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EDEXCEL INTERNATIONAL GCSE MATHS: QUESTIONS BY TOPIC

Circle Theorems (G6)

International GCSE Mathematics A (4MA1) · Higher tier

35 questions · 136 marks · Mark schemes included

Past papers June 2018 - June 2025 · 1 sub-topic

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HOW TO USE THIS BOOKLET

Questions are grouped by sub-topic and run from the lowest to the highest mark tariff, then oldest paper first.

Click any row below to jump straight to that sub-topic or question.

On every question page, click ANSWER in the top-right corner to jump to its mark scheme; < QUESTION brings you back.

A gold bar in the left margin marks exactly where the question sits on the original page.

Diagrams, graphs, grids and tables are printed exactly as they appear in the original papers.

Each question is filed under its main topic; longer questions may also test skills from other topics.

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G6a CIRCLE THEOREMS		35 Qs · 136 marks	3
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4. Q12	IGCSE Paper 2HR June 2018	3	7
5. Q16	IGCSE Paper 2HR November 2020	3	8
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35. Q22	IGCSE Paper 2H June 2018	6	40



QUESTIONS

35 questions · 136 marks · 1 sub-topic

SUB-TOPIC G6a

CIRCLE THEOREMS

Sub-topic G6a · 35 questions · 136 marks · lowest tariff first



(c) Find $n(C' \cap D')$

.....
(1)

(Total for Question 17 is 6 marks)

18

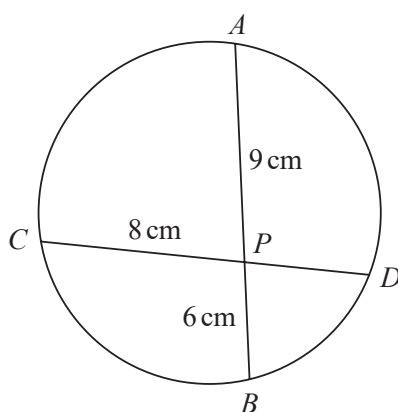


Diagram **NOT**
accurately drawn

APB and CPD are chords of a circle.

$AP = 9 \text{ cm}$ $PB = 6 \text{ cm}$ $CP = 8 \text{ cm}$

Calculate the length of PD .

..... cm

(Total for Question 18 is 2 marks)



20 A , B and C are points on a circle.

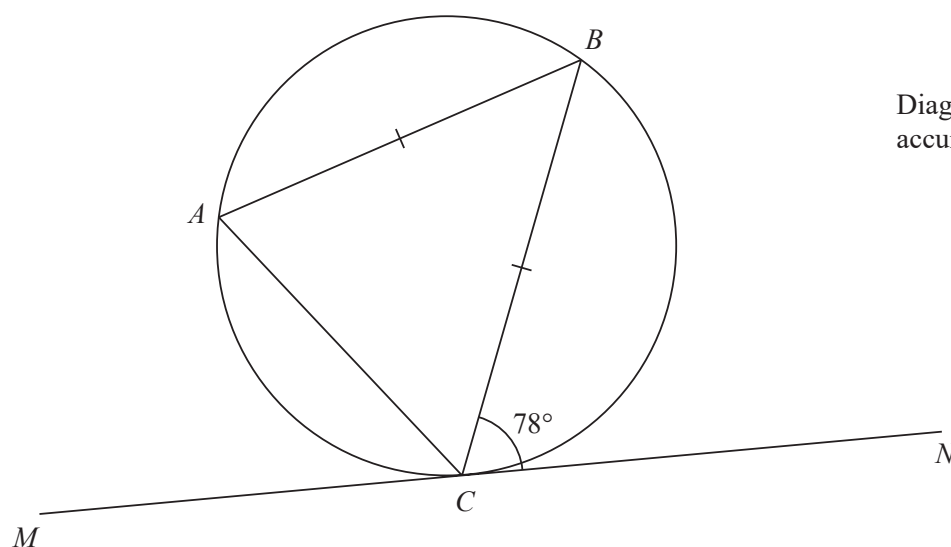


Diagram **NOT**
accurately drawn

MN is the tangent to the circle at C

$AB = CB$

Angle $BCN = 78^\circ$

Find the size of angle ABC

(Total for Question 20 is 2 marks)

- 5 In the diagram below, P and Q are points on a circle with centre O .

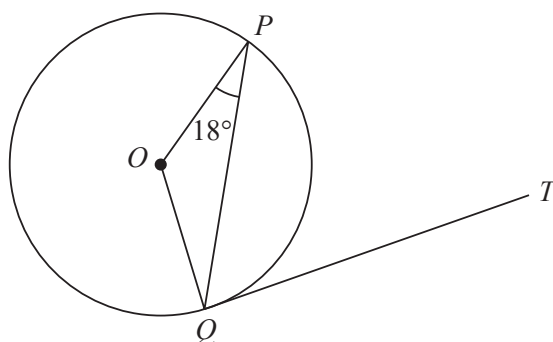


Diagram **NOT**
accurately drawn

QT is a tangent to the circle.
Angle $OPQ = 18^\circ$

Work out the size of angle PQT .
Give a reason for each stage of your working.

(Total for Question 5 is 3 marks)

12

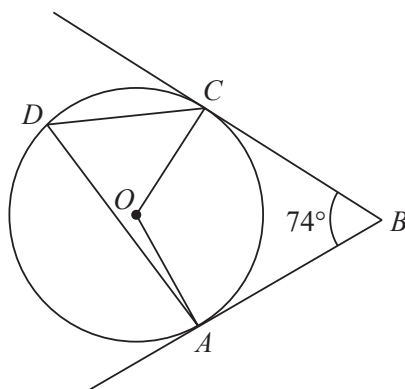


Diagram **NOT**
accurately drawn

A , C and D are points on a circle, centre O .
 AB and CB are tangents to the circle.

Angle $ABC = 74^\circ$

Work out the size of angle ADC .
Show your working clearly.

.....
(Total for Question 12 is 3 marks)



16

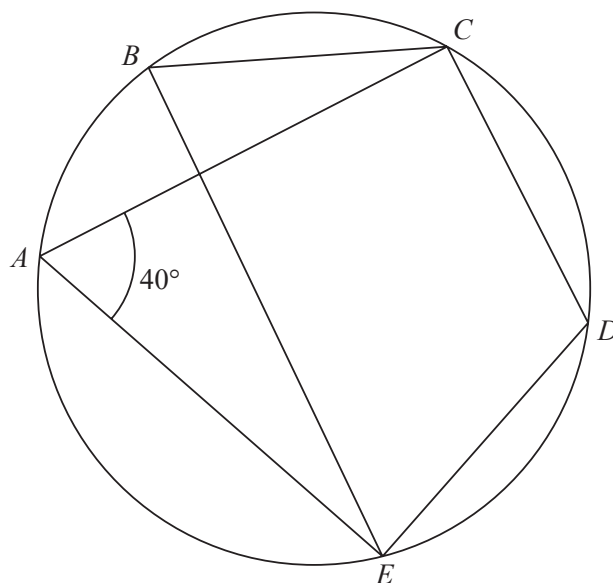


Diagram **NOT**
accurately drawn

A, B, C, D and E are points on a circle.

Angle $EAC = 40^\circ$

(a) (i) Write down the size of angle EBC .

.....
(1)

(ii) Give a reason for your answer.

.....
(1)

(b) Find the size of angle EDC .

.....
(1)

(Total for Question 16 is 3 marks)

13 P, Q, R, S and T are points on a circle with centre O .

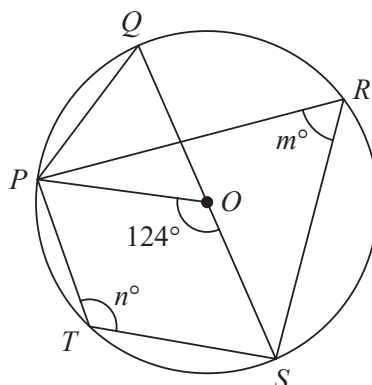


Diagram **NOT**
accurately drawn

QOS is a diameter of the circle.

angle $POS = 124^\circ$

angle $PRS = m^\circ$

angle $PTS = n^\circ$

(a) Find the value of

(i) m

(ii) n

(2)

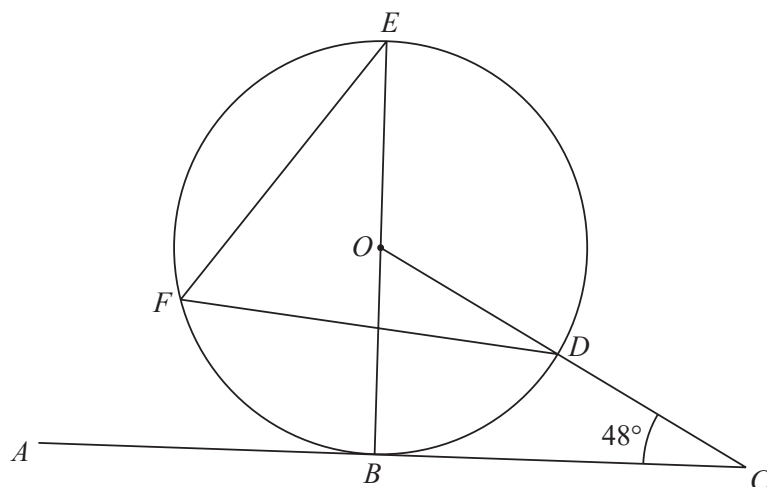
(b) Find the size of angle QPO .

(1)

(Total for Question 13 is 3 marks)



11

Diagram **NOT**
accurately drawn

B, D, E and F are points on a circle, centre O .
 ABC is a tangent to the circle.
 ODC is a straight line.

BOE is a diameter of the circle.

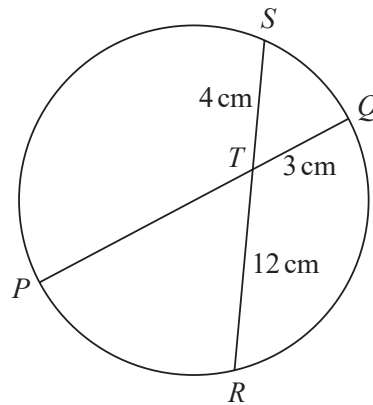
Angle $BCD = 48^\circ$

Find the size of angle DFE .

(Total for Question 11 is 3 marks)



19

Diagram **NOT**
accurately drawn

PTQ is a diameter of a circle.
 RTS is a chord of the circle.

$$TQ = 3 \text{ cm}$$

$$ST = 4 \text{ cm}$$

$$TR = 12 \text{ cm}$$

Calculate the radius of the circle.

..... cm

(Total for Question 19 is 3 marks)



15

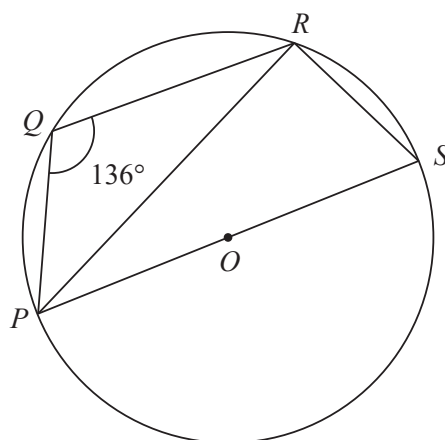


Diagram **NOT**
accurately drawn

P , Q , R and S are points on a circle with centre O

PS is a diameter of the circle.

