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EDEXCEL INTERNATIONAL A LEVEL MATHS: QUESTIONS BY TOPIC

Discrete Random Variables (S1.4)

International A Level Mathematics · S1: Statistics 1 (WST01)

18 questions · 217 marks · Mark schemes included

Past papers October 2020 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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QUESTIONS

18 questions · 217 marks · 1 sub-topic

SUB-TOPIC S1.4a

PROBABILITY DISTRIBUTIONS, EXPECTATION & VARIANCE

Sub-topic S1.4a · 18 questions · 217 marks · lowest tariff first



4. The cumulative distribution function of the discrete random variable W , which takes only the values 6, 7 and 8, is given by

$$F(W) = \frac{(w+3)(w-1)}{77} \quad \text{for } w = 6, 7, 8$$

Find $E(W)$

(4)

(Total for Question 4 is 4 marks)



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1. The discrete random variable X takes the values $-1, 2, 3, 4$ and 7 only.

Given that

$$P(X = x) = \frac{8 - x}{k} \text{ for } x = -1, 2, 3, 4 \text{ and } 7$$

find the value of $E(X)$

(5)

Question 1 continued

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Lined area for writing the answer to Question 1.

(Total 5 marks)

Q1

Question 1 continued

(Total for Question 1 is 9 marks)



Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)



Question 7 continued

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Question 7 continued

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Q7

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END



Question 3 continued

(Total for Question 3 is 11 marks)





Question 4 continued

Question 4 continued

(Total for Question 4 is 12 marks)







(Total for Question 2 is 12 marks)



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4. The random variable W has a discrete uniform distribution where

$$P(W = w) = \frac{1}{5} \quad \text{for } w = 1, 2, 3, 4, 5$$

- (a) Find $P(2 \leq W < 3.5)$

(1)

The discrete random variable $X = 5 - 2W$

- (b) Find $E(X)$

(3)

- (c) Find $P(X < W)$

(2)

The discrete random variable $Y = \frac{1}{W}$

- (d) Find

- (i) the probability distribution of Y
- (ii) $\text{Var}(Y)$, showing your working.

(5)

- (e) Find $\text{Var}(2 - 3Y)$

(2)

Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total 13 marks)

Q4



Question 5 continued

Lined area for writing the answer to Question 5.



Question 5 continued

Lined area for writing the answer to Question 5.



(Total for Question 3 is 15 marks)



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5. The discrete random variable Y has the following probability distribution

y	-9	-5	0	5	9
$P(Y=y)$	q	r	u	r	q

where q , r and u are probabilities.

- (a) Write down the value of $E(Y)$ (1)

The cumulative distribution function of Y is $F(y)$

Given that $F(0) = \frac{19}{30}$

- (b) show that the value of u is $\frac{4}{15}$ (3)

Given also that $\text{Var}(Y) = 37$

- (c) find the value of q and the value of r (4)

The coordinates of a point P are $(12, Y)$

The random variable D represents the length of OP

- (d) Find the probability distribution of D (6)



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Question 5 continued

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(Total 14 marks)

Q5



5. A red spinner is designed so that the score R is given by the following probability distribution.

r	2	3	4	5	6
$P(R = r)$	0.25	0.3	0.15	0.1	0.2

- (a) Show that $E(R^2) = 15.8$

(1)

Given also that $E(R) = 3.7$

- (b) find the standard deviation of R , giving your answer to 2 decimal places.

(2)

A yellow spinner is designed so that the score Y is given by the probability distribution in the table below. The cumulative distribution function $F(y)$ is also given.

y	2	3	4	5	6
$P(Y = y)$	0.1	0.2	0.1	a	b
$F(y)$	0.1	0.3	0.4	c	d

- (c) Write down the value of d

(1)

Given that $E(Y) = 4.55$

- (d) find the value of c

(5)

Pabel and Jessie play a game with these two spinners.

Pabel uses the red spinner.

Jessie uses the yellow spinner.

They take turns to spin their spinner.

The winner is the first person whose spinner lands on the number 2 and the game ends.

Jessie spins her spinner first.

- (e) Find the probability that Jessie wins on her second spin.

(2)

- (f) Calculate the probability that, in a game, the score on Pabel's first spin is the same as the score on Jessie's first spin.

(3)





Question 5 continued

Lined area for writing the answer to Question 5.



Question 7 continued



Question 7 continued



Question 7 continued

TOTAL FOR PAPER: 75 MARKS

6. A biased die with six faces is rolled. The discrete random variable X represents the score which is uppermost. The **cumulative** distribution function of X is shown in the table below.

x	1	2	3	4	5	6
$F(x)$	0.1	0.2	$3k$	$5k$	$7k$	$10k$

- (a) Find the value of the constant k (1)
- (b) Find the probability distribution of X (3)

A biased die with eight faces is rolled. The discrete random variable Y represents the score which is uppermost. The probability distribution of Y is shown in the table below, where a and b are constants.

y	1	2	3	4	5	6	7	8
$P(Y=y)$	a	a	a	b	b	b	0.11	0.05

Given that $E(Y) = 4.02$

- (c) form and solve two equations in a and b to show that $a = 0.15$
You must show your working.

(Solutions relying on calculator technology are not acceptable.)

- (d) Show that $E(Y^2) = 20.7$ (2)
- (e) Find $\text{Var}(5 - 2Y)$ (3)

These dice are each rolled once. The scores on the two dice are independent.

- (f) Find the probability that the sum of these two scores is 3 (2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 14 marks)



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6. The random variable A represents the score when a spinner is spun. The probability distribution for A is given in the following table.

a	1	4	5	7
$P(A = a)$	0.40	0.20	0.25	0.15

- (a) Show that $E(A) = 3.5$ (2)

- (b) Find $\text{Var}(A)$ (3)

The random variable B represents the score on a 4-sided die. The probability distribution for B is given in the following table where k is a positive integer.

b	1	3	4	k
$P(B = b)$	0.25	0.25	0.25	0.25

- (c) Write down the name of the probability distribution of B . (1)

- (d) Given that $E(B) = E(A)$ state, giving a reason, the value of k . (1)

The random variable $X \sim N(\mu, \sigma^2)$

Sam and Tim are playing a game with the spinner and the die.

They each spin the spinner once to obtain their value of A and each roll the die once to obtain their value of B .

Their value of A is taken as their value of μ and their value of B is taken as their value of σ . The person with the larger value of $P(X > 3.5)$ is the winner.

- (e) Given that Sam obtained values of $a = 4$ and $b = 3$ and Tim obtained $b = 4$ find, giving a reason, the probability that Tim wins. (2)

- (f) Find the largest value of $P(X > 3.5)$ achievable in this game. (4)

- (g) Find the probability of achieving this value. (2)

Question 6 continued

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Question 6 continued

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Question 6 continued

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Question 6 continued

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Q6

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END



Question 5 continued

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Question 5 continued

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Question 5 continued

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(Total 15 marks)

Q5



1. Jen has one fair 4-sided red die and one fair 4-sided blue die.

- The red die has sides numbered 1, 2, 3 and 4
- The blue die has sides numbered 1, 3, 5 and 7

The discrete random variable R represents the score from one roll of the red die.

The discrete random variable B represents the score from one roll of the blue die.

(a) Write down the name of the distribution of R (1)

(b) Find $P(R < 3)$ (1)

(c) Write down the value of (2)

(i) $E(R)$

(ii) $E(B)$

(d) Showing your working, find $\text{Var}(B)$ (3)

Jen rolls each die once.

(e) Find $P(R + B \leq 5)$ (2)

(f) Find $P(R^2 < B)$ (3)

The random variable D is defined as the magnitude of the difference between the score on the red die and the score on the blue die.

The table below shows the cumulative distribution function of D

d	0	1	2	3	4	5	6
$F(d)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{16}$	$\frac{3}{4}$	p	$\frac{15}{16}$	1

(g) Showing your working, find the value of p (3)

Question 1 continued



Question 1 continued

(Total for Question 1 is 15 marks)



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4. A spinner can land on the numbers 10, 12, 14 and 16 only and the probability of the spinner landing on each number is the same.
The random variable X represents the number that the spinner lands on when it is spun once.

