} \omega > 0$ with the same conditions.

A1: $k = 21$ following a correct equation.

Correct answer following correct equation with no working scores both marks

Alt (b)

M1: Correctly identifies $\frac{dx}{dt} = k - 3x = 0$ and substitutes in $x = 7$ leading to a value for k

A1: $k = 21$

(c)

M1: Substitutes $x = 5$ and their value for k into their equation from (a), then rearranges to $e^{\pm 3t} = \dots$

This is dependent upon having a **solvable** equation of the form $x = \pm p \pm qe^{-3t}$ o.e such as $x = \frac{\pm \mu e^{3t} \pm \kappa}{e^{3t}}$

Condone work on equations of the same form where they have lost the 3 and used another positive value,

usually 1. So, score for equations $x = \pm p \pm qe^{-\omega t}$ or $x = \frac{\pm \mu e^{\omega t} \pm \kappa}{e^{\omega t}} \omega > 0$ with the same conditions.

dM1: Solves an equation of the form $e^{\pm 3t} = k, k > 0$ to find a value for t. Condone work in solving equations of the form $e^{\pm \omega t} = k, k > 0$ following the loss of the 3

A1: **cso** 0.42 (seconds)

Alt (c)

M1: Substitutes $x = 5$ into their **solvable** equation of the form $\pm \beta \ln(k - 3x) = t \pm \delta \ln k$ with their k

dM1: ...And finds a value for t

A1: **cso** awrt 0.42 (seconds)