Angle $PQR = 136^\circ$

Work out the size of angle RPS

(Total for Question 15 is 3 marks)



7

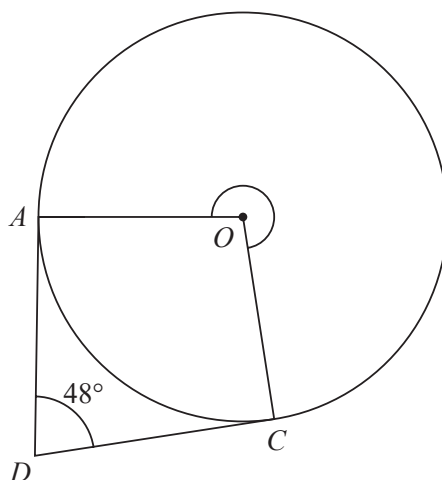


Diagram **NOT**
accurately drawn

A and C are points on a circle, centre O

DA is the tangent to the circle at A and DC is the tangent to the circle at C

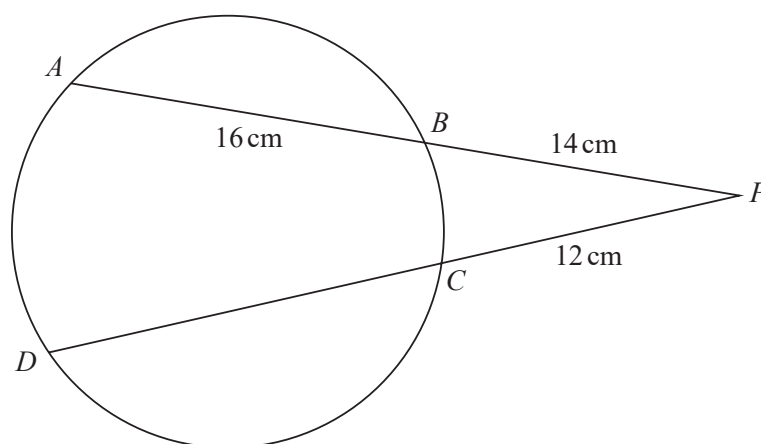
Angle $ADC = 48^\circ$

Work out the size of reflex angle AOC

(Total for Question 7 is 3 marks)



18

Diagram **NOT**
accurately drawn A, B, C and D are points on a circle. ABP and DCP are straight lines.

$AB = 16 \text{ cm}$

$BP = 14 \text{ cm}$

$CP = 12 \text{ cm}$

Work out the length of DC

..... cm

(Total for Question 18 is 3 marks)



14

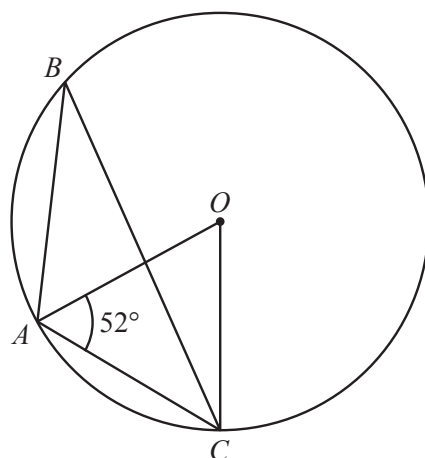


Diagram **NOT**
accurately drawn

A , B and C are points on a circle, centre O

Angle $OAC = 52^\circ$

Find the size of angle ABC

Give reasons for your working.

(Total for Question 14 is 3 marks)



15 A , B and C are points on a circle, centre O

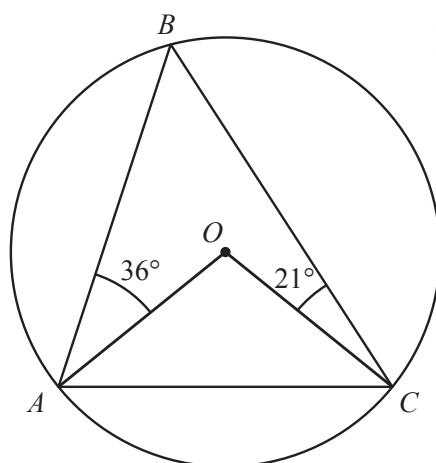


Diagram **NOT**
accurately drawn

Angle $BAO = 36^\circ$

Angle $BCO = 21^\circ$

Work out the size of angle ACO

(Total for Question 15 is 3 marks)



13

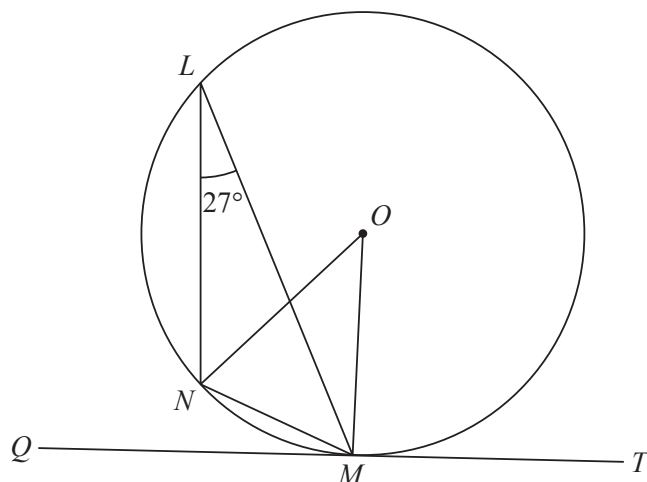


Diagram **NOT**
accurately drawn

L , M and N are points on a circle, centre O .
 QMT is the tangent to the circle at M .

(a) (i) Find the size of angle NOM .

.....
○

(ii) Give a reason for your answer.

.....
.....
(2)

(b) (i) Find the size of angle NMQ .

.....
○

(ii) Give a reason for your answer.

.....
.....
(2)

(Total for Question 13 is 4 marks)

23

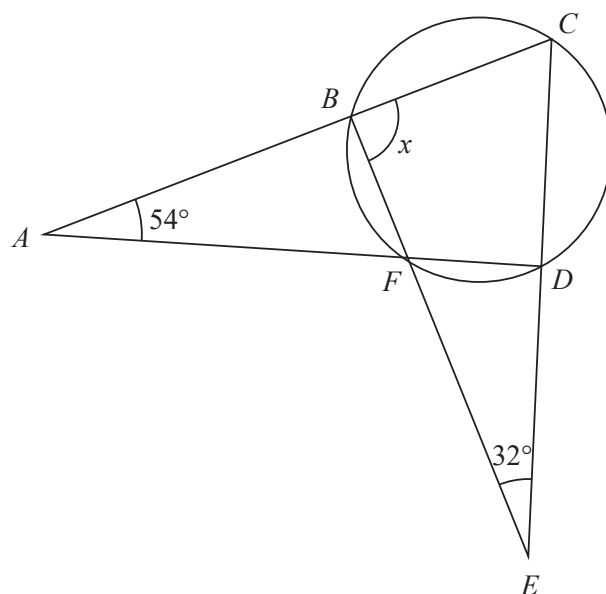


Diagram **NOT**
accurately drawn

B, C, D and F are points on a circle.
 ABC, AFD, BFE and CDE are straight lines.

Work out the size of angle x .
Show your working clearly.

$x = \dots\dots\dots$

(Total for Question 23 is 4 marks)



15 A, B, C and D are points on a circle, centre O .

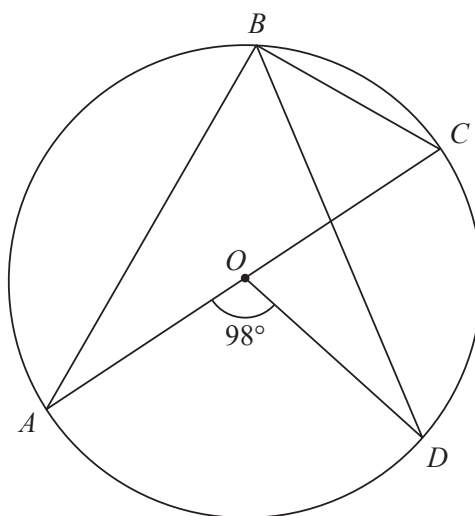


Diagram **NOT**
accurately drawn

AOC is a diameter of the circle.

Angle $AOD = 98^\circ$

Work out the size of angle DBC .

Give a reason for each stage in your working.

(Total for Question 15 is 4 marks)



14

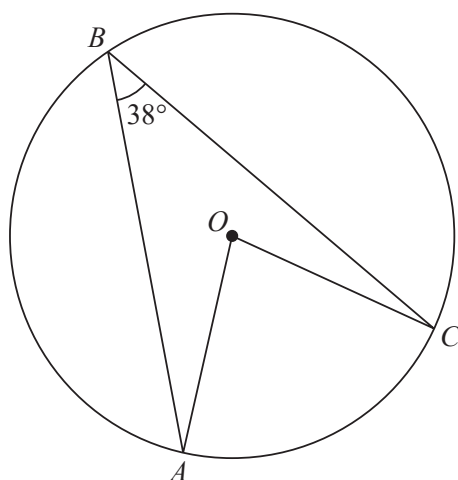


Diagram **NOT**
accurately drawn

A , B and C are points on a circle, centre O .
Angle $ABC = 38^\circ$

Work out the size of angle OAC .
Give a reason for each stage of your working.

(Total for Question 14 is 4 marks)



13

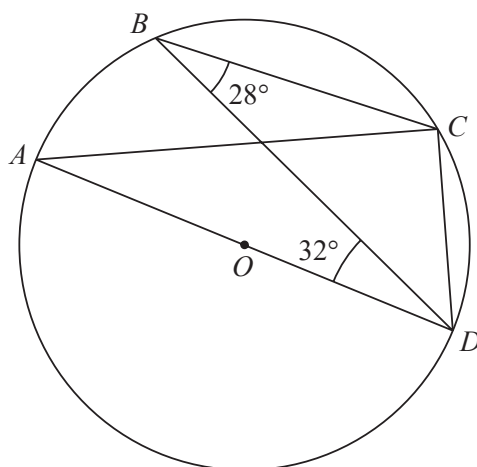


Diagram **NOT**
accurately drawn

A , B , C and D are points on a circle, centre O .
 AOD is a diameter of the circle.

Angle $CBD = 28^\circ$

Angle $BDA = 32^\circ$

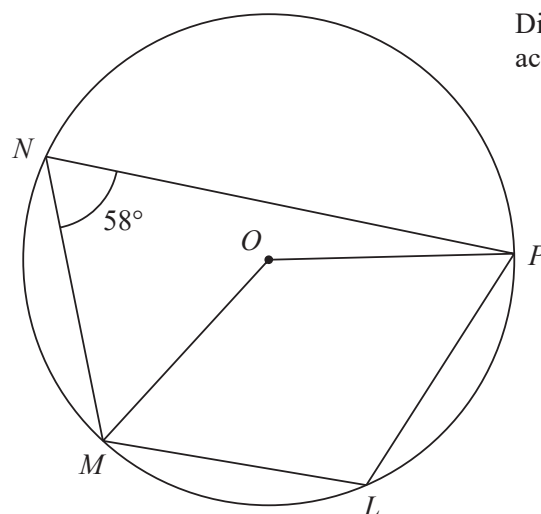
Find the size of angle BDC .