(a) State the name of the probability distribution of X .

(1)

(b) (i) Write down the value of $E(X)$

(1)

(ii) Find $\text{Var}(X)$

(2)

A second spinner can land on the numbers 1, 2, 3, 4 and 5 only.
The random variable Y represents the number that this spinner lands on when it is spun once. The probability distribution of Y is given in the table below

y	1	2	3	4	5
$P(Y = y)$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{6}{30}$	$\frac{5}{30}$	$\frac{6}{30}$

(c) Find (i) $E(Y)$

(2)

(ii) $\text{Var}(Y)$

(3)

The random variable $W = aX + b$, where a and b are constants and $a > 0$
Given that $E(W) = E(Y)$ and $\text{Var}(W) = \text{Var}(Y)$

(d) find the value of a and the value of b .

(5)

Each of the two spinners is spun once.

(e) Find $P(W = Y)$

(2)

Question 4 continued

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Question 4 continued

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(Total 16 marks)

Q4



MARK SCHEMES

official mark schemes, in the same order as the questions



Question Number	Scheme						Marks												
1.	<table><tr><td>x</td><td>-1</td><td>2</td><td>3</td><td>4</td><td>7</td></tr><tr><td>$P(X=x)$</td><td>$\frac{9}{k}$</td><td>$\frac{6}{k}$</td><td>$\frac{5}{k}$</td><td>$\frac{4}{k}$</td><td>$\frac{1}{k}$</td></tr></table>	x	-1	2	3	4	7	$P(X=x)$	$\frac{9}{k}$	$\frac{6}{k}$	$\frac{5}{k}$	$\frac{4}{k}$	$\frac{1}{k}$						M1
	x	-1	2	3	4	7													
	$P(X=x)$	$\frac{9}{k}$	$\frac{6}{k}$	$\frac{5}{k}$	$\frac{4}{k}$	$\frac{1}{k}$													
	$\sum P(X=x)=1 \Rightarrow \frac{25}{k}=1$						M1												
	$k=25$						A1												
$E(X)=\frac{1}{25}[-1 \times 9+2 \times 6+3 \times 5+4 \times 4+7 \times 1]$						M1													
$=\frac{41}{25}$						A1													
							[5]												
	Notes																		
	1 st M1 for at least 3 correct probabilities in terms of k (may be seen used in expression for $E(X)$) 2 nd M1 for attempting to use sum of 5 probs = 1 (ft their probabilities) 1 st A1 for $k=25$ (stated or used correctly) 3 rd M1 for attempt at a correct expression at least 3 products (ft their k – value or letter) 2 nd A1 for $\frac{41}{25}$ or exact equivalent e.g. 1.64 Correct answer with no incorrect method marks scores 5/5																		

Question	Scheme	
1 (a)	0.85	B1
		(1)
(b)	$3X - 3 < X + 2 \rightarrow X < 2.5$	M1
	$[P(X < 2.5) =] 0.5$	A1
		(2)
(c)	$[E(X) =] 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.35 + 4 \times 0.15 =$	M1
	2.35	A1
		(2)
(d)	$[E(X^2) =] 1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.35 + 4^2 \times 0.15 [= 6.65]$	M1
	$[Var(X) =] "6.65" - "2.35"^2$	M1
	$= 1.1275^*$	A1*
		(3)
(e)	$[Var(5 - 2X) =] 4.51$	B1
		(1)
		Total 9
	Notes	
	Look out for answers given next to the questions	
(a)	B1 oe Must be identified as answer to part (a) (not just seen anywhere in the script). Ignore their label.	
(b)	M1 for $X < 2.5$ (condone $X > 2.5$, $X = 2.5$) or $X \leq 2$ or for identifying $X = 1$ and $X = 2$ (may be circled, ticked, or otherwise clearly identified as the only 2 x-values that satisfy the inequality) or $0.3 + 0.2$	
	A1 oe Correct answer scores 2 out of 2.	
(c)	M1 for an attempt to find $E(X)$ with at least 3 correct products or terms with intention to add $0.3 + 0.4 + 1.05 + 0.6$ (may be seen in table at the start of the question) Division by 4 or any k ($k \neq 1$) is M0	
	A1 oe Correct answer scores 2 out of 2	
(d)	M1 for an attempt to find $E(X^2)$ with at least 3 correct products or terms with intention to add $0.3 + 0.8 + 3.15 + 2.4$ (may be seen in table at the start of the question or elsewhere) Division by 4 or any k ($k \neq 1$) is M0 6.65 on its own does not imply this mark	
	M1 for $6.65 - 2.35^2$ ft their $E(X)$ and $E(X^2)$. A full calculation scores M1M1 e.g. $1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.35 + 4^2 \times 0.15 - 5.5225$	
	A1*cso dependent on both M marks for a fully correct expression for $Var(X)$ leading to correct given answer 1.1275 with no incorrect working seen. $6.65 - 2.35^2 = 1.1275$ on its own scores M0M1A0	
(e)	B1 for 4.51 oe	

Question Number	Scheme						Marks									
7 (a)	$\frac{k+4}{8} = 1 \quad [k = 4^*]$						B1*									
							(1)									
(b)	x	1	2	3	4		M1 M1 A1									
	P(X = x)	$\frac{1}{13}$	$\frac{7}{26} - \frac{1}{13} = \frac{5}{26}$	$\frac{15}{26} - \frac{7}{26} = \frac{4}{13}$	$1 - \frac{15}{26} = \frac{11}{26}$											
							(3)									
(c)	4						B1ft									
							(1)									
(d)	$E(X) = 1 \times \frac{1}{13} + 2 \times \frac{5}{26} + 3 \times \frac{4}{13} + 4 \times \frac{11}{26} \left[= \frac{40}{13} \right]$			<table><tr><td>Y</td><td>7</td><td>20</td><td>33</td><td>46</td></tr><tr><td>P(Y = y)</td><td>$\frac{1}{13}$</td><td>$\frac{5}{26}$</td><td>$\frac{4}{13}$</td><td>$\frac{11}{26}$</td></tr></table>		Y	7	20	33	46	P(Y = y)	$\frac{1}{13}$	$\frac{5}{26}$	$\frac{4}{13}$	$\frac{11}{26}$	M1
	Y	7	20	33	46											
	P(Y = y)	$\frac{1}{13}$	$\frac{5}{26}$	$\frac{4}{13}$	$\frac{11}{26}$											
	$E(X^2) = 1^2 \times \frac{1}{13} + 2^2 \times \frac{5}{26} + 3^2 \times \frac{4}{13} + 4^2 \times \frac{11}{26} \left[= \frac{135}{13} \right]$			$E(Y) = 7 \times \frac{1}{13} + 20 \times \frac{5}{26} + 33 \times \frac{4}{13} + 46 \times \frac{11}{26} \left[= 34 \right]$		M1										
	$\text{Var}(X) = \frac{135}{13} - \left(\frac{40}{13} \right)^2 \left[= \frac{155}{169} \right]$			$E(Y^2) = 7^2 \times \frac{1}{13} + 20^2 \times \frac{5}{26} + 33^2 \times \frac{4}{13} + 46^2 \times \frac{11}{26} \left[= 1311 \right]$		M1										
$\text{Var}(13X - 6) = 13^2 \times \frac{155}{169}$			$\text{Var}(13X - 6) = '1311' - '34'^2$		M1											
$= 155$					A1											
							(5)									
	Notes						Total 10									
(a)	B1*	$\frac{k+4}{8} = 1$ oe Allow verification method $\frac{4+4}{8} = 1$ provided they conclude $k = 4$														
(b)	M1	For a correct method to find one probability from $x = 2$, $x = 3$ or $x = 4$ (implied by any one correct probability from $x = 2$, $x = 3$ or $x = 4$)														
	M1	For a correct method to find a second probability from $x = 2$, $x = 3$ or $x = 4$ (implied by any two correct probabilities from $x = 2$, $x = 3$ or $x = 4$)														
	A1	For a fully correct probability distribution. Need not be in a table, but 1, 2, 3 and 4 must be associated with correct probability														
(c)	B1ft	Must be consistent with the highest probability in their distribution in part (b). If no distribution is found, then the answer must be 4														
(d)	M1	For a correct method to find $E(X)$ (implied by awrt 3.08) ft their table use of $\sum xF(x)$ is M0 or for a correct probability distribution for $13X - 6$ ft their probabilities in (b)														
	M1	For a correct method to find $E(X^2)$ (implied by awrt 10.4) ft their table use of $\sum x^2 F(x)$ is M0 or for a correct method to find $E(Y)$ ft their table														
	M1	Use of $E(X^2) - E(X)^2$ ft their $E(X^2)$ and their $E(X)$ or for a correct method to find $E(Y^2)$ ft their table														
	M1	Use of $13^2 \text{Var}(X)$ ft their $\text{Var}(X)$ or use of $E(Y^2) - E(Y)^2$ ft their $E(Y^2)$ and their $E(Y)$														
	A1	Cao														

