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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Vectors (P4.7)

International A Level Mathematics · P4: Pure Mathematics 4 (WMA14)

20 questions · 182 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

20 questions · 182 marks · 1 sub-topic

SUB-TOPIC P4.7a

VECTORS IN 3D: LINES, INTERSECTIONS & ANGLES

Sub-topic P4.7a · 20 questions · 182 marks · lowest tariff first



Question 2 continued

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(Total 5 marks)

Q2



3.

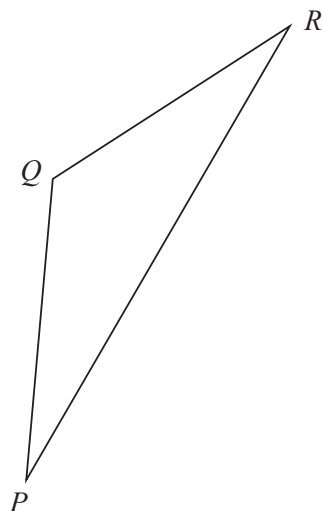


Figure 1

Figure 1 shows a sketch of triangle PQR .

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

- $\overrightarrow{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

(a) Find $\vec{RQ}$

(2)

(b) Find the size of angle PQR , in degrees, to three significant figures.

(3)

Question 3 continued

(Total for Question 3 is 5 marks)



9. With respect to a fixed origin O , the equations of lines l_1 and l_2 are given by

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2: \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$$

where λ and μ are scalar parameters.

Prove that lines l_1 and l_2 are skew.

(5)



Question 9 continued

(Total for Question 9 is 5 marks)



- $$l_1: \mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \qquad l_2: \mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ b \end{pmatrix}$$

Prove that for all values of $b \neq 7$, the lines l_1 and l_2 are skew.

(6)

Question 8 continued

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Q8

(Total 6 marks)



2. With respect to a fixed origin, O , the point A has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point B .

(2)

The point C has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where a is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of a .

(4)

Question 2 continued

(Total for Question 2 is 6 marks)



6. Relative to a fixed origin O .

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B .

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)



Question 6 continued

(Total for Question 6 is 8 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 8 marks)



Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total 9 marks)

Q7



Question 6 continued

Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 9 marks)



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Question 8 continued

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Lined area for writing the answer to Question 8 continued.



Question 8 continued

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Q8

(Total 10 marks)



Question 7 continued

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Question 7 continued

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Lined area for writing the answer to Question 7 continued.



Question 7 continued

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(Total 10 marks)

Q7





Question 4 continued

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Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 10 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 10 marks)



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6 continued.



Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 10 marks)



Question 8 continued

Lined area for writing the answer to Question 8 continued.





Question 8 continued

(Total for Question 8 is 10 marks)



Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total 11 marks)

Q8



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8 continued.



Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 11 marks)



Question 8 continued



Question 8 continued

(Total for Question 8 is 12 marks)



Question 5 continued

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Question 5 continued

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Lined area for writing the answer to Question 5 continued.



Question 5 continued

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Lined area for writing the answer to Question 5.

(Total 13 marks)

Q5

6. Relative to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (3\mathbf{i} + p\mathbf{j} + 7\mathbf{k}) + \lambda(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters and p is a constant.

Given that l_1 and l_2 intersect,

- (a) find the value of p ,

(4)

- (b) find the position vector of the point of intersection.

(2)

- (c) Find the acute angle between l_1 and l_2

Give your answer in degrees to one decimal place.

(3)

The point A lies on l_1 with parameter $\lambda = 2$

The point B lies on l_2 with $\overrightarrow{AB}$ perpendicular to l_2

- (d) Find the coordinates of B

(5)

Question 6 continued



Question 6 continued

(Total for Question 6 is 14 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

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Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

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Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
1	Attempts to get t in terms of x or y. E.g. $x = \frac{t}{t-3} \Rightarrow t = \frac{3x}{x-1}$ Forms a Cartesian equation and makes progress to making y the subject E.g. $y = \frac{1}{t} + 2 \Rightarrow y = \frac{x-1}{3x} + 2$ $\Rightarrow y = \frac{7x-1}{3x}$	M1 dM1 A1 (3) (3 marks)

M1: For an attempt to get t or $\frac{1}{t}$ in terms of x or y .

Condone poor attempts here. It is the intention that is important

dM1: For a complete attempt to form an equation of the form $y = f(x)$

Look for

- getting t or $\frac{1}{t}$ in terms of x or y
- using this to form a Cartesian equation and makes some progress towards making y the subject.

E.g.I. $x = \frac{t}{t-3} \Rightarrow t = \frac{3x}{x-1}$ **followed by** $y = \frac{1}{t} + 2 \Rightarrow y = \frac{1}{\frac{3x}{x-1}} + 2$

E.g.II. $y = \frac{1}{t} + 2 \Rightarrow t = \frac{1}{y-2}$ **followed by**

$$x = \frac{\frac{1}{y-2}}{\frac{1}{y-2} - 3} \Rightarrow x = \frac{1}{1-3(y-2)} \Rightarrow (7-3y)x = 1$$

Condone slips, for example in the signs and miscopies/misreads, but the overall mechanics of the attempt should be sound.

In E.g.II. an attempt must be made to make progress towards $y = f(x)$, so allow for example when an intermediate line such as $g(y) \times h(x) = b$ or $g(y) = h(x)$ where $g(y)$ is a linear expression in y

A1: Correct equation $y = \frac{7x-1}{3x}$ but allow $y = \frac{-1+7x}{3x}$

Alt Method:

M1: Substitutes both $y = \frac{1}{t} + 2$ and $x = \frac{t}{t-3}$ into $y = \frac{ax-1}{bx}$ and makes progress to a point in which sides can be compared

$$\frac{1}{t} + 2 = \frac{a \times \frac{t}{t-3} - 1}{b \times \frac{t}{t-3}} = \left(\frac{a-1}{b} \right) + \frac{3}{bt} \quad \text{or} \quad b + 2bt = (a-1)t + 3$$

dM1: Compares terms to set up and solve simultaneous equations in both a and b .

FYI $\frac{3}{b} = 1, \frac{a-1}{b} = 2$ or $b = 3, 2b = a - 1$

A1: $y = \frac{7x-1}{3x}$ NOT just for correct values of a and b

Question Number	Scheme	Marks
2 (a)	$\frac{3x}{(2x-1)(x-2)} \equiv \frac{A}{2x-1} + \frac{B}{x-2} \Rightarrow \text{Values for } A \text{ and } B$ One correct value, either $A = -1$ or $B = 2$ Correct PF form $\frac{-1}{2x-1} + \frac{2}{x-2}$	M1 A1 A1 (3)
(b)	$\int \frac{-1}{2x-1} + \frac{2}{x-2} dx = -\frac{1}{2} \ln(2x-1) + 2 \ln(x-2) + (c)$ $\int_5^{25} \frac{3x}{(2x-1)(x-2)} dx = \left[-\frac{1}{2} \ln(2x-1) + 2 \ln(x-2) \right]_5^{25}$ $= \left(-\frac{1}{2} \ln 49 + 2 \ln 23 \right) - \left(-\frac{1}{2} \ln 9 + 2 \ln 3 \right)$ $\ln \frac{529}{21}$	M1, A1ft dM1 A1 (4) (7 marks)

(a)

M1: Attempts a correct PF form leading to values for A and B .

Condone $\frac{3x}{(2x-1)(x-2)} \equiv C + \frac{A}{2x-1} + \frac{B}{x-2} \Rightarrow \text{Values for at least } A \text{ and } B$

A1: One correct value for A or B or one correct termA1: Correct PF form $\frac{-1}{2x-1} + \frac{2}{x-2}$.This is not just for correct values for A and B but it may be awarded from work seen in (b)

(b)

M1: $\int \frac{A}{2x-1} + \frac{B}{x-2} dx = C \ln(2x-1) + D \ln(x-2)$ o.e. Condone missing brackets here

A1ft: Correct ft on their values for A and B $\int \frac{A}{2x-1} + \frac{B}{x-2} dx = \frac{A}{2} \ln(2x-1) + B \ln(x-2) + (c)$

Do not condone missing brackets but the brackets can be implied by subsequent working, E.g. $\ln 49$

dM1: Attempts to substitute the limits 5 and 25 (no misreads on the limits but condone slips, so don't accept a consistent lower limit of 0 for instance) into an expression of the correct form (with brackets which may be implied) and subtracts, either way around. Condone attempts where the substitutions were made into incorrectly combined $\ln$ terms.

A1: cso $\ln \frac{529}{21}$ Also allow in mixed number form $\ln 25 \frac{4}{21}$



Question Number	Scheme	Marks
9	$\begin{pmatrix} 2-\lambda \\ 8+2\lambda \\ 10+3\lambda \end{pmatrix} = \begin{pmatrix} -4+5\mu \\ -1+4\mu \\ 2+8\mu \end{pmatrix}$ Attempts to solve any two of the three equations Either (1) and (2) $\begin{cases} 2-\lambda = -4+5\mu \\ 8+2\lambda = -1+4\mu \end{cases} \Rightarrow \lambda = -\frac{3}{2}, \mu = \frac{3}{2}$ (1) and (3) $\begin{cases} 2-\lambda = -4+5\mu \\ 10+3\lambda = 2+8\mu \end{cases} \Rightarrow \lambda = \frac{8}{23}, \mu = \frac{26}{23}$ (2) and (3) $\begin{cases} 8+2\lambda = -1+4\mu \\ 10+3\lambda = 2+8\mu \end{cases} \Rightarrow \lambda = -10, \mu = -\frac{11}{4}$ Substitutes their values of λ and μ into both sides of the "third" equation E.g. $\lambda = -\frac{3}{2}$ into $10+3\lambda = \frac{11}{2}$ and $\mu = \frac{3}{2}$ into $2+8\mu = 14$ Concludes that lines don't intersect with correct calculations and minimal reason Additionally states that $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ is not parallel to $\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$ with a minimal reason So lines are skew CSO *	M1, A1 dM1 A1 A1* (5) (5 marks)

Notes:

Main method seen

M1: Attempts to solve two of the three equations.

Accept as an attempt, writing down two of the three equations (condoning slips) followed by values for

both λ and μ

A1: Solves two of the three equations to find correct values for both λ and μ , Allow equivalent fractions

dM1: Either: Substitutes their values of λ and μ into both sides of the third equation....or into the equations of both lines to find both coordinates

A1: Having achieved correct values for λ and μ , the values for the third equation are found to enable a comparison to be made. E.g. solving equations (1) and (2) and using equation (3) stating $10+3 \times -\frac{3}{2} \neq 2+8 \times \frac{3}{2}$ is sufficient. If the values are found they must be correct.

Important: Additionally, to score this mark, a minimal statement must be made that states that the lines do not intersect /cross. Condone statements such as $l_1 \neq l_2$

Stating that the lines are skew at this point is not sufficient to score this mark

In the alternative stating that "as the values are not the same, the lines cannot intersect" is sufficient.

A1*: CSO . Hence all previous marks must have been scored.

In addition to not intersecting there must be a statement, with a minimal reason, that the lines are not parallel and hence skew. Accept statements like, not intersecting, not parallel (with reason), hence proven.



Question Number	Scheme	Marks
6(a)	$4y^2 + 3x = 6ye^{-2x}$	
	$4y^2 + 3x \rightarrow 8y \frac{dy}{dx} + 3$	B1
	$6ye^{-2x} \rightarrow -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$	M1 A1
	$8y \frac{dy}{dx} + 3 = -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ oe	M1 A1
		(5)
(b)	Sets $x = 0$ in $4y^2 + 3x = 6ye^{-2x} \Rightarrow y = \frac{3}{2}$ oe	B1
	Substitutes $\left(0, \frac{3}{2}\right)$ in their $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y} = \left(\frac{7}{-2}\right)$	M1
	$m_N = -1 \div \frac{7}{-2} \Rightarrow y = \frac{2}{7}x + \frac{3}{2}$	dM1
	$y = \frac{2}{7}x + \frac{3}{2}$ oe e.g. $y = \frac{6}{21}x + \frac{3}{2}$	A1
		(4)
		(9 marks)

(a)

B1: Differentiates $4y^2 + 3x$ to obtain $8y \frac{dy}{dx} + 3$. Allow unsimplified forms such as $4 \times 2y \frac{dy}{dx} + 3$

M1: Uses the product rule on $6ye^{-2x}$ to obtain an expression of the form $Aye^{-2x} + Be^{-2x} \frac{dy}{dx}$

A1: Differentiates $6ye^{-2x}$ to obtain $-12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$

M1: Collects two terms in $\frac{dy}{dx}$ (one from attempting to differentiate $4y^2$ and one from attempting to differentiate $6ye^{-2x}$) and proceeds to make $\frac{dy}{dx}$ the subject.

A1: $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ or equivalent e.g. $\frac{dy}{dx} = \frac{2e^{-2x} \times 6y + 3}{6e^{-2x} - 8y}$ or $\frac{dy}{dx} = \frac{12y + 3e^{2x}}{6 - 8ye^{2x}}$

You can ignore any spurious " $\frac{dy}{dx} =$ " at the start and allow y' for $\frac{dy}{dx}$.

(b)

B1: Uses $x = 0$ to obtain $y = \frac{3}{2}$ oe e.g. $\frac{6}{4}$ (ignore any reference to $y = 0$)

M1: Substitutes $x = 0$ and their y at $x = 0$ which has come from substituting $x = 0$ into the original equation into their $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ to find a numerical value. Working is normally shown here but you may need to check for evidence. Use of $x = 0$ and $y = 0$ is M0.



dM1: Uses the negative reciprocal of " $\frac{7}{-2}$ " for the gradient of the normal and uses this and their value of y at

$x = 0$ to form the equation of the normal. **Depends on the previous M.**

A1: $y = \frac{2}{7}x + \frac{3}{2}$ oe e.g. $y = \frac{6}{21}x + \frac{6}{4}$

Note that the use of $(0, 0)$ for P will generally lose the final 3 marks in (b)



Question Number	Scheme	Marks
7(a) Way 1	$\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \int \frac{1}{2} e^{2x} \cos x \, dx$	M1
	$= \dots - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$	dM1
	$\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$	A1
	$\frac{5}{4} \int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x \Rightarrow \int e^{2x} \sin x \, dx = \dots$	ddM1
	$= \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$	A1
		(5)
7(a) Way 2	$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + \int 2e^{2x} \cos x \, dx$	M1
	$= \dots + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$	dM1
	$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$	A1
	$5 \int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x \Rightarrow \int e^{2x} \sin x \, dx = \dots$	ddM1
	$= \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$	A1
		(5)
(b)	$\left(\frac{2}{5} e^{2\pi} \sin \pi - \frac{1}{5} e^{2\pi} \cos \pi \right) - \left(\frac{2}{5} e^0 \sin 0 - \frac{1}{5} e^0 \cos 0 \right) = \dots$	M1
	$= \frac{1}{5} e^{2\pi} + \frac{1}{5} = \frac{e^{2\pi} + 1}{5} *$	A1*
		(2)
		(7 marks)

Note that you can condone the omission of the dx's throughout.

(a) Way 1

M1: Attempts integration by parts with $u = \sin x$ and $v' = e^{2x}$ to obtain

$$\int e^{2x} \sin x \, dx = A e^{2x} \sin x \pm B \int e^{2x} \cos x \, dx \quad A > 0$$

dM1: Attempts integration by parts again with $u = \cos x$ and $v' = e^{2x}$ on $B \int e^{2x} \cos x \, dx$ to obtain

$$B \int e^{2x} \cos x \, dx = \pm C e^{2x} \cos x \pm D \int e^{2x} \sin x \, dx$$

Depends on the previous mark.

A1: For $\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$



Allow unsimplified e.g. $\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \left\{ \frac{1}{4} e^{2x} \cos x + \int \frac{1}{4} e^{2x} \sin x \, dx \right\}$

ddM1: Dependent upon having scored both M's.

It is for collecting $\int e^{2x} \sin x \, dx$ terms together and making it the subject of the formula

A1: $\int e^{2x} \sin x \, dx = \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$ (allow with or without “+ c”)

(a) **Way 2**

M1: Attempts integration by parts with $u = e^{2x}$ and $v' = \sin x$ to obtain

$$\int e^{2x} \sin x \, dx = \pm A e^{2x} \cos x \pm B \int e^{2x} \cos x \, dx$$

dM1: Attempts integration by parts again with $u = e^{2x}$ and $v' = \cos x$ on $B \int e^{2x} \cos x \, dx$ to obtain

$$B \int e^{2x} \cos x \, dx = \pm C e^{2x} \sin x \pm D \int e^{2x} \sin x \, dx$$

Depends on the previous mark.

A1: For $\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$

Allow unsimplified e.g. $\int e^{2x} \sin x \, dx = -e^{2x} \cos x - \left\{ -2e^{2x} \sin x - \int -4e^{2x} \sin x \, dx \right\}$

ddM1: Dependent upon having scored both M's.

It is for collecting $\int e^{2x} \sin x \, dx$ terms together and making it the subject of the formula

A1: $\int e^{2x} \sin x \, dx = \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$ (allow with or without “+ c”)

(a) **Way 3**

M1: Attempts integration by parts with $u = \sin x$ and $v' = e^{2x}$ to obtain

$$\int e^{2x} \sin x \, dx = A e^{2x} \sin x \pm B \int e^{2x} \cos x \, dx \quad A > 0$$

or attempts integration by parts with $u = e^{2x}$ and $v' = \sin x$ to obtain

$$\int e^{2x} \sin x \, dx = \pm A e^{2x} \cos x \pm B \int e^{2x} \cos x \, dx$$

dM1: Attempts integration by parts with $u = \sin x$ and $v' = e^{2x}$ to obtain

$$\int e^{2x} \sin x \, dx = \pm A e^{2x} \sin x \pm B \int e^{2x} \cos x \, dx$$

and attempts integration by parts with $u = e^{2x}$ and $v' = \sin x$ to obtain

$$\int e^{2x} \sin x \, dx = \pm A e^{2x} \cos x \pm B \int e^{2x} \cos x \, dx$$

A1: $I_1 = \int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \int \frac{1}{2} e^{2x} \cos x \, dx$ **AND** $I_2 = \int e^{2x} \sin x \, dx = -e^{2x} \cos x + \int 2e^{2x} \cos x \, dx$

ddM1: E.g. $4I_1 + I_2 = 2e^{2x} \sin x - e^{2x} \cos x = 5I \Rightarrow I = \dots$ Correct attempt to eliminate $\int e^{2x} \cos x \, dx$ term.