Give a reason for each stage of your working.

(Total for Question 13 is 4 marks)



16

Diagram **NOT**
accurately drawn L, M, N and P are points on a circle, centre O Angle $MNP = 58^\circ$ (a) (i) Find the size of angle MLP

o

(ii) Give a reason for your answer.

(2)

(b) Find the size of the reflex angle MOP

o

(2)

(Total for Question 16 is 4 marks)



15

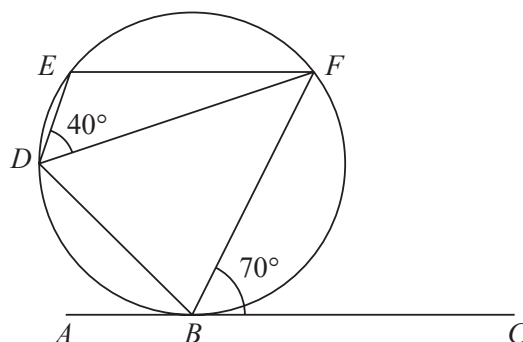


Diagram **NOT**
accurately drawn

B, D, E and F are points on a circle.
 ABC is the tangent to the circle at B .

Angle $EDF = 40^\circ$

Angle $FBC = 70^\circ$

Prove that the tangent ABC is parallel to EF .
 Give a reason for each stage of your working.

(Total for Question 15 is 4 marks)



14

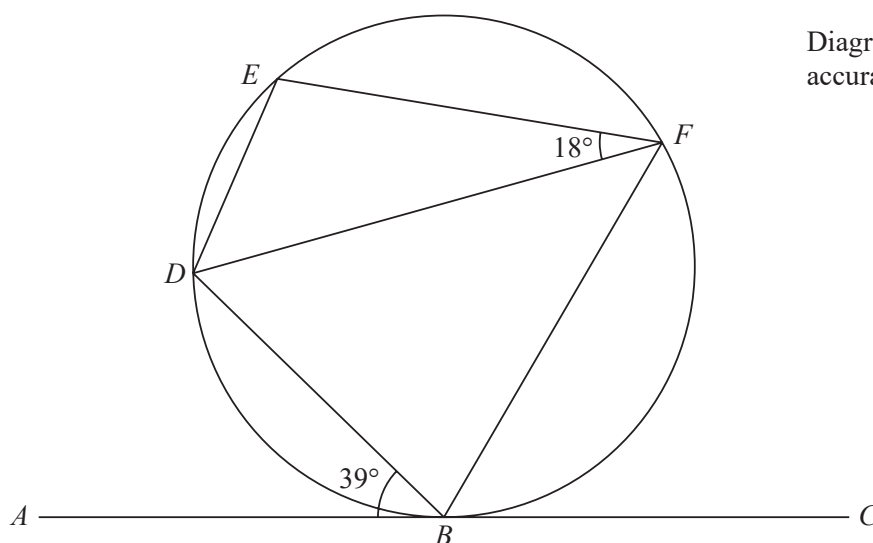


Diagram **NOT**
accurately drawn

B, D, E and F are points on a circle.

ABC is the tangent at B to the circle.

Angle $ABD = 39^\circ$

Angle $EFD = 18^\circ$

Work out the size of angle BDE .

Give reasons for your working.

(Total for Question 14 is 4 marks)



- 15 P , Q and R are points on a circle, centre O .
 TRV is the tangent to the circle at R .

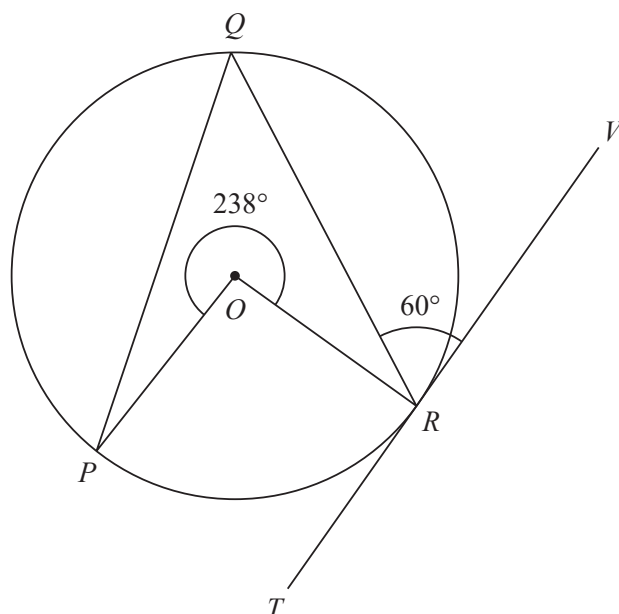


Diagram **NOT**
 accurately drawn

Reflex angle $POR = 238^\circ$

Angle $QRV = 60^\circ$

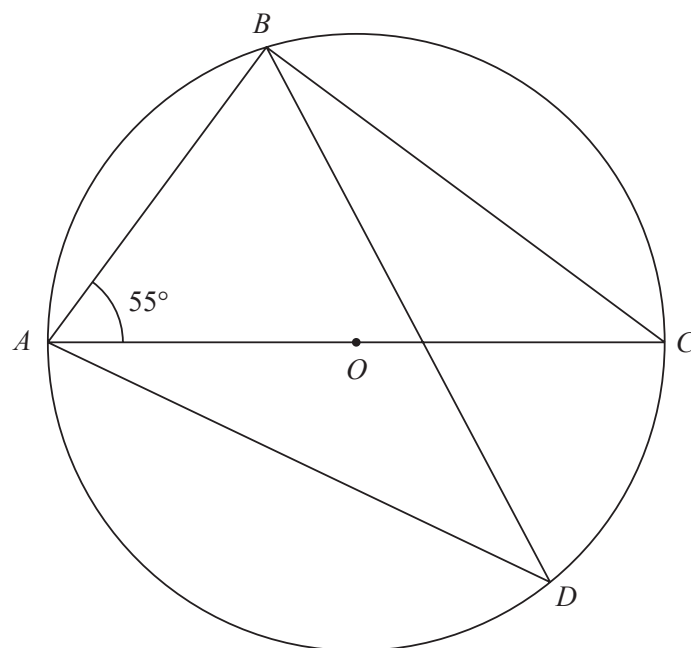
Calculate the size of angle OPQ .

Give a reason for each stage of your working.

(Total for Question 15 is 4 marks)



14

Diagram **NOT**
accurately drawn

A , B , C and D are points on a circle, centre O
 AOC is a diameter of the circle.

Angle $BAC = 55^\circ$

Work out the size of angle ADB
 Give a reason for each stage of your working.

(Total for Question 14 is 4 marks)



16 D, E, F and G are points on a circle, centre O

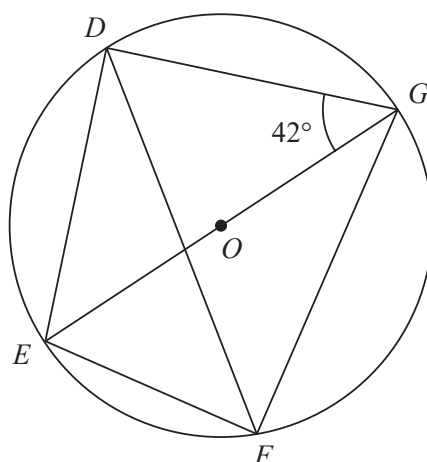


Diagram **NOT**
accurately drawn

EOG is a diameter of the circle.

Angle $EGD = 42^\circ$

Calculate the size of angle DFG

Give a reason for each stage of your working.

Angle $DFG = \dots\dots\dots^\circ$

(Total for Question 16 is 4 marks)



14

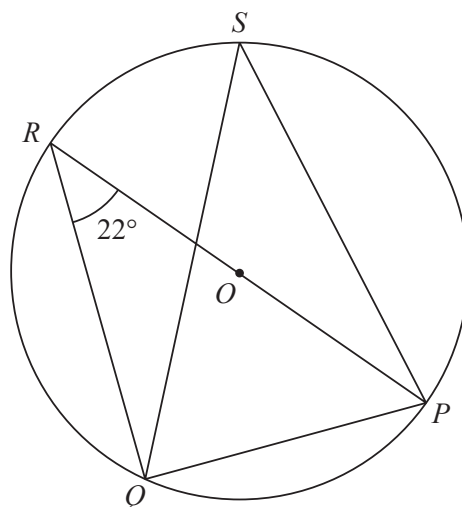


Diagram NOT
accurately drawn

P , Q , R and S are points on a circle, centre O
 ROP is a diameter of the circle.
 Angle $PRQ = 22^\circ$

(a) (i) Find the size of angle RQP

.....
 (1)

(ii) Give a reason for your answer.

.....

 (1)

(b) (i) Find the size of angle PSQ

.....
 (1)

(ii) Give a reason for your answer.

.....

 (1)

(Total for Question 14 is 4 marks)

19

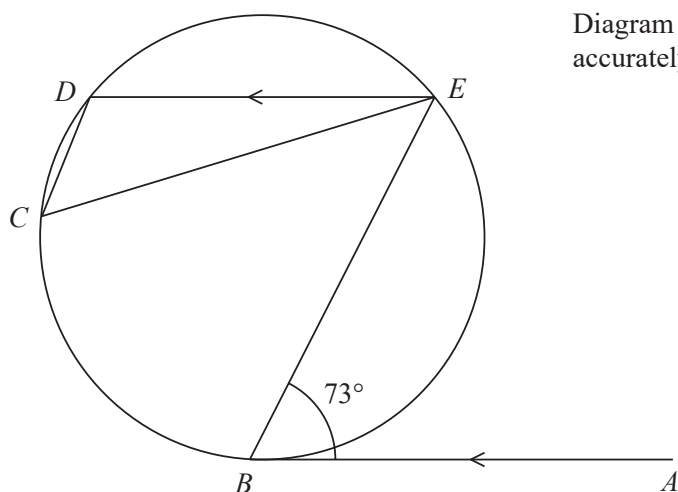


Diagram **NOT**
accurately drawn

B , C , D and E are points on a circle.

AB is the tangent at B to the circle.

AB is parallel to ED .

Angle $ABE = 73^\circ$

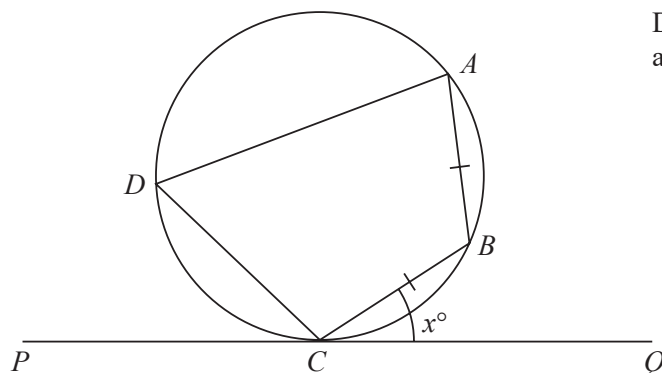
Work out the size of angle DCE .

Give a reason for each stage of your working.

(Total for Question 19 is 5 marks)



20

Diagram **NOT**
accurately drawn

A, B, C and D are points on a circle.

PCQ is a tangent to the circle.

$AB = CB$.