Question Number	Scheme	Marks
7. (a)	$P(X=3) = F(3) - F(2) = \frac{1}{38}$ $P(X=3) = \frac{7}{n} \times \frac{6}{n-1} \times \frac{5}{n-2}$ $\frac{7}{n} \times \frac{6}{n-1} \times \frac{5}{n-2} = \frac{1}{38} \rightarrow n(n-1)(n-2) = 7980 \quad (*)$	M1 M1 M1 A1cso (4)
(b)	$21 \times 20 \times 19 = 7980$	B1cso (1)
(c)	$a = F(0) = P(X=0) = \frac{14}{21} \times \frac{13}{20} \times \frac{12}{19}$ $a = \frac{26}{95}$ $P(X=1) = 3 \times \frac{14}{21} \times \frac{13}{20} \times \frac{7}{19} \left[= \frac{91}{190} \right] \text{ or } P(X=2) = 3 \times \frac{7}{21} \times \frac{6}{20} \times \frac{14}{19} \left[= \frac{21}{95} \right]$ $b = F(1) = P(X=0) + P(X=1) = \frac{26}{95} + \frac{91}{190} \text{ or } b = \frac{37}{38} - \frac{21}{95}$ $b = \frac{143}{190}$	M1 A1 M1 M1 dM1 A1 (6) [11]
Notes		
(a)	1 st M1 for use of $F(3) - F(2)$ Accept $\frac{1}{38}$ 2 nd M1 product of 3 probabilities where the denominators are n , $(n-1)$ and $(n-2)$ and the numerators are decreasing k , $(k-1)$ and $(k-2)$ This may be seen as a single term in a longer expression. 3 rd M1 setting up equation for $P(X=3) =$ product of correct 3 probabilities without replacement A1cso fully correct solution with no errors seen	
(b)	B1cso correctly evaluated product. Allow $21(21-1)(21-2) = 7980$	
(c)	1 st M1 product of 3 probabilities for $P(X=0)$ The three probabilities can be in any arrangement May be implied by $\frac{26}{95}$ 1 st A1 $a = \frac{26}{95}$ oe must be clear this is the value for a 2 nd M1 product of 3 probabilities for $P(X=1)$ or $P(X=2)$ or $\frac{91}{190}$ or $\frac{91}{570}$ or $\frac{21}{95}$ or $\frac{7}{95}$ oe seen. Condone incorrect labelling. The three probabilities can be in any arrangement 3 rd M1 $\times 3$ or adding the 3 sets of the 3 fractions or $\frac{91}{190}$ or $\frac{21}{95}$ Condone incorrect labelling 4 th dM1 their $P(X=0) +$ their $P(X=1)$ or $F(2) - P(X=2)$ (dep on 2 nd M1 being scored) 2 nd A1 $b = \frac{143}{190}$ oe must be clear this is the value for b NB if $a = 0.273\dots$ and $b = 0.7526$ implies the method marks.	

Question	Scheme		Marks
3 (a)	$E(X) = 2a + 3 \times 0.4 + 4(0.6 - a) [= 3.6 - 2a]$		M1 A1
			(2)
(b)	$0 < a < 0.6$ oe		B1
	$2 \times 0.6 + 3 \times 0.4 [= 2.4]$ or $3.6 - 2 \times 0.6 [= 2.4]$ and $3 \times 0.4 + 4 \times 0.6 [= 3.6]$ or $3.6 - 2 \times 0 [= 3.6]$	Alternative $0 > -2a > -1.2$ $3.6 > 3.6 - 2a > 2.4$	M1
	$2.4 < E(X) < 3.6$		A1
			(3)
(c)	$\text{Var}(X) = E(X^2) - E(X)^2$		
	$[E(X^2) =] 4a + 3.6 + 9.6 - 16a [= 13.2 - 12a]$		M1 A1
	$\text{Var}(X) = '(13.2 - 12a)' - ('3.6 - 2a')^2$		M1
	$-4a^2 + 2.4a - 0.32 = 0$		A1
	$a = \frac{-'2.4' \pm \sqrt{'2.4'^2 - 4 \times '-'4' \times '-'0.32'}}{2 \times '-'4'}$		M1
	$a = \frac{1}{5} \quad a = \frac{2}{5}$		A1
			(6)
Notes			Total 11
(a)	M1	for an attempt to find $E(X)$ with 2 out of the 3 products correct	
	A1	for $2a + 1.2 + 4(0.6 - a)$ oe	
(b)	B1	This may be seen as two separate parts e.g. $a > 0$ and $a < 0.6$, Allow the use of $\leq$ or $\geq$ for $<$ or $>$ We allow this to be written in words e.g. a is between 0 and 0.6	
	M1	for a correct method for finding the lower and upper end of the range. May be implied by $2.4 < E(X) < 3.6$ or sight of 2.4 and 3.6	
	A1	Allow e.g. 2.4 ., $3.6 - 2a$., 3.6	
		NB $2.4 < E(X) < 3.6$ or 2.4 ., $3.6 - 2a$., 3.6 scores 3/3	
(c)	M1	An attempt at an expression for $E(X^2)$ with 2 terms correct. May be seen in an attempt at $\text{Var}(X)$	
	A1	a correct expression for $E(X^2)$ May be seen in an attempt at $\text{Var}(X)$ Does not have to be fully simplified, allow $4a + 3.6 + 9.6 - 16a$ or better	
	M1	use of $\text{Var}(X) = E(X^2) - E(X)^2$ ft their $E(X^2)$ and their part (a)	
	A1	a correct 3TQ e.g. $25a^2 - 15a + 2 = 0$	
	M1	correct method for solving their 3TQ e.g. $(5a - 2)(5a - 1) = 0$ May be implied by $a = \frac{1}{5}$ and $a = \frac{2}{5}$ If the 3TQ is incorrect then a correct substitution of their values into the quadratic formula (If a and c are both negative, allow the omission of negatives in $4ac$ and allow a correct single value in the denominator) or a complete method using completing the square or a correct factorisation must be seen before their values of a	
	A1	$a = \frac{1}{5}$ oe and $a = \frac{2}{5}$ oe Allow any letter for a	