A1: $\int e^{2x} \sin x \, dx = \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$ (allow with or without “+ c”)

(b)

M1: For applying the limits 0 and π to an expression containing at least one term of the form $Ae^{2x} \sin x$ and at least one term of the form $Be^{2x} \cos x$. **There must be some evidence that both limits have been used.**

A1*: $\frac{e^{2\pi} + 1}{5}$ found correctly **from the correct answer in part (a)** via at least one intermediate line

which could be $\frac{e^{2\pi}}{5} + \frac{1}{5}$

Note a correct answer in (a) and evidence of use of the limits 0 and π followed by $\frac{e^{2\pi} + 1}{5}$ with no intermediate line scores M1A0



Question Number	Scheme	Marks
8	$\begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ b \end{pmatrix} \Rightarrow \begin{matrix} -1+2\lambda = 2+4\mu & (1) \\ 5-\lambda = -2-3\mu & (2) \\ 4+5\lambda = -5+\mu b & (3) \end{matrix}$	
	Uses equations (1) and (2) to find either λ or μ e.g. $(1) + 2(2) \Rightarrow \mu = \dots$ or $3(1) + 4(2) \Rightarrow \lambda = \dots$	M1
	Uses equations (1) and (2) to find both λ and μ	dM1
	$\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$	A1
	$4+5\lambda = -5+\mu b \Rightarrow 4+5 \times -\frac{19}{2} = -5 - \frac{11}{2}b$ or $4+5\lambda = -5+7\mu \Rightarrow 4+5 \times -\frac{19}{2} = -5 - \frac{11}{2} \times 7$	ddM1
	$\Rightarrow 11b = 77 \Rightarrow b = 7$ or obtains $-\frac{87}{2} = -\frac{87}{2}$	A1
	States that when $b = 7$, lines intersect or when $b \neq 7$, lines do not intersect Lines are not parallel so when $b \neq 7$ lines are skew. *	A1 Cso
		(6)
	Alternative assuming $b = 7$:	
	$\begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ 7 \end{pmatrix} \Rightarrow \begin{matrix} -1+2\lambda = 2+4\mu & (1) \\ 5-\lambda = -2-3\mu & (2) \\ 4+5\lambda = -5+7\mu & (3) \end{matrix}$	
	Uses any 2 equations to find either λ or μ	M1
	Uses any 2 equations to find both λ and μ	dM1
	$\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$	A1
	Checks in the 3 rd equation e.g. equation 3: $4+5\left(-\frac{19}{2}\right) = -5+7\left(-\frac{11}{2}\right) = \dots$ equation 1: $-1+2\left(-\frac{19}{2}\right) = 2+4\left(-\frac{11}{2}\right) = \dots$ equation 2: $5-\left(-\frac{19}{2}\right) = -2-3\left(-\frac{11}{2}\right) = \dots$	ddM1
	Equation 3: $-\frac{87}{2}$ Equation 1: -20 Equation 2: $\frac{29}{2}$	A1
	States that when $b = 7$, lines intersect or when $b \neq 7$, lines do not intersect Lines are not parallel so when $b \neq 7$ lines are skew. *	A1 Cso
		(6 marks)

M1: For attempting to solve equations (1) and (2) to find **either** λ **or** μ

dM1: For attempting to solve equations (1) and (2) to find **both** λ **and** μ **Depends on the first M.**

A1: $\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$

ddM1: Attempts to solve $4 + 5\lambda = -5 + \mu b$ for their values of λ and μ . Or uses $b = 7$ with their λ and μ in an attempt to show equality. **Depends on both previous M's.**

A1: Achieves (without errors) that they will intersect when $b = 7$

Note that the previous 3 marks may be scored without explicitly seeing the values of both parameters e.g.

$\mu = -\frac{11}{2}, (2) \rightarrow \lambda = 3\mu + 7 \rightarrow 4 + 5(3\mu + 7) = -5 + \mu b \rightarrow b = 7$

A1*: **Cso** States that when $b = 7$, lines intersect and since lines are not parallel it shows that when $b \neq 7$ lines are skew.

Alternative:

M1: Uses $b = 7$ and attempts to solve 2 equations to find **either** λ **or** μ

dM1: For attempting to solve 2 equations to find **both** λ **and** μ **Depends on the first M.**

A1: $\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$

ddM1: Attempts to show that the 3rd equation is true for their values of λ and μ

Depends on both previous M's.

A1: Achieves (without errors) that the 3rd equation gives the same values for (or equivalent)

A1*: **Cso** States that when $b = 7$, lines intersect and since lines are not parallel it shows that when $b \neq 7$ lines are skew.

To score the final mark there must be some statement that the lines intersect (or equivalent e.g. meet at a point, cross, etc.) when $b = 7$ or that they do not intersect if $b \neq 7$ **and** that the lines are not parallel which may appear anywhere (reason not needed but may be present) so lines are skew when $b \neq 7$.

Ignore any work attempting to show that the lines are perpendicular or not.



General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.



Question Number	Scheme	Marks
6 (a)(i)	$\overline{AB} = (\pm) \left[(8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}) - (2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}) \right] = \dots$	M1
	$\overline{AB} = 6\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$	A1
(ii)	$\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ o.e. such as $\mathbf{r} = \begin{pmatrix} 8 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 6 \\ -12 \end{pmatrix}$	B1ft
		(3)
(b)	Attempts $\pm \overline{CP} = \pm \begin{pmatrix} 2 + \lambda - 3 \\ -3 + \lambda - 5 \\ 5 - 2\lambda - 2 \end{pmatrix}$	M1
	$\overline{CP} \cdot \mathbf{k} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} \lambda - 1 \\ \lambda - 8 \\ -2\lambda + 3 \end{pmatrix} \cdot \mathbf{k} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = 0 \Rightarrow 1(\lambda - 1) + 1(\lambda - 8) - 2(-2\lambda + 3) = 0$ Alt: $(\lambda - 1)^2 + (\lambda - 8)^2 + (-2\lambda + 3)^2 = 6\lambda^2 - 30\lambda + 74 = 6\left(\lambda - \frac{5}{2}\right)^2 + \frac{73}{2}$	dM1
	$\Rightarrow \lambda = \frac{5}{2}$ [use of $\overline{AB}$ in $\overline{CP}$ gives $\lambda = \frac{5}{12}$, use of $\overline{OB}$ $\lambda = \frac{-7}{2}$ or $\frac{-7}{12}$]	A1
	$\overline{OP} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \frac{5}{2} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \frac{9}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$	ddM1, A1 (5)
		(8 marks)

Notes:

Accept either form of vector notation throughout. Accept with $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ in their column vectors.

(a)(i)

M1: Attempts to subtract vectors $\overline{OA}$ and $\overline{OB}$ either way around. May be implied by two correct components.

A1: $\overline{AB} = 6\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$ o.e.

(a)(ii)

B1ft: Any correct equation for the line, may use a correct or follow through multiple of $\overline{AB}$ for direction and with any point on the line. Must start $\mathbf{r} = \dots$ or accept $x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \dots$ ($l = \dots$ is B0).

(b)

M1: Attempts $\pm \overline{CP}$ using point C and a general point on their l

dM1: Sets the scalar product of their $\overline{CP}$ (either direction) and their direction of l (or $\overline{AB}$) to 0 and proceeds to an equation in λ . Condone sign slips in components if the intention is clear.

Alternatively attempts to minimise the distance CP (by completing square as shown, or by differentiation) to obtain a linear equation in λ .



A1: Finds a correct value of λ for their l . Note if they use $\overline{AB}$ the correct value is $\frac{5}{12}$

ddM1: Substitutes their λ (from a correct method) into their l

A1: $\overline{OP} = \frac{9}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$ Accept as coordinates, and accept $P = \dots$ instead of $\overline{OP}$.



Question Number	Scheme	Marks
6(a)	Attempts $\pm((5\mathbf{i} + 2\mathbf{j} + \mathbf{k}) - (\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})) = \pm(4\mathbf{i} + 4\mathbf{k})$	M1
	e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(4\mathbf{i} + 4\mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(4\mathbf{i} + 4\mathbf{k})$ oe	A1
		(2)
(b)	$\overrightarrow{AC} = 2\mathbf{i} + (\alpha - 2)\mathbf{j} + 8\mathbf{k}$	B1
	$ \overrightarrow{AB} = \sqrt{4^2 + 4^2}$ or $ \overrightarrow{AC} = \sqrt{2^2 + (\alpha - 2)^2 + 8^2}$	M1
	$ \overrightarrow{AB} = \sqrt{32}$ and $ \overrightarrow{AC} = \sqrt{68 + (\alpha - 2)^2}$	A1
	$\cos 45^\circ = \frac{8 + 32}{\sqrt{32} \times \sqrt{68 + (\alpha - 2)^2}}$	dM1
	$\cos 45^\circ = \frac{40}{\sqrt{32} \times \sqrt{68 + (\alpha - 2)^2}} \Rightarrow \alpha^2 - 4\alpha - 28 = 0$	ddM1
	$\alpha = 2 \pm 4\sqrt{2}$	A1
		(6)
		(8 marks)

Allow vectors to be written in any acceptable form. E.g. column vectors are fine

(a)

M1: Attempts to find the difference between the two vectors.

If no method is shown look for two correct components from $\pm(4\mathbf{i}), (0\mathbf{j}), \pm(4\mathbf{k})$

A1: Any correct equation for the line in the form given.

Examples of correct equations are;

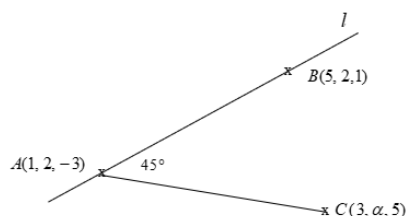
$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(4\mathbf{i} + 4\mathbf{k}), \mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + \mathbf{k})$$

$$\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 0 \\ -4 \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} + k \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \delta \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$$

Note that the LHS must be correct so $l = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(4\mathbf{i} + 4\mathbf{k})$ is A0

Condone the vectors not being bold or underlined

Condone incorrect vector notation for the M but not the A mark. E.g. $\mathbf{r} = \begin{pmatrix} 5\mathbf{i} \\ 2\mathbf{j} \\ \mathbf{k} \end{pmatrix} + \lambda \begin{pmatrix} -4\mathbf{i} \\ 0 \\ -4\mathbf{k} \end{pmatrix}$ is M1A0



(b) There are many ways to attempt this question.

The following is via the scalar product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Note that it could be attempted via the scalar product of $\pm(\mathbf{i} + \mathbf{k})$ and $\overrightarrow{AC}$ instead of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Attempts using an incorrect vector pair, e.g. $\overrightarrow{AB}$ and $\overrightarrow{BC}$, can only score a maximum of 1 mark (1st M1)

Condone incorrect vector notation throughout (b). E.g. $\overrightarrow{AC} = (2\mathbf{i}, (\alpha - 2)\mathbf{j}, 8\mathbf{k})$

B1: A correct $\overrightarrow{AC} = 2\mathbf{i} + (\alpha - 2)\mathbf{j} + 8\mathbf{k}$ (or $\overrightarrow{CA} = -2\mathbf{i} - (\alpha - 2)\mathbf{j} - 8\mathbf{k}$)

It must be simplified but may be implied by further work.

Allow in coordinate form $(2, \alpha - 2, 8)$ if used correctly

M1: Attempts to find the magnitude of either their $\overrightarrow{AB}$ or their $\overrightarrow{AC}$

For example, for an attempt at the magnitude of $\overrightarrow{AC}$

- there should be attempt to subtract each component of $\overrightarrow{OA}$ and $\overrightarrow{OC}$
- the three resulting values should then be squared and added
- the answer to this is then square rooted

If the method is not apparent then look for two correct components

A1: Correct magnitude of both $|\overrightarrow{AB}| = \sqrt{32}$ and $|\overrightarrow{AC}| = \sqrt{68 + (\alpha - 2)^2}$ ($= \sqrt{72 - 4\alpha + \alpha^2}$)

They must be simplified (as above) but can be scored following sign slips on either vector.

You can imply this mark from further work

dM1: Attempts the scalar product of $\pm\overrightarrow{AB}$ and $\pm\overrightarrow{AC}$

- $\pm\overrightarrow{AB}$ and $\pm\overrightarrow{AC}$ must be attempted via a correct method. If the method is not directly seen it may be implied by two correct components
- There must be a similar correct attempt at the magnitudes of $\pm\overrightarrow{AB}$ and $\pm\overrightarrow{AC}$
- All should be connected by the equation $\pm\overrightarrow{AB} \cdot \pm\overrightarrow{AC} = |\pm\overrightarrow{AB}||\pm\overrightarrow{AC}|\cos 45$

ddM1: Allowable method to find at least one value of α from a quadratic equation.

For example,

- $(\alpha - 2)^2 = 32 \Rightarrow \alpha = \dots$
- $\alpha^2 - 4\alpha - 28 = 0 \Rightarrow \alpha = \dots$

This can only be awarded if the equation has real roots.

It is dependent upon both previous M's

A1: $\alpha = 2 \pm 4\sqrt{2}$ or exact equivalent

Note: Alternative method via cosine rule

B1: $\overrightarrow{AC} = 2\mathbf{i} + (\alpha - 2)\mathbf{j} + 8\mathbf{k}$ (or $\overrightarrow{CA} = -2\mathbf{i} - (\alpha - 2)\mathbf{j} - 8\mathbf{k}$) May be implied by further work.

M1: Attempts to find the magnitude of either $\overrightarrow{AB}$ or $\overrightarrow{AC}$

A1: $|\overrightarrow{AB}| = \sqrt{32}$ and $|\overrightarrow{AC}| = \sqrt{68 + (\alpha - 2)^2}$ ($= \sqrt{72 - 4\alpha + \alpha^2}$)

dM1: Attempts $|\overrightarrow{BC}|^2 = |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 - 2|\overrightarrow{AB}||\overrightarrow{AC}|\cos 45$

$$\text{FYI } (2^2 + (\alpha - 2)^2 + 4^2) = (2^2 + (\alpha - 2)^2 + 8^2) + (4^2 + 4^2) - 2\sqrt{(2^2 + (\alpha - 2)^2 + 4^2)}\sqrt{(4^2 + 4^2)} \times \frac{\sqrt{2}}{2}$$

ddM1: Rearranges and proceeds to at least one value for α . This can only be awarded if the equation is quadratic and has real roots

A1: $\alpha = 2 \pm 4\sqrt{2}$ or exact equivalent



Question Number	Scheme	Marks
6	$u = 3 + 4 \sin x \Rightarrow \frac{du}{dx} = 4 \cos x$ $\int \frac{16 \sin 2x}{(3 + 4 \sin x)^2} dx = \int \frac{32 \sin x \cos x}{(3 + 4 \sin x)^2} dx = \int \frac{2(u-3)}{u^2} du$ $= \int \frac{2}{u} - \frac{6}{u^2} du = 2 \ln u + \frac{6}{u}$ Uses limits of 5 and 7 $\Rightarrow 2 \ln 7 + \frac{6}{7} - 2 \ln 5 - \frac{6}{5} = -\frac{12}{35} + \ln \frac{49}{25}$	B1 M1 A1 dM1 A1 M1 A1 (7 marks)

B1: States or uses $\frac{du}{dx} = 4 \cos x$ o.e. This may be seen within the integrand.

M1: Attempts to write all in terms of u .

Look for $\int \frac{16 \sin 2x}{(3 + 4 \sin x)^2} dx = \int \frac{\dots \sin x \cos x}{(3 + 4 \sin x)^2} dx = \pm \int \frac{\dots(u \pm 3)}{u^2} du$

It is common to lose the square on the u^2 which would be M0.

Only the penultimate mark would then be available.

A1: Correct $\int \frac{2(u-3)}{u^2} du$ o.e. Allow this to be unsimplified.

Condone a missing du this can be implied by further work.

dM1: Integrates $\int \frac{\dots(u \pm 3)}{u^2} du$ to $\dots \ln u \pm \frac{\dots}{u}$ o.e. Condone a missing du this can be implied by further work.

Note that $\int \frac{\dots(u \pm 3)}{u^2} du \rightarrow \dots \ln u^2 \pm \frac{\dots}{u}$ is acceptable

You may see attempts using parts.

$$\int \frac{1}{u^2} \times (2u-6) du = -\frac{1}{u} \times (2u-6) - \int -\frac{1}{u} \times 2 du = -\frac{1}{u} \times (2u-6) + 2 \ln u$$

A1: Correct $2 \ln u + \frac{6}{u}$. Note that $\ln u^2 + \frac{6}{u}$ or $-\frac{1}{u} \times (2u-6) + 2 \ln u$ is also completely correct

M1: Uses limits 5 and 7 within their attempted integral and subtracts. Condone poor attempts at the integration. Alternatively converts their answer in u back to x 's using the correct substitution and uses the given limits

A1: $-\frac{12}{35} + \ln \frac{49}{25}$ or equivalent such as $\ln 1.96 - \frac{12}{35}$

ISW after a correct answer.