Angle $BCQ = x^\circ$

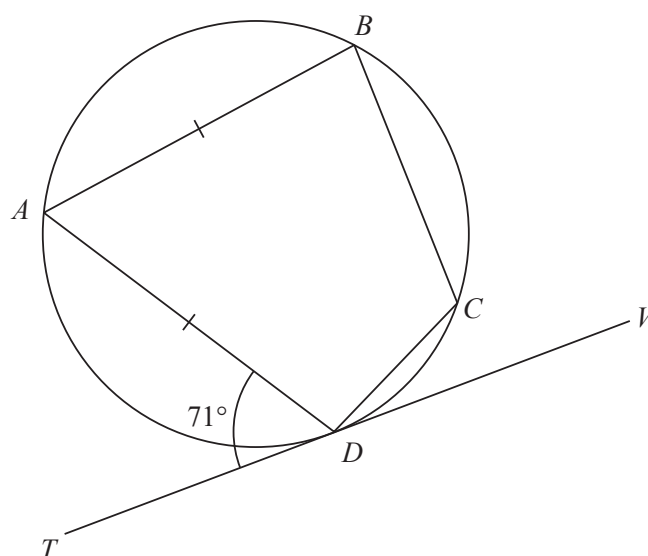
Prove that angle $CDA = 2x^\circ$

Give reasons for each stage in your working.

(Total for Question 20 is 5 marks)



20

Diagram **NOT**
accurately drawn

A , B , C and D are points on a circle.
 TDV is the tangent to the circle at D .

$AB = AD$
 Angle $ADT = 71^\circ$

Work out the size of angle BCD .
 Give a reason for each stage of your working.

(Total for Question 20 is 5 marks)



- 26 A, B, D and E are points on a circle.
 ABC and EDC are straight lines.

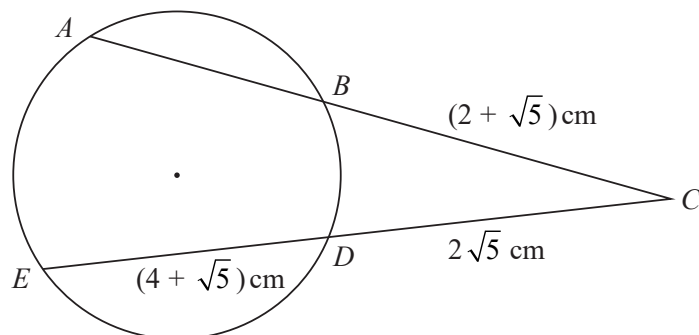


Diagram **NOT**
 accurately drawn

$$BC = (2 + \sqrt{5}) \text{ cm}$$

$$ED = (4 + \sqrt{5}) \text{ cm}$$

$$DC = 2\sqrt{5} \text{ cm}$$

Show that the length of AB is $(p\sqrt{5} + q) \text{ cm}$, where p and q are integers whose values are to be found.

Show your working clearly.

(Total for Question 26 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS



14 A, B, C and D are points on a circle, centre O

EBF is the tangent to the circle at B

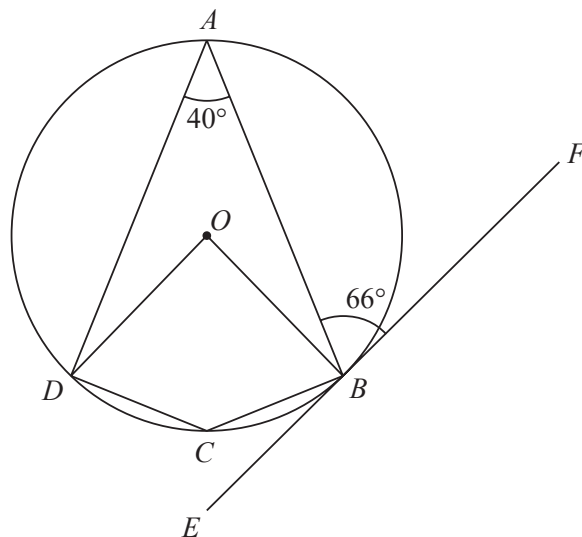


Diagram **NOT**
accurately drawn

(a) (i) Work out the size of angle DCB

(1)

(ii) Give a reason for your answer to (a)(i)

(1)

(b) Work out the size of angle ADO

(3)

(Total for Question 14 is 5 marks)



23 AEC and BED are chords of a circle.

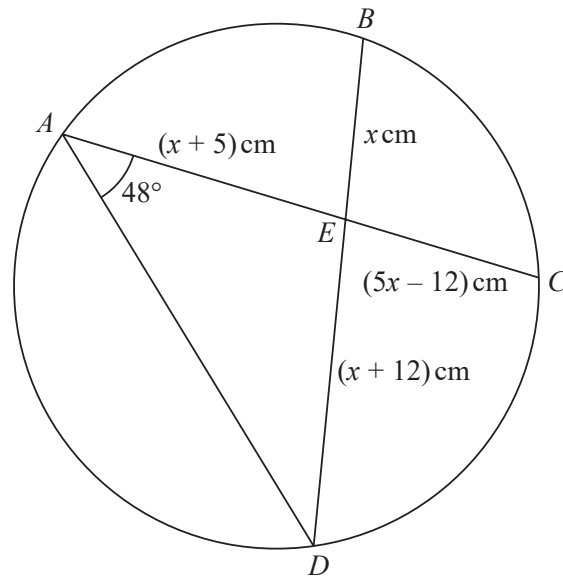


Diagram **NOT**
accurately drawn

$$AE = (x + 5) \text{ cm} \quad BE = x \text{ cm} \quad CE = (5x - 12) \text{ cm} \quad DE = (x + 12) \text{ cm}$$

$$\text{Angle } DAE = 48^\circ$$

Work out the size of angle ADE

Give your answer correct to one decimal place.



(Total for Question 23 is 5 marks)

Turn over for Question 24



13

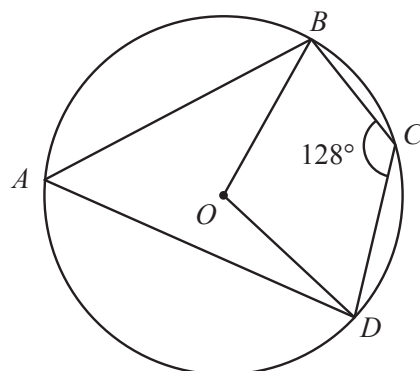


Diagram **NOT**
accurately drawn

A , B , C and D are points on a circle, centre O

Angle $BCD = 128^\circ$

Work out the size of angle OBD

Give a reason for each stage of your working.

angle $OBD = \dots\dots\dots^\circ$

(Total for Question 13 is 5 marks)



16 A, B, C and D are points on a circle, centre O

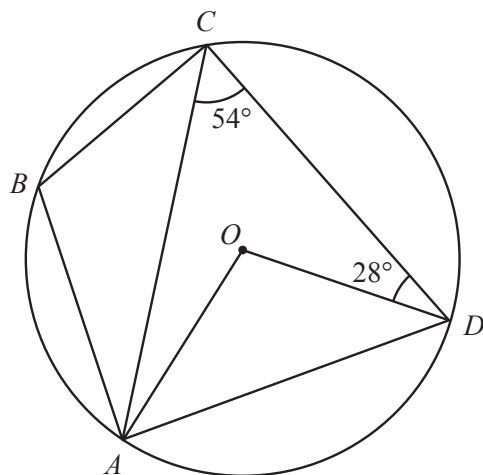


Diagram **NOT**
accurately drawn

(a) (i) Work out the size of angle AOD

.....
(1)

(ii) Give a reason for your answer to part (a)(i)

.....
.....
(1)

(b) Work out the size of angle CAO

.....
(1)

(c) Work out the size of angle ABC

.....
(2)

(Total for Question 16 is 5 marks)



23

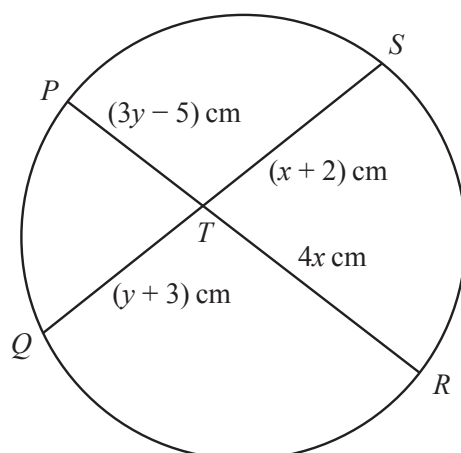


Diagram **NOT**
accurately drawn

PTR and QTS are chords of a circle.

$$PT = (3y - 5) \text{ cm} \quad QT = (y + 3) \text{ cm} \quad RT = 4x \text{ cm} \quad ST = (x + 2) \text{ cm}$$

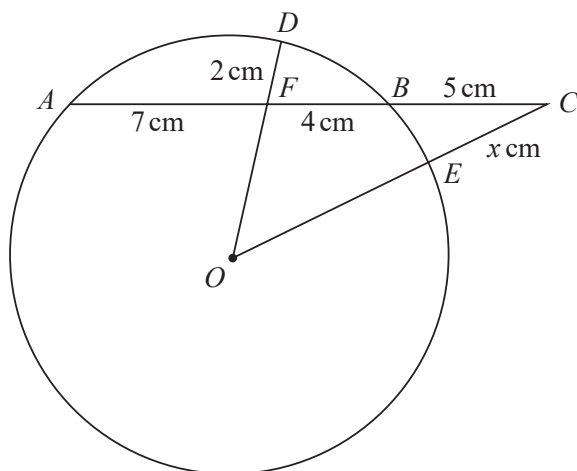
Find an expression for y in terms of x

$$y = \dots\dots\dots$$

Total for Question 23 is 5 marks)



22

Diagram **NOT**
accurately drawn

A , D , B and E are points on a circle, centre O .

$AFBC$, OEC and OFD are straight lines.

$AF = 7$ cm, $FB = 4$ cm, $BC = 5$ cm, $FD = 2$ cm and $CE = x$ cm.

Work out the value of x .

Show your working clearly.