Question Number	Scheme		Marks
4 (a)	$E\left(\frac{1}{X}\right)=1\times\frac{1}{10}+\frac{1}{2}\times\frac{1}{5}+\frac{1}{3}\times\frac{3}{10}+\frac{1}{4}\times\frac{2}{5}=\frac{2}{5}^*$		B1*
			(1)
(b)	$E\left(\left(\frac{1}{X}\right)^2\right)=1^2\times\frac{1}{10}+\left(\frac{1}{2}\right)^2\times\frac{1}{5}+\left(\frac{1}{3}\right)^2\times\frac{3}{10}+\left(\frac{1}{4}\right)^2\times\frac{2}{5}\left[=\frac{5}{24}\right]$		M1
	$\text{Var}\left(\frac{1}{X}\right)=\frac{5}{24}-\left(\frac{2}{5}\right)^2=\frac{29}{600}$		M1 A1
			(3)
(c) (i)	$[E(Y)=]12$		B1
(ii)	$[\text{Var}(Y)=]30^2\text{Var}\left(\frac{1}{X}\right)=\frac{87}{2}$ or If $y : 30 \ 15 \ 10 \ 7.5$ then $[\text{Var}(Y)=]\frac{375}{2}-12^2=\frac{87}{2}$		M1 A1
			(3)
(d)	$[Y < 20 \Rightarrow]\frac{30}{X} < 20 \Rightarrow X > 1.5 \quad \text{or } y : 30 \ 15 \ 10 \ 7.5$		M1
	$P(Y < 20) = P(X > 1.5) = \frac{9}{10}$		A1
	$[P(X < 3 Y < 20)]=\frac{P(X = 2)}{P(X > 1.5)}=\frac{\frac{1}{5}}{\left(\frac{9}{10}\right)}=\frac{2}{9}$ or $[P(X < 3 Y < 20)]=\frac{P(Y = 15)}{P(Y < 20)}=\frac{\frac{1}{5}}{\left(\frac{9}{10}\right)}=\frac{2}{9}$		dM1 A1 A1
			(5)
Notes			Total 12
(a)	B1*	Value given, so must see sight of a correct expression, with no incorrect working seen. (Allow equivalent expressions.)	
(b)	M1	For attempt at an expression for $E\left(\left(\frac{1}{X}\right)^2\right)$ with at least 3 correct terms (Allow equivalent expressions.) May be embedded in a correct expression for $\text{Var}(X)$	
	M1	For a correct expression for $\text{Var}\left(\frac{1}{X}\right)$ (Need not be simplified) ft a stated value of $E\left(\left(\frac{1}{X}\right)^2\right)$	
	A1	Cao Allow awrt 0.0483	
(c) (i)	B1	For $[E(Y)] = 12$	
(ii)	M1	For correct use of $30^2\text{Var}\left(\frac{1}{X}\right)$ ft their $\text{Var}\left(\frac{1}{X}\right)$ or $\frac{375}{2}-12^2$ (May be implied by $\frac{87}{2}$ oe)	
	A1	For $[\text{Var}(Y)]=\frac{87}{2}$ oe	
(d)	M1	For a correct inequality for $Y < 20$ or all 4 values of Y found (these may be seen in part (c))	
	A1	For $P(Y < 20) = \frac{9}{10}$ (May be seen as the denominator (e.g $0.2 + 0.3 + 0.4$ oe) in a ratio of probabilities and scores M1A1)	
	dM1	Dependant on 1 st M1 For $\frac{P(X = 2)}{P(X > 1.5)}$ or $\frac{P(Y = 15)}{P(Y < 20)}$ Allow $\frac{P(1.5 < X < 3)}{P(X > 1.5)}$ or a correct ratio of probabilities ft $P(Y < 20)$	
	A1	For a correct numerator	
	A1	For $\frac{2}{9}$ oe (Allow a decimal answer that is 3sf or better e.g. 0.222)	

Question	Scheme		Marks
2 (a)	$\left[P(2X - 3 > 5) = \right] 0.45$		B1
			(1)
(b)	$E(X^2) = 2^2 \times 0.25 + 4^2 \times 0.3 + 5^2 \times 0.2 + 7^2 \times 0.1 + 8^2 \times 0.15 [= 25.3]$		M1
	$\text{Var}(X) = 2^2 \times 0.25 + 4^2 \times 0.3 + 5^2 \times 0.2 + 7^2 \times 0.1 + 8^2 \times 0.15 - 4.6^2$ or $\text{Var}(X) = "25.3" - 4.6^2$		M1
	$= 25.3 - 4.6^2 = 4.14^*$		A1*
			(3)
(c)	$[E(Y) =] 13.4 = a \times 4.6 - b$		M1
	$[Var(Y) =] a^2 \times 4.14 = 66.24$		M1
	$a = 4$		A1
	$b = 5$		A1
			(4)
(d)	Sam throws 8 and Alex throws 2, 4 or 5 Sam throws 7 and Alex throws 2 or 4 Sam throws 5 or 4 or 2 and Alex throws 2		M1
	$0.15 \times (0.25 + 0.3 + 0.2)$ or $0.1 \times (0.25 + 0.3)$ or $(0.2 + 0.3 + 0.25) \times 0.25$		M1
	$0.15 \times (0.25 + 0.3 + 0.2) + 0.1 \times (0.25 + 0.3) + (0.2 + 0.3 + 0.25) \times 0.25$		M1
	$= 0.355 \quad \left(= \frac{71}{200} \right)$		A1
			(4)
	ALT Alex throws 2 Alex throws 4 and Sam throws 7 or 8 Alex throws 5 and Sam throws 8		M1
	0.25 or $0.3 \times (0.1 + 0.15)$ or 0.2×0.15		M1
	$0.25 + 0.3 \times (0.1 + 0.15) + 0.2 \times 0.15$		M1
	$= 0.355 \quad \left(= \frac{71}{200} \right)$		A1
	Notes		Total 12
(a)	B1	0.45 oe	
(b)	M1	for a correct method to find $E(X^2)$ At least 3 terms correct and added This is not implied by 25.3 on its own	
	M1	for use of correct equation ft their $E(X^2)$	
	A1*	for a correct expression, with all terms seen, leading to the given answer	
(c)	M1	for writing or using a correct equation for $E(Y)$	
	M1	for writing or using a correct method for $\text{Var}(Y)$	
	A1	for $a = 4$ may be seen as part of the expression $4X \pm \dots$	
	A1	for $b = 5$ may be seen as part of the expression $\dots X - 5$	
(d)	M1	for recognising all the required combinations – implied by 3 correct probabilities Ignore any repeats but do not ignore any incorrect combinations	
	M1	for any one correct calculation from the 3 given	
	M1	for any 2 correct calculations from the 3 given	
	A1	0.355 oe	

Question Number	Scheme	Marks												
4. (a)	$\frac{2}{5}$	B1 (1)												
(b)	$E(W) = 3$ $E(5 - 2W) = 5 - 2E(W)$ $E(X) = -1$	B1 M1 A1 (3)												
(c)	$P(X < W) = P(5 - 2W < W) = P(W > \frac{5}{3})$ or $P(W \geq 2)$ $= \frac{4}{5}$	M1 A1 (2)												
(d)(i)	<table border="1"><tr><td>[y]</td><td>1</td><td>$\frac{1}{2}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{5}$</td></tr><tr><td>[p]</td><td>$\frac{1}{5}$</td><td>$\frac{1}{5}$</td><td>$\frac{1}{5}$</td><td>$\frac{1}{5}$</td><td>$\frac{1}{5}$</td></tr></table>	[y]	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	[p]	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	B1
[y]	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$									
[p]	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$									
(ii)	$E(Y) = \frac{1}{5} \left(1 + \frac{1}{2} + \dots + \frac{1}{5} \right)$ or $\frac{1}{5} + \frac{1}{10} + \frac{1}{15} + \frac{1}{20} + \frac{1}{25} \left[= \frac{137}{300} = 0.4566... \right]$ $E(Y^2) = \frac{1}{5} \left(1^2 + \left(\frac{1}{2} \right)^2 + \dots + \left(\frac{1}{5} \right)^2 \right)$ or $\frac{1}{5} + \frac{1}{20} + \frac{1}{45} + \frac{1}{80} + \frac{1}{125} \left[= \frac{5269}{18000} = 0.2927... \right]$ $\text{Var}(Y) = '0.2927...' - ('0.4566...')^2$ awrt <u>0.0842</u>	M1 M1 M1 A1 (5)												
(e)	$\text{Var}(2 - 3Y) = (-3)^2 \text{Var}(Y)$ awrt <u>0.758</u>	M1 A1ft (2)												
Notes [13]														
(a)	B1 oe													
(b)	B1 sight of $E(W) = 3$ or the x values 3, 1, -1, -3, -5 (they may be added) M1 use of $E(5 - 2W) = 5 - 2E(W)$ or $\frac{1}{5}(3 + 1 + \dots + -5)$ Condone use of X instead of W A1 cao and labelled $E(X)$													
(c)	M1 for identifying $W > \frac{5}{3}$ or $W \geq 2$ eg $1 - P(W = 1) \geq 2$ or $1 - P(W \leq 1) \geq 2$ A1 oe													
(d)(i)	B1 Correct distribution (probabilities may be implied by correct use). May be seen in any part													
(ii)	M1 attempt at expression for $E(Y)$ using their values of y and p (at least 2 terms seen) or awrt 0.457 (0.45 if have 0.3 rather than 1/3) Condone incorrect labelling M1 attempt at expression for $E(Y^2)$ using their values of y and p (at least 2 terms seen) or awrt 0.293 (0.2885 if have 0.3 rather than 1/3) Condone incorrect labelling M1 For use of " $E(Y^2)$ " - " $(E(Y))^2$ " ft their values for $E(Y^2)$ and $E(Y)$ A1 awrt 0.0842 or $\frac{947}{11250}$													
(e)	M1 for use of $(-3)^2 \text{Var}(Y)$ with their $\text{Var}(Y) > 0$ condone $(3)^2 \text{Var}(Y)$ A1ft $\frac{947}{1250}$ or $9 \times$ "their part (d) > 0" evaluated correctly to 3sf or exact fraction													