Question Number	Scheme	Marks
7 (a)	Co-ordinates or position vector of a point on $l = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$ $\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$ Uses $\overrightarrow{AX} \cdot \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0 \Rightarrow 20 + 16\lambda - 15 + 9\lambda - 25 + 25\lambda = 0 \Rightarrow \lambda = \frac{2}{5}$ (i) Substitutes their λ into $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} \Rightarrow X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$ (ii) $\overrightarrow{AX} = -\frac{33}{5}\mathbf{i} + \frac{19}{5}\mathbf{j} - 3\mathbf{k}$ $\text{Shortest distance} = \sqrt{\left(-\frac{33}{5}\right)^2 + \left(\frac{19}{5}\right)^2 + (-3)^2} = \sqrt{67}$	M1 dM1 A1 dM1 A1 M1 A1 (7)
(b)	Uses $\overrightarrow{AX} = \overrightarrow{XB}$ or similar correct method point B has position vector $-\frac{21}{5}\mathbf{i} + \frac{23}{5}\mathbf{j} - 4\mathbf{k}$	M1 A1 (2) (9 marks)
7 (a) ALT way	Co-ordinates or position vector of a point on $l = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$ $\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$ Uses $d^2 = (-5-4\lambda)^2 + (5-3\lambda)^2 + (-5+5\lambda)^2$ which is smallest when its gradient $= 0 \Rightarrow -8(-5-4\lambda) - 6(5-3\lambda) + 10(-5+5\lambda) = 0 \Rightarrow \lambda = \frac{2}{5}$ (Note that this could be done by completing the square) (i) Substitutes their λ into $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} \Rightarrow X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$ (ii) Shortest distance $= \left(-5 - 4 \times \frac{2}{5}\right)^2 + \left(5 - 3 \times \frac{2}{5}\right)^2 + \left(-5 + 5 \times \frac{2}{5}\right)^2 \Rightarrow d = \sqrt{67}$	M1 dM1 A1 dM1 A1 M1 A1 (7)

Question	Scheme	Marks
4(a)	$16x^3 - 9kx^2y + 8y^3 = 875$	
	$(8)y^3 \rightarrow (8 \times) 3y^2 \frac{dy}{dx}$	B1
	$-9kx^2y \rightarrow \dots kxy \pm \dots - 9kx^2 \frac{dy}{dx}$	M1
	$48x^2 - 18kxy - 9kx^2 \frac{dy}{dx} + 24y^2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} (24y^2 - 9kx^2) = 18kxy - 48x^2 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2} \quad *$	A1*
		(4)
(b)	$\frac{dy}{dx} = 0, x = \frac{5}{2} \Rightarrow \frac{6k\left(\frac{5}{2}\right)y - 16\left(\frac{5}{2}\right)^2}{8y^2 - 3k\left(\frac{5}{2}\right)^2} = 0 \quad \text{or}$ $x = \frac{5}{2} \Rightarrow 16\left(\frac{5}{2}\right)^3 - 9k\left(\frac{5}{2}\right)^2 y + 8y^3 = 875$	M1
	$15ky - 100 = 0 \quad \text{or} \quad 250 - \frac{225}{4}ky + 8y^3 = 875$	A1
	E.g. $16\left(\frac{5}{2}\right)^3 - 9k\left(\frac{5}{2}\right)^2\left(\frac{20}{3k}\right) + 8\left(\frac{20}{3k}\right)^3 = 875 \Rightarrow k^3 = \dots \left(= \frac{64}{27} \right) \Rightarrow k = \dots$	M1
	$k = \frac{4}{3}$	A1
		(4)
		(8 marks)



Notes

(a)

B1: For $y^3 \rightarrow 3y^2 \frac{dy}{dx}$. Allow if seen in aside working without the 8.**M1:** Correct attempt at implicit differentiation on the $-9kx^2y$. Look for $-9kx^2y \rightarrow \dots kxy \pm \dots - 9kx^2 \frac{dy}{dx}$ **M1:** Collects both of their $\frac{dy}{dx}$ terms together, collects non $\frac{dy}{dx}$ terms the other side of the equation, factorises and divides to achieve $\frac{dy}{dx} = \dots$. Must have two $\frac{dy}{dx}$ terms, one from the attempt at differentiating $-9kx^2y$ and one from the attempt at differentiating y^3 , but condone if an extra $\frac{dy}{dx} = \dots$ term has been included.**A1*:** Achieves $\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$ with no errors

(b)

M1: Uses the information to produce one equation in k and y , e.g. sets the $\frac{dy}{dx}$ equal to 0 and substitutes $x = \frac{5}{2}$, or substitutes $x = \frac{5}{2}$ into the given equation. Allow one slip substituting.**A1:** A correct equation without fraction and with simplified coefficients, so $15ky - 100 = 0$ oe or $250 - \frac{225}{4}ky + 8y^3 = 875$ oe**M1:** For a complete method to find k so solves the equations simultaneously to achieve a value for k . May find y first e.g. substitutes their $k = \frac{20}{3y}$ into the original equation, solves to find y and substitutes this back into

$$k = \frac{20}{3y} \text{ to find } k \text{ via } 250 - 375 + 8y^3 = 875 \Rightarrow y = 5 \Rightarrow k = \frac{20}{3 \times 5} = \dots$$

A1 $k = \frac{4}{3}$

Alt:

If they do not substitute $x = \frac{5}{2}$ initially then score**M1:** Uses numerator of $\frac{dy}{dx}$ equal to 0 to find y in terms of x and k and substitute into original equation (allowing one slip)**A1:** Correct equation:

$$6kxy - 16x^2 = 0 \Rightarrow y = \frac{8x^2}{3kx} \Rightarrow 16x^3 - 9kx^2 \left(\frac{8x^2}{3kx} \right) + 8 \left(\frac{8x^2}{3kx} \right)^3 = 875 \text{ oe}$$

M1: Substitutes $x = \frac{5}{2}$ and solves to find k

A1: $k = \frac{4}{3}$



Question	Scheme	Marks
5(a)	$1 = 2 \sin u \Rightarrow p = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$	B1
	$x = 2 \sin u \Rightarrow \frac{dx}{du} = 2 \cos u$ oe	M1
	$\int \frac{3x+2}{(4-x^2)^{\frac{3}{2}}} dx = \int \frac{6 \sin u + 2}{(4-4 \sin^2 u)^{\frac{3}{2}}} 2 \cos u du = \int \frac{6 \sin u + 2}{(4 \cos^2 u)^{\frac{3}{2}}} 2 \cos u du$	M1
	$= \int \frac{12 \sin u}{8 \cos^2 u} + \frac{2}{4 \cos^2 u} du = \int_0^{\frac{\pi}{6}} \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du$ *	A1*
		(4)
(b)	$\int \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du = \frac{3}{2} \sec u + \frac{1}{2} \tan u$	M1A1
	$\left[\frac{3}{2} \sec u + \frac{1}{2} \tan u \right]_0^{\frac{\pi}{6}} = \left(\frac{3}{2} \sec\left(\frac{\pi}{6}\right) + \frac{1}{2} \tan\left(\frac{\pi}{6}\right) \right) - \left(\frac{3}{2} \sec 0 + \frac{1}{2} \tan 0 \right) = \dots$	M1
	$= \sqrt{3} + \frac{\sqrt{3}}{6} - \frac{3}{2} = \frac{7\sqrt{3}}{6} - \frac{3}{2} \left(= \frac{7\sqrt{3}-9}{6} \right)$	A1
		(4)
		(8 marks)
Notes		
(a)		
B1: $p = \frac{\pi}{6}$ Allow if seen anywhere, even in (b). $p = 30$ is B0.		
M1: $x = 2 \sin u \Rightarrow \frac{dx}{du} = \pm \dots \cos u$ or any rearrangement of this equation.		
M1: Full substitution from an integral in terms of x to an integral in terms of u and uses the identity $\sin^2 u + \cos^2 u = 1$ in the denominator. Do not be concerned with the limits for this mark.		
A1*: Achieves given answer include du (with their p) with no errors and at least one intermediate step with the fractional power simplified. Condone missing du in intermediate lines.		
(b)		
M1: $\int \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du = \dots \sec u + \dots \tan u$		
A1: $\frac{3}{2} \sec u + \frac{1}{2} \tan u$ ignore any constant c		
M1: Depends on having one term of the correct form, attempts to substitute in their p ($\neq 1$) and 0, subtracting either way round. The substitution must be seen or clearly implied, e.g. by correct values for each term in an intermediate step before the answer (allowing missing 0's).		
A1: $\frac{7\sqrt{3}}{6} - \frac{3}{2}$ or exact equivalent eg $\frac{7\sqrt{3}-9}{6}$ Allow if $p = 30^\circ$ was used.		

2) Alternatively sets $2t^2 - 6t = 20 \Rightarrow t = -2$ and substitutes into $y = t^3 - 4t \rightarrow y = 0 \Rightarrow B = (20, 0)$

This is a show that and there must be some evidence for the "0", not just sight of $t = -2$

(b)

M1: Attempts to differentiate $x(t)$ and $y(t)$ and calculates $\frac{dy}{dx}$ by using $\frac{dy/dt}{dx/dt}$ in part (b)

A1: $\frac{dy}{dx} = \frac{3t^2 - 4}{4t - 6}$

M1: Sub 'their' $t = -2$ into their $\frac{dy}{dx}$ to find the gradient of the tangent at B .

M1: Uses $(20, 0)$ and their $\frac{dy}{dx}$ to find an equation of the tangent.

If they use $y = mx + c$ they need to proceed as far as $c = \dots$

A1*: Achieves given answer. $cs0 \Rightarrow 7y + 4x - 80 = 0$ The $= 0$ must be seen

ISW after sight of this

(c) **The demand here is to use algebra which is satisfied by the setting up of the cubic equation.**

M1: Attempts to substitute $x = 2t^2 - 6t$, $y = t^3 - 4t$ into $7y + 4x - 80 = 0$

A1: Correct simplified equation in t : $7t^3 + 8t^2 - 52t - 80 = 0$ The $= 0$ may be implied by further work

dM1: Uses a correct method to find x

- Using a calculator this can be scored for finding a value of $t \neq -2$ and substituting this in $x(t)$. You may have to check this on a calculator if their equation is incorrect. An accuracy of 2 sf is required for non exact solutions. Note that the equation may be in different forms under this method so $7t^3 + 8t^2 - 52t = 80 \Rightarrow t = \frac{20}{7}$ is fine

Note: $7(t^3 - 4t) + 4(2t^2 - 6t) - 80 = 0 \Rightarrow t = \frac{20}{7} \Rightarrow x = -\frac{40}{49}$ would score M1 A0 dM1 A0

- Via factorisation look for factors where the ends work,
E.g. " $7t^3 + 8t^2 - 52t - 80$ " = $(Pt + a)(Qt + b)(Rt + c)$ with $PQR = "7"$ and $abc = "\pm 80"$
followed by substitution of a value of $t \neq -2$ in $x(t)$

A1: CSO $x = -\frac{40}{49}$ Do not accept decimals here but remember to isw following a correct answer.

Ignore any references to the y coordinate. Penalise extra solutions, incorrect factorisation etc on this mark.



Question Number	Scheme	Marks
5(a)	$\int \frac{\ln x}{x^2} dx = \int x^{-2} \ln x dx = -x^{-1} \ln x + \int x^{-1} \times \frac{1}{x} dx$ $= -x^{-1} \ln x - x^{-1} (+c)$	M1 dM1 A1 (3)
(b)	$\frac{3+2x+\ln x}{x^2} = 3x^{-2} + 2x^{-1} + x^{-2} \ln x \text{ oe}$ $\text{Area} = \left[-\frac{3}{x} + 2 \ln x - \frac{\ln x}{x} - \frac{1}{x} \right]_2^4$ $= \left(-\frac{3}{4} + 2 \ln 4 - \frac{\ln 4}{4} - \frac{1}{4} \right) - \left(-\frac{3}{2} + 2 \ln 2 - \frac{\ln 2}{2} - \frac{1}{2} \right) = 1 + 2 \ln 2$	B1 M1 M1 A1 (4) (7 marks)

(a)

M1: Attempts by parts to reach a form $\int x^{-2} \ln x dx = \pm ax^{-1} \ln x \pm b \int x^{-1} \times \frac{1}{x} dx$ where $a, b \neq 0$

If a formula is stated it must be correct.

dM1: Integrates again to reach a form $\pm ax^{-1} \ln x \pm bx^{-1}$

A1: $-x^{-1} \ln x - x^{-1} (+c)$ o.e such as $-\frac{\ln x}{x} - \frac{1}{x} + c$

Condone the omission of the constant of integration. Condone $-x^{-1} \ln x + (-x^{-1})$

(b)

B1: $\frac{3+2x+\ln x}{x^2} = 3x^{-2} + 2x^{-1} + x^{-2} \ln x$ or $\frac{3}{x^2} + \frac{2}{x} + \frac{\ln x}{x^2}$.

The $\frac{2}{x}$ may be implied by later work. That is if they state $\int \frac{2x}{x^2} dx = 2 \ln x$ or $\ln x^2$

M1: Area = $\left[-\frac{3}{x} + 2 \ln x - \frac{\ln x}{x} - \frac{1}{x} \right]_2^4$ following through on their part (a)

M1: For substituting 2 and 4 **into an expression of the form** $\left[\frac{p}{x} + q \ln x + r \frac{\ln x}{x} + \frac{s}{x} \right]_2^4$ and attempts to

simplify by using one log law correctly. (Note that the two terms in $\frac{1}{x}$ may be combined before substitution)

A1: Exact answer: Likely to be $1 + 2 \ln 2$ or $1 + \ln 4$

.....

For those who don't read the instruction in (b) to use (a) and do via parts score



$$B1: \int \frac{3+2x+\ln x}{x^2} dx = -x^{-1}(3+2x+\ln x) + \int \left(2 + \frac{1}{x}\right) \times \frac{1}{x} dx$$

$$M1: \text{Area} = \left[\pm x^{-1}(3+2x+\ln x) \pm 2 \ln x \pm x^{-1} \right]_2^4$$

Question Number	Scheme	Marks
6 (a)	$y = x^{\sin x} \Rightarrow \ln y = \sin x \ln x$ Differentiates $\frac{1}{y} \frac{dy}{dx} = \frac{\sin x}{x} + \ln x \cos x$ $\frac{dy}{dx} = \frac{y \sin x}{x} + y \ln x \cos x$ oe	B1 M1 M1 A1 A1 (5)
(b)	Puts $\frac{dy}{dx} = 0 \Rightarrow \frac{\sin x}{x} + \ln x \cos x = 0$ $\frac{\sin x}{\cos x} + x \ln x = 0 \Rightarrow \tan x + x \ln x = 0^*$	M1 A1* (2) (7 marks)

(a)

$$B1: y = x^{\sin x} \Rightarrow \ln y = \sin x \ln x \text{ o.e. Condone this written } \log y = \sin x \log x$$

$$\text{This may have been found via } \log_x y = \sin x \rightarrow \frac{\ln y}{\ln x} = \sin x$$

$$M1: \text{For differentiation of } \ln y \rightarrow \frac{1}{y} \times \frac{dy}{dx} \text{ or } \log_e y \rightarrow \frac{1}{y} \times \frac{dy}{dx}$$

M1: Attempts the product rule on $\sin x \ln x$ If the rule is quoted it must be correct.

If candidates set $u = \dots, u' = \dots, v = \dots, v' = \dots$ it may be implied by their $vu' + uv'$

$$\text{If it is neither quoted nor implied score for } \frac{\sin x}{x} \pm \ln x \cos x$$

$$A1: \text{Correct differentiation to } \frac{1}{y} \frac{dy}{dx} = \frac{\sin x}{x} + \ln x \cos x$$

$$A1: \text{Achieves } \frac{dy}{dx} = \frac{y \sin x}{x} + y \ln x \cos x \text{ or exact equivalent such as } \frac{dy}{dx} = y \left(\frac{\sin x + x \ln x \cos x}{x} \right)$$

$$\text{Accept this in terms of just } x. \text{ For instance } \frac{dy}{dx} = \frac{x^{\sin x} \sin x}{x} + x^{\sin x} \ln x \cos x$$

ISW after sight of a correct answer.

.....
 Note that you will see candidates who write $\frac{1}{y} \frac{dy}{dx} = \frac{\sin x}{x} dx + \ln x \cos x dx$ which can be marked in the same way



(b)

M1: Sets their $\frac{dy}{dx} = 0$, divides by y or $x^{\sin x}$ to form a simplified equation in x .

This may be implied by $\frac{dy}{dx} = \frac{y \sin x}{x} + y \ln x \cos x = 0 \Rightarrow \sin x + x \ln x \cos x = 0$ o.e.

For this to be scored $\frac{dy}{dx}$ **must be in the form**

$$\frac{y \sin x}{\dots} \pm y \ln x \cos x$$

$$\frac{x^{\sin x} \sin x}{\dots} \pm x^{\sin x} \ln x \cos x \text{ where } \dots \text{ could be } 1$$

A1*: CSO Proceeds to the given answer of $\tan x + x \ln x = 0$

showing at least the intermediate line $\frac{\sin x}{\cos x} + x \ln x = 0$

It can be implied by $\sin x + x \ln x \cos x = 0 \quad (\div \cos x) \Rightarrow \tan x + x \ln x = 0$

6 (a)	$y = x^{\sin x} \Rightarrow y = e^{\sin x \ln x}$	B1
	Differentiates $y = e^{\sin x \ln x} \rightarrow \frac{dy}{dx} = e^{\sin x \ln x} \times \dots$	M1
	$\frac{d}{dx}(\sin x \ln x) = \frac{\sin x}{x} + \ln x \cos x$	M1
	$\frac{dy}{dx} = e^{\sin x \ln x} \times \left(\frac{\sin x}{x} + \ln x \cos x \right)$	A2
		(5)

B1: $y = x^{\sin x} \Rightarrow y = e^{\sin x \ln x}$

M1: Differentiates $e^{\sin x \ln x} \rightarrow e^{\sin x \ln x} \times \dots$ where ... could be 1

M1: Attempts the product rule on $\sin x \ln x$. **See main scheme on how to apply this mark.**

A2: Correct differentiation $= \frac{\sin x}{x} + \ln x \cos x$

Question	Scheme	Marks
7 (i)	Chooses a suitable substitution for $\int_1^5 \frac{3x}{\sqrt{2x-1}} dx$	
	E.g. $u = \sqrt{2x-1}$ $u = \sqrt{2x-1} \Rightarrow \frac{dx}{du} = u$ $\int \frac{3x}{\sqrt{2x-1}} dx$ $= \int \frac{3}{2}u^2 + \frac{3}{2} du$ $= \frac{1}{2}u^3 + \frac{3}{2}u$ Limits $= \left[\frac{1}{2}u^3 + \frac{3}{2}u \right]_1^3 = \dots$ $\left(\frac{1}{2} \times 3^3 + \frac{3}{2} \times 3 \right) - \left(\frac{1}{2} \times 1^3 + \frac{3}{2} \times 1 \right) = 16$	B1
	E.g. $u = 2x-1$ $u = 2x-1 \Rightarrow \frac{dx}{du} = \frac{1}{2}$ $\int \frac{3x}{\sqrt{2x-1}} dx$ $= \int \frac{3}{4}u^{\frac{1}{2}} + \frac{3}{4}u^{-\frac{1}{2}} du$ $= \frac{1}{2}u^{\frac{3}{2}} + \frac{3}{2}u^{\frac{1}{2}}$ $= \left[\frac{1}{2}u^{\frac{3}{2}} + \frac{3}{2}u^{\frac{1}{2}} \right]_1^9 = \dots$ $\left(\frac{1}{2} \times 9^{\frac{3}{2}} + \frac{3}{2} \times 9^{\frac{1}{2}} \right) - \left(\frac{1}{2} \times 1 + \frac{3}{2} \times 1 \right) = 16$	M1 dM1 A1
		M1
		A1
		(6)
7 (ii)	$\frac{6x^2-16}{(x+1)(2x-3)} = 3 + \dots$	B1
	$\frac{6x^2-16}{(x+1)(2x-3)} = 3 + \frac{2}{x+1} - \frac{1}{2x-3}$	M1 A1
	$\int \frac{6x^2-16}{(x+1)(2x-3)} dx = \int 3 + \frac{2}{x+1} - \frac{1}{2x-3} dx$	
	$= 3x + 2 \ln x+1 - \frac{1}{2} \ln 2x-3 + c$	M1 A1ft A1
		(6)
		(12 marks)

(i)

B1: States or uses a suitable substitution. That is one that will work.