$x = \dots\dots\dots$

(Total for Question 22 is 6 marks)



MARK SCHEMES

official mark schemes, in the same order as the questions



Question	Working	Answer	Mark	Notes
18	eg $9 \times 6 = 8 \times PD$ oe		2	M1 A correct equation involving PD
		6.75		A1 oe
Total 2 marks				
19 (a)	eg $\frac{2(4-3x)}{10} - \frac{5(3x-5)}{10} = -3$ oe or $2(4-3x) - 5(3x-5) = -3 \times 2 \times 5$ $8 - 6x - 15x + 25 = -30$ oe		3	M1 Correct fractions over common denominator as an equation or Multiplying both sides by 10
				M1 A correct equation with no denominators or brackets
		3		A1 dep on M1
(b)	$(5y+8)(y-5) (\leq 0)$ or $(y =) \frac{-17 \pm \sqrt{(-17)^2 - 4 \times 5 \times -40}}{2 \times 5}$ -1.6, 5 oe		3	M1 Correct method to solve 3 term quadratic – factorising or correct use of formula
				A1 Correct critical values
		$-1.6 \leq y \leq 5$ oe		A1 Condone change of variable in place of y throughout this question.
Total 6 marks				
20	$(ASF =) \frac{13^2}{9^2}$ or $\frac{9^2}{13^2}$		4	M1 Correct SF for area. Accept $1.44^2 (= 2.07 \text{ or } 2.09)$ or better for ASF or $0.69^2 (= 0.47 \text{ or } 0.48)$ or better for ASF
	eg $A + \left(\frac{13^2}{9^2}\right) A = 1800$			M1ft Dep on previous M1
	eg $\frac{250}{81} A = 1800$			M1ft
		583.2		A1 Awrt 583
Total 4 marks				

20	180 – 78 – 78 oe or (90 – 78) × 2 oe		2	M1 for a complete correct method to find angle <i>ABC</i> . This is not awarded if the angles are incorrectly labelled unless they have clearly started again (Ignore incorrect angles on the diagram if a student shows a correct method leading to the required answer)
	<i>Correct answer scores full marks (unless from obvious incorrect working)</i>	24		A1 award full marks if 24 is seen in the correct place on the diagram unless contradicted on the answer line
				Total 2 marks



Question	Working	Answer	Mark	Notes
5	$\angle OQT = 90^\circ$ and $\angle OQP = 18^\circ$ or $90 - 18$		3	M1 For 90° and 18° correctly identified in the working or on the diagram or for $90 - 18$ or for other fully correct method
		72		A1
	Angle between tangent and radius(or diameter) is 90 degrees			B1 Correct reason for 90° angle [If used alternate segment theorem]
				Total 3 marks
6 (a)	$2 \times \pi \times 0.56 \times 1.6$		2	M1 Award even if part of a calculation including 1 or 2 circles
		5.63		A1 awrt 5.63
(b)	$\frac{0.6}{1.6} (=0.375)$ or $\frac{1.6}{0.6} (= \frac{8}{3} = 2.\dot{6})$ or $\frac{r}{0.56} = \frac{0.6}{1.6}$ or $(r =) \frac{0.56 \times 0.6}{1.6}$ or $0.56 \div 2.\dot{6}$ oe		2	M1 Correct scale factor (given as a fraction or ratio) or correct equation in r or a correct expression for r . Allow 2.6666... to 1 dp rounded or truncated
		0.21		A1 Allow 21 cm oe if units shown
				Total 4 marks
7 (a)	$(28 + 32) \times 72.6 (=4356)$ or $28 \times 75 (=2100)$		4	M1 Expression for total of both classes together or total for class A
	$(28 + 32) \times 72.6 - 28 \times 75 (=2256)$			M1 Expression for total of class B
	$\frac{(28+32) \times 72.6 - 28 \times 75}{32}$ (= "2256" $\div$ 32)			M1 Correct calculation for mean of class B
		70.5		A1
(b)	Highest in A = $39 + 57 (=96)$ Highest in B = $33 + 60 (=93)$		3	M1 for $39 + 57 (=96)$ or $33 + 60 (=93)$
	$(39 + 57) - 33$			M1 or for $33 - "96"$ or 33 to "96" oe
		63		A1
				Total 7 marks

Question	Working	Answer	Mark	Notes
12	eg ($COA=$) $360 - (2 \times 90 + 74)$ (=106) or ($COA=$) $180 - 74$ (=106) or $OAB = 90$ or $OCB = 90$	53	3	M1 Fully correct method to find COA or OAB or OCB
	"106" $\div 2$			M1
				A1 values may be seen on diagram throughout
				Total 3 marks
13	eg $m = \frac{1}{2}$ or $y = \frac{1}{2}x + c$		4	M1 for gradient = $\frac{1}{2}$
	eg $7 = \frac{1}{2} \times 4 + c$ or $y - 7 = \frac{1}{2}(x - 4)$			M1 for substituting (4,7) into an equation with gradient = $\frac{1}{2}$
	eg $\frac{1}{2}x + 5 = 0$ or $-7 = \frac{1}{2}(x - 4)$			M1 Inputting $y = 0$ into their correct equation
		(-10, 0)		A1 SC B2 for an answer of (18,0) or (0.5,0) oe or (7.5,0) oe
				Total 4 marks



16	(a) (i)		40	2	B1	cao (may be written on the diagram)
	(ii)		Angles in same segment (are equal)		B1	or angles at the circumference from the same arc of the circle or angles on the same arc of the circle Alternatively: (two applications of) Opposite angles of a cyclic quadrilateral sum to 180°
	(b)		140	1	B1	cao (may be written on the diagram)
Total 3 marks						

17	$yn^2 = n^2 + d$ or $y = 1 + \frac{d}{n^2}$		4	M1
	$yn^2 - n^2 = d$ or $-d = n^2 - yn^2$ or $y - 1 = \frac{d}{n^2}$			M1
	$n^2(y - 1) = d$ or $-d = (1 - y)n^2$			M1 for factorising n^2 from a suitable expression. or $n^2 = \frac{d}{y - 1}$
		$n = \sqrt{\frac{d}{y - 1}}$		A1 Accept $n = \sqrt{\frac{-d}{1 - y}}$ Penalise $\pm\sqrt{\quad}$
Total 4 marks				



13	(a) (i)		62	3	B1
	(a) (ii)		118		B1ft 180 – their (a)(i)
	(b)		62		B1
					Total 3 marks



11	$BOC(BOD) = 180 - 48 - 90 (= 42)$ oe or $EOC = 180 - (90 - 48)$ or $90 + 48 (= 138)$ oe		3	M1 for method to find angle BOC or EOC (may be shown in the correct place on the diagram)
	$\frac{180 - '42'}{2}$ oe or '138' $\div 2$ oe			M1 a fully correct method to find angle DFE
	Correct answer scores full marks (unless from obvious incorrect working)	69		A1
				Total 3 marks

18	eg $(2n+1)^2 + (2n-1)^2$ or $(2n+1)^2 + (2n+3)^2$ oe		3	M1 for setting up a correct algebraic expression (any letter can be used) must have intention to add (may come after expanding)
	Eg $4n^2 + 4n + 1 + 4n^2 - 4n + 1$ or $8n^2 + 2$ or $4n^2 + 4n + 1 + 4n^2 + 12n + 9$ or $8n^2 + 16n + 10$ oe			M1 correct expansion of brackets and correct signs or a correct result.
	eg $8 \times n^2 + 2$ $\frac{8n^2 + 16n + 10}{8} = n^2 + 2n + \frac{10}{8}$ which shows a remainder of 2 or $10 - 8 = 2$ or $\frac{8n^2 + 16n + 10}{8} = n^2 + 2n + 1$ remainder 2 oe $\frac{8n^2 + 16n + 10}{8} = n^2 + 2n + 1 + \frac{2}{8}$ oe $8(n^2 + 2n + 1) + 2$ oe	shown clearly		A1 conclusion dep on M2 for eg $8n^2 + 2$ and a suitable conclusion (may be shown as a calculation/in numbers). The conclusion must be an intention to show that the result is a multiple of 8 and there is 2 remaining.
				Total 3 marks
19	$(PT =) \frac{12 \times 4}{3} (= 16)$		3	M1 NB: 16 from $12 + 4$ is incorrect working
	$(r =) ("16" + 3) \div 2$			M1
		9.5		A1 oe
				Total 3 marks

13	12 (–) 3		2	M1	for both values unambiguously identified
		9		A1	
				Total 2 marks	

14	Elimination eg $9x - 15y = 75$ $20x + 15y = 70$ + $(29x = 145)$ or $12x - 20y = 100$ $12x + 9y = 42$ – $(-29y = 58)$	Substitution eg $4\left(\frac{25 + 5y}{3}\right) + 3y = 14$ or $4x + 3\left(\frac{25 - 3x}{-5}\right) = 14$ or $3\left(\frac{14 - 3y}{4}\right) - 5y = 25$ or $3x - 5\left(\frac{14 - 4x}{3}\right) = 25$		4	M1 for a correct method to eliminate x or y: coefficients of x or y the same and correct operation to eliminate selected variable (condone 1 arithmetical error) or for correctly writing x or y in terms of the other variable and correctly substituting
					A1 dep on M1 for $x = 5$ or $y = -2$
	eg $3x - 5 \times “-2” = 25$ or $4x + 3 \times “-2” = 14$ or $3 \times “5” - 5y = 25$ or $4 \times “5” + 3y = 14$				M1 dep on M1 for substitution of found variable or repeating the steps in first M1 for the second variable
			$x = 5$ $y = -2$		A1 cao, dep on M1 a correct answer without working scores no marks
					Total 4 marks



15	$PRS = 90$ or $PQS = 90$ or $PSR = 180 - 136 (= 44)$		3	M1	may be seen on diagram. Must be labelled on diagram or identified using 3 letter notation.
	$RPS = 180 - 90 - "44"$ oe or $RQS = 136 - 90 (= 46)$			M1	for a complete method
		46		A1	
				Total 3 marks	



6	(a)		7.6×10^7	1	B1
	(b)		0.000 54	1	B1
Total 2 marks					

7	$DCO = 90$ (or right (angle)) or $DAO = 90$ (or right (angle)) Could also be awarded for $CAO + CAD = 90$ or $DAC + CAO = 90$		3	M1 may be marked on diagram – also allow right angle 'square' symbol on diagram	M2 implied by $360 - 90 - 90 - 48$ or $360 - 228$
	Obtuse $AOC = 360 - 90 - 90 - 48 (= 132)$ oe or Obtuse $AOC = 2(180 - (0.5 \times 48) - 90) (= 132)$ or Obtuse $AOC = 180 - "24" - "24" \text{ or } 180 - 48$ (if working with $\triangle DAC$ and $\triangle OAC$) or Reflex $AOC = 90 + 90 + 48$			M1 dep on M1 being awarded may be marked on diagram	
	Correct answer scores full marks (unless from obvious incorrect working)	228		A1 SC if no other marks awarded 132 gains B1	
Total 3 marks					

18	$DP \times 12 = 30 \times 14$ or $DP \times 12 = 420$ or $(DC + 12) \times 12 = 30 \times 14$ or $(DC + 12) \times 12 = 420$ or $12DC + 144 = 420$ or $DC + 12 = 35$ or $(DP) = \frac{30 \times 14}{12} (= 35)$		3	M1
	$"35" - 12$ or $23 + 12 = 35$ or $(DC =) \frac{"420" - "144"}{12}$ or $(DC =) \frac{"276"}{12}$			M1
	Correct answer scores full marks (unless from obvious incorrect working)	23		A1
Total 3 marks				

14	$(AOC =) 180 - (2 \times 52) (= 76)$		3	M1 must not be contradicted on diagram
	<i>Working not required, so correct answer scores full marks (unless from obvious incorrect working)</i>	38		A1 If labelled on the diagram then it must be in the correct place
		2 correct reasons		B1 dep on M1 for at least 2 valid reasons for their method including a correct circle property <u>angle at the centre</u> is $2 \times$ (double/twice) <u>angle at circumference</u> or <u>angle at circumference</u> is $\frac{1}{2}$ (half) <u>angle at centre</u> and one from (i) Base angles in an <u>isosceles triangle</u> (are equal) (ii) <u>Angles</u> in a <u>triangle</u> sum to 180° or angles in a <u>triangle</u> sum to <u>180°</u>
				Total 3 marks