Qu	Scheme	Marks							
5(a)	<table><tr><td>P(Y = y)</td><td>2k</td><td>k</td><td>k</td><td>8k</td><td>17k</td><td>k</td></tr></table>	P(Y = y)	2k	k	k	8k	17k	k	M1
P(Y = y)	2k	k	k	8k	17k	k			
	$2k + k + k + 8k + 17k + k = 1$ or $30k = 1 \Rightarrow k = \frac{1}{30}$ *	A1*							
		(2)							
(b)	$k + k + 8k$ or $1 - (2k + 17k + k)$	M1							
	$= \frac{1}{3}$ oe awrt 0.333	A1							
		(2)							
(c)	$(1 \times 2k) + (2 \times k) + (3 \times k) + (4 \times 8k) + (5 \times 17k) + (6 \times k) =$	M1							
	$\frac{13}{3}$ oe awrt 4.33	A1							
		(2)							
(d)	$P(Y \geq 15 - 2Y)$ or $[X =] 13 \ 11 \ 9 \ 7 \ 5 \ 3$ only or $[Y =] 5$ or 6 only	M1							
	$[P(Y \geq 5) = P(Y = 5) + P(Y = 6)] = \frac{17}{30} + \frac{1}{30}$	M1							
	$= \frac{3}{5}$ oe	A1ft							
		(3)							
(e)	$\text{Var}(X) = 4\text{Var}(Y)$	M1							
	$[E(Y^2) =](1 \times 2k) + (2^2 \times k) + (3^2 \times k) + (4^2 \times 8k) + (5^2 \times 17k) + (6^2 \times k)[= \frac{302}{15}$ or awrt 20.1]	M1							
	$[\text{Var}(Y) =] \frac{302}{15} - \left(\frac{13}{3}\right)^2 [= \frac{61}{45}$ or awrt 1.36]	M1d							
	$[\text{Var}(X) =] \frac{244}{45}$ oe awrt 5.42	A1							
	ALT for 1 st 3 marks	(4)							
	$[E(X) =] (13 \times 2k) + (11 \times k) + (9 \times k) + (7 \times 8k) + (5 \times 17k) + (3 \times k)[= \frac{19}{3}$ or awrt 6.33]	M1							
	$[E(X^2) =](13^2 \times 2k) + (11^2 \times k) + (9^2 \times k) + (7^2 \times 8k) + (5^2 \times 17k) + (3^2 \times k)[= \frac{683}{15}$ or awrt 45.5]	M1							
	$[\text{Var}(X) =] \frac{683}{15} - \left(\frac{19}{3}\right)^2$	M1d							
	Notes	Total 13							
(a)	M1 for finding the probabilities in terms of k. The individual probabilities must be seen either in a table or in the calculation (but do not need to be simplified)								
	A1* Method mark must be awarded. For a correct equation which would lead to $k = 1/30$ *								
	NB Verification - $2\left(\frac{1}{30}\right) + \left(\frac{1}{30}\right) + \left(\frac{1}{30}\right) + 8\left(\frac{1}{30}\right) + 17\left(\frac{1}{30}\right) + \left(\frac{1}{30}\right) = 1$ gains M1 A0								
(b)	M1 for using $P(Y = 2) + P(Y = 3) + P(Y = 4)$ or $1 - P(Y = 1) + P(Y = 5) + P(Y = 6)$ Allow in terms of k or with $k = 1/30$ subst or with their probabilities. Do not allow in terms of y								
	A1 awrt 0.333								
(c)	M1 for using $\sum xP(x)$ At least 3 terms given Allow with $k = 1/30$ subst or ft their probabilities.								
	A1 awrt 4.33								
(d)	M1 forming correct inequality in Y or 13,11,9,7,5,3 seen anywhere or for 5 and 6 only. Implied by 2 nd M1								
	M1 finding their $P(Y = 5) +$ their $P(Y = 6)$ or $P(X = 5) + P(X = 3)$ eg $17k + k$								
	A1ft ft their probabilities								
(e)	M1 written or used $4\text{Var}(Y)$ (may come at the end of the calculation) or written or used $E(X)$ allow awrt 6.33 NB condone $-2^2\text{Var}(Y)$ if used $4\text{Var}(Y)$								
	M1 Correct method, at least 3 products correct, to find $E(Y^2)$ or $E(X^2)$ condone incorrect labels								
	M1d dep on the 2 nd M mark being awarded. For correct use of $E(Y^2) - [E(Y)]^2$ or $E(X^2) - [E(X)]^2$ For the ALT In addition to the 2 nd M1 the 1 st M1 must be awarded. Condone incorrect labelling								
	A1 awrt 5.42								

Question Number	Scheme	Marks								
5. (a)	[By symmetry $E(Y) = \underline{0}$	B1 (1)								
(b)	$q + r + u = \frac{19}{30}$ $2(q + r) + u = 1$ [and attempt to solve e.g. $q + r = \dots$] $u = \frac{8}{30} = \frac{4}{\underline{\underline{15}}} (*)$	M1 M1 A1*cso (3)								
(c)	$E(Y^2) = (-9)^2 \times q + (-5)^2 \times r + 5^2 \times r + 9^2 q$ or $162q + 50r$ $\text{Var}(Y) = 37 = E(Y^2) - "0"^{n2} \Rightarrow 37 = 162q + 50r$ oe Solving with $q + r = \frac{11}{30}$ oe e.g. $(162 - 50)q = 37 - \frac{55}{3}$ or $q = \frac{1}{\underline{\underline{6}}}$ and $r = \frac{1}{\underline{\underline{5}}}$	M1 dM1 M1 A1 (4)								
(d)	$Y = 0 \Rightarrow D = 12, D = \sqrt{12^2 + Y^2}; Y = \pm 5 \Rightarrow D = 13$ or $Y = \pm 9 \Rightarrow D = 15$ <table border="1"><tr><td>d</td><td>12</td><td>13</td><td>15</td></tr><tr><td>$P(D = d)$</td><td>$\frac{4}{15}$</td><td>$\frac{6}{15}$ or $\frac{2}{5}$</td><td>$\frac{5}{15}$ or $\frac{1}{3}$</td></tr></table>	d	12	13	15	$P(D = d)$	$\frac{4}{15}$	$\frac{6}{15}$ or $\frac{2}{5}$	$\frac{5}{15}$ or $\frac{1}{3}$	B1, M1;A1 M1A1ftA1ft (6)
d	12	13	15							
$P(D = d)$	$\frac{4}{15}$	$\frac{6}{15}$ or $\frac{2}{5}$	$\frac{5}{15}$ or $\frac{1}{3}$							
Notes										
(a)	B1 for 0									
(b)	1 st M1 for a correct equation in q, r and u using $F(0)$ 2 nd M1 for a second equation clearly based on sum of probs = 1 and an attempt to solve these 2 equations A1*cso correct value for u found with no incorrect working									
(c)	1 st M1 for an attempt at $E(Y^2)$ with at least 3 correct products seen. The negative numbers do not need to be in brackets 2 nd dM1 for attempt at correct equation in q and r using $\text{Var}(Y)$ [ft their $E(Y)$ and $E(Y^2)$] Condone missing subtraction of 0^2 if 0 in part(a) 3 rd M1 using $q + r = 11/30$ (awrt 0.37) to attempt to solve two linear equations in q and r leading to equation in one variable. May be implied by correct answers. A1 for $q = \frac{1}{6}$ and $r = \frac{1}{5}$ or exact equivalents									
(d)	B1 for $D = 12$ 1 st M1 for use of Pythagoras to work out $D = 13$ or 15 1 st A1 for $D = 13$ and 15 2 nd M1 for a correct value of D and an associated probability. Allow two occurrences (for 15 and 13) which add to the appropriate probability. 2 nd A1ft for two correct values of D and associated probs ft their +ve q and r if $q + r = \frac{11}{30}$ Allow two occurrences (for 15 and 13) which add to the appropriate probability. 3 rd A1ft for a fully correct probability distribution ft their +ve q and r if $q + r = \frac{11}{30}$									