M1: Attempts to change the integral in x to an integral in ' u 'Expect to see all aspects changed including the dx For $u = \sqrt{2x-1}$ expect to see $\int Au^2 + Bdu$

For $u = 2x - 1$ expect to see $\int Pu^{\frac{1}{2}} + Qu^{-\frac{1}{2}} du$

dM1: For correct method of integration. Allow for correct powers. Dependent upon previous M

A1: See scheme. Correct integration which may be left unsimplified

M1: Uses correct limits for their **valid** substitution.

A1: CSO 16. Look for 18 -2 or such before the 16.

16 with no working scores no marks. This is evidence of use of a calculator

(ii) **Note that open scoring is M1 M1 A1 M1 A1 A1**

B1: For a quotient of 3.

This can be scored by setting $6x^2 - 16 = A(x+1)(2x-3) + B(x+1) + C(2x-3)$ and reaching $A = 3$

or by division with 3 in the "correct place"

M1: A full attempt to find A , B and C .

Either sets $6x^2 - 16 = A(x+1)(2x-3) + B(x+1) + C(2x-3)$ condoning slips leading to values of A , B and C

or divides and uses partial fractions on the remainder to form an expression of the correct form.

A1: Correct partial fractions seen or implied. $\frac{6x^2 - 16}{(x+1)(2x-3)} = 3 + \frac{2}{(x+1)} - \frac{1}{(2x-3)}$

M1: Two of $A \rightarrow \dots x$, $\int \frac{B}{(x+1)} dx \rightarrow \dots \ln(x+1)$ $\int \frac{C}{(2x-3)} dx \rightarrow \dots \ln(2x-3)$

A1ft: Two correct of $A \rightarrow Ax \int \frac{B}{(x+1)} dx \rightarrow B \ln(x+1)$ and $\int \frac{C}{(2x-3)} dx \rightarrow \frac{C}{2} \ln(2x-3)$

Please note that answers such as $\int \frac{1}{2(2x-3)} dx \rightarrow \frac{1}{2} \ln(4x-6)$ are correct

A1: $3x + 2 \ln|x+1| - \frac{1}{2} \ln|2x-3| + c$ but condone $3x + 2 \ln(x+1) - \frac{1}{2} \ln(2x-3) + c$ **The +c is required.**

You should isw after a correct answer.

Note that it is possible to do **part (i)** by parts. This does not satisfy the demand of the question but can score some marks

B1: $\int \frac{3x}{\sqrt{2x-1}} dx = 3x\sqrt{2x-1} - \int 3\sqrt{2x-1} dx$

M1: $\int \frac{3x}{\sqrt{2x-1}} dx = ax\sqrt{2x-1} - b(2x-1)^{\frac{3}{2}}$ FYI: Correct when $a = 3, b = 1$

third M1 (5th mark on open): applies limits to a valid expression $\int_1^5 \frac{3x}{\sqrt{2x-1}} dx = \left[ax\sqrt{2x-1} - b(2x-1)^{\frac{3}{2}} \right]_1^5 = \dots$

second A1 (6th mark on open) $\int_1^5 \frac{3x}{\sqrt{2x-1}} dx = \left[3x\sqrt{2x-1} - (2x-1)^{\frac{3}{2}} \right]_1^5 = (45 - 27) - (3 - 1) = 16$ o.e.

to score on e-open 110011

Note that in **part (ii)** some candidates will write



$$\frac{6x^2-16}{(x+1)(2x-3)} = \frac{B}{x+1} + \frac{C}{2x-3} \text{ and via substitution will get "a correct" } \frac{2}{x+1} - \frac{1}{2x-3}$$

$$\int \frac{6x^2-16}{(x+1)(2x-3)} dx = \int \frac{"B"}{x+1} + \frac{"C"}{2x-3} dx = "B" \ln(x+1) + \frac{"C"}{2} \ln(2x-3)$$

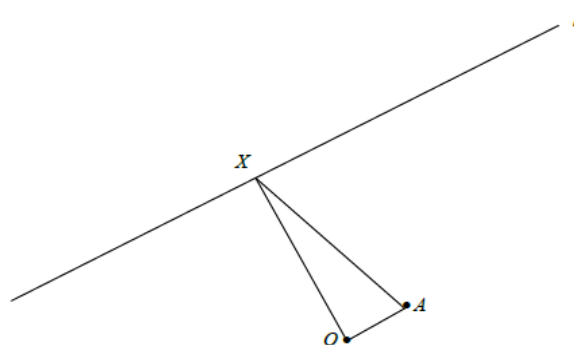
Scores SC B0 M0 A0 M1 A1ft A0 for 2 out of 6 if ft integration is correct

Question Number	Scheme	Marks
8 (a)	$\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \Rightarrow \begin{matrix} 4+3\lambda=2+2\mu \\ -3-2\lambda=0-1\mu \\ 2-1\lambda=-9-3\mu \end{matrix} \text{ any two of these}$	M1
	Full method to find either λ or μ eg $(1)+3(3) \Rightarrow \mu = \dots$	M1
	Either $\lambda = -4$ or $\mu = -5$	A1
	Position vector of intersection is $\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - 4 \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = \text{OR} \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} - 5 \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} =$	dM1
	$= \begin{pmatrix} -8 \\ 5 \\ 6 \end{pmatrix}$	A1
(b)	Co-ordinates or position vector of point $Q = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix}$	
	$\overrightarrow{PQ} = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu-8 \\ 7-1\mu \\ -9-3\mu \end{pmatrix}$	M1
	Uses $\overrightarrow{PQ} \cdot \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = 0 \Rightarrow 4\mu - 16 - 7 + \mu + 27 + 9\mu = 0 \Rightarrow \mu = -\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} \Rightarrow Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7} \right)$	ddM1 A1
		(5) (10 marks)



Question Number	Scheme	Marks
7 (a)	Finds $\left\ \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \right\ = \sqrt{4^2 + 4^2 + 2^2} = 6$ and attempts $\frac{1}{6} \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$ $\overrightarrow{OA} = \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$ oe	M1 A1 (2)
(b)	Co-ordinates or position vector of point $X = \begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix}$ $\overrightarrow{OX} \cdot \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = 0 \Rightarrow 4(1+4\lambda) + 4(-10+4\lambda) + 2(-9+2\lambda) = 0$ $36\lambda = 54 \Rightarrow \lambda = 1.5$ $X = (7, -4, -6)$	M1 dM1 ddM1 A1 A1 (5)
(c)	Finds $OX = \sqrt{7^2 + (-4)^2 + (-6)^2} = \sqrt{101}$ and $OA = 1$ $\text{Area } OXA = \frac{1}{2} \times 1 \times \sqrt{101} = \frac{\sqrt{101}}{2}$	M1 dM1 A1 (3)
		(10 marks)

Handy diagram



(a)

M1: Correct attempt at the unit vector. Look for an attempt at $\sqrt{4^2 + 4^2 + 2^2}$ and use of $\frac{\mathbf{r}}{|\mathbf{r}|}$ where $\mathbf{r} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$

or equivalent vector form such as $\mathbf{r} = (4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$. Condone an attempt to find the coordinates of A



A1: $\overrightarrow{OA} = \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$ or equivalent such as $\overrightarrow{OA} = \frac{1}{6} \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$ or $\overrightarrow{OA} = \frac{1}{6}(4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ but **must be in vector form and**

not in coordinate form.

(b)

M1: For an attempt at the co-ordinates or position vector of point $X = (1+4\lambda, -10+4\lambda, -9+2\lambda)$ or $\begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix}$

dM1: For using $\overrightarrow{OX} \cdot \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = 0$ to set up an equation in λ

Alternatively finds $OX^2 = (1+4\lambda)^2 + (-10+4\lambda)^2 + (-9+2\lambda)^2$ and attempts to differentiate and set = 0

May use $OX^2 + OA^2 = AX^2$ to set up an equation in λ

ddM1: Solves for λ This is dependent upon having scored both previous M's

A1: Correct value for $\lambda = 1.5$

A1: Correct coordinates for $X = (7, -4, -6)$. Condone position vector form $7\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$ o.e.

(c)

M1: Finds all elements required to calculate the area.

In the main scheme this would be the distance OX or OX^2 AND distance OA or OA^2 (which is 1)

dM1: Correct method of finding the area of $OXA = \frac{1}{2} \times OX \times 1$

A1: Area = $\frac{\sqrt{101}}{2}$ o.e

.....
There are various alternatives for part (c). Amongst others are;

Alt I via vector product.

$$\frac{1}{2} \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & -6 \\ 2/3 & 2/3 & 1/3 \end{vmatrix} = \frac{1}{2} \times \left| 8\mathbf{i} - \frac{19}{3}\mathbf{j} + \frac{22}{3}\mathbf{i} \right| = \frac{1}{2} \times \sqrt{\left(\frac{8}{3}\right)^2 + \left(-\frac{19}{3}\right)^2 + \left(\frac{22}{3}\right)^2} = \frac{\sqrt{101}}{2}$$

$$\text{M1: For an attempt at } \frac{1}{2} \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & -6 \\ 2/3 & 2/3 & 1/3 \end{vmatrix} = \frac{1}{2} \times \left| 8\mathbf{i} - \frac{19}{3}\mathbf{j} + \frac{22}{3}\mathbf{i} \right|$$

dM1: Followed by an attempt at finding the modulus of the resulting vector multiplied by 1/2

Alt II via scalar products

M1: Attempts to find all three components required to find the area triangle OXA

E.g. Angle OXA with length of side OX and XA

Alternatively angle OAX with length of side OA and XA

For this to be scored

- appropriate gradient vectors need to be attempted by subtracting
- a correct attempt at using $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ to find $\cos\theta$ or θ

dM1: Full attempt at area of triangle using $\frac{1}{2}|OX||XA|\sin(OXA)$ or equivalent

(c) **Otherwise:** Applies binomial expansion to $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}}$

B1: Correct simplified expression $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \frac{1}{2}(1 - 2x)^{\frac{1}{2}}$

M1: Correct structure for 3rd term: $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-2x)^2$ but allow $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm kx)^2$ with or without brackets. Even allow with $k=1$

A1: $\frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$ Ignore extra terms of x^3 and above

Direct expansion in (a)

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{3}{2}} = \left(\frac{1}{4}\right)^{\frac{3}{2}} + \left(-\frac{3}{2}\right)\left(\frac{1}{4}\right)^{\frac{5}{2}}\left(-\frac{1}{2}x\right) + \frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{\frac{7}{2}}\left(-\frac{1}{2}x\right)^2 + \frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right) \times \left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{\frac{9}{2}}\left(-\frac{1}{2}x\right)^3$$

B1: Obtains an expansion with a constant term of 8

M1: Attempts the binomial expansion of $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.

Look for $\frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{\frac{7}{2}}\left(\pm\frac{1}{2}x\right)^2$ or $\frac{\left(-\frac{3}{2}\right) \times \left(-\frac{5}{2}\right) \times \left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{\frac{9}{2}}\left(\pm\frac{1}{2}x\right)^3$

with or without the brackets on the $\left(\pm\frac{1}{2}x\right)$

A1: Correct simplified or unsimplified expansion .

Expression at the top of the page is acceptable

A1: Two correct and simplified terms of $8 + 24x + 60x^2 + 140x^3$

A1: All correct and simplified $8 + 24x + 60x^2 + 140x^3$

Direct expansion in (c)

B1: First two terms which may be unsimplified $\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} + \frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}\left(-\frac{1}{2}x\right)^1 + \dots$

M1: Correct form for the third term $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}\left(-\frac{1}{2}x\right)^2$

A1: Correct simplified expansion $\frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$



Question Number	Scheme	Marks
2(a)	States or uses $S = 6x^2$ or $\frac{dS}{dt} = 4$	B1
	States or uses $S = 6x^2$ and $\frac{dS}{dt} = 4$	B1
	Attempts to use $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} \Rightarrow 4 = 12x \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \dots$	M1
	$\Rightarrow \frac{dx}{dt} = \frac{1}{3x}$ oe	A1
		(4)
(a) Alt	States or uses $S = 6x^2$ or $\frac{dS}{dt} = 4$	B1
	States or uses $S = 6x^2$ and $\frac{dS}{dt} = 4$	B1
	$\frac{dS}{dt} = 4 \Rightarrow S = 4t + c \Rightarrow S = 6x^2 \Rightarrow 6x^2 = 4t + c \Rightarrow 12x \frac{dx}{dt} = 4 \Rightarrow \frac{dx}{dt} = \dots$ or $S = 6x^2 \Rightarrow 6x^2 = 4t + c \Rightarrow 12x = 4 \frac{dt}{dx} \Rightarrow \frac{dx}{dt} = \dots$	M1
	$\Rightarrow \frac{dx}{dt} = \frac{1}{3x}$ oe	A1
(b)	States or uses $\frac{dV}{dx} = 3x^2$	B1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \Rightarrow \frac{dV}{dt} = 3x^2 \times \frac{1}{3x} = x$	M1
	$\Rightarrow \frac{dV}{dt} = V^{\frac{1}{3}}$	A1
		(3)
(b) Alt	$V = x^3, S = 6x^2 \Rightarrow S = 6V^{\frac{2}{3}} \Rightarrow \frac{dS}{dV} = 4V^{-\frac{1}{3}}$	B1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dS} \times \frac{dS}{dt} = \frac{1}{4V^{\frac{1}{3}}} \times 4$	M1
	$\Rightarrow \frac{dV}{dt} = V^{\frac{1}{3}}$	A1
		(7 marks)



(a) **Condone the use of different variables provided the intention is clear.**

This will most likely be the use of A rather than S for surface area.

B1: States or uses $S = 6x^2$ or $\frac{dS}{dt} = 4$ (ignore any units associated with either)

Note that $S = 6x^2$ may be implied by $\frac{dS}{dx} = 12x$

B1: States or uses $S = 6x^2$ and $\frac{dS}{dt} = 4$ (ignore any units associated with either)

Note that $S = 6x^2$ may be implied by $\frac{dS}{dx} = 12x$

M1: Attempts to use $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$ or equivalent with their $\frac{dS}{dx}$ and $\frac{dS}{dt}$ where $\frac{dS}{dx} = kx$ and $\frac{dS}{dt}$ is a constant, in an attempt to find $\frac{dx}{dt}$. May be implied by their working.

A1: $\frac{dx}{dt} = \frac{1}{3x}$ or e.g. $\frac{1/3}{x}$, $\frac{4}{12x}$ from correct work and apply isw once a correct answer is seen.

Allow correct work leading to e.g. $k = \frac{1}{3}$.

The " $\frac{dx}{dt} =$ " must appear at some point in their working for this mark.

(a) **Alt.**

B1: States or uses $S = 6x^2$ or $\frac{dS}{dt} = 4$ (ignore any units associated with either)

Note that $S = 6x^2$ may be implied by $\frac{dS}{dx} = 12x$

B1: States or uses $S = 6x^2$ and $\frac{dS}{dt} = 4$ (ignore any units associated with either)

Note that $S = 6x^2$ may be implied by $\frac{dS}{dx} = 12x$

M1: Integrates $\frac{dS}{dt} = 4$ to obtain $S = 4t + c$ or $S = 4t$ and replaces S in terms of x and differentiates wrt t or wrt x to obtain $\alpha x \frac{dx}{dt} = \beta$ or $\alpha x = \beta \frac{dt}{dx}$

A1: $\frac{dx}{dt} = \frac{1}{3x}$ or e.g. $\frac{1/3}{x}$, $\frac{4}{12x}$ from correct work and apply isw once a correct answer is seen.

Note that $\frac{dx}{dt} = \frac{1}{3x}$ sometimes comes from incorrect work so check their method.

This can be scored from obtaining $S = 4t + c$ or $S = 4t$ earlier.

The " $\frac{dx}{dt} =$ " must appear at some point in their working for this mark.

(b)

B1: States or uses $\frac{dV}{dx} = 3x^2$ Allow this mark to score anywhere in the question.

M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ or equivalent with their $\frac{dV}{dx}$ and $\frac{dx}{dt}$ which may be implied.

A1: $\frac{dV}{dt} = V^{\frac{1}{3}}$ which may be implied by e.g. $\frac{dV}{dt} = x$, $x = V^p \Rightarrow p = \frac{1}{3}$.

The " $\frac{dV}{dt} =$ " must appear at some point in their working for this mark.