14 ALT	tangent drawn at A with a right angle shown or 38° shown between the line AC and the tangent at A		3	M1 for a correct tangent drawn with right angle shown or 38° shown between the line AC and the tangent at A
	<i>Working not required, so correct answer scores full marks (unless from obvious incorrect working)</i>	38		A1 If labelled on the diagram, then it must be correct
		38 and correct reason		B1 dep on M1 for 38 and <u>alternate</u> <u>segment</u> theorem and angle between <u>radius</u> /diameter and <u>tangent</u> = 90
				Total 3 marks

15	$ABC = 21 + 36 (= 57)$ or angle $ABO = 36$ and angle $CBO = 21$ and $21 + 36 (= 57)$ or (reflex) $AOC = 360 - (21 + 21 + 36 + 36) (= 246)$ or (obtuse) $AOC = 2 \times (21 + 36) (= 114)$ or $BAX = 90 - 36 (= 54)$ with the tangent at A drawn oe or $BCY = 90 - 21 (= 69)$ with the tangent at C drawn oe or angle $ADB = 180 - 90 - 36 (= 54)$ OR eg $x = \text{angle } ACO$ and $y = ABC$ and $x + x + 2y = 180$ oe and $y + (x + 36) + (x + 21) = 180$ oe or eg $180 - 2x = 2(180 - 2x - 21 - 36)$		3	M1 for a method to find one of the angles on the scheme; for this mark, values and calculations must be linked to the correct angle by notation or by being marked on the diagram. May also be awarded for a correct method to set up and solve an equation to find one of these angles; the variable must be clearly defined where X is a point on the tangent at A where Y is a point on the tangent at C where AD is a diameter OR for any correct pair of simultaneous equations with clearly defined variables one of which must be angle ACO or any correct equation in terms of ACO only
	$(ACO) = \frac{180 - 2 \times "57"}{2}$ or $(ACO) = \frac{180 - (360 - "246")}{2}$ or $(ACO) = \frac{180 - "57" - 21 - 36}{2}$ or $(ACO) = "54" - 21$ or $(ACO) = "69" - 36$ or $(ACO) = 90 - "57"$ OR eg $180 - 2x = 2(180 - 2x - 21 - 36) \Rightarrow x = \dots$			M1 for a complete method to find angle ACO OR forms a correct equation in terms of ACO only and solves to get a value (condone arithmetic errors) implies the 1st M mark (provided no incorrect working seen)
	Correct answer only scores full marks (unless from obviously incorrect working)	33		A1
Total 3 marks				

Question	Working	Answer	Mark	Notes
12 (a)		3, 4	1	B1
(b)		see graph at end of mark scheme	3	B3 for correct region identified If not B3 then award B2 for $x + y = 4$ drawn (with no additional lines drawn) and a region identified that satisfies at least 3 of the 5 given inequalities If not B2 then award B1 for line $x + y = 4$ drawn NB. May shade wanted or unwanted regions; lines may be solid or dashed
13 a (i)		54	1	B1 cao
(ii)		<u>angle at centre is twice angle at circumference</u>	1	B1 dep on B1 in (a)(i) accept alternative reasons eg. angle at circumference is half the angle at the centre
b (i)		27	1	B1 ft from (a)(i) for $\frac{54}{2}$
(ii)		<u>alternate segment theorem</u>	1	B1 dep on B1 in (b)(i) accept alternative reason angle between <u>tangent</u> and <u>radius</u> is 90° If answer for (b)(i) is ft from (a)(i) then reason must be angle between <u>tangent</u> and <u>radius</u> is 90°



23	$ABF = 180 - x$ or $CDF = 180 - x$		4	M1 for finding an expression for ABF or CDF . May be seen on diagram.
	$FDE = 180 - (180 - x) (= x)$ AFB or $ACE = 180 - (180 - x) - 54 (= x - 54)$ DFE or $ACE = 180 - x - 32 (= 148 - x)$ e.g. $54 + y + 180 - x = 180$ where $AFB = y$ $32 + y + (180 - (180 - x)) = 180$ where $DFE = y$			M1 method to find FDE and AFB or method to find FDE and DFE or method to find ACE or method to find FDE and an equation for AFB e.g. $54 + y + 180 - x = 180$ where $AFB = y$ or method to find FDE and an equation for DFE e.g. $32 + y + (180 - (180 - x)) = 180$ where $DFE = y$ May be seen on diagram.
	e.g. $32 + x + x - 54 = 180$ or $54 + 180 - x + 148 - x = 180$ or $x - 54 = 148 - x$ oe or $54 + y + 180 - x = 180$ and $32 + y + (180 - (180 - x)) = 180$ where $AFB = DFE = y$			M1 for setting up an equation or a pair of correct simultaneous equations to solve for x
		101		A1 dep on at least M1
				Total 4 marks

Question	Working	Answer	Mark	Notes
15	$ABD = 98^\circ \div 2 (= 49^\circ)$ or $ABC = 90^\circ$ <u>Angle at centre / middle is twice angle at circumference</u> <u>Angle in a semicircle / from a diameter is 90° / right angle</u> $DBC = (90 - 49) = 41$	41°	4	M1 Correct angle stated or seen on diagram B1 Dep M1 B1 Dep M1 A1 Correct answer + no reasons = M1A1
	Alt: $180 - 98 (= 82^\circ)$ $OAD = 82 \div 2 (= 41^\circ)$ Base / bottom angles in an <u>isosceles triangle</u> are equal $DBC = 41^\circ$ <u>Angles in the same segment / from the same chord (DC) are equal</u>	41°		M1 Correct angle stated or seen on diagram B1 Dep M1 B1 Dep M1 A1 Correct answer + no reasons = M1A1
	Alt: <u>$DOC = 180 - 98 (= 82^\circ)$</u> <u>Angles on a straight line = 180°</u> $DBC = 41^\circ$ <u>Angle at centre / middle is twice angle at circumference</u>	41°		M1 Correct angle stated or seen on diagram B1 Dep M1 B1 Dep M1 A1 Correct answer + no reasons = M1A1
				Total 4 marks

16	(a)	$y = \frac{k}{x^2}$ condone proportion symbol in place of = $16 = \frac{k}{1.5^2}$ or $9 = \frac{k}{2^2}$ or $4 = \frac{k}{3^2}$ or $2.25 = \frac{k}{4^2}$		3	M1 Setting up a correct equation " k " $\neq$ 1 M1 Using the values from the table to find the value of the constant or " k " = 36 A1 $\frac{36}{x^2} = M2 A0$
	(b)	$x^2 = \frac{36}{144}$ or $x = \sqrt{\frac{36}{144}}$			M1 Substituting $y = 144$ into the correct equation and making x^2 or x the subject.
			0.5 oe	2	A1 cao
					Total 5 marks

14	$(AOC =) 38 \times 2 (= 76)$		4	M1
		52		A1
				B2 (dep on M1) for all reasons relevant to their method – underlined words must be seen. <u>angle at the centre</u> is $2 \times$ (double) <u>angle at circumference</u> / <u>angle at circumference</u> is $\frac{1}{2}$ <u>angle at centre</u> <u>angles</u> in a <u>triangle</u> add to 180° or angles in a <u>triangle</u> add to 180° base angles in an <u>isosceles</u> triangle (are equal) If not B2 then award B1 (dep on M1) for a correct circle theorem
				Total 4 marks
15	e.g. $(EF =) 12\cos 40 (= 9.19\dots)$ or $(FD =) 12\sin 40 (= 7.71\dots)$ and $(EF =) \sqrt{12^2 - 7.71^2} (= 9.19\dots)$			M2 complete method to find EF (if not M2 then M1 for a correct statement involving EF e.g. $\frac{EF}{12} = \cos 40$)
	e.g. $\frac{9.19}{EG} = \tan 28$ or $\tan 62 = \frac{EG}{9.19}$ or $\frac{9.19}{FG} = \sin 28 (= 19.5\dots)$ and $19.5^2 - 9.19^2 (= 298.9\dots)$			M1 (dep on M2) for a correct trig statement involving EG or complete method to find FG and a correct start to Pythagoras process
		17.3	4	A1 accept $17.2 - 17.3$
				Total 4 marks



13	Angle $CAD = 28^\circ$ or angle $ACB = 32^\circ$ or angle $ACD = 90^\circ$ or angle $ABD = 90^\circ$		4	M1
		30°		A1 For a correct answer of 30
	<u>Angles</u> in the <u>same segment</u> are equal, <u>angle</u> in a <u>semicircle</u> is 90° (or <u>angle</u> at centre is <u>double</u> angle at <u>circumference</u> oe) angles in a <u>triangle</u> add up to <u>180°/angles</u> in a <u>triangle</u> <u>isosceles</u> triangle <u>alternate</u> angles vertically <u>opposite angles</u> (or <u>vertically</u> <u>opposite</u>) <u>angles</u> at a <u>point</u> <u>opposite angles</u> in a <u>cyclic quadrilateral</u> angle between <u>tangent</u> and <u>radius (diameter)</u> <u>alternate segment</u> theorem <u>angles subtended</u> by the <u>same arc</u> (or <u>chord</u>) at the <u>circumference</u> (or <u>on the circle</u>)			B2 Dep on M1 for all correct reasons for their method used (if not B2 then award B1(dep on M1) for a correct circle theorem reason)
				<i>Total 4 marks</i>



15	(a)		$\frac{2}{5}, \frac{3}{5}$ oe	2	B1 correct probabilities for spinner A
			$\frac{4}{5}, \frac{1}{5}, \frac{4}{5}, \frac{1}{5}$ oe		B1 correct probabilities for spinner B
	(b)	$\frac{2}{5} \times \frac{4}{5} \left(= \frac{8}{25} \right)$ or $\frac{2}{5} \times \frac{1}{5} \left(= \frac{2}{25} \right)$ or $\frac{3}{5} \times \frac{4}{5} \left(= \frac{12}{25} \right)$ or $\frac{3}{5} \times \frac{1}{5} \left(= \frac{3}{25} \right)$ oe		3	M1 ft from (a) provided $0 < \text{probability} < 1$
		$1 - \frac{8}{25}$ or $\frac{2}{25} + \frac{12}{25} + \frac{3}{25}$ or $\frac{2}{25} + \frac{3}{5}$ oe			M1 ft from (a) for a complete method
			$\frac{17}{25}$		A1 oe
Total 5 marks					

16	(a)(i)		122	1	B1
	(a)(ii)		reason	1	B1 (dep on a correct answer or a correct method seen for (i)) <u>Opposite angles</u> in a <u>cyclic quadrilateral</u> sum to 180°
	(b)	$360 - 2 \times 58$ or $2 \times '122'$		2	M1 ft from (a)
			244		A1
Total 4 marks					



14	$(4x+1)(x-3) = 4x^2 - 12x + x - 3 (= 4x^2 - 11x - 3)$ $(4x+1)(5x+6) = 20x^2 + 24x + 5x + 6 (= 20x^2 + 29x + 6)$ $(x-3)(5x+6) = 5x^2 + 6x - 15x - 18 (= 5x^2 - 9x - 18)$		3	M1 for multiplying 2 brackets with at least 3 out of 4 terms correct
	$(5x+6)(4x^2 - 11x - 3) = 20x^3 - 55x^2 - 15x + 24x^2 - 66x - 18$ $(x-3)(20x^2 + 29x + 6) = 20x^3 + 29x^2 + 6x - 60x^2 - 87x - 18$ $(4x+1)(5x^2 - 9x - 18) = 20x^3 - 36x^2 - 72x + 5x^2 - 9x - 18$			M1 (dep) for multiplying the product of the first 2 brackets (ft from the 1 st stage) by the 3 rd bracket, and getting at least 3 out of 6 or 4 out of 8 terms correct
		$20x^3 - 31x^2 - 81x - 18$		A1
	Alternative			
	$20x^3 + 24x^2 - 60x^2 + 5x^2 - 15x + 6x - 72x - 18$			B2 for at least 6 out of 8 terms correct (B1 for 4 or 5 out of 8 correct terms)
		$20x^3 - 31x^2 - 81x - 18$		A1
				Total 3 marks