Question Number	Scheme		Marks
5(a)	$\left[E\left(R^2\right)\right]=2^2 \times 0.25+3^2 \times 0.3+4^2 \times 0.15+5^2 \times 0.1+6^2 \times 0.2$ (= 15.8*)		B1cso*
			(1)
(b)	$\left[s d\left(R\right)\right]=\sqrt{15.8-3.7^2}$		M1
	$=\sqrt{2.11}$		
	Standard deviation = 1.4525....	awrt 1.45	A1
			(2)
(c)	$d=1$		B1
			(1)
(d)	$0.1+0.2+0.1+a+b=1$ oe		M1
	$2 \times 0.1+3 \times 0.2+4 \times 0.1+5 a+6 b=4.55$ oe		M1
	$5(0.6-b)+6 b=3.35$ or $5 a+6(0.6-a)=3.35 \Rightarrow a=0.25$ or $b=0.35$		M1
	$c=0.4+"0.25"$ or $c=1-"0.35"$		M1
	$c=0.65$ oe		A1
			(5)
(e)	$0.9 \times 0.75 \times 0.1$		M1
	$=0.0675$		A1
			(2)
(f)	For identifying that if Jessie scores 2, Pabel has no spin oe may be implied		M1
	$[0.10 \times 0+] 0.2 \times 0.3+0.1 \times 0.15+"0.25" \times 0.1+"0.35" \times 0.2$		M1
	$=0.17$		A1
			(3)
Notes			Total 14
(a)	B1	Correct calculation with all products seen (allow $1+2.7+2.4+2.5+7.2$) Figures may be seen in table before part (a). Condone missing addition signs if products seen in table.	
(b)	M1	Use of formula including the square root	
	A1	awrt 1.45 (correct answer with no working scores M1A1)	
(c)	B1	For 1	
(d)	M1	Allow equivalents eg $a+b=0.6$	
	M1	Allow equivalents eg $5 a+6 b=3.35$	
	M1	Correct method to eliminate a or b (implied by a correct value for a or b) This mark can still be scored even if the method leads to a value of a or b which is not a probability. May see $a=c-0.4$ to eliminate a or $b=1-c$ used to eliminate b	
	M1	A complete method for finding the value of c (condone using any value of a and b for this mark)	
	A1	0.65 oe	
(e)	M1	For the product of 3 probabilities	
	A1	0.0675 or exact equivalent fraction eg $\frac{27}{400}$	
(f)	M1	Identifying that if Jessie scores 2, there is only one spin or the 4 correct possibilities only	
	M1	At least 3 correct non-zero probability products ft their a and b (an answer of 0.195 scores M0M1A0)	
	A1	0.17	

Question Number	Scheme		Marks										
7. (a)	$\left[\frac{0.1}{0.8}\right] = \frac{1}{8}$		B1 (1)										
(b)	$[0^2 \times 0.1 +] 5^2 \times 0.2 + 10^2 \times 0.7 = 75^*$		B1*cso (1)										
(c)	$E(X) = [0 \times 0.1 +] 5 \times 0.2 + 10 \times 0.7 [= 8]$ $\text{Var}(X) = 75 - ('8')^2$ $\text{Var}(X) = \underline{11}$		M1 M1 A1 (3)										
(d)	$\text{Var}(4 - 3X) = 3^2 \times \text{Var}(X) [= 3^2 \times "11"]$ $= \underline{99}$		M1 A1ft (2)										
(e)	$P((0, 5), (0, 10), (5, 10))$ $= 0.1 \times 0.2 + 0.1 \times 0.7 + 0.2 \times 0.7$ $[= 0.02 + 0.07 + 0.14]$ $= \underline{0.23}$	$P((0, 0), (5, 5), (10, 10))$ $= 1 - (0.1^2 + 0.2^2 + 0.7^2)$ $[= 0.5(1 - (0.01 + 0.04 + 0.49))]$ $= \underline{0.23}$	M1M1 A1 (3)										
(f)	Products: 0, 25, 50, 100 $P(D = 0) = 0.1 + 0.1 - 0.1 \times 0.1 [= 0.19]$ $P(D = 25) = 0.2^2$ $P(D = 50) = 2 \times 0.2 \times 0.7$ $P(D = 100) = 0.7^2$ <table><tr><td>D</td><td>0</td><td>25</td><td>50</td><td>100</td></tr><tr><td>$P(D = d)$</td><td>0.19</td><td>0.04</td><td>0.28</td><td>0.49</td></tr></table>		D	0	25	50	100	$P(D = d)$	0.19	0.04	0.28	0.49	B1 M1 M1 A1 (4)
D	0	25	50	100									
$P(D = d)$	0.19	0.04	0.28	0.49									
Notes			Total 14										
(a)	B1 $\frac{1}{8}$ oe Allow 0.125 Do not ISW												
(b)	B1*cso correct expression and 75 with no errors seen. Allow $5^2 \times 0.2 + 10^2 \times 0.7 = 75$												
(c)	M1 correct method to find mean. If no method seen award if 8 is seen M1 attempt at expression for variance ie $75 - (\text{their } E(X))^2$ A1 11 cao												
(d)	M1 Use of $(-3)^2 \times \text{Var}(X)$ (condone 3 rather than -3 and missing bracket if final answer is correct) or $4^2 \times 0.1 + (-11)^2 \times 0.2 + (-26)^2 \times 0.7 - (-20)^2$ condone 11, 26 and 20 A1ft 99 or ft $9 \times \text{'their (c)'}^2$												
(e)	1 st M1 for at least one correct product. NB may be combined eg 0.3×0.7 but not in the numerator or denominator of a fraction 2 nd M1 a fully correct expression oe e.g. $0.1 \times 0.2 + 0.3 \times 0.7$ A1 0.23 oe												
(f)	B1 all 4 correct products with no incorrect extras unless they have a probability of 0 associated with them 1 st M1 A correct method to find 1 of the 4 probabilities. Does not need to be associated with the correct product ALT $P(D = 0) = 0.1 \times 0.1 + 2 \times 0.1 \times 0.2 + 2 \times 0.1 \times 0.7 [= 0.19]$ 2 nd M1 A correct method to find 3 of the 4 probabilities or 2 of the 4 probabilities if the total of the 4 probabilities is 1 Must be associated with the correct product A1 all four correct probabilities (oe) associated with the correct products												