(b) **Alt:**

B1: States or uses $\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ (from $S = 6x^2$, $V = x^3 \Rightarrow S = 6V^{\frac{2}{3}}$)

M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dS} \times \frac{dS}{dt}$ or equivalent with their $\frac{dV}{dS}$ and $\frac{dS}{dt}$ which may be implied.

A1: $\frac{dV}{dt} = V^{\frac{1}{3}}$ which may be implied by e.g. $\frac{dV}{dt} = x$, $x = V^p \Rightarrow p = \frac{1}{3}$

The " $\frac{dV}{dt} =$ " must appear at some point in their working for this mark.

Note that a common incorrect response involves using $S = x^2$ rather than $S = 6x^2$, giving:

$$(a) S = x^2, \frac{dS}{dt} = 4, \frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} \Rightarrow 4 = 2x \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{2}{x} \text{ scoring B1B0M1A0}$$

$$(b) \frac{dV}{dx} = 3x^2, \frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \Rightarrow \frac{dV}{dt} = 3x^2 \times \frac{2}{x} = 6x = 6V^{\frac{1}{3}} \text{ scoring B1M1A0}$$



Question Number	Scheme	Marks
3(i)	$\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx$	M1
	$= \frac{1}{2} x^2 e^{2x} - \left\{ \frac{1}{2} x e^{2x} - \int \frac{e^{2x}}{2} dx \right\}$	<u>M1</u>
	$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{e^{2x}}{4}$	A1
	$\int_0^4 x^2 e^{2x} dx = \left[\frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{e^{2x}}{4} \right]_0^4 = 8e^8 - 2e^8 + \frac{e^8}{4} - \frac{1}{4}$	M1
	$= \frac{25e^8}{4} - \frac{1}{4}$	A1
		(5)

(i) Condone the omission of “dx” throughout

M1: Attempts to integrate by parts once to achieve $Px^2 e^{2x} - Q \int x e^{2x} dx$, $P, Q > 0$

M1: Attempts to integrate $x e^{2x}$ to achieve $\lambda x e^{2x} \pm \mu \int e^{2x} dx$, $\lambda > 0$

A1: $\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{e^{2x}}{4}$ which may be unsimplified and isw once a correct answer is seen.

Watch for $\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \left(\frac{1}{2} x e^{2x} - \frac{e^{2x}}{4} \right) (+c)$ which scores all 3 marks. ISW after sight of this.

Watch for D & I method which may be seen. You will just see the answer here.

D	I
x^2	$+$ e^{2x}
$2x$	$-$ $\frac{1}{2} e^{2x}$
2	$+$ $\frac{1}{4} e^{2x}$
0	$\frac{1}{8} e^{2x}$

$$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x}$$

Score M2 for $Ax^2 e^{2x} \pm Bx e^{2x} \pm C e^{2x}$, $A > 0$

and then A1 for $\frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{e^{2x}}{4} (+c)$

M1: Attempts to substitute **both** limits into an expression of the form $Ax^2 e^{2x} \pm Bx e^{2x} \pm C e^{2x}$, $A > 0$,

subtracts either way round, and simplifies to the form $\alpha e^8 + \beta$, $\alpha, \beta \neq 0$ where α and β are numeric.

(Allow if the e^8 terms are not combined)

A1: $\frac{25e^8}{4} - \frac{1}{4}$ or exact simplified equivalent e.g. $\frac{1}{4} (25e^8 - 1)$, $\frac{25}{4} \left(e^8 - \frac{1}{25} \right)$. Do **not** condone “+ c” here.

(ii)	1. $u = 2x - 1 \Rightarrow \frac{du}{dx} = 2$ or e.g. $du = 2dx$ OR 2. $u = (2x - 1)^2 \Rightarrow \frac{du}{dx} = 4(2x - 1)$ or e.g. $du = 4(2x - 1)dx$ OR 3. $u = 2x \Rightarrow \frac{du}{dx} = 2$ or e.g. $du = 2dx$	B1
	1. $u = 2x - 1 \Rightarrow \int \frac{4x}{(2x - 1)^2} dx = \int \frac{2u + 2}{u^2} \times \frac{1}{2} du$ OR 2. $u = (2x - 1)^2 \Rightarrow \int \frac{4x}{(2x - 1)^2} dx = \int \frac{2\sqrt{u} + 2}{u} \times \frac{1}{4\sqrt{u}} du$ OR 3. $u = 2x \Rightarrow \int \frac{4x}{(2x - 1)^2} dx = \int \frac{2u}{(u - 1)^2} \times \frac{1}{2} du$	M1 A1
	1. $\int \left(\frac{1}{u} + \frac{1}{u^2} \right) du = \ln u - \frac{1}{u}$ OR 2. $\frac{1}{2} \int \left(\frac{1}{u} + \frac{1}{u^{\frac{3}{2}}} \right) du = \frac{1}{2} \ln u - \frac{1}{\sqrt{u}}$ OR 3. $\int \frac{u}{(u - 1)^2} du = -u(u - 1)^{-1} + \int \frac{1}{u - 1} du = -u(u - 1)^{-1} + \ln(u - 1)$	dM1 A1
	1. Uses limits $u = 5$ to $u = 20$: $\left(\ln 20 - \frac{1}{20} \right) - \left(\ln 5 - \frac{1}{5} \right)$ OR 2. Uses limits $u = 25$ to $u = 400$: $\left(\frac{1}{2} \ln 400 - \frac{1}{20} \right) - \left(\frac{1}{2} \ln 25 - \frac{1}{5} \right)$ OR 3. Uses limits $u = 6$ to $u = 21$: $\left(\ln 20 - \frac{21}{20} \right) - \left(\ln 5 - \frac{6}{5} \right)$	M1
	$= \frac{3}{20} + \ln 4$	A1
		(7)
		(12 marks)

Note that ' u ' = $2x - 1$ is by far the most common approach.



(ii)

B1: Uses an appropriate substitution and differentiates **correctly**. See scheme for 3 examples.

Note that for case 2. $u = (2x-1)^2 \Rightarrow x = \frac{\sqrt{u}+1}{2} \Rightarrow \frac{dx}{du} = \frac{1}{4}u^{-\frac{1}{2}}$ is also correct and scores B1

M1: Valid attempt to change the integral to one in "u" to obtain an integrand of the correct form:
This requires e.g.

$$1. k \int \frac{au+b}{u^2} du, \quad a, b \neq 0 \quad \text{OR} \quad 2. k \int \frac{a\sqrt{u}+b}{u} \frac{1}{\sqrt{u}} du, \quad a, b \neq 0 \quad \text{OR} \quad 3. k \int \frac{u}{(u-1)^2} du$$

Condone the omission of du as long as the form of the integrand is correct.

Note that the u 's may appear "combined" e.g. as $u^{\frac{3}{2}}$ in the denominator of case 2.

A1: A correct integrand in any form in terms of u only.

Condone the omission of du as long as the integrand is correct.

dM1: Correct form of the integration for their substitution: This requires e.g.

$$1. k \int \frac{au+b}{u^2} du \rightarrow p \ln u + qu^{-1} \quad \text{OR} \quad 2. k \int \frac{a\sqrt{u}+b}{u} \frac{1}{\sqrt{u}} du \rightarrow p \ln u + \frac{q}{\sqrt{u}}$$

$$\text{OR } 3. k \int \frac{u}{(u-1)^2} du \rightarrow \alpha u(u-1)^{-1} + \beta \ln(u-1)$$

Depends on the previous method mark.

A1: Fully correct integration. Ignore any limits for this mark.

Note that there may be other acceptable attempts to integrate e.g. in case 1.:

$$\int \frac{u+1}{u^2} du = \int (u+1)u^{-2} du = -(u+1)\frac{1}{u} + \int \frac{1}{u} du = -(u+1)\frac{1}{u} + \ln u$$

In such cases, award M1 for a fully correct method and A1 if correct.

M1: Attempts to use the correct limits for their substitution.

Need to see evidence that both correct changed limits have been substituted and the resulting expressions subtracted either way round.

There must have been some attempt to integrate however poor having used substitution.

The substitution must involve a transformation so e.g. $u = x$ is not acceptable.

Alternatively, reverses the substitution and uses the x limits 3 and $\frac{21}{2}$.

A1: $\ln 4 + \frac{3}{20}$ or e.g. $\ln 4 + 0.15$ and apply isw if necessary.



Special Case: Candidates who answer part (b) without any form of substitution can score a special case of B0M0A0 (no substitution) then M1A1dM1A1

e.g. Partial fractions:

$$\frac{4x}{(2x-1)^2} \equiv \frac{A}{2x-1} + \frac{B}{(2x-1)^2} \equiv \frac{2}{2x-1} + \frac{2}{(2x-1)^2}$$

$$\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2} dx = \ln(2x-1) - \frac{1}{(2x-1)}$$

$$\left[\ln(2x-1) - \frac{1}{2x-1} \right]_3^{21} = \ln 20 - \frac{1}{20} - \ln 5 + \frac{1}{5} = \frac{3}{20} + \ln 4$$

Mark as B0M0A0 then:

M1 for $\int \frac{A}{2x-1} + \frac{B}{(2x-1)^2} dx = \alpha \ln(2x-1) + \beta \frac{1}{(2x-1)}$

A1 for $\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2} dx = \ln(2x-1) - \frac{1}{(2x-1)} (+c)$

dM1 Substitutes both correct limits and subtracts either way round. **Depends on first M.**

A1: $\ln 4 + \frac{3}{20}$ or e.g. $\ln 4 + 0.15$ and apply isw if necessary.

or Parts:

$$\int \frac{4x}{(2x-1)^2} dx = -2x(2x-1)^{-1} + \ln(2x-1)$$

$$= \left[-2x(2x-1)^{-1} + \ln(2x-1) \right]_3^{21} = -\frac{21}{20} + \ln 20 + \frac{6}{5} - \ln 5 = \frac{3}{20} + \ln 20$$

M1 for $\int \frac{4x}{(2x-1)^2} dx = \alpha x(2x-1)^{-1} + \beta \ln(2x-1)$

A1 for $\int \frac{4x}{(2x-1)^2} dx = -2x(2x-1)^{-1} + \ln(2x-1) (+c)$

dM1 Substitutes both correct limits and subtracts either way round. **Depends on first M.**

A1: $\ln 4 + \frac{3}{20}$ or e.g. $\ln 4 + 0.15$ and apply isw if necessary.

If you find any attempts using a combination of methods that include substitution e.g.

$$\frac{4x}{(2x-1)^2} \equiv \frac{A}{2x-1} + \frac{B}{(2x-1)^2} \equiv \frac{2}{2x-1} + \frac{2}{(2x-1)^2}$$

$$\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2} dx = [\ln(2x-1)] + \int \frac{2}{u^2} \frac{du}{2} \quad (u = 2x-1)$$

$$\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2} dx = [\ln(2x-1)]_3^{21} + \left[-\frac{1}{u}\right]_5^{20} = \ln 20 - \ln 5 - \frac{1}{20} + \frac{1}{5} = \frac{3}{20} + \ln 4$$

OR

$$\int \frac{4x}{(2x-1)^2} dx = [-2x(2x-1)^{-1}] + \int \frac{2}{2x-1} dx$$

$$= [-2x(2x-1)^{-1}] + \int \frac{2}{u} \frac{du}{2} \quad (u = 2x-1)$$

$$= [-2x(2x-1)^{-1}]_3^{21} + [\ln u]_5^{20} = -\frac{21}{20} + \frac{6}{5} + \ln 20 - \ln 5 = \frac{3}{20} + \ln 20$$

if fully correct as above then score full marks otherwise send to review.



Question Number	Scheme	Marks
4(a)	Assume that there exists a positive number k such that $k + \frac{9}{k} < 6$	B1
	$k + \frac{9}{k} < 6 \Rightarrow k^2 + 9 < 6k \Rightarrow k^2 - 6k + 9 < 0$ or $k + \frac{9}{k} < 6 \Rightarrow k + \frac{9}{k} - 6 < 0 \Rightarrow (\sqrt{k}...)(\sqrt{k}...) < 0$ or $k + \frac{9}{k} < 6 \Rightarrow \left(k + \frac{9}{k}\right)^2 < 36 \Rightarrow k^2 + 18 + \frac{81}{k^2} - 36 < 0$	M1
	$\Rightarrow (k-3)^2 < 0$ or $\Rightarrow \left(\sqrt{k} - \frac{3}{\sqrt{k}}\right)^2 < 0$ or $\Rightarrow \left(k - \frac{9}{k}\right)^2 < 0$	A1
	But numbers squared are ≥ 0 , hence $k + \frac{9}{k} \geq 6$	A1*
		(4)
(b)	E.g. When $k = -3$, $-3 + \frac{9}{-3} (= -6)$ which is not ≥ 6	B1
		(1)
(b) Alt	$k < 0 \Rightarrow \frac{9}{k} < 0 \Rightarrow k + \frac{9}{k} < 0$ which is not ≥ 6	B1
		(5 marks)

(a)

B1: For setting up the correct contradiction. It must include a word/words such as “assume” or “let” and must be a strict inequality. As a minimum accept just “assume/let $k + \frac{9}{k} < 6$ ”.

Condone e.g. Assume that for all positive numbers k , $k + \frac{9}{k} < 6$

M1: Starting from $k + \frac{9}{k} < 6$ or $k + \frac{9}{k} \leq 6$ or $k + \frac{9}{k} = 6$, either

- multiplies by k and attempts to reach $k^2 \dots < 0$ or $k^2 \dots \leq 0$ or $k^2 \dots = 0$
- collects terms to one side to reach $k + \frac{9}{k} \pm 6 \leq / < / = 0$ and attempts to factorise to $(\sqrt{k}...)(\sqrt{k}...) \leq / < / = 0$
- squares both sides and collects terms to reach $k^2 \dots < 0$ or $k^2 \dots \leq 0$ or $k^2 \dots = 0$

A1: Reaches $(k-3)^2 \dots 0$ or e.g. $\left(\sqrt{k} - \frac{3}{\sqrt{k}}\right)^2 \dots 0$ or e.g. $\left(k - \frac{9}{k}\right)^2 \dots 0$ where ... is $<$ or $\leq$ or $=$

A1*: For a fully correct proof.

Requires

- correct calculations/algebra
- a reason: “numbers squared are ≥ 0 “this is impossible”, “numbers squared are not negative”
- a minimal conclusion. E.g. hence $k + \frac{9}{k} \geq 6$, hence proven, QED, etc. but not just “contradiction”

Alternatives for A1A1*:

1. Using discriminant:

A1: $b^2 - 4ac = 6^2 - 4 \times 9 = 0$ so one root and ‘positive’ quadratic

A1*: For a fully correct proof.

Requires

- correct calculations/algebra
- a reason e.g. so $k^2 - 6k + 9 \geq 0$ or curve is on or above the x/k axis
- a minimal conclusion. E.g. hence $k + \frac{9}{k} \geq 6$, hence proven, QED, etc. but not just “contradiction”

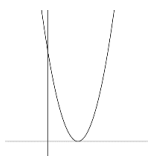
2. Via sketch:

A1: Sketches $y = k^2 - 6k + 9$:

A1*: For a fully correct proof.

Requires

- correct calculations/algebra
- a reason e.g. graph always on or above (or never below) the x -axis so $k^2 - 6k + 9 \geq 0$
- a minimal conclusion. E.g. hence $k + \frac{9}{k} \geq 6$, hence proven, QED, etc. but not just “contradiction”



3. Via differentiation:

A1: $\frac{d(k^2 - 6k + 9)}{dk} = 2k - 6 = 0 \Rightarrow k = 3 \Rightarrow k^2 - 6k + 9 = 0$ so minimum at (3, 0)

A1*: For a fully correct proof.

Requires

- correct calculations/algebra
- a reason e.g. so $k^2 - 6k + 9 \geq 0$ or curve is on or above the x/k axis
- a minimal conclusion. E.g. hence $k + \frac{9}{k} \geq 6$, hence proven, QED, etc. but not just “contradiction”

If candidates use another variable for k , e.g. x or m then withhold the final mark unless they revert to k in their conclusion.

This is also acceptable (working backwards):

Assume $k + \frac{9}{k} < 6$ B1

$$(k-3)^2 \geq 0 \Rightarrow k^2 - 6k + 9 \geq 0 \Rightarrow k - 6 + \frac{9}{k} \geq 0 \text{ M1}$$

Condone > for $\geq$

From $(k-3)^2 \geq 0$ expands and divides by k

$$\Rightarrow k + \frac{9}{k} \geq 6 \text{ A1}$$

Condone > for $\geq$

This contradicts the assumption and so $k + \frac{9}{k} \geq 6$ A1*

Note: Attempts that use the contradiction: “Assume there are negative numbers for which $k + \frac{9}{k} \geq 6$ ” generally score no marks but use review if necessary.

(b)

B1: Chooses a negative number, shows the result of substituting into $k + \frac{9}{k}$ and gives a

minimal conclusion e.g. It is not necessary to evaluate e.g. $-3 + \frac{9}{-3}$ is sufficient.

- which is not ≥ 6
- is < 6

(b)Alt:

B1: States that if $k < 0$ then $k + \frac{9}{k} < 0$ and gives a minimal conclusion e.g.

- which is not ≥ 6
- is < 6

**There must be no incorrect work and no contradictory/incorrect statements.
Do not allow arguments that refer to $k = 0$ being not valid.**

Question Number	Scheme	Marks
5(a)	$y^3 - x^2 + 4x^2y = k$	
	$y^3 \rightarrow 3y^2 \frac{dy}{dx}$	M1
	$4x^2y \rightarrow 8xy + 4x^2 \frac{dy}{dx}$	M1
	$y^3 - x^2 + 4x^2y = k \Rightarrow 3y^2 \frac{dy}{dx} - 2x + 8xy + 4x^2 \frac{dy}{dx} = 0$	A1
	$\Rightarrow \left(3y^2 + 4x^2\right) \frac{dy}{dx} = 2x - 8xy$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{2x - 8xy}{3y^2 + 4x^2}$	A1
		(5)
(b)	$\frac{dy}{dx} = -1$ at P	M1
	Uses $\frac{dy}{dx} = \pm 1$ and $y = x$ to set up and solve equation in x, y or ' p '	M1 A1
	$\Rightarrow -1 = \frac{2p - 8p^2}{3p^2 + 4p^2} \Rightarrow p = 2$	
	e.g. $k = 2^3 - 2^2 + 4 \times 2^3$	ddM1
	$k = 36$	A1
		(5)
		(10 marks)

(a) Allow equivalent notation for the $\frac{dy}{dx}$ e.g. y'

M1: Differentiates $y^3 \rightarrow 3y^2 \frac{dy}{dx}$

M1: Uses the product rule to differentiate $4x^2y$ and obtains $8xy + 4x^2 \frac{dy}{dx}$

A1: Correct differentiation including the “= 0” which may be implied by subsequent work.