15		$BDF = 70^\circ$	4	B1 may be marked on diagram
		<u>Alternate segment</u> theorem		B1 reason, the angle between a tangent and a chord is equal to the angle subtended in the <u>alternate segment</u>
		$EFB = 180 - (70 + 40) = 70$ <u>opposite angles</u> in a <u>cyclic quadrilateral</u>		B1 Angle EFB with reason, <u>opposite angles</u> in a <u>cyclic quadrilateral</u> sum to 180°
		$CBF = EFB$ <u>alternate</u> angles therefore EF is parallel to ABC		B1 conclusion, <u>alternate</u> angles are equal
				Total 4 marks



13	e.g. $\begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$		2	M1 or for $\begin{pmatrix} 7 \\ a \end{pmatrix}$ where $a \neq -1$ or $\begin{pmatrix} b \\ -1 \end{pmatrix}$ where $b \neq 7$
		$\begin{pmatrix} 7 \\ -1 \end{pmatrix}$		A1
Total 2 marks				

14	$BFD = 39^\circ$	$BED = 39^\circ$		4	B1
	$BDE = 180 - (18 + 39)$	$EBD = 18^\circ$ and $BDE = 180 - (18 + 39)$			M1
			123		A1
					B1 dep on M1 for all correct circle theorems relevant for their method e.g. <u>alternate segment theorem and opposite angles</u> in a <u>cyclic quadrilateral</u> sum to 180° or <u>alternate segment theorem and angles in same segment</u> are equal
Total 4 marks					



15	$ORQ = 90 - 60 (=30)$ or $OQR = 30$ or $PQR = 0.5 \times (360 - 238) (= 61)$ or $QPR = 60$ or $OPR = \frac{180 - (360 - 238)}{2} (= 29)$		4	M1 The correct working or the correct angle for ORQ or OQR or PQR or QPR or OPR . Must be clearly stated as the correct angle or shown on the diagram in correct position. (eg just seeing 30 in working without a label is not sufficient for the award of this mark)
	<i>Working not required, so correct answer scores M1A1 (unless from obvious incorrect working)</i>	31		A1 if not on answer line, may be seen on diagram or clearly labelled
	<i>NB: degrees symbol not essential for reasons</i> We will allow the symbol Δ for 'triangle' $\angle$ for angle Σ for sum	full reasons for method used		B2 (dep on a fully correct method that should lead to the answer) for fully correct reasons for method used (underlined words must be seen) eg Angle between <u>tangent</u> and <u>radius</u> is 90° <u>Angles</u> around a <u>point</u> total 360° <u>Angle</u> at <u>centre</u> is <u>twice</u> angle at <u>circumference/edge</u> Total of <u>angles</u> in <u>triangle</u> is 180° / <u>triangle</u> 180° Base angles in an <u>isosceles</u> triangle (or <u>2 sides equal</u> , so <u>2 angles equal</u>) <u>Angles</u> in a <u>quadrilateral</u> total 360° or <u>quadrilateral</u> 360° / Accept "4-sided shape" or "quad" <u>Alternate segment</u> theorem (B1 dep on M1 for at least one reason for method used)
				Total 4 marks



14	$ABC = 90^\circ$ and $ACB (= ADB) = 180 - 90 - 55 (= 35)$ or $ABO = 55^\circ$ and $AOB = 180 - 2 \times 55 (= 70)$ or $BDC = 55^\circ$, $ADC = 90^\circ$ and $ADB = 90 - 55 (= 35)$		4	M1
		35		A1 for $ADB = 35$
	<u>Angles in a semicircle</u> are 90° <u>Angles in a triangle</u> add to 180° (Angles in a triangle add to 180°) <u>Angles in the same segment</u> (are equal) OR <u>angles at the circumference subtend(ed) from the same arc/chord</u> of the circle (are equal) or Angles in an <u>isosceles triangle</u> (are equal) <u>Angles in a triangle</u> sum to 180° (Angles in a triangle add to 180°) <u>Angle at the centre</u> is $2 \times$ (double) angle at <u>circumference</u> / <u>angle at circumference</u> is $\frac{1}{2}$ angle at <u>centre</u> or <u>Angles in the same segment</u> (are equal) OR <u>angles at the circumference subtend(ed) from the same arc/chord</u> of the circle <u>Angles in a semicircle</u> are 90°			B2 (dep on M1) for all 3 reasons appropriate to their method B1 (dep on M1) for one correct circle theorem appropriate to their method NB For the third method only 2 reasons are required
				Total 4 marks

12	216 or 2.16 or 10^{120} or 10^{122} or $6^3 \times 10^{40 \times 3}$		3	M1 or for digits 216
	216×10^{120} oe or or 2.16×10^n where $n \neq 122$			M1
		2.16×10^{122}		A1
				Total 3 marks
13		$x \geq -1$	1	B1 oe condone > in place of $\geq$
		$y \geq x$	1	B1 oe condone > in place of $\geq$
		$x + 2y \leq 8$	1	B1 oe condone < in place of $\leq$
				SCB1 if all inequalities reversed
				Total 3 marks
14	$12 \times \tan 5 (=1.05)$ or $\tan 5 = \frac{y}{12}$ or $12 \tan 5$ or $\tan 85 = \frac{12}{y}$ or $\frac{12}{\tan 85}$ $\frac{y}{\sin 5} = \frac{12}{\sin 85}$ oe or $(y =) 1.04986 \dots$ oe		3	M1 oe correct expression using tan or the sine rule or $\sqrt{\left(\frac{12}{\cos 5}\right)^2 - 12^2} (= 1.04986 \dots)$
	$(AB \Rightarrow) 2.6 + "1.05"$ oe			M1
		3.65		A1 allow awrt 3.65
				Total 3 marks
15	5 5 7 8 10 12 13 14 16 21 23		3	M1 For ordering the numbers Allow one error or omission in the list.
	16 & 7 identified for LQ and UQ			M1 For identifying 16 and 7 – may also have identified the median (12)
		9		A1
				Total 3 marks



16	$DFE = 42^\circ$ or $DOG = 180 - 2 \times 42 (= 96)$ or $EFG = 90^\circ$ or $EDG = 90^\circ$ or $DEG = 90 - 42 (= 48)$		4	M1 used or seen in diagram (must be clearly labelled if not in diagram)
		48°		A1 award 2 marks for 48 unless from an incorrect method
	<u>angles</u> in <u>same segment</u> or <u>angles</u> from <u>same chord</u> or <u>angles</u> at the circumference subtended from the <u>same arc</u> of the circle <u>angles</u> in a <u>semicircle</u> are 90° <u>angles</u> in a <u>semicircle</u> are 90° <u>angle subtended</u> by <u>diameter</u> is 90° <u>angle</u> at <u>centre</u> <u>twice</u> <u>angle</u> at <u>circumference</u> or <u>angles</u> in a <u>triangle</u> add to 180 <u>angles</u> in a <u>triangle</u> add to <u>180</u>			B2 Dep on a fully correct method to find angle DFG for a full set of reasons relevant to their method. B1 dep on M1 for at least one relevant circle theorem .
				Total 4 marks

14	(a) (i)		90	2	B1
	(a) (ii)		<u>Angle</u> in a <u>semicircle</u> is 90° oe Angle in a <u>semicircle</u> is 90° oe <u>Triangle</u> in <u>semicircle</u> is 90° oe <u>Angle</u> at <u>centre</u> is <u>double</u> (oe eg $\times 2$) angle at <u>circumference</u> oe <u>Angle</u> at <u>circumference</u> is <u>half</u> (oe) angle at <u>centre</u> oe		B1 dep on B1 in (a)(i) Valid reason given, underlined words give minimally acceptable answer.
	(b) (i)		22	2	B1
	(b) (ii)		Angles in the <u>same segment</u> (are equal) or <u>angles</u> at the circumference subtended from the same <u>arc</u> of the circle or angles on the <u>same chord</u>		B1 dep on B1 in (b)(i) Valid reason given, underlined words give minimally acceptable answer.
					Total 4 marks



Question	Working		Answer	Mark	Notes
18	$\frac{\sqrt{8}}{\sqrt{8}-2} \times \frac{\sqrt{8}+2}{\sqrt{8}+2}$ $\frac{\sqrt{8}(\sqrt{8}+2)}{8-4} = \frac{8+2\sqrt{8}}{4} = \frac{8+4\sqrt{2}}{4}$ $= 2 + \sqrt{2}$		Shown	3	M1 or $\frac{2\sqrt{2}}{2\sqrt{2}-2}$ or $\frac{\sqrt{2}}{\sqrt{2}-1}$ M1 or $\frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$ A1 (dep on M2) Conclusion - need not state the value of n
19	Angle $BCE = 73^\circ$ Angle $DEB = 73^\circ$ and Angle $DCB = 180 - 73 (=107^\circ)$ Angle $DCE = 34^\circ$ eg <u>Alternate segment</u> theorem Opposite angles of a <u>cyclic quadrilateral</u> sum to 180° <u>Alternate angles</u> are equal Angles in the <u>Same segment</u> are equal <u>Angles</u> in a <u>triangle</u> sum to 180	Angle $BDE = 73^\circ$ Angle $DEB = 73^\circ$ and Angle $DBE = 180 - 73 \times 2 (=34^\circ)$	34	5	M1 angles may be written on the diagram M1 A1 B2 for a full set of reasons relevant to their method (B1 for at least one relevant circle theorem)

Question	Working	Answer	Mark	Notes
20	angle $CDB = x$ or angle $CAB = x$ angle $CBA = 180 - 2x$ angle $CDA = 180 - (180 - 2x) = 2x$	proof with reasons	5	M1 M1 M1 B1 dep on M1 for any one appropriate circle theorem reason A1 for complete proof with full reasons <u>alternate segment</u> theorem, angles in a <u>triangle</u> sum to <u>180°</u> , <u>isosceles triangle</u> , <u>opposite</u> angles of a <u>cyclic quadrilateral</u> sum to <u>180°</u>
	Alternative method angle $CDB = x$ or angle $CAB = x$ angle $ACB = x$ angle $ACQ = 2x$ and angle $CDA = 2x$			M1 M1 M1
	Alternative method angle $OCB = 90 - x$ angle $BOC = 180 - 2(90 - x)$ ($= 2x$) angle $AOB = 2x$ and angle $CDA = 2x$	proof with reasons	5	B1 dep on M1 for any one appropriate circle theorem reason A1 for complete proof with full reasons <u>alternate segment</u> theorem, <u>isosceles triangle</u>
				M1 M1 B1 dep for any one appropriate circle theorem reason A1 for complete proof with full reasons angle between <u>tangent and radius</u> is <u>90°</u> oe, angles in a <u>triangle</u> sum to <u>180°</u> , <u>isosceles triangle</u> , angle at <u>centre</u> is <u>twice</u> angle at <u>circumference</u> oe