Question Number	Scheme							Marks												
6(a)	$[10k = 1 \Rightarrow] k = 0.1$							B1												
								(1)												
(b)	e.g. $P(X = 1) = 0.1$ and $P(X = 2)[= F(2) - F(1)] = 0.1$							B1												
	e.g. $P(X = 3)[= F(3) - F(2)] = 0.1$							M1												
	<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$P(X = x)$</td><td>0.1</td><td>0.1</td><td>0.1</td><td>0.2</td><td>0.2</td><td>0.3</td></tr></table>	X	1	2	3	4	5	6	$P(X = x)$	0.1	0.1	0.1	0.2	0.2	0.3					
X	1	2	3	4	5	6														
$P(X = x)$	0.1	0.1	0.1	0.2	0.2	0.3														
								(3)												
(c)	$a + a + a + b + b + b + 0.11 + 0.05 = 1 \quad [3a + 3b = 0.84]$							M1												
	$a + 2a + 3a + 4b + 5b + 6b + 0.77 + 0.4 = 4.02 \quad [\Rightarrow 6a + 15b = 2.85]$							M1												
	e.g. $9a = 1.35 \Rightarrow a = 0.15^*$							A1*												
								(3)												
(d)	$b = 0.13$																			
	$E(Y^2) = 1^2 \times 0.15 + 2^2 \times 0.15 + 3^2 \times 0.15 + 4^2 \times '0.13' + 5^2 \times '0.13' + 6^2 \times '0.13' + 7^2 \times 0.11 + 8^2 \times 0.05 = [20.7]^*$							M1 A1*												
								(2)												
(e)	$[Var(Y) =] 20.7 - 4.02^2 [= 4.5396]$							M1												
	$Var(5 - 2Y) = 4Var(Y) = 4 \times '4.5396' = 18.1584$							awrt 18.2 M1 A1												
								(3)												
(f)	$'0.1' \times 0.15 + '0.1' \times 0.15 = 0.03$							M1 A1												
								(2)												
	Notes							Total 14												
(a)	B1	For $k = 0.1$ oe																		
(b)	B1	For correct use of $F(x)$ to find 2 probabilities May be implied by two correct probabilities																		
	M1	For correct use of $F(x)$ to find one other probability. May be implied by one other correct probability																		
	A1	For a fully correct probability distribution. Need not be in a table but probabilities must be attached to the correct X values																		
(c)	M1	For use of the sum of the probabilities = 1 to form a linear equation in a and b (May be implied by $3a + 3b = 0.84$)																		
	M1	For use of $\sum y \times P(Y = y) = 4.02$ to form a linear equation in a and b (May be implied by $6a + 15b = 2.85$)																		
	A1*	Answer is given so there must be a correct line between the 2 equations and the given answer																		
(d)	M1	For finding $E(Y^2)$ with their b (At least 4 correct terms). Values for a and b must be substituted into their expression for $E(Y^2)$																		
	A1*	For a fully correct expression (no incorrect working seen) eg. $14 \times 0.15 + 77 \times 0.13 + 49 \times 0.11 + 64 \times 0.05$ Allow $0.15 + 0.6 + 1.35 + 2.08 + 3.25 + 4.68 + 5.39 + 3.2$																		
(e)	M1	For a correct expression for $Var(Y)$. May be implied by awrt 4.54 Allow $E(5 - 2Y) = -\frac{76}{25}$ oe and $E((5 - 2Y)^2) = \frac{137}{5}$																		
	M1	For use of $4Var(Y)$ ft their $Var(Y)$ provided $Var(Y)$ is not 20.7 or 4.02 Do not allow $5 + 4Var(Y)$ Allow $Var(5 - 2Y) = \frac{137}{5} - \left(-\frac{76}{25}\right)^2$ ft their $E(5 - 2Y)$ and their $E((5 - 2Y)^2)$																		
	A1	awrt 18.2																		
(f)	M1	For use of $P(X = 1) \times P(Y = 2) + P(X = 2) \times P(Y = 1)$ ft their X distribution probabilities																		
	A1	Cao																		



Question Number	Scheme	Marks
6. (a)	$[E(A) =] 1 \times 0.4 + 4 \times 0.2 + 5 \times 0.25 + 7 \times 0.15$ $= \underline{3.5} \quad (*)$	M1 A1cso (2)
(b)	$[E(A^2) =] 1 \times 0.4 + 4^2 \times 0.2 + 5^2 \times 0.25 + 7^2 \times 0.15 [= 17.2]$ $\text{Var}(A) = E(A^2) - [E(A)]^2 = 17.2 - 3.5^2$ $= \underline{4.95}$	M1 M1 A1 (3)
(c)	(Discrete) uniform (distribution)	B1 (1)
(d)	By symmetry $k = 6$	B1 (1)
(e)	$\left[\text{Sam has } Z = \frac{3.5 - 4}{3} = -\frac{1}{6} \text{ and } \right] \text{Tim needs } \frac{3.5 - A}{4} < -\frac{1}{6} \text{ so } A > 4.166..$ $\text{So prob} = 0.25 + 0.15 = \underline{0.4}$	M1 A1 (2)
(f)	Need largest possible $\mu = 7$ and smallest possible $\sigma = 1$ $P(X > 3.5) \text{ is then } P\left(Z > \frac{3.5 - 7}{1}\right) = P(Z > -3.5)$ $= \underline{0.9998} \text{ (tables) or } 0.999767... \text{ (calc)}$	B1, B1 M1 A1 (4)
(g)	$[\text{Need } A = 7 \text{ and } B = 1 \text{ (or ft from (f)) so}] P(A = 7) \times P(B = 1) \text{ or } "0.15" \times 0.25$ $= \underline{0.0375}$	M1 A1cso (2)
[15]		
Notes		
(a)	M1 for an attempt at $E(A)$ – at least 3 correct products seen A1cso for 3.5 or exact equivalent with no incorrect working seen and M1 scored	
(b)	1 st M1 for an attempt at $E(A^2)$ – at least 3 correct products 2 nd M1 for use of $E(A^2) - [E(A)]^2$ ft their value for $E(A^2)$	
ALT	M2 for $(-2.5)^2 \times 0.4 + (0.5)^2 \times 0.2 + (1.5)^2 \times 0.25 + (3.5)^2 \times 0.15$ (at least 3 correct products) A1 for 4.95 or an exact equivalent e.g. $\frac{99}{20}$	
(c)	B1 for uniform (continuous uniform is B0)	
(d)	B1 for stating $k = 6$ with a suitable reason e.g. mention of symmetry or full calculation	
(e)	M1 for a suitable calculation for A e.g. $\frac{3.5 - A}{4} < -\frac{1}{6}$ or stating $A = 5$ or 7 or $A > \text{awrt } 4.2$ (o.e.) A1 for 0.4 (must be based on some correct calculation seen)	
(f)	1 st B1 for $\mu = 7$ may be implied from a standardisation with 3.5 seen 2 nd B1 for $\sigma = 1$ may be implied from a standardisation with 3.5 seen M1 for attempting correct probability i.e. $P(Z \dots \text{ or } X \dots)$ ft standardisation using 3.5, their $\mu \neq 4$ and their $\sigma \neq 3$ but their μ and σ must be “possible” values or $P(Z > -3.5)$ A1 for 0.9998 or better	
(g)	M1 for “0.15” $\times 0.25$ ft their value of A from (f) A1cso for 0.0375 or exact equivalent e.g. $\frac{3}{80}$ (Must clearly come from $A = 7$ and $B = 1$ in (f))	

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Question Number	Scheme	Marks
5. (a)	$\{E(X) = \} -2a - b + 0 \times c + b + 4a$ <u>or</u> $2a$ $\{ 2a = 0.5 \text{ so } \} \underline{a = 0.25}$	M1 A1 (2)
(b)	$\{E(X^2) = \} (-2)^2 \times a + (-1)^2 \times b + 0 + 1^2 \times b + 4^2 \times a$ <u>or</u> $20a + 2b$ (o.e.) $\{\text{Var}(X) = \} "20a + 2b" - 0.5^2$ $20a + 2b - 0.25 = 5.01$ (o.e.) e.g. $"4.75" + 2b = 5.01$ $\{ 2b = 0.26 \text{ so } \} \underline{b = 0.13}$ $\{\text{Use of sum of probs} = 1 \text{ to calculate a 2}^{\text{nd}} \text{ value}\} \underline{c = 0.24}$	M1 M1 A1 A1 A1ft (5)
(c)(i)	$\{E(Y) = 5 - 8 \times 0.5 \} = \underline{1}$	B1
(ii)	$\{\text{Var}(Y) = \} (-8)^2 \times 5.01$ $= 320.64$ awrt <u>321</u>	M1 A1 (3)
(d)	$4X^2 > 5 - 8X$ $(2X - 1)(2X + 5) > 0 \Rightarrow X > 0.5$ So need $X = 1$ or 4 <u>or</u> probability of $a + b$ $= \underline{0.38}$	M1 M1A1 M1 A1 (5)
[15 marks]		
Notes		
(a)	M1 for any correct expression for $E(X)$ in terms of a (or a, b, c) A1 for $a = 0.25$	
(b)	1 st M1 for attempt at an expression for $E(X^2)$ with at least 3 correct non-zero terms 2 nd M1 for a correct expression for $\text{Var}(X)$ eg " $18a - c + 1" - 0.5^2$ Allow with their value of a substituted 1 st A1 for a correct equation for b (or possibly c) eg " $18a - c + 1" - 0.5^2 = 5.01$ Allow with their value of a substituted 2 nd A1 for either $b = 0.13$ or $c = 0.24$ 3 rd A1ft for using $c = 1 - 2 \times "0.25" - 2 \times "0.13"$ or $b = (1 - 2 \times "0.25" - "0.24") \div 2$ to gain the correct ft answer for their 2 nd value	
(c)	B1 for $\{E(Y) = \} 1$ M1 for correct use of $\text{Var}(aX + b) = a^2 \text{Var}(X)$ A1 for awrt 321	
(d)	1 st M1 for correct quadratic inequality (may be inside prob statement) or table of values 2 nd M1 for an attempt to solve or identifying correct X values 1 st A1 for $X > 0.5$ [may also have $X < -2.5$] 3 rd M1 for realising need $X = 1$ and 4 only or answer of their $(a + b)$ 2 nd A1 for 0.38 (or exact equivalent) only (correct ans only 5/5)	