Note that some candidates have a spurious $\frac{dy}{dx} = \dots$ at the start (as their intention to differentiate) and this

can be ignored for the first 3 marks so condone $\frac{dy}{dx} = 3y^2 \frac{dy}{dx} - 2x + 8xy + 4x^2 \frac{dy}{dx} = 0$

Allow versions such as $3y^2 dy - 2x dx + 8xy dx + 4x^2 dy = 0$

M1: Dependent upon having achieved two different terms in $\frac{dy}{dx}$, one from each of the terms y^3 and $4x^2y$.

Look for $(\dots \pm \dots) \frac{dy}{dx} = \dots \Rightarrow \frac{dy}{dx} = \dots$ which may be implied by their working.

For those candidates who had a spurious $\frac{dy}{dx} = \dots$ at the start, they may incorporate this in their

rearrangement in which case they will have 3 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

A1: $\frac{dy}{dx} = \frac{2x - 8xy}{3y^2 + 4x^2}$ or exact equivalent e.g. $\frac{dy}{dx} = \frac{8xy - 2x}{-3y^2 - 4x^2}$

(b)

M1: States or uses $\frac{dy}{dx} = -1$ which may be implied by their working e.g.

$$\frac{2x-8xy}{3y^2+4x^2} = -1, \frac{8xy-2x}{3y^2+4x^2} = 1, \frac{3y^2+4x^2}{8xy-2x} = 1 \text{ etc.}$$

Do not award for e.g. “Gradient = -1”

M1: Uses $y = x$ or $x = y$ or $y = x = “p”$ in their $\frac{dy}{dx}$ sets $= \pm 1$ and attempts to solve to obtain a value for their variable.

$$\text{e.g. } \frac{2x-8x^2}{3x^2+4x^2} = \pm 1 \Rightarrow x = \dots, \frac{2y-8y^2}{3y^2+4y^2} = \pm 1 \Rightarrow y = \dots, \frac{2p-8p^2}{3p^2+4p^2} = \pm 1 \Rightarrow p = \dots,$$

A1: For x, y or “ p ” = 2 at P

ddM1: Uses their non-zero value for x, y or “ p ” to obtain a value for k using the given equation of the curve.

Depends on both previous method marks.

A1: $k = 36$

Correct answer only with no working scores no marks.



Question Number	Scheme	Marks
6(a)	$(2, 3, -7)$	B1
		(1)
(b)	Attempts $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} = 1 \times 4 + 2 \times -1 + 2 \times 8 = (18)$	M1
	Attempts $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta: 18 = \sqrt{1^2 + 2^2 + 2^2} \times \sqrt{4^2 + (-1)^2 + 8^2} \cos \theta$	dM1
	$\cos \theta = \frac{2}{3}$	A1
		(3)
(b) Alt	e.g. $\pm \left(\begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right) = \pm \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}$	M1
	$3^2 + 3^2 + 6^2 = 1^2 + 2^2 + 2^2 + 4^2 + (-1)^2 + 8^2 - 2\sqrt{1^2 + 2^2 + 2^2} \times \sqrt{4^2 + (-1)^2 + 8^2} \cos \theta$	dM1
	$\cos \theta = \frac{2}{3}$	A1
(c)	Uses $\lambda = 6$ to find length PQ E.g. $\overline{PQ} = 6\mathbf{i} + 12\mathbf{j} + 12\mathbf{k} \Rightarrow PQ = \sqrt{6^2 + 12^2 + 12^2} = (18)$ Or $PQ = 6 \times \sqrt{1^2 + 2^2 + 2^2} = (18)$	M1
	Area QPR = $\frac{1}{2} ab \sin C = \frac{1}{2} \times 18^2 \times \sqrt{1 - \left(\frac{2}{3}\right)^2} = 54\sqrt{5}$	M1 A1
		(3)
(d)	Attempts a correct method of finding at least one value for μ e.g. $(4\mu)^2 + \mu^2 + (8\mu)^2 = 6^2 + 12^2 + 12^2$ $\Rightarrow \mu = \frac{"18"}{\sqrt{4^2 + (-1)^2 + 8^2}} = (\pm)2$	M1
	Attempts one correct position $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} \pm 2 \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$	dM1
	Possible coordinates $(10, 1, 9)$ and $(-6, 5, -23)$	A1
		(3)
		(10 marks)

M1: Differentiates one term in y to a correct form. Award for either $-3y^2 \rightarrow \pm ky \frac{dy}{dx}$ or

$2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$ Accept equivalents if they differentiate with respect to y instead.

dM1: Differentiates both y terms to a correct form

Award for **Both** $-3y^2 \rightarrow \pm ky \frac{dy}{dx}$ and $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$

A1: Fully correct differentiation. (o.e. if with respect to y). Allow y' for $\frac{dy}{dx}$

You may see $24x^2 dx - 6y dy + 2y dx + 2x dy = 0$ which is fine.

Note that $\frac{dy}{dx} = 24x^2 - 6y \frac{dy}{dx} + 2y + 2x \frac{dy}{dx} = 0$ is A0 unless the extra $\frac{dy}{dx}$ is subsequently omitted

M1: For

- either rearranging to find $\frac{dy}{dx}$ (or $\frac{dx}{dy}$) in terms of **both** x and y and then substituting **both** (2, 5)
- or substituting **both** (2, 5) then rearranging to find $\frac{dy}{dx}$ (or $\frac{dx}{dy}$)

FYI $\frac{dy}{dx} = \frac{24x^2 + 2y}{6y - 2x}$ o.e.

It is dependent upon having exactly two terms in $\frac{dy}{dx}$, one each coming from differentiating the $3y^2$ and the $2xy$.

A1: Correct value for either $\frac{dy}{dx} = \frac{53}{13}$ (accept awrt 4.08) or $\frac{dx}{dy} = \frac{13}{53}$ (awrt 0.245)

M1: Correct method for the equation of the normal at (2, 5). E.g. $y - 5 = -\frac{13}{53}(x - 2)$

It is dependent upon them having attempted differentiation (may be only one term in $\frac{dy}{dx}$) and using

$x = 2, y = 5$ to find the value of $-\frac{dx}{dy}$ which must be the negative reciprocal of their $\frac{dy}{dx}$

If the form $y = mx + c$ is used then the method must proceed as far as $c = \dots$

A1: Correct answer or any integer multiple of this. The $= 0$ must be seen

Question	Scheme	Marks
4(a)	$A = \frac{1}{2} 5^2 (\theta - \sin \theta) \Rightarrow \frac{dA}{d\theta} = \frac{25}{2} (1 \pm \cos \theta)$	M1
	$\frac{dA}{d\theta} = \frac{25}{2} (1 - \cos \theta)$	A1
		(2)
(b)	$\frac{d\theta}{dt} = 0.1$	B1
	$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt} \text{ (oe)}$	B1
	$\frac{dA}{dt} = \frac{25}{2} \left(1 - \cos \frac{\pi}{3} \right) \times 0.1$	M1
	$= \frac{5}{8} \text{ (cm}^2 \text{ per second)}$	A1
		(4)
(6 marks)		



(a)

M1: Attempts formula for area of segment with $r = 5$ and differentiates with respect to θ .Look for $\alpha 5^2 (\theta - \sin \theta) \Rightarrow \frac{dA}{d\theta} = \beta (1 \pm \cos \theta)$ Allow with either $\alpha = \frac{1}{2}$ or 1You may see $A = \alpha 5^2 \theta - \alpha 5^2 \sin \theta \Rightarrow \frac{dA}{d\theta} = \beta \pm \beta \cos \theta$ again with $\alpha = \frac{1}{2}$ or 1**A1:** Reaches correct result with no errors seen. Allow with 12.5.**Must follow M1 (the form is given). It cannot just appear without sight of some correct working**

(b)

B1: States or uses $\frac{d\theta}{dt} = 0.1$

This may be seen anywhere in the question, even in part (a) or next to the Figure

B1: States or uses a correct version of the chain rule for the given problem.

May be seen in (a) or in the question itself.

E.g. $\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$ or $\frac{d\theta}{dA} \times \frac{dA}{dt} = \frac{d\theta}{dt}$ are two correct versionsThis may be implied by an equation involving their $\frac{dA}{d\theta}$ or $\frac{dA}{d\theta} = K(1 - \cos \theta)$ and their $\frac{d\theta}{dt}$ **M1:** Attempts the chain rule with their (a), a correct $\frac{d\theta}{dt} = 0.1$ and $\theta = \frac{\pi}{3}$ to find a value for $\frac{dA}{dt}$ If they don't attempt (a), or use a made up K , condone $\frac{dA}{dt} = 0.1 \times \frac{1}{2} 'K' = \frac{1}{20} 'K'$, but $\frac{dA}{d\theta}$ must be of the form $K(1 - \cos \theta)$ via this route.**A1:** CSO. units not required. Decimal answer is 0.625.

Cannot be scored following an incorrect part (a) but allow if they have

(a) left as $\frac{dA}{d\theta} = \frac{r^2}{2} (1 - \cos \theta)$ followed by (b) $\frac{dA}{dt} = \frac{25}{2} \left(1 - \cos \frac{\pi}{3}\right) \times 0.1 = \frac{5}{8}$ which scores 00 1111.

Question	Scheme	Marks
5(a)	$y = \frac{2}{t(3-t)} = 1 \Rightarrow 2 = 3t - t^2 \Rightarrow t = \dots$	M1
	$(t^2 - 3t + 2 = (t-1)(t-2) = 0 \Rightarrow) a=1, b=2$	A1
		(2)
(b)	Area under curve $= \int_{t=1}^{t=2} y \, dx = \int_1^2 y \frac{dx}{dt} dt = \int_1^2 \frac{2}{t(3-t)} \times (2t+2) dt$	M1 A1
	$t=1 \Rightarrow x=3, \quad t=2 \Rightarrow x=8$	B1
	So area of R $= 1 \times (8-3) - \int_1^2 \frac{2}{t(3-t)} \times (2t+2) dt$	M1
	$= 5 - 4 \int_1^2 \frac{t+1}{t(3-t)} dt$	A1
		(5)
(c)(i)	$\frac{t+1}{t(3-t)} = \frac{A}{t} + \frac{B}{3-t}$	M1
	$\Rightarrow t+1 = A(3-t) + Bt$	
	E.g. $t=0 \Rightarrow 1=3A \Rightarrow A=\frac{1}{3}; t=3 \Rightarrow 4=3B \Rightarrow B=\frac{4}{3}$	M1
	$\frac{t+1}{t(3-t)} = \frac{1}{3t} + \frac{4}{3(3-t)}$	A1
	$\int \frac{t+1}{t(3-t)} dt = \int \frac{1}{3t} + \frac{4}{3(3-t)} dt = \frac{1}{3} \ln t - \frac{4}{3} \ln(3-t)$	M1
	Area $= 5 - 4 \left[\frac{1}{3} \ln t - \frac{4}{3} \ln(3-t) \right]_1^2 = 5 - 4 \left(\frac{1}{3} \ln 2 - \frac{4}{3} \ln 1 - \frac{1}{3} \ln 1 + \frac{4}{3} \ln 2 \right)$ $= 5 - 4 \left(\frac{1}{3} \ln 2 + \frac{4}{3} \ln 2 \right)$	dM1
	$= 5 - \frac{20}{3} \ln 2$	A1
(ii)		(6)
(13 marks)		

(a)

M1: Sets the y equation equal to 1 (condoning slips) and then attempts to solve the resulting quadratic equation (usual rules). May be implied by correct values if a calculator is used.

A1: Correct values. That is $a=1$ and $b=2$. Condone $(t=)$ 1 and 2 due to the demand of the question. Correct answers with no (incorrect) working scores both marks but must be seen in part (a).

If they do state $a=3$ and $b=8$ following $t=1$ and $t=2$ score A0

(b) M1: Attempts $\frac{dx}{dt}$ and multiplies by y condoning slips (e.g. missing brackets).

No requirement to see any integral.

A1: Correct unsimplified expression for the area under the curve $\int_{-1}^{+2} \frac{2}{t(3-t)} \times (2t+2) dt$

Allow with their a and b or even with a and b (as letters). May be seen within an expression.

Condone the omission of the dt for this mark. Allow missing brackets to be recovered.

B1: Sight of correct (x) values 3 and 8 at the ends of the given curve. May be awarded from work done

in (a). Award for sight of 5 as the width of the rectangle or for $5 - \int$ as their attempt at the area of R

M1: Full attempt at finding the area of region R . Be aware that the form is given.

Look for the area of the rectangle subtract the area under the curve. dt 's not required here

It is dependent upon

Use of the correct x_a and x_b or use of their t values from (a) to find x_a and x_b which may be implied by the width of the rectangle

Use of $1 \times (x_b - x_a) - \int_{-1}^{+2} y \frac{dx}{dt} dt$ to find area of R .

May be achieved by $\int_{t=-1}^{t=+2} (1-y) dx = \int_1^2 (1-y) \frac{dx}{dt} dt = \int_1^2 (2t+2) - \frac{4(t+1)}{t(3-t)} dt = \left[t^2 + 2t \right]_1^2 - \int_1^2 \frac{4(t+1)}{t(3-t)} dt$

A1: Correct answer achieved. All aspects including the dt must be present at least once before the final solution

(c)(i)

M1: Correct form for partial fractions which may be implied

M1: Correct method to find at least one of the constants.

If they use cover up rule etc it may be implied by one correct constant or one correct pf

A1: Correct partial fractions, not just correct constants. May be awarded in (c)(ii)

Allow fractions like $\frac{1/3}{t} + \frac{4/3}{(3-t)}$. Ignore any ... which may be set equal to this.

(c)(ii)

M1: Integrates to correct form $p \ln t + q \ln(3-t)$ o.e.

Be aware that $p \ln ct + q \ln k(3-t)$ where c and k are constants is also correct

dM1: Substitutes their t limits (either way around) and uses the answer from (b) to find the area.

There must be some correct $\ln$ work seen. E.g. $\ln 1 = 0$ or attempts to combine terms

To score this there must be non-zero values for M , k , a and b .

Condone with M and k leading to a correct expression in M and k . E.g. $M - K \left(\frac{1}{3} \ln 2 + \frac{4}{3} \ln 2 \right)$

It is dependent upon the previous M1 and must be of the form $M - K \left(\frac{1}{3} \ln 2 + \frac{4}{3} \ln 2 \right)$

A1: Correct answer or equivalent simplest form. E.g. $\frac{15-20 \ln 2}{3}$. ISW after a correct answer.

Condone other simple equivalents such as $5 - \frac{5}{3} \ln 16$ or $5 - \frac{4}{3} \ln 32$



Question	Scheme	Marks
6(a)	$ \overrightarrow{OA} ^2 = (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2$	M1
	$ \overrightarrow{OA} = 5\sqrt{10} \Rightarrow (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2 = 250$	M1
	$\Rightarrow 64\lambda^2 + 16\lambda + 1 + \lambda^2 - 4\lambda + 4 + 16\lambda^2 + 40\lambda + 25 = 250$ $\Rightarrow 81\lambda^2 + 52\lambda - 220 = 0^*$	A1*
		(3)
(b)	$81\lambda^2 + 52\lambda - 220 = 0 \Rightarrow (81\lambda - 110)(\lambda + 2) = 0 \Rightarrow \lambda = \dots \left(-2, \frac{110}{81}\right)$ $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1+8 \times \text{"their } \lambda \text{"} \\ 2 - \text{"their } \lambda \text{"} \\ 5+4 \times \text{"their } \lambda \text{"} \end{pmatrix}$	M1
	E.g. $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - 2 \begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}^*$; or $\overrightarrow{OA} = \begin{pmatrix} 1+8 \times \frac{110}{81} \\ 2 - \frac{110}{81} \\ 5+4 \times \frac{110}{81} \end{pmatrix} = \begin{pmatrix} \frac{961}{81} \\ \frac{52}{81} \\ \frac{845}{81} \end{pmatrix}$	A1*; A1
		(3)
(c)	$\cos \theta = \pm \frac{\begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix}}{\sqrt{15^2 + 4^2 + 3^2} \sqrt{8^2 + 1^2 + 4^2}} = \dots$	M1
	$= \pm \frac{-136}{45\sqrt{10}} (= \pm 0.9557\dots)$ or e.g. $\theta = \text{awrt } 0.299(\text{rad}) / 17.1^\circ$	A1
	$\text{Area } OAB = \frac{1}{2} 5\sqrt{10} \times 4\sqrt{10} \sin \theta = \dots$	M1
	$= \text{awrt } 29.4$	A1
		(4)
(10 marks)		

(a)

M1: Attempts to find the distance, or distance squared, from O to a general point on the line, or to A . Condone slips. It is acceptable to use a different variable rather than λ .

M1: Attempts to use $|\overrightarrow{OA}| = 5\sqrt{10}$ to set up a quadratic equation in λ , need not be expanded for this mark.

Condone slips on the 250 for OA^2 on this mark. Allow it to appear as $5 \times 10 = 50$

A1*: Expands and reaches the correct equation from correct work.

An intermediate line equivalent to $64\lambda^2 + 16\lambda + 1 + \lambda^2 - 4\lambda + 4 + 16\lambda^2 + 40\lambda + 25 = 250$ must be written out before the final given answer is seen.