Q20 contd				
Alternative method where students assume $CDA = 2x$ and must work to show that $BCQ = x$				
	eg angle $ABC = 180 - 2x$ Angle $CAB = \text{angle } ACB =$ $[180 - (180 - 2x)] \div 2 = x$ $BCQ = CAB = x$			M1 M1 M1 B1 Dep on M1 for any one appropriate circle theorem reason A1 For complete proof with reasons e.g. <u>opposite angles</u> of <u>cyclic quadrilateral</u> sum to 180° angles in <u>triangle</u> sum to <u>180°</u> <u>isosceles triangle</u> <u>alternate segment</u> theorem
21	$y = \frac{6}{4}x(+33)$ or (gradient =) $\frac{6}{4}$ oe $m \times \frac{6}{4} = -1$ or (gradient of M =) $-\frac{2}{3}$ oe $\frac{k-6}{-4-5} = -\frac{2}{3}$	12	4	M1 M1 M1 or complete method to find equation of line ($3y = -2x + 28$) dep and then substitution of $x = -4$ A1



Students can use other methods to gain the correct answer					
20		angle $ABD = 71$ or angle $ACD = 71$ or using O as centre of circle, angle $ADO = 90 - 71 (=19)$		5	M1 clearly labelled or stated
		angle $ADB = 71$ or angle $ACB = 71$ or angle $BAD = 19 \times 2 (=38)$ or reflex angle $BOD = 2 \times 142 (=284)$			M1 dep clearly labelled or stated
		angle $BCD = 142$	142		A1 Clearly labelled or stated, from no incorrect working for their method
					B2 dep on A1 for fully correct reasons for each stage of working, repeated if used more than once. eg <u>alternate segment</u> theorem, base angles in an <u>isosceles</u> triangle are equal, <u>angles</u> in a <u>triangle</u> sum to 180° , angle between <u>tangent</u> and <u>radius(diameter)</u> is 90° <u>congruent</u> triangles (<u>equal</u> triangles) oe opposite angles of a <u>cyclic quadrilateral</u> sum to 180° <u>angles</u> in the <u>same segment</u> <u>angle at the centre</u> is <u>2 ×</u> angle at <u>circumference</u> oe <u>equal chords</u> subtend <u>equal angles</u> at the <u>circumference</u> If not B2 then award B1 dep on M1 for any one correct circle theorem reason associated with angle(s) found
					Total 5 marks

26	$(2 + \sqrt{5}) \times AC = (2\sqrt{5}) \times (2\sqrt{5} + 4 + \sqrt{5})$ or $(2 + \sqrt{5}) \times AC = (2\sqrt{5}) \times (3\sqrt{5} + 4)$ or $(2 + \sqrt{5}) \times (AB + 2 + \sqrt{5}) = (2\sqrt{5}) \times (2\sqrt{5} + 4 + \sqrt{5})$		5	M1 for using the intersecting chord theorem correctly eg may label $AB = x$ or $AC = x$ oe
	$(AC =) \frac{(2\sqrt{5}) \times (2\sqrt{5} + 4 + \sqrt{5})}{(2 + \sqrt{5})}$ or $(AC =) \frac{(30 + 8\sqrt{5})}{(2 + \sqrt{5})}$			M1 dep 1st M1 for rearranging for AC may use $AB + 2 + \sqrt{5}$ on LHS
	$(AC =) \frac{(30 + 8\sqrt{5})}{(2 + \sqrt{5})} \times \frac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$ or $(AB =) \frac{(21 + 4\sqrt{5})}{(2 + \sqrt{5})} \times \frac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$			M1 indep for multiplying by the conjugate of the denominator of <i>their</i> fraction, so long as fraction in the form $\frac{a + b\sqrt{5}}{c + d\sqrt{5}}$
	$(AC =) \frac{60 - 30\sqrt{5} + 16\sqrt{5} - 40}{4 - 5} (= 14\sqrt{5} - 20)$ or $(AB =) \frac{42 - 21\sqrt{5} + 8\sqrt{5} - 20}{4 - 5}$			M1 dep 3rd M1 for multiplying out the numerator
	$(AB =) \frac{20 - 14\sqrt{5}}{-1} - (2 + \sqrt{5})$	$13\sqrt{5} - 22$		A1 allow $p = 13$ and $q = -22$
Total 5 marks				

Question	Working	Answer	Mark	Notes
14 (a)(i)		140	1	B1
(a)(ii)		opposite angles of a cyclic quadrilateral (add to 180°) oe	1	B1 dep on B1 in (a)(i) or seeing 180 – 40 with no contradiction oe eg angle at centre is double (2 ×) angle at circumference oe AND angles around a point (or point 360)
(b)	$ADB = 66$ or $ABO = 90 - 66 (=24)$ or $BAO = 90 - 66 (=24)$ or $ODB = \frac{180-80}{2} (=50)$ or $DOB \text{ reflex} = 280$		3	M1 Clearly labelled in working or shown on diagram
	For 2 of: $ADB = 66$ or $ABO = 90 - 66 (=24)$ or $BAO = 90 - 66 (=24)$ or $ODB = \frac{180-80}{2} (=50)$ $DOB \text{ reflex} = 280$			M1 (award M2 for $360 - (280 + 40 + 24)$ oe
	Correct answer scores full marks (unless from obvious incorrect working)	16		A1
Total 5 marks				

23	eg $(x + 5)(5x - 12) = x(x + 12)$		5	M1 for setting up a correct equation
	eg $4x^2 + x - 60 (= 0)$ oe allow $4x^2 + x = 60$			A1 for writing the correct quadratic expression in the form $ax^2 + bx + c (= 0)$ allow $ax^2 + bx = c$
	eg $(4x - 15)(x + 4) (= 0)$ or $\frac{-1 \pm \sqrt{1^2 - 4 \times 4 \times -60}}{2 \times 4}$ or $4 \left[\left(x + \frac{1}{8} \right)^2 - \left(\frac{1}{8} \right)^2 \right] = 60$ oe			M1 (dep on M1) for a complete method to solve their 3-term quadratic (allow one sign error and some simplification – allow as far as $\frac{-1 \pm \sqrt{1 + 960}}{8}$) Allow + instead of $\pm$ in quadratic formula
	eg $(ADE) \sin^{-1} \left(\frac{("3.75" + 5) \sin(48)}{"3.75" + 12} \right)$			M1 for a complete method for ADE. Allow use of $x = -4$ for this mark
	Correct answer scores full marks (unless from obvious incorrect working)	24.4		A1 accept 24.3 – 24.4
				Total 5 marks

13	$BAD = 180 - 128 (= 52)$ or (reflex) $BOD = 2 \times 128 (= 256)$		5	M1 angles to be identified either by notation or correctly positioned on the diagram.
	(obtuse) $BOD = 2 \times "52" (= 104)$ or (obtuse) $BOD = 360 - "256" (= 104)$			M1 (dep on M1) angles to be identified either by notation or correctly positioned on the diagram.
	Correct answer of 38 scores 3 marks (unless from obvious incorrect working)	38		A1
				B2 dep on a fully correct method to find OBD for correct reasons for method used. (B1 dep on M1 for a correct circle theorem for their method) <u>Opposite angles of a cyclic quadrilateral</u> sum to 180° <u>Angle at the centre</u> is $2 \times$ (double) angle at <u>circumference</u> / <u>angle at circumference</u> is $\frac{1}{2}$ angle at centre <u>Angles</u> around a <u>point</u> add up to 360° <u>Isosceles</u> triangle <u>Angles</u> in a <u>triangle</u> add to 180° <u>Angles</u> in a <u>triangle</u> add to <u>180</u>
				Total 5 marks



16 (a)(i)		108	1	B1 Accept 252
(ii)		Correct reason	1	dep on (a)(i) correct <u>Angle</u> at the <u>centre(midpoint/origin/middle)</u> is <u>twice</u> the angle at the <u>circumference(side/edge/arc)</u> oe B1 <u>Inscribed angle</u> is <u>half</u> of the <u>central angle</u> oe (accept $\angle$ for 'angle' and $2 \times$ or double for twice)
(b)		26	1	B1
(c)	$ABC = 180 - "64"$ or $\frac{"108" + "124"}{2}$		2	M1ft follow through their $(\frac{1}{2}(180 - "108"))$ in $28 + "36" = 64$ ONLY FOLLOW THROUGH THEIR 108
	Correct answer scores full marks (unless from obvious incorrect working)	116		A1 cao
Total 5 marks				



23	eg $(y+3)(x+2) = 4x(3y-5)$ or $\frac{3y-5}{x+2} = \frac{y+3}{4x}$ or $\frac{x+2}{3y-5} = \frac{4x}{y+3}$ or $\frac{3y-5}{y+3} = \frac{x+2}{4x}$ or $\frac{4x}{x+2} = \frac{y+3}{3y-5}$		5	M1 for correct use of intersecting chords theorem to form an equation
	eg $xy + 2y + 3x + 6 = 12xy - 20x$ oe			M1 for expanding the brackets or for removing the fractions and expanding the brackets, allow one error in one term we can ft $4x(y+3) = (x+2)(3y-5)$ oe for the 2 nd , 3 rd and 4 th method marks
	eg $20x + 3x + 6 = 12xy - xy - 2y$ oe or $23x + 6 = 11xy - 2y$ oe			M1ft dep on previous M1 for correctly collecting all the y terms on one side and non-y terms on the other side
	eg $23x + 6 = y(11x - 2)$ or $20x + 3x + 6 = y(12x - x - 2)$			M1ft dep on 2 nd M mark for factorising, for y, an equation in the form $ax + b = cy + dy$ (may not be simplified) the factorisation must be correct
	Correct answer scores full marks (unless from obvious incorrect working)	$\frac{23x+6}{11x-2}$		A1 oe
				Total 5 marks

Incorrect working giving the radius as 16 cm gains M0A0				
Question	Working	Answer	Mark	Notes
22	$7 \times 4 = 2(2r - 2)$ or $7 \times 4 = 2(d - 2)$		6	M1 Or a correct equation in r eg $5.5^2 - 1.5^2 = 4r - 4$
	$r = 8$ or $d = 16$			A1
	$5 \times (5 + 4 + 7) = x \times (2 \times "8" + x)$			M1 Accept $5 \times 16 = x(2r + x)$
	$x^2 + 16x - 80 (= 0)$			A1
	$(x - 4)(x + 20) (= 0)$			M1 Correct factors or evidence of correct use of quadratic formula.
	$\frac{-16 \pm \sqrt{16^2 - 4 \times 1 \times (-80)}}{2 \times 1}$ or $\frac{-16 \pm \sqrt{576}}{2}$			
		4		A1 dep on first 2 method marks
				Total 6 marks
23	$\frac{48}{2}(2a + (48 - 1)d)$ or $\frac{36}{2}(2a + (36 - 1)d)$ oe		5	M1 For a correct expression for the first 48 terms or the first 36 terms
	$\frac{48}{2}(2a + (48 - 1)d) = 4 \times \frac{36}{2}(2a + (36 - 1)d)$ oe			M1 For a correct equation.
	$96a + 1392d = 0$ oe eg $4a + 58d = 0$, $2a + 29d = 0$ or $a = -14.5d$ etc			M1
	$\frac{30}{2}(2a + (30 - 1)d)$			M1 Indep Allow substitution of any 'found' values of a and d
		0		A1
				Total 5 marks