Question Number	Scheme					Marks										
1 (a)	Discrete uniform					B1										
						(1)										
(b)	$\frac{1}{2}$					B1										
						(1)										
(c)(i)	$[E(R) =] 2.5$					B1										
(ii)	$[E(B) =] 4$					B1										
						(2)										
(d)	$[E(B^2) =] \frac{1}{4}(1^2 + 3^2 + 5^2 + 7^2) = 21$					M1										
	$\text{Var}(B) = E(B^2) - (E(B))^2 = "21" - "4" = 17$					M1										
	$= 5$					A1										
						(3)										
(e)	Possible combinations $(R, B) : (1,1) (1,3) (2,1) (3,1) (4,1) (2,3)$ <u>or</u> $\frac{1}{4} \times \frac{1}{4} \times 6$					M1										
	$P(R + B = 5) = \frac{6}{16}$					A1										
						(2)										
(f)	<table><tr><td>R^2</td><td>1</td><td>4</td><td>9</td><td>16</td></tr><tr><td>B</td><td>1</td><td>3</td><td>5</td><td>7</td></tr></table>	R^2	1	4	9	16	B	1	3	5	7					
	R^2	1	4	9	16											
	B	1	3	5	7											
	Possible combinations $(R, B) : (1,3) (1,5) (1,7) (2,4) (2,5) (2,7)$ <u>or</u> $\frac{1}{4} \times \frac{1}{4} \times 5$					M1A1										
$P(R^2 < B) = \frac{5}{16}$					A1											
						(3)										
(g)	$B = 5 \text{ and } R = 1, B = 7 \text{ and } R = 3 \rightarrow D = 4$		$B = 7 \text{ and } R = 2 \rightarrow D = 5$			M1										
	$P(D = 4) = \frac{1}{4} \times \frac{1}{4} \times 2 = \frac{1}{8}$		$P(D = 5) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$			A1										
	$p = F(4) = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$		$p = F(5) - P(D = 5) = \frac{15}{16} - \frac{1}{16} = \frac{7}{8}$			A1										
						(3)										
Notes						Total 15										
(a)	B1	Must include both words (in either order). Ignore extraneous non-contradictory words.														
(b)	B1	0.5 oe														
(c) (i)	B1	cao need not be labelled unless done in wrong order (blue then red)														
(ii)	B1	cao need not be labelled unless done in wrong order (blue then red)														
(d)	M1	Correct method to find $E(B^2)$ at least 3 terms correct (implied by $E(B^2) = 21$). Ignore label. $E(B^2) = \frac{21}{4}$ is M0														
	M1	Correct method to find $\text{Var}(B)$ fit their $E(B^2)$ and their $E(B)$														
	A1	cao an answer of 5 without working send to review														
(e)	M1	At least 4 correct combinations identified with no incorrect ones given. Ignore duplicates, but do not accept eg (1, 4) as a duplicate of (4, 1). If not labelled, combinations must be consistently ordered. <u>or</u> if no combinations given, correct probability calculation (implied by correct answer with no obvious incorrect working)														
	A1	0.375 oe must come from correct combinations or correct working														

Question Number	Scheme	Marks
4 (a)	(Discrete) uniform (distribution)	B1 (1)
(b)(i)	[By symmetry] $E(X) = \underline{13}$	B1 (1)
(ii)	$\frac{10^2 + 12^2 + 14^2 + 16^2}{4} - 13^2$ or $\frac{696}{4} - 169$ or $174 - 169$ $= \underline{5}$	M1 A1 (2)
(c)(i)	$E(Y) = \frac{1}{30}(1 \times 4 + 2 \times 9 + 3 \times 6 + 4 \times 5 + 5 \times 6); = \frac{90}{30} = \underline{3}$	M1; A1 (2)
(ii)	$E(Y^2) = \frac{1}{30}(1^2 \times 4 + 2^2 \times 9 + 3^2 \times 6 + 4^2 \times 5 + 5^2 \times 6) = \left[\frac{324}{30} \text{ or } 10.8 \right]$ $\text{Var}(Y) = "10.8" - "[3]^2"; = \underline{1.8}$	M1 M1; A1 (3)
(d)	$E(W) = E(Y) \Rightarrow aE(X) + b [= E(W) \text{ or } E(Y) \text{ or } "3"]; \text{ i.e. } "13" a + b = "3"$ $\text{Var}(W) = \text{Var}(Y) \Rightarrow a^2 \times "5" = "1.8"; \text{ so } a = \frac{3}{5} \text{ or } \underline{0.6}$ $b = \underline{-4.8}$	M1; A1ft M1; A1 A1 (5)
(e)	Values of w are: $10 \times "0.6" - "4.8" = 1.2$ or 2.4 or 3.6 or 4.8 i.e. all non integers [So no cases are possible when $W = Y$ so $P(W = Y) = \underline{0}$]	M1 A1 (2)
[16 marks]		
Notes		
(a)	B1 for "uniform" but if they say "continuous uniform" B0	
For all parts, correct answer with no incorrect working seen scores full marks		
(b)(i)	B1 for 13	
(ii)	M1 for a fully correct expression, can ft their 13 May use $E(X - \mu)^2 = \frac{3^2 \times 2 + 1^2 \times 2}{4}$ A1 for 5	
(c)(i)	M1 for an attempt at $E(Y)$ with at least 3 correct products seen A1 for 3	
(ii)	1 st M1 for an attempt at $E(Y^2)$ with at least 3 correct products seen or 10.8 o.e. 2 nd M1 for correct expression for $\text{Var}(Y)$ (ft their 10.8 and 3) [NB $\text{Var}(Y) = \dots = 10.8$ M1M0] A1 for 1.8 (or exact equivalent)	
$E(X - \mu)^2$	May see $0 \times \frac{6}{30} + 1 \times \left(\frac{9}{30} + \frac{5}{30} \right) + 2^2 \times \left(\frac{4}{30} + \frac{6}{30} \right)$ if in doubt send to review.	
(d)	1 st M1 for correct use of $E(aX + b)$ formula i.e. $aE(X) + b$ or "13" $a + b$ 1 st A1ft for a correct <u>equation</u> in a and b ft their $E(X)$ and their $E(Y)$ 2 nd M1 for correct use of $\text{Var}(Y) = \text{Var}(aX + b)$ formula with their $\text{Var}(X)$ and their $\text{Var}(Y)$ 2 nd A1 for $a = 0.6$ or exact equivalent 3 rd A1 for $b = -4.8$ or exact equivalent	
(e)	M1 for a clear attempt to find all possible values of w (ft their values of a and b and w values needn't be correct) or state that no integer values for w (if this is true) Can ft their values of a and b even if the values for w are integers A1 for an answer of 0 provided it's true for their a and b (which may be incorrect)	