(b)

M1: Correct attempt at solving the quadratic equation, (allow a calculator), and substitutes at least one of their values for λ into the equation for the line to find a possible position vector. The $(-15, 4, -3)$ is given so they cannot just write $\lambda = -2, A = (-15, 4, -3)$ without some evidence. However, sight of a correct second coordinate $\left(\frac{961}{81}, \frac{52}{81}, \frac{845}{81}\right)$ would imply M1

A1*: Correct value $\lambda = -2$ found, and substituted to reach the given position vector.

$$\text{Scored for sight of } \vec{OA} = \begin{pmatrix} 1+8 \times -2 \\ 2-(-2) \\ 5+4 \times -2 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix} \text{ o.e.}$$

Also allow the coordinate $(-15, 4, -3)$ but not an incorrect hybrid version. E.g. $(-15\mathbf{i}, 4\mathbf{j}, -3\mathbf{k})$

A1: Correct second possible position vector but condone in coordinate form. $\left(\frac{961}{81}, \frac{52}{81}, \frac{845}{81}\right)$ Must be exact. Do not allow hybrid versions but if this form has been penalised already, allow here.

(c)

M1: Attempts the scalar product of $\pm \vec{OA} = \pm(-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$ and the direction vector of the line $\pm(8\mathbf{i} - 1\mathbf{j} + 4\mathbf{k})$ to find $\cos\theta$ for the angle between $\vec{OA}$ and l_2 . Condone slips but the intention must be clear. If the method of finding modulus is not shown award if one of the two is correct. For example, $4\sqrt{10}$ is a common error/slip on the modulus of $\pm(8\mathbf{i} - 1\mathbf{j} + 4\mathbf{k})$ as this length is given in the question.

Note that it is acceptable to find the scalar product of $\pm \vec{OA}$ with $\pm(8\mu\mathbf{i} - 1\mu\mathbf{j} + 4\mu\mathbf{k})$ as long as μ is used consistently within the calculation on the numerator. Note any variable may be used here.

A1: Correct value for $\cos\theta$ (either sign acceptable). Implied by use of $\theta = \text{awrt } 0.299 \text{ rads or awrt } 17.1^\circ$

M1: Uses formula for area of triangle in a complete method to find the area required. It is dependent upon having found $\cos\theta$ via a scalar product of the two correct vectors.

A1: Correct area, awrt 29.4

Alt: There are alternative methods most of which involve finding the perpendicular distance between the two lines. An example of such a method is shown below.

Alt (c)	Midpoint of possible A's is $M = \frac{1}{81} \begin{pmatrix} -127 \\ 188 \\ 301 \end{pmatrix}$ and $OM = \frac{1}{81} \sqrt{127^2 + 188^2 + 301^2} = \dots$	M1
	$= 4.6534\dots$	A1
	$\text{Area} = \frac{1}{2} \times 4\sqrt{10} \times 4.6534\dots = 29.43\dots$	M1; A1
		(4)

M1: Finds the midpoint for the two possible positions for A and proceeds to find the distance from O to this point.

A1: Correct distance between lines.

M1: Uses formula for area of triangle in a complete method to find the area required.

A1: Correct area, awrt 29.4

There may be longer alternatives, such as finding point B first and using a different angle, but the scheme will apply in like manner.

Question Number	Scheme	Marks
8(a)	$\pm(-6\mathbf{i} + 4\mathbf{j} - \mathbf{k} - (-10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}))$	M1
	e.g. $\mathbf{r} = -10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k} \pm \lambda(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ e.g. $\mathbf{r} = -6\mathbf{i} + 4\mathbf{j} - \mathbf{k} \pm \lambda(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$	A1
		(2)
(b)	$3 + 3\mu = -6 \Rightarrow \mu = -3$	M1
	$p + 12 = 4 \Rightarrow p = \dots$ or $q - 3 = -1 \Rightarrow q = \dots$	dM1
	$p = -8, q = 2$	A1
		(3)
(c)	$(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) = 12 + 4 + 3$	M1
	$(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) = \sqrt{26}\sqrt{26} \cos \theta = 19 \Rightarrow \cos \theta = \dots$	M1
	$(\cos \theta) = \frac{19}{26}$	A1
		(3)
(d)	$AC = AB \sin \theta = \sqrt{26} \sqrt{1 - \left(\frac{19}{26}\right)^2}$	M1
	$\frac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

(a)

M1: Attempts the direction of l_1 . The expression is sufficient condoning one sign slip or may be implied by 2 correct components of $\pm(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$

A1: Any correct equation including ' $\mathbf{r} =$ '. Allow any parameter to be used including μ .

$$\text{Allow } \mathbf{r} = -10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k} \pm \lambda(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}), \quad \mathbf{r} = \begin{pmatrix} -10 \\ 5 \\ -4 \end{pmatrix} \pm \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}, \quad \mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} \pm \lambda \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix}$$

Do not accept i, j, k notation in either bracket using column vector notation

$$\text{e.g. } \mathbf{r} = \begin{pmatrix} -10 \\ 5 \\ -4 \end{pmatrix} \pm \lambda \begin{pmatrix} 4\mathbf{i} \\ -1\mathbf{j} \\ 3\mathbf{k} \end{pmatrix}$$

(b)

M1: Equates the x component of l_2 to -6 and proceeds to find a value for μ .

dM1: Uses the value of μ to find p or q . It is dependent on the previous method mark.

Alternatively,

- forms a correct equation for the x component using their l_1 and l_2
- proceeds to find λ
- uses this value of λ with the value of μ correctly to find p or q by forming an equation for the y or z component.

Do not be concerned by the mechanics of the rearrangement in solving.

A1: Correct values

Alt (b)

M1: Forms three correct equations $3 + 3\mu = -6$, $p - 4\mu = 4$, $q + \mu = -1$

dM1: Solves simultaneously to find a value for p or q . Do not be concerned by the mechanics of the rearrangement.

A1: Correct values

(c)

M1: Attempts the scalar product using their direction vectors or may find another direction vector which should be a multiple of the one for l_1 or l_2 (condoning slips e.g. if they restart using another point on l_2 to find a direction vector). Look for at least two correct products using their l_1 . May be implied by “ ± 19 ”. May find the obtuse angle which can still score this mark.

M1: Completes the scalar product method using their direction vectors to find $\cos \theta$ (acute or obtuse). Attempts the magnitude of both of their direction vectors and uses these values in the correct positions in the equation for $\cos \theta$

A1: $\frac{19}{26}$ or exact equivalent. Do not accept fractions where the numerator or denominator is not an integer (square roots must be evaluated). Isw if they proceed to find θ

Alt (c) There will be many alternatives such as the one below. Send to review if unsure.

M1: The first mark should be for finding the required components to use in the cosine rule e.g. attempts to find another point P on l_2 and attempts to find the lengths of AB , BP and PA

M1: The second mark should be using all the components required in the cosine rule. E.g. uses their lengths for AB , BP and PA in the correct positions in the cosine rule.

A1: As above in the main scheme notes.

(d)

M1: Fully correct method to find the exact value for the length of AC .
May alternatively attempt e.g.

$$\begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 13+3\mu \\ -13-4\mu \\ 6+\mu \end{pmatrix} = 0 \Rightarrow \mu = -\frac{97}{26} \Rightarrow \overline{AC} = \begin{pmatrix} \frac{47}{26} \\ \frac{25}{13} \\ \frac{59}{26} \end{pmatrix} \Rightarrow AC = \sqrt{\left(\frac{47}{26}\right)^2 + \left(\frac{25}{13}\right)^2 + \left(\frac{59}{26}\right)^2}$$

Note the coordinates for C are $\left(-\frac{213}{26}, \frac{90}{13}, -\frac{45}{26}\right)$ and they may use their coordinates for C

with point A to find the distance AC . Condone one sign slip in the distance formula. i.e. there must be sufficient evidence that they were attempting the differences between appropriate coordinates.

Look out for other fully correct methods to find the exact value for the length of AC .
e.g. attempts to find $|BC|$ and uses either trigonometry or Pythagoras' Theorem.

A1: Correct value. Allow $\sqrt{\frac{315}{26}}$ or $3\sqrt{\frac{35}{26}}$



Question	Scheme	Marks
7(a)	$V = (\pi) \int_{(0)}^{(4)} \left(\frac{1}{4}(4-x)e^x \right)^2 dx$	M1
	$\int (f(x))^2 dx = \frac{1}{16} \int (4-x)^2 e^{2x} dx = \dots$	M1
	$V = \frac{\pi}{16} \int_0^4 (x^2 - 8x + 16) e^{2x} dx$	A1
		(3)
(b)	$= \left(\frac{\pi}{16} \right) \left((x^2 - 8x + 16) \frac{e^{2x}}{2} - \int (2x - 8) \times \frac{e^{2x}}{2} dx \right)$	M1 A1
	$= \left(\frac{\pi}{16} \right) \left((x^2 - 8x + 16) \frac{e^{2x}}{2} - \left[(2x - 8) \frac{e^{2x}}{4} - \int 2 \times \frac{e^{2x}}{4} dx \right] \right)$ $= \left(\frac{\pi}{16} \right) \left((x^2 - 8x + 16) \frac{e^{2x}}{2} - (2x - 8) \frac{e^{2x}}{4} + \frac{e^{2x}}{4} \right)$ Depends on the first method mark.	dM1
	$V = \frac{\pi}{16} \left[(x^2 - 8x + 16) \frac{e^{2x}}{2} - (2x - 8) \frac{e^{2x}}{4} + \frac{e^{2x}}{4} \right]_0^4 = \frac{\pi}{16} \left(0 + 0 + \frac{e^8}{4} - \frac{16}{2} - \frac{8}{4} - \frac{1}{4} \right)$ Depends on both previous method marks.	ddM1
	$= \frac{\pi}{64} (e^8 - 41) \text{ (cm}^3\text{)}$	A1
		(5)
(b) Way 2	$= \left(\frac{\pi}{16} \right) \left((4-x)^2 \frac{e^{2x}}{2} + \int (4-x) e^{2x} dx \right)$	M1 A1
	$= \left(\frac{\pi}{16} \right) \left((4-x)^2 \frac{e^{2x}}{2} + \frac{1}{2} (4-x) e^{2x} + \int \frac{1}{2} e^{2x} dx \right)$ $= \left(\frac{\pi}{16} \right) \left((4-x)^2 \frac{e^{2x}}{2} + \frac{1}{2} (4-x) e^{2x} + \frac{1}{4} e^{2x} \right)$ Depends on the first method mark.	dM1
	$= \left(\frac{\pi}{16} \right) \left[(4-x)^2 \frac{e^{2x}}{2} + \frac{1}{2} (4-x) e^{2x} + \frac{1}{4} e^{2x} \right]_0^4 = \left(\frac{\pi}{16} \right) \left(\frac{1}{4} e^8 - 8 - 2 - \frac{1}{4} \right)$ Depends on both previous method marks.	ddM1
	$= \frac{\pi}{64} (e^8 - 41) \text{ (cm}^3\text{)}$	A1

(b) Way 3	$\int_0^4 (x^2 - 8x + 16)e^{2x} dx = \int_0^4 x^2 e^{2x} dx - \int_0^4 8xe^{2x} dx + \int_0^4 16e^{2x} dx$	
	$= \left(\frac{\pi}{16} \right) \left(x^2 \frac{e^{2x}}{2} - \int 2x \times \frac{e^{2x}}{2} dx + \dots \right)$	M1A1
	$\int_0^4 x^2 e^{2x} dx - \int_0^4 8xe^{2x} dx + \int_0^4 16e^{2x} dx$ $= \left(\frac{\pi}{16} \right) \left(x^2 \frac{e^{2x}}{2} - \left\{ x \times \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx \right\} - 8 \left\{ x \times \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx \right\} + 8e^{2x} \right)$	dM1
	Depends on the first method mark. $\left(\frac{\pi}{16} \right) \left[\frac{x^2 e^{2x}}{2} - \frac{9xe^{2x}}{2} + \frac{41e^{2x}}{4} \right]_0^4 = \frac{\pi}{16} \left(8e^8 - 18e^8 + \frac{41}{4}e^8 - \frac{41}{4} \right)$	ddM1
	Depends on both previous method marks. $= \frac{\pi}{64} (e^8 - 41) \text{ (cm}^3\text{)}$	A1

(8 marks)**Notes:****(a)**

M1: Sets up the volume using the volume formula, but allow limits and π and the “dx” to be missing. No need to square $f(x)$ for this mark as long as the intent to do so is clear.

There should be an integral

M1: Attempts to square $f(x)$ (need not be part of an integral). The power of e should be correct and the bracket expanded, but allow if the $\frac{1}{4}$ is not squared.

A1: Correct expression for the integral, π and limits must be included, and no errors seen in working so the “dx” must be present throughout.

(b)

M1: Attempts integration by parts the correct way at least once on the given integral to obtain

$$\alpha(x^2 - 8x + 16)e^{2x} \pm \beta \int (ax - b)e^{2x} dx$$

A1: Correct first application of parts: $(x^2 - 8x + 16) \frac{e^{2x}}{2} - \int (2x - 8) \times \frac{e^{2x}}{2} dx$ or equivalent.

dM1: Attempts integration by parts again in the same direction on $\int (2x - 8) \times \frac{e^{2x}}{2} dx$ and proceeds

to complete the integration. Look for $K \left(\dots + \alpha(ax - b)e^{2x} \pm \beta \int e^{2x} dx \right)$.

ddM1: Applies the limits to their integral and subtracts, (but this need not have the K or π for this mark.) May be done in stages, so look for reaching a value using correct limits on each part of their integral.

A1: Correct answer achieved from correct work and the π must have been introduced before the final step, it should have been part of their answer to (a). Condone lack of units.

Way 2

Question Number	Scheme	Marks
8(a)(i) (ii)	$\pm((3\mathbf{i}+4\mathbf{j}-6\mathbf{k})-(2\mathbf{i}-\mathbf{j}+5\mathbf{k}))$	M1
	$= \mathbf{i}+5\mathbf{j}-11\mathbf{k}$	A1
	e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i}+5\mathbf{j}-11\mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k} + \lambda(\mathbf{i}+5\mathbf{j}-11\mathbf{k})$ o.e.	A1ft
		(3)
(a) (i) M1: Attempts to find the difference between the two vectors either way round. The expression e.g. $((3\mathbf{i}+4\mathbf{j}-6\mathbf{k})-(2\mathbf{i}-\mathbf{j}+5\mathbf{k}))$ is sufficient. If no working is shown, it may be implied by 2 correct components of $\pm(\mathbf{i}+5\mathbf{j}-11\mathbf{k})$ or e.g. $\pm(1, 5, -11)$ A1: $\mathbf{i}+5\mathbf{j}-11\mathbf{k}$ o.e. e.g. $\begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$ and condone $\begin{matrix} 1 \\ 5 \\ -11 \end{matrix}$ but not $\begin{pmatrix} \mathbf{i} \\ 5\mathbf{j} \\ -11\mathbf{k} \end{pmatrix}$ and not $(1, 5, -11)$ But allow this mark once a correct form is seen. (ii) A1ft: Any correct vector equation for the line following through their $\overline{AB}$ but must follow M1. Must include “$\mathbf{r} =$” (not e.g. “$l =$”) and allow any parameter. e.g. $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$ or e.g. $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -5 \\ 11 \end{pmatrix}$ Do not allow e.g. $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} \mathbf{i} \\ 5\mathbf{j} \\ -11\mathbf{k} \end{pmatrix}$ unless the notation $\begin{pmatrix} \mathbf{i} \\ 5\mathbf{j} \\ -11\mathbf{k} \end{pmatrix}$ has already been penalised on the previous A mark and this is the only error in their notation. Do not allow e.g. $l = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$ or e.g. $l : \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$ But condone $l : \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$		

(b)	$\overrightarrow{CA} = (2-p)\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$ $\overrightarrow{CB} = (3-p)\mathbf{i} - 5\mathbf{k}$	M1
	$(2-p)(3-p) - 30 = 0$	dM1
	$\Rightarrow 6 - 5p + p^2 - 30 = 0 \Rightarrow p^2 - 5p - 24 = 0 \Rightarrow p = \dots$	ddM1
	$p = -3, \quad p = 8$	A1
		(4)
(c)	$ \overrightarrow{CA} = \sqrt{(2-8)^2 + (-5)^2 + 6^2}$ or $ \overrightarrow{CB} = \sqrt{(3-8)^2 + (-5)^2}$	M1
	$ \overrightarrow{CA} = \sqrt{97}$ or $ \overrightarrow{CB} = \sqrt{50}$	A1
	Area of triangle $= \frac{1}{2} \times \overrightarrow{CA} \times \overrightarrow{CB} = \frac{1}{2} \times \sqrt{50} \times \sqrt{97} = \text{awrt } 34.8 \text{ units}^2$ or e.g. $\frac{1}{2} \times \overrightarrow{CA} \times \overrightarrow{AB} \sin BAC = \frac{1}{2} \times \sqrt{50} \times \sqrt{147} \sin \left(\cos^{-1} \frac{\sqrt{50}}{\sqrt{147}} \right) = \text{awrt } 34.8 \text{ units}^2$	dM1A1
		(4)
		(11 marks)

(b) Allow all marks in (b) for correct work using $\pm \overrightarrow{CA}$ and/or $\pm \overrightarrow{CB}$

M1: Attempts to find either $\pm \overrightarrow{CA}$ or $\pm \overrightarrow{CB}$ using subtraction.

If no working is shown, it may be implied by 2 correct components.

dM1: Depends on the previous method mark.

Attempts both $\pm \overrightarrow{CA}$ and $\pm \overrightarrow{CB}$ using subtraction and attempts the scalar product between their $\pm \overrightarrow{CA}$ and their $\pm \overrightarrow{CB}$ and sets $= 0$ to obtain $\pm(2-p) \times \pm(3-p) \pm k = 0$ where $k \neq 0$

ddM1: Depends on both previous method marks.

Multiplies out, collects terms and attempts to solve their three term quadratic (usual rules apply – see general guidance). May be implied by correct roots for their 3TQ.

Do not condone work where they lose or ignore their k .

A1: Correct values $p = -3, \quad p = 8$

(c)

M1: Uses at least one of their values of p from part (b) (which may be negative) and attempts $|\overrightarrow{CA}|$ or $|\overrightarrow{CB}|$ using Pythagoras. May be implied.

A1: For $|\overrightarrow{CA}| = \sqrt{97}$ or $|\overrightarrow{CB}| = \sqrt{50}$ seen or implied.

dM1: Depends on the previous method mark.

Attempts to find the area of the triangle using a correct method e.g. $\frac{1}{2} \times \text{their } |\overrightarrow{CA}| \times \text{their } |\overrightarrow{CB}|$

It is dependent on the previous method mark and it is dependent on having used a **positive** value for p .

If they use a less direct method e.g. $\frac{1}{2} \times |\overrightarrow{CA}| \times |\overrightarrow{AB}| \sin BAC$ then it must be a correct method,

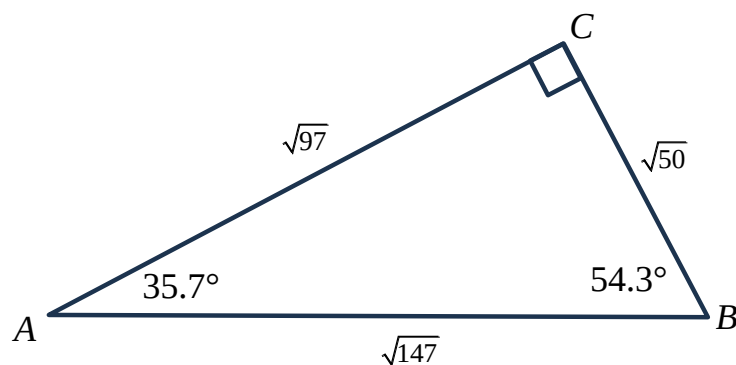
including an attempt to find $|\overrightarrow{AB}|$ using Pythagoras.

A1: awrt 34.8 This answer and no others. Do not allow for exact answers.

If they use the other value of p and give another area which is not rejected, score A0



For reference:



Note that other methods may be possible in (c) for the area e.g. using a vector product:

$$p = 8 \Rightarrow \overrightarrow{CA} = -6\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}, \quad \overrightarrow{CB} = -5\mathbf{i} - 5\mathbf{k}$$

$$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -5 & 6 \\ -5 & 0 & -5 \end{vmatrix} = \begin{pmatrix} 25 \\ -60 \\ -25 \end{pmatrix}$$

$$\text{Area of triangle} = \frac{1}{2} \sqrt{25^2 + 60^2 + 25^2} = 34.8$$

Score as:

M1: Attempts the vector product between 2 appropriate vectors e.g. $\overrightarrow{CA}$ and $\overrightarrow{CB}$
May be implied by 2 correct components.

A1: Correct vector

dM1: Attempts $\frac{1}{2} \times$ the magnitude of their vector

A1: For awrt 34.8



Question Number	Scheme	Marks
7	$x = 4 \sin \theta \Rightarrow \frac{dx}{d\theta} = 4 \cos \theta$	B1
	$\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx = \int \frac{4 \cos \theta}{(16-16 \sin^2 \theta)^{\frac{3}{2}}} d\theta = \int \frac{4 \cos \theta}{64 \cos^3 \theta} d\theta$	M1 A1
	$= \frac{1}{16} \tan \theta$	dM1
	Uses limits of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or 30° and 60° within their attempted integration and subtracts the right way round e.g. $f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)$ or $f(60^\circ) - f(30^\circ)$	M1
	$= \frac{1}{16} \tan \frac{\pi}{3} - \frac{1}{16} \tan \frac{\pi}{6} = \frac{\sqrt{3}}{24}$	A1
		(6 marks)

Notes:

B1: States or uses $\frac{dx}{d\theta} = 4 \cos \theta$ o.e. e.g. $dx = 4 \cos \theta d\theta$. This may be seen within the integrand.

M1: Simplifies $(16-x^2)^{\frac{3}{2}}$ using the given substitution to $k \cos^3 \theta$ using the Pythagorean identity

This may be implied if the work is correct and they proceed correctly to e.g. $\frac{1}{16} \int \cos^{-2} \theta d\theta$

A1: $\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx = \int \frac{4 \cos \theta}{64 \cos^3 \theta} d\theta$ or exact equivalent e.g. $\frac{1}{16} \int \cos^{-2} \theta d\theta$ or $\int \frac{1}{(4 \cos \theta)^2} d\theta$

or e.g. $\int \frac{4 \cos \theta}{(4 \cos \theta)(4 \cos \theta)(4 \cos \theta)} d\theta$

Condone if the "dθ" is missing but is otherwise correct.

dM1: For $\int k \frac{\cos \theta}{\cos^3 \theta} d\theta = \dots \tan \theta$

M1: Uses limits of $\frac{\pi}{6}(30^\circ)$ and $\frac{\pi}{3}(60^\circ)$ correctly with their attempted integration.

Condone poor integration here.

A1: $\frac{\sqrt{3}}{24}$ or exact equivalent e.g. $\frac{1}{8\sqrt{3}}$



Question Number	Scheme	Marks
8(a)	States or implies that $a = -3$ or $b = 10$	B1
	States or implies that $a = -3$ and $b = 10$	B1
		(2)
(b)	Attempts $= \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \dots$ either way around	M1
	$\overrightarrow{AB} = \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix}$ or $12\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
		(2)
(c)	Attempts $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	Attempts $\overrightarrow{AC} \cdot \overrightarrow{AB} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix} = 72 + 60 + 18$	dM1
	Attempts $\overrightarrow{AC} \cdot \overrightarrow{AB} = \overrightarrow{AC} \overrightarrow{AB} \cos \theta \Rightarrow 150 = \sqrt{172} \times \sqrt{189} \cos \theta \Rightarrow \theta = \dots$ (NB $\sqrt{172} = 2\sqrt{43}$, $\sqrt{189} = 3\sqrt{21}$)	ddM1
	$\theta = \text{awrt } 33.7^\circ$	A1
	(c) Alternative using the cosine rule:	
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$	dM1
	$\Rightarrow AB^2 = 12^2 + 6^2 + 3^2$, $BC^2 = 6^2 + 4^2 + 3^2$, $AC^2 = 6^2 + 10^2 + 6^2$	
	$BC^2 = AB^2 + AC^2 - 2AB \times AC \cos \theta$ $61 = 189 + 172 - 2\sqrt{189}\sqrt{172} \cos \theta \Rightarrow \cos \theta = \dots$	ddM1
	$\theta = \text{awrt } 33.7^\circ$	A1
		(4)
(a)		
B1: States or implies that $a = -3$ or $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)		
B1: States or implies that $a = -3$ and $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)		
In the rest of the question, you can condone slips in writing down vectors as long as the intention is clear.		
(b)		
M1: Attempts to subtract their vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ either way round. If no method is shown, it can be implied by at least 2 correct components for their vectors.		
A1: Correct vector using correct notation.		



Do **not** allow as coordinates and do **not** allow $\begin{pmatrix} 12i \\ 6j \\ -3k \end{pmatrix}$ but apply isw once a correct vector is seen.

(c)

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to find the scalar product of $\pm \overrightarrow{AC}$ and their vector $\pm \overrightarrow{AB}$

This may be implied by their value so you may need to check.

If the value is incorrect and no method is shown score M0

ddM1: Full attempt to find angle CAB using the scalar product of $\pm \overrightarrow{AC}$ and their vector $\pm \overrightarrow{AB}$

A1: $\theta = \text{awrt } 33.7^\circ$ and no other angles (Degrees symbol **not** required)

Note that in (a) they can use any multiples of $\pm \overrightarrow{AC}$ and $\pm \overrightarrow{AB}$ to find the required angle.

Alternative:

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around **and** attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OB}$ either way around and attempts the lengths or lengths² of AB , BC and AC

ddM1: Full attempt to find angle CAB using the cosine rule

A1: $\theta = \text{awrt } 33.7^\circ$ and no other angles (Degrees symbol **not** required)

(d)	Attempts a correct method of finding one position for D . See notes for possible approaches.	M1
	$(22, 9, -2)$ or $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for D . See notes for possible approaches.	dM1
	$(22, 9, -2)$ and $(-26, -15, 10)$	A1
		(4)
		(12 marks)

(d)

M1: Attempts a complete and correct method for finding one position for D .

(May be implied by at least 2 correct or correct ft components)

Examples:

$$\text{Starting from } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} : \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \text{ or } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\text{Starting from } A \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} : \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} + 6 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \text{ or } \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} - 6 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\text{Starting from } B \begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix} : \begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \text{ or } \begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix} - 9 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.



$$\text{Starting from } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} : \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \frac{5}{3}\overline{AB} \quad \text{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \frac{7}{3}\overline{AB}$$

$$\text{Starting from } A \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} : \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} + 2\overline{AB} \quad \text{or} \quad \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} - 2\overline{AB}$$

$$\text{Starting from } B \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} : \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} + \overline{AB} \quad \text{or} \quad \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - 3\overline{AB}$$

A1: One correct point $(22, 9, -2)$ or $(-26, -15, 10)$

Condone if given as a vector e.g. $22\mathbf{i} + 9\mathbf{j} - 2\mathbf{k}$ or $-26\mathbf{i} - 15\mathbf{j} + 10\mathbf{k}$

dM1: Attempts a complete and correct method for finding both possible positions for D .

(May be implied by at least 2 correct or correct ft components)

Examples:

$$\text{Starting from } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} : \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\text{Starting from } A \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} : \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} + 6 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} - 6 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\text{Starting from } B \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} : \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - 9 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

Note that some candidates may use their $\overline{AB}$ rather than the direction vector e.g.

$$\text{Starting from } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} : \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \frac{5}{3}\overline{AB} \quad \text{and} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \frac{7}{3}\overline{AB}$$

$$\text{Starting from } A \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} : \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} + 2\overline{AB} \quad \text{and} \quad \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} - 2\overline{AB}$$

$$\text{Starting from } B \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} : \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} + \overline{AB} \quad \text{and} \quad \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - 3\overline{AB}$$

A1: Gives both possible coordinates $(-26, -15, 10)$ and $(22, 9, -2)$

Condone if given as vectors e.g. $22\mathbf{i} + 9\mathbf{j} - 2\mathbf{k}$ and $-26\mathbf{i} - 15\mathbf{j} + 10\mathbf{k}$



Question Number	Scheme	Notes	Marks
5(a)	$4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $-5 + 6\lambda = 4 - 3\mu$	For writing down any 2 of these equations.	M1
	E.g. $4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $\Rightarrow \lambda = \dots$ or $\mu = \dots$	Full method for finding λ or μ	M1
	$\lambda = 2, \mu = -1$	Both correct values	A1
	$-5 + 6\lambda = -5 + 12 = 7$ $4 - 3\mu = 4 + 3 = 7$ So lines intersect	Shows that the parameters satisfy the third equation and makes a conclusion.	B1
	$\lambda = 2 \rightarrow (4 + 4)\mathbf{i} + (4 - 6)\mathbf{j} + (-5 + 12)\mathbf{k}$ or $\mu = -1 \rightarrow (13 - 5)\mathbf{i} + (-1 - 1)\mathbf{j} + (4 + 3)\mathbf{k}$	Uses their λ or μ to find A .	M1
	$8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$	Correct vector or coordinates	A1
			(6)
(b)	$\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} = 10 - 3 - 18 = \sqrt{2^2 + 3^2 + 6^2} \sqrt{5^2 + 1^2 + 3^2} \cos \theta$ Full attempt at the scalar product between the direction vectors		M1
	$\cos \theta = \pm \frac{11}{7\sqrt{35}}$	Correct magnitude for $\cos \theta$ (may be implied by e.g. $\theta = 105.4\dots$ or $74.6\dots$)	A1
	$\theta = 74.6^\circ$	Awrt 74.6	A1
			(3)
(c)	$ 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k} = \sqrt{2^2 + 3^2 + 6^2} = 7$	Finds the magnitude of the direction of l_1	M1
	$35 \div 7 = 5 \Rightarrow \lambda = 5$ $8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k} \pm 5(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	Correct strategy for one of the points	M1
	$(18, -17, 37)$ or $(-2, 13, -23)$	One correct point (ignore labels)	A1
	$P(18, -17, 37)$ and $Q(-2, 13, -23)$	Correct points with correct labels	A1
			(4)
			Total 13

Question	Scheme	Marks
2(a)	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$	
	$3x+4 = A(2x+1)^2 + B(x-2)(2x+1) + C(x-2)$ E.g. $x=2 \rightarrow 10 = A \times 25 \Rightarrow A = \dots$ or $x = -\frac{1}{2} \rightarrow \frac{5}{2} = C \times -\frac{5}{2} \Rightarrow C = \dots$ or $x^2: 0 = 4A + 2B, x: 3 = 4A - 3B + C, x^0: 4 = A - 2B - 2C \rightarrow A = \dots$	M1
	One of $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
	$\Rightarrow A = \dots, B = \dots$ and $C = \dots$	M1
	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{2}{5(x-2)} - \frac{4}{5(2x+1)} - \frac{1}{(2x+1)^2}$ or $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
		(4)
(b)	$\int \frac{p}{x-2} dx = \alpha \ln(x-2)$ or $\int \frac{q}{2x+1} dx = \beta \ln(2x+1)$	M1
	Alt: $\int \frac{8x+9}{(2x+1)^2} dx = \int \frac{8x+4}{(2x+1)^2} + \frac{5}{(2x+1)^2} dx = \gamma \ln(2x+1)^2 + \dots$	
	$\int \frac{k}{(2x+1)^2} dx = \frac{K}{2x+1}$	M1
	$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2} dx = \left[\frac{2}{5} \ln(x-2) - \frac{4}{5} \times \frac{1}{2} \ln(2x+1) + \frac{1}{4x+2} \right]_7^{12}$	A1ft
	$= \left(\frac{2}{5} \ln(10) - \frac{2}{5} \ln(25) + \frac{1}{50} \right) - \left(\frac{2}{5} \ln(5) - \frac{2}{5} \ln(15) + \frac{1}{30} \right) = \dots$	DM1
	$= \frac{2}{5} \ln \frac{10}{25} + \frac{1}{50} - \frac{2}{5} \ln \frac{5}{15} - \frac{1}{30} = \frac{2}{5} \ln \frac{10 \times 15}{25 \times 5} + \dots$	M1
	$\frac{2}{5} \ln \frac{6}{5} - \frac{1}{75}$ (oe)	A1
		(6)
(10 marks)		

Notes:**(a)****M1:** Attempts a correct method leading to one of the constants. E.g. Multiplies through to $3x+4 = A(2x+1)^2 + B(x-2)(2x+1) + C(x-2)$ (allowing for minor slips) and substitutes $x = -\frac{1}{2}$ or $x = 2$ or other method such as comparing coefficients. Implied by a correct value for A by cover-up rule if no incorrect working is seen. If e.g. multiplies through by a wrong term such as $(2x+1)^3$ then allow M1 here for the attempt.**A1:** Any one of the three constants correct (probably A or C).**M1:** For a full and correct method to obtain all three of the constants. If e.g. multiplies through by a wrong term such as $(2x+1)^3$ then M0 here. Implied by correct values if no method shown following a correct identity (otherwise M0 for just incorrect values stated).**A1:** For a correct full partial expression form or correct values stated.

Note: if by comparing coefficients allow both M's for achieving any 3 values following three equations from the attempt at comparing with at least one correct.

(b)**M1:** For integrating to obtain at least one $\ln$ term. Note $\alpha \ln(5x-10)$ and $\beta \ln(10x+5)$ are equally fine, as are other constant multiples of the inner brackets.**M1:** For $\frac{k}{(2x+1)^2} \rightarrow \frac{K}{(2x+1)}$ (any K) oe.**A1ft:** Fully correct integration following through on their non-zero constants.**DM1:** Depends on at least one the first two M's being scored. Substitutes the limits and subtracts either way round.**M1:** Achieves a single log term with evidence of a correct use of at least one log law (e.g. power or addition law). (Note log terms may initially have been combined earlier correctly, in which case look for reaching a single log term.)**A1:** Correct answer accept equivalents in the correct form (including if the modulus signs are left in).Use of constants (if no values for A , B and C are found) can score the M's but not the A's.

Question	Scheme	Marks
3(a)	$y^2x + 3y = 4x^2 + k$	
	$y^2x \rightarrow \dots \times y \frac{dy}{dx} (+\dots)$ or $3y \rightarrow 3 \frac{dy}{dx}$	M1
	$y^2x \rightarrow 2xy \frac{dy}{dx} + y^2$	A1
	$\dots + 3y = 4x^2 + k \rightarrow \dots + 3 \frac{dy}{dx} = 8x$	A1
	$2xy \frac{dy}{dx} + y^2 + 3 \frac{dy}{dx} = 8x \Rightarrow \frac{dy}{dx} = \dots$	dM1
	$\frac{dy}{dx} = \frac{8x - y^2}{2xy + 3}$ oe	A1
		(5)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 8x - 4 = 0 \Rightarrow x = \dots$	M1
	$p = \frac{1}{2}$	A1
	$k = 2^2 \times \frac{1}{2} + 3 \times 2 - 4 \times \frac{1}{4} = \dots$	M1
	$k = 7$	A1
		(4)
Alt (b)	$(p, 2) \text{ on } C \Rightarrow 4p + 6 = 4p^2 + k \Rightarrow 4p^2 - 4p + k - 6 = 0$ $\Rightarrow 4\left(p - \frac{1}{2}\right)^2 - 1 + k - 6 = 0$ or $b^2 - 4ac = 16 - 16(k - 6)$	M1
	(Min, single solution) $\Rightarrow p = \frac{1}{2}$ or $16 - 16(k - 6) = 0 \Rightarrow k = 7$	A1
	$\Rightarrow k = 7 - 4\left(\frac{1}{2} - \frac{1}{2}\right)^2$ or $4p^2 - 4p + 7 - 6 = 0 \Rightarrow p = \dots$	M1
	The other of $p = \frac{1}{2}$ and $k = 7$	A1
		(4)
(9 marks)		
Notes:		
(a) M1: Some evidence of an attempt at implicit differentiation. Look for one of the terms being differentiated with either the $3 \frac{dy}{dx}$ or a term with $kxy \frac{dy}{dx}$ or $y^2 \rightarrow ky \frac{dy}{dx}$ as part of a product rule seen appropriately. A1: y^2x differentiated correctly		